Theory of pulsar magnetospheres

F. Curtis Michel

Space Physics and Astronomy Department, Rice University, Houston, Texas

There is a wide range of fundamental physical problems directly related to how pulsars function. Some of these are independent of the specific pulsar mechanism. Others relate directly to the physics of the pulsar and already shed some light on the properties of matter at high density ($\sim 10^{15}$ g/cc) and in strong magnetic fields ($\sim 10^{12}$ G). Pulsars are assumed to be rotating neutron stars surrounded by strong magnetic fields and energetic particles. It is somewhere within this "magnetosphere" that the pulsar action is expected to take place. Currently there has been considerable difficulty in formulating an entirely self-consistent theory of the magnetospheric behavior and there may be rapid revisions in the near future, which is all the more surprising since many of the issues involve "elementary" problems in electromagnetism. One interesting discovery is that charge-separated plasmas apparently can support stable static discontinuities.

CONTENTS

Pulsar, we have pinned you down.
Neutron star that's spinning round.
Your magnet's crooked stellar ash.
How else could spinning make you flash?

Spewing plasma really fast,
Synchrotronning unsurpassed.
Gravitational radiator.
Positron annihilator.

Surface laser, maybe maser;
Comet grazer, giant phaser;
Radiating multipole
With plasma sheets that rock and roll.

Or maybe you are none of these—
Well just what are you, if you please?
Kindly read this little ode,
And blink the answer back in code.

M. A. Pelizzari (1975)

• — • • • — — • — —

I. INTRODUCTION

A. Why would pulsars interest a physicist?

What is there about pulsars (astronomical point sources of regularly pulsed radio emission) that would interest a physicist? Too often one finds that otherwise fascinating topics in astrophysics are cloaked in archaic and arcane terminology. Worse, observation of distant objects is inevitably imperfect (contrast our knowledge of the rings of Saturn before and after the Voyager Mission) whereas laboratory experiment can be refined, retested, and reverified. Worst of all, the resultant lacunae in information tend to be filled in by plausible speculation—a sort of scientific "scar tissue"—which, if left intact too long, firms and can only be excised by heroic measures. On the other hand, laboratory experiments can often be designed to specifically confront such issues. However, it has only been about fourteen years since the discovery of pulsars, and many of the above-mentioned impedimenta are not yet set in place.

Importantly, pulsars seem to be quite common in the sense that, as far as one can tell, almost every object that could be a pulsar, is a pulsar; pulsars are thought to be neutron stars left over from supernova explosions, and the number of active pulsars in the galaxy seems comparable to the rate of supernova explosions multiplied by a "typical" pulsar lifetime (each of these three quantities are, however, only roughly known at present). The point is that, for pulsars to be such common objects, they must function in some more-or-less standard way. This conclusion suggests that we should be able to come up with some sort of "standard model," analogous, say, to the harmonic oscillator or the Fermi-Thomas atom, which does not describe any one specific pulsar (oscillator, or atom), but which describes their properties in general. Thus one need know (or guess) only the essential properties of a pulsar, not the actual properties of any specific pulsar. The latter would be important, but first steps first.

We shall trace the efforts to construct such a standard model. What is surprising is the essential failure of these efforts. There remains no generally accepted model for pulsar emission. Why? To a physicist, such a situation should seem intolerable. "Classical" physics is supposed to be well understood. True, there are subtle points here and there (the three-body problem, etc), but here we lack even a basic model; there are numerous incomplete models with their loyal enthusiasts, but no counterpart to the "simple harmonic oscillator" that is specific, completely solved, representative.

A secondary goal of this review is to provide a reference resource for issues likely to be of interest to physicists. Even today, despite the lack of a satisfactory model for the functioning of pulsars, a large number of "pure" physical investigations have been launched. The high magnetic fields (10^{12} G) attributed to pulsars raise questions of how far energetic photons can travel in such fields without decaying into electron-positron pairs or into photon pairs (a sixth-order process in quantum electrodynamics!). In such fields, the electron cyclotron frequency corresponds to about ten kilovolts, and less energetic electrons are therefore quantized into the lowest ("nongyrating") eandau orbital. The transition rate for excited orbitals is quite rapid, and radiation plays an important role in the plasma dynamics. Atoms become distorted into needle-like objects, changing dramatically the chemistry and solid-state physics of matter in such fields (the magnetic field energy alone within the volume of an iron atom would exceed the entire rest mass energy of the atom itself). And the matter itself is compressed to nuclear densities by the strong gravitational field of the neutron star, opening a new region of solid-state physics at both high densities and high magnetic fields. One of the binary pulsars already has, for the first time, provided observational support for the existence of gravitational radiation, and more extreme binary systems may be discovered to give even more stringent observational information on the strong field limit side of gravitational theory. The mere existence of pulsars has provided important probes of the interstellar medium, its structure, density, and magnetic fields, and has even set limits on the photon rest mass. Favorably located pulsars can also probe the solar corona and the solar wind.

Returning to the physics of the pulsar phenomenon itself, there is a large body of pulsar data that is poorly if at all understood, and it is hard to predict what is actually hidden therein. If we could solve the puzzle of how these natural antennae function, some of these data should decode into new and unique observational information. We do not yet understand what we are looking at.

B. Historical notes

The discovery of distinctly periodic radio-frequency pulses (Fig. 1) from astrophysical sources has already produced a sizeable history of science and has touched scientists in far removed fields. It is now generally believed that such sources, *pulsars,* are rapidly rotating neutron stars. The Crab Nebula is a supernova remnant that is lit up by the pulsar in its center, as best one can tell. Fritz Zwicky had long ago (1938) proposed that such extreme objects might be found in supernova remnants, and, before the discovery of pulsars, it was theorized that a rotating magnetized neutron star could be the source of the nebular energy output (Pacini, 1967). See also Wheeler (1966). It was suggested in the discovery paper (Hewish *et al.,* 1968) that a neutron star might produce the phenomenon, and this interpretation was pursued strongly in the model by Gold (1968). Less familiar is the fact that (hard x-ray) data establishing the existence of a pulsar (in the Crab Nebula) were also in hand even before the discovery of pulsars themselves (Fishman *et al.,* 1969a, 1969b).

It is this rotating magnetized neutron star model that

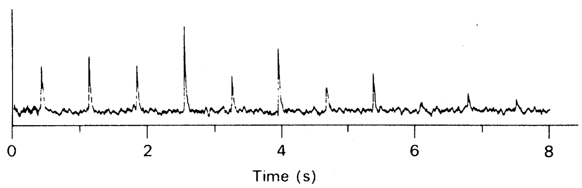

FIG. 1. Chart record of individual pulses from the 0.714 sec pulsar 0329 + 24 at 410 MHz (from Manchester and Taylor, 1977).

we shall discuss. For someone freshly entering the discussion and therefore of a possibly skeptical mind, we shall touch on the other models (and mechanisms) that have been suggested. However, it is worth stressing that the rotator model is intrinsically interesting in its own right in that it combines in a nontrivial way the interaction of rotation with magnetic fields in an astrophysical context. Thus it is a useful starting point whatever pulsars might be.

To begin with, we shall discuss the magnetic field and charged-particle distribution thought to exist about rotating neutron stars, the *magnetosphere* of a neutron star.

We have organized the review as follows: First, we give a minimal review of the pulsar phenomenon, followed by a discussion of the physics of and physical problems with the contemporary models of pulsar magnetospheres. Then we provide a more leisurely (but short) review of the observational evidence, and then explore other ideas of fundamental physical interest concerning pulsars. In this way we hope to satisfy first the reader who is simply curious about the subject and further on the reader inspired to make some contribution. Pulsars raise questions that lead to a surprisingly rich variety of investigations, some of which we shall explore in some detail.

II. PULSARS: A VERY BRIEF REVIEW

A. What are pulsars?

Pulsars are astronomical objects that populate the plane of our galaxy and therefore appear to be concentrated along the Milky Way. Hence they are at stellar distance (hundreds to thousands of light years), and the inverse-distance-squared decline in apparent brightness favors observation of those at hundreds of light years. Pulsars have not yet been observed in other galaxies, although that is, in principle, possible (Bahcall *et al.,* 1970). They emit regular radio-frequency pulses (i.e., at typical UHF television broadcast frequencies). At higher frequencies, they typically become faint rapidly and ultimately unobservable, while at lower frequencies they are difficult to observe for a variety of technical reasons (intrinsic turnover of the spectrum, scattering, ionospheric absorption, etc.). The pulsational periodicity (order of one second) is extremely regular with a "Q" of typically 10^{11}, a very pure tone. However, the pulses are better described in terms of a periodic "window" through which a pulse might or might not be observed; any given pulse might have a wide variety of shapes and amplitudes, as one can see in Fig. 1. All pulsars are observed to be either slowing down, with apparent time scales of typically a million years (this slowing down would limit their Q values to be about 10^{13}, were it not for a small but detectable level of "timing noise"), or changing period too slowly for detection. For these and a number of other reasons, it is thought that the basic pulse period is due to rotation of a star. Periodic astronomical objects generally fall into three classes: rotators, orbiters, and oscillators. The only known candidate possible for a one-second rotation period is the neutron star, namely a star that has collapsed to the point that only *nuclear* degeneracy pressure supports it against self-gravitation and prevents it from collapsing to a point (black hole). The Chandrasekhar mass limit, well known to astronomers, is the maximum mass star that *electron* degeneracy pressure can support. There are therefore three known stable endpoints for stars that have exhausted all internal sources of energy and can no longer be supported by thermal pressure, first as a white dwarf, then as a neutron star, and finally as a black hole, in order of increasing mass. Typical neutron star radii are estimated to be 10 km, and the canontical mass is taken to be just over the Chandrasekhar mass limit of about 1.4 times the mass of our own sun. As for the other possibilities, orbiting objects would have to speed up to lose energy, contrary to the observed slowing down behavior, and oscillating (e.g., radially pulsating) objects rarely have very high Q values.

Consequently it has come to be widely assumed that pulsars are neutron stars rotating at the observed radio-emission periodicity. We shall not neglect the various other suggestions; however, they will be discussed later. Table I lists some pulsars that have prototypical properties.

1. Pulsating x-ray sources

In this review, "pulsar" will refer to objects emitting strong radio pulses. There is another type of object, the pulsating x-ray source, which does not emit detectable radio pulses. The latter object is typically modeled (see Sec. VI) to be a neutron star onto which matter is falling, the x-rays being produced directly from heating as the result of the free-fall energy release and the beaming attributed to an intrinsic magnetic field which controls where the matter falls (e.g., at the magnetic poles). The infalling matter itself is often attributable to Roche lobe overflow from a giant star companion which is undergoing an expansive evolutionary stage. The pulsating x-ray sources are interesting objects in their own right, and may even provide valuable clues to how the radio pulsars function, as well as information about neutron stars, but at the moment the differences between them, both observational and theoretical, outweigh their similarities, and we shall not discuss them in much detail. See Vasyliu-

TABLE I. Some important pulsars.

Designation	Popular name	Notable feature(s)
0531+25	Crab Pulsar (NP 0532) (NP 0531)	Fastest known (0.033 sec) Pulsed emission from radio to γ ray Glitches Obvious supernova association Giant pulses Variations in dispersion measure
0833−45	Vela Pulsar	Supernova association Giant glitches Pulsed optical and γ rays
1913+16	Hulse-Taylor Binary	First binary pulsar discovered Evidence of gravitational radiation
1237+25		Exceptionally complicated pulse shape (5 components)
1641−45		First slow pulsar to show giant glitching
0809+74		Both nulls and drifts
0826−34		Exceptionally wide pulse profile ($\sim 145°$)
1919+21	CP 1919	First pulsar discovered (CP = Cambridge Pulsar, a discontinued designation now)

nas (1981) and Lewin and Joss (1981) for reviews of x-ray emission from neutron stars.

2. Comparison with planetary magnetospheres

Quite a bit of research has been done on the magnetosphere of the Earth, Jupiter, and Saturn. Indeed, Jupiter functions as a pulsar itself, albeit a very weak one (see Sec. VI.A for a discussion of the possible parallelism). Nevertheless there are a number of assumptions made about planetary magnetospheres that are thought to be inapplicable to the pulsar case.

The inner regions of the planetary magnetospheres are characterized by trapped particles (e.g., the Van Allen belts). These particles are trapped by magnetic mirroring, by which they spiral back and forth along the dipolar magnetic field lines, the spiral becoming tighter and finally reversing as the field strength increases toward each pole. In addition to this "bounce" motion, gradient, curvature, and gravitational drifts cause the particles to drift around the magnetic axis. In general the net drift is unrelated to either the direction or the speed of the planetary rotation. The particles themselves are typically quite energetic ($\sim$MeV).

The conventional view of a pulsar magnetosphere simply boosts the magnetic field from about 1 G at the surface to 10^{12}, but this shift has a profound effect on the particle motion because radiative losses, which are relatively unimportant in the planetary magnetosphere, become a dominant consideration. The spiraling particle then almost instantaneously radiates away its perpendicular energy, and without this component, the particle no longer mirrors (even if it did, it would soon radiate away even its parallel energy). To first order then, any "radiation belts" would be precipitated to the surface, leaving a vacuum. Now, however, the stellar rotation comes to play an unfamiliar role as well, by inducing an intense electrostatic field that the particles are pulled from the surface to neutralize. Our pulsar magnetosphere is now filled with cold particles that have been electrostatically lofted and which now corotate with the star, rather than having their own independent drift patterns. The number of particles is also controlled by the electrostatic field, rather than being a more-or-less free parameter. Thus a pattern of thinking has evolved for the pulsar magnetosphere that is quite different from that for the planetary magnetospheres.

B. A little numerology

For a pulsar to be readily detectable by existing radio telescopes it must have a radio luminosity of about 10^{28} erg/sec (our sun puts out 4×10^{33} erg/sec, but in "sunlight," not radio) at 100 parsecs (pc). For such a tiny object to be so luminous at radio frequencies indicates that it must have an important coupling to the electromagnetic field, and an intrinsic dipole magnetic field seems the most plausible assumption. If so, the field strength at the surface must be around 10^{12} G if the observed slowing down rates are to be matched. This calculation, essentially a dimensional analysis, was first given by Ostriker and Gunn (1969a). The torque is estimated from the observed slowing down rate, assuming a solar-mass sphere 10 km in radius, and simply equated to the classical magnetic dipole radiation from a rotating magnetized sphere.

Although calculating the radiation rate from a rotating or oscillating dipole is a standard textbook example, let us recalculate it using typical "astrophysical" approximations. The latter approach has the advantage that the overall physics is not obscured by mathematical details (spherical Bessel functions will not be found here). If the surface field of a magnetized sphere of radius a is B_0, then at a large distance r the field will be dominated by the dipole component, roughly,

$$B=B_0\left[\frac{a}{r}\right]^3 . \tag{2.1}$$

If the sphere rotates at angular rate Ω, rigid rotation of the external dipole field equals the speed of light at the distance

$$R_L=c/\Omega . \tag{2.2}$$

At this distance there must be a transition from the quasistatic near field to the wave field. The two fields must here be comparable, and one can simply estimate

$$B_{\rm wave}\approx B_0\left[\frac{a}{r}\right]^3 . \tag{2.3}$$

The energy loss rate is just the energy density (B_w^2/μ_0) times the wave velocity (c) times the area ($4\pi R_L^2$), or

$$\dot W\sim 4\pi B_0^2a^6\Omega^4/\mu_0c^3 . \tag{2.4}$$

(The exact result is smaller by a factor of $\frac{2}{3}$, simply owing to averaging over the $\cos^2\theta$ radiation pattern.) The quantity B_0a^3 is the magnetic moment, and for $P=1$ sec and $B_0a^3=10^{20}$ weber-meters one obtains (the factor of $\frac{2}{3}$ now included)

$$\dot W=5.77\times10^{31}\ \text{erg/sec} .$$

This is larger than the 10^{28} erg/sec of radio energy output, because only about 10^{-3} of the energy seems to be emitted in the form of radio waves (there is actually a range of radio efficiencies from 10^{-2} to about 10^{-8}). The total energy output seems to be largely invisible, but can be estimated from the rotational energy loss

$$\dot W=I\Omega\dot\Omega$$

of a rotator, which for $I=10^{45}$ g cm^2 (typical neutron star estimate), a one-second pulsar slowing down in 10^6 years ($\approx\Omega/\dot\Omega$) then has an energy output

$$\dot W=1.3\times10^{33}\ \text{erg/sec}.$$

We repeat this simple calculation because it emphasizes the very general nature of the argument. For example, the existence of plasma about the object cannot easily change the above estimate, which is basically a dimensional analysis argument. "How much" plasma one has is broadly parametrized by the Alfvén velocity:

$$V_A = B/(\mu\rho_0)^{1/2} . \quad (2.5)$$

In this form, the Alfvén velocity is the proper velocity $[\gamma\beta c$, where $\beta = v/c$, $\gamma = (1+\beta^2)^{-1/2}]$ and can be as large or small as one wishes. It is evident that our estimate (2.4) is mainly sensitive to where the light cylinder distance is taken to be, which in turn would be "corrected" to be $R_L = \beta c/\Omega$ to include inertial effects from the plasma. We shall see later (Sec. IV.A, Table V) that $V_A >> c$ in the conventional models, hence R_L is essentially independent of plasma density. If the pulsar object were, say, a pulsating white dwarf, a different scaling might well be appropriate, and we shall touch on the alternative theories even though they are not presently in vogue.

C. Observational situation

This section is provided only as a service to those who may understandably have only a vague acquaintance with the pulsar phenomenon. To those wishing a more detailed discussion than the thumbnail sketch here we recommend the books by Smith (1977) and Manchester and Taylor (1977); a much more up-to-date source is provided in the proceedings of the I.A.U. Symposium No. 95, "Pulsars." See also Manchester (1974) for a brief assessment of pulsar properties and problems of continuing interest.

The discovery of pulsars has recently provoked such a furor over who deserved the main credit (e.g., the Nobel Prize) that many readers are probably familiar with the fact that pulsars were discovered by accident by a research student, Jocelyn Bell, under the direction of Anthony Hewish at the Mullard Radio Astronomy Observatory. On 28 November 1967 the array (2 048 dipoles at 81.5 MHz) observed a train of pulses of varying amplitudes but quite regular spacing when "pointed" near $19^h 19^m$ right ascension and plus 21 degrees of declination, or about half a neck's length ahead of Cygnus the swan's head. One can immediately deduce the logic of pulsar designations, this first pulsar now being designated as PSR 1919 + 21.

1. The pulsar luminosity problem

One did not expect to find an intense source of low-frequency radiation that was also pulsed with short periods, but pulsars are just such objects.

Blackbody radiation at low frequencies has a surface emissivity that is well represented by the Rayleigh-Jeans formula,

$$J_0 = \frac{kT\omega^3}{3(2\pi c)^2} \quad (\hbar\omega << kT) . \quad (2.6)$$

Notice that, unlike the *total* blackbody radiation, which increases as T^4, the low-frequency radiation in any fixed-frequency interval $d\omega$ increases only linearly with T. At the same time, it is physically difficult for a source to pulse once a second and yet be much larger than one light second. It is widely taken as a rule of thumb that a source cannot have a pulse period (P) that is short compared to the light travel time across the source. More realistically, the limiting velocity is the longitudinal wave propagation time across the object, so the former criterion is conservative indeed. The radius of the object is therefore assumed to be

$$R \le cP/2 \quad (2.7)$$

and the total integrated luminosity below some observation frequency ω_0 is then

$$L \le L_0 = J_0 4\pi R^2 = kT\omega_0^3 p^2/12\pi . \quad (2.8)$$

As noted above, pulsars are typically at distances of hundreds of light years or more and therefore must have radio luminosities of the order of 10^{28} erg/sec or more. The observation at 81.5 MHz ($\omega_0 = 5.12\times 10^8$ rad/sec) of such luminosity from a pulsar with a period of 1.3 sec then gives a limiting luminosity

$$L_0 = kT(6\times 10^{24}/\text{sec}) \quad (2.9)$$

which requires $kT \approx 10^4$ ergs, hence a formal temperature of about 10^{20} K. And this temperature is certainly a minimum one; if the radiating surface is as small as a neutron star, we require 10^{29} K. Even astrophysicists were reluctant to accept such high temperatures as representing the actual kinetic temperatures of these sources. Instead, it must be assumed that they are somehow radiating coherently. Two common mechanisms for producing coherent radiation are (1) population inversion (e.g., masers and lasers) and (2) particle bunching. We shall discuss these mechanisms in more detail in Sec. IX.

2. Pulsar distances

The immediate question asked about any newly discovered astronomical object is, "How far away is it?" If the source moves across the sky relative to the background stars at an appreciable rate (its proper motion) it must be quite close. The same follows if it moves back and forth relative to very distant stars as the earth orbits the sun (its parallax). By and large, pulsars have neither detectable parallax nor proper motion, which immediately places them out among the "fixed" stars, i.e., at hundreds of light years distance. [Some pulsars now have very tiny observed proper motions, consistent with these large distances and also suggesting unusually high velocities (Manchester *et al.*, 1974.)]

Another distance indicator is association with other objects of known distance. The best example of such an association is the pulsar in the Crab Nebula, PSR 0531 + 21, which is in the center of that supernova remnant. The distance to this remnant is known, since it is young enough (928 years) to expand perceptibly over a number of years, and the Doppler shift of radiating filaments moving toward and away from us can be measured. Thus these two pieces of information immediately give the distance to the nebula, within some small uncertainty over its exact shape, which comes out to be about 2000 pc (Trimble, 1968).

Yet another indicator is the statistical distribution of pulsars in the sky. They are strongly concentrated in a band that is superimposed over the Milky Way. In other words, they are sources in the disk of our spiral galaxy and are seen at distances which are large compared to the thickness of that disk (several hundred parsecs; 1 pc = 1 arc sec of parallax = 3.262 light years).

Most of the above information is of little use for accurately determining the distance to a single isolated pulsar. However the pulsed nature of the emission and the slightly dispersive nature of the interstellar medium combine to provide an index (the dispersion measure, DM) which allows distance estimates to be made for individual pulsars.

The index of refraction of a plasma is just

$$n=(1-\omega_p^2/\omega^2)^{1/2}\approx 1-\omega_p^2/2\omega^2\ , \tag{2.10}$$

where ω_p is the plasma frequency

$$\omega_p^2=e^2 n_e/\varepsilon_0 m_e\ , \tag{2.11}$$

with n_e being the electron density. This form for the index of refraction neglects finite temperature and magnetic fields, but is, however, an excellent approximation for the low densities and weak field strengths appropriate for the interstellar medium. Typical values for n_e are now recognized to be about 0.03 cm^{-3}, giving $\omega_p=10^4$ rad/sec. Since the index of refraction is less than unity, the phase velocity exceeds slightly the velocity of light and the group velocity is slightly less,

$$V_g=nc\simeq c(1-\omega_p^2/2\omega^2)\ . \tag{2.12}$$

Consequently a sharp pulse of radio emission is dispersed so that the high-frequency components reach the earth before the low-frequency components. Pulsars are therefore interstellar whistlers. By measuring the pulse arrival times at different frequencies, one measures the accumulated time difference caused by the difference in group velocity over the path length; thus one directly determines the dispersion measure,

$$\text{DM}=\int n\, dl\ , \tag{2.13}$$

which is conveniently expressed in mixed units of parsecs per cc. Thus a DM of 30, combined with the above estimate for n, implies a distance of 1000 pc. The DM for the Crab pulsar, for example, is 57. The good agreement with independently determined distances to the nebula, about 2 kpc, is only slightly tempered by the fact that this pulsar is one of about 30 used to estimate the average value for n. Nowadays it is becoming possible to reliably correct for variations in n, but that is a separate industry.

For most pulsars, the DM is almost fully attributable to interstellar electrons. The one exception is the Crab pulsar, wherein variations have been observed (Rankin and Roberts, 1971). Evidently there is a variable contribution from the nebula (e.g., wisps moving across the line of sight; Apparao, 1974) or from plasma even nearer to the pulsar. Observed dispersion measures range roughly from about 3 to 300, with 100 being around the median.

Another index, the rotation measure, is a measure of the Faraday rotation experienced and is used to estimate the path-averaged line-of-sight component and distribution of the galactic magnetic field (order of 10^{-6} G; Manchester, 1972; Michel and Yahil, 1973; Simard-Normandin and Kronberg, 1980).

a. The photon mass

We can use pulsar dispersion to put a limit on a mass for the photon. Equation (2.10) can be rewritten in relativistic form [$ck=n\omega$, $\omega^\alpha=(\omega,ck\mathbf{n})$, where $\mathbf{n}$ is the propagation unit vector] to give

$$\omega_\alpha\omega^\alpha=\omega_p^2\ . \tag{2.14}$$

If the photon had a mass ($m_\gamma=\hbar\omega_\gamma$), we would have instead

$$\omega_\alpha\omega^\alpha=\omega_\gamma^2+\omega_p^2 \tag{2.15}$$

and therefore ω_γ must be less than the inferred plasma frequency, hence $m_\gamma\leq\hbar\omega_p\approx 10^{-9}$ eV. For a fuller discussion and refined estimates, see Feinberg (1969), Warner and Nather (1969), Synge (1969), Rawls (1972), and Goldhaber and Nieto (1971). Cole (1976) shows that pulsar observations provide (negative) tests of ether theories. Sadeh *et al.* (1968) similarly show that no "mass effect on frequency" is produced by the sun.

3. The pulsar object

The next question is "What does one look like?" Photographic plates of the best radio positions of pulsars show, at best, blank fields (those cluttered with candidates are, of course, less useful not more). There is nothing to be seen. Pulsars are evidently so faint optically as to be well below the plate limit. That limit is not a firm number but it is about 25th magnitude. The sun at 10 pc would about a 5th magnitude star, so a 25th magnitude star at that distance would be 20 magnitudes or 10^8 times fainter than the sun (5 magnitudes = 100 ×). At 100 pc it would still have to be 10^6 times fainter, and at 1000 pc it would be 10^4 fainter than the sun to appear as a 25th magnitude star.

The exception sorely tests the rule. It is again PSR

0531 + 21 in the Crab Nebula, which is in fact a visible object and appears to be a 16th magnitude star (at 2000 pc!). Moreover the optical emission is pulsed at the same rate as the radio, 33 msec, which is the shortest-period pulsar so far observed. As a visible star, the Crab pulsar has a peculiar featureless spectrum and was nominated many years ago by Minkowski (1942) as being the likely stellar remnant from the supernova explosion (see also Baade, 1942). This explosion was evidently observed and recorded by the Chinese, dating it at 1054 A.D., giving an independent determination of the nebula's age. The Vela pulsar has also been detected at optical wavelengths, at a very weak level of optical emission, close to the plate limit.

4. Pulsar formation in supernovae

A few words are appropriate on how and why supernovae are thought to form pulsars. It is now thought that stars somewhat more massive than the sun (three times or more) are the likely progenitors. Owing to their greater masses, the central pressure and hence temperature of these objects must be higher. Consequently, they evolve much more rapidly because they progressively exhaust internal sources of energy to maintain that temperature, burning first hydrogen to helium, then helium to carbon, oxygen, etc. (once one gets to iron, of course, there are no further exothermic nuclear reactions left). As a result, the central region condenses until the density becomes so great that electron degeneracy pressure, not thermal motion, supports what has now become essentially a white dwarf surrounded by a massive "atmosphere," i.e., the rest of the star. At the surface of this core, however, heat must continue to be produced, and the core steadily increases in mass until it approaches the Chandrasekhar mass limit (Chandrasekhar, 1957). At some point it simply collapses. A vast amount of energy is then released impulsively, owing, for example, to the formation of a shock when the core rebounds (Brown *et al.,* 1981). The rest of the star is thereby ejected leaving behind a roughly 1.4 $M_{\odot}$remnant. See, for example, Arnett (1969). The theoretical details and mechanisms of energy release, energy transfer, etc. are still strongly debated, but such fine tuning is beyond our purposes here. Fortunately the Crab and Vela pulsars (see Table I) are almost beyond reasonable doubt directly associated with supernovae remnants. On the average, pulsars seem to last much longer than the remnants ($\sim$ one million years versus ten thousand years), and the remaining known pulsars are not necessarily expected to be surrounded by a detectable remnant. It is still somewhat puzzling why more known remnants do not have young observable pulsars in them (only the Crab and Vela pulsars seem to be firm examples); however, most supernova remnants are at distances of several kiloparsecs, which are at the upper limit of the distance at which pulsars can be detected (unless they are very bright, like the Crab pulsar).

D. Pulsar properties and statistics

The natural expectation for a distinctive class of objects is that one can eventually learn something about them from their statistical distribution, once a large enough data base is built up. For most pulsars one can measure

- period,
- radio luminosity,
- pulse width (duty cycle),
- slowing down rate ($\dot{P}$),
- subpulse multiplicity (if any),
- radio-frequency spectral features (e.g., index, cutoff, etc.),

as well as other morphological features, many of which, however, may be possessed by only a few pulsars.

In general, nothing correlates very incisively with anything else.

Certainly, there are general trends (e.g., short period with large slowing down rate) that seem physically reasonable, but nothing like a Hertzsprung-Russell diagram. Classification schemes have been proposed (Taylor and Huguenin, 1971; Huguenin *et al.,* 1971; Backer, 1976; Roberts, 1976; Klyakotko, 1977; Kochhar, 1977). One of the more clear-cut distinctions is whether or not the pulsars *null,* a phenomenon in which the pulsar becomes undetectable for relatively long stretches of time (up to 10 to 100 pulse periods) before abruptly reappearing. So far neither warning nor aftereffects of the nulling have been reported in the pulse trains; they disappear and reappear unexpectedly. About 30% of all pulsars null. Another distinctive phenomenon is *drifting,* wherein the time-averaged pulsed is actually composed of one or more narrower subpulses that are seen within the pulse "window" at successively earlier times. About 5% of all pulsars are drifters. A drifter that also nulls (0809 + 74; Unwin *et al.,* 1978) has the interesting property that the drifting subpulse reappears after a null at the same position it had just before the null. Thus the sections containing nulls could be removed from the record and the drifting would appear to be uninterrupted. If such a phenomenon proves to be a general one among drifters that null, it would suggest a close relationship between the two phenomena. Manchester and Taylor (1977) further divide the pulsars into S (simple) and C (complex) according to their average pulse shapes. So far the record for complexity is held by 1237 + 25, which has five distinct subpulses evident in its overall pulse. (It is interesting that one does not see all five subpulses in any single pulse from 1237 + 25; they are apparent in the average waveform, Fig. 29 below, but are not "illuminated" simultaneously.)

1. Periods

One of the slowest pulsars is 0525 + 21 at 3.745 sec (slowest to date is 1845 − 19 at 4.308 sec), while the fastest is 0.033 sec (0531 + 21). They are almost companions in the sky (also having nearly the same dispersion measure), and a number of authors have suggested a common origin for these two pulsars, with 0525 being ejected in the event (Gott *et al.*, 1970; Morris *et al.*, 1978; but see Trimble and Rees, 1971a, 1971b). Wright (1979) discusses the general case for pulsar pair associations. It has been proposed that high spatial velocities for pulsars arise either in the collapse event (Michel, 1970a) or by radiation reaction (Tademaru and Harrison, 1975; Harrison and Tademaru, 1975; Tademaru, 1976, 1977; Morris *et al.*, 1976).

The fast pulsars are rare and the slow ones are common, of course, if the fast ones slow down rapidly. Although very slow pulsars are plausibly dimmer than fast pulsars, they are far too underabundant to be explained by, for example, an Ω^4 luminosity relation (Eq. 2.4). If one plots a P versus $\dot{P}$ diagram for the observed pulsars (Fig. 2) there seems to be a boundary across which pulsars vanish as they evolve to longer periods and (presumably) lower slowing down rates. Many pulsars near this boundary null, so the suggestion is that pulsars turn off not so much by getting dimmer but simply by being off longer and longer, at some point for good. Manchester *et al.*, (1981), however, note that the nulling pulsars tend systematically to be slower, whereas to explain the cutoff

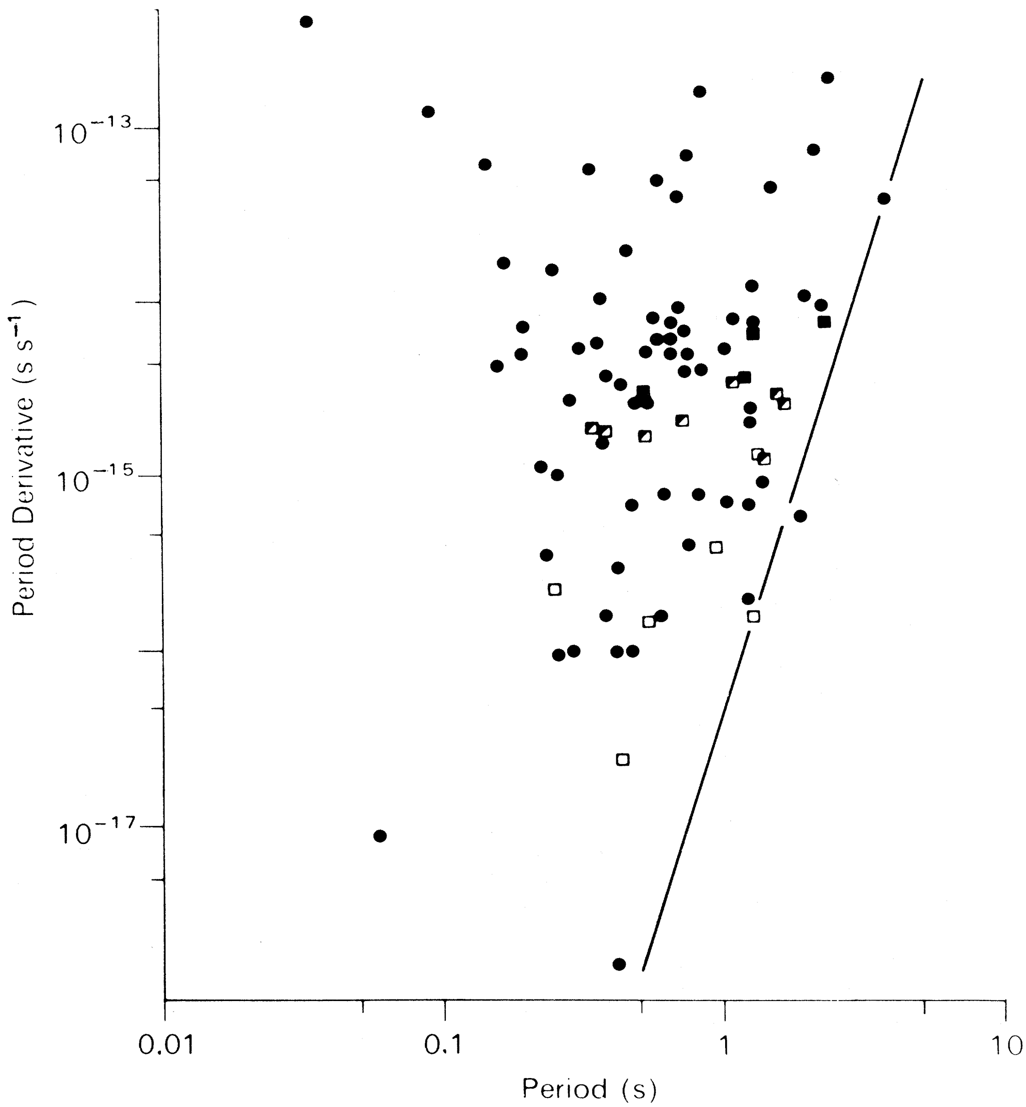

FIG. 2. Period derivative versus period for 87 pulsars (Manchester and Taylor, 1977). Note scattered appearance in form of a "*V*." Crab pulsar is in upper left-hand corner; an $n = 3$ evolutionary history would carry it on a trajectory passing slightly above the core of the distribution. In any event, pulsars are at least known to evolve to the right (and probably down), so the solid line locates an empirical cutoff. This line is consistent with $\dot{P}/P^5 =$ const, which in turn corresponds to a fixed magnetic field (about 2 G) at the light cylinder; the latter is variously taken as empirical evidence for emission at the light cylinder or for pair production to be essential.

in pulse periods, many pulsars must turn off at fairly short periods. Again, pulsar statistics never seem to be as clean as one would hope.

2. Period variations

There are three obvious variations to be expected of a rotating magnetized object: (1) precession or nutation of the rotation axis, (2) variations in the magnetic field, such as those seen in both the earth's field (major changes in millions of years) and the sun's field (the 11-year cycle), and (3) evolutionary changes in the size and shape of the solid body which would change its angular momentum.

It is not hard to find variations in pulsars. The total average intensity of pulsar emission can vary systematically over months or years, as well as the variation from pulse to pulse, the subpulse drift (if any), and the microstructure within the pulses (if any). And then there are the timing "residuals," which are the difference between the actual pulse arrival times and those predicted for a perfect, albeit slowing down, clock.

Nothing much seems to work. There is little in the pulsar data that connects any of the theoretically expected variations with observation, with the possible exception of variation (3), which is exemplified by the starquake hypothesis. Starquakes were originally designed to explain an abrupt drop in the period of the Vela pulsar (a "glitch"). The problem with all of these ideas is basically the expectation that the solid object—neutron star—of a pulsar ought to be very nearly spherical. This means that precession or nutation have little to act upon (a rotating sphere cannot nutate), and there is not enough potential moment-of-inertia change to explain the magnitude-plus-frequency of observed glitches in pulsars, unless we have been lucky in the sense of having seen a string of closely spaced glitches, which are actually rare on the average. Magnetic field changes (variation 2) are unpromising because the entire theory relies on the existence of a quasipermanent magnetic field, and one does not see any systematic change in mean pulse shapes, hence no evidence that this presumably vital factor waxes and wanes.

Table II lists observed glitches. Note that, even for the Vela pulsar, there is a serious problem because the glitches typically have a spin-up of $\Delta\Omega/\Omega \sim 2\times10^{-6}$ and are spaced a few years apart (see Fig. 3). This pulsar has a characteristic spin-down time ($P/\dot{P}$) of 22 700 years, which means that it would be expected to produce about 7000 more glitches. Since the relative moment-of-inertia change, $\Delta I/I$, equals the frequency change, this then implies a total change in moment of inertia of

$$\sum \Delta I/I \sim (2\times10^{-6}\text{per event})\times(7\times10^{3}\text{events}) = 1.4\times10^{-2}\ . \tag{2.16}$$

The centrifugal distortion from sphericity is easily calculated to be

TABLE II. Observed glitches.[a]

Observation	Interval (days)	$\Delta\Omega/\Omega$	$\Delta\dot{\Omega}/\dot{\Omega}$
Crab Pulsar (0531+21) ($P=0.0331^s$)			
1		5.99×10^{-9}	3.36×10^{-5}
2	670	2.99×10^{-9}	1.26×10^{-5}
3	1,285	38.5×10^{-9}	19.82×10^{-5}
4	476	1.04×10^{-9}	0.75×10^{-5}
5	745	2.5×10^{-9}	1.53×10^{-5}
Vela Pulsar (0833−45) ($P=0.0892^s$)			
1		2.34×10^{-6}	1.02×10^{-2}
2	900	1.96×10^{-6}	1.63×10^{-2}
3	1,500	2.02×10^{-6}	1.09×10^{-2}
4	1,000	3.06×10^{-6}	2.81×10^{-2}
1641−45 ($P=0.4550^s$)			
1		2.0×10^{-7}	
1325−43 ($P=0.5327^s$)			
1		1.0×10^{-7}	

[a]Adapted from Alpar *et al.* (1981).

$$\delta \equiv (r_E - r_p)/r_p \approx \Omega^2 a^3/2GM \tag{2.17}$$

where r_E and r_p are the equatorial and polar radii, and a is the average. For the Vela spin rate and nominal pulsar parameters, we then get $\delta = 1.35\times10^{-5}$. The maximum change in moment of inertia is, however, only 2δ, a factor of 500 too small to maintain the observed glitch rate. Clearly such a shortfall will persist even for a more refined analysis. It is possible that some distortion might be "fossil," left over from an earlier rapidly rotating phase. However, it is not expected that the crust could withstand the extreme stresses required to freeze in such huge distortions as would be required. Observation of a glitch in such a slow pulsar as 1641−45 (Manchester *et al.*, 1978) also poses severe constraints on any theory invoking a change in moment of inertia. Recent work therefore stresses coupling between components of the neutron star that might have different rotation rates (e.g., Pines *et al.*, 1980).

