Geophysical applications of very-long-baseline interferometry

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Very-long-baseline interferometry (VLBI) is a novel observing technique for measuring the relative positons of widely separated points on the surface of the Earth with centimeter-level accuracy. Such accuracy is two or three orders of magnitude better than was available with classical techniques only a few decades ago. This enormous improvement in accuracy has opened up for study a broad new spectrum of geophysical phenomena. The new measurements allow direct observation of the tectonic motions and deformations of the Earth's crustal plates, observations of unprecedented detail of the variations in the rotation of the Earth, and direct measurement of the elastic deformations of the Earth in response to tidal forces. These new measurements have placed significant constraints on models of the interior structure of the Earth; for example, measurements of the variations in the Earth's nutation have been shown to be particulary sensitive to the shape of the core-mantle boundary. The VLBI measurements, coupled with other space-based geodetic observing techniques such as the Global Positioning System, allow construction of a global reference frame accurate at the centimeter level. Such a frame will be essential to studying long-term global changes, especially those changes related to sea-level variations as recorded by tide gauge measurements.

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I. INTRODUCTION

The history of observational science is punctuated by the invention of revolutionary instruments such as the telescope and spectroscope for astronomy, the microscope for biology, the particle accelerator for physics, and the seismometer for geophysics. These critical inventions often redefined whole branches of science by allowing the study of entirely new classes of phenomena. The effect of many of these new instruments was to increase attainable measurement accuracies by several orders of magnitude. Very-long-baseline interferometry (VLBI) is a recent example of such a revolutionary development in instrumentation.

The invention of the telescope in the 17th century is perhaps the closest historical parallel to the development of VLBI. The telescope improved the angular resolution of optical observations by about two orders of magnitude, from roughly 1 minute of arc (achievable with the naked eye) to a fraction of a second of arc. By comparison, VLBI improved angular resolution by an additional three orders of magnitude, to the level of a fraction of a millisecond of arc. The telescope revolutionized astronomy in the past, and VLBI is having a similar effect on astronomy today, although the impact of VLBI on astronomy is beyond the scope of this article. [See Reid and Moran (1988, pp. 7–355) for recent papers.]

Less well known is the fact that the telescope was the critical instrument for a sequence of important discoveries in geophysics, including the discovery of nutation by Bradley and the measurement of the flattening of the figure of the Earth by Bouguer, both in the middle 1700s, the discovery of isostasy by a geodetic survey in India in the middle 1800s, and the discovery of polar motion by Chandler in the 1890s. By improving the accuracy and sensitivity of position determinations on the Earth by about three orders of magnitude over what is possible with telescopic observations, VLBI allows us to greatly refine some of these measurements, such as polar motion and nutation, and to observe entirely new categories of phenomena such as the tectonic motions and deformations of crustal plates, the effects of core-mantle interactions on the rotation of the Earth, and the details of the angular momentum interchanges between the atmosphere and solid crust of the Earth. Many of these uses of VLBI were first pointed out by Shapiro (1967; Shapiro and Knight, 1970). Continued and regular VLBI observing programs will be needed to make effective use of the extraordinary capacity of these observations to provide a wealth of new information about the structure and behavior of the Earth.

The basic idea that underlies geodetic VLBI observations is conceptually very simple: The fundamental measurement is the time delay or difference in the arrival times of a signal from an extragalactic radio source received at two (or more) radio observatories. Figure 1 shows a schematic illustration of the geometry of a VLBI observation. The extragalactic radio sources are at a sufficiently great distance from the Earth that their signals arrive as essentially plane-wave fronts. Thus to a first approximation the measured VLBI time delay is proportional to d, the component of the wave vector baseline between the VLBI stations in the direction of the radio source. (This is an oversimplified picture that will require a number of small corrections for effects such as atmospheric refraction and clock errors.) In Sec. III we shall show that a sufficient redundancy of these baseline-vector-component measurements can contain enough information to estimate not only all three components of the baseline vector, but also other parameters of interest including the coordinates of the radio sources, Earth rotation parameters, Earth tide strains, clock errors, and atmospheric refraction variations.

In making VLBI measurements, the signal from the extragalactic radio source received at each station is heterodyned, clipped, sampled, and recorded digitally on magnetic tape. This digitization of the signal is carefully done so as to preserve high-precision information concerning the signal phases (arrival times) measured with a hydrogen maser atomic clock (Rogers, 1970; Whitney, 1974). The tapes from all of the stations are then transported to a common location and analyzed using a cross-correlation algorithm to determine the VLBI time delay or differential arrival time. Section II will discuss this process in more detail.

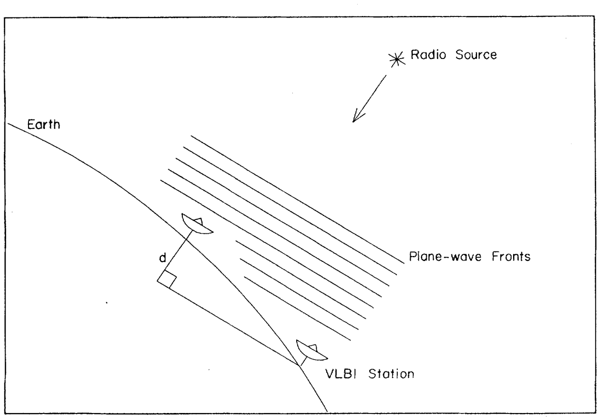

FIG. 1. Plane-wave fronts from an extragalactic radio source received at a pair of VLBI stations. The time delay is proportional to d, the component of the vector baseline in the direction of the radio source.

The observing frequencies used for geodetic VLBI observations are typically in the vicinity of 8.5 GHz, corresponding to wavelengths in the range of 2–3 centimeters. Since the correlation process can match the signals to a fraction of a wavelength, the theoretical uncertainty in the VLBI delay measurements is often less than a centimeter. Unfortunately there are unmodeled systematic errors present in the data, including especially atmospheric refraction effects, that are commonly larger than this and that often limit the accuracy attainable with VLBI measurements to about the centimeter level.

The celestial radio sources that are used for geodetic VLBI measurements are typically quasars. These peculiar astronomical objects have a number of properties that make them ideally suited for use as celestial fiducial marks. They are among the brightest objects in the radio sky, yet they are believed to be extremely distant, typically billions of light-years away; this extreme distance reduces or eliminates problems that would result from any proper motions the sources might have. (It is perhaps startling to realize that the radio signals being used to measure the Earth may in fact be substantially older than the Earth.) Quasars tend to have flat spectra over the frequency range used by VLBI observations; this facilitates the use of wide observing bandwidths as discussed in Sec. II. Finally, these sources tend to have very small apparent diameters, typically a millisecond of arc, and thus are well suited to functioning as fiducial marks for a reference coordinate frame. On the negative side, although they are very small, quasars are not always pointlike. Instead they commonly have complicated brightness structures that may be time variable as well. Such variability could pose a problem for the stability of the celestial reference frame defined by VLBI observations; however, it should be possible to minimize this problem by carefully selecting the sources to be observed, eliminating the more complex and variable sources from the observing schedules. Most of the observations discussed in this paper involve sources whose structures are small and simple on the scale of a millisecond of arc. For detailed discussions of the properties of some of the celestial radio sources observed with VLBI, see Reid and Moran (1988, pp. 7–355). Some caution is needed, of course, in interpreting the radio source structures cataloged in the astronomical literature as typical of the entire population of extragalactic radio sources: The astronomical literature tends naturally to focus on the sources that have the most structure and time variability.

There are two essentially independent problems to be addressed in the remainder of this paper. First is the development of the technology required to make the VLBI observations and determine the delay values as sketched in Sec. II. The second problem involves the use of the VLBI delay values for a variety of geodetic and geophysical measurements as discussed in Secs. III–VI. Readers who are primarily interested in the geophysical applications of VLBI may wish to skip directly to Sec. III.

II. TECHNOLOGY DEVELOPMENT

The history of the development of VLBI technology has often been recounted. (For overviews, see Klemperer, 1972; Cohen, 1973; Counselman, 1973; Moran, 1976; and Thompson *et al.*, 1986, pp. 29–33 and 247–249.) The development proceeded in three distinct stages starting with the Mark-I system developed by the National Radio Astronomy Observatory (NRAO), the National Aeronautics and Space Administration (NASA), and the Northeast Radio Observatory Corporation's Haystack Observatory. The Mark-I system was used extensively for geodetic observations in the 1960s and 1970s (Whitney *et al.*, 1976). The second stage, called Mark-II, was developed by NRAO and used extensively for astronomical observations, but it has seen only limited use for geodetic purposes and will not be discussed extensively here. The currently operational system, called Mark-III, was developed with NASA support by the Goddard Space Flight Center and Haystack Observatory in the 1970s and was steadily upgraded through the 1980s (Rogers *et al.*, 1983). It is in wide use throughout the world today. Work is progressing on the development of a Mark-IV system by the same team, but that system is not currently operational and will also not be discussed here.

Briefly, VLBI was first developed in the late 1960s by radio astronomers who sought to obtain greater angular resolution than could be achieved by conventional (connected-element) radio interferometers. In connected-element interferometry, the observations at all of the antennas are referred to a common clock. The antenna spacing is limited to the maximum distance over which timing signals can be transmitted by cables or microwave links without suffering serious degradation in phase stability; typically, this is a few tens of kilometers. In VLBI, each station is equipped with a highly stable atomic clock, and the observations are recorded completely independently at each station. Real-time processing of the observations is sacrificed; but, in compensation, the spacing of the stations is essentially unlimited.

