The search for direct CP violation

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The formalism necessary to understand the phenomenon of CP nonconservation as observed in the neutral-kaon system is presented. The distinction between indirect CP violation and direct CP violation is made, and the level of understanding of the phenomenon in the standard model is reviewed. Attention is placed on new experimental efforts that could definitely establish a first-order or direct effect. The authors analyze the potential for such an observation in both kaon and B-meson decays and give the range of predictions from the standard model. Other possible models are considered briefly.

CONTENTS

I. INTRODUCTION

Discrete symmetries have played a major role in particle physics. The three main symmetries are P, space inversion; T, time reversal; and C, particle-antiparticle interchange. Each of these symmetries relates one state or process to another (mirror) state or process. The test for the validity of these symmetries involves comparing the rate of a process and the mirror process.

Each of these symmetries is known to hold for quantum electrodynamics. It was originally assumed, either explicitly or implicitly, that these hold for the strong and weak interactions. The discovery in 1957 of the large parity violation in weak decay processes led to a reconsideration of the validity of all three symmetries (Lee, Oehme, and Yang, 1957). General principles of relativistic local quantum field theory yield the result that the product CPT should be a good symmetry (Luders, 1954). It soon become clear that, in weak processes like pion decay, there was large P and C violation but that CP and T symmetries seemed to hold. The universal V-A theory developed in 1957 and 1958 embodied these symmetries.

In 1964 Christenson, Cronin, Fitch, and Turlay (1964) discovered CP violation in the decay $K^0\rightarrow 2\pi$. To the present time the only evidence for CP violation comes from K^0 decay. From the point of view of strong interactions, the K^0 is a particle with strangeness $S=+1$ with an antiparticle $\bar{K}^0$ with $S=-1$. Because the weak interactions violate S, the eigenstates, that is, the states with a well-defined mass and width, are mixtures of K^0 and $\bar{K}^0$. If CP invariance held, these would be

$$K_1=(K^0+\bar{K}^0)/\sqrt{2}\ ,$$

$$K_2=(K^0-\bar{K}^0)/\sqrt{2}\ ,$$

with CP eigenvalues $+1$ and -1, respectively. (Here we choose $\bar{K}^0$ as the CP conjugate of K^0.)

Before 1964 the K^0 was observed to decay with a short-lived component K_S ($\tau_S=0.89\times 10^{-10}$ s) and a long-lived component K_L ($\tau_L=5.2\times 10^{-8}$ s). The component K_S, which decays predominantly into the CP-even states $\pi^+\pi^-$ and $\pi^0\pi^0$, was identified as K_1. The component K_L, which decays into 3π and $\pi l\nu$, was identified as K_2. The 1964 discovery was that K_L also decayed into $\pi^+\pi^-$ with a branching ratio of 2×10^{-3}. The analysis of the present data on $K^0\rightarrow 2\pi$ decays is given in Sec. II.

The theoretical requirement of CPT invariance leads to the belief that any CP-violating interaction also violates

 0034-6861/93/65(4)/1113(35)/$08.50

the *T* symmetry. In the theoretical analysis in this paper, we shall assume *CPT* invariance. In fact, it is possible to use the data on *CP* violation in K^0 decay to provide stringent tests of *CPT* invariance (Carosi *et al.*, 1990; Karlsson *et al.*, 1990). The agreement of the data with the assumption of *CPT* invariance provides indirect but compelling evidence that *T* is also violated.

There exist many other experiments that have searched unsuccessfully for further evidence on *CP* or *T* violation. In general, these experiments have lacked the sensitivity attainable in the $K^0 \to 2\pi$ decay experiments. One of the most sensitive involves the search for electric dipole moments of elementary particles such as the neutron or electron, which requires a violation of both the *P* and *T* symmetries. Present upper limits are 1×10^{-25} e cm for the neutron (Smith *et al.*, 1990) and about 10^{-26} e cm for the electron (Abdullah *et al.*, 1990).

After the discovery of *CP* violation, many theories were published the explain the result. With the advent of gauge theories, it is natural to formulate *CP* violation in terms of a gauge theory. An important observation is that the standard interaction of the gauge fields with other particles is *CP* invariant. This is because a choice of phase for the gauge boson fields allows real gauge couplings. Therefore *CP* violation is always introduced into the Higgs boson part of the theory. There are three possibilities: (1) The Higgs potential in the case of several Higgs fields may violate *CP*; (2) the Yukawa interaction between the Higgs bosons and the fermions may violate *CP*; and (3) *CP* is violated spontaneously, that is, by the vacuum expectation values of various Higgs fields (Lee, 1974).

In the standard $SU(3)\times SU(2)\times U(1)$ gauge model with only one Higgs doublet field, the only possibility for *CP* violation is in the complex coefficients of the Yukawa interaction. In the original form of the theory with only two generations of quarks, there is, in fact, no *CP* violation, because it can be shown that all the complex phases can be removed as a result of flavor symmetries. It was shown by Kobayashi and Maskawa (1973) that if there were three generations of quarks, then one phase would not be removed and the standard model allowed *CP* violation. Details of the KM model are given in Sec. III.

In the analysis of *CP* violation in the K^0 system, we shall distinguish *CP* violation that occurs in the mass matrix, K^0-$\bar{K}^0$ mixing, from that in the decay amplitude to a final state. The latter will be referred to as *direct CP* violation. The *CP* violation in the mass matrix has the consequence that the eigenstates K_S and K_L are mixtures of the *CP* eigenstates K_1 and K_2 given by (assuming *CPT* invariance)

$$K_S=(K_1+\tilde{\varepsilon}K_2)/\sqrt{1+|\tilde{\varepsilon}|^2}\ ,$$

$$K_L=(K_2+\tilde{\varepsilon}K_1)/\sqrt{1+|\tilde{\varepsilon}|^2}\ .$$

The parameter $\tilde{\varepsilon}$ is a measure of the *CP* violation associated with K^0-$\bar{K}^0$ mixing.

There exists a class of models called *superweak* in which essentially the only *CP* violation in the K^0 system is that due to K^0-$\bar{K}^0$ mixing measured by $\tilde{\varepsilon}$. The basic idea is that a new, very weak interaction (Wolfenstein, 1964) can make a significant contribution to the K^0-$\bar{K}^0$ mass matrix if it allows $\Delta S=2$ at tree level, that is, in the lowest order. The normal weak interaction only allows $\Delta S=1$, so that its contribution to K^0-$\bar{K}^0$ mixing, which determines the mass difference $(m_L$-$m_S)$, is second order. Assuming the new, very weak interaction violates *CP* invariance significantly, then $\tilde{\varepsilon}$ is given by the ratio of the very weak interaction to the second-order effect of the standard interaction. In the standard Kobayashi-Maskawa (KM) model of *CP* violation (in contrast to the superweak), direct *CP* violation is expected. In this review we are concerned with the experimental search for direct *CP* violation. Clear-cut evidence for direct *CP* violation would rule out superweak models as the only source of *CP* violation and could provide support for the KM model.

This discussion for the K^0-$\bar{K}^0$ system can be repeated for D^0-$\bar{D}^0$ and B^0-$\bar{B}^0$. In the case of D^0-$\bar{D}^0$, it is known empirically that there is very little mixing; for this and other reasons the D system is not a good place to look for *CP* violation. On the other hand, it is known experimentally that B^0-$\bar{B}^0$ mixing is comparable to that for the K^0 system; furthermore, *CP*-violating effects in the B^0 system are expected in the KM model to be much greater than for K^0.

II. PHENOMENOLOGY OF THE NEUTRAL-KAON SYSTEM

The propagation of the K^0 system may be given by

$$i\frac{d}{dt}K(t)=\left[M-i\frac{\Gamma}{2}\right]K(t)\ ,$$

where $\mathbf{K}$ is a two-component vector in the K^0-$\bar{K}^0$ space and M and Γ are 2×2 Hermitian matrices. In the Wigner-Weiskopf perturbation theory (Enz and Lewis, 1965; Kabir, 1968),

$$M_{\alpha\beta}=m_k\delta_{\alpha\beta}+\langle\alpha|H_{SW}|\beta\rangle -P\sum_k\frac{\langle\alpha|H_W|k\rangle\langle k|H_W|\beta\rangle}{E_k-m_k}\ ,$$

$$\Gamma_{\alpha\beta}=2\pi\sum_k\langle\alpha|H_W|k\rangle\langle k|H_W|\beta\rangle\delta(E_k-m_k)\ ,$$

where P denotes the principal part. Here H_W is the normal $\Delta S=1$ weak interaction, and H_{SW} is the $\Delta S=2$ superweak interaction.

To discuss *CP* violation, it is useful to use the K_1-K_2 basis for the vector K in place of K^0-$\bar{K}^0$; then

$$M-i\frac{\Gamma}{2}=\begin{bmatrix}M_1 & im'+\delta' \\ -im'+\delta' & M_2\end{bmatrix}-\frac{i}{2}\begin{bmatrix}\Gamma_1 & i\gamma' \\ -i\gamma' & \Gamma_2\end{bmatrix}\ . \tag{2.1}$$

The off-diagonal terms that mix K_1 and K_2 are measures of *CP* violation. The factor i and the antisymmetry indicate that the term m' violates not only *CP* but also *T*, as expected from the *CPT* theorem. On the other hand, the term δ' is *T* invariant and violates *CPT*, and we set it equal to zero in what follows. We also omit a similar *CPT*-violating term in Γ. All experimental results, in fact, are consistent with $\delta'=0$, and the limit on the magnitude of δ' provides the best test of *CPT* invariance. The *CP*-violating mixing parameter is then given for small mixing by

$$\tilde{\varepsilon}=\frac{-i(m'-i\gamma'/2)}{(M_1-M_2)-i(\Gamma_1-\Gamma_2)/2} . \tag{2.2}$$

The decays of most interest are those for K^0 going to two pions. The decay amplitudes may be written, by using *CPT* and unitarity,

$$\begin{aligned} &A(K^0\to\pi\pi(I))=A_I\exp(i\delta_I) ,\\ &A(\bar{K}^0\to\pi\pi(I))=A_I^*\exp(i\delta_I) , \end{aligned} \tag{2.3}$$

where I is the isospin of the $\pi\pi$ system and δ_I is the corresponding $\pi\pi$ phase shift for a center-of-mass energy equal to M_K. Electromagnetic final-state interactions have been neglected. Had *CP* invariance held, the parameter A_I would be real. The parameter γ' can be related to decay amplitudes by unitarity; to a good approximation, the unitarity sum is dominated by the $2\pi I=0$ state, in which case

$$\frac{i\gamma'}{\Gamma_1}\cong\frac{[A(K_1\to\pi\pi)]^*A(K_2\to\pi\pi)}{|A(K_1\to\pi\pi)|^2}\cong i\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} . \tag{2.4}$$

We are now left with the following parameters describing weak interactions for the K^0 system.

CP-conserving: M_1-M_2, Γ_1, Γ_2, $\mathrm{Re}A_0$, $\mathrm{Re}A_2$.
Indirect *CP* violation: m'.
Direct *CP* violation: $\mathrm{Im}A_0$, $\mathrm{Im}A_2$.

We can relate the *CP*-conserving parameters to well-known (Particle Data Group, 1992) observables, neglecting the small *CP*-violating effects:

$$\Gamma_1=\Gamma_S=(0.892\pm0.002\times10^{-10}\ \mathrm{sec})^{-1} ,$$

$$\Gamma_2=\Gamma_L=1.72\pm0.02\times10^{-3}\Gamma_S ,$$

$$M_1-M_2=M_S-M_L=-\Delta M=-(0.477\pm0.003)\Gamma_S ,$$

$$\omega=\mathrm{Re}A_2/\mathrm{Re}A_0=0.045 .$$

The value $\mathrm{Re}A_2$ is determined from the decay rate for $K^+\to\pi^+\pi^0$ using isospin relations, assuming $\Delta I=5/2$ transitions can be neglected. A parameter from the strong interactions that is of importance is the phase-shift difference

$$\theta'=\delta_2-\delta_0+\pi/2=(43\pm6)^\circ . \tag{2.5}$$

This value has been deduced (Ochs, 1991) from $\pi\pi$-scattering data, but the error may be underestimated. A somewhat lower value is obtained from the ratio of $\pi^+\pi^-$ to $\pi^0\pi^0$ decay for the K_S system using the value of $\mathrm{Re}A_2$ obtained from $K^+\to\pi^+\pi^0$.

There are four observables (two magnitudes and two phases) describing *CP* violation in $K_L\to\pi\pi$ and one describing *CP* violation in K_L semileptonic decays; these are

$$\eta_{+-}=\frac{A(K_L-\pi^+\pi^-)}{A(K_S\to\pi^+\pi^-)}=|\eta_{+-}|e^{i\phi}_{+-} ,$$

$$\eta_{00}=\frac{A(K_L\to\pi^0\pi^0)}{A(K_S\to\pi^0\pi^0)}=|\eta_{00}|e^{i\phi}_{00} ,$$

$$\delta=\frac{\Gamma(K_L\to\pi^-1^+\nu)-\Gamma(K_L\to\pi^+1^-\bar{\nu})}{\Gamma(K_L\to\pi^-1^+\nu)+\Gamma(K_L\to\pi^+1^-\bar{\nu})} .$$

Putting together previous equations, one finds (neglecting terms of order ε^2 or ε'^2)

$$\eta_{+-}=\varepsilon+\varepsilon'/(1+\omega e^{i\theta'})\approx\varepsilon+\varepsilon' , \tag{2.6a}$$

$$\eta_{00}=\varepsilon-2\varepsilon'/(1-\sqrt{2}\omega e^{i\theta'})\approx\varepsilon-2\varepsilon' , \tag{2.6b}$$

$$\begin{aligned}\varepsilon&=\tilde{\varepsilon}+i(\mathrm{Im}A_0/\mathrm{Re}A_0)\\ &=\sin\theta e^{i\theta}(m'/\Delta M+\mathrm{Im}A_0/\mathrm{Re}A_0) ,\end{aligned} \tag{2.7}$$

$$\varepsilon'=\frac{1}{\sqrt{2}}e^{i\theta'}(\mathrm{Im}A_2/\mathrm{Re}A_2-\mathrm{Im}A_0/\mathrm{Re}A_0)\omega , \tag{2.8}$$

$$\begin{aligned}\theta&=\tan^{-1}[2\Delta M/(\Gamma_1-\Gamma_2)]\simeq\tan^{-1}(2\Delta M/\Gamma_S)\\ &=43.67\pm0.14^\circ .\end{aligned} \tag{2.9}$$

It follows from the $\Delta Q=\Delta S$ rule that the lepton asymmetry δ is a result of K^0-$\bar{K}^0$ mixing and is given by $2\,\mathrm{Re}\varepsilon$.

From these equations we reach a number of conclusions.

(1) The quantity ε can be the result of K^0-$\bar{K}^0$ mixing (m') or of direct *CP* violation ($\mathrm{Im}A_0$), but there is no way to separate these. In fact, there exists the possibility of choosing a phase convention, via the transformation of the phase of the s quark ($s\to se^{i\alpha}$), such that $\mathrm{Im}A_0$ goes to zero. This is the Wu-Yang (1964) phase convention, which has the consequence that $\varepsilon=\tilde{\varepsilon}$. In a particular model of *CP* violation, one normally chooses a convenient phase convention in writing down the Hamiltonian. This is unlikely to be the Wu-Yang phase convention, so that, if the model is not superweak, ε will have contributions from m' and $\mathrm{Im}A_0$. This is the case in the standard model, where, for all common phase conventions, $\mathrm{Im}A_0$ arises from penguin graphs.

(2) A nonzero ε' is an unambiguous indication of direct *CP* violation. It is independent of m' (or $\tilde{\varepsilon}$) and measures the difference in the *CP*-violating phase of the $I=0$ and $I=2$ amplitudes. However, even if the theory has direct *CP* violation, ε' may still be zero. This occurs if the two *CP*-violating phases are equal. This may happen accidentally, which could possibly be the case in the standard

KM model for a value of top mass m_t between 200 and 300 GeV, as discussed in Sec. V. The phases could also be equal in a model in which all $K \rightarrow 2\pi$ amplitudes have a single phase even though other $\Delta S = 1$ amplitudes may have a different phase. An example is a theory in which the *CP*-violating phase in the Hamiltonian is a relative phase between *P*-conserving and *P*-violating terms; this is the case for a particular version of the left-right model of *CP* violation (Mohapatra and Pati, 1975). In such a theory direct *CP* violation could be detected by the difference between η_{+-} for $K \rightarrow 2\pi$ and the corresponding quantity η_{+-0} for $K \rightarrow 3\pi$.

(3) The inner product

$$\langle K_L | K_S \rangle = \frac{2\,\mathrm{Re}\varepsilon}{|+|\varepsilon|^2} = \frac{2\,\mathrm{Re}\bar{\varepsilon}}{|+|\bar{\varepsilon}|^2}$$

is independent of phase convention. A nonzero value of $\mathrm{Re}\varepsilon$ therefore provides an unambiguous measure of K^0-$\bar{K}^0$ mixing; however, depending on the phase convention, this may be due to m' or γ', as is evident from Eq. (2.2). If there is direct $\Delta S = 1$ *CP* violation, one naturally expects to find *CP* violation in second-order $\Delta S = 2 K^0$-$\bar{K}^0$ mixing contributing directly to γ' and virtually to m'.

(4) The experimental determination of ε' has been carried out so far by measuring the ratio

$$\begin{aligned} R \equiv \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 &\approx 1 - 6\ \mathrm{Re}(\varepsilon'/\varepsilon) \\ &= 1 - 6[\varepsilon'/\varepsilon]\cos(\theta - \theta') \\ &\cong 1 - 6[\varepsilon'/\varepsilon]\ , \end{aligned} \tag{2.10}$$

where $[\varepsilon'/\varepsilon]$ is the real number $(\varepsilon'/\varepsilon)e^{-i(\theta'-\theta)}$. The value of θ' given in Eq. (2.5) is quite uncertain, and values as disparate as 30° to 60° can be found in the literature. However, even within this large range $\cos(\theta - \theta')$ differs from unity by less than 5%. Thus, assuming *CPT* invariance, one can directly interpret the value of $|\eta_{00}/\eta_{+-}|$ as determining the magnitude of ε'/ε and the sign of $[\varepsilon'/\varepsilon]$.

The present experimental results on *CP* violation in the K^0 system are given by (Particle Data Group, 1992)

$$\eta_{+-} = (2.279 \pm 0.022) \times 10^{-3}\ ,$$

$$\varphi_{+-} = 46.5 \pm 1.2\ ,$$

$$|\varphi_{00} - \varphi_{+-}| < 2^0\ ,$$

$$\delta = (3.27 \pm 0.12) \times 10^{-3}\ .$$

A detailed discussion of the η_{00} measurements is given in Sec. V. Since it turns out that ε'/ε is very small, to a good approximation the value of η_{+-} determines that of ε,

$$|\varepsilon| = 2.3 \times 10^{-3}\ .$$

The agreement between the phase of η_{+-} and the theoretical phase of θ of ε given in Eq. (2.9) is the source of the most precise test of *CPT* invariance mentioned earlier. The lepton asymmetry δ agrees with the $\Delta Q = \Delta S$ rule prediction $\delta = 2\,\mathrm{Re}\varepsilon$. Thus the quantity $|\varepsilon|$ provides up to this time the single independent measure of *CP* violation.

III. THE STANDARD MODEL

A. The CKM matrix

In the standard electroweak model, the interactions of the quarks with the charged gauge bosons W are given by

$$g\bar{u}_j V_{ji} \gamma_\lambda (1 - \gamma_5) d_i W^\lambda + \mathrm{H.c.} \tag{3.1}$$

Here $u_j = (u,c,t)$ are the up-type quarks and $d_j = (d,s,b)$ are the down type. V is the unitary CKM (Cabibbo-Kobayashi-Maskawa) matrix, the 3×3 generalization of the Cabibbo mixing matrix. A convenient parametrization of V due to Maiani (1977) is

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} C_\sigma C_\theta & C_\sigma S_\theta & S_\sigma e^{-i\gamma} \\ -C_\tau S_\theta - C_\theta S_\sigma S_\tau e^{i\gamma} & C_\tau C_\theta - S_\theta S_\sigma S_\tau e^{i\gamma} & C_\sigma S_\tau \\ S_\theta S_\tau - C_\theta C_\tau S_\sigma e^{i\gamma} & -C_\theta S_\tau - C_\tau S_\theta S_\sigma e^{i\gamma} & C_\tau C_\sigma \end{bmatrix}, \tag{3.2}$$

where $C_\theta = \cos\theta$ and $S_\theta = \sin\theta$. As originally noted by Kobayashi and Maskawa (1973), it is possible by defining the phase of the quark fields to eliminate all but one of the phases in V. Thus all *CP* violation in this model depends on the phase γ. Experimental data on strange-particle and B decay rates can determine the magnitudes of V_{us}, V_{cb}, and V_{ub}. Given these magnitudes, there is the empirical observation (Wolfenstein, 1983) that the mixing angles have a hierarchical structure allowing expansion in powers of $\lambda = \sin\theta = 0.22$ with

$$\sin\tau = A\lambda^2\ , \tag{3.3a}$$

$$\sin\sigma e^{-i\gamma} = A\lambda^3(\rho - i\eta)\ . \tag{3.3b}$$

The analysis of experimental data from decay rates discussed in Sec. III.C is summarized by

$$A = 0.9 \pm 0.1\ , \tag{3.4}$$

$$(\rho^2 + \eta^2)^{1/2} = 0.4 \pm 0.2\ , \tag{3.5}$$

where the errors are primarily theoretical.

Expanding V in powers of λ to order λ^3, we see that the matrix has the simple form

$$V=\begin{bmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} . \tag{3.6}$$

We have chosen a phase convention (that is, a definition of the phases of quark fields) in Eqs. (3.2) and (3.6) such that V is manifestly *CP* invariant to order λ^2, and *CP* violation shows up first in order λ^3. Of course, the physics is independent of the phase convention.

