Breaking of isospin symmetry in compound-nucleus reactions

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The authors present an analysis of all extant data on isospin mixing in statistical compound-nucleus reactions. The analysis is based on a generalization of the Hauser-Feshbach formula allowing for isospin mixing. The strength of the mixing is described by a single parameter z. The theory is applicable when all compound-nucleus resonances overlap strongly. It is derived from a statistical theory of nuclear reactions allowing for the mixing of two classes of states. The parameter z comprises both internal mixing (via the Coulomb interaction) and external mixing (via the channels). The theory contains both the static criterion (Coulomb matrix elements compared with spacings) and the dynamical criterion (spreading widths compared with decay widths) for isospin symmetry breaking. The theory yields the dependence on z of observables like average cross sections, and auto- and cross-correlation functions. The data show that isospin symmetry breaking is neither so weak as to be altogether negligible, nor so strong as to reduce our theory to a Hauser-Feshbach formalism without any reference to the isospin quantum number. The authors argue in favor of a parametrization of isospin symmetry breaking in the data in terms of a spreading width rather than a Coulomb matrix element. They find that internal mixing dominates, and that the associated spreading width is nearly independent of mass number and excitation energy.

CONTENTS

I. INTRODUCTION

From a comparison of (γ,p) and (γ,n) cross sections, Morinaga inferred in 1955 that isospin is at least partially conserved in statistical nuclear reactions. Since then, many experimental and theoretical investigations have been devoted to the study of isospin symmetry in compound-nucleus reactions. This problem forms the topic of the present review.

Since we do not in this paper address the general subject of isospin mixing in nuclei [we refer the reader to Wilkinson (1969), Anderson *et al.* (1969), Auerbach (1983) and the numerous papers cited therein], it is perhaps useful to define the concept of a statistical nuclear reaction (or, equivalently, a compound-nucleus reaction) at this stage. We consider reactions in which an intermediate compound nucleus is formed at such a high excitation energy that *the average width of the compound-*

nucleus resonances is larger than their average spacing.

The implications of this definition are borne out as we consider Fig. 1. This figure shows an isolated isobaric analog resonance (Genz *et al.*, 1975) with $J^\pi = \frac{1}{2}^+$ in the compound nucleus ^{89}Y. The resonance has isospin $T_2 = \frac{13}{2}$. The fine structure visible in the excitation function of Fig. 1 is due to states with isospin $T_1 = \frac{11}{2}$ (the same isospin as the ground state of ^{89}Y). We refer to these states as class one states. The mean width of the class one resonances is larger than their mean spacing, and the fine structure visible in Fig. 1 can therefore not be resolved into the contributions of individual resonances. Rather, it is the manifestation of statistical fluctuations (Ericson fluctuations) of the cross section [see Ericson and Mayer-Kuckuk (1966)]. As the excitation energy of the compound nucleus ^{89}Y is increased beyond the domain shown in Fig. 1, the average spacing of the states with isospin $T_2 = \frac{13}{2}$ (the "class two states") decreases, and their average width increases. A few MeV above the excitation energy of Fig. 1 we encounter a situation where the average spacing of the class two states is also smaller than their average width. This is the domain of statistical nuclear reactions reviewed in this paper: The compound-nucleus resonances in classes one *and* two overlap strongly. The cross section for any reaction then no longer displays isolated resonance features but only statistical fluctuations.

In a statistical nuclear reaction, the spectroscopy of isolated levels is impossible by definition, and a statement on isospin symmetry breaking relates to *average* properties of the levels. Such a statement can, for instance, be inferred from a comparison of average cross sections of an isospin-forbidden and an isospin-allowed reaction which populate the same compound nucleus. One of the first comprehensive studies of this kind was published by Bizzeti and Bizzeti-Sona in 1968. These authors found a sizable suppression of the isospin-forbidden reaction, confirming Morinaga's conjecture. At first sight, this result is astonishing, for the following reason. The compound nucleus ^{26}Al studied by Bizzeti and Bizzeti-Sona (1968) at 20 MeV excitation energy has an average level spacing of

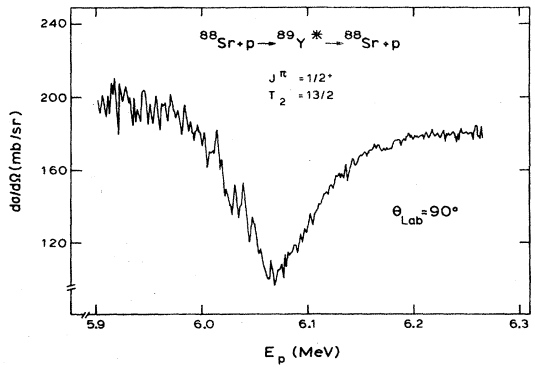

FIG. 1. Isobaric analog resonance (with spin and isospin $J^\pi; T_2 = \frac{1}{2}^+; \frac{13}{2}$) and fine structure (with isospin $T_1 = \frac{11}{2}$) in the compound nucleus ^{39}Y, excited by elastic proton scattering on ^{88}Sr [from Genz *et al.* (1975)].

about 1 keV for either class of states. Partial conservation of isospin symmetry seems to require that the isospin-breaking matrix elements have a root-mean-square value smaller than 1 keV. In actual fact (see Sec. VII), the root-mean-square value is about 2 keV in this case. That this is compatible with partial isospin conservation in spite of the small average level spacing was intuitively and correctly ascribed by Morinaga to the short lifetime of the compound nucleus, which prevents the states in one class from completely mixing with those in the other.

Statements such as this cannot be quantified, and the analysis and interpretation of the data remain incomplete, without an appropriate theory of isospin mixing in statistical nuclear reactions. Such a theory was first formulated by Grimes *et al.* in 1972. Since then, several groups have contributed to the growth of the theory in both completeness and complexity.

It is the primary purpose of the theory to describe the dependence of average cross sections and of cross-section correlation functions on level densities, compound-nucleus lifetimes, and the root-mean-square of the isospin-mixing matrix elements. However, recent theoretical developments in areas beyond the realm of nuclear physics have given the theory of statistical nuclear reactions a wider perspective. From the study of classical chaotic dynamical systems and their quantum counterparts, it now appears that the spectral fluctuation properties of nuclei (the fluctuations of level spacings about their average value) are manifestations of chaotic behavior [see Bohigas *et al.* (1984)]. Moreover, the same statement appears to apply [see Verbaarschot *et al.* (1984,1985); Weidenmüller (1984); Mello *et al.* (1985)] to the fluctuations in energy of the elements of the nuclear scattering matrix about their mean values—the very phenomenon studied in a statistical nuclear reaction. This connection suggests that fluctuation properties of nuclei are independent of specific details of the nuclear dynamics, are generic for small systems, and are therefore of general interest. In the light of these developments, the study of isospin symmetry breaking in statistical nuclear reactions gains a new dimension because it shows how a partially conserved quantum number affects the dynamics of an otherwise chaotic quantum system.

The theoretical framework used in this review was laid down in several publications (Harney *et al.*, 1977,1980,1983; Weidenmüller *et al.*, 1978). It is based on a diagrammatic expansion technique, the results of which are consistent with, but generalize, earlier findings (Grimes *et al.*, 1972). The expansion technique applies in the case considered here of strongly overlapping compound-nucleus resonances. Our faith in the results obtained by this method has been strengthened recently by developments in the (simpler) case of compound-nucleus reactions without isospin mixing. There, it has been possible to derive analytically closed-form expressions for average cross sections (Verbaarschot *et al.*, 1984,1985). It was then shown that the asymptotic expansion (valid for strongly overlapping resonances) of these formulas gives the same results as the diagram expansion technique of

Agassi *et al.* (1975). We expect that an analogous relation exists between the exact solution of the isospin-mixing problem in compound-nucleus reactions (which has not as yet been derived) and the diagram expansion results which we use.

In view of these facts, and since our theoretical framework yields expressions for all observables investigated so far to detect isospin mixing, we feel that this review is timely. In particular, we include some details of the theoretical derivation that have not been published before.

For pedagogical reasons we begin in Sec. II with an extension of the standard statistical model. We consider two classes of compound-nucleus states, differing in isospin, but we do not allow for any isospin mixing. Experimental data presented in Sec. III serve to illustrate some features of this theory. The data to be analyzed in Sec. VI show, however, that isospin mixing cannot be neglected in general. A suitable extension of the theoretical framework that allows for isospin mixing is described in Sec. IV. We pay particular attention to the statistical assumptions that we introduce. Relegating details of the theoretical derivation to Appendix A, we conclude Sec. IV with the general expression for the correlation function of a pair of elements of the nuclear scattering matrix. This result is discussed in Sec. V, where we devote special attention to the distinction between external and internal isospin mixing. In Sec. VI we discuss all experimental data known to us in the framework laid down in Secs. IV and V. This is done by specializing the general expressions of Sec. IV to seven different types of experiments. In Sec. VII the results are condensed into tables and figures. As stated above, we are concerned only with statistical reactions. We do not relate mixing parameters and matrix elements to nuclear structure properties. Rather, it is our purpose to organize the information available from the experiments in such a way that it can be compared for different nuclei and/or different excitation energies and is available for future nuclear structure studies. In spite of these self-imposed limitations, we note that the results given in Sec. VII nicely complement the information on isospin mixing obtained from studies of bound states and of isobaric analog resonances, which causes us to draw at least some qualitative conclusions on nuclear structure aspects.

The aim and layout of this article—a thorough examination of the experimental results in the framework of a single comprehensive theory—has prevented us from presenting in detail the theoretical work of Grimes *et al.* (1972), Shikazono and Terasawa (1975), Feinstein (1977), and Friedman *et al.* (1981), who have addressed various aspects of the problem. We apologize to these authors and to all those whose work we have overlooked.

II. ISOSPIN AS A CONSERVED QUANTUM NUMBER IN COMPOUND-NUCLEUS REACTIONS

Before discussing the subject proper of this review—isospin symmetry breaking in compound-nucleus reactions—it is advantageous to consider the idealized case of complete isospin conservation, as treated by Robson *et al.* (1973). This is done in the present section. We use the opportunity to introduce some basic notations and definitions. More importantly, we derive and/or display some fundamental physical concepts which pervade everything that is to follow.

A. Isospin in compound-nucleus reactions

We consider a reaction that leads from channel α to channel β via compound-nucleus formation. All states of the compound nucleus are assumed to have pure isospin T. The ground state has $T=T_1$ and isospin projection $T_z=T_1$. We consider excitation energies such that the average level spacing D_1 of levels with isospin T_1 ("class one" levels) is very small in comparison to the average width Γ_1 of such levels. (A precise definition of Γ_1 is given below.) We make the same assumption for levels in class two with isospin $T_2=T_1+1$ so that

$$D_m \ll \Gamma_m, \quad m=1,2 \; . \tag{2.1}$$

We neglect altogether compound-nucleus levels with isospin $T>T_2$. This assumption is reasonably realistic for those nuclei for which actual data exist; it simplifies the problem to a treatment of two classes of compound-nucleus levels.

If isospin is conserved, the reaction channels can also be labeled in terms of T. We accordingly write the channel labels in the form (a,T), where a comprises all quantum numbers necessary to characterize relative motion and internal structure of the two reaction partners (including their individual isospin values), and T is the total isospin, which, by assumption, is restricted to the two possible values T_1, T_2.

The channels (a,T) just introduced differ from the *physical channels* which are characterized by isospin projections of the reaction partners rather than by total isospin values. (For instance, a proton plus a target with isospin T_a is described by a linear combination of states with isospins $T_a \pm \frac{1}{2}$.) We label the physical channels by (a,t), where t is the isospin projection of the light fragment. [The fixed value $T_z=T_1$ of the isospin projection of the compound nucleus then fixes the isospin projection of the heavy fragment to be (T_1-t).] The transition from the (a,T) representation to the (a,t) representation obviously involves an orthogonal transformation, the elements $\langle aT \mid at \rangle$ of which are the Clebsch-Gordan coefficients,

$$\langle aT \mid at \rangle = (T_a, T_1-t; t_a, t \mid T, T_1) \; , \tag{2.2}$$

where T_a and t_a are the isospin values of heavy and light fragments, respectively. In what follows, only light fragments with $t_a=0$ (deuterons, alpha particles, etc.) or $t_a=\frac{1}{2}$ (protons, neutrons, tritons, etc.) will be considered. For $t_a=0$, the coefficient $\langle aT \mid at \rangle$ is either unity or zero, depending on whether $T_a=T$ or $T_a \neq T$. For $t_a=\frac{1}{2}$, the coefficients $\langle aT \mid at \rangle$ can be found in Fig. 2.

For $t_a=\frac{1}{2}$, the physical channels may be grouped into

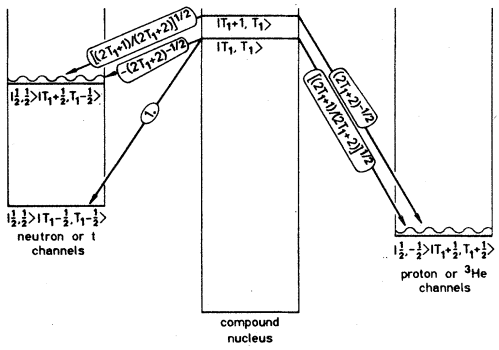

FIG. 2. Nucleon (or more generally $t_a=\frac{1}{2}$) decay channels from a compound nucleus having states with isospin configurations $|T,T_z\rangle=|T_1,T_1\rangle$ and $|T_1+1,T_1\rangle$. The arrows indicate transitions to channels for which the Clebsch-Gordan coefficients $\langle aT\,|\,at\rangle\neq 0$. The values of these coefficients are given on the arrows. In all three nuclei, states with isospin T_1+2 or greater are neglected.

pairs of mirror channels. Except for the sign of t these channels carry identical quantum numbers. In the absence of all isospin-breaking forces, the threshold energies of a pair of mirror channels are equal. In physical reality, there is a Coulomb energy difference between the ground-state energies of fragments forming a pair of mirror channels, as indicated by the wavy lines in Fig. 2. In consequence the threshold energy of a proton channel lies below that of its mirror neutron channel. This is disregarded in the present section. The actual error incurred in this way is not as serious as one might suspect because the energy of the Coulomb barrier of the proton precisely compensates for the difference in threshold energies. (This argument does not, of course, apply to mirror channels involving ^{3}He and ^{3}H, or heavier pairs with isospin $\frac{1}{2}$. At moderate excitation energies, however, the number of such channels is very small.)

Since isospin is conserved, the nuclear scattering matrix (S matrix) in the (a,T) representation is diagonal with respect to isospin,

$$S_{aT,bT'}=\delta_{TT'}\cdot S^T_{a,b}\ . \tag{2.3}$$

Usually the S matrix is written in a representation in which the physical channels (a,t) are used. The two S matrices $S_{at,bt'}$ and $S^T_{a,b}$ are related by

$$S_{at,bt'}=\sum_{T=T_1}^{T_1+1}\langle aT\,|\,at\rangle S^T_{a,b}\langle bT\,|\,bt'\rangle\ . \tag{2.4}$$

B. Compound-nucleus reactions without isospin breaking

We first recall a few general facts and results of compound-nucleus theory [see Mahaux and Weidenmüller (1979)]. Ignoring, for the moment, the question of isospin symmetry, we consider a general S-matrix element $S_{\alpha,\beta}(E)$ characterized by a complete set of quantum numbers, with E the excitation energy of the compound nucleus and α,β a pair of channels. In this review we deal exclusively with situations in which neither individual resonance structures (doorways) nor precompound processes obscure the pure compound-nucleus process. Then $S_{\alpha\beta}(E)$ can be written as the sum of two terms:

$$S_{\alpha\beta}(E)=\langle S_{\alpha\beta}(E)\rangle+S^{\rm fl}_{\alpha\beta}(E)\ , \tag{2.5}$$

where the angle brackets denote an average with respect to energy over an interval of length $I\gg\Gamma$, with Γ the average width of the compound-nucleus resonances. The average S matrix $\langle S_{\alpha\beta}(E)\rangle$ describes the fast processes (shape elastic scattering, or direct reactions), while the fluctuating part $S^{\rm fl}_{\alpha\beta}(E)$ stands for that part of the reaction amplitude due to the long-lived compound system. It is assumed that the separation (2.5) uniquely and completely describes the system, i.e., that no intermediate structure exists. Without loss of generality we assume that $\langle S_{\alpha\beta}(E)\rangle$ is diagonal,

$$\langle S_{\alpha\beta}(E)\rangle=\delta_{\alpha\beta}\langle S_{\alpha\alpha}(E)\rangle\ . \tag{2.6}$$

If $\langle S_{\alpha\beta}(E)\rangle$ does not obey Eq. (2.6), a unitary transformation can be found [see Engelbrecht and Weidenmüller (1973), Hofmann *et al.* (1975), and Nishioka and Weidenmüller (1985)] such that Eq. (2.6) is obeyed by the transformed matrix.

For $\Gamma\gg D$ where D is the average compound-nucleus level spacing, it is also known that $S^{\rm fl}_{\alpha\beta}(E)$ is a randomly fluctuating function of energy with a Gaussian probability distribution [see Agassi *et al.* (1975)]. This distribution is completely characterized by the first two moments:

$$\begin{aligned}&\langle S^{\rm fl}_{\alpha\beta}(E)\rangle=0\ ,\\ &\langle S^{\rm fl}_{\alpha\beta}(E_1)S^{\rm fl}_{\delta\gamma}(E_2)\rangle=0\ ,\\ &\langle S^{\rm fl}_{\alpha\beta}(E_1)S^{\rm fl*}_{\delta\gamma}(E_2)\rangle=(\delta_{\alpha\gamma}\delta_{\beta\delta}+\delta_{\alpha\delta}\delta_{\beta\gamma})\frac{T_\alpha\cdot T_\beta}{N+2i\pi\varepsilon/D}\ .\end{aligned} \tag{2.7}$$

The first equation follows from the definition of $S^{\rm fl}_{\alpha\beta}(E)$; the second equation expresses the fact that all poles of $S_{\alpha\beta}$ lie on the same side of the real E axis. The transmission coefficients T_α in the third equation measure the unitarity deficit of the average S matrix and are defined by

$$T_\alpha=1-|\langle S_{\alpha\alpha}\rangle|^2\ . \tag{2.8a}$$

The symbol ε denotes the energy difference E_2-E_1, and the "effective number of open channels" N is defined by

$$N=\sum_\alpha T_\alpha\ . \tag{2.8b}$$

The width Γ of the compound nucleus is given by

$$\Gamma=\frac{D}{2\pi}\cdot N\ ; \tag{2.9}$$

it is the correlation width of the S matrix. The quantity $\hbar/\Gamma$ is the average lifetime of the compound nucleus.

Equations (2.7) and the fact that $S^{\rm fl}_{\alpha\beta}$ is Gaussian distributed allow us to evaluate average compound-nucleus cross sections, as well as auto- and cross-correlation functions of nuclear cross sections. In particular, the combination of Kronecker symbols in the last of Eqs. (2.7) leads to an elastic enhancement factor of 2 (Kretschmer and Wangler, 1978; Mahaux and Weidenmüller, 1979).

We disregard all kinematical factors and define cross sections as squares of S-matrix elements:

$$\sigma_{\alpha\beta}(E)=|S_{\alpha\beta}(E)-\delta_{\alpha\beta}|^2 . \tag{2.10}$$

Using Eqs. (2.5) and (2.6) and the first of Eqs. (2.7), we write the average $\langle\sigma_{\alpha\beta}\rangle$ of $\sigma_{\alpha\beta}$ as the sum of a shape elastic part $\sigma^{\rm SE}$ and a compound-nucleus part $\sigma^{\rm CN}$:

$$\langle\sigma_{\alpha\beta}(E)\rangle=\delta_{\alpha\beta}\sigma^{\rm SE}_{\alpha\alpha}+\sigma^{\rm CN}_{\alpha\beta} \tag{2.11}$$

with

$$\sigma^{\rm SE}_{\alpha\beta}=|\langle S_{\alpha\alpha}\rangle-1|^2,\quad \sigma^{\rm CN}_{\alpha\beta}=\langle|S^{\rm fl}_{\alpha\beta}(E)|^2\rangle . \tag{2.12}$$

The cross-correlation function $C_{\alpha\beta,\gamma\delta}(\varepsilon)$ of the cross section is defined by

$$\begin{aligned}C_{\alpha\beta,\gamma\delta}(\varepsilon)&=\langle\sigma_{\alpha\beta}(E)\sigma_{\gamma\delta}(E+\varepsilon)\rangle-\langle\sigma_{\alpha\beta}\rangle\cdot\langle\sigma_{\gamma\delta}\rangle\\&=|\langle S^{\rm fl}_{\alpha\beta}(E)S^{\rm fl*}_{\gamma\delta}(E+\varepsilon)\rangle|^2 .\end{aligned} \tag{2.13}$$

The last equality follows from the Gaussian distribution of S-matrix elements and from Eqs. (2.7).

We now apply Eqs. (2.5)—(2.9) to the case of isospin-conserving compound-nucleus reactions. In keeping with the statistical model, we postulate that for $T_1\neq T_2$, $S^{T_1}_{ab}(E_1)$, and $S^{T_2}_{a'b'}(E_2)$ are uncorrelated random functions, each individually characterized by relations of the type (2.5)—(2.9). The transmission coefficients τ_{aT} are now defined by

$$\tau_{aT}=1-|\langle S^T_{a,a}\rangle|^2 ; \tag{2.14}$$

by a straightforward extension of the last of Eqs. (2.7) we have

$$\begin{aligned}\langle S^{T{\rm fl}}_{a,b}(E_1)S^{T'{\rm fl}*}_{c,d}(E_2)\rangle&=\delta_{TT'}(\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc})\\&\times\frac{\tau_{aT}\tau_{bT}}{N_T+2i\pi\varepsilon/D_T} .\end{aligned} \tag{2.15}$$

Here, D_T is the average level spacing of states with isospin T, $\varepsilon=E_2-E_1$, and

$$N_T=\sum_c\tau_{cT} \tag{2.16a}$$

is the effective number of decay channels open to the levels of class T. We note that Eq. (2.15) allows for the existence of different average widths

$$\Gamma_T=\frac{1}{2\pi}D_T\cdot N_T \tag{2.16b}$$

for the two classes. We also use the notation N_m and D_m with $m=1,2$ replacing the indices T_1 and T_2.

Using the transformation (2.4), we can express Eqs. (2.7) in terms of the S-matrix elements relating to the physical channels (a,t). We find

$$\begin{aligned}\langle S^{\rm fl}_{at,bt'}(E_1)S^{\rm fl*}_{ct'',dt'''}(E_2)\rangle&=\delta_{ac}\delta_{bd}\sum_T\frac{\tau^{tt''}_{aT}\tau^{t't'''}_{bT}}{N_T+2i\pi\varepsilon/D_T}\\&+\delta_{ad}\delta_{bc}\sum_T\frac{\tau^{tt'''}_{aT}\tau^{t't''}_{bT}}{N_T+2i\pi\varepsilon/D_T} .\end{aligned} \tag{2.17}$$

We have introduced the quantities

$$\tau^{tt'}_{aT}=\langle aT\,|\,at\rangle\tau_{aT}\langle aT\,|\,at'\rangle . \tag{2.18}$$

These will also be referred to as transmission coefficients in the sections that follow, although this term is a misnomer: The coefficients $\tau^{tt'}_{aT}$ are not related to the unitarity deficit of $\langle S_{at,bt'}\rangle$, in contradistinction to Eq. (2.14) connecting $\langle S^T_{a,a}\rangle$ and the τ_{aT}. The effective number of channels is now written as

$$N_T=\sum_{c,t}\tau^{tt}_{cT} . \tag{2.19}$$

C. Observables in compound-nucleus reactions with conserved isospin symmetry

Equation (2.17) shows that the average compound-nucleus cross section connecting two physical channels (a,t) and (b,t') is given by

$$\begin{aligned}\sigma^{\rm CN}_{at,bt'}&=\sum_T N_T^{-1}(\tau^{tt}_{aT}\tau^{t't'}_{bT}+\delta_{ab}\tau^{tt'}_{aT}\tau^{tt'}_{aT})\\&=\left[\sum_T N_T^{-1}\tau_{aT}\tau_{bT}\langle aT\,|\,at\rangle^2\langle bT\,|\,bt'\rangle^2\right](1+\delta_{ab}) .\end{aligned} \tag{2.20}$$

The last factor reveals that elastic scattering ($a=b,t=t'$) is characterized by an elastic enhancement factor of 2. At the same time, however, Eq. (2.20) shows that an enhancement factor of 2 arises also in a *charge exchange reaction* populating a mirror channel ($a=b,t=-t'$); see Harney *et al.* (1980). This is a natural consequence of the fact that, without isospin symmetry breaking, the reaction amplitudes populating mirror channels are identical, except for geometrical factors.

We turn now to the cross-section fluctuations. Specializing the cross-correlation function (2.13) to our present situation, and using Eq. (2.17), we find

$$\begin{aligned}C_{at,at';ct'',dt'''}(\varepsilon)=\Bigg|\sum_T\frac{\tau_{aT}\tau_{bT}}{N_T}\cdot\frac{\Gamma_T}{\Gamma_T+i\varepsilon}\langle aT\,|\,at\rangle\langle bT\,|\,bt'\rangle(\delta_{ac}\delta_{bd}\langle aT\,|\,at''\rangle\langle bT\,|\,bt'''\rangle\\+\delta_{ad}\delta_{bc}\langle aT\,|\,at'''\rangle\langle bT\,|\,bt''\rangle)\Bigg|^2 .\end{aligned} \tag{2.21}$$

We observe that the correlation function (2.21) is a coherent superposition of two Lorentzian terms. One retrieves a single Lorentzian—the result of Ericson's original theory of cross-section fluctuations (Ericson and Mayer-Kuckuk, 1966; Richter, 1974)—in two cases: (i) if the two classes of levels have the same correlation width and (ii) if the reaction proceeds via one of the two classes of levels only.

It is clear that the present way of including isospin in the statistical model applies to any conserved quantum number, e.g., the total angular momentum J of the system. The J dependence of the correlation width Γ was, however, always neglected in the analysis of fluctuating excitation functions. This can be justified by numerical calculations performed by Eberhard *et al.* (1969), Ernst *et al.* (1969), and Eberhard and Richter (1972). Hence Eq. (2.21) has been discussed by Robson *et al.* (1973), but not used, in connection with angular momentum conservation. In contradistinction, the two widths Γ_1 and Γ_2 may be very different in reactions involving two different isospins, owing to the fact that both the mean level spacing D_m and the effective numbers of open channels N_m often are very different for the two classes (see Secs. III and VI).

A basic result of fluctuation theory remains untouched by the present extension: The variance of compound-nucleus cross sections, i.e., the value of $C(\varepsilon=0)$, is equal to the square of the average compound-nucleus cross section. This follows from a comparison of Eqs. (2.20) and (2.21).