As noted in the previous section, the VLBI measurement entails determining the difference in the arrival time of the extragalactic signal at the two stations. This is accomplished by cross-correlating the signals from the two stations:

$$F(\tau)=\int_{-\infty}^{\infty} S_1(t)S_2(t+\tau)dt \ , \tag{1}$$

where $F(\tau)$ is the correlation function and $S_n(t)$ is the signal at station n. The special properties of the correlation function are fairly easy to understand heuristically. If the signals were infinite bandwidth white noise with zero mean, then the correlation function would be zero almost everywhere, because the product of zero-mean white-noise signals is also zero-mean white noise. However, at the value of τ for which the signals from the stations match (they differ only by a time delay), the integrand will be everywhere positive; the correlation function will have a singularity at that point. It is difficult to improve on the signal-to-noise ratio of a function that has a singularity at the point of interest and is zero everywhere else. Again, this discussion is oversimplified. The signals differ by more than just a time delay, because of effects such as different Doppler shifts at the two stations. However, these effects are easily dealt with analytically by the computer correlation software.

Of course, in the real world the integration limits are not carried to infinity, and the singularity is reduced to a finite spike. More seriously, the observing bandwidth is not infinite. The bandwidth of the first VLBI observations was restricted to a few hundred kilohertz by the capacity of the tape recorders available at that time. To understand the problem that the bandwidth limitation causes, it is useful to consider the limiting case of zero bandwidth, i.e., the case in which the incoming signal is a pure sinusoid with a period of Δt. When such signals are correlated, any time delay t for which the signals are found to match (i.e., for which the correlation function has a maximum) will have a corresponding and equally good match at $t+n\Delta t$, where n is any integer. This problem is referred to as the phase-delay ambiguity, and it can cause serious problems for geodetic observations that are typically made at observing wavelengths of about 2–3 cm. To resolve phase ambiguities of this magnitude it would be necessary to have *a priori* knowledge of the vector baselines and their variations, as well as of the other parameters affecting the delay, such as clock and atmospheric refraction variations, as well as of the other parameters affecting the delay, such as clock and atmospheric refraction variations, accurate to much better than 2–3 cm.

Rogers (1970) first realized that the availability of much wider bandwidth observations could alleviate this problem by allowing the use of group delay instead of phase delay. If we define the interferometer phase, ϕ, as the length d (from Fig. 1) measured in wavelengths (λ) at some observing frequency f,

$$\phi=\frac{d}{\lambda}=\frac{df}{c} \tag{2}$$

(where c is, of course, the speed of light), then the phase that is measured (ϕ_m) by correlation of a narrow-band signal at this frequency will be different from this phase by n cycles because of the phase ambiguity:

$$\phi_m=\frac{df}{c}\pm n \ . \tag{3}$$

If we then measure the phase at a second observing frequency, we can construct the group delay as

$$\tau_g=\frac{\Delta\phi_m}{\Delta f}=\frac{d}{c}\pm\frac{N}{\Delta f} \ , \tag{4}$$

where N is the difference between the (unknown) values of n at each frequency. The important thing about group delay is that we can control the size of the ambiguity spacing by proper choice of the frequency spacing Δf. The only tradeoff is that if Δf is chosen to be very small, in order to maximize the size of the ambiguity spacing, then the precision of the determination of the slope, $\Delta\phi/\Delta f$, is reduced. This problem can be alleviated by making simultaneous observations at more than two frequency channels. The problem of the optimum selection of the observing frequency channels will be discussed below.

The technique of constructing the group delay using observations made simultaneously at several narrow frequency bands is referred to as bandwidth synthesis. In Mark-I observations the different frequency channels were time-multiplexed on the recording tapes, whereas the Mark-III tape recorders allow the simultaneous recording of signals from up to 28 frequency channels (Hinteregger, 1980), although the number of distinct frequencies is normally limited to 14 by the number of available local oscillators.

The measurement error for bandwidth-synthesized group-delay observations, σ_g, can be expressed in the following form (Rogers, 1970, 1980b; Clark *et al.*, 1985, pp. 440–441; Thompson *et al.*, 1986, p. 394):

$$\sigma_g = \left[\frac{2k}{F}\right]\left[\frac{T_{s_1}T_{s_2}}{A_1 A_2 2Bt}\right]^{1/2}\left[\frac{1}{2\pi\eta\,\Delta f_{\rm rms}}\right], \qquad (5)$$

where

k= Boltzmann's constant;
F= Correlated flux density of the source, or power radiated by the source in units of 10^{-26} watts per square meter of antenna collecting area per hertz of observing bandwidth. The word "correlated" indicates that portion of the power that is emitted from a region small enough to avoid significant phase differences at the two receiving antennas;
T_S= System temperature at each station, a measure of the noise level of the receiver electronics;
A= Effective area of each antenna, equal to the total collecting area of the antenna corrected for losses caused by such things as antenna surface imperfections and aperture blockages;
B= Total recorded bandwidth;
t= Total integration time ($2Bt$ is the total number of bits at the Nyquist rate);
η= Loss factor to account for clipping and other signal processing losses;
$\Delta f_{\rm rms}$= root-mean-square (rms) bandwidth.

Much of the development of VLBI technology from the Mark-I to the Mark-III system over the past 20 years can be understood as a series of highly successful efforts to reduce σ_g by a systematic assault on the terms of this equation to the limits of technology and available funding. For example, the source flux densities F are maximized by utilizing the strongest sources that have acceptably small structure and time variation (and that are suitably distributed over the sky). T_S is presently minimized by using helium-cooled amplifiers that employ high-electron mobility transistors (HEMT's), which provide T_S values for modern VLBI systems of typically a few tens of kelvins. In contrast, the uncooled parametric amplifier systems that were employed in Mark-I observations ordinarily provided T_S values in the range of hundreds of kelvins. The effective antenna areas A are ordinarily maximized by using large, high-quality parabolic antennas. Of course, the desire to maximize the observing time t must be balanced against the desire to maximize the number of observations.

One area in which a great deal of technical progress has been made, and more is likely to be made in the future, involves the recorded bandwidth B, where the unprecedented progress made in data storage technology over the last few decades can be exploited. The Mark-I system employed conventional computer tape drives capable of recording a single channel of 0.36 MHz (Whitney *et al.*, 1976). In contrast, the Mark-III system tape recorders are capable of recording 28 separate channels simultaneously, each with a bandwidth of 4 MHz (Clark, 1980; Hinteregger, 1980; Rogers, 1980c; Rogers *et al.*, 1983), although the normal practice in geodetic observations is to use only 14 channels of 2 MHz each.

The remaining factor that needs to be optimized involves the rms spanned bandwidth, $\Delta f_{\rm rms}$. This term is maximized first by enlarging the total bandwidth of the antenna/receiver system. In the Mark-I system this total spanned bandwidth typically ranged from 20 to 40 MHz, while in the Mark-III system a bandwidth of 360 MHz has been fairly standard, and 720 MHz was recently employed experimentally. The celestial radio sources (quasars) that are observed tend to have fairly flat spectra over a much wider bandwidth than this; the observed bandwidth is limited much more by the capability of the receivers and recorders than it is by the properties of the signals being observed.

The rms bandwidth is a function of the particular selection of frequency channels employed. It is in the area of the selection of frequency channels that we encounter some interesting tradeoffs. Rogers (1970, 1976) showed that the problem of optimal placement of frequency channels is essentially identical to the problem of the optimal placement of antennas in a linear array. He showed that the effects of the selection of frequency channels can be expressed in terms of the ambiguity function or delay resolution function, which is simply the Fourier transform of a set of (zero-bandwidth) impulse functions spaced at the frequencies of the observing channels. This follows in a straightforward manner from the Fourier transform properties of the correlation integral. (The delay resolution function is modified slightly by the nonzero bandwidth of the individual frequency channels, but that small correction is not important for the discussion here.) For both the frequency selection problem and the antenna placement problem the desired response pattern is one that is as close to an impulse function as possible, that is, a response pattern that has a central lobe that is as narrow as possible (its width determines the precision of the group-delay observation) and sidelobes that are as small as possible. The sidelobes are important because, in the presence of noise, they could become large enough to be mistaken for the central lobe. As we have seen, the impulse function (with no sidelobes at all) is the limiting case in the direction of the infinite bandwidth and integration time.

These goals are somewhat conflicting. For example, the width of the central lobe can be minimized by placing all of the observing channels at the edge of the observing band. This provides the minimum width of the central peak at the expense of producing unacceptably large sidelobes close to the central peak. Thus part of the difficulty of optimizing the delay resolution function lies in specifying the desired combination of features to be maximized; e.g., how much central peak width should be traded off against sidelobe height.

In the absence of generally accepted criteria for optimizing the delay resolution function, a variety of heuristic approaches and simulations have been attempted. One approach that has been found to be useful involves the use of nonredundant difference sequences, i.e., integer sequences will have the property that the difference of each pair of integers is distinct from the difference of each of the other pairs. More formally, a sequence of integers such that $(a_j - a_i) \neq (a_l - a_k)$ unless $j = l$ and $i = k$ or else $j = i$ and $l = k$. These sequences, sometimes called "spanning rulers," are of interest in electrical engineering and applied mathematics; they have applications not only in antenna theory, but also in coding theory, radar pulse sequences, and a variety of other applications. The simplicity of the specification of a spanning ruler is deceptive—there is no known way of constructing general spanning rulers other than that of exhaustive computer search. As Taylor and Golomb (1985) put it: "... there are several different contexts in which the problem of finding efficient spanning rulers has been identified. In at least three cases it came as a surprise to the researchers that such rulers are not easily obtained." One of the rationales for using spanning rulers for group-delay observations is a natural consequence of Eq. (4): the spanning ruler provides the maximum number of distinct values of Δf.

The smallest possible spanning ruler for a given number of integers is sometimes called a "Golomb ruler," after S. Golomb, who pioneered much of the work on these sequences. Table I lists all of the known Golomb rulers and is complete up to rulers with 16 elements. Most of the sequences in this table were discovered by exhaustive computer search. Not only is there no other known way to construct these sequences, there is no easy way to determine even the magnitude of the largest element or how many distinct rulers exist for a given number of elements. The sequences given in Table I have been found to provide excellent delay resolution functions, as seen in Fig. 2.