All *CP*-violating observables are proportional to a quantity J (Greenberg, 1985; Jarlskog, 1985; Wu, 1986) which is independent of phase convention

$$J=\mathrm{Im}(V_{us}V_{cb}V_{ub}^*V_{cs}^*)$$
$$=\mathrm{Im}(V_{ud}V_{tb}V_{ub}^*V_{td}^*) . \tag{3.7}$$

There are nine different ways of writing J corresponding to crossing out one row and one column of V and then multiplying together the diagonal elements of the resulting 2×2 matrix by the complex conjugates of the off-diagonal elements. From our parametrization,

$$J=C_\sigma^2 C_\tau C_\theta \sin\theta \sin\tau \sin\sigma \sin\gamma \simeq A^2\lambda^6\eta . \tag{3.8}$$

The unitarity condition can be illustrated by a triangle (Fig. 1). Using the condition

$$V_{ub}^*V_{ud}+V_{cb}^*V_{cd}+V_{tb}^*V_{td}=0$$

and setting $V_{ud}\approx V_{tb}\cong 1$, we find the condition illustrated

$$(A\lambda^3)^{-1}(V_{ub}^*+V_{td})=1 . \tag{3.9}$$

The angles of the triangle are measures of *CP* violation. The quantity J is equal to $2A^2\lambda^6$ times the area of the triangle.

B. The calculation of ε

In the standard model in our phase convention (as well as others commonly used), the main contribution to ε comes from K^0-$\bar{K}^0$ mixing, that is, the m' term in Eq. (2.7). The $\mathrm{Im}A_0$ term in Eq. (2.7) will be discussed in Sec. V.A but it makes less than a 10% correction to the value of ε.

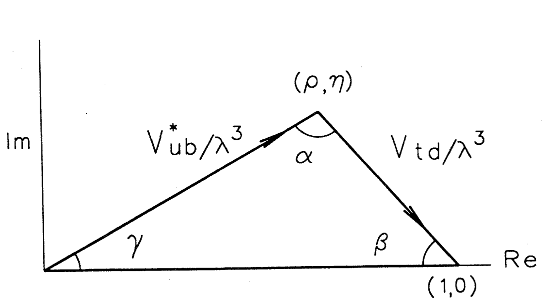

FIG. 1. The unitarity triangle.

The *CP*-violating part of the $(K^0$-$\bar{K}^0)$ mass matrix can be calculated (Ellis *et al.*, 1976) from the second-order box diagram (Fig. 2). The result of the calculation (Inami and Lim, 1981; Buras *et al.*, 1984), including QCD corrections (Gilman and Wise, 1983; Buras *et al.*, 1990; Flynn, 1990), is well represented for $m_t > m_w$ by

$$\varepsilon e^{-i\theta}=3.4\times 10^{-3}A^2\eta B\left[1+1.3A^2(1-\rho)\left[\frac{m_t}{m_w}\right]^{1.6}\right] . \tag{3.10}$$

The first term in the bracket corresponds to contributions that depend on m_c (we use $m_c=1.4$ GeV) and are approximately independent of m_t for $m_t > m_w$. The second term is the contribution that goes asymptotically as m_t^2; the form given is accurate to a few percent for $m_t < 200$ GeV. The greatest uncertainty in the theory comes from the evaluation of B, which is defined by

$$\langle \bar{K}^0|\bar{s}\gamma_\lambda(1-\gamma_5)d\bar{s}\gamma^\lambda(1-\gamma_5)d|K^0\rangle = B[4f_K^2 m_K/3] .$$

The value $B=1$ corresponds to the vacuum insertion method (Gaillard and Lee, 1974), whereas there exists a calculation based on PCAC (partial conservation of axial-vector current) and SU(3) (Donoghue *et al.*, 1983) connecting B to $\mathrm{Re}A_2$ that gives the value $B=\frac{1}{3}$. However, there are indications that there are large corrections to this calculation (Bijnens, Sonoda, and Wise, 1984). Various lattice calculations (Kilcup *et al.*, 1990; Kilcup, 1992) give values of B in the range 0.7 to 0.9, but often these same calculations give too large a value for $\mathrm{Re}A_2$.

Equation (3.10) must be used to try to determine η, since ε is the only *CP*-violating parameter we know. There exist three uncertainties (besides the value of m_t), each of the order of a factor of 2: (1) Reasonable values of B are probably between $\frac{1}{2}$ and 1; (2) there is an uncertainty in A of about 10% mainly due to the theoretical problem of deriving V_{cb} from data on semileptonic B decay; and (3) the value of ρ can be anywhere between about -0.5 and $+0.5$.

C. Constraints on the CKM parameters

The predictions of the standard model depend on the values of the CKM parameters, A, ρ, and η. At present the value of A (or V_{cb}) is constrained from the rate of $B\to D$ semileptonic decays, while the value of $(\rho^2+\eta^2)$ (or V_{ub}) is constrained from semileptonic decay rates to

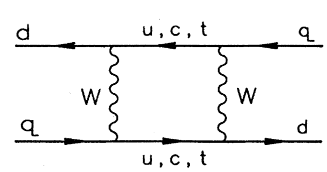

FIG. 2. Box diagram for K^0-$\bar{K}^0$ or B^0-$\bar{B}^0$ mixing. For the K^0 case, $q=s$; for the B^0 case, $q=b$.

noncharmed states.

From the B lifetime $\tau_B=(1.29\pm0.05)\times10^{-12}$ sec (Particle Data Group, 1992) and the semileptonic branching ratios for $B\rightarrow xl\nu$ of $(11\pm1)\%$, one can calculate A by using the spectator approximation, assuming the decay is primarily

$$b\rightarrow cl\nu\ ,$$

with a small correction for $b\rightarrow ul\nu$. Kim and Martin (1989) have fitted the inclusive spectrum using a method due to Altarelli *et al.* (1982) for treating the spectator; their result corresponds to

$$A=1.00\pm0.13\ .$$

The theoretical error in using the spectator model is quite uncertain. In fact, one expects the majority of the decays to go to a few states of the D system, particularly the ground states D and D^*. Therefore it would seem better to try to analyze exclusive decays.

The decays $B\rightarrow De\nu$ and $B\rightarrow D^*e\nu$ can be analyzed using the heavy-quark symmetry (Isgur and Wise, 1989). The idea is to look at the kinematic limit in which the $D(D^*)$ does not recoil. Assuming very heavy (relative to the QCD scale) b and c quarks, the weak matrix elements are essentially unity in this limit, since the transition $b\rightarrow c$ does not change the color field. Neubert (1991) has attempted to extrapolate the spectrum for $B\rightarrow D^*e\nu$ to the zero-recoil point; if the branching ratio for this decay is 4.7%, his result is

$$A=0.9\pm0.15\ .$$

An alternate analysis by Burdman (1992) yields

$$A=0.85\pm0.1\ .$$

In our later fits we use a range of A between 0.8 and 1.0.

At this time there is limited information on decays corresponding to the quark transition

$$b\rightarrow ul\nu\ .$$

This is distinguished from the predominant $b\rightarrow cl\nu$ by observing the end of the lepton spectrum. Various assumptions about the lepton spectrum (Cassel, 1992) lead to the range

$$\frac{V_{ub}}{V_{cb}}=0.085\pm0.045\ ,$$
$$\sqrt{\rho^2+\eta^2}=0.4\pm0.2\ , \tag{3.11}$$

which we use in our later fits.

The magnitude of V_{td} can be determined only from virtual processes involving the td vertex. The only available process is B_d^0-$\bar{B}_d^0$ mixing, which yields $x_d=\Delta m(B_d)/\Gamma(B_d)=0.7\pm0.1$. This can be calculated from the second-order box diagram with the top quark in the legs (Fig. 2). For m_t between 100 and 200 GeV, the theoretical result (Ellis *et al.* 1977; Inami and Lim, 1981) can be written to a good approximation as

$$\frac{\Delta m(B_d)}{\Gamma(B_d)}=\frac{1}{6}A^2[(1-\rho)^2+\eta^2]\left[\frac{m_t}{m_w}\right]^{1.6}\times\left[\frac{\sqrt{B_B\eta_2}f_B}{120\ \text{MeV}}\right]^2 . \tag{3.12}$$

Here B_B is the parameter that is unity if the vacuum insertion is used to evaluate the matrix element of the effective $\Delta B=2$ operator; values between 0.75 and 1 are given in the literature. The QCD correction factor η_2 was calculated by Hagelin (1981) as 0.85, but there may be significant corrections to this for values of $m_t>m_w$ (Buras *et al.*, 1990). The main uncertainty, however, is the value of the B-decay constant f_B; values as disparate as 75 MeV (Suzuki, 1985) and 300 MeV (Allton *et al.*, 1991) have been derived and are summarized by Geng and Turcotte (1992). We use in our fits the range

$$\sqrt{B_B\eta_2}f_B=100-200\ \text{MeV}\ .$$

This gives the result

$$A^2[(1-\rho)^2+\eta^2]=(1.5\ \text{to}\ 6)(m_t/m_w)^{-1.6}\ . \tag{3.13}$$

From Eq. (3.10), using the empirical value of ε, we have, assuming a range of B_K between 0.5 and 1.0,

$$A^2\eta=\left[\frac{3}{4}\ \text{to}\ \frac{3}{2}\right](1+1.3A^2(1-\rho)(m_t/m_w)^{1.6})^{-1}\ . \tag{3.14}$$

The constraints from Eqs. (3.11), (3.13), and (3.14) are shown in Fig. 3 for $m_t=140$ and 180 GeV. The allowed

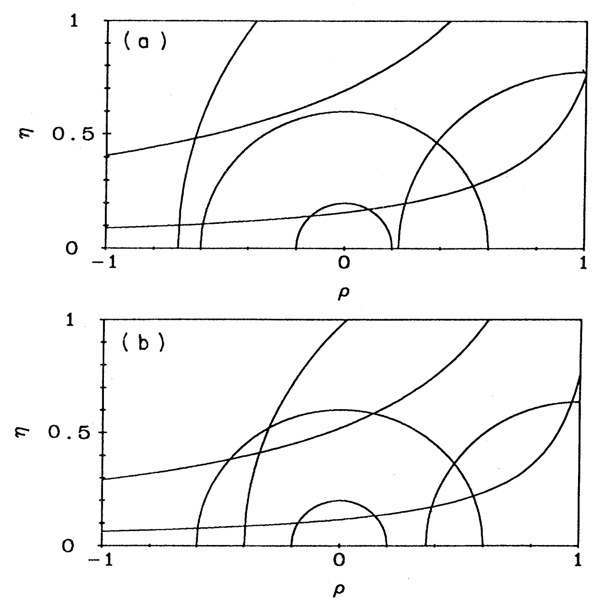

FIG. 3. Allowed regions in the (ρ,η) plane. Points between the circles centered at (0,0) are favored from determinations of V_{ub}. Points between the circles centered at (1,0) are favored from B^0-$\bar{B}^0$ mixing measurements. Points between the two hyperbolas are favored from the value of ε. (a) $m_t=140$ GeV; (b) $m_t=180$ GeV.

values of η are seen to range from 0.1 to 0.6 fairly independently of m_t. This large range arises from Eq. (3.14), because A is allowed to range from 0.8 to 1.0 and because $(1-\rho)$ is poorly constrained by Eq. (3.13).

IV. CONSTRAINING η TO BE NONZERO BY MEASURING MAGNITUDES

Within the context of the standard model, η is constrained as indicated by the *CP*-violating parameter ε. However, if we allow the alternative of superweak *CP* violation, ε does not tell us anything about η. Nevertheless, it is conceivable, assuming only three generations, to discover a nonzero η from experiments that do not measure *CP* violation. This is because the unitary 3×3 matrix is completely determined by the magnitudes of its elements. Specifically, it is clear from Eq. (3.6) that a measurement of $|V_{ub}|$ and $|V_{td}|$ determines both ρ and η, once A is known. This is illustrated in Fig. 3 by the intersection of the two circles. Thus if we used the central values of (3.11) and (3.13), we would find $\eta^2 \approx 0.16$.

The problem, of course, lies in the uncertainties in $|V_{ub}|$ and $|V_{td}|$, which are mainly theoretical in origin. With the uncertainties as large as indicated, the allowed region in Fig. 3 includes $\eta=0$, if one ignores the ε constraint as one must to establish a direct effect. Furthermore, even if the uncertainties were reduced so that $\eta=0$ were not allowed, this would not rule out the superweak alternative, because the constraint from $\Delta m(B_D)$ that we have used [Eq. (3.13)] would not be valid if $\Delta m(B_D)$ had a superweak contribution (Liu and Wolfenstein, 1987b) in addition to that of the standard model. Thus, to rule out the superweak alternative, it is necessary to use data *exclusively* from decay amplitudes. We discuss here the future possibilities for doing this, although the conclusion is that they are not, in fact, very promising.

A. The determination of $|V_{td}|$

For the foreseeable future the element V_{td} will not be determined directly from top decays, since the $t\to d+W$ branching ratio is much too small. Thus one is dependent on higher-order processes involving a virtual t quark. The most relevant processes are the induced neutral-current decays

$$b\to d+l+\bar{l}\ , \tag{4.1}$$

$$s\to d+l+\bar{l}\ . \tag{4.2}$$

While (4.1) has a larger branching ratio than (4.2), the best hope seems to be the decays of type (4.2). There are three decays of this type of particular interest,

$$K^+\to\pi^+\mu^+\mu^-\ , \tag{4.3a}$$

$$K_L\to\mu^+\mu^-\ , \tag{4.3b}$$

$$K^+\to\pi^+\nu\bar{\nu}\ . \tag{4.3c}$$

The decay (4.3a) is expected to have a branching ratio of about 10^{-7} and is presumably dominated by one-photon exchange, a weak-electromagnetic process. The transition (4.2), proportional to V_{td}, is calculated to provide only a small additional amplitude. However, as pointed out by Savage and Wise (1990), the interference between the two amplitudes is detectable by measuring the longitudinal polarization of one of the muons. We discuss this possibility below.

The decay (4.3b) has a branching ratio of 7×10^{-9} and is known to arise largely from the virtual transition

$$K_L\to\gamma+\gamma\to\mu^++\mu^-\ . \tag{4.4}$$

The absorptive part of (4.4) corresponding to the real intermediate state can be calculated accurately (Sehgal, 1969) and by itself explains a branching ratio approximately equal to the experimental rate. To this part must be added $(D+S)^2$, where D is the dispersive part of (4.4) corresponding to virtual $\gamma+\gamma$ and S is the contribution of (4.2) proportional to V_{td}. (To be more accurate, S is proportional to $\mathrm{Re}V_{td}$, since $\mathrm{Im}V_{td}$ contributes to $K_S\to\mu^+\mu^-$.) Unfortunately, the calculation of D is quite model dependent; and so, even though S may be significant, one cannot determine V_{td} with any accuracy in this way. However, various authors have tried to use this decay to place limits on V_{td} (Belanger and Geng, 1991).

Most of the emphasis has been placed on the decay (4.3c), since there is no electromagnetic contribution, and so the decay is expected to be entirely due to the second-order weak process. As discussed below, the branching ratio is only of the order 10^{-10} and the detection is difficult.

The rare decay $K^+\to\pi^+\nu\bar{\nu}$ can be calculated for large values of m_t in terms of the CKM elements without any problem of hadronic matrix elements. The calculation involves a box diagram with intermediate u,c,t quarks on one side and e, μ, or τ on the other. One must also include the flavor-changing Z exchange, which occurs in second-order weak interactions. The diagrams are shown in Fig. 4. The result is an effective matrix element of the form (Hagelin and Littenberg, 1989)

$$\frac{G_F}{\sqrt{2}}\frac{e^2}{4\pi}C\bar{s}_\alpha\gamma_\mu(1-\gamma_5)d_\alpha\bar{\nu}_l\gamma^\mu(1-\gamma_5)\nu_l\ , \tag{4.5}$$

where α is a color index summed over and l is the lepton flavor. The hadronic matrix element is directly known from the decay $K^+\to\pi^0e^+\bar{\nu}_e$.

The coefficient C has the general form

$$C=\sum_{u,c,t}U_{qs}^*U_{qd}\tilde{C}_{\nu,q}\ ,$$

where $\tilde{C}_{\nu,q}$ have been calculated by Inami and Lim (1981). The $\tilde{C}_{\nu,q}$ are functions of $X_q=(m_q^2/m_w^2)$ and, because of the GIM (Glashow-Iliopolis-Maiani) cancellation, vanish as X_q goes to zero. Thus the t contribution dominates for large m_t even though it is suppressed by λ^4 compared to the others. However, the c contribution

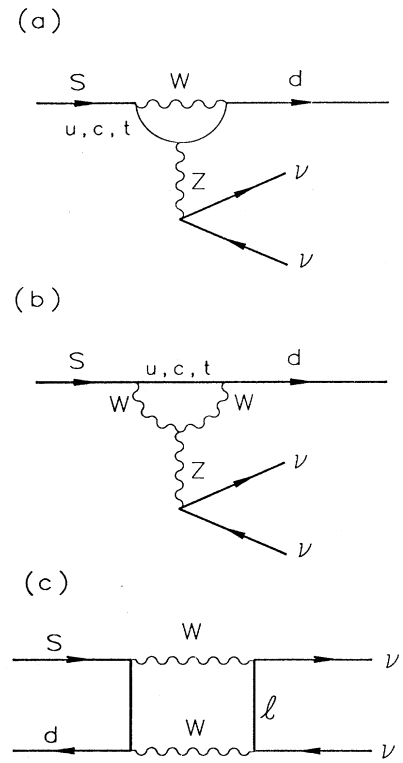

FIG. 4. Second-order electroweak diagrams contributing to $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L^0 \to \pi^0 \nu\bar{\nu}$.

remains important; it is proportional to

$$\tilde{C}_{\nu,c} \propto (X_c \ln X_c - \tfrac{1}{3} X_c) .$$

Assuming m_c lies between 1.3 and 1.7 GeV, we see there is an uncertainty of $\pm 22\%$ in the value of $\tilde{C}_{\nu,c}$. It is customary to set m_u and thus $\tilde{C}_{\nu,u}$ equal to zero; it is not clear whether this introduces a significant additional error. One must also include the QCD correction calculated by Dib *et al.* (1989a) that depresses $\tilde{C}_{\nu,c}$ by 30%.

The final result for the branching ratio given by Dib *et al.* (1989a) summed over three neutrino flavors can be written

$$B(K^{\pm} \to \pi^{\pm} \nu\bar{\nu}) = 6\alpha^2 B(K^+ \to \pi^0 e^+ \nu_e) |C_\nu|^2$$
$$= 1.5 \times 10^{-5} |C_\nu|^2 ,$$
$$C_\nu = 10^{-3} \{ (1 \pm 0.25) + 1.1 A^2 (1 - \rho - i\eta) X_t^{0.55} \} . \quad (4.6)$$

The first term in C_ν is the charm term, where we estimate a total uncertainty of 25% mainly due to m_c. The X_t dependence is a very good approximation to the exact form of $\tilde{C}_{\nu,t}$ for m_t between 100 and 200 GeV. For $m_t \sim 140$ GeV ($X_t = 3$), the m_t term is 1 to 3 times larger than the first term, depending mainly on the value of ρ. The measurement of the branching ratio primarily determines ρ with a small dependence on η^2. For example, if ρ is near zero, a measurement of the branching ratio to $\pm 20\%$ would determine $A^2(1-\rho)$ with a 15% uncertainty due to m_c and another 15% uncertainty due to the experimental error, assuming m_t has been determined to be 140 GeV. To use this result to form the unitarity triangle (Dib, 1992) requires a knowledge of V_{td}/V_{cb} and thus of $(1-\rho)$ rather than $A^2(1-\rho)$. It follows that a 10% error in A gives another 20% uncertainty. For the optimum case of ρ near zero, therefore, the error in ρ will be at least ± 0.3.

The predicted branching ratio is about 1.5×10^{-10} within a factor of about 3 either way, depending on m_t and ρ. The present limit on the branching ratio is 5×10^{-9}, and an ongoing experiment (E787) at Brookhaven should see a few events, if the branching ratio is 2×10^{-10}. A measurement of the branching ratio to an accuracy of 20% probably awaits a much more intense source of kaons.

The same electroweak diagrams that contribute to $K^+ \to \pi^+ \nu\bar{\nu}$ also contribute to the decay $K^+ \to \pi^+ \mu^+ \mu^-$. However, in this case the primary contribution to the decay arises from the single-photon exchange, a weak-electromagnetic process, giving a branching ratio of order 10^{-7}. It was pointed out by Savage and Wise (1990) that the interference between the electroweak amplitude and the leading amplitude produces a polarization of either of the muons. For these purposes the effective matrix element can be written

$$\frac{G_F}{\sqrt{2}} \frac{e^2}{4\pi} \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha \{ A \bar{\mu} \gamma_\mu \mu + C \bar{\mu} \gamma^\mu \gamma_5 \mu \} ,$$

where A is the dominant photon-exchange contribution whose magnitude can be determined from the measured rate. The factor C is the same as in Eq. (4.5); only the axial coupling of the μ is relevant for the polarization. Unfortunately, the expected polarization is 0.5% at most.

While the branching ratio for decays of form (4.1) seem too small, Soares (1992) and Ali and Greub (1992) have analyzed the possibility of using the branching ratio for $B \to \rho + \gamma$, which is proportional to $|V_{td}|^2$ and expected to be 10^{-6}. Soares has analyzed the problems in relating this decay to the observed decay $B \to K^* + \gamma$ in order to reduce the uncertainty of the hadronic matrix element.

B. The determination of $|V_{ub}|$

The element V_{ub} is to be determined by direct observation of decays of the form $b \to u + l + \nu$. There are three possibilities: (1) inclusive decays; (2) exclusive semileptonic decays like $B \to \pi l \nu$ or $B \to \rho l \nu$; and (3) the pure leptonic decays $B^+ \to \tau^+ \nu_\tau$ or $B^+ \to \mu^+ \nu_\mu$.

