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Recent Developments in Architectural Acoustics

VERN O. KNUDSEN, *University of California at Los Angeles*

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§1. INTRODUCTION

ARCHITECTURAL acoustics is no longer enveloped by mystery or uncertainty; it is securely grounded on physical facts and principles. These facts and principles have evolved from the contributions of a relatively small number of physicists and engineers. Without these contributions, which have come largely from physicists during the past thirty-five years, architectural acoustics would yet be empirical, based upon generalizations from geometrical acoustics and a few qualitative observations regarding the reflective or absorptive properties of building materials and interior furnishings. The practical accomplishments which have come from recent studies in architectural acoustics furnish a conspicuous example of the value of applying physicists and their technique to neglected or undeveloped branches of technology, even in these days of intense interest in atomic physics. Thirty-five years ago the acoustical designing of buildings, if considered at all, was no more than an attempt, which was often unsuccessful, to imitate certain designs which had gained public acclaim or to avoid others which were reputedly poor. Today acoustical designing is based upon reliable formulas and quantitative data regarding the sound-absorptive and sound-insulative properties of building materials, as well as the shape of the building, so that the outcome in the acoustics of a building can be predicted in advance of construction; or, what is more important, any specified acoustical requirements can be practically realized in the completed structure.

Architectural acoustics is thus approaching the status of an applied art and eventually it will become a properly endowed member of the engineering family. But, unfortunately, there are as yet no well-developed departments of acoustical engineering in our educational system; and therefore, although a number of large corporations, interested in the manufacture and sale of acoustical instruments or materials, have made important contributions, especially in the development of instruments for investigating the acoustical properties of rooms, the immediate development of architectural acoustics is largely dependent upon the researches of physicists. Until architectural acoustics becomes an established and fostered art in our engineering schools it is greatly to be desired not only that those physicists who are now working in this field will continue their researches but that additional workers will be attracted to the many unfinished and fruitful problems which await the interested and qualified investigator.

Because of this situation, it will be the purpose of the present paper not only to review recent developments which have resulted from the application of the tools and methods of physics to the acoustics of buildings but also to suggest unfinished or unsolved problems which hold promise of yielding important results.

§2. Early Developments in Architectural Acoustics

The author has given elsewhere[1] an account of the evolution of the auditorium and the early beginnings of architectural acoustics in the nineteenth century. Although many of the basic problems pertaining to the acoustics of Greek and Roman theatres are discussed intelligently in the famous *Ten Books on Architecture* by Vitruvius, it was not until the nineteenth century that serious study was given to the acoustical problems of architectural interiors and these studies were only of a qualitative nature. Notable among the investigators of the nineteenth century were (1) Dr. J. B. Upham[2] (an M.D.), who investigated the reverberatory properties of the main auditorium of the Boston Music Hall during the completion and furnishing of the room and who recommended the use of additional upholstery, or even canvas on the walls, as a means of reducing reverberation; (2) the distinguished Joseph Henry,[3] who described lucidly the factors which affect reverberation, echo and acoustical "flutter" and who designed a new lecture room for the Smithsonian Institution, based upon his theories and experiments, which was accredited as having highly satisfactory acoustics; and (3) Mr. T. Roger Smith,[4] an English architect, who recognized clearly the different acoustical requirements for speech rooms and music rooms. Mention also should be made of the customary prescience of Lord Rayleigh[5] who recognized the possibility of controlling reverberation in rooms by the use of carpets, hangings and furnishings. But all of these findings and speculations were only of a qualitative nature.

§3. The Modern Era of Architectural Acoustics

The quantitative development of architectural acoustics began with W. C. Sabine in 1895 when he undertook an investigation of the acoustics of Fogg Lecture Hall at Harvard University.[6] Sabine soon discovered that reverberation was the most important factor affecting the acoustical quality of a room and he devoted a major portion of his remaining days (until his untimely death in 1918) to a quantitative study of the growth and decay of sound in an enclosure. He derived an equation and determined coefficients of absorp-

[1] V. O. Knudsen, *Architectural Acoustics*, John Wiley and Sons (1932). See especially Secs. 4, 5 and 6. See also Bagenal and Wood, *Planning for Good Acoustics*, Methuen (1931); and P. E. Sabine, *Acoustics and Architecture*, McGraw-Hill (1932).

[2] J. B. Upham, *A Consideration of Some of the Phenomena and Laws of Sound, Their Application in the Construction of Buildings, Designed Especially for Musical Effects, Am. J. of Science and Art*, **65**, 215–226, 348–363; **66**, 21–23 (1853).

[3] Joseph Henry, *Acoustics Applied to Public Buildings*, Smithsonian Reports (1854 and 1856).

[4] T. Roger Smith, *Acoustics of Public Buildings* (1861).

[5] Rayleigh, *Theory of Sound*, Vol. II, p. 128, Macmillan (1926).

[6] An excellent account of his work is described in his *Collected Papers on Acoustics*, Harvard University Press, (1922).

tion of building materials, which made it possible to calculate the reverberatory properties of a room either before or after construction, and made fundamental contributions to a better understanding of the pertinent factors which affect the acoustical properties of all rooms. He is properly honored by his colleagues in all countries as the founder of architectural acoustics. His work not only made possible the design, construction or remodeling of rooms so as to provide them with good acoustics but it also attracted the attention of other investigators who have taken up his unfinished work, have improved his reverberation formula and have made other important discoveries regarding the insulation of sound, the amplification of sound, room resonance and the optimal reverberation for speech and music rooms. All these factors have an important bearing upon the problem of room acoustics but the most basic factor of all is the problem so simply and elegantly solved by W. C. Sabine, the growth and decay of sound in a room.

§4. The Growth and Decay of Sound in Rooms—General Considerations

Obviously, a rigorous treatment of the growth and decay of sound in an enclosure should be based upon the general theory of vibration in a bounded three-dimensional continuum. Historically, however, much simpler, but admittedly only approximate, treatments have been the vogue. But even these approximate theories, when used with *caution* and *understanding*, have served satisfactorily for the practical purposes of acoustical designing and they will continue so to serve until they are superseded by more exact theories.

These approximate theories have been based upon a faulty premise, one which is equivalent to assuming that sound, originating at some point in a room, propagates quanta or rays of vibratory energy, uniformly in all directions; that these rays are partially reflected by the boundaries of the room; and that even after the source of sound is stopped these rays persist with their original frequency but become feebler after each reflection, in a manner quite similar to a large number of billiard balls moving with constant speed on a billiard table but shrinking in size after each reflection until ultimately they become vanishing small. That is, in these approximate theories, at least in the later and better ones, it is assumed that the sound energy persists in rays or bundles; that during the decay the sound energy in the room remains constant in these rays or bundles for a short interval of time, equal to the time for a ray of sound energy to travel the average distance between successive reflections, called the "mean free path," and then the total sound energy in the room suddenly drops a certain amount, determined by the "average" absorption coefficient of the boundaries of the room; and this process of absorption by discreet steps continues until the sound energy is all converted into heat. The formulas to which such approximate theories lead are sufficiently valid for all practical purposes in rooms which are bounded by materials having the same coefficient of absorption. However, in rooms bounded by materials having widely different coefficients of absorption, such as are commonly found in practice, the formulas approach validity only as the decadent sound in the room is made to approach a completely diffuse state. It should be clearly recognized, therefore, that these formulas which we are now ready to derive must be used with *caution* and *understanding*, especially with reference to the average coefficient of absorption in rooms which are bounded partly by highly reflective materials and partly by highly absorptive materials.

§5. The Reverberation Theory of Sabine and Jaeger

The early experiments of W. C. Sabine[6] showed that the time of reverberation in a room (that is, the time required for the sound intensity to diminish 60 db, or to one-millionth of its initial intensity, during free decay) is proportional directly to the volume of the room and inversely to the total amount of absorption supplied by the boundaries of the room, namely $\Sigma\alpha s$, where α is the coefficient of sound absorption of any portion of the boundary having an area s and where the summation is extended to the entire boundary of the room. By a series of ingenious experiments in different rooms Sabine was able to determine the constant of proportionality k between reverberation time t_{60} and volume V divided by total absorption a and he gave to architects and builders the simple but epochal equation

$$t_{60}=kV/a. \quad (1)$$

The value of k as determined experimentally by Sabine for a large number of rooms of different shapes and sizes, at normal room temperature, was 0.05 in British units and 0.164 when V is in cu. m and a is in sq. m.

A few years later, Jaeger[7] obtained this same equation from theoretical considerations based upon a method which had been most fruitful in the kinetic theory of gases. He obtained an equation for the rate at which diffuse sound energy in an enclosure strikes unit area of the boundary, namely $\rho c/4$, where ρ is the average volume density of sound energy in the room and c is the velocity of sound.[8] He next assumed that the rate of absorption of sound energy by the boundaries of the room is $(\rho c/4)\Sigma\alpha s$, a plausible assumption provided the sound energy is completely diffuse during the decay and provided further that ρ changes continuously.[9] (This will be approximately realized when the decay occurs slowly, that is, when the boundaries of the room are highly reflective so that many reflections are required to degrade the sound energy into heat.) Jaeger then equates the rate of change of sound energy in the room $V(d\rho/dt)$ to the difference between the rate of emission of sound energy by the source E and the rate of absorption of sound energy by the boundaries of the room. That is,

$$V(d\rho/dt)=E-(\rho c/4)\Sigma\alpha s.$$

The solution of this equation, introducing the appropriate boundary conditions and remembering that $a=\Sigma\alpha s$, gives for the *growth* of sound energy

$$\rho=\rho_0(1-e^{act/4V}); \quad (2)$$

for the *decay* of sound energy

$$\rho=\rho_0 e^{-act/4V}; \quad (3)$$

and for the average value of the steady state energy density ρ_0

$$\rho_0=4E/ca. \quad (4)$$

If Eq. (3) be solved for t when $\rho/\rho_0=10^{-6}$, the result will give the time of reverberation, t_{60}, namely,

$$t_{60}=kV/a \quad (5)$$

and the value of k, which depends only on the velocity of the sound, is, at normal room temperature, 0.049 in British units and 0.161 in metric units, in very good agreement with the experimental value obtained by Sabine [see Eq. (1)]. The excellence of this agreement furnished such a convincing confirmation of Sabine's empirical equation that it was used for nearly 30 years for calculating the reverberation time of either contemplated or finished rooms. Even the fallacious conclusion to which the equation leads for a room with totally absorptive surfaces, namely, that $t_{60}=kV/S$ instead of zero (where S is the total surface area of the room) was overlooked or was not sufficiently disturbing to destroy confidence in the validity of the Sabine-Jaeger equation, until recently. It should be mentioned, however, that the equation is in practice satisfactory for frequencies between about 200 and 1000 cycles in the large majority of rooms in which the rate of decay of sound is slow; that is, the equation applies to "live" rooms, provided the frequencies are high enough to be well above the fundamental resonant frequency of the room but not high enough to involve a consideration of the attenuation in the medium.

