Problems in flow acoustics

Willi Möhring, Ernst-August Müller, and Frank Obermeier

Max-Planck-Institut für Strömungsforschung und Universität Göttingen, Böttingerstrasse 4-8, D 3400 Göttingen, Federal Republic of Germany

The research field "flow acoustics" is defined, and problems of current interest are discussed. The physical interpretation of fluid-mechanical sound sources for an acoustic medium, both at rest and in motion, is addressed. In the first case, it has been possible to relate the sound pressure field for low Mach numbers to sound sources that depend linearly on velocity; progress has been achieved by means of the causality correlation method with the application of laser-Doppler velocity measurements. In the theory of the acoustic medium in motion, it has been found that vortices downstream from a nozzle discharge can function as sound sinks; for a unidirectional mean flow, conditions for the sound sources are developed on the basis of the causality principle and the boundedness of the flow quantities. Experimental and theoretical results for flowfield oscillations in a high-velocity duct flow with sudden duct enlargement are discussed. Finally, some points concerning the influence of shear flow on sound propagation are described.

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LIST OF SYMBOLS

Some of the symbols, and their meanings, that appear several times in the text are listed below.

B	Stagnation enthalpy per unit mass
c	Speed of sound
D	Diameter of the nozzle
f	Frequency
$f(x_2)$	Mode shape function
G	Green's function
$\mathbf{G}$	Green's vector
I	Sound intensity
k	Axial wave number
M	Mach number
p	Pressure
q	Source distribution
s	Entropy per unit mass
S	Strouhal number
T	Absolute temperature
$\mathbf{u}$	Velocity vector
U	Characteristic mean flow velocity
$\mathbf{v}$	Velocity vector of the incompressible approximation of the flowfield
$\mathbf{w}$	Vorticity vector
W	Spectral intensity
Z	Impedance
γ	Ratio of specific heats
δ	Characteristic shear-layer thickness
λ	Characteristic wavelength of the generated sound field
ρ	Density
ω	Angular frequency

Subscript o refers to flow quantities in a medium at rest.
Subscripts i,j $(=1,2,3)$ refer to the components of the flow velocity.
Superscript $'$ denotes the fluctuating part of flow quantities.

I. INTRODUCTION

What is flow acoustics? It differs from other acoustic disciplines in that flows play an essential role in the acoustic phenomena. One may distinguish three essentially different processes:

(1) Generation of sound with essential participation of the flow,
(2) Propagation of sound through flowfields,
(3) Generation of flow by sound.

In all three cases one may be dealing with liquids or gases; the flow may be a one-phase or a multiple-phase flow; and thermal or chemical processes, as well as motions of arbitrarily deformable surfaces of bodies, may participate in the processes.

Flows are governed by nonlinear equations, a fact which is responsible for the complexity of fluid dynamics research, and thus also of flow acoustics. The nonlinearities of the equations are directly involved with the three processes mentioned above and their mutual interactions, often making it difficult to separate them from one another. With some care and caution, however, characteristic and fundamental processes can be isolated which allow us to study some aspects of flow acoustics and thus to gain a deeper physical understanding of the field.

In the following we shall describe basic issues which have engaged the interests of the flow acoustics community during the last few years and which are concerned with those characteristic and fundamental processes. Furthermore, typical methods for solving flow-acoustic problems, as well as actual solutions and the interpretation of these solutions, will be presented.

Even though the selection of our topics may be subjective and influenced by the personal interests of the authors, we nevertheless hope our presentation will shed some light on typical questions, methods, today's pressing problems, and the practical importance of flow acoustics.

In what follows we shall be concerned with details of only the first two processes introduced above, since Lighthill (1978) has fairly recently presented a survey on the production of flow by sound.

II. THE GENERATION OF SOUND BY FLOWS

Steady flows do not produce sound. Therefore we need only consider unsteady flows. Two causes for unsteady flows may be mentioned:

(1) The motion of solid bodies (for instance, propellers or pistons).

(2) The instability of flows. Here solid boundaries of flows may often be at rest, i.e., the unsteady flow is caused solely by the instability of the flow itself. Examples are turbulence—for instance, free jet turbulence behind the nozzle of a tube—or vortex sheet instability connected with the well-known phenomena of edge tones or aeolian tones.

Which basic problems related to these two processes are of special interest at the moment? We shall mention a few:

(a) *Lighthill's theory.* Thirty years ago Lighthill (1952) published his theory of aerodynamic sound production, which he had developed during the preceding three years (Lighthill, 1982). He rewrote the basic equations of fluid dynamics in such a way that a wave equation for the density fluctuations with a source term q on the right-hand side appeared. In the simplest case (which, in particular, implies a bounded region of sound production, isentropic medium, and characteristic Mach number $M \ll 1$), the corresponding equation for the pressure fluctuations reads

$$\Box p = q(\mathbf{x},t) = \frac{\partial^2(\rho_o v_i v_j)}{\partial x_i \partial x_j} . \tag{1}$$

Here, as in many other cases of low-Mach-number flow, it is admissible in the source region to approximate the actual sound-generating velocity field by an incompressible one which we denote by $\mathbf{v}=(v_1,v_2,v_3)$. The other symbols have the following meaning: x_i ($i=1,2,3$), Cartesian components of the radius vector $\mathbf{x}$; t, time; ρ, density; p, pressure; $M=U/c_o$; U characteristic mean flow velocity; c, speed of sound; subscript o, values of the ambient medium; $\Box=(1/c_o^2)(\partial^2/\partial t^2)-\Delta$ (Δ, Laplacian operator).

This means that the actual medium has been replaced by an ideal medium, in which the usual linear wave equation without convective terms is valid everywhere, and in which sources of sound q are present—sources which also comprise all convective and refractive effects of the actual medium. This theory may therefore be termed a *theory of the ideal acoustic medium at rest.* In this theory the source distribution q is of local quadrupole character, as is apparent from its mathematical representation (second spatial derivative).

Lighthill's (1952) derivation is generally valid, and may well be applied to many flow-acoustic problems. The following questions arise, however:

(1) In interpreting q as local quadrupole sources one has to keep in mind that other source distributions may produce the same sound field at large distances (see, for example, Ffowcs Williams, 1974). In other words, one can add nonradiating sources to a given source field without changing the far field. This well-known nonuniqueness in the source distribution (compare Baltes, 1978) leads to the question as to which is the physically most plausible representation of the sources.

(2) Is it possible to achieve improvements by considering the mean motion of the medium more explicitly? In other words, what can be gained by a *theory of an ideal acoustic medium in motion?*

(b) *Flight effects on sound fields.* In the study of jet engine noise, one is interested in flight effects on aircraft noise. One of the most pressing problems concerns the predictability of the sound field produced by a jet engine in flight when only the sound field generated by the same engine running on a static test stand is known. Here various discrepancies between theory and experiment have occurred. Are these discrepancies due to the motion of the sound sources, due to additional sound sources occurring under in-flight conditions, due to additional sound-scattering processes, due to changes in the properties of the jet turbulence inflight, or are they due to other factors?

(c) *Coherent structures.* In turbulent jet flows often so-called "coherent or orderly structures" are observed. They are initiated by large-scale vorticity fluctuations like interacting and pairing vortex rings. Now the question is: What role do these structures play in flow-acoustic sound production?

(d) *Complex processes.* There are many flow-acoustic processes for which general theories such as Lighthill's are valid, but for which, because of the complexity of the processes, these theories have given little detailed information. This is the case for many problems in which feedback and/or resonance play a role. The same applies to relaxation processes, for instance to oscillations in a flow with evaporation or condensation. Furthermore, one must differentiate between flows that are purely subsonic and those which are at least partially supersonic. Here additional types of instabilities occur—to mention but a few, the screech noise of supersonic jets, the Hartmann-Sprenger tube, oscillations in Laval nozzles, the buffeting phenomenon in flows around airfoils, and certain types of oscillations occurring in tubes with changes in cross section.

All these phenomena have one point in common: the theories describing them are more or less *ad hoc*, and have to be tested experimentally.

The problems listed under (a) to (d) above are—among others—the subject of intensive current research efforts. Progress has been considerable, so that all these problems could be extensively discussed in this paper. This is, however, impossible because of the prohibitive amount of material. We shall therefore concentrate on the problems mentioned under (a) and (b)—both from the theoretical and experimental points of view—and shall present a characteristic example of the problems mentioned under (d). With respect to the remaining questions the reader is referred to the literature [(a): Fuchs, 1978; Ribner, 1978; Crighton, 1979; Kibens, 1979. (d): Karamcheti *et al.*, 1969; Woolley and Karamcheti, 1974 (edge tones); Rockwell and Naudascher, 1979 (impinging free shear layers); Mørch, 1964; Brocher *et al.*, 1970; Sarohia and Back, 1979 (Hartmann-Sprenger tube); Meier, 1974; Jungowski, 1978; Seegmiller *et al.*, 1978; Marvin *et al.*, 1980 (instabilities in supersonic and transonic flow); Meier, 1976 (Laval nozzle)]. A general introduction to flow acoustics has been given by Goldstein (1976), a short survey on jet noise by Ribner (1981). Further papers of general interest were published by Crighton (1981) and Ffowcs Williams (1982).