The study of the inclusive decays depends on observations of the high-energy electron or muon beyond or near the end of the spectrum for $B \to D l \nu$. This part of the spectrum necessarily includes a significant contribution from low-mass final states like π, ρ, and A_2, but also important contributions from multipion "nonresonant" states. Some calculations model all the final states as resonances (Isgur *et al.*, 1989); some use a form of the parton model (Altarelli *et al.*, 1982); and some try to combine the two (Ramirez *et al.*, 1990). Because such a small part of the spectrum can be studied, there is a sizable model dependence on the theoretical predictions. Much of the error of 50% on $(\rho^2+\eta^2)^{1/2}$ in Eq. (3.11) is theoretical in origin, and it is not clear that it can be reduced to less than about 30%.

In the case of decays of the form $b\to cl\nu$, it is generally believed that the exclusive decays can be analyzed theoretically with considerable accuracy as a result of the heavy-quark symmetry. No such simple analysis is possible for the exclusive decays $B\to\pi l\nu$ or $B\to\rho l\nu$. A number of authors (Burdman and Donoghue, 1992b; Wise, 1992) have combined the heavy-quark symmetry with chiral symmetry to treat the decay $B\to\pi l\nu$, but it is not clear that quantitative results are possible. It has also been suggested that one could use the heavy-quark symmetry to relate decays such as $B\to\rho l\nu$ to $D\to\rho l\nu$, since the KM element V_{cd} is known from unitarity. However, it is not clear whether this relation is any better than the scaling relation of f_B to f_D, which is now quite suspect. Another possibility (Burdman and Donoghue, 1992a) is to look at the ratio of $B\to\rho l\nu$ to $B\to K^*\gamma$, since the matrix elements are related by SU(3) at one point in the Dalitz plot. All in all, it is not clear that exclusive semileptonic decays will prove better than inclusive.

The purely leptonic decays $B\to l\nu$ are directly proportional to $|V_{ub}|^2$. The rate is given by

$$\Gamma(B\to l\nu)=G^2f_B^2|V_{ub}|^2m_l^2(m_B^2-m_l^2)^2/8\pi m_B^3 \ .$$

This gives a branching ratio

$$B_\tau=B(B^+\to\tau^+\nu_\tau)=2.3(f_B/f_\pi)^2|V_{ub}|^2$$

of the order 10^{-4}. The decay $B^+\to\mu^+\nu_\mu$ is lower by another factor of 200. Such decays by themselves cannot improve the determination of V_{ub} because of the uncertainty in f_B.

V. $K\to 2\pi$

A. ε' in the standard model

Within the standard model, one expects nonleptonic K decay to be dominated by the tree graph (W exchange) producing the quark transition

$$s\to u+\bar{u}+d \ . \tag{5.1}$$

If this is the only amplitude, then there can be no direct CP violation and $\varepsilon'=0$, since all nonleptonic decay amplitudes would have the common phase factor $V_{us}V_{ud}^*$. Gilman and Wise (1979) noted that a significant value of ε' could arise from "penguin graphs." These involve a loop diagram (Fig. 5) leading to

$$s\to d+g \ , \tag{5.2}$$

where g is a gluon. Because the loop involves the c and t quarks as well as u quarks, the complex factors $V_{cs}V_{cd}^*$ and $V_{ts}V_{td}^*$ are involved. The penguin graphs lead to an

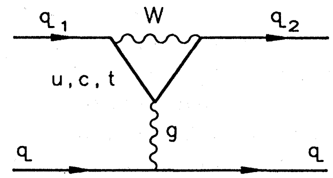

FIG. 5. Penguin diagram for direct CP violation in K and B decays. For K decays, $q_1=s$ and $q_2=d$. For B decays, $q_1=b$ and $q_2=d$ or s.

effective four-quark operator of the form

$$\bar{s}\lambda^\alpha\gamma^\lambda(1-\gamma^5)d\sum_q\bar{q}\lambda_\alpha\gamma_\lambda q \ .$$

This operator corresponds to gluon exchange and thus involves two currents that are octets under SU(3) (λ^α is the octet operator). The penguin operator has two important features: (1) It transforms as $\Delta I=\frac{1}{2}$, because the second factor involves a sum over quark flavors. (2) It is not purely left-handed, because the gluon coupling is pure vector, that is, left plus right. The right-handed current coupled to the left-handed current yields an effective scalar operator Q_6 after a Fierz transformation

$$Q_6=\sum_q\bar{s}(1+\gamma_5)q\bar{q}(1-\gamma_5)d \ . \tag{5.3}$$

The relatively large values expected for the matrix element of Q_6 enhance the importance of penguin graphs.

As a result of the $\Delta I=\frac{1}{2}$ property, the penguin operator contributes only to $\mathrm{Im}A_0$. A theoretical value of ε'/ε can then be obtained from Eq. (2.8) using empirical values for ε and $\mathrm{Re}A_0$. Many calculations have been performed over the years; for values of $m_t\lesssim m_w$, the main uncertainty concerns the matrix element of Q_6 (Donoghue, Golowich, and Holstein, 1986; Bardeen *et al.*, 1987). The calculation of Buras and his collaborators (Buchalla *et al.*, 1990; Buras *et al.*, 1993) yields for $m_t\approx m_w$

$$\varepsilon'/\varepsilon=(1 \text{ to } 6)\times10^{-3}A^2\eta \ . \tag{5.4}$$

In this calculation the matrix element depends on m_s (the strange-quark mass) and Λ_{QCD}, and the range of values corresponds to (m_s,Λ) between (200 MeV, 100 MeV) and (125 MeV, 300 MeV). Paschos and his collaborators (Paschos and Wu, 1991; Heinrich *et al.*, 1992) claim that the $\Delta I=\frac{1}{2}$ operators are enhanced in higher-order chiral perturbation theory, which can be interpreted as the final-state-interaction effect in the $I=0$ state. Their results for $m_t\sim m_w$ are about a factor of 2 higher,

$$\varepsilon'/\varepsilon=(4 \text{ to } 8)\times10^{-3}A^2\eta \ , \tag{5.5}$$

for m_s ranging from 175 to 125 MeV and $\Lambda_{\mathrm{QCD}}=200$ MeV. Since fits to the CKM matrix for $m_t\sim100$ GeV give $A^2\eta\sim0.3$ within a factor of 2, Eqs. (5.4) and (5.5) suggest that ε'/ε should be equal to 10^{-3} within a factor of about 4 either way.

From Eq. (2.8) it is evident that a value of $\mathrm{Im}\,A_2$ much smaller than $\mathrm{Im}\,A_0$ can be of importance, because $\mathrm{Re}\,A_2$ is much smaller empirically than $\mathrm{Re}\,A_0$. Taking into account isospin violations, particularly $\pi^0-\eta-\eta'$ mixing (Donoghue, Golowich, Holstein, and Trampetic, 1986), can result in a nonzero $\mathrm{Im}\,A_2$ starting with the operator Q_6. Calculations (Buchalla *et al.*, 1990) indicate this decreases ε'/ε by about 30%, and this correction has been included in Eqs. (5.4) and (5.5). A more important issue is the "electroweak penguin" corresponding to loop diagrams (Fig. 6) that yield the transitions

$$s \to d+\gamma \ , \tag{5.6a}$$

$$s \to d+Z \ . \tag{5.6b}$$

Because of the isovector coupling of the photon and the Z to the quarks, these transitions lead to effective $\Delta I=\frac{3}{2}$ operators. A complete gauge-invariant calculation includes box diagrams (Fig. 6) as well. When the effect of (5.6a) was introduced by Bijnens and Wise (1984), it was found to be not very important given the other uncertainty. However, as m_t increases above m_w, the contribution of (5.6b) begins to increase roughly as m_t^2. Calculations show that the resulting phase φ_2 (corresponding to $\mathrm{Im}\,A_2/\mathrm{Re}\,A_2$) has the same sign as φ_0 (corresponding to $\mathrm{Im}\,A_0/\mathrm{Re}\,A_0$), so that the two terms in Eq. (2.8) tend to cancel (Flynn and Randall, 1989c) as m_t increases.

Detailed calculations of the electroweak penguin have

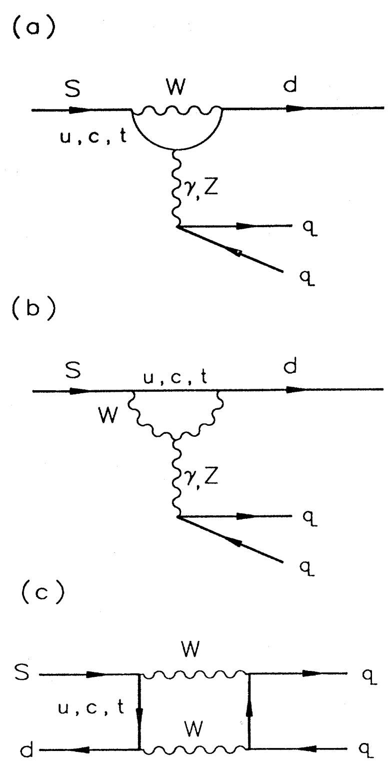

FIG. 6. Second-order electroweak diagrams contributing to ε'. These same diagrams contribute to $K_L\to\pi^0e^+e^-$ when qq is replaced by ee.

been given by a number of authors. The Buras group (Buchalla *et al.*, 1991) concludes that as m_t goes well above m_w, the result of Eq. (5.4) must be multiplied by

$$(1+P_yX_t^{0.65}+0.18P_zX_t^{0.93}) \ , \tag{5.7}$$

where $X_t=(m_t/m_w)^2$, $P_y\le 0.1$, and $P_z\approx -1.1$. It is the P_z term associated with Z exchange that is important and gives the result that ε'/ε actually is expected to vanish for m_t somewhere between 200 and 250 GeV. For $m_t\approx 2m_w$, there is a reduction by a factor of about 2. The Paschos group emphasizes that the cancellation is between the amplitude of the usual penguin going to the $I=0$ state and that of the electroweak penguin going to the $I=2$ state. In their calculation, final-state interactions enhance the amplitude to the $I=0$ state and depress that to the $I=2$ state. As a result, they predict a much smaller decrease with m_t, and ε'/ε does not vanish for any value of m_t. For a fixed value of $A^2\eta$, they find only about a 15% decrease due to the electroweak penguin for $m_t\approx 2m_w$.

In spite of some disagreements, three recent detailed calculations (Heinrich *et al.*, 1992; Buras *et al.*, 1993; Ciuchina *et al.*, 1993) give similar predictions with large theoretical uncertainties. In particular, given the present indications that m_t lies between 100 and 200 GeV, the standard model clearly predicts a nonzero positive value for ε'/ε. The present theoretical uncertainties in the evaluation of matrix elements together with the uncertainty in the value of η allow for a large range of values from as low as 10^{-4} to as high as 3×10^{-3}.

B. Experimental results on ε'/ε

Here we discuss what is currently known of the parameter ε'/ε and what is likely to be learned in the near future. The three active experiments will be described in this section. There are two recent determinations, by the E731 group at Fermilab and by the NA31 group at CERN, and a third group—CPLEAR (low-energy antiproton ring for the study of *CP* violation)—is taking data.

1. The CPLEAR experiment at CERN

This experiment (PS195) is a collaboration of Athens, Basel, Boston, CERN, Coimbra, Delft, Fribourg, Ioannina, Ljubljana, Liverpool, Marseille, Saclay, PSI (Paul-Scherrer-Institute, Switzerland), Stockholm, Thessaloniki, and IMP/ETH Zurich. The group is studying the tagged decays of $K^0\to\pi^+\pi^-$ and $\pi^0\pi^0$ through antiproton annihilations at rest to $K^-\pi^+K^0$ and its conjugate. Since the K^0 (anti K^0) $\propto K_S\pm K_L$, any *CP*-violating interference term in the particle decay will have opposite sign from the antiparticle decay. From a small amount of data taken in 1990 (Adler *et al.*, 1992), the group has clearly observed such an interference term. This leads to the results

$$\eta_{+-}=2.24(16)\times10^{-3} \text{ and } \Phi_{+-}=46.8^\circ+3.5^\circ ,$$

both in good agreement with the values reported by the Particle Data Group.

With about 200 days of running, the group expects errors of less than 1° for φ_{+-} and for the phase difference $\varphi_{+-}-\varphi_{00}$ and an error of about 1.5×10^{-3} on ε'/ε.

2. The E731 Fermilab and NA31 CERN approaches

The first Fermilab experiment on ε'/ε was E617. Proposed in 1979, it ran in 1982. The result (Bernstein *et al.*, 1985) was consistent with zero with a precision of 0.006. A BNL experiment (Black *et al.*, 1985) at about the same time had similar sensitivity. In 1983, the Fermilab group (E731, a collaboration from Chicago, Elmhurst, Fermilab, Illinois, and Saclay) proposed a new beam to make the measurement with a precision of 0.001. A year later, a CERN group (NA31, a collaboration from CERN, Edinburgh, Mainz, Orsay, Pisa, and Siegen) proposed a measurement with similar sensitivity. E731 had a brief test run in 1985, with the result $\varepsilon'/\varepsilon=0.0032\pm0.0030$ (Woods *et al.*, 1988). For that test run, it was required that one photon convert in a thin conversion plane in the middle of the decay region. This was done to give more information on the decay point. An extensive upgrade followed the 1985 run, which permitted the use of events with no conversion and thus much higher statistics. E731 then had one run, for five months, in the 1987/88 fixed-target run. The CERN experiment ran in 1986, 1988, and 1989; in 1987 they took data to measure Φ_{+-} and Φ_{00}. Results are available from both collaborations, and we shall make a comparative analysis here.

To extract ε'/ε, one clearly needs to *count* accurately decays of K_S and K_L into both $\pi^+\pi^-$ and $2\pi^0$. We can largely treat these experiments as counting experiments and discuss their systematics accordingly. To be sure, there are other systematic effects associated with calibration and with background corrections, but the experiments are not particularly different with respect to these effects.

The NA31 experiment uses purely calorimetric detectors for the reconstruction of the particle energies. The NA31 detector is shown in Fig. 7. This is the first attempt to do a high-precision kaon-decay experiment without a magnet. The K_S are derived from a close target which moves throughout the decay region, thereby very much reducing the acceptance corrections for K_S vs K_L decays. Both K_S decay modes are collected at once, alternating with the taking of both K_L modes. The two modes use essentially orthogonal detectors; so the stability of this technique in the presence of pileup effects, detector drifts, and changing accelerator and background conditions needs to be closely monitored.

The E731 experiment uses a more conventional magnetic spectrometer. The E731 detector is shown in Fig. 8. The K_S are derived from a (fixed) regenerator placed in one of two K_L beams. Decays from both beams (K_L

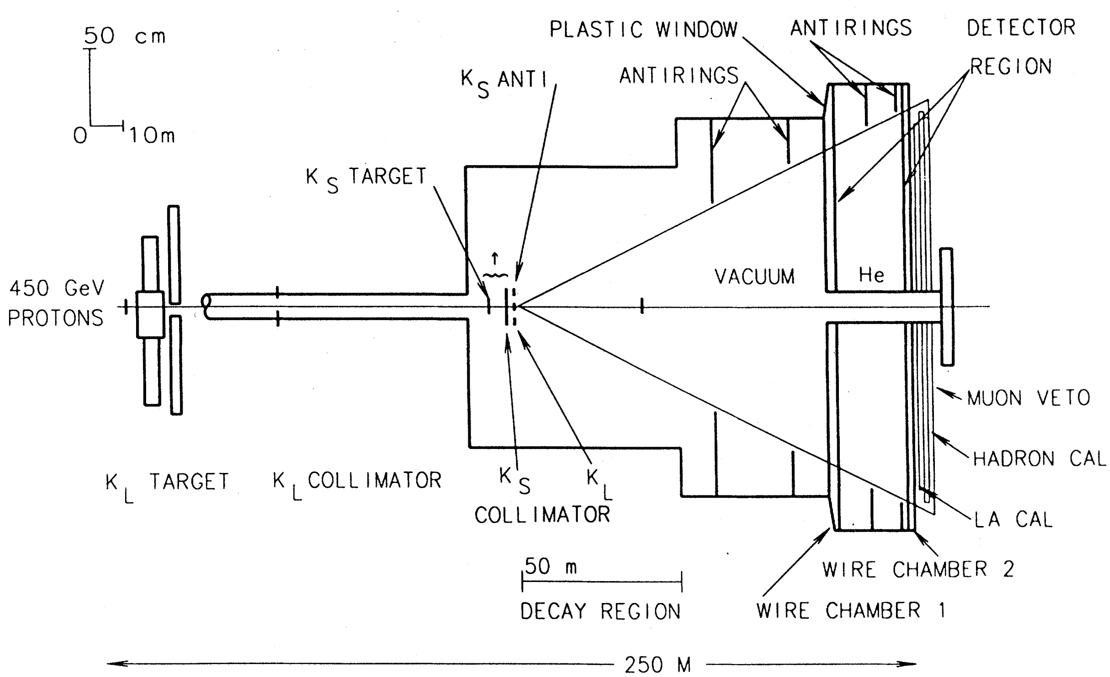

FIG. 7. Schematic of NA31 detector.

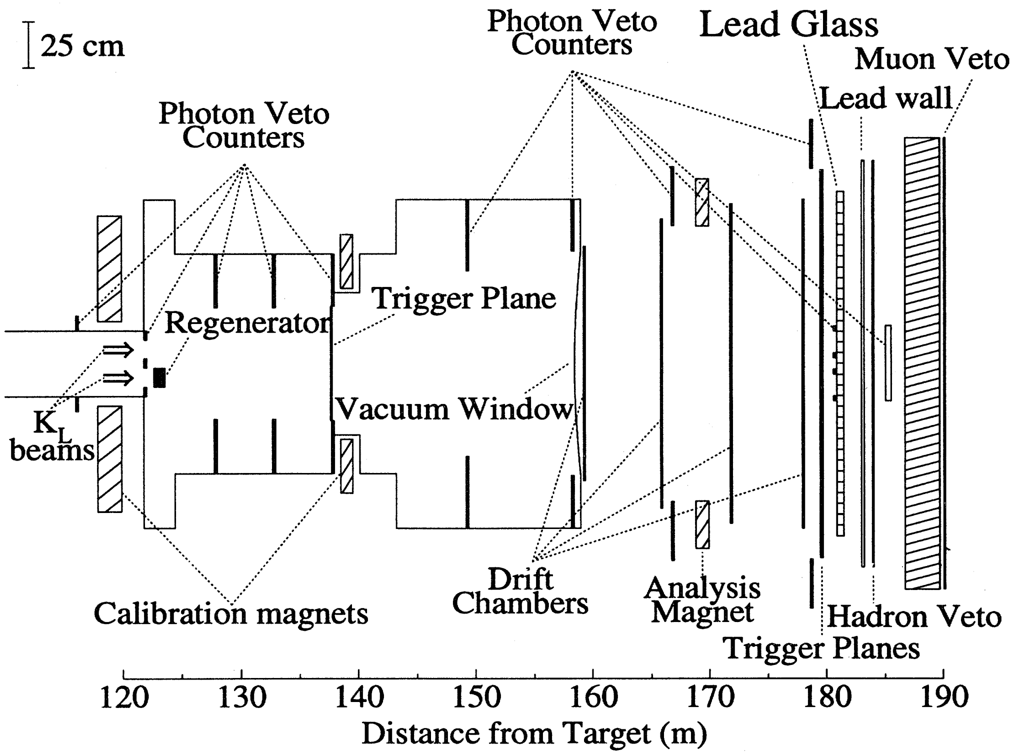

FIG. 8. Schematic of E731 detector.

and K_S) are collected simultaneously. Thus both $2\pi^0$ modes are collected, alternating with the taking of both $\pi^+\pi^-$ modes. As such, rate effects, detector and accelerator drifts become negligible. However, because of the difference in lifetimes (which translates into a difference in decay vertex position), one needs to use a Monte Carlo for the detector acceptance.

For accurate counting of decays, one needs to worry about losses. For NA31, the primary concern is losses at the data collection and analysis stages, while for E731 it is mostly in the acceptance corrections. These different sensitivities arise from the major differences in the experimental techniques for the measurement of the four decay rates.

Some other relevant factors in the comparison of the two experiments are the following. The E731 selection criteria are blind to K_S vs K_L, the distinction being made only at the level of background subtraction. In other words, track quality cuts, fiducial cuts, and kinematic reconstruction and selection are done without knowledge of from which beam the kaon decayed. The NA31 selection criteria discard about 40% of otherwise good $\pi^+\pi^-$ decays based upon hadron calorimetry. The lack of a magnet necessitates this rather large loss in order to keep the $\pi e \nu$ background at a manageable level. This loss must be absolutely stable; it has been checked using a transition radiation detector that was installed for the '88 run. The NA31 electromagnetic calorimeter (lead liquid Ar) is superior to that of E731 (lead glass): its energy and position resolution are $0.5\% + 8\%/\sqrt{E}$ and 0.8 mm, respectively, compared to $1.5\% + 5\%/\sqrt{E}$ and 3.5 mm. The E731 tracking chambers have 100 μm plane resolution compared to about 750 μm for those of NA31.

3. Results from the two experiments

Based upon the data taken in 1986, the NA31 group announced (Burkhardt *et al.*, 1988) a departure in the double ratio (2.10) of 2.0% with a statistical error of 0.4% and a systematic error of 0.5%. This gives the result

$$\mathrm{Re}(\varepsilon'/\varepsilon) = (33 \pm 6.7 \pm 8.3) \times 10^{-4} \quad (\mathrm{NA31,\ '86}) .$$

The first result from the E731 group was taken from an analysis of 20% of the data sample. For this sample, all four modes were collected simultaneously (Patterson *et al.*, 1990). The result was

$$\mathrm{Re}(\varepsilon'/\varepsilon) = (-4 \pm 14 \pm 6) \times 10^{-4} \quad (\mathrm{E731,\ 20\%\ sample}) .$$

The results were not in the best of agreement: the NA31 result indicated a significantly nonzero effect, while the E731 result was still consistent with zero.

The E731 result from the full data set is now available (Gibbons *et al.*, 1993a). As a result of improvements to the calibration techniques and in the understanding of the detector, the systematic error was reduced by a factor of 2. An enlarged fiducial region was also used for the final analysis. With the new techniques, the 20% data sample was reanalyzed with the result changing by 1.6×10^{-4}, well within the systematic error. The result is then

$$\mathrm{Re}(\varepsilon'/\varepsilon)=(7.4\pm5.2\pm2.9)\times10^{-4}\quad\text{(E731, final result)}\ .$$

This implies that $\mathrm{Re}(\varepsilon'/\varepsilon)\leq 17\times10^{-4}$ (95% confidence) and so does not support the rather large result from NA31.