The glitches and other irregularities raise serious questions on how pulsars actually slow down. It has proven very difficult to determine the *rate of change* of slowing down. Both ω and $\dot{\omega}$ can be determined to high precision, but $\ddot{\omega}$ is in every case masked by apparently erratic components that are comparable to or larger than any secular term. In the rotator models one expects

$$\dot{\Omega} \sim \Omega^n\ , \tag{2.18}$$

where n is called the deceleration or braking parameter and can be determined from observation, since

$$\ddot{\Omega}\Omega/\dot{\Omega}^2 = n\ . \tag{2.19}$$

From Eq. (2.4), one would expect to find $n=3$. The best determination for NP0532 (Groth, 1975a, 1975b) gives $n=2.5$, while for PSR 0833 the erratic components dominate and no sensible value for n has been inferred. Statistical analysis of many pulsars give $n \sim 4\pm1$ (Ellison, 1975), but such analysis necessarily assumes that all pul-

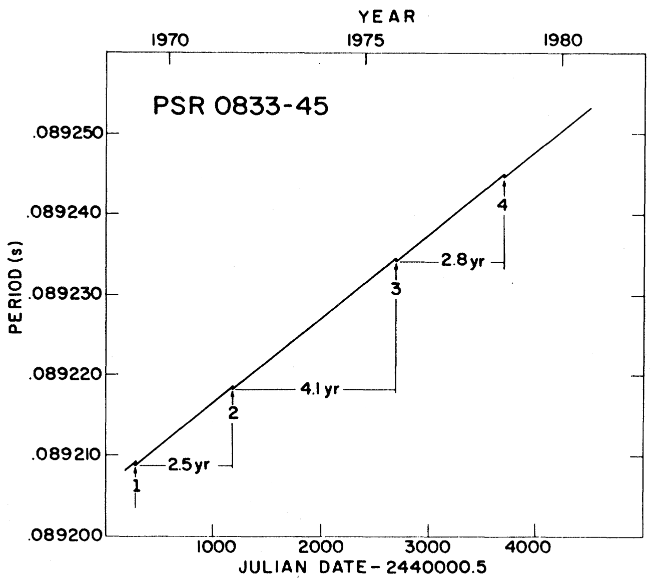

FIG. 3. The Vela glitches (Downs, 1981). For $n=3$ this curve should be a section of a parabola ($P^2 \sim t$) rather than a straight line; however, the deviation would amount to only about one part in 10^3, too small to be seen on this scale and obviously small compared to the variations introduced by the glitches.

sars evolve over a large range of periods, which need not be true (Michel, 1975d). It still seems premature to use observational deceleration parameters to test pulsar theories.

It was suggested that planets might be a source of timing irregularities (Hills, 1970; Michel, 1970c; Treves, 1971a), but long-term observation has not revealed the implied underlying periodicities (even the uncertainties in the solar system planetary masses is a source of concern in reducing pulsar periods to a stationary frame; Mulholland, 1971). Precession has been suggested (Vila, 1969) but discounted (Axford *et al.,* 1970).

3. Radio-frequency luminosity

Although the Crab and Vela pulsars are visibly pulsing (Vela is barely detectable), all the rest are only observed for certain at radio frequencies although gamma-ray pulsations have been tentatively reported from a few pulsars (see X.B. 2).

Pulsars seem to have comparatively little variation in radio-frequency luminosity. It is around 10^{29} erg/sec for the Crab pulsar and hardly less than 10^{26} erg/sec for the weakest known. Thus the radio luminosity, intense in blackbody terms though it may be, seems to be only a "dirt effect" insofar as the overall energy budget of pulsars is concerned.

4. Pulse width

The pulse width tends to be fairly constant. A duty cycle of 1 in 30 would be a serviceable generalization, corresponding to radiation through 12° of "longitude." It would be less model dependent to use the word "phase," but the practice is to use longitude. The exceptions (longer pulse widths, up to about 90°) are not yet well correlated with other properties, although there seems to be a movement towards associating complex—hence wide—pulse profiles with aging pulsars. An extreme exception is 0826−34, which has a double-humped pulse about 145° wide (Durdin *et al.,* 1979). Actually, the pulse seems to be on at almost all phases, although the pulsar itself nulls and is only active about 20% of the time.

5. Slowing down rates

If the rotating magnetized neutron star hypothesis is correct, then slow pulsars should have less energy to give up to radio-frequency emission. However, even here the radio output is estimated to account for only about 10^{-3} of the total power output. The quantity $P/2\dot{P}$ is sometimes called the characteristic "age" of a pulsar. This is the time it would take an initially rapid pulsar to slow down to its present period, assuming $n=3$. This estimate is insensitive to the initial spin period because the pulsar would slow down so rapidly at first. Consequently, the initial spin period of a pulsar could even have been comparable to the present value without greatly changing the pulsar "age." For the Crab pulsar, this age is 1240 years while the historical record gives 928 years. These ages are therefore ballpark estimates, and if the pulsar magnetic field were to have evolved, for example, they could be wildly misleading. Thus, these ages seem most reliable as indicators of the magnetic field strength.

6. Polarization

The radio pulses are in general significantly polarized, usually linearly, but often with important amounts of circular polarization as well. It is difficult to summarize the polarization characteristics; however, there are some general trends. (1) The polarization of individual subpulses is often quite high, approaching 100%. These high polarizations are sometimes averaged out in the time-average pulse shape. (2) Intensity tends to anticorrelate with polarization, with the wings of a pulse being strongly polarized but the core of the pulse only weakly polarized if at all (but there are counter examples, such as 1929 + 10). (3) Many pulsars display a "swing" in the position angle of the linear polarization, corresponding to a nearly constant rate of rotation during the pulse. Also observed are orthogonal mode changes wherein the polarization abruptly changes by 90°, rather than rotating smoothly as above. [A simple model for this effect would be to have two separate (i.e., uncorrelated) sources of emission with orthogonal polarization; then whichever happens to be strongest completely determines the net polarization; see Cheng and Ruderman, 1979.]

E. Mechanisms for coherent radio-frequency emission

The extremely high brightness temperatures of pulsar radio emission (Sec. I) require a coherent source for this radiation. Coherence on such a large scale is surprising, but it does occur elsewhere in nature. For example, the electromagnetic radiation from a lightning stroke is highly coherent, otherwise it would not cause radio interference. A more ominous source of coherent radio waves is the giant electromagnetic pulse (EMP) created by prompt gamma rays from a high-altitude nuclear explosion (the Compton-scattered electrons are all produced nearly in phase to gyrate in the earth's magnetic field; Broad, 1981). Another, less accessible, example is maser action in giant molecular clouds. Most sound waves and water waves are essentially coherent.

For pulsars, three basic mechanisms have been advanced to explain the high brightness temperatures (see, for example, Ginzburg *et al.,* 1968).

(1) particle anisotropy in physical space (e.g., bunches),
(2) particle anisotropy in velocity space (e.g., maserlike action),
(3) true masers.

The first mechanism acts in the case of lightning because the current is bunched, providing coherence by having many charges (electrons) radiating together in phase. Such a mechanism immediately favors low frequencies since, for a bunch of some characteristic size λ, wavelengths longer than λ will be in phase (coherent), and if N particles are in the bunch they radiate like a single particle with charge Ne, hence N times more intensely than the N single particles radiating incoherently. The brightness temperature can then be as high as N times the particle energy; $kT \sim NE$. Another example, possibly *a propos* to pulsars, is the radiation in an electron storage ring (Michel, 1982). Such rings produce useful quantities of incoherent synchrotron radiation at x-ray wavelengths. However, the electrons are stored as bunches (size of order of a centimeter) and consequently radiate coherently at wavelengths long compared to a centimeter. The coherent power output as such wavelengths can become the dominant power loss and causes some important problems in storage ring design.

The second mechanism assumes some peculiarity in the velocity distribution. Roughly speaking, if a Maxwellian fit to the *local* velocity distribution would require a *negative* temperature, then one has what amounts to a classical population inversion; the appropriate wave modes grow exponentially at first and continue to grow until the "local" (in velocity space) temperature becomes positive. The simplest example in plasma physics is the two-stream instability wherein, for example, counterstreaming electrons in a uniform, positive, background charge density excite plasma oscillations (simple longitudinal oscillations about local charge neutrality). These oscillations do not radiate, since they do not happen to propagate, owing to the simplicity and symmetry of this example. However, the negative temperature aspect is easy to see in the example of a one-dimensional velocity distribution of two cold counterstreaming (velocity V_0) beams. The velocity drops precipitously for $V > V_0$ or $V < -V_0$, assuming the beams are very cold. But for $V < V_0$ or $V > -V_0$ the population of fast particles is larger than for slow particles, hence negative temperatures (the only way a Maxwellian can mathematically match such a situation), hence maserlike exponential growth of waves.

A combination of the above two mechanisms is exemplified by the free-electron laser. The coherence basically comes from particle bunches (mechanism 1), but the bunching itself is caused by interaction of the electrons with the radiation field. Hence the exponential growth is from feedback, not population inversion. This type of mechanism apparently amplifies whistler emissions in the Earth's magnetosphere (see, for example, Helliwell *et al.,* 1980). Goldreich and Keeley (1971) have shown that a uniform beam in a storage ring is unstable to bunching, and have suggested that this mechanism may be active in pulsars. See also Asseo *et al.,* (1981), who confirm this analysis and generalize it.

The third mechanism is analogous not with population inversion, but with a true inversion. Here, instead of atoms, one imagines electrons in a strong magnetic field; each electron is in a quantized Landau orbital, one of an equally spaced set of levels. Population inversion could then lead to true maser emission. An early set of theories were based on such ideas (Sec. IX.B.1).

III. INTRODUCTION TO THE MODELS

We now begin our discussion of models for the pulsar object. From the above sketch we have attempted to motivate the assumptions that (1) pulsars are rotating neutron stars, (2) the rotation period is the pulsation period, and (3) the coupling to the radiation field is by a strong magnetic field in the neutron star. Moreover, pulsars are numerous in the sense that they are being born at about the same rate as their presumed progenitors, the supernovae. The neutron star is thus attributed to the explosion. The other two properties, rotation and magnetization, seem to be ubiquitous, hence plausible, properties of all astrophysical bodies. The theories bifurcate according to whether these properties, alone, are taken to be sufficient for pulsar action ("vacuum models"), or whether yet additional factors are deemed essential. Again, since pulsars are so populous, any supplementary requirements must have high *a priori* probabilities. For example, there might be additional matter near the pulsar, such as a disk, that is essential to pulsar action.

As mentioned in Sec. I., oscillating and orbiting models have fallen into disfavor, and with them the idea of extraneous matter near pulsars. Careful timing of the pulse arrivals has ruled out the presence of significant companions (e.g., planets; Lamb and Lamb, 1976) which could neither be close to the pulsar, since tidal forces would disrupt them, nor far, since then the motion of the neutron star relative to the barycenter would be large and therefore detectable as a cyclic arrival-time shift in the pulses. For example, the Crab pulsar has a pulse only a millisecond or so wide, and the precise location of a long string of such pulses can be determined to, say, one percent of that value. Consequently a periodic displacement of the pulsar by only three kilometers would be detectable. A planet one hundredth the mass of the Earth placed as close as the Roche limit (about 1.7×10^6 km) could thus be detectable, and more distant objects would have to be proportionately less massive not to be seen. Furthermore, it has been plausibly suggested (Ostriker and Gunn, 1971) that intense radiation from a rapid, newly born pulsar might even create the supernova explosion by accumulating in the collapse cavity, hence driving away and exciting the envelope of the preexisting star. At the same time, more and more pulsars have been discovered, few of which have companions (it appears that about 1% of the radio pulsars are in binary systems, 3 out of 300 to date). Pulsars also seem to be much-higher-velocity objects than the other stars that populate the galactic disk. Taken together, these various ideas and observations suggest that the supernova explosion was so violent and the resultant pulsar so energetic, possibly even recoiling from the event, that the neutron star would be stripped clean. The popularity of the vacuum models (vacuum in the above sense; it seems necessary that some material be continuously ejected from the neutron star to explain the pulsar action) therefore seems well founded. Nevertheless, it has recently been proposed that material does remain about the neutron star in the form of a disk and that this disk is essential for pulsar action (Michel and Dessler, 1981). We shall therefore discuss first the vacuum models and then touch on the disk model.

Table III lists some early classic papers. Pacini (1967, 1968) anticipated that a rotating neutron star could power the Crab Nebula, before the discovery of pulsars. Gold (1968) suggested that a bunch of electrons ($\sim10^{22}$ within a one-meter sphere) corotated with the pulsar, trapped by the pulsar's magnetic field and located near the aforementioned light cylinder (i.e., where corotation would be at the speed of light, $R_L=c/\Omega$). Unfortunately, radiation reaction would cause the bunch either to be ejected or to retreat from the light cylinder, even if it could be formed in the first place. See also Good (1969). Nevertheless the idea of bunching as a mechanism for coherence seemed sound and influenced future work. Goldreich and Julian (1969) initiated work on the aligned rotator and showed that electrostatic forces would pull plasma off the pulsar surface to fill the magnetosphere (see also Michel, 1969a). This space-charge density estimate appears frequently, even in quite different models, and is called the Goldreich-Julian density. Their model concentrates on the physics of the aligned rotator and is not a model for pulsar action *per se.* Ostriker and Gunn (1969a) (or Gunn and Ostriker, 1969) basically introduced the idea of intense magnetic fields ($\sim10^{12}$ G) by equating the energy loss from pulsars to that of a rotating magnetic dipole, again not a pulsar model *per se.* Sturrock (1970, 1971a) introduced the first "modern" pulsar model: particles continuously ejected from the magnetic poles *(polar caps;* an idea implicit already in the Goldreich-Julian model) at a controlled rate *(space-charge-limited flow)* with electrons radiating gamma rays because they must follow curved field lines *(curvature radiation),* and with the gamma rays pair-producing in the magnetic field *(pair-production cascade).* Coherence is attributed to bunching in the counterstreaming electron-positron plasma. Ruderman and Sutherland (1975) elaborated and improved upon the Sturrock model (the original way of handling the space-charge limitation is now known to be flawed: see Michel (1974c), Fawley *et al.* (1977), and Sec. III.B.5, below). Goldreich (1969) seems to have first suggested the possible importance of pair production, but never elaborated upon it. The Ruderman-Sutherland reformulation has received much more attention than did Sturrock's model in its time, possibly because it addresses some of the observational puzzles that arose in the interim (e.g., drifting subpulses are attributed to localized discharges, "sparks," that drift systematically about the polar caps). See Table IV for a sketch of current theory versus expectation.

We shall first discuss the simplest version of the above models.

A. Observational constraints

The central problem of pulsar theory is to explain how the mechanism for such intense coherent radio emission would arise spontaneously in the neutron star formation

TABLE III. Classic papers on pulsar theory.

Author(s)	Date	Model
Pacini	1967	Prediscovery prediction
Gold	1968	Circulating particle bunch
Goldreich and Julian	1969	Aligned rotator and plasma source
Ostriker and Gunn	1969	Oblique rotator and magnetic field estimate
Sturrock	1970	Aligned rotator and pair production
Ruderman and Sutherland	1975	Extension of Sturrock model (gap, sparks)

TABLE IV. Theory versus expectation in pulsar models.

Consideration	Standard model	"Actual" pulsar[a]
Rotation = Period	Yes	Yes
Strong magnetic field	Yes	Yes
Alignment	Yes	No
Coherent radio emission	Not necessarily	Yes
Pair production	No[b]	Yes
Ions from surface	Yes	No consensus

[a]According to conventional wisdom, namely a consensus based on lack of better information.
[b]Pair production is sometimes discussed, but not in any self-consistent model as of yet.

event (hence what mechanism it is). The next problem is to decipher the large amount of observational data to see if one can understand how pulsars differ from one another and to organize this data. Before either of these tasks can be performed, however, it is necessary to account for the overall gross energetics. The Crab pulsar has become a prototype, and the feeling is that if this pulsar can be explained, all pulsars can be explained (although there are those who worry that any one specific object could have very unusual morphological properties and hence could not really be representative). Thus one needs to explain the slowing down of pulsars, which translates into accounting for the loss of rotational energy in the rotating neutron star model. Moreover, it would be nice to account for the excitation of the nebula around the Crab pulsar, the energy output from which is, within uncertainties, comparable to the rotational energy output from the pulsar. The nebula contains electrons too energetic to have been left over from the supernova explosion and a magnetic field too strong to be simply the adiabatically expanded remnant of any conceivable internal stellar field. The pulsar is therefore the plausible source of the excitation, the particles, and the magnetic field.

The sign of the magnetic moment determines whether electrons or positive particles tend to flow out along field lines from the polar caps. Since there is no *a priori* reason for one sign or the other sign, can this dichotomy show itself in two distinguishable pulsar families? This interesting question promises to be with us for some time. The observed pulsars do not seem to fall neatly into two distinct groups, and perhaps one of the two cases does not pulse significantly (or this sign makes no difference).

B. Dimensional analysis

A conducting star rotating through its own (dipole) magnetic field creates an induction (quadrupole) electric field in a vacuum which would initially give

$$E_{\|}=(\underline{E}\cdot\underline{B})/B\sim a\Omega B_0\left[\frac{a}{r}\right]^4\cos^3\theta\ . \tag{3.1}$$

However, Eq. (3.1) gives a force greatly exceeding gravity on any charged particle near the pulsar surface (see Table V below). Thus the electrostatic forces are expected to ultimately create a space charge such that $\underline{E}\cdot\underline{B}\rightarrow 0$, which in turn suggests that the magnetic field lines become equipotentials. Figure 4(a) shows the magnetic (solid) and electric (dotted) field lines for a rotator in a vacuum, while Fig. 4(b) shows the modified electric field lines, if the magnetic lines become equipotentials and the appropriate space charge surrounds the rotator. If the magnetic field lines are equipotentials, then it immediately follows that the plasma motion (which is just the $\underline{E}\times\underline{B}/B^2$ drift velocity, neglecting any motion along field lines) corresponds to *rigid corotation* of the plasma with the pulsar. This result is easily demonstrated as shown in Fig. 5. Here we have two field lines, differing in electrostatic potential by $\Delta\Phi$ and confining a magnetic flux Δf between their respective surfaces of revolution (often termed "magnetic shells"). If the perpendicular distance between the two is S (at some arbitrary point), then it follows that

$$E\sim\Delta\Phi/S \tag{3.2}$$

and

$$B\sim\Delta f/Sr\sin\theta\ . \tag{3.3}$$

Thus the azimuthal drift velocity is just

$$V_\phi=E/B\sim r\sin\theta\ , \tag{3.4}$$

and the constant of proportionality is determined at the surface by the condition that any free conduction electrons move with the surface material. Thus we obtain the important results

$$V_\phi=\Omega r\sin\theta\quad\text{(rigid corotation)} \tag{3.5}$$

and, moreover,

$$\Delta\Phi=\Omega\Delta f\ . \tag{3.6}$$

(Note that the magnetic vector potential is just $A_\phi=f/r\sin\theta$.) These results are sometimes termed "Ferraro's law of isorotation" (see Ferraro and Plumpton, 1961). Equation (3.6) looks innocuous enough, but it has proven to be a difficult boundary condition to

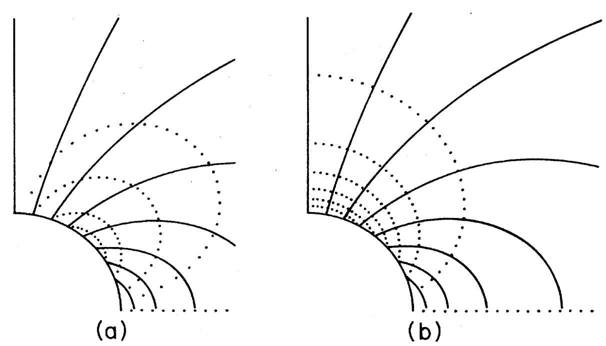

FIG. 4. Electric field lines (dotted) about an aligned rotator (solid lines are the dipole magnetic field lines) for the vacuum case (a) and for the Goldreich-Julian case (b).

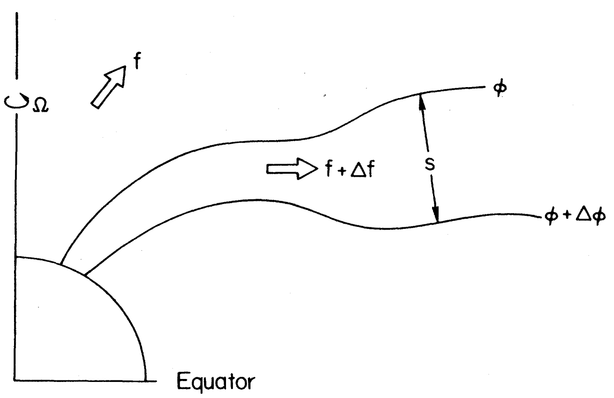

FIG. 5. Magnetic field lines as equipotentials. The field lines can also be labeled, given axial symmetry, by the total magnetic flux (f). The flux Δf between two field lines of potential difference $\Delta\Phi$ is therefore geometrically in a fixed ratio along each field line.

satisfy, as we shall detail.

Rigid corotation must fail, even for massless particles, if $V_\phi > c$ or at an axial distance

$$\rho = r\sin\theta \geq c/\Omega \,, \tag{3.7}$$

which defines the *light cylinder* distance (Gold, 1968).

The most elementary and fundamental requirement of any electrodynamical system is that it conserve current. In a steady-state system, then, the net current to or from an object must be zero. It turns out to be quite difficult, however, to theoretically model a rotating magnetized star, particularly if it has an aligned dipolar field, to give a closed current flow. The basic problem is that, while electrons (say) could easily be ejected to infinity along polar field lines, the positive particles are instead injected on the strong closed field lines near the star. Evidently, then, the system would become positively charged. One would expect this positive charge to modulate the emission over the polar caps; however, once the stellar charge became large enough to do that, electrons would be unable to escape the system (they could be ejected from the surface, but not with sufficient energy to escape the system). We shall explore in much more detail, below, this current closure problem. There is growing support, in fact, for the view that it has no solution because the system simply ends up trapping both electrons and ions (or positrons).

If $E > cB$, it follows that the particles would cross equipotential surfaces. The canonical assumption of the standard model is that this does not happen (a point which we shall return to), and therefore the field lines are obliged to acquire an azimuthal component with distance, which adds to B^2 without changing $\underline{E}\times\underline{B}$ and thereby keeps V_ϕ less than c. If this did not happen, the strong currents from particles crossing field lines would presumably act to create such a component. Such a magnetic field component requires currents flowing in the meridional plane and therefore flow parallel to the magnetic field lines in that plane. However, these currents cannot close (e.g., they cannot flow out of one hemisphere, follow a closed dipolar field line back to the other hemisphere, and finally flow through the star to the original hemisphere) in the absence of electromotive force to counter ohmic dissipation in the solid body of the pulsar. It would then seem to follow that currents flow only on open field lines, and that all field lines that cross the light cylinder are open. We mention below some counterarguments.

It is quite traditional to separate radiation fields from static fields, even though the distinction is not always sharp. For example, one calculates the structure of the hydrogen atom using just the static Coulomb field and then one calculates the lifetimes of excited states by treating the coupling to the radiation field as a perturbation. A few pulsar models are, in effect, based on the argument that such separation is unphysical. In the model of Mestel, Phillips, and Wang (1979), for example, it is proposed that a closed current can flow out from the neutron star and return. To do this, the particles cross field lines near the light cylinder as a result of radiation there. If the coupling to the radiation field could be "turned off" (e.g., by reducing e/m of the particles), this mechanism would be ruled out. Jackson (1981) has also proposed a model with closed currents flowing as a consequence of radiation, but not necessarily radiating just at the light cylinder. See also Jackson (1976a, 1976b) and Rylov (1978). Such models pose an interesting challenge to physical intuition. Is it possible to find examples of such radiational "bootstraps," namely dynamic systems that function and radiate only because they radiate?

Figure 6 shows schematically the resultant structure of the standard model, with a corotation zone of closed field lines and a magnetized stellar wind flowing out of the polar caps. Mestel (1966) gave a quite similar qualitative picture.

Several important results can now be gotten without more detailed analysis.

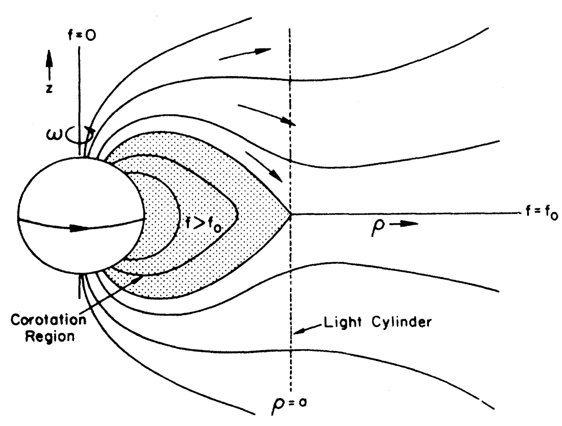

FIG. 6. Schematic of supposed magnetic field structure around an aligned rotator. Dashed vertical line locates the light cylinder. The field line f_0 is the "last open field line." Shaded region contains the closed field lines (Michel, 1974b).

1. Magnetized stellar wind

It is clear from Fig. 6 that, as a first approximation, we could take the corotation zone boundary to be dipolar, in which case the magnetic flux may be estimated from

$$f = f_0 \rho^2 / r^3 \ , \tag{3.8}$$

and for the first field line to the light cylinder $\rho = c/\Omega = r$; thus

$$f_E = \Omega f_0 / c \tag{3.9}$$

is the magnetic flux escaping to infinity from each polar cap. The polar magnetic field strength at the pulsar is

$$B_p = 2 f_0 / a^3 \ , \tag{3.10}$$

where $r = a$ is the radius of the pulsar. If we look down from a pole at the field lines, we see that they must spiral (to keep $V_\phi \le c$) as shown in Fig. 7, since the plasma must flow radially outward at velocity slightly less than c.

Since the magnetic flux f_E is trapped between consecutive sprirals, it is clear that the magnetic field becomes azimuthal and

$$B \to B_\phi \sim \Omega f_E / cr$$
$$= \Omega^2 B_p a^3 / 2c^2 r \ , \tag{3.11}$$

where cr/Ω is roughly the area through which the flux must pass (we are here neglecting, of course, factors of order unity which describe the distortion of the corotation region and the detailed distribution of the flux in the meridional plane).

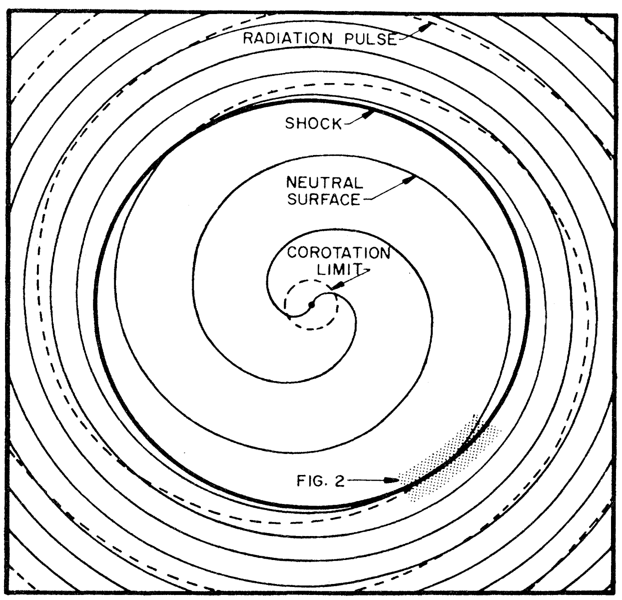

FIG. 7. Asymptotic magnetic field structure projected on equatorial plane, showing spiraling of the magnetic field lines (Michel and Tucker, 1969).

Thus, for example, the pulsar in the Crab Nebula should contribute a nebular field of about

$$B \sim 2 \times 10^{-5}\ \mathrm{G} \tag{3.12}$$

for $B_p = 10^{12}$ G, $a = 10^4$ m, $\omega = 200$ rad/sec, and $r = 1$ 1 yr$=10^{16}$ m. The magnetic field should be greatly enhanced by shock and magnetohydrodynamic compression as the wind interacts with the nebular material (Michel, 1969a; Piddington, 1969; Rees, 1971b; Rees and Gunn, 1974) and seems consistent with observational estimates for the nebular field. There, the nebular magnetic field is estimated from the spectrum and intensity of the continuum synchrotron radiation from the nebula; the total energy (particles plus field) is a minimum for $B \sim 10^{-3}$ g and rapidly rises for either very much weaker or very much stronger fields (Burbidge, 1956). Moreover, the above analysis for the nebula concludes that the radiating electrons must have an energy of about 10^{11} eV, which is also that obtained below (Eq. 3.29), reinforcing confidence in the general approach.

2. Torque on the pulsar

We have implicitly ignored the energy carried by the particles; thus the total energy loss rate is at least that in the Poynting flux away from the pulsar, or

$$P = \frac{1}{\mu_0} \int (\underline{E} \times \underline{B}) \cdot d\underline{S} \approx \frac{1}{\mu_0} c B_\phi^2 4\pi r^2$$
$$= \pi \Omega^4 B_p^2 a^6 / \mu_0 c^3 \ , \tag{3.13}$$

which is, apart from a factor of $\frac{2}{3}$, also the electromagnetic power that would be radiated by a dipole moment $B_p a^3$ rotating orthogonal to the moment axis. Separate calculations (Sec. VII.A) indicate that the particles may in fact carry rather little energy. Thus one result is that the net torque is largely independent of the moment-spin orientation and is proportional to Ω^3. For an orthogonal magnetic moment the outflow is in the form of large-amplitude electromagnetic waves, while for the aligned case it is supposedly in the form of a stellar wind. (The same scaling argument follows for a stellar wind as for the dipole radiation case, since in either case the asymptotic flux to infinity would be those field lines crossing the light cylinder; however we shall see that there are some problems with existing pictures of wind production.) The expected slowing down behavior is then

$$\dot{\Omega} \propto \Omega^n, \ \text{with } n = 3 \ . \tag{3.14}$$

Kaplan *et al.* (1974) have argued against this view, citing effects from turbulent, ultrarelativistic plasma near the pulsar and obtaining $n = 3.4$.

3. Nebular excitation

In the frame of reference of the (essentially stationary) nebular material, the pulsar wind contains an electrostatic field

$$E \sim cB_{\phi} \ , \tag{3.15}$$

and consequently dissipative currents will be excited which act to stop the flow (and accelerate the nebular material). As a result, a substantial fraction (if not all) of the pulsar wind energy should be dissipated in the nebula.

In the case of the Crab pulsar, there is in fact good agreement between the total luminosity of the nebula itself and the inferred total luminosity ($I\Omega\dot{\Omega}$) of the pulsar, both being about 10^{38} erg/sec. A comparable amount of energy may be deposited in the kinetic energy of expansion of the nebula (Trimble and Rees, 1970). (The total pulsed luminosity of the pulsar, largely in x and γ rays, is about 10^{-1} of this grand total, whereas the pulsed radio luminosity is only about 10^{-8} of the grand total.)

4. Particle injection rate

If the field lines are equipotentials, the plasma corotates with

$$\underline{V} = \underline{\Omega} \times \underline{r} \ . \tag{3.16}$$

Thus we can immediately write

$$\underline{E} = -\underline{V} \times \underline{B} \ , \tag{3.17}$$

and

$$q = \epsilon_0 \underline{\nabla} \cdot \underline{E} = 2\epsilon_0 \underline{\Omega} \cdot \underline{B} \tag{3.18}$$

is the space charge, corresponding to a number density of

$$n = q/e \ , \tag{3.19}$$

where e is the elementary charge of the plasma particles. Since the particle rest-mass energies are small compared to the electrostatic potential differences, the particles should flow at essentially c in response to these strong electromagnetic forces. Thus the particle flux is nc, and this flux flows along the open field lines. A dipole field line satisfies

$$f = \sin^2\theta / r = \text{const} \tag{3.20}$$

so that the polar cap radius $\rho = a \sin\theta$ at $r = a$ is intercepted by the same field line that extends to the light cylinder, $R_L = c/\Omega$, if $\rho^2 = \Omega a^3 / c$. Thus the *polar cap* area is just

$$A = \pi\rho^2 = \pi\Omega a^3 / c \ . \tag{3.21}$$

Altogether then (two polar caps contribute) a particle flux

$$\dot{N} = 4\pi\varepsilon_0 \omega^2 B_p a^3 / e$$
$$= 1.3 \times 10^{34} \text{ particles/sec} \tag{3.22}$$

would result from the Crab pulsar (here B_p is taken to be 6×10^{12} g, as required to give the correct total Crab luminosity). A much larger flux of 10^{41}/sec (Shklovsky, 1968, 1970, 1977) is frequently quoted, which is obtained if one assumes that the energy input into the nebula is entirely in the form of energetic electrons. Here, to the exact contrary, most of the energy would be in the Poynting flux of the magnetized stellar wind (Michel, 1975d). In this latter case the energetic electrons could be produced by local reacceleration, energy being transferred from the electromagnetic field to the particles as a result of the wind stagnation upon interaction with the surrounding nebular shell.

5. Electron energies: Space-charge-limited flow

As we have just seen, a plausible estimate can be given for the injection rate from the pulsar. Given this flux, we can estimate the particle energies.

For a vacuum tube diode, there is a fixed current that can flow in response to a fixed plate potential, regardless of how hot the filament is. In the same way, the current flowing from a pulsar is directly related to the accelerating potential; the assumption that plasma be freely available from the surface does not imply that unlimited currents can flow. We can simply invert this fact here and use the current [Eq. (3.22), essentially] to calculate the accelerating potential. In other words, the field lines are equipotentials only if sufficient space-charge plasma is present, and if not, an appropriate acceleration potential develops to provide it. Knowing the loss rate then allows one to estimate the required potential and hence the particle energy.

a. Vacuum tube analogy

Sturrock (1971a) originally used exactly the above analogy to (over) estimate the electron energy. If in a one-dimensional problem one has a current J_0 of relativistic electrons, the space-charge density is just J_0/c, and the accelerating potential is just

$$\Phi = J_0 h^2 / 2c\varepsilon_0 \ , \tag{3.23}$$

where h is the linear distance (i.e., height above the surface). One now estimates h to be the radius of the polar cap, using Eq. (3.21). Here the argument is that geometric divergence limits the validity of Eq. (3.23) to this distance, which is just an approximate way of solving the actual three-dimensional electrostatics (another "astrophysical" approximation).

b. Differential space charge

Michel (1974c) pointed out that method (a) above greatly overestimates the potential because the "current" J_0 was estimated in the first place by multiplying the Goldreich-Julian density by c. But this density is already consistent with $\underline{E} \cdot \underline{B} = 0$, hence yielding *zero* accelerating potential (Tademaru, 1974). Thus the accelerating potential arises only from deviations from this density, not from the density itself. The basic consequence, as discussed below, is that the potential does not increase quadratically, but only linearly. In fact, the differential space charge is almost all confined to a thin sheath (Buckley, 1975) at the surface, within which the particles are accelerated to relativistic velocities. Fawley *et al.* (1977) have carefully reviewed and reconfirmed this result.

The derivation is straightforward, using the ansatz that it is only a charge difference from the Goldreich-Julian density (now q_0) that leads to an accelerating potential. We have then, in one dimension, Poisson's equation for the accelerating component of the electric field,

$$\frac{d^2\Phi}{dx^2}=(q_0-q)/\varepsilon_0 . \tag{3.24}$$

If we write the particle energy as $e\Phi=(\gamma-1)mc^2$ and q_0 as $2\varepsilon_0\Omega B$, we have a natural length scale

$$\lambda=(mc^2/4e\Omega B)^{1/2} , \tag{3.25}$$

which is less than about one millimeter using Crab pulsar parameters. Electrons gain an energy of mc^2 in traversing this distance. The resultant equation reads, writing $d\gamma/dx$ as p and $d^2\gamma/dx^2$ as $\frac{1}{2}dp^2/d\gamma$,

$$\lambda^2 dp^2/d\gamma=q/q_0-1 . \tag{3.26}$$

If we set $q=q_0$ at large distances, the particles having attained an asymptotic Lorentz factor γ_0, the right-hand side vanishes, which gives, asymptotically (Michel, 1974c), using $q/q_0=\beta_0/\beta$,

$$\begin{aligned}(\lambda p)^2&=\int_1^{\gamma_0}(\beta_0/\beta-1)d\gamma\\ &=1-\frac{1}{\gamma_0}\approx 1 .\end{aligned} \tag{3.27}$$

What has happened is that the acceleration of electrons from rest to relativistic velocities always leaves behind a charge layer wherein $|q|>|q_0|$, and the electric field from this charge layer is not cancelled out because $q\rightarrow q_0$. In three dimensions this field is not constant, but vanishes as $r\rightarrow\infty$, while in one dimension one must limit the validity of Eq. (3.27) somehow, and a reasonable estimate is to take, as before, a height of about one polar cap radius

$$h_0\sim(a^3\Omega/c)^{1/2} , \tag{3.28}$$

which is about 10^4 cm for the Crab pulsar. Then the maximum Lorentz factor is estimated from (3.27) by writing $p\sim\gamma_0/h_0$ and solving for γ_0,

$$\gamma_0=h_0/\lambda\sim 10^5 , \tag{3.29}$$

which completes the argument and gives the particle energy.

c. Effect of pair production

Here we show that pair production simply limits γ to a value below that of Eq. (3.29) above. We add pair production to the model by assuming that at some height h_2 pairs are formed owing to *curvature radiation* as the electrons follow the strong magnetic field lines (synchrotron radiation being, in contrast, that caused by electrons circling the field lines). This process provides a downward flux of relativistic positrons of density

$$\delta q\equiv -q/2\gamma_2 , \tag{3.30}$$

where γ_2 is an unknown constant to this point (but we shall find that the particles will end up with a Lorentz factor of this order).

If one imagines that one has "copious" pair production as Ruderman and Sutherland (1975) discuss (i.e., $\delta q>>q_0$), one finds an impossible situation if any significant fraction of these particles return to the surface, since the space charge would now have the wrong sign above the surface, a problem already recognized by Sturrock (1971a). On the other hand, if one increases q_0 to keep the space charge negative, the charge density above h_2 is now not only larger than q_0, but much larger, and we cannot asymptotically approach q_0 outside of the acceleration region as before. Thus for a steady-state solution, one can only tolerate a small downward positron flux (here assuming upward electron primaries); hence we must have $\gamma_2>>1$, and indeed we find that the system quite naturally achieves such a condition.

