Consistent interpretations of quantum mechanics

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Within the last decade, significant progress has been made towards a consistent and complete reformulation of the Copenhagen interpretation (an interpretation consisting in a formulation of the experimental aspects of physics in terms of the basic formalism; it is consistent if free from internal contradiction and complete if it provides precise predictions for all experiments). The main steps involved decoherence (the transition from linear superpositions of macroscopic states to a mixing), Griffiths histories describing the evolution of quantum properties, a convenient logical structure for dealing with histories, and also some progress in semiclassical physics, which was made possible by new methods. The main outcome is a theory of phenomena, viz., the classically meaningful properties of a macroscopic system. It shows in particular how and when determinism is valid. This theory can be used to give a deductive form to measurement theory, which now covers some cases that were initially devised as counterexamples against the Copenhagen interpretation. These theories are described, together with their applications to some key experiments and some of their consequences concerning epistemology.

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I. INTRODUCTION

A. About interpretation

1. What is interpretation?

The goal of an interpretation, when a theory is so abstract that our intuition fails to encompass it, is to reexpress the phenomena, the setup of an experiment and its data, in terms of the basic formalism. This, at least, is how the first interpretations appeared in physics with the advent of relativity.

The problem is deeper in the case of quantum mechanics because of a conflict between the intrinsically probabilistic character of the theory and the factual data one uses when verifying its predictions. The point is that an experimental device is made of atoms and, as such, one expects it to be submitted to quantum laws, which can only deal with probabilities. But a datum is a fact. Its certainty is taken for granted, and here is the conflict.

The program of an interpretation can therefore be defined as (i) reexpressing the phenomena and the data within the conceptual framework of theory; and (ii) reconciling the probabilistic character of the theory with the certainty of facts and the existence of macroscopic determinism (which may be not universal but is anyway inescapable).

The Copenhagen interpretation answered the first question by its axioms of measurement theory. It gave up solving the second one when it assumed that the measured object should be described by quantum mechanics and the experimental data by unmitigated classical physics.

2. Which interpretation?

One may wonder what criteria should be used to judge the quality of an interpretation. Interpretation is an intrinsic part of the theory. This is obvious, since it uses only the means that are provided by the theory in order to reach phenomenology. So, as a part, it must share the criteria of a theory, namely, to agree with experiment and to be consistent. Consistency is understood here in its usual logical sense, a theory being consistent when it contains no internal contradiction.

An interpretation should also be complete. One does not mean by that a philosophical notion of completeness such as was put forward, for instance, by Einstein *et al.* (Einstein, Podolsky, and Rosen, 1935). One demands only that the theory give a precise and unambiguous prediction in the case of every experimental situation. This is not a trivial condition, since it is not always clear what are the predictions of the Copenhagen interpretation in quite a few cases. Leggett (1980, 1987a, 1987b) was the first to stress that the Copenhagen interpretation is incomplete, and he proposed specific experiments to probe these weak points. Similarly, the interest shown in the observation of isolated atoms is not only due to the fact that it is a technical feat leading to useful data, but also because the predictions of the Copenhagen interpretation are somewhat unclear in that case. The same is true in the case of delayed-choice experiments.

Finally, an interpretation must make it clear how it describes facts. Bohr (1958) spoke of phenomena as being described in terms of classical physics. Landau and Lifshitz (1958a) formulated quantum mechanics in terms of a separate classical physics. Heisenberg (1958) and others (London and Bauer, 1939; Peierls, 1985) stressed the central role of an external, essentially classical observer. Accordingly, the Copenhagen interpretation does not really describe phenomena in the language of the theory; it only puts them side by side with theory. The question of consistency cannot even be stated in these conditions.

If, on the contrary, one assumes that there is only one physics, which must then be quantum physics, the theory must account for the existence of the factual data entering in its experimental verification. This means that, *if one assumes the universality of quantum mechanics, the theory must be consistent enough to encompass the conditions of its own experimental verification.*

This is of course an unthinkable goal in the older version of the Copenhagen interpretation, which is, however, the only one we have. Its formulation is too well known to be repeated here, and one may consult d'Espagnat (1976) or Primas (1983) for a specific account, see also the short and lucid discussion by Van Kampen (1988). There is no serious alternative to it, since the approach through hidden variables, whatever its interest, has not been developed to the point of giving a theory but only the preliminaries of a theory (Bell, 1964, 1966; Bohm, 1966; Belinfante, 1973).

The agreement of the Copenhagen interpretation with a vast number of experiments is excellent, despite the slight ambiguities that were mentioned before. It resists criticism beautifully, and recent progress has only made its real depth more obvious. Why then did not the Copenhagen interpretation look really satisfactory, to an extent that has kept research going on the subject for six decades?

The reasons have already been mentioned: the Copenhagen interpretation is incomplete, its consistency is very questionable, and its treatment of phenomena is much too superficial. It must necessarily be completed by solving at least two basic problems, namely, why one does not observe the linear superpositions of classically different states as they are exemplified by Schrödinger's cat (Schrödinger, 1935) and why classical determinism holds at larger scales despite the probabilistic behavior of atoms.

As far as determinism is concerned, one cannot give it up, even if it cannot be held as universal (as shown by the existence of chaotic systems) nor as absolute because of quantum fluctuations. Determinism is necessary to predict how a device in a laboratory is going to work. Without it, no apparatus would have any useful purpose and no experiment could be checked. It also tells us how to reconstruct the past from a record. Without it, one would not be able to keep records of a series of experiments, and it would even be impossible to check the probabilities that are predicted by quantum mechanics.

The shortcomings of the Copenhagen interpretation mentioned above should not, of course, be misconstrued as a lack of depth. It constitutes a wonderful achievement, considering when it was put forward. Its empirical success provides good reason to believe that there should exist a version of it enjoying completeness and consistency, while providing a satisfactory description of phenomena and justifying the use of common sense notions.

The present review is devoted to some recent attempts at finding this kind of consistent Copenhagen interpretation and to the claim that these goals have been met.

B. Orientation

The kind of progress one needs to obtain a satisfactory version of the Copenhagen interpretation does not demand a drastic revision in the foundations of the theory. Rather, it asks for the solution of a few specific problems, and this can be accomplished by purely technical advances.

Quite a few such advances have been made during the last decade. They took place after an episode during which much interest was focused upon hidden variables because of a test for their existence that was discovered by John Bell (1964). The experiments have now decided against hidden variables (Aspect *et al.*, 1981, 1982), although it would be excessive to say that all the conceivable possibilities have been excluded. The remaining ones, however, either assume action at a distance, or they have no precise formulation, so that they offer little grasp for research. The time is therefore ripe for a better understanding of the Copenhagen interpretation.

The renewal of this interpretation began when decoherence became understood (Zurek, 1982). Decoherence means essentially that the wave functions of the internal electrons and atoms are orthogonal for two clearly different situations of a macroscopic body, or at least they rapidly tend to become orthogonal. This is due to the interaction between the collective and internal motions that is also responsible for dissipation. Quantum density operators describing the macroscopic properties then become almost diagonal, so that the linear superpositions of quantum states are broken, together with the quantum interferences they go with. For instance, an initial quantum superposition of two states for Schrödinger's cat, say $2^{-1/2}|\text{dead}\rangle+2^{-1/2}|\text{alive}\rangle$, becomes through dynamics a simple statistical alternative for the two disjoint events, either "dead" or "alive" with equal probabilities.

Another significant step forward was made when Griffiths (1984) introduced the notion of history for a quantum system. It is defined as a sequence of properties holding at a sequence of times, and it was found to be an encompassing notion, wide enough to describe all of phenomenological physics. Among all the conceivable histories, only a few make sense, insofar as one can assign a probability to them. The meaningful ones can be selected according to well defined algebraic criteria. One can best appreciate the interest of this result by means of a simple example: when the possibility of observing a photon behind an interferometer is stated by mentioning the various regions where it can hit a screen, there is no meaningful history stating that the photon also goes through only one arm of the interferometer. Accordingly, histories give us a grasp of what is meaningful or meaningless in quantum mechanics.

Gell-Mann and Hartle brought together the ideas of history and decoherence. They proposed a theory that can be considered as a good candidate for a consistent and complete interpretation, although some parts of it still remain in the state of a program (Gell-Mann and Hartle, 1990).

In the meantime, the author had proposed another similar theory (Omnès, 1988a). Although it relied upon some new principles, it soon turned out to be in fact a reformulation of the Copenhagen interpretation well suited to the treatment of consistency. It stressed strongly the logical aspects of quantum mechanics.

The logical structure of quantum mechanics was found to be a useful guide in organizing the various problems occurring in the construction of an interpretation. It led first to a theory of classical phenomenology containing the proof that classical determinism is a consequence of quantum mechanics (Omnès, 1989). Together with decoherence, it generated an apparently complete and consistent interpretation (Omnès, 1990).

Altogether, therefore, there may now exist at least two complete and consistent interpretations of quantum mechanics. They are mostly equivalent. Their construction requires in one form or another four basic ingredients, namely,

(i) histories,
(ii) logic,
(iii) decoherence,
(iv) advanced semiclassical physics.

The purpose of the present review is to describe these theoretical ideas together with their application to a few significant experiments. The strategy of this approach is best described as follows, at least in the version advocated by the present author:

(a) We shall assume that quantum mechanics is a universal theory, standing as usual upon some principles stating its mathematical framework and its dynamics, which is based upon the Schrödinger equation.

(b) In order to avoid all kinds of uncontrolled prejudice concerning the interpretation of the theory, we shall try to rely upon a firm logical basis, stating what can be considered as a property of a quantum system and how one can deal with these properties within a consistent logical setup. This can be done by working with histories, a unique, well defined, and universal rule for interpretation replacing all the usual axioms of measurement theory.

(c) Then the theory itself will be used to extract from its background all that one usually takes for granted from a phenomenological observation. For macroscopic systems, this includes their classical description and ordinary determinism, together with explicit criteria asserting when and how these classical ideas are allowed and when they do not apply. One might describe this essential step in a nutshell as distinguishing when common sense applies to the description of a quantum system and when it does not apply.

(d) Having recovered completely and limitedly classical physics, we shall reconstruct measurement theory, basing it upon the principles established above for working with histories.

The strategy used by Gell-Mann and Hartle is somewhat different, and it is described in Sec. II.H.

Section II will deal with the four ingredients (i)–(iv) of a theory, listed above. They are organized in a logical order that was advocated by the author, which is different from that followed by Gell-Mann and Hartle. This is why their theory is discussed only later on; the two approaches do not rely upon exactly the same lines of argument. Then measurement theory is treated in Sec. III. It is in principle completely deductive. It essentially recovers the prescriptions of the Copenhagen interpretation, except for a few more or less minor changes. Section IV is devoted to a discussion of some experiments to which the Copenhagen interpretation does not apply trivially. They include, together with some refined neutron interference experiments, the observation of a unique atom, delayed-choice experiments, and the macroscopic quantum behavior of superconducting interference devices. Many other beautiful experiments are not reviewed because they are conveniently and directly covered by the Copenhagen interpretation. Some epistemological comments are finally added in Sec. V. The Einstein-Podolsky-Rosen problem is included in Sec. IV because it occurs as a canonical case in a deductive measurement theory.

This review does not try to cover all the recent significant contributions to our understanding of quantum mechanics. This is because there has been progress in too many directions, and the corresponding literature is too prolific to be reviewed usefully. We shall be concerned here only with attempts at a synthesis and with the notions entering them. Even so, there is much to cover, and there have been particularly many significant results in the theory and applications of decoherence. Fortunately one can expect a forthcoming review by Zurek on this specific subject, so that it will only be sketched here as needed for the present purpose. See in the meantime Zurek (1991).

II. FOUNDATIONS

A. Preliminaries

Phenomenology is concerned with the properties of a physical system; it is important to make clear what a property of a quantum system is. This question goes back to the beginnings of the theory. Von Neumann (1932) showed that properties can be associated with projectors in Hilbert space. Later on, Gleason (1957) proved that the state of a system together with the associated quantum probabilities is uniquely defined by a few simple and logical requirements bearing upon the statement of properties. These results will be needed later on, so we shall review them briefly here for the sake of clarity.

1. Mathematical foundations

One can assume that, with the possible exception of spacetime, all of physics is controlled by quantum mechanics. Gell-Mann and Hartle also envision a cosmological version of the theory in which spacetime is included, at least in principle.

This assumption implies that the theory must deal with individual systems rather than statistical ensembles, since discussing, for instance, statistical ensembles of the solar system would be rather awkward. The rules of quantum mechanics are therefore stated for an isolated system, albeit if necessary an arbitrarily large one. A physical system consists of particles, either fixed in number or with fixed values for the conserved quantum numbers (in the case of field theory).

The concepts of fields or particles are given a content by some specific observables, which may be the fields themselves or the momentum and position of particles (position being easy to define only in the nonrelativistic case). These observables can be more precisely defined with the help of invariance properties, momenta being for instance the generators of space translations. They can also be thought of as functions in Feynman's path-integration approach, (Feynman, 1948; Feynman and Hibbs, 1965), as elements of an abstract algebra (Dirac, 1930; Haag and Kastler, 1964), or as self-adjoint operators in a Hilbert space (Von Neumann, 1932). It is well known that quantum mechanics has several such equivalent formulations, one or the other being best suited to a specific problem. The Hilbert-space approach will be used here because it is most widely known.

An individual isolated physical system is therefore associated by the theory with a definite Hilbert space and a definite algebra of observables (i.e., self-adjoint operators). The theory will never leave this framework, which is enough to provide an account of dynamics, logic, and interpretation.

2. The properties of a system

A property of a system is given when one can assert that the value of an observable A is in some given set D of real numbers. This set may be just one number when the spectrum of A is discrete, or an interval or a more complicated domain. Von Neumann (1932) associated a property with a projector in Hilbert space, namely,

$$E=\sum_{a\in D}\left[\sum_r |a,r\rangle\langle a,r|\right] , \tag{2.1}$$

where $|a,r\rangle$ is a normalized eigenvector of A with eigenvalue a in Dirac's notation, and r is a degeneracy index.

The following features of this correspondence are worth noticing (Von Neumann, 1932; Mackey, 1963):

(i) Only the values of the observable A belonging to the spectrum of the associated self-adjoint operator contribute to the sum (2.1).

(ii) When a change of observable amounts to a straightforward computation, there is no change of projector. For instance, it amounts to the same thing to say that the value of A is in the interval $[-1,+1]$ or that the value of A^2 is in the interval [0,1]. The projectors are the same.

(iii) Every projector E is associated with some property. This can be shown by taking $A=E$ and $D=\{1\}$.

From here on, no explicit distinction will be made between properties and projectors, no more than between observables and self-adjoint operators.

Although the properties are only sentences with no empirical meaning at this early stage in the theory, we note with Von Neumann that they are at least sensible as sentences: Keeping the observable A fixed and denoting by E and E' the projectors associated with different domains D and D', one sees that

(i) E and E' commute;

(ii) One can identify the properties saying that "the value of A is in D and/or in D'" with the simpler ones saying that "the value of A is in $D\cap D'$ (or $D\cup D'$, respectively)," having for their respective projectors EE' and $E+E'-EE'$. The negation "the value of A is not in D" also says that "the value of A is in $\bar{D}$," $\bar{D}$ being the complement of D and the associated projector $\bar{E}$ being $I-E$. The conventional rules for the use of the logical functions (and, or, not) are consistent with this formulation of properties.

When noncommutative observables are considered, the associated projectors do not commute, and there is no possibility of preserving these simple logical aspects of the construction. Birkhoff and Von Neumann tried to dispose of this difficulty by using nonconventional logics (Birkhoff and Von Neumann, 1936; Mackey, 1963; Mittlestaedt, 1978, 1986; Jauch, 1968; Primas, 1983; Mittlestaedt and Stachow, 1985). It does not seem, however, that this approach yields an explicitly consistent interpretation, and accordingly the properties associated with noncommuting projectors will never be considered simultaneously in the following.

3. Probabilities and states

One can think of the *state* of a system as a datum allowing one to define a probability for each property. The probability of a property with projector E will be written as $p(E)$. Once again, at this early stage in the construction, the probability is not yet given an empirical content; it is only a convenient mathematical device to be used for constructing an interpretation. Accordingly, one can only submit it to conditions arising from the meaning of properties and from the axioms of probability calculus. These are the following:

(i) The probability $p(E)$ depends only upon the projector E and nothing else. For instance, it is insensitive to the choice of a basis in the subspace of the Hilbert space having E for its projector.

(ii) Positivity: $p(E)\geq 0$.

(iii) Normalization: $p(I)=1$, the identity operator I corresponding to a trivial property saying that the value of an observable is anything whatever.

(iv) Additivity: if two properties (E,E') are such that E and E' commute and the product EE' is zero, one has

$$p(E+E')=p(E)+p(E') . \tag{2.2}$$

This assumption corresponds, for instance, to the case of two *disjoint* domains D and D' for the values of an observable A. Additivity is clearly necessary for the consistency of the language of properties in that case.

An important theorem, due to Gleason (1957), states that these conditions are sufficient to define completely the mathematical expression of $p(E)$: There must exist a positive operator ρ with unit trace such that

$$p(E)=\mathrm{Tr}(\rho E) . \tag{2.3}$$

This operator ρ defines, therefore, the state of the system, since it constitutes a datum allowing a knowledge of probabilities. It may be called a state operator, although it is more frequently referred to as the density operator. The system is said to be in a pure state ψ (ψ being a normalized vector in the Hilbert space) when ρ is the projector upon ψ,

$$\rho=|\psi\rangle\langle\psi| . \tag{2.4}$$

When the property E states that the value of A is in D and the state is pure, one gets from Eq. (2.3)

$$p(E)=\sum_{a\in D}\left[\sum_r |\langle a,r|\psi\rangle|^2\right] , \tag{2.5}$$

i.e., Born's probability rule, which therefore appears as a consequence of straightforward logical considerations.

Gleason's theorem holds only for a dimension of the Hilbert space larger than 2. It was first stated for a separable Hilbert space and the proof was made clearer by Jost (1976). It has been extended to a large class of nonseparable Hilbert spaces. For a recent survey, see Matsuda (1990).

4. Dynamics

Time evolution is expressed by a unitary operator $U(t)$ generated by a Hamiltonian operator H,

$$U(t)=\exp(-iHt/\hbar) . \tag{2.6}$$

This is of course equivalent to the Schrödinger equation.

One can use either the Schrödinger representation, in which the observables are kept fixed and the density operator evolves according to

$$\rho(t)=U(t)\rho U^{-1}(t) , \tag{2.7}$$

or the Heisenberg representation, in which the density operator is kept fixed and the observables change with time according to

$$A(t)=U^{-1}(t)AU(t) . \tag{2.8}$$

The time-dependent Heisenberg operator $E(t)$ associated with a given projector E can be used to give a meaning to a time-dependent property, stating for instance that "the value of A is in D at time t." The associated probability can be written as

$$p=\mathrm{Tr}\{\rho(t)E\}=\mathrm{Tr}\{\rho E(t)\} . \tag{2.9}$$

B. Histories

1. What is a history?

The idea of histories, extending significantly the notion of properties, was introduced by Griffiths (1984). Basically, a history is simply a series of properties occurring at an ordered sequence of times $t_1, t_2, \ldots, t_n$ $(t_k < t_{k+1})$. A typical history states some property at time t_1, then another property at time t_2, and so on. It may give, for instance, the position of a particle within some well defined range at time t_1, then its momentum under similar conditions at time t_2, its position again at time t_3, and so on.

As a matter of fact, one can find earlier considerations of histories in several works (Gell-Mann, 1963; Aharonov *et al.*, 1964; Houtappel *et al.*, 1965; Carmichael and Walls, 1976; Cohen-Tannoudji, 1975; Cohen-Tannoudji and Reynaud, 1979; Walls, 1979). The breakthrough made by Griffiths was his discovery of the concept of consistent histories, as will be explained below.

One may envision a history as a sequence of snapshots resulting in a motion picture of the system. It contains of course much more than the unique photograph given by a Von Neumann property. What is perhaps surprising is that histories are versatile enough to provide a complete description for all of physics, as will be seen.

For definiteness, we shall consider that the property holding at time t_k states that the value of an observable A_k is in a range D_k. No commutativity of the various observables A_k is assumed, but one can use several commuting observables $A_k^{(r)}$ together at the same time in place of just one, asserting for instance that the values for the three components of a position are in some given volume. The time-dependent projectors associated with the properties occurring in the history will be denoted by $E_k(t_k)$.

2. Probability of histories

The probability of a history is given by the quantity

$$p=\mathrm{Tr}\{E_n(t_n)\cdots E_2(t_2)E_1(t_1)\rho E_1(t_1)E_2(t_2)\cdots E_n(t_n)\}, \tag{2.10}$$

where ρ is the state operator at an initial time zero earlier than $t_1 (0 \le t_1)$. Once again, this probability is only a convenient mathematical tool with no empirical content for the time being. Its main purpose is to provide a consistent language for talking about physics with a view towards interpretation.

Equation (2.10) is the one that was used by Gell-Mann and Hartle and by the author. Griffiths used a slightly different notion, what he called a conditional probability, in which the state operator is specifically assumed to be given by a property, i.e., by a projector E_0. Then he defined the probability as

$$p_G=\mathrm{tr}\{E_n(t_n)\cdots E_1(t_1)E_0E_1(t_1)\cdots E_n(t_n)\}/\mathrm{Tr}\{E_0E_{n(t_n})\} . \tag{2.11}$$

The main difference between these two definitions is that p_G is time-reversal invariant whereas p is not. This was the main reason for Griffiths's choice, but it turns out that the choice (2.10) is more convenient in spite of that. Moreover, there is no reason to assume that the initial state is always defined by a property. It depends in fact upon the complete previous history of the system, which tells how it was prepared, and this has no correspondence with a time-reversed situation.

One must of course justify this choice of probabilities: Eq. (2.10) was not new when Griffiths proposed it. It had already been found to describe the outcome of several successive measurements in the Copenhagen interpretation (Aharonov *et al.*, 1964), and Griffith's approach was essentially similar, although he did not refer to the properties as being necessarily checked by a measurement.

This kind of argument is no longer satisfactory when the history is considered as a pure assumption and the notion of history itself becomes a building block for an interpretation to come. One must then find better reasons for adopting Eq. (2.10).

The simplest approach uses Feynman paths integrals (Gell-Mann and Hartle, 1990; Omnès, 1988a). It is restricted to the case in which each observable A_k is either a position or a momentum observable. One writes down the probability amplitude for the last property at time t_n, given the initial state at time zero, as a Feynman path integral. If then the property at time t_k states that A_k is, for instance, the position, and it lies in a domain D_k, one only keeps the paths crossing this domain D_k at time t_k

in the Feynman sum. When the same is done for all intermediate times, one gets Eq. (2.10).

There is also a formal approach more akin to the logical foundations of Gleason's theorem. It consists once again in giving a meaning to the language of properties. One assumes that

(i) The probability of a history depends only upon the properties occurring in it, i.e., upon the corresponding projectors.