Of course there is no absolute requirement to use the shortest possible spanning ruler, and other rulers have been found that are sometimes better matched to a given total bandwidth and number of channels. For example, the 8-element spanning ruler 0, 1, 4, 10, 21, 29, 34, 36 has been used to conveniently span the 360 MHz bandwidth of recent Mark-III observations, and the 6-element spanning ruler 0, 1, 8, 19, 23, 25 has been used for six-channel systems. These sequences were also constructed by extensive computer search.

TABLE I. All of the known minimum-length spanning rulers ("Golomb rulers").

3	0	1	3													
4	0	1	4	6												
5 (a)	0	1	4	9	11											
(b)	0	2	7	8	11											
6 (a)	0	1	4	10	15	17										
(b)	0	1	8	12	14	17										
(c)	0	1	8	11	13	17										
(d)	0	1	4	10	12	17										
7 (a)	0	1	4	10	18	23	25									
(b)	0	1	7	11	20	23	25									
(c)	0	1	11	16	19	23	25									
(d)	0	2	7	13	21	22	25									
(e)	0	2	3	10	16	21	25									
8	0	1	4	9	15	22	32	34								
9	0	1	5	12	25	27	35	41	44							
10	0	1	6	10	23	26	34	41	53	55						
11(a)	0	1	4	13	28	33	47	54	64	70	72					
(b)	0	1	9	19	24	31	52	56	58	69	72					
12	0	2	6	24	29	40	43	55	68	75	76	85				
13	0	2	5	25	37	43	59	70	85	89	98	99	106			
14	0	4	6	20	35	52	59	77	78	86	89	99	122	127		
15	0	4	20	30	57	59	62	76	100	111	123	136	144	145	151	
16	0	1	4	11	26	32	56	68	76	115	117	134	150	163	168	177

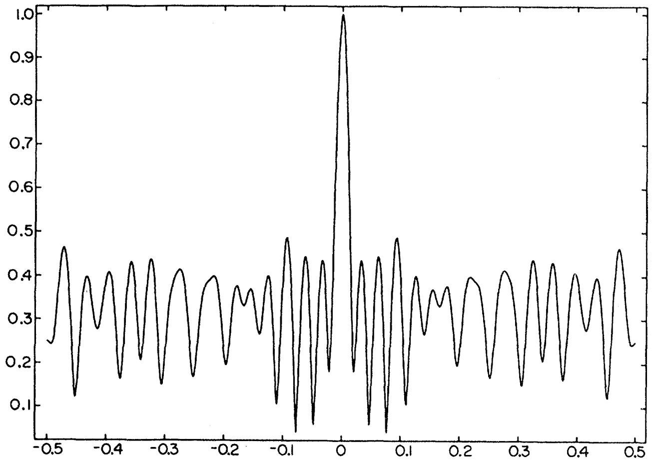

FIG. 2. Delay resolution function for an eight-channel observing system with a channel spacing of 0, 1, 4, 9, 15, 22, 32, and 34. If the channel spacing has units of MHz, then the horizontal scale has units of μsec.

In addition to providing group-delay observations with unprecedented precision, the Mark-III system includes a number of features that are used to calibrate or eliminate a variety of important sources. Such features include the use of dual-frequency observations to eliminate dispersive refractive delays of the sort that can be caused by charged particles in the path of the incident radio waves. In other words, the observing frequency channels have been grouped into two widely separated synthesized bands, one at X band ($\sim$8500 MHz) and the other at S band ($\sim$2500 MHz). The additional delay at each station caused by the refractivity of the Earth's ionosphere can vary by an order of magnitude between day and night, and can amount to about 30 cm at X band and several meters at S band in the daytime. Other features that improve the accuracy of VLBI observations include a calibration system that eliminates relative phase drifts of signals recorded at the different frequency channels ("phase calibration" system), and a system that calibrates the thermal and mechanical variations in the cables used to transmit the calibrating signals ("cable calibration" system; see Rogers, 1980a).

With these advances in the technology of VLBI hardware the accuracy of the group-delay observations has improved from the meter level obtained with the early Mark-I observations (Hinteregger *et al.*, 1972) to the centimeter level obtained with Mark-III today. Still further advances can be expected in the future. NASA is funding a development program whose objective is to provide millimeter-level-accuracy VLBI observations in the future. But there are many interesting investigations that can be undertaken with the current centimeter-level observing accuracies, as will be outlined in the remaining portion of this paper.

III. INFORMATION CONTENT OF VLBI OBSERVATIONS

Shapiro (1976) provides an analysis of the information content of VLBI delay observables. He points out in his review article that for a given baseline and celestial radio source the behavior of the group delay as a function of time will have a diurnal sinusoidal component because the baseline rotates with the Earth. In addition, the function will have an offset from zero resulting from the component of the baseline parallel to the spin axis, and will have a further offset and slope resulting from clock errors. Measurements of this function could provide us with four observables: the amplitude and phase of the sinusoidal component, and the offset and slope. At the same time, there are seven parameters that we wish to estimate: three components of the baseline, two source coordinates, and the clock offset and rate. The number of parameters reduces to six, because there is no natural origin in right ascension—we can only solve for differences in source right ascension. Clearly we cannot estimate six parameters from four observables, but we can improve the situation by adding new sources: each source adds three new observables (offset, amplitude, and phase; the slope is common to all sources) and only two unknowns (the right ascension and declination of the source). Thus a minimum of three sources, well distributed in the sky, is needed to estimate all of the components of the baselines and sources that we wish to observe. In practice there is no need to operate close to this minimum: about 25 sources are typically observed during a 24-hour geodetic observing session.

This calculation needs a number of refinements. The clock errors are never as simple as an offset and a rate, and there are atmospheric refraction effects that currently must be estimated from the data. In addition, the motion of the Earth is more complicated than a simple rotation about a fixed axis. Variations in both the direction and magnitude of the spin vector, and departures of the behavior of the Earth's surface from that expected of a simple rigid body, all combine to complicate the picture. The atmospheric refraction effects, as discussed in the next section, are particularly troublesome. But detailed numerical experiments have shown that with a sufficient redundancy of VLBI data it is possible to separate many of these effects and produce detailed determinations of geophysically interesting effects such as baseline variations, polar motion, UT1, and nutation at unprecedented levels of spatial and temporal resolution.

IV. GEOPHYSICAL PARAMETER ESTIMATION

VLBI observations are commonly processed using generalized least-squares techniques to estimate the geodetic and geophysical parameters of interest. In the analyses discussed in this paper the theoretical model for the observed delay used in the least squares fitting process employs an algorithm developed by Shapiro (1983a, 1983b; see also Herring, 1989 and Ryan, 1989), which includes effects of the positions and velocities of the stations and the relativistic effects of the gravity field of both the Sun and the Earth. This algorithm yields estimates of theoretical delays within 2 picoseconds (<1 mm equivalent path length) of the algorithm developed by Hellings (1986), when gravitational effects on the radio wave propagation in the vicinity of the Earth are added to the latter.

The motions and deformations of the Earth's surface are modeled according to the algorithms outlined in the International Earth Rotation Services (IERS) standards document (McCarthy, 1989). Briefly, these standards specify the use of the International Astronomical Union (IAU) 1980 nutation model (Wahr, 1981b), an Earth tide model, and a model for station displacements caused by ocean loading. Wahr's model is used for the solid-Earth deformations resulting from the motion of the pole (Wahr, 1985). Plate motion is modeled using the no-net-rotation model (AM0-2) of Minster and Jordan (1978), with the reference epoch for the motion defined to be Jan 1, 1988. Much of the theory used to model the behavior of a realistic, nonrigid Earth was developed by Wahr in a series of classic papers that began in 1981. Using a normal-mode expansion technique developed by Smith (1974), Wahr worked out the response of a nonrigid, stratified, elliptical, oceanless, rotating Earth with a fluid core to effects such as nutation (1981b), Earth tides (1981a), and the pole tide (1985). Recent work has focused on the effects of barometric pressure loading on the surface of the Earth (Van Dam and Wahr, 1987) and the effects of the Earth's solid inner core on its rotation (De Vries and Wahr, 1991; Matthews *et al.*, 1991a, 1991b; Herring *et al.*, 1991).

Atmospheric refraction variations and clock errors both exhibit stochastic time variability at a significant level. The total additional delay due to refraction in the atmosphere is about 1.5 m in the vertical direction, and it scales roughly as $1/\sin(\varepsilon)$ (where ε is the elevation angle of the observation). The clocks typically have slow variations with a magnitude of about 10^9 sec (~ 30 cm) over a day. In the past these time variations were modeled by fitting the coefficients of simple polynomials. More recent models use continuous, piecewise linear models with frequent rate breaks, often constrained with *a priori* constraint equations to remove singularities in the solution. A more sophisticated approach to handling the stochastic variability of these parameters employs Kalman filter techniques pioneered by Herring and his colleagues (Herring, Davis, and Shapiro, 1990). Detailed analyses with Kalman filter techniques have shown that subcentimeter repeatability in the determination of transcontinental baselines is attainable for observing sessions in which the data have been carefully scheduled to include a wide range of elevation angles, including especially low-elevation angles which are of critical importance to the separation of atmosphere parameters from the station coordinates (Davis *et al.*, 1985, 1988, 1990).

The effective use of low-elevation observations is strongly dependent on the use of accurate models for the behavior of atmospheric refraction effects as a function of elevation angle. Davis and his colleagues have used a continued fraction expressions of the following form:

$$m(\varepsilon)=\cfrac{1}{\sin\varepsilon+\cfrac{a}{\tan\varepsilon+\cfrac{b}{\sin\varepsilon+c}}}\ , \tag{6}$$

where ε is the elevation angle (angle above the horizon) of the observation, and the parameters a, b, and c are linear functions of meteorological variables such as surface barometric pressure. The coefficients that determine the values of a, b, and c were determined by fitting to results from computerized ray-tracing algorithms that numerically integrate the refractivity of a detailed atmosphere model. The differences between the total refractivity values produced by this analytic model and the ray-trace algorithm were typically less than 1.5 mm rms for $\varepsilon > 5°$. Using an algorithm of this form, Davis and his colleagues were able to demonstrate subcentimeter repeatability for the determinations of the length of a transcontinental baseline. Nevertheless, atmospheric refraction remains a difficult problem; these corrections will probably need further refinement.