Equation (2.21) contains the autocorrelation function $C_{at,bt'}(\varepsilon)$ as a special case, obtained by setting $c=a$, $d=b$, $t''=t$, and $t'''=t'$. For a pair of mirror channels (b,t') and $(b,-t')$, Eq. (2.21) implies that the cross-correlation function $C_{at,bt';at,b-t'}(\varepsilon)$ is different from zero. This is most clearly exhibited by a special example. Let the entrance channel (a,t) populate only the class one states and let $T_1=0$. Then we find from Eq. (2.21) and Fig. 2

$$C_{at,bt'}(\varepsilon)=C_{at,b-t'}(\varepsilon)=C_{at,bt';at,b-t'}(\varepsilon)\ . \tag{2.22}$$

Hence, in this case, the autocorrelation functions of a pair of mirror channels are equal. Moreover, the cross-correlation function between the pair of mirror channels is different from zero and equal to either of the two autocorrelation functions. Such cross-correlation functions were first measured experimentally by Détraz *et al.* (1971) and by Simpson *et al.* (1978), and treated theoretically by Weidenmüller *et al.* (1978; see Sec. VI of the present paper).

The discussion in this section shows that any use of the statistical model requires a careful analysis of the underlying statistical assumptions. Assuming isospin to be a conserved quantum number, we are naturally led to consider S-matrix elements pertaining to different isospin values as uncorrelated. This in turn implies a strong correlation of fluctuations in mirror channels. In the limit of strong isospin symmetry breaking, on the other hand, one would expect $S^{\rm fl}_{at,b\,1/2}$ and $S^{\rm fl}_{at,b\,-1/2}$ to be uncorrelated, and this is indeed what we shall find (see Sec. IV).

III. REACTIONS IN WHICH ISOSPIN SEEMS NEARLY CONSERVED: THE CASE OF TWO CORRELATION WIDTHS

The experimental data presented in Sec. VI show convincingly that the theory developed in Sec. II is too simple, and that isospin mixing cannot be neglected altogether. However, some data can be understood semiquantitatively with the theory of Sec. II, as we now demonstrate.

Robson *et al.* (1973), Kildir and Huizenga (1973), and Kreische *et al.* (1976) noticed that (p,p') and (p,α) reactions on the same target nucleus and at the same bombarding energy may have widely differing correlation lengths Γ_p and Γ_α, respectively. An example is shown in Fig. 3. An explanation of this observation in terms of a difference of angular momenta in the exit channels can be ruled out. The effect has been interpreted with the help of the theory in Sec. II as a manifestation of the different isospin states contributing to the two reactions.

As an example [analyzed by Robson *et al.* (1973) and Kildir and Huizenga (1973) on the basis of data reported by Ernst *et al.* (1969) and Katsanos *et al.* (1970)], we consider the reactions

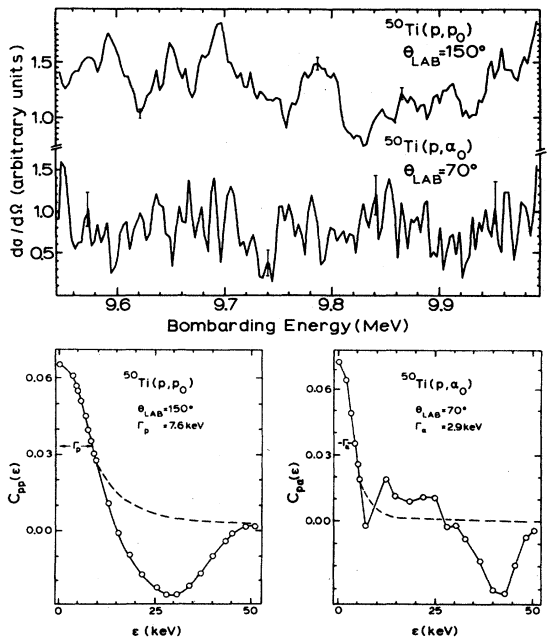

FIG. 3. Cross-section fluctuations in the reactions $^{50}\mathrm{Ti}+p\rightarrow{}^{51}\mathrm{V}^*\rightarrow{}^{50}\mathrm{Ti}+p$ and $^{50}\mathrm{Ti}+p\rightarrow{}^{51}\mathrm{V}^*\rightarrow{}^{47}\mathrm{Sc}+\alpha$, measured by Kreische *et al.* (1976). The correlation widths are different in the proton- and alpha-particle channels. This is obvious from the excitation functions and is measured by the width at half maximum of the autocorrelation functions $C(\varepsilon)$.

$$^{54}\mathrm{Cr}+p\rightarrow{}^{55}\mathrm{Mn}^{*}\rightarrow\begin{cases}^{54}\mathrm{Cr}(T=T_1+\frac{1}{2})+p' , \\ ^{51}\mathrm{V}(T=T_1)+\alpha . \end{cases} \tag{3.1}$$

The target nucleus ^{54}Cr has isospin $T=3$, so that $T_1=\frac{5}{2}$ and $T_2=\frac{7}{2}$. The states populated in the (p,α) reaction have the isospin $\frac{5}{2}$ of the ^{51}V nucleus. Under strict isospin conservation, the (p,α) reaction therefore proceeds only via compound states with $T=T_1=\frac{5}{2}$ (the class one states), while the (p,p') reaction populates both class one and class two levels. Adapting Eq. (2.21) to these two situations, we find, using a simplified notation

$$C_{p\alpha}(\varepsilon)=\frac{\tau_{p1}^2\cdot\tau_{\alpha1}^2}{N_1^2}\left|\frac{\Gamma_1}{\Gamma_1+i\varepsilon}\right|^2 . \tag{3.2}$$

Here, $\tau_{p1}=\tau_{p1}^{1/2}{}^{1/2}$ and correspondingly for $\tau_{\alpha1}$. Analogously, the expression for $C_{pp'}(\varepsilon)$ reads

$$C_{pp'}(\varepsilon)=\left|\frac{\tau_{p2}\tau_{p'2}}{N_1}\frac{\Gamma_1}{\Gamma_1+i\varepsilon}+\frac{\tau_{p2}\tau_{p'2}}{N_2}\frac{\Gamma_2}{\Gamma_2+i\varepsilon}\right|^2(1+\delta_{pp'})^2 . \tag{3.3}$$

We note that $C_{pp'}(\varepsilon)$ depends upon both Γ_1 and Γ_2, in contrast to $C_{p\alpha}(\varepsilon)$. Because of finite-range-of-data errors, it is usually not possible to extract more than one datum of information from the correlation function, typically the width at half maximum[1] (see Fig. 3). For the (p,α) reaction, this yields $\Gamma_\alpha=\Gamma_1$. The width Γ_2 can then be obtained from the width at half maximum of $C_{pp'}(\varepsilon)$, provided the ratio $(\tau_{p2}\tau_{p'2}/N_2)/(\tau_{p1}\tau_{p'1}/N_1)$ of compound-nucleus cross sections is known. This ratio can be determined semiempirically by calculating the transmission coefficients τ_{pi} $(i=1,2)$ and $\tau_{\alpha1}$ for the actual reaction channels (see footnote 2 in Sec. VI.A), and by determining N_1 and N_2 from the measured cross sections

$$\sigma_{p\alpha}=\frac{\tau_{p1}\cdot\tau_{\alpha1}}{N_1} , \tag{3.4}$$

$$\sigma_{pp'}=\left[\frac{\tau_{p1}\tau_{p'1}}{N_1}+\frac{\tau_{p2}\tau_{p'2}}{N_2}\right](1+\delta_{pp'}) . \tag{3.5}$$

For the case of the reactions (3.1), the excitation energy of ^{55}Mn* was 17.6 MeV, and the correlation widths were found (Robson *et al.*, 1973) to be $\Gamma_1=\Gamma_\alpha=8.2\pm0.6$ keV and $\Gamma_p=13.9\pm0.7$ keV. [The quantity Γ_p is the width at half maximum of $C_{pp'}(\varepsilon)$.] From these values and with the above-mentioned analysis, the correlation width Γ_2 of class two levels was calculated as $\Gamma_2=19^{+38}_{-1.7}$ keV. In the analysis, the effective numbers of open channels were found to be $N_1=890^{+890}_{-180}$ and $N_2=11^{+30}_{-5}$. The level density for either class of states can be calculated from Γ_m and N_m using Eq. (2.16b).

In the example of ^{55}Mn*, we thus have $\Gamma_2>\Gamma_1$ (or $N_2D_2>N_1D_1$) in spite of the fact that $N_2\ll N_1$. This is typical for systems with $T_1>0$: The inequality $N_2\ll N_1$, caused by the fact that there are many more open neutron channels for class one states than for class two states (see Fig. 2), is overcompensated by the difference between the level densities of the two classes. For $T_1>0$ we have $D_2\gg D_1$. We thus expect that in heavy compound nuclei we have $\Gamma_2\gg\Gamma_1$. This feature is in fact observed in isolated isobaric analog resonances with widths typically 100 keV for nuclei with mass numbers $A\gtrsim100$, while the fine structure due to the $T=T_1$ levels is so narrow that it usually escapes detection.

This large difference between Γ_1 and Γ_2 is expected to disappear when isospin mixing is complete. Conversely, a large value of Γ_2 alone suffices to demonstrate qualitatively the (approximate) conservation of isospin in the domain of overlapping isobaric analog resonances (Berg *et al.*, 1975). In proton scattering on ^{90}Zr, the compound nucleus ^{91}Nb* was excited to about 20 MeV, and the correlation width $\Gamma_p=13.6\pm2.0$ keV was found. For complete isospin mixing one estimates, following Eberhard *et al.* (1969) and using Eq. (5.6) below, the correlation width to be about 0.2 keV. This suggests that Γ_p is close to the correlation width Γ_2 of the pure class two states.

IV. COMPOUND-NUCLEUS REACTIONS WITH BROKEN ISOSPIN SYMMETRY

The presentation in Sec. II, based on isospin conservation, is at variance with many data. This calls for an extension of the statistical theory that would allow for isospin mixing and would permit us to deduce the relevant Coulomb mixing matrix elements from the data. Such a theory is presented in the present section. We confine ourselves here to a discussion of the underlying physical assumptions and a presentation of the resulting formulas, and relegate the formal derivation to Appendix A. Formulas for compound-nucleus reactions with isospin symmetry breaking were first suggested by Grimes *et al.* (1972) and later derived from a quantum-statistical model by Harney *et al.* (1977) and Weidenmüller *et al.* (1978). In describing the physical input, we start with the compound-nucleus resonances and subsequently turn to the channel wave functions.

A. Isospin mixing between compound-nucleus levels: physical assumptions

For the compound-nucleus levels, we introduce a basis set of states $|m\mu\rangle$ with pure isospin. Here $m=1,2$ labels the two classes, and μ is a running index labeling the levels in a given class. The states $|m\mu\rangle$ are orthogonal. We assume the full Hamiltonian H (including all isospin-symmetry-breaking terms) to have been diagonalized

[1]Although there exist more precise ways to obtain the correlation length—see Ernst *et al.* (1969) and Spijkervet (1978)—simplicity and general use favor the method applied here.

within each class; the resulting eigenfunctions are the $|m\mu\rangle$. The eigenvalues are denoted by $E_{m\mu}$. The mean spacing of the $E_{m\mu}$ with m fixed is denoted by D_m, $m=1,2$, in keeping with the notation of Sec. II. Finer details of the distribution of the $E_{m\mu}$ turn out to be unimportant (Hofman *et al.*, 1981; Weidenmüller, 1984; Müller and Harney, 1985). For the statistical reaction theory in the domain of strongly overlapping resonances, see condition (2.1).

We denote by H_C (where C stands for Coulomb) the totality of isospin-symmetry-breaking terms in the Hamiltonian, and we consider the matrix elements $\langle 1\mu|H|2\nu\rangle=\langle 1\mu|H_C|2\nu\rangle$. Following arguments given, for example, by Krieger and Porter (1963), these matrix elements, taken as functions of the index μ or the index ν, are assumed to have a Gaussian distribution centered at zero. It is advantageous in the statistical theory to replace running averages (over the index μ) by ensemble averages (over a distribution of Hamiltonian matrices). This is permissible in view of an ergodicity theorem considered by Moldauer (1964), Richert and Weidenmüller (1977), and French *et al.* (1978), and will be done throughout this review. Denoting the ensemble average by a horizontal bar, we thus assume

$$\overline{\langle 1\mu|H_C|2\nu\rangle}=0\ ,$$

$$\overline{\langle 1\mu|H_C|2\nu\rangle\langle 1\mu'|H_C|2\nu'\rangle}=\delta_{\mu\mu'}\delta_{\nu\nu'}\overline{H_C^2}\ . \tag{4.1}$$

Using time-reversal invariance, we have assumed that the matrix $\langle 1\mu|H_C|2\nu\rangle$ is chosen to be real and symmetric,

$$\langle 1\mu|H_C|2\nu\rangle=\langle 1\mu|H_C|2\nu\rangle^*=\langle 2\nu|H_C|1\mu\rangle\ .$$

The average $\overline{H_C^2}$ of $\langle 1\mu|H_C|2\nu\rangle^2$ measures the strength of isospin mixing. The square root of this expression is generally called the average Coulomb matrix element and is the quantity of spectroscopic interest. As displayed below, the matrix elements $\langle 1\mu|H_C|2\nu\rangle$ are not the only source of isospin mixing in compound-nucleus reactions. To distinguish this source from symmetry-breaking effects caused by coupling to the channels, we refer to isospin mixing due to $\overline{H_C^2}$ as "internal mixing," a term first coined in the theory of isobaric analog resonances.

We now turn to the channels and introduce the channel wave functions $|at\rangle$, which denote, in contrast to Sec. II, the physical channels *including all the isospin mixing* not caused by coupling to the compound nucleus.

Let $S^{(0)}$ denote the S matrix that describes nuclear scattering without intermediate compound-nucleus formation. We assume that direct reactions do not contribute and that accordingly $S^{(0)}$ is diagonal in the physical channels,

$$S^{(0)}_{at,bt'}=\delta_{ab}\delta_{tt'}\exp(2i\varphi_{at})\ . \tag{4.2}$$

Here, φ_{at} is the elastic scattering phase shift. We emphasize that differences in threshold energies of mirror channels, as discussed in Sec. II, are contained in the definition of the channel wave functions $|at\rangle$ and of the associated phase shifts φ_{at}.

We pause a moment to reflect upon what assumption (4.2) implies in the limit in which all isospin-breaking parts in the Hamiltonian tend to zero, $H_C\to 0$. Continuity shows that $S^{(0)}$ remains diagonal in this limit. On the other hand, for $H_C=0$, the considerations of Sec. II apply, and $S^T_{a,b}$ is the proper S matrix to use. A glance at Eq. (2.4) shows that $S^{(0)}$ can be diagonal for $H_C=0$ only if $S^T_{a,b}$ is independent of T. This is the limitation imposed by assumption (4.2), and it is not completely realistic. Indeed, we recall the existence of the Lane potential (Lane, 1962a,1962b), a term in the optical-model potential that accounts for direct transitions between mirror channels. This potential depends explicitly on total isotopic spin. In the absence of all isospin-breaking forces, it will yield an S matrix S^T that is different for different values of T. The argument just given can be made less roundabout by observing that the diagonality assumption (4.2) is at variance with the existence of the Lane potential.

Why do we use assumption (4.2) if it is not totally realistic? Because a more general theory which would allow for both a direct coupling between channels t,t' with $t\neq t'$ and isospin mixing in the compound nucleus has not yet been worked out. We point to this gap as an opportunity for further investigations. How serious is the neglect of direct transitions between mirror channels for applications of the theory? This neglect probably has little importance for general inelastic compound-nucleus reactions. For the specific case of transitions or correlations between mirror channels, however, we expect that direct isospin-conserving reactions (i.e., transitions caused by the Lane potential) between such channels reduce the effect of isospin-breaking forces. Put differently, when we apply the formulas of this review, derived under the neglect of such direct reactions, to the analysis of reactions involving mirror channels, we expect to obtain an underestimate of the actual strength of isospin-breaking forces.

Returning to the formulation of our problem, we have to specify how the physical channels $|at\rangle$ are coupled to the compound levels $|m\mu\rangle$. We denote the relevant matrix elements by

$$V^{at}_{m\mu}=\langle at|V|m\mu\rangle\ . \tag{4.3}$$

These are evaluated by assuming what might be called "minimal isospin violation" as follows. Let P_{at} be a suitably defined penetration factor in channel $|at\rangle$. We take P_{at} to be a slowly varying function of energy which takes account of the threshold and Coulomb barrier effects discussed in Sec. II, and we write

$$V^{at}_{m\mu}=(P_{at})^{1/2}\cdot\tilde{V}^{at}_{m\mu}\ . \tag{4.4}$$

It will become apparent later that a precise definition of P_{at} is not needed. All we have to assume is that P_{at} can be chosen in such a way that the statistical properties of the matrix elements $\tilde{V}^{at}_{m\mu}$ can be taken to be independent of energy. We notice that the elements $\tilde{V}^{at}_{m\mu}$ are defined and different from zero even below the threshold of channel (a,t), where P_{at} and therefore $V^{at}_{m\mu}$ both vanish.

The statistics of the $V^{at}_{m\mu}$ are now defined by making

statistical assumptions on the distribution of the $\tilde{V}^{at}_{m\mu}$. These latter assumptions strictly follow the reasoning of Sec. II and therefore do not invoke breaking of isospin symmetry. We transform from the (a,t) representation to the representation involving total isospin,

$$\tilde{V}^{aT}_{m\mu} \equiv \sum_t \langle aT \mid at \rangle \tilde{V}^{at}_{m\mu} ; \tag{4.5}$$

we assume that $\tilde{V}^{aT}_{m\mu}$ vanishes unless isospin is conserved,

$$\tilde{V}^{aT}_{m\mu} = \delta_{TT_m} \tilde{V}^{aT}_{m\mu} \equiv \delta_{TT_m} \tilde{V}^{a}_{m\mu} , \tag{4.6}$$

and we assume—following Krieger and Porter (1963)—that the $\tilde{V}^{a}_{m\mu}$ are Gaussian-distributed random variables with zero mean and with a second moment given by

$$\overline{\tilde{V}^{a}_{m\mu} \tilde{V}^{b}_{n\nu}} = \delta_{ab}\delta_{mn}\delta_{\mu\nu} \overline{(\tilde{V}^{a}_{m})^2} . \tag{4.7}$$

Here and in Eq. (4.1) the assumption of Gaussian statistics is consistent with the results of Brody *et al.* (1981) and Harney (1984) from statistical spectroscopy. The independence of the right-hand sides of both Eqs. (4.1) and (4.7) of the running indices μ and ν is necessary to ensure stationarity of the statistics with respect to energy [see Agassi *et al.* (1975)] and, ultimately, ergodicity [see Moldauer (1964), Richert and Weidenmüller (1977), and French *et al.* (1978)].

Since the transformations (4.4) and (4.5) are both linear and homogeneous, it follows that the matrix elements $\tilde{V}^{at}_{m\mu}$ and $V^{at}_{m\mu}$ also have Gaussian distributions with mean values zero. The second moments are given, respectively, by

$$\overline{\tilde{V}^{at}_{m\mu} \tilde{V}^{bt'}_{n\nu}} = \delta_{ab}\delta_{mn}\delta_{\mu\nu} \langle aT_m \mid at \rangle \langle aT_m \mid at' \rangle \overline{(\tilde{V}^{a}_{m})^2} , \tag{4.8a}$$

$$\overline{V^{at}_{m\mu} V^{bt'}_{n\nu}} = \delta_{ab}\delta_{mn}\delta_{\mu\nu} P^{1/2}_{at} P^{1/2}_{at'} \langle aT_m \mid at \rangle \times \langle aT_m \mid at' \rangle \overline{(\tilde{V}^{a}_{m})^2} . \tag{4.8b}$$

Equation (4.8a) suggests that the present approach predicts correlations between mirror channels very similar to those in Sec. II. Equation (4.8b) indicates that the *only* source of isospin symmetry breaking in the matrix elements $V^{at}_{m\mu}$ is the penetration factor P_{at} (hence the term "minimal isospin violation"). It is shown below that this symmetry breaking leads to an additional mixing of the compound levels of different isospin over and above that caused by the matrix elements of H_C in Eq. (4.1). Since this mixing comes about by an intermediary transition to the channels, it is referred to as "external mixing," again in analogy to the situation encountered in isolated isobaric analog resonances. If one wishes to obtain information on the matrix elements of H_C from the data, it is necessary to have reliable theoretical estimates on the magnitude of external mixing, since, as shown below, external and internal mixing always appear jointly in the formulas. In the framework of the assumptions specified above, Harney and Tang (1981) have estimated the size of external mixing effects. It was found that these are usually small in comparison with internal mixing (see Sec. VII). This supports the hope that other isospin-mixing effects neglected in the analysis given above may be negligible, too: assumption (4.6), for instance, excludes processes in which a $T=0$ channel wave function is coupled, via H_C, to $T=1$ compound levels or vice versa. (Scattering of an alpha particle by a $T=0$ target nucleus with the population of intermediate $T=1$ compound-nucleus resonances would be a case in point.) Although low-lying states of isolated nuclei are known to have rather pure isospin, it does not follow that assumption (4.6) holds true, and estimates of the relevant matrix elements do not seem to exist.

We turn now to the form of the S matrix with the inclusion of coupling between channels and compound-nucleus levels. We recall Eq. (4.2) and define the diagonal matrix Ω (in the physical channels) by

$$\Omega_{at,bt'} = \delta_{ab}\delta_{tt'} \exp(i\varphi_{at}) . \tag{4.9}$$

Using matrix notation and applying techniques explained by Mahaux and Weidenmüller (1969), we find that the nuclear scattering matrix S has the form

$$S = \Omega^2 - 2i\Omega t \Omega , \tag{4.10}$$

with t given by

$$t_{at,bt'} = \pi \sum_{m\mu n\nu} V^{at}_{m\mu} (D^{-1})_{m\mu,n\nu} V^{bt'}_{n\nu} . \tag{4.11}$$

The matrix D is a level matrix and is given by

$$D_{m\mu,n\nu} = \delta_{mn}\delta_{\mu\nu}(E - E_{m\mu}) - \langle m\mu \mid H_C \mid n\nu \rangle \cdot (1-\delta_{mn}) + i\pi \sum_{ct''} V^{ct''}_{m\mu} V^{ct''}_{n\nu} . \tag{4.12}$$

The reader will observe that in Eq. (4.12) the shift functions have been suppressed. This is done because in practical applications they have never been evaluated, but rather have been put equal to zero relying on arguments given by Mahaux and Weidenmüller (1969) and Harney and Tang (1981). They are included in the formulas of Weidenmüller *et al.* (1978).

The S matrix (4.10) is a function of the matrix elements $V^{at}_{m\mu}$ and $\langle m\mu \mid H_C \mid n\nu \rangle$, both of which are Gaussian-distributed random variables. We assume these two sets of random variables to be uncorrelated, so that, for instance,

$$\overline{V^{at}_{m\mu} \langle m'\mu' \mid H_C \mid n\nu \rangle} = 0 , \tag{4.13}$$

again in keeping with the statistical model. By virtue of the ensemble character of the matrix elements $V^{at}_{m\mu}$ and $\langle m\mu \mid H_C \mid n\nu \rangle$, we also deal with an ensemble of S matrices of the form (4.10), and it is our aim to calculate ensemble averages of the S matrix, of cross sections, and of cross-section correlation functions in a manner analogous to the scheme sketched in Secs. II.B and II.C. This problem is similar to, but not the same as, the problem solved by Agassi *et al.* (1975) because, in the present case, the statistically uncorrelated quantities are not the coupling matrix elements between compound levels and physical

channels [this is the assumption used by Agassi *et al.* (1975)], but rather the transformed matrix elements $\tilde{V}^a_{m\mu}$ [see Eqs. (4.4)—(4.8)]. This situation requires a generalization of the work of Agassi *et al.*, which was given by Weidenmüller *et al.* (1978). Results obtained in this way are presented in Sec. IV.B; the details of the calculation are given in Appendix A.

We conclude this section with a comment on the origin of isospin mixing in the framework of the present approach. We have already commented on the important role of internal mixing caused by the matrix elements of H_C that couple directly compound states of different isospin symmetry [see Eqs. (4.1)]. The role of "external mixing" is somewhat more subtle in that it is not caused directly by mixing elements of H_C, but rather by the difference between threshold energies and penetrabilities in mirror channels. This is seen most clearly by combining Eqs. (4.4)—(4.6),

$$V^{at}_{m\mu}=(P_{at})^{1/2}\langle aT_m \mid at\rangle \tilde{V}^a_{m\mu} . \tag{4.14}$$

One might consider introducing another set of matrix elements $V^{aT}_{m\mu}$ by

$$V^{aT}_{m\mu}=\sum_t \langle aT \mid at\rangle V^{at}_{m\mu}=X_a^{TT_m}\cdot\tilde{V}^a_{m\mu} . \tag{4.15}$$

We have used Eq. (4.14) and the definition

$$X_a^{TT_m}=\sum_t \langle aT \mid at\rangle P^{1/2}_{at}\langle aT_m \mid at\rangle . \tag{4.16}$$

The quantity $X_a^{TT_m}$ is nondiagonal in isospin. This fact causes the matrix elements $V^{aT_1}_{m\mu}$ and $V^{aT_2}_{m\mu}$ to be correlated, and this is the cause of isospin symmetry breaking via external mixing [see also Eq. (4.8b)]. The matrix $X_a^{TT_m}$ is nondiagonal and is isospin breaking only if the penetration factors $P_{a+1/2}$ and $P_{a-1/2}$ are not equal; otherwise, the orthogonality of the Clebsch-Gordan coefficients in Eq. (4.16) yields $T_m=T$. We therefore expect that isospin-breaking effects due to external mixing vanish together with the difference $(P_{a+1/2}-P_{a-1/2})$, and this is indeed what we shall find. External mixing in the present framework is thus identified as a threshold phenomenon.

Statistical assumptions that differ from those introduced above have been discussed by Weidenmüller *et al.* (1978) and have been shown to be less plausible.

B. Isospin mixing between compound-nucleus levels: formal results

In this section we present the results of the model formulated in Sec. IV.A. The derivation is given in Appendix A. We confine ourselves here to a display of the relevant formulas. A thorough discussion of these results, and a physical interpretation, are given in Sec. V.

