Electroweak measurements and the top quark

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The remarkable agreement of electroweak measurements with theory places limits on the masses of the top quark and the W boson. It is shown how these limits arise and what constraints each set of measurements provides within the context of the theory.

CONTENTS

I. INTRODUCTION

The successful unification of the electromagnetic and weak interactions (Glashow, 1961; Weinberg, 1967; Salam, 1968) is one of the great achievements of elementary-particle physics in the second half of the 20th century. This *electroweak* theory has been able to describe a wide variety of data in terms of a small number of parameters. Its successful predictions have included charge-preserving interactions of neutrinos (Hasert *et al.*, 1973, 1974; Benvenuti *et al.*, 1974), new processes involving parity violation (Prescott *et al.*, 1978, 1979; Noecker *et al.*, 1988, and references therein), and the existence of the carriers of the weak force, the W and Z bosons (Arnison *et al.*, 1983a, 1983b, 1983c; Bagnaia *et al.*, 1983; Banner *et al.*, 1984).

The initial successes of the lowest-order theory have now been joined by a host of precision tests that are sensitive to the next order in perturbation theory (so-called *radiative corrections)* and that therefore can shed light on unexplored physics.

It is our aim to describe how this sensitivity arises and to point out its main implications for the masses of the top quark, Higgs boson, and W boson. The top and Higgs are yet to be found, while improved measurements of the W-boson mass are a key feature of programs at the Fermilab Tevatron $\bar{p}p$ Collider and the e^+e^- LEP Collider at CERN. If the top quark is discovered in the near future, one will be able to see at a glance the implications of measurements of its mass for a number of other electroweak observables.

This article is intended for an audience of nonspecialists as well as for those working in the field who wish a quick overview of the subject. Hence we shall necessarily oversimplify some points, while hoping to remain true to the spirit of the theory. Several comprehensive analyses of electroweak experiments have appeared recently (Altarelli *et al.*, 1992; del Aguila *et al.*, 1992; Kennedy, 1992; Langacker, Luo, and Mann, 1992; Peskin and Takeuchi, 1992). These analyses are particularly tailored to exhibiting the sensitivity of experiments to various types of "new physics" beyond the electroweak theory. Our aim here is more modest; we wish to show *within the context of the minimal electroweak theory* which experiments probe similar ranges of parameters and which experiments provide complementary information. At the same time, some comments at the end are intended for practitioners in the field, in the event that the long-sought top quark still eludes us by the time this article is in print.

Section II gives a brief overview of the electroweak theory, with an eye to quickly displaying the sensitivity of the theory to the top-quark and Higgs-boson masses m_t and M_H. We do not spend much time on the lowest-order theory; for an excellent discussion, consult Quigg (1983). We present all constraints among m_t, M_H, and the W-boson mass M_W as curves in the (m_t, M_W) plane for various values of M_H. We then turn in Sec. III to a discussion of the various electroweak observables that indirectly shed light on m_t, M_W, and M_H. We consider both the effects of a global fit and the information provided by each set of observables. Our conclusions are given in Sec. IV.

II. ELECTROWEAK THEORY AND RADIATIVE CORRECTIONS

A. Lowest order

The theory of beta decay elaborated by Fermi (1934a, 1934b) involved an interaction among four fermions characterized by a coupling constant G_F with dimensions of $(\text{mass})^{-2}$. The lifetime of the muon gives us a very precise value $G_F = 1.166\,37 \pm 0.000\,02 \times 10^{-5}$ GeV^{-2}. (Here and subsequently we shall take units in which $\hbar = c = 1$, so that mass, momentum, energy, inverse length, and inverse time are all equivalent.) The fermions may be grouped in pairs; one pair exchanges charge with the other in the Fermi interaction.

Superficially the Fermi interaction looks very different from the electromagnetic one, in which particles affect one another by exchange of a photon. (The Coulomb field is attributed here to the exchange of virtual longitudinal and scalar photons.) The electromagnetic scattering amplitude for particles of charges Q_1 and Q_2 may be written in the momentum representation as $Q_1 Q_2/q^2$, where q is the momentum transfer (a four-vector), and q^2 is its invariant square. In the units we have adopted, charges are dimensionless coupling constants.

In order to draw a closer analogy between the weak and electromagnetic interactions, it was proposed by many people, including Yukawa (1935), Klein (1939), and Schwinger (1957), that the Fermi interaction also was associated with particle exchange. The four-fermion interaction for charge-changing weak interactions was then to be understood as the low-energy limit of a scattering amplitude involving the exchange of a charged particle, the W. This scattering amplitude is proportional to $g^2/(M_W^2-q^2)$, where g is a new dimensionless coupling constant, analogous to (but independent of) the electric charge. Whereas particles may have different electric charges, the value of g for any weakly interacting particle is found experimentally to be universal. In the limit $q^2 \ll M_W^2$, the scattering amplitude then reduces to a constant proportional to g^2/M_W^2, which has just the same dimensions as the Fermi coupling constant G_F.

In the W boson theory of weak interactions we then identify

$$\frac{G_F}{\sqrt{2}}=\frac{g^2}{8M_W^2} , \tag{2.1}$$

where the factors of $\sqrt{2}$ and 8 come from the way G_F and g were introduced into the theory (see, e.g., Quigg, 1983).

Equation (2.1) does not look as though it represents much progress. It replaces one known quantity, G_F, with two unknown ones, g and M_W. Moreover, it is not enough to introduce just charged W bosons if one wishes to have a self-consistent *gauge theory,* i.e., one in which the interactions arise from gauge invariance, as an electromagnetism. One must also have a neutral W. The photon cannot be identified with this particle. While the charged W bosons couple only to matter spinning left-handedly, the photon couples equally to left-handed and right-handed particles. The simplest solution to this problem, proposed by Glashow (1961), involved the introduction of another neutral boson B in addition to the W^0. The constant describing the coupling of B to matter is called g'.

The photon can be identified as a linear combination of the W^0 and the B. We shall denote its field by the symbol A. There will also be another particle, represented by a linear combination orthogonal to the first, which we may call Z. Thus we may write

$$A=B\cos\theta+W^0\sin\theta , \quad Z=-B\sin\theta+W^0\cos\theta . \tag{2.2}$$

The *weak mixing angle* θ turns out to be related to the ratio of g' to g: $\tan\theta=g'/g$. Moreover, g and g' are related to the electric charge by

$$\frac{1}{e^2}=\frac{1}{g^2}+\frac{1}{g'^2} . \tag{2.3}$$

The simplicity of the relation (2.3) is remarkable. It comes about when one expresses the interactions of W^0 and B with matter in terms of those of the physical photon and Z fields. One may also write

$$e=g\sin\theta=g'\cos\theta , \tag{2.4}$$

relating g and g' to the equivalent quantities e and θ.