§6. Recent Modifications of the Reverberation Formula

A more satisfactory reverberation formula can be obtained by assuming that the decay of sound takes place discontinuously, at time intervals equal to the time required for sound to travel the mean free path.[9] It is apparent that this discontinuous process of absorption accords

[7] A. Jaeger, *Zur Theorie des Nachhalls*, Sitzungsber. d. Kaisl. Akad. d. Wiss. in Wien, Math.-Natur. Klasse, **120** (1911).

[8] Compare with the Clausius formula for the rate at which gas molecules in an enclosure strike unit area of the boundary, namely $nc/4$ where n is the number of molecules per cc and c the average velocity of the molecules.

[9] See for example, Eyring, J. Acous. Soc. Am. **1**, 217 (1930); Fokker, Physica, **7**, 198 (1927); Schuster and Waetzmann, Ann. d. Physik **1**, 671 (1929).

more nearly with the actual process than does the continuous process, since each sound ray travels a finite distance (equal on the average to the mean free path) without absorption and then experiences a finite absorption at each reflection. According to the findings of Sabine[6] and Jaeger[7] the mean free path for a room is $4V/S$. Strictly, the mean free path is dependent upon the shape of the room and the location of the source, especially during the first few reflections, but for most rooms of conventional shape, it does not differ from $4V/S$ by more than ± 8.0 percent.[10] In the following discussion the mean free path will be assumed equal to $4V/S$.

Let Δt be the time required for sound to travel a distance equal to the mean free path, that is, $\Delta t = 4V/Sc$, and suppose that the decay is sufficiently slow and uniform so that the average density of sound energy during each time interval Δt is the arithmetical mean of the initial and final densities, an assumption which is sufficiently rigorous for the rates of decay encountered in all rooms in practice. At the beginning of the decay the density is ρ_0, that is, the average steady state density. After a time Δt, that is, after the first reflection, assuming all path lengths equal to the mean free path, the density will be diminished approximately to $\rho_0(1-\bar{\alpha})$, where $\bar{\alpha} = \Sigma \alpha s/S$ is the arithmetical mean of the absorption coefficients of all boundaries of the room. Then the average density during the first interval Δt will be $\frac{1}{2}[\rho_0 + \rho_0(1-\bar{\alpha})]$. Therefore, the amount of sound energy absorbed in this first interval Δt will be approximately $\Sigma \alpha s \Delta t \frac{1}{2}[\rho_0 + \rho_0(1-\bar{\alpha})]c/4$. Consequently, a more nearly correct value for the energy density at the end of the first time interval will be

$$\rho_0 - \frac{[\rho_0 + \rho_0(1-\bar{\alpha})]c}{8V} \sum \alpha s \Delta t,$$

or $\rho_0 - \frac{1}{2}[\rho_0 + \rho_0(1-\bar{\alpha})]\bar{\alpha}$, where $c\Delta t$ is the mean free path, that is, $4V/S$. The approximate value of the energy density after successive time intervals Δt, $2\Delta t$, etc., is given in Table I. After a time t, $n = t/\Delta t$ and therefore the value of

TABLE I.

t	ρ
0	ρ_0
Δt	$\rho_0 - \left[\frac{\rho_0 + \rho_0(1-\bar{\alpha})}{2}\right]\bar{\alpha}$
$2\Delta t$	$\rho_0 - \left[\frac{\rho_0 + \rho_0(1-\bar{\alpha})}{2} + \frac{\rho_0(1-\bar{\alpha}) + \rho_0(1-\bar{\alpha})^2}{2}\right]\alpha$
$\vdots$	$\vdots$
$n\Delta t$	$\rho_0 - \left[\frac{\rho_0 + \rho_0(1-\bar{\alpha})}{2} + \cdots \frac{\rho_0(1-\bar{\alpha})^{n-1} + \rho_0(1-\bar{\alpha})^n}{2}\right]\bar{\alpha}$

ρ at this time t is

$$\rho = \rho_0\left\{1 - \bar{\alpha}\left[\frac{1}{2} + \sum_{n=1}^{n=Sct/4V}(1-\bar{\alpha})^n - \frac{(1-\bar{\alpha})^{Sct/4V}}{2}\right]\right\}. \tag{6}$$

For the rate of decay encountered in most rooms, in fact, in all rooms except those which are bounded by materials having a coefficient of absorption of nearly unity, the first and third terms in the brackets are small compared with the summation of the second term and they are significant only when the decay is made up of a very few reflections. Therefore, approximately,

$$\begin{aligned}\rho &= \rho_0\left\{1 - \bar{\alpha}\sum_{n=1}^{n=Sct/4V}(1-\bar{\alpha})^n\right\} \\ &= \rho_0\left\{1 - \bar{\alpha}\,\frac{1-(1-\bar{\alpha})^{Sct/4V}}{1-(1-\bar{\alpha})}\right\} \\ &= \rho_0(1-\bar{\alpha})^{Sct/4V}.\end{aligned} \tag{7}$$

This same result can be obtained by a number of methods, the simplest of which assumes that the energy density is reduced the same fractional amount $\bar{\alpha}$ after each reflection; that is, after each interval of time Δt, so that the energy densities after times Δt, $2\Delta t$, $\cdots n\Delta t$ will be $\rho_0(1-\bar{\alpha})$, $\rho_0(1-\bar{\alpha})^2$, $\cdots \rho_0(1-\bar{\alpha})^n$, respectively.[11] Or,' in

[10] V. O. Knudsen, *Architectural Acoustics*, pp. 132–141, John Wiley and Sons, 1932. For a cruciform type of church the mean free path is 4.24 V/S; for a large rectangular public office with a low ceiling it is 3.75 V/S.

[11] The method was proposed by R. F. Norris at a meeting of the Acoustical Society of America, New York, May 11, 1929. The assumption that the energy density is reduced by the same fractional amount after each reflection is, of course, inherent in the derivation of Eq. (6) but the notion of *continuous* absorption is partially preserved in the derivation of Eq. (6), whereas it is not in the case of Eq. (7').

general

$$\rho = \rho_0(1-\alpha)^{Sct/4V}. \tag{7'}$$

If we let $(1-\overline{\alpha}) = e^x$, Eq. (7) becomes similar to Eq. (3), namely,

$$\rho = \rho_0 e^{S \ln(1-\overline{\alpha}) ct/4V} \tag{8}$$

and reduces to Eq. (3) when $\overline{\alpha}$ is very small, that is, when the boundaries of the room are highly reflective. Both Eqs. (3) and (8) are only approximate formulas; Eq. (3) is satisfactory only for very live rooms, whereas Eq. (8) meets the practical requirements for both live and "dead" rooms, provided the absorptive materials in the room are not concentrated on one or two surfaces of the room. For frequencies above 1000 cycles both Eqs. (3) and (8) are invalid.

Eqs. (3) and (8) are based upon the assumption that all the absorption of sound energy in a room occurs at the boundaries, that is, that the absorption in the air is negligible. Stokes, Kirchhoff and Rayleigh have calculated the absorption of sound in air owing to the effects of viscosity and heat conductivity and their calculations would indicate that for frequencies up to 6000 or even 8000 cycles, the absorption is so small as to be negligible in comparison with the surface absorption encountered in actual rooms. Experimentally, however, this is not found to be true.[12] The absorption of sound in air is not only much greater than is predicted by the classical theory of Stokes, Kirchhoff and Rayleigh, but it depends upon temperature and humidity in a characteristic way which is accounted for only by considering the absorption which results from collisions between gas molecules.[13, 14] Recent experiments on the absorption of sound in air show that the coefficient of absorption is of the order of 10 to 100 times greater than that predicted by classical theory, so that for high frequencies (10,000 cycles, for example) the absorption in the air in a room may be greater than the surface absorption, especially in large rooms. The curves in Fig. 1 give the absorption coefficient m per foot (or per cm) for plane waves

[12] V. O. Knudsen, J. Acous. Soc. Am. 3, 126 (1931).
[13] V. O. Knudsen, J. Acous. Soc. Am. 5, 112 (1933).
[14] H. O. Kneser, J. Acous. Soc. Am. 5, 122 (1933).

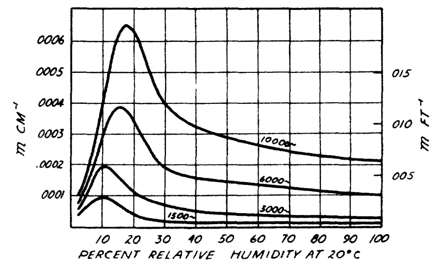

FIG. 1. Coefficients of absorption of sound in air containing different amounts of water vapor, for frequencies of 1500, 3000, 6000 and 10,000 cycles.

in air at 20°C and at relative humidities indicated by the abscissae, for frequencies of 1500, 3000, 6000 and 10,000 cycles sec. The coefficient m is defined by $\rho_x = \rho_0 e^{-mx}$, where ρ_0 is the energy density in the plane wave at $x=0$ and ρ_x is the density after the wave has been propagated a distance x. It will be seen from the curves in Fig. 1 that m has a maximum for a certain concentration of water vapor, different for each frequency. Further, the magnitudes of these maxima are proportional to the first power of the frequency and not the second power as is required by the classical theory of absorption. It will be noted, for example, that at a relative humidity of 18 percent, m is 0.020 ft.$^{-1}$ for a frequency of 10,000 cycles. Hence, such a wave will have its intensity (or energy density) reduced to $1/e$ of its initial value after travelling a distance (1/0.020), or 50 ft. This is equivalent to a rate of decay of 96 db/sec.—which is much in excess of the most desirable rate of decay for music rooms.