We recall Eqs. (4.2) and (4.9) and define a transformation of the S matrix

$$\tilde{S}=\Omega^{-1}S\Omega^{-1} \tag{4.17}$$

which removes the elastic background scattering phase shifts. Decomposing $\tilde{S}$ into an average and a fluctuating part in complete analogy to Eq. (2.5), we find for the second moment of $\tilde{S}^{\rm fl}$ the result

$$\langle \tilde{S}^{\rm fl}_{at,bt'}(E)\tilde{S}^{\rm fl*}_{ct'',dt'''}(E+\varepsilon)\rangle=\delta_{ac}\delta_{bd}\sum_{m,n}\tau^{tt'}_{am}\Pi_{mn}\tau^{t''t'''}_{bn}+\delta_{ad}\delta_{bc}\sum_{m,n}\tau^{tt'''}_{am}\Pi_{mn}\tau^{t't''}_{bn} . \tag{4.18}$$

The matrix Π_{mn} with $m,n=1,2$ is a 2×2 matrix in the space of isospin classes, the inverse of which is given by

$$\Pi^{-1}=\begin{bmatrix} N_1+z+2i\pi\varepsilon/D_1 & -z \\ -z & N_2+z+2i\pi\varepsilon/D_2 \end{bmatrix} . \tag{4.19}$$

The effective numbers N_m of open decay channels are defined in terms of the transmission coefficients τ^{tt}_{am} as in Eq. (2.19). The transmission coefficients themselves, however, are now given as follows:

$$\tau^{tt'}_{am}=A^t_{am}A^{t'}_{am} \tag{4.20}$$

with $(m=1,2)$

$$A^t_{am}=2\cdot F_a\cdot\langle aT_m \mid at\rangle\cdot(\tilde{x}^a_m P_{at})^{1/2}\cdot(1+\tilde{x}^a_{3-m}P_{a-t}) , \tag{4.21}$$

where

$$\tilde{x}^a_m=\pi^2\overline{(\tilde{V}^a_m)^2}/D_m \tag{4.22}$$

and

$$F_a^{-1}=1+(2T_1+2)^{-1}\{P_{a1/2}[\tilde{x}^a_1+(2T_1+1)\tilde{x}^a_2]+P_{a-1/2}[\tilde{x}^a_2+(2T_1+1)\tilde{x}^a_1]\}+\tilde{x}^a_1\tilde{x}^a_2P_{a1/2}P_{a-1/2} . \tag{4.23}$$

The isospin-mixing parameter has the form

$$z=4\pi^2\cdot\overline{H_C^2}/(D_1D_2)+4\sum_a{}' F_a^2\cdot\frac{(2T_1+1)}{(2T_1+2)}\tilde{x}^a_1\tilde{x}^a_2\cdot(P_{a1/2}-P_{a-1/2})^2 , \tag{4.24}$$

where the restricted sum $\sum'$ in the last term extends only over pairs of mirror channels. For channels with $t=0$, both $P_{a1/2}$ and $P_{a-1/2}$ have to be replaced by P_{a0} in Eqs. (4.21) and (4.23).

A great simplification can be achieved by introducing an assumption that has been used at least implicitly in all investigations of isospin mixing published so far, and that is discussed in Sec. V: We assume the quantities $\tilde{x}^a_m$ to be independent of the class index m, so that

$$\bar{x}^a_1=\bar{x}^a_2\equiv\bar{x}^a \ . \tag{4.25}$$

It is then convenient to define the quantities

$$x_{at}=\bar{x}^a P_{at} \ . \tag{4.26}$$

In this way, the definitions (4.20)—(4.24) simplify to

$$\tau^{tt'}_{am}=\langle aT_m \mid at\rangle\langle aT_m \mid at'\rangle \frac{4(x_{at}x_{at'})^{1/2}}{(1+x_{at})(1+x_{at'})} \tag{4.27}$$

and

$$z=4\pi^2\overline{H_C^2}/(D_1D_2) + 4\sum_a{}' \frac{2T_1+1}{(2T_1+2)^2}\cdot\frac{(x_{a\,1/2}-x_{a\,-1/2})^2}{(1+x_{a\,1/2})(1+x_{a\,-1/2})} \ . \tag{4.28}$$

Equations (4.18), (4.19), (4.27), and (4.28) form the basis of our comparison with experimental work.

V. DISCUSSION OF THE THEORETICAL FRAMEWORK

In this section we give a physical interpretation of Eqs. (4.17)—(4.19), (4.27), and (4.28), as well as of assumption (4.25). We mention once again that formulas for isospin symmetry breaking in compound-nucleus reactions were first given by Grimes *et al.* (1972), without, however, any detailed theoretical justification. This work was followed by the work of the present authors (Harney *et al.*, 1977; Weidenmüller *et al.*, 1978), in which a derivation was given that essentially is reproduced, with minor modifications, in Appendix A. A third approach, stimulated by the question of isospin mixing in giant resonances, was formulated by Shikazono and Terasawa (1975). It was shown by Lane (1978) that all three approaches are *formally* equivalent in the sense that if one identifies the mixing parameters of the three approaches, the cross-section formulas can be transformed into one another. This, of course, leaves open the question of any *microscopic* equivalence between the three theories.

We begin with assumption (4.25). Aside from the fact that it is badly needed in order to reduce the complicated expressions (4.20)—(4.24) to the simpler forms (4.27) and (4.28), and in order to reduce the number of independent parameters to obtain an unambiguous fit to the data, this assumption is also physically eminently reasonable. Indeed, the average level spacings D_m may be very different for the two classes, and so may be the values $(\overline{V_m^a})^2$ of the averaged squares of matrix elements. However, the ratio $(\overline{V_m^a})^2/D_m$ is nearly independent of m since, as D_m gets smaller, the square of each matrix element will get correspondingly smaller, too, because the strength is spread over a larger number of states.

A. The isospin-mixing parameter z

Equation (4.18) shows that the amount of isospin mixing in a compound-nucleus reaction is governed by the nondiagonal element $\Pi_{12}=\Pi_{21}$ of the matrix Π. The latter, given by Eq. (4.19), becomes diagonal for $z=0$. Therefore z, as specified in Eq. (4.28), is referred to as the isospin-mixing parameter. According to Eq. (4.28), it consists of two contributions: the term $4\pi^2\overline{H_C^2}/(D_1D_2)$, which represents internal mixing and which is the quantity of spectroscopic interest in the study of isospin-forbidden reactions, and a second term given by the sum over mirror channels, which represents external mixing [it disappears when we set $P_{a\,1/2}=P_{a\,-1/2}$; see Eq. (4.26)]. We observe that internal mixing and external mixing both contribute to z. From the analysis of the data, it is possible in many cases to determine z. In order to obtain information on $\overline{H_C^2}$, it is necessary to estimate theoretically the amount of external mixing and the level spacings D_1 and D_2. Estimates of these parameters are presented in Sec. VII.

It turns out that internal mixing dominates over external mixing. The sum $\sum'$ in Eq. (4.28), which yields external mixing, extends over all pairs of mirror channels. These are the channels marked by wavy lines in Fig. 4. The neutron channels $|\frac{1}{2}\,\frac{1}{2}\rangle\,|T_1-\frac{1}{2},T_1-\frac{1}{2}\rangle$ leading to the lowest possible isospin in the residual nucleus formed by neutron emission are, however, not included in the sum, since they have no mirror images. There are only relatively few channels that significantly contribute to the sum. For those pairs of channels $(a\,\frac{1}{2}),(a-\frac{1}{2})$, where the channel energy is much larger than either threshold ener-

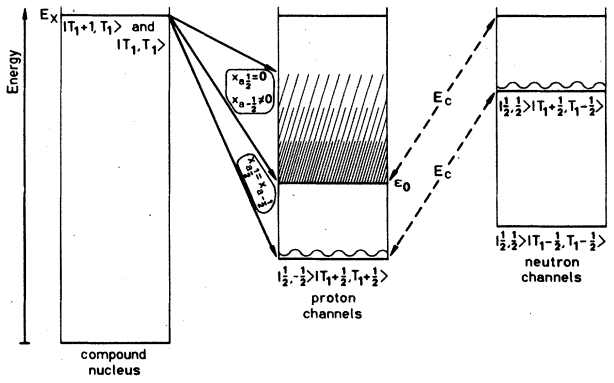

FIG. 4. Nuclear decay channels and their contribution to external isospin mixing. We consider the proton channels leading to states in the residual nucleus which lie between the ground state $|T_1+\frac{1}{2},T_1+\frac{1}{2}\rangle$ and the energy ε_0: For every one of them there is an open mirror channel. The final nuclear state $|T_1+\frac{1}{2},T_1-\frac{1}{2}\rangle$ reached by neutron decay is the isobaric analog state of the state $|T_1+\frac{1}{2},T_1+\frac{1}{2}\rangle$ formed by proton decay. These pairs of states are separated by the Coulomb energy difference E_C. Such pairs of channels do not contribute to external mixing, since their absorption coefficients $x_{a\,1/2}$ and $x_{a\,-1/2}$ are equal. For the states with energy above ε_0 reached by proton decay, the neutron mirror channel is closed, i.e., $x_{a\,1/2}=0$ and $x_{a\,-1/2}\neq 0$. These channels—indicated by the shaded area—essentially contribute to external mixing. The decreasing intensity of shading indicates the decreasing penetrability due to the Coulomb barrier.

gy, the transmission coefficients $\tau_{a\,1/2}$ and $\tau_{a\,-1/2}$ will both be close to unity, hence $x_{a\,1/2}\simeq x_{a\,-1/2}$ and these terms are negligibly small. On the other hand, there are pairs of channels with threshold energies so large that the channel with $t=\frac{1}{2}$ is closed, while the channel with $t=-\frac{1}{2}$ is open. The difference between the two threshold energies is the Coulomb energy difference between the residual states $|\,T_1+\frac{1}{2},T_1-\frac{1}{2}\rangle$, and $|\,T_1+\frac{1}{2},\,T_1+\frac{1}{2}\rangle$ (see Fig. 4). This is roughly equal to the Coulomb barrier for proton emission from the compound nucleus. Therefore the nucleon channels contributing to the sum in Eq. (4.28) essentially comprise the proton channels with channel energies below the Coulomb barrier. This is also the most important class of channels responsible for external mixing, since there are very few channels with isospin-$\frac{1}{2}$ particles heavier than nucleons. The channels dominating the sum in Eq. (4.28) are indicated by the shaded area in Fig. 4. If the Coulomb interaction were turned off, one would have $P_{a\,1/2}=P_{a\,-1/2}$ for all a, and external mixing would vanish together with internal mixing.

We recall that N_1, the effective number of open channels for states in class one, can also be written as $2\pi\Gamma_1^{\uparrow}/D_1$ [see Eq. (2.16)], in keeping with the standard notation of compound-nucleus theory [see, for example, Agassi *et al.* (1975)]. Here, $\Gamma_1^{\uparrow}$ is the "escape width" for the decay into the open channels. Looking at the contributions to z from internal mixing, we note that $2\pi\overline{H_C^2}/D_2$ is the usual expression for the "spreading width" $\Gamma_{1,\mathrm{int}}^{\downarrow}$ (due to internal mixing) of states in class one, so that the internal mixing contribution to z can be written as $2\pi\Gamma_{2,\mathrm{int}}^{\downarrow}/D_1$. Analogously, the external mixing contribution to z is formally written as $2\pi\Gamma_{1,\mathrm{ext}}^{\downarrow}/D_1$, and the (1,1) element of the matrix in Eq. (4.19) acquires the simple and obvious form $2\pi(\Gamma_1^{\uparrow}+\Gamma_{1,\mathrm{int}}^{\downarrow}+\Gamma_{1,\mathrm{ext}}^{\downarrow}+2i\varepsilon\pi)/D_1$. An analogous form holds for the (2,2) elements because of the symmetry of z in the class index.

B. The correlation functions

For $z=0$, the correlation function (4.18) becomes diagonal in the isospin indices, as it should, and agrees with the results given in Sec. II [see Eq. (2.17)], except for the transformation (4.17) from S to $\tilde{S}$, which involves the physical phase shifts in the channels. This point was discussed extensively in Sec. IV.A. Both for $z=0$ [where Eq. (2.17) demonstrates this fact explicitly] and for $z\neq 0$, the correlation function (4.18) contains two Lorentzians. The correlation lengths λ_1,λ_2 are found as the roots at $\varepsilon=i\lambda_1,i\lambda_2$ of the determinant of Π^{-1}. We obtain

$$\lambda_{1,2}=\tfrac{1}{2}(\Gamma_1^{\uparrow}+\Gamma_1^{\downarrow}+\Gamma_2^{\uparrow}+\Gamma_2^{\downarrow}) \pm\tfrac{1}{2}[(\Gamma_1^{\uparrow}+\Gamma_1^{\downarrow}-\Gamma_2^{\uparrow}-\Gamma_2^{\downarrow})^2+4\Gamma_1^{\downarrow}\Gamma_2^{\downarrow}]^{1/2}\,. \tag{5.1}$$

Here, $\Gamma_m^{\downarrow}$ is the sum of the internal and external spreading widths $\Gamma_{m,\mathrm{int}}^{\downarrow}$ and $\Gamma_{m,\mathrm{ext}}^{\downarrow}$ introduced above. For $z=0$, i.e., $\Gamma_1^{\downarrow}=\Gamma_2^{\downarrow}=0$, this yields $\lambda_m=\Gamma_m^{\uparrow}$, as it should. Note that $\lambda_m>0$ for all values of $\Gamma_m^{\uparrow}>0$, as is to be expected. Only for $z\gg N_1$ *and* $z\gg N_2$, i.e., $\Gamma_m^{\downarrow}\gg\Gamma_m^{\uparrow}$, do we encounter an unexpected situation: one of the roots of $\det(\Pi^{-1})$ remains finite,

$$\lambda_1\underset{z\to\infty}{\to}\Gamma_{\mathrm{corr}}\,, \tag{5.2}$$

while the other one tends to infinity as

$$\lambda_2\underset{z\to\infty}{\to}z(N_1+N_2)/2\pi\,. \tag{5.3}$$

Further analysis of this limit [see Harney *et al.* (1983)] shows that the correlation function (4.18) is characterized by a single Lorentzian, and that it attains the form of a standard Hauser-Feshbach expression, generalized to $E_1\neq E_2$, namely,

$$\langle\tilde{S}^{\mathrm{fl}}_{at,bt'}(E_1)\tilde{S}^{\mathrm{fl}*}_{ct'',dt'''}(E_2)\rangle\underset{z\to\infty}{\to}(\delta_{ac}\delta_{bd}\delta_{tt''}\delta_{t't'''} +\delta_{ad}\delta_{bc}\delta_{tt'''}\delta_{t't''}) \times\frac{\Gamma_{\mathrm{corr}}}{\Gamma_{\mathrm{corr}}+i\varepsilon}\cdot\frac{\tau_{at}\tau_{bt'}}{\sum_{e,s}\tau_{es}}\,, \tag{5.4}$$

where $\varepsilon=E_2-E_1$. The transmission coefficients τ_{at} are given by

$$\sum_m\tau_{am}^{tt'}=\delta_{tt'}\tau_{at}\,, \tag{5.5}$$

and the correlation width Γ_{corr} by

$$\Gamma_{\mathrm{corr}}=(2\pi)^{-1}(N_1+N_2)(D_1^{-1}+D_2^{-1})^{-1}\,. \tag{5.6}$$

We note that

$$N_1+N_2=\sum_{amt}\tau_{am}^{tt}=\sum_{at}\tau_{at}\,. \tag{5.7}$$

The result (5.4) is to be expected, as for $z\to\infty$ the mixing between the classes becomes so strong as to make any distinction between them physically meaningless. We note that Eq. (5.6), too, has the expected form in view of Eqs. (5.7) and the fact that $D_1^{-1}+D_2^{-1}$ equals the total level density. The coefficients

$$\tau_{at}=\frac{4x_{at}}{(1+x_{at})^2} \tag{5.8}$$

have the form of physical transmission coefficients. Complete isospin symmetry breaking is also reflected in the arrangement of Kronecker symbols appearing on the right-hand side of Eq. (5.4): It leads to uncorrelated amplitudes in physically different channels, and it leads to an enhancement factor of 2 in elastic, but not in charge exchange, reactions.

Since our results essentially reduce to those of Sec. II for $z\to 0$, we expect that the general case ($z\neq 0$, but z not large in comparison to N_1 and/or N_2) also has the peculiarities displayed in Sec. II: an "elastic" enhancement for charge exchange reactions, and correlations in mirror channels. This is dealt with in Secs. VI and VII. In Sec. VI we show how z can be determined experimentally.

VI. EXPERIMENTAL EXAMPLES OF ISOSPIN MIXING

In the first part of this section four groups of experiments are discussed, which all have one point in common. The average cross section of a reaction forbidden by the isospin selection rule is compared to the cross section of an allowed reaction. This results in a reduction factor f, which can be related to the average decay probability of the isospin class two levels into the levels of class one. Because of its conceptual simplicity, the first example below is used to define the reduction factor. Three further possibilities of investigating isospin mixing are presented in Secs. VI.E–VI.G. Here, we deal with correlations between mirror channels, with the correlation widths $\Gamma_m^{\downarrow}$, and with the enhancement factor in charge exchange reactions.

A. First example: isospin-allowed and isospin-forbidden average cross sections in the reaction ^{28}Si(d,α) ^{26}Al

The comparison of allowed and forbidden reactions of the type $A+a\rightarrow C^*\rightarrow B+b$ is illustrated with the help of experimental data by Bizzeti and Bizzeti-Sona (1968) and by Richter *et al.* (1970). The reaction

$$^{28}\mathrm{Si}+d\rightarrow{}^{30}\mathrm{P}^*\rightarrow{}^{26}\mathrm{Al}+\alpha \tag{6.1}$$

leading to a residual state with isospin $T_B=T_b=0$ is *isospin allowed,* since the total isospins in the entrance and exit channels agree ($T_A=T_a=0$). The compound nucleus ^{30}P* has $T_1=0$ and $T_2=1$. The entrance channel couples only to class one states with $T_1=0$ and so does the exit channel labeled α:

$$\tau_{d1}\neq 0,\quad \tau_{\alpha 1}\neq 0,\quad \tau_{d2}=0,\quad \tau_{\alpha 2}=0\ . \tag{6.2}$$

As long as the upper indices t and t' are equal, we use a simplified notation for the transmission coefficients of Eq. (4.27). (Coefficients with $t\neq t'$ will appear only in Sec. VI.E, and the full notation is used there.) We accordingly write

$$\tau_{am}^{tt}=\tau_{at,m} \tag{6.3}$$

and indicate the physical channel (a,t) by the symbol for the light reaction partner, e.g., p, d, or α.

To calculate the average compound-nucleus cross section, we invert the matrix of Eq. (4.19),

$$\Pi=\begin{pmatrix} N_2+z+2\pi i\varepsilon/D_2 & z \\ z & N_1+z+2\pi i\varepsilon/D_1 \end{pmatrix}\frac{1}{(N_1+z+2\pi i\varepsilon/D_1)(N_2+z+2\pi i\varepsilon/D_2)-z^2} \tag{6.4}$$

and obtain from Eq. (4.18)

$$\sigma_{d\alpha}(\text{allowed})=\tau_{d1}\frac{1+z/N_2}{1+z/N_1+z/N_2}\frac{1}{N_1}\tau_{\alpha 1}\ . \tag{6.5}$$

We recall that isospin mixing is controlled by the parameter z. In the limit of no mixing ($z\rightarrow 0$), Eq. (6.5) becomes the usual Hauser-Feshbach formula

$$\sigma_{d\alpha}(\text{allowed, no mixing})\simeq\tau_{d1}\frac{1}{N_1}\tau_{\alpha 1}\ , \tag{6.6}$$

where the levels of class two do not occur. In the limit of very strong mixing ($z\rightarrow\infty$), Eq. (6.5) tends toward

$$\sigma_{d\alpha}(\text{complete mixing})\simeq\tau_{d1}\frac{1}{N_1+N_2}\tau_{\alpha 1}\ , \tag{6.7}$$

which is the Hauser-Feshbach formula without distinction between the levels of class one and class two—an obvious result. For arbitrary z, the interpretation of Eq. (6.5) is also straightforward: only the fraction $(1+z/N_2)(1+z/N_1+z/N_2)^{-1}$ of the levels of class one is available for decay into the exit channel $\alpha 1$, which is populated with the probability $\tau_{\alpha 1}/N_1$.

Consider now the excitation of the first excited $T_B=1$ state of ^{26}Al through the reaction (6.1). This reaction is *isospin forbidden.* Again the entrance channel couples to the class one states, but the exit channel only to the class two states, i.e.,

$$\tau_{d1}\neq 0,\quad \tau_{\alpha 1}=0,\quad \tau_{d2}=0,\quad \tau_{\alpha 2}\neq 0\ . \tag{6.8}$$

Equation (4.18) then gives

$$\sigma_{d\alpha}(\text{forbidden})=\tau_{d1}\frac{z/N_2}{1+z/N_1+z/N_2}\frac{1}{N_1}\tau_{\alpha 2}\ . \tag{6.9}$$

Bizzeti and Bizzeti-Sona (1968) have measured the cross sections $\sigma_{d\alpha}$ between 7 and 11 MeV incident energy corresponding to a mean excitation energy of 20 MeV in ^{30}P. They are reproduced here in Fig. 5 for seven different final states as functions of the scattering angle. The expressions $\tau_{dm}\tau_{\alpha n}$ with $m=n=1$ for the allowed reaction and $m=1$, $n=2$ for the forbidden transition were calculated.[2] Subsequently, the factor $(1+z/N_2)(1+z/N_1+z/N_2)^{-1}N_1^{-1}$ was adjusted in such a way that

[2]Note that the actual Hauser-Feshbach cross sections are sums over terms like those of Eqs. (6.5) or (6.9), weighted with Racah coefficients and angular functions. The sums extend over orbital angular momenta and channel spins of both entrance and exit channels, and over the total spin of the system. The expression is given explicitly, for example, by Bizzeti and Bizzeti-Sona (1968) and von Witsch *et al.* (1966). Figure 5 shows that the transition to the first excited $0^+, T_B=1$ state in ^{26}Al is weaker by about a factor of 30 than the transition to the $3^-, T_B=0$ ground state. Most of this suppression is due to the different spins of the final states, as is borne out by the solid curves.

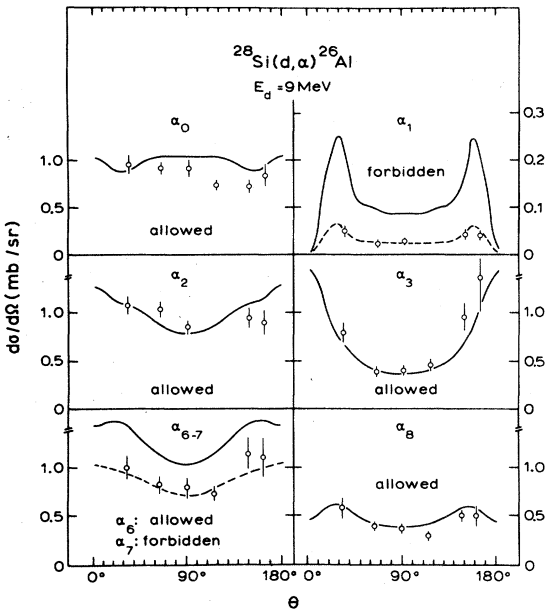

FIG. 5. Average differential cross sections of ^{28}Si$(d,\alpha)^{26}$Al. The label α_0 designates the transition to the ground state of ^{26}Al, the labels α_k, $k=1,\ldots,8$ the transitions to the kth excited state. Both the data points and the curves are from Bizzeti and Bizzeti-Sona (1968). The transitions to the sixth and seventh excited states are experimentally unresolved. The solid curves are Hauser-Feshbach calculations normalized to the set of isospin-allowed transitions. The dashed curves for the isospin-forbidden cases α_1 and α_7 result from a suppression of the Hauser-Feshbach calculations by a factor of $f=0.25$. Note the expanded scale for α_1.

all allowed transitions described by Eq. (6.5) were simultaneously reproduced. This resulted in the solid curves in Fig. 5. If isospin mixing is complete, one expects to reproduce all transitions with this normalization. The experimental results for the forbidden transitions α_1 and α_7 in Fig. 5 are, however, suppressed with respect to the solid curves by the factor $f=0.25\pm0.05$. Inspecting Eqs. (6.5) and (6.9), we see that this suppression factor is

$$f=\frac{z/N_2}{1+z/N_2}\ . \tag{6.10}$$

In principle the quantities $z/N_1, z/N_2$ could depend upon spin and parity of the compound nucleus. This dependence was neglected by Bizzeti. We discuss this assumption in Sec. VII.

The suppression factor f is the central quantity of this article. It is related through Eq. (6.10) to the microscopic theory described in Sec. IV. As Eq. (6.5) shows, the fraction of states of class one that decay into the levels of class two is given by

$$\frac{z/N_1}{1+z/N_1+z/N_2}\ . \tag{6.11}$$

The complementary expression

$$\nu=\frac{z/N_2}{1+z/N_1+z/N_2} \tag{6.12}$$

is a measure of the fraction of states in class two decaying into class one. The quantity ν agrees with the one introduced by Lane (1978). If the number N_1 of decay channels is much larger than N_2, Eq. (6.12) approaches the definition of f in Eq. (6.10). This is always the case if the lowest possible isospin T_1 in the compound nucleus is larger than zero. For self-conjugate nuclei[3] $N_1\simeq N_2$, and therefore the expression ν in Eq. (6.12) can be converted into Eq. (6.10).

We interpret the quantity z/N_2 in the following way. As in Sec. V we define the mixing or spreading widths

$$\Gamma_m^{\downarrow}=D_m z/2\pi\ . \tag{6.13}$$

They govern the decay rate of the levels of class m into those of class $n\neq m$ [see Agassi *et al.* (1975) and Harney *et al.* (1977)]. The quantity

$$\frac{z}{N_2}=\frac{\Gamma_2^{\downarrow}}{\Gamma_2^{\uparrow}} \tag{6.14}$$

is then seen to be the ratio of decay probabilities of the class two levels into class one and into the open channels.

One may wonder why the suppression factor of a forbidden reaction depends upon the parameters of the class two levels rather than depending symmetrically on both classes. This is due to an "asymmetric definition" of f. It measures the strength of a forbidden reaction relative to an allowed one that passes through the class one levels. This is brought about by the experimental situation: An allowed reaction proceeding through the class two levels is—in the above example (6.1)—not accessible.

The structure of the quantity z/N_2 and the size of $\overline{H_C^2}$ that can be deduced will be discussed in detail in Sec. VII. We note that the example just given stands for a whole class of (d,α), (d,d'), (α,α'), $(^6\text{Li},\alpha)$,... reactions. The most extensive experiment is the study of ^{32}S$(d,\alpha)^{30}$P by Spijkervet (1978), including more than twenty final states with five forbidden transitions (see Fig. 10 below and the discussion in Sec. VI.F). Two relatively recent examples are the reactions ^{12}C$(^6\text{Li},\alpha)^{14}$N and ^{14}N$(d,d')^{14}$N. In both cases the isospin-forbidden excitation of the first excited $(J^\pi;T)=(0^+;1)$ state in ^{14}N has been investigated [see Schwenzel *et al.* (1981) and Aoki *et al.* (1979)]. For

[3]This argument can be verified with the help of Fig. 2. If $T_1=0$, the residual states reached by neutron emission *all* have their isobaric analog among the states reached by proton emission. Due to the Coulomb barrier in the proton channel there is essentially a one-to-one correspondence between the states populated by emission of either nucleon (see Sec. V). Within the approximation that the nucleon channels exhaust the decay channels, one obtains $N_1\simeq N_2$.

further references, the reader is referred to these papers. The results[4] for the suppression factor f are listed in Table I in Sec. VII below. They are 0.3—0.5 for excitation energies relatively close to neutron threshold and apparently decrease to only a few percent at high excitation energies. This fact, which was first observed by Richter *et al.* (1970) in the reaction ^{28}Si$(d,\alpha)^{26}$Al, is discussed in Sec. VII in more detail. It shows that in compound-nucleus reactions isospin symmetry seems to be restored at high excitation energy.

B. Second example: isospin-forbidden dipole radiation from alpha and heavy-ion capture reactions

As in the preceding section we consider a nuclear reaction $A+a\rightarrow C^*\rightarrow B+b$, but this time b refers to an electric dipole γ ray carrying the isospin $T_b=1$. (Recall that the selection rule for $E1$ transitions is $\Delta T=0,1$, with transitions from $T=0$ to $T=0$ being forbidden.) We treat two cases.