A. The ideal acoustic medium at rest

1. Theory

In later sections we shall often use the vorticity vector $\mathbf{w}=\mathrm{curl}\mathbf{v}$, so it may be appropriate to say a few words here about its most important properties. One form of Helmholtz's vortex equation reads, for incompressible inviscid flows,

$$\frac{\partial \mathbf{w}}{\partial t}-\mathrm{curl}\mathbf{A}=0$$

($\mathbf{A}=\mathbf{w}\times\mathbf{v}$), which states a vortex conservation law with an antisymmetric vorticity flux tensor. It shows that the total vorticity in a region of space can change only by a vorticity flux through its boundary. From the definition of $\mathbf{A}$ one concludes that the vorticity flux vanishes if the vorticity vanishes, which implies that the total vorticity in a region of space can change only if the vorticity on its boundary is nonzero.

The Helmholtz vortex equation can also be written as

$$\frac{\partial \mathbf{w}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{w}=\mathbf{w}\cdot\nabla\mathbf{v}\ .$$

This is a form which focuses attention on a material particle. One observes that the vorticity of a particle changes if the velocity varies in the direction of $\mathbf{w}$. The direction of $\mathbf{w}$ defines a vortex line, and varying velocity implies in general a stretching of this line. These two versions of the Helmholtz vortex equation illustrate the way the vorticity changes at a fixed point in space as well as for a fixed material particle. In the important special case of two-dimensional flow, it is not possible for a particle to change its vorticity. Only minor modifications are required for compressible flows with constant entropy. One finds that $\mathbf{w}$ in the preceding equation should be replaced by $1/\rho\,\mathrm{curl}\mathbf{u}$ [$\mathbf{u}=(u_1,u_2,u_3)$ is the velocity field which, in contrast to $\mathbf{v}$, includes compressibility as well], which shows that a material compression is accompanied by a vorticity increase.

Let use now return to question (1) of section (a), concerning the physically most plausible source representation. In addition to Lighthill's distribution q [Eq. (1)], two further equivalent source distributions q are commonly used to determine the radiated sound,

$$q=\rho_o\mathrm{div}\mathbf{A} \tag{2}$$

(local dipoles) (Powell, 1964), and

$$q=-\frac{1}{c_o^2}p_{tt}^{(0)} \tag{3}$$

(local monopoles), where $p^{(0)}$ means the so-called "pseudosound" pressure in the flow (Ribner, 1962), i.e., the pressure in the flow incompressible approximation. How can one proceed from these different representations? A particularly simple and physically instructive solution to the problem was recently developed by Möhring (1978a) and Obermeier (1977), which we shall present here. Möhring proceeds from Powell's representation (2) and the well-known solution of an inhomogeneous wave equation by means of a Green's function G,

$$p(\mathbf{x},t)=\int G(\mathbf{x},\mathbf{y},t-t')q(\mathbf{y},t')d^3y\,dt'$$
$$=-\rho_o\int \mathbf{A}\cdot\mathrm{grad}G\,d^3y\,dt'\ . \tag{4}$$

He replaces the Green's function G (this is the principle idea) by a Green's vector $\mathbf{G}$ with the help of the definition

$$\mathrm{grad}G=\mathrm{curl}\mathbf{G} \tag{5}$$

and obtains

$$p(\mathbf{x},\mathbf{t})=-\rho_o\int \mathbf{G}\cdot\mathrm{curl}\mathbf{A}\,d^3y\,dt'\ . \tag{6}$$

The integrability condition of Eq. (5) ($\mathrm{div}\,\mathrm{grad}_y G=0$) is often fulfilled, if $\mathbf{x}$ is a far-field point. By invoking Helmholtz's vorticity transport equation one obtains the general result for the far field,

$$p(\mathbf{x},\mathbf{t})=\rho_o \frac{\partial}{\partial t} \int \mathbf{G}(\mathbf{x},\mathbf{y},t-t')\cdot \mathbf{w}(\mathbf{y},t')d^3y\, dt' \; . \qquad (7)$$

Example for application. Consider the far-field sound pressure of a low-Mach-number flowfield in free space, i.e., in the absence of solid bodies (for instance, in the far field of a turbulent flow region). The computations after Lighthill and Möhring, respectively, yield the following results:

$$p(\mathbf{x},t)=\frac{x_i x_j}{4\pi c_o^2 x^3}\frac{\partial^2}{\partial t^2}\int \rho_o v_i v_j(\mathbf{y},t-x/c_o)d^3y \; , \qquad (8a)$$

$$p(\mathbf{x},\mathbf{t})=\frac{x_i x_j}{12\pi c_o^2 x^3}\frac{\partial^3}{\partial t^3}\int \rho_o y_i[\mathbf{y}\times\mathbf{w}(\mathbf{y},t-x/c_o)]_j d^3y \; , \qquad (8b)$$

where $x=|\mathbf{x}|$. The main conclusions are as follows:

(1) In contrast to all previous representations [see, for example, Eq. (8a)], the sound pressure appears here for the first time as a quantity that is *linear* in the vorticity vector and, therefore, linear in the velocity. The contributions of the vortices to the sound pressure are *additive.*

(2) Only those components of the vorticity vector $\mathbf{w}$ contribute to the radiated sound which are normal to the observation direction.

(3) By representing the sound sources in terms of vorticity the process of aerodynamic sound production is now based on a fundamental aerodynamic quantity for which conservation equations exist, viz., the vorticity theorems.

(4) The calculation of the sound production by flows which can be modeled by vortex motions becomes simple and clear.

On the basis of Möhring's general results, explicit formulas have been derived for the sound radiation of prototypical flows (e.g., Obermeier, 1977, 1979; Möhring, 1978a; Kambe and Minota, 1981). For instance, Möhring (1978a) calculated the far-field sound created by two identical coaxial vortex rings spinning around each other. This is the first example in which a full explicit determination of the aerodynamic sound produced by a three-dimensional flow was achieved. Kambe and Murakami (1979) have reported on first steps comparing Möhring's results with measurements of the sound produced by the head-on collision of two vortex rings. An extension of the theory to higher Mach numbers is desirable, but not yet available.

Based on Lighthill's acoustic analogy approach, analytical descriptions of the influence of solid bodies in the flow upon aerodynamics sound generation were made by Curle (1955). Later Ffowcs Williams and Hawkings (1969) and Möhring *et al.* (1969) independently generalized Curle's equation to include bodies with arbitrarily moving boundaries. As the integral representations of the sound field in these theories were based on free-space Green's functions, they include volume integrals as well as surface integrals.

An alternative representation of such sound fields can be obtained if the free-field Green's function is replaced by Green's functions especially tailored to the geometry of the bodies in question. In these cases the complete sound field is expressed in terms of volume integrals only, whereby the adjusted Green's functions may be evaluated by means of a reciprocity theorem as proposed by Howe (1975b). Applying this method, Obermeier (1980) determined quite generally the corresponding Green's vector function used in Eq. (7) for low-Mach-number flows.

From that solution, simple and plausible results are obtained, provided the mean flow (not the actual unsteady flow) is either a two-dimensional potential flow, for instance, a potential flow around "infinite" wings, or a potential flow around a sphere or through an axisymmetric duct with varying cross section. In all these cases the aerodynamically generated sound field radiating into an arbitrary observation direction $\mathbf{x}$ is determined by the rate at which unsteady vorticity components normal to $\mathbf{x}$ cross the streamlines of a hypothetical steady potential flow around the body with $\mathbf{x}$ direction at infinity.

This outcome can be regarded as a generalization of a result obtained already by Howe (1975a). He found that in a two-dimensional flowfield the sound generated by a line vortex moving in the vicinity of the trailing edge of a half-infinitely extended solid plate is determined by the rate at which the line vortex cuts across streamlines of a potential flow around the plate.

2. Experiments

On the experimental side of things—particularly from the aspect of noise reduction—one seeks information as to the distribution of flow-acoustic sources of sound. To achieve this, various experimental methods have been developed and applied. In the present survey we confine the discussion to the "causality correlation method" as developed by Lee and Ribner (1972) and Siddon (1973).

While it is often useful for both theoretical considerations and practical computations to proceed from vorticity (see, for example, Morfey, 1979), the measurement of this quantity turns out to be difficult, as it requires the measurement of velocity gradients. This is more difficult than measuring the velocities themselves. Therefore, measurements of the source distributions have up to now almost exclusively proceeded from Eq. (1) or similar equations. On decomposing the velocity v_i into a time-independent (V_i) and a time-dependent part (v_i'), one obtains four source terms:

$$\rho_o v_i v_j=\rho_o V_i V_j+\rho_o V_i v_j' +\rho_o v_i' V_j+\rho_o v_i' v_j' \; . \qquad (9)$$

The first term is time independent, and therefore does not contribute to sound generation. The second and third term define the so-called "shear noise," and the fourth term the so-called "self-noise." This nomenclature was first used by Lilley (1958), who wrote: "The calculation is divided into the contribution from 'self-noise' in the turbulence and the interaction between the turbulence and the mean shear."