(ii) When $n=1$, the probability is given by Gleason's theorem.

(iii) When $E_{k+1}(t_{k+1})=E_k(t_k)$, i.e., when an immediately previous property is stated again, this is a tautology and the probability is the same as for the history mentioning the property only once.

For the same reason, when the first property is tautological with respect to the initial state, i.e., when

$$\rho E_1(t_1)=\rho \ , \tag{2.12}$$

one can suppress the first property.

(iv) When two successive properties $E_{k+1}(t_{k+1})$ and $E_k(t_k)$ contradict each other, i.e., when they commute and their product vanishes, this is nonsense and the probability is zero.

(v) Probabilities are additive for two disjoint properties occurring at the same time.

These simple conditions imply Eq. (2.10) for the probabilities. Accordingly, this formula is unique, and it can be used safely as a basis for constructing an interpretation. It may be mentioned that not all the additivity conditions (v) are satisfied by the formula, and one obtains Eq. (2.10) by using additivity only for the last property.

3. Additivity and consistency

As will become clearer when quantum logics are introduced, one should not consider a unique history alone but a complete family of possible histories together, just as one considers a complete family of possible events in probability calculus. To do so, one specifies once and for all the initial state, the sequence of times $\{t_k\}, k=1,\ldots n$, and the observables $\{A_k\}$. The spectrum of each observable, say A_k, is divided into a complete family of disjoint sets $D_k^{\alpha_k}$. When replacing the sets $\{D_k\}$ in the previous definition of a history by all the possible sets $\{D_k^{\alpha_k}\}$, one obtains a complete family of histories. Figure 1 shows how this can be represented graphically in the case $n=2$. Each property $[A_k, D_k^{\alpha_k}]$ is associated with a projector $E_k^{\alpha_k}(t_k)$. All these histories produce not only one motion picture but a complete family of motion pictures with different scenarios. One can also think of them as different events as in probability calculus.

Several disjoint histories can be put together to produce another less detailed history. This is shown in Fig. 1 where the shaded rectangular region corresponds to a history; this rectangle is made of several smaller rectangles, i.e., the larger history comprises several more detailed histories. Additivity requires that the probability of the larger history is the sum of the probabilities for the more detailed ones entering it.

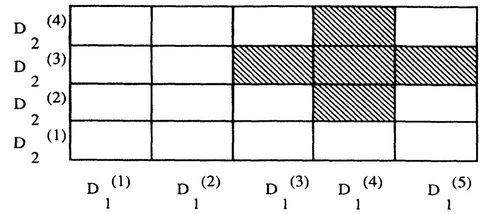

FIG. 1. Elementary histories are associated with small boxes. A larger box at the lower left is associated with a larger history. The shaded region at the upper right is associated with a more general proposition.

Not all complete families of histories satisfy additivity. If one goes back to the expression of probabilities in terms of Feynman path integrals, one finds that the windows through which the paths must go are quite severely restricted for this to be true. They must be so well chosen that the additivity of probabilities is consistent with the quantum additivity of amplitudes. This condition can also be expressed by some mathematical equations or *consistency conditions* (Griffiths, 1984).

Gell-Mann and Hartle (1990) have proposed a convenient set of consistency conditions. They read as follows:

$$\mathrm{Tr}\{E_{n-1}^{\alpha_{n-1}}(t_{n-1})\cdots E_1^{\alpha_1}(t_1)\rho E_1^{\alpha'_1}(t_1) \cdots E_{n-1}^{\alpha'_{n-1}}(t_{n-1})E_n^{\alpha_n}(t_n)\}=0 \ , \tag{2.13}$$

when the sequence $\{\alpha_k\}$ is different from the sequence $\{\alpha'_k\}(k=1,\ldots,n-1)$. These conditions are sufficient for additivity but they are not necessary. This means that they might be in some cases too restrictive, leaving aside families of histories for which additivity holds whereas the stringent conditions (2.13) do not.

Necessary and sufficient conditions have been given by Griffiths (1984) and by the author in another form (Omnès, 1988a), and one then speaks of consistent families of histories. It will be enough to give an example in the simplest case where $n=2$, when each spectrum is divided into two complementary sets (see Fig. 2). There is

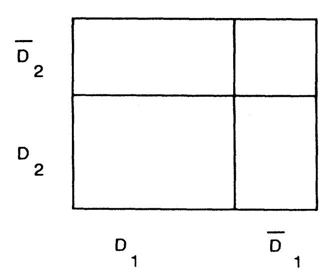

FIG. 2. The simplest family of histories giving rise to a consistency condition.

only one condition in that case, which was written by Griffiths as

$$\mathrm{Re}\,\mathrm{Tr}\{E_1(t_1)\rho\bar{E}_1(t_1)E_2(t_2)\}=0 \tag{2.14}$$

and by the author as

$$\mathrm{Tr}\{[E_1(t_1),[\rho,\bar{E}_1(t_1)]]E_2(t_2)\}=0\ . \tag{2.15}$$

Each of these forms can be extended in a precise manner to the general case. They should be compared with condition (2.13), which reads in that case

$$\mathrm{Tr}\{E_1(t_1)\rho\bar{E}_1(t_1)E_2(t_2)\}=0\ . \tag{2.16}$$

Clearly, the conditions (2.14) or (2.15), which are equivalent, are less stringent than condition (2.13).

For instance, in the case of spin $\frac{1}{2}$, if the initial state specifies the value of a spin component along a direction n_0, whereas the properties at time t_1 and t_2, respectively, state the values of the spin components along two directions n_1 and n_2, the conditions (2.14) or (2.15) give the following geometric condition:

$$(n_0\times n_1)\cdot(n_1\times n_2)=0\ .$$

The more stringent condition (2.13) demands in that case that $n_1=\pm n_0$ or $n_1=\pm n_2$.

It turns out that, in most cases of practical interest, one can use families of histories for which the simpler condition (2.13) holds. The Griffiths conditions (2.14) are, however, necessary when it comes to proving beyond any doubt that a statement is meaningless whatever its context, such as saying for instance through which arm of an interferometer a photon goes. In that case, one must make sure that no possibility has been left out, and one must use necessary and sufficient conditions.

One can also use more general families of histories that are sometimes useful for making calculations simpler (Omnès, 1988a). Their use is seldom imperative, except when discussing delayed-choice experiments.

C. The logical structure of quantum mechanics

1. What is the problem?

Griffiths histories can be used to build up a consistent logical structure of quantum mechanics. This is essential for an interpretation because the difficulty one meets when relating the mathematical structure of the theory to the empirical description of phenomena is not so much a quantitative matter as a logical one. The language of Hilbert spaces, operators, and so on and the language of phenomena look very far apart, but they have at least one common feature, which is their submission to logic. The problem of interpretation can therefore be made clearer if one is able to find a firm logical structure in quantum mechanics.

The language of properties was a first step in that direction. It shows how some statements having a rather clear meaning in empirical physics can also be given a perfectly clear meaning within the mathematical language of the theory. The main problem is then to use this language of properties for reasoning and not only for talking.

This can be made clearer with an example: Consider an experiment in which a neutron coming from a reactor produces fission in a target containing uranium; some decay products enter a mass spectrometer and are then detected. As an example of empirical reasoning, one might say that a counter has registered, and this corresponds to a xenon mass, therefore it detected a xenon nucleus. Therefore fission has taken place. Therefore a neutron collided with a uranium nucleus. Therefore this neutron came from the reactor and crossed a velocity selector belonging to the experimental device. From this one can deduce the neutron momentum and the kinematics of the collision.

Similar reasoning is used when one derives physical data from an experiment, or when computing statistics and estimating systematic errors. This reasoning essentially relies upon common sense and makes only a limited use of quantum notions. The problem is to found this reasoning consistently upon quantum mechanics, which is assumed to be the only basis of physics.

One sees from the example that it is far from enough to consider only one property, which would be the actual detection of the xenon nucleus as the Copenhagen interpretation would have it. Many other properties are worth mentioning for a complete understanding of the process. Furthermore, logical implications are everywhere needed, each time one might say "therefore" or "if. . . ,then."

Accordingly, one needs a logical structure for the theory extending the language of properties in order to allow for all the different properties occurring at different times to be mentioned. It should give an explicit recipe for deciding when one can infer a logical consequence of some known property via an implication.

This need was clearly perceived by Von Neumann, but he stumbled upon the rigidity of quantum rules and of formal logic. His proposal for using unconventional logics (Birkhoff and Von Neumann, 1936) was brilliant but *a priori* unable to recover common sense, despite the efforts made in that direction (Mittlestaedt, 1978, 1986).

Griffiths consistency conditions have shown where the difficulty lies: One cannot put together arbitrary properties at various times. Among the histories that one can build from them, only those satisfying the consistency conditions can be meaningful. Once this is realized, so that simultaneously meaningful properties can be sorted out, it becomes possible to reconcile quantum-mechanical concepts with conventional logic.

The logical structure of quantum mechanics is therefore based upon consistent families of histories together with logical rules allowing one to infer their consequences. Although this logical structure is only a part of a complete and consistent interpretation, it is nevertheless very useful because it provides a guideline for connecting all the other constituent parts of the interpretation.

The content of the present section is due to the author (Omnès, 1988a, 1990). Its partial extension to the framework of C^* algebras and to relativistic situations has been worked out by Blencowe (1991).

2. What is logic?

What one needs to know about logic for interpreting quantum mechanics is very little, but it should at least be stated precisely. One must first distinguish between a logic that deals with a specific subject and logic as such, the general theory. What is called here a logic is something peculiar to the language of physics, what a logician would rather call an interpretation of formal logic. This name was introduced by Birkhoff and Von Neumann because the word "interpretation" is used in physics for something else.

A logic consists of three ingredients: (i) a field of propositions, (ii) a few logical tools, and (iii) a criterion for truth.

A field of propositions is a family of sentences. For instance, all the quantum properties $[A,D]$, where the observable A is kept fixed and the range of values D can vary, constitute a field of propositions. In classical physics, one can consider as another example a given physical system with canonical coordinates (q,p). Given a domain C in phase space, a typical proposition in classical physics would say that the coordinates (q,p) of the system are in the cell C at time t. The propositions in that field can be denoted by $[C,t]$.

The logical tools are of two kinds, operations and relations. There are three operations: not, and, or. The negation of a proposition a will be denoted by $\bar{a}$. Given two propositions (a,b), one can consider "a and b" as being another proposition, similarly for "a or b." In the examples just given, it is clear how "not, and, or" can be associated, respectively, with the familiar operations in set theory, complementation, intersection, and union. For instance, the proposition "(q,p) is in C or in C'" is equivalent to the proposition "(q,p) is in $C \cup C'$."

There are only two logical relations, namely, implication ($\Longrightarrow$) and equivalence ($=$). Given two propositions (a,b), a meaning is supposed to be given to the sentence "if a, then b," also denoted by $a \Longrightarrow b$. As for logical equivalence ($a=b$), it simply means that $a \Longrightarrow b$ and $b \Longrightarrow a$.

One can give a nontrivial example of these notions in the case of classical mechanics. Consider two domains C and C' in phase space and the two propositions (a,b) stating, respectively, that (q,p) is in C (C') at time t (t'). Assume Hamilton's equations for classical motion. These equations define how a point (q,p) in phase space moves during the time interval $[t,t']$. Accordingly, they define how the points in C are transformed by classical motion, so that C is transformed into another domain $g(C)$. Then one can say that a implies b when $g(C)$ is included in C'.

We note that the two propositions are logically equivalent when $g(C)$ coincides with C'. Incidentally, this shows that *classical determinism is essentially a logical equivalence.* How the logical tools are explicitly defined upon a given field of propositions is left up to us. Whatever they may be, the tools should in any case satisfy some twenty or so formal axioms that are listed in textbooks on logic (Van Heijenoort, 1967; Manin, 1977).

Although the need for a criterion for truth has been mentioned, it will have to be left aside momentarily, except for mentioning that in classical physics a proposition is true when it expresses an actual fact *or* it is logically equivalent to an actual fact. The consistency of the two possibilities in this criterion lies upon the validity of classical physics and particularly upon determinism. The notion of truth in quantum mechanics is much more subtle, and one will have to wait until Sec. III.C before having the means to give it a content. Apologies are made to the reader for this unavoidable inconvenience.

3. Quantum logics

A quantum logic must first specify a definite field of propositions. This field is based upon a family of histories: One considers a physical system in a given initial state at time 0, a sequence of times $(t_1, \ldots, t_n)$, and a family of observables $(A_1, \ldots, A_n)$, the spectrum of each observable A_k being divided into a complete collection of disjoint sets $D_k^{\alpha_k}$. It will be simpler to consider explicitly the case in which $n=2$ because it can be made clearer by using some drawings.

Any history in this family can be represented by a two-dimensional set $D_1^{\alpha_1} \times D_2^{\alpha_2}$ (as in Fig. 1) to be called a box. Using these boxes as building blocks, one can construct various sets, such as the shaded region in Fig. 1. Expressing that the values of the observables A_1 and A_2 at times t_1 and t_2 belong to such a set constitutes a proposition. The logical operations (not, and, or) will then be associated as usual with the operations among sets consisting of taking a complement, an intersection, or a union of sets.

Then, one must define implication. To do so, we shall assume that the family of histories satisfies the consistency conditions, so that a sensible probability is well defined for each proposition as a sum of elementary probabilities (2.10). Given two propositions (a,b), it will be said that a implies b when the conditional probability for b given a is unity,

$$p(b|a)=p(a \text{ and } b)/p(a)=1 . \qquad (2.17)$$

It can be proved that all the conventional axioms of formal logic are satisfied by these conventions when the underlying collection of histories is consistent. Moreover, consistency is a necessary and sufficient condition for this to be true.

A given logic can be simplified, for instance, by taking a coarser graining for the sets dividing a spectrum or by suppressing every reference to some specific time t_k in the sequence (this amounts to not dividing the spectrum of A_k at all; i.e., to taking the coarsest graining). One can also extend a logic by using a finer graining or by adding further reference times together with their associated observables, as long as the necessary supplementary consistency conditions are satisfied.

4. The complementarity principle

In classical physics, one might proceed analogously by introducing an arbitrary sequence of times and completely dividing phase space into cells, assuming that an initial probability distribution $f(q,p)$ is given. There are many such logics because of the arbitrariness in the choice of reference times and of the graining of phase space. This is, however, of no consequence: given two logics L and L' (i.e., two fields of propositions), one can always find a finer-grained logic L'' containing both L and L'.

This insensitivity to the arbitrariness of a choice of logic no longer holds in quantum physics because, when two logics L and L' are given, there does not exist in general a larger consistent logic L'' containing both of them. The two logics L and L' can be said in that case to be *complementary,* which is essentially what Bohr meant by the complementarity principle.

As an example, one can consider the following two logics: The system is a spin $\frac{1}{2}$. One considers two times t_1 and t_2. The initial state is a pure state with $\sigma_x=1$. A logic L is obtained by taking $A_1=\sigma_z$ and $A_2=\sigma_z$, each spectrum being naturally divided into its two components $\{1\}$ and $\{-1\}$. Another logic L' can be obtained similarly by taking $A_1=\sigma_x$ and $A_2=\sigma_z$. It is easily shown that L and L' are both consistent, but they are complementary to each other.

One might be afraid that the existence of different logics could raise the spectre of paradoxes, if a paradox is a situation where the same assumption a implies a conclusion b through one mode of reasoning whereas it does not imply b through another mode. This is fortunately not so, because a no-contradiction theorem shows that, whatever the consistent logics L and L' both containing (a,b) in their fields, an implication $a \Longrightarrow b$ holds necessarily in both logics together. So, at least in a restricted but a precise sense, quantum mechanics is in principle immune to paradoxes.

This does not mean of course that quantum mechanics must agree with common sense, which may be defined in its learned version as the use of logic with purely classical concepts. On the contrary, quantum mechanics must predict when and why one is allowed to use common sense.

5. A rule of interpretation

One can then introduce a rule for the interpretation of quantum mechanics, which reads as follows:

Any description of a physical system should consist of propositions belonging to a common consistent quantum logic and any reasoning about it should consist of valid implications.

This rule can replace all the axioms of measurement theory as they were set forth by the Copenhagen interpretation and it can also be used as a guideline for constructing a complete and consistent interpretation.

No specific example of its use will be given now, but a few will soon occur when the rule is used to produce a theory of phenomena in Secs. II.E and II.F, and others will be detailed in Sec. IV, where experiments are discussed. Some elementary examples can be found in Omnès (1988a, 1988b, 1990).

6. Approximate logic

The overall consistency between quantum mechanics and the description of phenomena by common sense cannot be perfect, and one must be prepared to allow for some errors. To take an extreme example, the Earth could leave the Sun and go revolving around another star by a tunnel effect. So, when common sense says that the sun will rise tomorrow, this is only correct up to a very small error in probability from the standpoint of quantum mechanics. Accordingly, the logical structure of quantum mechanics cannot be absolutely precise. Some amount of error must be allowed for.

Given a very small number ϵ, a logic will be said to be consistent to order ϵ when the trace in the consistency condition (2.13) is not strictly zero but of relative order ϵ as compared with a typical relevant probability. Similarly, an implication $a \Longrightarrow b$ holds with error ϵ when the conditional probability $p(b|a)$ is larger than $1-\epsilon$. These approximations have been justified, as far as the use of incompletely consistent probabilities is concerned, by Gell-Mann and Hartle (1990).

D. Macroscopic systems

Classical physics deals mainly with macroscopic systems and, more properly, with macroscopic objects. It will be convenient to review briefly the relevant concepts for the sake of clarity and particularly the notion of collective observables describing the classical behavior of such a system. Then, we shall see how to express a classical property as a quantum property by associating it with a projector. This will give a consistent representation of classical kinematics. The next task will consist in finding necessary and sufficient conditions for a close correspondence between quantum and classical dynamics. When this is done, it becomes possible to reformulate the logic of classical physics as a special case of an approximate quantum logic and to give a precise meaning to classical determinism, to find when it applies and what are its limitations. This programme will be discussed here in the ideal case when there is no friction, the case of friction having to be treated by other techniques, to be given in Sec. II.G.

1. The notion of collective observables

One can consider the case of an ordinary pendulum as an example of a simple physical object. It is macroscopic, i.e., made up of a large number of particles. It is, moreover, a specific object. Many different objects can be made from the same particles; for instance, two smaller pendula can be obtained from the matter of the pendulum. Accordingly, many objects and many nonobjects can occur as different manifestations of the same physical system.

There are good reasons to assume that an object is represented by a family of states corresponding to a subspace in the overall Hilbert space of the physical system. This has been discussed, for instance, by Bohr and Mottleson (1975) in the case of nuclei.

One can define a set of commuting observables acting in this subspace, to be called the collective position observables. They are essentially the variables one uses in classical mechanics. In the case of the pendulum, they specify its position. One might go farther and also consider the elastic deformations of the pendulum and particularly the wire as being described by other collective variables. In principle, one can even go so far as to treat the position of atoms as collective variables by using the Born-Oppenheimer approximation (Born and Oppenheimer 1927; Hagedorn, 1980b). This is more or less a matter of convenience, since the results one will obtain are quantitative, and it is found that the more detailed the description of the object by collective observables, the larger are the errors involved in the classical approximation. One can finally complete the collective position observables by other observables in order to get a complete commuting set. These are called the microscopic observables. They contain detailed information about the particles in the object.

There is no general theory of collective observables, i.e., no known way of finding out directly from first principles all the objects that can be obtained from a given system of particles and how they should be characterized and described. There may be some hints of such a theory in a work of Feffermann (1983), but not a general answer. In any case, one knows how to define the classical observables explicitly when one consider a specific object.

It is often convenient to treat formally an object as being made of two interacting physical systems, the collective system and the environment. The collective system is associated with the collective position coordinates $Q=(Q_1,\ldots,Q_n)$, where n is the number of collective degrees of freedom. Its states can be described in terms of wave functions $\psi(q)$, which are defined upon a configuration space Γ. They are square-integrable functions with a scalar product

$$(\psi_1,\phi_2)=\int_\Gamma \psi_1^*(q)\psi_2(q)\mu(q)d_nq \ . \qquad (2.18)$$

The collective system is also associated with a collective Hamiltonian H_c, and the weight function $\mu(q)$ is chosen so that the scalar products are invariant under a time evolution, which is given by the Schrödinger equation with a collective Hamiltonian H_c depending only upon these observables.

The environment is described by microscopic coordinates. It consists of an internal environment taking into account the particles constituting the matter of the object. There can also be an external environment. For instance, when the collective observables of a pendulum describe only the motion of the ball, the internal environment is the formal physical system that is made up of the particles in the pendulum, after separating out the center of mass. The external environment is made of air molecules around the pendulum or of the photons in an external light.

The total Hamiltonian consists of three parts,

$$H=H_c+H_e+H_{\text{int}} \ , \qquad (2.19)$$

where H_c depends only upon the collective observables and H_e does not depend upon them (it represents an internal energy). The interaction Hamiltonian H_{int} couples the collective system and the environment and is responsible for their exchange of energy, i.e., dissipation.

2. Collective observables and classical dynamical variables

A general collective observable can be defined by a self-adjoint operator A acting in the Hilbert space of the collective system. A classical dynamical variable, to be compared with it, is a real function $a(p,q)$ defined in classical phase space. It is possible to get a one-to-one correspondence between them satisfying the following conditions:

(i) The correspondence $A \rightleftarrows a(p,q)$ is linear.

(ii) The position observable Q_j and the corresponding momentum observables P_j are associated, respectively, with the phase space coordinates (q_j,p_j).

(iii) Let U be a unitary transformation belonging to Heisenberg's group, i.e., having the form $\exp\{i\,F(Q,P)\}$, where $F(Q,P)$ is a homogeneous second-order polynomial. It transforms an observable A into $A'=U^{-1}AU$. These transformations are such that the observables (Q',P') are related to (Q,P) by a linear canonical transformation. They are therefore the simplest transformations allowing a correspondence between unitary transformation and canonical transformations. One assumes that the dynamical variable $a'(q,p)$ associated with A' can also be obtained by performing the same canonical transformation upon the variables (q,p) in the function $a(q,p)$.

(iv) When A is self-adjoint, $a(p,q)$ is real.

Condition (i) asserts that a change of scale acts in the same way in the quantum and classical cases and it ensures that a conservation law looks the same in both cases. The correspondence between observables and dynamical variables is unique under these conditions

(Hörmander, 1979a, 1985). Its simplest form occurs when the configuration space is a Euclidean space $\mathbb{R}^n$. Then one has explicitly

$$a(x,p)=\int \langle x'|A|x''\rangle \delta(x-[x'+x'']/2) \times \exp\{ip\cdot(x''-x')/\hbar\}dx'\,dx'' . \quad (2.20)$$

This formula was first given by Wigner (1932) in the case of the density operator and was made systematic by Weyl (1950).