1. Alpha capture reactions

We compare (α,γ) cross sections for two reactions, one with a self-conjugate target nucleus, the other with a target nucleus that has isospin $T_A\neq 0$. In the first case, the $E1$ γ decay is isospin forbidden; in the second, it is not. By comparing the γ yields one can deduce the suppression factor f. Examples are the (α,γ) reactions on the pairs ^{20}Ne-^{22}Ne and ^{24}Mg-^{26}Mg [see Kuhlmann (1979) and Kuhlmann *et al.* (1983)].

It was noted that the integrated $E1$ strength of the (α,γ_0) ground-state transition in self-conjugate nuclei in the region of the $E1$ giant resonance is systematically weaker than in non-self-conjugate nuclei, while the competing $E2$ strength is about the same and can be disentangled from the $E1$ strength. Furthermore, the α capture reactions are shown to be predominantly of compound-nucleus character.

We first write down—in keeping with Sec. VI.A—the capture cross section for the *predominantly isospin-allowed* reaction on $T_A=1$ targets, for instance, on ^{22}Ne, ^{26}Mg, ^{30}Si. We label the entrance and exit channels α and γ, respectively. The α channel couples to the class one levels only, while the γ channel couples to both classes. The transmission coefficients are therefore

$$\tau_{\alpha 1}\neq 0,\quad \tau_{\gamma 1}\neq 0,\quad \tau_{\alpha 2}=0,\quad \tau_{\gamma 2}\neq 0\ . \tag{6.15}$$

With the help of Eqs. (4.18) and (6.4) one then finds

$$\sigma_{\alpha\gamma}(\text{nonconjugate})=\tau_{\alpha 1}\frac{1+z/N_2}{1+z/N_1+z/N_2}\frac{1}{N_1}\tau_{\gamma 1}+\tau_{\alpha 1}\frac{z/N_2}{1+z/N_1+z/N_2}\frac{1}{N_1}\tau_{\gamma 2}\ . \tag{6.16}$$

This formula has two terms. The first term is entirely analogous to the isospin-allowed expression (6.5) discussed in Sec. VII.A. The second term is analogous to the isospin-forbidden case (6.9). Here, both terms appear together, since the γ ray couples to both classes of levels in the compound nucleus.

The *isospin-forbidden* capture cross section on self-conjugate $T_A=0$ targets, such as ^{20}Ne, ^{24}Mg, ^{28}Si, has the transmission coefficients

$$\tilde{\tau}_{\alpha 1}\neq 0,\quad \tilde{\tau}_{\gamma 1}=0,\quad \tilde{\tau}_{\alpha 2}=0,\quad \tilde{\tau}_{\gamma 2}\neq 0\ . \tag{6.17}$$

Note that we have to distinguish the parameters entering the Hauser-Feshbach expression for self-conjugate targets by a tilde, since target, compound, and residual nuclei are different in the α capture on self-conjugate and non-self-conjugate nuclei. The cross section is then [as in Eq. (6.9)]

$$\sigma_{\alpha\gamma}(\text{self-conjugate})=\tilde{\tau}_{\alpha 1}\frac{\tilde{z}/\tilde{N}_2}{1+\tilde{z}/\tilde{N}_1+\tilde{z}/\tilde{N}_2}\frac{1}{\tilde{N}_1}\tilde{\tau}_{\gamma 2}\ . \tag{6.18}$$

Certainly, to a good approximation, isospin mixing is the same in the different compound-nucleus pairs, for instance, ^{24}Mg-^{26}Mg, ^{28}Si-^{30}Si, ^{32}S-^{34}S, and we therefore omit henceforth the tildes in the expression $(\tilde{z}/\tilde{N}_2)/(1+\tilde{z}/\tilde{N}_1+\tilde{z}/\tilde{N}_2)$. The numbers N_1 and $\tilde{N}_1$ of open channels are, however, different in the two compound nuclei considered. This is taken care of by calculating numerically the following Hauser-Feshbach expressions[5]

$$\sigma_{\mathrm{HF}}(\text{non-self-conjugate})=\tau_{\alpha 1}\frac{1}{N_1}\tau_{\gamma 1}\ ,$$
$$\sigma_{\mathrm{HF}}(\text{self-conjugate})=\tilde{\tau}_{\alpha 2}\frac{1}{\tilde{N}_1}\tilde{\tau}_{\gamma 2}\ . \tag{6.19}$$

The measured cross sections $\sigma_{\alpha\gamma}$ and the calculated cross section σ_{HF} are then combined in the ratio

[4]The suppression factor in the reaction ^{12}C$(^6$Li$,\alpha)^{14}$N has been estimated from Fig. 6 of Schwenzel *et al.* (1981): It is the ratio of the data to the Hauser-Feshbach calculation labeled α_1. In the case of the ^{14}N$(d,d')^{14}$N reaction by Aoki *et al.* (1979), the quantity ε^2 given in their Table 2 is identical with our factor f.

[5]Actually, Kuhlmann (1979) used the total number of decay channels N_1+N_2 and $\tilde{N}_1+\tilde{N}_2$ instead of N_1 and $\tilde{N}_1$. From Fig. 8 of Kuhlmann *et al.* (1983) one finds $\tilde{N}_1+\tilde{N}_2\simeq 1.7\tilde{N}_1$. We assume $N_1\gg N_2$, since these numbers refer to non-self-conjugate nuclei [see the discussion following Eq. (6.12)]. The suppression factors derived in the paper by Kuhlmann (1979) and Kuhlmann *et al.* (1983) should then be divided by 1.7.

$$\frac{\sigma_{\alpha\gamma}(\text{non-self-conjugate})/\sigma_{\text{HF}}(\text{non-self-conjugate})}{\sigma_{\alpha\gamma}(\text{self-conjugate})/\sigma_{\text{HF}}(\text{self-conjugate})}=\frac{1+z/N_2}{z/N_2}+\frac{\tau_{\gamma 2}}{\tau_{\gamma 1}}\ . \tag{6.20}$$

The first term on the right-hand side of this equation is the inverse of the suppression factor f defined in Eq. (6.10). The second term can be expressed by the ratio of isospin Clebsch-Gordan coefficients [see Eq. (4.27)]. In the particular cases considered here one has

$$\frac{\tau_{\gamma 2}}{\tau_{\gamma 1}}=\frac{(1,1;1,0\,|\,2,1)^2}{(1,1;1,0\,|\,1,1)^2}=1\ .$$

Note that in the analysis of Kuhlmann (1979) the term $\tau_{\gamma 2}/\tau_{\gamma 1}$ is absent.[6] In order to convert the suppression factor quoted there in Table 1, we use the inverse of Eq. (6.20),

$$\frac{\sigma_{\alpha\gamma}(\text{self-conjugate})/\sigma_{\text{HF}}(\text{self-conjugate})}{\sigma_{\alpha\gamma}(\text{non-self-conjugate})/\sigma_{\text{HF}}(\text{non-self-conjugate})}=\frac{f}{1+f}\ .$$

The left-hand side of this equation was named f by Kuhlmann (1979). Using the figures given in this reference, applying the correction factor explained in footnote 5, and calculating our factor f, we find $f=0.16\pm0.07$ for ^{24}Mg-^{26}Mg at $E_x=17.6$ MeV, and $f=0.14\pm0.06$ for ^{32}S-^{34}S at $E_x=15.7$ MeV. These suppression factors are of the same magnitude as the one derived in the ^{28}Si$(d,\alpha)^{26}$Al reaction at a similar excitation energy of the compound nucleus ^{30}P.

2. Heavy-ion capture reactions

The second case involves radiative capture of self-conjugate heavy ions A and a, leading to the statistical emission of high-energy γ rays into $T_B=0$ final states (Snover, 1984; Harakeh *et al.*, 1985). These $E1$ γ rays are strongly inhibited with respect to isospin-allowed $E1$ γ rays of lower energy feeding $T_B=1$ final states. This again can be related to the suppression factor f.

In order to explain this more in detail, we consider the heavy-ion capture reaction $^{12}\text{C}+{}^{16}\text{O}$ leading to the highly excited compound nucleus ^{28}Si*. In the specific example by Snover (1984) and Harakeh *et al.* (1985), ^{28}Si is excited at $E_x=34$ MeV. The γ-ray spectrum observed after capture is shown in Fig. 6. Two parts of the spectrum can be distinguished, a low-energy part including γ rays up to about 12 MeV energy and a high-energy part. The low-energy spectrum is composed of γ rays emitted from the compound nucleus ^{28}Si and of secondary γ rays from residual nuclei formed by first chance particle emission. The high-energy part, however, results only from γ rays emitted from the compound nucleus. Those transitions feed almost entirely $T=0$ final states. The $E1$ part of the high-energy γ rays is suppressed through the isospin selection rule, and the emission of such γ rays is only possible via isospin mixing.

The amount of isospin mixing is estimated by comparing the measured γ-ray spectrum with a CASCADE model calculation [see Pühlhofer (1977)], modified by Harakeh *et al.* (1985) to include isospin. The modifica-

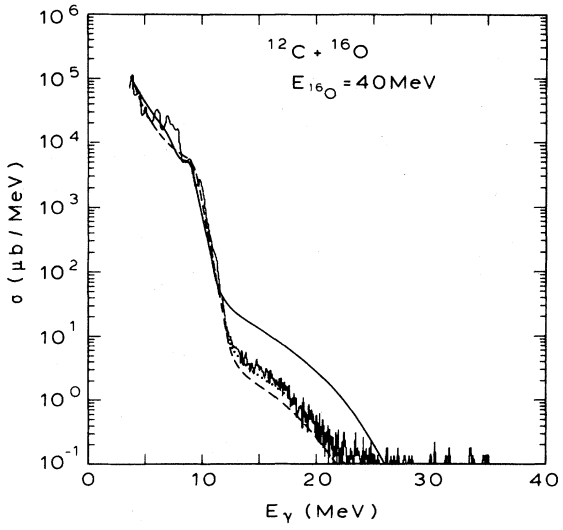

FIG. 6. Spectrum of γ rays following capture of ^{16}O by ^{12}C leading to ^{28}Si at $E_x=34$ MeV excitation energy. The data and CASCADE calculations are from Snover (1984) and Harakeh *et al.* (1985). The dashed curve shows the $E1+E2$ intensity if isospin is conserved. The solid curve is obtained with completely mixed isospin, while the dotted curve (adjusted to fit the data) requires—as explained in the main text—an isospin-mixing parameter $f\lesssim 0.05$.

[6]Indeed $\tau_{\gamma 2}/\tau_{\gamma 1}=1$ is the upper limit of this quantity, since in the non-self-conjugate nuclei the giant dipole resonance is split into a T_1 and a T_2 component [see Fultz *et al.* (1971)]. This causes the ratio $\tau_{\gamma 2}/\tau_{\gamma 1}$ to be different from the corresponding ratio of vector coupling coefficients, since the assumption (4.25) no longer applies. Since the experiments on the non-self-conjugate nuclei by Kuhlmann (1979) have covered more the location of the T_1 component than that of the T_2 component of the giant resonance, one expects $\tau_{\gamma 2}/\tau_{\gamma 1}<1$. However, the splitting is not large when compared to the widths of the components; we therefore expect $\tau_{\gamma 2}/\tau_{\gamma 1}\simeq 1$ to be a reasonable approximation. Note that the situation is different in the example discussed in Sec. VI.C.

tions are threefold. First, the transmission coefficients are factored into an isospin Clebsch-Gordan coefficient and an isospin-independent transmission coefficient entirely consistent with our Eq. (4.27). Second, γ-ray strength functions for $E1$ isovector, $E2$ isoscalar, and $E2$ isovector giant resonances are incorporated into the transmission coefficients. Third, isospin-dependent level densities are introduced for the compound nucleus and the residual nuclei [see, for example, Jensen (1977)].

These modifications are appropriate to describe the γ-ray spectrum under the assumption that isospin is conserved (dashed curve in Fig. 6). The calculation satisfactorily describes the low-energy part of the spectrum as well as the low- and the high-energy parts of the spectrum of γ rays following the capture of $^3\mathrm{He}+{}^{25}\mathrm{Mg}$, where both classes of levels with isospins $T_1=0$ and $T_2=1$ are populated. These results strongly suggest that—apart from a small isospin mixing—the standard statistical model correctly describes the process. The small isospin mixing is reflected in the difference between the dashed curve and the data in the high-energy part of the γ-ray spectrum in Fig. 6: The calculation that assumes isospin conservation underestimates the high-energy part of the spectrum. To cure this problem, a parameter α^2 has been introduced in the level densities entering CASCADE, which measures the fraction of states of a given isospin class that decays via mixing into the other class. The value $\alpha^2=0.05$ is necessary to bring the calculation into agreement with the data (dotted curve in Fig. 6). This is an upper limit for isospin mixing in the highly excited compound nucleus, since some mixing may occur also in the final states reached by its decay.

The quantity α^2 may be equated with Eq. (6.12):

$$\alpha^2 \equiv \nu = \frac{z/N_2}{1+z/N_1+z/N_2} , \tag{6.21}$$

since we have argued in Sec. VI.A that this is the fraction of states of class two decaying into class one. In the present example, Eq. (6.21) becomes symmetric in the two classes, since we have $N_1 \simeq N_2$ in self-conjugate nuclei. Transforming α^2 into the suppression factor f yields

$$f = \frac{\alpha^2}{1-\alpha^2} . \tag{6.22}$$

The small value of $f \lesssim 0.05$ implied by $\alpha^2 \lesssim 0.05$ for isospin mixing in the compound nucleus ^{28}Si excited at $E_x=34$ MeV supports the statement made at the end of Sec. VI.A that isospin is quite pure at high compound-nucleus excitation energies.

The arguments just given ascribe the isospin mixing only to the compound-nucleus levels at 34 MeV. The high-energy γ decay with typical energies around 15 MeV populates states in the same nucleus around 20 MeV excitation energy. It was argued in Sec. VI.B.1 that these latter levels are isospin-mixed with a parameter f around 0.15. This in turn should enhance the high-energy γ rays. This fact is presently not understood, but is perhaps related to the difference in total angular momentum governing the two reactions.

C. Third example: isospin-forbidden neutron decay of the giant dipole resonance

In the previous section we discussed two examples for the determination of isospin mixing in the giant dipole resonance in self-conjugate nuclei. For nuclei with a neutron excess ($T_1 \neq 0$), the giant dipole resonance is split into a component with isospin T_1 and a higher-lying one with isospin $T_2=T_1+1$. If both components are excited by photon absorption, the isospin selection rule forbids neutron decay of the T_2 component to the low-lying residual states [see Morinaga (1955)] labeled $|\, T_1-\frac{1}{2}, T_1-\frac{1}{2}\rangle$ in Fig. 2. Moreover, neutron decay into the $|\, T_1+\frac{1}{2}, T_1-\frac{1}{2}\rangle$ states is hindered: Because of the nuclear symmetry energy, such states occur at fairly high excitation energy in medium-weight nuclei. The proton decay of both classes with isospin T_1 and T_2 into the states $|\, T_1+\frac{1}{2}, T_1+\frac{1}{2}\rangle$ is allowed. This fact is well

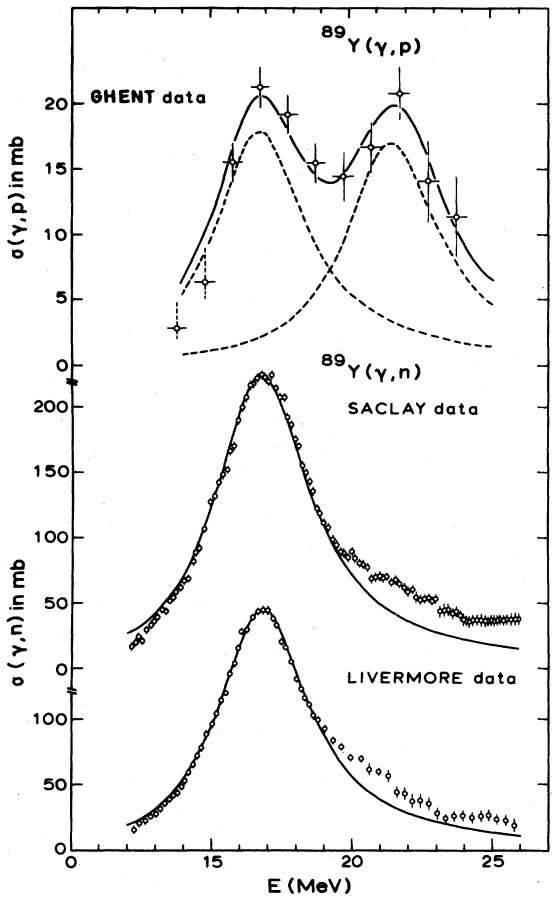

FIG. 7. Photoproton (upper part) and photoneutron (lower part) cross section of ^{89}Y measured by Van Camp *et al.* (1984; Ghent data), Leprêtre *et al.* (1971; Saclay data), and Berman *et al.* (1967; Livermore data) in the region of the giant dipole resonance. The resonance is split into components with isospin T_1 at $E_1=16.8$ MeV and T_2 at $E_2=21.5$ MeV. Only the lower component decays strongly by neutron emission. The solid curves are Lorentzians fitted to the data.

illustrated by the example of the photonuclear cross sections measured in the $^{89}Y(\gamma,n)$ reaction by Berman *et al.* (1967) and Leprêtre *et al.* (1971) and the $^{89}Y(\gamma,p)$ reaction by Van Camp *et al.* (1984) (Fig. 7). The double hump in the (γ,p) cross section reflects the two isospin components of the giant dipole resonance located at excitation energies $E_1=16.8$ MeV and $E_2=21.5$ MeV. The (γ,n) cross section, however, peaks only at the lower energy and exhibits a small amount of cross section at E_2, just visible as a deviation from the Lorentzian curve fitted to the T_1 component of the giant dipole resonance.[7]

For a quantitative estimate of isospin mixing at the excitation energy $E_2=21.5$ MeV of the T_2 state in ^{89}Y we follow Van Camp *et al.* (1984). The recipe is again—as in the two foregoing sections—a comparison of an isospin-forbidden and an isospin-allowed cross section, i.e., of the photoneutron cross section and the photoproton cross section from the T_2 component. The method is based on the assumption that the giant dipole resonance mainly spreads into the underlying compound-nucleus states. That this is the case was shown for ^{89}Y by Van Camp *et al.* (1984).

As in Sec. VI.B, the *predominantly isospin-allowed* photonuclear proton cross section for the isospin class T_2 levels—called here $\sigma_{\gamma 2,p}$—is written as

$$\sigma_{\gamma 2,p}(\text{allowed})=\tau_{\gamma 2}\frac{1+z/N_1}{1+z/N_1+z/N_2}\frac{1}{N_2}\tau_{p2} + \tau_{\gamma 2}\frac{z/N_2}{1+z/N_1+z/N_2}\frac{1}{N_1}\tau_{p1}\,. \quad (6.23)$$

Note that we have totally omitted the part of the reaction corresponding to the absorption of the photon into the class one states. We have therefore deliberately chosen the transmission coefficients as follows:

$$\tau_{\gamma 1}=0,\quad \tau_{p1}\neq 0,\quad \tau_{\gamma 2}\neq 0,\quad \tau_{p2}\neq 0\,. \quad (6.24)$$

Correspondingly, the transmission coefficients for the photoneutron channel are

$$\tau_{\gamma 1}=0,\quad \tau_{n1}\neq 0,\quad \tau_{\gamma 2}\neq 0,\quad \tau_{n2}=0\,. \quad (6.25)$$

The compound-nucleus photoneutron cross section then reads

$$\sigma_{\gamma 2,n}(\text{inhibited})=\tau_{\gamma 2}\frac{z/N_2}{1+z/N_1+z/N_2}\frac{1}{N_1}\tau_{n1}\,. \quad (6.26)$$

As in Sec. VI.B [see Eq. (6.19)], the measured cross sections have to be normalized to the following Hauser-Feshbach cross sections:

$$\sigma_{\rm HF}(\text{allowed})=\tau_{\gamma 2}\frac{1}{N_2}\tau_{p2}\,,\qquad \sigma_{\rm HF}(\text{inhibited})=\tau_{\gamma 2}\frac{1}{N_2}\tau_{n1}\,. \quad (6.27)$$

We emphasize here that unlike all reactions considered so far, the allowed reaction proceeds primarily through the class two levels. The experimental and the calculated cross sections are lumped together in the ratio

$$\frac{\sigma_{\gamma 2,p}(\text{allowed})/\sigma_{\rm HF}(\text{allowed})}{\sigma_{\gamma 2,n}(\text{inhibited})/\sigma_{\rm HF}(\text{inhibited})}=\frac{1+z/N_1}{z/N_1}+\frac{\tau_{p1}}{\tau_{p2}}\,. \quad (6.28)$$

In contradistinction to Eq. (6.20), the first term on the right-hand side depends upon z/N_1 and the second term is determined by the ratio of the respective Clebsch-Gordan coefficients, which are given in Fig. 3, i.e., $\tau_{p1}/\tau_{p2}=2T_1+1$.

The quantity z/N_1 can be converted into z/N_2 if the ratio N_2/N_1 is calculated. This has been done by Van Camp *et al.* (1984) for ^{89}Y and for ^{60}Ni, ^{88}Sr, ^{90}Zr, and ^{92}Mo. The corresponding suppression factors[8]—listed in Table I in Sec. VII—range from $f=0.48$ to 0.84, indicating a fairly large isospin mixing at excitation energies $E_x\simeq 20$ MeV. The Coulomb matrix elements derived[9] from this study are listed in Table I.

D. Fourth example: ratios of evaporation spectra

An important part of the existing information on isospin mixing in highly excited compound nuclei is derived from studies of evaporation spectra of (α,α'), (p,p'), (p,α'), and (α,p') reactions (Vaz *et al.*, 1972; Wiley *et al.*, 1973; Lux *et al.*, 1977) proceeding through the same compound nucleus. The idea behind these studies is as follows. If the isospin selection rule does not play any role, then the experimental ratio

$$R=\frac{\sigma_{\alpha\alpha'}\sigma_{pp'}}{\sigma_{ap'}\sigma_{p\alpha'}} \quad (6.29)$$

is approximately unity because of Bohr's hypothesis: Formation and decay of the compound nucleus are independent. Using the Hauser-Feshbach expression (6.7) for complete isospin mixing one may write R in this case

[7] In Fig. 7, $^{89}Y(\gamma,n)^{88}Y$ cross sections from Livermore (Berman *et al.*, 1967) and Saclay (Leprêtre *et al.*, 1971) are shown in order to demonstrate the strong suppression of the T_2 component in two independent experiments. For the estimate of isospin mixing, the Livermore data have been used.

[8] The isospin-mixing parameter is called μ^2 by Van Camp *et al.* (1984). It is related to the quantities defined in the present paper through $\mu^2=\nu$ [see Eq. (6.12)]. Since for all compound nuclei studied $N_1 \gg N_2$, we have $\mu^2\simeq f$, i.e., μ^2 nearly equals the suppression factor used throughout this review.

[9] We have changed the values of the Coulomb matrix elements given by Van Camp *et al.* (1984) in their Table II. Their values follow from Eqs. (2.16), (4.28), and (6.14) if z/N_2, D_1, and $\Gamma_2^\downarrow$ are known and external mixing is neglected. Van Camp *et al.* (1984) calculated $\Gamma_2^\downarrow$ and D_1. We have inferred $\Gamma_2^\downarrow=20$ keV from the work of Berg *et al.* (1975) for ^{88}Sr, ^{89}Y, ^{90}Zr, and ^{92}Mo (for ^{60}Ni there is no measurement of $\Gamma_2^\downarrow$) and recalculated D_1 with the help of Gilbert and Cameron (1965).

$$R(\text{complete mixing}) \equiv R_{\mathrm{CM}} = \left[\tau_\alpha \frac{1}{N_1+N_2} \tau_{\alpha'}\right]\left[\tau_p \frac{1}{N_1+N_2}\tau_{p'}\right]\left[\tau_\alpha \frac{1}{N_1+N_2}\tau_{p'}\right]^{-1}\left[\tau_p \frac{1}{N_1+N_2}\tau_{\alpha'}\right]^{-1} \simeq 1 \,. \tag{6.30}$$

Here, τ_α and τ_p are the usual transmission coefficients as defined in Eqs. (5.8). The quantity R_{CM} in (6.30) is not expected to be exactly unity because the angular momenta involved in the four reactions may not be the same, even though the excitation energy of the compound nucleus is chosen to be the same in the four reactions. One can, however, calculate R_{CM} using the full angular-momentum-dependent Hauser-Feshbach expression (see footnote 2).

In the opposite case, when isospin is conserved, R is larger than R_{CM} since the (p,p') reaction can proceed through both classes of levels, while the other three reactions are restricted to the levels of class one. From Eq. (6.6) one obtains now

$$R(\text{no mixing}) \equiv R_{\mathrm{NM}} = \left[\tau_{\alpha 1}\frac{1}{N_1}\tau_{\alpha' 1}\right]\left[\tau_{p1}\frac{1}{N_1}\tau_{p'1}+\tau_{p'2}\frac{1}{N_2}\tau_{p'2}\right] \times \left[\tau_{\alpha 1}\frac{1}{N_1}\tau_{p'1}\right]^{-1}\left[\tau_{p1}\frac{1}{N_1}\tau_{\alpha' 1}\right]^{-1} . \tag{6.31}$$

Introducing the values of the Clebsch-Gordan coefficients that appear in the transmission coefficients $\tau_{\alpha m}, \tau_{pm}$ [see Eq. (4.27) and Fig. 2], one can relate R_{NM} to R_{CM} and finds

$$R_{\mathrm{NM}} = R_{\mathrm{CM}}\left[1+\frac{1}{(2T_1+1)^2}\frac{N_1}{N_2}\right] . \tag{6.32}$$

With Eqs. (4.18) and (4.27) the general case of arbitrary mixing can be worked out (see Appendix B):

$$R/R_{\mathrm{CM}} = 1+\left[(1-\nu)\frac{N_1}{N_2}-\nu\right] \times \left[\left[1-\nu\frac{N_2}{N_1}\right](2T_1+1)+\nu\right]^{-2} . \tag{6.33}$$

Here, the quantity ν of Eq. (6.12) has been used. Equation (6.33) may be employed to determine ν from experimental data, provided the ratio N_1/N_2 is obtained from level densities and R_{CM} from Hauser-Feshbach expressions [see Vaz *et al.* (1972), Wiley *et al.* (1973), Lux *et al.* (1977), and Li and Harney (1982)]. We have calculated the quantity ν from the information given by Lux *et al.* (1977) in their Table II for 17 cases by observing that the quantities G and $G_{\max}$ of that paper are related to the expressions of the present article via

$$G = R/R_{\mathrm{CM}} \quad \text{and} \quad G_{\max} = R_{\mathrm{NM}}/R_{\mathrm{CM}} . \tag{6.34}$$

The results are listed in Table I of Sec. VII, together with the results of all experiments discussed in the present section. The isospin-mixing parameter extracted by Lux *et al.* (1977) is different from the one in Eq. (6.33). The relations between the various mixing parameters used in the literature have been given by Lane (1978), together with a discussion of the similarities and differences between the corresponding theories treating isospin mixing in the highly excited compound nucleus.