This excessive absorption of sound in air makes it necessary to introduce an appropriate attenuation factor in Eq. (8), so that the decay equation becomes,

$$\begin{aligned}\rho &= \rho_0 e^{-mct}\left[e^{\ln(1-\overline{\alpha})\cdot Sct/4V}\right]\\ &= \rho_0 e^{[(\ln(1-\overline{\alpha})\cdot S/4V)-m]ct}.\end{aligned} \tag{8a}$$

The time of reverberation, t_{60}, is obtained by solving Eq. (8a) for t when $\rho_0/\rho = 10^{-6}$. Thus,

$$t_{60} = \frac{55.3V}{c[4mV - S\ln(1-\overline{\alpha})]}. \tag{9}$$

At room temperature, 21°C, $c=1125$ ft. sec., so that for most working conditions

$$t_{60}=\frac{0.049\,V}{4mV-S\cdot\ln\,(1-\bar{\alpha})]} \qquad (10)$$

in British units, or

$$t_{60}=\frac{0.161\,V}{4mV=S\cdot\ln\,(1-\bar{\alpha})]}, \qquad (11)$$

where V and S are in cu.m and sq.m, respectively. For frequencies below about 1000 cycles, m is so small that the first term in the denominator of Eqs. (9), (10) or (11) can be neglected, that is, the absorption in the air is inappreciable; whereas at high frequencies (above 10,000 cycles), this term may become larger than the second (or surface absorption) term. At sufficiently high frequencies (above the audible range), the second term will become negligible, in which case the rate of decay and consequently the time of reverberation will be independent of the size of the room.

The foregoing reverberation formula, Eq. (9), is sufficiently valid for practical purposes provided the sound in the room is thoroughly diffuse throughout the decay. This condition is realized for frequencies above about 250 cycles in all but very small rooms, provided all the boundaries of the room have approximately the same absorptivity, or provided suitable rotating paddles or "warble tones" be used to "mix" the sound in the room. Suitable precautions, such as those just mentioned, can be taken in making measurements in an acoustical laboratory, such as a reverberation chamber. But in many rooms encountered in practice, the absorptive material may be concentrated on a single surface, as when a carpet, upholstered seats and audience are all located on the floor and the other surfaces in the room are highly reflective. In such rooms, especially if the opposite walls are parallel and not too far apart, the decay of sound will not conform to the approximately exponential decay predicted by Eq. (9) but will consist, first, of a rapid rate of decay, while the sound is relatively diffuse and, second, of a much slower rate of decay, made up largely of a horizontal flow of sound energy between the parallel and highly reflective walls. The time of reverberation in such a room will be longer than that calculated by means of Eq. (9) with an arithmetical mean for $\bar{\alpha}$. This is borne out by some oscillograms of the rate of decay obtained in a small room, $8'\times 8'$ $\times 9.5'$ (high), with the floor covered with a material having a rated absorption coefficient of 0.60 at 512 cycles and with the walls and ceiling finished with painted concrete. The first part of the decay (15 to 17 db) was relatively rapid, 95 to 100 db per sec. This was followed by a much slower decay, 38 to 40 db/sec. If the first part of the decay be used for calculating the time of reverberation t_{60} and the absorptivity of the floor material α, we obtain $t_{60}=0.61$ sec., and $\alpha=0.55$. If the latter part be used, $t_{60}=1.54$ sec. and $\alpha=0.22$. It will be seen that the first part of the decay, while the sound is relatively diffuse, yields a value for α which agrees fairly well with the rated value of 0.60 and therefore conforms reasonably well with the requirements of Eq. (9), whereas the latter part of the decay, which is made up largely of the horizontal flow (or resonance) of sound energy between parallel, highly reflective surfaces, is much slower than would be predicted by Eq. (9).

Fortunately, however, for the best acoustical quality in a room the absorptive material should be distributed on all surfaces of a room so that the rate of decay will be at least approximately the same in all directions and under these circumstances, Eq. (9) will yield results which, as a rule, do not differ more than 10 percent from the observed values. Furthermore, in very large rooms, as in theatres, school auditoriums and churches, there is very little tendency for the reverberation to persist in two dimensions, even though most of the absorption is concentrated on the floor or on the floor and in the ceiling, (1) because the dimensions of the room are large compared with the wave-lengths of the sound and (2) because the architectural treatment of large rooms usually involves structural forms and ornamentations which tend to diffuse the sound during free decay. In such rooms, provided there are no curved surfaces giving rise to concentration of sound, the first 30 db (or more) of decay conforms very closely to Eq. (9); and it is this portion of the decay that is pertinent to the acoustical quality of speech and music in rooms. Stated otherwise, the rate of decay after the first 30 db of decay is of little consequence, since in articulated speech or music such residual sounds will be so weak as to be completely masked by the primary (and much louder) sounds which follow. It is apparent, therefore, that Eq. (9) is satisfactorily valid for the practical calculations of reverberation in most rooms.

§7. Room Resonance

It is to be expected that the reverberation theory described in the two preceding sections will not apply when the wave-length of the sound is not short compared with the dimensions of the room, for under such conditions one or more of the lower modes of vibration of the room may be prominently stimulated and the presence of these modes of vibration would preclude the possibility of a diffuse state of sound. As a matter of fact, whether the wave-length be short or long, reverberation in a room should be regarded as the free, damped vibration of the air confined in the room, as has been pointed out by Strutt[15] and by Waetzmann and Schuster.[16] The author has demonstrated that, at least at low frequencies, reverberation is precisely these free damped vibrations of the air in the room.[17] The natural frequencies of a rectangular room are obtained readily by imposing suitable boundary conditions on the wave equations.[18] The frequencies n are given by

$$n=\frac{c}{2}\left[\frac{p^2}{l_1^2}+\frac{q^2}{l_2^2}+\frac{r^2}{l_3^2}\right]^{\frac{1}{2}}, \tag{12}$$

where l_1, l_2 and l_3 are the dimensions of the room and p, q, r are integers, 0, 1, 2, 3$\cdots$. The fundamental mode in the l_1 direction is given when $p=1$ and $q=r=0$; that is, $n_{1,0,0}=c/2l_1$, which corresponds to the fundamental mode of vibration of a pipe of length l_1, either closed at both ends or open at both ends. Thus, for a rectangular room in which $l_1=12.5$ ft., $n_{1,0,0}=1125/25=45$. If 12.5 ft. be the longest dimension of the room, then the gravest mode of vibration will have a frequency of 45 cycles. In general, however, there will be a triple infinity of overtones. The first twenty characteristic frequencies of a room $8'\times 8'\times 9.5'$, as calculated by Eq. (12), were readily recognized (1) by their resonant effect on the steady state intensity of a tone of slowly varying frequency (the intensity of the tone was greatly enhanced whenever the frequency of the tone coincided with one of the natural frequencies of the room); and (2) by an oscillographic investigation of the decay of low pitched tones in the room.[17] In the author's investigation of resonance in rooms, it was found that the frequency of a reverberating tone is never the same as that of the exciting tone except when the exciting tone is accurately tuned to one of the room's characteristic frequencies. The frequency (or frequencies) of the reverberating tone always consists of one or more of the free modes of vibrations of the room. Thus, one of the most prominent of the free vibrations in a small room was the fundamental in a horizontal direction, 71 cycles per second. When a tone of any frequency between 66 and 76 cycles was produced in the room and then stopped, the decay would be made up of a tone having a frequency of 71 cycles. Fig. 2 contains a series of oscillograms which exhibit clearly the resonant character of reverberation in a small room. The frequency of the steady state tone is indicated above each oscillogram. Although seven different pure tones, having frequencies between 90.8 and 100.6 cycles, were separately employed for producing the steady state of vibration in the room, the decay in each case is seen to consist predominantly of two characteristic room frequencies, namely 92.8 and 99.8 cycles, the two natural frequencies which are in the closest proximity to the impressed frequency. In each case the beat frequency of 7.0 cycles per second is clearly discernible. Many other oscillograms, obtained in this and other rooms, confirm the conclusion that reverberation always consists of the free damped vibration of the air in the room. These findings give support to the theoretical predictions of Waetzmann and Schuster and of Strutt and consequently a valid theory of reverberation must be based upon a consideration of the resonant vibrations of a three-dimensional bounded space. However, as Strutt[15] has shown, the formal law governing the free damped

[15] M. J. O. Strutt, Zeits. ang. Math. Mech. 10, 360 (1930).
[16] Schuster and Waetzmann, Ann. d. Physik 1, 671 (1929).
[17] V. O. Knudsen, J. Acous. Soc. Am. 4, 20 (1932).
[18] See, for example, Rayleigh, *Theory of Sound*, Vol. II, p. 69.

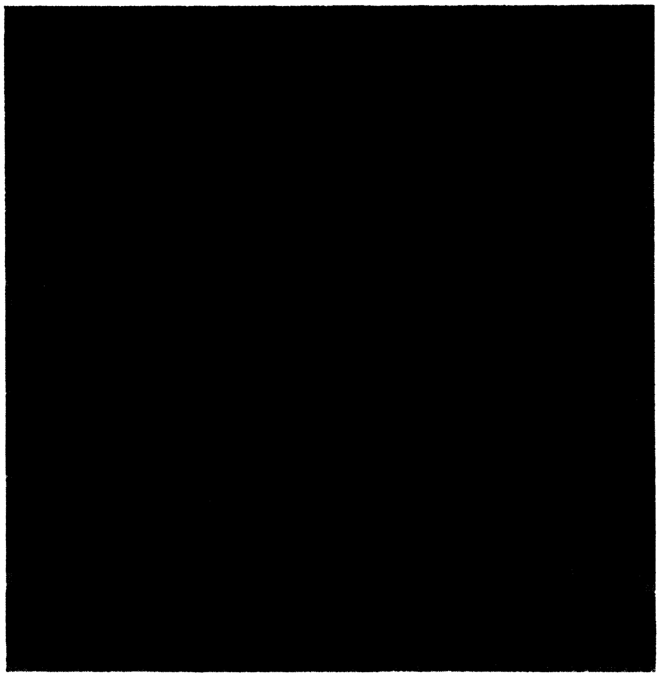

FIG. 2. Oscillograms of the decay of sound in a small rectangular room, showing that the decay of sound consists of the damped free vibrations of the room.

vibration of a three-dimensional continuum approaches asymptotically the simple Sabine and Jaeger law of reverberation as the wave-length of the exciting sound becomes small in comparison with the wave-length of the gravest mode of vibration for the room. Thus, in a room in which the longest dimension is 10 feet (which is about the smallest room in which acoustics is a factor of importance) the wave-length of the gravest mode is 20 feet. In such a room, sound having a wave-length of one-tenth of that of the fundamental, namely 2.0 feet, would be sufficiently short to conform reasonably well to the Sabine law. In other words, when the room is filled with sound having a wave-length shorter than 2.0 feet, that is, a frequency greater than about 560 cycles, the modes of vibration which are excited are so numerous and the frequencies so close together that the sound in the room is essentially diffuse and, therefore, the requirements are fulfilled for the approximate theories of reverberation described in the two preceding sections, especially if the absorptive material be distributed uniformly over the boundaries of the room or if some means be provided for mixing or diffusing the sound during the decay. In addition, the pitch of the decadent tone is indistinguishable from the pitch of the tone during the steady state, a condition which is not realized for frequencies near the lowest modes of vibration of a room. In larger rooms, such as concert halls, church or school auditoriums and theatres, the lowest modes of vibration are usually in the subaudible range of frequencies, so that the elementary theory of reverberation applies with adequate rigor in such rooms for all frequencies above 100 cycles and the effects of room resonance usually can be neglected.