The contributions of both the shear noise terms and the self-noise terms to the total sound generation cannot be determined from acoustic measurements outside of the flow alone; one has rather to correlate measurements within the sound-radiating flowfield with measurements in the far field. The method of "causality correlations" has turned out to be useful here. This technique proceeds from the autocorrelation $\overline{p_s(\mathbf{x},t)p_s(\mathbf{x},t-\tau)}$ of the far-field pressure p_s, in which $p_s(\mathbf{x},t)$ (the "effect") is represented—by means of Eq. (8a)—by the terms $\rho_o v_i v_j$ within the flow region (the "cause" of p_s). The bar stands for the time average. One obtains by straightforward calculation and using Eq. (9)

$$\overline{p_s(\mathbf{x},t)p_s(\mathbf{x},t-\tau)}=\frac{\rho_o}{4\pi c_o^2 x}\int \frac{\partial^2}{\partial\tau^2}[2V_x(\mathbf{y})\overline{v_x'(\mathbf{y},t)p_s(\mathbf{x},t+x/c_o-\tau)}+\overline{v_x'^2(\mathbf{y},t)p_s(\mathbf{x},t+x/c_o-\tau)}]d^3y \ , \tag{10}$$

where the first term in square brackets is the shear noise term and the second is the self-noise term and where the subscript x indicates the component of the velocity vector in the direction from the flow region to the observer. Thus the autocorrelation, and therefore the spectrum of the far-field pressure, are fully determined by the two correlations in Eq. (10). The integrand vanishes outside the flow region.

A serious source of error for measurements within the flow region is the interaction of the measuring probe with the flow itself. Therefore, a good deal of effort has been expended on nonobtrusive measuring techniques. Considerable progress was recently made by Schaffar (1979; Schaffar and Hancy, 1982) and Richarz (1979), who succeeded in using LDV (laser-Doppler velocimetry) for determining the causal correlations [right-hand side of Eq. (10)] for jet noise. Among their results are the following:

(1) For $M=0.98$ (Schaffar, 1979), where M is the mean Mach number in the nozzle exit, the zone of the jet generating most of the radiated noise at $\theta=20°$ and 30° to the jet axis ($\theta=0°$ is the flow direction) is located in a cylindrical domain about the jet axis within the transition region $5\leq z/D\leq 10$, where D is the diameter of the nozzle and z the axial coordinate of the jet ($z=0$ at the nozzle). This is in good agreement with results obtained earlier by other authors (e.g., Grosche, 1979). For $\theta=30°$ and $M\approx 0.97$ (Schaffar and Hancy, 1982), the intensity of the shear noise generated by the jet in this domain was found to be 8 dB greater than that of the self-noise.

(2) Comparison with the pressure spectrum obtained by autocorrelation at the observation point showed good agreement for $\theta=20°$ and 30° (see Fig. 1). With the then-used measuring technique a disparity developed for larger angles θ (Schaffar, 1979; Schaffar and Hancy, 1982).

(3) For $M\approx 0.3$ (Richarz, 1979) and an angle 40° the shear noise and self-noise spectra [right-hand side of Eq. (10)] determined from the sound which was emitted from the zone $3\leq z/D\leq 7$ were similar in shape. They exhibited a frequency shift somewhat smaller than the expected one-octave shift, yet having comparable absolute amplitudes.

(4) The mean-square source strength per unit length of the jet decreased with increasing z/D in the zone $3\leq z/D\leq 7$ for all Strouhal numbers (see Fig. 2) (Richarz, 1979). [The Strouhal number is a frequency made nondimensional by a characteristic length and velocity, respectively, and was for the first time used by Strouhal (1878) to describe the frequency of vortex shedding behind a cylinder in an airstream.]

These results leave a number of questions still unanswered. In the work of Schaffar (1979; Schaffar and Hancy, 1982) refraction—at least for smaller wavelengths—could have modified the results, since the Mach number is rather high; the measurements became less accurate for $\theta>30°$. For $\theta\lesssim 30°$, however, a physical interpretation of the experimental data is complicated by the observation (Lush, 1971; Tanna, 1977) that the measured noise spectra no longer scale with the Strouhal number fD/U but with the nondimensionalized frequency fD/c_o. Explanations of this finding have been suggested, for instance, by Lilley *et al.* (1974) and Goldstein (1975), who investigated theoretically the sound field produced by point multipoles convected downstream in slightly diver-

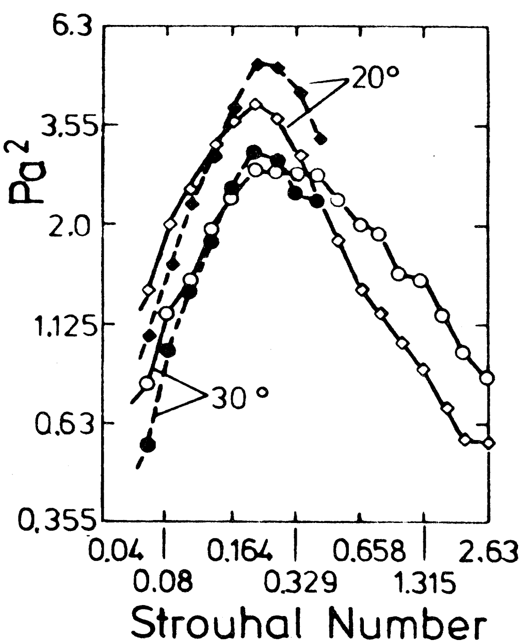

FIG. 1. Pressure spectra in $\frac{1}{3}$ octave bands, with microphone at 20° and 30° to the jet axis: ◇ and ○, far-field measurements (autocorrelation technique at the observation point); ◆ and ●, causality correlation technique [right-hand side of Eq. (10)]. From Schaffar (1979).

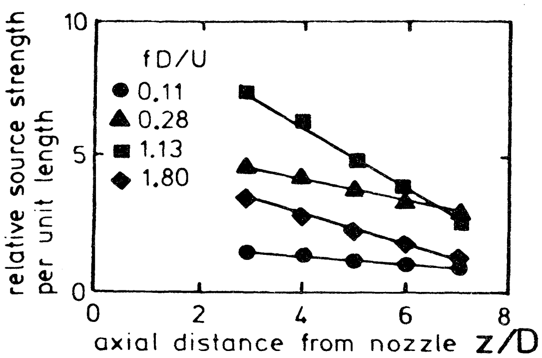

FIG. 2. Mean-square source strength distribution per unit length: D, nozzle diameter; U, mean flow velocity at nozzle exit; f, frequency; fD/U, Strouhal number. Microphone at 40° to the jet axis. From Richarz (1979).

gent shear flows. [In the case of Richarz (1979) the measurements do not give the point of maximum source strength (see Fig. 2); the phase information is neglected.] Further, there exist unexplained discrepancies between the results obtained by the causality correlation method and those gained earlier from far-field measurements only, for instance by acoustic mirrors (Grosche, 1973; Chu *et al.*, 1972), acoustic telescopes (Billingsley and Kinns, 1976), or polar correlation technique (Fisher *et al.*, 1977).

It is, however, obvious that great progress may be made by unobtrusive measuring techniques. In, say, ten years, the accuracy of aerodynamic noise prediction should have been considerably improved by means of them.

B. The ideal acoustic medium in motion

There is no doubt that the ideal acoustic medium at rest is a correct model for flow-acoustic sound generation (see, for example, Ribner, 1977). One might, however, argue that it is not very sensible to model convection effects in terms of sources. One would prefer very much to rewrite Eq. (1) in a form in which the linear fluctuating terms appear on the left-hand side of the equation. This would reduce the problem of flow-acoustic sound production to the problem of sound generation in an ideal acoustic medium moving with a time-independent flow. Pursuing this, one finds that it is impossible in general to derive one single equation for a fluctuating quantity; one has to accept a complicated system of equations. Simplifications are, however, possible in two important special cases where one single equation could be derived. The first special case is that of a time-independent potential flow, the second that of a time-independent unidirectional flow. We shall consider both cases.

1. Howe's model

a. Theory

On the basis of the gas dynamic equations, Howe (1975a) derives

$$\left[\frac{d}{dt}\left[\frac{1}{c^2}\frac{d}{dt}\right]+\frac{1}{c^2}\frac{d\mathbf{u}}{dt}\cdot\nabla-\Delta\right]B=\operatorname{div}\mathbf{L}-\frac{1}{c^2}\frac{d\mathbf{u}}{dt}\cdot\mathbf{L}\ . \tag{11}$$

B is the stagnation enthalpy $h+\frac{1}{2}|\mathbf{u}|^2$ per unit mass, s is the entropy per unit mass, T the absolute temperature,

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\mathbf{u}\cdot\nabla \tag{12a}$$

and

$$\mathbf{L}=-\mathbf{u}\times\operatorname{curl}\mathbf{u}-T\nabla s\ . \tag{12b}$$

Equation (11) is exact in the absence of dissipative processes. Note that the material derivative d/dt involves the actual (possibly rotational) fluid velocity, and not merely that of the mean flow. Equation (11) shows that B obeys a kind of convective wave equation with sources which are related to $\mathbf{L}$. As B reduces in regions without flow to $(1/\rho_o)p$ and can therefore be used to determine the acoustic pressure, one concludes that aerodynamic sound generation requires vorticity [compare Eq. (7)] or entropy gradient fluctuations. At points of the flow exterior to regions of entropy inhomogeneities and vorticity, the right-hand side of Eq. (11) vanishes identically. In these regions the left-hand side agrees fully with that of the equation which describes the propagation of small irrotational disturbances in a steady irrotational flow [c and $\mathbf{u}$ in (11) being replaced by their steady values]. Howe's model, therefore, satisfies the above-mentioned condition of having all linear terms in fluctuation quantities on the left-hand side in any potential flow region.

b. Application: Vortices as sinks of sound in low-Mach-number flow

Howe's model has been applied to many flow-acoustic problems, e.g., to the theory of the flute (Howe, 1975a), to trailing-edge noise (Howe, 1978), and to the transmission of sound through a perforated screen (Howe, 1979a). We should here like to discuss one interesting problem in some detail, namely that of the attenuation of a sound wave (angular frequency ω) by a low-Mach-number nozzle flow (see Fig. 3).