3. Microlocal analysis

The previous correspondence between A and $a(q,p)$ has been known for a long time in physics. Mathematicians have also turned it more recently into a powerful and precise tool, known as the Weyl calculus. It belongs to a vast branch of mathematics, the microlocal analysis or pseudo-differential calculus (Taylor, 1981; Hörmander, 1985).

Among the main results of this mathematical theory, it may be mentioned that

(i) One can obtain many properties of the operator A by looking directly at its so-called *symbol* $a(q,p)$, e.g., one can find when the spectrum of A is discrete or one can estimate its Hilbert norm $\|A\|$ with known errors.

(ii) One can estimate the so-called trace norm, i.e., the quantity $\mathrm{Tr}|A|$, where the absolute-value operator A is defined as the square root $(AA^+)^{1/2}$. The quantity $\mathrm{Tr}|A-B|$ is often a much better measure of the proximity of two operators (A,B) than the Hilbert norm $\|A-B\|$. It was little used up to very recently because of the difficulty of computing it by conventional Hilbert-space techniques. Nevertheless, it plays an important technical role in the interpretation of quantum mechanics, particularly because of the so-called master inequality

$$|\mathrm{Tr}(AB)|\le\|A\|\cdot\mathrm{Tr}|B| . \quad (2.21)$$

(iii) One can perform algebraic calculations upon operators. For instance, the symbol $c(x,p)$ of a product of operators $C=AB$ is given formally by

$$c(x,p)=a(x,p)\exp\left[-i\frac{\hbar}{2}\left[\frac{\overleftarrow{\partial}}{\partial x_j}\frac{\overrightarrow{\partial}}{\partial p_j}-\frac{\overleftarrow{\partial}}{\partial p_j}\frac{\overrightarrow{\partial}}{\partial x_j}\right]\right] \times b(x,p) , \quad (2.22)$$

the direction of an arrow showing upon which factor a derivative acts. More precisely, this is a series in powers of $\hbar$ when the exponential is expanded. It can be cut off at any given power of $\hbar$ and there are good estimates for the neglected terms. The usual correspondence between commutators and Poisson brackets is exact for X and P and it is valid up to the leading order in Planck's constant in all cases.

4. Semiclassical physics

Semiclassical physics establishes under what conditions the properties of classical physics are valid and it gives the corrections in terms of Planck's constant. Its oldest result is Ehrenfest's theorem and its oldest technique is the BKW approximation.

It can be nowadays approached with the help of two powerful methods, one of them being microlocal analysis. The other one came to be used earlier. It makes use of coherent states (Glauber, 1963), or in the present case of Gaussian wave functions:

$$g_{qp}(x)=\mathrm{const}\times\exp\left[ip\cdot x/\hbar-\frac{1}{2}\sum_{j,k}A_{jk}(x_j-q_j)(x_k-q_k)\right] . \quad (2.23)$$

The $n\times n$ matrix A determines the uncertainties and correlations for the various position coordinates, the average values of X and P being equal to q and p, respectively.

Hepp (1974) gave a rigorous proof of Ehrenfest's theorem when the initial state is coherent by using coherent states as a tool, showing that the leading correction behaves like the square root of Planck's constant. It was then shown that semiclassical physics amounts to an asymptotic expansion in powers of $\hbar^{-1/2}$, which is Borel summable (Ginibre and Velo, 1979). The most useful formulation was given by Hagedorn (1980a, 1981); see also Heller (1976).

E. Classical properties as quantum properties

A classical property states that the classical coordinates (q,p) lie within a given domain C in phase space. When the system has n degrees of freedom and C is a rectangular $2n$-dimensional box, this means that the values of (q,p) are given up to well defined upper errors. Since motion transforms a rectangular box into a domain having almost any possible shape when various dynamics are considered, it is better to consider in general a more or less arbitrary domain C.

The first question one can ask is whether such a classical property can be given a meaning in quantum mechanics. In other words: is it possible to associate a projector in the Hilbert space of collective wave functions with a classical property? As a matter of fact, the question goes back to Von Neumann (1932).

1. Which cell?

Simple-minded semiclassical considerations can show that the domain C entering in the statement of a classical property must be somewhat restricted. Elementary considerations (Landau, 1958) suggest that one can associate essentially one quantum state with a small rectangular box in phase space having the volume h^n. A projector will be more or less well defined if one can pile up many such boxes in C, the rank of the projector being given by the number of semiclassical states,

$$N=\int_C h^{-n} dq\, dp \ ; \tag{2.24}$$

N should be large and therefore C big enough.

The shape of C should also be simple enough. If, for instance, it is the kind of filamentary and tortuous region one obtains from chaotic motions (see Fig. 3), it will be impossible to tile it with rectangular boxes. The boxes would have to become very tortuous themselves, and this is inconsistent with semiclassical physics (Feffermann, 1983).

So, one must be content with a big bulky cell. This can be made more precise by mathematical conditions to be given now in some detail for the sake of definiteness.

Let C be a connected (in one piece) and simply connected cell (without holes). One can introduce reference scales (L,P) for the coordinates and momenta in such a way that

$$N=(PL/h)^n \ . \tag{2.25}$$

They will be made more precise later on but, when the shape of C is simple, L and P are characteristic scales for its geometry (see Fig. 3).

To describe the geometric features of the boundary S of C, one must introduce a metric on phase space. The simplest one, when the configuration space is $\mathbb{R}^n$, is given by

$$ds^2=dq^2/L^2+dp^2/P^2 \ . \tag{2.26}$$

It is dimensionless when the coordinates and momenta are measured with unit scales (L,P). The volume $V(C)$ of the cell and the area $\Sigma(S)$ of its boundary are then also dimensionless quantities and one may completely fix L and P so that the area is minimal under condition (2.25).

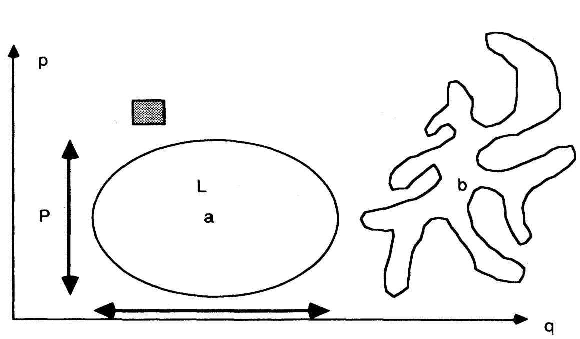

FIG. 3. Some regions in phase space are associated with classically meaningful properties. Cell (a) with typical dimensions (L,P) is regular. The region (b) is irregular. The shaded box has an area h.

Introducing the parameters

$$\epsilon=(\hbar/LP)^{1/2} \ , \tag{2.27}$$

$$\theta=\Sigma(S)/V(C) \ , \tag{2.28}$$

$$\eta=\epsilon\theta \ , \tag{2.29}$$

We shall consider only cells for which η is much smaller than 1. In that case, ϵ is also small. Such cells are said to be *regular.* The parameter ϵ will be called the *classicity parameter* and η the *effective classicity parameter.* They control how well the classical property is also a quantum property.

2. What kind of a projector?

One can pile up boxes of volume h^n in C in many different ways. Although a change of piling within the bulk of C essentially amounts to a change of basis among the vectors defining the projector, different arrangement near the boundary S will lead to slightly different projectors. Accordingly, one cannot anticipate getting a uniquely defined projector but rather a class of more or less equivalent ones. Furthermore, since the validity of classical physics, including classical logic, cannot be perfect, one must also be ready to deal with operators that are nearly but not exactly projectors.

These remarks lead to the introduction of *quasiprojectors.* A quasiprojector is defined as a self-adjoint operator having only discrete eigenvalues belonging to the interval [0,1]. Most of its eigenvalues are near 1 or 0. This condition can be given a precise meaning in the following way: A quasiprojector F of rank N and order η is defined by the conditions

$$\mathrm{Tr}F=N \ , \tag{2.30}$$

$$\mathrm{Tr}(F-F^2)=N\,O(\eta) \ . \tag{2.31}$$

Two quasiprojectors F and F' are equivalent to order η when

$$\mathrm{Tr}|F-F'|=N\,O(\eta) \ . \tag{2.32}$$

One must also say how F is related to the cell C. To do so, one can consider a wave function $\psi_1(q)$ with average values (q_0,p_0), for position and momentum, and uncertainties Δq, Δp. When the $2n$-dimensional rectangular box having its center at (q_0,p_0) and half-sides $(\Delta q, \Delta p)$ is well inside C, one will say that the state ψ_1 is well inside C. Similarly let ψ_2 be a state well outside C. One will say that the quasiprojector F is associated with the cell C when

$$F\psi_1\simeq\psi_1 \ , \qquad F\psi_2\simeq 0 \ . \tag{2.33}$$

This convention clearly covers the case of quasiclassical states as being essentially eigenstates of F.

3. Which projectors?

One can explicitly construct the projectors or quasiprojectors associated with a given cell C in two different ways: The first one uses coherent states (Daubechies, 1987), and a quasiprojector is given by the simple formula

$$F=\int_C |g_{qp}\rangle\langle g_{qp}|dq\,dp/h^n\ . \qquad (2.34)$$

The other approach uses microlocal analysis, and F is defined in terms of its symbol $f(q,p)$. This symbol is essentially the characteristic function of the cell (equal to 1 in C and to 0 outside), smoothed down near the boundary of C upon a region of width ϵ, in order to get a well behaved operator.

The outcome of this process is a theorem according to which one can associate with a regular cell C a family of quasiprojectors of rank N and order η, these parameters being given by Eqs. (2.25) and (2.29). The operators belonging to this family, including some true projectors, are all equivalent to each other to order η.

This theorem goes back to Hörmander (1979b). The estimates were obtained by Omnès (1989, 1990). It may be noticed that the errors are proportional to $\hbar^{1/2}$, as expected from the results by Ginibre and Velo, and Hagedorn. It is proportional to the ratio between the area of the boundary of C and its volume, as expected from intuitive arguments suggested by the piling up of boxes in the cell.

It is also interesting to compare the quasiprojectors that are associated with two clearly separated cells C and C' of similar size. Let δ denote the smallest distance between them in the nondimensional metric (2.26). The associated projectors F and F' are such that

$$FF'\simeq F'F\simeq 0\ , \qquad (2.35)$$

as expected from independent properties. More precisely, the trace norm $\mathrm{Tr}|FF'|$ is of order $N\exp(-\delta/\epsilon^2)$. This means that the projectors associated with two clearly separated classical properties satisfy Eq. (2.35) to a very high precision. The corresponding classical properties are therefore sharply separated, even when expressed in the conceptual language of quantum mechanics.

F. Determinism

1. What is the problem?

It has already been mentioned how classical determinism, or at least a sufficient amount of determinism, is needed to describe and to understand phenomena, to use common sense confidently, and to get ultimate consistency in quantum mechanics when one must describe within its framework the experiments that are used to check the probabilities it predicts. Accordingly, any synthetic approach to physics taking its roots in quantum mechanics must be able to prove that classical determinism is a consequence of quantum mechanics and to assert when it holds.

A convenient formulation of classical determinism is the following: let C_0 be a domain in classical phase space and C_t the transform of C_0 by classical motion during a time t. The two propositions stating that "(q,p) is in C_0 at time 0" and "(q,p) is in C_t at time t," respectively, are logically equivalent. Determinism can be expected to hold even in quantum mechanics when this logical equivalence is also valid in a consistent quantum logic. Of course, it cannot be exact but only approximate, if only because of wave-packet spreading.

This question has been investigated under the following restrictions:

(i) C_0 and C_t are both regular cells.

(ii) There is no dissipation.

The cells describe some classical properties involving only collective degrees of freedom. They must be regular in order to be associated with quantum properties. It may sometimes happen that C_t is in several pieces when C_0 consists of one piece. This occurs when there is a potential barrier that is crossed by some trajectories issuing from C_0, whereas some other trajectories are reflected upon it. Such exceptional cases must be discussed separately for their own sake. As for dissipation, its neglect means that one neglects the coupling Hamiltonian in Eq. (2.19). There are good reasons for believing that this restriction can be removed by using the results of decoherence theory, but this remains to be proved.

When everything starts from quantum mechanics, a preliminary question is of course to decide what is meant by classical motion. To answer it, one may consider the dynamical variable $h(q,p)$ associated with the collective Hamiltonian H_c, treating it as a Hamilton function and writing down the corresponding Hamilton equations. This procedure defines classical motion and it gives a content to the statement of determinism, at least as far as classical physics is concerned.

Then one must give a quantum version of this determinism: let F_0 and F_t be two quasiprojectors, associated with the two cells. The corresponding quantum properties, occurring at times 0 and t, respectively, would be logically equivalent in a trivial way if one had

$$U^{-1}(t)F_0U(t)=F_t\ . \qquad (2.36)$$

where $U(t)$ is the quantum evolution operator associated with the collective Hamiltonian H_c,

$$U(t)=\exp(-iH_ct/\hbar)\ . \qquad (2.37)$$

The question of determinism therefore amounts to finding out when Eq. (2.76) is valid and evaluating the errors it involves. Since quasiprojectors are concerned, there are again two possible approaches, according to whether one uses coherent states or microlocal analysis. The first method shows more clearly the physical meaning of the corrections, whereas the second one is more general and more precise.

2. The coherent-states approach

Given a coherent (Gaussian) wave function as an initial state, one may wonder how well it remains Gaussian under time evolution and, as far as the leading Gaussian part is concerned, how its center moves and how it spreads.

This question has been investigated in detail by Hagedorn (1980a) in the case of the Hamiltonian

$$H_c = P^2/2m + V(x)$$

for any dimension of space, the potential being bounded from below and satisfying some regularity conditions such as the existence of second derivatives. The Kepler problem has also been treated analytically and numerically by Nauenberg (1989).

In order to describe Hagedorn's results, it is convenient to write down the Gaussian wave functions of interest as

$$\psi(x,t) = (\det A)^{-1/2} \exp\{-Q(x,t)\} , \qquad (2.38)$$

where $Q(x,t)$ is a quadratic quantity

$$Q(x,t) = -(1/4\hbar)(x-q|BA^{-1}|x-q) + ip\cdot(x-q)/\hbar . \qquad (2.39)$$

The quantities q and p evolve according to Hamilton's equations

$$\dot{q} = p/m, \quad \dot{p} = -\partial V(q)/\partial q . \qquad (2.40)$$

The matrices A and B describing the spreading of the wave packet obey the equations

$$\dot{A} = iB/2m, \quad \dot{B} = 2iV''(q)A , \qquad (2.41)$$

where V'' is the matrix whose coefficients are the second derivatives of the potential. One may start from a given correlation matrix $A(0)$ at time zero and take $B(0) = I$.

When the exponential is a polynomial (second-degree at most), time evolution is completely given by Eqs. (2.38)–(2.41), up to a phase factor. In the case of a more general potential, non-Gaussian corrections to the wave function appear. These corrections are of order $\hbar^{3/2}$ and they are controlled to leading order by the third derivatives of the potential. These results can be used to give a precise formulation for Eq. (2.36) (Omnès, 1989).

3. The microlocal-analysis approach

This approach does not restrict the configuration space to being Euclidean, nor does it restrict the form of the Hamiltonian function $h(q,p)$. It assumes, however, some rather stringent regularity conditions, because $h(q,p)$ must be infinitely differentiable and satisfy some bounds upon its derivatives (Omnès, 1990). The results one obtains will be given in that case.

Equation (2.36) takes the more exact form

$$\mathrm{Tr}|U^{-1}(t)F_0U(t) - F_t| = N\,O(\eta_t) , \qquad (2.42)$$

where N is still given by Eq. (2.25). The dynamical classicity parameter η_t has a rather complicated expression in which one can recognize the effects of cell deformation, wave-packet spreading, and some effects analogous to the non-Gaussian corrections encountered by Hagedorn [in the present case they are controlled by the third derivatives of $h(q,p)$]. For all practical purposes, however, one can simply assimilate η_t into the upper bound of the effective classicity parameter $\eta(t')$ for all cells that are the transforms of C_0 by classical motion during the time interval $0 < t' < t$.

4. The meaning of the results

Equation (2.42) gives a precise form of Ehrenfest's theorem, in terms of quasiprojectors expressing classical properties. It contains an explicit estimate of errors and it enables one to say that the dynamics of an object (or several objects) is *regular* or *deterministic* when the dynamical classicity parameter is much smaller than unity. This crucial property explicitly refers to a given initial cell C_0 and to a finite time t, which may sometimes be very large. For this kind of determinism to represent a physical situation, the classical property associated with the cell C_0 must be valid for the initial state. This means that

$$F_0\rho F_0 = \rho , \qquad (2.43)$$

where F_0 is a quasiprojector associated with C_0, or more precisely

$$\mathrm{Tr}|F_0\rho F_0 - \rho| = O(\eta_t) . \qquad (2.44)$$

There are cases in which this condition is not satisfied, when a macroscopic object is in a quantum state with no classical description (Leggett, 1987b; see also Sec. IV).

Much more frequently, when determinism does not hold, this is due to a deformation of the cell C_t so extreme that one cannot even associate a quasiprojector with it. This occurs for chaotic systems for which the geometric surface-to-volume ratio $\theta(t)$ entering in the parameter η_t increases exponentially with time (Cornfeld *et al.*, 1982). The correspondence between classical and quantum mechanics is then lost. This remarkable result does not seem, however, to open up an experimental check of the theory because the statistical predictions of the two dynamics essentially agree (Omnès, 1990).

Restricting oneself to a regular object in a classically meaningful initial state, one can use Eq. (2.42) to obtain a clearcut proof of common sense as it applies to the object: One selects as many discrete times $(t_1, \ldots, t_n)$ as needed for an argument, together with the associated cells $C_{t_1} \cdots C_{t_n}$. Each cell is associated with a quasiprojector F_k, and the various projectors $F_k(t_k)$ may be used to build up a history. A quantum logic is obtained by also allowing the complementary projectors $I-F_k$ to enter into other histories.

It is then a straightforward matter to prove that this logic is approximately consistent, i.e., the consistency conditions (2.13) are satisfied up to an error of relative order η_t with $t=t_n$. This is a direct consequence of Eqs. (2.31) and (2.42) and the inequality (2.21). Then one can prove that the logical implications expressing determinism in classical physics also hold in the quantum logic up to an error at most of order η_t.

In other words, *one gets a proof of classical determinism as a consequence of quantum mechanics.* It is worth stressing once again that this holds for objects having a regular dynamics that are initially in a state compatible with a classical property. Furthermore, dissipation has been neglected.

Determinism is not absolute, and one can find bounds for the errors it involves. They are most often extremely small. Most objects have a regular dynamics; even chaotic systems are regular during a finite time, and the regularity of dynamics can be checked by essentially classical calculations.

Except for very special systems to be discussed in Sec. IV, the most frequent case in which the state of a macroscopic object does not agree with a classical property occurs after a quantum measurement. The state of a measuring device is then a linear superposition of different macroscopic situations. What happens in that case will be treated in next section: it concerns decoherence and it also sheds some light upon the role of dissipation.

G. Decoherence

Decoherence is here presented last but not least among the building blocks necessary to obtain a satisfactory interpretation. It has become by now an important subject by itself, and only its most salient features will be mentioned here.

1. What is decoherence?

Decoherence is a dynamical effect through which the states of the environment associated with different collective states become rapidly orthogonal. It comes from a loss of local phase correlation between the corresponding wave functions of the environment, which is due to the interaction between the collective system and the environment, also responsible for dissipation. It depends essentially upon the fact that the environment has a very large number of degrees of freedom.

The effect can be described more easily for a specific example. Let us consider again the case of a pendulum: It is released at time zero from a position x_1 with zero velocity. This can be represented by an initial wave function $\psi_1(x)$ with an average value x_1 for the position and a vanishing average value for the momentum. The internal environment, i.e., the matter of the pendulum, will be assumed to be initially in thermal equilibrium. When the temperature is zero, the environment is in its ground state. This will be assumed to be the case for simplicity, so that the initial state is pure, given by the state vector

$$\Psi_1=|\psi_1\rangle_c\otimes|0\rangle_e \ . \tag{2.45}$$

Let ψ_2 be another similar initial state, differing from ψ_1 only by the initial average position x_2 of the pendulum, and let Ψ be a quantum superposition of these two states:

$$\Psi=a\Psi_1+b\Psi_2 \ . \tag{2.46}$$

This kind of state represents what one gets when, for instance, the pendulum is assimilated to a pointer registering the outcome of a quantum measurement.

As time goes on, this state evolves into another pure state $\Psi(t)$ according to a Schrödinger equation in which the Hamiltonian is given by Eq. (2.19). The associated density operator is given by

$$\rho(t)=|\Psi(t)\rangle\langle\Psi(t)| \ . \tag{2.47}$$

If the position of the pendulum (or any other collective observable) is observed at a time t, the probabilities of the results are given by the so-called reduced or collective density operator $\rho_r(t)$, which is obtained from $\rho(t)$ by a partial trace upon the environment:

$$\rho(t)=\mathrm{Tr}_e\rho(t) \ . \tag{2.48}$$

This is well known in the Copenhagen interpretation, and it will also come out of the measurement theory to be given in Sec. III.

The Hamiltonian H_{int} coupling the collective system and the environment produces energy exchanges between them, i.e., dissipation. The wave functions of the environment are sensitive to the collective motion, and they become very different in the states $\Psi_1(t)$ and $\Psi_2(t)$ as time increases. As a result, the overlap integral between the two environment wave functions tends rapidly to vanish. The nondiagonal terms of the reduced density operator are proportional to this overlap integral and they also vanish. As a result, the reduced density operator, which had initially the form of a pure state

$$\rho_r(0)=(a|\psi_1\rangle+b|\psi_2\rangle)(a^*\langle\psi_1|+b^*\langle\psi_2|) \ , \tag{2.49}$$

becomes a mixed state. More, exactly, it becomes a diagonal density operator representing a situation in which the object has a classical probability $|a|^2$ of being in state 1 and another $|b|^2$ of being in state 2. No interference effect survives: this is the decoherence effect.

Since decoherence has the same physical origin as dissipation, it disappears with it. This means that it depends upon the dissipation coefficient λ entering in the average collective motion of the pendulum, as given by the classical equation

$$\frac{d^2x}{dt^2}+\lambda\frac{dx}{dt}+\omega^2x=0 \ . \tag{2.50}$$

The theory of decoherence gives in the present case

$$I(t)=\exp\{-(1/4\hbar)m\omega(x_1-x_2)^2[1-\exp(-\lambda t)]\} , \quad (2.51)$$

where $I(t)$ is a factor entering in the nondiagonal elements of $\rho_r(t)$. With a mass of the pendulum of the order of 1 gram, a period of 1 second, and a damping time of ten minutes, one finds that for an initial distance x_1-x_2 of 1 micron, after a time t of 1 nanosecond, $I(t)$ is of the order of $\exp(-10^5)$. The effect is therefore among the most efficient ones known in physics, and it is even stronger at a finite temperature.