At this time, these are the only published results available. In the summer of 1991, the NA31 group gave preliminary results for their '88 and '89 runs (Barr, 1992). The following were presented:

$$\mathrm{Re}(\varepsilon'/\varepsilon)=(17\pm6.5\pm7)\times10^{-4}\quad\text{(NA31,'88 preliminary)}$$

and

$$\mathrm{Re}(\varepsilon'/\varepsilon)=(21\pm5.3\pm7)\times10^{-4}\quad\text{(NA31,'89 preliminary)}\ .$$

The combined result, taking into account correlated systematic errors, is

$$\mathrm{Re}(\varepsilon'/\varepsilon)=(23\pm3.5\pm6)\times10^{-4}\quad\text{(NA31,'86, '88, '89 preliminary)}\ .$$

This is more than three standard deviations from zero and thus would appear to rule out the superweak model with very high confidence. The error is dominantly systematic.

The E731 group has also reported (Gibbons *et al.*, 1993b) precise values for other parameters of the neutral-kaon system determined from the 2π data samples. The results are

$$\tau_S=(0.8929\pm0.0016)\times10^{-10}\ \text{sec}\ ,$$

$$\Delta m=(0.5286\pm0.0028)\times10^{10}\ \text{sec}^{-1}\ ,$$

$$\Phi_{+-}-\Phi_{00}=(-1.6\pm1.2)^\circ\ ,$$

$$\Phi_{+-}=(42.2\pm1.4)^\circ\ .$$

We give the statistical data samples in Table I and background levels in Table II for the three runs of NA31 and the one run of E731.

TABLE I. Statistical samples (10^3 events) for NA31 and E731.

	NA31 '86	NA31 '88	NA31 '89	E731
$K_L\to2\pi^0$	109	110	180	410
$K_L\to\pi^+\pi^-$	295	290	470	327
$K_S\to2\pi^0$	932	560	630	800
$K_S\to\pi^+\pi^-$	2300	1380	1530	1060
Error in R (2.10)	0.38%	0.39%	0.32%	0.28%

TABLE II. Background levels (%) for NA31 and E731.

	NA31 '86	NA31 '88	NA31 '89	E731
$K_L\to2\pi^0$ ($3\pi^0$)	4.0	3.2	2.6	1.78
$K_L\to\pi^+\pi^-$	0.6	0.9		0.32
% shift in R (2.10) from "accidentals"	−0.34	0.05	−0.48	−0.10

4. Discussion of the two experiments

From the above tables, we can discern the following. The statistical precision for the E731 data run exceeds that of any of the NA31 runs. However, combining the three runs of NA31 gives a statistical precision about 30% better than E731.

Mostly because of the shorter decay region, the residual background from $3\pi^0$ under the $K_L\to2\pi^0$ signal in E731 is about 50% smaller than in NA31. This is important, since the number of the residual $3\pi^0$ events is determined by a complex simulation, requiring, for example, knowledge of the efficiency of the many veto counters for very low-energy photons. E731 also has the "crossover" background, where incoherent K_S from the regenerator reconstruct in the vacuum beam. While this background is at the 2.5% level, it is argued that it is nearly canceled by incoherent events which remain in the regenerator beam. In addition, this effect is precisely measured, with the well-reconstructed incoherent events decaying into $\pi^+\pi^-$; and it is this distribution which is used for the subtractions in the $\pi^0\pi^0$ samples.

The background in the $K_L\to\pi^+\pi^-$ sample is significantly smaller in E731. This is mostly because of the magnetic spectrometer. The determination of the level of residual events is more complex in NA31 because of the poorer mass resolution: 23 MeV/c^2 in NA31; 3.5 MeV/c^2 in E731.

The shift in the double ratio from accidental activity is generally greater in NA31 than in E731. This arises simply from the fact that, in E731, the two neutral or charged modes are collected at the same time.

The most significant differences between the two experiments are likely the following. NA31 has to be concerned that collecting the two $\pi^+\pi^-$ ($\pi^0\pi^0$) samples at different times under different rate conditions will introduce shifts in the overall efficiency. This is not an issue for E731. On the other hand, E731 has to be concerned about the large relative acceptance corrections that need to be made. This is not an issue for NA31 because of the motion of the K_S source throughout the K_L decay region. To address this problem, E731 has made heavy use of the decays $K_L\to3\pi^0$ and $K_L\to\pi e\nu$, which provide checks of the understanding of the detector; the vertex distributions (data and Monte Carlo) for these as well as for the *CP*-violating modes are given in Gibbons *et al.* (1993a).

The recent ε'/ε results are given in Table III.

TABLE III. Results on ε'/ε (10^{-4}).

Data set	ε'/ε	Statistical error	Systematic error	Total error
BNL-Yale	17	72	43	83
Chicago-Saclay	−46	53	24	58
E731 Test	32	28	12	30
NA31 '86	33	7	8	11
E731 20%	−4	14	6	15
E731 Full	**7.4**	**5.2**	**2.9**	**5.9**
NA31 '88[a]	17	6.5	7	10
NA31 '89[a]	21	5.3	7	9
NA31 Full[a]	**23**	**3.5**	**6**	**7**

[a]Preliminary.

5. Summary of current knowledge of ε'/ε

The question arises as to what to use for the result or, in other words, how to average the information from the two experiments. There are always difficulties in the averaging of results from different experiments. In this particular case, we first have the problem that the E731 result is final while the NA31 results from 1988 and 1989 data are still preliminary. Second, the E731 error is dominated by statistical uncertainty, whereas, as it now stands, those for NA31 are systematics dominated; and the procedure for averaging in such a case is somewhat cloudy. Third, when the central values are not in the best of agreement, there are again difficulties.

As is standard practice, we shall use all data whose results are within a factor of 3 in accuracy of the best result. Then we need only use the NA31 and E731 results in the averaging. We shall adopt the approach of the Particle Data Group, which has a well-known procedure for error inflation when central values are in disagreement. First consider only the published data from both groups; then the average value is 13.1, with a χ^2 value of 4.2. The procedure is then to inflate the error by $\sqrt{\chi^2}$, so that we have

$$\mathrm{Re}(\varepsilon'/\varepsilon)=(13\pm11)\times10^{-4} \quad \text{(published results)} .$$

The same procedure, using the NA31 preliminary result, gives an average value of 13.9 with a χ^2 value of 2.9. We then have

$$\mathrm{Re}(\varepsilon'/\varepsilon)=(14\pm8)\times10^{-4}$$
$$\text{(E731 published, NA31 preliminary)} .$$

In any case, while the average is well within the range expected in the standard model, the evidence for a nonzero effect is less than two standard deviations.

6. Upgrade plans of NA31 and E731

Each of the two groups has proposed to make improved measurements, and both groups are now approved to do so. These measurements should commence sometime in 1995, and the goals are a determination of ε'/ε with a precision in the range $1-2\times10^{-4}$. Fermilab E832 (Arisaka *et al.*, 1990) will use the identical technique of E731 with a new beam and detector; CERN NA48 (Barr *et al.*, 1990b) will use a new technique to be discussed below, again with a new beam and detector.

With the attainment of this precision, we have seen from the previous section that if the standard model is the right model for *CP* violation, there is a very good chance that a nonzero effect will be established. In this section, we shall discuss the necessary modifications in order to reach this level of precision.

First, without distinguishing the two experiments, we shall make some general observations. In order to achieve 10^{-4} precision, at a minimum one must collect 3 million $K_L\rightarrow2\pi^0$ decays. These decays are usually the mode with the limiting statistics; however, since a part of the statistical error will result from the three other more copious decays, a number more like 4 or 5 million is actually required. This is about one order of magnitude greater than either group has collected. The increase results from the building of beams that can deliver higher intensity, choosing detector elements that can handle higher rates, and improving the data-acquisition hardware in order to have greater live-time.

We now turn to sources of systematic error. In general, there are four classes of systematic uncertainty.

The first is due to backgrounds; each of the four modes will have some residual background which must be determined, modeled, and subtracted. It is the uncertainty in the subtracted events that is the source of the systematic error.

The second is due to energy calibration. The (double) ratio of decay rates is measured in the laboratory; however, it is required *in the center-of-mass system.* In order to boost back to the center-of-mass system, the reconstructed kaon energy must be accurately known. In fact, it is the *relative* energy scale, between the charged decay products and the neutral decay products, that is needed.

The third is due to acceptance. The K_S and K_L have different decay distributions, and an apparatus with a vertex-dependent acceptance will introduce a bias which needs to be determined, modeled, and corrected.

The fourth source is due to residual activity in the detector not associated with the event in question; it goes by the name "accidentals." Such activity can reject an event or cause an otherwise unacceptable event to be counted, and these gains and losses can be mode dependent. Such effects need again to be modeled and corrected for.

We shall now treat the approaches of each of the two groups regarding these classes of systematics.

Regarding backgrounds, for E731 and NA31, the worst backgrounds were at the level of a few percent, and so these needed to be modeled carefully: a 1% shift in the double ratio corresponds to 1.6×10^{-3} in ε'/ε. For the new efforts, the background levels will be substantially reduced, so much so that detailed modeling of the residual effects will likely not be necessary.

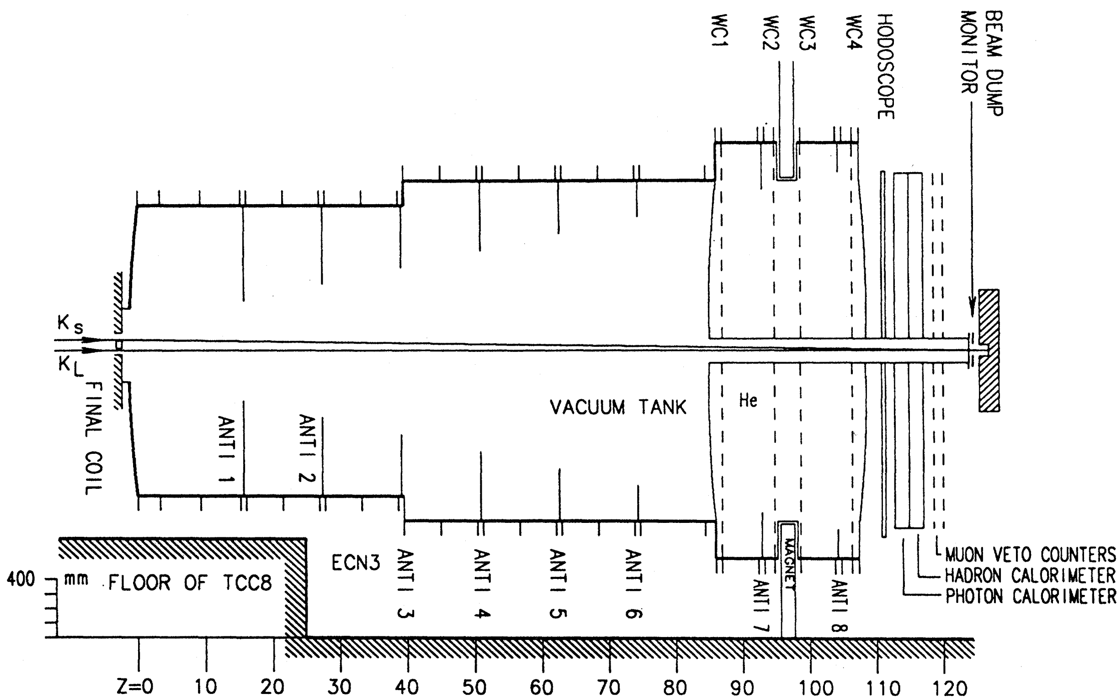

FIG. 9. The layout of the CERN NA48 experiment.

The reduction of the backgrounds is a direct result of the new electromagnetic calorimeters used by both groups for photon and electron energy measurements. These devices will offer the greatest possible precision. The $\pi^0\pi^0$ decays will be reconstructed with excellent resolution, and this will result in greatly increased background rejection. Background in $\pi^+\pi^-$ comes largely from $\pi e\nu$ decays, and the new calorimeters, with much better "E/p" resolution, will greatly reduce this as well. Figure 9 shows the new NA48 detector; Fig. 10 shows the new E832 detector.

The E832 group will use crystals of pure CsI, while NA48 will use a detector of liquid krypton. Both of these devices are quite sophisticated, and their construction and installation are critical path items for the respective detectors. Although successful prototype detectors for both technologies have been built, there is no experience with large detectors of either type at present. Both give

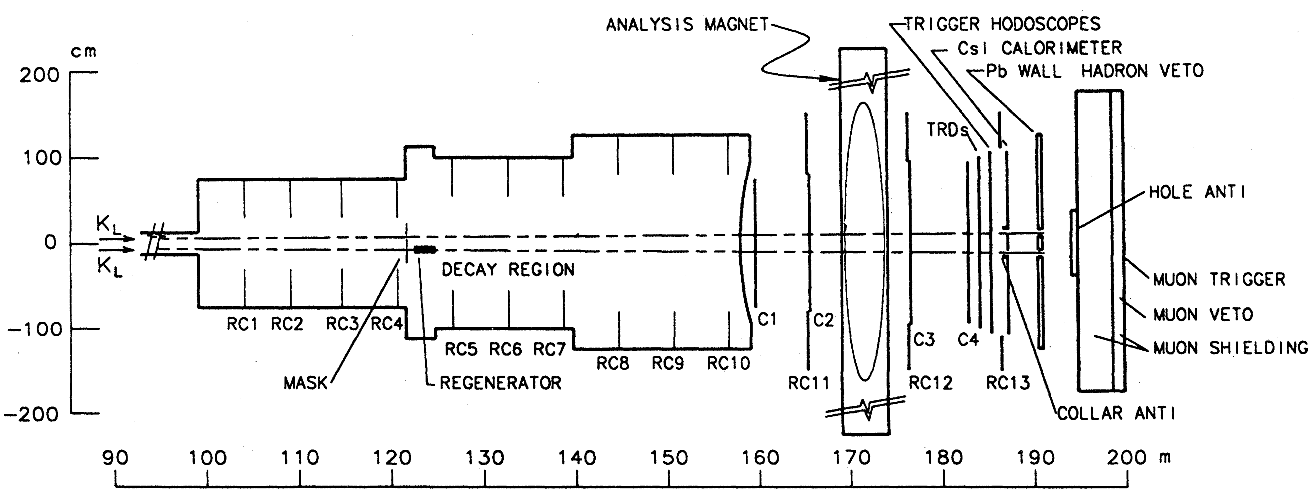

FIG. 10. The layout of the Fermilab E832/799 experiment.

excellent energy and position resolution. Liquid krypton is slower and as well has a larger Molier radius and more interaction lengths, so that photon showers will be fatter and more hadrons will interact relative to pure CsI. However, CsI is harder to calibrate and probably more prone to radiation damage.

The new electromagnetic calorimeters will address the second class of systematic error, the (relative) energy scale. NA48 will use a magnetic spectrometer, so both groups will have the charged energy scale with negligible uncertainty. The positions and bend angles of charged pions are very reliably measured in drift chambers, and the magnetic field of the analyzing magnets can be accurately determined, at least relatively. This leaves one overall scale on the magnetic field which can be adjusted to the accurately known kaon mass. This procedure can also be checked with the Λ mass. In contrast, the neutral energy scale is much more problematic in that one has to deal with possible nonlinearities in the response of the electromagnetic detector. Calibration with electrons is very helpful, but the final adjustment on the overall energy scale relies on the sharp edge in the reconstructed vertex for the K_S decays. This can be done with a residual uncertainty below the statistical one.

The acceptance issue is addressed differently by the two groups. The E731/E832 group has used a Monte Carlo technique to determine the relative difference in acceptance for K_S vs K_L. This requires attention to minimize sources of the difference and a detailed understanding of the detector in its response to charged and neutral particles. To achieve this understanding, a very large number of $\pi e \nu$ and $3\pi^0$ decays are collected simultaneously with the 2π decays, so that the relevant detector properties can be accurately determined with a very large, orthogonal data set.

Formerly, NA31 moved the source of K_S along the beam, thus simulating the distribution of K_L decays, and this made their reliance on Monte Carlo minimal. However, now they will not do so and will rather have a fixed source of K_S decays. In NA48, the plan is to use an event-weighting scheme whereby K_L decays are weighted by the expected K_S distribution. This gives up statistical power, but in principle avoids the use of a Monte Carlo. This technique is equivalent to determining the ratio of K_S to K_L events in small vertex bins, small enough so that any change in acceptance over the bin is negligible. However, one is then very sensitive to the way in which events get moved from one vertex bin to another due to resolution smearing, and one needs to understand the response of the electromagnetic detector, that is, resolution tails and non-Gaussian effects. So, in effect, either one understands the average detector responses well enough to make the acceptance corrections or one understands the resolution-smearing effects well enough to use very small vertex bins.

To treat the final class of systematic uncertainty, accidental activity, we shall describe the two different techniques used to derive a K_S beam.

E731 has used two simultaneous beams, one in which K_L decays are recorded and the other in which K_S decays are recorded. The beams are side-by-side and identical in physical dimensions. The important principle in this approach is that one is recording K_S and K_L to $\pi^+\pi^-$ (or $\pi^0\pi^0$) *simultaneously;* this guarantees that one is relatively immune to the inevitable small changes in detector or accelerator performance. Accidental activity in the detector will largely affect K_L and K_S decays the same. E731 also took a part of its data recording all four modes simultaneously. While not essential, this ensures, for example, that the incident momentum spectrum is identical for all four decays, and this is a valuable constraint in comparing the performance of the detector for charged vs neutral decays. E832 will use two beams and record all four modes simultaneously.

An important element of the new NA48 approach is that they will now use two beams. In addition, they will record all four modes at the same time. Thus the issue of accidental activity appears to be adequately addressed by both groups.

However, there remain significant differences in the way in which the K_S beams are derived, and these can have an impact on the performance of the experiment. E731 has used a thick regenerator in the second K_L beam to provide the necessary K_S component through coherent regeneration; the same technique will be used in E832. NA48 will target a very low-intensity beam on a nearby target to give their source of K_S decays. The choice of the best technique is not obvious; here we shall list the relative advantages and disadvantages.

(1) For E832, the two beams are identical in shape, whereas, by necessity, for NA48 the K_S beam has larger divergence.

(2) For E832, the K_S/K_L distinction is made on the basis of where the event reconstructs; the beams are separated by several cm in order that this can be made cleanly. For NA48, the distinction is made on the basis of tagging counters in the primary proton beam incident upon the K_S target. Although this scheme has never been tried, it should work even at the high rates envisioned, and this allows the beams to be physically closer.

(3) For E832, the roles of the two beams will be interchanged on a pulse-by-pulse basis, thereby eliminating a possible source of asymmetry. This cannot be done with the NA48 technique.

(4) The use of a regenerator allows the study of K_L-K_S interference, and this can provide independent corroborating evidence for an ε'/ε signal.

(5) The regenerator will scatter some kaons inelastically so that they reconstruct in the wrong beam. This "crossover" background was at the few percent level in E731. In a subsequent run (E773), a new "active" regenerator made of plastic scintillator was used, and the level of inelastic regeneration was reduced by a factor of 10. Such a regenerator will be used in E832.

(6) The incident neutrons and kaons interacting inelast-

ically in the regenerator give a higher ambient level of background in the E832 detector. Thus the accidental activity will be greater.

7. Is 10^{-4} sensitivity the limit?

At the current accelerators (Fermilab, CERN), the limiting factor in the precision on ε'/ε may very well be simply statistics. So, if it is necessary to continue measuring ε'/ε after the next round of experiments, a new facility will likely be required.

The Main Injector ring at Fermilab can be used as a source of very high intensity, high duty-cycle 120 GeV protons, which can be used to provide a much more intense neutral-kaon beam. A study (Arisaka *et al.*, 1991) indicates that a measurement with a precision in the range 2×10^{-5} is possible. The KAON facility (Gill, 1990) at TRIUMF can also provide very high-intensity kaons, although at a lower energy, and a measurement of ε'/ε could likely be done there as well. However, a necessary ingredient will be experience with the next generation of measurements at the 10^{-4} level.

8. The Frascati Φ factory

We briefly discuss the prospects for determining ε'/ε at a Φ factory (Franzini, 1992). Such a project has been approved for Frascati, and there are similar plans at both UCLA and Novosibirsk. The Φ's are produced by e^+e^- collisions, and the final state of $K_LK_S\to\pi^+\pi^-4\gamma$ is studied. One needs very high statistics and a detector that can collect as many K_L decays as is practical, given that the K_L lifetime is about 340 cm. The overall acceptance (acc) must be high, and its variation over the fiducial radius (R) of the detector must be very well understood. Very good electromagnetic calorimetry is required. A straightforward calculation shows that the (statistical) error on ε'/ε can be expressed as

$$\sigma_{\varepsilon'/\varepsilon}=1.9\times10^{-4}\times\sqrt{10^{40}/\int L\,dt}\times\sqrt{50\%/\mathrm{acc}}\times\sqrt{1m/R}\ . \tag{5.8}$$

The systematic issues with this technique are very different from those using accelerators; so a result in this range would be very interesting. It is clear that integrated luminosities on the order of 10^{40} are required.

VI. DIRECT *CP*-VIOLATING EFFECTS IN OTHER STRANGE-PARTICLE DECAYS

The same physics that contributes to ε' also induces direct *CP* violation in other well-known strange-particle decays. However, while ε' has been determined to be less than 10^{-5} and, as discussed, will be measured to a level below 10^{-6}, no comparable accuracy exists for any other strange-particle decay. Nevertheless, it may be worth considering other decays for the following reasons.

(1) ε' is suppressed by a factor of $\omega=(\mathrm{Re}A_2/\mathrm{Re}A_0)\cong0.045$, as shown in Eq. (2.8), because of the predominance of one amplitude in $K\to2\pi$ decay ($\Delta I=\frac{1}{2}$ rule). This suppression need not occur in some other strange-particle decays.

(2) For large values of m_t there may be an additional suppression factor for ε', as shown in Eq. (5.7), and this suppression factor may not be present in other decays.