The refractive effects of the dispersive portion of the atmosphere (resulting from the effects of charged particles in the Earth's ionosphere and elsewhere along the line of sight to the radio sources) are largely eliminated by the use of dual frequency observations, at S band (~ 2200 to 2300 MHz) and at X band (8200 to 8600 MHz). Because to first order the dispersive refraction scales inversely as the square of the observing frequency, the X- and S-band observations can be combined to eliminate most of the ionospheric effects. The delay at S or X band ($\tau_{s,x}$) can be written as

$$\tau_{s,x}=\tau_c+\frac{k}{f_{s,x}^2}\ , \tag{7}$$

where τ_c is the corrected or ionosphere-free delay, k is the constant related to the total count of charged particles along the line of sight, and $f_{s,x}$ is the observing frequency. Simultaneous observations at S and X band can then be combined algebraically to eliminate the unknown k and produce an observable with ionosphere effects removed:

$$\tau_c=\tau_x-(\tau_x-\tau_s)\left[\frac{f_s^2}{(f_x^2-f_s^2)}\right]\ . \tag{8}$$

In addition to atmospheric refraction there is a variety of other small systematic error sources whose effects present some difficulty for VLBI data reduction. Effects of source structure, the time-variable spatial distribution of radio emission brightness of the source, for example, are straightforward to correct if the brightness distribution is known (Cotton, 1980; Ulvestad, 1988), but the necessary brightness distribution maps are seldom available. Similarly, thermal, gravity, and wind-loading effects can produce motions of the antenna structures at the centimeter level. Removal of these effects would require extensive modeling calculations, which have not generally been done. In addition, the barometric pressure variations of the atmosphere have been shown to cause vertical motions of the Earth's crust at the level of a few centimeters (Van Dam and Wahr, 1987). The necessary atmospheric data for calculating these effects are available from meteorological centers, but are not presently used in VLBI data reduction. Thus, although the accuracy of the individual VLBI observations is generally in the subcentimeter range, unmodeled systematic errors are somewhat larger than this and may dominate the statistical errors. Further improvements in the determination of geodetic parameters are likely to depend critically on improvements in our ability to model these systematic effects.

V. VLBI OBSERVING PLANS AND PROGRAMS

A. Fixed station

Many of the observations discussed below were obtained under the National Oceanic and Atmospheric Administration's (NOAA's) POLARIS and IRIS projects, whose objectives are to monitor the rotation of the Earth and to coordinate the development and operation of a global network of geodetic VLBI observatories. The stations in such a network could serve as fiducial points for regional or local studies (using other observing techniques; see Sec. VII) of crustal motion, deformation, and other tectonic processes that complicate the measurement of important geophysical phenomena such as tide gauge measurements of sea level (Carter *et al.*, 1985). The NOAA programs began in 1980 with the implementation of two observatories in the United States—in Westford, Massachusetts, and in Fort Davis, Texas—which performed routine 24-hour observing sessions at seven-day intervals. In 1984 a third station was completed in Florida, and a fourth station, at Wettzell in Germany, began routine operations in cooperation with the U.S. stations. Since 1984 the IRIS program has operated 24-hour observing sessions involving these four stations every five days. The station at Onsala, Sweden, has participated in these campaigns about once a month since 1980. In 1986 a station in Hartebeesthoek, South Africa, became operational and performed about two months of observing each year until 1989, at which point regular monthly 24-hour observing sessions involving Hartebeesthoek, Wettzell, Richmond, Westford, and Mojave were started. Another IRIS observing campaign, called IRIS-P, involving stations in Australia, Japan, Alaska, California, and Florida, is coordinated by the National Astronomical Observatory in Japan. Yet another IRIS project involves a program using stations in Australia and South Africa to observe radio sources low in the celestial Southern Hemisphere. Finally, there is a daily 45-minute IRIS observing campaign involving Westford and Wettzell for the purpose of making determinations of UT1 for the intervals between the 5-day observing sessions (Carter *et al.*, 1985, 1988; Carter and Robertson, 1986).

The NASA Crustal Dynamics Project (CDP) organized observing campaigns through the 1980s for the purpose of measuring the motions and deformations of the Earth's crustal plates (Coates *et al.*, 1985; Coates, 1988). The CDP operates a variety of observing campaigns at irregular intervals that generally employ observing sessions of 24-hour or longer duration and involve a variety of stations in the United States, Europe, the Pacific islands, Japan, China, and Australia (Coates *et al.*, 1985).

The high-precision group-delay observations available from these projects can be used to study a variety of geodetic and geophysical phenomena including tectonic plate motions and variations in the rotation of the Earth.

B. Mobile VLBI

By the late 1970s it had become clear that mobile VLBI stations could be enormously useful for surveying and for studying regional tectonic problems, such as crustal motions in the highly faulted and tectonically active regions of the western United States. The Jet Propulsion Laboratory constructed three mobile VLBI units using Mark-II recorders (MacDoran, 1979; Davidson and Trask, 1985). In the middle 1980s these mobile units were upgraded to Mark-III capability and transferred to NOAA for a series of joint NASA-NOAA observing campaigns in the western United States, Alaska, Canada, and Europe (Lyzenga *et al.*, 1986; Lyzenga and Golombek, 1986; Clark *et al.*, 1987; Ma *et al.*, 1990). They are also being used to establish a network of stations within the continental United States to serve as base stations for regional Global Positioning System (GPS) satellite observation networks (Strange and Mader, 1988).

VI. RESULTS

A. Relativistic deflection of radio signals by the solar gravity field

One of the most interesting phenomena that can be explored using the unprecedented angular resolution of VLBI is the deflection of electromagnetic radiation by the solar gravity field, as was first suggested by Shapiro (1967). Will (1990) has recently outlined the significance of such measurements for testing the general theory of relativity and for deciding between various alternative theories.

A generalized formulation of the deflection is given by Misner *et al.* (1973) as

$$\delta\alpha=\frac{(1+\gamma)M_s}{r_E}\left[\frac{1+\cos\alpha}{1-\cos\alpha}\right]^{1/2}, \qquad (9)$$

where α is the angle between the Sun and the observed object, M_s is the mass of the Sun, r_E is the Earth-Sun distance, and γ is a parameter whose value ranges from 1 in general relativity to -1 (no deflection). The magnitude of this deflection approaches 1750 milliseconds of arc at the solar limb, while the uncertainties in determining the positions of radio sources with VLBI are typically only a fraction of a millisecond of arc. In practice, fluctuations in the refractivity of the solar corona disrupt radio signals very close to the solar limb; this places a limit on the magnitude of the maximum observed deflection. However, observations in the range of about 2.5 degrees from the Sun, where the deflection is larger than 150 milliseconds of arc, are routinely achieved. Figure 3 (reproduced from Robertson *et al.*, 1991) shows the magnitude of the deflection through the solar year for the sources that are routinely observed in the IRIS-A observing campaign.

A total of 342,810 observations from the IRIS and CDP projects was analyzed to determine γ. The closest observation to the Sun was an observation of the radio source 2128-123, which was observed at 2.53 degrees from the Sun with the 7832 km baseline from Hartebeesthoek (South Africa) to Wettzell (Germany). A total of 214 observations were obtained within three degrees of the Sun, where the deflection is larger than 155 milliseconds of arc; an additional 592 observations were obtained within six degrees, where the deflection is larger than 77 milliseconds of arc.

Analysis of these observations has produced an estimate of γ of 1.0002 with a formal standard error of 0.00096 and an estimated total error of 0.002. The total error estimate was based on numerical studies of the effects of errors resulting from effects such as atmospheric refraction, nutation, Earth orientation, source structure, Earth tides and tectonic displacement, and other geophysical effects (Robertson *et al.*, 1991). The estimate is comparable in accuracy and in good agreement with the result from Mars-Viking time-delay measurements (Reasenberg *et al.*, 1979). Its importance stems partly from being completely independent of that result and partly from the fact that the measurement programs are ongoing. With improvements to the VLBI observing systems and data processing algorithms this determination of γ could improve substantially with time.

B. Baselines

One of the obvious uses of centimeter-accuracy transcontinental baseline-length measurements is to monitor global tectonic processes, to determine the motions and deformations of the continental-scale plates that form the Earth's crust. Classical determinations of plate motions have employed a variety of indirect geological/geophysical information that spans millions of years (magnetic data, transform fault strikes, etc.) to determine the long-term behavior of the crustal plates (Minster and Jordan, 1978; DeMets *et al.*, 1990), but VLBI observations have the capability of determining these motions directly from observations on time scales of a decade or less. Figure 4 shows the determinations of the length of the baseline from Westford, in Massachusetts, to Wettzell, in Germany. The observed increase in baseline length, 18.0 mm/yr, is in good agreement with the predictions of both the Minster-Jordan plate model (18.8 mm/yr; see Minster and Jordan, 1978) and the NUVEL

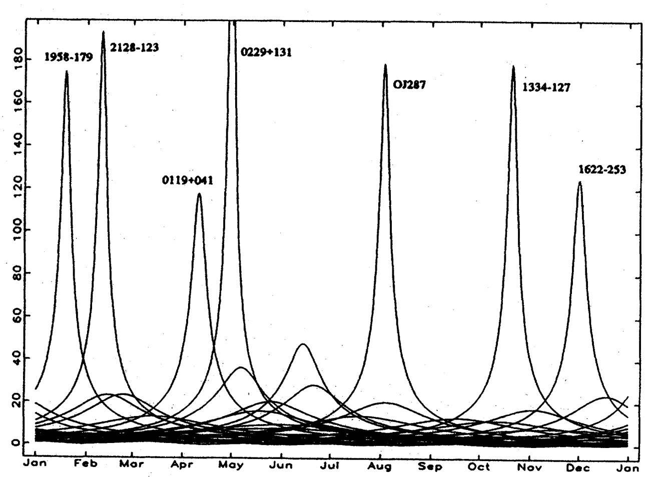

FIG. 3. Relativistic deflection in milliseconds of arc for the 26 sources that are currently observed in the IRIS program. Seven sources give peaks larger than 100 milliseconds of arc.