In Eq. (3.27) we assumed that at $\gamma=\gamma_0$ the asymptotic condition has been reached. With pair production at h_2, we have a charge density $q-\delta q$ below (e.g., primary electrons minus secondary positrons) but a charge density $q+\delta q$ above (primary plus secondary electrons). It therefore follows that we can apply the "asymptotic" condition simply by setting $q+\delta q=q_0$ (hence no acceleration above h_2), which in turn requires the charge density in the accelerating region to be

$$q-\delta q=q_0-2\delta q=q_0\left[1-\frac{1}{\gamma_2}\right] . \tag{3.31}$$

As a consequence, we get a mixture of the first two theoretical treatments, with some of the accelerating field due to the surface sheath and the rest due to a small, fixed, non-Goldreich-Julian charge density below h_2 from the down-flowing secondaries, namely,

$$\lambda^2p^2=1-\gamma/\gamma_2 , \quad \gamma<\gamma_2 \quad (h<h_2) . \tag{3.32}$$

Equation (3.32) can immediately be integrated to give

$$\gamma_2=h_2/2\lambda . \tag{3.33}$$

Now, however, the relationship between h_2 and γ_2 is determined by the physics of the pair production, as shown in Fig. 8. The dashed line is the curve $\gamma=h/\lambda$, namely, the acceleration away from the negative-current sheet, while the curved line indicates the moderating effects of the intervening excess positrons. If $h_2>>h_0$, the basic limitation is geometrical, pairs are not important since they all escape, and $\gamma=\gamma_0$. If $h_2<<h_0$, pair pro-

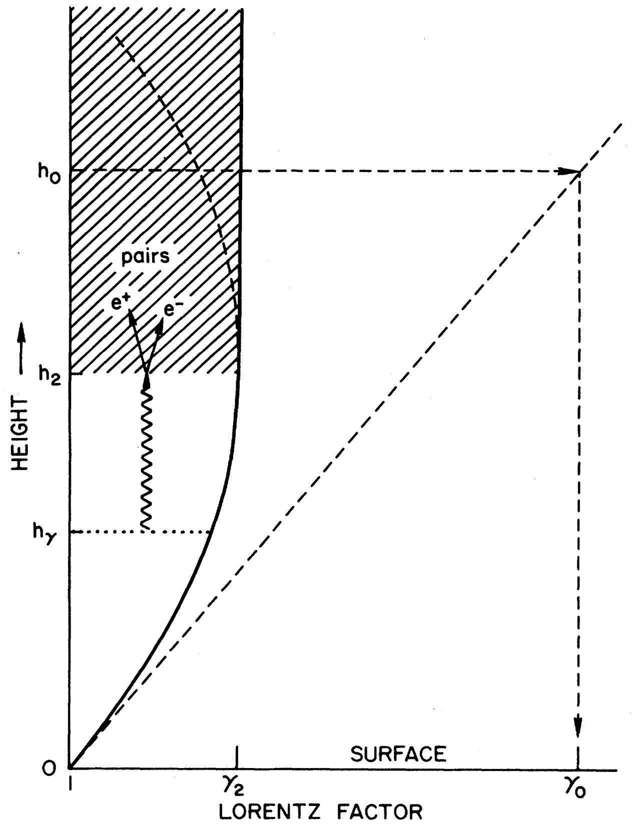

FIG. 8. Effect of pair production. Electrons gain energy $(\gamma-1)mc^2$ in being accelerated to height h (solid curve). At height h_γ the curvature radiation first produces photons energetic enough to be converted into pairs at h_2. Since only a very small downward flux of positrons can be tolerated, the accelerating field must essentially vanish quite close to h_2 in order that positrons produced at and above h_2 are not returned. Consequently the electron energy never reaches the value γ_0 it would have obtained if acceleration all the way to h_0 had been possible, as illustrated. For $h_0 < h_2$, pair production becomes unimportant in limiting γ, and for $h_0 < h_\gamma$, there is little or no pair production.

duction is the controlling factor and $\gamma \approx \gamma_2$, and pair production reduces, if anything, the particle energy. The above is a heuristic version of the work by Arons and Scharlemann (1979).

In summary, Sturrock's (1971a) way of handling space charge is to assume that the particle sees progressively more space charge between itself and the star as it departs, hence $\Phi \sim h^2$. With the $\underline{E}\cdot\underline{B}=0$ correction, the particle sees only a fixed sheath of space charge covering the stellar surface, hence $\Phi \sim h$. If pair production is included, one simply imposes an additional limitation owing to a tiny excess volume charge from down-flowing charges of sign opposite to the sheath, namely that Φ will increase only to the point where pair production is initiated, and is then terminated ["poisoned" in the view of Arons and Scharlemann (1979)].

d. Distributed space charge

In the above discussion, we have emphasized the role of a thin polar cap surface charge whose main effects are confined within a small distance comparable to the size of the polar cap regions (typically a few hundred meters). Arons and Scharlemann (1979) develop the role of a quite different, widely distributed component of non-Goldreich-Julian charge density which results from the curvature of the magnetic field lines. As pointed out below, a relativistic flow of space charge along magnetic field lines is only able to satisfy the static space charge required by $\underline{E}\cdot\underline{B}=0$ for field lines that are straight. For field lines that curve away from the rotation axis, the case for all the field lines in a perfectly aligned rotator, the flowing space charge exceeds more and more that required for the magnetosphere to be force free. The natural conclusion is that the resultant electric field acts to halt the flow in this case. For field lines of the opposite curvature, one expects an accelerating field, and in this way it is proposed that the particles end up crossing a net potential comparable to that across the polar cap, rather than the very small potential implied by the local conditions near the polar caps. Naturally larger potential drops imply larger particle energies and thereby enhance the potential importance of pair production, etc.

The above consideration is important particularly because it emphasizes the global nature of magnetospheric physics, despite the natural hope of breaking the problem down into manageable elements. It is therefore essential that the full global problem be solved. Only tentative preliminary steps have been taken in this direction, as will be discussed.

6. Pair production

We can now calculate the condition for pair production to be important, since (Fig. 8) the electrons must create hard enough photons at h_γ to produce pairs at h_2. The electron Lorentz factor at h_γ is just

$$\gamma_\gamma = \gamma_2(1-\Delta^2/h_2^2) \tag{3.34}$$

where $\Delta \equiv h_2 - h_\gamma$, and such an electron will radiate photons with energy (in units of $m_e c^2$) up to

$$\gamma_p = 3\xi\gamma_\gamma^3 \lambda\!\!\!^{-}_c/\rho_c \tag{3.35}$$

where $\lambda\!\!\!^{-}_c$ is the electron Compton wavelength ($=\hbar c/mc^2 = 4\times10^{-11}$ cm), and ρ_c is the field line curvature. The factor ξ is of order unity and parametrizes a slight uncertainty over what constitutes a "significant" flux of photons, because the synchrotron spectrum still extends somewhat beyond the critical frequency. In principle, we could calculate ξ self-consistently, but an exact value is not essential here (ξ is most probably between 1 and 3).

Magnetic opacity has the property (see Sec. X.B.2) that absorption goes almost discontinuously from zero to infinity at the critical condition

$$\gamma_p B \sin\theta = B_{\text{crit}} = 2\times 10^{12}\ \text{G}\ . \tag{3.36}$$

Here B is the surface magnetic field and θ is the angle between the photon propagation direction and the local field orientation. Photons are created with $\theta \sim 1/\gamma_p$ and hence would never be reabsorbed in a typical pulsar field were it not for the field line curvature ρ. As a consequence, after going the distance Δ they satisfy

$$\sin\alpha = \Delta/\rho_c \tag{3.37}$$

and Eq. (3.32) then becomes

$$y(1-y^2)^3 = \rho^2/l^2\ , \tag{3.38}$$

where $y = \Delta/h_2$, and using Eq. (3.33) to eliminate γ_2,

$$l^2 = 3\xi B \lambda\!\!\!^{-}_c h_2{}^4/8\lambda^3 B_{\text{crit}}\ . \tag{3.39}$$

Since the left-hand side of Eq. (3.38) has a maximum value of 0.238... at $y = 1/\sqrt{7}$, we have a condition on the curvature, namely that if

$$\rho_c \lesssim l/2,\ \ \text{pair production obtains}\ ,$$
$$\rho_c \gtrsim l/2,\ \ \text{no pair production obtains} \tag{3.40}$$

(we have approximated the square root of the maximum, 0.48787. . ., by $\frac{1}{2}$). Since $h_2 < h_0$ if pair production is to be important, we can replace h_2 with h_0 (hence with the polar cap radius) to obtain a liberal condition for pair production (i.e., pair production will certainly not be important if the condition is not met). Inserting Crab pulsar parameters into Eq. (3.39) then gives $l \sim 4\times 10^2$ m, which would require rather curved magnetic field lines (i.e., highly multipolar, corresponding to magnetic "spots," etc.) even for the Crab pulsar. As noted above, Sturrock (1971a) originally contemplated much more energetic particles.

This analysis gives an unfavorable assessment for pair production to be important, since the whole accelerating voltage is assumed to come from the differential space charge created by particle inertia at the stellar surface [Eq. (3.29)]. The effect of field line curvature (Sec. III.B.5 above) is to allow the existence of a much stronger parallel electric field, which can lead to pair creation in the Crab and other short-period pulsars without having to hypothesize nondipolar radii of curvature (Arons and Scharlemann, 1979; Arons, 1981). However, if pair creation is significant in all observed pulsars, it is consistent with space-charge-limited flow only if the surface magnetic fields are substantially nondipolar in the long-period objects (Barnard and Arons, 1981). It should be noted that this assessment is implicitly based on at least a part of the Goldreich-Julian model. Ruderman and Sutherland do not assume space-charge-limited flow because they propose that pulsar action is only gotten if the spin axis is anti-aligned with the magnetic moment, thereby pulling ions from the polar caps. They then argue that ions are not available and that consequently huge accelerating potentials like those originally proposed by Sturrock become possible (See Sec. IV.D.3.). In its simplest form, this model seems to give no space-charge limitation (see Fig. 19), because one has equal numbers of upward-moving positrons and downward-moving electrons. However this cancellation is not exact at the edges of the gap (just above the surface and just below h_2), and we have already seen that the acceleration sheath at the surface alone can exercise an important limitation. See Cheng and Ruderman (1977).

7. Radiation reaction

Because the electrons radiate copiously, it is sometimes supposed that radiation reaction will be the fundamental limiting process, robbing energy as fast as it can be delivered to the particles. This does not seem to be the case for the parameters under consideration. The radiation loss due to curvature radiation is

$$mc^2\frac{d\gamma}{dx} = \frac{e^2\gamma^4}{6\pi\varepsilon_0\rho_c^2}\ , \tag{3.41}$$

whereas the input rate is just mc^2/λ. Thus equating the two gives the asymptotic Lorentz factor

$$\gamma_R^4 = 6\pi\varepsilon_0\rho_c^2 mc^2/\lambda e^2\ , \tag{3.42}$$

and for a conservative estimate ($\rho_c \sim 10^6$ cm), one finds

$$\gamma_R \approx 3\times 10^6\ . \tag{3.43}$$

This value is significantly higher than the space-charge-limited flow value (i.e., radiation reaction is unimportant for the selected parameters). Even if we replace the left-hand side of Eq. (3.41) with the full corotationally induced field in Eq. (3.1), we still find a limiting Lorentz factor of 7×10^7. It is therefore evident that radiation reaction is poised only to suppress acceleration to much higher energies than those already imposed by space-charge limitation or pair production.

8. Frozen-in flux

As noted above, the field lines are taken to be equipotentials, an assumption worthy of comment. Within the context of the aligned rotator, where the system is simply acting to zero the parallel field at the surface by emitting particles, *one* plausible end result is the Goldreich-Julian—like inner magnetosphere where one has a static space-charge distribution with $E_{\parallel} = 0$. Even this special case may not be unique, as discussed below (Sec. IV. B.7).

The frozen-in flux (FIF) concept actually embraces two assumptions: (1) that the field lines are equipotentials, and (2) that the plasma only experiences $\underline{E}\times\underline{B}$ drift. When these two conditions are met, the magnetic flux through any arbitrary closed loop is constant even though the loop, each point of which is taken to be fixed in the local rest frame of the plasma, is distorted and displaced as the plasma circulates.

Frozen-in flux is frequently assumed in treatments of the pulsar magnetosphere, and we have no specific criticism of that assumption. However, a few cautionary notes should be made.

a. Equipotential field lines

For the field lines to literally be equipotentials from the stellar surface is not possible; some potential drop is necessary to accelerate plasma from the surface. In the space-charge-limited theories, this potential drop is tiny compared to the effective (cross-polar-cap) potential. Thus FIF seems a reasonable approximation. In the free-flow pair-production theories (e.g., Ruderman and Sutherland, 1975) the full cross-cap potential drop is also taken as the acceleration potential, and thus FIF is less plausible except asymptotically at large distances. Moreover, in both cases the particles are highly relativistic, and the field lines will no longer necessarily act as if they were highly conducting because changing the potential now has negligible effect on relativistic particle motion and cannot easily stimulate the formation of neutralizing space charge.

Finally, it is not obvious that the surface potential is not significantly modified by resistive effects. It is often assumed that both the surface and the magnetic field lines are perfectly conducting. Although this picture poses no apparent difficulties when the field lines extend away to infinity, it leads to immediate absurdities if, for example, a conducting disk orbited the pulsar. In that case at least one of the elements (surface, field lines, or disk) must act as a resistive element. Or the assumed geometry is simply wrong.

b. $\underline{E} \times \underline{B}$ drift

As noted previously, this assumption requires $E < cB$, which is an assumption expected to be valid near the star (where $E \sim \Omega a B$, and $\Omega a << c$) but not necessarily at large distances.

IV. THE ALIGNED VACUUM ROTATOR

We now turn to quantitative magnetospheric models, seeking to confirm the dimensional analysis above. There are several motivations for studying the case where the magnetic field is axisymmetric about the rotation axis, the model originally suggested by Goldreich and Julian (1969): (a) it presents a nontrivial physical system, (b) analysis of this problem is of intrinsic theoretical interest, and (c) it is possible that spin-period modulation (i.e., the pulsed emission) can be introduced as a simple modification to the basic model, either by (1) tilting the spin axis relative to the field symmetry axis, or by (2) introducing azimuthal asymmetries into the magnetosphere (hot spots, etc.). In the latter cases, we would essentially have achieved our goal of understanding the physics of the pulsar phenomenon.

The aligned rotator models fall into several classes according to the specific assumptions made. It is most convenient to contrast these with what has come to be known as the "standard model".

A. The standard model (Goldreich-Julian)

The standard model basically consists of making those simplifying assumptions that still promise to keep the problem interesting. These assumptions are

(a) The magnetosphere is filled with a plasma such that $\underline{E}\cdot\underline{B} \approx 0$ everywhere,

(b) The particle motion consists of $\underline{E}\times\underline{B}$ drift across field lines plus free "sliding" along field lines,

(c) Stationary ($\partial/\partial t = 0$) and axisymmetric ($\partial/\partial\phi = 0$) solutions exist.

A number of corollary assumptions then follow naturally. The magnetic field source is taken to be a centered magnetic dipole moment aligned either parallel or antiparallel to the spin axis [assumption (c)]. Assumption (a) requires some source of plasma, which is assumed to be free-field emission from the surface (zero work function); pair production is typically ignored, since it is a supplementary plasma source potentially important only for high spin rates and strong fields. (The idea here is to solve the basic physics of what happens if one rotates a spherical magnet that can freely emit plasma, not necessarily to model an active pulsar; thus the critical role assigned to pair production in some models is irrelevant insofar as this more modest goal is concerned, but could well be the key to resolving some of the paradoxes below). Assumption (b) usually includes the neglect of gravity and centrifugal forces near the star, since these are all small compared with the electrostatic forces. It is sometimes forgotten, however, that such neglect only makes sense if the plasma is entirely charge separated, in which case a weak parallel electric field component (parallel to $\underline{B}$) suffices to resist these inertial forces. However, for a two-component plasma (both + and − charges), one or the other component cannot be so supported. In this case one requires thermal support. Thus the temperatures would have to be large enough to resist the neutron star gravity ($\sim 10^9$ K for electrons, and m_+/m_e times larger if it is the positive charge carrier that must be thermally supported), and the large radiation loss that would ensue does not square with observation. An electron radiates its perpendicular energy in a time of the order of 2.58×10^8 sec$/B^2$ (gauss) (Bekefi, 1966), which is essentially instantaneous in a 10^{12} G field. Therefore, the electron pressure tensor would become totally anisotropic, having only a component along the field. If the electrons collide frequently, they also radiate away this parallel component, and if they collide rarely, they conduct efficiently to the surface, which in turn cannot plausibly be at 10^9 K (the neutron star blackbody luminosity would then be 6×10^{44} erg/sec!). Thus the corollary assumptions are made that, for static regions of the magnetosphere, the plasma is charge separated and the particles have negligible thermal

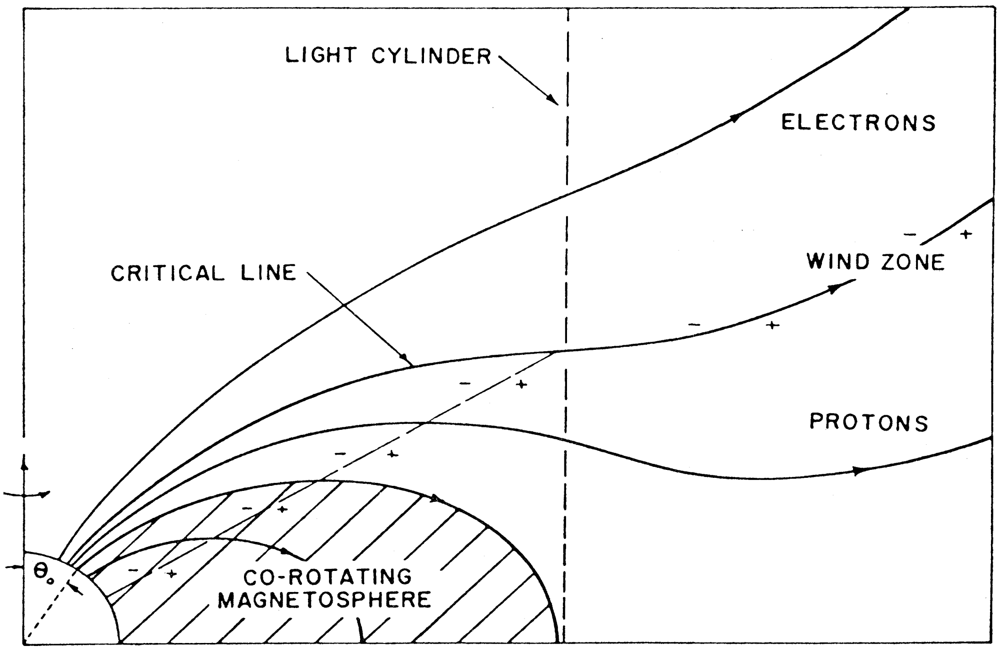

FIG. 9. Goldreich-Julian magnetosphere. Near the neutron star one find only electrons above latitude 35° and only positive charges below. In the wind zone, positive particles supposedly flow away at low latitudes, but, as is easily seen, the field lines on which they are constrained to flow lead to the *negative* polar cap region (Goldreich and Julian, 1969).

motion. Assumption (b) is not a good approximation for the Earth's magnetosphere, for example, because the particles have perpendicular energy, and, consequently, conservation of the moment invariance leads to mirroring which traps particles in regions of weak field; the particles do not slide freely along field lines but are instead accelerated toward the weak field regions. Wang (1978) has proposed that anomalous resistivity might be invoked to retard the gravitational segregation discussed above. Endean (1972b) has questioned the drift approximation, asserting that $E > cB$ zones exist. But see Buckley (1977a) and Burman (1977b).

Goldreich and Julian (1969) were the first to formulate the model in essentially these terms, and they suggested the general qualitative solution still adopted by many today, as shown in Fig. 9. In their paper, they repeatedly touch on the issue of whether their proposed model was "unique." The figure illustrates what was bothersome about the solution, namely that a straight line [the locus of $B_z = 0$; see Eq. (3.18)] starts out from the star separating regions of positive space charge from negative regions. The awkward thing is that "open" field lines must thread from the star to infinity across this "null" line. In other words, the very field lines on which one hopes to find positive particles being injected to produce a net neutral wind (one cannot very well charge the star indefinitely) are rooted deep in polar cap regions having everywhere *negative* space charge. How can positive charges be pulled from the surface electrostatically without collapsing the entire negative space charge? And if it must collapse, what does a steady-state solution mean? [See Gilinsky *et al.* (1970) for a point-by-point analysis of the Goldreich and Julian theory, and also Goldreich *et al.* (1971).]

In the next section, we shall see that, nevertheless, the standard vacuum model was found to have many plausible physical properties. On closer examination, however, one after another has proven to be flawed. At this point, the standard model still remains the consensus model. Its deficiencies are recognized, but many hold out hope that inclusion of pair production, oblique alignment, or some other consideration will relax an unsuspected unphysical constraint and permit one to assemble a fully self-consistent model. These deficiencies have not escaped the notice of the observers, and Sir Anthony Hewish (1981) has dubbed this the "current closure problem." Even the more popular contemporary models (e.g., Ruderman and Sutherland, 1975) have yet to prove that they satisfy this elementary requirement, namely that the average net current from the pulsar be zero.

Table V summarizes a number of quantities pertinent to the pulsar problem.

1. The vacuum solution

It is useful to start at the beginning and write down the solution for an aligned rotator without any magneto spheric plasma. We shall take the magnetic moment to be a point dipole at the center of the star. It is then elementary electrostatics to calculate all of the potentials. Inside the star, we shall have the Goldreich-Julian potential (and "space" charge)

TABLE V. Physical parameters of the standard model.

Quantity (at surface)	Estimator	Crab	Typical
Magnetic field (B)	— —[a]	4×10^{12} G[c]	10^{12} G[c]
Rotation Rate (Ω)	— —[a]	200 rad/sec[c]	6 rad/sec
Radius (a)	— —[a]	10^{6} cm	10^{6} cm
Mass (M)	— —[a]	1.4 M	1.4 M
Moment of Inertia	— —[a]	10^{45} gcm^2	10^{45} g cm^2
$E_{\parallel}$ (vacuum)	ΩaB	8×10^{12} V/cm	6×10^{6} V/cm
Polar cap area	$\pi\Omega a^3/c$	2×10^{10} cm^2	6×10^{8} cm^2
Polar cap radius	$(\Omega a^3/c)^{1/2}$	8×10^{4} cm	1.4×10^{4} cm
grav./elect. force	$mg/eE_{\parallel}$[b]	1.5×10^{-9}	2×10^{-7}
Pole to equator potential	$\Omega a^2B/2$	4×10^{18} V	3×10^{16} V
Potential across polar cap	$\Omega^2a^3B/2c$	3×10^{16} V	6×10^{12} V
Electron Cyclotron frequency (ω_c)	eB/m_e	7×10^{19} rad/sec	1.8×10^{19} rad/sec
Proton Cyclotron frequency	eB/m_H	4×10^{16} rad/sec	1.0×10^{16} rad/sec
Iron Cyclotron frequency	eB/m_{Fe}	8×10^{14} rad/sec	2×10^{14} rad/sec
electron concentration (n_e)	$2\epsilon_0\Omega B/e$	9×10^{12}/cc	7×10^{10}/cc
Electron plasma frequency[d]	$(e^2n_e/\epsilon_0m_e)^{1/2}$	1.7×10^{11} rad/sec	1.5×10^{10} rad/sec
Alfvén velocity[e,b]	$(2\omega_{ci}/\Omega)^{1/2}c$	$4\times10^{7}c$ (Fe)	$10^{7}c$ (Fe)
Particle flux[f]	$4\pi\epsilon_0\Omega^2Ba^3/e$	1.1×10^{34}/sec	2.5×10^{30}/sec
Slowing down rate ($\Omega/\dot{\Omega}$)	$3I\mu_0c^3/8\pi B^2a^6\Omega^2$	1340 yr	2×10^{7} yr
Particle flux[f]	$4\pi\epsilon_0\Omega^2Ba^3/e$	1.1×10^{34}/sec	2.5×10^{30}/sec
Slowing down rate ($\Omega/\dot{\Omega}$)	$3I\mu_0c^3/8\pi B^2a^6\Omega^2$	1340 yr	2×10^{7} yr

[a]Input assumption or observation.
[b]For singly ionized iron ions (case where gravitation would be most important).
[c]Rounded values.
[d]An equivalent expression is $\omega_p^2=2\Omega\omega_c$, hence $\omega_p<<\omega_c$. B (or n_e) should be decreased by about $(c/a\Omega)^3=3.4\times10^{6}$ to give the value at the light cylinder.
[e]Here one uses the ion cyclotron frequency.
[f]A simple equivalent is $L/I^2=15W/A^2$, where L is the total power output and I is the current (Michel, 1978a), hence e times the particle flux.

$$\Phi=\Phi_0 a\sin^2\theta/r\ ,$$

where $\Phi_0=\Omega a^2B/2$. Since all quantities are scaled by this voltage, we shall normalize it to unity in the following expressions. The electric field and space-charge density are immediately given from the potential as given in Table VI. Matching the potential to vacuum monopole and quadrupole moments then gives the external fields.

Although this solution is elementary, there are a number of noteworthy points to be made. Firstly, the net space charge in the system is zero; both the surface charge and the internal volume charges are distributed as $(1-3\cos^2\theta)$, which integrates to zero over a spherical surface. Nevertheless, there is a net charge on the star because it has a monopole moment, and therefore this charge must be located at the magnetic dipole point source.

a. Surface charge

Secondly, there is a surface charge. Basically it is this surface charge that is pulled from the surface to form the magnetosphere. Note that $\underline{E}\cdot\underline{B}$ changes sign with σ (surface-charge contribution), thus acting to pull electrons from the polar caps and positive particles from the equatorial regions. A modest surface charge can be maintained on conductors under laboratory conditions, since the work function is nonzero, but there is no way that a pulsar could maintain such large negative surface-charge concentrations. As noted below, in connection with the Ruderman-Sutherland model, there is the possibility that the ion work function might be sufficient to maintain a positive surface charge, but we assume both species are freely available for the moment.

Any solution of the standard model will require setting $\sigma=0$ everywhere on the surface, since the assumption of free emission precludes binding charges to form a surface charge. Note, in this respect, that image charges in a conductor are actually surface charges; thus, despite the fact that the neutron star is treated as a perfect conductor, the external magnetospheric charge distribution does not produce an additional "image" contribution [$\underline{E}$ is not even normal to the surface, as one can see from Table VI(a)].

The Goldreich-Julian solution simply corresponds to extending the interior solution to infinity, with the space-charge density continuing smoothly through the surface (rigid corotation of the magnetosphere is then obvious), the discontinuity there being only in the neutral component of the neutron star.

TABLE VIa. The vacuum solution (point dipole field).

Quantity	Expression	Surface values Equator[a]	Pole[b]
	Inside star		
Φ	$\sin^2\theta/r$	$+1$	0
E_r	$\sin^2\theta/r^2$	$+1$	0
E_θ	$-2\sin\theta\cos\theta/r^2$	0	0
q/ε_0	$2(1-3\cos^2\theta)/r^3$	$+2$	-4
$\underline{E}\cdot\underline{B}$	0	0	0
	Outside star		
Φ	$\frac{2}{3r}+\frac{1}{3r^3}(1-3\cos^2\theta)$	$+1$	0
E_r	$\frac{2}{3r^2}+\frac{1}{r^4}(1-3\cos^2\theta)$	$+\frac{5}{3}$	$-\frac{4}{3}$
E_θ	$-2\sin\theta\cos\theta/r^4$	0	0
q/ε_0	0	0	0
$\underline{E}\cdot\underline{B}$	$4\cos\theta(1-3\cos^2\theta/r^2)/3r^5$	0	$-\frac{8}{3}$
	Surface		
σ/ε_0[c]	$2(1-3\cos^2\theta)/3$	$+\frac{2}{3}$	$-\frac{4}{3}$
$\underline{E}\cdot\underline{B}$ (average)	$2\cos\theta(1-3\cos^2\theta)/3$	0	$-\frac{4}{3}$
	Everywhere		
B_r	$2\cos\theta/r^3$	0	$+2$
B_θ	$\sin\theta/r^3$	$+1$	0

[a]Here $f=1$, $\theta=\pi/2$.
[b]Here $f=0$, $\theta=0$.
[c]The surface charge density σ is given from $E_r(\text{out})-E_r(\text{in})$.

b. The central charge

Let us now ask why there should be a huge charge associated with a point dipole. See, for example, Cohen *et al.* (1975). From E_r (outside) and Gauss's law we have a positive central charge

$$Q=8\pi\varepsilon_0 a\Phi_0/3 ,$$

which is of the order of 10^{12} C (or about 10^7 moles of electrons!) for the Crab pulsar. This charge is actually distributed throughout the magnetic field source region, as we can see by replacing the dipole with a uniform interior magnetization as shown in Table VI(b). Here we see that a uniform magnetic field in the interior actually becomes negatively charged while the surface becomes positively charged, by factors of, respectively, two and three times the net charge. In a more realistic field model that did not have a discontinuity in B_θ at the surface, the surface charge would, of course, be distributed over a finite volume.

TABLE VIb. The vacuum solution (uniformly magnetized interior)

Quantity	Expression	Surface values Equator	Pole
B_r	$2\cos\theta$	0	$+2$
B_θ	$-2\sin\theta$	-2	0
Φ	$r^2\sin^2\theta$	$+1$	0
E_r	$-2r\sin^2\theta$	-2	0
E_θ	$-2r\sin\theta\cos\theta$	0	0
q/ε_0	-4	-4	-4
$\underline{E}\cdot\underline{B}$ (inside)	0	0	0
σ/ε_0 (surface)	$2+7(1-3\cos^2\theta)/6$	$-\frac{1}{3}$	$\frac{19}{6}$

Of particular importance to the theory is the fact that, for a pulsar, unlike the laboratory situation, the potential and charge are not free parameters (in the zero-work-function limit). Any attempt to alter the charge Q would produce a net surface charge, and any such charges would be lost into the magnetosphere. Thus, if the total system charge were to be more or less neutral, the magnetosphere would have to contain a negative charge excess of $-Q$.

2. The detailed model

Here we abandon the vacuum solutions, suppose that the star is surrounded by a Goldreich-Julian neutralizing plasma, and assume that this plasma is corotating in the equatorial zones near the star and is flowing outward along polar field lines.

If one specifies the magnetic field lines by the enclosed magnetic flux (f) between any given field line and the axis of rotation, then one has in general a total current defined to be $\mu_0 A(f)$ flowing out of the pole within the surface of rotation bounded by the field line f. It follows immediately that the azimuthal field is given by

$$B_\phi = \Omega A(f)/\rho c \ . \tag{4.1}$$

Moreover, since both the current and the flux magnitude between any two field line surfaces are conserved, it follows that (subscript m stands for "meridional" and indicates a vector in that plane)

$$J_m = \mu_0 \left[\frac{dA}{df} \right] \Omega \underline{B}_m / c \ . \tag{4.2}$$

These relations, together with the condition that the system be in electromagnetic force balance,

$$q\underline{E} + \underline{J} \times \underline{B} = 0 \ , \tag{4.3}$$

or

$$qE_m + J_m B_\phi - B_m J_\phi = 0 \ , \tag{4.4}$$

where

$$E_m = \Omega \rho B_m \ , \tag{4.5}$$

allow one to factor out the (nonzero) factor B_m to obtain

$$\Omega \rho q + \Omega^2 \mu_0 A A'/c - J_\phi = 0 \ . \tag{4.6}$$

In labeling field lines according to magnetic flux (f), one obtains (in cylindrical coordinates)

$$B_\rho = -f_z/\rho \ , \tag{4.7}$$

$$B_z = f_\rho/\rho \ , \tag{4.8}$$

$$\mu_0 J_\phi = (\underline{\nabla} \times \underline{B})_\phi = \left[f_{zz} + f_{\rho\rho} - \frac{1}{\rho} f_\rho \right] \frac{1}{\rho} \ , \tag{4.9}$$

and

$$q/\varepsilon_0 = -\nabla^2 \Phi = -\Omega \nabla^2 f = -\Omega \left[f_{zz} + f_{\rho\rho} + \frac{1}{\rho} f_\rho \right] \ , \tag{4.10}$$

and if one defines a scale length

$$a = c/\Omega \ , \tag{4.11}$$

one obtains (Michel, 1973a)

$$f_{zz} + f_{\rho\rho} - \frac{1}{\rho} \left[\frac{a^2+\rho^2}{a^2-p^2} \right] f_\rho - \frac{A'A}{a^2-\rho^2} = 0 \ . \tag{4.12}$$

This equation is sometimes referred to as the "pulsar equation". So far nothing has been specified about the function A. An obvious simplification would be for $A \sim f$, in which case the pulsar equation would be soluble by conventional eigenvalue techniques (see Scharlemann and Wagoner, 1973, who independently derived this equation). Other independent derivations of Eq. (4.12) are given by Julian (1973) and by Cohen *et al.* (1973). The latter are more general, in that the plasma was taken to have both signs of charge carrier present. See also Cohen and Rosenblum (1972, 1973), Endean (1974), and Schmalz *et al.* (1979, 1980).

Unfortunately, we see from Eq. (4.2) that $A(f)$ must have at least three zeros in the standard model, two at each pole (no line current along the spin axis) and one on the equatorial plane (since $B_\phi = 0$ there by symmetry); hence Eq. (4.12) apparently must be nonlinear. As a result, only a few special cases have been worked out, as discussed below.

3. Restricted exact solutions (A = constant)

Since the space charge required for $\underline{E} \cdot \underline{B} = 0$ rigidly corotates, it in turn generates a current which modifies $\underline{B}$ from a pure dipole configuration. This effect was evaluated (Michel, 1973b) and shown to lead to a cusp-like configuration at the light cylinder which separates the closed field lines from open ones. This solution corresponds to the choice $A = 0$ in Eq. (4.12), in which case one can solve for $f(z,\rho)$ by conventional separation-of-variable methods (Michel, 1973b). Figure 10 shows the resultant field line configuration. Mestel *et al.* (1979) have repeated this analysis and have found exactly the same results, although they claim their method (expanding the z dependence as cos (λz) rather than $e^{\pm\lambda z}$) to be superior. Poor convergence is often a technical difficulty with such eigenvalue expansions, but this can be overcome, and one can show that rotation distortion increases the amount of flux crossing the light cylinder, compared to that for an undistorted dipole field, by a factor

$$f_0 = 1.591\,842\,8 \pm 0.000\,000\,4 \ ,$$

which seems adequate precision for a "flawed" method (Mestel *et al.*, 1979, obtain 1.592). Hinata and Jackson (1974) have found unusual solutions for this same case,

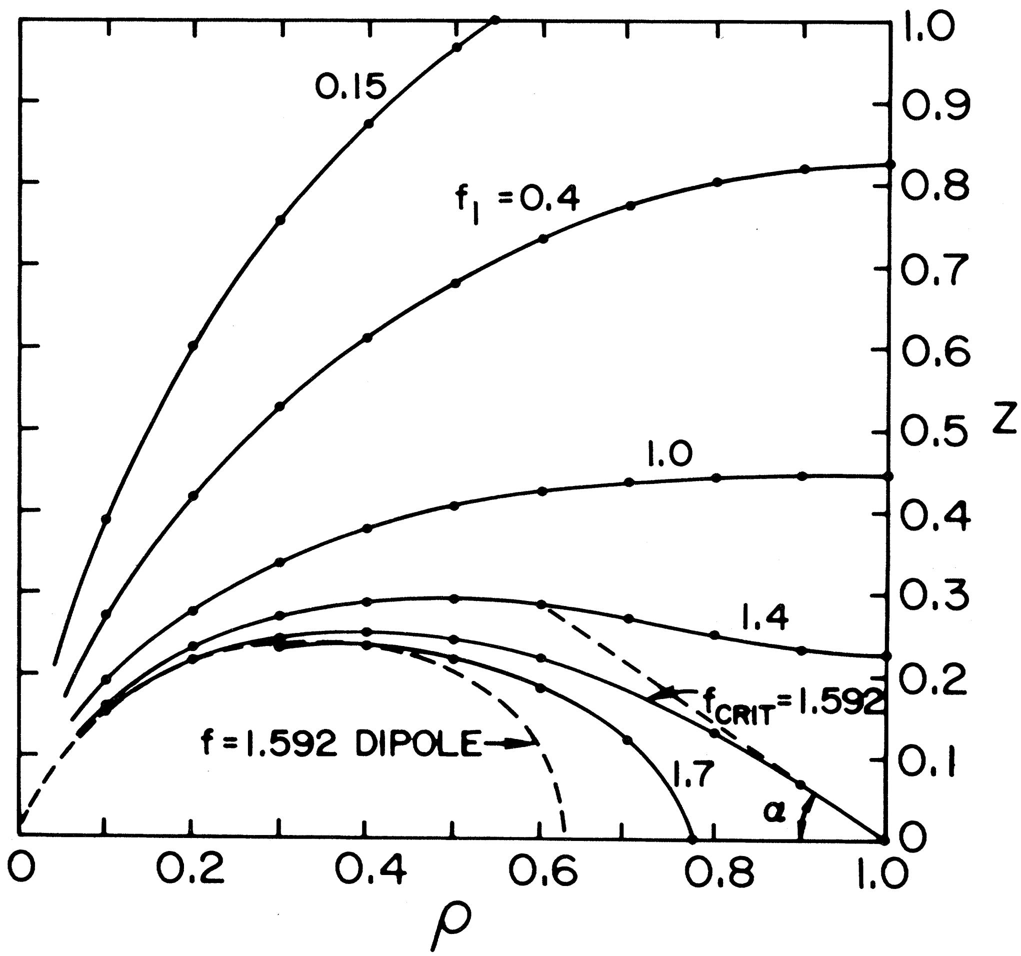

FIG. 10. Exact solution to the "pulsar equation" if outward flow is neglected. Here corotation of the space charge produces currents modifying the dipole magnetic field, which in turn modifies the space charge. Magnetic field lines are labeled such that the $f=1$ flux line of the undistorted dipole would cross the equator at light cylinder distance ($\rho=1$). Distortion causes an increased (factor of 1.592) flux to cross the light cylinder. Note cusp [angle $\alpha=\tan^{-1}(1/\sqrt{2})$] reminiscent of Fig. 6 (Michel, 1973b).

corresponding to the presence of strong external magnetic fields surrounding the object.

One can see that $A=$ constant also satisfies the same equations. This choice corresponds to a line current along the z axis, hence simply the superposition of an azimuthal field, $B_\phi=\text{const}/\rho$. Scharlemann and Wagoner (1973) discuss the parallel solution for $A(f)=\text{const}\times f$, which again can be solved by separation of variables. As later noted by Michel (1975c), this choice for A requires a discontinuity joining two separate asymptotic solutions; thus one cannot simply "solve" Eq. (4.13) for this choice, because the solutions are not global but must be patchwise matched across supplementary current discontinuities. See Sec. IV 5.c. below. Similar considerations probably follow for the nonlinear versions.

4. Exact monopole solution

An encouraging special solution to the "pulsar equation" (4.12) is the case for a monopole magnetic field. Here one has an exact solution (Michel, 1973a), given by

$$V_m=c,\quad V_\theta=0\,,\qquad \tan\xi=\Omega\rho/c\,,\qquad B_\phi=\Omega f_0\rho/r^2c\,, \tag{4.13a}$$

with the monopole field

$$f=f_0z/r\,,\qquad B_r=f_0/r^2,\quad B_\theta=0\,. \tag{4.13b}$$

Here ξ is the "garden hose" angle (imagine watering the lawn while spinning on one's heels), the angle between the magnetic field line and the radial direction, as shown in Fig. 11. Thus the field lines are wrapped backwards by the rotation, whereas the plasma streams radially outward. These are precisely the properties expected for stellar-wind-type solutions (see below), and they considerably reinforced confidence in this general approach.