§8. Reverberation Measurements in Small Rooms; The Determination of Absorption Coefficients of Building Materials

The accurate measurement of reverberation in small rooms is of prime importance because such measurements have been and are used almost entirely, since the pioneer work of W. C. Sabine, as the practical method of determining the coefficients of sound absorption of building materials and furnishings and, especially, such materials as acoustical felts, tiles and plasters. The total absorption of a room, such as a reverberation room, is determined by measuring either the rate of decay or the time of reverberation, first when the room contains a certain area of the acoustical material and again when the material is removed from the room. By means of Eq. (8a) or Eq. (9) the value of $\bar{\alpha}$ and consequently the total absorption $\bar{\alpha}S$ can be calculated. The absorption of the acoustical material in the room is assumed to be equal to the difference between the absorption of the room with the material in it and the absorption of the room with the material removed. This is equivalent to assuming that $\bar{\alpha}$ is the arithmetical mean of all absorptive surfaces in the room, an assumption which is justifiable provided the sound in the room is kept thoroughly diffuse during the steady state and the decay. Warble tones at least 100 cycles in breadth[19] with a warble frequency of at least 4 or 5 per second should be used for test tones below about 500 cycles, and large rotating vanes should be used for test tones of all frequencies. The test tones should be pure. When these precautions are taken the rate of decay will conform satisfactorily to the theoretical rate given by Eq. (8a), and if the test area be as large as about 72 sq. ft. in a room having a volume less than 10,000 cu. ft., the difference between the rates of decay with and wi.hout the acoustical material in the room will be large enough to yield coefficients of absorption accurate to about ± 0.03 for frequencies up to 2000 cycles.[19] At higher frequencies the absorption in the air, which may change during the time required for the completion of the test, is so large a factor that errors of the order of ± 0.10 are unavoidable unless the reverberation room is carefully air-conditioned. Even with an air-conditioned room the accuracy is not satisfactory at frequencies above 4000 cycles, because the absorption in the air is such a large factor that the difference between the rates of decay with and without the acoustical material in the room is not appreciable unless the test area be prohibitively large.

So serious is this source of error in making measurements of the absorptivity of acoustical materials at high frequencies that the author is preparing a test chamber which can be filled with a non-absorptive gas, as nitrogen,[13] instead of air. In nitrogen, at room temperature, the absorption is only slightly greater than that due to viscosity and heat conduction and is, therefore, almost negligible for frequencies up to 8000 cycles. Preliminary tests indicate that this expedient will greatly improve the accuracy of absorption measurements for frequencies above 2000 cycles.

The rate of decay is measured by some type of reverberation meter which in general consists of (1) a suitable source of steady or warble tones, usually a vacuum tube oscillator, an electrical low-pass filter, a power amplifier and an electrodynamic loudspeaker; (2) a high quality microphone and amplifier; (3) an electrical attenuator for varying the gain of the amplifier; and (4) either a recorder which registers continuously, on a moving paper chart or on a light-sensitive medium, a graphic record of the decay, or some type of indicator, usually a relay and chronograph, by means of which the rate of decay can be determined.

The Bell Telephone Laboratories, Inc., have developed an automatic level recorder[20] which gives a response proportional to the logarithm of the actuating current and the instrument is so adjusted that the record gives directly, when the paper tape is moving with constant speed, the rate of decay of the sound in decibels per second. If the decay follows the exponential law, the curves will be straight lines. However, since the decay consists of several contiguous frequencies

[19] F. V. Hunt, J. Acous. Soc. Am. 5, 127 (1933) has shown that the frequency of the warble tone should vary about 20 percent. See also Meyer and Just, E. N. T. 5, 293 (1928), and Barrow, J. Acous. Soc. Am. 3, 562 (1932).

[20] E. H. Bedell and K. D. Swartzel, J. Acous. Soc. Am. 5, 34 (1933).

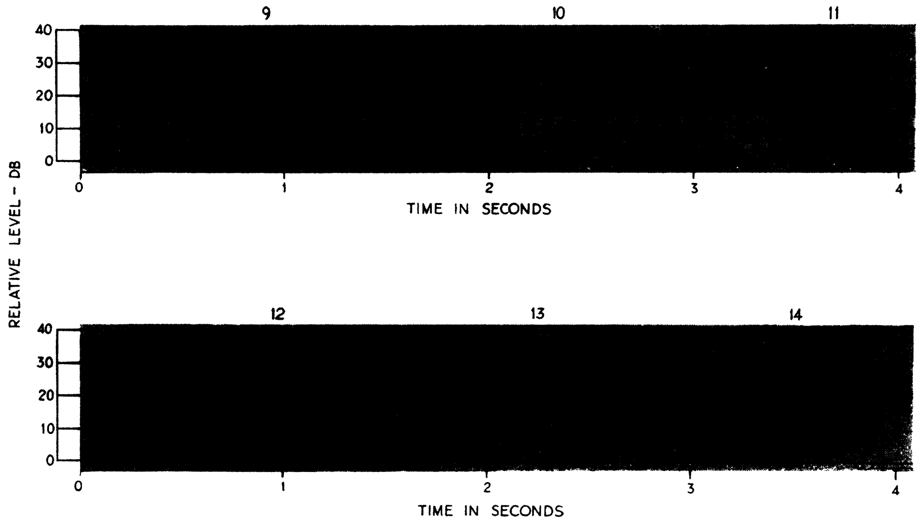

FIG. 3. Curves showing the decay of sound in rooms, obtained by an automatic sound recorder developed by Bell Telephone Laboratories, Inc. Curves 9, 10 and 11 are for a pure tone with the recorder adjusted to speeds of 240, 120, and 60 db/sec., respectively. Curves 12, 13 and 14 were made with corresponding recorder speeds but with warble tones.

in close proximity to the frequency of the exciting tone, there will be interference between these several frequencies (each of which may decay exponentially) so that the resultant decay curve generally will be quite irregular. Typical decay curves obtained with this instrument in the reverberation room of Electrical Research Products in New York are reproduced in Fig. 3.[21] As these records show, the decay is not strictly exponential but, except for minor fluctuations which can be attributed to the resonant or interfering phenomena discussed in the preceding section, the general trend of the decay conforms very satisfactorily to the exponential law, over a range of 40 db; and if a straight line be fitted to the recorded curve of decay, the slope of this line will give the rate of decay with sufficient accuracy for practical purposes.

The curves labeled 9, 10 and 11 were made with a pure tone and a single microphone and with the recorder adjusted to "follow" maximal speeds of decay of 240, 120 and 60 db/sec., respectively. In 9, for example, the recorder is capable of following the actual decay much more closely than it is in 11, where only the slower variations of decay are recorded. The curves labeled 12, 13 and 14 were made at the same recorder speeds, respectively, but a warble tone was used instead of a single pure tone. The advantage of the warble tone for reverberation measurements is obvious from these decay curves.

In the indicating type of reverberation meter, in use in most acoustical laboratories in this country, the attenuator is successively set to give different amounts of attenuation in the amplifier circuit. Then, for each setting of the attenuator, the time required for the decay to proceed to a certain predetermined level, such as that required to operate a relay and give an indication on a chronograph, or just barely to flash a neon lamp, is measured for each setting of the attenuator. Then, if the readings of the attenuator (which is generally calibrated in the decibel scale) be plotted as a function of these observed times, the resulting curve will give a typical decay curve. Fig. 4 shows a set of decay curves

[21] These records were furnished through the courtesy of S. K. Wolf of Electrical Research Products, Inc., and the Bell Telephone Laboratories, Inc.

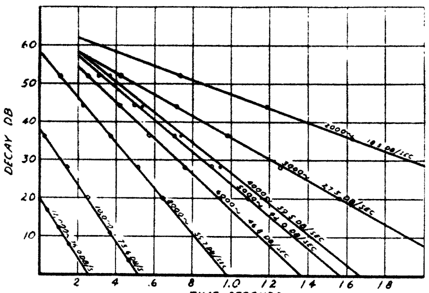

FIG. 4. Decay curves in a 6-ft. cubical chamber, showing (1) that the decay is exponential and (2) that the rate of decay increases with the frequency. The increase in the rate of decay at higher frequencies is attributable to absorption in the medium and not to any marked increase in the absorption of boundaries.

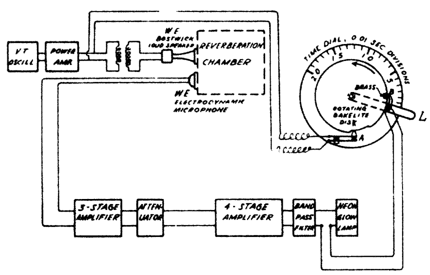

FIG. 5. Block diagram of apparatus used at the University of California at Los Angeles for measuring the rate of decay of sound in small chambers.

obtained by this method (with a neon lamp indicator) in a six-foot steel plate cubical chamber at the University of California at Los Angeles.[22] The circuit arrangement for obtaining these decay curves is indicated in Fig. 5. The setting of lever L on the time dial determines the time, after the decay commences, at which the neon lamp is connected in the output of the amplifier. For each setting of the attenuator (which determines the ordinates in Fig. 4) the lever L is adjusted until the lamp just barely flashes, when the contacts at B are closed by the rotating brass inlay. This determines the abscissae in Fig. 4. The large increase in the rate of decay at higher frequencies, shown in Fig. 4, is due almost entirely to the increased absorption in the air at the higher frequencies. If the chamber were filled with a non-absorptive gas (nitrogen, for example), the rate of decay would be nearly constant for all frequencies and not more than the rate for the 2000 cycle tone, that is, not more than 18 db/sec. The decay curves in Fig. 4 were obtained with the use of pure tones but a large rotating paddle was used to keep the sound in the chamber thoroughly diffuse.