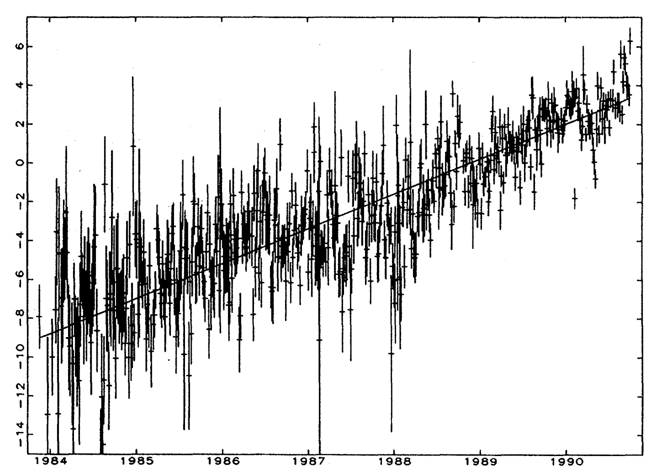

FIG. 4. Westford-Wettzell baseline-length determinations in centimeters, plotted as differences from a weighted mean. The vertical bars represent one-sigma formal errors. The best-fit straight line that is shown has a slope of 18.0 mm/yr.

model (18.9 mm/yr; see DeMets *et al.*, 1990).

There are other baselines that cross plate boundaries with faster relative velocities than that between North America and Europe, showing similar agreement with the expected plate motions. Figure 5 depicts a baseline between Hawaii and Alaska having a larger baseline rate but an order of magnitude fewer data than on the transatlantic baseline. The best-fit slope to these data is −42.5 mm/yr, in fairly good agreement with the expected plate motion from the NUVEL model of −45.4 mm/yr. Several detailed studies have shown that the VLBI observed plate motions are generally in very good agreement with the geological models whose data span so radically different a time interval, thus indicating that the long-term geological rates provide an excellent model for the current motions of the plates. These studies have begun to refine the geological models by exploring the behavior of the crust in the vicinity of plate boundaries, such as in California and Alaska, and by investigating other tectonic motions such as vertical rebound. The studies in California are particularly interesting because California is one of the rare locations where an extensive plate boundary can be studied on land. The VLBI observations in California have been used to refine the knowledge of the motion in the vicinity of the boundary and to place constraints on tectonic deformation in nearby regions such as the Basin and Range province. The motion of sites such as Vandenberg and Ft. Ord on the Pacific plate (west of the San Andreas fault) relative to sites in North America was found to be consistent with recent geological calculations for plate motions (DeMets *et al.*, 1987), and the spreading of the Basin and Range province was found to be at the level of 9-10 mm/yr (Herring, 1986a, 1986b; Herring, Shapiro, *et al.*, 1986; Clark *et al.*, 1987; Gordon, 1988; Ryan and Clark, 1988; Sauber *et al.*, 1988; Argus and Gordon, 1990).

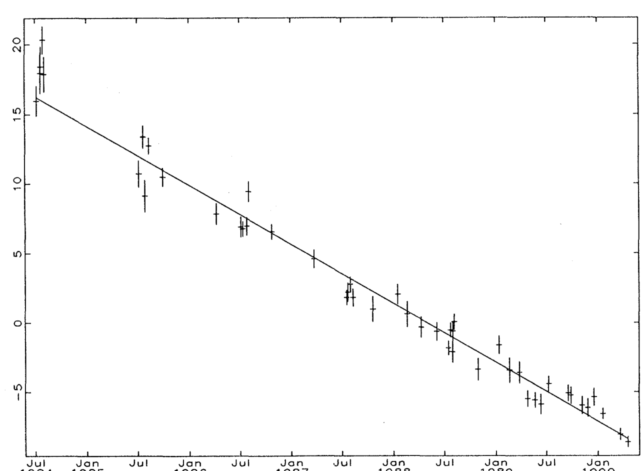

FIG. 5. Same as Fig. 3, for a Hawaii-Alaska baseline.

C. Observations of coseismic deformations associated with earthquakes

Three stations had been occupied by mobile VLBI observatories in the San Francisco area prior to the earthquake at Loma Prieta on October 17, 1989. Immediately following the earthquake the mobile observatories were dispatched to these sites for new observations. No significant offsets were observed for the Presidio and Pt. Reyes sites, the interpretation being that these sites were too far from the rupture zone. However, the third station, at Ft. Ord near Monterey, showed a motion of about 4.5 cm in a north-south direction, as seen in Fig. 6, which shows the N-S coordinates of the Ft. Ord site relative to stations on the North American plate. The epoch of the earthquake is indicated by a vertical line. The slope that is seen in the station position before and after the earthquake results from the motion of the Pacific plate relative to the North American plate. (Ft. Ord, as noted above, is on the Pacific plate.) Of course, because there were no observations between May and October, it is not possible from these data to define the epoch of the motion more precisely than a 5.5-month window. However, the observations are consistent with the offset expected at this site from calculations of the coseismic slip model (see Clark *et al.*, 1990).

D. Earth rotation

VLBI observations have revolutionized the measurement and study of fluctuations in the Earth's orientation. Classical observations of the Earth's orientation that used optical observations of galactic stars had errors typically at the level of a few tens of milliseconds of arc or about a meter of equivalent displacement at the Earth's

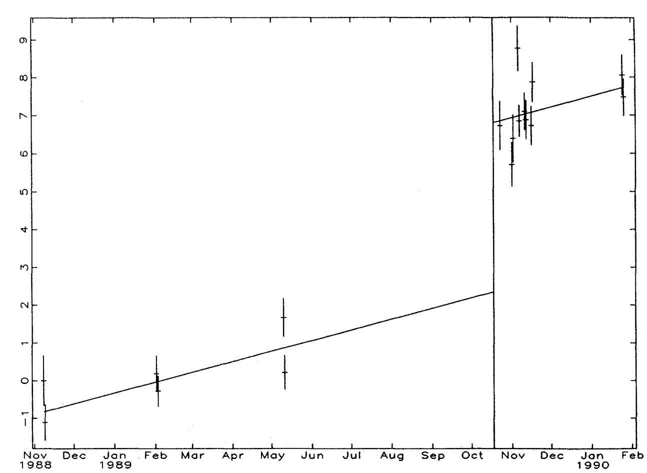

FIG. 6. Mobile VLBI determinations of the *N-S* component of the position of Ft. Ord, California, in cm, showing the offset seen at the time of the Loma Prieta earthquake, whose epoch is indicated by the vertical line in October.

surface. In contrast, VLBI determinations routinely achieve accuracies of a fraction of a millisecond of arc, or about a centimeter of displacement at the Earth's surface.

The geophysical implications of variation in the Earth's rotation have been surveyed in monographs by Munk and MacDonald (1960) and Lambeck (1980), while less detailed but more recent surveys can be found in Wahr (1986) and Lambeck (1988). The variations in the Earth's rotation are usually classified into three distinct components. The first component is termed polar motion, or wobble, and is the change in the orientation of the Earth's spin axis relative to its crust. The second component involves the Earth's total rotational phase angle, which, for historical reasons, is called universal time or UT1. [This terminology dates from the period before the early 20th century when the spinning Earth was the most accurate clock available. Fluctuations in UT1 are sometimes expressed in terms of the length of the day (LOD), which is the negative of the time derivative of UT1.] The third component of the Earth's rotation reflects changes in the orientation of the Earth's spin axis relative to an inertial reference frame in space. (The best currently available approximation to such an inertial reference frame can be defined in terms of the locations of extragalactic radio sources observed with VLBI.) These three rotation components have different sensitivities and responses to a variety of different geophysical phenomena, such as mass shifts of the atmosphere and ocean and interactions between the mantel and fluid core. Each component provides a different probe of the magnitude of these interactions.

E. Polar motion

In his classical study of the rotation of rigid bodies, Euler showed that if the spin axis of a rigid Earth were not perfectly aligned with its axis of symmetry, then the rotation axis would exhibit a free nutation about that symmetry axis with a period of about ten months. In the 1890s S. C. Chandler discovered that this polar motion actually had a period of about 14 months and an amplitude of a few tenths of an arc-second, and that it was mixed with an annual motion with a comparable amplitude. Simon Newcomb quickly showed that the increase in the period from 10 to 14 months could be explained qualitatively as resulting from the effects of the nonrigidity of the Earth.

The three fundamental geophysical problems that are associated with the Earth's wobble relate to the value of its period, the value of its damping factor, Q, and the nature of the excitation mechanism that maintains the motion against damping over geological time scales. These problems are tightly coupled, as it is difficult, for example, to determine the damping factor without knowledge of the excitation mechanism and vice versa. Jeffreys (1958) and Smith and Dahlen (1981) noted that the Chandler wobble provides a probe of the anelastic properties of the Earth at a period ($\sim$435 days) far removed from the seismic frequencies that are commonly employed to determine the structure and rheological properties of the Earth. Recent determinations of the Q of the Chandler wobble using classical observations are in the range of 50–400 (Jeffreys, 1968; Currie, 1974, 1975; Wilson and Haubrich, 1976; Ooe, 1978). A full exploitation of the geophysical information provided by the motion of the Earth will require an accurate determination of the excitation mechanism(s).