As a result of the possibility of Z exchange, the electroweak theory predicts the presence of four-fermion charge-*preserving* interactions. For example, in such interactions, a neutrino can interact with matter without changing its charge. The discovery of such interactions (Hasert *et al.*, 1973, 1974; Benvenuti *et al.*, 1974) was the first piece of experimental evidence in favor of this attempt at unifying weak and electromagnetic interactions. Furthermore, the strengths of such interactions (especially when comparing neutrino-induced and antineutrino-induced processes) gave the first indication of the value of θ. This, through the relation (2.4) between e and g, led to a value of g and hence, through (2.1), to a prediction for the value of M_W:

$$M_W^2=\frac{\pi\alpha}{\sqrt{2}G_F\sin^2\theta} , \tag{2.5}$$

with $\alpha\equiv e^2/4\pi$.

In the low-energy limit, the exchange of the Z gives rise to an interaction with the same overall strength as the ordinary weak interaction (with specific differences for various types of quarks and leptons depending on electric charge and spin projection). Since the Z is made partly from the W and partly from the B [Eq. (2.2)], both g and g' contribute to the couplings of the Z to matter. The relation analogous to Eq. (2.1) is

$$\frac{G_F}{\sqrt{2}}=\frac{g^2+g'^2}{8M_Z^2} . \tag{2.6}$$

Using (2.4), one has

$$M_Z^2=\frac{\pi\alpha}{\sqrt{2}G_F\sin^2\theta\cos^2\theta} . \tag{2.7}$$

The relation $M_Z=M_W/\cos\theta$ holds in the lowest-order version of the theory.

B. Photon vacuum polarization

An important higher-order effect in electromagnetism [Fig. 1(a)] is the polarization of the vacuum through the creation of virtual fermion-antifermion pairs by the photon. The presence of these fermions leads to a

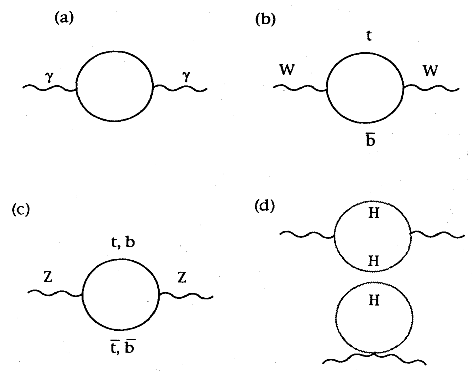

FIG. 1. Vacuum polarization graphs for gauge bosons: (a) photon vacuum polarization; (b) W producing a $t\bar{b}$ pair; (c) Z producing $t\bar{t}$ and $b\bar{b}$ pairs; (d) W and Z loop diagrams involving Higgs bosons.

strengthening of the effective electric charge at short distances or large momentum transfers. The appropriate value of α to use in Eqs. (2.5) and (2.7) then turns out to be not 1/137 (the low-energy value), but $\alpha(M_Z^2)\simeq 1/128$, where we have exhibited the square of the momentum scale at which α is evaluated.

The Z mass has been measured very precisely at LEP (LEP, 1991): $M_Z=91.175\pm0.021$ GeV$/c^2$. Inserting this value into Eq. (2.7) along with $\alpha(M_Z^2)$, we obtain a value of θ. This may then be used to infer a value of M_W via Eq. (2.5). This value is very close to the measured one:

$$M_W=(\text{measured})\begin{cases}79.92\pm0.39\ \text{GeV}/c^2 & (\text{CDF, 1991}) \quad (2.8\text{a})\\ 80.35\pm0.37\ \text{GeV}/c^2 & (\text{UA2, 1992}) \quad (2.8\text{b})\\ 80.14\pm0.27\ \text{GeV}/c^2 & (\text{average})\ . \quad (2.8\text{c})\end{cases}$$

In Eq. (2.8b) we have recalibrated the quoted value in terms of the known Z mass.

C. Top quark

Just as in the case of the photon, the W and Z bosons can create fermion-antifermion pairs, with appreciable effects. For the photon, gauge invariance prohibits contributions quadratic in fermion masses; but for the W and Z, no such prohibition applies, and heavy top quarks can lead to a substantial correction of the lowest-order results. The key contributions stem from the diagrams in Figs. 1(b) and 1(c). They lead to a modification of Eq. (2.6) for neutral-current exchanges:

$$\frac{G_F}{\sqrt{2}}\rho=\frac{g^2+g'^2}{8M_Z^2}\ , \tag{2.9}$$

where

$$\rho\simeq 1+\frac{3G_F m_t^2}{8\pi^2\sqrt{2}}\ . \tag{2.10}$$

Equation (2.7) is now approximately

$$M_Z^2=\frac{\pi\alpha}{\sqrt{2}G_F\rho\sin^2\theta\cos^2\theta}\ . \tag{2.11}$$

We have omitted some small terms logarithmic in m_t. Now, a precise measurement of M_Z specifies θ through (2.10) and (2.11) only if m_t is known; so we have $\theta=\theta(m_t)$ and hence, via (2.5), $M_W=M_W(m_t)$. The prediction for M_W acquires a dependence on the top-quark mass.

The factor of ρ in (2.9) will multiply every neutral-current four-fermion amplitude in the electroweak theory. Thus, for example, cross sections for charge-preserving interactions of neutrinos with matter will be proportional to ρ^2, while parity-violating neutral-current amplitudes will be proportional to ρ. Partial decay widths of the Z, since they involve the combination $g^2+g'^2$, will be proportional to ρ.

D. Higgs boson

In unifying the electromagnetic and weak interactions, one has to explain why the photon remains massless but the W and Z are so heavy. In other words, what breaks the *electroweak symmetry*?

A photon has two polarization states (which we can call left circular and right circular, or horizontal and vertical). So does *any* massless spinning particle. These polarization states are transverse to the direction of the particle's motion. A theory of a massless photon and massless W's and Z's turns out to be completely tractable to all orders in perturbation theory and is an example of a general class of theories studied by Yang and Mills (1954). Putting in W and Z masses "by hand" destroys the tractability of the theory; quantities calculated to higher orders in perturbation theory turn out to be infinite.

Now, a massive spin-1 particle (our eventual goal in the theory for the W and Z) must have *three* polarization states. The third corresponds to polarization *along* the direction of motion, or *longitudinal* polarization. A spin-1 particle with this polarization has many features in common with a spinless particle (whose field is analogous to a sound wave). Higgs (1964a, 1964b) discovered that a massless, spinless particle could act as a suitable substitute for the longitudinal polarization state of the spin-1 meson. Thus it was possible to combine a massless, spinless Higgs particle (with one polarization state) and a massless W or Z (with two polarization states) into a massive W or Z (with three polarization states).