In this problem the sound wave propagates in the nozzle and approaches the exit from the left-hand side. At the edge of the nozzle, vortices are shed by the sound wave. These vortices occur as sound sources in Eq. (11). For low frequencies, Howe (1979b, 1979c) solved Eq. (11) to first order in Mach number using the incompressible limit of the results of Munt's (1977) calculations of the motion of the vortices. Essential for the calculations is the use of a reciprocity relation by which one is able to interchange the roles of source and observer.

This solution enables one to calculate the acoustic energy W_F radiated into the far field, and to compare it with the "transmitted" energy W_T (this is the difference between the energy of the incident wave and the wave reflected back into the nozzle). For thin vortex sheets,

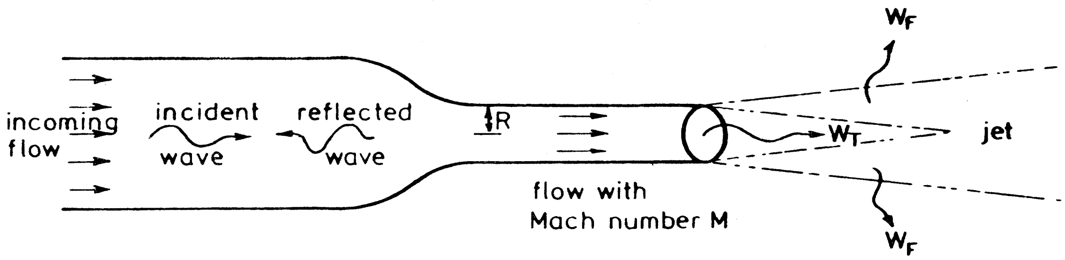

FIG. 3. Symbolic diagram for the attenuation of a sound wave by vortex shedding.

Howe finds

$$\frac{W_F}{W_T}=\frac{(kR)^2}{4M\nu+(kR)^2},\quad k=\frac{\omega}{c_o}, \tag{13}$$

if the density and the speed of sound in the jet agree with their free-space values. kR is proportional to the ratio of the tube radius to the wavelength of the incident sound wave. ν depends monotonically on the Strouhal number $\omega R/U$ (U exit flow velocity) and is equal to 1 for Strouhal numbers approaching 0, and $\frac{1}{2}$ for Strouhal numbers approaching infinity. The ratio of the two energy fluxes W_F and W_T is unity, if the acoustic energy is conserved. Equation (13) shows that this is the case only for no flow ($M=0$). The ratio would be greater than one if additional sound were generated by the sources on the right-hand side of Eq. (11), and smaller than one if the sources were to act as sinks, thereby attenuating sound. Equation (13) reveals the latter is true. One can interpret this phenomenon as a "sucking off" of acoustic energy and transformation of it into flow energy, a process which is linear [no amplitude dependence in Eq. (13)!] and in which no dissipative effects are involved.

An earlier experimental investigation by Bechert *et al.* (1977) shows (see Fig. 4) attenuations of as much as 20 dB. This work agrees satisfactorily with theory within the validity range of the theory ($M \ll 1$ and $kR \ll 1$). Recently Bechert (1979) has given a simple derivation of Howe's result [Eq. (13)] with the $(kR)^2$ missing from the denominator and $\nu=1$. There is good agreement with the experimental values for a large range of parameter combinations M and kR. Howe (1980) himself reconfirmed his low-Strouhal-number result by an elegant integral method. For further discussion see Cargill (1982).

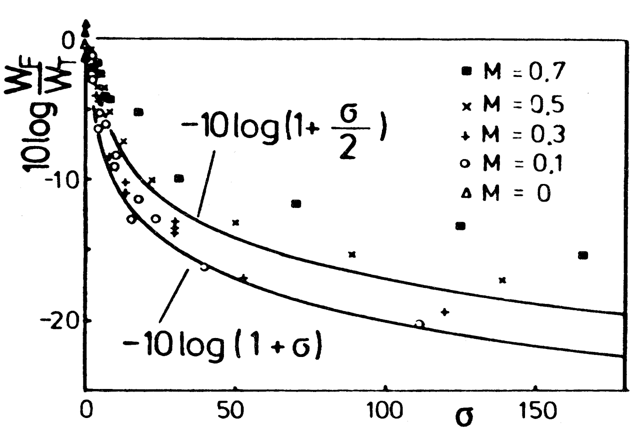

FIG. 4. The ratio of the energy W_F radiated into the far field to transmitted energy W_T. Experiments from Bechert (1977). Upper curve, theory for Strouhal number$\to\infty$; lower curve, theory for Strouhal number$\to 0$ (from Howe, 1979b). $\sigma=4M/k^2R^2$.

These results are important not only for nozzle flows but also for other situations where separation of a mean flow from the edge of a rigid surface occurs (for examples see Bechert, 1979). Absorption of sound by vorticity shedding might provide a useful complement to conventional broad-band sound absorption systems, which are usually inefficient for low frequencies.

c. Generalization: A self-adjoint form of Eq. (11)

Howe's calculations are restricted to the lowest order in Mach number. The same is true for many other applications of Eq. (11). One of the reasons for this restriction is the lack of a reciprocity relation for Eq. (11) for higher Mach numbers. Equation (11) admits such a relation only to first order in Mach number or, what amounts to the same thing, it is not self-adjoint. Recently Möhring (1979) found a modification of Eq. (11) that is self-adjoint for B for arbitrary functions c, $\mathbf{u}$, and ρ, and therefore admits a reciprocity relation for all Mach numbers. For an ideal gas, it reads

$$\rho\frac{d}{dt}\left[\frac{1}{c^2}\frac{dB}{dt}\right]-\nabla\cdot\rho\nabla B=\mathrm{div}\rho\mathbf{L}+(\gamma-1)\frac{\rho T}{c^2}\frac{d}{dt}\frac{\partial s}{\partial t}, \tag{14}$$

where γ denotes the ratio of specific heats.

2. Unidirectional flow

a. Theory

The potential flow model described in the preceding section does not seem too well suited for application to problems with extended regions of rotational flow—such as jets, for example. In many cases the unidirectional flow model might constitute an improvement. If the mean flow velocity and the mean speed of sound are given by $U_1(x_2)$ (see Fig. 5) and $\bar{c}(x_2)$ and the velocity fluctuations are assumed to be incompressible, then the perturbation pressure p' obeys the equation

$$\frac{1}{\bar{c}^2}\frac{D^3p'}{Dt^3}-\frac{D}{Dt}\Delta p'-\frac{d}{dx_2}[\ln(\bar{c}^2)]\frac{D}{Dt}\frac{\partial p'}{\partial x_2}+2\frac{dU_1}{dx_2}\frac{\partial^2 p'}{\partial x_1\partial x_2}=\frac{\rho_o c_o^2}{\bar{c}^2}\left[\frac{D}{Dt}\frac{\partial^2(v_i'v_j'-\overline{v_i'v_j'})}{\partial x_i\partial x_j}-2\frac{dU_1}{dx_2}\frac{\partial}{\partial x_1}(v_k'\partial v_2'/\partial x_k-\overline{v_k'\partial v_2'/\partial x_k})\right], \quad (15)$$

where $D/Dt=\partial/\partial t+U_1\partial/\partial x_1$. Equation (15), given by Tester and Burrin (1974), is a simplified version of a relation due to Lilley (1973). Here we want to emphasize two points: (i) As in Lighthill's theory, the right-hand side of Eq. (15) usually is regarded as a source distribution, even though it still depends on unknown flow quantities that one has to evaluate, and (ii) Eq. (15) is a third-order equation. The latter point may seem somewhat surprising for an acoustic problem. The reason, however, is that Eq. (15) also contains hydrodynamic disturbances, and it seems in general impossible to separate them from the acoustic perturbations. There is one exception, that for a linear profile (see Sec. III). The incompressible limit of Eq. (15) is none other than the Orr-Sommerfeld equation for an inviscid flow, sometimes called the Rayleigh equation, studied intensively in the context of hydrodynamics stability theory. One therefore expects Eq. (15) to have unstable solutions (see, for example, Doak, 1974). Such unstable solutions do exist, in general, if one admits arbitrary sources on the right-hand side and requires the solutions to be "causal," i.e., requires that p' arises from the excitations on the right-hand side.

Alternatively, one may argue that one never finds exponentially growing solutions in reality, and therefore one has to look for bounded solutions of Eq. (15). They exist, but they are no longer causal: p' shows precursors to the excitation of the right-hand side.