2. The theory of decoherence

It is difficult to give a simple explanation for the existence of decoherence. Basically, it occurs because one is dealing with an environment that is very complex, with too many degrees of freedom. Decoherence is therefore a complex effect concerning the phase of a many-body wave function. This is very difficult to obtain from a theoretical standpoint.

The first quantitative evaluation was given by Feynman and Vernon (1963), who were mostly interested in a quantum calculation of dissipation. They assumed an environment made up of an infinite number of oscillators.

Equation (2.51) for the decoherence effect in the case of a pendulum was obtained by Hepp and Lieb (1973) as the outcome of a thorough mathematical analysis, which, however, relied heavily upon the fact that the collective motion is harmonic. They also considered the corrections coming from the finite number of oscillators in the environment, whereas, using simpler and somewhat less realistic models, Zurek (1981, 1982) suggested the universality of the effect; thus he was the first to clarify its basic importance in measurement theory. See also Unruh and Zurek (1989).

The most complete analysis to date is due to Caldeira and Leggett (1983a, 1983b). They use the following model:

(i) The environment is treated as a collection of harmonic oscillators. This might look like a very restrictive assumption, had not Caldeira and Leggett shown that an internal environment is reliably described mathematically by such a model, one oscillator with frequency ω_k representing faithfully one energy level E_k of environment with $\omega_k=(E_k-E_0)\hbar$, at least in some cases.

(ii) The environment is *initially* in a state of thermal equilibrium.

(iii) The coupling Hamiltonian $H_{\rm int}$ has a special form, linear in the positions of the environment oscillators and not involving their momenta. The authors claim that this is a correct assumption in most cases of physical interest by reviewing many different cases.

Then they proceed to a calculation of the reduced density operator $\rho_r(q,q',t)$ by using Feynman path integrals. The trace upon the environment degrees of freedom can be explicitly performed because of assumptions (i) and (ii), since quadratic Lagrangians can be integrated exactly. What they find is a density operator vanishing exponentially when t increases, except when $q-q'$ is very small. Again this is the decoherence effect, since the density operator is becoming diagonal in the q basis.

Looking at the correlation between the midpoint $(q+q')/2$ and a given initial position, Caldeira and Leggett find a generalization of Eq. (2.50), i.e., they get a general form of classical motion in which friction enters but is not necessarily instantaneous, as would otherwise be expected from general causality arguments (Landau and Lifshitz, 1958b). One therefore completely recovers classical mechanics, including the friction effects.

Whereas all this is concerned with the internal environment, Joos and Zeh (1985) also showed the existence of a decoherence effect coming from the external environment. When, for instance, a macroscopic ball is hit by air molecules or photons, the phase of the ball-particle collision S matrix depends upon the position of the ball center. One has

$$S_x(k,k')=S(k,k')\exp[i(k-k')\cdot x] , \quad (2.52)$$

where S is the S matrix for the center of the ball at the origin and S_x for the center at a position x, and where k and k' are the initial and final wave numbers of a colliding particle. It is found that many such collisions produce a complete loss of phase coherence in the environment for different positions of the center of the ball. Coherence still decreases exponentially in a very short time when the ball is macroscopic, the time being inversely proportional to the number of collisions per second.

One can give an amusing example of this effect. The center of mass of the Moon is decoupled from its internal environment so that no decoherence can be due to it (except for tide effects, to be ignored here). Light coming from the Sun, however, represents an external environment leading to a very efficient decoherence effect taking place in about 10^{-35} sec. This means that whatever quantum effect happens to the Moon, it must be seen in one place if only because of light. The calculation takes into account an exponential decrease $e^{-t/T}$ in the nondiagonal elements in the density operator, the time T behaving like $\sigma N v$ where σ is the cross section of the Moon, N the density of photons, and $v=c$ their velocity. Of course, this example is just given for the fun of it; it would be sufficient to use the deterministic character of the Moon's motion to make sure that it is in only one place.

3. Schrödinger's cat

Although the famous example of Schrödinger's cat is far from containing all the basic problems of interpretation, its pedagogical value makes it worth a special comment (Schrödinger, 1935).

As is well known, it goes as follows: A cat is enclosed in a box where a radioactive source can trigger a system liberating some poison. Because of internal decoherence, it can be said that the cat is necessarily either alive or dead. It is not in a quantum superposition state. When the box is open, if one finds the cat to be dead, an autopsy using classical determinism can tell what was actually the time of its death. These are the true consequences of quantum mechanics, and the use of retrodiction (here by an autopsy) tells much more than what could be said in the older form of the Copenhagen interpretation. Of course, these conclusions completely agree with common sense. Moreover, one can consistently state that, at any time, the cat is *actually* dead, alive, or dying, as will later be shown.

4. Is decoherence a complete answer?

Although nobody denies the existence and the importance of decoherence, a criticism has been raised against its basic significance for the interpretation of quantum mechanics (Bell, 1975; Zurek, 1982; d'Espagnat, 1990). Although the reduced density operator becomes diagonal, the full density operator $\rho(t)$ still represents a pure state with a permanent superposition, as long as the system remains isolated. Is it not therefore possible in principle to perform a very refined measurement upon the environment, revealing the existence of quantum interferences?

Zurek (1982) gave a pragmatic answer to this objection, namely, that such a measurement is impossible in practice. One can, however, go further and show that it is also impossible as a matter of *principle*.

It can be shown that no measurement, except for testing an individual oscillator in the environment, can exhibit interferences revealing a superposition. Since, according to Caldeira and Leggett, each oscillator formally represents an eigenstate of the environment Hamiltonian, one must therefore perform a measurement upon such an eigenstate. This can be done for instance by a yes/no measurement telling whether a specific eigenstate is occupied some given time t after preparation. The probability p for a positive answer is very small because of the very high density of energy levels, which behaves exponentially with the number of atoms in the pendulum. Let ν be the number of degrees of freedom for all particles in the pendulum and ν' the corresponding number for the apparatus to be used for the measurement. It is explicitly assumed that the degrees of freedom are continuous (which excludes the case of spins).

By necessity, classical physics is only approximate. When stating the result of a measurement, it is open to an error of order η (as given in Sec. VI), which has a lower bound when the size of the apparatus is given.

The experiment will be significant, which means that it can be reliably considered as giving a "yes" answer to the question rather than as being due to a malfunctioning of the apparatus (e.g., a photodetector firing in the dark), only if $\eta \ll p$. When a pendulum is the object tested for superposition and one uses another measuring device made of ordinary metal to perform the test, everything can be computed explicitly by elementary solid-state physics, and this condition becomes

$$\nu' > \kappa' \exp(\kappa \nu^{2/3}) , \tag{2.53}$$

where κ and κ' are of order unity for all practical purposes.

Taking for example $\nu = 10^{24}$ would mean that the measuring apparatus would have to contain about $\exp 10^{16}$ atoms! This is clearly impossible in practice, and even in principle, because the apparatus would be too big to act at a fixed time t because of its sheer size and of the finite velocity of light, not to mention the finiteness of the universe unable to provide enough matter to build up this monster.

This remark has several noticeable consequences:

(i) The Von Neumann chain, consisting of a measuring apparatus to be used for measuring another apparatus while both remain in a state of superposition, is meaningless.

(ii) There are propositions in physics that can be formulated but cannot be tested empirically, even in principle. Therefore many observables can never be measured.

(iii) One can, of course, answer Bell's objection by saying that decoherence is not only a practical effect but also a matter of principle. This answer does not rely, however, uniquely upon quantum mechanics, but calls for relativity or the finiteness of the universe.

A word of caution should be added. The proof of Eq. (2.53) assumes that the quantum fluctuations making a "no" effect look like a "yes" have a probability of order η and not $\exp(-\delta/\epsilon^2)$ (see Sec. II.E), which happens when some pointer in the measuring apparatus takes clearly distant positions for the two cases, yes and no. It was assumed that such a good apparatus is impossible when one must isolate an eigenenergy of the environment from extremely nearby ones. This is most probably true but not rigorously proved.

As for the origin of the exponential in Eq. (2.53), it comes from the exponential decrease in the separation between the energy eigenvalues of a solid when the number of atoms increases.

Finally, this result also shows that incomplete decoherence is after all most easily seen by looking at the collective degrees of freedom, as shown in practical examples in Sec. IV.C.

5. The direction of time

Decoherence is a time-directed effect, as is dissipation, which occurs along with it. It follows the usual pattern of irreversible thermodynamics: one starts from an ordered state (here a pure state), and one obtains a disordered (mixed) state. To restrict the discussion to the observation of collective observables plays essentially the same role as making a coarse graining.

The reverse process is mathematically meaningful: One can consider the formal time-reversed density operator that is obtained from $\rho(t)$ as being an initial state and let the Schrödinger equation act on it. The outcome will be the initial state $\rho(0)$ after a time t.

It is worth noticing, however, that it is usually impossible to prepare this time-reversed density operator, *as a matter of principle*. The argument is the same as that just given above: the preparing device would be too large to work according to the full laws of physics. This impossibility may, by the way, point towards a new understanding of the second principle of thermodynamics.

The most important point in the present context is that the logical direction of time must coincide with the thermodynamical direction. This is most obvious when the initial state ρ is well described by a property having a projector E_0, so that ρ can be written as $E_0/\mathrm{Tr}E_0$. A unique projector E_0 describes the initial situation, whereas several projectors must be used to describe the possible final situations. This is exactly what happens in thermodynamics, where irreversibility comes from an ordered preparation of the initial state, yielding disordered states.

H. Discussion

The interpretation of quantum mechanics that one can get by using all these ideas together should by now be rather clear, as far as the organization of the basic ideas is concerned. It has been presented here in a form whose whole construction is ordered by logic, which corresponds to the author's inclination. Gell-Mann and Hartle's approach attributes the main role to decoherence, so that their theory looks rather different at first sight. An important step in the discussion, therefore, will be to compare these two formulations and to show that they are almost equivalent.

Having thus made sure that there is, at least now, essentially only one candidate for a consistent and complete interpretation, it will become possible to compare the new theory with the older Copenhagen interpretation.

1. The theory of phenomena

A few basic results of the modern approach play a central role in all these discussions. They are concerned with the description of phenomena (Gell-Mann and Hartle, 1990) or equivalently the so-called theory of facts (Omnès, 1990).

Phenomena may be defined as the classical properties of macroscopic objects. From what was obtained in the previous sections, it can be asserted that

(i) When conveniently specified, these properties have a clearcut meaning in the phenomenological language of classical physics, as well as in the formal language of quantum mechanics.

(ii) Many of them, which can be made explicit, obey determinism up to a very small error in probability. Moreover, they can be described and discussed according to common-sense logic.

(iii) When it happens that some initial macroscopic state does not agree with classical physics by involving some quantum superpositions, this feature is very rapidly lost in most cases by decoherence, so that the phenomena become clearly separated.

(iv) Furthermore, phenomena are inescapable insofar as no conceivable experiment can exhibit a property going against this separation.

(v) Finally, explicit dynamical calculations can be used to assert when a macroscopic system cannot be expected to behave classically.

There is little doubt that this theory of phenomena is the main key to interpretation. It should be noticed that it contains words of caution and it avoids sweeping statements. This kind of caution does not hide ignorance, but it refers to known conditions for the applicability of some theorems and to known error bounds. This is why the theory can be said to be complete by offering a precise prediction in every experimental situation.

2. Gell-Mann and Hartle's interpretation

The Gell-Mann–Hartle (GMH) theory is still partly in the form of a program, which is more ambitious than what has been presented here since it aims at extending the interpretation of quantum mechanics in several directions, namely, towards cosmology, towards a quantum account of observers, and towards a systematic treatment of the classical domain (Gell-Mann and Hartle, 1990, 1991a, 1991b).

It also relies upon consistent families of histories, which are histories of the universe, at least when the quantum effects of gravitation have become negligible. Consistency is written in the form of Eq. (2.13). Its logical consequences are not much developed, but conditional probabilities are used for retrodiction so that the basic ingredients of logic are already present. The direction of time is defined by the evolution of the universe, starting from a state that is far from equilibrium because of expansion.

Rather than assuming the existence of classically behaving objects, Gell-Mann and Hartle proceed in a more systematic way. They consider so-called full sets of decohering histories of the universe as being a deeper preliminary concept.

A property occurring in a history specifies a domain of values for an observable. This may be considered as some sort of coarse graining, and there exists, at least intuitively, an optimal graining so that a finer graining would spoil consistency and a coarser one would only increase ignorance with no appreciable gain in consistency. It is not, however, objective as long as the observables it refers to can be chosen arbitrarily.

One should notice at this point that the consistency conditions (2.13), as they are using them, can be satisfied for various reasons: When one is interested in a microscopic system, e.g., a spin-$\frac{1}{2}$ system, the consistency conditions can be viewed exactly in the same light as in Griffith's theory, and the necessary and sufficient conditions represented by Eqs. (2.11) or (2.12) are to be preferred. For a macroscopic system, when the properties to be considered are also macroscopic, one may expect that consistency is a consequence of decoherence. It takes in that case the form of Eq. (2.13).

The authors assign paramount importance to consistency through decoherence. According to their views, it should be the key to the existence of classical physics, from which one should obtain the full extent of the classical domain, namely, classical properties as such but also an objective criterion for the right choice of collective coordinates.

When microscopic systems interact with macroscopic ones during some measurements, Gell-Mann and Hartle assume that the full consistent decohering histories contain what they call a generalized record. This is a projector for a macroscopic property, which multiplies a projector for a microscopic property. It is called "generalized" because the authors cannot rely upon the theorems of Sec. II.F in their approach, so that the persistence of the record is not proved. The corresponding families of histories, whether in the case of a measurement or only for macroscopic systems, are called full sets of decohering histories (Gell-Mann and Hartle, 1991b).

Gell-Mann and Hartle try to give an objective content to the notion of full sets of decoherent histories by combining decoherence with maximal information: Let $\{E_k^{\alpha_k}(t_k)\}$, $k=1,2,\ldots,n,\ldots$ be the properties occurring in such a set of alternative consistent histories for the universe. This formalism satisfies the consistency conditions (2.13) with very small (formally zero) values for the right-hand side.

One can then define a coarse-grained density matrix $\tilde{\rho}$ that is sufficient for a faithful description of these histories. It satisfies the conditions

$$\mathrm{Tr}\{E_n^{\alpha_n'}(t_n)\cdots E_1^{\alpha_1'}(t_1)(\rho-\tilde{\rho})E_1^{\alpha_1}(t_1)\cdots E_n^{\alpha_n}(t_n)\}=0 , \tag{2.54}$$

which means that it gives the same probabilities and the same amount of consistency as the complete density matrix of the universe ρ. The most economical coarse-grained density operator gives a maximum for the entropy functional

$$S(\tilde{\rho})=-\mathrm{Tr}(\tilde{\rho}\log\tilde{\rho}) , \tag{2.55}$$

subject to the conditions (2.54). Any other one, including ρ, would only contain useless information that cannot be extracted from the decoherent histories.

One can then look, in principle, for the best set of alternative histories. Consistency increases rapidly when the graining is coarser, and one rapidly reaches a point where a negligible gain in consistency must be paid for by too large an increase in entropy. Introducing more intermediate times does not necessarily decrease entropy, because these times may just happen to yield tautologies (this is the place where determinism shows up). Finally, among all the possible sequences of projectors, one of them certainly gives a minimal entropy, at a fixed level of consistency. In principle, this should provide an algorithm for extracting the full sets of decoherent histories.

A reasonable guess is then that many properties occurring in these privileged histories are in some sense classical. As long as the search for full sets of decoherent histories is uncompleted, the existence of this "classical domain" remains postulated as a good guess. Its "classicity" cannot of course be perfect, and some parameters should express how well it is achieved (one can think of them as the parameters η in Secs. V and VI). The relevant observables should be determined directly by the full sets of decoherent histories, but Gell-Mann and Hartle give some arguments to show that one may expect the particle densities obtained from quantum field theory to be among them, or at least to enter in their construction. Finally, one expects the tautological relations existing between these properties to be well represented by the classical equations of motion.

Gell-Mann and Hartle also suggest that the algorithmic complexity of some parts of the classical domain should increase as time goes on. Algorithmic complexity, which is a well-defined notion in information theory (Solomonoff, 1964; Kolmogorov, 1965; Chaitin, 1966), can measure the complexity of a dynamical behavior or its virtualities and therefore distinguish a part of the universe as being potentially an acting observer or an inert chunk of matter.

The fact that several parts of the program have not yet been worked out makes it less easy to describe it faithfully, and nothing can replace a reading of the inspiring paper by Gell-Mann and Hartle.

A criticism of this theory by d'Espagnat (1990) looks at first sight as if it comes from a misunderstanding. It may, however, be viewed in another light (d'Espagnat, 1991): When assuming that the evolution of the universe generates some classically meaningful alternative histories, and when considering an observer as an "Information Gathering and Utilizing System" (IGUS), Gell-Mann and Hartle seem to deny a free will to the observer or, at least, some people guess that one would then have to find free will's ultimate origin in some fluctuations in the brain of a quantum or a chaotic origin. If this were so, any criticism relying on a free choice would not apply, but, of course, this conceptual framework is not suited to everybody's taste. Conversely, one should also say that the point of view of the critics excludes *a priori* that the theory of the IGUS and of their self-programmation might lead to nontrivial results, which is of course not to the taste of the workers in the field. Anyway, this kind of criticism does not apply to the less ambitious interpretation advocated by the present author.

3. One or two interpretations?

There are, therefore, two theories, by Gell-Mann and Hartle and by the author, aiming at a complete and consistent interpretation of quantum mechanics. Notwithstanding the divergences in their programs, in what they stress and in what has actually been achieved by each of them on specific points, it is important to determine whether they are comparable formulations of a basically unique theory or whether they are significantly different.

A careful examination clearly decides in favor of their close relationship. The same basic ideas occur in both of them: the essential role of histories, the logical aspects that are the same when they can be compared, the clearly limited but widely valid domain of classical physics, the tautological nature of determinism when it holds and, finally, the necessary role of decoherence to disentangle classically meaningful phenomena from highly intricate quantum correlations. These ideas were not developed in the same order, nor with the same emphasis; they did not use the same word for the same notion (though an effort in that direction has been made in the present review), nor did they use the same mathematical techniques. Nonetheless, they are close enough for each of them to benefit from what the other has achieved or the openings offered by the other.

They both incorporate seminal ideas first published by Griffiths (1984), whose work is not presented here as yielding a possibly complete and consistent interpretation because it lacks a theory of phenomena. Griffiths's insightful work provides, in fact, a better understanding of the conventional Copenhagen interpretation rather than full logical consistency. Whatever it may be, it remains in any case an essential landmark.

4. Where do we stand?

If one agrees that there is basically only one framework of interpretation, which is approached from two different standpoints, the two approaches should now be compared if only to find out the most promising directions of future research.

The so-called logical approach by the author seems to yield more easily immediate results; it already includes a theory of phenomena with precise results, and it gives explicit estimates for a classicity criterion. This can be ascribed to the use of microlocal analysis, which provides an extremely powerful tool for semiclassical physics. One wonders therefore, how large a part of Gell-Mann and Hartle's program could be realized by using the same techniques.

The "logical" approach is rather simple-minded as far as the notion of objects or their description by collective observables are concerned. The program set up by Gell-Mann and Hartle is perhaps deeper, and it also takes the construction of the collective coordinates as one of its targets. One might therefore consider it as more ambitious in principle and concentrate the discussion on how it might be fully realized. Rather than talking about full sets of decohering alternative histories, it may be clearer to speak of objects, Hilbert subspaces of objects, and collective observables, even if the two notions do not necessarily completely coincide.

The approach advocated by Gell-Mann and Hartle is very sound as far as the fundamental reasons for classicity are concerned. One may wonder, however, what are its chances of success considering the techniques they propose. These techniques are essentially based upon decoherence, even if adding considerations of information theory. The point is that the theory of decoherence, at least as it stands now, is a rather blunt mathematical tool with little grip and little delicacy. Explicit calculations of Feynman path integrals are either very difficult to perform or they have a very limited domain of application. It looks attractive, therefore, to retain the program while looking for more adequate mathematics.

It might very well be that microlocal analysis is again the proper tool; we refer here to a beautiful paper by Feffermann (1983). He was able, for instance, to prove that the eigenstates of energy for a system of fermions interacting via Coulomb forces, when the energy is near enough to the ground-state energy, represent atoms, molecules, or at least clustered particles. This is a very strong indication of the ability of microlocal analysis to show the existence of objects by using only the fundamental interactions.

Another result by Feffermann concerns a system with an arbitrary number of degrees of freedom and a Hamiltonian with a quadratic kinetic energy and a rather arbitrary potential energy. He has shown that one can then microlocalize the Hilbert space by quasiprojectors of a special type. Their definition must use a local canonical transformation of phase space, i.e., one chooses convenient observables by means of a specific algorithm. The total Hamiltonian is found to be approximately diagonal in the associated subspaces of the Hilbert space. One wonders whether these purely mathematical results do not in fact contain a theory of objects and an objective construction for a hierarchy of observables. It would be most useful if the publication of Feffermann's proofs ultimately resulted in their being used for a more practical purpose.

Finally, the question "where does one stand?" can be answered by saying that there is now a workable complete and consistent interpretation of quantum mechanics. What is lacking is a proper definition of objects and an objective algorithmic definition of collective observables. A program has been formulated for bridging this gap, and a technique looks promising for fulfilling the program. Not much more can be said, at this stage, about this last remaining weak point.

5. Relation with the Copenhagen interpretation

A significant difference between the logical form of the present interpretation and the Copenhagen interpretation lies in their axiomatics: a unique rule of interpretation now replaces the full set of axioms concerning measurement. This is not, however, essential.

The real difference lies in the treatment of phenomena. They become a consequence of quantum mechanics in the modern theory, so that a consistent and complete interpretation becomes possible. The Copenhagen interpretation, as a matter of fact, cannot even be considered as translating the language of phenomenology into the formal language of the theory. It just keeps them apart, so that the notion of consistency cannot even be stated in its framework.

As for completeness, there are cases clearly calling for a completion of the Copenhagen interpretation. There exist, for instance, macroscopic objects behaving in accordance with quantum mechanics rather than classical physics. This is, by the way, a case in which a proposal that was initially put forward as a criticism against the Copenhagen interpretation (Leggett, 1980, 1987a, 1987b) confirms the present one. Other examples will be met in Sec. IV.

Except for well controlled exceptional cases, however, one recovers as consequences of the new theory the main prescriptions of the Copenhagen formulation. This will now be shown by considering measurement theory.

III. MEASUREMENT THEORY

This third section is devoted to measurement theory. It follows mainly the treatment given by the present author (Omnès, 1990), taking into account later improvements. In Sec. III.A measurements are defined in a precise manner. This is to give a clearcut setup for the theory and also to answer a very general criticism by Bell (1987) against the fuzziness of too many statements concerning measurement. Section III.B will show how the usual Copenhagen axioms of measurement theory reappear as theorems in a consistent approach, though wave-packet reduction is now found to be a convenient calculation recipe with no specific physical content. In Sec. III.C, one considers what can actually be said to be true when an experiment has been performed. This allows a very simple way out of the apparent difficulties that were stressed long ago by Einstein, Podolsky, and Rosen (1935), as shown in Sec. III.D.