It was conjectured by Weinberg (1967) and Salam (1968; see also Salam and Ward, 1964) and proved by 't Hooft (1971a, 1971b) that the above *Higgs mechanism* led to a self-consistent electroweak theory. At this point both the theory and the Higgs mechanism began to attract serious attention.

The simplest set of Higgs particles that provides a satisfactory description of W and Z masses turns out to have one field for each of W^+, W^-, and Z, but necessarily one neutral particle left over. It is this particle that will be called *the* Higgs boson, and we shall refer to it as H.

The need for a particle with the properties of H can be seen by considering the scattering of W's and Z's off one another (Lee *et al.*, 1977). If such a particle did not exist (with mass below about 2 TeV/c^2), there would be more scatterings at energies above about 1 TeV than there were incident particles!

All we know at present is a lower bound on the Higgs boson mass: $M_H \gtrsim 60$ GeV/c^2 (from searches at LEP). For present purposes we shall consider the effects of a (hypothetical) Higgs boson with a mass somewhere between 50 and 1000 GeV/c^2. We shall be concerned with the radiative corrections such a boson introduces, as examples of the sorts of effects that could arise from particles that have yet to be discovered directly. Indeed, we shall see that precise electroweak tests can shed some light on the properties of a hypothetical Higgs boson.

The W and Z would be affected by virtual Higgs-boson states, as shown in Figs. 1(d). Small corrections, logarithmic in M_H, would appear on the right-hand sides of Eqs. (2.1), (2.5), (2.9), and (2.11), and a more important term,

$$\Delta\rho = -\frac{3}{8\pi\cos^2\theta}\ln\frac{M_H}{M_W} , \tag{2.12}$$

would be added to the right-hand side of (2.10). Since $\rho=\rho(m_t,M_H)$, a measurement of M_Z implies $\theta=\theta(m_t,M_H)$ via Eq. (2.11), so that $M_W=M_W(m_t,M_H)$ through (2.5).

No terms quadratic in the Higgs-boson mass appear in $\Delta\rho$. This is a special feature of the single-Higgs-doublet mode; the two-doublet model (Denner *et al.*, 1990) is discussed briefly at the end of Sec. III.

E. Smaller effects

The weak charge-changing and neutral-current interactions are probed under a number of different conditions, corresponding to different values of momentum transfer. For example, muon decay occurs over a range of momentum transfers, all of which are small with respect to M_W, while the decay of a Z boson into fermion-antifermion pairs imparts a momentum of approximately $M_Z/2$ to each member of the pair. Now, the right-hand sides of Eqs. (2.1) and (2.9) contain quantities (masses and coupling constants) that vary fairly rapidly with momentum transfer. The quotients of these quantities, however, vary much less rapidly, as was initially pointed out by Veltman (1977a, 1977b). Thus it is not a bad approximation to neglect such variation altogether. Small corrections to (2.1) and (2.9), logarithmic in m_t and M_H, can be taken into account, and we shall do so. We shall take account of what have to come to be known as the "oblique" corrections, which occur in the photon, W, and Z propagators as illustrated in Fig. 1 (Lynn *et al.*, 1986; Kennedy *et al.*, 1989; Kennedy and Lynn, 1989; Peskin and Takeuchi, 1990, 1992; Golden and Randall, 1991). The effects of other (smaller) "direct" radiative corrections are discussed by some of these authors and by others such as Degrassi *et al.* (1991), Hioki (1991), and Hollik (1990). These papers should be consulted for actual values of observables. The reader interested only in our results and not in details may skip directly to Sec II.F at this point.

We may then replace (2.1) and (2.9) [for more extensive discussions see, e.g., Rosner (1991, 1992)] by

$$\frac{G_F}{\sqrt{2}} = (1+\Delta Z_W)\frac{g^2}{8M_W^2} \tag{2.13}$$

and

$$\frac{G_F\rho}{\sqrt{2}} = (1+\Delta Z_Z)\frac{g^2+g'^2}{8M_Z^2} , \tag{2.14}$$

where ΔZ_W and ΔZ_Z represent the effects of variation with momentum transfer mentioned above (Peskin and Takeuchi, 1990, 1992; Golden and Randall, 1991). They may be expressed in terms of coefficients S_W and S_Z of order 1, defined by Peskin and Takeuchi (1990) as

$$\Delta Z_W = \frac{\alpha S_W}{4\sin^2\theta} , \quad \Delta Z_Z = \frac{\alpha S_Z}{4\sin^2\theta\cos^2\theta} . \tag{2.15}$$

Similarly, the ρ parameter discussed earlier can be related to a parameter T of order 1 by setting

$$\rho = 1+\alpha T . \tag{2.16}$$

The parameters S_W, S_Z, and T allow one to express a wide variety of electroweak observables in terms of quantities sensitive to new physics (Marciano and Rosner, 1990; Kennedy and Langacker, 1990, 1991; Peskin and Takeuchi, 1990, 1992; Altarelli and Barbieri, 1991; Golden and Randall, 1991; Rosner, 1991, 1992; Altarelli *et al.*, 1992).

It is convenient to express the effects of new physics in terms of *deviations* from some nominal values of top-quark and Higgs-boson masses, so that one may utilize precise calculations for particular values of these parameters and then expand about them. Accordingly, we choose $m_t = 140$ GeV/c^2 and $M_H = 100$ GeV/c^2. We then have

$$T \simeq \frac{3}{16\pi\sin^2\theta}\left[\frac{m_t^2-(140\ \mathrm{GeV})^2}{M_W^2}\right] - \frac{3}{8\pi\cos^2\theta}\ln\frac{M_H}{100\ \mathrm{GeV}} , \tag{2.17}$$

while contributions of Higgs bosons and of possible new fermions U and D with electromagnetic charges Q_U and Q_D to S_W and S_Z are (Kennedy and Langacker, 1990)

$$S_W = \frac{1}{6\pi}\left[\ln\frac{M_H}{100\,\text{GeV}} + \sum N_C\left[1 - 4\bar{Q}\ln\frac{m_U}{m_D}\right]\right], \tag{2.18}$$

$$S_Z = \frac{1}{6\pi}\left[\ln\frac{M_H}{100\,\text{GeV}} + \sum N_C\left[1 - 4Q_D\ln\frac{m_U}{m_D}\right]\right]. \tag{2.19}$$

The equations for S_W and S_Z are written for doublets of fermions with N_C colors and $m_U \geq m_D >> m_Z$, while $\bar{Q} \equiv (Q_U + Q_D)/2$. The sums are taken over all doublets of new fermions. In the limit $m_U = m_D$, one has equal contributions to S_W and S_Z. For a single Higgs boson and a single heavy top quark, (2.18) and (2.19) become

$$S_W = \frac{1}{6\pi}\left[\ln\frac{M_H}{100\,\text{GeV}} - 2\ln\frac{m_t}{140\,\text{GeV}}\right], \tag{2.20}$$

$$S_Z = \frac{1}{6\pi}\left[\ln\frac{M_H}{100\,\text{GeV}} + 4\ln\frac{m_t}{140\,\text{GeV}}\right]. \tag{2.21}$$

We shall use these equations, together with Eqs. (2.13)–(2.17), to plot M_W as a function of m_t for various values of M_H.