The only way out of this dilemma is to require the right-hand side to be constituted in such a way that exponentially growing solutions are not excited (Möhring and Rahman, 1979). This leads to conditions for the incompressible velocity fluctuations v_i', and hence gives information about the sound-generating flow, e.g., turbulent jet flow. Physically this means that the flow adjusts itself in such a way that no exponentially growing disturbances are produced. The same conditions for velocity fluctuations are obtained if one requires the bounded solutions of Eq. (15) not to show precursors.

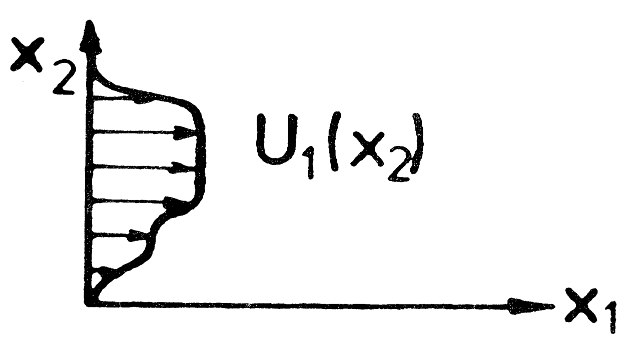

FIG. 5. Unidirectional flow.

To elucidate these problems, which are directly related to the fact that the source term in Eq. (15) depends on unknown flow quantities, Müller (1980) suggested the following equation as an example:

$$y''(t)-k^2y(t)=y^2(t)+f(t)\equiv Q(t)\,. \quad (16)$$

f is a given function, triggering the process $y(t)$, vanishing for $t<0$, and nonzero only for a short time; $k>0$. The term y^2 on the right-hand side stands for the nonlinearity of the equation. If one requires causality (no precursors), the equation

$$y(t)=\int_o^t\frac{e^{k(t-\tau)}}{2k}Q(\tau)d\tau-\int_o^t\frac{e^{-k(t-\tau)}}{2k}Q(\tau)d\tau \quad (16a)$$

is valid, as one can derive by using the method of variation of parameters. This fulfills the causality condition, because only Q values with $\tau<t$ contribute to the solution. If one additionally requires the solution to be bounded, then the condition

$$\int_o^\infty(y^2+f)e^{-k\tau}d\tau=0 \quad (16b)$$

follows. If one requires only $y(t)$ to be bounded, then one obtains

$$y(t)=\int_t^\infty\frac{e^{k(t-\tau)}}{2k}Q(\tau)d\tau-\int_o^t\frac{e^{-k(t-\tau)}}{2k}Q(\tau)d\tau \quad (16c)$$

in place of Eq. (16a). One sees from the first term that there are precursors. If one additionally requires causality, one again obtains condition (16b). Whatever the details of the solution may be, if the conditions both of causality and of boundedness are required, then Eq. (16b) must hold. Applications to turbulence are given in Dowling *et al.* (1978), Möhring (1979), Ffowcs Williams and Purshouse (1981). It is interesting to note that although this consideration has an acoustic origin, it throws light on the problematics of turbulent flow—even for the incompressible case.

For practical flow-acoustic calculations it is often difficult to fulfill the conditions analogous to Eq. (16b). This means that one has to decide whether one prefers the causal (no precursors) or the noncausal (precursors) formulation. Because small errors and disturbances grow exponentially in the causal case and remain bounded in the noncausal case, one might often choose the latter.

b. Applications

Equation (15) is rarely amenable to analytic treatment, and in general one has to use approximate or numerical methods. High- and low-frequency approximations have been made, for example, by Balsa (1976), Tester and Morfey (1976), and Goldstein (1975, 1982). Exact solutions in terms of parabolic cylinder functions are known for a linear velocity profile $U_1(x_2)$ (Goldstein and Rice, 1973). They have been applied to sound propagation problems through a linear shear layer by Scott (1979), and Koutsoyannis *et al.* (1980). An analytic solution is also possible for a piecewise constant-velocity profile ("vortex sheet"). In the case of a jet, for example, it seems worthwhile to study a cylindrical vortex sheet model where $U_1(x_2)$ is constant inside and vanishes outside of the jet. Such an acoustic model was developed by Dowling *et al.* (1978). One of the many interesting results of this investigation relates to the sound generated by very light jets of density ρ_j. Dowling *et al.* (1978) show that, for $\rho_j \ll \rho_o M^2 S^2 \cdot |\ln(SM)|$ and $(SM) \ll 1$, the spectra level W of the radiated sound is given by

$$W \sim \frac{(\rho_o-\rho_j)^2}{\rho_o} U_1^3 \frac{1}{MS}\frac{R^2}{x^2},\quad S=\frac{\omega R}{U_1}\ , \tag{17}$$

where R is the radius of the vortex sheet and S is the Strouhal number. Here the low power of the Mach number is surprising; in fact, Eq. (17) differs from the usual scaling law by a factor $(MS)^{-4}$.

The same model can also be applied to the experiment of Bechert *et al.* (1977). In this case it is possible to ignore all the sources on the right-hand side of Eq. (15), and to regain only the incident sound wave in the nozzle as source. Munt (1977) discussed this problem using the causal solution. Satisfactory agreement was found with the directivity pattern measured by Pinter and Bryce (1976) for the noise radiated by a hot jet. In a later paper Munt (1978) achieved a very favorable comparison with measurements obtained by Bechert *et al.* (1977), again using the causal solution. However, Munt had to ignore a contribution from the unstable wave.

C. In-flight effects

In recent years there has been a growing interest in prediction schemes for aeroacoustic noise generation by aircraft in flight or—more fundamentally—in-flight effects upon noise production by a turbulent jet exhausting from a nozzle into an outer, coflowing stream. Experimental investigations have shown that simply allowing for Doppler effects—applied to the known sound field of a free jet in a medium at rest, in the following often called a static jet—does not yield convincing results. Moreover, most alternative methods were not very successful in explaining measured flight effects satisfactorily; on the contrary, existing theories and data in the literature are still controversial (Crighton *et al.*, 1976; Ffowcs Williams, 1977; Michalke and Michel, 1979; and Michel, 1981). Those uncertainties have led to speculations as to what happens hydrodynamically to a jet in flight compared to a static jet, i.e., a jet running on a test stand (see Cocking and Bryce, 1974, 1975; Bushell, 1975; and Plumblee, 1976).

In a recent paper Obermeier (1981) critically reviewed existing prediction schemes and looked for the roots of their deviations from experimental data. He then suggested the following new method, which takes advantages of scaling laws and similarity arguments.

A quite general representation of the sound intensity I_{jet} radiated by a static jet can be given in terms of multipole expansions,

$$I_{\text{jet}}=N\frac{\rho_j^2}{\rho_o}U^4\frac{D^2}{R_o^2 c_o}\sum_{m=0}\left[\frac{D}{\lambda}\right]^{2m}\phi_m(\theta,T_j/T_o) \tag{18a}$$

or, alternatively,

$$I_{\text{jet}}=N\frac{\rho_j^2}{\rho_o}M^4\frac{c_o^3 D^2}{R_o^2}\sum_{m=0}M^{2m}\tilde{\phi}_m(\theta,T_j/T_o)\ . \tag{18b}$$

Here the characteristic flow velocity U is the jet exit velocity, ρ_j is the density of the medium exhausting from the nozzle, T_j the corresponding temperature, λ a characteristic wavelength of the generated sound field, R_o the distance between the "source" in the jet and an observer ($\lambda \ll R_o$), and θ the observation angle. The functions ϕ_m or $\tilde{\phi}_m$, respectively, are directivity functions of the sound field. N is a measure for the "number" of sound sources in the jet, and m marks the multipole order of the single terms ϕ_m or $\tilde{\phi}_m$; i.e., $m=2$ corresponds to quadrupolelike sources (cold mixing flow), $m=1$ to dipolelike sources (temperature and density effects, interaction between solid bodies and the unsteady jet flow), $m=0$ to monopolelike sources (mass sources, real gas effects in very hot mixing flows). Since, for most observation directions, the spectrum of the radiated jet noise scales with the Strouhal number fD/U, one finds that D/λ is proportional to $U/c_o=M$, which explains the equivalence between Eqs. (18a) and (18b). The directivity functions ϕ_m and $\tilde{\phi}_m$ depend on the multipole order and the geometry of the flow. Furthermore, it is assumed that the jet consists of N sound sources, which generate sound independently of each other, and whose sizes are proportional to D. Therefore, the overall intensity is obtained by adding the intensities of the sound fields of the single sources.

In order to apply Eq. (18) to a jet in a coflowing stream (velocity U_{flight}), one first needs to know how the outer flow affects the characteristic flow quantities U, λ, R_o, the number N of the sources, and the characteristic shear-layer thickness δ of the jet. Relationships applied by Obermeier are summarized in Table I. These results were obtained from dimensional analysis combined with a

TABLE I. Characteristic flow quantities for calculation of in-flight effects: U, mean flow velocity; δ, shear-layer thickness of the jet; λ, wavelength of the generated sound field; N, number of sources; R_0, distance between source and observer at emission time; $\tilde{R}_0$, distance between source and observer at receiving time; $\eta = U_{\text{flight}}/U$. Based on Obermeier (1981).