In terms of $A(f)$, this solution corresponds to

$$A(f)=(f_0^2-f^2)/f_0\,, \tag{4.14}$$

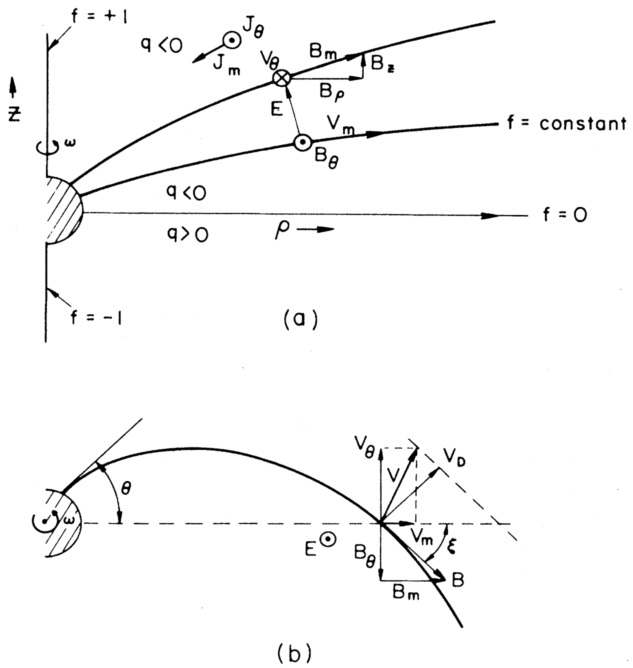

FIG. 11. Monopole field line geometry. (a) Meridional projection. Symbol are standard except q (electric charge density) and subscript m (vector quantity in meridional projection). (b) Orthogonal projection onto plane normal to local E, namely, the plane defined by the meridonal vectors (all are parallel) and the θ direction. The plasma is obliged to have a specific V_m and $V_D=\underline{E}\times\underline{B}/B^2$ drift velocity, thereby resulting in the net velocity V constructed as shown (Michel, 1973a).

where f_0 scales the magnetic flux. Thus the exact monopole solution corresponds to a quadratic (nonlinear) choice for A. In the previous choice, Eq. (4.12) was linear, and hence the field multipolarity was mathematically irrelevant since the dipole solution can be gotten by differentiating the monopole solution, etc. Once A is nonlinear, however, the solutions no longer can be expanded by superposition.

There has been no systematic mathematical analysis of the pulsar equation. The only known analytic solutions are those given above for $A=0$ or constant, the linear equation $A=-2f$, and the nonlinear choice [Eq. (4.14)] appropriate for a monopole field. Since $A(f)$ physically represents the current flowing on field lines poleward of the f-field line, it is evident that $A(0)$ must be zero, and since the total current from the star must be zero, we also must have $A(f_c)=0$, where f_c is the last open field line. Thus $A(f)$ must at least be quadratic to have two zeros, and $A'A$ is therefore at least cubic in f. The monopole solution (4.14) has this basic property, for example, except that there is no f_c and the two zeros are therefore at the two polar caps [note that f is normalized differently for a monopole field (4.14) than for a dipole field (3.20), f being zero in the equatorial plane for the monopole rather than on the polar axis as for the dipole].

5. Wind-zone solutions

a. The far-zone limit

The monopole case above is basically a wind-zone solution since it has no transition point from nonrelativistic to relativistic flow. More realistic wind solutions have been derived, first for the general case of a neutral plasma being driven away from a pulsar by the rotating magnetic field (Michel, 1969a, 1969b) and then for the charge-separated case above (Michel, 1974c). Here one assumes the meridional fields to be asymptotically radial, in which case (4.12) simplifies to

$$(1-\mu^2)f''-2\mu(1-\mu^2)f'+\tfrac{1}{2}dA^2/df=0\ , \qquad (4.15)$$

where $\mu=\cos\theta$, $f\to f(\mu)$, and $f'=df/\,d\mu$, with $A=A(f)$ as before. However, a solution of this equation is seen, by direct substitution, to be

$$f'=-A(1-\mu^2)\ , \qquad (4.16)$$

from which one obtains again

$$\tan\xi=\Omega\rho/c\ , \qquad (4.17)$$

giving asymptotically a perfect Archimedian spiral, as well as $V_\theta\to 0$, $V_m\to c$, and $q\to\varepsilon_0\Omega A(dA/df)/\rho^2$. When the equation is in this form, one need only to choose a form for A to obtain a possible solution (albeit not necessarily one appropriate to a dipole source at the origin).

b. The current and charge-density paradox

The asymptotic solution above was eventually noticed to have an intrinsic flaw (Michel, 1975b). Indeed Eq. (4.16) has the simple physical interpretation

$$E_\theta/B_\phi=c\ , \qquad (4.18)$$

using $d\Phi=\Omega df$ as the imposed surface condition, Eq. (3.6). In other words, the particles are simply drifting outward in the $\underline{E}\times\underline{B}$ motion dictated by the dominant fields at large distance. However, it turns out that J_ϕ [Eq. (4.9)] and q [Eq. (4.10)] do not, in general, vanish simultaneously on the same field line, the field lines now being designated by $\mu=\cos\theta$ instead of f. For a charge-separated system, J_ϕ and q must vanish together, and therefore a serious restriction is placed on the function A.

c. Determination of A(f)

It was then shown (Michel, 1975c) that in fact the linear choice

$$A(f)=-2f$$

avoided the above difficulty (or, more precisely, hid it). Here one has solutions reminiscent of the original monopole solution, but with a current sheet discontinuity in the equatorial plane. Although such a sheet current might well be considered artificial, it nevertheless resembled the qualitatively expected structure at large distances as shown in Fig. 6. The current sheet itself could be imagined to be the consequence of the mathematical idealizations, representing in fact a distributed volume current, possibly representing shock-heated plasma. Such a current sheet would map into the edges of the polar caps to form an "auroral" zone (see also Lovelace, 1973). Although this field structure seems at least promising, there are even more severe problems remaining. Moreover, it is not possible to connect this asymptotic solution to the corresponding near-zone solution (Scharlemann and Wagoner, 1973).

B. Problems with the standard model

1. The uncharged field line

We can rewrite Eqs. (4.9) and (4.10) (Okamoto, 1974; Pelizzari, 1975) to give

$$\frac{\Omega\rho}{c^2}J_\phi=-q-2\Omega\varepsilon_0 B_z \ , \tag{4.19}$$

which leads to an essential paradox if we wish to adopt a picture such as Fig. 6 for the magnetic field configuration.

Since the only currents in the standard model result from motion of the space charge, it follows that if $q=0$ then $J_\phi=0$, which from Eq. (4.19) gives the condition

$$B_z=0 \quad (\text{if } q=0) \ , \tag{4.20}$$

which in turn means that the uncharged field line must extend parallel to the equatorial plane. The dipolar field line that starts from the surface with $B_z=0$ is buried deep within the corotation region, however. Thus there would be no plausible way to detach that field line from the pulsar. For the monopole magnetic field case discussed above, there is no difficulty, since there are no closed field lines and the field line in question is simply the one in the equatorial plane, hence automatically satisfying constraint (4.20). Thus the same difficulty in the wind zone proves equally vexing near the star.

Perhaps the simplest way out would be to have $\Omega=\Omega(f)$ and not constant, in which case additional terms would enter and possibly relieve the requirement that $B_z=0$ along the null charge-density line. However, this suggestion would mean that the field line potentials have a different value from that imposed at the surface and therefore $\underline{E}\cdot\underline{B}\neq 0$, violating a basic assumption of the model. Resistive effects within the star could strongly modify the surface potential. However, the current densities are actually not all that large for pulsars, but are rather comparable to those in electrical wires to home appliances, so the resistivity would have to be about 10^8 times higher than for normal metals for the resistive potential drops to become significant in altering Ω. Observationally, the consequent heat dissipation and blackbody radiation (soft x rays) are not seen. For the Crab, this component is less than about 10^{-5} of the total output (Harnden *et al.*, 1980). Thus the internal potential drops along field lines are probably much less than 10^{-3} of the accelerating potential, hence introducing only a small correction to the standard model surface potential.

Another suggestion is that the plasma is not charge separated (Okamoto, 1975), in which case $J_\phi=0$ and $q=0$ need not occur at the same point. It is difficult, as discussed above, to see how such a plasma can be pulled from the pulsar surface by an electrostatic field, but possibly the $\partial/\partial t=0$ assumption prevents one from investigating that process (e.g., the accelerating field might alternate, first pulling negative particles, then positive, etc.). Pair production could admix some pairs into the primary beam and avoid complete charge separation; however, it is not obvious that the paradox is thereby resolved. The $q=0$ line at the surface is a site of zero particle emission, hence no local pair production, so pairs are unavailable just where they are most needed. Another problem is that the interstreaming between two charged species is well known to be unstable, and it seems doubtful that it could be maintained. But if both components had the same velocity, the plasma would again behave as if charge separated. Salvati (1973) also examined this limit, concluding that either charge separation could not be complete or that the magnetic field configuration would somehow not fill the entire space around the rotator. Similar considerations were raised by Buckley (1978) and Endean (1976). Jackson (1978b) discusses the effects of perturbations to the aligned magnetosphere, arguing that such solutions are unstable.

2. Monotonic field lines

A related problem is that a field line cannot curve over and approach the equatorial plane within the light cylinder, as would be required for the wind field lines adjacent to the corotation region (Fig. 9). Such a field line would have $B_z=0$ where it turns over, and therefore

$$V_\phi=\frac{J_\phi}{q}=\frac{c^2}{\Omega\rho}>c \quad (B_z=0) \tag{4.21}$$

since ρ is inside the light cylinder.

3. Boundary conditions at the light cylinder

Scharlemann and Wagoner (1973) noted that the singular nature of the pulsar equation at $\rho=a$ meant that the B_z component was fixed there by the function $A(f)$. As a result, the equation is independently soluble inside and outside of the light cylinder. As pointed out by Ingraham (1973), this independence means that for an arbitrary $A(f)$, i.e., current flow pattern, the field lines need not match up, violating $\nabla\cdot B=0$, or if they do match up, may do so with a "kink," requiring a current sheet, and he suggested that $A(f)$ could be determined by the condition that the field lines match without kinking. Numerical calculations by Pelizzari (1975) confirm the difficulty in matching field lines across the light cylinder, as shown in Fig. 12. Pelizzari numerically solved the pulsar equation, Eq. (4.12), for a number of trial functions, including ones of the form (see discussion in thesis)

$$\frac{dA}{df} = -2(1-f^n/f_0^n) , \tag{4.22}$$

where f_0 has the same significance as f_{crit} in Fig. 10. Although the $n=1$ case comes close to appearing continuous, one sees that the physical behavior of the two solutions is quite different near the light cylinder. (As a test case, Fig. 10 was accurately reproduced by the numerical code.)

One must also ask what would happen if an obstacle were presented to the pulsar wind just outside of the light cylinder. Certainly the flow pattern inside the light cylinder would be modified, and as the pulsar equation stands, this modification could result only by changing $A(f)$.

4. Wind-zone problems

It is reasonable to suppose that the pulsar wind flows radially away from the pulsar at large distances, in

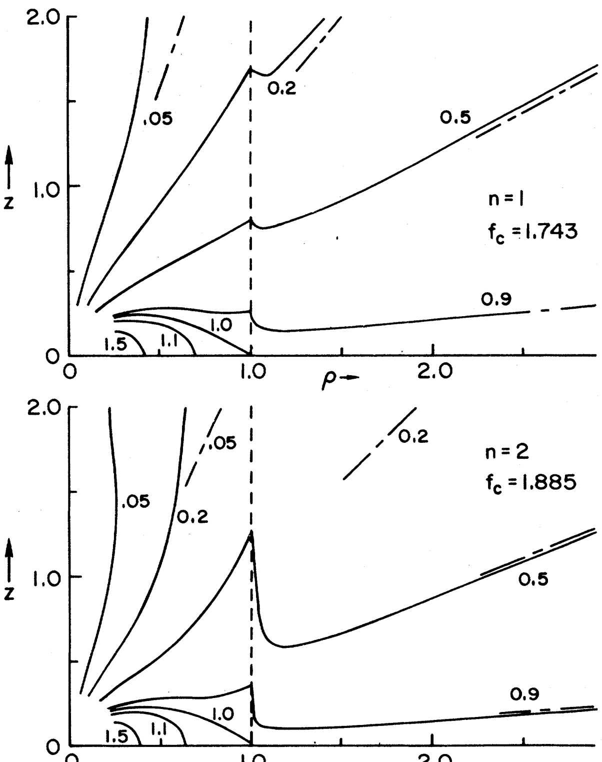

FIG. 12. Attempts to solve the "pulsar equation" numerically for plausible functions of the form $dA/df \propto 1-f^n/f_c^n$, where f_c is the critical field line (see Fig. 10) leading to the cusp at the light cylinder distance ($\rho=1$). Note kinks and reversal of slope at the light cylinder (vertical dashed line).

which case

$$\begin{aligned} &V_m \to c , \\ &E,\ B_\phi,\ V_\phi \to 1/\rho , \\ &q,\ J_m,\ B_m \to 1/\rho^2 , \end{aligned} \tag{4.23}$$

and

$$J_\phi \to 1/\rho^3 ,$$

and, asymptotically, it is clear that the force-balance equation reads

$$qE_m = -J_m B_\phi , \tag{4.24}$$

which is just Eq. (4.16). Unfortunately, Morris (1975) has found that this balance requires the asymptotic condition $V_m \to c$, which is impossible for particles with finite mass. Heuristically, this condition results because q is the source of E_m while J_m is the source of B_ϕ. Thus dimensionally we have $q^2 \sim J_m^2 \times$const, and the constant is simply $1/c^2$, so the reduction of the flow problem to one of pure electromagnetism automatically requires c to be the characteristic velocity. Although the plasma from a pulsar might plausibly be highly relativistic, $V_m < c$ and therefore the magnetic forces cannot quite cancel the electrostatic forces in Eq. (4.24). See Buckley (1977).

An analogous problem arises in the theory of the solar wind, except there it is the qE_m term that is negligible in this nearly neutral plasma, and instead pressure gradient terms must counter $J_m B_\phi$. Since the pressure gradients decline faster than ρ^{-3}, asymptotically, the assumption is then that there must be nonradial flow away from the equator to reduce B_ϕ and J_m. In the ultrarelativistic version of the standard (pulsar) model, this problem is concealed, since the qE_m term exactly cancels the $J_m B_\phi$ term, but only in the limit $V_m \to c$. For flows of real particles, the cancellation is not exact (Morris, 1975), and consequently some nonradial flow may result. A fully satisfactory analysis of this point remains to be made. See Sec. VIII for more details.

5. The transition region

Several authors have suggested that the physics is incomplete unless a shock transition is included. Such a magnetohydrodynamic discontinuity is not included in the pulsar equation. A series of papers by Ardavan (1976a—1976e) has suggested that a shock wave exists at the light cylinder. Aspects of this calculation have been challenged, however (Burman, 1977a, 1980a, 1980b). Most recently, it has been concluded that the discontinuities are not shocks (Ardavan, 1981), suggesting instead a possible internal inconsistency in the underlying assumptions. A sizable literature exists pointing up problems encountered near the light cylinder. Scharlemann (1974) thought in this connection that the complete charge separation might be an unrealistic assumption. Others have also been unable to avoid discontinuities at the light cylinder (Buckley, 1976; Henricksen and

Norton, 1975a) or related problems (Mestel, 1973). Steady-state flow equations are rather subtle to solve, however, because once one has imposed the steady state assumption one is no longer free to arbitrarily choose the boundary conditions as well. Errors in the latter choice generally result in mathematical peculiarities (i.e., infinities, etc.).

6. Curved field lines

Another elementary defect in the pulsar equation was noted by Scharlemann *et al.* (1978), namely that, if along a given field line one had $J_m = qV_m$ where $V_m \simeq c$ (close to the surface, for example), then in general V_m cannot be c elsewhere and still have q be the local Goldreich-Julian charge density (q_0). In other words, the idea that the plasma flows at roughly c everywhere cannot be maintained, consistent with the assumption of $\underline{E}\cdot\underline{B} << |E||B|$ *everywhere*. Indeed, in the scenario of Mestel *et al.* (1979), it is assumed that $V_m << c$ well inside the light cylinder with q almost exactly equal to the Goldreich-Julian value. The price then paid is that $|J_m|$ is small compared to $q_0 c$, and the torque on the star is much less than the observed torque (alternatively, the magnetic moment is $\sim c/V_m$ greater than the conventionally estimated value).

On the other hand, if $V_m \simeq c$, as is consistent with the attribution of pulsed γ rays in the Crab and Vela pulsars to radiation from particles accelerated along polar field lines, then *in general* q cannot be the Goldreich-Julian density along a given field line at more than one point. In one special case (the monopole) both q and the Goldreich-Julian density vary as r^{-2} and can be held in the fixed ratio $V_m/c \simeq 1$. But if the field line curves, J_m/q_0 cannot stay in the fixed ratio $V_m \simeq c$ because the Goldreich-Julian density has an additional dependence on the direction of a field line [the proportionality to B_z, Eq. (3.18)], and consequently $\underline{E}\cdot\underline{B} \neq 0$ must appear if the field line curves. This has led to the idea of "favorable" and "unfavorable" curvature, depending on whether $\underline{E}\cdot\underline{B}$ has the correct or incorrect sign to accelerate charges of the same sign as Eq. (3.18). Scharleman *et al.* (1978) concluded on this basis that the aligned rotator cannot have a steady, charge-separated flow solution (with $V_m \simeq c$). They also pointed out that the favorably curved part of the polar flux tube of the *oblique* rotator could have consistent, ultrarelativistic, charge-separated flow out to distances of order the light cylinder radius, and noted that at these radii the particle energy density can become comparable to the magnetic energy density, with current closure conceivably occurring through inertial forces. This drastic departure from the assumptions of the standard model has not received a consistent quantitative development as yet. Figure 13 illustrates a pulse model (*with* pair creation) based on these considerations (Arons and Scharlemann, 1979). Here, no current flow and pair creation whatsoever is proposed along the unfavorably curved field lines, while pair production (see Sec. IV.D) occurs at and above a well defined surface along the favorably curved field lines.

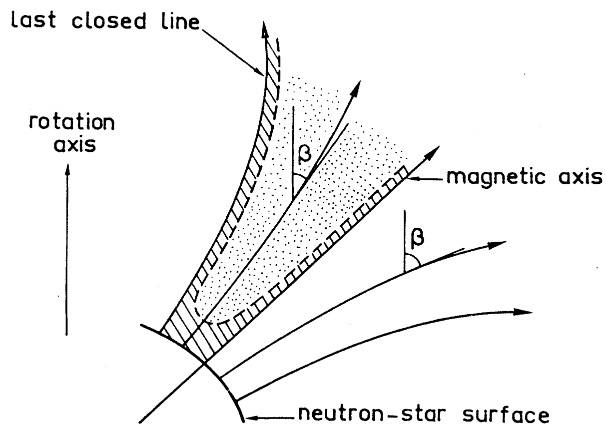

FIG. 13. Pair production discharge on "favorably curved" field lines (after Arons, 1979). Since flow is assumed to be choked off owing to excess space-charge accumulation along downward-curved field lines, only the upward ones are assumed to be able to maintain particle injection. Dotted region represents a pair-dominated outward-flowing plasma, while the shaded area is the acceleration region (see Fig. 8).

7. Vacuum gaps

In addition to the above difficulties, it has slowly become evident that even the static (corotating) charge distribution near the star, in the Goldreich-Julian model (Fig. 9), makes no sense, plausible as it might seem at first. As one can see in the figure, some field lines requiring positive space charge in the equatorial regions lead to negative space charge in the polar regions. Holloway (1973) argued that, if some of the positive equatorial charge were to be removed, it could not plausibly be replaced, there being no way to accelerate new charges from the polar regions without first driving all of the negative charges to the surface. He concluded that the system would respond by splitting open along the $q=0$ surface in the Goldreich-Julian distribution, and a vacuum gap would separate the two charge populations as shown in Fig. 14. He did not, however, give any mathematical models. Ruderman and Sutherland (1975) pointed out that for a suitable *charged* rotating star in a vacuum, an $\underline{E}\cdot\underline{B}=0$ surface exists above the polar caps. They proposed then that the gap between this surface and the star could be vacuum, while beyond, one again had a Goldreich-Julian—type solution. (This solution, however, still has the same pathologies as the gapless solution: Michel, 1979b.) Their emphasis was on the way in which the gap modulated the particle acceleration above the polar caps. In a later paper (Cheng *et al.*, 1976) a Holloway-type gap is proposed as well, but again this type of gap is not explicitly modeled. Jackson (1976a, 1976b) discusses a gap solution similar to that of Ruderman and Sutherland, but makes a quite different

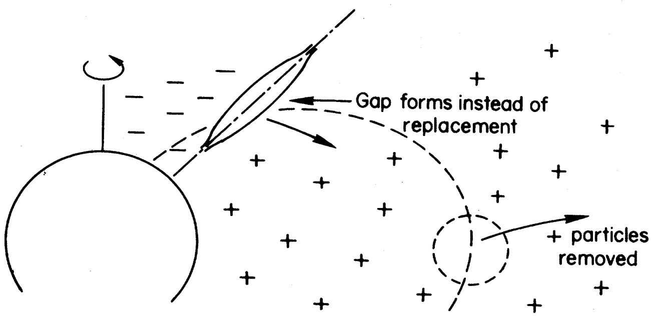

FIG. 14. Holloway's gap. If positive particles are removed from the equatorial regions of the Goldreich-Julian model, as illustrated, replacement from the surface seems impossible. Holloway proposed that the zero-charge surface splits to create a vacuum gap as shown.

interpretation. He regards the $\underline{E}\cdot\underline{B}=0$ surface as an accumulation point for charges, which he proposes to be only partially filled. Pelizzari (1976) has examined the Størmer-like escape of particles from plasma-free rotators (i.e., all "gap") and was unable to eject both positive and negative charges at once. Salvati (1973) considered the complementary possibility of regions devoid of magnetic fields.

It should be emphasized that such gaps have a remarkable property. Here charges and magnetic fields are arranged so that one passes through a region of space swarming with charged particles, following a magnetic field line along which the particles can move freely because it is an equipotential, and then one abruptly finds oneself in a vacuum. In other words, one has a true discontinuity in charge density (at zero temperature), going from a finite value to zero in an infinitesimal distance, as illustrated in Fig. 15. The remarkable fact of such true discontinuities is ambiguous in the above

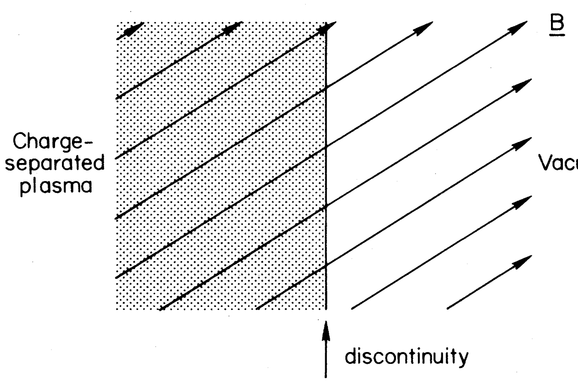

FIG. 15. Discontinuity. The plasma density falls abruptly to zero to separate regions of finite space charge (but $\underline{E}\cdot\underline{B}=0$) from regions of nonzero parallel field (but zero density). The parallel field returns particles to the discontinuity. (Such discontinuities can not be stable for a two-component plasma, since one or the other component would be accelerated away.) At finite temperatures the discontinuity would be "fuzzed" out over a few thermal scale heights.

works. Jackson assumed that his trapped plasma had a density that declined exponentially toward zero. In Holloway's model one is free to assume the same thing, since no explicit model is given, and the zero-density interface is the proposed site of gap formation. The Ruderman-Sutherland gap is really a transition from the acceleration zone to pair production (Fig. 8). Michel (1979b) reexamined the general question of how it is that vacuum gaps exist and developed a general formulation for constructing them in certain axisymmetric geometries, proposing that the field line potential distribution, which has been the source of so many difficulties as described above, might be modulated by the formation of appropriate gaps over the polar caps.

Jackson (1976a, 1976b) treats the $\underline{E}\cdot\underline{B}=0$ surface as an accumulation region, and dubs it an FFS (force-free surface). In Fig. 16 we illustrate the respective roles of the FFS's, both as discontinuities and as accumulation regions. Here, we treat a charged magnetized *nonrotating* sphere that emits some of its charge. Thus we start with an FFS in the equatorial plane at which the particles accumulate. The resulting discontinuities between the particles and vacuum are the new FFS's. Thus the FFS appears in two distinct contexts: (1) a place where charged particles of a certain sign can congregate, which is a property of the system in the absence of local space charge, and (2) a plasma-vacuum discontinuity. We shall continue to use the term "discontinuity" to so designate the latter. Accumulation of charges at an FFS [sense (1)] splits it into a pair of discontinuities [FFS's in sense (2)]. We cannot, of course, split a discontinuity to end up with two more FFS's; adding particles to a discontinuity simply shifts its location. In the charged rotating star model, however, the FFS can be of either type. Thus Jackson regarded the FFS as a "dome" over the polar cap which has accumulated (trapped) some charged particles. Alternatively, one is free to regard this same FFS as a discontinuity if one wants to model a "gap" over the polar caps similar to that of Ruderman and

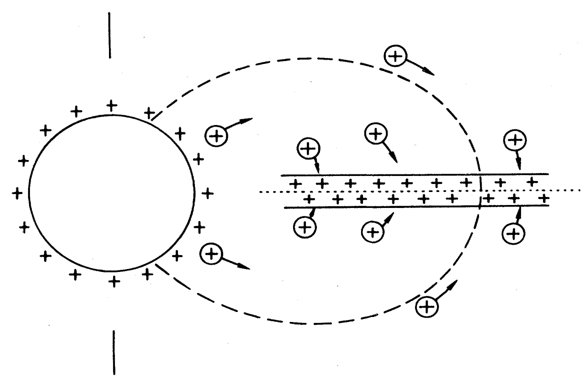

FIG. 16. Example of discontinuity formation. A charged *nonrotating* magnetized star loses some charge along field lines. These charges are repelled from the star to form a disk. The disk has finite thickness owing to self-repulsion of the charges. The disk is therefore bounded by two discontinuities (each a force-free surface) and is formed by splitting of the original vacuum force-free surface in the equatorial plane.

Sutherland. Basically, one has a hierarchy of special cases. The most general case is given by treating FFS's in sense (1), in which case they would each be split into a pair of discontinuities by accumulated space charge, namely, a lenticular dome over the polar caps (plus also a possible equatorial disk of opposite charge). These structures resolve the uncertainty of how the particles might distribute themselves about the FFS, since the Jackson paper indicated that a continuous distribution was expected, and no suggestion whatsoever was made that the FFS also act as a discontinuity. The dome and disk can then be expanded to fill in and give the Ruderman and Sutherland "gap." Yet further filling would eliminate the gap to give the Goldreich and Julian solution.

Later Michel (1980) concluded that the gap served not to modulate the plasma flow but to open-circuit the entire magnetosphere (a result implicit in Rylov, 1976). Then the magnetospheric structure around an aligned rotator would consist of a dome of charge over the polar caps and an equatorial disk of opposite charge which envelopes the entire surface but does not fill the magnetosphere (see Fig. 17). This possibility could totally defeat the Goldreich-Julian model, because now there need be no plasma loss beyond the light cylinder. Such pulsars would be "dead." Actually, Rylov had earlier (1976, 1977) come to the same conclusions for the same reasons, and even calculated approximate shapes for the charged clouds. This early work seems to have been neglected because Rylov went on to postulate an unknown mechanism to allow the equatorial particles to escape, which served only to make this model appear to be a peculiar version of the Goldreich-Julian model, rather

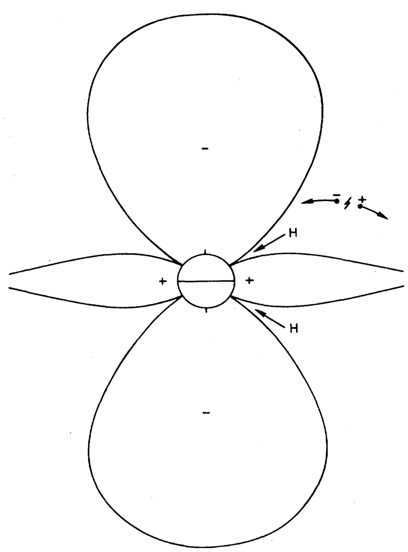

FIG. 17. Sketch of the proposed space-charge configuration. A Holloway gap (H) separates the electron dome from the positive disk. Pair production would reduce the gap between the two $E_{\parallel}=0$ surfaces, and hence nullify the pair creation field (even assuming that a discharge could be maintained without a source of primaries).

than a refutation of that model. There is no doubt, however, that Rylov appreciated the existence of stationary (dead) aligned magnetospheres.

The existence of a trapping region over the polar caps can be seen from the basic electrostatics of the aligned rotator. In Table VI(a), the external potential outside of the star is given by

$$\Phi=\frac{2}{3r}+\frac{1}{3r^3}(1-3\cos^2\theta) \tag{4.25}$$

Of course this result corresponds to a large surface charge, and as charges are released from the surface, they see a potential well (given the constraint to follow field lines) along the polar axis, with a minimum at $r=\sqrt{3}$ ($r=1$ is the stellar surface in these normalized units). Thus this potential well can be at least partially filled with electrons, while the positive particles are trapped in the closed magnetic field geometry.

It is interesting to go back to the full Goldreich-Julian charge distribution and decompose the potential ($\sin^2\theta/r$ in these normalized units) into multipole moments. The result is, at $r=1$,

$$\Phi=\frac{2}{3r}+\frac{1}{15}\left[\frac{3}{r^3}+2r^2\right](1-3\cos^2\theta)\ , \tag{4.26}$$

where the first term is again the central charge, the second term is the external quadrupole moment produced by the space charge inside $r=1$, and the third term is the internal quadrupole moment produced by the space charge outside of $r=1$. Potential (4.26) satisfies $\underline{E}\cdot\underline{B}=0$ at $r=1$; thus such a solution would not inject further charge and would not require a surface charge. The r dependences are here only formal, however, serving to remind us of the multipole behavior of each contribution. If we go to a larger radius, there is now more space charge inside and less outside, which increases the "3" (coefficient of the external quadrupole) and decreases the "2." In fact, of course, each term declines as $1/r$, and the sum is just $\sin^2\theta/r$. However, it is easy to see, from Eq. (4.26), the requirements for an alternative magnetospheric structure. First note that the first two terms are boundary conditions, fixed by the star; neither the central charge nor the internal space charge of the conducting star can be modified. Moreover, the same external quadrupole component must also be present, otherwise $\underline{E}\cdot\underline{B}\neq 0$ inside the star. In the vacuum case, this component is provided by the surface charge; in the Goldreich-Julian case it is provided by the external (magnetospheric) charge distribution. For the vacuum (surface-charge) case, potential (4.25) corresponds to a point above the surface-charge layer, and (4.26) to a point just below it (same coefficient but $r^{-3}\rightarrow r^{+2}$ charge in r dependence). It follows therefore that every solution satisfying $\underline{E}\cdot\underline{B}\approx 0$ internally, owing to presumed high conductivity, must give the same third term contribution in Eq. (4.26) at the surface. If no surface charge can be maintained, then all the charges must be in the magnetosphere. Thus the internal electrostatics are satisfied by *any magnetosphere* that provides a potential

$$\Phi_{\rm mag}=\frac{2r^2}{15}(1-3\cos^2\theta) \quad (r\le 1)\ . \tag{4.27}$$

As a simple example, suppose the magnetosphere consisted of two huge negative point charges over each polar cap. These charges $(-Q)$ see the potential of the star

$$Q_* = \frac{2}{3r}+\frac{1}{5r^3}(1-3\cos^2\theta) \tag{4.28}$$

and one another; thus along the polar axis each charge sees an electric field

$$E=\frac{2}{3r^2}-\frac{6}{5r^4}-\frac{Q}{4r^2}\ . \tag{4.29}$$

Note that, as advertised, there is a natural trapping region above each polar cap at $r^2=\frac{9}{5}$ $(Q=0)$. The full shape of this region (i.e., the force-free surface) is that of a sphere centered on the polar axis and crossing the axis at $r^2=\frac{9}{5}$ and 0 (the stellar center; Michel, 1979b). [Note that Eq. (4.25) overestimates the quadrupole potential of the star itself because here the surface-charge contribution is also included, and there is no such contribution once the particles are ejected to form the magnetosphere.]

This "magnetosphere" is too crude to reproduce potential (4.27) at the surface, except as the leading term in the multipole expansion of the field of two symmetrically located point charges. The latter condition is just

$$Q/r^2=\tfrac{1}{15}\ ,$$

which, when inserted into (4.29) with the condition $E=0$, gives $r=1.889\ldots$ and $Q=0.238\ldots$, hence a total system charge of $\frac{2}{3}-2Q=0.191\ldots$. Thus we obtain a crude model for Fig. 17, collapsing each dome into a point charge and ignoring the equatorial torus.

In this picture, then, the aligned rotator electrostatically traps negative particles above the polar caps and magnetically confines positive particles to an equatorial torus. The precise structure and stability of such configurations has not yet been shown, but that is in the process of being checked directly by numerical modeling. Jackson (1979, 1980a, 1980b) has shown several explicit closed magnetospheric configurations and proposed their possible relevance to the pulsar problem. These configurations do not satisfy the surface potential distribution $(\Phi=\sin^2\theta)$ given by a rigidly rotating neutron star, but correspond to differentially rotating stars. They nevertheless demonstrate that finite magnetospheres exist.

Pilipp (1974) has shown that such solutions (with vacuum gaps) cannot have the magnetosphere linked to the star along magnetic field lines. One requires noncorotating regions of space charge, such as that shown in Fig. 16. This result can be seen in a simple way if one neglects perturbations to the magnetic field. Then the connected regions would all have Goldreich-Julian charge densities, and a finite "solution" could be subtracted from the Goldreich-Julian infinite-extent solution. One would have a supposedly $E=0$ cavity surrounded by the remaining external quadrupolar charge distribution, but then the quadrupole moment (and field) within the cavity could not be zero, contradicting the supposed existence of a null solution. Note in this respect that once the vacuum interface is specified on any finite surface element, it is specified everywhere (Kellogg, 1967; Sunyach, 1980). For example, the $\Phi=\sin^2\theta$ surface potential, plus $\underline{E}\cdot\underline{B}=0$ there, uniquely requires the vacuum solution (4.26). Hence, if one had a vacuum region above the surface at some point, the entire vacuum region would have to satisfy (4.26) and could not be truncated until another force-free surface (FFS) was encountered. Thus one can immediately discount the possibility of a vacuum gap immediately above the polar caps, because (4.26) has no other FFS along the polar axis (it does have a conical FFS extending to infinity; Michel, 1979b).

Mestel *et al.* (1976) have argued that, in the aligned case considered here, field lines may not cross the light cylinder. The model suggested here is certainly consistent with that conclusion, since there need be no light cylinder. Holloway (1975) and Scharlemann *et al.* (1978) concluded that steady unidirectional flow of completely charge-separated plasma is impossible in the aligned rotator, which also points to the existence of static solutions. If this is correct, the version of the aligned rotator model having charge-separated plasma is faced with severe difficulties that may be insurmountable. Pelizzari (1976) generalized the Størmer theory (see, for example, Rossi and Olbert, 1970) to include accelerated motion in the strong electrostatic field outside a vacuum rotator. He too found that positive charges could not escape, but were trapped in a torus, whereas negative particles could be ejected to infinity at first, but then became electrostatically trapped as the stellar charge grew owing to this loss. Figure 18 shows these results where η is the dimensionless charge on the star in the same units as Eq. (4.25), which corresponds to the case $\eta=\frac{2}{3}$. Note, in

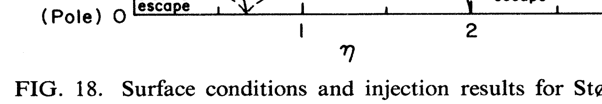

FIG. 18. Surface conditions and injection results for Størmer theory generalized to include electrostatic fields (Pelizzari, 1976). The electrostatic field is that of Eq. (4.25), where the fixed charge of $\frac{2}{3}$ has been replaced by a variable charge η. For $\eta=0$ electrons are pulled from everywhere on the surface, but can only escape for $\theta<15°$. For $\eta\gtrsim 2$, corresponding to a highly positive stellar charge, one obtains a similar behavior for the ions. In no case can both species escape at the same time, and for η between $\frac{2}{3}$ and 2 the ions cannot be injected, even though they could escape (thus we cannot obtain an oscillatory solution about $\eta=\frac{2}{3}$).

fact, that it is this value of the charge, which was taken to be a free parameter in Pelizzari's work, that corresponds to the nonescape condition for both electrons and "ions" (positive charges). If the star could have a smaller charge, electrons could escape and vice versa, but in no case can both be ejected at once (i.e., electrons escaping nearly along the polar field lines while neighboring ions are accelerated to sufficient rigidity to escape weak distant field lines).

a. *Inertial gaps*

Holloway and Pryce (1981) have pointed out that, even in the Goldreich-Julian model itself, there must be gaps, owing to gravitation and centrifugal forces. These inertial forces are neglected relative to the electromagnetic forces, but still exist. Thus $\underline{E}\cdot\underline{B}$ cannot be exactly zero. To balance gravity, a slightly different central charge is needed, whereas to balance centrifugal force, an additional uniform charge density must be added. However, these perturbation terms have opposite signs for the electrons versus the ions, even in the corotating portions of the magnetosphere. Consider ions in the corotating region. Centrifugal force will move them outward along field lines. To compensate, a uniform negative (attractive) background charge must be added. As a result the null surface ($q=0$) will be shifted equator-ward in Fig. 9. But for electrons the effect will be just the opposite, suggesting again that a vacuum gap opens up along the null surface as sketched in Fig. 14. Such gaps may not be too important, but they once again illustrate the discontinuous nature of a charge-separated plasma.

8. Numerical modeling

Given these many difficulties in finding steady-state solutions, why not just set up the full time-dependent equations on a computer and let the program run? Preliminary calculations of this sort have been done (Kuo-Petravic *et al.*, 1974, 1975; Petravic, 1976). These ambitious calculations numerically followed a linear spin-up of a rotator from rest. The unexpected finding was that *the magnetic field lines remained closed beyond the light cylinder.* Unfortunately a number of compromises had to be made in this calculation; to avoid grid size problems the calculations were begun not at the stellar surface but at $R_L/5$, and a rather large particle mass was adopted (order of 10^{16} eV). These two assumptions conspire to allow the particles to escape the magnetic field, owing partially to the injection scheme (Michel, 1974a; see, however, Kuo-Petravic and Petravic, 1976) and partially to the fact that the gyroradii of these massive particles becomes comparable to the light cylinder distance (Michel, 1974a, 1975a). The latter results because massive particles do not readily screen out the rotationally induced electric field, and consequently they achieve high energies (Sec. III.B.5). Thus closure of the field lines beyond the light cylinder would simply result from the free escape of energetic massive particles, a feature which is not expected to accurately simulate a pulsar magnetosphere. Although there was talk at the time of performing calculations with more realistic particle masses, these calculations have not yet materialized, possibly owing to the expense and difficulty involved. Nevertheless, the unexpected result, once again a closed magnetosphere, may also be suggestive of the static solutions of the previous section.

9. Summary of problems

The standard model seems beset by difficulties in all regions (near zone, transition region, far zone). No numerical solutions are at hand, and it has now been suggested that the rotator may in fact not emit particles. Clearly confusion still persists as to the simple classic behavior of a rotating magnet. This confusion is not diminished by the regular appearance of semiqualtitative models which assume that the Goldreich-Julian model is "basically" correct. Such models are often appealing because they seem more realistic and relate more directly to the observational data. Consistency with observation is then turned around and the observations are claimed to support the model. Unfortunately, this approach is a false parallel to the practice in physics of choosing between two internally self-consistent but alternative theories on the basis of observation. Here the basic theory is yet to be shown to be self-consistent, leaving no real choice at all.