F. M_W versus m_t plots

Combining the dependence of ρ on m_t and M_H with the small corrections to Eqs. (2.1) and (2.9) mentioned in Sec. II.E, we find the results shown in Fig. 2. Shown too, are the 1σ limits on M_W, based on Eq. (2.8c), and the 95% confidence-level lower limit on m_t from the CDF analysis (CDF, 1992): $m_t \geq 91$ GeV/c^2. For $M_H \leq 1000$ GeV/c^2, a top quark lighter than about 200 GeV/c^2 already is favored. We shall show how this conclusion is affected by other present and future electroweak data.

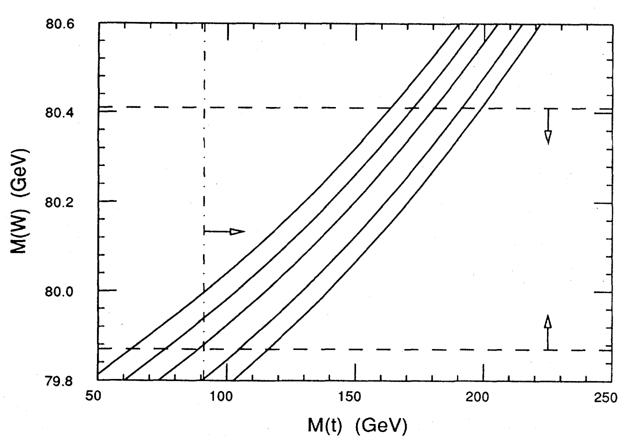

FIG. 2. Families of curves predicted in the standard electroweak theory with one Higgs doublet for the dependence of M_W on m_t. From left to right, the curves correspond to M_H=50, 100, 200, 500, and 1000 GeV/c^2. The vertical dot-dashed line corresponds to the 95% confidence-level lower limit on the top-quark mass obtained by CDF (1992), while the horizontal dashed lines correspond to the 1σ limits on M_W obtained from an average of the values measured by CDF (1991) and UA2 (1992).

III. ELECTROWEAK OBSERVABLES

A. A collection of observables

In Table I we present a set of electroweak observables, along with their values predicted by the electroweak theory with a single Higgs boson for $m_t = 140$ GeV/c^2 and $M_H = 100$ GeV/c^2. The agreement is remarkable. We shall briefly explain each of these quantities, returning to the impact of individual measurements in the following subsections.

In the lowest-order theory presented in Sec. II.A, there are several equivalent ways to express $x \equiv \sin^2\theta$ in terms of observable quantities. One can use Eq. (2.5) for M_W, Eq. (2.7) for M_Z, the ratio of the two [leading to $\sin^2\theta = 1 - (M_W^2/M_Z^2)$], or relations among coupling constants derived from measurements of the decay properties of the Z.

When radiative corrections are taken into account, the various definitions of x differ from one another. For example, we may denote the value of x derived from the precise measurement of the Z mass via Eq. (2.7) as x_0. Our discussion in Sec. II, in which successive radiative corrections are applied, remains valid when θ is defined in terms of coupling constants derived from the study of Z decays: $\tan^2\theta = g'/g$. Henceforth, we shall denote the corresponding value of $\sin^2\theta$ by the symbol $\bar{x}$. Strictly speaking, this definition is to be made at a particular value of q^2, which we take to be M_Z^2.

Many experiments, such as forward-backward asymmetries in the reaction $e^+e^- \to Z \to f\bar{f}$, where f denotes a fermion, probe $\bar{x}$ directly, aside from small corrections. Others, such as polarized electron-deuteron ($\vec{e}D$) and electron-carbon ($\vec{e}C$) scattering, and the ratio $\sigma(\nu_\mu e)/\sigma(\bar{\nu}_\mu e)$, probe coupling constants at lower values of q^2; but for the present purposes we neglect the variation with q^2 of coupling-constant ratios in these processes and use them also as sources of information about $\bar{x}$.

The nominal value of $\bar{x}$ is that which would lead to the observed Z mass, $M_Z = 91.175 \pm 0.021$ GeV/c^2, for $m_t = 140$ GeV/c^2, and $M_H = 100$ GeV/c^2. This value has been estimated (Marciano and Rosner, 1990) to be

$$\bar{x} \equiv x_0 = 0.2323 \pm 0.0002 \pm 0.0005\,, \tag{3.1}$$

where the first error comes from M_Z and the second from uncertainties in photon vacuum polarization effects. Equation (3.1) results from Eq. (2.9) with $\rho = 1$ when $\alpha(M_Z^2) = [128.48 \pm 0.18]^{-1}$. One can just think of this as the effective fine-structure constant for the present set of electroweak calculations; depending on different definitions, one will see variations of about ± 0.5 in $\alpha^{-1}(M_Z^2)$ in the literature. In obtaining Eq. (3.1), *all*

TABLE I. Electroweak observables incorporated into a fit to the standard electroweak theory.