Static jet	Jet in flight
U	$(1-\eta)U$
δ	$\delta_{\text{flight}} = \frac{1-\eta}{1+\eta}\delta$
λ	$\lambda_{\text{flight}} = \frac{1+M_{\text{flight}}\cos\theta}{1+\eta}\lambda$
N	$N_{\text{flight}} = \left[\frac{1+\eta}{1-\eta}\right]^2 N$
R_0	$\tilde{R}_0 = (1-M_{\text{flight}}\cos\theta)R_0$

substantial use of experimental results. Provided all quantities in Eq. (18) are known, then the in-flight effects on jet noise may be expressed by

$$\Delta\text{OASPL} = 10\log\frac{I_{\text{flight}}}{I_{\text{jet}}} = 10\log\left[(1+\eta)^2\alpha^2\frac{\sum_{m=0}(\alpha M)^{2m}\phi_m}{\sum_{m=0}M^{2m}\phi_m}\right], \tag{19}$$

where

$$\alpha = \frac{1-\eta}{1+M_{\text{flight}}\cos\theta}.$$

This result is equivalent to a formula derived by Michalke and Michel (1979) and Michel (1981). Their derivations, however, are more complex than Obermeier's. Furthermore, some of the essential assumptions proposed in their investigations are at complete variance with those of Obermeier (1981). This could be a starting point for further research on the topic.

Comparing the prediction scheme of Eq. (19) with experimental data, it turns out that the latter are surprisingly well described by the theory. A typical example is shown in Fig. 6, where measured in-flight effects on the noise field generated by a GE-J85 turbojet engine (Bertin Aérotrain) (Drevet *et al.*, 1977; Hoch and Berthelot, 1977) are compared with the predicted effects.

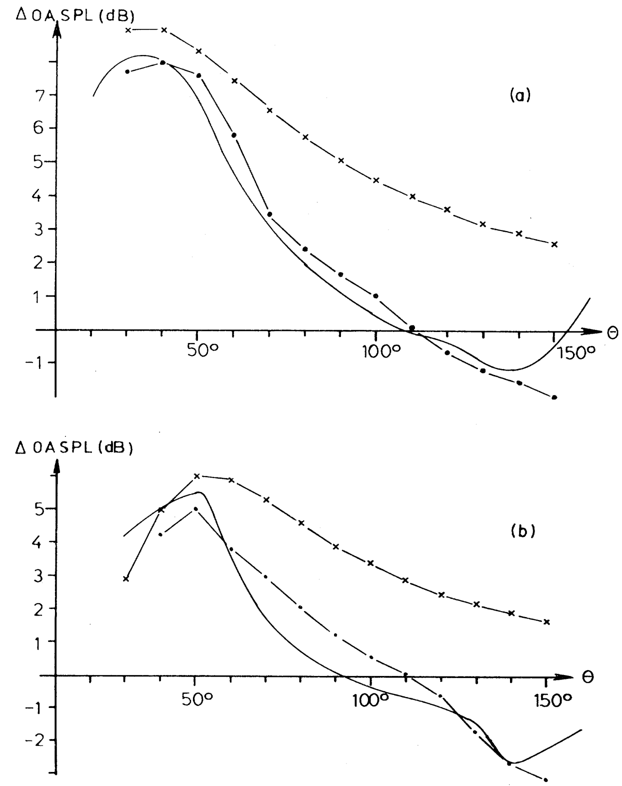

FIG. 6. Reduction of sound generation by a GE-J 85 turbojet engine due to in-flight effects (OASPL, overall sound pressure level). (a) Solid curve, experimental data (Drevet, 1977; $U=505$ m/s, $T_j=802$ K); dot-dashed curve, theory, Eq. (19); ×-dashed curve, theory, Ffowcs Williams (1963). (b) Solid curve, experimental data (Drevet, 1977; $U=625$ m/s, $T_j=940$ K); dot-dashed curve, theory, Eq. (19); ×-dashed curve, theory, Ffowcs Williams (1963).

D. A typical flow oscillation

We should now like to consider one of the complex flow-acoustic processes belonging to group (d) in the list of problems outlined at the beginning of Sec. II, namely, the generation of flow oscillations behind a sudden enlargement of cross section in an air duct. Behind the enlargement a supersonic flow regime is assumed to exist, bounded downstream by a shock.

There are two reasons why this example should prove of interest. First, due to the existence of both subsonic and supersonic flow regimes typical flow oscillations may be observed; second, these oscillations are among the few examples of flow oscillations which have been investigated both experimentally and theoretically to some extent. We shall not deal with the sound produced, since we are here interested only in the oscillation mechanism.

The above-mentioned oscillations in the duct (see Fig. 7) are observed for certain values of the pressure ratio p_a/p_e (Albers, 1979). For other values of this ratio the flow may either be steady, or it may execute other oscillations, for which as yet there exists no theory.

Figure 7 shows in the center two interferograms of the plane flowfield. The middle figures show the extrema of a full cycle of the oscillation, which, in a duct of height 3.3 cm and length 24 cm, have a frequency of several hundred hertz. As may be seen from the sketch at the top

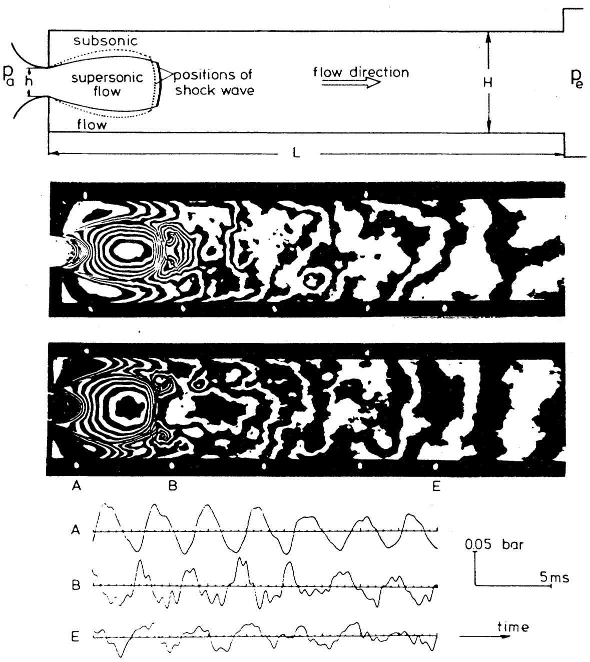

FIG. 7. Interferograms and surface pressure fluctuations: $h=10$ mm; $H=33.2$ mm; $L=240$ mm; $p_e/p_a=0.364$; depth$=100$ mm. See text for discussion.

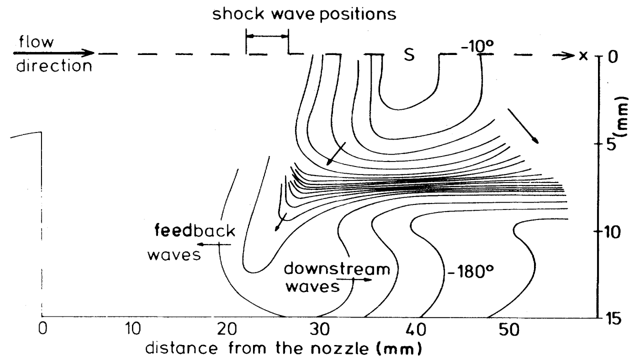

FIG. 8. Constant-phase contours of the local velocity fluctuations (x component): $f=390$ Hz (fundamental oscillation); the angle increment between adjacent lines is 10°; the arrows point in the direction of decreasing phase; S is source of velocity fluctuations. From Albers (1979).

of the figure, the shape of the supersonic flow regime changes periodically with the oscillation cycles, such that a channel of variable width is created, connecting the downstream part of the flowfield (to the right of the shock) and the dead air in the corners of the duct. In this channel a fluctuating flow exists, constituting a feedback from the shock to the upstream part of the flow, i.e., to the whole supersonic flow region in the enlarged duct. This channel is thus the feedback path. It is essential that the supersonic flow does not completely fill the downstream cross section of the duct enlargement, because if it did, with supersonic flow everywhere in the duct cross section, no waves, i.e., no information, could propagate upstream. Because of the important role of the permanently existing separation region in the duct corners immediately downstream of the jump in cross section, these oscillations are termed "dead-air" or base-pressure oscillations. The lower part of the figure shows temporal wall pressure fluctuations, recorded by piezoelectric microphones at stations A, B, and E of the duct wall.

The physical model of the oscillation, as obtained from the interferograms, has been improved by LDV measurements. Figure 8 shows a phase diagram of the downstream component of the fluctuating flow velocity. The phase distribution of these velocity fluctuations shows the region S immediately downstream of the normal shock terminating the supersonic regime to be the source of the fluctuations, and it shows further that the fluctuations may be split into a feedback part and a blowdown part. Anderson and Meier (1982) concluded on the basis of further measurements that the whole length of the duct may be considered as a resonator coupled to the oscillating system, and quantitatively confirmed the feedback mechanism in the channel mentioned above with LDV measurements.