[22] See reference 13.

Reverberation meters are not only useful for determining the coefficients of sound-absorption of acoustical materials in a reverberation chamber but are equally useful for determining the reverberatory properties of all rooms. In general, a reverberation meter should be capable of making measurements at all frequencies between about 128 and 4096 cycles. In special cases, as in music rooms and theatres, it may be desirable to make measurements at frequencies as high as 8192 cycles. But in the large majority of rooms it is sufficient to make measurements only at low, medium and high frequencies, as 128, 512 and 2048 cycles.

§9. The Determination of Absorption Coefficients by Direct Reflection

The reverberation method of measuring coefficients of sound-absorption is an indirect method, subject to the errors and limitations mentioned in the preceding sections. Furthermore, the coefficients thus obtained are for random angle of incidence; and in some instances, both of practical and theoretical interest, it is desirable to know the coefficient at any specified angle of incidence. Obviously, the simplest means of making such measurements would consist of directing a beam of plane parallel sound against the specimen of acoustical material and measuring the intensities of the incident and reflected beams.[23] The practical difficulties in the use of this method are: (1) the necessity for reflecting surfaces which are large in comparison with the

[23] F. R. Watson (*Acoustics of Buildings*, p. 102, John Wiley and Sons (1930)) has used this method for measuring both the reflection and transmission of sound by building materials and partitions. More recently, Kühl and Meyer (Berl. Akad. Ber. Math. Phys. Kl. **26**, 416, (1923)) and Cremer (E. N. T. **10**, 242 and **10**, 302 (1933)) have developed this method of measuring coefficients of sound-absorption.

wave-length of the sound (even for a frequency of 512 cycles the reflecting area should be at least $12' \times 12'$); (2) the necessity for no reflections from other surfaces in the test room, that is, all other surfaces in the room should be so far removed or so non-reflective that the reflections from these surfaces will not contribute an appreciable amount of sound to the beam reflected from the test specimen; and (3) the microphone or detector should be small in comparison with the wave-length of the sound, so that it will not distort the sound field in which it is placed. If these difficulties are adequately met, the coefficient of absorption α is given by

$$\alpha = 1 - (\bar{P}_r/\bar{P}_i)^2, \tag{13}$$

where $\bar{P}_i$ is the measured root mean square pressure amplitude in the incident beam and $\bar{P}_r$ that in the reflected beam.

If the acoustical impedance z of the reflecting material be known,[24] a simple formula is readily deduced which gives the coefficient of reflection (or absorption) for any angle of incidence. Thus, let the reflecting surface be in the plane $x=0$ and let plane parallel sound (from a parabolic mirror, large in comparison with the wave-length of the sound) strike the surface at an angle of incidence θ. For the incident waves, travelling in the $+x$ direction, the velocity potential ϕ_i may be represented by

$$\phi_i = A e^{jk(ct - x\cos\theta + y\sin\theta)} \tag{14}$$

and the velocity potential for the reflected waves ϕ_r by

$$\phi_r = B e^{jk(ct + x\cos\theta + y\sin\theta)}, \tag{15}$$

where A and B are proportional to the pressure amplitudes (velocity amplitudes would serve as well, but most microphones are pressure indicating instruments); k is the wave-length function $2\pi/\lambda$, where λ is the wave-length; and c is the velocity of sound in free air. The resultant velocity potential is related to the particle velocity by

$$u = \partial\phi/\partial x, \quad v = \partial\phi/\partial y, \quad w = \partial\phi/\partial z, \tag{16}$$

where u, v and w are the components of the particle velocity. Also, we can write the force equation,

$$\delta p = -\rho_0 \partial\phi/\partial t, \tag{17}$$

where ρ_0 is the undisturbed density of the air.

The acoustical impedance of the reflecting medium is defined as the ratio of the instantaneous change of pressure, δp, to the rate of volume displacement in the x direction. That is,

$$z = \delta p/(\partial q/\partial t)_x = \delta p/u = -(\rho_0/u)\partial\phi/\partial t. \tag{18}$$

Hence, at the surface of the reflecting medium

$$u = \partial(\phi_i + \phi_r)/\partial x = -(\rho_0/z)\partial(\phi_i + \phi_r)/\partial t. \tag{19}$$

Substituting the ϕ_i and ϕ_r, as given in Eqs. (14) and (15), respectively, in Eq. (19) and solving for z, there results

$$z = \frac{\rho_0 c(A+B)}{(A-B)\cos\theta}. \tag{20}$$

Whence,

$$B/A = \bar{P}_r/\bar{P}_i = \frac{z\cos\theta - \rho_0 c}{z\cos\theta + \rho_0 c}, \tag{21}$$

and since

$$\alpha = 1 - |\bar{P}_r/\bar{P}_i|^2$$

we have finally,

$$\alpha = 1 - \left|\frac{z\cos\theta - \rho_0 c}{z\cos\theta + \rho_0 c}\right|^2. \tag{22}$$

The coefficient of absorption is thus seen to depend upon the angle of incidence in a characteristic manner and if z (which is complex) is largely real, as is usually the case for many porous materials, the coefficient of absorption is a maximum when $\cos\theta = \rho_0 c/z$. For most acoustical materials, as felts, tiles or plasters, z is of the order of 60 to 200 c.g.s. units, and $\rho_0 c$ is approximately 41 at room temperature. Hence the absorption coefficient is usually a maximum and approaches unity, for some angle of incidence between 50° and 80°, and vanishes at grazing incidence.[25] But usually it is necessary to take into account both the real and imaginary parts of z and when this is done it is found that α is

[24] Several methods have been developed for measuring the acoustical impedance of a material. See, for example, Crandall, *Theory of Vibrating Systems and Sound*, p. 100, Van Nostrand (1926) or Kühl and Meyer, reference 23.

[25] See E. T. Paris, *On the Reflection of Sound from a Porous Surface*, Proc. Roy. Soc., **A115**, 407 (1927); and Kühl and Meyer, reference 23.

influenced by several factors, such as porosity, frequency, angle of incidence, thickness of the reflecting medium and flexural yielding. In general, α is quite small (usually below 0.20 for the most absorptive acoustical materials) for frequencies of 100 cycles or less and increases to about 0.80 or 0.90 at frequencies above 2000 cycles; and α usually increases as the angle of incidence increases from zero to 70° or 80° and then decreases to zero at an incidence of 90°.

§10. The Absorption of Sound by Porous Materials

The theory of the absorption of sound by porous materials received the early attention of Lord Rayleigh[26] and also his latest attention.[27] Rayleigh's later work was followed and extended by Crandall who obtained a working formula for a honeycombed material, with small pores or channels leading perpendicularly inward from the exposed surface.[28] This formula, applied to a structure like hairfelt, assuming the pores to be closely packed and circular with a diameter of 0.02 cm, gives results which are in fair agreement with measurement for frequencies above 400 cycles. Kühl and Meyer,[29] following closely the analysis of Rayleigh, obtain formulas for the absorption coefficients of both finitely and infinitely thick porous media. For an infinitely thick porous medium, the absorptivity is a simple function of the porosity P (defined as the ratio of the volume of voids to the total volume of the absorptive material), namely

$$\alpha = 1 - \left(\frac{\cos\theta - P}{\cos\theta + P}\right). \tag{23}$$

Kühl and Meyer are able to check the approximate validity of this equation by making measurements, by the beam method, on a stacked pile of corrugated paper, the porosity of which could be varied by applying pressure transversely to the channels of the paper. Fig. 6

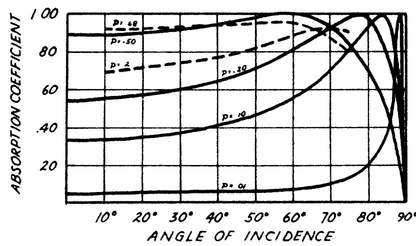

Fig. 6. The solid curves show theoretical values of the absorption coefficient of porous materials for different angles of incidence and for porosities of 0.01, 0.10, 0.20 and 0.50. The dashed curves are experimental values obtained by Kühl and Meyer for corrugated paper with porosities of 0.20 and 0.68.

shows four curves calculated from Eq. (23) for four porosities, namely, 0.01, 0.10, 0.20, and 0.50; and two experimental curves obtained by Kühl and Meyer for corrugated paper having porosities of 0.20 and 0.68.

The theory is extended by Zwikker[30] and by Cremer[31] to apply to porous materials not only of the type considered by Kühl and Meyer but also to materials in which the pores are unordered and more or less randomly distributed in direction, thus conforming to actual materials, such as acoustical tiles, felts or plasters. Cremer shows, for example, that for the ordered (*geschichtetes*), porous medium, that is, where the pores extend only perpendicularly into the surface of the absorptive medium, the absorption coefficient increases with increasing angle of incidence, reaches a maximum, and then decreases to zero at an incidence of 90° (thus agreeing with the conclusions of Rayleigh, of Paris and of Kühl and Meyer). Further, for the ordered arrangement of pores, the angle of incidence at which the maximum occurs increases as the porosity decreases and also as the frequency decreases. The value of the maximum is independent of the porosity and approaches 1.00 at high frequencies and diminishes to 0.83 at low frequencies.

For the unordered arrangement of pores (*ungeschichtetes medium*), Cremer shows that for low frequencies the absorption depends upon the

[26] Rayleigh, *Theory of Sound*, II, 328–333 (1896).

[27] Rayleigh, *The Resonant Reflection of Sound from a Perforated Wall*, Phil. Mag. **39**, 225 (1920). This article was written in 1919 but was published after his death.

[28] Crandall, *Theory of Vibrating Systems and Sound*, pp. 186–191, Van Nostrand (1926).

[29] Kühl and Meyer, reference 23.

[30] C. Zwikker, *Gelindsabsorptie door Porenze Wanden*, de Ingenieur, 20, (1932).