A. $K\to3\pi$

The decay $K_S\to3\pi^0$ is *CP* violating just as $K_L\to2\pi$ is. The decay $K_S\to\pi^+\pi^-\pi^0$ can occur without *CP* violation only if the final state is the $I=2$ state involving pions in p waves; as a result, it is expected to be highly suppressed. In either case, the best chance of seeing *CP* violation is to observe an interference effect producing a difference in the decays $K^0\to3\pi$ and $\bar{K}^0\to3\pi$ as a function of time. At the center of the Dalitz plot (or averaged over the Dalitz plot) this difference measures the *CP*-violating quantities

$$\eta_{000}=\frac{A(K_S\to3\pi^0)}{A(K_L\to3\pi^0)}\ ,$$

$$\eta_{+-0}=\frac{A(K_S\to\pi^+\pi^-\pi^0)}{A(K_L\to\pi^+\pi^-\pi^0)}\ .$$

(Note in this interference term η_{+-0} there is no contribution from the *CP*-conserving $K_S\to\pi^+\pi^-\pi^0$ amplitude.) As in the $K\to2\pi$ case, we can write

$$\eta_{000}=\varepsilon+\varepsilon'_{000}\ ,\qquad \eta_{+-0}=\varepsilon+\varepsilon'_{+-0}\ . \tag{6.1}$$

Our interest here is in the quantities ε'_{000} and ε'_{+-0}, which are signs of direct *CP* violation. So far experiments on $K^0\to3\pi$ have provided limits on η_{000} and η_{+-0} of about 0.3 and so are not sensitive enough even to measure at the level of ε.

The dominant amplitude A_1 for the decay $K^0\to\pi^+\pi^-\pi^0$ is a transition to the $I=1$ symmetric state of three pions. Then

$$\varepsilon'_{+-0}\cong i[\mathrm{Im}A_1/\mathrm{Re}A_1-\mathrm{Im}A_0/\mathrm{Re}A_0]\ , \tag{6.2}$$

corresponding to the relative *CP*-violating phase between the dominant $K^0\to2\pi$ and $K^0\to3\pi$ amplitudes. It was pointed out by Li and Wolfenstein (1980) that in the soft-pion limit in the standard model the two phases are the same, and so in this approximation ε'_{+-0} vanishes. A nonzero value results when $\Delta I=3/2$ amplitudes are included, in which case one predicts ε'_{+-0} has the same order of magnitude as ε'; both are suppressed by the $\Delta I=\frac{1}{2}$

rule. However, the dominant contribution to ε'_{+-0} may really come from Eq. (6.2) by going beyond the soft-pion limit (Donoghue *et al.*, 1987c). The result is then suppressed by a factor $(m_K/\Lambda)^2$, where Λ is the "scale of chiral symmetry breaking," but the result is not so small as implied by the Li-Wolfenstein argument. It has also been argued that the electroweak penguin (Z exchange) contribution, which suppresses ε' for large m_t, actually somewhat enhances ε'_{+-0}. A detailed analysis (Cheng, 1991) leads to the result that $\varepsilon'_{+-0}/\varepsilon$ and $\varepsilon'_{000}/\varepsilon$ have an order of magnitude of about 10^{-2} in the standard model.

Direct *CP*-violating effects could also be found by comparing K^+ and K^- decays. Two measures are

$$\Delta\Gamma(\tau)=[\Gamma(K^+\to\pi^+\pi^+\pi^-)-\Gamma(K^-\to\pi^-\pi^-\pi^+)]/[\Gamma(K^+\to\pi^+\pi^+\pi^-)+\Gamma(K^-\to\pi^-\pi^-\pi^+)] ,$$

and

$$\Delta g(\tau)=[g(\pi^+\pi^+\pi^-)-g(\pi^-\pi^-\pi^+)]/[g(\pi^+\pi^+\pi^-)+g(\pi^-\pi^-\pi^+)] ,$$

where g is the slope parameter of the Dalitz plot, which measures the dependence on the energy of the odd pion. In either case it is necessary to see an interference effect between the amplitude for the dominant symmetric $I=1$ state and a second amplitude; the result is proportional to a final-state phase shift, likely to be small because of the low energy of the pions. Rough estimates of the expectations in the standard model are $\Delta\Gamma(\tau)<10^{-6}$ (Grinstein *et al.*, 1986) and $\Delta g(\tau)<10^{-4}$ (D'Ambrosio *et al.*, 1991), whereas present limits on either are a few times 10^{-3}.

To experimentally address *CP* violation in 3π decays of the neutral kaon, it is necessary to have a source of K_S mesons. There are three techniques that are currently possible. The first uses a tagged K^0 or $\bar{K}^0$ and looks for the interference in the first few K_S lifetimes downstream of the source. The interference conveniently changes sign as one goes from particle to antiparticle, allowing important systematic studies. This is the approach of the CPLEAR Collaboration, operating at the LEAR facility at CERN. Because of difficulties in the detection of an all-photon final state, there is little sensitivity to the parameter η_{000}. It is estimated that the *CP*-violating parameter η_{+-0} can be determined with a precision of about 6×10^{-4} (Adler, 1992).

The second uses an enriched K^0 beam at a high-energy accelerator and again studies the decay distribution in the first few lifetimes downstream of the target. Here a comparison must be made with a pure K_L beam, ideally running simultaneously. This was the approach of the Fermilab E621 collaboration. Here a result has been given in conference for $|\eta_{+-0}|$ (Border *et al.*, 1987); with the data in hand, a result with a sensitivity of 5×10^{-3} is expected, and it is thought that the technique could be pushed to a sensitivity of 4×10^{-4} in a new experiment (Sannes *et al.*, 1988).

The third technique uses a pure K_S source and looks directly for the *CP*-violating decays. This is the approach of the Φ factories, since the Φ meson decays purely into K_LK_S and one can therefore cleanly tag the K_S by the observation of a K_L. Here one expects about 30 $3\pi^0$ decays in a one-year run at design luminosity, permitting the determination of η_{000} with a precision of about 2×10^{-4} (Fukawa *et al.*, 1990). The branching ratio for the $\pi^+\pi^-\pi^0$ decay will also be determined, but it will not be possible to distinguish the *CP*-conserving from the *CP*-violating parts.

Thus the techniques presently on the horizon are sensitive to the expected level of *CP* violation in the mass matrix but will not likely be able to detect any direct effects.

For studies with charged kaons, one needs copious sources of $K^\pm$-meson decays. Again there are three possible avenues on the horizon.

The first is the NA48 collaboration, which has considered using its apparatus for charged-K decay studies. They estimate that in a one-year run, about 2×10^8 $\pi^+\pi^+\pi^-$ and $\pi^-\pi^-\pi^+$ decays could be collected, permitting a determination of $\Delta g\approx5\times10^{-4}$.

The second is the Φ factory, using the K^+K^- decays of which about 10^9 would be collected. The symmetry of the final state is exploited to minimize systematic uncertainty, so that it is estimated that precisions $\Delta\Gamma\approx5\times10^{-5}$ and $\Delta g\approx5\times10^{-4}$ would be obtained.

Finally, the E865 experiment at Brookhaven (Zeller, 1991) has considered such a rate comparison. This experiment, which seeks charged-K decay sensitivity to three-bodies at the 10^{-12} level, could collect about 10^{10} decays in only a few weeks. It is clear that systematic uncertainties would dominate.

B. Hyperon decays

The decays $\Lambda\to p\pi^-$ and $\Lambda\to n\pi^0$ are usually analyzed in terms of the amplitudes $S_1e^{i\delta_1}$, $P_1e^{i\delta_{11}}$, $S_3e^{i\delta_3}$, and $P_3e^{i\delta_{31}}$, where $S(P)$ indicate $s(p)$ waves in the final state; 1(3) indicate $I=\frac{1}{2}$ $(\frac{3}{2})$ final states, and the δ's are the strong final-state phase shifts. The observables are the rates Γ, the asymmetry α of the outgoing particles relative to the Λ spin, and the β parameter measuring the polarization of the final nucleon transverse to the plane of the Λ spin and the final momentum.

CP violation can be detected by comparing these parameters for Λ and $\bar{\Lambda}$. The *CP*-violating observables for

$\Lambda \to p\pi^-$ are given approximately by

$$\Delta = \frac{\Gamma - \bar{\Gamma}}{\Gamma - \bar{\Gamma}} \approx \sqrt{2} \frac{\mathrm{Im} S_3^* S_1}{|S_1|^2} \sin(\delta_1 - \delta_3) \approx 7 \times 10^{-3} \sin\phi \; ,$$

$$A = \frac{\alpha + \bar{\alpha}}{\alpha - \bar{\alpha}} \approx \frac{\mathrm{Im} S_1^* P_1}{\mathrm{Re} S_1^* P_1} \tan(\delta_1 - \delta_{11}) \approx -0.1 \tan\theta_1 \; ,$$

$$B = \frac{\beta + \bar{\beta}}{\beta - \bar{\beta}} \approx \frac{\mathrm{Im} S_1^* P_1}{\mathrm{Re} S_1^* P_1} \frac{1}{\tan(\delta_1 - \delta_{11})} \approx -10 \tan\theta_1 \; ,$$

where ϕ is the *CP*-violating relative phase of S_3 and S_1, and θ_1 the *CP*-violating relative phase of S_1 and P_1.

Note that the rate asymmetry Δ is suppressed both by the $\Delta I = \frac{1}{2}$ rule and by the small values of the final-state phase shifts. The *CP*-violating phases ϕ and θ_1 arise from the same physics as ε' and are calculated by Donoghue, He, and Pakvasa (1986) to be of the order 10^{-4} in the standard model. They find Δ to be less than 10^{-6}, A less than 10^{-4}, and B between 10^{-2} and 10^{-3}. The large value of B is deceptive, since the parameter β is difficult to measure and less than 0.1 in magnitude. They also calculate these observables for other hyperon decays with similar results.

To measure the parameter A at the 10^{-4} level or better requires a copious source of Λ and $\bar{\Lambda}$ and a detector whose inherent asymmetries can be minimized and understood. The best technique (Hamann *et al.*, 1992; Hsueh and Rapidis, 1992) appears to be the use of the exclusive reaction $\bar{p}p \to \bar{\Lambda}\Lambda \to \bar{p}\pi^+ p\pi^-$, where an intense source of $\bar{p}$ is used on a gas jet target. The hyperons are produced with a mean polarization P of 46% within the angular range $-0.75 \le \cos\theta^* \le 0.30$, and it is the difference in decay asymmetries with respect to the hyperon polarization vectors that is the signature of direct *CP* violation. Specifically, it is helpful to use the difference in asymmetries of the final-state protons (Donoghue, Holstein, and Valencia, 1986):

$$\tilde{A} = \frac{[N(p,\uparrow) - N(\bar{p},\downarrow)] + [N(\bar{p},\uparrow) - N(p,\downarrow)]}{[N(p,\uparrow) + N(\bar{p},\downarrow)] + [N(\bar{p},\uparrow) + N(p,\downarrow)]} = \frac{\bar{\alpha} + \alpha}{2} P \; ,$$

where the arrows indicate the direction of the p ($\bar{p}$) with respect to the hyperon production plane. To reach a precision of 10^{-4}, about 2×10^9 decays are needed.

To date, the best result on the parameter A is

$$A = 0.013 \pm 0.029 \; ,$$

and this is based on about 60 000 detected events in PS185 at LEAR (Fischer, 1992).

To reach the required statistical level, a luminosity of about 10^{32} cm^{-2} sec^{-1} is needed, and it appears that this could be obtained either at Fermilab or at CERN, using the intense $\bar{p}$ sources at these laboratories. An extensive report on the feasibility of the measurement at the proposed LEAR-2 (Hamann *et al.*, 1992) addresses a number of systematic issues mostly having to do with biases from possible hidden detector inefficiencies and misalignments. It appears that such biases could indeed be controlled at the 10^{-4} level.

Very recently a proposal was submitted to Fermilab by Gidal *et al.* (1993), a group with a great deal of experience in hyperon physics. They propose to study *CP* violation in both Ξ and Λ decays with the following technique. Ξ^- hyperons, produced at 0° in proton interactions on a fixed target, will have no polarization. Their decay into $\Lambda\pi^-$ gives the Λ a polarization equal to the α parameter in Ξ decay: -0.046 ± 0.014. Then the asymmetry in the subsequent Λ decay can be compared to the asymmetry for the same reaction chain with antihyperons. What one then measures is a combined decay asymmetry: $A = A_\Xi + A_\Lambda$. It is claimed that 2×10^9 decays in each chain can be collected in a 200-day run, giving sensitivity to a 10^{-4} asymmetry. Many sources of uncertainty can be reduced by frequent alternation of particle vs antiparticle running; nevertheless, it will be quite a challenge to keep the overall systematic error to the 10^{-4} level.

C. $K^0 \to \gamma\gamma$

The final $\gamma\gamma$ states for the decay of a spin-zero particle can be classified by their *CP* eigenvalues

$$|+\rangle = (LL\rangle + |RR\rangle)/\sqrt{2} \; ,$$

$$|-\rangle = (|LL\rangle - |RR\rangle)/\sqrt{2} \; ,$$

where $L(R)$ defines the photon polarization. There are then two *CP*-conserving amplitudes,

$$A_+ = A(K_S \to +), \quad A_- = A(K_L \to -) \; ,$$

and two *CP*-violating amplitudes,

$$B_+ = A(K_L \to +), \quad B_- = A(K_S \to -) \; .$$

We then have two measures of *CP* violation,

$$\eta_+ = B_+ / A_+ = \varepsilon + \varepsilon'_+ \; ,$$

$$\eta_- = B_- / A_- = \varepsilon + \varepsilon'_- \; .$$

These can be studied by looking for interference effects (Sehgal and Wolfenstein, 1967; Decker *et al.*, 1984) that are different for initial K^0 and $\bar{K}^0$ beams. Early discussions were based on the possibility that the rate for $K_S \to \gamma\gamma$ was much larger than for $K_L \to \gamma\gamma$; it is now known that the rates have the same order of magnitude, so that measurement of η_+ and η_- are both very difficult.

The decays $K^0 \to \gamma\gamma$ are expected to be dominated by virtual (or real) intermediate states of pions (and other nonstrange particles), which then convert to photons by electromagnetic interactions. Thus *CP* nonconservation will arise from the same physics that causes *CP* violation in the decays of kaons to pions. Theoretical estimates have concentrated on η_- calculated using pole diagrams (Chau and Cheng, 1985; Donoghue, Holstein, and Valencia, 1987b),

$$K^0 \to \pi \text{ or } \eta \text{ or } \eta' \to 2\gamma \ .$$

It is argued that the suppression factor $\omega = 0.045$, which enters Eq. (2.8) for ε', does not occur here. Thus it is concluded that

$$\frac{\varepsilon'_-}{\varepsilon} \sim (20 \text{ to } 30)(\varepsilon'/\varepsilon) \ .$$

There could also be short-distance contributions to $K^0 \to \gamma\gamma$ corresponding to box graphs leading to the transition $s \to d + \gamma + \gamma$. It has been suggested (Eeg *et al.*, 1990) that these give a still larger *CP*-violating effect, but a complete calculation shows that these are unimportant (Liu, 1992).

From the available experimental information and the above discussion, we can construct the following table of approximate branching ratios.

	$\gamma\gamma(+)$	$\gamma\gamma(-)$
K_L	7×10^{-9}	6×10^{-4}
K_S	2×10^{-6}	5×10^{-12}

If one could determine the *CP* state of the final state $\gamma\gamma$, then the observation of the time dependence for initially pure K^0 and $\bar{K}^0$ to decay into the state $\gamma\gamma(-)$ would contain the best information for seeing the direct component. With only the mass mixing, one expects of order 10^{-3} asymmetries in this decay curve, due to the $K_S \to \gamma\gamma(-)$ amplitude, and these reverse for particle/antiparticle. If $\varepsilon'_-/\varepsilon$ is of order 0.1, then there would be significant departures from the superweak expectations. Unfortunately, there is too large an acceptance loss in the detection of the polarization of one of the final-state photons. The difficulty of the experiment is a result of the very small branching ratio for $K_S \to \gamma\gamma(-)$.

The best technique, then, for observing *CP* violation in $\gamma\gamma$ decays is the comparison of the polarization-averaged decay rates from initially pure K^0 and $\bar{K}^0$. Again, asymmetries of order ε are expected in the comparison of K^0 and $\bar{K}^0$ in the first five K_S lifetimes (Chau and Cheng, 1985). One needs more than 10^5 such events to detect even the asymmetry due to mixing, and more than 10^6 to have a chance of seeing a direct effect. The ideal facility for such an experiment is LEAR; however, more than 10^{11} produced K^0's are needed, where LEAR provides about 10^{10} per year. At a "kaon factory" there would be ample flux; however, there are severe difficulties with the detection of decays very close to the production target, as would be necessary.

D. The decay $K^0 \to \mu^+\mu^-$

In the standard model the decay $K^0 \to \mu^+\mu^-$ is believed to arise primarily via

$$K^0 \to \gamma + \gamma \to \mu^+\mu^- \tag{6.3}$$

with real or virtual intermediate photon pairs. Since this amplitude is order α^2, this may prove to be a good place to look for *CP* violation as a result of competing amplitudes that could interfere with (6.3).

There are two possible final states,

$$|+\rangle = (LL + RR)\sqrt{2} \ ,$$

$$|-\rangle = (LL - RR)/\sqrt{2} \ ,$$

where the states are designated by their *CP* eigenvalue and $L\,(R)$ define the muon helicity. *CP* violation due to the transition (6.3) is expected to be small, since it is determined by *CP* violation in the transition to $\gamma\gamma$ (Sec. VI.C). The two *CP*-conserving decay amplitudes are

$$A(K_1 \to +) = A_{1+}, \quad A(K_2 \to -) = A_{2-} \ .$$

If there is some direct *CP* violation, it will show up as either

$$A(K_1 \to -) = B_{1-} \text{ or } A(K_2 \to +) = B_{2+} \ .$$

The simplest effects to look for is the interference between B_{2+} and A_{2-}, which can cause an excess of (LL) with respect to RR, or vice versa, in the decay $K_L \to \mu^+\mu^-$. This then shows up as a net longitudinal polarization P_L of *either* muon; in practice, experiments can measure the μ^+ polarization. In the absence of final-state interactions, the *CP*-violating amplitudes B are imaginary and the *CP*-conserving amplitudes A are real, so that there is no interference in the rates and thus P_L is 0. However, in fact the experimental rate for $K_L \to \mu^+\mu^-$ is consistent with just the absorptive part (real intermediate state) of the transition (6.3), and so we believe $\mathrm{Im}\,A_{2-} > \mathrm{Re}\,A_{2-}$. Neglecting $\mathrm{Re}\,A_{2-}$ and including the indirect *CP* violation due to ε, one finds to a good approximation (Herczeg, 1983)

$$P_L = [2\,\mathrm{Im}B_{2+} + \sqrt{2}|\varepsilon|(\mathrm{Re}\,A_{1+} + \mathrm{Im}\,A_{1+})]/|A_{2-}| \ . \tag{6.4}$$

It is much harder to try to detect *CP* violation due to B_{1-}. The decay $K_s \to \mu^+\mu^-$ has never been seen. The rate for $K_s \to \mu^+\mu^-$ is expected to be similar to that of $K_L \to \mu^+\mu^-$, but this means the branching ratio is of order 10^{-11}. To see the interference of B_{1-} with A_{2-}, one would have to look at the time dependence of $K^0 \to \mu^+\mu^-$ at short times before K_s disappear.

Within the standard model the major *CP*-violating amplitude for $K^0 \to \mu^+\mu^-$ comes from the box diagram discussed for $K_L \to \pi^0 e^+ e^-$ in Sec. VII. The chiral structure of the W interactions tells us that the effective matrix element has the form

$$A_{\rm Box} \sim P^{\mu}\bar{\mu}\gamma_{\mu}(1-\gamma_5)\mu$$
$$=2m_{\mu}\bar{\mu}\gamma_5\mu \ .$$

This corresponds to a *P*-odd, *C*-even, *CP*-odd final state. Thus the *CP*-violating part of the box contributes only to B_{1^-}. Even though this may be significant compared to A_{1^+}, there seems to be no practical way to observe what, in principle, could be a sizable *CP*-violating effect.

Since the branching ratio for $K_L \to \mu^+\mu^-$ is of order 10^{-8}, large numbers of these decays could be observed at a "kaon factory," and it might be possible to measure P_L to better than 1%. Within the standard model the direct *CP* violation [B_{2^+} in Eq. (8.1)] is expected to be practically negligible (Botella and Lim, 1986), and P_L is directly proportional to $|\varepsilon|$. The expected value (Ecker and Pich, 1991), approximately 2×10^{-3}, is probably too small to be measured. Thus the major goal of this measurement is a search for direct *CP* violation beyond the standard model. This can occur in special versions of the superweak theory.

E. The decays $K\to\pi\pi\gamma$ and $K\to\pi\pi e^+e^-$

The decay $K\to\pi\pi\gamma$ occurs predominantly as an $E1$ bremsstrahlung proportional to the $K\to\pi\pi$ decay rate. It is possible also that there are direct decay amplitudes that could be *CP* violating.

In the case of the decay $K^+\to\pi^+\pi^0\gamma$, there has been a long-term interest in the possibility of direct decays (Costa and Kabir, 1967), since the bremsstrahlung is suppressed by the $\Delta I=\frac{1}{2}$ rule. A *CP*-violating direct decay amplitude can arise from the electromagnetic penguin amplitude $s\to d+\gamma$. As a result, there can be an asymmetry in the rates for $K^+\to\pi^+\pi^0\gamma$ and $K^-\to\pi^-\pi^0\gamma$. This asymmetry will be a maximum for intermediate photon momenta where the bremsstrahlung and direct amplitudes interfere. Dib and Peccei (1990) calculate that the average asymmetry could be as large as 10^{-3}. Such an asymmetry would be a clear indication of direct *CP* violation.