E. Fifth example: cross-section fluctuations in isobaric mirror channels

It was pointed out in Sec. II that the cross sections for the pair of reactions

$$(a,t) \rightarrow (\text{compound nucleus}) \rightarrow \begin{cases} (b,\tfrac{1}{2}), \\ (b,-\tfrac{1}{2}) , \end{cases} \tag{6.35}$$

leading to the isobaric mirror channels $(b,\frac{1}{2})$ and $(b,-\frac{1}{2})$ are correlated, if isospin is a good quantum number. Indeed, the cross-correlation function of the two cross sections $\sigma_{at,b\,1/2}$ and $\sigma_{at,b\,-1/2}$ is expected from Sec. II to be nonzero. If isospin is disregarded, one would expect these two cross sections to fluctuate in an uncorrelated way, as does any pair of cross sections pertaining to different final states [see Ericson and Mayer-Kuckuk (1966) and Richter (1974)]. Figure 8 qualitatively shows that isospin conservation introduces the expected correlation. Excitation functions by Détraz *et al.* (1971) of the reactions

$$^3\mathrm{He}+{}^{19}\mathrm{F} \rightarrow {}^{22}\mathrm{Na}^* \rightarrow \begin{cases} {}^{15}\mathrm{O}+{}^{7}\mathrm{Li}, \\ {}^{15}\mathrm{N}+{}^{7}\mathrm{Be} , \end{cases} \tag{6.36}$$

are displayed. Apparently, the pair of mirror reactions leading to ^{15}O(ground state) + ^{7}Li(ground state) and ^{15}N(ground state) + ^{7}Be(ground state) reveals correlated structure. This is also true for the mirror reactions with the exit channel ^{15}O(ground state) + ^{7}Li ($E_x = 0.48$ MeV, $\frac{1}{2}^-$) and ^{15}N(ground state) + ^{7}Be ($E_x = 0.43$ MeV, $\frac{1}{2}^-$). Correlations between any other pair of the four excitation functions seem to be absent. The quantitative analysis confirms this, although the finite-range-of-data errors are large due to the short energy range covered [see Richter (1974)].

A considerably more extensive investigation has been published by Simpson *et al.* (1978) on the reactions

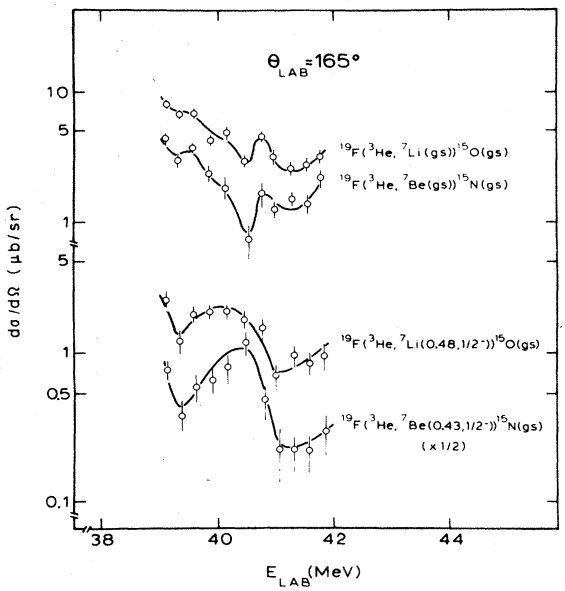

FIG. 8. Excitation functions of two pairs of compound-nucleus mirror reactions from Détraz *et al.* (1971). The entrance channel ^{3}He+^{19}F leads to the compound nucleus ^{22}Na at the mean excitation energy of 53.6 MeV. The exit channels form pairs of mirrors, ^{15}O+^{7}Li and ^{15}N+^{7}Be. The cross sections of mirror reactions are visibly correlated.

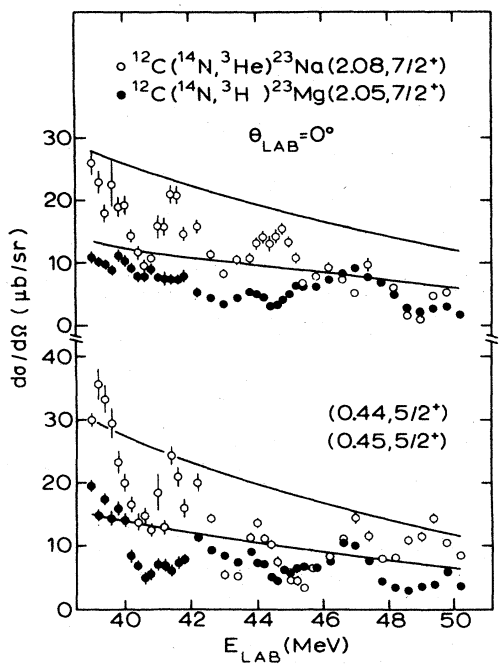

FIG. 9. Excitation functions of two pairs of mirror reactions from Simpson *et al.* (1978). The entrance channel ^{14}N+^{12}C leads to the compound nucleus ^{26}Al at a mean excitation energy of 35.7 MeV. The exit channels form pairs of mirrors, ^{23}Mg+^{3}H and ^{23}Na+^{3}He. The curves show a long-range variation of the excitation function that has to be removed before the correlation coefficients are evaluated. The normalized correlation coefficient is 0.23 for the upper pair and 0.41 for the lower pair of reactions.

$$^{14}\mathrm{N}+^{12}\mathrm{C}\rightarrow{}^{26}\mathrm{Al}^*\rightarrow\begin{cases}^{23}\mathrm{Mg}+^{3}\mathrm{H},\\ ^{23}\mathrm{Na}+^{3}\mathrm{He}\,.\end{cases}\tag{6.37}$$

In Fig. 9 the excitation functions for the pair of mirror channels ^{23}Mg (E_x=0.45 MeV, $\frac{5}{2}^+$) + ^{3}H and ^{23}Na (E_x=0.44 MeV, $\frac{5}{2}^+$) + ^{3}He, as well as for the pair ^{23}Mg (E_x=2.05 MeV, $\frac{7}{2}^+$) + ^{3}H and ^{23}Na (E_x=2.08 MeV, $\frac{7}{2}^+$) + ^{3}He, are shown. The correlations are less apparent than in the previous example. They may, however, be determined quite precisely from the excitation functions of Fig. 9 and those of several more final states. We use the reactions in Eq. (6.37) as the model case for a discussion of cross-section fluctuations in isobaric mirror channels.

The analysis of the data is carried out in terms of the correlation coefficients,

$$\begin{aligned}C_{tt'}&=C_{tt'}(0)\\&=\langle\sigma_{a,bt}(E)\sigma_{a,bt'}(E)\rangle-\langle\sigma_{a,bt}(E)\rangle\langle\sigma_{a,bt'}(E)\rangle\\&=|\,\langle S^{\mathrm{fl}}_{a,bt}(E)S^{\mathrm{fl}*}_{a,bt'}(E)\rangle\,|^2\,.\end{aligned}\tag{6.38}$$

Here, the entrance channel a is composed of ^{14}N + ^{12}C. The exit channel $(b,\frac{1}{2})$ denotes one of the residual states of ^{23}Mg + ^{3}H. The channel $(b,-\frac{1}{2})$ characterizes the mirror residual states in ^{23}Na + ^{3}He. Note that the values $+\frac{1}{2}$ and $-\frac{1}{2}$ that the indices t,t' can assume will simply be written as plus and minus in what follows. For $t=t'$, the quantity of Eq. (6.38), i.e., C_{++} or C_{--}, is the variance of the excitation function $\sigma_{a,bt}(E)$. For $t\neq t'$, C_{+-} measures the degree of correlation between $\sigma_{a,b+}(E)$ and $\sigma_{a,b-}(E)$.

The experiment yields the normalized correlation coefficient

$$r(\text{mirror})=\frac{C_{+-}}{(C_{++}C_{--})^{1/2}}\,,\tag{6.39}$$

which ranges between zero and unity. From altogether six pairs of mirror excitation functions, Simpson *et al.* (1978) find

$$r(\text{mirror})=0.41^{+0.06}_{-0.07}\,.\tag{6.40}$$

This value is significantly larger than the correlation between cross sections that do not belong to mirror reactions. For these pairs, it was found that on the average

$$r(\text{random})=0.10\pm0.04\,.\tag{6.41}$$

We turn now to the interpretation of this result. The transmission coefficients for the entrance channel couple

to the levels of class one with $T_1=0$ only, i.e.,

$$\tau_{a1}=\tau_a,\quad \tau_{a2}=0\ . \tag{6.42}$$

The transmission coefficients for the exit channels, however, couple to both classes of levels and one has—with the proper Clebsch-Gordan coefficients from Fig. 2 already inserted into Eq. (4.27)—for the triton channel

$$\tau_{b1}^{++}=\tfrac{1}{2}\tau_{^3\mathrm{H}}=\tau_{b2}^{++}\ . \tag{6.43}$$

Note that we are using the convention (5.8), with the channel index (at) replaced by the symbol for the emitted light particle. Then $\tau_{^3\mathrm{H}}$ is the usual isospin-independent transmission coefficient for the channel $^{23}\mathrm{Mg}+{}^3\mathrm{H}$. For the ^{3}He channel one finds similarly

$$\tau_{b1}^{--}=\tfrac{1}{2}\tau_{^3\mathrm{H}}=\tau_{b2}^{--}\ . \tag{6.44}$$

Equations (4.18) and (6.38) yield for the variance

$$C_{++}=|\,\tfrac{1}{2}\tau_a(\Pi_{11}+\Pi_{12})\tau_{^3\mathrm{H}}\,|^2\ . \tag{6.45}$$

The result for C_{--} is the same, with $\tau_{^3\mathrm{H}}$ replaced by $\tau_{^3\mathrm{He}}$.

In order to express C_{+-}, one needs the "off-diagonal" transmission coefficients of Eq. (4.27), namely,

$$\tau_{b1}^{+-}=-\tfrac{1}{2}(\tau_{^3\mathrm{H}}\tau_{^3\mathrm{He}})^{1/2}=-\tau_{b2}^{+-}\ , \tag{6.46}$$

and finds

$$C_{+-}=|\,\tfrac{1}{2}\tau_a(\Pi_{11}-\Pi_{12})(\tau_{^3\mathrm{H}}\tau_{^3\mathrm{He}})^{1/2}\,|^2\ . \tag{6.47}$$

Hence, the correlation coefficient of Eq. (6.39) is

$$r(\text{mirror})=\left[\frac{\Pi_{11}-\Pi_{12}}{\Pi_{11}+\Pi_{12}}\right]^2\ . \tag{6.48}$$

Taking the elements of the matrix Π from Eq. (6.4), we express this result in terms of z/N_2,

$$r(\text{mirror})=\left[\frac{N_2}{N_2+2z}\right]^2=(1+2z/N_2)^{-2}\ . \tag{6.49}$$

As expected, the correlation coefficient r(mirror) is unity if isospin is conserved, i.e., $z=0$, and it approaches zero for very strong mixing, i.e., $z/N_2\to\infty$. In this case we recover the conventional theory of nuclear cross-section fluctuations.

Through Eq. (6.49), we can relate the experiment on correlations in mirror channels to all the other experiments described in the previous sections by calculating the suppression factor of Eq. (6.10),

$$f=(1-\sqrt{r}\,)/(1+\sqrt{r}\,)\ . \tag{6.50}$$

The result is given in Table I of Sec. VII.

This analysis contains a simplification that is not obviously correct, but that has been shown by Li and Harney (1982) to be justified. As mentioned in footnote 2, the actual cross sections are not simply squares of a single S-matrix element. In general, they are an incoherent sum of terms, each of which is the square of a linear combination of S-matrix elements [see Ericson and Mayer-Kuckuk (1966)]. Therefore the cross section σ_t is not just of the form of Eq. (2.12), but rather an expression like

$$\sigma_t=\sum_\mu\left\langle\left|\sum_{a,b}g_{ab}^{\mu}S_{a,bt}^{\mathrm{fl}}(E)\right|^2\right\rangle\ . \tag{6.51}$$

Here, the g_{ab}^{μ} are geometric coefficients and angular functions, and μ denotes the spin projections of the reaction partners. Using Eq. (6.38) and the statistical independence of any two S-matrix elements differing in the indices a,b, one finds the correlation functions measured in the experiment,

$$C_{tt'}=\sum_{\mu\nu}\left|\sum_{ab}g_{ab}^{\mu}g_{ab}^{\nu*}\left\langle S_{a,bt}^{\mathrm{fl}*}(E)S_{a,bt'}^{\mathrm{fl}}(E)\right\rangle\right|^2\ . \tag{6.52}$$

This expression replaces Eq. (6.38). It gives for $t=t'$

$$C_{tt}=\sum_{\mu\nu}\left|\tfrac{1}{2}\sum_{ab}g_{ab}^{\mu}g_{ab}^{\nu*}\tau_a(\Pi_{11}+\Pi_{12})\tau_{bt}\right|^2 \tag{6.53}$$

and for $t\neq t'$

$$C_{+-}=\left[\frac{\Pi_{11}-\Pi_{12}}{\Pi_{11}+\Pi_{12}}\right]^2\times\sum_{\mu\nu}\left|\tfrac{1}{2}\sum_{ab}g_{ab}^{\mu}g_{ab}^{\nu*}\tau_a(\Pi_{11}+\Pi_{12})\right|^2\times(\tau_{^3\mathrm{H}}\tau_{^3\mathrm{He}})^{1/2}\exp i(\varphi_{^3\mathrm{H}}-\varphi_{^3\mathrm{He}})\ . \tag{6.54}$$

The phases $\varphi_{^3\mathrm{H}}$ and $\varphi_{^3\mathrm{He}}$ are the potential scattering phase shifts appearing in Eqs. (4.9) and (4.10). In Eq. (6.54) we have introduced the assumption that the mixing parameter z/N_2 does not depend on spin and parity of the compound-nucleus levels and that therefore $(\Pi_{11}-\Pi_{12})^2/(\Pi_{11}+\Pi_{12})^2$ can be factored out of the summations [see Eqs. (6.48) and (6.49)]. This assumption is ubiquitous and is to some extent confirmed by the results compiled in Sec. VII, showing that z/N_2 does only weakly depend on the compound-nucleus level density. Equation (6.49) must now be replaced by

$$r(\text{mirror})=\left[\frac{1}{1+2z/N_2}\right]^2\hat{r}\ , \tag{6.55}$$

where $\hat{r}$ is formed by the ratio of the sums in Eqs. (6.53) and (6.54) according to the definition of r(mirror) in Eq. (6.39). The quantity $\hat{r}$ can be calculated, and Eq. (6.55) again allows one to extract the isospin-mixing parameter from the measured correlation coefficient r(mirror). We note that $\hat{r}$ does not depend on the isospin-mixing parameter, since one has in the self-conjugate compound nucleus ^{26}Al about equal numbers of decay channels for both classes of levels, $N_1\simeq N_2$, and therefore

$$\Pi_{11}+\Pi_{12}=\frac{N_2+2z}{N_1N_2+z(N_1+N_2)}\simeq\frac{1}{N_1}\ . \tag{6.56}$$

In order to estimate $\hat{r}$, we neglect momentarily the phase differences in Eq. (6.54). The cross sections $\sigma_{^3\mathrm{He}}$ are larger by about a factor of 2 than the cross sections $\sigma_{^3\mathrm{H}}$ for the corresponding mirror channels. Assuming that all transmission coefficients $\tau_{^3\mathrm{He}}$ are larger than $\tau_{^3\mathrm{H}}$ by a constant factor, one immediately finds $\hat{r}=1$. The full numerical calculation carried out by Li and Harney (1982) yields $\hat{r}=0.93$ and thus essentially confirms this estimate and the simplified discussion leading to Eqs. (6.49) and (6.50).

In their original papers, Simpson and co-workers (1978,1979) arrived at a different estimate of $\hat{r}$ starting from a different assumption.[10] Since they did not perform a numerical calculation of $\hat{r}$, we believe that their original analysis should be modified in this respect.

We have found three more examples of measured mirror correlations in the literature. Glasner *et al.* (1983) have studied

$$^6\mathrm{Li}+^{16}\mathrm{O}\rightarrow{}^{22}\mathrm{Na}^*\rightarrow\begin{cases}^{21}\mathrm{Na}+n,\\ \\ ^{21}\mathrm{Ne}+p\ ,\end{cases}\tag{6.57}$$

at an excitation energy of 18.9 MeV in ^{22}Na. They obtain $r(\text{mirror})=0.49\pm0.09$ and set $\hat{r}=1$ to extract the isospin-mixing parameter. This should be a very good approximation in their case, since they have $\sigma_+\simeq\sigma_-$, which suggests $\tau_{b+}=\tau_{b-}$ for all nucleon channels b in Eqs. (6.53) and (6.54), whence $\hat{r}=1$. Finally, Assmann (1981) and Evers (1982) have studied the compound nuclei ^{24}Mg ($E_x=27.1$ MeV) and ^{28}Si ($E_x=29.8$ MeV) via nucleon decay as in the reactions (6.57). In their cases the average proton cross sections σ_- are larger than the average neutron cross sections σ_+ by about a factor of 2. This is similar to the reactions (6.37). The quantity $\hat{r}$ has not been calculated. We have included their results in Table I of Sec. VII assuming that the calculation reported by Li and Harney (1982) is representative and we may put $\hat{r}=1$ here, too.

F. Sixth example: correlation widths

We have emphasized in Sec. V that the familiar Lorentzian form (5.4) of the correlation function is only obtained in the limit of very strong isospin mixing. In the case of strict isospin conservation, one finds a superposition of two Lorentzians. This case was considered in Sec. III. We now give the general result. It has a complicated structure, and we therefore discuss it in the limit of small isospin mixing. We apply it to the analysis of measured correlation widths. Here, the aim will not be to extract the isospin-mixing parameter z (which is known from experiments described above), but rather to see whether the data confirm the structure of the correlation function obtained theoretically.

The determinant of the matrix Π^{-1} in Eq. (4.19) is a quadratic form in ε and can be written as

$$\det(\Pi^{-1})=-\frac{4\pi^2}{D_1D_2}(\varepsilon-i\lambda_1)(\varepsilon-i\lambda_2)\ .\tag{6.58}$$

The roots λ_m are given in Eq. (5.1). From Eqs. (4.18), (6.4), and (6.13) one finds the general form of the autocorrelation function [see Harney *et al.* (1983)]:

$$\langle S^{\mathrm{fl}}_{at,bt'}(E)S^{\mathrm{fl}*}_{at,bt'}(E+\varepsilon)\rangle=[(\lambda_1+i\varepsilon)(\lambda_2+i\varepsilon)]^{-1}$$
$$\times\left[\tau_{\alpha1}\frac{\Gamma^{\uparrow}_1}{N_1}(\Gamma^{\uparrow}_2+\Gamma^{\downarrow}_2+i\varepsilon)\tau_{\beta1}+\tau_{\alpha1}\frac{\Gamma^{\uparrow}_2\Gamma^{\downarrow}_1}{N_2}\tau_{\beta2}+\tau_{\alpha2}\frac{\Gamma^{\uparrow}_1\Gamma^{\downarrow}_2}{N_1}\tau_{\beta1}+\tau_{\alpha2}\frac{\Gamma^{\uparrow}_2}{N_2}(\Gamma^{\uparrow}_1+\Gamma^{\downarrow}_1+i\varepsilon)\tau_{\beta2}\right].\tag{6.59}$$

Equation (6.59) applies for $a\neq b$; the right-hand side has to be multiplied by 2 for elastic scattering, i.e., $a=b$, $t=t'$. The case of charge exchange scattering, i.e., $a=b$, $t\neq t'$, will be considered in Sec. VI.G.

To simplify the discussion, we assume $z\ll N_1,N_2$, expand the matrix Π in powers of the matrix

$$\begin{pmatrix}0 & z\\ z & 0\end{pmatrix},$$

and keep only terms of up to first order. The result is

$$\langle S^{\mathrm{fl}}_{\alpha\beta}(E)S^{\mathrm{fl}*}_{\alpha\beta}(E+\varepsilon)\rangle=\frac{\tau_{\alpha1}\tau_{\beta1}}{N_1}\frac{\Gamma^{\uparrow}_1}{\Gamma^{\uparrow}_1+\Gamma^{\downarrow}_1+i\varepsilon}+\left[\frac{\tau_{\alpha1}\tau_{\beta2}}{N_2}\Gamma^{\uparrow}_2\Gamma^{\downarrow}_1+\frac{\tau_{\beta1}\tau_{\alpha2}}{N_1}\Gamma^{\uparrow}_1\Gamma^{\downarrow}_2\right]\frac{1}{\Gamma^{\uparrow}_1+\Gamma^{\downarrow}_1+i\varepsilon}\frac{1}{\Gamma^{\uparrow}_2+\Gamma^{\downarrow}_2+i\varepsilon}$$
$$+\frac{\tau_{\alpha2}\tau_{\beta2}}{N_2}\frac{\Gamma^{\uparrow}_2}{\Gamma^{\uparrow}_2+\Gamma^{\downarrow}_2+i\varepsilon}\ .\tag{6.60}$$

[10] In Eq. (4) of Simpson *et al.* (1978) and Eq. (6) of Simpson and Wilson (1979), the difference between the ^{3}He and ^{3}H cross sections is accounted for by splitting the ^{3}He reaction amplitudes $f(^3\mathrm{He})=f(^3\mathrm{H})+g$ into the triton reaction amplitudes $f(^3\mathrm{H})$ and a *statistically independent* term g. This results in $\hat{r}=\sigma_+/\sigma_-$, which is given by experiment as 0.54 ± 0.02. The present discussion then suggests that g is *not independent* of f.

This expression contains four terms. The first and the last—proportional to $\tau_{\alpha m}\tau_{\beta m}$—are due to the allowed reactions via classes one and two, respectively. Their form is familiar. The other two terms—proportional to $\tau_{\alpha m}\tau_{\beta n}$ with $m \neq n$—are connected with a forbidden change of isospin. In the limit $z \to 0$, i.e., $\Gamma_m^{\downarrow} \to 0$, Eq. (6.60) agrees with the isospin-conserving version (2.17). For $z \neq 0$ and z small, the autocorrelation function of an isospin-forbidden reaction is proportional to the product of the Lorentzians of both isospin-allowed transitions—a result already anticipated by Bizzeti (1964). If a case could be found in which the transitions $T_1 \to T_1$, $T_2 \to T_2$, and $T_1 \to T_2$ were measured separately, this behavior could be observed, since the autocorrelation functions of the allowed reactions would have the form of Eq. (3.2) and the one of the forbidden reaction would be given by

$$C_{\text{forb}}(\varepsilon) = \left[\frac{\tau_{\alpha 1}\tau_{\beta 2}}{N_2}\right]^2 \left|\frac{\Gamma_2^{\uparrow}\Gamma_1^{\downarrow}}{(\Gamma_1^{\uparrow}+\Gamma_1^{\downarrow}+i\varepsilon)(\Gamma_2^{\uparrow}+\Gamma_2^{\downarrow}+i\varepsilon)}\right|^2 . \tag{6.61}$$

The experimental precision is usually not sufficient to distinguish the form (6.61) from a pure Lorentzian; however, we would certainly expect the width at half maximum of C_{forb} to be smaller than the correlation widths of both isospin-allowed reactions.

There are indeed two sets of data which corroborate this claim. We discuss them in turn.

1. The reaction ^{32}S$(d,\alpha)^{30}$P

Spijkervet (1978) has studied the isospin-allowed reaction ^{32}S$(d,\alpha)^{30}$P ($T=0$) for 18 final states and the forbidden reaction ^{32}S$(d,\alpha)^{30}$P ($T=1$) for five final states in the range $2.0 \leq E_d \leq 5.7$ MeV of incident energy E_d at five different scattering angles. Unfortunately this extensive work has never been published. In Fig. 10 we reproduce a set of Spijkervet's excitation functions. The coherence width Γ_{all} found in the allowed transitions is indeed somewhat larger than the coherence width Γ_{forb} in the forbidden transitions (see Fig. 11). (The precise definitions of Γ_{all} and Γ_{forb} are given below.) We now investigate whether the ratio $\Gamma_{\text{all}}/\Gamma_{\text{forb}}$ is in agreement with theory.

The analysis described in Sec. VI.A yields the suppression factor f displayed in Fig. 12. This value is one of the largest found. Adopting the value $f=0.41\pm0.02$, one finds[11] [see Eqs. (6.10) and (6.14)]

$$\frac{z}{N_2} = \frac{\Gamma_2^{\downarrow}}{\Gamma_2^{\uparrow}} = 0.69 \pm 0.05 . \tag{6.62}$$

[11]We note that there is a misprint in Table II.10 on p. 2-49 of Spijkervet's (1978) work, where the last entry should read $\Gamma_{2,\text{int}}^{\downarrow} = 0.59\Gamma$.

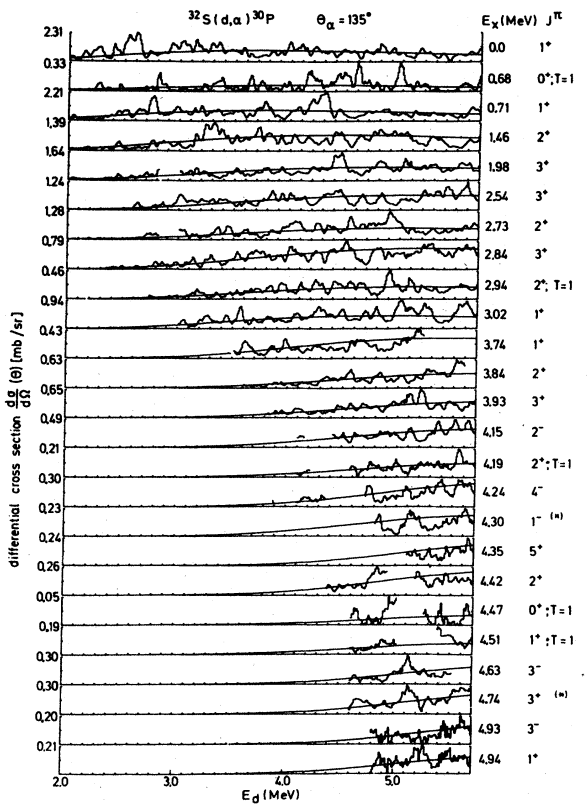

FIG. 10. Excitation functions of ^{32}S$(d,\alpha)^{30}$P from Spijkervet (1978). Twenty-five transitions are shown. In two of them (denoted by an asterisk), the spin-parity assignment to the final state is uncertain. They have been omitted in the analysis. The smooth curves are average cross sections calculated with the help of Eq. (4.18).