Quantity	Reference	Experimental value	Nominal theory[a]	Expt. ÷ theory
$Q_W(\mathrm{Cs})$	b	-71.04 ± 1.81	-73.20	0.970 ± 0.025
M_W (GeV/c^2)	c	80.14 ± 0.27	80.20	0.999 ± 0.003
$3\Gamma(Z\to\nu\bar{\nu})$ (MeV)	d	496 ± 9	499	0.994 ± 0.018
$\Gamma(Z\to l^+l^-)$ (MeV)	d	83.2 ± 0.4	83.6	0.995 ± 0.005
$\Gamma(Z\to\mathrm{all})$ (MeV)	d	2487 ± 10	2488 ± 6	1.000 ± 0.005
$\bar{x}$(lept. asym.)	d	0.2327 ± 0.0022	0.2323	1.002 ± 0.010
$\bar{x}(q\bar{q}$ asym.)	d	0.2310 ± 0.0035	0.2323	0.994 ± 0.015
$\bar{x}(A_{\mathrm{FB}}^{(b)})$	d	0.2265 ± 0.0038	0.2323	0.975 ± 0.016
$\bar{x}(\vec{e}D)$	e	0.224 ± 0.020	0.2323	0.965 ± 0.086
$\bar{x}(\vec{e}C)$	f	0.20 ± 0.05	0.2323	0.86 ± 0.22
$\bar{x}[\sigma(\overset{(-)}{\nu}_\mu)]$	g	0.233 ± 0.014	0.2323	1.00 ± 0.06
R_ν	h	0.307 ± 0.004	0.310	0.990 ± 0.013
$R_{\bar{\nu}}$	h	0.387 ± 0.009	0.376	1.02 ± 0.02

[a]For $m_t=140\ \mathrm{GeV}/c^2$, $M_H=100\ \mathrm{GeV}/c^2$.
[b]Noecker *et al.*, 1988; Dzuba *et al.*, 1989; Blundell *et al.*, 1990, 1992.
[c]CDF, 1991; UA2, 1992.
[d]LEP, 1992. This analysis is based on 1989 and 1990 data. An analysis of 1991 data has been performed but was still preliminary at the time of writing.
[e]Prescott *et al.*, 1978, 1979.
[f]Souder *et al.*, 1990.
[g]Geiregat *et al.*, 1991.
[h]Bogert *et al.*, 1985; Allaby *et al.*, 1987; Blondel *et al.*, 1990; Reutens *et al.*, 1990.

relevant radiative corrections (not just the loop diagrams of Fig. 1) are taken into account. *Deviations* from this minimal value of $\bar{x}$ are then ascribed either to new physics or to deviations of m_t and M_H from their nominal values. We find, with the help of the above discussion, that

$$\bar{x}=x_0+\frac{\alpha}{1-2x_0}\left[\frac{1}{4}S_Z-x_0(1-x_0)T\right], \tag{3.2}$$

or

$$\bar{x}=x_0+(3.65\times10^{-3})S_Z-(2.61\times10^{-3})T\ . \tag{3.3}$$

Other electroweak observables are generally functions of ρ and $\bar{x}$. For example, the quantity Q_W, describing parity violation in an atom whose nucleus has Z protons and N neutrons, is

$$Q_W(Z,N)=\rho(Z-N-4Z\bar{x})\ , \tag{3.4}$$

aside from small radiative corrections. This quantity is linear in ρ, since it represents a weak amplitude, measured via interference with photon exchange. It was found (Marciano and Rosner, 1990; Sandars, 1990) that when one substitutes $\rho=1+\alpha T$ in Eq. (3.4) and takes account of Eq. (3.3), there is almost complete cancellation of the T dependence in Q_W for nuclei around the best-studied case, cesium.

The W mass is an implicit function of $\bar{x}$ and S_W through the relation (2.5). It is the only quantity that depends on S_W in Table I; all the others depend only on S_Z and T alone. Thus, in discussions where S_W, S_Z, and T are treated as free parameters, the value of the W mass is not implied by other electroweak measurements. In the present treatment, where all quantities are functions of m_t and M_H, the W mass is very closely tied to other electroweak measurements, as we shall see.

It is only in the lowest-order theory that one can write $\sin^2\theta=1-(M_W/M_Z)^2$. One sometimes sees this relation as a definition of θ, but we shall reserve θ for the mixing angle as defined in terms of coupling constants.

The partial decay width of the Z to a fermion-antifermion pair is

$$\Gamma(Z\to f\bar{f})=\frac{G_F M_Z^3\rho}{12\pi\sqrt{2}}F_{f\bar{f}}(\bar{x})\ , \tag{3.5a}$$

where (e.g.) $F_{\nu\bar{\nu}}=1$, $F_{e^+e^-}=[1+(1-4\bar{x})^2]/4$, and so on. The partial width to $\nu\bar{\nu}$ is sensitive just to ρ, while others involve a combination of ρ and $\bar{x}$. An alternate equation, utilizing the relation (2.14), is

$$\Gamma(Z\to f\bar{f})=\frac{\alpha(M_Z^2)M_Z}{24\bar{x}(1-\bar{x})}(1+\Delta Z_Z)F_{f\bar{f}}(\bar{x})\ . \tag{3.5b}$$

Equation (3.5b) depends not just on $\bar{x}$, but also on ΔZ_Z.

The ratios of neutrino and antineutrino neutral-current (NC) to charged-current (CC) cross sections, R_ν and $R_{\bar{\nu}}$, are predicted to be (Llewellyn Smith, 1983)

$$R_\nu\equiv\frac{\sigma_{\mathrm{NC}}(\nu N)}{\sigma_{\mathrm{CC}}(\nu N)}=\rho^2\left[\frac{1}{2}-\bar{x}+\frac{5}{9}\bar{x}^2(1+r)\right], \tag{3.6}$$

$$R_{\bar{\nu}}\equiv\frac{\sigma_{\mathrm{NC}}(\bar{\nu}N)}{\sigma_{\mathrm{CC}}(\bar{\nu}N)}=\rho^2\left[\frac{1}{2}-\bar{x}+\frac{5}{9}\bar{x}^2\left[1+\frac{1}{r}\right]\right], \tag{3.7}$$

where $r \equiv \sigma_{CC}(\bar{\nu}N)/\sigma_{CC}(\nu N)$. Equations (3.6) and (3.7) are proportional to ρ^2, since they involve neutral-current *cross sections* (squares of amplitudes). The quantity in square brackets in (3.6) is quite sensitive to $\bar{x}$, while that in (3.7) turns out to be almost independent of $\bar{x}$ for the experimental values $\bar{x} \simeq x_0 = 0.2323$, $r \simeq 0.4$. It turns out that the $\bar{x}$ and ρ dependences in (3.6) combine in such a way that R_ν actually depends on m_t and M_H in very much the same way as does M_W, as we shall see presently.

B. Global fit

With this introduction to the observables, we now present the results of a fit to the 13 quantities listed in Table I, where m_t and M_H are allowed to vary. We plot these results on a graph of m_t versus M_W in Fig. 3.

The contours of χ^2 are very flat with respect to m_t, while they are strongly dependent on M_W. They are not very different, indeed, if one omits the explicit M_W information from the fit. The errors on M_W from present direct measurements are comparable to those inferred (in the context of the electroweak theory) from other electroweak measurements such as Z properties and R_ν.