Anderson *et al.* (1978) formulated a theoretical model for this oscillating system, which contains two flows as the essential elements: first, the *supersonic* flow in the center of the duct, described by the strongly simplified basic equations for momentum and energy in integral form; second, the *subsonic* periodic flow through the channel of variable cross section, close to the wall, as calculated from Bernoulli's equation on the basis of the pressure difference between the dead-air region and regions further downstream. The oscillation frequency obtained by the solution of this coupled system of equations is in satisfactory agreement with the observed frequencies for both rectangular and circular cross sections (see Fig. 9). Consideration of the resonant frequency of the part of the duct further downstream shows that an optimal interaction between the oscillating system and the coupled reso-

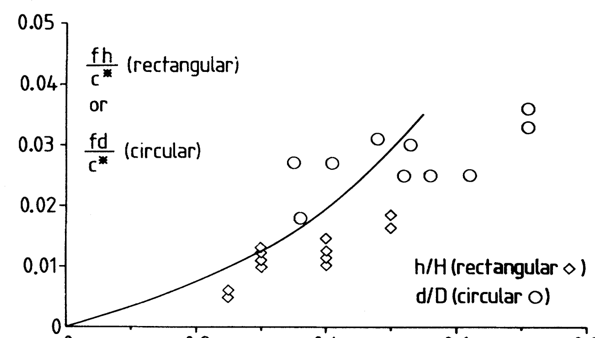

FIG. 9. Frequency f of base pressure oscillations for ducts with rectangular and circular cross sections: $h=10$ mm; $H=33.2$ mm; d, D, diameters of circular duct cross section before and behind enlargement; c^*, critical (sonic) speed of sound; solid curve, theory, evaluated from Anderson (1982).

nator exists only for certain duct lengths. This agrees with the experimental result that large-amplitude oscillations are observed only for certain duct lengths.

III. SOUND PROPAGATION IN SHEAR FLOW

Having described the generation of sound by unsteady flow in the preceding sections, we shall now give a review of the propagation of sound in a steady shear flow. Results of analytical character will be given preference to more numerically or experimentally oriented work; the main point is to illustrate how the presence of flow can affect fundamental acoustic concepts. Other papers surveying this topic are those of Lighthill (1972), Nayfeh *et al.* (1975) (especially for flow ducts), Ribner (1975), and Piercy *et al.* (1977) (especially for atmospheric sound propagation).

It is assumed that the unsteady perturbations $\mathbf{u}', p', \ldots$ of a steady flowfield with a mean velocity profile are small enough that a linearized treatment is sufficient. The governing equation can be obtained from Eq. (15), which describes sound generation and propagation in an arbitrary unidirectional flow. The right-hand side of this equation contains the sources. If this side is replaced by zero, i.e., if there are no sources in the flow, then Eq. (15) describes sound propagation. With appropriate boundary conditions, scattering and diffraction of sound are also governed by this equation, which replaces the familiar wave equation in this more general situation. The most noticeable difference between these two equations is that they are of different order. Equation (15) is of third order, while the wave equation is of second. A second-order equation is also valid for sound propagation in a constant-velocity flow $U_1 = \text{const}$ where the pressure obeys the convective wave equation

$$\frac{1}{c_o^2}\frac{D^2 p'}{Dt^2} - \Delta p' = 0\ . \tag{20}$$

It may seem surprising that the order of the governing equation for sound propagation in unidirectional shear flow is higher than the order for the constant-velocity case, implying that many more solutions occur. Additional light is shed on this fact if one observes that the velocity $\mathbf{u}'$ even for $U_1 = \text{const}$ obeys, not a second-order equation, but the third-order equation

$$\frac{1}{c_o^2}\frac{D^3\mathbf{u}'}{Dt^3} - \frac{D\mathbf{u}'}{Dt} = 0\ . \tag{21}$$

This implies that the set of velocity fields is larger than the set of pressure fields. These additional solutions are the gust solutions, which happen to show no pressure fluctuations in a constant-velocity flow. They always have a nonvanishing vorticity. Thus a decoupling of the vortical gust solutions and the irrotational sound waves has been achieved in constant-velocity mean flow.

The more complicated relation between vorticity and sound waves in shear flow can best be illustrated with the Beltrami vorticity diffusion equation. Let us assume for simplicity the two-dimensional isentropic case. Then the vorticity vector has only one component w, and the Beltrami equation reads

$$\frac{d}{dt}\frac{1}{\rho}w = 0\ . \tag{22}$$

Linearization of Eq. (22) leads to

$$\frac{D\left[\dfrac{w}{\rho}\right]'}{Dt} + \mathbf{u}'\cdot\nabla\frac{\bar{w}}{\bar{\rho}} = 0\ . \tag{23}$$

In a sound wave, $\mathbf{u}'$ does not vanish and propagates with approximately the speed of sound. Hence the second term of Eq. (23) requires that some part of $(w/\rho)'$ propagate with that velocity if $\bar{w}/\bar{\rho}$ is not constant. Sound waves, therefore, generate vorticity fluctuations. For constant $\bar{w}/\bar{\rho}$ Eq. (23) shows that $(w/\rho)'$ is convected with the mean flow velocity. Perturbations propagating with a different velocity, e.g., sound waves, then necessarily have

$$\left[\frac{w}{\rho}\right]' = \frac{w'\bar{\rho} - \bar{w}\rho'}{\bar{\rho}^2} = 0\ . \tag{24}$$

The most important example with constant $\bar{w}/\bar{\rho}$ is the linear shear flow with constant density $\bar{\rho} = \rho_0$. In this case mean flow particles keep their value of $(w/\rho)'$. One would then expect there to be a second-order partial differential equation for perturbations obeying Eq. (24), and one can actually show that the velocity component u_1' obeys

$$\frac{1}{c_0^2}\frac{D^2 u_1'}{Dt^2} - \Delta u_1' = \frac{\partial}{\partial x_2}\rho_0\left[\frac{w}{\rho}\right]'\ . \tag{25}$$

Perturbations fulfilling Eq. (24) are solutions of the second-order convective wave equation. This result has been extended to three-dimensional perturbations and a potential representation for the perturbations derived by Möhring (1976). These general relations are illustrated by an example described in Müller (1976; see Fig. 10). It is shown there that a wave propagating in a linear velocity profile in the x_2 direction is given by

$$u_1' = -\bar{w}F(x_2 - c_o t),\quad u_2' = c_0 F',\quad \rho' = \rho_0 F'\ , \tag{25'}$$

where F is arbitrary and F' denotes the derivative of F with respect to its argument. One observes that the fluc-

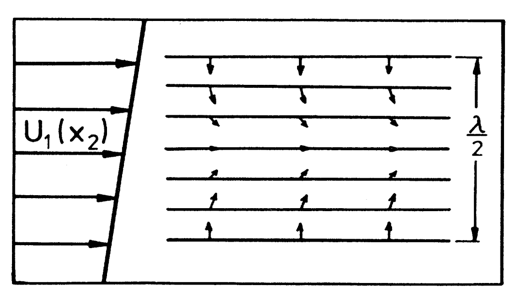

FIG. 10. Phase fronts of the sound wave described by Eq. (25'). The arrows indicate the particle velocities. From Müller, (1976).

tuating vorticity $w'=\bar{w}F'$ obeys Eq. (24), and u_1' obeys Eq. (25).

Equations (23)—(25) show that the linear shear flow is exceptional insofar as a separation of sound waves and gust can be achieved. It may also serve to illustrate the notoriously difficult question of distinguishing between sound and turbulence in a flow. One might be inclined to assume Eq. (24) as a definition of sound in a linear shear flow, which implies the perhaps surprising fact that sound waves are in general not irrotational. Definitions assuming the opposite (Doak, 1972; Yates, 1977) imply that turbulence in low-Mach-number flow may partly propagate with the speed of sound—a rather inconvenient consequence.

The fact that sound and gust are contained in the same differential equation implies a strong coupling between them. An example of this coupling has been studied by Goldstein (1979), who discussed the generation of sound by gust in a shear layer impinging on a semi-infinite rigid plate.

Most applications of Eq. (15) refer to sound propagation in ducts (Fig. 11), with jet engines in mind. For sound propagation problems the sources on the right-hand side of Eq. (15) vanish, and one is left with

$$\frac{1}{\bar{c}^2}\frac{D^3p'}{Dt^3}-\frac{D}{Dt}\Delta p'-\frac{d}{dx_2}[\ln(\bar{c}^2)]\frac{D}{Dt}\frac{\partial p'}{\partial x_2}+2\frac{dU_1}{dx_2}\frac{\partial^2 p'}{\partial x_1\partial x_2}=0\ . \quad (26)$$

A. The modes

The modes

$$p'=f(x_2)e^{i(kx_1-\omega t)} \quad (27)$$

form the main tool for solving Eq. (26), where ω is angular frequency, k is axial wave number, and $f(x_2)$ is mode shape function. The assumption (27) is possible for an arbitrary duct cross section (f then becomes a function of the two variables in the duct cross section), but most work has been done either on plane ducts and on circular or on annular ones. We shall restrict ourselves here to plane ducts. Inserting Eq. (27) into Eq. (26) leads to an ordinary differential equation of second order for f,

$$\frac{d^2f}{dx_2^2}-\frac{2}{\Gamma}\frac{d\Gamma}{dx_2}\frac{df}{dx_2}+(\Gamma^2-k^2)f=0 \quad (28)$$

with

$$\Gamma=\frac{\omega-kU_1(x_2)}{\bar{c}(x_2)}\ .$$

Equation (28) has to be supplemented by boundary conditions which describe the wall behavior. Often a linear relation $p_w=Zv_w$ between the pressure at the wall p_w and the normal wall velocity v_w with an impedance Z is assumed. The effect of liners can often be very well described by this assumption (Nayfeh *et al.*, 1975). Z depends on the frequency, but is independent of k for locally reacting liners. Bulk reacting liners require Z to depend also on the axial wave number k. In terms of the mode shape function f, one then obtains the boundary condition

$$f=\pm\frac{Z\omega}{ikp_o\Gamma^2}\frac{df}{dx_2} \quad (29)$$

at $x_2=\pm b$.