Computing $\Gamma_m^{\uparrow}$ and N_m, $m=1,2$ from level-density expressions, Spijkervet gets

$$\Gamma_1^{\uparrow} = \Gamma_2^{\uparrow} \equiv \Gamma \tag{6.63}$$

and

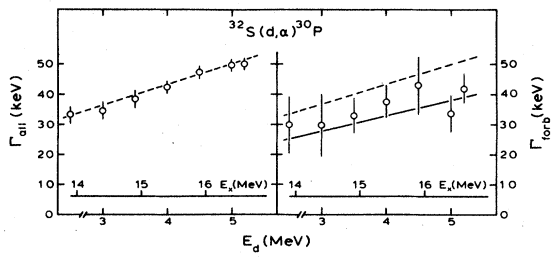

FIG. 11. Coherence widths of the isospin-allowed transitions ^{32}S$(d,\alpha)^{30}$P ($T=0$) and of the isospin-forbidden transitions ^{32}S$(d,\alpha)^{30}$P ($T=1$) as functions of the excitation energy E_x in the compound nucleus ^{34}Cl. The data are from Spijkervet (1978). The dashed lines fit the data points on the left-hand part. The solid line on the right-hand part indicates the prediction of Eq. (6.71).

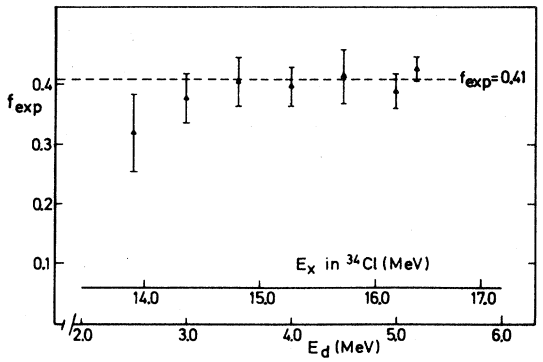

FIG. 12. The suppression factor f defined in Eq. (6.10) for the compound nucleus ^{34}Cl. Results from Spijkervet (1978).

$$N_1 = 1.30 N_2 \tag{6.64}$$

[see Sec. II.2 of Spijkervet (1978)] and introduces these relations as reasonably well-established assumptions into the further analysis. This yields

$$\frac{\Gamma_1^{\downarrow}}{\Gamma_1^{\uparrow}} = 0.53 \pm 0.04 \ . \tag{6.65}$$

For the allowed reaction with transmission coefficients as in Eq. (6.1), one finds from Eq. (6.59) the autocorrelation $C_{\rm all}$ to be

$$C_{\rm all}(\varepsilon) = \left| \frac{\tau_{\alpha 1}\tau_{\alpha 1}}{N_1} \frac{\Gamma_1^{\uparrow}(\Gamma_2^{\uparrow}+\Gamma_2^{\uparrow}+i\varepsilon)}{(\lambda_1+i\varepsilon)(\lambda_2+i\varepsilon)} \right|^2 . \tag{6.66}$$

Equations (5.1) and (6.63) yield

$$\lambda_1 = \Gamma, \quad \lambda_2 = \Gamma + \Gamma_1^{\uparrow} + \Gamma_2^{\downarrow} , \tag{6.67}$$

hence

$$C_{\rm all}(\varepsilon) = \left| \frac{\tau_{\alpha 1}\tau_{\alpha 1}}{N_1} \frac{\Gamma(\Gamma+\Gamma_2^{\downarrow}+i\varepsilon)}{(\Gamma+i\varepsilon)(\Gamma+\Gamma_1^{\downarrow}+\Gamma_2^{\downarrow}+i\varepsilon)} \right|^2 . \tag{6.68}$$

The quantity taken from experiment is the width at half maximum $\Gamma_{\rm all}$ of this curve:

$$C_{\rm all}(\Gamma_{\rm all}) = \tfrac{1}{2} C_{\rm all}(0) \ . \tag{6.69}$$

We turn now to the forbidden reaction. With transmission coefficients as in Eq. (6.8), one obtains from Eq. (6.59) the autocorrelation function

$$C_{\rm forb}(\varepsilon) = \frac{\tau_{\alpha 1}\tau_{\alpha 2}}{N_2} \frac{\Gamma\Gamma_1^{\uparrow}}{(\Gamma+i\varepsilon)(\Gamma+\Gamma_1^{\downarrow}+\Gamma_2^{\downarrow}+i\varepsilon)} \ . \tag{6.70}$$

The width at half maximum $\Gamma_{\rm forb}$ of this curve is defined in analogy with Eq. (6.69). Using the results (6.62) and (6.65), it is straightforward to calculate

$$\left[\frac{\Gamma_{\rm all}}{\Gamma_{\rm forb}} \right]_{\rm theory} = 1.32 \pm 0.01 \ . \tag{6.71}$$

The average experimental ratio of the results of Fig. 11 is

$$\left[\frac{\Gamma_{\rm all}}{\Gamma_{\rm forb}} \right]_{\rm expt} = 1.22 \pm 0.06 \ , \tag{6.72}$$

in satisfactory agreement with the result (6.71).

2. Reactions populating the compound nucleus ^{30}P

The second available set of data consists of three different reactions populating the compound nucleus ^{30}P at 20 MeV excitation energy.

(a) In the reaction ^{28}Si$(d,\alpha)^{26}$Al ($T=0$)—already discussed in Sec. VI.A—the correlation length was found by Bizzeti and Bizzeti-Sona (1968) to be

$$\Gamma_{\rm all} = 75 \pm 5 \text{ keV} \ . \tag{6.73}$$

This reaction represents the allowed transition $T_1 \rightarrow T_1$. The definition of $\Gamma_{\rm all}$ is given in Eqs. (6.66) and (6.69).

(b) Kildir (1980) investigated the reaction ^{29}Si$(p,\alpha)^{26}$Al ($T=1$). This essentially represents the allowed transition $T_2 \rightarrow T_2$. The transmission coefficients are

$$\tau_{p1} \neq 0, \quad \tau_{p1} \neq 0, \quad \tau_{\alpha 1} = 0, \quad \tau_{\alpha 2} \neq 0 \ . \tag{6.74}$$

With the help of Eqs. (6.59) and (6.74) one can write down the autocorrelation function $C_b(\varepsilon)$ and define the coherence width Γ_b in analogy with Eq. (6.69). The experimental result is

$$\Gamma_b = 104 \pm 35 \text{ keV} \ . \tag{6.75}$$

(c) The correlation width $\Gamma_{\rm forb}$ of the forbidden reaction ^{28}Si$(d,\alpha)^{26}$Al ($T=1$) was measured by Bizzeti and Bizzeti-Sona (1968) to be

$$\Gamma_{\rm forb} = 55 \pm 6 \text{ keV} \ . \tag{6.76}$$

This value is indeed smaller than the correlation widths (6.73) and (6.75) of both allowed reactions as expected from Eqs. (6.59) and (6.60). One can predict $\Gamma_{\rm forb}$ using Eqs. (6.70), (6.73), and (6.75), the isospin-mixing parameter z/N_2 given in Sec. VI.A, and the assumption

$$N_1 = N_2 \tag{6.77}$$

(see footnote 3 in Sec. VI.A). The details have been described by Harney *et al.* (1983). This yields

$$\Gamma_{\rm forb}(\text{predicted}) = 65^{+5}_{-8} \text{ keV} \ , \tag{6.78}$$

in reasonable agreement with Eq. (6.76).

We conclude that the experimental results described under (a) and (b) at least do not contradict the structure of the correlation function (6.59).

G. A suggested experiment: enhancement of charge exchange scattering

Charge exchange reactions like (p,n) or $(^3\text{He},^3\text{H})$ that lead to the isobaric analog state of the target can be considered as a form of elastic scattering in isospin space if isospin is conserved. Compound elastic scattering is

enhanced over inelastic channels: Statistical theories—as reviewed by Mahaux and Weidenmüller (1979)—show that the elastic enhancement factor W is equal to 2 in the limit of strongly overlapping resonances. [This has been confirmed by an experiment of Kretschmer and Wangler (1978), who find $W=2.09\pm0.14$ in the scattering of protons on ^{30}Si.] Through the combination of Kronecker symbols in Eq. (4.18), the formalism of Sec. IV also yields an elastic enhancement factor of 2: The compound nuclear cross section of a reaction going from channel (at) to channel (bt') is

$$\sigma_{at,bt'}=\langle\,|S^{\rm fl}_{at,bt'}|^2\rangle =[1+\delta_{ab}(W_{tt'}-1)]\sum_{mn}\tau^{tt}_{am}\Pi_{mn}\tau^{t't'}_{bn}\,, \tag{6.79}$$

where for $t=t'$ one has $W_{tt}=2$. But what about the case of charge exchange scattering, when $a=b$ but $t\neq t'$? We expect that $W_{tt'}$ takes the value of 2 if there is no isospin mixing and that it drops to unity for very strong mixing, when the conventional statistical theory applies. The result of Harney *et al.* (1980),

$$W_{tt'}=1+\left[1+\frac{(2T_1+2)^2}{2T_1+1}\frac{z}{N_1+N_2}\right]^{-1}\quad\text{for } t\neq t'\,, \tag{6.80}$$

indeed shows this behavior. The proof of Eq. (6.80) is given in Appendix C.

In all the examples given so far, the effects of isospin conservation are destroyed whenever the mixing parameter z of Eq. (4.24) becomes large compared to $\min(N_1,N_2)=N_2$. Equation (6.80), however, shows that W_{+-} approaches unity only if z becomes large compared to both N_1 and N_2. This practically never happens, as one can see from the results compiled in Table I below. Put differently, measurements of W_{+-} are inadequate to determine the *amount* of isospin mixing, since W_{+-} depends on z very weakly; they would, however, be stringent tests of the *theory*, since W_{+-} is almost invariably predicted to be close to 2. This is borne out by the predictions for W_{+-} listed in Table I.

No experimental study of the enhancement factor W_{+-} in compound-nucleus charge exchange scattering has as yet been published.

VII. SUMMARY OF EXPERIMENTAL RESULTS

A summary of all the data on isospin mixing in highly excited compound nuclei which have come to our attention is given in Table I. In this section, we discuss the entries in this table and draw conclusions on isospin mixing in nuclei.

The first column of Table I characterizes the compound nucleus, the second the excitation energy at which the experiment was performed. In the third column we list the value of T_1, the isospin of the compound nucleus in its ground state. In the next three columns we list the values of f, ν, and r [as defined in Eqs. (6.10), (6.12), and (6.55), respectively] determined from the data. In the seventh column one finds the values of the isospin-mixing parameters $\Gamma_2^{\downarrow}/\Gamma_2^{\uparrow}=z/N_2$ deduced from the values of f, ν, or r. We note that, in all cases, $\Gamma_2^{\downarrow}/\Gamma_2^{\uparrow}$ is neither negligibly small nor very large compared to unity. This shows that isospin mixing is neither negligible nor so strong that the standard compound-nucleus theory is applicable. A theory allowing for partial isospin symmetry breaking like that formulated in Sec. IV is obviously necessary to account for the data.

In order to deduce further information from the ratio $\Gamma_2^{\downarrow}/\Gamma_2^{\uparrow}=z/N_2$ we recall that the isospin-mixing parameter z contains contributions from both internal and external mixing. As described in Sec. V, internal mixing is due to the direct mixing of states in the two classes via isospin-breaking forces (essentially the Coulomb interaction), while external mixing is caused by an indirect coupling of the states in the two classes via channels in which isospin symmetry is not conserved. We accordingly have [see Eqs. (2.16), (4.28), and (6.13)]

$$\Gamma_2^{\downarrow}/\Gamma_2^{\uparrow}=z/N_2=\Gamma_{2,\rm int}^{\downarrow}/\Gamma_2^{\uparrow}+\Gamma_{2,\rm ext}^{\downarrow}/\Gamma_2^{\uparrow}\,, \tag{7.1}$$

where

$$\Gamma_{2,\rm ext}^{\downarrow}/\Gamma_2^{\uparrow}=2\pi\overline{H_C^2}/(D_1\Gamma_2^{\uparrow}) \tag{7.2}$$

and

$$\Gamma_{2,\rm ext}^{\downarrow}/\Gamma_2^{\uparrow}=\frac{4}{N_2}{\sum_a}'\frac{2T_1+1}{(2T_1+2)^2}\frac{(x_{a\,1/2}-x_{a\,-1/2})^2}{(1+x_{a\,1/2})(1+x_{a\,-1/2})}\,. \tag{7.3}$$

Experimental investigations aim at the determination of the strength of internal mixing and of the associated Coulomb matrix elements $\overline{H_C^2}$. This is possible provided $\Gamma_{2,\rm ext}^{\downarrow}/\Gamma_2^{\uparrow}$ can be estimated reliably. Harney and Tang (1981) have calculated both the sum over pairs of mirror channels and N_2, the effective number of open decay channels for class two levels appearing in Eq. (7.3), for many of the cases listed in Table I. The results are given in column eight. Comparing this with the previous column, we note that, in most cases, internal mixing dominates over external mixing. The calculations listed in column eight have a universal feature which, following the discussion in Sec. V, should be intuitively obvious: External mixing attains its maximum value for energies at or close to the threshold of the (p,n) charge exchange reaction. This is exemplified in Fig. 13. Indeed, the main contribution to the sum in Eq. (7.3) arises from those proton channels the isobaric mirror channels of which are closed, as indicated by the shaded area in Fig. 4. On the other hand, all open channels contribute to $\Gamma_2^{\uparrow}$, which therefore grows monotonically with excitation energy. Therefore $\Gamma_{2,\rm ext}^{\downarrow}/\Gamma_2^{\uparrow}$ is maximal at or near the threshold of the (p,n) charge exchange reaction, i.e., of the channel $n+|\,T_1+\frac{1}{2},T_1-\frac{1}{2}\rangle$.

Values of the root-mean-square Coulomb matrix element $(\overline{H_C^2})^{1/2}$ are listed in column nine. These are ob-

TABLE I. Summary of the experiments discussed in Sec. VI. Given are the compound nucleus, its excitation energy E_x, and the isospin T_1 of the class one states; the quantity f, ν, or r [see Eqs. (6.10), (6.12), (6.55)], whichever was primarily determined from experiment, and the isospin-mixing parameter $\Gamma_2^{\downarrow}/\Gamma_2^{\uparrow}$ derived from it; the external mixing $\Gamma_{2,\text{ext}}^{\downarrow}/\Gamma_2^{\uparrow}$ whenever it has been calculated; the average Coulomb matrix element $(\overline{H_C^2})^{1/2}$ and the spreading width $\Gamma_{2,\text{int}}^{\downarrow}$, both deduced from internal isospin mixing; the enhancement factor W_{+-} predicted for charge exchange scattering [see Eq. (6.80)]; the reference to the experiment; and finally the place in the present review where this experiment is discussed. Blanks in the column of references mean that the last reference above is relevant.

Compound nucleus	E_x (MeV)	T_1	f	ν	r	$\Gamma_2^{\downarrow}/\Gamma_2^{\uparrow}$
^{16}O	29.60	0	0.025			0.026
	30.93		0.017			0.017
	39.71		0.008			0.008
	33.39		0.005			0.005
^{18}F	15.84	0	0.29 ± 0.15			$0.41^{+0.38}_{-0.25}$
	16.78		0.10 ± 0.05			0.11 ± 0.06
	21.56		0.06 ± 0.03			0.07 ± 0.04
	26.55		$(9\pm4)\times10^{-4}$			$(9\pm4)\times10^{-4}$
^{22}Na	18.90	0			0.49 ± 0.09	$0.21^{+0.08}_{-0.05}$
^{24}Mg	27.10	0			0.33 ± 0.05	$0.37^{+0.07}_{-0.06}$
^{24}Mg, ^{26}Mg	17.60	0, 1	0.16 ± 0.07			$0.19^{+0.11}_{-0.09}$
^{26}Al	35.70	0			$0.41^{+0.06}_{-0.07}$	$0.25^{+0.07}_{-0.05}$
^{28}Si	29.80	0			0.32 ± 0.06	$0.38^{+0.10}_{-0.07}$
	34	0				0.05
^{28}Si, ^{30}Si	18	0, 1	0.17 ± 0.07			$0.20^{+0.11}_{-0.09}$
^{30}P	20.4	0	0.25 ± 0.05			$0.33^{+0.10}_{-0.08}$
	25.4		0.06 ± 0.03			0.63 ± 0.03
^{32}S, ^{34}S	15.7	0, 1	0.14 ± 0.06			0.17 ± 0.08
^{34}Cl	15.4	0	0.41 ± 0.02			0.69 ± 0.05
^{49}V	20.5	$\frac{3}{2}$		0.20 ± 0.17		$0.25^{+0.35}_{-0.22}$
^{52}Cr	24.2	2		0.37 ± 0.07		$0.59^{+0.29}_{-0.16}$
^{55}Mn	21.8	$\frac{5}{2}$		0.33 ± 0.20		$0.49^{+0.64}_{-0.34}$
^{56}Fe	23.9	2		0.44 ± 0.22		$0.81^{+1.30}_{-0.52}$
^{60}Ni	21.0	2		0.86 ± 0.10		≥ 3.2
	23.3			0.37 ± 0.21		$0.60^{+0.86}_{-0.41}$
^{63}Cu	19.9	$\frac{5}{2}$		0.46 ± 0.21		$0.86^{+1.20}_{-0.52}$
^{64}Zn	17.6	2		0.70 ± 0.15		$2.70^{+5.4}_{-1.4}$
	19.0			0.55 ± 0.16		$1.30^{+1.5}_{-0.64}$
	20.5			0.44 ± 0.11		$0.82^{+0.48}_{-0.32}$
	22.0			0.34 ± 0.15		$0.53^{+0.48}_{-0.29}$
	23.5			0.40 ± 0.12		$0.69^{+0.46}_{-0.29}$
^{66}Zn	22.7	3		0.68 ± 0.10		$2.20^{+1.5}_{-0.80}$
^{69}Ga	17.4	$\frac{7}{2}$		0.41 ± 0.04		0.70 ± 0.12
	18.4			0.45 ± 0.03		0.82 ± 0.10
	20.4			0.39 ± 0.10		$0.64^{+0.33}_{-0.23}$
	22.4			0.34 ± 0.09		$0.52^{+0.24}_{-0.19}$
^{88}Sr	21.6	6		0.73 ± 0.1		$2.7^{+2.2}_{-1.0}$
^{89}Y	21.8	$\frac{11}{2}$		0.63 ± 0.1		$1.7^{+1.0}_{-0.6}$
^{90}Zr	20.4	5		0.62 ± 0.1		$1.6^{+1.0}_{-0.5}$
^{92}Mo	20.5	4		0.48 ± 0.1		$0.92^{+0.46}_{-0.31}$
^{111}In	20.9	$\frac{13}{2}$		0.68 ± 0.14		$2.1^{+2.5}_{-0.8}$

tained as follows. From the values of $\Gamma_2^{\downarrow}/\Gamma_2^{\uparrow}$ listed in column seven we subtract the corresponding entry in column eight whenever it is available. Otherwise, $\Gamma_{2,\text{ext}}^{\downarrow}/\Gamma_2^{\uparrow}$ is neglected. The result is identified with $2\pi\overline{H_C^2}/(D_1\Gamma_2^{\uparrow})$. The parameters D_1 and $\Gamma_2^{\uparrow}$ were either inferred from experimental data or inferred from calculations. They differ strongly for different entries. Details of their estimation may be found in the original articles.

TABLE I. (*Continued.*)

$\Gamma^{\downarrow}_{2,\text{ext}}/\Gamma^{\uparrow}_2$	$(\overline{H_C^2})^{1/2}$ (eV)	$\Gamma^{\downarrow}_{2,\text{int}}$ (keV)	W_{+-}	Ref.	Discussed in Section
				Aoki *et al.* (1979)	VI.A
				Schwenzel *et al.* (1981)	VI.A,VII
				Sokol and Browne (1978)	
				Glasner *et al.* (1983)	VI.E
				Assmann (1981)	IV.E
				Evers (1982)	
	8500±2200			Kuhlman (1979)	VI.B
0.04	1100^{+100}_{-300}	44^{+16}_{-11}	1.75	Simpson *et al.* (1978)	VI.E
				Li and Harney (1982)	
				Assmann (1981)	VI.E
				Evers (1982)	
				Snover (1984)	VI.B
				Harakeh *et al.* (1985)	
				Kuhlmann *et al.* (1983)	VI.C
0.073	1660±640	33±16	1.60	Bizzeti and Bizzeti-Sona (1968)	VI.A
0.079			1.89	Richter *et al.* (1970)	
	3800±800	4.4±2.1		Kuhlman (1979)	VI.B
0.10	3970±160	20.7±1.8	1.77	Spijkervet (1978)	VI.A,VI.F
0.075	66^{+48}_{-66}	$5.5^{+10.7}_{-5.5}$	1.95	Porile *et al.* (1975)	VI.D
0.089	98±18	13.6±5.4	1.92		VI.D
0.095	32^{+19}_{-16}	8.0^{+12}_{-6}	1.97		VI.D
0.095	100^{+67}_{-46}	29^{+53}_{-20}	1.82	Wiley *et al.* (1973)	VI.D
				Van Camp *et al.* (1984)	VI.C
0.099	114^{+76}_{-63}	21^{+38}_{-17}	1.87	Wiley *et al.* (1973)	VI.D
0.10	26^{+15}_{-11}	13^{+21}_{-9}	1.90	Wiley *et al.* (1973)	VI.D
0.10	290^{+220}_{-94}	57^{+117}_{-31}	1.51	Lux and Porile (1975)	VI.D
0.097	160^{+80}_{-48}	42^{+53}_{-21}	1.70		
0.094	100^{+32}_{-24}	42^{+28}_{-17}	1.79		
0.099	70±34	43^{+51}_{-30}	1.84		
0.11	61^{+23}_{-19}	79^{+69}_{-41}	1.81		
0.10	50^{+15}_{-11}	133^{+95}_{-51}	1.83	Wiley *et al.* (1973)	VI.D
0.095	22±2	9.2±1.5	1.97	Porile *et al.* (1974)	VI.D
0.092	18.1±1.3	11.8±1.6	1.97		
0.090	8.2±2.0	$10.1^{+5.5}_{-3.7}$	1.97		
0.10	4.2±1.0	8.9±4.2	1.97		
	10^{+35}_{-2}	54^{+40}_{-20}		Van Camp *et al.* (1984)	VI.C
	3.8±0.8	34^{+20}_{-12}			VI.C
	9.0±2.0	32^{+20}_{-10}			VI.C
	6.4±1.5	18^{+10}_{-6}			VI.C
0.029	$0.18^{+0.08}_{-0.04}$	13.5^{+16}_{-6}	1.93	Lux *et al.* (1977)	VI.D

Blanks in column nine indicate that the available information on D_1 and/or $\Gamma^{\uparrow}_2$ is insufficient to calculate $(\overline{H_C^2})^{1/2}$. The errors given in column nine refer *only* to the experimental uncertainty in $\Gamma^{\downarrow}_2/\Gamma^{\uparrow}_2$ but not to the uncertainties in the theoretical estimation of D_1 and $\Gamma^{\uparrow}_2$. These latter uncertainties are difficult to assess and may be important.

It is obvious that the values of $(\overline{H_C^2})^{1/2}$ span a range of

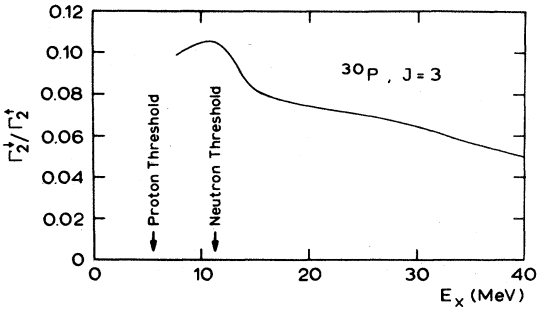

FIG. 13. External isospin-mixing parameter of Eq. (7.3) as a function of the excitation energy of the compound nucleus ^{30}P. The calculation is taken from Harney and Tang (1981). It was done for the states of spin three, which dominate the reaction of Bizzeti and Bizzeti-Sona (1968) cited in Table I. The spin dependence of this function is, however, quite weak.

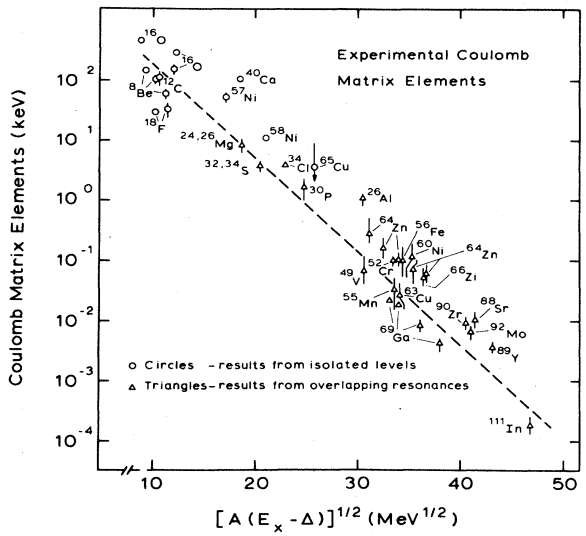

FIG. 14. Coulomb matrix elements as functions of the square root of compound-nucleus mass number and excitation energy. The triangles are data from Table I. The circles are from spectroscopic investigations of isolated states: see Oodthoudt and Garvey (1977) for ^{8}Be; Adelberger *et al.* (1976) for ^{12}C; Rolfs and Rodney (1974), Miska *et al.* (1975), Harney (1977), and Wagner *et al.* (1977) for ^{16}O; Rolfs *et al.* (1973) and Berka *et al.* (1977) for ^{17}F; Gräf *et al.* (1977) for ^{40}Ca; Spangenberger *et al.* (1985) for ^{58}Ni; Atkinson *et al.* (1968) for ^{57}Ni and ^{65}Cu.

many orders of magnitude and do not scatter around a "typical" value. This is a consequence of the fact that $\Gamma_2^\downarrow/\Gamma_2^\uparrow$ lies essentially between zero and one: $(\overline{H_C^2})^{1/2}$ therefore reflects the exponential dependence of the level density D_1^{-1} on excitation energy and mass number. This suggests plotting $(\overline{H_C^2})^{1/2}$ vs $[A(E_x-\Delta)]^{1/2}$ where A is the mass number of the compound nucleus, E_x the excitation energy, and Δ the pairing correction taken[12] from Gilbert and Cameron (1965). Such a plot, originally suggested by Kuhlmann (1979), is shown[13] in Fig. 14, which also includes some Coulomb mixing matrix elements from the spectroscopy of isolated levels (references are given in the figure caption). The dashed line is the fit function proposed by Kuhlmann. It has a slope of $-(7.5\ \text{MeV})^{-1/2}$, and it reproduces the overall behavior of $(\overline{H_C^2})^{1/2}$ very well. Now, if $\overline{H_C^2}$ is roughly proportional to the level spacing D_1, then

$$(\overline{H_C^2})^{1/2} \propto \exp[-(a(E_x-\Delta))^{1/2}] ,$$

where a is the level density parameter and $a \sim A$. The slope of $-(7.5\ \text{MeV})^{-1/2}$ yields $a = A/(7.5\ \text{MeV})$, a reasonable result [see Richter (1974)].