A. The conditions for a measurement

Rather than trying to give a fully general treatment of quantum measurements, we shall consider here a few specific cases that may be used as references for further extensions if necessary. An effort will be made, however, to cover enough of them to make these generalizations easier.

A measurement can be grossly defined as an interaction between a quantum system and a macroscopic object producing an actual fact, yielding also as a result a value of an observable pertaining to the quantum system. The latter will be called the measured system and it is denoted by Q, whereas the macroscopic object acting as a measuring device is denoted by M. The measured observable will be called A.

Bell (1987) has stressed how poorly defined are the notions too often entering measurement theory, so that a few comments and distinctions will be needed to avoid this criticism.

1. Conditions concerning the measured system

One will have to distinguish between an ordinary classical measurement and a quantum measurement, the latter being the only one of interest here.

The measured system Q is supposed to be isolated before the measurement. It may be microscopic or macroscopic. As a matter of principle, it can be prepared in a state involving linear superpositions of the eigenstates $|a,r\rangle$ of A with different eigenvalues a. This is possible when Q consists of a very small number of particles. As another example, one may consider the case of a macroscopic crystal containing a magnetic impurity. The spin of this impurity can sometimes be prepared in a quantum state: it can be oriented along the z direction by a magnetic field and left in that state before performing a measurement of its value along the x direction by a resonance device or other means. A counterexample occurs when Q is a macroscopic object and A is a collective observable. Then, because of decoherence, the state of Q cannot be a linear superposition of the type to be considered. A measurement of this observable would most often be an interaction between two macroscopic systems obeying classical physics.

2. Conditions concerning the measuring device

It is essential that the measurement be signaled by a phenomenon, a fact. Before elaborating upon this point, it will be useful to consider counterexamples.

Consider the case in which a spin-$\frac{1}{2}$ atom goes through a Stern-Gerlach device oriented along the z direction and let T_+ and T_- be the two trajectories behind the magnet corresponding to $S_z=\pm\frac{1}{2}$. One can very well assert that the atom goes along T_+ by using the notions of classical physics. A quasiprojector covering completely the values of position and momentum along T_+ at some time can do the job. This is, however, a property of a classical nature, not a fact. Although this distinction is pretty obvious, it can be made still clearer by using an argument due to Wigner (1963).

Let us assume that, by some cleverly devised magnetic fields, the two trajectories T_+ and T_- are recombined into a unique beam T' (see Fig. 4). It can be shown that, if a measurement of S_x is made after recombination, the results will be the same as if the first Stern-Gerlach device had not been present. No factual phenomenon has kept track of the trajectory that was followed, and to state the value of S_z has no more meaning than telling through which arm of an interference device a photon went (see Sec. IV).

The phenomena signaling the result of a measurement can sometimes be subtle. Consider again the Stern-

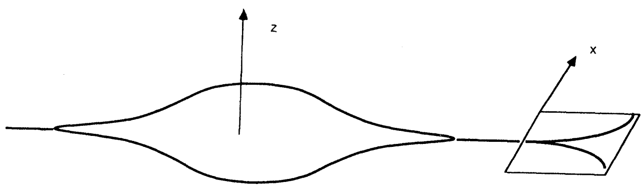

FIG. 4. The Wigner gedanken experiment: The beams issued from a first Stern-Gerlach device oriented along the z direction are recombined before entering another Stern-Gerlach device oriented along the x direction.

Gerlach device with no recombination of the beams. The atoms are not detected before entering other Stern-Gerlach devices arranged along T_+ and T_- and measuring S_x. This time, the atom is detected when coming out of the second apparatus. The methods to be explained in next sections will allow one to assert not only the value of S_x at the time of the second measurement but also along which trajectory T_+ or T_- the atom went beforehand. This proceeds by logical implications. In any case, it should be clear that nothing can be asserted without the help of an actual fact.

A fact occurs when a property of the measuring apparatus changes in a significant way. As a fact, it concerns a collective observable (or several of them). The change of properties usually goes with an expenditure in energy, coming, for example, from latent heat in a bubble chamber or from the electric field in a Geiger counter. It must be an irreversible process, not so much for dissipation itself but for decoherence, which does not exist without some dissipation.

The outcome of the measurement can be registered in many ways, for instance by a pointer on a dial or a number displayed on a counter. It can also be a track in a rock nobody has ever seen. Considering the case of a pointer, its final position is a classically meaningful property, which can be described by a projector (acting in the collective Hilbert space of M). It is convenient to denote by an index n ($n=1,2,\ldots$) the various final positions of the pointer and by the index 0 its initial neutral position. Let E_n^M be the associated classically meaningful projectors. The mathematics of the discussion is a bit simpler if one defines an observable

$$B=\sum_{n=0} b_n E_n^M \ , \tag{3.1}$$

all the b_n's being different. It is then equivalent to say that the pointer is in position n or that B has the value b_n.

Before the measurement, the pointer is in the neutral position or $B=b_0$. This means that the initial state of the apparatus is consistent with this initial property or, more formally,

$$E_0^M \rho_M E_0^M = \rho_M \ , \tag{3.2}$$

ρ_M being the initial-state operator of the apparatus. For the sake of rigor, one should take into account that the projector is not uniquely defined but belongs to an equivalence class of quasiprojectors. There is accordingly an unavoidable error η in the statement of a classical property, η depending upon the apparatus, so that Eq. (3.2) should rather be written as

$$\mathrm{Tr}|E_0^M \rho E_0^M - \rho| = 0(\eta) \ . \tag{3.3}$$

This is, however, a bit pedantic because η is extremely small, and this kind of refinement can be ignored here for the sake of clarity.

3. The conditions for a measurement

The interaction between Q and M must be very special if it is worth being called a measurement. It will be assumed to take place between an initial time t_m and a final time t'_m. It takes most often some more time for the complete working of the apparatus to take place, but this is inessential, and it will be assumed that the pointer immediately shows the final data at time t'_m. The evolution operator $U(t'_m, t_m)$ for the interacting $Q+M$ system will be denoted by S, a notation analogous to that used for a collision operator: in some sense, Q and M collide. As a matter of principle, this operator can be computed from the complete Schrödinger equation describing the interacting system $Q+M$.

Consider first the case in which the measured observable A has discrete eigenvalues a_n. A perfect measuring apparatus has the following property: when the initial state of Q is an eigenstate of A with the value a_n, then the final state of M is in an eigenstate of B with the value b_n. It should be stressed that this is only a property of the interaction dynamics, something coming from the Schrödinger equation. It can be written as a condition upon the matrix elements of S [Eq. (3.5) to follow].

In many cases, the measured system Q is lost. It has decayed or it has been irretrievably lost in the bulk of the apparatus. When it survives, it may happen that it is still in an eigenstate of A with the same value a_n. The measurement is then said to be of Type I (Pauli, 1933), but this case is at present much less important than in the Copenhagen interpretation.

When the spectrum of A is continuous, one can often proceed along the same lines by cutting the spectrum into pieces $\{D_\alpha\}$ and agree that one is in fact measuring the various projectors associated with the properties $[A, D_\alpha]$. There is no difficulty in extending all these considerations to the simultaneous measurement of several commuting observables.

An important distinction must be made concerning the outcome of the measurement. The classically meaningful property $B=b_n$, as shown by a pointer or a counter, will be called the *data*. It can be read at time t'_m but, because of the deterministic character of the apparatus, it also constitutes a record that can be read at any later time. However, what one is really interested in is the *result* of the experiment. This is a quantum property, namely $A=a_n$, stating something about the measured system (for instance, a value for a spin component). It is a property holding at the time t_m when the interaction begins. The goal of the measurement is to assert the result; its tool is the actual occurrence of the data.

Finally, it should be said that all this is a theorist's dream of a measuring apparatus. No one is really perfect and, even so, the data itself suffers from the quantum corrections to classical logic These systematic errors can, however, be treated for their own sake and they will be ignored here.

B. The theorems of measurement theory

The Copenhagen axioms of measurement theory become theorems in a consistent interpretation. This point of view has been developed at some length by the author (Omnès, 1988a, 1990). Gell-Mann and Hartle (1990) limited themselves to a few indications, but this is quite enough to make sure that the notions coincide.

1. General results

Let us begin with the results that are universally valid. It will be assumed that, at the initial time 0, the measured and the measuring systems are uncorrelated, so that the total state operator for $Q+M$ can be written as a product,

$$\rho=\rho_Q\otimes\rho_M . \tag{3.4}$$

When there is correlation, a special analysis is needed.

The systems Q and M do not interact before time t_m, and thereafter interaction takes place until a time t'_m when the data have become classically meaningful and decoherent. It will be assumed that the data are $B=b_n$. It does not matter whether this is simply a possibility as envisioned by classical probability calculus (since the possible final states of M are described by this calculus because of decoherence) or whether it is an actual fact. In order to seriate the problems, it is better to stick to the first point of view for the time being. The result is the quantum property stating that $A=a_n$ at time t_m.

The technique consists in building up histories based upon the initial state (3,4). The most economical one only mentions the various possible results at time t_m and the various possible data at time t'_m. It must be shown that they build up a consistent logic in which some relevant implications can be exhibited.

The essential tool for this, the "measure" character of the interaction, has already been discussed and it can be formally written as

$$SE_0^M(t_m)E_n(t_m)=E_n^M(t'_m)SE_0^M(t_m)E_n(t_m) , \tag{3.5}$$

where E_n is associated with the property $A=a_n$ for the measured system. The projector $E_n(t_m)$ is associated with the result of the measurement. One also uses condition (3.2) for the initial state of M and the fact that the classical properties stating that $B=b_0$ (neutral position of the pointer) at time 0 and at time t_m are logically equivalent because of the deterministic behavior of the measuring apparatus, which remains isolated prior to interaction with Q.

The first theorem one gets is that the data d is logically equivalent to the result r:

$$d\Longrightarrow r, \quad r\Longrightarrow d . \tag{3.6}$$

It shows how knowledge of the data given by the pointer implies knowledge of the result, which is what one is looking for.

The second theorem has to do with probabilities. What one actually gets from a repeated set of individual measurements are the probabilities for the data. They are given by

$$p_n=\mathrm{Tr}[\rho(t'_m)E_n^M] , \tag{3.7}$$

the trace being taken over the $Q+M$ Hilbert space. It turns out that, because of the measure property (3.5), this probability can be expressed much more simply in terms of quantities depending only upon the Q system, namely,

$$p_n=\mathrm{Tr}[\rho_Q(0)E_n(t_m)]=\mathrm{Tr}[\rho_Q(t_m)E_n] . \tag{3.8}$$

The trace and the operators are now defined much more simply in the Hilbert space of Q.

This is Born's rule. It states that the statistics of the results do not depend upon the details of the measuring device that is used. Of course, this might have been expected from Gleason's theorem and the logical equivalence (3.6), which is the really profound result.

The last general result concerns successive measurements: At a time t later than t'_m, a measurement of another observable A' is made by another apparatus M'. The two observables A and A' may be the same or be different. One looks for the probabilities for the second measurement, once the first one has given the result a_n. These probabilities can now be expressed in terms of histories beginning at the time t'_m when Q becomes isolated again after the first interaction. In these later histories, it is found that one must take for the density operator of Q at time t'_m the expression

$$\rho_{Q,\mathrm{after}}=\frac{\mathrm{Tr}_M\{E_n^M(t'_m)\rho_Q\otimes\rho_M E_n^M(t'_m)\}}{\mathrm{Tr}\{E_n^M(t'_m)\rho_Q\otimes\rho_M E_n^M(t'_m)\}} . \tag{3.9}$$

The trace in the numerator is taken over the Hilbert space of M, while in the denominator it is taken over the whole Hilbert space of $Q+M$.

This formula, which appears in the approaches of both Gell-Mann and Hartle and the author, is the most general expression of wave-packet reduction. It does not mean, as will be seen, that reduction is an actual effect. It only appears here as a useful tool for computing the probabilities for later interactions, including measurements.

That this is wave-packet reduction can be seen most clearly when the result of the measurement is a nondegenerate eigenvalue a_n. By using Eq. (3.5) together with the unitarity of the S matrix, we greatly simplify Eq. (3.9), which becomes

$$\rho_{Q,\text{after}} = |a_n\rangle\langle a_n| \ . \tag{3.10}$$

In a measurement that is strongly of type I, so that an initial eigenstate $|a_n, r\rangle$ for a degenerate eigenvalue a_n remains the same in the final state, one gets (Lüders, 1951)

$$\rho_{Q,\text{after}} = \frac{E_n \rho_Q(T_m) E_n}{\text{Tr}\{E_n \rho_Q(t_m) E_n\}} \ . \tag{3.11}$$

The meaning of the general expression (3.9) is best understood by writing down explicitly the propagators entering in the time-dependent projectors. The trace in the numerator on the right-hand side of Eq. (3.9) becomes

$$\text{Tr}_M\{U^{-1}(t'_m) E_n^M U(t_m) \rho U^{-1}(t'_m) E_n^M U(t'_m)\} \ . \tag{3.12}$$

Taking into account the absence of interaction between times 0 and t_m, one has

$$U(t'_m) = S U_0(t_m) \ , \tag{3.13}$$

where U_0 is the free propagator for the two systems Q and M with no interaction. The trace (3.12) being only partial, the two propagators $U(t'_m)$ and $U^{-1}(t'_m)$ on the extreme right and left do not trivially cancel each other, but they do so when due care is taken of Eq. (3.5) together with the unitarity of S. Then the trace becomes

$$\text{Tr}_M\{E_n^M S \rho(t_m) S^+ E_n^M\} \ . \tag{3.14}$$

The factors S and S^+ express the details of the interaction between Q and M when the measurement is not of type I. Consider, for instance, a case in which a charged particle crosses a bubble chamber without stopping in it. One can, however, sometimes get at the momentum of the particle by looking at the ionization rate, i.e., the density of bubbles along the track. When going out, the particle has lost some momentum, and the probabilities of its later interactions can be computed from the effective state operator given by Eq. (3.10). The S matrix represents the effect of the interaction of the particle with the chamber. This is why wave-packet reduction is so complicated generally.

2. Time monitoring

It has been assumed that the measurement begins at a well defined time t_m. This is what occurs, for instance, when Q contains an atom standing permanently in the vicinity of apparatus M. In a nuclear-magnetic-resonance experiment, the measurement begins when the microwave magnetic field is set up. This is controlled from the outside and, if it takes place automatically, the setup is completely described by the collective classical properties of the apparatus. Then it is clear that t_m is well defined.

In a Stern-Gerlach experiment, the interaction begins when the atom penetrates the region of the magnetic field. This is controlled by the initial wave packet of the atom, and the beginning is loosely defined. One might in principle describe what happens by the techniques of collision theory but, so long as the measured observable is a constant of motion, this is of no importance. In any case, there is in principle no difficulty in extending the previous results to a case in which the interaction between Q and M builds up continuously. This can be considered as a technical problem to be treated for its own sake if an experiment is so devised as to need it.

3. Correlated measurements

When several measurements are performed one after the other, one can also encounter the following situation: An apparatus M measures an observable A upon a system Q while another apparatus M' measures an observable A' on another system Q', but the initial states of Q and Q' are correlated. This occurs when the density operator $\rho_{Q+Q'}$ does not turn out to be the tensor product of ρ_Q and $\rho_{Q'}$, the three operators being computed by partial traces upon the state of the laboratory.

There is no special difficulty: one has only to consider $Q+Q'$ as the measured system, and the recipe (5.9) still holds for it when the two measurements are made at different times. Otherwise, it is a simultaneous measurement of two commuting observables. Of course, the correlation between Q and Q' results in correlations among the data: The probability $p(a,a')$ for the results $A=a$ and $A'=a'$ is not the product $p(a)p'(a')$. Such correlations are often used to get useful information: for instance, one can get at the spin and parity of a particle by this kind of measurement performed upon the decay products.

Another aspect of this situation is of a more logical nature. It has to do with possible paradoxes first considered by Einstein, Podolsky, and Rosen. This will be considered in Sec. III.D.

4. The status of wave-packet reduction

To get wave-packet reduction in the form (3.9), one has to describe the two measurements taking place one after the other. The physical systems is made up of Q, M, and M'. One uses a rather large family of histories involving the various possible data from M and M' as well as the various results, which are properties of Q occurring at the time when one or the other measurement begins. The logic one thus obtains can be shown to be consistent, and one can therefore compute the conditional probability for some data shown by M', given that M shows data n. It turns out that this probability can be written in a form analogous to Eq. (3.8) in which the initial state of Q is now given at time t'_m (the end of the first measurement) in the form (3.9). The presence of a denominator in Eq. (3.9) comes from the fact that there is always a denominator in a conditional probability.

However, there is no physical effect that might be called a reduction effect. Reduction is nothing but a convenient shortcut to avoid keeping track of all measurements by dealing explicitly with histories involving many outside devices, when one needs only to compute some probabilities. It comes from a theorem stating how one can conveniently express these probabilities and nothing else. It has no physical content, just as there is no physical content when one writes down Newton's equations of motion and then integrates them: No formula resulting from a mathematical analysis is supposed to have a physical content, and wave-packet reduction is only a formula expressing the result of a calculation in logic.

Of course, the whole construction depends upon the theory of phenomena expressing data, and it has been shown that decoherence breaks forever the quantum correlations between different phenomena and the different possible data. This *is* a physical effect and it has been observed, as discussed in Sec. IV. So, one can find in it a physical reason for the simplicity of wave-packet reduction. It is, however, not a physical effect occurring in the measured system Q, but a physical effect occurring solely in the measuring device M.

Finally, it should be stressed that real physics is described by histories, not by reduction. One can talk about an atom Q from time to time, when it becomes isolated for a moment, and this is convenient to concentrate upon what is relevant when an experiment is done. It should not, however, be so pervasive a point of view as to make all of us think of the real world as a series of quantum measurements. In reality, there are many objects, some of them being apparata devised for experiments in physics. An atom may have had a long and complex past history with all sorts of interactions with its surroundings. Only a Griffiths history can described completely what they were and what became lost and forgotten because of decoherence. In some cases this preparation process, either spontaneous or organized by a physicist, is so peculiar that the state of the atom becomes clearcut and is completely expressed by a property. In most practical cases, only a few relevant characteristics of the state are well determined, but this is enough to proceed with measurement theory.

The best one can say is that there are parts of the universe well enough isolated to be described by a state operator. Long ago, this was only the whole universe itself (Gell-Mann and Hartle, 1990). The state of a momentarily isolated subsystem is obtained from the state operator of the sufficiently isolated system to which it belongs (e.g., the solar system) by a partial trace over the surroundings.

Experiments in physics are so well done that sometimes in practice one can arrive at this state operator and a series of measurements can be made. It turns out that they can be represented by the simple formula (3.9) without having to invoke over and over again the holistic universe. This is most fortunate; no angel has to touch a particle to change its wave function every time a counter makes a noise.

5. The state of the universe

When one wants to describe reality rather than what happens within a corner of a laboratory, it can be done as follows (Gell-Mann and Hartle, 1990, 1991a): Let ρ_i be some "initial" state of the universe at an "initial" time zero. Let $E^{\alpha_n}(T_n)$ be the projectors representing all past actual facts in the history of the universe. Then one can take the present state of the universe as given by

$$\rho = M\rho_i M/\mathrm{Tr}(M\rho_i M)\ , \tag{3.15}$$

where M is the time-ordered product of all the projectors representing past facts. This is enough to take into account present facts and to predict the probabilities of future ones. Of course, due caution concerning relativity should be exercised. There is no difficulty as long as observables belonging to spacelike separated regions commute.

Gell-Mann and Hartle have also considered all the possible histories of the universe, which leads them to set up their interpretation within Everett's (1957) many-worlds framework. The author preferred to start from a given factual present state of the form

$$\rho = B/\mathrm{Tr}B\ , \tag{3.16}$$

where B is the (commuting) product of all the projectors of existing present phenomena. He then proceeded to show that a large part of past facts can be known and shown to be uniquely defined by retrodiction from present records (Omnès, 1990). Later evolution resulting in the occurrence of new actual facts preserves the form (3.16).

Both formulations lead to the same conclusion, namely, that all observations of present phenomena and all predictions of probabilities for future ones can be obtained from either Eq. (3.15) or (3.16) for the state of the universe.

6. About information

The present formulation relies only upon factual phenomena, whether they came to the knowledge of an observer or not. It does not mean of course that an observer should not have to allow for his limited information in order to make the best of it. This has nothing to do with the extreme positivistic point of view according to which the only content of physics is information: Information theory is a very useful superstructure of science, but it cannot be its foundation. True enough, the principles of physics are essentially economical summaries of experimental information, but they cannot be put within the framework of information theory because one cannot formulate the class of all conceivable laws of physics and assign them *a priori* probabilities that would be necessary to write down the information. So, basic physics definitely does not rely upon information theory.

This being said, when an observer knows only the average values of a few observables A_k, including the square of some of them to assert uncertainties, the best he can do for fixing the relevant density operator is to minimize the information

$$I = \mathrm{Tr}(\rho \log\rho) \tag{3.17}$$

under the constraints

$$\mathrm{Tr}(\rho A_k) = a_k, \quad \mathrm{Tr}\rho = 1 \ . \tag{3.18}$$

This gives him the best possible density operator,

$$\rho = \exp\left[-\alpha - \sum_k \beta_k A_k\right] , \tag{3.19}$$

where the Lagrange parameters α and λ_k are fixed by the conditions (3.18).

C. The notion of truth

Asserting what can be said to be true or real when the laws of physics are those of quantum mechanics is an old question that was first considered by Heisenberg (for discussion, see Jammer, 1966, 1974; d'Espagnat, 1976). The experimental data, though not clearly distinguished from experimental results, were supposed to be true by the Copenhagen interpretation exclusively of anything else.

It will be necessary to discuss this question again, if only because of the role of truth in logic. Moreover, it was mentioned in Sec. II.C.2 that the full definition of a logic must involve a criterion for truth. This condition can at last be considered now. As a matter of fact, the renewal of interest in these questions lying at the margin of physics came from an objection by d'Espagnat (1989) against the interpretations that are reviewed here. He criticized the legitimacy of Griffiths's use of retrodiction, and this would also apply to what Gell-Mann and Hartle have since published on the subject. He also denounced the existence of a gap in Omnès's (1988a) logical construction where a criterion of truth was still missing, contrary to the definition of a logic given in Sec. II.C.2.

To discuss this rather delicate point, we shall first show where the difficulty lies by an example, to find that it comes from complementarity. Then a criterion for truth allowing us to go beyond the multiplicity of complementary logics will be given (Omnès, 1991). These notions will then be applied to the Einstein-Podolsky-Rosen situation, where their full power will appear.