In the case of the decay $K_L\to\pi^+\pi^-\gamma$, experiment E-731 (Ramberg *et al.*, 1993a) indicates that there are two amplitudes of comparable magnitude: (1) the *CP*-conserving $M1$ transition $K_2\to\pi^+\pi^-\gamma$ and (2) the indirect *CP* violation proportional to ε corresponding to the $E1$ transition $K_1\to\pi^+\pi^-\gamma$. Thus this is an example of a decay in which there is a large *CP* violation, but this cannot be detected without measuring the photon polarization. There is a *CP*-violating asymmetry that does not depend on the photon polarization, that between π^+ and π^- across the Dalitz plot. This asymmetry requires the $E2$ amplitude, which would correspond to direct *CP* violation, but this is expected to be very small. The (*CP*-violating) interference between K_S and K_L decays was recently observed for the first time (Ramberg *et al.*, 1993b); however, this is an indirect effect only.

It is possible indirectly to measure the photon polarization through its internal conversion to an e^+e^- pair giving the process $K_L\to\pi^+\pi^-e^+e^-$, which has recently been observed (Wah, 1992). The *CP*-conserving $M1$ transition corresponds to the case in which the virtual ρ (leading to $\pi^+\pi^-$) and the virtual photon (leading to e^+e^-) have perpendicular planes of polarization. This leads to a characteristic angular distribution $1+2\cos^2\phi$, where ϕ is the angle between the $\pi^+\pi^-$ and e^+e^- planes. Interference between the *CP*-conserving $M1$ and *CP*-violating $E1$ transitions leads to a characteristic $\sin\phi\cos\phi$ term. Sehgal and Wanninger (1992) calculated this to be a 4% effect, but their later correction raised this to about 15%. The calculated effect is entirely due to indirect *CP* violation, and it is doubtful whether a small direct *CP*-violation contribution could be identified even if it were present.

VII. THE *CP*-FORBIDDEN DECAYS $K_L\to\pi^0 l\bar{l}$

Instead of searching for very small *CP*-violating direct decay effects like ε' in common K decays, it is possible to look for decays that are to a good approximation forbidden by *CP* invariance. The decay $K\to\pi e^+e^-$ is expected to occur via single-photon exchange, a process first order in the weak-times-electromagnetic interactions. However, for K^0's this leads to $K_1\to\pi^0e^+e^-$, so that in this approximation $K_L\to\pi^0e^+e^-$ is *CP* forbidden. As discussed in Sec. IV, the decay $K\to\pi\nu\bar{\nu}$ occurs as a second-order weak process. In the case of K^0, the *CP*-conserving part of this second-order amplitude leads to $K_1\to\pi^0\nu\bar{\nu}$, so that the decay $K_L\to\pi^0\nu\bar{\nu}$ is essentially a signal of direct *CP* violation.

A. The decay $K_L\to\pi^0e^+e^-$

Combining the weak plus electromagnetic interactions, one can obtain the transition $K\to\pi+\gamma^*$, where γ^* is a virtual photon. This leads to the decays $K^+\to\pi^+e^+e^-$ and $K^0\to\pi^0e^+e^-$. By Lorentz invariance, the transition amplitude has the form

$$A(K^0)=[p(K^0)+p(\pi^0)]_{\mu}\bar{\psi}_e\gamma^{\mu}\psi_e \ . \tag{7.1}$$

This form is invariant under *P* but changes sign under *C*. If we define $|\bar{K}^0\rangle=CP|K^0\rangle$ and note that $CP|\pi^0\rangle=-|\pi^0\rangle$, it then follows that if *CP* invariance holds,

$$A(\bar{K}^0)=+A(K^0) \ .$$

As a result the one-photon contribution vanishes for $K_2\to\pi^0e^+e^-$, unless *CP* is violated. Thus the decay $K_L\to\pi^0e^+e^-$ appears as a possible place to look for direct *CP* violation.

There are, in general, three contributions to this decay: (1) *CP*-conserving decays arising from the intermediate transition $K_2\to\pi^0+\gamma+\gamma$; (2) indirect *CP* violation proportional to the K^0-$\bar{K}^0$ mixing $\tilde{\varepsilon}$, which is nearly equal to

ε; and (3) direct *CP* violation. One originally thought of this as coming from the "electromagnetic penguin" $s \to d + \gamma$. For large values of the top mass m_t, however, dominant contributions come from *CP*-violating second-order weak effects, in particular, the "electroweak penguin" $s \to d + Z^*$, where Z^* is a virtual Z^0.

If the decay $K_L \to \pi^0 \gamma\gamma$ were governed by short-distance effects, one might expect that the final state could be described as a π^0 in an s state relative to a $(\gamma\gamma)$ pair in a 0^+ state. [This corresponds to the two gammas having parallel polarization in contrast to the 0^- state of $(\gamma\gamma)$ arising from π^0 decay.] In the c.m. system of the e^+e^- pair coming from this $(\gamma\gamma)$ state, the e^+ and e^- must have the same helicity to give spin zero; however, in the $m_e = 0$ limit, electromagnetic couplings always yield e^+ and e^- with opposite helicities. Thus in this approximation the *CP*-conserving decay vanishes, and early calculations by Donoghue *et al.* (1987a) and others led to a very low branching ratio for the *CP*-conserving contribution of the order of 10^{-12} or less. However, it was pointed out by Sehgal (1988) that the decay $K_L \to \pi^0 \gamma\gamma$ has an important contribution from long-distance effects associated with virtual vector mesons, which lead to final-state d waves. In this case the resulting $K_L \to \pi^0 e^+ e^-$ decay does not vanish in the $m_e = 0$ limit. Sehgal predicted a branching ratio for the *CP*-conserving decay of the order 10^{-11}. Various predictions are shown in Table IV.

This large range of predictions can be narrowed by experimental observation of the decay $K_L \to \pi^0 \gamma\gamma$. The first observations of this decay (Barr *et al.*, 1990a; Papadimitriou *et al.*, 1991) gave a branching ratio between 1.5 and 2.0×10^{-6}, which is larger than the predictions that neglect the vector-meson effects. On the other hand, the spectrum (Barr *et al.*, 1992) shows no evidence of the low-mass $\gamma\gamma$ pairs predicted by vector-meson dominance. On the basis of the spectrum, it is concluded that the rate for $K_L \to \pi^0 e^+ e^-$ due to the intermediate 2γ state is less than 10^{-12}. Future measurements of the spectrum should make it possible to calculate quite accurately the rate of $K_L \to \pi^0 e^+ e^-$ due to the absorptive part of the $K_L \to \pi^0 \gamma\gamma \to \pi^0 e^+ e^-$ transition. There will still be an uncertainty as large as a factor of 2 from the dispersive part.

TABLE IV. Theoretical branching ratios for $K_L \to \pi^0 e^+ e^-$.

	$K_2 \to \pi^0 \gamma\gamma \to \pi^0 e^+ e^-$ *CP* conserving	$K_L \to K_1 \to \pi^0 e^+ e^-$ $\lvert\varepsilon\rvert^2$ term $= B_{\text{ind}}$
Sehgal	2×10^{-11} [a]	
Ko *et al.*	$< 3 \times 10^{-12}$ [b]	$\sim 2 \times 10^{-10}$ [g]
Morozumi & Isawaki	$> 2 \times 10^{-11}$ [c]	
Flynn & Randall	8×10^{-12} [d]	
Pich	$< 10^{-12}$ [e]	$(1.5 \text{ or } 15) \times 10^{-12}$ [e]
Donoghue	2×10^{-13} [f]	5×10^{-13} [f]

[a]Sehgal (1988, 1990).
[b]Ko and Truong (1991).
[c]Morozumi and Iwasaki (1989).
[d]Flynn and Randall (1989a).
[e]Pich (1990).
[f]Donoghue *et al.* (1987a).
[g]Ko (1991).

The *CP* violation due to K^0-$\bar{K}^0$ mixing depends on the rate of the *CP*-conserving process $K_1 \to \pi^0 + e^+ + e^-$. A first guess would be that the rate for K_1 is the same as for $K^+ \to \pi^+ e^+ e^-$, so that

$$\Gamma(K_1 \to \pi^0 + c^+ + e^-) = \Gamma(K^+ \to \pi^+ + e^+ + e^-) = (22 \pm 4)\,\text{sec}^{-1} . \tag{7.2}$$

This then gives the branching ratio for the indirect *CP* violation,

$$B_{\text{ind}}(K_L \to \pi^0 e^+ e^-) \approx |\varepsilon|^2 \times 22 \times (5.2 \times 10^{-8}) = 6 \times 10^{-12} .$$

In theoretical calculations, many of the same diagrams contribute to the K^+ and K^0 decays; however, there are some unique contributions associated with K^+ because of its charge. Donoghue *et al.* (1987a) find as a result that the K^+ rate is much larger than the K^0. On the other hand, a detailed calculation by Ko (1991) gives, for reasons that are not clear, a much larger rate for K^0. Some typical results are shown in Table IV. This could unambiguously be pinned down if it were possible to measure the branching ratio for $K_S \to \pi^0 e^+ e^-$. If (7.2) is correct, this branching ratio is 2×10^{-9}, which is 10^4 times smaller than the present experimental limit.

We now turn to the direct *CP* violation, which is the contribution that interests us. If $m_t < m_w$, the main contribution comes from the electromagnetic penguin $s \to d + \gamma^*$, where γ^* is the virtual photon. This goes roughly as $\log(m_t/m_w)^2$ and has a large QCD correction that reduces the amplitude by a factor of about 2. For $m_t < m_w$, there is a rough proportionality between the *CP*-violating amplitude for $K_L \to \pi^0 e^+ e^-$ and ε'.

The situation is significantly different, when m_t becomes larger than m_w (Dib *et al.*, 1989b; Flynn and Randall, 1989b). One then begins to get a significant contribution from the box diagram and a contribution from the electroweak penguin $s \to d + Z^*$ that grows asymptotically as m_t^2. These are the same diagrams that contribute to ε' (Fig. 6), with $q\bar{q}$ replaced by $e\bar{e}$. In the case of ε', the main effect of the Z^* is to give a $\Delta I = \frac{3}{2}$ *CP* violation, which tends to cancel out the $\Delta I = \frac{1}{2}$ *CP* violation of the normal penguin. In the case of $K_L \to \pi^0 e^+ e^-$, the main effect of the Z^* is to add an axial-vector term, which adds incoherently to the contribution of the γ^*. Thus for large values of m_t, like 200 GeV where ε' may be approaching zero, the direct *CP*-violating $K_L \to \pi^0 e^+ e^-$ amplitude is rising almost like m_t^2.

The effective quark amplitude may be written as

$$(e^2/4\pi)(\bar{s}\gamma_\lambda(1-\gamma_5)d)\bar{e}(c_v \gamma^\lambda + c_A \gamma^\lambda \gamma_5)e . \tag{7.3}$$

For $m_t < m_w$, where the γ^* dominates, $c_v > c_A$. However, the Z^* mainly contributes to C_A, since its contribution to c_v is reduced by the factor $(1 - 4\sin^2\theta_w)$. For

$m_t > 150$ GeV, it is the c_A term that dominates.

A major advantage of the process $K_L \to \pi^0 + e^+ + e^-$ is that, unlike the case of ε', there is no problem in going from the quark amplitude to the mesons for a semileptonic process. The matrix element $\langle \pi^0 | \bar{s}\gamma_\lambda d | K^0 \rangle$ is related by isospin to that for $K^+ \to \pi^0 + e^+ + \nu_e$, which can be determined from experiment. Using this, Dib, Dunietz, and Gilman (1989b) find that the branching ratio for direct *CP* violation is

$$B_{\rm dir}(K_L \to \pi^0 e^+ e^-) = 10^{-5}(A^2\lambda^4\eta)^2(c_v^2 + c_A^2)$$
$$\approx (5\times 10^{-11})A^4\eta^2(c_v^2 + c_A^2) \ . \qquad (7.4)$$

For $m_t \sim m_w$, the dominant term comes from c_v^2, which is about 0.2. In this case there is a direct relation between $B_{\rm dir}$ and $(\varepsilon'/\varepsilon)$, which arises from analogous penguin graphs; and one finds, using Eq. (5.4),

$$B_{\rm dir}(K_L \to \pi^0 e^+ e^-) \approx 10^{-6}(\varepsilon'/\varepsilon)^2$$

within a factor of about 5.

As m_t goes from m_w to 180 GeV, c_v^2 grows from 0.2 to 0.4 while c_A^2 grows to 0.6. Using typical values of η fitted to the ε parameter, one finds

$$B_{\rm dir}(K_L \to \pi^0 + e^+ + e^-) \sim 2\text{--}4\times 10^{-12} \ ,$$

with a range between 10^{-11} and 10^{-13} for allowed values of η for all m_t up to 180 GeV.

Neglecting the *CP*-conserving term, we combine the two *CP*-violating contributions to give a branching ratio

$$B_{CP\ \rm odd}(K_L \to \pi^0 e^+ e^-)$$
$$= \{e^{i\pi/4}[B_{\rm ind}]^{1/2} \pm iA^2\eta c_v(5\times 10^{-11})^{1/2}\}^2$$
$$+ A^4\eta^2 c_A^2 5\times 10^{-11} \ , \qquad (7.5)$$

where $e^{i\pi/4}$ is the phase of ε; we have neglected the contribution of $\mathrm{Im}A_0/\mathrm{Re}A_0$ to ε. To determine the sign of the interference requires theoretical calculation of the sign of the $K_S \to \pi^0 e^+ e^-$ amplitude.

Given the uncertainties in the *CP*-conserving contribution and in $B_{\rm ind}$ (Table IV), the measurement of the branching ratio by itself, while very interesting, cannot be taken as proof of direct *CP* violation. It was pointed out by Sehgal (1988) that interference between the *CP*-conserving and *CP*-violating amplitudes could be detected by observing the *CP*-violating asymmetry between the e^+ and e^- spectra. However, this could be due to either the direct or the indirect *CP* violation and, in fact, is insensitive to the C_A term which will dominate the direct *CP* violation at high m_t.

B. Status and prospects for $K_L \to \pi^0 e^+ e^-$

The current experimental sensitivity is still far from the expected levels. The two best 90%-confidence-level limits are 5.5×10^{-9} from BNL 845 (Ohl *et al.*, 1990) and 7.5×10^{-9} from FNAL E731 (Aihara *et al.*, 1990). Sensitivity to this mode at about 10^{-9} is expected from the first run of FNAL E799, which ended in January 1992 (Barker *et al.*, 1988; Wah, 1992).

The Kyoto, KEK collaboration KEK 162 (Miyake *et al.*, 1988) has as its aim a sensitive search for $K_L \to \pi^0 ee$, at a level of about 10^{-10}; it should begin taking data sometime in 1993. This will be the first *K*-decay experiment to use very high-precision calorimetry.

The last phase of the E799 experiment is scheduled for 1995. The experiment plans an exposure that would yield a single-event sensitivity better than 10^{-11} for a four-body decay, in the absence of background. For this effort, the simulation of a variety of backgrounds has been made (Barker *et al.*, 1988). As the rates in such a rare decay experiment are large, so-called accidental events have been overlaid upon pure Monte Carlo events in order to correctly simulate the effect of the high level of ambient beam activity. The most serious such backgrounds have been found to be either single or double Dalitz decays in $2\pi^0$ or $3\pi^0$ decays with accidental photons. However, these processes will only contaminate the signal at the 10^{-12} level. More serious is the radiative Dalitz decay background, $K_L \to e^+ e^- \gamma\gamma$ discussed by Greenlee (1990), which satisfies all kinematic discrimination except for the criterion that the invariant mass of the $\gamma\gamma$ system be that of the π^0. Thus the very best gamma-ray energy (and position) resolution is required and can be obtained with the pure CsI calorimeter that will be used in E799. Further background rejection is obtained by selectively cutting into the four-body phase space at the sacrifice of some signal: the distributions for signal and background are not the same. After optimization (with a 30% loss in signal), it is found that the single-event sensitivity is 1×10^{-11} and that seven background events remain. This implies that a 90%-confidence upper limit of 7×10^{-11} could be set. While this is significantly better than the current sensitivity, there still remains the issue of untangling the *CP*-conserving and indirect contributions.

During the same experimental run, it is expected that about 10^4 $K_L \to \pi^0\gamma\gamma$ will be collected with background at the 20% level; this will allow a branching-ratio determination at the few-percent level and a good measurement of the Dalitz plot, which should greatly help in determining the *CP*-conserving contribution.

To reach the level of the standard model, one will very likely need even more sensitive experiments, those at a dedicated kaon facility or factory. The experiment has been considered using the Main Injector at Fermilab. A conceptual design study (Arisaka *et al.*, 1991) estimated that a determination of the branching ratio could be made at the 25% level if the central value were 4×10^{-12}. In addition, the decay $K_S \to \pi^0 ee$ could be seen with a sensitivity below 10^{-11}, which should be more than enough to determine accurately the indirect contribution.

Assuming the measurement of $K_L \to \pi^0\gamma\gamma$ restricts the *CP*-conserving contribution to a value below 10^{-12} and that the measurement of $K_S \to \pi^0 e^+ e^-$ provides a good measure of $(B_{\rm ind})$, then it would be possible that a large value for the branching ratio would be a clear indication of direct CP violation. It will be much easier to find direct CP violation in this way if the interference in Eq. (7.5) is constructive (positive sign) than if it is destructive. Thus, depending on the actual values of B_{ind}, η, and the relative sign, the measurement discussed above could have a chance of finding direct CP violation.

C. The decay $K_L \to \pi^0 \nu\bar{\nu}$

The process $K_L \to \pi^0 \nu\bar{\nu}$ has been considered by a number of authors. There are potentially two contributions: a direct CP-violating one from the dominant K_2 and indirect CP violation from the K_1 contamination in the K_L. The CP-violating contribution arises from the diagrams of Fig. 4 and is given by the same equation as that for $K^+ \to \pi^+ \nu\bar{\nu}$ [Eq. (4.6)], provided one includes only the η term and corrects for the difference in K^+ and K_L rates. The resulting branching ratio summed over three neutrino flavors is

$$B(K_2 \to \pi^0 \nu\bar{\nu}) = 8\times 10^{-11}\,(m_t/m_w)^{2.2} A^4 \eta^2 \; . \qquad (7.6)$$

We note that the branching ratio increases approximately like the square of the top mass, and the CP violation is manifest in the proportionality to η^2.

The CP-conserving contribution from the K_1 piece of the K_L can be limited using the upper limit for the corresponding $K^+ \to \pi^+ \nu\bar{\nu}$ transition. In fact, the rate for the K_1 transition is expected to exceed that for the K_2 by a factor of roughly 25, but the very small admixture of K_1 in the K_L makes this contribution negligible. We then have the branching ratio for $K_L \to \pi^0 \nu\bar{\nu}$ given by Eq. (7.6).

From the discussion in Sec. III, one expects $A^2\eta$ to lie between 0.1 and 0.5. The mass of the top quark has been determined from precise data from LEP to be about 140 ± 40 GeV/c^2. Thus $B(K_L \to \pi^0 \nu\bar{\nu})$ could be as large as about 10^{-10}. The central value is about 2×10^{-11}.

D. Status and prospects for $K_L \to \pi^0 \nu\bar{\nu}$

Since this decay is pure direct CP violating, it should be seriously considered (Littenberg, 1989). The experimental signature is a single unbalanced π^0. The lack of a more distinct signature makes for severe difficulties in background rejection. One of the most important backgrounds is likely to be $K_L \to 2\pi^0$, where two photons are missing. To reach the level of the standard model, one would have to reject this background by about a factor of 10^8, or about 10^4 for each photon; this would require a nearly perfectly hermetic detector. At Brookhaven, for the experiment searching for $K^+ \to \pi^+ \nu\bar{\nu}$, an inefficiency below 10^{-3} has been achieved for relatively low-energy photons (20–220 MeV); the main contribution to the inefficiency appears to be due to the photonuclear effect where a photon interacts *strongly* with a nucleus in the veto counters, leaving little signal (Marlow, 1990). A study of this process for the higher-energy photons of 30 to 700 MeV has been made, and it appears that the necessary rejection could be achieved in an experiment using the Fermilab Main Injector as a source of very high-intensity and high-energy neutral kaons (Arisaka *et al.*, 1991). At other laboratories there have also been studies of the potential to study this decay (Bryman, 1991; Inagaki, 1991).

In spite of the somewhat optimistic projections, there are many orders of magnitude that need to be covered. (Of course, if a signal emerged well above the level of the standard model, that would be significant.) The only limit to date [branching ratio, $B(K_L \to \pi^0 \nu\bar{\nu}) < 2.2\times 10^{-4}$, 90% confidence] comes from Graham *et al.* (1992), where, in order to better define the final state, a Dalitz decay of the π^0 is required. Thus the final state is $ee\gamma$, and the vertex of the decay can be reconstructed, allowing the invariant mass of the π^0 to be calculated. Two additional backgrounds were uncovered in this search. The first results from radiative semileptonic decays, $K_L \to \pi e \gamma \nu$, where the pion is misidentified as an electron, and the second from $\Lambda \to n\pi^0$, $\pi^0 \to e^+ e^- \gamma$ decays. For future searches of this type, the first can be further reduced with excellent electromagnetic calorimeter resolution together with TRDs (transition-radiation detectors) for particle identification, while the second can be significantly reduced by detecting the neutron (which almost always remains within the kaon beam) and by demanding sufficient transverse momentum for the π^0. It is likely that a sensitivity in the range of 10^{-8} will be reached in the second phase of E799.

VIII. CP VIOLATION IN B DECAYS

Soon after the discovery of the third generation, many authors discussed the possibilities for studying CP violation in the decays of B mesons. With the measurement of the B-meson lifetime, it became apparent that the CKM matrix had a hierarchical structure, as noted in Eqs. (3.3). As a result, it became apparent that the CKM matrix in the form (3.6) could contain relatively large phases in the V_{ub} and V_{td} elements. Large CP-violating effects resulting from these large phases are uniquely expected in B decays.

There exists now a large literature on the expected CP violation in B decays. For reviews, see Jarlskog (1989), Stone (1992), and Nir and Quinn (1992). Most of the papers are directed at reconstructing the unitarity triangle (Fig. 1) by determining the angles α, β, and γ. Here we limit our discussion to the simpler, but still difficult, task of detecting direct CP violation via the B system.