The excitation of the Earth's wobble generally results from mass shifts within the Earth. (External torques will also excite a wobble motion, but generally at a much smaller level.) If we start with an Earth whose spin axis is perfectly aligned with its principal axis of rotational symmetry, so that it exhibits no wobble motion, then virtually any mass shift within the Earth (other than the exceptional case of a mass shift that is symmetric with respect to the spin axis) will change the orientation of the axis of symmetry and move it away from the spin axis. The spin axis will then begin its slowly damped wobble about the new principal axis.

Geophysicists have speculated for years about the nature of the mass shifts that are required to maintain the wobble against damping. The mass shifts associated with large earthquakes were ruled out by Munk and MacDonald (1960, pp. 163, 164) using a simple argument based on crustal block movement that indicated that the tensor changes were about a factor of 100 to 10 000 too small. But Press (1965) pointed out that earthquake strain fields were much larger than had been realized. Mansinha and Smylie (1967, 1968) claimed to have found a correlation between the epochs of large earthquakes and corresponding shifts in the pole centroid. However, in Stacey's words, "Both convincing observations and realistic calculations indicating adequacy of the earthquake excitation have been difficult to produce" (Stacey, 1977, p. 69). Chao and Gross (1987) concluded that "The computed changes in the Earth's global geodetic/gravitational parameters induced by the earthquakes during 1977–1985 are in general two orders or magnitude smaller than the observed values that are available." Meteorological shifts of atmospheric masses are another possible source of the excitation mechanism that has received extensive study (Munk and Hassan, 1961; Wilson and Haubrich, 1976; Wahr, 1982, 1983; Barnes *et al.*, 1983; Hide, 1984). A related possibility involves changes in the distribution of surface and ground water resulting from changes in rainfall and snow cover (Wilson and Hinnov, 1985; Chao *et al.*, 1987; Hinnov and Wilson, 1987; Chao, 1988; Chao and O'Connor, 1988). Lambeck noted that these meteorological effects are highly detrimental to the use of Earth rotation measurements for studying the properties of the solid Earth. He writes: "Without corrections for this meteorological noise, high-precision and high-resolution observations of the Earth's rotation loose [*sic*] much of their interest," (Lambeck, 1988, p. 15). Interpretation of all of these excitation mechanisms is hampered in various ways by the difficulty in obtaining the necessary raw data, such as data concerning the detailed strain fields associated with earthquakes or snow cover depths over central Asia. The nature of the excitation mechanism for the Chandler motion remains one of the major unsolved problems in geophysics.

The major effect of the new VLBI observations on studies of the excitation of the Earth's wobble is to reduce the observational errors to the point that their magnitude is negligible compared to the effects of the forcing function being studied. This is no way reduces the difficulty of separating the various meteorological and solid-Earth effects, but it does eliminate one large component of the analysis of the earlier pole-position data. The VLBI determinations of the pole position from the IRIS five-day sessions from January, 1984 to December, 1990 are shown in Fig. 7. The formal errors of the determinations are in the range of 0.5 millisecond of arc and are shown as barely visible crosses in the figure. The spiral motion seen in the figure is not related to damping of the Chandler motion, but rather results from a 6-yr beat phenomenon between the 14-month Chandler motion and a comparable-magnitude 12-month motion caused by seasonal atmospheric effects.

To place a bound on the true accuracy of the VLBI pole-position determinations, we have compared them with comparable determinations from satellite laser ranging (SLR). The rms differences between the two series have been found to be less than 2 milliseconds of arc (Robertson, Carter, Tapley, *et al.*, 1985). The differences are too small to display on the scale of Fig. 7, but Fig. 8 shows the VLBI and SLR residuals to a generalized pole-position model, described by Eqs. (10) and (11). The model assumes two elliptical contributions with annual and Chandler frequencies, respectively, and two terms of a polynomial to account for possible long-term drift.

$$x=\sum_{k=1}^{2}[A_{x,k}\cos(\omega_k t)+B_{x,k}\sin(\omega_k t)]+x_0+x_1 t , \quad (10)$$

$$y=\sum_{k=1}^{2}[A_{y,k}\sin(\omega_k t)+B_{y,k}\cos(\omega_k t)]+y_0+y_1 t . \quad (11)$$

The summation is taken over the annual and Chandler frequencies. For a perfectly axisymmetric Earth, this model could, in principle, be written as circular rather than elliptical motion, eliminating the out-of-phase (B) coefficients. Departures from axisymmetry in both the structure of the Earth and the excitation functions necessitate the use of the extra terms. It is clear from Fig. 8 that the residual (irregular) motion of the pole, presumably resulting at least in part from the poorly known stochastic excitation process(es), is at least an order of magnitude larger than the differences between the two separate determinations that bound the total observational errors. However, the amplitude of these irregular motions is about 30 milliseconds of arc, or about 1 m. This is roughly the size of the errors in the classical optical determinations of the pole position. With the new observations, the investigation of these irregular excitations can now proceed virtually unhampered by observational errors.

F. UT1

The physical causes of the variations in UT1 can best be understood in the context of the total angular momentum of the Earth, **L**. If we write **L**=**I**ω, where **I** is the inertia tensor of the Earth and ω is the spin vector, then it is clear that UT1 variations, which entail changes in the magnitude of ω, can be caused by either changes in the total angular momentum of the solid Earth **L** or changes in the inertia tensor **I**. Both types of effect have

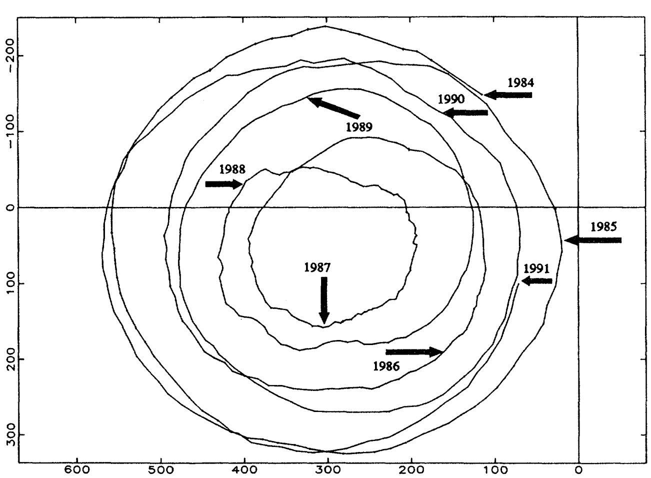

FIG. 7. VLBI determinations of the pole position, 1984–1990. Scales are in milliseconds of arc. The linear scale of the spiral figure is about 15 m across its widest diameter.

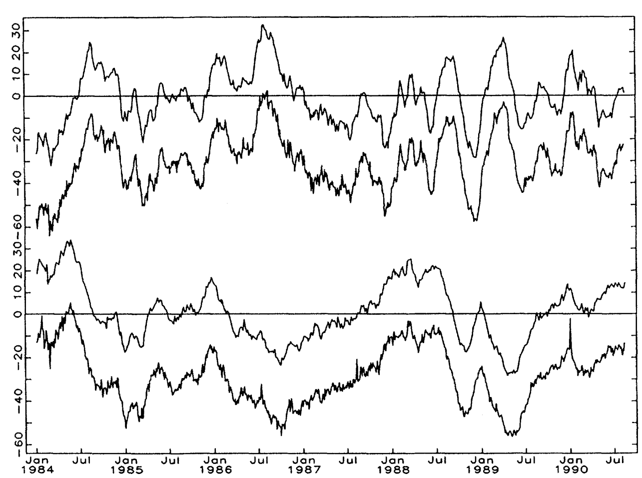

FIG. 8. X-component (top) and Y-component (bottom) pole position residuals in milliseconds of arc from VLBI observations and SLR observations. The SLR residuals have been offset downward by 30 milliseconds for plotting clarity.

important consequences for the observed variations in UT1.

The largest short-period changes in the inertia tensor of the Earth are caused by tidal effects. Both the Sun and Moon raise a tidal bulge in the solid Earth. Because those bodies are not restricted to the Earth's equatorial plane, the symmetry axes of the tidal bulges will follow the motion of those bodies into and out of the equatorial plane. Such motions of the tidal bulge will cause significant changes in the moment of inertia of the Earth and will produce UT1 variations with amplitudes up to a millisecond of time with periods ranging from a few days to a year.

Figure 9 shows the theoretical tidal variations in UT1 as calculated by Yoder *et al.* (1983), superimposed on measurements of UT1 from daily 45-min observing sessions using the Westford-Wettzell baseline (see Robertson, Carter, Campbell, and Schuh, 1985). The observed UT1 values have had a linear drift removed and have been high-pass filtered to remove variations longer than 100 days. The signature of the tidal variations, sinusoidal terms with amplitudes up to 1 millisecond of time ($\sim$40 cm) and periods ranging from a few days to months, can be clearly seen in the observations. It is equally clear that other variations (largely meteorological in origin; see below) are also present in the time series.

The geophysical significance of these tidal variations in UT1 lies in the fact that the driving force (tidal gravity) is well known, and the amplitude of the response to this force is sensitive to the rheological properties of the Earth. In particular, anelastic effects will tend to amplify the response of the Earth, because, other factors being equal, an anelastic Earth will experience slightly less restoring force than a purely elastic one, and its response to a disturbing force will therefore be slightly greater. Wahr has calculated the effects of anelasticity on the

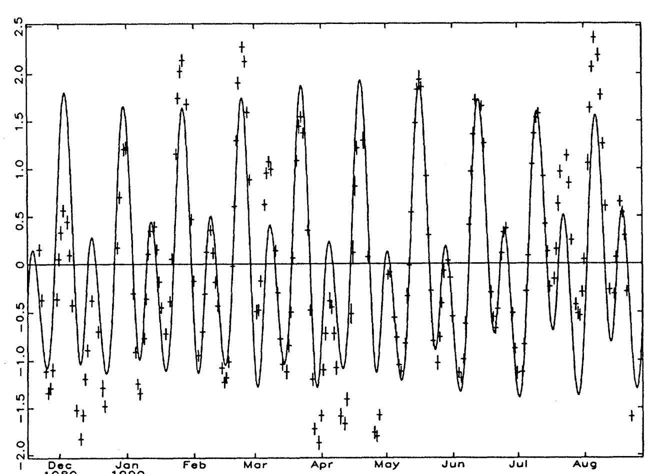

FIG. 9. Predicted Earth tide effects (solid curve) on UT1 in milliseconds of time. The crosses show the observed values from the IRIS daily observing sessions. The vertical bars show one-sigma formal errors.