The flatness of the contours in Fig. 3 with respect to m_t implies that within the standard electroweak theory one has little information on m_t and M_H separately; they are highly correlated (see, e.g., Schaile, 1992). Thus a direct measurement of m_t will provide unique and valuable information that may be applied indirectly to constrain the Higgs-boson mass.

C. Improvements in the M_W and m_t measurements

It is hoped that with forseeable improvements in experimental accuracy, one may be able to measure M_W to ± 50 MeV/c^2 and m_t to ± 5 GeV/c^2. At Fermilab, such a precise measurement of M_W would require the proposed Main Injector (Holmes and Winstein, 1989), an upgrade project that would lead to an improvement in luminosity by a factor of 5. Without such increased luminosity, one could probably achieve $\Delta M_W = \pm 120$ MeV/c^2. At LEP, one might be able to attain an accuracy of $\Delta M_W \lesssim \pm 100$ MeV/c^2 in each individual experiment through the reaction $e^+e^- \rightarrow W^+W^-$ (Böhm and Hoogland, 1987), leading to $\Delta M_W \lesssim \pm 50$ MeV/c^2 if each of the four experiments' errors is dominated by statistical limitations.

The impact of measurements with $\Delta M_W = \pm 50$ MeV/c^2, $\Delta m_t = 5$ GeV/c^2 is shown in Fig. 4. The plotted point lying on the family of curves associated with various Higgs-boson masses indicates that one can begin to learn about the Higgs sector if these accuracies really are achieved. At the same time, there is much potential for uncovering new physics if the values of m_t and M_W lie outside the limits of the predictions (as indicated by the plotted point $\times$).

D. Values of $\sin^2\theta$

For a given value of m_t and M_H, one predicts not only M_W but also $\theta(m_t, M_H)$, as mentioned in Sec. II. Contours of $\sin^2\theta$ are shown on the (m_t, M_W) plane in Fig. 5. They slope upward noticeably.

Present accuracies of direct measurements on $\sin^2\theta$, as shown in Table I, are about ± 0.002. One can anticipate improvements to ± 0.001 or better in the near future through asymmetry measurements at LEP and possibly at the Fermilab Collider. The effect of a measurement to ± 0.001 is shown by the arrows, centered for illustration on $\sin^2\theta = 0.232$. Because of the upward slope of the

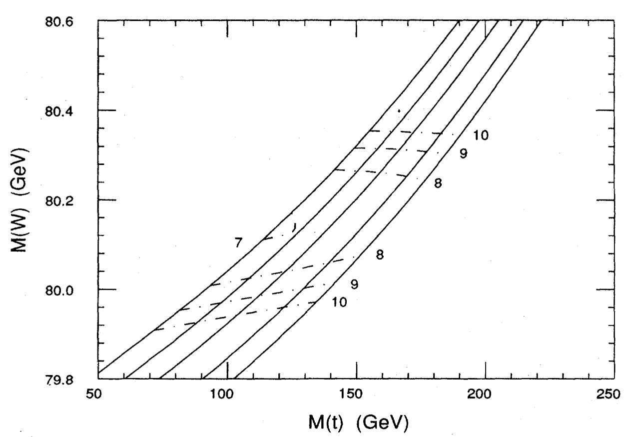

FIG. 3. Contours of χ^2 for a fit to the 13 electroweak observables in Table I based on Standard Model parameters, displayed as functions of m_t and M_W (dot-dashed curves). The five diagonally sloping solid curves are the same as in Fig. 2 and correspond to the prediction of various Higgs-boson masses.

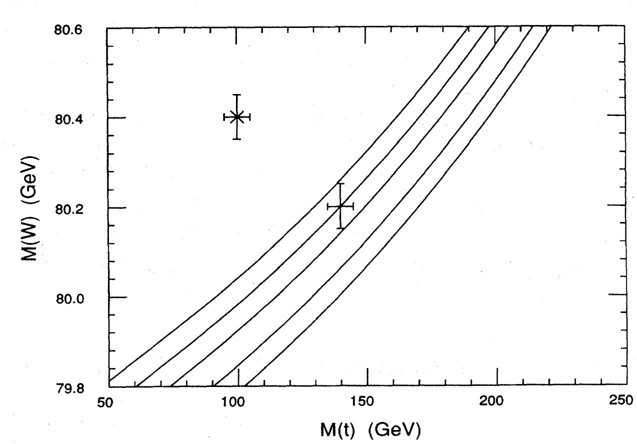

FIG. 4. Examples of the impact of measurement of m_t to ± 5 GeV/c^2 and M_W to ± 50 MeV/c^2. One plotted point lies within the range of Standard Model predictions and serves to provide information on the Higgs-boson mass. Another plotted point (the symbol $\times$) lies outside Standard Model predictions and requires some new physics for its explanation. Diagonally sloping lines are the same as those in Fig. 2.

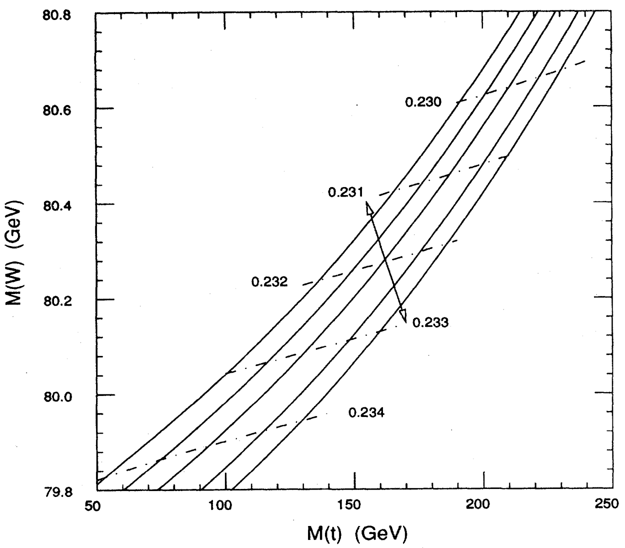

FIG. 5. Contours of $\sin^2\theta$ on a plot of M_W vs m_t. Arrows show the impact of a hypothetical measurement $\sin^2\theta=0.232\pm0.001$. Diagonally sloping lines are the same as those in Fig. 2.

contours, these measurements have a *correlated* impact on implied M_W and m_t values. For a *fixed* value of m_t, $\Delta(\sin^2\theta)=\pm0.001$ is equivalent to about $\Delta M_W=140$ MeV. This is not precise enough to learn much about the Higgs-boson mass. On the other hand, precise values of $\sin^2\theta$ are very helpful in detecting *departures* from standard electroweak predictions, especially when combined with improved measurements of m_t and M_W.