1. What is the problem?

Let us consider the following example: A spin-$\frac{1}{2}$ system is initially in the state $s_x = +\frac{1}{2}$ and a measurement shows that $s_z = +\frac{1}{2}$ at a later time t. One considers an intermediate time t' $(0 < t' < t)$ and two logics L_x and L_z. Both of them include a description of the preparation and the measurement processes and they also state some properties of the spin $-\frac{1}{2}$ system at time t'. The logic L_x (L_z) envisions histories in which s_x (s_z) can take the two possible values $\pm\frac{1}{2}$ at time t'.

It can be shown that both logics are consistent. They are also obviously complementary, since they assert the values of two noncommuting observables at the same time. In L_z, one can prove that the property for the value of s_z to be $+\frac{1}{2}$ at time t' follows by a logical implication from the measurement made at time t. Let a_z denote the corresponding proposition. One can also prove that in L_x, the property for the value of s_x to be $+\frac{1}{2}$ at time t' is implied by the preparation data. Let a_x denote the corresponding proposition.

The argument by d'Espagnat is quite simple. It says that a_x and a_z cannot be considered to be true. The reason is that, when two propositions a_x and a_z are both true, the joint proposition "a_x and a_z" should also be true (Manin, 1977). So, d'Espagnat argued, the properties mentioned in histories are not true and have no actual physical meaning.

Before discussing this issue, one can give another example that is also useful: Consider a particle that is produced at the origin in an outgoing isotropic S wave with a rather well defined velocity v. At a time t, it is measured to be in a small volume δV around a point x. Consider again an intermediate time t' and three logics L_x, L_p, and L_c. They all involve the same isotropic initial state and the same detected property at time t. They also involve one property at time $t'(0 < t' < t)$ and its negation. This property is the following (Omnès, 1988b, 1990):

(i) In L_x: One considers the point x' on the straight line going from the origin to x, such that $|x - x'| = v(t - t')$. A sphere V' has its center at x' and a radius R'. The property is that the position of the particle is in V' at time t'. It can be shown that the logic L_x is consistent if the radius R' of V' is conveniently chosen.

(ii) In L_p: One defines a region B' in momentum space. This region is contained inside a cone having its axis in the direction of the measured position x and its apex at the origin of p space. It is also contained between two spheres defined by $|p| = mv - \Delta p$ and $|p| = mv + \Delta p$. The property is that the momentum of the particle is in B' at time t'. Here again, it can be shown that the associated logic L_p is consistent after a convenient choice for the width $2\Delta p$ of the region B and the angle under which it is seen from the origin.

(iii) In L_c: This last logic relies upon a classical property according to which (x,p) is in some region C' of classical phase space at time t'. This region contains the point associated with classical motion going along a straight line from the origin to point x. The property is defined by a quasiprojector, which can also be chosen to give a consistent logic.

It may be noticed that these three logics assert in three different ways that the particle goes essentially along a straight line from the origin to the final position x. This is well known to be true from both experiment and theory (Mott, 1929) when the track of the particle is detected by a medium, as for instance in a bubble chamber or a Wilson chamber. The logics one has introduced say that some kind of straight-line motion also occurs when there is no detector to show it.

Contrary to the previous example, the three logics L_x, L_p, and L_c are still consistent, but they are not complementary (i.e., they are consistent with each other), at least up to a small error, when the regions V', B', and C' are well chosen (essentially big enough to be classical). In all of them the property occurring at time t' is implied by the measurement made at time t. Nevertheless, the previous difficulty is still present: Let one of these logics, for instance L_c, be imbedded in a larger logic L_C^M involving also the properties of the measuring device detecting the particle near point x at time t and of a preparing device, triggering, for instance, the production of the initial state by the decay of a source particle. It turns out that one might as well construct another logic stating that the particle is still in an isotropic S state at time t', and one would obtain exactly the same difficulty as in the case of a spin $\frac{1}{2}$: The previous role of property "$s_x=\frac{1}{2}$" would be replaced by the property now stating that the angular momentum is zero at time t', whereas the previous role of the property "$s_z=\frac{1}{2}$" is replaced by the quasiclassical statement expressing that the particle is going along a straight line. Therefore d'Espagnat's criticism is serious and it must be answered seriously.

2. A criterion for truth

It is possible to give a criterion for the truth of a property that goes beyond complementarity and satisfies the simple conditions on truth that logicians are asking for.

One must first restrict oneself to a special class of logics: those containing all the actual facts, i.e., all the real classical phenomena. Alternatively, one might deal with the class of histories containing a unique sequence of consistent phenomenological properties of the universe in Gell-Mann and Hartle's formulation (i.e., properties belonging to the classical domain). These logics do not contain only the phenomena, they may also involve many other properties, as needed, for instance, for a thorough discussion of an experiment, straight-line motion, or anything else. They are assumed to be consistent. Being consistent and in accordance with the facts, they may be said to be *sensible*.

One can then assert what should be said to be true. To begin with, actual fact will be taken to be true. Some other properties, which are not necessarily of the same type, will also be said to be true when they satisfy the two following criteria:

(i) One can add them to *any* sensible logic while preserving consistency. Given a sensible logic L and a property a of that kind, this means that one can augment the logic L by adding a and its negation to its field of propositions, the supplementary consistency conditions that come from this extension being automatically satisfied because of some dynamical property.

(ii) In all these augmented logics, a is logically equivalent to a factual phenomenon.

Measurement theory, as it was described previously, can be used to prove that the result of an experiment is always true. Another example comes from determinism: a past classical property that can be reconstructed logically in a deterministic way from present records can be said to be true, even when the sensible logics one is using involve only the present facts.

This theory of truth, therefore, covers the two main avenues of knowledge provided by physics: measurement, in quantum mechanics, and, in a classical framework, inference from present records concerning, for instance, the past history of the Earth, or the solar system and so on.

It seems that the two examples just given are the only ones. Thus one recovers essentially Heisenberg's point of view as far as quantum events are concerned, except for a deeper understanding of the meaning of truth among the phenomena themselves. The second example also answers an old question: It shows that a standing object at which nobody is looking is still nevertheless at the same place, and this can be taken to be true despite the fact that classical physics relies upon quantum mechanics.

3. Reliable properties

A *reliable* property partakes of a bit of truth because it never leads to self-contradiction, but it is not universally valid and is limited by complementarity so that it is not fully true. More precisely, it is defined as a property entering in *some but not all* sensible logics and in the logics where it has a meaning, it is implied logically by a fact (Omnès, 1991). Reliable properties have also been called "trustworthy" by d'Espagnat (1990), who analyzed them afterwards.

This notion of reliability can be used to discuss some matters of principle such as the Einstein-Podolsky-Rosen experiment or the separability of quantum mechanics. They also have a more practical use. In Sec. II.C.1, we discussed briefly an experiment in nuclear physics. When an experimentalist considers the corresponding systematic errors, he will have to take into account, for instance, the possibility that a neutron has suffered a collision with the shielding before hitting the target. He will, of course, use a classical description of the motion, for instance by a Monte Carlo calculation. This is complementary with another conceivable though impractical description in which one would use the complete wave function of the neutron. Finally, he will retain only the trajectories that are consistent with the final data, which are the logical consequences of this fact in the logic where they enter.

They represent, therefore, possible reliable properties, and their probabilities can be evaluated although they cannot be held to be true.

Thus one can say that the estimate of systematic errors relies upon reliable properties. If only for that purpose, reliable properties are therefore useful tools of physics, whereas it is not clear whether it was already noticed that this important aspect of experimental physics goes beyond the limits set up by the Copenhagen interpretation. Anyway, it can be justified within the present interpretation, and the corresponding probabilities, as used in a Monte Carlo approach, follow directly from a correct use of histories.

D. The Einstein-Podolsky-Rosen experiment

The notion of truth will now be applied to a discussion of the Einstein, Podolsky, and Rosen (1935) experiment. It will be found that the "elements of reality" that were considered by these authors do not correspond to true properties, though they are reliable. As a consequence, they are arbitrary, since they depend upon an arbitrary choice of logic, and this is enough to deny them any real value for knowledge. This result will then be generalized to show that, as far as true properties are concerned, quantum mechanics is separable. Finally, we shall go back to the state of the universe to consider up to what point some arbitrariness occurs in its definition.

1. The Einstein-Podolsky-Rosen experiment

In a form due to Bohm, the EPR experiment is the following: A spin-zero particle decays into two spin-$\frac{1}{2}$ particles 1 and 2, their total spin state being given by

$$|\alpha\rangle = 2^{-1/2}(|s_z^{(1)}=\tfrac{1}{2}\rangle\langle|s_z^{(2)}=-\tfrac{1}{2}| - |s_z^{(1)}=-\tfrac{1}{2}\rangle\langle|s_z^{(2)}=\tfrac{1}{2}|) \ . \tag{3.20}$$

The spin component of particle 1 along a direction n is measured at time t, and the spin component of particle 2 along a direction n' is measured at time $t'(t \le t')$. It will first be assumed that $t < t'$.

The discussion hinges upon the properties of the nonmeasured particle 2 at the time of the first measurement. In the two-dimensional Hilbert space for the spin of particle 2, every property asserts the value of the spin component of particle 2 along some direction n'', so one considers a logic containing the properties:

$$\begin{aligned} &s^{(1)}\cdot n = \pm\tfrac{1}{2} \text{ at time } t \ , \\ &s^{(2)}\cdot n'' = \pm\tfrac{1}{2} \text{ at time } t \ , \\ &s^{(2)}\cdot n' = \pm\tfrac{1}{2} \text{ at time } t \ . \end{aligned} \tag{3.21}$$

Preparation and measurements could be explicitly introduced together with the relevant properties of the measuring devices as in Secs. III.A and III.B, but this is cumbersome and unnecessary. We shall simply assume that this has been done so that all the logics to be considered are sensible. It will also be assumed for definiteness that the results of the two measurements are $s^{(1)}\cdot n = +\frac{1}{2}$ and $s^{(2)}\cdot n' = +\frac{1}{2}$. These are true properties, as one can check by an explicit calculation.

To be consistent, a logic including the properties (3.21) must satisfy some consistency conditions. There is only one condition, which is analogous to Eq. (2.14) and can be transformed algebraically into a condition upon the unit vectors (n, n', n''), namely,

$$(n \times n'')\cdot(n'' \times n') = 0 \ . \tag{3.22}$$

It is satisfied in particular when n'' is collinear with n or n', and these will be the only cases of interest here. They are, by the way, the only cases satisfying the consistency conditions in the stronger Gell-Mann–Hartle form (2.13).

There are therefore two logics worth considering for this experiment. Both contain the results of the experiments (and their formal negation). The first one, say L, also states that $s^{(2)}\cdot n = \pm\frac{1}{2}$ at time t, whereas the other, L', states that $s^{(2)}\cdot n' = \pm\frac{1}{2}$ at time t. They are obviously complementary when n and n' are not collinear. (By the say, one can say that n and n' are collinear for all practical purposes when their angle is smaller than the errors of classical logic for the measuring devices.) In the logic L, the property

$$a: \ s^{(2)}\cdot n = -\tfrac{1}{2} \tag{3.23}$$

is found to be the logical consequence of the first result, itself logically equivalent to the first data. Therefore it is reliable. The property

$$a': \ s^{(2)}\cdot n' = +\tfrac{1}{2} \tag{3.24}$$

is also reliable in logic L'.

Property a follows also from wave-packet reduction after the first measurement. However, it has been found that reduction is not a logical necessity but only a mathematical convenience, allowing one to predict the probabilities of later measurements. Here one is not considering a series of measurements giving rise to a probability, but an individual system where all the facts are known, so that reduction is not only unnecessary but useless. The meaning of property a' is that it anticipates at time t what will be the result of the second measurement at the later time t'.

There is no good reason for choosing one logic over the other, and the choice between them is arbitrary. It might even be said that, if two different experimentalists were to perform the measurements and they knew from theory what could be said in a consistent way, each one of them might prefer what was implied by his own result.

Anyway, property a is the one selected by reduction, and it was called an *element of reality* by Einstein, Podolsky, and Rosen for reasons that are not essential here. This gave rise to a large number of papers, far too many to cite here. The main point resulting from the previous remarks is that *the elements of reality have nothing to do with reality,* since they result from an arbitrary logical choice, i.e., from something that is left entirely to the freedom of the speaker. For more details, see Omnès (1991), where the case $t=t'$ is also considered. The extension to measurements made in relativistically moving frames is straightforward, and spacelike separated measurements have the same logical relation as nonrelativistic simultaneous measurements.

2. The separability of quantum mechanics

A theory is said to be *separable* when it has the following property: The properties of a system S cannot be changed by an action that is performed upon another system S' not interacting with it. The EPR situation was usually considered to show that quantum mechanics is nonseparable because wave-packet reduction resulting from a measurement upon S' changes the properties of S, if reduction is supposed to be a physical effect, S and S' being the particles 1 and 2 one considered earlier (see, for example, d'Espagnat, 1976).

It should be clear from the discussion of the EPR experiment that one must be careful about the characters of the properties to be considered when asserting separability. If one allows for reliable properties, nonseparability holds, but if one keeps to true properties, one gets separability. Since reliable properties are only logical artifacts, one can say that quantum mechanics is truly separable. Similar conclusions have been endorsed by d'Espagnat (1990).

It should, however, be mentioned that separability has also been defined as a statistical property by Bell (1964). It was thought for a long time to be essentially the same kind of separability as the one just mentioned, but they are in fact quite different. Bell's criterion refers to parameters acting as hidden variables for which the probabilities of two separate measurements are correlated with the parameters of the measured objects, though not correlated directly with each other. This statistical separability, which leads to Bell's inequalities, cannot even be formulated in quantum mechanics. It belongs to another world of theory. One can, however, use quasiclassical logic to show why it holds when macroscopic systems are concerned, which is probably why incautious common sense can believe in it too easily (Omnès, 1991).

IV. EXPERIMENTS

Many recent and not-so-recent experiments have been devised with the purpose of testing the interpretation of quantum mechanics. Some of them are not easy to interpret within the conventional Copenhagen framework. As a matter of fact, many experiments have given rise here and there to some technical improvements aimed at giving more precision to the basic interpretation, for practical purposes. It is, however, difficult to do justice to this kind of work, which is disseminated in the specialized literature dealing with many fields of physics.

This is a review about some aspects of theoretical physics and, as such, it is highly concerned with what experiments can bring out but, on the other hand, it is not a review of experimental physics. Our attention will therefore be focused upon experiments that may be considered as particularly significant from the standpoint of interpretation, leaving aside the many experiments fitting easily with the Copenhagen point of view. References in which specific details can be found will be quoted but, despite an effort towards a reasonable amount of completeness, some important contributions have certainly escaped. The author can only apologize for that.

These experiments have been classified into families. Section IV.A deals with those requiring most obviously the use of histories. They are concerned with decaying particles, continuous measurements, and a limiting case known as the Zeno effect; the most interesting deal with the observation of an isolated atom. Section IV.B deals with interference experiments and the quantum behavior of some superconducting systems. Finally, it may be recalled that the description of the straight-line motion of particles, so useful for the evaluation of systematics error in experiments, has already been discussed.

A. Experiments requiring histories

1. The decay of a particle

This is the simplest example one can think of. The system one considers has two types of states. One of them is a particle; let E_g denote the associated projector, corresponding to the property stating that the particle has not decayed. There is also a whole family of states having a continuous energy spectrum, which represent the decay products. Let E_d be the associated projector. One may restrict it by specifying the total energy within fixed bounds around the available decay energy, or a range of momenta for the decay products and so on, but this is a trivial modification and one does not need to consider it in detail. The projector E_d states that the particle has decayed. This property can be tested by the detection of a decay product.

The elementary theory of decay usually considers the probability $p(t,t+\Delta t)$ that the unstable particle existing at time zero (initial state) will still be intact at time t and will decay during the time interval $(t,t+\Delta t)$. The meaning of this double probability is rather questionable in the Copenhagen interpretation, since the existence of the particle at time t is not the result of a measurement. This difficulty can be avoided by a thorough two-channel treatment of the complete wave function (see, for example, Goldberger and Watson, 1967).

It is simpler to use a history with two times, t and $t+\Delta t$, in which the particle is intact at time t and the decay products are present at time $t+\Delta t$. The Hamiltonian includes the terms that are responsible for the decay.

One must, however, be careful to forbid the regeneration of the initial particle from its decay products, which is allowed by this Hamiltonian. This can be done in two alternative ways: (i) If a decay product is actually detected at time $t+\Delta t$, the detecting apparatus is included in the total physical system together with its factual data, as was done in measurement theory. (ii) When there is no actual detection, one can take into account the outgoing character of the decay products in the projector E_d. This is found to be enough to insure the consistency of the simplest logic including the decay history, but the time interval Δt must satisfy the inequality

$$\Delta t \gg \hbar/E \ , \tag{4.1}$$

where E is the available energy for the decay products.

In both cases, one finds that the history belongs to a consistent logic, so that the use of probability $p(t,t+\Delta t)$ is legitimate. If $p(t)$ is the probability for the particle to be intact at time t, which is well defined by the history, one then gets

$$p(t+\Delta t)-p(t)=-f(\Delta t)p(t) \ . \tag{4.2}$$

The calculation of the function $f(\Delta t)$ proceeds exactly as in textbooks, and one gets as usual

$$f(\Delta t)=\Delta t/\tau \ , \tag{4.3}$$

where τ is the lifetime ensuing from dynamics. This formula is valid when $\tau \gg \Delta t \gg \hbar/E$. Equations (4.2) and (4.3) give immediately the well known exponential decay law. There are slight corrections for large times, when one takes into account the finite width of the initial energy level allowing in principle a finer preparation of the initial state, but this is easy to deal with and without interest in practice.

It is worth recalling that Eq. (4.2) is modified when Δt is of the order of $\hbar/E$ or smaller. One finds in that case

$$f(\Delta t)<\Delta t/\tau \ , \tag{4.4}$$

and $f(\Delta t)$ is of order $(\Delta t)^2$ when Δt is very small. This remark will be useful in the discussion of the Zeno effect; it can be used only when some decay product is actually detected, since otherwise the logic one is using is not consistent.

2. Continuous measurements

Systematically repeated measurements have been the subject of much discussion (Chiu *et al.*, 1977; Misra and Sudarshan, 1977; Peres, 1990; Singh and Whittaker, 1982; Joos, 1984; Zurek, 1984; Braunstein and Caves, 1988; Caves and Milburn, 1988). They will first be discussed in a rather formal way before going to the observation of isolated atoms.

Let us consider a specific ideal example in which a measuring device M checks repeatedly at time Δt, $2\Delta t$, $3\Delta t, \ldots$ that an unstable particle is still intact. Another measuring device M', also acting at the same times, checks for the presence of decay products.

A history stating that the particle survives up to time $n\Delta t$ and has decayed at time $(n+1)\Delta t$ is associated with a sequence of properties having the projectors $\{E_g(\Delta t), \ldots, E_g(n\Delta t), E_d((n+1)\Delta t)\}$. The family consisting of all the possible histories of that kind turns out to be consistent if one is careful to forbid a decay with regeneration between two times in the sequence. If this is done by excluding regenerating particles (e.g., photons) in the initial state and by restricting E_d to outgoing waves, one gets a consistent logic as long as condition (4.1) is satisfied. Alternatively, one can include the measuring devices in the system and use the projectors E_g^M, $E_d^{M'}$ expressing the factual data in place of the projectors E_g, E_d expressing directly the properties of the decaying system. One again gets a set of consistent logics in which the elementary properties of the decaying system follow by logical induction from the data: they are true, even if condition (4.1) is not satisfied.

The consistency and truth of the results can be expressed in a striking way by saying that the logical structure of quantum mechanics has for a consequence the existence of *quantum jumps*, i.e., a change of state that can occur during a very short time interval.

When there is no actual checking of the undecayed state at times $\Delta t, \ldots, n\Delta t$ and consistency is only ensured by the absence of decay products in the initial state and their outgoing character at time $(n+1)\Delta t$, the quantum jump is only complete when $\Delta t \gg h/E$. Otherwise, one finds that the system is in a linear superposition of decayed and undecayed states and the logic is not consistent. On the other hand, when all actual measurements are made, there can only be complete quantum jumps.

3. The Zeno effect

The Zeno effect is what occurs when the time interval between the measurements is of the order of $\hbar/E$ or smaller. It was usually considered as promising to show in principle an actual reduction of the wave packet, i.e., its manifestation as a physical effect, and this is why it was thought to be particularly interesting. Of course, it is extremely difficult to observe because of the finite duration of a practical measurement. Nevertheless, let us see what is predicted.

Using the consistent logic based upon the measurement data, one can still get Eq. (4.2), where now t is equal to $n\Delta t$, the probability for a decay between times $n\Delta t$ and $(n+1)\Delta t$ satisfying the inequality (4.4). From that, one gets for the probability of observing the undecayed particle at time t

$$p_z(t)=\exp\{-t[F(\Delta t)/\Delta t]\} \ . \tag{4.5}$$

In principle, when Δt becomes very small, $f(\Delta t)/\Delta t$ vanishes, and it looks as if the decaying particle had been frozen by the successive measurements and acquired a much larger lifetime.

The Zeno effect exists in principle in the present interpretation, and it has of course nothing to do with an actual physical reality of wave-packet reduction, since it has been found by relying directly upon histories.

Another remarkable effect has been experimentally observed that has sometimes been considered as a kind of Zeno effect. It is the narrowing of resonance lines in nuclear-magnetic-resonance experiments performed upon gaseous or liquid samples (Abragam, 1964).

The origin of the effect is the following: Resonance lines are displaced by the fields generated locally by neighboring atoms. These fields behave randomly, giving a width of the resonance line, since the various resonating nuclei see different local fields. This is indeed what is observed in a solid sample, and it is well explained by the theory, the width of the line $\Delta\omega$ being related to the fluctuating local magnetic field ΔB by

$$\hbar\Delta\omega=\mu|\Delta B| \ , \tag{4.6}$$

where μ is the magnetic moment of the resonating nuclei.

In a gaseous or liquid sample, one may observe a narrowing of the resonance line. If τ_c is the collision time with neighboring atoms or molecules, the effect occurs when

$$\Delta\omega\tau_c < 1$$

and the width is found to be of the order of

$$\gamma=(\Delta\omega)^2\tau_c \ . \tag{4.7}$$

The theory of this effect is well known (Abragam, 1962), and it is based upon a description of the fluctuating local field as a random classical signal. This makes a comparison with the Zeno effect difficult. However, there is no essential difficulty in recasting Abragam's calculations within the framework of decoherence, using the results of Caldeira and Leggett (1983b).

The outcome of this analysis is that the narrowing of NMR lines and the Zeno effect have analogies but they are nevertheless quite distinct physical effects. As a matter of fact, in the Zeno effect, decoherence occurs in the measuring device, whereas the measured system is unaffected by it. In contrast, in the narrowing of spectral lines, the sample of resonating nuclei has a directly decoherent density operator. One can therefore conclude that the Zeno effect has not as yet been observed.