UT1 tidal amplitudes (Wahr and Bergen, 1986; Wahr, 1988). He comments that (similar to the polar motion effects) the tidal signatures in UT1 provide a probe of the anelastic properties of the Earth at a frequency range that is far from the range probed with seismic observations (or that of the Chandler wobble).

A study of the tidal amplitudes seen in the daily VLBI observations showed that the formal uncertainty of the estimated tidal amplitudes is at least an order of magnitude smaller than Wahr's calculated effects of anelasticity in the mantle (Robertson *et al.*, 1988). In other words, the uncertainties in the VLBI observations are a negligible part of the problem. Unfortunately, the study also showed that the variability of the tidal amplitudes estimated from different subsets of the data is large compared with the formal uncertainties and compared with the magnitude of the anelastic effects that are of interest. At least part of these large variations have been tentatively ascribed to effects of ocean water motions on UT1. The oceans obviously respond strongly at the tidal frequencies. Separation of ocean effects from solid-Earth effects will require detailed studies of ocean tidal motions, which have only recently been attempted (Brosche *et al.*, 1989).

The other important component of UT1 variation is caused by changes in the angular momentum of the solid Earth; these changes, of course, represent an exchange of angular momentum with other physical bodies. The most important of these exchanges involve interactions with the Sun and Moon through tidal torques, and momentum exchanges with fluid components of the Earth, especially the fluid core, the oceans, and the atmosphere. The tidal torques produce a very slow decrease in the rotation of the Earth, important on time scales of centuries and best studied using a variety of historical and paleontological evidence. These effects are of little interest for the short time span over which VLBI observations are available. Similarly, effects of the momentum of the fluid core are expected to have characteristic time scales of decades and are not yet of much interest in the analysis of the VLBI UT1 results (see Stacey, 1977, pp. 65–67).

Momentum exchanges with the atmosphere are a different story. The atmosphere has been found to control most of the variability of the rotation of the Earth on time scales significantly shorter than a decade (Barnes *et al.*, 1983; Rosen and Salstein, 1983); Fig. 10 shows the variations in length of day (LOD) contrasted with the variations implied by changes in the total angular momentum of the atmosphere as calculated from detailed numerical models by the U.S. National Meteorological Center. The agreement between the curves is striking. The 50-day periodicity that was first found in Earth rotation studies has been shown to result from atmospheric effects (Langley *et al.*, 1981). Important atmospheric phenomena such as El Niño are also reflected in the rotation of the Earth (Carter *et al.*, 1984; Rosen *et al.*, 1984; Chao, 1989). Recent studies have shown that nontidal

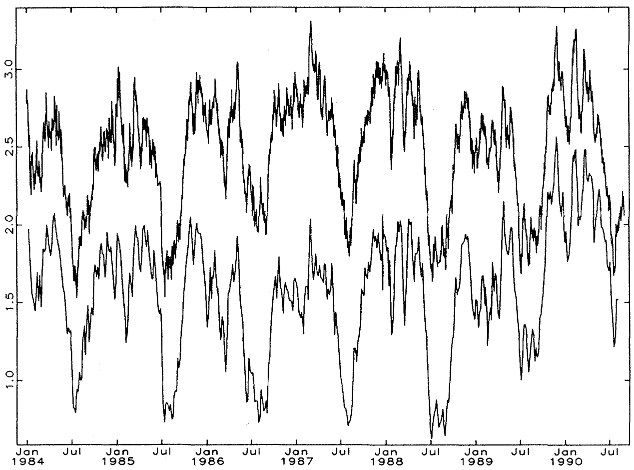

FIG. 10. LOD variations inferred from atmospheric angular momentum calculations (top) and IRIS VLBI observations (bottom) in milliseconds of time. The curves are offset by about a millisecond for plotting clarity.

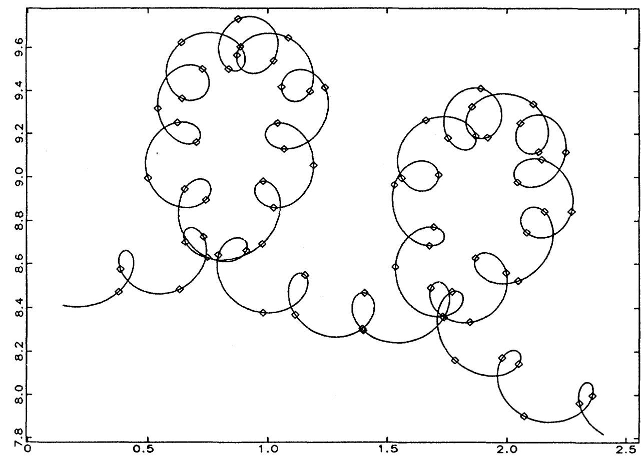

FIG. 11. One-year trace of the path of the celestial pole. The scales are in seconds of arc, with longitude on the horizontal axis and obliquity on the vertical. The diamonds mark the IRIS 24-hour observing sessions spaced at 5-day intervals.

variations in LOD with periods ranging from two years down to about 14 days are well correlated with observed changes in the angular momentum of the atmosphere. Below 14 days the correlation disappears (Rosen *et al.*, 1990). Studies are ongoing to try to determine whether the lack of correlation at periods less than 14 days is due to inaccuracies in the numerical atmosphere models on which the angular momentum calculations are based, or whether some nonatmospheric phenomena, perhaps related to ocean effects, are beginning to dominate the rotational spectrum of the Earth at these periods.

G. Nutation

The Earth's nutations are similar to tidal variations in UT1 in that the driving mechanisms are well known. The nutations are caused by tidal torques of the lunar and solar gravity fields acting on the Earth's equatorial bulge. The theoretical effects of these torques on realistic models of the Earth have been calculated in great detail by Wahr (1981b), whose model for nutation has been adopted by the IAU as the standard to be used for the reduction and analysis of precise astrometric measurements. Figure 11 shows a 1-year sample of the Wahr model. The large loops have a semiannual period and result from the shift of the Sun from the northern to the southern hemisphere. The smaller loops have a fortnightly period and result similarly from the orbital motion of the Moon. The dates of the IRIS 5-day VLBI observing sessions are shown as diamonds on the plot.

Figure 12 depicts the residuals of the VLBI determinations from the Wahr model. Notice that the scale, in milliseconds of arc, is about three orders of magnitude smaller than the scale in Fig. 11. In effect, the errors in the Wahr model calculations are much smaller than the width of the line shown in that figure. Nevertheless, small as they are, the residuals shown in Fig. 12 contain a wealth of information about the structure and behavior of the Earth.

The dominant and obvious feature of the residuals seen in Fig. 12 is an annual signature that results from a resonance with a natural nutation mode of the Earth, sometimes called the free-core nutation (FCN). This mode is caused by the presence of a fluid core in the Earth constrained within an ellipsoidal cavity defined by the core-mantle boundary (CMB), as shown schematically in Fig. 13. The outer ellipse represents the surface of the Earth, and the inner ellipse represents the CMB. The positions of the symmetry axes of the ellipses are shown with small tick marks. If the rotation axis of the fluid core were misaligned with the symmetry axis of the CMB (as indi-

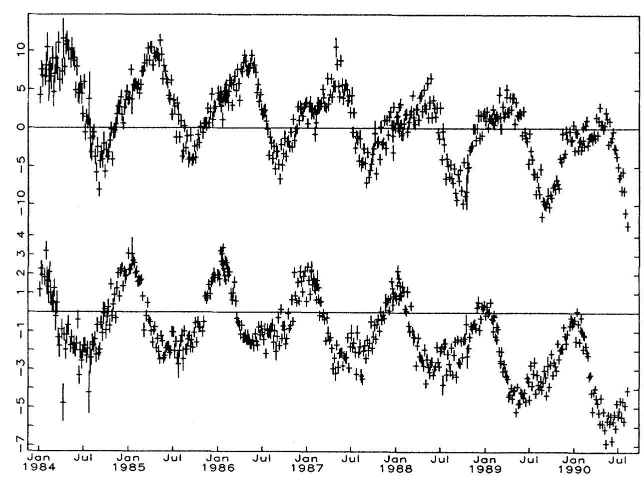

FIG. 12. VLBI corrections to the IAU 1984 nutation series, in milliseconds of arc. Corrections in longitude at the top, obliquity at the bottom.

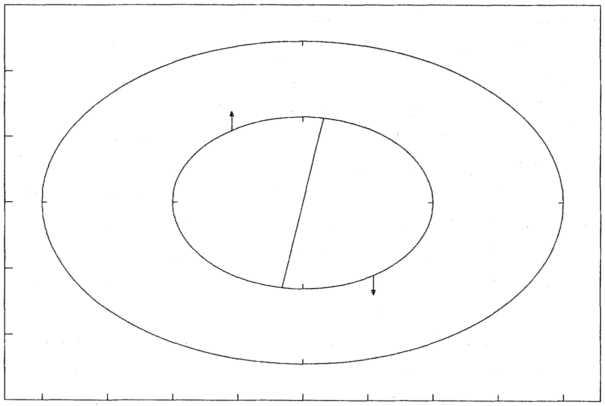

FIG. 13. Schematic diagram illustrating the effect of a rotating fluid core inside an ellipsoidal core-mantle boundary. The rotation axis of the fluid core is shown by the diagonal line.

cated by the diagonal line in Fig. 13), then the rotating fluid would exert a torque on the boundary, as indicated by the arrows shown in the figure. This torque would cause a precession-like motion of the mantle with a period that was calculated to be about 460 days. (Because of conservation of angular momentum, the fluid core would experience an equivalent motion 180 degrees out of phase with the mantle and with an amplitude that scales by the ratio of the moments of inertia of the core and mantle. See Toomre, 1974 and Rochester *et al.*, 1974.)