C. Attempts to salvage the standard model

A number of suggestions have been made criticizing the basic assumptions in the standard model while proposing modifications that could still be implemented within the overall picture.

1. Two-component plasma

A popular suggestion is that a two-component plasma is present (electrons plus ions or positrons to give a partially neutralized plasma). The apparent advantage of this suggestion is that electric current and charge density can, in principle, be decoupled so that one can be zero without obligating the other to vanish as well. The severe difficulty is that a two-component plasma is inconsistent with the model itself. One cannot pull both signs of a particle from the surface with a static electric field. It is conceivable that the accelerating field at the surface oscillates, first pulling electrons and then ions out to produce a quasineutral plasma. If so, it is not obvious that the $\partial/\partial t=0$ assumption then means anything. In other words, it would make more sense to simply seek oscillatory solutions than to impose a two-component plasma on a steady-state system, since the latter two assumptions are mutually inconsistent. Moreover, many of the difficulties with the standard model seem to persist even if a two-component plasma is assumed. (The corotating regions, for example, would still be segregated by the inertial forces.)

2. Pair production

An alternative hope is that pair production (which we discuss in more detail below) automatically provides for the neutralizing charges. See, for example, Cheng *et al.* (1976). See Sec. IX.A.3 for a survey of such models. This suggestion essentially begs the question, since the standard model ignores pair production. After all, one can choose parameters such that pair production cannot be important, which then leaves the same problems in explaining what happens. Indeed, one of the successes claimed for the pair production models is that cessation of this process could account for the lack of slow pulsars (Sturrock *et al.,* 1976). But one should then be able to conclude from the insolubility of the aligned rotator (a classical mechanics problem) that pair production must exist in pulsars, and at what rate (a quantum electrodynamical phenomenon). This eventuality would seemingly be profound, but still leaves open the question of how these systems function when "off." Alternatively, one might argue that the aligned rotator is a separate unsolved problem of no immediate relevance. Nevertheless, no one has even shown that pair production solves the fundamental underlying current closure problem.

3. Closed current loops

The standard model neglects radiation from the particles. If we approximate the plasma to be continuous fluid, then a steady-state system need not radiate. However, it has been suggested that radiation plays an essential role (Mestel *et al.,* 1979; Mestel and Wang, 1979). Here, the basic defect of the Gold (1968) model, namely that charged particles would drift across field lines as a result of radiation reaction (hence not be trapped as supposed) is taken to be an advantage. In this model it is supposed that the particles come from the poles and move out to the light cylinder, where they radiate and consequently move across field lines toward the equator. There they then return along field lines to the star. Thus radiation reaction is argued to provide the emf to drive a closed current loop out from the star, down along the light cylinder, and back. It seems premature to evaluate this new suggestion, which is only now being investigated. As noted before, this picture is uniquely at odds with the approach, up-to-now successful, of treating radiation as a perturbation. However, Endean (1980) has pointed out that a nonaxisymmetric charge-separated magnetosphere must radiate as a result of rotation, and has proposed this effect as a pulsar model (Endean, 1972a, 1981). Again, radiation reaction would require the magnetospheric charge distribution to circulate, and if loss were impossible, these currents would presumably be closed within the system. See also Holloway (1977) and Wang (1978).

4. Formal solutions

Some recent formal advances have been made in which self-consistent charge-separated solutions are claimed to exist (Schmalz *et al.,* 1979, 1980). At present these programs have not yet been fully carried through, so the nature of the solutions cannot yet be examined.

D. The Sturrock-Ruderman-Sutherland model

Some researchers have preferred to "leap frog" the issue of self-consistency. In their models the functioning of the aligned rotator model is taken as a given, and the analysis is built upon assumed properties of the standard model. The intention is basically to confront observation by designing a functioning pulsar, but based on physics not yet shown to be self-consistent. As a result, there is considerable confusion as to which model is "best." The aligned rotator models are typically directed at sorting out the self-consistency problem, while the extended models are aimed at explaining the data, but unfortunately in terms of a basic model not yet shown to work. The model originally proposed by Sturrock (1970, 1971a; see also Komesaroff, 1970) and refined by Ruderman and Sutherland (1975) is one of the more important examples of the latter genre. An extensive discussion of this model can be found in the review by Sutherland (1979). (See also Sturrock, 1971b; Roberts and Sturrock, 1972a, 1972b; Ruderman, 1976; Cheng and Ruderman, 1977a—1977c, 1979.)

1. Pairs

In the corotating plasma of the standard model, there is always a local frame of reference in which locally $\underline{E}=0$, at least within the light cylinder. In the vacuum case, however, there are strong parallel electrostatic fields that cannot be removed by change of reference frames ($\underline{E}\cdot\underline{B}$ is a relativistic invariant), introducing the possibility that a cascade breakdown could occur in these strong fields. The idea, first developed by Sturrock (1971a) and apparently also suggested by Goldreich (1969), is that electrons accelerated to high energies by the electric field would emit hard gamma rays owing to their curvature radiation in following the dipolar magnetic field lines. These gamma rays would in turn pair produce in the intense magnetic field and thereby provide yet more electrons and positrons, and so on. Actually, the particle energies rapidly degrade with each step, and only for extreme parameters would the secondary pairs result in tertiary pairs; hence the "cascade" terminates quickly. For example, a primary electron of energy γ_0 produces a gamma ray of energy [Eq. (3.34)], in units of $m_e c^2$,

$$\gamma_p \sim 3\gamma_0^3 \lambda\!\!\!^{-}_c/\rho_c \ ,$$

TABLE VII. Energy degradation in pair cascade.

$\log_{10}$: γ_0	γ_1	γ_2	γ_3	γ_4
$\eta=10^{-1}$	(10^{-3})[a]			
(14)[c]	(13)	10	1	b
(12)	(11)	8	−1	
(10)	(9)	6	−3	
(8)	(7)	4	−5	
(6)	(5)	2	−7	
$\eta=10^{-2}$	(10^{-6})			
(14)	(12)	6	−12	
(12)	10	4	−14	
(10)	8	2	b	
(8)	6	0		
6	4	−2		
$\eta=10^{-4}$				
(14)[c]	10	−2		
(12)	8	−4		
(10)	6	−6		
8	4	−8		
6	2	−10		

[a]Reading this column as γ_0 instead of γ_1, γ_1 instead of γ_2, etc. lent to $\eta=10^{-3}$ instead of $\eta=10^{-1}$, etc.
[b]Frequencies below 10^{-15} cannot propagate through interstellar space (see Sec. II.C.2.a).
[c]Values in parentheses correspond to $\eta\gamma_0$ products in excess of 10^4, which violate Eq. (4.32) and are therefore unlikely from radiation reaction considerations.

which in turn pair produces, if above threshold, to give an electron-positron pair, each particle of which has a Lorentz factor $\gamma_1 \approx \gamma_p/2$. Thus the successive Lorentz factors are γ_0, $\gamma_1 \sim k\gamma_0^3$, $\gamma_2 \sim k(k\gamma_0^3)^3, \ldots$, where k represents the above coefficients. We can rewrite this series in dimensionless form

$$\gamma_k = \gamma_0 \eta^{p(k)} \tag{4.30}$$

where

$$p(k) = (3^k - 1)/2 \quad \text{and} \quad \eta = k\gamma_0^2 \ . \tag{4.31}$$

Thus p takes on the successive values $0,1,4,13,40,121,\ldots$, and it is evident that, unless η is very nearly unity, the pair production quickly falls below any plausible threshold ($\gamma \geq 2$ being the absolute limit). A straightforward estimate for the Crab pulsar would (Table V), for $\gamma_0 \approx 10^5$, give $\eta \sim 10^{-6}$. If instead we regard η as a free parameter, we would require $\eta \approx 10^{-3}$ to obtain Crablike visible photons as the last conversion step. For example, primary electrons with $\gamma_0 = 10^6$ would produce $\gamma_1 = 10^3$ secondaries, which would in turn produce radiation with "γ_2"$=10^{-6}$ (~ 1 eV photons, but no accompanying pairs of course). See Table VII. Thus, even with these rather extreme parameters, there would be only one stage of pair production, with those pairs then radiating visible light. If the injection energy could be boosted to $\gamma_0 \sim 10^{10}$, then $\eta \approx 10^{-4}$ (easily obtained at that energy) would cascade first to $\gamma_1 = 10^6$ and then once again $\gamma_2 = 10^{-6}$. If the absorption threshold were placed near γ_1, we would then obtain a spectrum extending from the hard gamma rays to the visible, as observed for the Crab. This possibility is therefore an attractive one, but at odds with the limitations imposed by radiation reaction (Sec. III.B.7).

The role of radiation reaction can be estimated by requiring the accelerating field to be at least of the order of γ_0/ρ_c and repeating the above analysis, which then gives the parameter-free limits

$$\eta\gamma_0 \lesssim 9\pi\varepsilon_0 mc^2 \lambda_c / e^2 = 308 \ , \tag{4.32}$$

the latter figure being just a numerical factor $(\frac{9}{4})$ times the reciprocal of the fine-structure constant. Equation (4.32) is not really a firm limit, since the accelerating field could be larger than estimated; however, very much larger values of $\eta\gamma_0$ clearly become suspect and demand very special geometries. In Table VII we have indicated those values of $\eta\gamma_0$ is excess of 10^4. Since the first cascade gives photons of energy $\eta\gamma_0$, it is clear that a secondary cascade is obtained only if the system operates near the radiation reaction limit. A tertiary cascade would occur only if $\gamma_0\eta^4$ exceeded unity, hence requiring $\eta \sim 10^{-1}$ but restricted to low values of γ_0 ($\lesssim 10^5$), an implausible physical circumstance. The above analysis assumes the radiation to be curvature radiation; the presumably weak contribution from synchrotron radiation, owing to the finite pitch angles, has not been included. See Daugherty and Harding (1981).

Figure 19 shows the (one-stage) cascade actually expected, in which a downward-moving electron (remember the Ruderman-Sutherland polar caps have the opposite

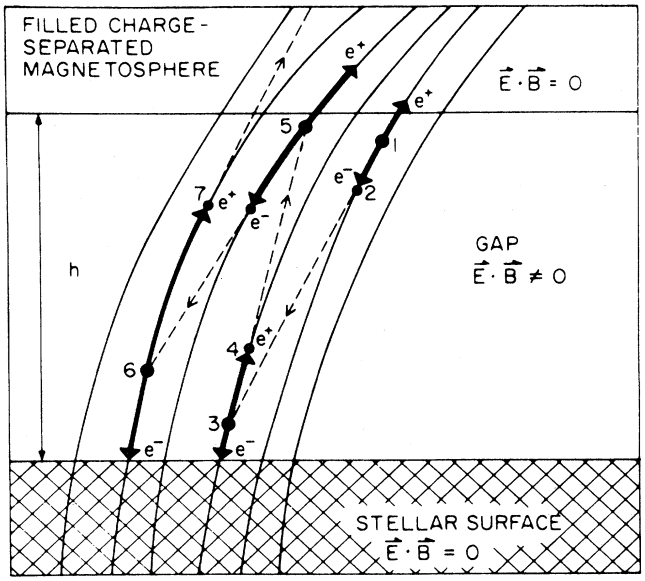

FIG. 19. Nature of a pair-production discharge. All the potential drop must appear in the gap (h), otherwise the system would be flooded by downward-accelerated electrons. Above the gap the energetic primaries would continue to radiate and produce yet more pairs, resulting in a relatively dense pair plasma. The gap width would be maintained so that pair production within the gap is kept just at threshold (otherwise the average number of particles there would exponentiate). Pair production takes place at points 1, 3, 5, . . . and gamma radiation at points 2, 4, 6, ... Because the process requires curved field lines, it automatically "marches" toward the least-curved field line, suggesting that it exhibits a relaxation-type oscillation (Cheng and Ruderman, 1977b).

sign to the Goldreich-Julian choice) formed at point (1) gains sufficient energy at point (2) to produce a gamma ray that is abosrbed at (3) to produce a pair, the electron almost immediately being absorbed by the surface and the positron reaccelerated in the opposite direction. This process must run just at threshold with one "particle" (downward-moving electron or upward-moving positron) being conserved in the gap. Exponentiation (one electron→two positrons→4 electrons, etc.) would close the gap and vice versa. Note that this process is also self-extinguishing, since the cascade automatically seeks the least-curved field line, moving toward the poles in this figure. The space charge is largely zero in this model except for two sheaths at the top and bottom of the gap, and the current flow is modulated by keeping pair production near threshold in the gap rather than through space-charge limitation. Note also that the multiplicity (pairs produced for each oscillation of the "primary" particles in the gap) is just γ_1/γ_0, from energy conservation, hence of the order η^{-1}. Figure 13 illustrates a more realistic geometry for the gap.

The cascade model is, in the Ruderman-Sutherland version, based on calculations showing that iron ions would be too tightly bound to escape from the crystalline lattice, owing to the expected strong magnetic fields, which we discuss in the next section.

2. Atoms in strong magnetic fields

The entire issue of what structure atoms would have in strong magnetic fields is an interesting pure theoretical question in its own right. And a considerable literature has already been devoted to the states of the atoms in such fields: Cohen *et al.* (1970), R. G. Newton (1971), Barbieri (1971), Kadomtsev and Kudryavtsev (1971), Mueller *et al.* (1971), Wilson (1974), Banerjee *et al.* (1974), Mueller *et al.* (1975), Rau *et al.* (1975), Rau and Spruch (1976), Angelie *et al.* (1980), and Hylton and Rau (1980). The more difficult problem of calculating the lattice binding energy has been approached by Ruderman (1971), Witten (1974), and Chen *et al.* (1974). The early estimates now seem to have been too large (Hillebrandt and Müller, 1976) and have since been revised downward (Flowers *et al.*, 1977). See also Constantinescu and Moruzzi (1978) and Endean (1973). These later estimates give a binding energy of about 2.6 keV for iron ions in a 10^{12} G magnetic field (the density at the neutron star surface is estimated to be 10^4 to 10^5 g/cc under these circumstances; Ruderman, 1979). Alternatively, it has been suggested that helium might be available from the pulsar crust (Michel, 1975d; Burdyuzha, 1977), which would significantly enhance the availability of weakly bound ions. The uncertainty is how ideal a real neutron star might be. See Glasser (1975) for a discussion of the effect of the field on the conduction electrons.

It is interesting to note that the contemporary explanation for x-ray bursts (Woosley and Taam, 1976; Maraschi and Cavaliere, 1977; Joss, 1977, 1978; Lamb and Lamb, 1978; Taam and Picklum, 1978) postulates, in fact, that sizable amounts (about 10^{21} g) of helium can accumulate on the neutron star surface. Such quantities are just those necessary to supply ions to a pulsar wind over the pulsar lifetime (Michel, 1975d). However, if all pulsars are born at high spin rates (where they spend a negligible fraction of their lives), these light ion sources would be depleted (Ruderman, 1981).

3. Electric current production

A particularly attractive feature of the cascade model is the possibility that the cascades are localized within "sparks" rather than uniformly filling the entire polar cap. This model is thereby endowed with additional morphological detail which could well elucidate the rich variety of behavior in observed pulsars. Cheng and Ruderman (1980) have expanded their model to include ion production from the polar caps, owing to the electron bombardment (see also discussion by Matese and Whitmire, 1980). Arons (1981a) has recently undertaken an analysis of how the cascade process operates. At the moment, however, pair production complicates the physics without greatly clarifying it. Current closure has not been demonstrated. Benford and Buschauer (1977) argue that bunching would grow too slowly in this model to account for coherent radio emission.

E. Variations on the standard model

1. Massive magnetospheres

One could add *neutral* plasma to the magnetosphere of the standard model in addition to the space-charge plasma. By and large there has been little enthusiasm for such an approach, since one cannot use the induced electrostatic field to lift a neutral component off the surface, and it takes about 100 MeV per nucleon to lift ions off a neutron star. (For example, if the 10^{41} electrons/sec proposed by Shklovsky to be injected into the Crab Nebula were to come from a neutron star and were accompanied by a neutralizing component of nonrelativistic helium ions, then an energy comparable to the entire luminosity of the nebula would have to go into simply lifting those ions from the neutron star.) Nevertheless, there has been some motivation for studying the consequences of models having excess density (Roberts and Sturrock, 1972a, 1972b, 1973; Henriksen and Rayburn, 1974; Pustil'nik, 1977; Evangelidis, 1977). The massive magnetosphere models are interesting in that the critical distance separating open field lines from closed field lines is moved inward from the light cylinder. As a result the stellar wind magnetic field is proportionately enhanced (i.e., less surface field would be required to give the same torque). Roberts and Sturrock (1973) also emphasize that the massive magnetospheres can give $n=\frac{7}{3}$, instead of 3, which is in closer agreement with observation (Groth, 1975a, 1975b) of NP 0532 ($n\approx 2.5$). Also, the massive magnetosphere could be a source of plasma injection into the circumpulsar space (Scargle and Pacini, 1971). In contrast, the total mass of a standard model magnetosphere would typically amount to only a few kilograms. Ozernoi and Usov (1973b) propose, nevertheless, that timing fluctuations could be caused by plasma accumulation and ejection from the magnetosphere.

2. Pulsar extinction

Within the standard model, which does not embrace any specific emission mechanism, there is no particular distinction between slow pulsars and rapid pulsars. Yet observation suggests that pulsars cease to radiate at periods longer than a few seconds. A number of proposals have been made. The most popular seems to be magnetic field decay (Ostriker and Gunn, 1969b; Fujimura and Kennel, 1980), although there have been theoretical difficulties in justifying the high stellar resistivity implied (Ewart *et al.,* 1975; see Sec. X.A.2 and counterarguments by Holt and Ramaty, 1970). The next most popular supposition is that pair production is essential (Sturrock, 1971a) and that extinction occurs when the pulsar rotates too slowly to produce pairs (Sturrock *et al.,* 1976; this suggestion appears, in the form of an incidental remark, in many places). It has also been pointed out that if iron ions were not available from the surface, pulsars could run until they ran out of fossil surface helium (Michel, 1975d), and, conversely, if iron ions were available they would no longer be confined within a slow pulsar magnetosphere (Michel, 1975a; Hill, 1980), suggesting transitions in the magnetospheric properties.

V. THE OBLIQUE ROTATOR

The aligned rotator problem may be regarded as a warming-up exercise (Michel, 1970b) for the oblique rotator (magnetic field axis at an appreciable angle to the spin axis). If one cannot solve the former, then how can one solve the latter case with its added degree of complexity? Here the pioneering work is that of Deutsch (1955), who originally solved for the full $\underline{E}$ and $\underline{B}$ distribution in the vacuum case, suggesting that cosmic rays might be accelerated by the $E_{\|}$ fields. See also Soper (1972).

The oblique rotator is of particular interest because its structure is manifestly time dependent at the spin frequency and is therefore a natural pulsar possibility. One idea is that the presumed polar cap emission from an aligned rotator continues for the oblique case, and it is this emission, sweeping past the observer, that produces the pulse. Of course, if the aligned case does not function to begin with, one is hardly perturbing a viable solution to obtain pulses. However, no one has come up with any other convincing reason for such a system's producing the sharp pulses actually observed. One approach is to regard the radiation and wind from such a rotator as a beam which interacts with surrounding material (Sec. VI.C.) to form the sharp pulses. Another is to invoke plasma effects to "shape" the electromagnetic

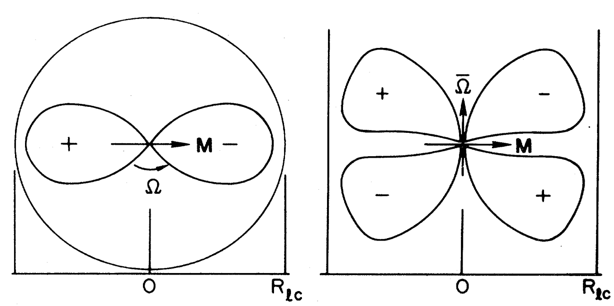

FIG. 20. Charge distribution about an orthogonal rotator (Parish, 1974). Left-hand figure shows the charged "clouds" as seen looking along the rotation axis. In this view the clouds (actually just one isodensity contour is shown for simplicity) appear to be concentrated over the two polar caps (**M** is the magnetic moment axis). The right-hand figure shows, as seen from "below" on the left, the actual structure of the clouds, each of which is actually two oppositely charged clouds on either side of the magnetic poles.

pulse, which would otherwise simply be a sine wave in the vacuum case (Michel, 1971).

Some basic preparatory work has been accomplished. Ostriker and Gunn (1969a) treated the vacuum case and regarded particles emitted from the surface as a perturbation, showing that the particles would absorb the wave energy. See also Eastlund (1968, 1970), Cohen and Toton (1971), and Krivdik and Jukhimuk (1977). Mestel (1971) and Cohen and Rosenblum (1972) extended the Goldreich-Julian model (as a source of surface plasma) to the oblique case. Others have argued that these particles nevertheless strongly influence the electromagnetic field structure (Michel, 1971). The near-zone equations have been written down by Cohen *et al.* (1973), Parish (1974), Pfarr (1976), and Kaburaki (1978). Cohen *et al.* and Kaburaki give general relativistic formulations. There has been considerable development toward a full dynamic model: Endean (1972b), Henriksen and Norton (1975b), Mestel *et al.* (1976), Burman and Mestel (1978), Jackson (1978a), Scharlemann *et al.* (1978), Avetisyan (1979), Kaburaki (1980a, 1980b), Davila *et al.* (1980), and Ray (1980).

At the moment, the overall status of the oblique rotator model seems to be in a curious sort of limbo. It is not clear whether or not the difficulties with the aligned rotator carry forward to the oblique case or whether those difficulties in fact result from the imposed geometrical constraint of alignment. In the extreme case of an orthogonal rotator (Fig. 20) the positive and negative space-charge regions above the magnetic poles are the same, and consequently outward flow is conceptually much more plausible than in the aligned case. The lesson from the last decade or so of study of the aligned rotator, however, has been that first appearances can be misleading. The extra dimension introduced by obliquity might also simply make it more difficult to analyze and to discover possible inconsistencies. It seems likely, nevertheless, that much more attention will be devoted to this model in the near future.

VI. A SURVEY OF OTHER PULSAR MODELS

In view of the problems with the basic physics of the rotating magnetized neutron star model, it is probably worthwhile to review briefly various other suggestions that have been made.

A. Jupiter as a pulsar

The similarity between the pulsar phenomenon and the Jovian decametric emissions was noted almost immediately (Burbidge and Strittmatter, 1968; Dowden, 1968; Warwick, 1969). At the time it was thought that Io was entirely responsible for exciting the Jovian emissions, owing to the unexpected correlation between that satellite's orbital position and the radio emissions (Bigg, 1964), which was theoretically attributed entirely to the satellite (Goldreich and Lynden-Bell, 1969). Actually it was known all along that there was a "non-Io" component to the radiation (see the reviews by Gehrels, 1976; Carr and Gulkis, 1969), which is now thought to be due to a torus (or disk) of plasma more-or-less filling Io's orbital path, but much less startling than the Io component. The preoccupation with Io may have been misleading (see Sec. VIII); in any event the analogy itself was quickly dispatched by the observation that such a body would have to be within the Roche limit (Douglas-Hamilton, 1968), assuming that it was at the orbital period of such a satellite that the pulsation took place (in the Jupiter analogy). Since then, the Jupiter parallel has surfaced repeatedly (Mertz, 1974; Kennel and Coroniti, 1975; Michel, 1979a; Braude and Bruk, 1980), although these later efforts tended to try to force Jupiter to conform to the "standard model." Actually Kennel and Coroniti (1975) thought that a convection-type model might be promising in view of the "grave difficulties" with the standard model, while Michel (1979a) suggested that some essential physical element might be missing, since other energetic radio sources seem to contain magnetized plasmas in relative motion [Sedrakyan (1970a, 1970b), suggests differential rotation as an energy source; see also Akasofu (1978) and Lu (1976)].

Orbiting (binary) neutron stars would not suffer the Roche limit objection and were proposed (Saslaw *et al.*, 1968; McIlraith, 1968; Aldrige, 1968) and dismissed (Pacini and Salpeter, 1968). Again, orbiting systems should speed up, not slow down as energy is radiated.

B. Oscillating objects

1. White dwarfs

It is ironic that there is fairly little literature on this model, considering that it was the one most heavily favored to begin with. Many influential astrophysicists at the time were loath to immediately adopt an object, the neutron star, that might well not exist. But white dwarfs, although known to exist, simply could not oscillate or rotate rapidly enough. Heroic assumptions were required to get a period even approaching one second (which is about average for a pulsar). Higher harmonics were suggested as a mechanism to get shorter periods (Ostriker and Tassoul, 1968), but as it became clear that the pulsars were such excellent clocks, the idea of a pure high-harmonic oscillation came to seem forced, and any remaining pockets of resistance were more or less swept away with the discovery of a 33 msec pulsar in the Crab Nebula (see Henry, 1968; Israel, 1968; Durney *et al.*, 1968; Faulkner and Gribben, 1968; Van Horn, 1968; Cocke and Cohen, 1968; Kundu and Chitre, 1968; Simon and Sastri, 1971; and Black, 1969). Even at the time, this program seemed doomed (Skilling, 1968; Bland, 1968). It was also suggested that the oscillations might be confined to the white dwarf atmosphere (Black, 1969).

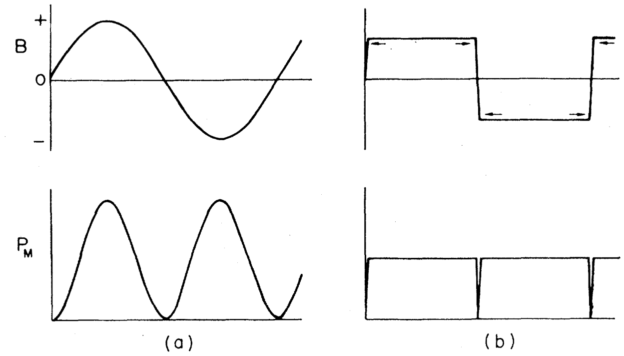

FIG. 21. Rearrangement of a sinusoidal large-amplitude wave to form a step-function wave. The wave is shown in the plasma rest frame, wherein the average electric field over each *half* cycle is zero. (a) The sinusoidal case wherein the total pressure is unbalanced if the plasma is cold. The large magnetic field pressure can only be balanced by dynamic pressure, namely, the acceleration of plasma towards the zero-field (neutral sheet) regions. (b) The result of the above acceleration process. Plasma streams into the neutral sheets and a step-function structure would be in static equilibrium were some magnetic energy dissipated to heat the plasma at the neutral sheet. In actuality the approach to equilibrium would be complicated by propagation of shock waves into the uniform field regions, heating of the plasma at other than the neutral sheet, etc., but (b) above illustrates a physically consistent simplification, unlike (a). Such a transition would presumably take place near the light cylinder, where the electromagnetic fields first begin to become wavelike (Michel, 1971).

2. Neutron stars

An oscillating neutron star suffers from the opposite problem of a white dwarf; it would oscillate too fast (~ 1 msec at small amplitudes) and would become unbound at large amplitudes (Thorne and Ipser, 1968). However, such models have been discussed (Hoyle and Narlikar, 1968; Israel, 1968; Baglin and Hayvaerts, 1969; Harrison, 1970; Stothers, 1969; Papoyan *et al.,* 1973). Heintzmann and Nitsch (1972) have estimated the damping times for such oscillations to be of the order of months to years. Finzi and Wolf (1968) proposed even before the discovery of pulsars that the Crab Nebula could be excited by a vibrating neutron star.

3. Plus rotation

Chiu and Canuto (1969a, 1969b) proposed a pulsar model, basically the rotating magnetized neutron star, in which, at the same time, oscillation of the star was proposed to excite maser action in the near magnetosphere (see Sec. IX; there is an extensive literature). See also Kumar (1969), Vladimirskii (1969), Yukhimuk (1971), and Van Horn (1980).

C. Sheet discontinuities

A number of theories propose radiation from sheetlike structures such as standing shock waves or magnetic neutral sheets.

1. Shock waves and neutral sheets

Michel and Tucker (1969) proposed that the pulsar generates a magnetically driven supersonic wind, which eventually must shock and become subsonic in response to external pressure, requiring a standing shock wave. Neutral sheets convected outward were proposed to be sites of coherent particle excitation upon crossing the standing shock. This model would have to operate at large distances from the pulsar and at quite low particle densities, making sufficient coherence a problem. Arons (1979) has analyzed the beaming properties of this model.

The transition from near zone to stellar wind is an alternative region that has attracted considerable attention because such a zone could quite plausibly contain a shock wave (see Fig. 21) and, in a charge-separated flow, such a shock could produce coherent motions of charged particles over a scale of the order of the shock thickness. For the Crab pulsar, for example, this circumstance is not implausible, since the magnetic field at the light cylinder is $\sim 10^6$ g, while the electron energies are $\sim 10^{11}$ eV [either from space-charge-limited flow (Michel, 1974c) or independently from inference regarding the nebular synchrotron radiation (Shklovsky, 1968)], which gives a cyclotron radius of ~ 3 m. The number of particles in a cubic meter at the light cylinder (Crab) should be $\sim 10^{14}$, giving a limiting effective temperature of $NE \sim 10^{29}$ K which, at the light cylinder, would be about adequate to account for the brightness temperature of the Crab pulsar (Michel, 1971). Note that the brightness temperature limitation is considerably relaxed because the radiating elements are expanded from small bunches over the polar caps to a broad sheet. In fact, Eq. (2.9) is essentially scaled to emission from the light cylinder. See also Bertotti *et al.* (1969a, 1969b), Kardashev (1970), Endean and Allen (1970), Ferrari and Trussoni (1973), and Stewart (1977). Fujimura and Kennel (1979) have given numerical solutions for such shock transitions.

2. Current sheets

A closely related model was suggested by Lerche (1970a−1970c, 1971), who imagined that the neutron star's electromagnetic field was essentially in a vacuum, separated from external plasma by a current sheet which oscillated as the neutron star rotated, producing coherent emission. See also Dokuchaev *et al.* (1976). Tademaru (1971), on the other hand, proposed that outwardly expanding charge sheets were the emission source. See also Grewing and Heintzmann (1971).

D. Unconventional

1. Volcanoes

Dyson (1969b) suggested that matter might be ejected from the interior of a neutron star, giving an inhomogeneous plasma distribution about the star. Related proposals were recently made by Kovalev (1979, 1980), wherein coherent radiation escapes through cracks in the pulsar crust. Kaplan and Eidman (1969) suggested that parts of the surface might move with relativsitic velocities. See also Kumar (1969).

2. Starspots or flares

See Fujimoto and Murai (1972, 1973), Apparao and Chitre (1970), Ostriker (1968), Glencross (1972), and Smoluchowski (1972). The first paper also proposes ejection of matter from the neutron star.

3. Other

See Tsygan (1977), Karpman *et al.* (1975), and Lu (1976).

E. Discussed elsewhere

See Sec. IX for a discussion of maser emission mechanisms, some of which are applied to models other than the rotating magnetized neutron star model.

F. Quasars as pulsars

If anything, quasars are more poorly understood than pulsars. Thus this section does not strictly qualify as supplying an alternative model for pulsars, but the parallel may be of interest. Basically, two suggestions have been made; (1) that a quasar is, in effect, one huge pulsar (Morrison, 1969; Cavaliere *et al.*, 1969; Sturrock, 1971c; Stecker, 1971; Fowler, 1971; Ozernoi and Usov, 1973a; but see Bisnovatyi-Kogan and Blinnikov, 1973), or (2) that a cloud of pulsars excites the quasar activity (Rees, 1971a; Arons *et al.*, 1975; Kulsrud, 1975; Kulsrud and Arons, 1975), as an alternative to the multiple supernovae model (see Petschek, Colgate, and Colvin, 1976).

VII. THE DISK MODEL

A. The model

It has recently been argued (Michel and Dessler, 1981) that the radio pulsars and the x-ray pulsars differ mainly in the fact that the latter are surrounded by an accretion disk, while the former are surrounded by a fossil collapse disk presumably left over from the formation event. Specifically, they suggest that active radio pulsars are rotating neutron stars surrounded by fossil disks left over from the collapse event and that energy is extracted from the rotation of the neutron star by interaction with the disk to produce pulsar luminosity. A disk might well result from angular momentum conservation in the collapse phase, some matter being shed to carry off excess angular momentum. Parallels between the radio and x-ray pulsars were previously suggested by Tucker (1969).

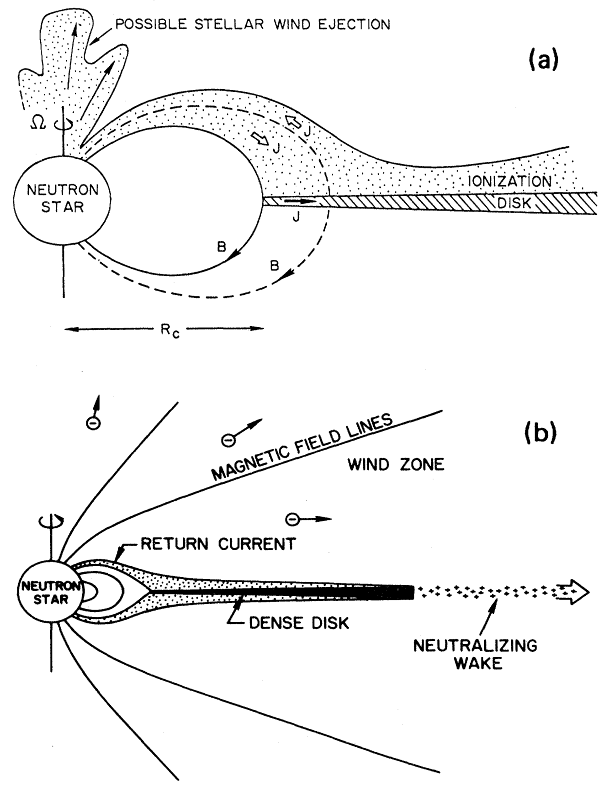

FIG. 22(a). Sketch of disk and neutron star interaction. A current loop couples the two systems dissipatively, thereby heating the disk to maintain electrical coupling, and the polar caps to provide a stellar wind (at least in the case of more energetic pulsars such as the Crab). A Pedersen current shown in the disk exerts a torque on the disk and star, while an axial component into the disk segment shown resists centrifugal forces. The arc drawn between the disk and the star is presumed to be the source of the coherent radiaton. (b). "Strong Coupling" limit. If finite conductivity is insufficient to limit the current flow, the resultant $\mathbf{j}\times\mathbf{B}$ forces should eject the magnetic field lines. The steady state solution would then not be that shown in Fig. 22(a), wherein most of the stellar flux penetrates the disk, but that shown here where only a small flux penetrates. Because $\Delta\Phi=\omega\Delta f$, the flux reduction also reduces the available emf. Thus we return to the standard model geometry of Fig. 6, with the disk acting as a "neutral sheet" in the equatorial plane. In this geometry, the field structure is identical to that proposed by Roberts and Sturrock (1973), which gives $n=\frac{7}{3}$.

As we have seen, in the "standard model" for pulsars, the rotationally induced electric field of a rotating, magnetized neutron star pulls plasma off the surface and ejects it beyond the light cylinder to form a relativistic stellar wind. Pulsar emission is then associated with the acceleration of new particles to maintain this wind, assuming that rotation self-excites the pulsar emission. However, there is some doubt that the standard model can function this way, as has been discussed in Sec. IV.B. It appears that the charge-separated magnetosphere may even have finite extent and not reach out to the light cylinder at all (Rylov, 1976, 1977; Michel, 1980; Michel and Pellat, 1981). If so, the Goldreich-Julian (1969) picture does not obtain for an aligned rotator. If the same consequences follow for the oblique rotator as well (this has yet to be demonstrated, but seems plausible for small angles of obliquity), even the Ostriker and Gunn (1969a) picture of particle acceleration by giant dipole waves fails, because there are no particles in the wave zone to be accelerated. The standard model would then do no more than emit unobservably-low-frequency waves, as was originally supposed (Pacini, 1967).

In essence, one would have two coupled unipolar generators: the neutron star and the disk, as shown in Fig. 22(a), the latter acting here as a load. As we have seen, corotation of plasma with the stellar surface gives a potential that, if projected along dipolar magnetic field lines to the equator, would give

$$\Phi_{ns}=\Omega B_0 a^3/2r+\Phi_{os} \tag{7.1}$$

where ϕ_{os} is the system potential relative to zero at infinity. The disk potential itself is, however, assuming Keplerian rotation,

$$\Omega^2=GM/r^3\ , \tag{7.2}$$

$$\Phi_d=(GM)^{1/2}B_0a^3/5r^{5/2}+\Phi_{od}\ . \tag{7.3}$$

It is clear that there can be only one radial distance at which the two potentials agree. Thus, if sufficient plasma exists in such a system to electrically couple the neutron star and the disk, closed currents will flow. The most obvious change is one of scale; with a disk, the natural scale length is no longer the light cylinder distance, but is now the "corotation" distance where a particle in Keplerian orbit rotates with the same period as the star:

$$R_c\equiv(GM/\Omega^2)^{1/3}\ . \tag{7.4}$$

No electromagnetic forces are required to enforce corotation at this distance. The pulsation period is taken to be locked to that of the neutron star, and the interaction with the disk is assumed to take place just outside the corotation distance. It is assumed, moreover, that the interaction region on the disk is a "spot" projected along field lines from the star to the disk, this spot rotating (on the average) with the neutron star regardless of the local disk velocity. Such localization of the interaction region is observed in radio emission from both Jupiter and Saturn (Dessler *et al.*, 1981; Kaiser *et al.*, 1980). Such spots, or more generally, longitudinal inhomogeneities, have a theoretical justification in terms of magnetic field variations on the neutron star surface. These variations map outward into the magnetosphere (Dessler, 1980a, 1980b; Dessler *et al.*, 1981; Hill *et al.*, 1981), and the degree of localization could be nonlinearly enhanced [e.g., concentrated locally, similar to the "sparks" suggested by Ruderman and Sutherland (1975)]. The point stressed is that pulsed emissions are produced by even such highly symmetric rotators as Saturn (Kaiser *et al.*, 1980), so the localization phenomenon exists independent of whether a generally accepted theoretical model is at hand.

A disk not only modifies several parameters important to the possible emission mechanisms, but it also introduces a richer variety of plausible physical circumstances: (a) if plasma is derived from the disk as well as from the pulsar surface, the current flow need not be entirely charge separated (a troublesome theoretical constraint), (b) moving the characteristic distance in from the light cylinder rescales most of the pulsar parameters, decreasing the minimum required magnetic field strength, for example, and (c) the disk itself becomes a potential site for particle radiation. One can estimate the maximum torque on the disk, since the maximum shear exerted by the magnetic field cannot be greater than about $B^2/2\mu_0$. Integrating this maximum torque over the entire disk (from R_c to infinity) and multiplying by the rotation rate then gives the maximum power output from the neutron star:

$$L\lesssim\pi B^2a^6\Omega/3\mu_0R_c^3\ . \tag{7.5}$$

Hence $L\sim\Omega^3$ instead of $L\sim\Omega^4$ as implied by Eq. (3.14). The disk torque would be comparable to the wind torque, given the same B, for $R_L\sim R_c$ (rotation at breakup).

A disk model does not necessarily require a new set of emission mechanisms, it merely rescales the parameters somewhat and fixes the geometry more narrowly. No specific radiation mechanism is endorsed. The prediction is that, even if the spin, magnetic, and disk axes are all parallel (a circumstance sometimes argued to be incompatible with pulsed emission), pulsar action will occur, in analogy with the emissions from Jupiter and Saturn.