[31] Cremer, *Theorie der Shallabsorption in porösen Wanden*, E. N. T. **10**, 242 (1933).

angle of incidence in a manner similar to that for the ordered arrangement but at high frequencies the absorption coefficient is independent of the angle of incidence; and also the absorption increases with the porosity. Further, for low frequencies, the coefficient at normal incidence is lower than the coefficient obtained by the reverberation method, which gives the coefficient for the mean of random incidence; whereas, for high frequencies, the coefficient at normal incidence is practically the same as that obtained by the reverberation method. Complete experimental verification of these conclusions is yet lacking, although Paris and also Kühl and Meyer have obtained at least sustaining evidence.

Penman and Richardson[32] recently have tested the Rayleigh theory for the absorption of porous materials at normal incidence, and sum up their findings by stating that their experimental tests confirm the general character of the Rayleigh theory but that "other sources of damping of sound waves in narrow tubes must be sought before quantitative agreement with the theory can be expected."

§11. Optimal Reverberation Time for Speech and Music Rooms

The reverberatory properties of rooms are so patent in determining the acoustical quality of either speech or music rooms that it has become customary to rate these rooms by means of their reverberation times. Historically, undue importance has attached to the reverberation time at one frequency, 512 cycles; indeed, when one speaks of the time of reverberation of a room as being so many seconds, one always means that this reverberation time has been calculated or measured for a frequency of 512 cycles. Obviously, however, since speech and music are made up of frequencies between about 30 and 15,000 cycles, it is necessary to give regard to the reverberatory properties of a room throughout this very wide range of frequencies. But as mentioned in Section 8, it is usually sufficient to limit consideration to the range between 128 and 4096 cycles. This is especially true in speech rooms, since frequencies outside this range have but very little influence on the quality of speech. Fortunately, if the reverberation time be adjusted by the installation of the most commonly used absorbents, including the audience, to the most favorable time for a frequency of 512 cycles, the reverberation time at other frequencies will be quite satisfactory. Hence it is feasible, at least from the practical viewpoint, to consider the reverberation of rooms at this single frequency of 512 cycles, provided the absorbent materials used are of such a nature as to give the proper balance of absorption between low, medium and high frequencies. In the immediately following paragraphs we shall restrict the consideration of reverberation time to this single frequency; later we shall consider the most favorable relation between reverberation time and frequency.

A certain amount of reverberation is desired in speech rooms (1) because it enhances loudness, which is a prime necessity in large rooms; and (2) because we approve of the blending or fusing effect of that amount of reverberation which unites the separate sounds of speech into an artistically articulated whole.

Let us consider this first factor, which is of a physical nature and which can be evaluated quantitatively. The intensity of sound in a room for a constant sound source is inversely proportional to the amount of absorption in the room (see Eq. (4)) and therefore nearly proportional to the time of reverberation (see Eq. (9)). The sound intensity of average speech in large rooms is only about one-hundredth of that which is required for the most distinct audition of speech. Thus, measurements of the average intensity of speech in typical auditoriums indicate that the speech level for the average speaker does not exceed 50 db,[33] whereas the level should be about 70 db for the most favorable hearing conditions[34] (see Fig. 7). Any gain in the loudness

[32] Penman and Richardson, *Absorption of Sound by Porous Materials at Normal Incidence*, J. Acous. Soc. Am. **4**, 322 (1933).

[33] V. O. Knudsen, *Architectural Acoustics*, p. 377, John Wiley and Sons (1932). A *speech level* of 50 db means that the intensity is 50 db above the intensity of barely audible speech. The terms *sound level*, *noise level*, etc. will be used in a similar manner, when referring to the intensities of sound, noise, etc.

[34] H. Fletcher, *Speech and Hearing*, p. 272, Van Nostrand (1929).

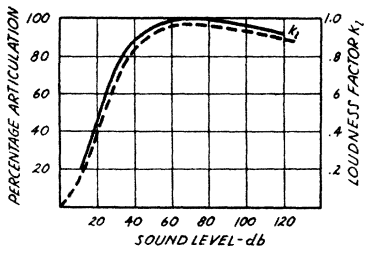

FIG. 7. The dashed line gives the percentage speech articulation at different loudness levels as determined by Fletcher and Steinberg. The solid line gives the loudness-reduction-factor for different sound levels.

of speech, therefore, such as may be gained from an increase of reverberation time, is greatly to be desired. But this gain from increased reverberation cannot be pushed far before the deleterious effects of the commingling of the successive words of speech offset, or more than offset, this gain.

If we can determine the functional relations between loudness and distinctness of speech and between reverberation time and distinctness, it is a simple matter to determine the optimal time of reverberation for any specified loudness or, what amounts to the same thing, the optimal time for a room of any specified size. These relations have been determined experimentally and the results foretell not only the optimal time of reverberation for speech rooms of specified size but also, quantitatively, how well average speech can be heard in a room of specified size and time of reverberation.

The dashed line in Fig. 7 gives the results obtained by Fletcher and Steinberg in an investigation on the effect of loudness on the hearing of speech. The solid line in Fig. 7 is called the loudness-reduction-factor curve; it is given arbitrarily a value of 1.0 at the optimal sound level of 70 db (where the articulation is 96 percent) and its value at any other sound level is the ratio of the ordinate of the dashed curve at that level to the ordinate at the level of 70 db. Thus, if there is no disturbance from noise, echoes or reverberation, the percentage speech articulation is simply 96 k_l, where k_l is the loudness-reduction-factor given by the solid line curve in Fig. 7.

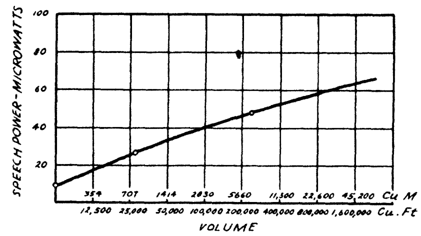

FIG. 8. Probable speech power in microwatts of average speakers in auditoriums.

Fig. 8 gives the probable speech power, in microwatts, of the average speaker in auditoriums of different sizes between a small room of 6000 cu. ft. and a large auditorium of more than 10^6 cu. ft.[35] It appears, and this is borne out by the experience of speakers, that in a large auditorium the speaker raises his voice in an attempt to provide adequate loudness for distinct hearing. In general, however, he falls far short of actual requirements. The data in Table II are for 8 speakers in an auditorium having a volume of 6790 cu. m (240,000 cu. ft.). The average sound level was measured with a sound meter (microphone, amplifier and electrostatic voltmeter), the microphone being located near the middle of the auditorium. The average power of the speaker's voice was calculated from data on the threshold of audibility, the average sound level of the speech and Eq. (4). It is of interest to note the wide variation in the power output of different speakers. This accounts for the difficulty fre-

TABLE II.

Speaker	Observed average sound level (db)	Total absorption in room (sq. m)	Average power of speaker's voice (mw)
1 (man)	49.4	335	65.5
2 (man)	45.6	335	27.5
3 (man)	46.1	413	37.8
4 (woman)	43.0	632	28.4
5 (man)	43.5	531	26.8
6 (man)	51.0	502	142.0
7 (man)	42.7	560	23.4
8 (woman)	44.3	629	38.2
Average	45.7		48.9

[35] V. O. Knudsen, *The Hearing of Speech in Auditoriums*, J. Acous. Soc. Am. **1**, 72 (1929).

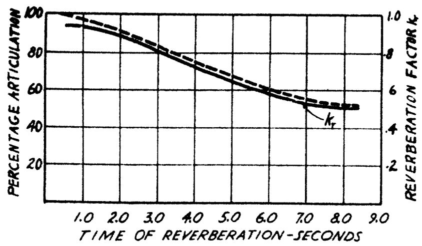

FIG. 9. The dashed line gives the percentage speech articulation for different times of reverberation in large auditoriums when the speech level is 70 db. The solid line gives the reverberation-reduction-factor for different times of reverberation.

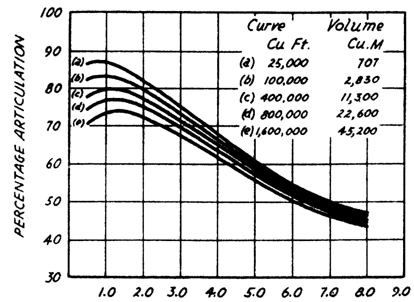

FIG. 10. Curves showing the probable percentage speech articulation in rooms of different sizes, with different times of reverberation.

quently encountered in failing to hear many speakers, especially in large auditoriums. By means of Fig. 8 and data similar to those in Table II we can calculate the probable sound level of the average speaker in a room of specified size and time of reverberation.

Experimental data on the effect of reverberation on the hearing of speech are summarized in Fig. 9. The dashed line gives the percentage articulation for average speech amplified without distortion to a sound level of 70 db, in large auditoriums (about 300,000 cu. ft.) having different times of reverberation between 0.5 sec. and 8.0 sec. The articulation diminishes about 7 percent for each additional second of reverberation, between 1.0 and 5.0 seconds. Since an articulation of 75 percent is requisite for satisfactory hearing, the reverberation should not exceed 4.0 seconds, even when the sound level is as high as 70 db and when there is no disturbance from noise. The solid line in Fig. 9 gives the reverberation-reduction-factor k_r which is obtained in a manner similar to that for k_l. The value of k_r is taken arbitrarily as 1.0 for a time of reverberation of 0.5 sec., but is a justifiable choice, from the practical standpoint, since the interfering effect of so little reverberation is almost negligible.

Now when a sound level less than 70 db as well as reverberation in excess of 0.5 sec. exert their combined influence on speech, the percentage articulation is given by 96 $k_l k_r$ and both k_l and k_r can be determined by means of Figs. 7 and 9 for a room of specified size and reverberation time. Owing to the presence of unavoidable noise in auditoriums, there is also a noise-reduction-factor, which can be determined in a manner similar to that which we have just described for loudness and reverberation. For a moderately quiet room, the noise-reduction-factor k_n is about 0.96, so that in such rooms the percentage articulation is 92 $k_l k_r$. The curves in Fig. 10 were calculated by means of this relation and the values of k_l and k_r as given in Figs. 7 and 9, respectively. These curves give the probable percentage articulation for the average speaker in auditoriums of different sizes and with different times of reverberation and thus provide a means for determining quantitatively the acoustical merit of either contemplated or completed speech rooms. Limitations which should be imposed upon the design of speech rooms with respect to both size and reverberation time are clearly indicated by these curves. Thus, average, unamplified speech can never be heard satisfactorily in a room having a volume as large as 45,200 cu. m (1,600,000 cu. ft.), no matter what time of reverberation be provided for the room, simply because the average speaker does not comprise an adequate power supply. Suitable amplification of speech in large auditoriums is thus seen to be an inescapable requirement for good acoustics.