The Earth models used by Wahr to calculate the period of the FCN assumed that the shape of the CMB was such that the boundary was in hydrostatic equilibrium. Gwinn and his colleagues showed that if the CMB deviated from hydrostatic equilibrium by about 0.5 km, then the frequency of the FCN would shift to about 430 days, and this shift would bring the resonant frequency close enough to the 365-day nutation to significantly alter the response of the Earth at that frequency and produce the residual signature seen in Fig. 12. Part of the significance of this work lay in the fact that geophysicists concerned with convection processes in the Earth's mantle had suggested that such convection would perturb the shape of the CMB away from hydrostatic equilibrium by about 1.5 km (Hager *et al.*, 1985). The VLBI observations seem to completely rule out such a large deviation. The consequent change in the annual nutation would have been far outside the errors in the VLBI determinations of that amplitude (Gwinn *et al.*, 1986; Herring, Gwinn, and Shapiro, 1986).

Fourier analysis of the residuals shown in Fig. 12 shows other periodic structures, especially at semiannual and 13-day periods. Investigations are currently underway to determine what modifications are needed in the detailed Earth structure models used in the theoretical nutation calculations to account for these deviations from the theory. The 13-day terms may result from effects of tidal motions of ocean water, which were not considered in the nutation theory, and part of the remaining discrepancies may result from effects of the Earth's solid inner core (de Vries and Wahr, 1991; Herring *et al.*, 1991; Matthews *et al.*, 1991a, 1991b).

H. High-frequency variations of Earth rotation parameters

Recent work has begun to focus on the essentially unexplored realm of subdiurnal variations in the rotation of the Earth. VLBI observations have been shown to have sufficient sensitivity and time resolution to probe these variations and provide useful interesting information. The observed subdiurnal variations have amplitudes of a few hundredths of a millisecond of arc and frequencies characteristic of tidal interactions. Preliminary analysis seems to indicate that the effects may be caused by the mass shifts associated with tidal motions of the water in the oceans, although atmospheric effects cannot yet be ruled out (Dong and Herring, 1990).

I. Earth tide Love numbers

The elastic Earth is perturbed by the same component of the disturbing gravitational potential of the Sun and Moon that raises tides in the ocean. The response of the Earth to this disturbing potential is characterized by a set of scalar values called Love numbers (named for the English geophysicist A.E.H. Love) that scale the response: h scales the vertical displacement of the surface and l scales the horizontal displacement. (A third Love number g which scales the gravitational response, has no effect on VLBI observables.) The magnitude of the solid-Earth displacement due to these tidal stresses is about 30 cm in the vertical direction and 1–3 cm in the horizontal directions. Studies of estimates of the Love numbers from VLBI observations (Robertson, 1975; Herring *et al.*, 1983; Sovers *et al.*, 1984; Carter *et al.*, 1985; Herring, 1986a; Ryan, *et al.*, 1986) have shown values that are in good agreement with theoretical values calculated from the known rheological parameters of the Earth (Wahr, 1981a); but, as Herring put it, "The geophysically interesting aspects of Earth tide measurements . . . seem to be below the level where they can be reliably studied by using the VLBI . . . . Improvements in the VLBI models (e.g., atmospheric refraction models) could allow such small displacements to be measured in the future" (Herring, 1986a, p. 183).

VII. FUTURE WORK

A. Global reference frames

The revolutionary capability of VLBI observations to determine the vector components of transcontinental baselines with accuracies at the centimeter level provides the promise of producing a global geodetic reference frame (i.e., a set of stations whose coordinates and velocities are known) accurate to this level. The existence of such a frame would open the possibility of studying a variety of important phenomena including sea-level variations (see below) and regional and global tectonic motions which previously were unobservable because of the inaccuracies of global surveying techniques. There are in fact two observing systems that could produce such a reference system—VLBI and SLR observations of the LAGEOS and satellite. But VLBI has substantial advantages over SLR observing systems, primarily the ability to connect to a nearly inertial reference frame defined in terms of distant, extragalactic radio sources. In contrast, the SLR frame is tied to the dynamics of the satellite orbit, which is perturbed by forces (e.g., from gravitational effects of ocean tides, or from radiation pressure of light reflected off cloud cover on the Earth) that are not well known at the level of interest here. VLBI also has the ability to take measurements in all but the most extreme weather conditions (SLR is hampered by cloud cover), and the ability to observe a variety of radio sources that cover the sky fairly uniformly, rather than being restricted to observations of a satellite that is below the horizon most of the time. In counterbalance, the SLR determinations are tied to the location of the center of mass of the Earth, to which the VLBI observations are insensitive. (VLBI observations of Earth satellites could, in principle, be used to determine the center of mass. However, such observations are not presently being made routinely.) The translational origin of the VLBI reference frame is tied to some subset of stations on the surface of the Earth.

Only two problems hinder the immediate establishment of a global VLBI reference network. First, the distribution of extant VLBI observing stations is not presently uniform across the globe; the distribution is particularly weak in the Southern Hemisphere and in the Central–Asia-Indian Ocean region. This problem is being partially addressed by a number of projects that should produce new observatories in South America and in the Antarctic, but it will be a number of years before reasonably uniform global coverage of VLBI stations exists.

The second problem is that VLBI stations are not very portable. Although both mobile and transportable VLBI systems have been constructed and used, they tend to be large and are expensive to transport and use. One possible solution to this problem involves the use of GPS satellite observations, as discussed in the next section.

B. Global positioning system

The basic reason that VLBI observatories tend to be large, expensive, and not very portable is that the extragalactic radio signals are extremely feeble, typically 0.5 jansky (1 jansky $=10^{-26}$ watts $\mathrm{m}^{-2}\,\mathrm{Hz}^{-1}$), and thus require large antennas and sophisticated data recording systems. The obvious cure for the problem is to produce artificial signals from satellites that would serve the same purpose as the natural signals but that would be strong enough to be picked up on small, inexpensive antenna/receiver systems. This is the basic idea behind the Global Positioning System (GPS) satellite network, which is designed to deploy a constellation of 18 satellites in three orbits in such a way that at least four satellites will always be visible at any point on the Earth.

Some of the problems with the GPS network are similar to SLR: the coordinate frame is tied to the satellite orbits, which must be precisely known. Moreover, the satellites transmit at frequencies around 1200 and 1500 MHz, where the highly variable ionosphere can introduce refractive effects at the meter level. The coordinate frame problems can be addressed by determining the satellite orbits from data collected at tracking stations collocated with a global network of VLBI stations. And the ionosphere problems are partly dealt with by the use of two frequency channels. Recent work has shown that this system, using small, inexpensive, and portable ground stations, can determine baselines of several hundred kilometers length with standard errors of a centimeter or better (Davis *et al.*, 1989). For a full discussion of the GPS system and its current accuracy levels, see, e.g., Leick (1990). Further refinements of this system may allow it to effectively densify the geodetic network formed from a sparse grid of VLBI stations.

C. Sea level and global change

One of the major concerns in the problems involving global climate change, greenhouse effect, and global warming involves the change in sea level. The Antarctic and Greenland glaciers contain enough ice to raise sea level by about 100 meters if they were to melt, which would produce catastrophic consequences: a large fraction of the population of the world presently lives at elevations less than 100 meters above sea level. But measuring the present rate of change of ice volume or, alternatively, of sea level is not an easy problem, and one of the fundamental difficulties in such measurements involves vertical tectonic motions of the land surface that affect the readings of tide gauges that measure changes in sea level.

Douglas and Herbrecthsmeier (1989) have already shown that correction of tide gauge records for the effects of postglacial rebound in North America and Europe substantially improves the agreement in the long-term secular sea-level trends from those gauges. But postglacial rebound is only one component of the tectonic motion that might affect the station position. The only way to be certain that all tectonic motions have been correctly removed is to directly measure the vertical motion of each station, and doing that will require the global geodetic network described in the previous section. For a full discussion of the use of VLBI/GPS observations to correct tide gauge observations, see Carter *et al.* (1986; 1989).

VIII. CONCLUSIONS

VLBI observations have opened new windows onto a variety of interesting global phenomena that were too small to be observed with classical geodetic observations. Although the interpretation of many of these observations is still at a preliminary stage, many important discoveries have already been made, especially concerning the effects of the shape of the core-mantle boundary on the nutation of the Earth. But perhaps the most important and exciting feature of the VLBI observations is that their accuracy has generally far outstripped our present ability to interpret them. They thus provide great incentives for the theoreticians to construct the more detailed models of the Earth and its atmosphere and oceans that will be needed to explain the variety of phenomena seen with the VLBI observations. Unraveling the details of the complex interactions of the elastic and fluid components of the Earth is a difficult and daunting problem, but it is a problem that is well matched to the rapidly expanding number-crunching capability of modern computers. In some areas a great deal of progress has already been made. One of the more difficult tasks, computer modeling of the total angular momentum of the atmosphere, has already provided insight into the spectrum of the interactions between the atmosphere and the solid Earth. There is every reason to expect that, as the detailed numerical models of the interactions between the atmosphere, oceans, and solid Earth become more sophisticated, further insight will be provided into the dynamics of the Earth and its constituent parts. In time the models should allow us to calculate the effects of these interactions at a level commensurate with the accuracy of the present VLBI observations.

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