Figure 14 suggests that the characteristic parameter for internal isospin mixing is not the rms Coulomb matrix element $(\overline{H_C^2})^{1/2}$, but rather the spreading width for internal mixing, $\Gamma_{2,\text{int}}^\downarrow = 2\pi\overline{H_C^2}/D_1$, and that $\Gamma_{2,\text{int}}^\downarrow$ changes only slowly (if at all) with mass number and excitation energy. This supports the assumption introduced in Sec. VI.A that $\Gamma_2^\downarrow/\Gamma_2^\uparrow$ is also nearly independent of spin [the escape width $\Gamma_2^\uparrow$ is only weakly spin dependent; see Ernst *et al.* (1969)]. We list $\Gamma_{2,\text{int}}^\downarrow$ in column ten of Table I. It has been calculated from $\Gamma_2^\downarrow/\Gamma_2^\uparrow$, whenever $\Gamma_2^\uparrow$ was available. If $\Gamma_{2,\text{ext}}^\downarrow/\Gamma_2^\uparrow$ was known, we subtracted it from $\Gamma_2^\downarrow/\Gamma_2^\uparrow$; if it was not available, we neglected it. Following a suggestion by Kuhlmann (1985), we plotted the resulting data on $\Gamma_{2,\text{int}}^\downarrow$ in Fig. 15, together with many spreading widths $\Gamma_2^\downarrow$(IAR) of isolated isobaric analog resonances. The data concerning $\Gamma_2^\downarrow$(IAR) and references to the published literature are given in Table II. The quantities $\Gamma_2^\downarrow$ and $\Gamma_2^\downarrow$(IAR) are physically equivalent, as one can see from theories of isobaric analog resonances (e.g., Mahaux and Weidenmüller, 1969). Figure 15 contains data from light to heavy nuclei with extremely different level densities and yet $\Gamma_2^\downarrow$ lies essentially between 5 and 80 keV. This near constancy in turn can be understood semiquantitatively as the manifestation of a sum rule: The expression

$$\sum_\mu |\langle 1\mu | H_C | 2\nu\rangle|^2 = \langle 2\nu | H_C^2 | 2\nu\rangle , \tag{7.4}$$

when averaged over a group $\{\nu\}$ of class two states, is constant. On the other hand,

$$\sum_\mu \overline{|\langle 1\mu | H_C | 2\nu\rangle|^2} \simeq \int \rho_1(\varepsilon) d\varepsilon \cdot \overline{H_C^2}$$
$$\simeq (\overline{H_C^2}/D_1)\cdot L , \tag{7.5}$$

[12]For the mass numbers $A = 8$, 12, 16 the value $\Delta = 5.13$ MeV suggested by Kuhlmann (1979) was used.

[13]Figure 14 differs from Fig. 2 of Kuhlmann (1979) because he used another set of pairing corrections. The values from Gilbert and Cameron (1965) have a better justification, although they lead to a wider scatter of the results.

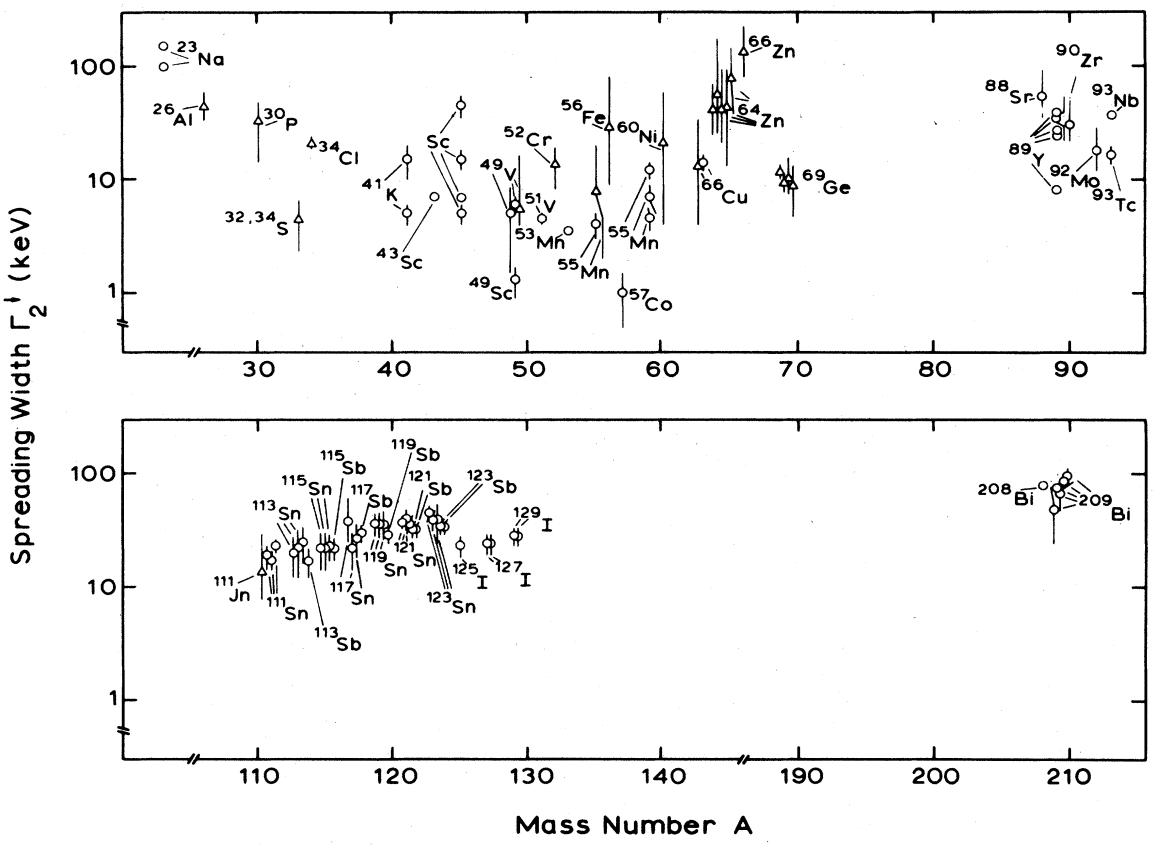

FIG. 15. Spreading widths $\Gamma_2^{\downarrow}$ plotted against the mass number A of the compound nucleus. Triangles represent $\Gamma_{2,\text{int}}^{\downarrow}$ from statistical reactions (see Table I), and circles represent $\Gamma_2^{\downarrow}$(IAR) from isobaric analog resonances (see Table II). Points without error bars do not imply that the error is small, but rather that this information is missing in the original publication.

where L is the length of the energy interval over which coupling between class one states and class two states occurs. We see that the main energy and mass-number dependence of $\overline{H_C^2}$ arises from the exponential dependence of D_1.

Table I also contains evidence that isospin mixing (or $\Gamma_2^{\downarrow}/\Gamma_2^{\uparrow}$) decreases with increasing excitation energy E_x. (Qualitatively, this would be expected if $\Gamma_2^{\downarrow}$ were indeed constant, since $\Gamma_2^{\uparrow}$ increases strongly with increasing E_x.) Figure 16 further illustrates this statement and the limitations of the data presently available, for the case of the compound nucleus ^{18}F*. We reproduce the excitation functions by Schwenzel *et al.* (1981) and by Sokol and Browne (1978) of the isospin-allowed reactions ^{12}C(^{6}Li,α) populating the ground and second excited states of ^{14}N (upper part of the figure). The cross sections for both transitions are about equal, and are reasonably well reproduced by Hauser-Feshbach calculations (Kuhlmann, 1984) (uppermost solid curve). The lower solid curve is the prediction of Hauser-Feshbach theory with full breaking of isospin symmetry for the transition to the first excited state in ^{14}N. This transition is isospin forbidden, and the data accordingly lie below the Hauser-Feshbach prediction. The distance from the data to the Hauser-Feshbach curve is an indicator of isospin conservation: The larger the distance, the better isospin is conserved. The dashed line merely summarizes the trend of the data. There is a gap that widens with excitation energy, especially at the data point with highest excitation energy. A similarly strong effect was observed by Richter *et al.* (1970) in the compound nucleus ^{30}P. A quantitative analysis of this behavior in terms of the expected dependence of $\Gamma_2^{\uparrow}$ has not been performed, however.

VIII. CONCLUSIONS

In this review we have compared the statistical theory of isospin breaking in statistical compound-nucleus reactions developed in Secs. IV and V, and in Appendix A, with the available body of data. As emphasized in the In-

TABLE II. Spreading widths $\Gamma_2^\downarrow$(IAR) from isobaric analog resonances. Given are nucleus and excitation energy E_x where the resonance is found; the isospin T_1 of the class one states (we note that the isospin of the isobaric resonance is T_1+1); spin and parity of the resonance; the nuclear reaction in which it was observed; the spreading width $\Gamma_2^\downarrow$(IAR); and the reference to the publication. Blanks in this column mean that the last reference above is relevant.

Nucleus	E_x (MeV)	T_1	Spin parity	Reaction	$\Gamma_2^\downarrow$(IAR) (keV)	Reference
^{23}Na	10.94	$\frac{1}{2}$	$\frac{3}{2}^-$	^{22}Ne(p,p); E_p=2.15 MeV	150	Keyworth *et al.* (1968)
	11.37		$\frac{3}{2}^+$	=2.58 MeV	100	
^{41}K	9.68	$\frac{3}{2}$	$\frac{3}{2}^-$	^{40}Ar(p,p); E_p=1.87 MeV	15±5	Bilpuch *et al.* (1976)
	10.26		$\frac{1}{2}^+$	=2.45 MeV	5±1	
^{43}Sc	6.75	$\frac{1}{2}$	$\frac{1}{2}^-$	^{42}Ca(p,p); E_p=1.82 MeV	7	Browne *et al.* (1968)
^{45}Sc	8.54	$\frac{3}{2}$	$\frac{3}{2}^-$	^{44}Ca(p,p); E_p=1.65 MeV	6.9	Bilpuch *et al.* (1976)
	8.93		$\frac{1}{2}^-$	=2.04 MeV	45±10	
	9.13		$\frac{1}{2}^+$	=2.24 MeV	5±1	
	9.52		$\frac{3}{2}^-$	=2.63 MeV	15±3	
^{49}Sc	11.59	$\frac{7}{2}$	$\frac{3}{2}^-$	^{48}Ca(p,p); E_p=1.97 MeV	1.3±0.4	Wilhjelm *et al.* (1969)
^{49}V	9.61	$\frac{3}{2}$	$\frac{1}{2}^-$	^{48}Ti(p,p); E_p=2.86 MeV	6±1	Bilpuch *et al.* (1976)
	9.71		$\frac{3}{2}^-$	=2.95 MeV	5±3.5	
^{51}V	10.60	$\frac{5}{2}$	$\frac{1}{2}^-$	^{50}Ti(p,p); E_p=2.55 MeV	4.5±0.5	
^{53}Mn	9.33	$\frac{3}{2}$	$\frac{3}{2}^-$	^{52}Cr(p,p); E_p=2.77 MeV	3.5±0.5	
^{55}Mn	10.68	$\frac{5}{2}$	$\frac{3}{2}^-$	^{54}Cr(p,p); E_p=2.61 MeV	4±1	
^{57}Co	8.56	$\frac{3}{2}$	$\frac{1}{2}^-$	^{56}Fe(p,p); E_p=2.53 MeV	1.0±0.5	
^{59}Co	9.59	$\frac{5}{2}$	$\frac{3}{2}^-$	^{58}Fe(p,p); E_p=2.22 MeV	7±2	
	9.89		$\frac{1}{2}^-$	=2.52 MeV	12±2	
	10.27		$\frac{3}{2}^-$	=2.98 MeV	4.5±1	
^{63}Cu	9.17	$\frac{5}{2}$	$\frac{3}{2}^-$	^{62}Ni(p,p); E_p=3.05 MeV	14±2	
^{89}Y	12.12	$\frac{11}{2}$	$\frac{5}{2}^+$	^{88}Sr(p,p); E_p=5.06 MeV	8	Cosman *et al.* (1966)
	13.13		$\frac{1}{2}^+$	=6.02 MeV	24	Cosman *et al.* (1966), Genz *et al.* (1975)
	14.06		$\frac{5}{2}^+$	=6.99 MeV	39	Cosman *et al.* (1966)
	14.13		$\frac{3}{2}^+$	=7.06 MeV	27	
^{93}Nb	11.99	$\frac{11}{2}$	$\frac{3}{2}^+$	^{92}Zr(p,p); E_p=5.95 MeV	37±3	Robson *et al.* (1965)
^{93}Tc	9.40	$\frac{7}{2}$	$\frac{1}{2}^+$	^{92}Mo(p,p); E_p=5.30 MeV	16.5±3	Bilpuch *et al.* (1976)
^{111}Sn	10.507	$\frac{11}{2}$	$\frac{9}{2}^+$	^{112}Sn(p,d)	17±3	Taketani *et al.* (1980)
	11.076		$\frac{1}{2}^-$		19±4	
	11.339		$\frac{3}{2}^-$		23±3	
^{113}Sn	11.826	$\frac{13}{2}$	$\frac{9}{2}^+$	^{114}Sn(p,d)	22±10	
	12.254		$\frac{1}{2}^-$		20±8	
	12.513		$\frac{3}{2}^-$		25±9	
^{115}Sn	13.317	$\frac{15}{2}$	$\frac{9}{2}^+$	^{116}Sn(p,d)	22±8	
	13.694		$\frac{1}{2}^-$		22±8	
	13.948		$\frac{3}{2}^-$		23±6	
^{117}Sn	14.151	$\frac{17}{2}$	$\frac{9}{2}^+$	^{118}Sn(p,d)	22±8	
	14.496		$\frac{1}{2}^-$		38±13	
	14.773		$\frac{3}{2}^-$		27±9	
^{119}Sn	14.995	$\frac{19}{2}$	$\frac{9}{2}^+$	^{120}Sn(p,d)	36±9	
	15.329		$\frac{1}{2}^-$		36±10	
	15.622		$\frac{3}{2}^-$		36±8	
^{121}Sn	15.953	$\frac{21}{2}$	$\frac{9}{2}^+$	^{122}Sn(p,d)	40±8	
	16.304		$\frac{1}{2}^-$		36±7	
	16.610		$\frac{3}{2}^-$		37±4	

TABLE II. (*Continued.*)

Nucleus	E_x (MeV)	T_1	Spin parity	Reaction	$\Gamma_2^{\downarrow}$(IAR) (keV)	Reference
^{123}Sn	16.943	$\frac{23}{2}$	$\frac{9}{2}^+$	^{124}Sn(p,d)	39±8	
	17.306		$\frac{1}{2}^-$		45±7	
	17.656		$\frac{3}{2}^-$		39±15	
^{113}Sb	13.629	$\frac{11}{2}$	$\frac{1}{2}^+$	^{112,114}Sn(^{3}He,t)	17±5	Becchetti *et al.* (1976)
^{115}Sb	13.593	$\frac{13}{2}$	$\frac{1}{2}^+$	^{114,116}Sn(^{3}He,t)	22±5	
^{117}Sb	13.473	$\frac{15}{2}$	$\frac{1}{2}^+$	^{117}Sn(^{3}He,t)	30±5	
^{119}Sb	13.670	$\frac{17}{2}$	$\frac{1}{2}^+$	^{119}Sn(^{3}He,t)	29±5	
^{121}Sb	13.720	$\frac{19}{2}$	$\frac{3}{2}^+$	^{120,122}Sn(^{3}He,t)	32±5	
	13.780		$\frac{1}{2}^+$		32±5	
^{123}Sb	13.779	$\frac{21}{2}$	$\frac{3}{2}^+$	^{122,124}Sn(^{3}He,t)	34±5	
	13.905		$\frac{1}{2}^+$		34±5	
^{125}I	14.050	$\frac{19}{2}$	$\frac{1}{2}^+$	^{125}Te(^{3}He,t)	23±5	
^{127}I	14.079	$\frac{21}{2}$	$\frac{3}{2}^+$	^{126,128}Te(^{3}He,t)	24±5	
	14.140		$\frac{1}{2}^+$		24±5	
^{129}I	14.105	$\frac{23}{2}$	$\frac{3}{2}^+$	^{128,130}Te(^{3}He,t)	28±5	
	14.285		$\frac{1}{2}^+$		28±5	
^{208}Bi	15.21	21	0^+	^{207}Pb(p,p); E_p=11.49 MeV	78±8	Lenz and Temmer (1968), Melzer *et al.* (1985)
^{209}Bi	18.63	$\frac{43}{2}$	$\frac{9}{2}^+$	^{208}Pb(p,p); E_p=14.83 MeV	75±5	Melzer *et al.* (1985)
	19.19		$\frac{11}{2}^+$	=15.38 MeV	66±19	
	20.10		$\frac{15}{2}^-$	=16.30 MeV	48±25	
	20.19		$\frac{5}{2}^+$	=16.39 MeV	86±9	
	20.68		$\frac{1}{2}^+$	=16.87 MeV	96±16	

troduction, this theory is based on our present understanding of chaotic motion in quantum systems in the presence of a partially conserved quantum number. The theory is accordingly footed on a set of statistical assumptions. The degree of isospin symmetry breaking is characterized by a single parameter z. This theory is, to the best of our knowledge, the only one presently available capable of accounting for a large number of different experimental observables—cross sections, correlation functions, reactions populating mirror channels, etc.—in terms of this single parameter.

From the theoretical point of view, it is of course very interesting to query whether this theory is *consistent* with observation. This question is all the more important since the field of isospin symmetry breaking seems to provide the most stringent testing ground for any statistical theory that goes beyond the pure compound-nucleus reaction. [Due to its greater complexity, the field of precompound or nonequilibrium reactions, for which a similar transport theory has been formulated by Agassi *et al.* (1975), appears to be less suitable.] While all the data discussed in Secs. VI and VII can be understood in the framework of the theory, a stringent test like the one proposed in Sec. VI.G has not as yet been performed, although the analysis of correlation lengths in Sec. VI.F goes a long way in this direction. Such a stringent test would consist in the comparison of an experiment with a theoretical prediction based on the present theory and input data obtained solely from *other* observables. The column in Table I containing the W_{+-} values offers such a possibility.

The mixing parameter $\Gamma_2^{\downarrow}/\Gamma_2^{\uparrow}$ incorporates both the static and the dynamic criterion for isospin mixing and consists of two contributions. These are due to external and internal mixing, respectively. Whenever isospin mixing is important overall, external mixing is comparatively small, and the knowledge of $\Gamma_2^{\downarrow}/\Gamma_2^{\uparrow}$ allows the determination of the internal mixing parameters $(\overline{H_C^2})^{1/2}$ or $\Gamma_{2,\text{int}}^{\downarrow}$. The rms Coulomb matrix element $(\overline{H_C^2})^{1/2}$ has been found to change drastically with excitation energy and mass number, and does not appear to be a suitable measure for isospin mixing. A better measure by far is provided by the internal spreading width $\Gamma_{2,\text{int}}^{\downarrow}$. This quantity appears to be fairly independent of excitation energy and mass number. We have given qualitative arguments why this should be so. These arguments also lead us to expect that isospin symmetry breaking decreases with increasing excitation energy, owing to the growth of $\Gamma_2^{\uparrow}$ with energy. The available data seem to support this expectation.

The internal spreading widths $\Gamma_{2,\text{int}}^{\downarrow}$ derived from statistical reactions are consistent with the spreading widths found in isolated isobaric analog resonances, where again

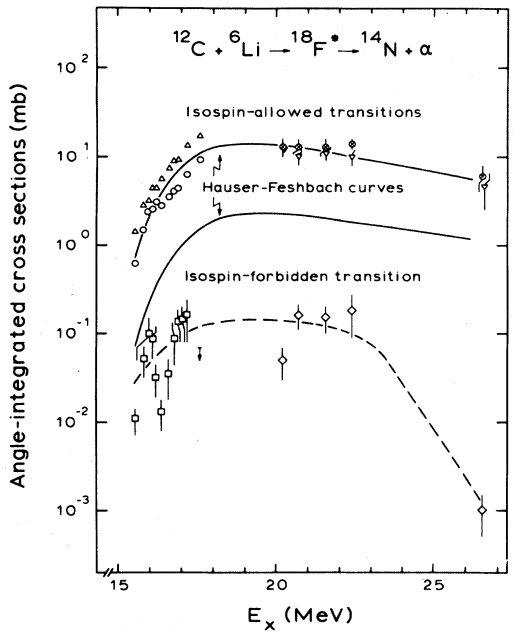

FIG. 16. Comparison of isospin-allowed and isospin-forbidden $^{12}C(^{6}Li,\alpha)^{14}N$ reactions. Circles stand for the allowed reaction populating the $T=0$ ground state of ^{14}N and triangles are for the allowed reaction leading to the $T=0$ state at 3.95 MeV. Squares indicate the forbidden reaction that leads to the $T=1$ state at 2.31 MeV. Circles with crosses are from Schwenzel *et al.* (1981), and open symbols from Sokol and Browne (1978). There is a common Hauser-Feshbach curve for the allowed reactions, since their cross sections are very similar. The lower solid curve is the prediction of the isospin-independent Hauser-Feshbach theory for the forbidden transition (maximum isospin symmetry violation). The dashed curve is to guide the eye.

$\Gamma^{\downarrow}_{2,\mathrm{int}}$ is fairly constant over a wide range of excitation energy and mass number. The detailed understanding of this behavior of $\Gamma^{\downarrow}_{2,\mathrm{int}}$ and of its mean value is a challenge for nuclear structure theory.

ACKNOWLEDGMENTS

In the course of this work we have benefited from helpful discussions with many colleagues. In particular, we are grateful to the following persons for providing us with relevant data (partly unpublished): W. Assmann, E. Van Camp, D. Evers, E. Kuhlmann, S. Skorka, K. A. Snover, and A. L. Spijkervet. A.R. was partly supported by a Grant from the Bundesministerium für Forschung und Technologie.

APPENDIX A: TECHNICAL DETAILS OF THE THEORY OF ISOSPIN MIXING IN THE COMPOUND NUCLEUS

In this appendix we outline the derivation of Eq. (4.18), since it cannot be found it the published literature.

To avoid a cumbersome notation with many indices, we write the matrix t of Eq. (4.11) in the form

$$t=\pi V^{+}D^{-1}V\ , \tag{A1}$$

where V is the rectangular matrix with elements

$$V_{m\mu,at}=V^{at}_{m\mu} \tag{A2}$$

and V^{+} is its transpose. The level matrix D of Eq. (4.12) is

$$D=E-H-H_C+i\pi VV^{+}\ , \tag{A3}$$

where H denotes the diagonal matrix of the compound-nucleus level energies $E_{m\mu}$, and H_C is the Coulomb energy matrix with elements $\langle m\mu \mid H_C \mid n\nu\rangle$ breaking isospin. We rewrite Eq. (4.4) in matrix form,

$$V=\tilde{V}P^{1/2}\ , \tag{A4}$$

where the notation is obvious. We introduce the matrix C of an orthogonal transformation from the physical channels (at) to the isospin channels (aT) with elements

$$C_{at,bT}=\delta_{ab}\langle at \mid aT\rangle \tag{A5}$$

[see Eq. (2.2)]. We calculate the first and second moments of the matrix

$$t'=C^{+}P^{-1/2}tP^{-1/2}C=\pi C^{+}\tilde{V}^{+}D^{-1}\tilde{V}C \tag{A6}$$

and transform back afterward. Defining the rectangular matrix γ with

$$\gamma_{m\mu,aT}=\sqrt{\pi}(VC)_{m\mu,aT}=\sqrt{\pi}\delta_{TTm}\tilde{V}^{a}_{m\mu} \tag{A7}$$

[see Eqs. (4.5) and (4.6)], we have for t'

$$t'=\gamma^{+}D^{-1}\gamma\ . \tag{A8}$$

The matrix D can be written

$$D=E-H-H_C+i\gamma y\gamma^{+}\ , \tag{A9}$$

where

$$y=C^{+}PC\ . \tag{A10}$$

The structure of t' is similar to that of Eqs. (3.9) and (3.10) of Agassi *et al.* (1975), except that the present matrix y is not diagonal. This is the reason why one cannot immediately use their results. [We note that y becomes diagonal if $P_{a,1/2}=P_{a,-1/2}$; see the discussion following Eq. (4.16).] We can, however, calculate the quantities of interest by application of the ideas of the contraction technique used by Agassi *et al.* (1975). Since a generalized version of this technique has recently been described by Müller and Harney (1985) in detail, we shall often refer to the latter work.

We note that inclusion of the shift functions defined by Weidenmüller *et al.* (1978) results in a complex matrix y. The subsequent formalism is entirely suited for treating this case, too. For notational simplicity we restrict ourselves to a real matrix y.

The decay amplitudes γ are assumed to be Gaussian

random variables with properties described in Sec. IV. We still have to specify the statistics of the energies $E_{m\mu}$ and use the "disordered picket fence" introduced by Müller and Harney (1985): The average values $\varepsilon_{m\mu}$ of $E_{m\mu}$ form a regular lattice,

$$\varepsilon_{m\mu}-\varepsilon_{m\nu}=(\mu-\nu)D_m \ , \tag{A11}$$

with level distance D_m, the $E_{m\mu}$ are statistically independent of each other, and their distribution is given by $p_m(\varepsilon_{m\mu}-E_{m\mu})$. The function p_m is even and has a maximum at zero. The experimentally observed rigidity of nuclear spectra requires that the variance σ_m of $E_{m\mu}$ be of the order of the mean level distance D_m, as discussed by Müller and Harney (1985). We shall see that this is the only information needed about p_m. The approximations

$$\sum_\mu p_m(\varepsilon_{m\mu}-E)\simeq\frac{1}{D_m}\int p_m(\varepsilon-E)dE=1/D_m \tag{A12}$$

and

$$\begin{aligned}\sum_\mu p_m(\varepsilon_{m\mu}-E_1)p_m(\varepsilon_{m\mu}-E_2)&\\ \simeq\frac{1}{D_m}\int p_m(\varepsilon-E_1)p_m(\varepsilon-E_2)d\varepsilon&\\ =g_m(E_1-E_2)&\end{aligned} \tag{A13}$$

will be used. The distribution g_m has the variance $\sqrt{2}\,\sigma_m$; the integral over $g_m(z)$ is equal to D_m.