The point of inflection in each of the curves in Fig. 10 gives the optimal time of reverberation for a speech room of the size for which that curve applies. The optimal time for small rooms is seen to be slightly less than 1.0 sec. and the optimal

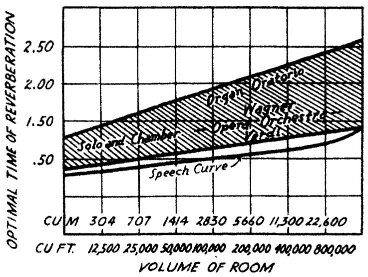

FIG. 11. The charted area gives a summary of optimal reverberation times for music rooms. The optimal curve for speech rooms is shown by the solid line.

time increases with the size of the room, reaching approximately 1.5 sec. for very large rooms. The maxima in these curves, however, are rather broad, so that a deviation of ±0.25 sec. from the optimal time is of little consequence. This allows a certain amount of freedom to satisfy the second factor bearing upon reverberation time and mentioned near the beginning of this section, namely, that our experience in listening to speech in interiors has accustomed us to (or developed in us an appreciation for) a certain amount of reverberation, sufficient to sustain a pleasing continuity and smoothness in the flow of articulated speech. Such data and experiences as are available[36] would seem to indicate that cultivated taste prefers a time of reverberation which is about 0.3 to 0.5 sec. longer than is indicated by the maxima in Fig. 10. Fortunately, it is possible to satisfy this aesthetic requirement without seriously sacrificing the optimal conditions for hearing the sounds of speech. If a compromise be made by adding 0.2 sec. to the optimal times indicated by Fig. 10, the resulting times of reverberation will serve as satisfactory criteria for designing the acoustics of speech rooms.

It is generally recognized that music requires longer reverberation times than does speech, and different kinds of music require different amounts of reverberation. The times indicated by the chart in Fig. 11 summarize the findings and recommendations of those who have investigated this subject and provide practical guidance in the design of music rooms.[37] Heinrich Benecke[38] determines the optimal absorption for rooms from a consideration of optimal reverberation times recommended by Lifschitz and Watson He concludes that the absorption in a room should be of such an amount that the "time of growth" (time required for the sound to grow to 0.63 of its steady state value) will be 0.06 sec.

At a recent meeting of the Acoustical Society of America, December 4, 1933, J. P. Maxfield presented a paper on the acoustics of sound recording rooms in which he shows that the ratio of the intensities of the direct sound and the total reflected or reverberant sound is an important quantity in determining the acoustical condition of a room.

§12. Variation of Reverberation Time with Frequency, for Speech or Music Rooms

In the preceding section we presented the results of experiments and inductive reasoning which determine the optimal times of reverberation for speech rooms, at a frequency of 512 cycles. The process of arriving at these optimal times is simple, direct and limited in accuracy only by errors in the experimental technique. Further, the correctness of these optimal times is well supported by the public acclaim of the acoustics of those speech rooms which have times of reverberation in agreement with the predicted optimal times. In the present section we shall consider the more difficult problem of determining how the reverberation time should vary with the frequency in order to provide the best acoustical condition for listening to speech or to music. We shall refer to this condition as the "optimal reverberation characteristic." Specifically, we wish to determine the particular

[36] See, for example, Lifschitz, Phys. Rev. **27**, 618 (1926) and Watson, J. Am. Inst. Arch. **16**, 259 (1928).

[37] Hope Bagenal has given us valuable data and recommendations on this subject, based upon a study of European concert halls. See Bagenal and Wood, *Planning for Good Acoustics* (Methuen, 1931). See also G. v. Bekesy, Ann. d. Physik **8**, 851 (1931), who arrives at the optimal reverberation time for music rooms from a consideration of what he calls the "Präsenz-Zeit" (the time interval in which a changing sound phenomenon, as the decay of sound in a room, is perceived as a whole), which is equal to about 0.8 sec. His conclusion is that the reverberation time should be about 1.0 sec. in small music rooms, increasing to 1.4 sec. in large concert halls.

[38] Heinrich Benecke, Ann. d. Physik **15**, 259 (1932).

functional relation which should exist between reverberation time and frequency in the ideal speech or music room.

Several criteria have been proposed for determining the optimal reverberation characteristic, two of which we shall now consider. The first criterion, proposed by MacNair,[39] is that the loudness level of all frequency components in speech or music should decay at the same constant rate, and a second criterion, proposed by the author,[40] is that all frequency components of a complex sound should decay at such rates as will make all components reach the threshold of audibility at the same instant. When the first criterion is realized, the second one is approximately satisfied, at least for frequencies up to 2000 cycles. Both criteria require a reverberation time at low frequencies (around 100 cycles) which is approximately twice as long as the time at 512 cycles. According to the first criterion, the reverberation time should remain approximately constant for frequencies above 1000 cycles; according to the second criterion it should increase for frequencies above 2000 cycles. The first criterion depends upon a functional relation between loudness, sound level and frequency and is the same for speech, music or any other sound. The second depends upon the "spectral distribution" of the vibrations which make up speech or music and therefore is different for each and every sound. From the practical standpoint it seems desirable to the author to give preference to the reverberation characteristic which conforms to the second criterion, based upon average spectral distribution of speech and music, particularly because this criterion requires a slight increase in reverberation time for frequencies above 1000 cycles. This tends to emphasize the high frequencies, namely, those frequencies which are most important for the correct recognition of the sounds of speech and for the preservation of the richest quality of music; and these also are the frequencies which are most likely to suffer excessive attenuation as they are propagated through the air.

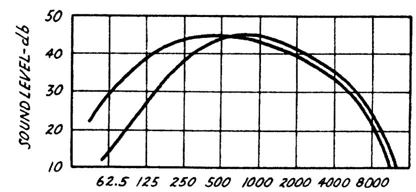

FIG. 12. Energy distribution of speech of men and women (after Fletcher). The curve to the left is for men, that to the right for women.

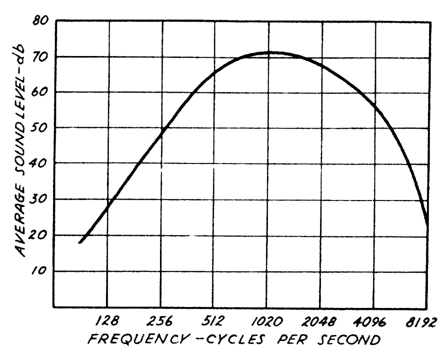

FIG. 13. Approximate energy distribution of music, as played by piano or orchestra.

Data giving the approximate spectral distribution of the speech of men and of women have been obtained by Fletcher[41] and are shown in Fig. 12 in a form suitable for the present purpose. The sound level at different frequencies is about that which it would be for unamplified speech in a large auditorium. Fig. 13 is a similar curve for music. The sound level is a maximum at about 500 to 1000 cycles, both for speech and for music, and falls off quite rapidly at both lower and higher frequencies. If the rates of decay at the different frequencies are such that all

[39] W. A. MacNair, *Optimum Reverberation Time for Auditoriums*, J. Acous. Soc. Am. 1, 242 (1930). Loudness depends upon frequency in a significant manner. Thus, a tone of 100 cycles 30 db above threshold sounds as loud, to the average person, as a tone of 1000 cycles which is 60 db above threshold. See Fletcher and Munson, J. Acous. Soc. Am. 5, 82 (1933).

[40] V. O. Knudsen, *Acoustics of Music Rooms*, J. Acous. Soc. Am. 2, 434 (1931). This is essentially the criterion proposed by Lifschitz (reference 36) for determining the optimal time of reverberation in rooms of different sizes.

[41] H. Fletcher, *Physical Characteristics of Speech and Music*, Bell Sys. Tech. J. 10, 349 (1931); Rev. Mod. Phys. 3, 258 (1931).

components will reach the threshold of audibility at the same instant, then, obviously, the rate of decay (in db per second) at a certain frequency must be directly proportional to the sound level at that frequency, that is, proportional to the ordinates as given by such a distribution as is shown in Figs. 12 or 13. Or, since the reverberation time is inversely proportional to the rate of decay, the criterion we have proposed will be satisfied, provided $St = \text{constant}$, where S is the sound level at a certain frequency and t is the time of reverberation at that frequency. This criterion is to be regarded only as a temporary working rule, which provides a satisfactory reverberation characteristic between frequencies of about 100 and 4000 cycles, rather than a valid law based upon physical and psychological principles. At very low and very high frequencies, for example, the proposed criterion would require the reverberation time to become infinitely long. In this respect, the criterion proposed by MacNair, namely, that during decay the loudness level of all components should diminish at a constant rate, is a more reasonable one; but the criterion $St = \text{constant}$ is here preferred, primarily because it provides for an increase in the reverberation time at frequencies above about 1000 cycles. Wente[42] also has called attention to certain facts which would seem to favor an increase in reverberation time at high frequencies. He emphasizes especially the necessity for (1) preserving or enhancing the high frequency components of speech because of their importance in consonants and (2) suppressing the low frequency components because of their masking effect on the higher frequencies. These arguments are worthy of attention from the practical standpoint, particularly because most materials used for the absorption of sound in rooms are much more absorptive for high frequencies than for low and there is therefore a real problem which arises from the over-absorption of the high frequencies.

The criteria here reviewed suggest that the optimal reverberation characteristic is one in which the reverberation time is about twice as long at 128 cycles as it is at 512 cycles, decreasing uniformly from 128 to 512 cycles; it then remains constant to frequencies of 1000 to 2000 cycles; and then increases again at higher frequencies. It is premature to formulate the exact shape of the optimal reverberation characteristic: it would appear at present that this can be determined only by a series of rather laborious experiments, requiring the cooperation of acousticians, phoneticians, musicians and possibly aestheticians.[43]

[42] E. C. Wente, Am. Arch. August 20 (1928).