B. The disk

The disk is proposed to have a mass $\sim10^{-5}M_p$ (the satellites about the sun and the satellites about the major planets are of this order) and a density comparable to that of the presupernova core ($\sim10^6$ g/cc), hence electron degenerate and highly conducting. If the disk is essential for pulsar action, then such interaction is not likely to "blow away" the disk, despite what one might imagine. The disk would simply move out to a position where the interaction is sufficiently reduced (it is somewhat difficult to blow away a $\rho\sim10^6$ g/cc disk with radiation pressure, regardless of where it is). For a young pulsar such as the Crab pulsar, the inner edge of the disk would now be at about 1.7×10^7 cm ($\sim17R_p$), about ten times closer than the conventional light cylinder distance (1.5×10^8 cm) and well within the Roche limit (1.7×10^{11} cm) for normal matter. The neutron star couples magnetically to the disk plasma, which acts to force the latter into corotation, whereas neutral matter in the disk is in Keplerian motion, leading to heating of the disk and maintainance of electrical contact between the neutron star and the disk. Because the interaction is well within the light cylinder, radiation *per se* would carry away rather little angular momentum. Thus the system could evolve at essentially constant angular momentum, with the neutron star slowing down while simultaneously driving the disk away to greater distances. It seems likely, however, that much of the energy output will be in the form of a stellar wind, just as for the standard model. See Fig. 22(b).

It may seem rather surprising that such a first-order model was not suggested earlier, especially since it does not yet seem to be excluded on observational grounds. Regardless of whether the model proves to be viable, the reason it was not previously suggested exemplifies the inhibiting role of "conventional wisdom." As noted above, Burbidge and Strittmatter (1968) recognized the parallelism between Jupiter's emissions and those from a pulsar. But at that time it was thought that Io along excited the emission and it was immediately noted (Douglas-Hamilton, 1968) that a satellite with an orbital period equal to a pulsar period would be inside the Roche limit and hence be broken up. Such a system would speed up, not slow down as energy was radiated. Then came the failure to detect any objects whatsoever in orbit about a pulsar. Finally, it was theorized that newly formed pulsars are so active as to account for the supernova outburst itself (Ostriker and Gunn, 1971; Bodenheimer and Ostriker, 1974). All of this seemed to amount to an internally consistent picture wherein the pulsar simple "blew" its surroundings clear of matter at birth. The only firm observational fact, however, is that pulsars do not seem to have planets. Disks were not ruled out, the Roche limit argument is irrelevant to disks and the idea that newly born pulsars are so fiercely bright as to be capable of ejecting any nearby matter is simply that—an idea yet to be supported by observation; quite to the contrary, the main puzzle has been that most young supernova have no detectable pulsars in them, not that they contain exceptionally energetic pulsars. Obviously, the disk proposal is here only circumstantial, but much the same situation obtains for the theory of x-ray burst sources (Lewin and Joss, 1981), which invokes not only an accretion disk about a neutron star but also a low-mass nearby companion star, neither of which is readily detectable (since the bursters are not seen as pulsars, one loses the fine timing information that would make a binary system evident).

C. Particle acceleration

If pulsars are neutron stars with disks, it seems quite natural to suppose that the neutron star is formed at maximum rotational velocity with an "attached" disk. The potential differences between the neutron star and disk is of the order of $\Omega B a^2$, where $\Omega \sim 6000$ rad/sec, $B \sim 10^{12}$ g, and $a \sim 15$ km, hence particle energies of order 1.3×10^{20} eV. Since the magnetic fields drops rapidly with distance, particles with energy $\sim \frac{1}{4}$ of this value (Michel, 1982) cannot be retained in the magnetosphere and escape. Jupiter, for example, is a source of electrons with energies ~ 30 MeV, comparable to its 170 MeV pole-to-equator potential drop (Chenette *et al.*, 1974; L'Heureux and Meyer, 1976). The composition of the disk should be representative of the processed stellar material in the vicinity of the precollapse core. Thus even if the neutron star itself were pure iron, there would be plentiful supplies of much more ordinary ("solar") ions.

D. Consequences

From an observational point of view, a disk system (Fig. 22) provides a potentially rich variety of morphological variation, which one can compare with the similarly rich variety of pulsar properties. Michel and Dessler discuss several such points—for example, that alignment of spin and magnetic axes need not affect pulsar activity, that ion confinement in the pulsar surfaces need not hinder current flow, that long nulling periods could be attributed to long time scales associated with the disk, that the magnetic field may be markedly reduced from previous estimates (typically to as low as 10^9 G, as inferred from the drifters), that the braking index would be characterized by a limit near $n=2$ [see Eq. (7.4) and (7.5)] rather than a fixed value of three, and that some "residuals" in the time data would naturally arise in a disk model, owing to small variations in the location of the "spot." Finally, the disk model may even explain the lack of a one-to-one association between pulsars and supernovae in terms of plasma obscuration, and further such obscuration suggests an explanation for the giant pulses from the Crab pulsar and its comparative dimness as a radio source.

The model in its present form does not assess the possible thermal emissions from the disk itself. Of particular interest would be detection of weak x-ray emission (presumably the bulk energy output is in the form of a stellar wind; otherwise the slowing down rates would imply luminosities, if all in electromagnetic radiation, not seen by existing uv or x-ray satellites).

VIII. THE PULSAR WIND

A. Steady-state solutions

As noted above, there is substantial direct observational evidence that the Crab pulsar excites the nebulosity, contributing both energetic particles and magnetic fields. The emission of such plasma, a generalized "wind" analogous to the solar wind, but much more powerful, has been a subject of investigation from early on. One of the early pulsar theories proposed radio emission excited when such a wind is shocked (Michel and Tucker, 1969).

This model stimulated the extension of the nonrelativistic pressure-dominated solar wind solutions (Dicke, 1964; Weber and Davis, 1967; Modisette, 1967) into the relativistic magnetically driven regime (Michel, 1969a, 1969b; see Goldreich and Julian, 1970, for an elaboration; Okamoto, 1978; Nakamura, 1980; Nerney, 1980; and Fujimura and Kennel, 1981).

It is a bit complicated to sketch through the wind derivation, although the results are fairly easily understood. Basically, the magnetic field pattern rotates rigidly until the corotational velocity approaches the transverse Alfvén velocity (i.e., a radially propagating shear wave). This provides one critical point (Dicke, 1964). A second critical point is where the radial velocity equals the compressional Alfvén velocity; for pressure-free flow (negligible temperature), the latter point is asymptotic to infinity (Goldreich and Julian, 1970). A crucial role in these calculations is played by the dimensionless parameter (see Fig. 23)

$$\sigma = e\Omega B a^2/mc^2 ,$$

which is much larger than unity for relativistic flows, and vice versa. Physically, σ is what the Lorentz factor of the particles would be if, asymptotically, all the Poynting flux went into the particles. However, the flow equations imply that the particles get a Lorentz factor of only $\sigma^{1/3}$. The total luminosity scales as σ^2 and the particle flux as σ; thus most of the energy output is in the Poynting flux in the relativistic limit.

In the calculations described above, the outstreaming plasma was taken to be neutral, rather than charge separated, which leads to some differences ($\gamma \sim \sigma^{1/2}$ in the latter case; Michel, 1974b), although the physical idea is simply to treat the plasma as a conducting fluid entraining magnetic field lines. See also Henriksen and Rayburn (1972).

B. Waves in the wind

For an oblique rotator, the stellar wind should in fact be a large-amplitude wave (Ostriker and Gunn, 1969a), as well as a wind. The important case of vacuum solutions was already covered in Sec. V; here we comment on the plasma effects and point out that the waves themselves do not appear to be stable. Indeed, the original Ostriker and Gunn (1969a) proposal was that energy was systematically transferred from the wave to the particle. (See Fischer and Straumann, 1972; Usov, 1975; Drake *et al.*, 1976; Buckley, 1977.) In their treatment, however, the particles were treated as a perturbation (i.e., the particles are simply accelerated from rest by the radiation pressure), raising the question of whether or not relativistic waves might be formed (once the particles became sufficiently energetic). This question has been studied by Max and Perkins (1971, 1972) and Max (1973a), who concluded that relativistic waves would indeed propagate in an electron-ion plasma. See also Ferrari (1972), Ferrari and Trussoni (1971, 1974, 1975a, 1975b), Onishchenko (1979), Stenflo (1980), and Luheshi and Stewart (1979). Kennel and Pellat (1976) modeled a simple relativistic wave in an electron-positron plasma, but concluded that radiation reaction by the particles would damp the wave in about ten wavelengths (Asseo, Kennel, and Pellat, 1978). Moreover, it was later discovered that the two-stream instability of the counterflowing electrons and positrons should destroy the wave within one oscillation (Asseo, Llobet, and Schmidt, 1980). Strong instabilities were also found by Max (1973b) for strong waves in a cold neutral plasma, and by Arons *et al.* (1977) for such waves in a hot neutral plasma.

There are basically two possible types of wave; either the wave propagates *through* a plasma with phase velocity greater than c, or it convects the plasma along with it (phase velocity less than c; Kulsrud, 1972). In the first case, the particles are obliged not only to become relativistic but also to reverse transverse flow direction as the wave cycles past them, hence the radiation reaction loss and, in the case of an electron-positron plasma, the strong two-stream instability. For phase velocity less than c, one has really a wind in disguise, since one can now physically move with the flow, and the "wave" can actually be any combination of plasma and magnetic field such that the total pressure is constant. A simple example is to put all the particles in sheets separating compartments of alternating magnetic field (Michel, 1971)—a set of marching neutral sheets. It can be shown easily, however, that this system cannot propagate far either. To separate two compartments of field strength B and $-B$, a surface current density

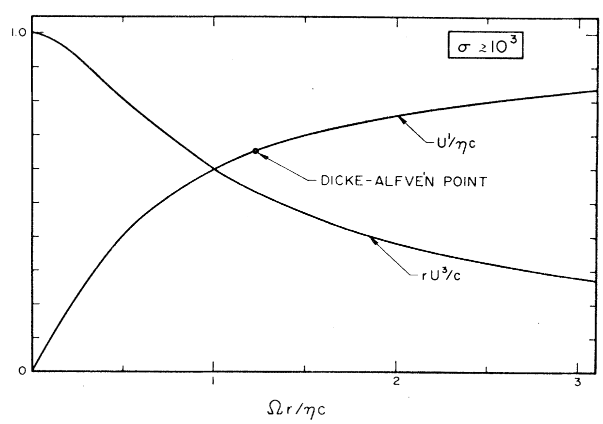

FIG. 23. Universal curve for acceleration by magnetic "slinging" to extreme relativistic velocities. The radial proper velocity is plotted in units of its asymptotic value ($U^1 \rightarrow \eta c$, where $\eta^3 = \sigma$), while the distance scale is in units of $\eta c/\Omega$ (c/Ω = corotation limit). The critical point is located at the corotation limit and is therefore lost into the origin in the limit $\eta \rightarrow \infty$. Note that the azimuthal proper velocity is plotted in units of c (not ηc as for the radial velocity), and consequently the rise from zero to about 1 in the vicinity of the critical point is not shown in this limit. The Dicke-Alfvén point remains fixed in this figure, located as shown. The second solution, which crosses at this point, is essentially a vertical line and has been omitted.

$$j = 2B/\mu_0 \equiv \int J\,dx \tag{8.1}$$

must flow (dx is the distance through the sheet). But the surface density of relativistic particles required is then

$$\sigma \geq j/ec \ . \tag{8.2}$$

For radial expansion away from the pulsar, the surface density σ declines as r^{-2}, whereas the azimuthal magnetic field entrained between two sheets declines only as r^{-1}. Thus σ and j cannot be in fixed ratio (once more a current and charge-density problem!). Presumably the magnetic fields would simply reconnect and transfer magnetic energy into particle energy once Eq. (8.2) was violated.

There does not seem to be a consensus as to what exactly a pulsar emits (other than the radio emissions), yet there is apparently a fairly strong magnetic field to be explained in the Crab nebula. If this field cannot be transported from the pulsar in the form of waves, it must get there in the form of a wind. The charge-separated wind, however, also has theoretical difficulties. A quasineutral wind seems indicated, but it is not easy to see how the standard model would produce such a wind.

IX. EMISSION MECHANISMS (COHERENT RADIO FREQUENCY)

A. Bunching mechanisms

1. The standard vacuum model

As noted earlier, the simplest model involving *ad hoc* particle bunches is the model proposed by Gold (1968). What, however, would produce the bunches in a self-consistent model? In the standard vacuum model, there is simply a direct acceleration and ejection of the particles from the neutron star, so there is no beam instability to be excited (e.g., by the two-stream instability). Goldreich and Keeley (1971) argue that particles moving in an arc (actually a circle, for simplicity of analysis, but here simulating curved dipolar magnetic field lines) are unstable to clumping owing to the radiation reaction of one upon the other. Buschauer and Benford (1978) and Asseo *et al.* (1981) have carefully reconfirmed the analysis of this effect.

The growth rate S is given from [Goldreich and Keeley, 1971, Eq. (27)]

$$\frac{S}{\omega_0} \approx \left[\frac{\Gamma(\frac{2}{3}) r_c N_0}{2(3)^{1/3} a \gamma^3}\right]^{1/2} n^{2/3}(1+i\sqrt{3}) \ ,$$

where $r_c = 2.82\times10^{-13}$ cm is the classical electron radius, N_0 the number of particles in a bunch, a the radius of curvature, γ the Lorentz factor, n the circular harmonic corresponding to the bunch size, and ω_0 the circulation frequency (for a ring). For a polar cap ejection model, $a \approx R_L$, $\omega_0 \approx \Omega$, $n \approx 2\pi a/\lambda$ (where λ=bunch size), $\gamma = \gamma_0$ [Eq. (3.29)], and $N_0 \approx q_0 \lambda^3/e$ [$q_0 = 2\varepsilon_0 \Omega B$; see Eq. (3.25)]. Altogether we have for $\lambda = 1$ m, $\Omega = 200$ rad/sec, $B = 10^{12}$ G, and $\gamma_0 = 10^5$,

$$S/\Omega \approx 1.5\times10^{-4} \ ,$$

which seems too slow, because the particles are expected to be ejected into the wind zone before traversing more than about a radian of arc (i.e., $S/\Omega >> 1$ required). The functional behavior is

$$S/\Omega \sim (\lambda^2/B\Omega^6)^{1/4}$$

so growth rates are actually improved for slower pulsars (mainly because γ is reduced), but not by enough ($\sim 20\times$). However, these estimates are somewhat model dependent and should only be taken as illustrative. In fact, the situation may be more favorable in laboratory storage rings, where S/ω_0 is still small but one has long storage times giving $ST >> 1$ and hence the potential of observing such bunching (Michel, 1982).

Alternatively, it has been proposed that radiation reaction at the pulsar surface immediately causes bunching by nonlinearly suppressing particle emission from the surface (Michel, 1978a), but the details of this mechanism have not yet been followed through. See also Rylov (1976) and Sturrock (1971a).

The radiation characteristics expected from such bunches are discussed by Saggion (1975), Eidman (1971), Cox (1979), Epstein (1973), Sturrock *et al.* (1975), Buschauer and Benford (1976, 1977), Shklovsky (1970a), Pacini and Rees (1970), Tademaru (1973), Michel (1978). See Fig. 30 below. Rylov (1978) discussed a model with pair production (see below) wherein the return beam is excited by passing through a trapped space-charge region.

2. Turbulence

The distinction between bunching of particles and plasma "turbulence" (the latter is surely one of the all-time favorite buzz words in astrophysics) is often one of style. However, the term makes sense if the radiation is really from a spectrum of inhomogeneities as opposed to being from a few discrete ("monochromatic") bunches. The following authors specifically refer to radiation from *turbulence* excited in the plasma: Layzer (1968), Coppi and Ferrari (1970a, 1970b, 1971), Ichimaru (1970), Coppi (1972), Buckee *et al.* (1974), Khakimova *et al.* (1976), and Hinata (1976a—1976c, 1977b, 1978, 1979).

3. Pair production models

In the models where positron-electron plasma is invoked, one has counterstreaming of the two equal-mass particles, hence some version of the two-stream instability, hence bunching (Cheng and Ruderman, 1977a; Hardee and Rose, 1976, 1978; Arons and Smith, 1979; Mikhailovskii, 1980). In general, such beam-plasma instabilities have received considerable attention (Elsässer and

Kirk, 1976; Elsässer, 1976; Hinata, 1976a, 1976b; Melrose and Stoneham, 1977; Buschauer, 1977; Rylov, 1978; Lominadze *et al.*, 1979a, 1979b; Lominadze and Mikhailovskii, 1979; Hardee and Morrison, 1979; and Asseo, Pellat, and Rosado, 1980). Benford (1975) has argued that the beam is unstable to filamentation. Other discussions, based on the assumption of pair production, are given by Parker and Tiomno (1972a, 1972b), Hinata (1973), Al'ber *et al.* (1975), Daugherty and Lerche (1975, 1976), Jones (1977b, 1978, 1979, 1980b, 1980c), Kirk and ter Haar (1978), Lominadze *et al.* (1979), Sturrock and Baker (1979), and Kundt and Krotscheck (1980).

4. Other mechanisms

Ruderman (1981) has explored the role of instabilities between counterstreaming Fe ions and backstreaming electrons. These electrons could either come from a pair production region or from stripping of incompletely ionized Fe primaries. See Cheng and Ruderman (1980). Arons and Smith (1979) propose a shearing instability which would operate even in a completely charge-separated magnetosphere.

B. Maser mechanisms

1. Chiu and Canuto

One of the first mechanisms proposed was a true maser mechanism through population inversion. Here the states in question are the Landau orbitals. The "pumping" to produce this population inversion was assumed to be driven by an oscillation of the neutron star, with rotation providing the timing mechanism (Chiu and Canuto, 1969a, 1969b; Chiu and Occhionero, 1969; Chiu *et al.*, 1969; Chiu and Canuto, 1970; Chiu, 1969, 1971; Chiu and Canuto, 1971; Canuto, 1971). The high brightness temperatures asserted were criticized at the time (Roberts and Fahlman, 1969; Simon and Strange, 1969), although at least one attempt has been made to resuscitate the model (Virtamo and Jauho, 1973, 1975). See also Casperson (1977) and Mertz (1974).

2. The Russian school

Except for the efforts of Chiu and Canuto (above), research in the U.S.A. has tended to assume that coherence is due to particle bunching. The Russians, however, have proven to be quite interested in the maser mechanisms. Broadly speaking, the idea is that electrons in a strong magnetic field quickly radiate away their transverse energy. This leaves behind a very anisotropic distribution function which exhibits certain instabilities. See Ginzburg *et al.* (1969a, 1969b), Takakura (1969), Ginzburg and Zheleznyakov (1970a, 1970b), Coppi and Ferrari (1970a, 1970b), Eastlund (1970), Tsytovich and Kaplan (1972), Zheleznyakov and Suvarov (1972), Kaplan and Tsytovich (1973a, 1973b), Sazonov (1973), Suvarov and Chugunov (1973), Ginzburg and Zheleznyakov (1975) (this paper is mainly a review paper), Machabeli and Usov (1979). See also Blandford (1975) for a critical, but not necessarily unfavorable assessment of maser processes, and Kawamura and Suzuki (1977) for a rather different maser process. Kaplan *et al.* (1970) have even examined the possibility that maser action in the Compton down-scattering of energetic photons (by energetic electrons) into radio photons might work.

However, Zheleznyakov and Shaposhnikov (1979) note that if the energy density of the radiating particles is less than that of the magnetic field (the usual assumption, consistent with dimensional estimates), then coherent curvature radiation is ineffective for maser action. See also Blandford and Scharlemann (1976).

3. Stimulated linear acceleration radiation

Cocke (1973) found a maserlike action in the electron acceleration region, assuming simply a uniform acceleration of electrons to ultrarelativistic velocities while following curved magnetic field lines. Melrose (1978) extended this work, finding it a promising emission mechanism, whereas Kroll and McMullin (1979) argue that the amplified emission is almost completely suppressed by propagation effects.

4. Čerenkov emission

See Charugin (1975), Kolbenstveldt (1977), and Mikhailovskii (1981).

C. Propagation

Although the propagation of signals can often be treated separately from the emission of the signal, at high brightness temperature the question of reabsorption and nonlinear effects becomes quite important. Thus we touch here on the literature regarding propagation.

1. Inhomogeneous, moving media

Since the pulsar magnetosphere, at least in the standard vacuum model, is in relative motion near and beyond the light cylinder, there is the question of how such motion would modify outgoing pulsed radio waves. Lerche initiated a considerable dialogue here (Lerche, 1974a—1974i; Lee, 1974; Lee and Lerche, 1974, 1975; Lerche, 1975a—1975d, 1976; also Elitzur, 1974; and Harding and Tademaru, 1979, 1980). See also Dorman *et al.* (1973). Ko and Chuang (1978) have disputed some aspects of this work.

2. Strongly magnetized media

See Ockelkov *et al.* (1972), Novick *et al.* (1977), Heintzmann and Schrüfer, (1977), Pavlov and Shibanov (1978, 1979), Ventura (1979), Fang and Liu (1976), and Cocke and Pacholczyk (1976).

3. Streaming plasma

See Elitzur (1974), Heintzmann *et al.* (1975a, 1975b), Ko (1979), Onishchenko (1975), and Melrose (1979).

4. General relativistic effects

See Evangelidis (1979), Galtsov and Petukhov (1978), and Ignat'ev (1975).

5. Caveat

To quote Hewish (1981): "Until these atmospheric [magnetospheric] problems have been solved, it may be rash to consider detailed radiation mechanisms." It seems to us that this caution is well taken; if, in ten years say, the correct theory is at hand, 90% or more of the above work will probably be irrelevant because the physical environment postulated will not be that appropriate to actual pulsars. Accordingly, we have not been sufficiently motivated to try to glean the likely contributions, and have only provided the above shopping list, lumping together the good, the flawed, and the indifferent contributions. For the reader, this lack is something of a disservice, and one hopes that a complementary review will be written to illustrate how observational data may be used to more specifically constrain pulsar models. Ruderman (1981) provides a fine example of such analysis. Our interest here has been in seeing how far one can go from first principles.

X. OTHER ISSUES OF PHYSICAL INTEREST

A. The white dwarf hypothesis

The possibility that pulsars might be neutron stars was mentioned in the discovery paper. However, there was considerable reluctance among the more influential astrophysicists at that time, understandably, to seize on a hypothetical object before exhausting the known stellar objects that might be pulsar candidates. The known object then could hardly be other than a white dwarf. A white dwarf is simply a star in which electron degeneracy pressure supports the matter (ions, essentially) against self-gravitation. Although such a star would have to be hot to be luminous and visible, the temperatures are usually small compared to the Fermi temperature throughout most of the star, and consequently the structure can be calculated in first approximation simply by ignoring the temperature altogether. The mass, radius, or central density determines uniquely the other two, so one basically has a one-parameter family of stars. A burnt-out sun would become a dwarf star about the size of the earth. But for only a slightly heavier dwarf, about 1.4 solar masses, the radius shrinks to zero! What happens is that for more massive stars the degeneracy pressure must be higher to hold the outer layers, hence the Fermi temperature (as well as the density) must be higher to provide that pressure. The electrons begin to become relativistic when required to support a large enough stellar mass, which in turn makes them too compressible to resist gravitation. It is not difficult to prove that a self-gravitating fluid satisfying the adiabatic law

$$P=k\rho^{\gamma} ,$$

with

$$\gamma=\tfrac{4}{3}$$

(i.e., relativistic particles or photons) is neutrally bound—a radially inward push causes it to shrink to a point, and an outward push disperses it to infinity; consequently, the electrons lose their ability to support the star as they become relativistic. However, when the electrons were nonrelativistic, the star had a finite *binding* energy, and there is no way for the star to restore that deficit when the electrons become relativistic (i.e., the star cannot become neutrally bound); thus at a certain point the star dynamically collapses, stopping at the point where the *nucleons* become degenerate—the neutron star—and the same game is replayed.

The limiting masses of the two objects are essentially determined by the heaviest mass particle (m)

$$M(\max)\sim 1/m^{2}$$

and therefore, even before correcting for nuclear forces, etc., the maximum mass of white dwarfs and neutron stars are necessarily similar, despite the quite different mechanism for support (in both cases, it is essentially the nucleons which must be supported against gravity). Note that the existence of very massive particle states beyond the neutron and proton state would not, in itself, provide a more massive stable stellar object; hence the neutron star is usually regarded as being the final "normal" state of stars. A more massive object is presumed to collapse to become a black hole. It is amusing to note that neutron stars, viewed as endpoints of stellar evolution, are actually endothermic, insofar as the "chemical" potential of the nucleons is concerned. In other words, the highly publicized role played by nuclear "burning" serves not so much (in the end) as an energy source but as a delaying action to stave off as long as possible the conversion of hydrogen into neutrons, the net energy output all coming, therefore, from gravitational binding, not nuclear binding.

Since white dwarfs are so easily modeled, there is little doubt about what their minimum vibrational periods might be, and the minimum rotation period is of the same order of magnitude. This period is easy to estimate, since the characteristic propagation velocity of a compressional wave is of the order

$$V^2 \sim P/\rho \sim kT/m_i$$

and we have see that the white dwarf pressure is from nonrelativistic electrons, while the mass density is from the nuclei. Thus, if we take the maximum Fermi temperature that the electrons could have and yet not be fully relativistic,

$$kT_F \lesssim m_e c^2 ,$$

we find for the propagation velocity

$$V \lesssim 7\times 10^6 \text{ m/s} .$$

Ordinary white dwarfs have radii the size of the earth ($\sim 7\times 10^6$ m) and therefore a vibrational period of about one second. Pulsar periods are typically one second, and a lot of effort was devoted to refining (and reducing) this figure so that the shortest-period pulsar then known (PSR 0950+08 with a period of 0.253 sec) could also be explained. However, the discoveries of the Vela (PSR 0832−45) and Crab (PSR 0531+21) pulsars with periods of 0.089 and 0.033 sec, respectively, dashed these hopes. These pulsars, if white dwarfs, would have to be 10 to 30 times smaller than normal dwarfs and the electrons would unquestionably be highly relativistic (i.e., this would not be a stable configuration, but would be collapsing to become a neutron star).

B. The neutron star

1. Internal structure

Observation of period variations, and in particular the attribution of the "glitch" in the Crab pulsar to a starquake (Baym *et al.*, 1969) has stimulated an enormous literature on this subject. For the most part, it is assumed that the pulsars function more-or-less independently of what is happening in their interiors, and observation seems to suuport that view, since no dramatic changes in radio output or character (pulse shape, etc.) seem to be associated with period variations. It is not even clear that the period variations have anything to do with the pulsar interior, but this view has been extensively promoted. Roberts and Sturrock (1973) and Pustil'nik (1977) did propose that mass loading and ejection in the magnetosphere might be involved, and Scargle (1969) and Scargle and Harlan (1970) reported activity in the "wisps" near the Crab pulsar following the first reported glitch. Most workers, however, seem to have adopted the Goldreich-Julian density as characteristic of the magnetosphere, which is far too tiny to be kinematically significant. Disk models are, of course, a complete break from this view.

The literature is too vast and the subject too distant from magnetospheric issues for us to treat in detail here, so we shall just sketch the theoretical situation and supply a few introductory references. Oppenheimer and Volkoff (1939) first treated the structure of the neutron star assuming simply a degenerate Fermi gas equation of state. See Baade and Zwicky (1934). Here the problem is quite sensitive to detail, and the maximum possible mass of a neutron star is quite sensitive to the equation of state. The general relativistic corrections are important, since the central pressure of a star of incompressible matter goes to infinity before the Schwarzchild radius equals the stellar radius (Møller, 1952). At the same time, the ability of the nuclear matter to hold up a star vanishes as the constituent particles become relativistic, and so the behavior is delicately sensitive to fine details of the equation of state (see Arnett and Bowers, 1974). Observationally, the binary x-ray sources are providing neutron star masses near 1.4 $M_\odot$, so supernova remnant neutron stars may tend to be standardized objects (Rappaport *et al.*, 1976), probably because their masses are limited by the maximum possible for the presupernova core, rather than by what their own maximum mass might be. At the moment, it appears as if a neutron star mass-versus-radius relationship would tell us more about nuclear physics than vice versa (Pines, 1980a). Shang-Hui *et al.* (1981) summarize previous estimates and give 1.7 $M_\odot$ as the maximum neutron star mass.

For quite some time there was no "direct" evidence that pulsars were either highly magnetized or neutron stars, or that neutron stars even existed as tiny objects with radii of about 10 km. The power balance in the Crab Nebula (Pacini, 1967; Finzi and Wolf, 1969; Gold, 1969a) however points to a collapsed object with the correct moment of inertia, which therefore requires Ma^2 to be of the correct magnitude. In earlier days, it was thought that thermal neutron stars were the most plausible candidates for astrophysical x-ray sources. However, early rocket-borne x-ray detectors failed to observe a point source in the Crab Nebula, using lunar occultation to provide sufficient spatial resolution. Actually the point source was there, but just below the sensitivity of the experiments. Subsequent confirmed x-ray objects proved to be just about anything other than thermal radiation from a neutron star. Now, however, some of the x-ray burst sources seem to be neutron stars heated impulsively. The subsequent cooling of the neutron star by thermal radiation seems consistent with $a \sim 10$ km radius (Hoffman *et al.*, 1977; Swank *et al.*, 1977; van Paradijs, 1978), providing some support for the present models of neutron star structure. Finally, an x-ray feature in Her X-1 (Trümper *et al.*, 1978) has been interpreted as cyclotron emission or absorption from iron ions in a 10^{12} G field. See also Wheaton *et al.* (1979). However, Nagel (1981) interprets this "line" as a neighboring absorption feature. In any case, it now seems to be generally accepted that neutron stars typically have 10^{12} G fields. Pines (1980b) reviews the observational evidence for neutron star properties.

A recent review by Cline (1981) presents evidence for gamma-ray burst sources being neutron stars; these sources often display both a 50 keV absorption feature and a 400 keV emission line. The latter is attributed to red-shifted positron annihilation near the surface of a neutron star, and the former to the above cyclotron resonance seen in absorption.

Except possibly during the formation event, thermal pressure is not expected to play an important role in neutron star structure (at 10^{12} K, the blackbody luminosity of a single neutron star would exceed that of all the stars in the universe). Since the nucleons must therefore be degenerate, they have rather low heat capacity and high heat conductivity. These factors, combined with the small size of the object, point to an initial period of rapid cooling, typically on the order of a thousand years (see Tsuruta *et al.*, 1972; Tsuruta, 1974, 1975; Maxwell, 1979; Lamb and Van Riper, 1981).

The original starquake hypothesis has evolved considerably since its inception (Baym and Pines, 1966; Pines and Shaham, 1974). Recent work on giant glitches (Alpar *et al.*, 1981) downgrades starquakes and have focused more on "vortex unpinning," namely the decoupling of an inferred neutron superfluid core and crust (assumed to be magnetically pinned at the crustal nuclei) to temporarily change its apparent moment of inertia. A similar idea (decoupling between the core and the crust) was originally forwarded by Greenstein and Cameron (1969). Other mechanisms for glitches have been proposed, such as changes in the magnetosphere (Scargle and Pacini, 1971; Ozernoi and Usov, 1972; Roberts and Sturrock 1973), a change of state in the crust (Bisnovatyi-Kogan 1970), or evaporation of material from a planet (Rees *et al.*, 1971). Cordes and Greenstein (1981) have recently reviewed the timing noise processes.

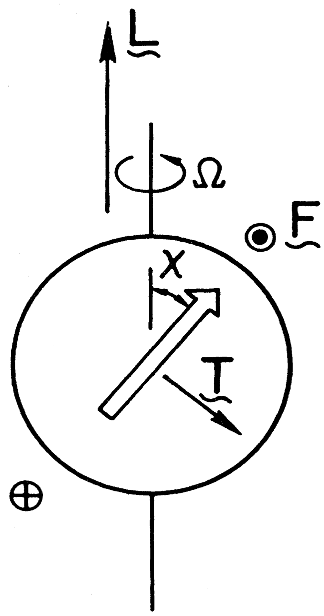

FIG. 24. Origin of the alignment and slowing down torques. Sweeping back of field lines by retardation effects gives a force component opposite to rotational velocity and concentrated near the magnetic pole. Resultant torque vector (**T**) has a component opposite the angular momentum vector, which produces the slowing down (L) and another toward the nearest pole, which produces the alignment.

2. Magnetic field

It is evident that pulsar theory leans heavily on the expectation that neutron stars are highly magnetized. Why should they be? There is a general plausibility argument to the effect that, if internal magnetic flux in a star is conserved owing to high conductivity, then the surface magnetic field increases as $1/a^2$ where a is the radius. Thus, if the sun collapsed to neutron star dimensions (roughly a factor of 7×10^4 in radius), one would magnify the general 1 G solar fields to about 5×10^9 G. This value is a bit shy of 10^{12} G, but the sun is not necessarily the best example of the presupernova object (Ostriker and Gunn, 1969a; Imoto and Kanai, 1971). The implication, then, is that the presupernova core, with a radius of say 10^9 cm, will have a "surface" field (actually deep within a massive giant star) of 10^6 G. Note that the magnetic moment, which is proportional to Ba^3, actually decreases in such a flux-conserving collapse, and would vanish for collapse to a point (nonrotating black holes are predicted to be unmagnetized: Anderson and Cohen, 1970). One can formulate an empirical "magnetic Bode's law" which shows a rough proportionality between spin angular momentum and magnetic moment (Hill and Michel, 1975; Ahluwalia and Wu, 1978; see also Greenstein, 1972). Such a phenomenological proportionality does not even have to be "extrapolated" to pulsars, since the planet Jupiter has both a magnetic moment and an angular momentum only slightly less than those typically attributed to pulsars. Such empiricism, however, has not yet helped to elucidate which physical mechanisms might be important, although it broadly suggests that mechanisms such as differential rotation of the core (as proposed for planetary magnetic fields) may be relevant.

Ferromagnetism has been examined as a possible magnetic field source (Silverstein, 1969; O'Connell and Roussel, 1972; Pfarr, 1972; Schmid-Burgk, 1973), but it seems unattractive. Differential rotational between, say, superfluid protons and normal electrons has been another suggestion (Sedrakyan, 1970a, 1970b; Sedrakyan and Shakhabasyan, 1972; Sedrakyan *et al.*, 1975, 1977; Woodward, 1978). The fossil magnetic field picture suggests eventual decay of the field, and Ostriker and Gunn (1969b) argued for such decay on empirical grounds to explain the paucity of long-period pulsars, although this view is not without its counterarguments (Pacini, 1969; Holt and Ramaty, 1970; Setti and Woltjer, 1970; Chanmugam and Gabriel, 1971; Heintzmann and Grewing, 1972). Flowers and Ruderman (1977) argue that internal processes, not resistivity, can reduce the field. Other causes of field variation, such as by evolution of the internal source field have also been examined (Vandakurov, 1972; Chanmugam, 1973; O'Connell, 1975;

Jones, 1976a—1976c; Chanmugam, 1978). Unfortunately, none of the observed pulsar variations can confidently be attributed to magnetic variations, and their overall evolutionary pattern still remains somewhat obscure.

It has been repeatedly suggested that pulsar turnoff at long periods be the result of magnetic field decay (see Gunn and Ostriker, 1970; Fujimura and Kennel, 1980); however, the theoretical estimates consistently give conductivities that are orders of magnitude too large (see Ewart *et al.*, 1975).

Observational evidence for neutron star magnetic fields (beyond the interference from the pulsar phenomena itself) is discussed in Sec. I above.

3. Alignment of spin and magnetic moment

It was recognized by several groups simultaneously (Michel and Goldwire, 1970; Davis and Goldstein, 1970; see also Mestel, 1968) that the torque on an oblique rotator acted not only to brake the rotation but also to align the magnetic moment (which is assumed to be "frozen" into the neutron star) with the rotation axis (which is not). Figure 24 illustrates in a simple heuristic way how this alignment torque comes about, although it can be calculated directly from the near-zone fields (Soper, 1972; Imoto and Kanai, 1972). Aligning mechanisms have also been identified in neutrino emission (Tennakone, 1972) and generalized polar wander, in analogy with the earth (Macy, 1974). The electromagnetic alignment idea has been refined (Chau *et al.*, 1971) and applied to observation (Henriksen, 1970; Chau and Henriksen, 1970; Jones, 1975, 1976a, 1976b, 1977a). The braking index is then given by

$$n = 3 + 2\cot^2\chi ,$$

where χ is the angle between spin axis and magnetic moment (Fig. 24). As already noted, the only determination of n available at present gives 2.5, an impossible value in the model.

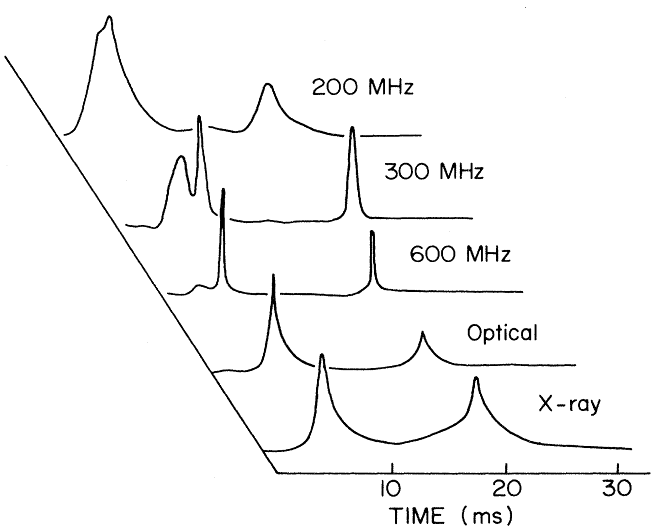

FIG. 25. The Crab pulse profile as seen from radio to x ray. The arrival time phase shift has been removed, and the pulse alignment over 11 decades of frequency suggests a simultaneous emission at all frequencies. At frequencies below about 100 MHz (not shown), scattering within the nebula blurs the radio pulse into a sine wave and much of the pulsed amplitude is converted into a powerful steady source (since the spectrum continues to rise). At about 10 MHz, only the steady component is detectable and not the pulsed emission (although one presumes that the pulsar itself is actually emitting sharp pulses at this frequency). Smith (1977).

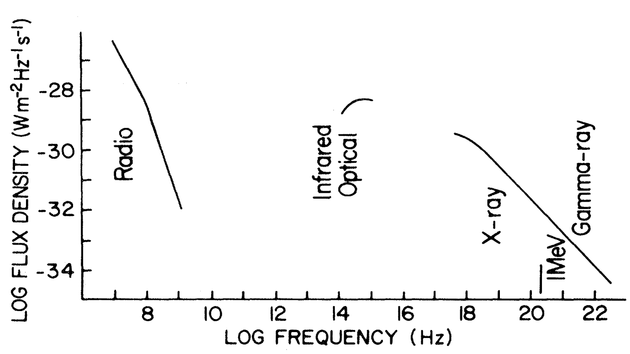

FIG. 26. Power output from the Crab pulsar at observable frequencies (from Smith, 1977). It seems plausible that the optical to gamma-ray emission is from a single mechanism and probably distinct from that of the radio emission. The total nebular luminosity roughly equals that of the pulsar at the lowest frequency shown, is roughly flat out to the optical, then declines to parallel and to roughly equal the pulsed gamma-rays luminosity.

Goldreich (1970) pointed out, however, that for certain assumptions regarding the shape of the neutron star alignment need not take place. It is easy to understand this argument if one assumes instead that alignment has taken place and looks for a contradiction. The contradiction would be to have the body axes at an angle to the (presumed) aligned spin and moment axes. There would not be any alignment torque, but at the same time the body axes would have to precess, leading therefore to a nonalignment. It is, however, easy to see that the two most obvious sources of nonspherical body shape, centrifugal forces and deformation due to the internal magnetic field stresses, produce nonspherical distortions that are, however, aligned. Thus, as Goldreich (1970) points out, it is necessary to suppose some hysteresis wherein the star is still distorted along an earlier spin axis direction. Precession due to triaxiality of the moment of inertia has also been suggested from time to time to explain features of pulsar observations; see Høg and Lohsen (1970), Burns (1970), Chiuderi and Occhionero (1970), Chau and Henriksen (1971), Chau and Srulovicz (1971), Avakyan *et al.* (1972), and Pines and Shaham (1974).