Let us first calculate the *average S matrix*. This and the subsequent calculation of the correlation functions are done by expanding S into a geometrical series, averaging term by term, and resumming afterward to obtain a closed expression. The propagator

$$b=i(H-E)^{-1} \tag{A14}$$

is introduced, and D is written in the form

$$iD=b^{-1}(1+ibH_C-b\gamma y\gamma^+) \ . \tag{A15}$$

In order to avoid the singularities of b, one goes to the complex energy

$$E^+=E+i\eta \tag{A16}$$

and takes the limit $\eta\to 0$, after the ensemble average with respect to all statistical variables is carried out. The limit will be trivial, since the statistical assumptions have been chosen such that the problem is stationary, i.e., the average is independent of E^+. One finds the expansion

$$\overline{-it'}=\sum_{q\geq 0}\overline{\gamma^+[b(\gamma y\gamma^+-iH_C)]^q b\gamma} \ . \tag{A17}$$

Here, the overbar denotes the ensemble average with respect to all statistical variables. It is assumed that the problem is ergodic, i.e., that the ensemble average is equal to the desired energy average. We calculate the simplest term of Eq. (A17) with $q=0$ and obtain

$$\begin{aligned}\overline{(\gamma^+b\gamma)}_{aT,bT'}&=i\sum_{m\mu}\overline{\gamma_{m\mu,aT}\gamma_{m\mu,bT'}b_{m\mu}}\\ &=i\delta_{ab}\sum_m\overline{(\gamma_{m\mu,aT})^2}\delta_{TT_m}\delta_{T'T_m}\sum_\mu\overline{b_{m\mu}} \ ,\end{aligned} \tag{A18}$$

where Eq. (A7) was used. Note that, due to Eq. (4.7), the second moment of $\gamma_{m\mu,aT}$ is independent of μ. Because of Eq. (A12), the sum over the $\bar{b}$ is

$$\begin{aligned}\sum_\mu\overline{b_{m\mu}}&=\sum_\mu i\int(E'-E^+)^{-1}p_m(\varepsilon_{m\mu}-E')dE'\\ &=iD_m^{-1}\int(E'-E^+)^{-1}dE'=-\pi/D_m \ ,\end{aligned} \tag{A19}$$

where we assume that E is sufficiently close to the center of the lattice $\varepsilon_{m\mu}$ to make logarithmic terms that arise from edge effects negligible. One defines a matrix x, which is diagonal in the channels (aT) and has the elements

$$x_T^a=\pi\overline{(\gamma_{m\mu,aT})^2}/D_m \quad \text{with } T_m=T \ . \tag{A20}$$

This is the same as the quantity of Eq. (4.22). For simplicity, the tildes are suppressed here. One obtains the average

$$\overline{(\gamma^+b\gamma)}_{aT,bT'}=\delta_{ab}\delta_{TT'}x_T^a \ . \tag{A21}$$

With the arguments used in Sec. III of Müller and Harney (1985) one shows that all terms of Eq. (A17) containing factors H_C vanish and the remaining terms factorize:

$$\overline{\gamma^+b[\gamma y\gamma^+]^q b\gamma}=\overline{\gamma^+b\gamma}\cdot\overline{[y\gamma^+b\gamma]^q} \ . \tag{A22}$$

Hence one obtains

$$\overline{it'}=-\overline{\gamma^+b\gamma}\sum_{q\geq 0}\overline{[y\gamma^+b\gamma]^q}=x(1+yx)^{-1} \tag{A23}$$

and inverts the transformation (A6) to find the average S matrix

$$\begin{aligned}\bar{S}&=\Omega[1-2i\bar{t}]\Omega\\ &=\Omega[1-2P^{1/2}Cx(1+yx)^{-1}C^+P^{1/2}]\Omega \ .\end{aligned} \tag{A24}$$

The matrix $\bar{S}_{at,bt'}$ is diagonal in a and b, since C and therefore also y have this property; it therefore decomposes into blocks of 2×2 matrices for the channels a with an emitted particle of isospin $\frac{1}{2}$ and into "blocks" of dimension 1 for the channels a with an ejectile of isospin 0. The transmission matrix has these properties, too. It can be expressed as

$$\begin{aligned}1-\bar{S}\bar{S}^+&=4\Omega P^{1/2}C(1+xy)^{-1}\\ &\quad\times x^{1/2}[\Omega P^{1/2}C(1+xy)x^{1/2}]^+ \ .\end{aligned} \tag{A25}$$

If one inserts the explicit form of C for the 2×2 blocks,

$$\begin{aligned}C&=(C_{at,bT})\\ &=\delta_{ab}(2T_1+2)^{-1/2}\begin{bmatrix}1 & (2T_1+1)^{1/2}\\ -(2T_1+1)^{1/2} & 1\end{bmatrix}\end{aligned} \tag{A26}$$

for $t=+\frac{1}{2},-\frac{1}{2}$ and $T=T_1,T_1+1$ (in this order), then one obtains by a lengthy but straightforward calculation the elements of the transmission matrix,

$$(1-\bar{S}\,\bar{S}^{+})_{at,bt'}=\delta_{ab}\exp[i(\varphi_{at}-\varphi_{at'})]\sum_{m}\tau_{am}^{tt'}\ , \quad \text{(A27)}$$

with the coefficients τ defined by Eqs. (4.20)—(4.23).

We turn now to the *calculation of the correlation functions* of Eq. (4.18). Again one expresses the elements of S by the elements of t' [see Eq. (A6)] and inverts the transformations after averaging. Hence one has to calculate

$$\overline{t'^{\,\mathrm{fl}}_{aT,bT'}(E)t'^{\,\mathrm{fl}*}_{cT'',dT'''}(E+\varepsilon)} = \overline{t'_{aT,bT'}(E)\vdots t'^{*}_{cT'',dT'''}(E+\varepsilon)}-\overline{t'_{aT,bT'}}\;\overline{t'^{*}_{cT'',dT'''}}\ . \quad \text{(A28)}$$

The technique described by Agassi *et al.* (1975) and by Müller and Harney (1985) is used: The t-matrix elements are expressed by their expansions (A17); both terms on the right-hand side of Eq. (A28) are multiple sums over channels and levels. The sums are averaged term by term and subsequently resummed to obtain Eq. (4.18). This is done under the assumption that the effective numbers N_1 and N_2 of open channels [see Eq. (2.16) and note that we write the index m for T_m] are large compared to unity. Hence Eq. (4.18) is the leading first-order term of an expansion in powers of $1/N_m$, $m=1,2$. Physically speaking, it is assumed that in both classes of levels the resonances strongly overlap—as pointed out in the Introduction.

Keeping track of the terms to be averaged is greatly simplified by the following observation. In the first term on the right-hand side of Eq. (A28), a dotted line has been inserted between the elements of t' and t'^{*}, called the center line. Imagine it in every term of the multiple sum. Note that, whenever the parameters occurring to the left of the center line are statistically independent from the parameters to the right, the term is canceled by the appropriate term within the second expression on the right-hand side of Eq. (A28). This independence is given whenever all level indices that occur to the left of the center line are different from the ones occurring to the right. The contrary is true if at least one factor b to the left of the center line [see expansion (A17)] carries the same index as at least one factor b to the right. The two parts of Eq. (A28) then do not cancel each other. The simplest term in which this arises is

$$\overline{\gamma_{m\mu,aT}b_{m\mu}\gamma_{m\mu,bT'}\vdots\gamma_{m\mu,cT'''}b^{*}_{m\mu}\gamma_{m\mu,dT''}} - \overline{\gamma_{m\mu,aT}b_{m\mu}\gamma_{m\mu,bT'}}\;\overline{\gamma_{m\mu,cT'''}b^{*}_{m\mu}\gamma_{m\mu,dT''}}\ .$$

We call this expression, when summed over m and μ, a contraction and introduce the notation

$$\overline{(\gamma^{+}b\gamma)_{AB}\vdots(\gamma^{+}b^{*}\gamma)_{CD}}-\overline{(\gamma^{+}b\gamma)_{AB}}\,\overline{(\gamma^{+}b^{*}\gamma)_{CD}} = (\gamma^{+}\underbrace{b\gamma)_{AB}\vdots(\gamma^{+}b^{*}}\gamma)_{CD}\ , \quad \text{(A29)}$$

where we have simplified the channel indices via $A=(aT)$, $B=(bT')$, $C=(cT'')$, $D=(dT''')$. In these expressions, the energy arguments that appear in Eq. (A28) have been suppressed. One must keep in mind that b is taken at E^{+} and b^{*} at $(E^{+}+\varepsilon)^{*}$.

The task is now to sum all contractions that cross the center line. This is done in three steps, as described by Müller and Harney (1985); details are discussed here only insofar as they are specific to the problem at hand.

(i) The contraction pattern of Eq. (A29), as well as any other one, may carry statistically independent *end factors* leading, for example, to

$$\sum_{q\geq 0}(\overline{[\gamma^{+}b\gamma\gamma}\gamma]^{q}\gamma^{+}\underbrace{b\gamma)_{AB}\vdots(\gamma^{+}b^{*}}\gamma)_{CD} = ([1+xy]^{-1}\gamma^{+}\underbrace{b\gamma)_{AB}\vdots(\gamma^{+}b^{*}}\gamma)_{CD}\ , \quad \text{(A30)}$$

by using the results obtained when averaging t'. One defines the matrix

$$\hat{\gamma}=\gamma(1+yx)^{-1} \quad \text{(A31)}$$

and the sum of the simple contractions (A29) with all possible end factors becomes

$$(\hat{\gamma}^{+}\underbrace{b\hat{\gamma})_{AB}\vdots(\hat{\gamma}^{+}b^{*}}\hat{\gamma})_{CD}\ .$$

From the statistical properties of the γ's one derives the second moment of $\hat{\gamma}$,

$$\pi D_m^{-1}\overline{\hat{\gamma}_{m\mu,aT}\hat{\gamma}_{m\mu,bT'}} = \delta_{ab}x_m^{a}(1+yx)^{-1}_{aT_m,aT}(1+yx)^{-1}_{aT_m,aT'}\ , \quad \text{(A32)}$$

and the fourth moment

$$\overline{\hat{\gamma}_{m\mu,aT}\hat{\gamma}_{m\mu,aT'''}\hat{\gamma}_{m\mu,bT'}\hat{\gamma}_{m\mu,bT''}} = (1+2\delta_{ab})\overline{\hat{\gamma}_{m\mu,aT}\hat{\gamma}_{m\mu,aT'''}}\;\overline{\hat{\gamma}_{m\mu,bT'}\hat{\gamma}_{m\mu,bT''}}\ . \quad \text{(A33)}$$

In the last equation, the term with $a=b$ is enhanced by a factor of 3 over the term with $a\neq b$ because we require Gaussian statistics for the γ's.

(ii) The next step is the summation of all *contractions with branchings* [see Sec. IV C of Müller and Harney (1985)]. The simplest term contains one "branch," i.e., two connected b's on the left-hand side with any number of independent factors bracketed by the branch:

$$\sum_{q\geq 0}(\gamma^{+}\underbrace{b(\gamma y\gamma^{+}-iH_C)[b(\gamma y\gamma^{+}-iH_C)]^{q}b\gamma)_{AB}\vdots(\gamma^{+}b^{*}}\gamma)_{CD}\ .$$

It turns out that the matrix elements of H_C contribute only to the term with $q=1$. If $q=0$, only the diagonal elements of H_C are required, and these vanish (see Sec. IV.A). If $q>1$, the average removes all H_C because the b's in the square brackets are independent of each other and of the contracted b's. As a result, one finds Eq. (A34) to be equal to

$$(\gamma^+ b\tfrac{1}{2}\Gamma b\gamma)_{AB}\vdots(\gamma^+ b^*\gamma)_{CD} ,$$

where we have introduced the diagonal level matrix Γ with elements

$$\Gamma_{m\mu}=2[\gamma y(1+x)^{-1}\gamma^+]_{m\mu,m\mu}+2\pi\overline{H_C^2}/D_{3-m} . \tag{A34}$$

The sum over all branches is found as

$$\sum_{q,r\geq 0}(\gamma^+ b[\tfrac{1}{2}\Gamma b]^q\gamma)_{AB}\vdots(\gamma^+ b[\tfrac{1}{2}\Gamma b^*]^r\gamma)_{CD} =(\gamma^+\tilde{b}\gamma)_{AB}\vdots(\gamma^+\tilde{b}^*\gamma)_{CD} , \tag{A35}$$

where the "dressed propagator"

$$\tilde{b}=b(1-\tfrac{1}{2}\Gamma b)^{-1} \tag{A36}$$

with the elements

$$\tilde{b}_{m\mu}=i(E_{m\mu}-E^+-i\Gamma_{m\mu})^{-1} \tag{A37}$$

has been introduced. Since $\Gamma_{m\mu}$ involves a sum over all open channels, the number of which we assume to be large, we replace $\Gamma_{m\mu}$ by its average,

$$\Gamma_m=\overline{\Gamma_{m\mu}} . \tag{A38}$$

The dressed propagators $\tilde{b}$ are then statistically independent of the γ's very much as are the propagators b. Evaluation of Eq. (A35) yields zero unless the indices a,b,c,d coincide pairwise. For the case $a=d$, $b=c$, one finds

$$\begin{aligned}(\hat{\gamma}^+\tilde{b}\hat{\gamma})_{aT,bT'}\vdots(\hat{\gamma}^+\tilde{b}^*\gamma)_{bT'',aT'''}&=2\pi\sum_m\overline{\hat{\gamma}_{m,aT}\hat{\gamma}_{m,aT'''}\hat{\gamma}_{m,bT'}\hat{\gamma}_{m,bT''}}D_m^{-1}(\Gamma_m+i\varepsilon)^{-1}\\&\quad-\sum_m\overline{\hat{\gamma}_{m,aT}\hat{\gamma}_{m,bT'}}\,\overline{\hat{\gamma}_{m,bT''}\hat{\gamma}_{m,aT'''}}2\pi i\int g_m(z)(z-\varepsilon+i\Gamma_m)^{-1}dz\\&\simeq(1+\delta_{ab})\overline{\hat{\gamma}_{m,aT}\hat{\gamma}_{m,aT'''}}2\pi D_m^{-1}(\Gamma_m+i\varepsilon)^{-1}\overline{\hat{\gamma}_{m,bT'}\hat{\gamma}_{m,bT''}} .\end{aligned} \tag{A39}$$

The subscript μ at the quantity $\hat{\gamma}$ has been suppressed, since the averages do not depend on it. The last part of this equation is obtained with the help of the statistical properties (A32) and (A33) of the $\hat{\gamma}$ and from the fact that the present theory applies to the regime of overlapping levels, characterized by $\Gamma_m\gg D_m$. Therefore the width of the function g_m is small compared to D_m [see Eq. (A13) and the discussion there].

The case with $a=c,b=d$ is covered by Eq. (A39) because of the symmetry of $(\hat{\gamma}^+\tilde{b}\hat{\gamma})_{AB}$ with respect to A and B. The contraction (A35) yields zero for $a=b,c=d$ [see Sec. VI of Müller and Harney (1985)]. These statements hold also for the subsequent multiple contractions.

(iii) The last step of the contraction procedure is to sum over all *multiple contractions* [see Sec. IV D of Müller and Harney (1985)]. The simplest term is

$$(\hat{\gamma}^+\tilde{b}(\gamma y\gamma^+-iH_C)\tilde{b}\hat{\gamma})_{AB}\vdots(\hat{\gamma}^+\tilde{b}^*(\gamma y\gamma^++iH_C)\tilde{b}^*\hat{\gamma})_{CD} .$$

It differs from any of the branched contractions in that the common summation index of the first and the last b is independent of the common summation index of the second and the third b. We hope by now to have given enough explicit details of the technique so that the interested reader can verify that the sum over all double contractions for $a=d,b=c$ is

$$\sum_{q,r\geq 0}(\hat{\gamma}^+\tilde{b}(\gamma y\gamma^+-iH_C)[b\gamma y\gamma^+]^q b\hat{\gamma})_{aT,bT'}(\hat{\gamma}^+\tilde{b}^*(\gamma y\gamma^++iH_C)[b\gamma y\gamma^+]^r\tilde{b}^*\hat{\gamma})_{bT'',aT'''} =2\pi\sum_{m,n}\overline{\hat{\gamma}_{m,aT}\hat{\gamma}_{m,aT'''}}\mathbf{A}_{mn}D_m^{-1}(\Gamma_m+i\varepsilon)^{-1}\overline{\hat{\gamma}_{n,bT'}\gamma_{n,bT''}} . \tag{A40}$$

Here, the 2×2 matrix $\mathbf{A}$ in the class indices has been introduced, with the elements

$$\mathbf{A}_{mn}=2\pi D_m^{-1}(\Gamma_n+i\varepsilon)^{-1}\left[\sum_c\overline{\gamma_{m,cT_m}^2}[(1+xy)x]^2_{cT_m,cT_n}\overline{\gamma_{n,cT_n}^2}+(1-\delta_{mn})\overline{H_C^2}\right] . \tag{A41}$$

If Eq. (A40) is contracted in the following way

$$(\hat{\gamma}^+\tilde{b}\cdots\tilde{b}\hat{\gamma})_{aT,bT'}\vdots(\hat{\gamma}^+\tilde{b}^*\cdots\tilde{b}^*\hat{\gamma})_{bT'',aT'''} ,$$

which we call an "interlaced contraction," then for $a \neq b$ the result is of higher than first order in $1/N_1, 1/N_2$ and can be neglected. Compare the discussion of Sec. IV D of Müller and Harney (1985). However, if $a = b$, the interlaced contraction can be converted into one of the type of Eq. (A40) by reversing the order of the factors on one side of the center line.

The final result is obtained by summing over the q-fold contractions that are not interlaced. This sum can be expressed as a geometric series in the matrix $\mathbf{A}$:

$$\overline{t'^{\mathrm{fl}}_{aT,bT'}(E) t'^{\mathrm{fl}*}_{cT'',dT'''}(E+\varepsilon)} = \delta_{ac}\delta_{bd} \sum_{q \geq 1} (\hat{\gamma} + \underbrace{\tilde{b} \cdots \tilde{b}\hat{\gamma})_{aT,bT'} \vdots (\hat{\gamma} + \tilde{b}^* \cdots \tilde{b}^* \hat{\gamma})_{bT''',aT''}}$$

$$+ \delta_{ad}\delta_{bc} \sum_{q \geq 1} (\hat{\gamma} + \underbrace{\tilde{b} \cdots \tilde{b}\hat{\gamma})_{aT,bT'} \vdots (\hat{\gamma} + \tilde{b}^* \cdots \tilde{b}^* \hat{\gamma})_{bT'',aT'''}}$$

$$= \delta_{ac}\delta_{bc} 2\pi \sum_{mn} \overline{\hat{\gamma}_{m,aT}\hat{\gamma}_{m,aT''}} ((1-\mathbf{A})^{-1})_{m,n} D_n^{-1} (\Gamma_n + i\varepsilon)^{-1} \overline{\hat{\gamma}_{n,bT'}\hat{\gamma}_{n,bT'''}}$$

$$+ \delta_{ad}\delta_{bc} 2\pi \sum_{mn} \overline{\hat{\gamma}_{m,aT}\hat{\gamma}_{m,aT'''}} ((1-\mathbf{A})^{-1})_{m,n} D_n^{-1} (\Gamma_n + i\varepsilon)^{-1} \overline{\hat{\gamma}_{n,bT'}\hat{\gamma}_{n,bT''}} \,. \tag{A42}$$

Now one uses the transformation (A6) to convert from the transition matrix t' to the transition matrix t. [Do not confuse this matrix with the isospin projection t that appears in the channel indices (at), etc.] This transforms the factors $\pi\overline{\hat{\gamma}_{m,aT}\hat{\gamma}_{m,aT''}}/D_m$, etc., into the transmission coefficients $\tau^{tt''}_{am}$, etc., of Eq. (4.20), and one obtains

$$\overline{S^{\mathrm{fl}}_{at,bt'}(E) S^{\mathrm{fl}*}_{ct'',dt''}(E+\varepsilon)} = \delta_{ac}\delta_{bd} \sum_{mn} \tau^{tt''}_{am} \Pi_{mn} \tau^{t't''}_{an}$$

$$+ \delta_{ad}\delta_{bc} \sum_{mn} \tau^{tt'''}_{am} \Pi_{mn} \tau^{t't''}_{an} \,. \tag{A43}$$

Here the matrix Π has been introduced. Its inverse has the elements

$$(\Pi^{-1})_{mn} = 2\pi(\Gamma_m + i\varepsilon)(\delta_{mn} - \mathbf{A}_{mn}) D_n^{-1} \,. \tag{A44}$$

A straightforward calculation shows that Eq. (A44) yields Eq. (4.19). This completes the proof of Eq. (4.18).

APPENDIX B: THE RELATION BETWEEN RATIOS OF EVAPORATION SPECTRA AND THE ISOSPIN-MIXING PARAMETER

In the following we derive Eq. (6.33).

It is assumed that the alpha-particle channels couple to the levels of class one only, i.e., one neglects possible transitions to states in the residual nucleus B after α emission that have an isospin larger than the isospin $T_B = T_1$ of the ground-state of B (see Fig. 17). The transmission coefficients then are

$$\tau_{\alpha 1} = \tau_\alpha, \quad \tau_{\alpha 2} = 0 \,. \tag{B1}$$

The proton channels couple to both classes of states, although a similar approximation is introduced. The transmission coefficients read

$$\tau_{p1} = \tau_p (2T_1 + 1)/(2T_1 + 2) \,,$$
$$\tau_{p2} = \tau_p / (2T_1 + 2) \,. \tag{B2}$$

From Eqs. (4.18), (4.27), and (6.3) one obtains for the cross sections

$$\sigma_{\alpha\alpha'} = \tau_\alpha \frac{1}{N_1} \frac{1 + z/N_2}{1 + z/N_1 + z/N_2} \tau_{\alpha'} \,, \tag{B3}$$

$$\sigma_{pp'} = \left[\frac{2T_1 + 1}{2T_1 + 2} \right]^2 \tau_p \frac{1}{N_1} \frac{1 + z/N_2}{1 + z/N_1 + z/N_2} \tau_{p'}$$
$$+ \frac{2(2T_1 + 1)}{(2T_1 + 2)^2} \tau_p \frac{1}{N_1} \frac{z/N_2}{1 + z/N_1 + z/N_2} \tau_{p'}$$
$$+ \frac{1}{(2T_1 + 2)^2} \tau_p \frac{1}{N_2} \frac{1 + z/N_1}{1 + z/N_1 + z/N_2} \tau_{p'} \,, \tag{B4}$$

and

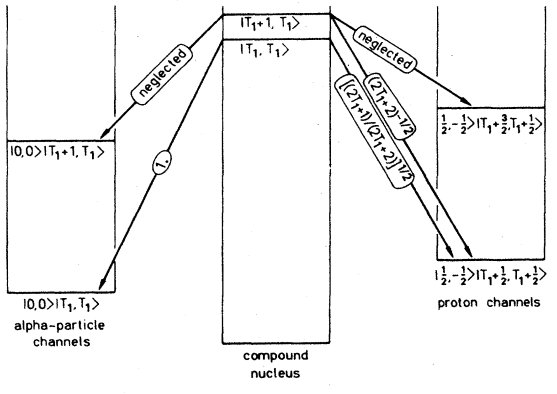

FIG. 17. Proton- and alpha-particle decay channels of a compound nucleus having two classes of levels with different isospins. The values of the Clebsch-Gordon coefficients coupling the compound-nucleus levels to the channels are given on the arrows. In the analysis of evaporation spectra, transitions to residual states that have isospins larger than those of the corresponding ground states are neglected. There are relatively few such states available if $T_1 > 0$.

$$\sigma_{\alpha p'}=\frac{2T_1+1}{2T_1+2}\tau_\alpha\frac{1}{N_1}\frac{1+z/N_2}{1+z/N_1+z/N_2}\tau_{p'}+\frac{1}{2T_1+2}\tau_\alpha\frac{1}{N_1}\frac{z/N_2}{1+z/N_1+z/N_2}\tau_{p'}\,. \quad \text{(B5)}$$

Finally, $\sigma_{p\alpha'}$ is obtained from $\sigma_{\alpha p'}$ if one replaces τ_α by τ_p and $\tau_{p'}$ by $\tau_{\alpha'}$ in the last equation.

The (α,α') reaction is an allowed one, proceeding through the class one levels, and Eq. (B3) therefore has the same structure as Eq. (6.5). Several processes contribute to $\sigma_{pp'}$. The first term on the right-hand side of Eq. (B4) describes the allowed part of the reaction, where the channels are coupled to the class one levels. The second term takes care of the forbidden part, where the isospins in the entrance and exit channels are different—very much as in Eq. (6.9). The third term gives the allowed reaction coupled to the class two levels. The three terms appear since both entrance and exit channels couple to both classes of levels. The (α,p') and (p,α') reactions are analogous to the (α,γ) reaction of Eq. (6.16).

If one assumes that N_1 and N_2 have the same dependence on the total spin [which, for the example of Fig. 2 of Li and Harney (1982), has been numerically shown to hold], then

$$\left[\tau_\alpha\frac{1}{N_1}\tau_{\alpha'}\right]\left[\tau_p\frac{1}{N_1}\tau_{p'}\right]\left[\tau_\alpha\frac{1}{N_1}\tau_{p'}\right]^{-1}\left[\tau_p\frac{1}{N_1}\tau_{\alpha'}\right]^{-1}$$

is equal to $R_{\rm CM}$ defined in Eq. (6.30). Inserting Eqs. (B3)—(B5) into the definition (6.29) or R, we get

$$R=R_{\rm CM}(1-\nu N_2/N_1)[(2T_1+1)^2(1-\nu N_2/N_1)+2(2T_1+1)\nu+(1-\nu)N_1/N_2][(2T_1+1)(1-\nu N_2/N_1)+\nu]^{-2}\,, \quad \text{(B6)}$$

where the isospin-mixing parameter ν defined in Eq. (6.12) has been used. Equation (6.33) follows immediately from Eq. (B6).

APPENDIX C: THE ENHANCEMENT FACTOR OF CHARGE EXCHANGE SCATTERING

We prove Eq. (6.80) of Sec. VI.F. With Eq. (4.18) and the explicit form of the matrix II as given in Eq. (6.4), one finds the cross section of Eq. (6.79) to be

$$\sigma_{at,bt'}=[(N_1+z)(N_2+z)-z^2]^{-1}[N_2\tau_{a1}^{tt}\tau_{b1}^{t't'}+N_1\tau_{a2}^{tt}\tau_{b2}^{t't'}+\delta_{ab}(N_2\tau_{a1}^{tt'}\tau_{a1}^{t't}+N_1\tau_{a2}^{tt'}\tau_{a2}^{t't})+z(\tau_{a1}^{tt}+\tau_{a2}^{tt})(\tau_{b1}^{t't'}+\tau_{b2}^{t't'})+\delta_{ab}z(\tau_{a1}^{tt'}+\tau_{a2}^{tt'})(\tau_{a1}^{t't}+\tau_{a2}^{t't})]\,. \quad \text{(C1)}$$

Using the assumption introduced by Eq. (4.25) and the expression (4.27) for the transmission coefficients, one observes that with $t\neq t'$ the last term in Eq. (C1) vanishes due to the orthogonality of the vector coupling coefficients. The remaining terms give

$$\sigma_{at,bt'}=[(N_1+z)(N_2+z)-z^2]^{-1}[(1+\delta_{ab})(N_1+N_2)(2T_1+1)(2T_1+2)^{-2}+z]\tau_{at}\tau_{bt'}\,, \quad \text{(C2)}$$

where the notation of Eq. (5.8) and the explicit form (A26) of the vector coupling coefficients have been used. This yields the enhancement factor

$$W_{tt'}=[2(N_1+N_2)(2T_1+1)(2T_1+2)^{-2}+z]\times[(N_1+N_2)(2T_1+1)(2T_1+2)^{-2}+z]^{-1}\,, \quad \text{(C3)}$$

which is the same as Eq. (6.80).

If we were not to use assumption (4.25), the last term in Eq. (C1) would not vanish. Let us consider the limit of very strong isospin mixing and retain only the terms proportional to z. The enhancement factor then becomes

$$1+(\tau_{a1}^{tt'}+\tau_{a2}^{tt'})^2(\tau_{a1}^{tt}+\tau_{a2}^{tt})^{-1}(\tau_{a1}^{t't'}+\tau_{a2}^{t't'})^{-1}>1$$

and therefore does not tend to unity. Since this seems physically unreasonable, we believe that Eq. (4.25) is the proper choice.

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