§13. The Transmission of Sound Through Building Materials

From the theory of reflection and absorption considered in Section 9, it is apparent (see Eq. (21)) that a plane sound wave incident upon a rigid, non-porous material, as plaster, stone or wood, is nearly all reflected and that only a very small portion of the incident sound wave continues as a refracted ray in the new medium. The acoustical impedance z of most rigid materials (given by the product of the density of the medium and velocity of sound in the medium) is very large compared with the acoustical impedance of the air, so that for rigid, non-porous materials, which form the boundaries of most rooms, not more than one-millionth of the incident sound energy is refracted into the boundary material. This small amount of refracted energy is insufficient to account for the observed intensity of sound transmitted through

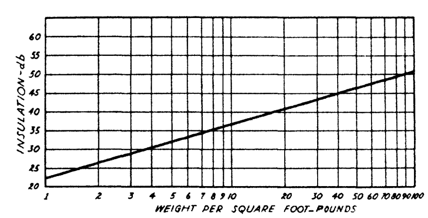

FIG. 14. The insulation of sound by solid, non-porous partitions. The transmission loss in db is directly proportional to the logarithm of the mass per sq. ft. of the partition.

[43] The author's experience would indicate, at least in the case of speech rooms, that the reverberation characteristic is not so critical as some recent writers on this subject would have it. (See reference 35.) M. J. O. Strutt, Rev. d'Acoustique 2, 1 (1933) expresses the opinion that distortion introduced by selective absorption does not seriously impair the perception of sound.

building partitions and, indeed, it is necessary to seek other explanations. The amount of insulation (measured in db loss) furnished by a rigid wall is found to be very closely proportional to the logarithm of the mass per unit area of the wall (see Fig. 14) which indicates that the wall reacts to the alternating force of the impinging aerial vibrations essentially as would a mass. In fact, Meyer[44] has shown that the root mean square acceleration of a rigid partition (from quarter-inch wood panels to heavy brick or concrete walls) is proportional directly to the alternating pressure of the incident sound and is essentially independent of the frequency of the sound, at least for frequencies above 100 cycles. The fundamental frequency of rigid panels, such as are used in buildings, is well below 100 cycles (usually of the order of 20 to 50 cycles) and therefore it is to be expected that the mass reaction, which is proportional to the frequency, will be dominant at frequencies well above the fundamental frequency of the partition. In the case of thin, flexible panels, the stiffness, the internal damping, the size of the panel and the manner in which it is fastened or clamped around the edges, all affect the amount of vibration which will be imparted to the partition. In general, it is necessary to take these factors into account only for low frequencies, below about 200 cycles for materials or structures used in buildings.

An entirely different mode of sound transmission occurs in very porous material, as in loose or felted wool or cotton or in supported panels of pumice, cinders, etc. In porous media of this type, the refracted ray becomes appreciable and the loss of sound energy is the result of viscous forces within the small pores, friction of fibers or other small ingredients rubbing against each other and the internal damping of flexural vibrations of component parts of the structure. The refracted ray therefore suffers a large attenuation and the total reduction (or insulation) in db will be proportional to the thickness of the porous partition. The insulation supplied by felted materials is of the order of 3 to 5 db per inch thickness for a frequency of 512 cycles. As would be expected, the attenuation, and therefore the insulation, increases rapidly with an increase of frequency.

§14. Calculation of Sound-Insulation and Noise-Reduction in Buildings

The insulation of sound is an almost universally neglected feature in the design of buildings. Although satisfactory insulation of sound in buildings often is incompatible with ventilation by means of open windows, much can be done to alleviate the excessive noise from which most of our metropolitan structures suffer. It is possible, at least, to calculate the insulation-value of any proposed design and thereby determine which elements in the proposed building are most responsible for the faulty or inadequate insulation.

If Eq. (4) be rewritten in terms of the average intensity I, defined by the average rate of flow of sound energy against unit area of the boundary, namely, $\rho_0 c/4$ in the steady state, then

$$I = E/a, \tag{24}$$

where E is the rate of supply of sound energy to the room and a is the total absorption in the room. Suppose a single-room structure be situated where its walls and ceiling are surrounded by a sound field of uniform intensity I'. Then

$$E = I'(\tau_1 s_1 + \tau_2 s_2 + \cdots) = I'\Sigma\tau s, \tag{25}$$

where $s_1, s_2 \cdots$ are the areas of the different types of boundary, and $\tau_1, \tau_2 \cdots$ are the corresponding coefficients of transmission (defined by the ratio of the transmitted to the incident sound energy). Substituting Eq. (25) in Eq. (24) and defining the noise-reduction-factor as I'/I, where I is the resultant intensity in the room, we get

$$I'/I = a/\Sigma\tau s. \tag{26}$$

That is, the noise-reduction-factor, which states how many fold the intensity of the outside noise is reduced after transmission into the room, is proportional directly to the amount of absorption in the room and inversely to the total transmission. Since the coefficients of absorption and transmission of most building materials and structures are known, it is possible by means of Eq. (26) to calculate the noise-reduction-factor for nearly all proposed or completed buildings.

[44] Meyer, *Grundlegende Messungen zur Schallisolation von Einfach.* Trennwänden, Sitzungsber. d. Preuss. Akad. d. Wiss., Phys.-Math. Klasse, IX, (1931).

It is customary to express the noise-reduction in db, in which case Eq. (26) becomes

$$\text{Noise-Reduction (in db)} = 10 \log_{10} (a/\Sigma\tau s). \quad (27)$$

This equation, with obvious modifications, is applicable not only to single-room structures but to all types of rooms encountered in practice. For example, if two adjacent rooms are separated by a uniform partition, except for a door, the $\Sigma\tau s$ is made up only of two terms, namely, $\tau_1 s_1 + \tau_2 s_2$, where τ_1 and s_1 refer to the partition and τ_2 and s_2 to the door. In most rooms τ_2 will exceed τ_1 by a much greater ratio than s_1 exceeds s_2 and, therefore, most of the transmission is through the door. In such a case it would be futile to improve the insulation of the partition without providing an even greater improvement in the insulation of the door. Other examples, such as the transmission through windows, openings, ducts, etc., will reveal the necessity for making calculations of sound-insulation in connection with the entire design of a building. Unfortunately, this is rarely, if ever, done. It should be mentioned, however, that many architects, and especially manufacturers of sound-absorbents, do recognize the significance of the numerator of Eq. (27) and therefore use absorptive materials for the ceilings (or both the ceilings and walls) in offices, hospitals, restaurants and many other public buildings, for the purpose of reducing outside as well as inside noise. By this means, it often is possible to reduce the noise in a room by as much as 7 or 8 db, which, happily, under the conditions most frequently encountered in practice, is judged by the average person to correspond to a reduction of loudness of about one-half and this often means the difference between satisfactory and unsatisfactory acoustics.

In conclusion, if noise measurements have been made in the vicinity of the site of a proposed building and if the amount of noise which can be tolerated inside the building has been determined (or agreed upon), it is possible to design the building in such a manner as will fulfill the specified requirements and, in many instances, this can be done in a practical manner without adding appreciably to the cost of the building.

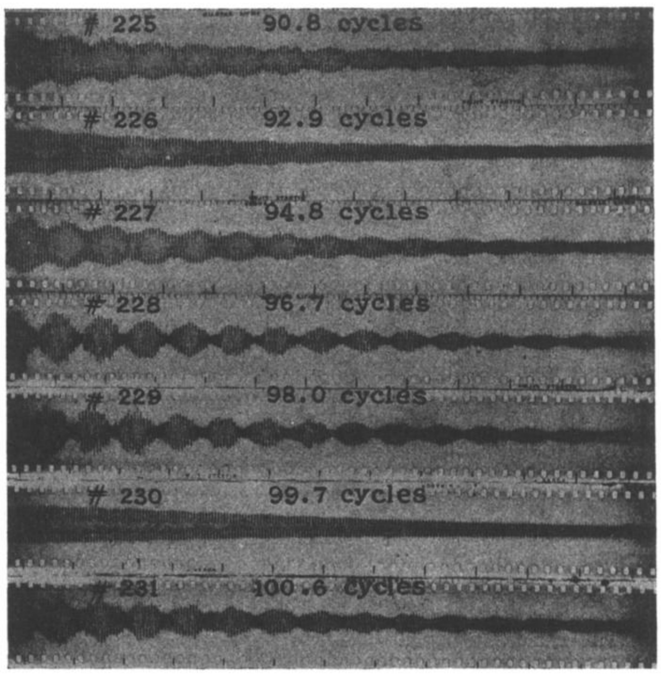

FIG. 2. Oscillograms of the decay of sound in a small rectangular room, showing that the decay of sound consists of the damped free vibrations of the room.

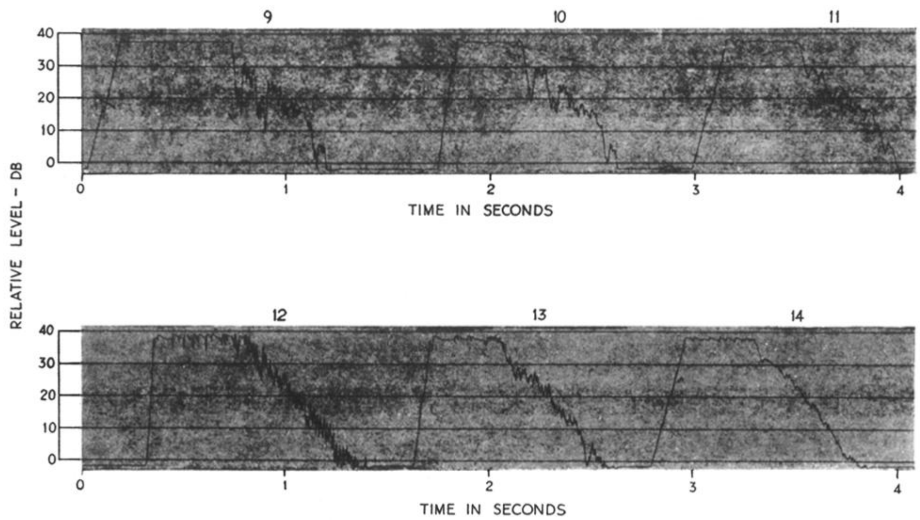

FIG. 3. Curves showing the decay of sound in rooms, obtained by an automatic sound recorder developed by Bell Telephone Laboratories, Inc. Curves 9, 10 and 11 are for a pure tone with the recorder adjusted to speeds of 240, 120, and 60 db/sec., respectively. Curves 12, 13 and 14 were made with corresponding recorder speeds but with warble tones.