Broadly speaking, there seem to be no forceful observational data that point to either alignment or precession, which is not to say that these physically plausible effects do not take place.

C. Other pulsar emissions

1. Optical

The Crab and Vela pulsars are the two known pulsars in the optical, Vela being quite faint, but the Crab corresponding to roughly a 16th magnitude star. Unlike the radio emission from pulsars, the Crab optical pulse is extremely stable in shape and amplitude (Jones *et al.*, 1980) and shows no flickering (Hegyi *et al.*, 1969; Jelley and Willstrop, 1969; Horowitz *et al.*, 1972; Miller *et al.*, 1975), although some modulation has been predicted (Sturrock *et al.*, 1971). Figure 25 shows the pulse shape over a wide range of frequencies. The consensus view seems to be that these high-frequency emissions are incoherent and arise from a distinct mechanism (albeit not necessarily from a distinct emission region). As can be seen in Fig. 26, the high-frequency emissions appear to have a quite distinct spectrum from that of the radio. Tsytovich *et al.* (1970) suggest a maser mechanism, while Elitzur (1979) suggests Compton boosting of the radio photons off relativistic electrons. (For other models involving the Compton effect, see Tystovich and Chikachev, 1969; Apparao and Hoffman, 1970; Arons, 1972; Sweeney and Stewart, 1974; Stewart, 1974, 1975; Bonometto and Scarscia, 1974; Shaposhnikov, 1976.) Sturrock *et al.* (1975) suggest that, contrary to the above general view, the optical may in fact be a coherent emission phenomenon. See also Epstein and Petrosian (1973) and Eastlund (1971).

2. Gamma rays

Only the Crab and Vela pulsars are firmly observed to radiate detectable fluxes of gamma rays. The early reports that 1747-46 and 0740-28 emitted detectable levels of gamma rays have not been reconfirmed (Masnou, 1980), although the pulsar 1822-09 now holds some promise (Mandrou *et al.*, 1980). There are a number of point sources ("COS-B" sources, named after the observing satellite; Swanenburg, 1981) that do not seem to be pulsed but could conceivably be a steady component from as yet unidentified pulsars. Such energetic photons are interesting probes of the pulsar magnetosphere, since they would be absorbed in the strong magnetic fields usually proposed. The gamma-ray pulses seem to be in phase with the radio pulses, which either suggests that these two emission regions are close together or, if they are widely separated, there must be a rather specific geometric constraint imposed. For example, emission near the surface, as is often suggested for the coherent radio emission, places the gamma rays in the strongest possible absorbing fields, whereas locating just the gamma-ray source at, say, the light cylinder would apparently introduce a phase shift of the order of a radian between it and a radio source near the surface (Arons, 1981a).

The role of pair production and photon splitting in a pulsar magnetosphere is largely model independent. For all practical purposes, gamma rays of a given energy can only escape from beyond a roughly spherical region surrounding the pulsar. Those produced inside are all absorbed. The size of this sphere is not especially sensitive to the precise parameters (owing, paradoxically, to the extreme sensitivity of the absorption coefficients), and one finds that photons in excess of $\sim 10^6$ eV coming from near the surface would be reprocessed via pair production and reradiation in the magnetosphere. The observation of a cutoff energy above these energies would therefore be indicative of where in the magnetosphere the γ rays are emitted. Gamma rays from the Crab and Vela pulsars have now been observed (Kanbach *et al.*, 1977) with energies in excess of 2×10^9 eV, which puts some interesting restrictions on where these photons could have been generated.

a. Pair production

The (pair formation) attenuation length for a photon traversing a magnetic field is given by (Erber, 1966; Erber and Spector, 1973; Tsai and Erber, 1974)

$$\kappa=\alpha\omega_c \sin\theta T(\lambda)/2c \ , \tag{10.1}$$

where $\lambda=3(B/B_c)(\hbar\omega/mc^2)\sin\theta$, θ is the angle between the field direction and the photon direction, $B_c=m^2c^3/e\hbar=4.41\times 10^{13}$ G, α is the fine-structure constant ($\approx\frac{1}{137}$), and $\omega_c=eB/m$. The factor $T(\lambda)$ is extremely sensitive to λ at small values:

$$T(\lambda)\sim C_p e^{-4/\lambda}, \ \ \lambda<<1 \tag{10.2}$$

but only slowly varying at large values

$$T(\lambda)\sim D_p \lambda^{-1/3}, \ \ \lambda>>1 \tag{10.3}$$

and has a maximum value of order unity for λ of order unity (for p=parallel polarization, $C_{\|}=0.612$, $D_{\|}=1.04$, and $T(\max)=0.17$, while for p=perpendicular, $C_{\perp}=0.316$, $D_{\perp}=0.69$, and $T(\max)=0.27$). However, much of this detail is irrelevant, since for $B=10^{12}$ G, $\omega_c=1.8\times 10^{19}$/sec, and therefore $\kappa=2\times 10^6\ T(\lambda)$ cm^{-1}. Clearly then this attenuation coefficient is huge, even considering the small scales associated with the pulsar object ($\sim 10^6$ cm neutron star). Consequently, attenuation at large values of λ is entirely irrelevant; the photon will have interacted almost immediately to produce an electron-positron pair, which in turn almost immediately radiates away its energy in the form of new photons. As illustrated in Table VII, the gamma rays are quickly reprocessed and reduced in frequency until finally all the photons at the end of the $e\bar{e}$ cascade can freely escape. Consequently $T(\lambda)\sim e^{-4/\lambda}$. Now, however, the attenuation coefficient is exquisitely sensitive to λ because λ is going to be small even at the point of most probable absorption; thus the exponential is going to be large, and consequently even a tiny change in λ will produce a large change in T. What this means is that the magnetosphere absorbs like the surface of an opaque solid object—no attenuation whatsoever and then suddenly complete absorption. Consequently the spatial variation of B and θ becomes irrelevant—they can be replaced by their values at the "surface." As for the location of the surface, it will simply be placed near the point where $\kappa r \sim 1$. Thus

$$1 \approx (r\alpha\omega_c \sin\theta/2c)e^{-4/\lambda} \tag{10.4}$$

or

$$\lambda^{-1} \approx +\tfrac{1}{4}\ln(r\alpha\omega_c \sin\theta/2c) \approx 7.5 \; . \tag{10.5}$$

This later step is a "reasonable" approximation as opposed to a "good" approximation, namely the value of 7.5 is probably only good to 10–20 %, but that is entirely adequate for our purposes. Thus the fact that r, ω_H, and $\sin\theta$ are variables really makes little difference in estimating λ itself; that step is the insensitive one. It then follows that the absorption surface is located where

$$\gamma b \sin\theta \approx 4.4\times 10^{-2} \; . \tag{10.6}$$

Here, we write $\gamma = \hbar\omega/mc^2$ and $b = B/B_c$ to obtain this dimensionless relationship. Essentially the same estimate was given by Sturrock (1971a). Given a specific photon energy, viewing angle, and magnetic field model (e.g., dipolar), Eq. (10.6) defines three-dimensional surface surrounding the pulsar. Energetic photons, if produced inside this surface, would be absorbed and degraded into lower-energy photons (plus electron-positron pairs) until the daughter photons had too little energy to convert into pairs.

b. Photon splitting

The only other known process for photon absorption in the vacuum is photon splitting (Adler *et al.*, 1970; Adler, 1971), where the incident photon converts in the magnetic field into two outgoing photons. Here to a good approximation

$$\kappa \sim 0.1158(b\sin\theta)^6\gamma^5\,\mathrm{cm}^{-1} \; . \tag{10.7}$$

Thus again simply setting $\kappa r \sim 1$ with $r \sim 10^6$ cm gives

$$(b\sin\theta)^6\gamma^5 \approx 10^{-5} \; . \tag{10.8}$$

One can now solve for the photon energy and $b\sin\theta$, for which the two processes are comparable, obtaining

$$\gamma_* \sim 10^{-3}$$

$$(10.9)$$

$$(b\sin\theta)_* \sim 35 \; .$$

The small value of γ_* alerts one to the fact that we have exceeded the limits of applicability of Eq. (10.2), since pair production vanishes for $\gamma < 2$. Thus Eq. (10.9) tells us that photon splitting is unimportant if in competition with pair production; that is, for any γ of interest (certainly one greater than this tiny value for γ_*), and for any likely value $(b\sin\theta)_*$, since B is expected to be much less than B_c, the absorption surface will be found at much stronger field strengths ($\sim 10^3\times$) than the pair production absorption surface, and therefore photon splitting becomes unimportant because the photons would have already pair-produced. If, on the other hand, $\gamma < 2$, only photon splitting is operative, and from Eq. (10.8) we find that for $\gamma = 2$ and $\sin\theta = 1$, a minimum field of 4×10^{12} G would be required (to split 10^6 eV photons). Thus a pulsar such as the Crab would be a marginal candidate for splitting the photons emitted near this critical energy. (Since only photons with electric vectors perpendicular to $\underline{B}$ are split, there could be spectral ranges of 100% polarization.) However, since the Crab spectrum continues to much higher energies, the expected onset of absorption by pair production is not seen, and therefore it does not seem too promising to look for photon splitting effects in this pulsar.

c. Structure of the absorbing surface

Equation (10.6) describes, for a fixed gamma-ray energy, a cloverleaf pattern in a dipole magnetic field (Massaro and Salvati, 1979; Fig. 27). The deep dips over the polar caps are suggestive of a gamma-ray beaming mechanism, as pointed out by Salvati and Massaro (1978). However, the modulation largely vanishes if the observer does not happen to view exactly along the polar field line, and no interpulse would be seen unless the alignment were nearly orthogonal with the observer in the equatorial spin plane. There may therefore be statistical difficulties with such a beaming mechanism, although Massaro *et al.* (1979) point out that the nearly equal-strength interpulse and pulse emissions actually ob-

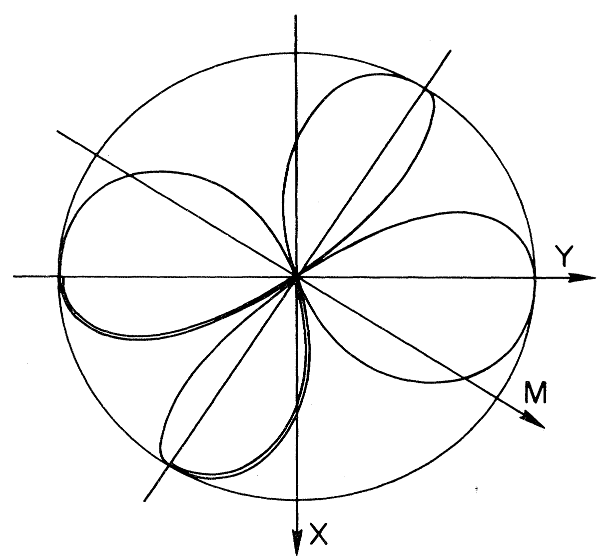

FIG. 27. The gamma-ray absorption "cloverleaf" (after Massaro and Salvati, 1979). $\underline{M}$ is the magnetic dipole axis and, for an observer viewing along the x axis, gamma rays are visible if viewed almost exactly along the local field lines. Gamma rays are not visible from the other three dips because they would have to cross absorbing regions to reach the observer, and hence are shadowed.

served for the Crab and Vela could be understood in such a model.

Curvature radiation produces photons with $\theta \approx 0$, but, owing to just this same curvature, the photon is soon crossing magnetic field lines at significant values of θ. It is therefore difficult ever to see gamma rays except from beyond the "cloverleaf" viewing pattern. The characteristic size of this pattern is just

$$r/a \sim (\gamma/2)^{1/3}$$

in a 10^{12} G field. Thus for us to see 2×10^9 eV photons ($\gamma=4\times10^3$) from the Crab nebula they must be emitted on the order of 13 stellar radii away. The light cylinder is only about 150 radii away here. This consideration is one of the motivating considerations in the "outer gap" model of Cheng *et al.* (1976), namely to find a plausible site for gamma radiation not too close to the star.

d. Spectrum of the radiation

It is rather difficult to account for the observed gamma-ray spectrum (of the Crab), since it falls as a power law with index ~ -1 (energy flux per unit energy window), whereas the spectrum from the mechanism usually suggested, curvature radiation (Bertotti *et al.*, 1969a, 1969b; Sturrock, 1971a; Treves, 1971b; Ozernoi and Usov, 1977; Hinata, 1977a; Hardee, 1977; Salvati and Massaro, 1978; Harding *et al.*, 1978; Hardee, 1979; Ayasli and Ögelman, 1980), rises with an index of $+\frac{1}{3}$ and then cuts off exponentially beyond a critical frequency. Earlier works suggested synchrotron radiation (i.e., perpendiculr motion across field lines) as the source of the gamma radiation (Apparao, 1969; Dean and Turner, 1971). Since the accelertaion mechanisms are typically just transport of particles across a potential drop, almost all of the radiation comes from the particles with maximum energy, which continue, moreover, to radiate upon leaving the acceleration region. Thus the spectrum is hardly modified at all from what one would expect from a monoenergetic beam. Reasonable fits can be made near the turnover portion of the synchrotron spectrum (Massaro and Salvati, 1979; Ayasli and Ögelman, 1980; Harding, 1981), but the resultant spectra become quite deficient at lower energies. The observed power-law spectrum extends to the optical, a range of 10^9, and "minor" differences in spectral index become quite pronounced. The gamma rays have also been attributed to inverse Compton scattering of radio photons by energetic electrons (Cheng and Ruderman, 1977c; Schlickeiser, 1980). It has been suggested that photon-photon collisions could *absorb* gamma rays (Pollack *et al.*, 1971), and that gamma-ray lasing might be observed (Rivlin, 1980).

3. X rays

Since the x-ray emission from the Crab pulsar is apparently part of a general power-law spectrum extending from the optical to gamma rays, it is not clear that a special explanation of just the x-ray portion is required. X rays have been attributed to curvature radiation (Ochelkov and Usov, 1980), synchrotron radiation (El-Gowhari and Arponen, 1972; Aschenbach and Brinkman, 1975), plasma instabilities (Hardee and Rose, 1974), and differential magnetic absorption of thermal surface emission (Daishido, 1975). Silk (1971) proposed that the diffuse x-ray background came from young pulsars.

As noted before, there is an important class of intrinsic x-ray pulsars which are not radio pulsars and appear to function by accretion. In fact, it was proposed some time ago that old "dead" radio pulsars might become active x-ray objects owing to accretion (Shvartsman, 1970; Ostriker *et al.*, 1970; see also Michel, 1972).

4. Cosmic rays

As putative generators of highly relativistic particles, pulsars are a natural candidate for sources of cosmic rays (Gunn and Ostriker, 1969; Ruderman, 1969; Gold, 1969b, 1974; Arnett and Schramm, 1973; Kennel *et al.*, 1973), and a rather extensive literature now exists. A recent workshop and review have summarized the state of theory here (Osborne and Wolfendale, 1975; Cesarsky, 1980). Broadly speaking, a number of difficulties must be reconciled in such theories. Naive versions of the standard model give more or less monochromatic particle fluxes, not a power law as observed. It is difficult to get the highest-energy cosmic rays ($\sim 10^{21}$ eV) with believable pulsar parameters (see Table V). Even if young pulsars could produce sufficiently energetic particles, these particles must be able to diffuse rapidly through the surrounding supernova remnant so as not to be adiabatically deenergized by expansion of the remnant, and not to experience too many collisions (i.e., traverse matter equivalent to 3 g/cm^2 or less of integrated exposure). Arons (1981b) points out that the expected production of gamma rays at the high-energy end of the spectrum would give too much gamma-ray background, at least according to existing theoretical views.

Although at least some of the cosmic-ray spectrum may contain particles directly accelerated from pulsars, there are presently too many uncertainties surrounding the propagation and modification of any input spectrum to provide definitive constraints on magnetospheric theory.

5. Gravitational waves

Pulsars have been viewed both as possible sources themselves of gravitational waves (Melosh, 1969; Chau, 1970; Ipser, 1971; Ruffini, 1971; Bertotti and Anile, 1973; Zimmerman, 1978) and as probes for indirect detection of such radition from binary systems (Brecher, 1975; Will, 1975, 1976; Wheeler, 1975; Wagoner, 1975; Esposito and Harrison, 1975; Blandford and Teukolsky, 1976; Epstein, 1977), as well as for a stochastic background of gravitational waves (left over from the big bang, for example: Rosi and Zimmerman, 1976; Sazhin, 1978; Detweiler, 1979). Photoproduction of gravitons has also been investigated (Papini and Valluri, 1975). Direct detection with tuned cylinders (Hirakawa *et al.*, 1978; Oide *et al.*, 1979), laser interferometry (Levine and Stebbins, 1972; Lu and Gao, 1976), and seismic measurements (Dyson, 1969; Wiggins and Press, 1969; Mast *et al.*, 1972, 1974) have not yet been successful (see however, Sadeh, 1972). Lunar mascons have also been suggested to serve as detectors (de Sabbata, 1970).

The binary pulsar 1913 + 16, discovered by Hulse and Taylor (1975; see also Taylor *et al.*, 1976; Fowler *et al.*, 1979) shows the expected behavior if the system is radiating gravitational radiation (Wagoner, 1975; Taylor *et al.*, 1979). Since then, two other binary pulsars have been detected, 0820 + 02 (Manchester *et al.*, 1980) and 0655 + 64 (see Table VIII and Damshek *et al.*, 1981). All three are consistent with a pulsar mass of about 1.4 $M_\odot$. That about 1% of all pulsers , might be binary was predicted (Guseinov and Novruzova, 1974). See also Trimble and Rees (1971a, 1971b), Shvartsman (1971), Bisnovatyi-Kogan and Komberg (1974), Barker and O'Connell (1975, 1976), and Hari Dass and Radhakrishnan (1975).

Since the magnetosphere is not implicated in gravitational radiation, binary pulsars are presently of interest mainly as probes of gravitational theory (Esposito and Harrison, 1975; Eardley, 1975; Barker and O'Connell, 1975; Blandford and Teukolsky, 1975; Nordtvedt, 1975; Will and Eardley, 1977; Will, 1977; Rosen, 1978; Schweizer and Straumann, 1979). Baroni *et al.* (1980) instead attribute the orbital period change to a diffuse gas cloud. Symbalisty and Schramm (1981) point out that 1913 + 60 should collide (or fuse) with its companion in about twelve million years, possibly ejecting neutron-rich material into the interstellar medium.

6. Neutrinos

Neutrino fluxes have not yet been observed from any astrophysical object (barely, if at all, even from the sun). The general possibility of such detection has been studied by several groups (Sato, 1977; Eichler, 1978a; Margolis *et al.*, 1978; Eichler and Schramm, 1978; and Helfand, 1979). The favorite mechanism seems to be energetic particle interaction with surrounding material (Eichler, 1978b, 1978c; Shapiro and Silberberg, 1979); however, Hara and Sato (1979) believe any such flux to be too weak to be detectable by the Dumand project. Direct neutrino production in the pulsar acceleration regions has also been examined by Skobelev (1976) and Loskutov and Skobelev (1976, 1980).

XI. PHENOMENOLOGY

In many areas of science the systems are so complex that they are difficult to model faithfully. The systematics of many-electron atoms and their excited levels forms a coherent physical picture even though one cannot yet calculate, from first principles, accurate excitation energies for every given state. In the same way, many have hoped to construct satisfactory models of pulsar magnetospheres without first having to solve the structure of the pulsar magnetosphere, phenomenology and theory going hand in hand, the former fleshing out what are necessarily simplified pictures offered by the former.

A. The hollow cone model

One of the most influential models remains the model proposed by Radhakrishnan and Cooke (1969), as shown in Fig. 28, which is a specific proposal for the radiation from an oblique rotator. It is quite common to find observational papers couched in terms of this model. Complex pulse shapes are often parametrized in terms of coaxial nests of emitting cones, whereby almost any conceivable pulse shape can be modeled. A single pulse is attributed to grazing the cone, a double pulse to cutting across the cone, a triple pulse to cutting across an outer cone and grazing an inner cone, etc. Drifting subpulses could be regions of enhanced emission circling the surface of the cone. The swing of polarization in Sec. II.D.6 is also naturally explained in this model. The broad pulse of 0826-34 (Table I) would require a viewing angle

TABLE VIII. The binary pulsars.

Property	1913 + 16	0820 + 02	0655 + 64
Pulse period (sec)	0.0590	0.8648	0.1956
Orbital period	7^h45^m	1233^d	24^h41^m
Projected semimajor axis	7.00×10^5 km	4.86×10^7 km	12.4×10^5 km
Mass function $(M_\odot)$[a]	0.131	0.023	0.0712
Eccentricity	0.6	0.01	<0.0001
Longitude of periastron	179°	332°	
DM	167	22.2	9.8

[a]Defined as $(M_2 \sin i)^3/(M_1+M_2)^2$, where M_1 is the pulsar, M_2 is the companion, and i is inclination of orbital plane.

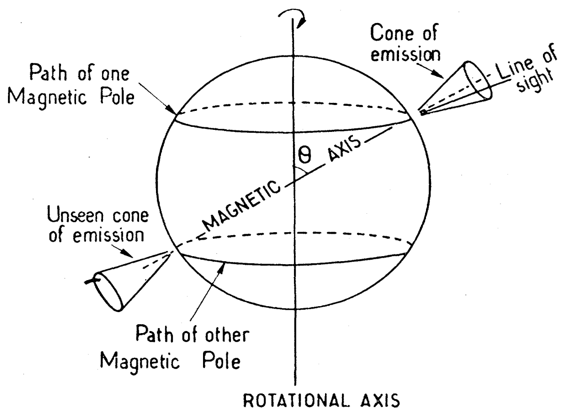

FIG. 28. The hollow cone model (Radhakrishnan and Cooke, 1969). Emission is assumed to take place in the form of a cone which rotates with the star, sweeping the pattern past the observer. Interpulses are observed if θ is near 90° and the observer is near the equatorial plane, in which case both poles will be visible.

very nearly along the spin axis. The hollow cone model, or similar proposals, has been refined by a number of authors (Böhm-Vitense, 1969; Radhakrishnan, 1971; Komesaroff *et al.*, 1971; Manchester *et al.*, 1973; Oster, 1975; Oster and Sieber, 1976a, 1976b, 1978; Backer *et al.*, 1976; Backer, 1976; Oster *et al.*, 1976a, 1976b; Sieber and Oster, 1977; Manchester, 1978; Cordes, 1978; Ochelkov and Usov, 1979; Proszynski, 1979; Jones, 1980a), and disputed by others (Izvekova *et al.*, 1977).

B. The corotating source

In this model (sometimes called the *Smith model)* the emission is attributed to a detached region in the magnetosphere that corotates with the star, typically rather close to the light cylinder so that relativistic effects would cause even an isotropically radiating source to appear pulsed. Geometrically the model is similar to that of Gold (1968). Consequently many of the same problems are there to be faced; how is the region held in place and how is it powered? The original proposal by Smith (1969) has been followed up (Smith, 1970, 1971a—1971d, 1973a, 1973b, 1974, 1976) and have attracted a considerable amount of attention and elaboration (Manchester and Tademaru, 1971; Zheleznyakov, 1971; McCrea, 1972; Zheleznyakov and Shaposhnikov, 1972, 1975; Cocke *et al.*, 1973, 1974; Ferguson, 1973, 1976a, 1976b, 1979; Ferguson *et al.*, 1974; Zlobin and Udal'tsov, 1975; Malov and Malofeev, 1977; Malov, 1979; Lyne and Smith, 1979).

Both the corotating source and the hollow cone model, though mutually incompatible, enjoy a considerable following, with possibly a slight edge to the hollow cone model because of its similarity with the standard model (except for the concept of multiple nested cones, which is not a natural, unforced aspect of that theory). The corotating source model, however, has provided some rather impressive quantitative accounts for the variation of polarization and intensity in pulse profiles.

C. Neutral sheets

Finally, in order of distance from the pulsar, comes the idea that the radiation comes from sheetlike discontinuities (shocks, neutral sheets) that form at or beyond the light cylinder. We have already discussed this idea in Sec. VI.C. The attractive features are (1) the relatively large surface areas for emission (the speed-of-light argument for the size pertains only to the thickness of the sheet), (2) the natural relativistic beaming by the portions of the sheet moving toward the observer, and (3) the natural preservation of the pulse because the radiation cannot overlap the next earlier sheet (at the high expansion γ's expected) until the sheet has undergone a huge expansion and thus presumably has ceased to radiate.

The major difficulties, however, are that (1) the distinction between the north and south poles of an oblique rotator, which are quite pronounced close to the object (Fig. 28), virtually vanish by the time one gets to the light cylinder, suggesting naively at least that one should get pairs of nearly identical pulses rather than the single pulses so often observed (observers have checked to be sure that the "single"-pulse pulsars are not actually producing a nearly identical pulse-plus-interpulse by averaging data over twice the period: The two average pulse shapes, one representing an average over the even numbered pulses and the other being the average over the odd pulses, are found to be identical within statistical uncertainties, and hence are concluded *not* to be produced separately—i.e., their similarity is too good to be true), (2) the stable but complex multiple subpulse patterns (Fig. 29) seem more difficult to account for in a straightforward way than a single simple pulse would be, and (3) the actual interpulse spacings are not always 180° from the main pulse, although again naive theory would argue that only the dipole magnetic field component should be dominant at the light cylinder, and hence one would expect rather precise alternation of poles even if the amplitude differences could be explained away.

No doubt some of these difficulties may be overstated (as well as the attractive features). Indeed the highest-energy emissions (γ ray) from both the Crab and Vela pulsars have rather equally spaced and rather similar amplitude pulse and interpulse (cf. Maraschi and Treves, 1974). The possibility of pulsed radiation being formed far from the pulsar may not yet be closed, although it is not presently in vogue.

D. Spectrum

Some phenomenology concentrates on accounting for the pulsar spectrum instead of (or in addition to) the pulse shape. Here the idea that coherent radiation comes from bunches is easiest to model, since a bunch can be described with one or two parameters, whereas a maser-

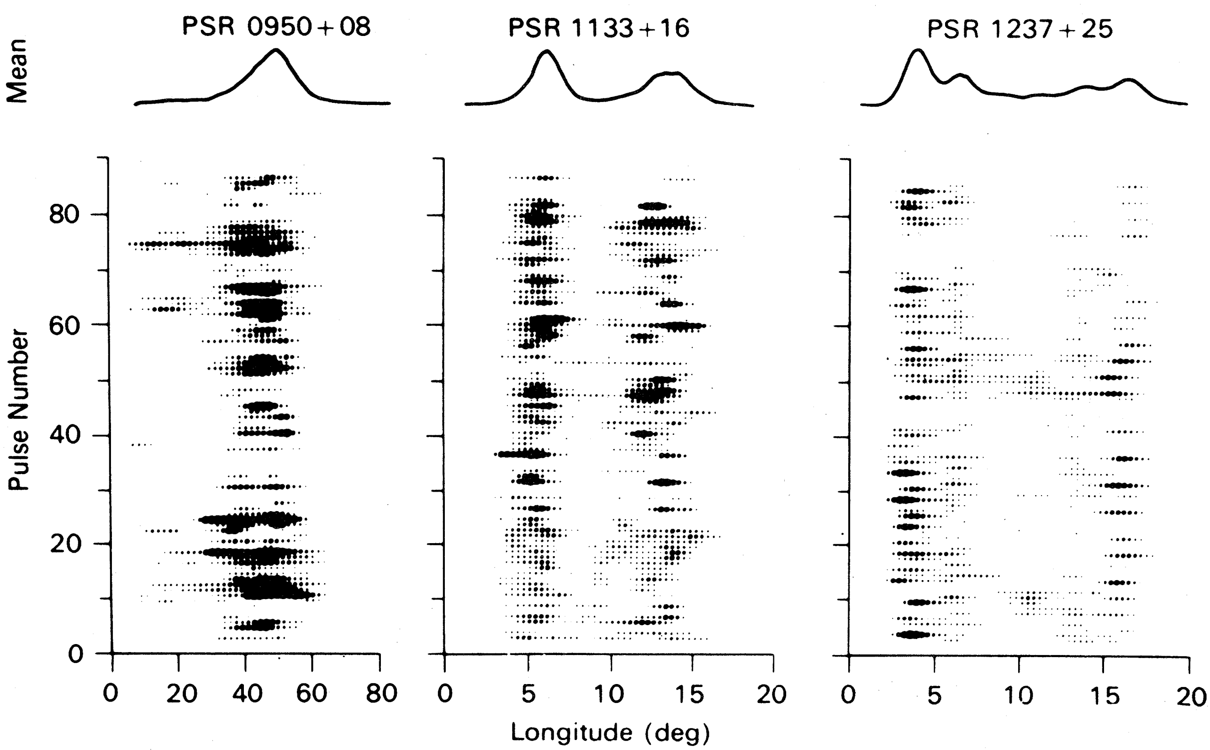

FIG. 29. Complex pulse exhibited by several pulsars (from Manchester and Taylor, 1977). Note 1237 + 25, which exhibits at least five distinct subpulses, not all of which are necessarily active at the same time. This would require three nested hollow cones, each of which is only partially lit up at any one time.

active region, for example, poses a much more amorphous picture. Figure 30 illustrates the simplest picture that results from parametrizing the bunch with a single scale λ. For wavelengths long compared to λ, all N_R particles in the bunch radiate together, and the incoherent single-particle synchrotron or curvature radiation is proportionately amplified. For wavelengths very much longer than λ, only the incoherent radiation is obtained. In between, there is diminished coherence where the "effective" size of the bunch that can radiate without destructive interference shrinks proportionally to the wavelength of observation until only a single particle remains, on the average. Sturrock *et al.* (1975) discuss in more detail the expected spectrum for specific realistic bunch shapes. Cordes (1979a, 1979b) discusses how one might infer the bunch shape from observation. See also Rickett (1975) and Cordes (1976). Similar spectra are presumably present in electron storage rings (Michel, 1982), where the energetic electrons are stored in the form of small circulating bunches.

E. Other

A number of other phenomenological proposals have been made, often to explain a single feature observed in only one or a few pulsars. Unfortunately, insofar as attempts to construct a basic theory go, the value of phenomenology decreases rapidly with the number of versions among which one is free to select.

Once again, we admit here a bias towards a first-principles effort in solving the pulsar mechanism. The number of known pulsars may well have passed 400 before the time of publication of this review. Yet neither the statistics of this rather extensive sample nor the properties of any specific pulsar seems yet adequate to reveal the underlying mechanism at work. Experience to date is therefore not too encouraging that pulsars will be more or less solved simply by examining the data in the context of general physical principles. One possible resolution of this impasse would be the formulation of an idealized, simple, self-consistent, physically complete model to compare with the available data. Another resolution, of course, would be the discovery of a pulsar so exceptional that it was obvious how it must function. Number 401, say.

XII. CONCLUSION

If the usual view presented to physics graduate students is correct, the analysis of pulsar models mainly involves well-known microphysics. It is also often presented to these graduate students that the real mysteries of physics only reside in the remaining unknown microphysics (quark confinement, etc.). If that is true, then how do pulsars work?

We have tried to be reasonably complete in this review

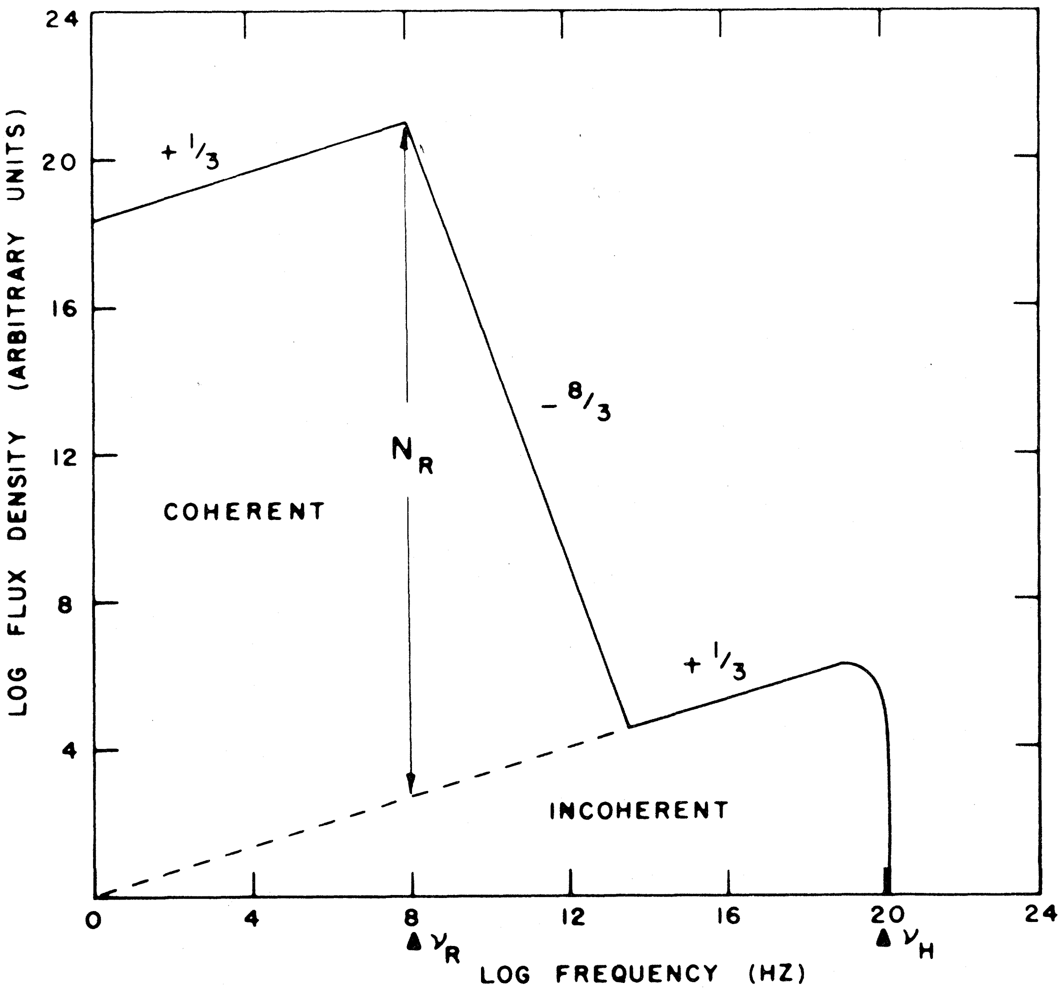

FIG. 30. Idealized spectrum expected from bunched particles emitting curvature radiation. The maximum coherent amplification (N_R, the number of particles in a bunch) is obtained at wavelengths long compared to the bunch (which then acts as a single particle of very large charge). No coherence is obtained at wavelengths shorter than the mean spacing between particles, and the behavior in between these limits is simply interpolated. The steep spectral drop with increasing frequency is an observed property typical of pulsars. However the Crab pulsar falls off faster than the $-\frac{8}{3}$ law shown, more nearly a $-\frac{10}{3}$ law.

insofar as the rotating magnetized neutron star theories go, and the immediately relevant ancillary subjects such as cosmic-ray acceleration. But there is no natural cutoff because the pulsar phenomenon has opened wide a door into distantly related subjects. Pulsar dispersion and rotation measures (DM and RM) provide new measurements of the interstellar medium. Fluctuations in pulsar periodicities may give insight into their interiors, the applicability of solid state physics, the properties of matter at high densities and at high fields, and on and on. These subjects are neither entirely relevant nor entirely irrelevant to the question of pulsar magnetospheres and therefore have been only touched upon, in many cases, if at all.

The sociology of pulsar physics deserves a few words, because it differs at least superficially from that of more traditional fields such as elementary particle physics. It is our perception that progress is rather rapid in particle physics, owing to the often deplored "bandwagon" effect. One year Regge poles or some similar idea will occupy center stage and be examined, discussed, modified, reformulated, and in general be bandied about, only later to be pushed aside to make room for the next fad. This activity may appear "wasted" when seen from certain senatorial viewpoints, because so much time has been spent on what is now not thought to be the final answer. But in the process an entire aspect of the problem has come to be rather well understood. Equally importantly, for a field heavily supported by grants and contracts, the progress at least has a semblance of order to it, and those making significant advances can often be identified. In pulsar theory, in contrast, there has been no such bandwagon effect. True, the Goldreich-Julian model has enjoyed wide adoption, but beyond that the territory is fragmented into tiny fiefdoms constantly at odds with one another, yet each too small to be very effective. (As an index of effectiveness, one might note that at present there are about 13 huge expensive experiments in progress to measure the lifetime of the proton, roughly comparable to the number of individual theorists who are actively working on the pulsar problem at any one time.) One subgroup feels that pair production is *the* answer to pulsar phenomena. Another finds discontinuities at the light cylinder and is convinced that this must have soemthing to do with the pulsar phenomenon. A third thinks it a waste of time to solve simplified models ("since pulsars are surely much more complicated objects") and searches for quick penetrating phenomenological insights. Pulsar theory is a field in which a competent referee will comment that, "this aligned rotator model is irrelevant because it can't pulse as pulsars do, owing to axial symmetry." As a result, it has taken a decade to critically analyze the physical properties of the very model that most workers have adopted, and even now important answerable uncertainties remain. We hope that this review will stimulate renewed critical investigation of the pulsar problem.

ACKNOWLEDGMENTS

I should like to express my appreciation to my colleagues for having scrutinized this review: J. Arons, A. F. Cheng, M. A. Ruderman, E. T. Scharlemann, J. H. Taylor, and A. J. Dessler, not to mention the unsung referees and E. E. Salpeter. Special appreciation also to former students, M. A. Pelizzari and C. D. Morris, who struggled courageously to make "simple" extensions to existing theory. Morris Aizemann and the National Science Foundation (AST79-14379) are to be commended for their tolerance of the unproductivity entailed in writing this review.