E. Measurement of Z decays

As we have mentioned, $\Gamma(Z\to l^+l^-)$ measures the combination $\rho(1+[1-4\bar{x}]^2)$. The effect of such a measurement is depicted parametrically by the contours in Fig. 6. A measurement $\pm0.5\%$ (the present error, shown by the larger limits) is equivalent to $\Delta M_W=\pm270$ MeV. A measurement to an accuracy of $\pm0.1\%$ (the smaller error bar) would lead to $\Delta M_W\simeq\pm50$ MeV and thus is what is needed to be comparable to other, more direct, determinations. Improved systematics and precise monitoring of luminosity (see, e.g., OPAL, 1991) may make this goal attainable if the results of the four LEP experiments are combined.

F. Neutrino deep-inelastic scattering

In Fig. 7 we show the effects of present and anticipated accuracies in measurements of R_ν.

The world average of experiments before 1992 implied an accuracy of $\Delta R_\nu/R_\nu=\pm1.3\%$, corresponding to $\Delta M_W\simeq\pm300$ MeV. Preliminary values from a recent Fermilab experiment (Bernstein, 1992) are tantamount to a value of $R_\nu/R_\nu^{\rm std}=1.018\pm0.015$, nearly equivalent in statistical power to the previous world average of

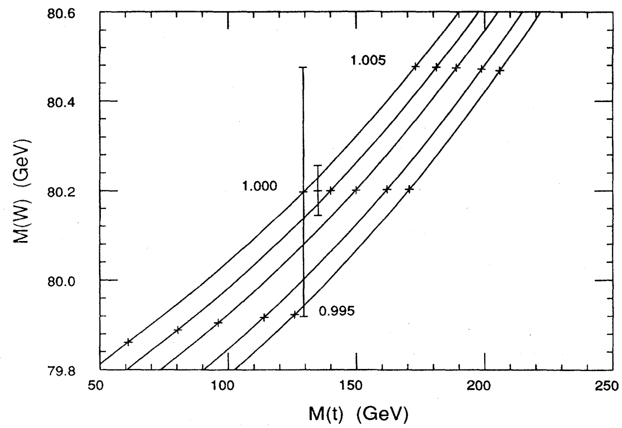

FIG. 6. Contours of $\Gamma(Z\to l^+l^-)/\Gamma(Z\to l^+l^-)^{\rm std}$ on a plot of M_W vs m_t. Error bars denote measurements of accuracy $\pm0.5\%$ (present) and $\pm0.1\%$ (proposed). Diagonally sloping lines are the same as those in Fig. 2.

0.990 ± 0.013 (Table I) but favoring slightly higher W and top-quark masses. (The favored top-quark mass for any given M_H in our global fit moves upward by 2 to 3 GeV$/c^2$ when these data are added.) The next round of experiments at Fermilab anticipates an accuracy of $\Delta R_\nu/R_\nu=\pm0.5\%$, implying $\Delta M_W=\pm120$ MeV. With further upgrades at Fermilab, one could achieve $\Delta R_\nu/R_\nu=\pm0.25\%$, or $\Delta M_W=\pm60$ MeV. All these possibilities are shown as error bars.

G. Parity violation in atoms

When small radiative corrections are applied to Eq. (3.4), the predicted result for an arbitrary isotope of Cs ($Z=55$) with N neutrons is (Marciano and Sirlin, 1983, 1984)

$$Q_W(^{55+N}\mathrm{Cs})=0.986\rho[-N+55(1-4.01\bar{x})]\ . \qquad (3.8)$$

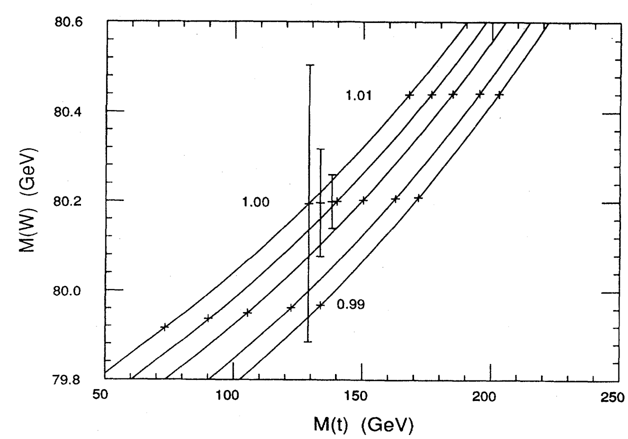

FIG. 7. Contours of $R_\nu/R_\nu^{\rm std}$ on a plot of M_W vs m_t. Error bars denote measurements of accuracy $\pm1.3\%$ (present), $\pm0.5\%$ (proposed), and $\pm0.25\%$ (proposed, with Fermilab Main Injector). Diagonally sloping lines are the same as those in Fig. 2.

If we substitute Eq. (3.3) and $\rho=1+\alpha T$ into (3.8), we find for the isotope studied at present ($N=78$) that

$$Q_W=-73.20\pm0.13-0.80S_Z-0.005T\ , \tag{3.9}$$

where the second term represents an estimated error in electroweak radiative corrections. As mentioned, the T dependence is almost absent, and Q_W(Cs) is a good probe of S_Z.

In the Standard Model, the contributions (2.21) to S_Z are very small. They lead to the predicted contours shown in Fig. 8. There is no room in the standard electroweak theory for substantial deviations from these predictions. The experimental value, $Q_W(\mathrm{Cs})=-71.04 \pm1.58(\mathrm{stat.}) \pm0.88(\mathrm{syst.})$, is compatible with the predictions at present, but an anticipated reduction of the statistical errors by a factor of 3 could have substantial implications for new physics if the central value were to remain the same.

H. Excuses for not finding the top quark

So far we have shown results (see, e.g., Fig. 3) that indicate the top quark should lie below about 200 GeV. What if it doesn't?

(1) *A very heavy Higgs boson* is not a likely solution. One could imagine further curves in Figs. 2–8 for $M_H=2$, 5, 10, etc. TeV, but these do not makes sense. If one does not put an elementary Higgs boson with mass less than about 1 TeV/c^2 into the theory, the interactions of W's and Z's become strong at high energies (Lee *et al.*, 1977), with the result that a Higgs boson of mass in the 1–2 TeV range is generated *dynamically*. In this case, it is likely that a good deal else also happens in the 1–2 TeV range, and this is a prime reason for studying this energy regime.