A. ε for the B system

It is possible to calculate B^0-$\bar{B}^0$ mixing for the B system using the box diagram just as is done for the K^0-$\bar{K}^0$ system. There are two important differences for the B^0 case.

(1) For the K^0 case, the real part of the box diagram (and related long-distance effects) that determines Δm is proportional to

$$(V_{cs}^* V_{cd})^2 = \lambda^2 \; ,$$

while the imaginary part, the m' term in Eq. (2.7), is proportional to $A^2\lambda^6\eta$, as discussed in Sec. III.A. Thus

$$\varepsilon \sim \frac{m'}{\Delta m} \sim A^2\lambda^4 \; . \tag{8.1}$$

In contrast, in the B^0 case, no matter which intermediate quark (u, c, or t) is on the legs of the box diagram (Fig. 2), there is always a factor λ^6. Since it is the top quark that dominates, the result of the calculation of the B^0-$\bar{B}^0$ mixing, calculated from the box diagram, yields the factor

$$(V_{tb}^* V_{td})^2 = (1-\rho-i\eta)^2 A^2\lambda^6 \; .$$

It follows that the CP-violating term m' is related to Δm by

$$m'/\Delta m = \frac{1}{2}\sin 2\beta = \frac{\eta(1-\rho)}{(1-\rho)^2+\eta^2} \; , \tag{8.2}$$

where β is the phase of V_{td}^*.

Thus the CP violation in the mass matrix for B^0 is not suppressed by any power of λ, whereas for the K^0 system it is suppressed by $\lambda^4 (=2.3\times 10^{-3})$.

It should be noted that this result holds in the phase convention we are using in which the dominant B decays involving $b \to c$ transitions have no CP violation in the decay amplitude itself. As noted for the K^0 case, it is possible to choose a phase convention so that ε is all due to m' or all due to the decay amplitude [Eq. (2.7)]. We discuss below the question of definitively finding direct CP violation.

(2) Because of the large number of CP-even and CP-odd final states available, one expects $\Delta\Gamma$, the difference in the widths of the two B^0 eigenstates, to be much smaller than Δm, whereas for K^0, $\Delta\Gamma \approx 2\Delta m$. As a result, ε for the B^0 system is almost purely imaginary [Eq. (2.2) with $\gamma'=0$, $\Gamma_1=\Gamma_2$]. It follows that the detection of CP violation via semileptonic decays (analogous to the parameter δ for the K^0 system), which is proportional to Reε, is expected to be less than 1% (Hagelin, 1981; Altomari *et al.*, 1988). In addition, one cannot separate B_L from B_S, as one does for K_L, and so one cannot duplicate the K_L-type experiments.

The best way to observe CP violation due to mixing in the B^0 system is to study the decays of B^0 and $\bar{B}^0$ to a CP eigenstate (Bigi and Sanda, 1981, 1987). Assuming there is no CP violation in the decay amplitude, the time dependence of the decays has the form

$$P_a(t)\alpha e^{-\Gamma t}[1\pm\alpha(a)\eta_a \sin\Delta m t] \; , \tag{8.3}$$

where the plus sign is for B^0 and the minus sign for $\bar{B}^0$. The factor $\eta_a = \pm 1$ is the CP eigenvalue of the final state denoted as a. The asymmetry parameter $\alpha(a)$ is a measure of CP violation.

One expects that the B decays that are not suppressed by the CKM matrix, that is, $b \to c$ decays, will be well represented by tree amplitudes. In our phase convention these amplitudes are real, so that the only CP violation is due to mixing, which yields the eigenstates

$$\begin{aligned} |B_h\rangle &\sim \cos\beta|B_1\rangle + i\sin\beta|B_2\rangle \; , \\ |B_l\rangle &\sim i\sin\beta|B_1\rangle + \cos\beta|B_2\rangle \; , \end{aligned} \tag{8.4}$$

where B_1, B_2 are the CP eigenstates and β is defined in Eq. (8.2).

The transition of most interest is

$$b \to c + \bar{c} + s \; , \tag{8.5}$$

and the corresponding final state most discussed is ψK_S, which has $\eta=-1$. The asymmetry parameter α for these decays calculated from Eq. (8.4) is

$$\alpha(\psi K_S) = \sin 2\beta \; . \tag{8.6}$$

Note that the asymmetry parameter depends only on the CKM matrix and is independent of hadron dynamics. The value of $\sin 2\beta$ varies between 0.1 and 1 over the allowed CKM matrix values shown in Fig. 3.

Since this asymmetry by itself can be explained solely in terms of mixing, it could result from a superweak CP-violating $\Delta B=2$ interaction. While, in general, one does not expect a superweak interaction to give much larger CP violation in the B system than in the K system, as one expects in the standard model, the possibility cannot be ruled out, particularly in multi-Higgs models (Gérard and Nakada, 1991). Thus it is necessary to look for analogs of ε' for the B system to rule out the superweak alternative.

B. ε' for the B system

If the only CP-violating effect is due to mixing, then the asymmetry parameter $\alpha(a)$ will be independent of the final state a. Thus one can find a sign of direct CP violation by finding decays for which α is different from that for ψK_S. The obvious choice is the $b \to u$-type decays, which at tree level have a different phase from the $b \to c$ decays. We consider, in particular, the decays

$$b \to u + \bar{u} + d \; . \tag{8.7}$$

The simplest decay of this type is $\bar{B}^0 \to \pi^+\pi^-$, which has $\eta=+1$.

In our phase convention, considering only the tree amplitude, the decays of the form (8.7) have a phase $\theta_{ub} = -\gamma$. One must combine this CP-violating phase with that due to mixing, giving

$$\alpha(\pi^+\pi^-) = \sin 2(\beta+\gamma) \; . \tag{8.8}$$

In general, this will be quite different from $\alpha(\psi K_S)$, thus proving there is direct CP violation. Note that the observation of $\alpha(\pi^+\pi^-)$ by itself could be explained by mixing. By choosing a different phase convention for the b

quark, V_{ub} could be taken as real; but then V_{tb} would acquire the phase γ, so that the CP violation ascribed to B^0-$\bar{B}^0$ mixing would be modified from that given by Eq. (8.6) to that given by Eq. (8.8). As is well known from the K system (see Sec. II), there is no way to separate the mixing and the direct CP violation contributions to ε. However, the relative phase between the amplitudes (8.5) and (8.7) given by γ is a direct CP-violating effect revealed by the difference between $\alpha(\psi K_S)$ and $\alpha(\pi^+\pi^-)$.

The ratio of these two asymmetry parameters (Harris and Rosner, 1992; Soares and Wolfenstein, 1993) can be written

$$\frac{\alpha(\pi^+\pi^-)}{\alpha(\psi K_S)}=\frac{\sin(2\beta+\gamma)}{\sin 2\beta}=\frac{\rho-K}{K(1-\rho)}\ , \tag{8.9}$$

where $K=\rho^2+\eta^2$. This describes the variation of the ratio as one moves along one of the central circles of Fig. 3. The ratio has a large range of possible values, including the value unity which occurs for (Winstein, 1992)

$$\eta=(1-\rho)\sqrt{\rho/(2-\rho)}\ . \tag{8.10}$$

If (η,ρ) should approximately satisfy this equation, $\alpha(\pi^+\pi^-)\approx\alpha(\psi K_S)$ in the standard model, so that this measurement might not distinguish the standard model from superweak models. This possibility requires a relatively large value of f_B, around the upper limit given in Sec. III.C.

C. Penguin contributions

As in the case of K^0 decay, it is possible to see direct CP violation in a single decay, provided there is an admixture of contributions that come from loop diagrams besides those from tree diagrams. For nonleptonic decays, these involve a quark loop from which a gluon is emitted (Fig. 5), often called penguin graphs.

For the case of decays like $b\to c+\bar{c}+s$, one expects negligible CP violation from loop graphs, because the tree amplitude is of order $A\lambda^2$ and CP-violating amplitudes require $A\lambda^3$. There are three cases of particular interest.

(1) $b\to u+\bar{u}+d$, yielding decays like $B\to\pi\pi$. In this case the main contribution is the tree diagram of order $A\lambda^3$; but the CP-violating penguin contribution (Grinstein, 1989; Gronau, 1989; London and Peccei, 1989), which is also of order $A\lambda^3$, can be of order 10%, depending on the particular decay.

(2) $b\to u+\bar{u}+s$, yielding decays like $B\to K\pi$. In this case the tree diagram is suppressed by a factor $A\lambda^4$, whereas the penguin graphs with t or c quarks in the loop have a factor $A\lambda^2$. In this case one expects the dominant contribution to come from penguin graphs, but there is still a significant tree-graph contribution.

(3) $b\to d+\bar{s}+s$, yielding decays like $B\to K\bar{K}$. In this case there is no tree contribution, but the different quarks (u,c,t) in the penguin loops come in with different phases.

The most direct signal of CP violation in a decay amplitude is a difference in the partial decay rates of B^+ and B^-. This is discussed in the next section.

In the case of B^0 decays, the effect of adding penguin diagrams tot tree diagrams is to modify the time dependence from the form of Eq. (8.3) to

$$P_a(t)\sim e^{-\Gamma t}[1\pm X_a\cos(\Delta m)t\pm\alpha(a)\eta_a\sin(\Delta m)t]\ . \tag{8.11}$$

The new term proportional to $\cos(\Delta m)t$ gives a difference between B^0 and $\bar{B}^0$ decay rates at time $t=0$ before any mixing takes place. Therefore it has the same origin as the difference between B^+ and B^- partial rates. The other effect is to modify the coefficient of the $\sin(\Delta m)t$ term. In particular, for the case of decays of the form (8.7), the asymmetry parameter α is no longer given by Eq. (8.8). Instead, assuming X_a is very small,

$$\alpha(b\to u+\bar{u}+d)=\sin 2(\beta+\psi_a)\ ,$$

$$\tan\psi_a=\frac{\eta}{\rho}\left[1+\frac{R_a}{\rho(1-R_a)}\right]^{-1}\ ,$$

where R_a is the ratio of the penguin matrix element to the tree matrix element for the final state a. Theoretical estimates based on the work of Bauer, Stech, and Wirbel (1985) suggest that R_a is quite different for different decays. As a result, in principle, even if $\alpha(\pi^+\pi^-)$ were equal to $\alpha(\psi K_S)$, the ambiguity discussed in Eq. (8.10), this equality would not hold for all $b\to u+\bar{u}+d$ decays. This is illustrated in Table V (Soares and Wolfenstein, 1992).

TABLE V. Theoretical values of the asymmetry parameters α and branching ratios for several decays where (η,ρ) have been chosen so $\alpha(\pi^+\pi^-)=\alpha(\psi K_S)$.

		$\rho=0.2,\ \eta=0.3$		$\rho=0.4,\ \eta=0.3$	
Final state a	R_a	α	BR	α	BR
ψK_S	0	**0.66**		**0.82**	
$\pi^+\pi^-$	0.06	0.66	10^{-5}	0.82	2×10^{-5}
$\pi^0\pi^0$	0.19	0.93	6×10^{-7}	0.95	1×10^{-6}
$\pi^0\rho^0$	0.09	0.76	2×10^{-6}	0.86	3×10^{-6}
$\rho^0\rho^0$	0.12	0.82	10^{-6}	0.89	2×10^{-6}

D. *CP* violation in B^+, B^- decays

A measure of direct *CP* violation is the difference in the rates of a particle P and its antiparticle $\bar{P}$ to a state or set of states $F\,(\bar{F})$,

$$\Delta_F = \Gamma(\bar{P} \to \bar{F}) - \Gamma(P \to F) .$$

The asymmetry is then given by

$$a_F = \Delta_F / \Sigma_F = \frac{\bar{\Gamma} - \Gamma}{\bar{\Gamma} + \Gamma} .$$

It follows from *CPT* invariance that the total rate of decay of P and $\bar{P}$ are equal. In order to obtain a significant difference in partial rates, one must have a strong final-state interaction linking different final states. In addition, of course, there must be two contributions to the decay amplitude with a weak *CP*-violating phase difference between them.

The decays that have been most discussed in this regard (Gérard and Hou, 1991; Simma, Wyler, and Eilam, 1991; Wolfenstein, 1991) are $B^+(B^-)$ decays corresponding to the quark transitions

$$b \to s\bar{u}u \ , \tag{8.12a}$$

$$b \to s\bar{s}s \ , \tag{8.12b}$$

$$b \to d\bar{u}u \ , \tag{8.12c}$$

$$b \to d\bar{s}s \ , \tag{8.12d}$$

where, as discussed in the last section, there are large penguin contributions. Calculations have concentrated on the inclusive decays, with the final-state interaction corresponding to the absorptive part of a quark diagram (Bander, Silverman, and Soni, 1979). For example, in the $b \to s\bar{u}u$ amplitude, there would be a term

$$b \to s\bar{c}c \to s\bar{u}u$$

corresponding to final-state scattering from states like $D\bar{D}_S$ to states like $K^+ n\pi$. For the inclusive decays (8.12a) and (8.12b), the asymmetries a_F are found to be of the order of 1% or less, and the asymmetries are even smaller for (8.12c). For the rare decays (8.12d) with an inclusive branching ratio of the order 10^{-4}, the inclusive asymmetry a_F may lie between 3% and 40% depending on the values of ρ and η. The calculation for exclusive decays is much less certain, because there is important final-state scattering between states with the same quark content. In one particular calculation (Simma and Wyler, 1991), the asymmetry for $B^- \to K^- K^0$ $(B^+ \to K^+ K^0)$ is found to be about the same as for the inclusive decay (8.12d). However, the exclusive branching ratio is expected to be between 10^{-6} and 10^{-7}. Large asymmetries are also possible in the decay $B \to \rho + \gamma$ (Soares, 1991).

It is possible to get interference between two amplitudes even without penguins in a very special type of decay (Bigi and Sanda, 1988; Gronau and Wyler, 1991). One considers the two decay amplitudes

$$A(b \to c\bar{u}s) \propto V_{cb} V_{us}^* = A\lambda^3 ,$$

$$A(b \to u\bar{c}s) \propto V_{ub} V_{cs}^* = A\lambda^3(\rho - i\eta) .$$

These lead to the decays $B^- \to D^0 K^-$ and $B^- \to \bar{D}^0 K^-$, respectively:

$$A(B^- \to D^0 K^-) = A_1 e^{i\delta_1} ,$$

$$A(B^- \to \bar{D}^0 K^-) = A_2 e^{-i\gamma} e^{i\delta_2} ,$$

where γ is the phase of V_{ub}^*, and δ_1 and δ_2 are strong-interaction phase shifts, mainly related to inelastic scattering. It is now possible to imagine looking at D decays into a *CP* eigenstate. Since one expects negligible *CP* violation in the D decay, one would in this way distinguish the states D_+ and D_- with well-defined *CP* properties; for example,

$$\frac{D^0 + \bar{D}^0}{\sqrt{2}} = D_+ \to K^+ K^- \ \text{or} \ \pi^+\pi^- ,$$

$$\frac{D^0 - \bar{D}^0}{\sqrt{2}} = D_- \to K_S \pi^0 .$$

We then find a rate asymmetry

$$\frac{\Gamma(B^- \to D_+ + K^-) - \Gamma(B^+ \to D_+ + K^+)}{\Gamma(B^- \to D_+ + K^-) + \Gamma(B^+ \to D_+ + K^+)} = \left[\frac{2A_1 A_2}{A_1^2 + A_2^2} \right] \sin\gamma \sin(\delta_2 - \delta_1) \tag{8.13}$$

and the same asymmetry with opposite sign for a D_- in the final state.

The ratio A_1/A_2 can be experimentally determined by observing the decays to D^0 or $\bar{D}^0$, the flavor eigenstates. We expect A_2/A_1 to equal $\sqrt{\eta^2 + \rho^2}$ times a color-suppression factor of the order of $\frac{1}{4}$. Thus the first factor on the right-hand side in Eq. (8.13) is of order 0.2. Most of the (η, ρ) solutions have $\sin\gamma > \frac{1}{2}$. The great uncertainty is $\sin(\delta_1 - \delta_2)$. In any case it is unlikely the asymmetry is greater than 10%, and it is probably considerably smaller. Unfortunately, the largest cascade branching ratios, such as $B^- \to D^0 K^- \to (K_S \pi^0) K^-$, are expected to be only of order 10^{-6}.

E. Experimental prospects

Although the asymmetries expected in the standard model are generally large, it is still not trivial to be able to detect them and then measure them with enough precision to rule out superweak models. This is primarily a result of the fact that those modes that will show appreciable asymmetry have relatively small branching ratios. It is generally believed that one must produce and be able to observe the decays from of order $\gtrsim 10^8$ B mesons in order to cleanly observe the expected effects.

Aside from hadronic interactions to which we shall re-turn later, the most copious source of B mesons is through the $\Psi(4s)$ resonance. The $\Psi(4s)$ decays into B^+B^- or $B^0\bar{B}^0$ with a definite angular momentum with $C=-1$ and $P=-1$. The cross section to produce this resonance in e^+e^- is 1.15 nb; this is the highest cross section for B-meson production at any energy below the Z^0 resonance. The signal-to-noise ratio is $\frac{1}{3}$, highest of any B production process.

To study the expected particle/antiparticle decay distributions in neutral-B decays to CP eigenstates, it is necessary to tag the B mesons. This is readily accomplished at the $\Psi(4s)$ by observation of an appropriate decay of the other B meson. However, because the initial state has $C=-1$, the time-integrated asymmetries vanish whether or not CP is violated. Hence it is necessary to measure the time difference between the decay of the tagging B and the B decay to the CP eigenstate.

The mean decay length of a B meson is only 23 μm when produced with equal energy e^+ and e^- beams, and it is now beyond detector technology to measure the vertex difference in such a situation. Even if the B-meson decay vertices were reconstructed with sufficient resolution, the distance between the two decay vertices gives the *sum* of the decay times rather than the *difference.* Hence one needs to give the center of mass a boost, and this is accomplished by running the collider asymmetrically (Dunietz and Nakada, 1987; Oddone, 1987; Aleksan *et al.*, 1989), for example, an 8-GeV electron beam colliding with a 3.5 GeV positron beam, which seems to be close to the optimal condition for the experiment (Nakada, 1989). Both decays are then boosted in the direction of the electron beam, and, to a very good approximation, the decay time difference is proportional to the vertex separation along that axis, which would be, in the example given, about 160 μm.

Such asymmetric colliders are under design at Stanford (SLAC, 1991), Cornell (Cornell, 1991), and KEK (KEK, 1990). The design luminosity is 3×10^{33} cm^{-2}sec^{-1}. Operating for three full years (10^7 sec/year) at design luminosity will accumulate about 100 fb^{-1} and will produce about 10^8 B-meson pairs. The design luminosity is an order of magnitude greater than is available today, and the machine poses significant challenges, including meeting the requirement of a clean environment around the vertex detectors, which must be close to the interaction point. At present none of the proposals has official approval.

For our purposes, perhaps the easiest measurement to make with such a facility is the asymmetry in the decay to ψK_S. The branching ratio is known (4×10^{-4}) and the decay has been reconstructed with little background. With the 100 fb^{-1} exposure, it is estimated that the asymmetry into this mode can be determined with a precision of 0.05 to 0.07. Other similar decays, such as ψK^* (Kayser *et al.*, 1990), can also be used to determine the same (quark level) asymmetry; however, it will be prudent to regard these as "insurance" against possible shortfalls elsewhere.

As was mentioned earlier, it is necessary to tag the other B decay. To reach the precision in the ψK_S mode, it is necessary to tag the other B with kaons (requiring particle identification) as well as with semileptonic decays. In addition, it is desirable to observe the $K_S\to2\pi^0$ decays requiring a high-precision electromagnetic calorimeter. Such a detector is already in place in CLEO.

Thus the asymmetry in the ψK_S mode ought to be determined with reasonable precision after a few years of operation of a B factory. To rule out the superweak model, it is necessary to see a different size effect in another transition, and thus we now turn to the experimental feasibility of the $\pi^+\pi^-$ decay. Unfortunately, there is as yet no measurement of the branching ratio, with only an upper limit of a few $\times10^{-5}$ (Drell, 1992). In addition, there is background with which to contend, coming largely from the continuum. The background can be substantially reduced by making cuts using the vertex detector. Assuming a branching ratio of 1×10^{-5}, a precision of 0.10 for the $\pi^+\pi^-$ asymmetry would be obtained with the 100 fb^{-1} exposure. While other modes, even non-CP eigenstates, can likely be used (e.g., $\rho\pi$, Aleksan *et al.*, 1991), we shall again adopt a conservative approach, keeping these other options in the background as insurance.

In conclusion, with a B factory delivering an integrated luminosity of 30 fb^{-1} per year and an appropriate detector, the asymmetry in the ψK_S mode should be able to be determined with a precision of better than 10% after three years. The asymmetry in $\pi\pi$ could also be determined with comparable precision if the branching ratio is of order 1×10^{-5}. Depending upon the actual values of the asymmetries, for a substantial portion of the allowed values of the CKM matrix parameters, one could establish direct CP violation and thereby rule out the superweak model; however, as discussed in Sec. VIII.B, there are values of the parameters for which the two cannot be distinguished (Winstein, 1992).

The technique (Gronau and Wyler, 1991) proposed for measuring the third angle of the unitarity triangle, γ, using decays of the $\psi(4S)$, can be used to detect a direct effect through the rate asymmetry given in Eq. (8.13) above, allowing the distinction between the standard and superweak models. A recent (Witherell, 1992) estimate has been made for the capability of a B factory experiment to observe such an asymmetry. It was found that with an 100 fb^{-1} exposure, the statistical error on the asymmetry in Eq. (8.13) would be about 35%, a figure far from what is needed. However, since the degree of color suppression is unknown, this remains an experimental question.

Charged decays, in general, offer some significant advantages in the search for direct CP effects. First, the "self-tagging" decays, for example, $B^-\to D^0\pi^-$, are useful, since the decay products determine uniquely the sign of the decaying B meson. Second, there are charmless decays without a complicated decay chain, such as ρK or $\rho\pi$, which can be reconstructed with high efficiency. The charged B's are produced in equal numbers, so one simply needs to count decays to the state and to the CP conjugate state. (This feature is probably unique to the e^+e^- colliders: pp colliders are not charge symmetric, and the production ratio has to be measured.)