4. Observing a single atom

One can now monitor a single atom (Bergquist *et al.*, 1986; Nagourney *et al.*, 1986; Sauter *et al.*, 1986), and this is enough to show that an interpretation of quantum mechanics that would be strictly limited to the consideration of statistical ensembles cannot be complete.

As an example, consider the following experiment (Nagourney *et al.*, 1986; Dehmelt, 1990): A singly ionized barium atom is confined in a Paul trap, i.e., a cleverly devised combination of static and oscillating electric fields forbidding the charges to escape. A laser (blue) light provokes transitions between the ground state g and a well defined excited state e of the barium atom. When in its excited state, the atom can fall back to the ground state by stimulated emission of a photon, in which case the emitted photon is indistinguishable from the laser light. The atom can also emit a fluorescence photon with essentially the same frequency, when the photon escapes in a direction different from the laser-beam direction, and it can be detected. If τ is the natural lifetime of the excited atom, one can observe an average of one fluorescence photon per time interval 2τ (the factor 2 coming from the probability $\frac{1}{2}$ for the atom to be in state e rather than g). This fluorescence emission is strong enough for the atom to be seen with the naked eye or to be photographed as a point source.

One can also add another (red) laser light, which can provoke a transition from the state e towards another excited state f (see Fig. 5). The transition from e to f is forbidden, so that a rather large average time τ' is needed for its occurrence. Before that, one can observe a large amount of fluorescence when the atom is flipping back and forth between the states e and g. Once the state f has been reached under the action of the red laser, fluorescence stops. It turns out that the lifetime of the state f is rather large, because the transition from f to g is also forbidden. Eventually state f ends up by a deexcitation and the atom is back in its ground state g, from which it is reset to flipping by the blue laser, and fluorescence reappears until a new transition to f occurs and so on.

The description that has just been given is quite intuitive, but it does not fit trivially with the Copenhagen interpretation. Is there a reduction of the atom wave function each time a fluorescence photon is detected? What occurs when these photons are not detected? How can one describe the detailed statistical signals that are obtained during a long time of observation? The reaction time of a photomultiplier is finite; does the reduction of the wave function wait that long or does it take place before the measurement is completed, which would go against the spirit of the Copenhagen interpretation but would better fit the statistics of the events, particularly the intensity of fluorescence light and its fluctuations?

None of these problems occur when one uses histories:

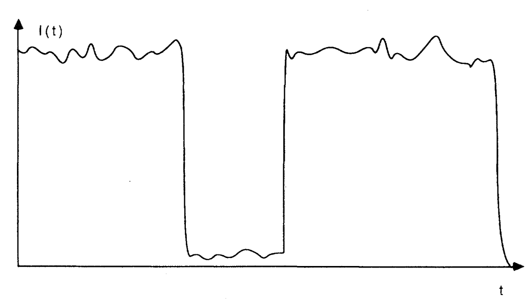

FIG. 5. A typical fluorescence signal from a single atom under the influence of two lasers, as explained in the text.

What is actually seen is just a history of the system. Consistency does not even require the fluorescence light to be detected, and it is enough to take into account the outgoing character of the photons. There is no problem of principle in predicting the statistical properties of the fluorescence signal. As a matter of fact, the detailed theories of this kind of experiment rely, explicitly or implicitly, upon histories (Cohen-Tannoudji and Dalibard, 1986; Kim *et al.*, 1987; Zoller *et al.*, 1987; Reynaud *et al.*, 1988).

Finally, it may be noticed that, when saying that the atom goes from the state e to state g when a photon is emitted, one asserts two true properties. The range of truth, or reality, is therefore much wider in quantum mechanics than what was previously believed in the conventional interpretation.

B. Interferences

One can observe interferences with a single photon (Grangier *et al.*, 1986) or currently with a single neutron. This is of course in agreement with quantum mechanics.

One also observes much more often interferences with "classical" light. It contains many photons and it can therefore be considered as a macroscopic system. Nevertheless, photon-photon interactions are so weak that there is neither dissipation nor decoherence, which is why interferences can be observed. This classical light is well described by coherent states (Glauber, 1963), which are eigenstates of the amplitude and phase of the electromagnetic field, and one can measure in that case a local intensity. A sensible logic including the factual properties of preparation and detection leads in that case, through implications, to the knowledge of some properties of light that must be expressed in terms of fields.

Conversely, when interferences at low luminosity are observed with photodetectors, the detectors react statistically one by one to photons, and a factual single detection leads through implication to a property of light that is expressed in terms of an observable for the photon. Accordingly, the present interpretation completely agrees with Bohr's considerations about wave-particle duality: the true statement of an experimental result is determined by the experimental device. There is no other consistent complementary choice. This, however, is not imposed from above by some kind of supplementary axiom but follows directly from the theory itself.

1. Meaningless statements

May it be said that a photon goes along a single arm of an interferometer or through a single hole in Young's experiment? This is an old question about which much has been written.

Its answer is quite simple if one accepts the universal rule of interpretation: a property is meaningful if and only if it can be included in a consistent logic.

It will be convenient to consider the case of an experiment with single photons coming one by one, the initial wave packet of the photon being sharp enough to be at some time t' completely inside the interferometer. Of course, it is equally divided between the two arms. Other cases could be considered, as in Young's experiment, but they are essentially equivalent, except sometimes for a more tedious mathematical formulation.

For definiteness, let it be assumed that the interferences can be seen in the focal plane of a converging lens where there is a photographic plate. Each (classically behaving) grain of the emulsion acts as a position detector for the photon. So, in a sensible logic where all the facts are mentioned (each grain being either reacting or nonreacting, only one of them reacting), one must include all the properties stating the presence of the photon in every grain as possible at the time t when the wave packet reaches the focal plane. One can also, with the help of projectors for position, introduce two properties holding at the intermediate time t'. Each of them states that the photon is in a definite arm of the interferometer.

Let us consider a logic involving both kinds of properties (those expressing the result of a detection and those stating which path the photon takes). One then finds as many consistency conditions as there are grains in the emulsion.

For a grain centered at a point x_n, the corresponding consistency condition turns out to be given, after computation of the trace, by

$$\cos\Delta\varphi_n = 0 \,, \tag{4.8}$$

where $\Delta\varphi_n$ is the phase difference between two semiclassical paths going through each arm, respectively, and ending at point x_n.

Not all these conditions can be satisfied, so that one must conclude that the statement that a photon goes through only one arm is meaningless. This has nothing to do with the presence or absence of a photon detector in the arms, although of course one also finds that such actual detectors would destroy interferences.

N.B.: This is a case in which one needs necessary and sufficient consistency conditions in the form (2.14) and not conditions (2.16), which are only sufficient. This is because all opportunities for a possible meaning must be considered when a statement is to be rejected as meaningless.

2. The Badurek-Rauch-Tuppinger experiment

Some refined interference experiments are not so easily disposed of. A very interesting one was proposed by Vigier (see Dewdney *et al.*, 1984; Vigier, 1986) and, despite the technical difficulties, it was beautifully realized by Badurek, Rauch, and Tuppinger (1985).

This is a single-neutron interference experiment using polarized neutrons. A neutron can follow two separate paths, along each of which there is a spin-flipper reversing the direction of the spin. Since the spin is finally the same whatever the path, interferences are possible. The interest of the experiment comes from elsewhere: A spin-flipper is a magnetic-resonance device usually involving a stationary magnetic field and a microwave with a definite frequency ω. One can then say that a spin flip corresponds to the absorption or emission of a photon with frequency ω. The essential point in the experiment is that the two spin-flippers have different, well separated frequencies ω_1 and ω_2.

When the experiment is performed, interferences are found. So, Vigier asked the question, since there was only one neutron and therefore only one photon to be absorbed (or emitted) this photon must have either the frequency ω_1 or ω_2, and its frequency is a signature of the path that was followed by the neutron. Is this not, therefore, a case in which there are interferences and in which one might nevertheless say which path the neutron took?

The question can be answered within the framework of the Copenhagen interpretation by using it with delicacy. A more systematic approach consists in using once again consistent logics. One introduces again, as intermediate properties, the two statements of which arm the neutron is in at a time t' when the full wave packet is separated into two disjoint parts, the final properties expressing possible detections of the neutron and allowing one to detect interferences if they exist. The time dependence of the projectors associated with the properties involves the neutron propagators $U(t-t')$. The Hamiltonian entering in this propagator involves the interaction of the neutron with the spin-flippers, which is linear in the creation and annihilation operators for the microwaves photons, these operators being different for the two spin-flippers since the waves are in two different modes.

In a sensible logic where all the devices interacting with the neutron are included, the traces occurring in the consistency conditions involve subtraces upon the electromagnetic degrees of freedom. To perform them, one must take into account the state of the microwaves, which are produced by a classical current in a coil. Their state is, therefore, a coherent Glauber state. The matrix elements of the creation and annihilation operators then become pure numbers factoring out of the traces. This means, in more physical terms, that the device is in principle unable to signal which kind of photon has been emitted or absorbed, and this is also the reason why the Copenhagen interpretation can be used.

In any case, one falls back upon the consistency conditions in the form (4.8), from which one can again conclude that it is meaningless to assert that the neutron followed a single path.

3. Delayed-choice experiments

According to Wheeler, the origin of delayed-choice experiments goes back to von Weizäcker (1931) and to a single sentence by Bohr (Schillp, 1949). They have been discussed at length by Wheeler (1978).

The principle is the following: one decides to active a device that is able to detect a photon in one arm of an interferometer only at a time when the full wave packet is already completely inside the two arms. If wave-packet reduction were a genuine physical effect, the decision would come too late to cancel the packet that is following the other arm, so that one might perhaps see interferences and nevertheless assert through which arm the photon went, or the collapsing of the wave packet would be an effect propagating faster than light. This kind of experiment therefore checks the physical character of wave-packet reduction, if one excludes the possibility of action at a distance.

These experiments have now been performed (Alley *et al.*, 1987; Hellmuth *et al.*, 1986). For the sake of definiteness, we shall describe only the one by Alley *et al.* It is essentially a single-photon experiment. A laser pulse triggers an electronic signal before entering an emitting diode, which emits a very weak pulse (reduced in intensity by a factor 10^{-16} with respect to the incident laser intensity, the delay being about 2.5 ns). The electronic signal triggers a random yes/no output signal from a randomizer. This output triggers in turn two Pockels cells located in the arms of the interferometer.

A Pockels cell is based upon the Kerr effect, i.e., the generation of birefringence in a crystal by an applied electric field. The light from the diode goes through a linear polarizer before entering the interferometer. When activated, a Pockels cell turns the polarization by 90°. Everything takes place in a time interval of 13.5 ns at which the 3-cm-long photon wave packet is far inside the interferometer.

A detector is located at a position where the existence of interferences can be checked by the number of counts. One gets statistics showing interferences when the two cells or none have been activated and no interference when only one of them is activated. This agrees with Bohr's prediction according to which it does not matter "whether our plans for handling the instruments are fixed beforehand or whether we postpone the completion of our planning until a later moment when the particle is already on its way from one instrument to another" (Schillp, 1945).

A theory of these experiments using the Birkhoff-Von Neumann approach has been given by Mittlestaedt (1987). It relies upon refinements of this approach involving the restrictions imposed upon the use of a language by pragmatic observations (Mittlestaedt, 1978). It seems that, when these restrictions hold, one can use a consistent logic in the sense meant here, although it would be interesting to know whether this equivalence also goes the other way round. In the present case, both "logical" approaches yield the same results.

The present interpretation can treat in principle any type of delayed-choice experiment. One introduces as before the properties stating through which arm a photon goes, these properties being now actualized as facts (or not) by a detector that can be randomly triggered when the wave packet is already inside the interferometer. In order to stay clearly within the bounds of quantum mechanics, the detector may be assumed to be triggered randomly by a quantum event, for instance the decay of a particle. This is only a technical bypass allowing us to avoid a precise description of an actual randomizer.

Basically, the system to be described consists of one photon, whose wave packet at a time t' is split between the two arms of the interferometer, and of an unstable particle in its undecayed state. When one-half of the photon wave packet crosses a region V farther along on one arm of the interferometer, it is factually detected in a nondestructive measurement if and only if the unstable particle has decayed at that time. One must of course use a logic involving the properties of the photon, of the unstable particle, and of the detectors inside and outside, the latter detectors being used to find out interferences.

The techniques that must be used for this analysis combine the methods that were explained in the previous section and the present one, and their outcome is quite simple: For the class of events in which inside detection of the photon takes place, the probability of detection outside is the sum of the probabilities for the photon going through only one arm or the other. For the class of events with no inside detection, interferences, i.e., addition of amplitudes rather than probabilities, should be observed. This is completely in agreement with Bohr's statement and with experiments.

C. Macroscopic quantum systems

A macroscopic system does not necessarily behave according to classical physics. The proofs of classicity that were given in Sec. II allowed for such exceptions. Two conditions must, however, be satisfied for observing a nonclassical behavior of a collective degree of freedom, namely, an initial state that is not a state of fact and the absence or ineffectiveness of decoherence. This last condition suggests that we consider nondissipative systems, the most obvious ones being light, superconductors, or superfluids.

Light is a macroscopic system consisting of many photons. The semiclassical theorems given in Secs. V and VI correspond in that case to geometrical optics, although the quantum aspects are more difficult to analyze by using these theorems because of the infinite number of degrees of freedom in quantum field theory. Classical behavior can be violated by a Young interference device, because the slits are so narrow that passage through them is incompatible with a classical property. The interferences are therefore observed under conditions excluding the assumptions leading to classical motion, as explained in Sec. II. Other interferometers rely upon other devices, all of them being based upon some wave properties' holding because of coherence (such as, for instance, semireflecting mirrors).

Whatever it may be, light is so well understood and of such a primary concern for physicists that few people are ready to put it on the same level as a macroscopic tunnel effect occurring in a superconducting device big enough to be plainly touched and seen.

The possibility of such a behavior in semiconductors was first put forward by Leggett (1980). He made it plain how this would be contrary to the assumptions made by some adherents of the Copenhagen interpretation (Landau and Lifshitz, 1958a), according to whom a collective degree of freedom is always expected to behave classically. For discussion of the Copenhagen interpretation on this point, see Leggett, 1980, 1987a, 1987b.

1. A superconducting quantum interference device

Superconducting quantum interference devices (or SQUIDs) have been used for a long time in fundamental and applied physics. It is important to see how they work in order to understand how they can be used for testing the interpretation of quantum mechanics.

A SQUID consists of a superconductive ring containing a Josephson junction. Leads are connected to the ring at two points in order to allow the injection of an electric current from the outside or the measurement of a voltage (see Fig. 6).

This is a rather ordinary electrical circuit, and it is easy to write down its dynamical equations. The variables are the electric charge Q across the Josephson junction (acting like a capacitor), the electric current I in the ring, the voltage V across the junction, and the magnetic flux φ across the ring. The internal parameters are the capacitance C, the inductance L, and the resistance R of the junction. The external varying parameters are the externally imposed magnetic flux φ_{ex} and the injected current I_{ex}. The characteristics of the junction are also well known: Let I_0 be the critical current, i.e., the maximal current the junction can sustain. Remembering that a superconductor contains many Cooper pairs, which all have essentially the same wave function, and denoting by δ the jump in the phase of this wave function across the junction, one has

$$I = I_0 \sin\delta \ . \tag{4.9}$$

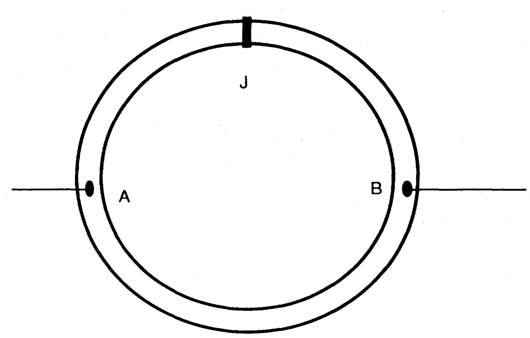

FIG. 6. A superconducting quantum interference device, or SQUID. The superconductor ring contains a Josephson junction J. Voltages can be measured and current can be injected with the contacts A and B.

The voltage is also related to the phase by

$$V=2\pi\varphi_0\frac{d\delta}{dt}\ , \tag{4.10}$$

where $\varphi_0=h/2e$ (e being the charge of an electron) is a convenient unit of magnetic flux. Finally, the Schrödinger equation for the wave function of the Cooper pairs gives a relation between δ and φ, namely,

$$\delta=2\pi\varphi/\varphi_0\ . \tag{4.11}$$

Using the characteristics (4.9)–(4.11) of the junction, together with elementary circuit theory, one easily gets an equation giving the time evolution of the flux in the form

$$Cd^2\varphi/dt^2=-\partial U/\partial\varphi\ , \tag{4.12}$$

where

$$U(\varphi)=(2L)^{-1}(\varphi-\varphi_{ex})^2-(I_0/2\pi)\cos(2\pi\varphi/\varphi_0) -(I_{ex}\varphi_0/2\pi)\varphi\ . \tag{4.13}$$

The resistance of the circuit has been neglected, as well as the thermal (Nyquist) noise. Shot noise is negligible.

One sees that Eq. (4.12) is entirely analogous to the classical equation of motion for a particle with a mass C in a potential $U(\varphi)$. This analogy is very convenient for getting a clearer picture of what is going on. Note also that the functioning of the junction relies upon quantum mechanics, but everything ends up in a perfectly classical equation of motion for the collective degree of freedom φ.

2. Quantum aspects of the SQUID

Because of the essentially quantum character of a Josephson junction, it is often said that the working of a SQUID is already by itself a macroscopic quantum effect. This is also true of superconductivity. It may be recalled here that the first experimental check of the Aharonov-Bohm effect (Bohm and Aharonov, 1957) was made with a SQUID involving two Josephson junctions (Jaklevic *et al.*, 1964). This quantum origin of superconductivity is manifested by the occurrence of the unit flux φ_0 proportional to Planck's constant in Eq. (4.10), and it also appears in the explicit expression for the critical current I_0.

Notwithstanding, Leggett (1980) proposed to consider the classical equation of motion (4.12) as a classical limit of a quantum dynamics to be described by the Schrödinger equation,

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2C}\frac{\partial^2}{\partial\varphi^2}\psi+U\psi\ , \tag{4.14}$$

where ψ is a wave function depending upon the variable φ. This is best understood as coming from quantization of the electromagnetic field, which leads to the introduction of a quantum observable for the magnetic flux. It also comes from quantum fluctuations in the number of Cooper pairs, i.e., from their description by a quantized field. So, from whatever approach, the Schrödinger equation (4.14) is associated with quantum field theory.

The potential $U(\varphi)$, as shown by Eq. (4.13), consists of an oscillating part coming from the junction together with a simpler contribution upon which one can play, in principle, by a suitable choice of the constitutive parameters I_0, L, C and the control parameters φ_{ex}, I_{ex}. Two special cases have particularly attracted attention (see Fig. 7): Case (a) shows a double well. It should make possible the observation of changes in the localization of a state, symmetric and antisymmetric states, and transitions between them. Its actual realization is, however, very difficult (Chakravarty, 1986; Tesche, 1986; Tesche, 1990).

The second case has been realized experimentally, and the results will now be considered. Before doing so, we note that all dissipation effects have been neglected in the present discussion. Despite the dissipationless character of superconductivity, this is very questionable because the junction has a resistance and there is also dissipation outside the superconductivity circuit, the impedance of which enters in a complete description of the effects. These dissipation corrections are much more difficult to compute than the simple dissipationless motion in Eq. (4.12). They rely upon the general approach in Caldeira and Leggett (1983a), which also takes into account finite-temperature effects.

3. Experiments

After a series of experiments showing the existence of the predicted quantum effects (den Boer and de Bruyn

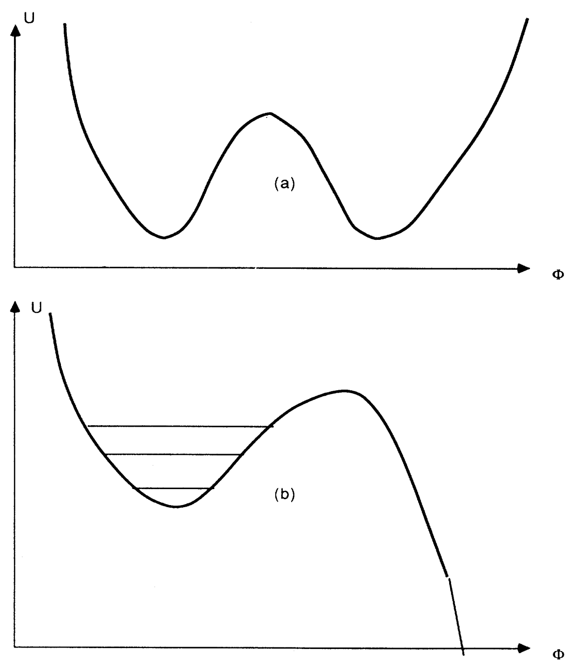

FIG. 7. Two examples of potentials that may be realized with a SQUID: (a) A double well; (b) a well with a barrier allowing bound states that are unstable under a tunnel effect.

Outboter, 1980; Jackel *et al.*, 1981; Prance *et al.*, 1981; Voss and Webb, 1981; Bol *et al.*, 1983, 1985; Dmitrenko *et al.*, 1985; Washburn *et al.*, 1985), quantitative checks of the Schrödinger equation (4.14) together with the quantum theory of decoherence were realized (Schwartz *et al.*, 1985; Martinis *et al.*, 1987; Devoret *et al.*, 1984, 1987). For a lucid review, see Clarke *et al.*, 1988.

The most complete experiments to date have been made with the potential configuration shown in Fig. 7(b). All the parameters can be measured in separate monitoring experiments, so that the theory contains no adjustable parameter. The potential $U(\varphi)$ shows a well where there can be a finite number of bound states. It will be convenient to describe what happens in terms of an analogous "particle" with mass C in the potential $U(\varphi)$.

In the ground state of the well, the average velocity $d\varphi/dt$ of the model particle is zero, so that, according to Eqs. (4.10) and (4.11), the voltage V is zero. When the model particle crosses the barrier, it aquires a kinetic energy by rolling down the potential hill, so that a voltage appears and then increases. This is a clear signal of the decay of the bound state.