(2) *The assumptions underlying electroweak symmetry breaking might be at fault.* The result $\rho=1$ in lowest order arises in the electroweak theory from an assumption that electroweak symmetry breaking takes place through

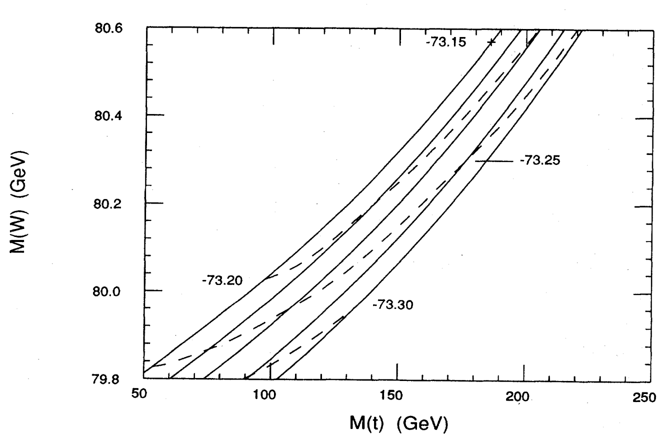

FIG. 8. Contours of Q_W(Cs) on a plot of M_W vs m_t. Diagonally sloping lines are the same as those in Fig. 2.

vacuum expectation values of Higgs fields with weak isospin $I\leq1/2$, the lowest value that can have any effect on W and Z masses. Thus, in the conventional picture, the spinless particles mentioned in Sec. II.D consist of a weak isospin doublet field

$$\phi=\begin{Bmatrix}\phi^+\\ \phi^0\end{Bmatrix} \tag{3.10}$$

and its conjugate

$$\phi^\dagger=\begin{Bmatrix}\phi^-\\ \bar\phi^0\end{Bmatrix}\ . \tag{3.11}$$

The ϕ^+, ϕ^-, and the combination proportional to $\phi^0-\bar\phi^0$ are the sources of the longitudinal components of the W^+, W^-, and Z, while the combination proportional to $\phi^0+\bar\phi^0$ is the Higgs boson H. The Fermi coupling constant arises as a result of a nonzero vacuum expectation value of this last combination.

There could be small contributions to W and Z masses from higher weak isospin values. If one has a small vacuum expectation value $V=\langle\Phi^0\rangle$ of the neutral member Φ^0 of a triplet ($I=1$) Higgs field with charges

$$\Phi=\begin{Bmatrix}\Phi^{++}\\ \Phi^+\\ \Phi^0\end{Bmatrix}\ , \tag{3.12}$$

the lowest-order ρ (call it ρ_0) is given instead by

$$\rho_0\cong1-\sqrt{2}G_FV^2\ ; \tag{3.13}$$

so we may compensate the effects of m_t in

$$\rho=\rho_0\left[1+\frac{3G_Fm_t^2}{8\pi^2\sqrt{2}}\right] \tag{3.14}$$

by letting $V\simeq\sqrt{3}m_t/4\pi$. Although this is an artificial solution, it might be more natural in models where the top-quark mass is correlated with electroweak symmetry breaking (Bardeen *et al.*, 1990; Nambu, 1991; Yamawaki, 1991). No such natural compensation for a heavy top has yet been demonstrated, however. For top quark masses large enough, the effects of m_t show up in other processes, particularly through the $t\bar t$ loop in $Z\to b\bar b$, and an upper limit $m_t\leq350$ GeV still can be set (Langacker, 1992). The main constraints prohibiting higher values of m_t turn out to be the total Z width, as affected by the prediction for $Z\to b\bar b$, and additional $\ln m_t$ terms in such expressions as S_W and S_Z.

(3) *An extra Higgs doublet* introduces new degrees of freedom which can be adjusted in order to partially compensate for a heavy top quark (Denner *et al.*, 1990). With two Higgs doublets, there are now *two* neutral scalars with masses M_1, M_2, a pseudoscalar with mass M_3, and positively and negatively charged scalars with masses M_+. The greatest compensation occurs when $M_1\sim M_2\sim0$, $M_+\sim0.56M_3\equiv0.56M$, or when $M_3\sim0$, $M_+\sim0.56M_{1,2}\equiv0.56M$, in which case one can have

$$\Delta\rho_{2H} = -\frac{G_F}{8\pi^2\sqrt{2}}(0.216)M^2 \ . \tag{3.15}$$

The largest region of parameter space for two-Higgs models, however, corresponds to positive contributions to ρ and hence to *improved* upper bounds on the top-quark mass.

IV. CONCLUSIONS

We have shown that a measurement of the top-quark mass in the near future will provide unique and valuable information regarding the standard picture of electroweak interactions. A measurement with $\Delta m_t \leq 5$ GeV/c^2, coupled with any other electroweak measurement equivalent to $\Delta M_W \leq 50$ MeV/c^2, will begin to shed light on the Higgs-boson sector. Such an error on M_W can be achieved either directly (e.g., via a proposed upgrade of the luminosity at Fermilab), or through indirect means *if one assumes the validity of the electroweak theory.* Such means could include measurement of R_ν (the neutral- to charged-current ratio in deep-inelastic neutrino scattering) to $\pm 0.2\%$. Alternatively, a measurement of $\Gamma(Z \to l^+ l^-)$ to 0.1% (5 times its present accuracy) could provide comparable information.

It is important to reiterate the *complementarity* of various electroweak measurements. Each different measurement provides different information, especially when searching for new physics. Our purpose in this article has been to collect all the predictions of the *minimal* electroweak model, to see the impact of each different measurement on *standard* parameters. The most interesting situation, of course, would be if different measurements yielded different inferred results in the (m_t, M_W) plane. One would then make use of more general parameterizations (e.g., in terms of the parameters S_W, S_Z, and T) to search for the type of new physics implied by such measurements.

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The abbreviations appearing in this article and in its Reference Section are defined as follows: ALEPH—Apparatus for LEP Physics; CCFR—University of Chicago, Columbia University, Fermilab, University of Rochester (and the University of Wisconsin on the latest experiment); CCFRR—University of Chicago, Columbia University, Fermilab, University of Rochester, Rockefeller University; CDF—Collider Detector at Fermilab; CDHSW—CERN, University of Dortmund, University of Heidelberg, Saclay (DPhPE-CEN), University of Warsaw; CERN—Conseil Européen pour la Recherche Nucleaire, also known as the European Laboratory for Particle Physics; CHARM—CERN, Hamburg, Amsterdam, Rome, Moscow; DELPHI—Detector with Lepton, Photon, and Hadron Identification; FMM—Fermilab, Massachusetts Institute of Technology, Michigan State University; L3—CERN-LEP-L3 experiment (internal CERN numbering of experiment); LEP—Large Electron-Positron; OPAL—Omni-Purpose Apparatus for LEP; UA—Underground Area, CERN $\bar{p}p$ collider.

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