One needs two interfering amplitudes (a penguin and a Cabibbo-suppressed spectator) together with a strong phase difference and, of course, a weak phase difference. Unfortunately, there are large uncertainties at present on the size of the penguin contributions. These uncertainties will be limited when the branching ratios are better determined. It is generally true that the channels with the smallest branching ratios are expected to give the largest asymmetries.

Given the cross section and the fraction of charged B's produced at the $\Psi(4S)$, one can calculate the statistical error on a determination of the asymmetry in a charged-B decay to a final state with a branching ratio B and detection efficiency ε from a 100 fb^{-1} exposure. The result is

$$\sigma(A)=6\%\sqrt{10^{-5}/B}\,\sqrt{0.25/\varepsilon}\ .$$

Given the present status of the predictions, it appears that one would have to be "lucky" to have a branching ratio at the level of 10^{-5} and an asymmetry of order 20%, allowing a three-standard-deviation observation of direct CP violation. Given the large number of channels over which to search, a four- or five-standard-deviation effect would be more desirable.

A copious source of B mesons exists at hadron machines. For example, at the Fermilab Tevatron, the cross section for the production of B-meson pairs appears to be around 60 μb. The Main Injector upgrade for that machine will provide a peak luminosity of 5×10^{31} cm^{-2} sec^{-1}. Thus, in two years of running (10^7 sec/year), a total of 1 fb^{-1} could be collected; within this sample there would be 6×10^{10} B-meson pairs produced. The rates far exceed those at a dedicated e^+e^- machine; however, the signal to noise is far worse, about 10^{-3}. This is roughly the level for charm production in Fermilab fixed-target experiments, and the success of the latter lead one to be optimistic regarding studies of B mesons at the collider.

Several groups (Castro *et al.*, 1990, 1991; Ellett *et al.*, 1991) have analyzed the possibilities; these have recently been summarized (Fermilab, 1992). There are a few key ingredients that a detector would need in order to make meaningful measurements of the expected asymmetries in the neutral-B decays. These are a three-dimensional vertex detector, the ability to trigger on secondary vertices, and K/π particle identification.

The vertex detector is needed to give background rejection and to allow the study of the time dependence of the B-meson decays. The ability to trigger on a secondary vertex is perhaps the most challenging requirement. This is necessary, in particular, for the $B\to\pi^+\pi^-$ decay; for ψK_S and related modes, the $\mu\mu$ signature provides an easy trigger. The particle identification is needed for two reasons: first, to increase the tagging efficiency, since the other B gives a K meson much more often than a lepton; and, second, to allow the discrimination of the $\pi\pi$ decay from the $K\pi$ decay.

The major focus of the CDF (Collider Detector at Fermilab) group is the continuing search for the top quark and other high mass scale physics. Nevertheless, they are gaining valuable experience in B physics now at the collider with a recently installed silicon vertex detector (Muller, 1993). Issues related to how cleanly and how efficiently one can separate vertices can only be addressed with such experience. They have presented (Bedeschi, 1993) a curve of reconstructed lifetime for inclusive J/Ψ events, which shows a prompt signal (85%) and a signal (15%) having a lifetime consistent, with good precision, with the world average b lifetime. In addition, they have reconstructed $J/\psi K_S$ and $J/\psi K^\pm$ decays of B mesons with little background. Based upon their current yields, it is estimated that by the end of Fermilab's collider Run I in late 1994 with an accumulated luminosity of 100 pb^{-1}, they will have about 200 $J/\psi K_S$ and 1000 $J/\psi K^\pm$ events. With respect to the tagging efficiency, little is known experimentally regarding the correlations in the B production processes. The collider detector at Fermilab will be able to study these correlations to better determine the tagging efficiency; from simulations, it is estimated to be in the range of 3%. False tagging and the cleanliness of the signals will also be studied. A substantial upgrade is in the works for the following collider Run II with a more powerful vertex detector having extended coverage. In Run II, to begin around 1997 with about 150 pb^{-1}, CDF should collect about 1000 $J/\psi K_S$ events with a tagging efficiency in the 5–10% range, perhaps permitting a measurement of $\sin(2\beta)$ at the level of 20%. Some trigger improvements are necessary to handle the higher luminosity.

To push beyond this level will require even more intensity; the Main Injector ring should give an additional factor of 5. A dedicated B experiment at the Main Injector might be a new one or a redirected collider experiment (CDF or $D0$). It is estimated that with the CDF detector, by adding silicon detectors in the forward direction perhaps another factor of 10 tagged $J/\psi K_S$ events could be collected with 1 fb^{-1}, permitting a measurement of $\sin(2\beta)$ with a precision of 0.07.

The $\pi^+\pi^-$ state poses significant additional problems, which will be addressed to some extent in the current runs. The background level in this mode is not known. As was mentioned earlier, one must develop a trigger on a displaced vertex and make sure that this is sufficiently stable and will not significantly bias the sample.

At the SSC, there are two possibilities for CP studies in B decays: a collider option and a fixed-target option. The same is true for the LHC.

At the collider, the B production cross section is likely in the range of 500 μb with a corresponding signal to noise of about 0.5%. Running at a reduced luminosity of 10^7 interactions per second, the collider still produces about 5×10^{11} B pairs per year of running, about one order of magnitude more than at the upgraded Tevatron. Thus the prospects are good that one could make detailed studies at the SSC, if a dedicated B-meson experiment were constructed.

An extracted 20-TeV external beam can also be used to study B physics at the SSC (Cox, 1990). The cross section is expected to be in the 10-μb range, so that, with an interaction rate of 10^7/sec, about 5×10^{10} B pairs are produced per year of running, comparable to the upgraded Fermilab Tevatron. Since the B-decay products have much higher momentum and are boosted into a smaller solid angle, there are potentially several advantages. With a "live target" of silicon planes, the (charged) B tracks are directly registered: on average, a B meson travels about 10 cm before decaying. In addition, at such momenta, multiple Coulomb scattering of the decay products is nearly negligible, and lepton triggering is likely more efficient. The proponents estimate that $\sin(2\beta)$ could be determined at the percent level in one year of running.

The B_S meson will likely not be studied at an e^+e^- factory. If it could be isolated at a hadron collider, a measurement of its decay asymmetry to $J/\psi\varphi$ could give information on the question at hand. This is because the standard model predicts negligible asymmetry for such a decay, whereas a superweak model could have a sizable asymmetry (Gérard and Nakada, 1991).

It should be emphasized that many systems in an SSC B experiment, either at the collider or in fixed-target mode, need to perform well beyond what has been achieved in existing experiments.

IX. OTHER MODELS OF *CP* VIOLATION

The standard KM model provides the most economical theory of CP violation, because it requires no particles or interactions beyond those already known or needed for understanding electroweak interactions. The CP violation is introduced explicitly into the Yukawa interaction of the quarks with the Higgs field. When the Higgs field acquires a vacuum expectation value, this yields CP-violating mass matrices; the diagonalization of these matrices results in the CP-violating CKM mixing matrix. Our interest in looking at theories that go beyond the standard model arises from (1) the lack of evidence yet available confirming the standard model, (2) the hope that one may gain greater insight into the origins of CP violation, and (3) the sensitivity of CP violation to new physics.

Alternative gauge theories require an extension of the gauge group or at least of the particle content of the standard model. In general, even though such theories provide new CP-violating interactions, they still have the CP violation of the KM type; that is, the parameter η will not be zero unless some new symmetry of the model requires it. In this paper we have contrasted two extremes: (1) All CP violation is due to a new superweak interaction; (2) all CP violation is explained by the standard model. Given the present experimental situation, we believe this is useful. In fact, most theories with a superweak interaction also have a nonzero η, although the value of η may be too small to explain the value of ε in the K^0 system.

A superweak interaction relevant to K^0 and B^0 physics leads to an effective interaction of the form

$$C_b\bar{b}O_i d\bar{b}O_i d + C_s\bar{s}O_i d\bar{s}O_i d + \text{H.c.}\ , \tag{9.1}$$

where O_i is a Dirac operator. If the superweak contribution to the K^0-$\bar{K}^0$ or B^0-$\bar{B}^0$ mixing matrix is called μ, then

$$\frac{\mu(B)}{\mu(K)}\sim\frac{f_B^2 m_B C_b}{f_K^2 m_K C_s} \tag{9.2}$$

using vacuum insertion to calculate the matrix elements in (9.1). The observables Q related to the mass matrix [for example, $\tilde{\varepsilon}$ in Eq. (2.2)] are proportional to $\mu/\Delta m$ where Δm is the measured mass difference between the eigenstates. Therefore

$$\frac{Q(B)}{Q(K)}\sim\frac{f_B^2 m_B C_b \Delta m(K)}{f_K^2 m_K C_s \Delta m(B)}\ .$$

Since $\Delta m(B)\sim10^2\Delta m(K)$,

$$\frac{Q(B)}{Q(K)}\sim\frac{1}{10}\frac{f_B^2}{f_K^2}\frac{C_b}{C_s}\ . \tag{9.3}$$

There are at least four possibilities discussed in the literature.

(1) The superweak interaction has a large CP violation, so that it contributes to ε_K but not to $\Delta m(K)$. The magnitude of $Q(K)$ is then of the order 10^{-2} to 10^{-3}. If we assume

$$(C_b/C_s)\le(m_b/m_s) \tag{9.4}$$

and $(f_B/f_K)^2<3$, then

$$Q(B)\le10Q(K)\le(10^{-2}\text{ to }10^{-1})\ . \tag{9.5}$$

Even though the superweak CP violation in the B systems given by (9.5) may be somewhat bigger than in the K system, it still is at least an order of magnitude smaller than expected in the standard model. With these assumptions, the superweak interaction is not important for B decays.

(2) The superweak interaction may make a significant contribution to $\Delta m(K)$. Letting Q_R represent CP-conserving observables, we let $Q_R(K)\sim(10^{-1}$ to $1)$; then, assuming (9.4) again,

$$Q_R(B)\le1\text{ to }10\ .$$

In this case $\Delta m(B)$ can arise from the interplay of superweak and standard model contributions (Liu and Wolfenstein, 1987b; Soares and Wolfenstein, 1992), although the superweak CP violation is not important for the B system.

(3) If we assume (C_b/C_s) is larger than (m_b/m_s), then there can be a large superweak CP-violating contribution to B^0-$\bar{B}$ mixing as well as the CP-conserving one (Gérard and Nakada, 1991).

(4) C_b is much larger than C_s, so there are significant

superweak effects in the B system but not in the K system (Nir and Silverman, 1990).

The simplest extension of the standard model is to add extra Higgs doublets. There is no basic reason to assume there is only a single scalar field in the theory. In general, theories with more than one Higgs doublet will have flavor-changing neutral scalar-boson exchange (FCNE). There are then two possibilities: (1) adjoin a discrete symmetry to forbid the FCNE leading to the Weinberg (1976) model of *CP* violation or (2) allow FCNE, but sufficiently small, leading to a superweak interaction.

The superweak *CP*-violating scalar-boson-exchange models have been formulated by many authors (Sikivie, 1976; Lahanas and Vayonakis, 1979; Branco *et al.*, 1985; Liu and Wolfenstein, 1987a; Deshpande *et al.*, 1992). In general, it is assumed that the scalar boson that is exchanged has a mass much larger than the electroweak scale, so as to make the interaction superweak. Liu and Wolfenstein (1987a) assume the scalar boson has a mass of the electroweak scale, but tune the couplings to make the interaction sufficiently weak.

Quantitative predictions of superweak scalar-exchange models depend on the variety of parameters such models necessarily contain. The following are some general features.

(1) The value of ε is primarily determined by the superweak interaction. The papers that consider the KM parameter η (Liu and Wolfenstein, 1987a; Deshpande *et al.*, 1992) find that η determines the value of ε', which may be within an order of magnitude of the standard model prediction. On the other hand, it is possible to force η to zero by a discrete symmetry (Bigi and Sanda, 1989; Lavoura, 1992).

(2) The FCNE may provide significant *CP*-violating effects in rare processes such as $K_L \to \mu^+\mu^-$ (Liu and Wolfenstein, 1987a).

(3) Since Higgs bosons couple proportionately to mass, the FCNE interactions are stronger for the B^0 system than for the K^0 system. Thus it is possible, but not necessary, that the FCNE plays a significant role in B^0-$\bar{B}^0$ mixing.

In the Weinberg model of *CP* violation due to Higgs exchange, there are three Higgs doublets. As a result of a discrete symmetry, only φ_1 couples to up-type quarks and φ_2 to down-type quarks. It then follows that there is no FCNE. The model contains two physical charged Higgs particles H_1^+, H_2^+, and *CP* violation occurs due to the tree-level exchange of these particles. Thus *CP* violation occurs in the decay amplitude (direct *CP* violation) as well as in K^0-$\bar{K}^0$ mixing in second order. The calculation of ε'/ε (Donoghue and Holstein, 1985) is very uncertain, but the order of magnitude is 10^{-2} if one tries to explain the value of ε entirely from this cause. In this case, also, one tends to obtain too large a value for the electric dipole moment of the neutron (Anselm *et al.*, 1985; Khatsymovsky *et al.*, 1987; Bigi and Sanda, 1989). On the other hand, if one assumes this interaction occurs in addition to the standard KM *CP* violation, then it may be unimportant for ε but may make important contributions to ε'/ε and to electric dipole moments. A unique feature of such Higgs exchange models is *CP* violation in semileptonic decays such as $K \to \pi\mu\nu$ (Belanger and Geng, 1992; Garisto and Kane, 1992), but the effects are probably too small to be observed.

Another possibility is to add heavy quarks to the theory that are singlets under the usual SU(2), so that they do not have the normal weak interaction. As a result of mixing between the singlet quarks and the usual quarks, there will be FCNE involving the Z bosons, since the GIM (Glashow-Iliopolis-Maini) mechanism is violated. If the mixing is large enough, this Z exchange could provide a superweak mechanism for B^0-$\bar{B}^0$ mixing (Nir and Silverman, 1990).

Gauge theories that go beyond the standard model involve larger groups that can be decomposed to $SU(3)\times SU(2)\times U(1)\times G$. These models may be superweak if G involves a horizontal symmetry, so that the gauge bosons in G change flavor and may allow $\Delta S=2$ at tree level (Mohapatra *et al.*, 1975; Maehara and Yanagida, 1978, 1979; Shanker, 1981; Joshipura and Montvay, 1982). As in the standard model, the origin of the *CP* violation is in the quark mixing matrices. In most papers it is assumed that this superweak interaction is the only source of *CP* violation, but this requires some additional symmetry to force the KM *CP* violation to vanish. Unlike the case of Higgs models, there is little reason in these models to expect significant *CP* violation in the B^0 system. In general, in these models the new gauge boson mass is greater than 10^3 TeV (Decker *et al.*, 1984), but in a version by Hou and Soni (1987) the mass can be as low as 10 TeV.

Still another class of superweak models (Barr and Zee, 1985) involves the exchange of diquarks. These are constructed with spontaneous *CP* violation such that there is no KM *CP* violation.

One of the simplest extensions of the standard model gauge group is the "left-right" theory with the group $SU(2)_L\times SU(2)_R\times U(1)$. Mohapatra and Pati (1975) originally pointed out that even with two generations of quarks one could get *CP* violation in this model. With three generations of quarks, the model contains many parameters and many sources of *CP* violation. These include, in addition to the parameter η in the KM matrix, (1) six phases associated with the coupling of the W_R gauge boson, (2) a phase describing the mixing of the usual W boson with W_R, and (3) superweak scalar-boson exchange. The phenomenology of various versions of this model is given in reviews by Mohapatra (1989) and Ecker and Grimus (1985). Of most interest are those versions in which W_R is sufficiently light (less than 10 TeV), so that most *CP* violation is associated with W_R exchange. The order of magnitude of ε'/ε is 5×10^{-3} in these versions, but no real quantitative statements can be made. As in other models, it is possible (if W_R is heavier) that ε is primarily explained by the KM parameter η, but that ε' and dipole moments may be dominated by the new physics.

X. CONCLUSIONS

We have described the theoretical and experimental status relevant to the most likely avenues in the search for a direct *CP*-violating effect. We regard the firm establishment of such an effect as an important milestone in the already nearly 30-year experimental study of *CP* nonconservation.

If the effect is soon established, it will lend strong support to the hypothesis that the source of *CP* violation is in the normal charged current and is associated with physics at the scale of 100 GeV. In the absence of such an established effect, models that sense physics at much higher mass scales are still viable, in particular, left-right symmetric theories and various superweak models.

If the standard model correctly accounts for *CP* violation through a nonzero value of the parameter η, the most likely place where a direct effect will first be established is in the continuing measurements of ε'/ε. Two experiments already have measured ε'/ε to an accuracy below 10^{-3}. One of these experiments (NA31 at CERN) gives a result over three standard deviations away from zero, which is on the high side of the allowed range in the standard model. The other (E731 at Fermilab) finds a smaller value not significantly different from zero. Both groups are building new experiments with sensitivities close to 10^{-4}, which should run in the mid 1990s. With this sensitivity it is likely that a definitive nonzero value of ε'/ε will be found if the standard model is correct. However, a result consistent with zero will not rule out the standard model, because of the uncertainties in the prediction.

Still in the area of K decays, the $K_L \to \pi^0 e^+ e^-$ and $\pi^0 \nu\bar{\nu}$ channels are the most promising. The former may have a significant indirect contribution and a large background as well. The latter, while pure direct *CP* violating, is experimentally very challenging. Both of these studies will need a much more intense source of kaons than is now available, such as can be derived from the Main Injector ring at Fermilab. These investigations with the requisite sensitivity are not likely to commence before the year 2000.

The B meson promises to be a rich laboratory for the study of *CP*-violating effects. A B factory would provide an intense source of relatively clean B-meson decays. If the standard CKM model is correct, *CP* violation in the neutral-B decay to ψK_S can almost certainly be seen at an asymmetric B factory. The branching ratio for this mode is known and the channel is quite clean. The theoretical uncertainties in relating such an effect to the parameters of the CKM matrix are very small. Such an observation would constitute a major discovery, that of the first signature of *CP* violation outside of the neutral-kaon system. It could, however, be superweak in nature. To establish direct *CP* violation, a different size effect must be established in another channel. The $\pi\pi$ decay is a good candidate, and if the branching ratio is 10^{-5} or greater, then with a 100 fb^{-1} exposure a clear direct effect should be established over much of the allowed (ρ,η) parameter space.

The possibilities for such studies at a B factory are so attractive that it is likely that at least one will be built. Such an integrated luminosity will in all probability be accumulated sometime early in the next decade. Direct *CP* violation could also be demonstrated by observing the difference in B^+ and B^- decays to a particular channel, such as ρK. The sizes of the possible asymmetries are quite uncertain, in particular because of their dependence on final-state interactions. Such observations could be made at a symmetric $\psi(4S)$ B factory. On the other hand, reasonable estimates suggest that it is very unlikely that such an asymmetry could be definitely observed with even 100 fb^{-1}.

Although there are many modes that can be studied at a B factory, it is likely that, if direct *CP* violation remains an open question after the ψK_S and $\pi\pi$ studies, a much more intense source of B mesons will be needed. Such a source will exist at the Fermilab Tevatron with the Main Injector upgrade, and an even more intense source will exist when the SSC and LHC begin operation early in the next decade. It will be up to the ingenuity of a new generation of experimental physicists to fully exploit the capability of the hadron colliders in this area.

Note added in proof. A final result on ε'/ε from the NA31 Collaboration has been submitted for publication and will appear shortly [G. D. Barr *et al.*, Phys. Lett. B **317**, 233 (1993)]. They report

$$\mathrm{Re}(\varepsilon'/\varepsilon) = (20.3 \pm 4.3 \pm 5.0) \times 10^{-4} \ (\text{NA31 '88, '89 Final}) ,$$

where they combine the 1988 and 1989 data sets. They then combine this result with their 1986 one, taking into account correlated systematic errors, to give

$$\mathrm{Re}(\varepsilon'/\varepsilon) = (23 \pm 3.6 \pm 5.4) \times 10^{-4} \ (\text{NA31 Final}) .$$

Averaging now this result with the final one from E731, using the procedure in Sec. V.B. 5, gives

$$\mathrm{Re}(\varepsilon'/\varepsilon) = (14 \pm 8) \times 10^{-4} \ (\text{NA31, E731 average})$$

so that we cannot as yet claim that direct CP violation is established.

The CLEO Collaboration is now reporting a positive observation for the sum of the $B \to \pi\pi$ and $B \to K\pi$ branching ratios (M. Battle *et al.*, CLEO Report No. CBX 93-90). The value is 2.4×10^{-5} and is more than 4 standard deviations from zero. For the $\pi\pi$ branching ratio itself, they are reporting $B(B^0 \to \pi^+\pi^-) \approx (1.3 \pm 0.7) \times 10^{-5}$. More data is being collected and if this is confirmed, then the prospects for the study of the $\sin(2\alpha)$ decay modes at a B Factory are good. At this time, it appears that the U.S. Congress has approved the SLAC proposal for a B Factory. Sadly, it has terminated the SSC project.

ACKNOWLEDGMENTS

We wish to thank many colleagues for discussions, in particular, I. Bigi, E. Cheu, B. Cox, G. Crawford, P. Drell, D. Herzog, B. Kayser, K. Lingel, E. Paschos, M. Procario, J. Rosner, L. M. Sehgal, A. Snyder, J. M. Soares, S. Stone, M. Wise, and M. Witherell. In addition, we thank T. Nakada and J. Rosner for many comments on the manuscript. Bruce Winstein acknowledges the contributions of his colleagues in E731/773/799/832. The work of Bruce Winstein and Lincoln Wolfenstein was supported in part by NSF Grant 9100690 and the U.S. Department of Energy Contract No. DE-FG02-91ER40682, respectively. The work of Lincoln Wolfenstein was partly carried out at the Institute for Nuclear Theory at the University of Washington, supported by the U.S. Department of Energy. Finally, we thank Kathy Visak for her patience in typing the manuscript.

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