At high enough temperature, the model particle can go classically over the barrier because of thermal motion. When the temperature decreases, thermal motion is replaced by a quantum tunnel effect with a well-defined transmission coefficient. The lifetime is explicitly given at zero temperature under the conditions of the experiment (Caldeira and Leggett, 1983a, 1983b) by

$$\tau^{-1}=\frac{\omega_p}{2\pi}\left[120\pi\left[\frac{7.2\Delta U}{\hbar\omega_p}\right]\right]^{1/2} \times\exp\left[-7.2\frac{\Delta U}{\hbar\omega_p}\left[1+\frac{0.87}{Q}\right]\right] . \tag{4.15}$$

The plasma frequency ω_p can be understood as the frequency at which the "particle" hits the barrier. It is explicitly given by

$$\omega_p=(2\pi I_0/\varphi_0 C)^{1/2}[1-(I_{\text{ex}}/I_0)^2]^{1/4} . \tag{4.16}$$

The energy ΔU is representative of the effective height of the barrier. When the potential is approximated in the neighborhood of the wall by a third-degree polynomial, one can use

$$\Delta U=[2(2)^{1/2}I_0\varphi_0/3\pi](1-I_{\text{ex}}/I_0)^{3/2} . \tag{4.17}$$

Finally, the damping coefficient Q depends upon the resistance R and is given by

$$Q=\omega_p RC . \tag{4.18}$$

Equation (4.15) clearly shows the effect of decoherence through the occurrence of the damping coefficient. One can act upon the resistance R by adjusting the input impedance and thus check this crucial aspect of the theory. It should be stressed that Eq. (4.15) is obtained by keeping only the leading order in an expansion with parameter $1/Q$. It cannot be used for the ideal case where $R=0$, but it is a good representative of the situations that can be realized with present technological means. The agreement between theory and experiment is very good. A quantitative check of the quantized energy levels in the well was also made by perturbing the system with an applied microwave field.

In another series of experiments (Estève *et al.*, 1989), still with the same potential configuration, the junction is biased by an adjustable impedance $Z(\omega)$ [see Fig. 8(a)]. This can be realized by partially covering a SQUID consisting of two parallel superconducting wires by an absorbing load [Fig. 8(b)]. The uncovered part of the SQUID behaves like a transmission line with a finite time delay. Friction in the absorbing load is therefore retarded.

Büttiker and Landauer (1982) have shown that a tunnel effect is characterized by a finite time τ_b, which they interpret as the time needed by a "particle" to cross a barrier. This interpretation has been questioned (Low and Mende, 1991), and the effect can be understood less controversially as a time delay originating from the momentum dependence of the transmission amplitude, as it occurs in any kind of collision (Froissart *et al.*, 1963).

Whatever the origin of the delay, it is found that, when the transmission line delays the wave packet for a time longer than τ_b, there is little effect of the shunting impedance upon the rate of tunneling, whereas, in the opposite case, this rate is strongly reduced. This is a direct observation of the decoherence effect. From the point of view of Büttiker and Landauer, it would mean that decoherence has enough time in the latter case to freeze the position of the model particle. The particle then obeys essentially a classical equation of motion with an almost instantaneous friction, so that the tunnel effect becomes much reduced (Persson and Baratoff, 1988). It would be interesting to see whether this point of view agrees with a history approach, but this has not yet been done.

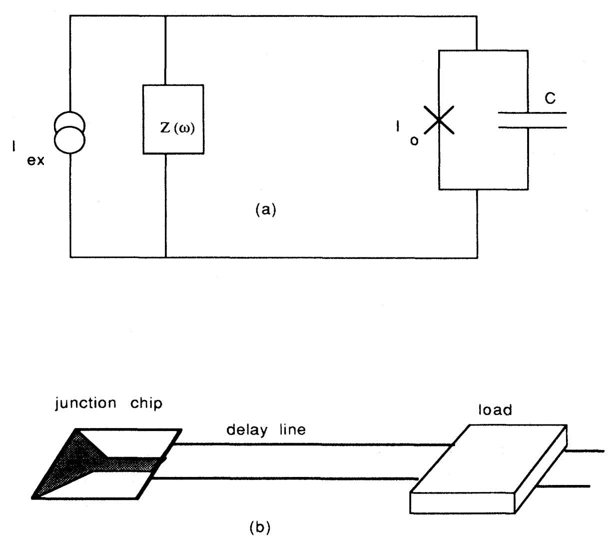

FIG. 8. A device realizing a tunnel effect: (a) The equivalent electric circuit; (b) schematic view of the actual device.

V. OBJECTIVITY, REALISM, AND SO ON

Since its beginnings, quantum mechanics has been the subject of intense philosophical discussions (Jammer, 1966, 1974). The interpretations presented here obviously answer some of these questions and they raise new ones, so that it may be convenient to sketch them briefly as some sort of a conclusion.

After some general comments about interpretation in Sec. V.A, Sec. V.B will briefly review the most obvious consequences of the theory, having to do essentially with its objective character. Section V.C is more speculative, centered on the connection between theory and reality, together with a related question concerning the actuality of facts.

A. Why an interpretation?

Since the purpose of physics is to arrange in order the facts belonging to the material world, one may wonder why it happens to end up with theories that are obscure enough to need an interpretation. Should not the facts speak for themselves?

The reason seems to lie in the potentialities that are offered by the scientific method, a description of which can be borrowed from Einstein (1952): Physics starts from empirical data but it goes through a conceptual stage in which the basic concepts are invented together with their ruling principles. They give rise to a mathematical construction representing the properties and the behavior of reality. Once this representation is obtained, its consequences can be worked out and compared with experiments.

When physics extended its range beyond ordinary phenomena by considering electromagnetic fields, or motions near the velocity of light, or the structure of atoms, there was no reason to expect that it would still conform to common sense.

Common sense could also be called phenomenology or classical physics. It has two main features, namely, intuition and plain language. Intuition consists in a representation of the world, both visual and active, which comes from everyday experience. It goes with perception and action. Ordinary language describes what exists, what happens, and how the facts are linked together in ordinary circumstances. This is done, of course, according to reason, i.e., with some due regard to logic. Common sense is undoubtedly as essential to physics as the conceptual and mathematical language of theory because it is the framework in which one must describe phenomena.

When science results in a theory that is apparently too far away from common sense, it is necessary to reconcile them, if only to link the theory with the empirical data lying at its origin or the experiments that are used to check it. This is where interpretation comes in.

As long as a theory can be expressed by using the language of common sense, there is no need for an interpretation. This was more or less the situation for classical physics. An interpretation becomes necessary when no pedagogical method can be found to reexpress the basic concepts of the theory within the categories of common sense. Then one must go the other way round by recasting these categories within the mathematical framework of the theory.

Although one may see *a posteriori* some hints of an interpretation when the electromagnetic field and statistical mechanics entered classical physics, the problem came to the forefront only with the advent of relativity. What an observer sees and measures had to be translated into some well defined mathematical notions belonging to the theory.

The role of the observer was, however, overstated. It provided a convenient pedagogical approach to an interpretation by making it easier to grasp. It resorted to intuition, and this is very helpful. Nevertheless, it should not mislead us into believing that observers have a compelling role in physics. One can also interpret relativity by translating its phenomenology directly into the conceptual language of the theory without ever mentioning observers but only physical systems.

Interpreting quantum mechanics is much more difficult than interpreting relativity, whether special or general. Not only is its mathematical language much farther from intuition, but it must also solve new problems that are not accounted for in the basic theory. This is because the theory is fundamentally probabilistic, whereas the phenomena one observes are endowed with certainty by observation and common sense: they are facts.

This is why an interpretation of quantum mechanics consists in reexpressing the phenomena and the data within the conceptual framework of the theory, as stated in the Introduction.

B. About objectivity

A theory is objective when it deals only with facts, whether they are observed or not, and not with our conscious knowledge of them. Niels Bohr (1958) made it plain that he considered quantum mechanics to be objective. For the sake of the argument, it will not be necessary to distinguish here in detail between Bohr's interpretation and the present one, which can be considered as its modern version, even if it differs in some respects.

It may be mentioned by the way that nothing can inspire a greater respect for Bohr's insights than rediscovering them with the help of a deductive method. One gets the feeling that he was able to keep in parallel all the necessary tenets by the sheer power of thought, a feat that would be broken by the linear rendering of a discourse. If this were true, it might explain why Bohr never wrote more than hints and landmarks and why he gave no complete explanation. It might also suggest that many later accounts of his views, essentially glosses over some of his quotations, were in fact contrary to his spirit (excepting, of course, direct witnessing from close collaborators). It may be true, as some people say, that everything is in Bohr, but this has been a matter for hermeneutics, with the endless disputes any scripture will lead to. It may also happen that he guessed the right answers, but the pedagogical means and the necessary technique details were not yet available to him. Science cannot, however, proceed by quotations, however elevated the source. It proceeds by elucidation, so that the feats of genius can become ordinary learning for beginners.

Rather than making long comments, it may now be simpler to list a few theses following more or less obviously from the previous parts of this review. They will also provide a clearcut summary. Although they follow closely the logical formulation, analogous statements might probably be reached by using Gell-Mann and Hartle's approach:

(1) Quantum mechanics is universal: Leaving aside some questions having to do with the structure of space and time, one can rely at present upon a unique set of physical laws, which is quantum mechanics. One does not need to assume anything special about the kind of physics taking place at a macroscopic level. All physical objects, whether microscopic or macroscopic, are completely subject to quantum mechanics.

(2) Quantum mechanics has two main structures. Within a unique mathematical framework, it can be thought of as consisting of two categories of laws and results running first in parallel and then intimately related, one being concerned with dynamics and the other with logic. Neither gives a complete formulation of physics without the other.

(3) Whatever is said about something physical, the statements in language can be replaced in principle by some mathematical objects, and the reasoning can be proved like an equation. The logical structure of quantum mechanics can be completely formulated in terms of mathematical entities, and its use can be reduced to mathematical equations. Its range is wide enough to cover in principle all that physics can reach.

(4) The whole interpretation of physics can be based upon a unique logical axiom stating how one can describe a physical system and what kind of reasoning one can make about it. This axiom is exclusive: any sentence that does not satisfy these conditions is deemed to be meaningless.

(5) Phenomena pre-exist in the theory. This means that the theory predicts the existence (in the mathematical sense of this word) of some mathematical objects, which are able to express the usual properties of phenomena as they exist (in reality). These properties can be described by educated common sense, i.e., by classical physics, as well as by quantum mechanics.

(6) The Moon is not fuzzy: Phenomena are clearly separated and, as alternative possibilities, they obey the conventional calculus of probability.

(7) One cannot beat phenomena: No measurement, however, costly in time and equipment, or whether it is just dreamed of as being possible, will ever deny the separation of phenomena, as long as the principles of this measurement are consistent with the basic laws of physics including relativity.

(8) Classical determinism is a direct consequence of quantum mechanics, despite the latter's probabilistic character. This determinism is, however, approximate, though generally with very small errors in probability. It also means that the existence of reliable records and memories is compatible with quantum mechanics.

(9) Aristotle is back again: Common sense logic, when applied to the macroscopic world, is consistent and it is useful in most cases, with a very good approximation. Its range of validity, however, is not universal and it is explicitly limited by the theory.

(10) One can think in one's own way. The logical structure of the theory allows for a wide number of different, so-called complementary, consistent logics. This multiplicity cannot, however, be responsible for any internal contradiction.

(11) One can give an explicit criterion for truth in physics. Once actual facts are taken to be true, a few other properties can also be said to be true according to well defined prescriptions. The most remarkable examples of true properties are the results of an individual measurement and the past facts, as they can be logically reconstructed from their present records or traces.

(12) There is no Einstein-Podolsky-Rosen paradox. The famous difficulty exhibited by these authors came from what turns out now to be a confusion between the true properties of a system and the so-called reliable properties, which are not self-contradictory as such but involve an arbitrary choice of logic outside which they cannot even be stated.

(13) As far as true properties are concerned, quantum mechanics is separable. This means that no true property of a system can be modified by an action that is performed upon another isolated system far away.

(14) There are many propositions that can be formulated within a consistent logical framework but to which one cannot assign a truth value (as being true or false). They cannot be verified or falsified by experiment, whatever the experiment. This implies that most observables cannot enter propositions within the range of truth. They can be used only for purposes of discussion, with no truthful meaning.

(15) Some very small probabilities, though theoretically meaningful, have no empirical meaning because they are below the confidence probability threshold for their factual measurement.

(16) There is a logical direction of time. It coincides with the direction in thermodynamics.

(17) An observer is only a part of the universe with no privileged role as compared with any other object that is able to detect a fact and to treat information.

(18) Perception is a special case of a quantum measurement.

These theses might warrant various comments but the arguments for most of them have already been given in some detail, so that it will be better to avoid undue repetition. The last one is the outcome of a rather lengthy exercise, which can be summarized as follows: When an object is seen, one must take into account the photons in the light allowing it to be seen As shown by Joos and Zeh (1985), the geometric characteristics of the object, its position and orientation, become separated, i.e., they become in a very short time alternative distinct possibilities as a result of decoherence. Only one of them can be realized, because the projectors describing two such different facts are exclusive. The light scattered by the object contains, in its emission pattern, a huge amount of information about this unique shape and position of the object, which can be expressed in terms of geometrical optics. The pigments in the retina are receptors for these scattered photons, and this is essentially a quantum measurement. The many data registered by various cells contain a large part of the information contained in the emission amplitude. Two eyes allow a reconstruction for the part of the surface of the object emitting towards the eyes. The details of this analysis are not very instructive, but the basic idea is quite simple. Analogous considerations can probably be made for the perception of sound and for touch. Their main interest is to show that a classical measurement is a special kind of quantum measurement.

Finally, no special comment is needed to advocate the objectivity of this interpretation. It has not been formally proved to be complete and consistent in the sense given in the introduction but, though below the level one requires in formal logic, the arguments that have been given make this more or less obvious.

Some other versions of an interpretation, if not necessarily inconsistent, can be considered as unnecessary, and they may better be avoided for philosophical reasons. The so-called standard interpretation (Von Neumann, 1932; London and Bauer, 1939; Wigner, 1963) attributes to consciousness the role of breaking the linear superpositions issuing from a measurement. This looks more than ever troublesome at a time when most experiments leave to consciousness only the modest role of reading the listings of a computer. Everett's conception of parallel universes also seems to answer a somewhat empty question (Everett, 1957; De Witt and Graham, 1973), though it is considered as a possible framework by Gell-Mann and Hartle. It assumes the simultaneous occurrence of several versions of the universe, each time a linear superposition is broken by decoherence.

There is nothing against envisioning the future evolution of the universe as alternative histories or versions, all of them being possible with definite probabilities. Because of decoherence, this is nothing but ordinary probability calculus. To say with Everett that something might remain, though inaccessible, of past unrealized possibilities is quite different. Such a point of view can be criticized on two grounds. As a matter of fact, most branchings of the universe are of a classical chaotic nature. Should one say that two universes are generated and separated from each other when a turbulent brook pushes a pebble on the right rather than on the left? Or should one say that this manifestation of chaos is a quantum effect hidden in the quantum origin of the motion uncertainties that are amplified by chaos?

A somewhat more cogent argument comes from logic: why not start the description of the universe from now rather than from a nebulous origin? There are at present a vast number of facts, some of them explored and many remaining unknown. When one starts from them and uses logical retrodiction, with the help of determinism, many past facts can be recovered. This is how science proceeds, and why should science rely upon something else it cannot get at directly? From the uniqueness of present facts results the uniqueness of many past facts, and of all of them in idealized models (Omnès, 1990), whereas the future must remain potential. This essential distinction between the characters of past and future is not only plain evidence, it also preexists in the logical theory of facts, so that, even there, interpretation regains common sense. For analogous considerations, see Haag (1990).

Of course, the basic question to which the Everett description is addressed is the reconciliation between the uniqueness of actual facts and the multiplicity of potential phenomena in the case of a quantum measurement. This question will have to be considered anyway, but parallel universes look a bit too much like what happened in the world where Caesar was born king of Persia; if one can avoid them, this will be for the best.

C. Realism and actuality

A basic tenet of realism is that there exists something, reality, that does not depend upon human consciousness (see, for instance, d'Espagnat, 1976, 1983, 1984, 1989). Except for the advocates of the standard interpretation of quantum mechanics and some idealist philosophers, most physicists accept it. A much stronger statement of realism asserts that reality is accessible to knowledge as a matter of principle. This is what d'Espagnat calls strong realism, while offering other versions as weak realism or empirical realism.

It is sometimes said that the main alternative to realism is positivism. This other approach to epistemology, as advocated by Stuart Mill, distinguishes some aspects of reality as not accessible to knowledge and others as knowable, with knowledge of the latter being obtained by agreement between sensible men. No physicist, at least before the advent of quantum mechanics, was a hard-boiled positivist.

Most classical physicists had a third position, which in one version or another stated that physics arrives at some principles giving an economical expression of empirical laws. It does not provide a complete knowledge of reality or a full explanation of it but, in most cases, a representation of it in terms of some mathematical models. This interpretation of physics as keeping all appearances is best analyzed by Duhem (1908, 1914), who quotes in its favor

Pascal, Newton, Ampère, Kelvin, Maxwell, Mach, and tens of others, as compared with only two clearcut realists, Descartes and Kepler, and two not-so-clear ones, Copernicus and Galileo. It would therefore be an error to consider realism as typical of classical physics, or at least this view ought to be strongly qualified.

Furthermore, the formal definition of realism is rather unclear, since it fails to specify the principles allowing reality to be known, if reality is not metaphysical, and the criterion for truth that applies to a knowledge of reality. The interpretation of quantum mechanics presented here is obviously not strongly realist, since it leaves some conceivable properties of a physical system as inaccessible to knowledge, restricting knowledge to what satisfies a criterion for truth. This leaves it, however, in good company, and perhaps one should not ask of quantum mechanics what Newton did not ask of classical dynamics. In less controversial terms, it offers a representation of reality in terms of a mathematical construction. It allows a notion of truth in which all objective facts can fit, and there is no reason to expect more from a theory.

The most startling (even if not quite new) feature of the present interpretation is the status it gives to empirical knowledge. Given a fact, its origin and its consequences are not absolutely certain, and there always remains a tiny uncertainty in them. Their account by common sense does not strictly obey the rules of logic, and even these have a slight risk of error. Of course, in the rare event that erratic quantum effects happen, one rather attributes them to an unknown ordinary cause and they go unnoticed. They cannot be reproduced and therefore they escape direct knowledge.

This fringe of uncertainty has its rewards: to think that physics has been able to recognize such limitations, to work within them by well-made experiments, and to express them economically by general principles is something remarkable. To find that, most probably, common sense rests upon more basic *raisons d'être* and that its own limitations can be understood, is worth some philosophical consideration. Conversely, it gives a warning against incautious philosophical principles, which after all are most often ennobled forms of common sense.

We shall not enter here into this kind of comment but rather reconsider the representation of reality by the mathematical model of physics, the main reason for that being to shed some light upon the difficult problem of actual facts.

Despite its constraints by experiments, a theory is essentially a purely mathematical and logical entity. How and why it can adequately represent reality is the prime mystery of science. Thus any consideration of physical knowledge cannot leave aside the nature of mathematics.

Realism is not restricted to physics. There is also a mathematical realism according to which a mathematician more or less freely invents, but, when he really succeeds, this is a discovery. He has encountered something having its own existence, its own reality. The wonderful coherence of modern mathematics and its harmonious architecture are often taken as an argument in favor of this point of view. One may also add its efficiency in physics. Whether this other kind of reality proceeds from the one we touch and see is far from clear, and it may be better to give it another name for clarity: Philosophy calls it *logos*.

As far as one can see, physics (together with other sciences) is a representation of reality, and mathematics might well be a representation of a logos. Both are secreted by man, growing, changing through exploration and confrontation, but they represent something outside man. Now, the representation occurring in physics demands for its construction the representation built up by mathematics. One may therefore wonder whether this necessity is also representative of a more elevated correspondence between reality and logos. It might be that science is possible because reality has order, a notion expressing that it is strongly connected with logos.

Most physicists, if not perhaps all of them, believe that there is order in reality; it is expressed by the laws of physics in their mathematical form. But if one assumes, with most philosophers after Leibniz, that mathematics is an arbitrary game invented by man, one is back to a science and even to an order in reality that are also human. This is why the present author advocates what might be called a *total realism,* where both the physical reality and the logos exist by themselves.

Total realism does not suffer from the pitfalls of ordinary realism. It gives meaning to ordinary language as being a special representation of reality; it is explicit about the meaning it gives to knowledge, since it has criteria for truth, and also about what it means when stating something "in principle." It does not need to assert what should be *a priori* the extent of knowledge, and it has no distinctions such as that between "strong realism" and "weak realism." Nevertheless, it does not reduce science to an "agreement between the observers" nor theory to the too modest goal of "keeping all appearances."

Whatever it may be, these remarks hint at the fact that physical realism remains incomplete. It does not state what kind of knowledge it refers to, and it even does not know what knowledge is, as long as it avoids stating what it assumes about mathematics and logic.

This apparent digression was meant to prepare the ground for the problem of interpretation and probably the only essential problem: what is an actual fact? Quantum theory envisions possible phenomena, for instance, that one detector will register rather than another. These are potential phenomena, about which the theory can give only some *a priori* probabilities. Now, all of a sudden, one of them becomes real and the others fade into oblivion. How is this?

It should be stressed that *actuality, whether in a quantum measurement or in plain classical situations, is the only point where theory and reality come into contact with each other.* All the rest is a matter of relations between phenomena and observations of their frequencies, which are obtained entirely within the framework of theory if one includes in theory its account of common sense. This is also the only point for which theory does not provide an explanation, nor a mechanism, nor a cause for what is observed.

Perhaps the best way to see what it is all about is to consider what would happen if a theory were able to offer a detailed mechanism for actualization. This is, after all, what the advocates of hidden variables are asking for. It would mean that everything is deeply determined. The evolution of the universe would be nothing but a long reading of its initial state. Moreover, nothing would distinguish reality from theory, the latter being an exact mathematical copy of the former. More properly, nothing would distinguish reality from logos, the time-changing from the timeless. Time itself would be an illusion, just a convenient ordering index in the theory.

So, one falls back upon a conception of physics going back to Bohr, who stated that reduction of the wave packet is a law of physics differing in its nature from all other laws. This idea was made somewhat obscure by its reference to reduction of the wave packet, and it can now be elucidated: physics is not a complete explanation of reality, which would be its insane reduction to pure unchanging mathematics. It is a *representation* of reality that does not cross the threshold of actuality.

To make this idea clearer, one might follow Bohr by stating it as a law of physics differing from all the other laws, because it is the only one referring directly to reality. It would be that reality is always unique. It evolves in time, in such a way that the different facts originating from identical initial phenomenological conditions show frequencies in accordance with the theoretical probabilities.

Finally, it is wonderful how quantum mechanics succeeds in giving such a precise and, as of now, such an encompassing description of reality, while avoiding the risk of an overdeterministic insanity. It does it because it is probabilistic in an essential way. This is not an accident, nor a blemish to be cured, since probability was found to be an intrinsic building block of logic long before reappearing as an expression of ignorance, as empirical probabilities. Moreover, and this is peculiar to quantum mechanics, theory ceases to be identical with reality at their ultimate encounter, precisely when potentiality becomes actuality. This is why one may legitimately consider that the inability of quantum mechanics to account for actuality is not a problem nor a flaw, but the best mark of its unprecedented success.

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