For a given frequency ω, Eqs. (28) and (29) constitute a complicated eigenvalue problem, with the axial wave number k being the eigenvalue. It is not of the classical Sturm-Liouville type, as k appears several times both in the equation and in the boundary conditions. Very few general results from the theory of ordinary differential equations apply to this type of problem. Exact solutions of Eq. (28) are known only for a linear velocity profile (Goldstein and Rice, 1973), where a reduction to parabolic cylinder functions has been achieved. This is certainly related to the above-mentioned exceptional character of the linear profile [compare Eq. (25)]. The influence of small perturbations $\delta U_1(x_2)$ and $\delta\bar{c}(x_2)$ on the eigenvalue can be determined analytically. Möhring and Rahman (1976) have derived the following relation:

$$\alpha\delta\omega+\beta\delta k+\gamma\delta Z=\int_{-b}^{b}A(x_2)\left[\frac{k}{\Gamma}\delta U_1(x_2)+\delta\bar{c}(x_2)\right]dx_2\ , \quad (30)$$

with constants α,β,γ, and a function $A(x_2)$, all expressible in terms of the unperturbed mode shape functions. In Möhring (1973a), α and β have been interpreted as average energy density and energy flux, such that Eq. (30) describes a connection between group velocity and energy-transport velocity. Of course, some of the approximate methods of applied mathematics can be, and have been, applied to Eqs. (28) and (29). Thus Pridmore-Brown (1958) used a high-frequency approximation, and Shankar (1971) assumed small deviations from a uniform profile. Unfortunately, neither method is very well suited to the problem of main interest, which covers comparatively low

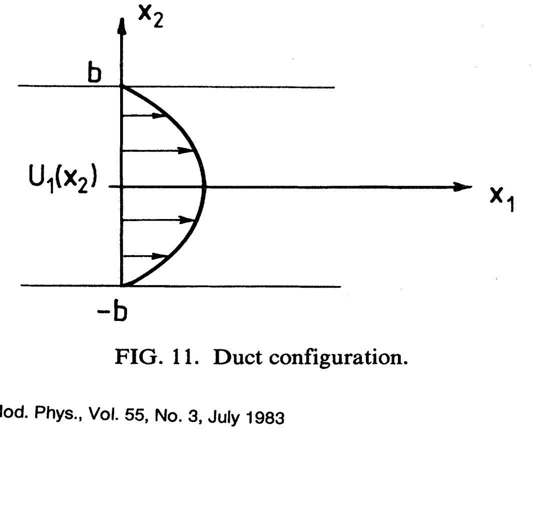

FIG. 11. Duct configuration.

frequencies and velocity profiles significantly different from zero in the inner duct region and vanishing at the wall. A thin-shear-layer approximation has been used by Eversman and Beckemeyer (1972). In addition, many numerical studies have been performed, e.g., Astley and Eversman (1979). The results of these calculations show a behavior that, for subsonic flow, is qualitatively similar to the much better understood no-flow case. There are a finite number of eigenvalues k, with a comparatively small imaginary part (meaning small attenuation), which would vanish for lossless liners (Z purely imaginary) corresponding to the cut-on modes. The higher modes show a rapidly increasing imaginary part corresponding to strong attenuation. They can often be ignored. Of course, significant differences can also be found. One has to assume that $\omega - kU_1(x_2)$ is different from zero, otherwise there would be a singularity in Eq. (8). This excludes a range of k values from the calculations. Another difference is found between upstream- and downstream-propagating modes. In a constant-velocity flow, upstream-propagating modes have the flow acting against them. This reduces their propagation speed and increases their attenuation; the effect is just the opposite for downstream-propagating modes. In reality, however, the large attenuation predicted by calculations based on a constant-velocity profile is not achieved because the counteracting effect of refraction tends to shift pressure fluctuations towards the duct center for upstream-propagating modes and towards the walls for downstream-propagating modes (Ko, 1971). This means that the mode shape functions f are small at the wall for upstream propagation and large for downstream propagation (Fig. 12).

Shankar (1972) observed in his calculations for rigid-walled ducts that the mode shape functions differ by their number of zeros. He found one mode for each given number of zeros, and he used this as a check that all modes had been found. It seems to be unknown whether this is generally true.

B. The representation problem

Once the modes have been determined, one may ask whether every sound field is a superposition of modes. In the no-flow case, where the modes are the trigonometric functions known to be complete and orthogonal, the expansion of an arbitrary, prescribed pressure profile into modes is straightforward. In an arbitrary shear flow no suitable orthogonality relation has been found, and the completeness and independence of the mode shape functions have not been shown. A very elegant formal method for obtaining expansion coefficients has been given by Mani (1980), but the question of whether this method actually gives an expansion of an arbitrary pressure profile into mode shape functions remains open. Shankar (1972) has taken a numerical least-squares approach and observed that the mode shape functions seem to be capable of representing arbitrary pressure profiles.

In view of these difficulties, it may be worthwhile to return to the partial differential equation (26) and to try to solve the initial-value problem. Once again, one is faced with the difficulty that Eq. (26) is of third order, and it cannot be expected that a boundary-value problem comparable to the no-flow case (in which a second-order equation is valid) possesses a unique solution. Fourier transform methods (Swinbanks, 1975) show that an arbitrary solution of Eq. (26) is not just a superposition of modes. One finds additional contributions from all wave numbers k that lead to a singularity in Eq. (28). A singularity appears if there are values of x_2 with $\omega - kU_1(x_2)=0$, i.e., if there are critical layers in the flow (cf. Mani, 1980). The latter contribution is well known in the hydrodynamic stability problem (Case, 1960), where it decays for a long time in cases without excitation. How this contribution behaves in problems with harmonic excitation seems to be unknown. Furthermore, one is faced with the existence of instabilities which have been shown to occur for certain velocity profiles (Lees and Lin, 1946) and for locally reacting walls (Möhring, 1973b). In order to surmount these difficulties one could use the argument of Sec. II.B.2 and assume that these instabilities are not excited.

Looking at the representation problem from this viewpoint raises the question of whether the concepts involved in the use of the second-order wave equation are adequate to describe sound propagation in flows. Perhaps the presence of flow requires a fundamental change in approaching the problems.

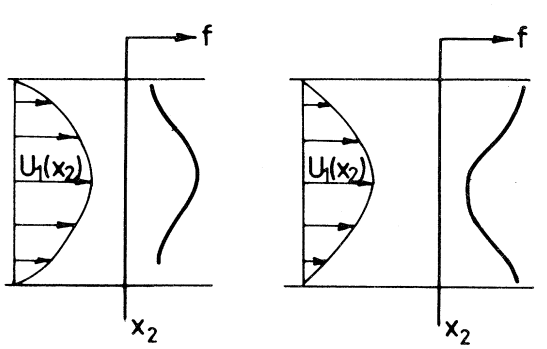

FIG. 12. Sketch of fundamental mode shape function of upstream (left) and downstream (right) propagation.

C. Inhomogeneous ducts

Problems that fall under this heading deal in most cases with sound propagation in ducts having an area change or a finite-length lining. These problems are not well understood, even in the no-flow case. Parker (1966) discovered acoustic resonances in a series of experiments with flat-plate cascades in a duct. Such resonances can lead to violent pressure fluctuations, with resultant structural vibrations and noise radiation. It is expected that similar resonances occur for many geometrical configurations with and without flow (Koch, 1982).

In variable-area ducts, the assumption of unidirectional flow has to be abandoned, and a mean steady flow has to be determined in advance (see, for example, Eversman and Astley, 1981). A one-dimensional treatment can be applied for low frequencies with only one mode cut-on (Davies, 1976). Special features arise when the mean flow approaches the speed of sound. Then a linear treatment is no longer appropriate, and nonlinear terms have to be retained (Myers, 1981). [Some general methods from scattering theory can be applied for sound propagation in rigid-walled, variable-area ducts with subsonic potential flow. For example, Möhring (1978b) has derived an energy-conservation relation and a reciprocity relation (with reversed flow), which often provide a valuable check for the numerical calculations.] Several studies have been performed on a finite-length liner in a constant-area duct with uniform velocity (Namba and Fukushige, 1980; Koch and Möhring 1981; Möhring and Eversman, 1982). They revealed another nonuniqueness, which can apparently be remedied only by carefully prescribing conditions at the edges of the liner. Koch and Möhring (1981) discuss four more or less plausible assumptions about the edge behavior. Carefully controlled experiments are urgently needed in order to clarify which of these assumptions applies.

This survey of problems related to sound propagation in flows shows that ideas from both acoustics and hydrodynamics are being used. The merger of these different disciplines has not as yet been completely achieved, although new light has been shed on both fields. The practical importance of these questions will certainly lead to further research in both areas.

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