REFERENCES

Adler, S., 1971, Ann. Phys. (N.Y.) **67**, 599.
Adler, S. L., J. N. Bahcall, C. G. Callan, and M. N. Rosenbluth, 1970, Phys. Rev. Lett. **25**, 1061.
Ahluwalia, D. V., and T. -Y. Wu, 1978, Lett. Nuovo Cimento **23**, 406.
Akasofu, S. -I., 1978, Space Sci. Rev. **21**, 489.
Al'ber, Ya. I., Z. N. Krotova, V. Ya. Eidman, 1975, Astrophys. **11**, 189.
Aldridge, F. T., 1968, Proc. Nat. Acad. Sci. USA **60**, 743.
Alpar, M. A., P. W. Anderson, D. Pines, and J. Shaham, 1981, University of Illinois preprint.
Anderson, J. L., and J. M. Cohen, 1970, Astrophys. Space. Sci. **9**, 146.
Angelie, C., C. Deutsch, and M. Signore, 1980, J. Phys. (Paris) Colloq. **41**, 133.
Apparao, K. M. V., 1969, Nature (London) **223**, 385.
Apparao, K. M. V., 1974 Astrophys. Space Sci. **31** L9.
Apparao, K. M. V., and S. M. Chitre, 1970, Proc. Indian Acad. Sci. A **72** 285.
Apparao, K. M. V., and J. Hoffman, 1970, Astrophys. Lett. **5**, 25.
Ardavan, H., 1976a, Astrophys. J. **203** 226.
Ardavan, H., 1976b, Astrophys. J. **204** 889.
Ardavan, H., 1976c, Mon. Not. R. Astron. Soc. **175** 645.
Ardavan, H., 1976d, Astrophys. J. **206**, 822.
Ardavan, H., 1976e, Mon. Not. R. Astron. Soc. **177**, 661.
Ardavan, H., 1981, unpublished.
Arnett, W. D., 1969, Nature (London) **222**, 359.
Arnett, D., and R. L. Bowers, 1974, Univ. Texas Publ. Astron. **9**.
Arnett, W. D., and D. N. Schramm, 1973, Astrophys. J. Lett. **184**, L47 (Erratum; **187**, L47).
Arons, J., 1972, Astrophys. J. **177**, 395.
Arons, J., 1979, Space Sci. Rev. **24**, 437.
Arons, J., 1981a, in *Pulsars* (IAU Conf. No. 95; Reidel, Dordrecht), p. 69.
Arons, J., 1981b, in *Origin of Cosmic Rays* (IAU Conf. No. 94; Reidel, Dordrecht), p. 175.
Arons, J., and E. T. Scharlemann, 1979, J. **231**, 854.
Arons, J., and D. F. Smith, 1979, Astrophys. J. **229**, 728.
Arons, J., R. M. Kulsrud, and J. P. Ostriker, 1975, Astrophys. J. **198**, 687.
Arons, J., C. A. Norman, and C. E. Max, 1977, Phys. Fluids **20**, 1302.
Aschenbach, B., and W. Brinkmann, 1975, Astron. Astrophys. **41**, 147.
Asseo, E., C. F. Kennel, and R. Pellat, 1978, Astron. Astrophys. **65**, 401.
Asseo, E., X. Llobet, and G. Schmidt, 1980, Phys. Rev. A **22**, 1293.
Asseo, E., R. Pellat, and M. Rosado, 1980, Astrophys. J. **239** 661.
Asseo, E., R. Pellat, and H. Sol, 1981, Phys. Rev. A **24**, 1063.
Avakyan, R. M., G. G. Arutyunyan, and G. S. Saakyan, 1972, Astrophys. **8**, 282.
Avetisyan, A. K., 1979, Astrophys. **15**, 80.
Axford, W. I., H. E. Johnson, D. A. Mendis, and T. Yeh, 1970, Comments Astrophys. Space Phys. **2**, 53.
Ayasli, S., and H. Ögelman, 1980, Astrophys. J. **237**, 227.
Baade, W., 1942, Astrophys. J. **96**, 188.
Baade, W., and F. Zwicky, 1934, Phys. Rev. **45**, 138.
Backer, D. C., 1976, Astrophys. J. **209**, 895.
Backer, D. C., J. M. Rankin, and D. B. Campbell, 1976, Nature (London) **263**, 202.
Baglin, A., and J. Heyvaerts, 1969, Nature **222**, 1258.
Bahcall, J. N., M. J. Rees, and E. E. Salpeter, 1970, Astrophys. J. **162**, 737.
Banerjee, B., D. H. Constantinescu, and P. Rehak, 1974, Phys. Rev. D **10**, 2384.
Barbieri, R., 1971, Nucl. Phys. A **161**, 1.
Barker, B. M., and R. F. O'Connell, 1975, Astrophys. J. Lett. **199**, L25.
Barker, B. M., and R. F. O'Connell, 1976, Phys. Rev. D **14**, 861.
Barnard, J. J., and J. Arons, 1981, unpublished.
Baroni, L., G. Callegari, C. Gualdi, and P. Fortini, 1980, Lett. Nuovo Cimento **27**, 509.
Baym, G., C. Pethick, D. Pines, and M. Ruderman, 1969, Nature (London) **224**, 872.
Baym, G., and D. Pines, 1966, Ann. Phys. **66**, 16.
Bekefi, G., 1966, *Radiation Processes in Plasmas* (Wiley, New York) p. 181.
Benford, G., 1975, Astrophys. J. **201**, 419.
Benford, G., and R. Buschauer, 1977, Mon. Not. R. Astron. Soc. **179**, 189.
Bertotti, B., and A. M. Anile, 1973, Astron. Astrophys. **28**,

429.
Bertotti, B., A. Cavaliere, and F. Pacini, 1969a, Nature (London) **221**, 624.
Bertotti, B., A. Cavaliere, and F. Pacini, 1969b, Nature (London)**223**, 1351.
Bigg, E. K., 1964, Nature (London) **203**, 1008.
Bisnovatyi-Kogan, G. S., 1970, Radiophys. Quantum Electron. **13**, 1441.
Bisnovatyi-Kogan, G. S., and S. I. Blinnikov, 1973, Sov. Astron.—AJ **17**, 304.
Bisnovatyi-Kogan, G. S., and B. V. Komberg, 1974, Sov. Astron.—AJ **18**, 217.
Black, D. C., 1969, Nature (London) **221**, 157.
Bland, B. H., 1968, Nature (London) **219**, 23.
Blandford, R. D., 1975, Mon. Not. R. Astron. Soc. **170**, 551.
Blandford, R. D., and E. T. Scharlemann, 1976, Mon. Not. R. Astron. Soc. **174**, 59.
Blandford, R., and S. A. Teukolsky, 1975, Astrophys. J. Lett. **198**, L27.
Blandford, R., and S. A. Teukolsky, 1976, Astrophys. J. **205**, 580.
Bodenheimer, P., and J. P. Ostriker, 1974, Astrophys. J. **191**, 465.
Böhm-Vitense, E., 1969, Astrophys. J. Lett. **156**, L131.
Bonometto, S., and L. Scrascia, 1974, Astron. Astrophys. **32**, 115.
Braude, S. Ya., and Yu. M. Bruk, 1980, Pis'ma V. Astron. Zh. (USSR) **6**, 301 (1980).
Brecher, K., 1975, Astrophys. J. Lett. **195**, L113.
Broad, W. J., 1981, Science **212**, 1116.
Brown, G. E., H. A. Bethe, and G. Baym, 1981, Nordita preprint 81/17.
Buckee, J. W., S. Grounds, L. C. M. Miranda, and D. Ter Haar, 1974, AGARD Conf. Proc. **138**, (AGARD, Nevilly sur seine), p. 18.
Buckley, R., 1975, Oxford University preprint, QUEL-1140/75.
Buckley, R., 1976, Mon. Not. R. Astron. Soc. **177**, 415.
Buckley R., 1977a, Mon. Not. R. Astron. Soc. **180**, 125.
Buckley, R., 1977b, Nature (London) **266**, 37.
Buckley, R., 1978, Mon. Not. R. Astron. Soc. **183**, 771.
Burbridge, G. R., 1956, Astrophys. J. **124**, 416.
Burbidge, G. R., and P. A. Strittmatter, 1968, Nature (London) **218**, 433.
Burdyuzha, V. V., 1977, Pis'ma V. Astron. Zh. (USSR) **3**, 121.
Burman, R., 1977a, Astrophys. Space Sci **51**, 239.
Burman, R., 1977b, Phys. Lett. A **60A**, 309.
Burman, R., 1980a, Astrophys. Space Sci. **72**, 251.
Burman, R., 1980b, Aust. J. Phys. **33**, 771.
Burman, R. R., and L. Mestel, 1978, Aust. J. Phys. **31**, 455.
Burns, J. A., 1970, Nature (London) **228**, 986.
Buschauer, R., 1977, Mon. Not. R. Astron. Soc. **179**, 99.
Buschauer, R., and G. Benford, 1976, Mon. Not. R. Astron. Soc. **177**, 109.
Buschauer, R., and G. Benford, 1977, Mon. Not. R. Astron. Soc. **179**, 99.
Buschauer, R., and G. Benford, 1978, Mon. Not. R. Astron. Soc. **185**, 493.
Canuto, V., 1971, in *The Crab Nebula* (IAU Symp. No. 46; Reidel, Dordrecht), p. 455.
Carr, T. D., and S. Gulkis, 1969, Annu. Rev. Astron. Astrophys. **7**, 577.
Casperson, L. W., 1977, Astrophys. Space Sci. **48**, 389.
Cavaliere, A., F. Pacini, and G. Setti, 1969, Astrophys. Lett. **4**, 103.
Cesarsky, C., 1980, Annu. Rev. Astron. Astrophys. **18**, 289.
Chandrasekhar, S., 1957, *An Introduction to the Study of Stellar Structure* (Dover, New York), p. 412.
Chanmugam, G., 1973, Astrophys. J. Lett. **182**, L39.
Chanmugam, G., 1978, Astrophys. J. **221**, 965.
Chanmugan, G., and M. Gabriel, 1971, Astron. Astrophys. **11**, 268.
Charugin, V. M., 1975, Astrophys. Space Sci. **37**, 449.
Chau, W. Y., 1970, Nature (London) **228**, 655.
Chau, W. Y., and R. N. Henriksen, 1970, Astrophys. J. Lett. **161**, L137.
Chau, W. Y., and R. N. Henriksen, 1971, Astrophys. Lett. **8**, 49.
Chau, W. Y., and P. Srulovicz, 1971, Phys. Rev. D **3**, 1999.
Chau, W. Y., R. N. Henriksen, and D. R. Rayburn, 1971, Astrophys, J. Lett. **168**, L79.
Chen, H. -H., M. A. Ruderman, and P. G. Sutherland, 1974, Astrophys. J. **191**, 473.
Chenette, D. L., T. F. Conlon, and J. A. Simpson, 1974, J. Geophys. Res. **79**, 3551.
Cheng, A. F., and M. A. Ruderman, 1977a, Astrophys. J. **212**, 800.
Cheng, A. F., and M. A. Ruderman, 1977b, Astrophys. J. **214**, 598.
Cheng, A. F., and M. A. Ruderman, 1977c, Astrophys. J. **216**, 865.
Cheng, A. F., and M. A. Ruderman, 1979, Astrophys. J. **229**, 348.
Cheng, A., and M. A. Ruderman, 1980, Astrophys. J. **235**, 576.
Cheng, A., M. Ruderman, and P. Sutherland, 1976, Astrophys. J. **203**, 209.
Chiu, H.-Y., 1971, in *The Crab Nebula* (IAU Symp. No. 46; Reidel, Dordrecht), p. 414.
Chiu, H.-Y., and V. Canuto, 1969a, Nature (London) **221**, 529.
Chiu, H.-Y., and V. Canuto, 1969b, Phys. Rev. Lett. **22**, 415.
Chiu, H.-Y., and V. Canuto, 1970, Nature (London) **225**, 1230.
Chiu, H.-Y., and V. Canuto, 1971, Astrophys. J. **163**, 577,
Chiu, H.-Y., V. Canuto, and L. Fassio-Canuto, 1969, Nature (London) **221**, 529.
Chiu, H.-Y., and F. Occhionero, 1969, Nature (London) **223**, 1113.
Chiuderi, C., and F. Occhionero, 1970, Nature (London) **226**, 337.
Cline, T. L., 1981, NASA Tech. Memo. 82141 (GSFC).
Cocke, W. J., 1973, Astrophys. J. **184**, 291.
Cocke, W. J., and J. M. Cohen, 1968, Nature (London) **219**, 1009.
Cocke, W. J., and D. C. Ferguson, 1974, Astrophys. J. **194**, 725.
Cocke, W. J., and A. G. Pacholczyk, 1976, Astrophys. J. Lett. **204**, L13.
Cocke, W. J., D. C. Ferguson, and G. W. Muncaster, 1973, Astrophys. J. **183**, 987.
Cohen, J. M., and A. Rosenblum, 1972, Astrophys. Space Sci. **16**, 130.
Cohen, J. M., and A. Rosenblum, 1973, Astrophys. J. **186**, 267.
Cohen, J. M., and E. T. Toton, 1971, Astrophys. Lett. **7**, 213.
Cohen, J. M., L. S. Kegeles, and A. Rosenblum, 1975, Astrophys. J. **201**, 783.
Cohen, R., J. Lodenquai, and M. Ruderman, 1970, Phys. Rev. Lett. **25**, 467.

Cohen, R. M., B. Coppi, and A. Treves, 1973, Astrophys. J. **179**, 269.
Cole, T. W., 1976, Mon. Not. R. Astron. Soc. **175**, 93P.
Constantinescu, D. M., and G. Moruzzi, 1978, Phys. Rev. D. **18**, 1820.
Coppi, B., 1972, Verh. Dtsch. Phys. Ges. **7**, 777.
Coppi, B., and A. Ferrari, 1970a, Nuovo Cimento Lett. **3**, 93.
Coppi, B., and A. Ferrari, 1970b, Astrophys. J. Lett. **161**, L65.
Coppi, B., and A. Ferrari, 1971, in *The Crab Nebula* (IAU Symp. No. 46; Reidel, Dordrecht), p. 460.
Cordes, J. M., 1976, Astrophys. J. **210**, 780.
Cordes, J. M., 1978, Astrophys. J. **222**, 1006.
Cordes, J. M., 1979a, Aust. J. Phys. **32**, 9.
Cordes, J. M., 1979b, Space Sci. Rev. **24**, 567.
Cordes, J. M., and G. Greenstein, 1981, Astrophys. J. **245**, 1060.
Cox, J. L., Jr., 1979, Astrophys. J. **229**, 734.
Daishido, T., 1975, Publ. Astron. Soc. Jpn. **27**, 181.
Damshek, M., P. R. Backus, and J. H. Taylor, 1981, unpublished.
Daugherty, J. K., and A. K. Harding, 1981, NASA Technical Memorandum 82161, GSFC.
Daugherty, J. K., and I. Lerche, 1975, Astrophys. Space Sci. **38**, 437.
Daugherty, J. K., and I. Lerche, 1976, Phys. Rev. D **14**, 340.
Davila, J., C. Wright and G. Benford, 1980, Astrophys. Space Sci **71**, 51.
Davis, L., and M. Goldstein, 1970, Astrophys. J. Lett. **159**, L81.
de Sabbata, V., 1970, Mem. Soc. Astron. Ital. **41**, 65.
Dean, A. J., and M. J. L. Turner, 1971, Astrophys. Lett. **8**, 145.
Dessler, A. J., 1980a, Planet Space Sci. **28**, 781.
Dessler, A. J., 1980b, Icarus **44**, 291.
Dessler, A. J., B. R. Sandel, and S. K. Atreya, 1981, Planet. Space Sci. **29**, 215.
Detweiler, S., 1979, Astrophys. J. **234**, 1100.
Deutsch, A. J., 1955, Ann. Astrophys. **18**, 1.
Dicke, R. H., 1964, Nature (London) **202**, 432.
Dokuchaev, V. P., V. V. Tamoikin, Yo. V. Chugunov, 1976, Sov. Astron.-AJ **20**, 299.
Dorman, L. I., M. Ye. Kats, and A. K. Yukhimuk, 1973, Geomagn. Aeron. (USSR) **13**, 171.
Douglas-Hamilton, D. H., 1968, Nature (London) **218**, 1035.
Dowden, R. L., 1968, Proc. Astron. Soc. Austral **1**, 159.
Downs, G. S., 1981 Astrophys. J. (in press).
Drake, J. F., Y. C. Lee, and N. L. Tsintsadze, 1976, Phys. Rev. Lett. **36**, 31.
Durdin, J. M., M. I. Large, A. G. Little, R. N. Manchester, A. G. Lyne, and J. H. Taylor, 1979, Mon. Not. R. Astron. Soc. **186**, 39P.
Durney, B. R., J. Faulkner, J. R. Gribbin, and I. W. Roxburgh, 1968, Nature (London) **219**, 20.
Dyson, F. J., 1969a, Astrophys. J. **156**, 529.
Dyson, F. J., 1969b, Nature (London) **223**, 486.
Eardley, D. M., 1975, Astrophys. J. Lett. **196**, L59.
Eastlund, B. J., 1968, Nature (London) **220**, 1293.
Eastlund, B. J., 1970, Nature (London) **225**, 430.
Eastlund, B. J., 1971, in *The Crab Nebula* (IAU) Symp. No. 46; Reidel Dordrecht), p. 443.
Eichler, D., 1978a, AIP Conf. Proc. **52**, 38.
Eichler, D. 1978b, Astrophys. J. **222**, 1109.
Eichler, D., 1978c, Nature (London) **275**, 725.
Eichler, D., and D. N. Schramm, 1978, Nature (London) **275**, 704.
Eidman, V. Ya., 1971, Astrophysics **7**, 78.
El-Gowhari, A., and J. Arponen, 1972, Nuovo Cimento B **11**, 201.
Elitzur, M., 1974, Astrophys. J. **190**, 673.
Elitzur, M. 1979, Astrophys. J. **229**, 742.
Ellison, D., 1975, M. S. Thesis, Rice University.
Elsässer, K., 1976, Astron. Astrophys. **52**, 177.
Elsässer, K., and J. Kirk, 1976, Astron. Astrophys. **52**, 449.
Endean, V. G., 1972a, Mon. Not. R. Astron. Soc. **158**, 13.
Endean, V. G., 1972b, Nature (London) Phys. Sci. **237**, 72.
Endean, V. G., 1973, Nature (London) **241**, 184.
Endean, V. G., 1974, Astrophys. J. **187**, 359.
Endean, V. G., 1976, Mon. Not. R. Astron. Soc. **174**, 125.
Endean, V. G., 1980, Mon. Not. R. Astron. Soc. **193**, 213.
Endean, V. G., 1981, Mon. Not. R. Astron. Soc. **195**, 55p.
Endean, V. G., and J. E. Allen, 1970, Nature (London) **228**, 348.
Epstein, R. I., 1973, Astrophys. J. **183**, 593.
Epstein, R., 1977, Astrophys. J. **261**, 92; **231**, 644(E).
Epstein, R. I., and V. Petrosian, 1973, Astrophys. J. **183**, 611.
Erber, T., 1966, Rev. Mod. Phys. **38**, 626.
Erber, T., and H. N. Spector, 1973, Astrophys. J. **184**, 301.
Esposito, L. W., and E. R. Harrison, 1975, Astrophys. J. Lett. **196**, L1.
Evangelidis, E., 1977, Astrophys. Space Sci. **51**, 319.
Evangelidis, E. A., 1979, Astrophys. Space Sci. **60**, 213.
Ewart, G. M., R. A. Guyer, and G. Greenstein, 1975, Astrophys. J. **202**, 238.
Fang. L.-Z., and Y.-Z. Liu, 1976, Acta Phys. Sin. Abstr. **25**, 521.
Faulkner, J., and J. R. Gribbin, 1968, Nature (London) **218**, 734.
Fawley, W. M., J. Arons, and E. T. Scharlemann, 1977, Astrophys. J. **217**, 227.
Feinberg, G., 1969, Science **166**, 879.
Ferguson, D. C., 1973, Astrophys. J. **183**, 977.
Ferguson, D. C., 1976a, Astrophys. J. **205**, 247.
Ferguson, D. C., 1976b, Astrophys. J. **209**, 606.
Ferguson, D. C., 1979, Nature (London) **278**, 331.
Ferguson, D. C., W. J. Cocke, and T. Gehrels, 1974, Astrophys. J. **190**, 375.
Ferrari, A., 1972, Mem. soc. Astron. Ital. **43**, 715.
Ferrari, A., and E. Trussoni, 1971, Nuovo Cimento Lett. **1**, 137,
Ferrari, A., and E. Trussoni, 1973, Astrophys. Space Sci **24**, 3.
Ferrari, A., and E. Trussoni, 1974, Astron. Astrophys. **36**, 267.
Ferrari, A., and E. Trussoni, 1975a, Phys. Lett. A **51**, 304.
Ferrari, A., and E. Trussoni, 1975b, Astrophys. Space Sci. **33**, 111.
Ferraro, V. C. A., and C. Plumpton, 1961, *An Introduction to Magneto-fluid Mechanics* (Oxford University, N.Y.); p. 28.
Finzi, A., and R. A. Wolf, 1968, Astrophys. J. **153**, 835.
Finzi, A., and R. A. Wolf, 1969, Astrophys. J. Lett. **155**, L107.
Fischer, W., and N. Straumann, 1972, Helv. Phys. Acta **45**, 1089.
Fishman, G. J., F. R. Harnden, and R. C. Haymes, 1969a, Astrophys. J. Lett. **156**, 107.
Fishman, G. J., F. R. Harnden, W. N. Johnson, and R. C. Haymes, 1968b, Astrophys. J. Lett. **158**, L61.
Flowers, E., and M. A. Ruderman, 1977, Astrophys. J. **215**, 302.
Flowers, E. G., J.-F. Lee, M. A. Ruderman, P. G. Sutherland,

W. Hillebrandt, and E. Müller, 1977, Astrophys. J. **215**, 291.
Fowler, L. A., J. M. Cordes, and J. H. Taylor, 1979, Aust. J. Phys. **32**, 35.
Fowler, W. A., 1971, in *The Crab Nebula* (IAU Symp. No. 46; Reidel, Dordrecht), p. 364.
Fujimura, F. S., and C. F. Kennel, 1979, Astron. Astrophys. **79**, 299.
Fujimura, F. S., and C. F. Kennel, 1980, Astrophys. J. **236**, 245.
Fujimura, F. S., and C. F. Kennel, 1981, UCLA preprint PPG-552.
Fujimoto, M., and T. Murai, 1972, Publ. Astron. Soc. Jpn. **24**, 269.
Fujimoto, M., and T. Murai, 1973, Publ. Astron. Soc. Jpn. **25**, 75.
Galtsov, D. V., and V. I. Petukhov, 1978, Phys. Lett. A **66**, 346.
Gehrels, T. (ed.), 1976, *Jupiter* (University of Arizona, Tucson).
Gilinsky, V., M. Hubbard, G. Modesitt, and P. Collas, 1970 in *Topics in High Energy Physics* (P-4494, The Rand Corp., Santa Monica), p. 188.
Ginzburg, V. L., and V. V. Zheleznyakov, 1970a, Comments Astrophys. Space phys. **2**, 167.
Ginzburg, V. L., and V. V. Zheleznyakov, 1970b Comments Astrophys. Space Phys. **2**, 197.
Ginzburg, V. L., and V. V. Zheleznyakov, 1975, Annu. Rev. Astron. Astrophys. **13**, 511.
Ginzburg, V. L., V. V. Zheleznyakov, and V. V. Zaitsev, 1968, Nature (London) **220**, 355.
Ginzburg, V. L., V. V. Zheleznyakov, and V. V. Zaitsev, 1969a, Astrophys. Space Sci. **4**, 464.
Ginzburg, V. L., V. V. Zheleznyakov, and V. V. Zaitsev, 1969b, Sov. Phys.—Uspekhi **12**, 378.
Glasser, M. L., 1975, Astrophys. J. **199**, 206.
Glencross, W. M., 1972, Nature (London) **237**, 157.
Gold, T., 1968, Nature (London) **218**, 731.
Gold, T., 1969a, Nature (London) **221**, 25.
Gold, T., 1969b, Nature (London) **223**, 162.
Gold. T., 1974, Philos. Trans. R. Soc. London Ser. A **277** 453.
Goldhaber, A. S., and M. M. Nieto, 1971, Rev. Mod. Phys. **43**, 277.
Goldreich, P., 1969, Proc. Astron. Soc. Australia **1**, 227.
Goldreich, P., 1970, Astrophys. J.. Lett. **160**, L11.
Goldreich, P., and D. Lynden-Bell, 1969, Astrophys. J. **156**, 59.
Goldreich, P., and W. H. Julian, 1969, Astrophys. J. **157**, 869.
Goldreich, P., and W. H. Julian, 1970, Astrophys. J. **160**, 971.
Goldreich, P., and D. A. Keeley, 1971, Astrophys. J. **170**, 463.
Goldreich, P., F. Pacini, and M. J. Rees, 1971, Comments Astrophys. Space Sci. **3**, 185.
Good, M. L., 1969, Nature (London) **221**, 250.
Gott, J. R., III, J. E. Gunn, and J. P. Ostriker, 1970, Astrophys. J. Lett. **160**, L91.
Greenstein, G., 1972, Astrophys. J. **177**, 251.
Greenstein, G. S., and A. G. W. Cameron, 1969, Nature (London) **222**, 862.
Grewing, M., and H. Heintzmann, 1971, Astrophys. Lett. **8**, 167.
Groth, E. J., 1975a, Astrophys. J. **200**, 278.
Groth, E., 1975b, Astrophys. J. Suppl. **29**, 453.
Gunn, J. E., and J. P. Ostriker, 1969, Nature (London) **221**, 454.
Gunn, J. E., and J. P. Ostriker, 1970, Astrophys. J. **160**, 979.
Guseinov, O. Kh., and Kh. I. Novruzova, 1974, Astrophysics **10**, 163.
Hara, T., and H. Sato, 1979, Prog. Theor. Phys. **62**, 969.
Hardee, P. E., 1977, Astrophys. J. **216**, 873.
Hardee, P. E., 1979, Astrophys. J. **227**, 958.
Hardee, P. E., and W. K. Rose, 1974, Astrophys. J. Lett. **194**, L35.
Hardee, P. E., and W. K. Rose, 1976, Astrophys. J. **210**, 533.
Hardee, P. E., and W. K. Rose, 1978, Astrophys. J. **219**, 274.
Hardee, P. E., and P. J. Morrison, 1979, Astrophys. J. **227**, 252.
Harding, A. K., 1981, Astrophys. J. **245**, 267.
Harding, A. K., and F. Tademaru, 1979, Astrophys. J. **233**, 317.
Harding, A. K., and E. Tademaru, 1980, Astrophys. J. **238**, 1054.
Harding, A. K., E. Tademaru, and L. W. Esposito, 1978, Astrophys. J. **225**, 226.
Hari Dass, N. D., and V. Radhakrishnan, 1975, Astrophys. Lett. **16**, 135.
Harnden, F. R., Jr., R. Giacconi, J. Grindlay, P. Hertz, E. Schreier, F. Steward, H. Tannenbaum, and L. Van Speybroeck, 1980, Bul. Am. Astron. Soc. **12**, 799.
Harrison, E. R., 1970, Nature (London) **225**, 44.
Harrison, E. R., and E. Tademaru, 1975, Astrophys. J. **201**, 447.
Hegyi, D., R. Novick, and P. Thaddeus, 1969, Astrophys. J. Lett. **158**, L77.
Heintzmann, H., and M. Grewing, 1972, Z. Phys. **250**, 254.
Heintzmann, H., and J. Nitsch, 1972, Astron. Astrophys. **21**, 291.
Heintzmann, H., and E. Schrüfer, 1977, Phys. Lett. A. **60**, 79.
Heintzmann, H., W. Kundt, and J. P. Lasota, 1975a, Phys. Lett. A **51**, 105.
Heintzmann, H., W. Kundt, and J. P. Lasota, 1975b, Phys. Rev. A **12**, 204.
Helfand, D. J., 1979, Nature (London) **278**, 720.
Helliwell, R. A., D. L. Carpenter, and T. R. Miller, 1980, J. Geophys. Res. **85**, 3360.
Henriksen, R. N., 1970, Astrophys. Lett. **7**, 89.
Henriksen, R. N., and D. R. Rayburn, 1972, Astrophys. Lett. **11**, 107.
Henriksen, R. N., and D. R. Rayburn, 1974, Mon. Not. R. Astron. Soc. **166**, 409.
Henriksen, R. N., and J. A. Norton, 1975a, Astrophys. J. **201**, 431.
Henriksen, R. N., and J. A. Norton, 1975b, Astrophys. J. **201**, 719.
Henry, G. R., 1968, Phys. Rev. Lett. **21**, 468.
Hewish, A., 1981 in *Pulsars* (I.A.U. Conf. No. 95; Reidel, Dordrecht), p. 1.
Hewish, A., S. J. Bell, J. D. M. Pilkington, P. F. Scott, and R. A. Collins, 1968, Nature (London) **217**, 709.
Hill, T. W., 1980, Astrophys. Lett. **21**, 11.
Hill, T. W., and F. C. Michel, 1975, Rev. Geophys. Space Phys. **13**, 967.
Hill, T. W., A. J. Dessler, and L. J. Maher, 1981, J. Geophys. Res., **86**, 9020.
Hillebrandt, W., and E. Müller, 1976, Astrophys. J. **207**, 589.
Hills, J. G., 1970, Nature (London) **226**, 730.
Hinata, S., 1973, Astrophys. J. **186**, 1027.
Hinata, S., 1976a, Astrophys. J. **203**, 223.
Hinata, S., 1976b, Astrophys. Space Sci. **44**, 389.
Hinata, S., 1976c, Astrophys. J. **206**, 282.
Hinata, S., 1977a, Astrophys. J. **216**, 101.

Hinata, S., 1977b, Astrophys. Space Sci. **51**, 303.
Hinata, S., 1978, Astrophys. J. **221**, 1003.
Hinata, S., 1979, Astrophys. J. **227**, 275.
Hinata, S., and E. A. Jackson, 1974, Astrophys. J. **192**, 703.
Hirakawa, M., K. Tsubono, and M.-K. Fujimoto, 1978, Phys. Rev. D **17**, 1919.
Hoffman, J. A., W. H. G. Lewin, and J. Doty, 1977, Astrophys. J. Lett. **217**, L23.
Høg, E., and E. Lohsen, 1970, Nature (London) **227**, 1229.
Holloway, N. J., 1973, Nature (London) Phys. Sci. **246**, 6.
Holloway, N. J., 1975, Mon. Not. R. Astron. Soc. **171**, 619.
Holloway, N. J., 1977, Mon. Not. R. Astron. Soc. **181**, 9P.
Holloway, N. J., and M. H. L. Pryce, 1981, Mon. Not. R. Astron. Soc. **194**, 95.
Holt, S. S., and R. Ramaty, 1970, Nature (London) **228**, 351.
Horowitz, P., C. Papaliolios, and N. P. Carleton, 1972, Astrophys. J. Lett. **172**, L51.
Hoyle, F., and J. Narlikar, 1968, Nature (London) **218**, 123.
Huguenin, G. R., R. N. Manchester, and J. H. Taylor, 1971, Astrophys. J. **169**, 97.
Hulse, R. A., and J. H. Taylor, 1975, Astrophys. J. Lett. **195**, L51.
Hylton, D. J., and A. R. P. Rau, 1980, Phys. Rev. A **22**, 321.
Ichimaru, S., 1970, Nature (London) **226**, 731.
Ignat'ev, Yu. G., 1975, Acta Phys. Pol. B **6**, 203.
Imoto, M., and M. Kanai, 1971, Publ. Astron. Soc. Jpn. **23**, 363.
Imoto, M., and M. Kanai, 1972, Rend. Accad. Nazion. dei Lincei **162**, 271.
Ingraham, R. L., 1973, Astrophys. J. **186**, 625.
Ipser, J. R., 1971, Astrophys. J. **166**, 175.
Israel, W., 1968, Nature (London) **218**, 1235.
Izvekova, V. A., I. F. Malov, and V. M. Malofeev, 1977, Sov. Astron. Lett. **3**, 442.
Jackson, E. A., 1976a, Nature (London) **259**, 25.
Jackson, E. A., 1976b, Astrophys. J. **206**, 831.
Jackson, E. A., 1978a, Astrophys. J. **222**, 675.
Jackson, E. A., 1978b, Mon. Not. R. Astron. Soc. **183**, 445.
Jackson, E. A., 1979, Astrophys. J. **227**, 266.
Jackson, E. A., 1980a, Astrophys. J. **237**, 198.
Jackson, E. A., 1980b, Astrophys. J. **238**, 1081.
Jackson, E. A., 1981, Preprint, U. of Illinois, Ill-(Ast)-81-11.
Jelley, J. V., and R. V. Willstrop, 1969, Nature (London) **224**, 568.
Jones, D. H., F. G. Smith, and J. E. Nelson, 1980, Nature (London) **283**, 50.
Jones, P. B., 1975, Astrophys. Space Sci. **33**, 215.
Jones, P. B., 1976a, Nature (London) **262**, 120.
Jones, P. B., 1976b, Astrophys. J. **209**, 602.
Jones, P. B., 1976c, Astrophys. Space Sci. **45**, 369.
Jones, P. B., 1977a, Mon. Not. R. Astron. Soc. **178**, 879.
Jones, P. B., 1977b, Nature (London) **270**, 37.
Jones, P. B., 1978, Mon. Not. R. Astron. Soc. **184**, 807.
Jones, P. B., 1979, Astrophys. J. **228**, 536.
Jones, P. B., 1980a, Astrophys. J. **236**, 661.
Jones, P. B., 1980b, Mon. Not. R. Astron. Soc. **192**, 847.
Jones, P. B., 1980c, Astrophys. J. **237**, 590.
Joss, P. C., 1977, Nature (London) **270**, 310.
Joss, P. C., 1978, Astrophys. J. Lett. **225**, L123.
Julian, W. H., 1973, Astrophys. J. **183**, 967.
Kaburaki, O., 1978, Astrophys. Space Sci. **58**, 427.
Kaburaki, O., 1980a, Astrophys. Space Sci. **67**, 3.
Kaburaki, O., 1980b, unpublished.
Kadomtsev, B. B., and V. S. Kudryavtsev, 1971, JETP Lett. **13**, 42.
Kaiser, M. L., M. D. Desch, J. W. Warwick, J. B. Pearce, 1980, Science **209**, 1238.
Kanbach, G., *et al.*, 1977, in *Proceedings of the 12th ESLAB Symposium Frascati* (ES SP-124), p. 21.
Kaplan, S. A., and V. Ya. Eidman, 1969, JETP Lett. **10**, 203.
Kaplan, S. A., and V. N. Tsytovich, 1973a, Nature (London) Phys. Sci **241**, 122.
Kaplan, S. A., and V. N. Tsytovich, 1973b, Phys. Rep. **7C**, 1.
Kaplan, S. A., V. N. Tsytovich, and A. S. Chikhachev, 1970, Astrophysics **6**, 253.
Kaplan, S. A., V. N. Tsytovich, and V. Ya. Eidman, 1974, Sov. Astron.—AJ **18**, 211.
Kardashev, N. S., 1970, Sov. Astron.—AJ **14**, 375.
Karpman, V. I., C. A. Norman, D. ter Haar, and V. N. Tsytovich, 1975, Phys. Scr. **11**, 271.
Kawamura, K., and I. Suzuki, 1977, Astrophys. J. **217**, 832.
Kellogg, O. D., 1967, *Foundations of Potential Theory,* (Springer, New York), p. 262.
Kennel, C. F., and F. V. Coroniti, 1975, Space Sci. Rev. **17**, 857.
Kennel, C. F., and R. Pellat, 1976, J. Plasma Phys. **15**, 335.
Kennel, C. F., G. Schmidt, and T. Wilcox, 1973, Phys. Rev. Lett. **31**, 1364.
Khakimova, M., F. K. Khakimov, and V. N. Tsytovich, 1976, Astrophysics **12**, 348.
Kirk, J. G., and D. ter Haar, 1978, Astron. Astrophys. **66**, 359.
Klyakotko, M. A., 1977, Sov. Astron. Lett. **3**, 129.
Ko, H. C., 1979, Astrophys. J. **231**, 589.
Ko, H. C., and C. W. Chuang, 1978, Astrophys. J. **222**, 1012.
Kochhar, R. K., 1977, Nature (London) **270**, 38.
Kolbenstvedt, H., 1977, Phys. Rev. D. **15**, 975.
Komesaroff, M. M., 1970, Nature (London) **225**, 612.
Komesaroff, M. M., J. G. Ables, and P. A. Hamilton, 1971, Astrophys. Lett. **9**, 101.
Kovalev, Yu. A., 1979, Astrophys. Space Sci. **63**, 3.
Kovalev, Yu. A., 1980, Astrophys. Space Sci. **67**, 387.
Krivdik, V. G., and A. K. Jukhimuk, 1977, Geofiz, Sb. **79**, 78.
Kroll, N. M., and W. A. McMullin, 1979, Astrophys. J. **231**, 425.
Kulsrud, R. M., 1972, Astrophys. J. Lett. **174**, L25.
Kulsrud, R. M., 1975, Astrophys. J. **198**, 709.
Kulsrud, R. M., and J. Arons, 1975, Astrophys. J. **198**, 709.
Kumar, N., 1969, Phys. Lett. A **30**, 199.
Kundu, M. R., and S. M. Chitre, 1968, Nature (London) **218**, 1037.
Kundt, W., and E. Krotscheck, 1980, Astron. Astrophys. **83**, 1.
Kuo-Petravic, L. G., M. Petravic, and K. V. Roberts, 1974, Phys. Rev. Lett. **32**, 1019.
Kuo-Petravic, L. G., M. Petravic, and K. V. Roberts, 1975, Astrophys. J. **202**, 762.
Kuo-Petravic, L. G., and M. Petravic, 1976, Phys. Rev. Lett. **36**, 686.
Lamb, D. Q., and F. K. Lamb, 1976, Astrophys. J. **204**, 168.
Lamb, D. Q., and F. K. Lamb, 1978, Astrophys. J. **220**, 291.
Landau, L., 1932, Phys. Z. Sowjetunion **1**, 88.
Layzer, D., 1968, Nature (London) **220**, 247.
Lee, M. A., 1974, Astrophys. J. **194**, 165.
Lee, M. A., and Lerche, I., 1974, Astrophys. J. **194**, 409.
Lee, M. A., and Lerche, I., 1975, Astrophys. J. **198**, 477.
Lerche, I., 1970a, Astrophys. J. **159**, 229.
Lerche, I., 1970b, Astrophys. J. **160**, 1003.

Lerche, I., 1970c, Astrophys. J. **162**, 153.
Lerche, I., 1971, in *The Crab Nebula* (IAU Symp. No. 46; Reidel, Dordrecht), p. 449.
Lerche, I., 1974a, Astrophys. J. **187**, 589.
Lerche, I., 1974b, Astrophys. J. **187**, 597.
Lerche, I., 1974c, Astrophys. J. **188**, 627.
Lerche, I., 1974d, Astrophys. J. **191**, 191.
Lerche, I., 1974e, Astrophys. J. **191**, 753.
Lerche, I., 1974f, Astrophys. J. **191**, 759.
Lerche, I., 1974g, Astrophys. J. **191**, 763.
Lerche, I., 1974h, Astrophys. J. **194**, 177.
Lerche, I., 1974i, Astrophys. J. **194**, 403.
Lerche, I., 1975a, Phys. Rev. D **11**, 740.
Lerche, I., 1975b, Astrophys. J. Suppl. **29**, 113.
Lerche, I., 1975c, Astrophys. Space Sci. **35**, 363.
Lerche, I., 1975d, Astrophys. J. **199**, 734.
Lerche, I., 1976, Astrophys. Space Sci **41**, 387.
Levine, J., and R. Stebbins, 1972, Phys. Rev. D **6**, 1465.
Lewin, W. H. G., and P. C. Joss, 1981, unpublished.
L'Heureux, J., and P. Meyer, 1976, Astrophys. J. **209**, 955.
Lominadze, J. G., and A. B. Mikhailovskii, 1979, Sov. Phys. JETP **49**, 483.
Lominadze, J. G., G. Z. Machabeli, and A. B. Mikhailovskii, 1979a, Fizika Plazmy **5**, 6 (1979).
Lominadze, J. G., G. Z. Machabeli, and A. B. Mikhailovskii, 1979b, J. Phys. (Paris) Colloq. **40**, C7/713.
Loskutov, Yu. M., and V. V. Skobelev, 1976, Theor, Math. Phys. **29**, 932.
Loskutov, Yu. M., and V. v. Skobelev, 1980, Sov. J. Nucl. Phys. **31**, 1279.
Lovelace, R. V., 1973, (abstract) Bull. AAS **5** 426.
Lu, D.-J., and J.-G. Gao, 1976, Acta Phys. Sin. Abst. **25**, 181.
Lu, K. U., 1976, Int. J. Theor. Phys. **15**, 411.
Luheshi, M., and P. Stewart, 1979, Astron. Astrophys. **75**, 185.
Lyne, A. G., and F. G. Smith, 1979, Mon. Not. Astron. Soc. **188**, 675.
Machabeli, G. Z., and V. V. Usov, 1979, Sov. Astron. Lett.
Macy, W. W. Jr., 1974, Astrophys. J. **190**, 153.
Malov, I. F., 1979, Sov. Astron. Lett. **5**, 177.
Malov, I. F., and V. M. Malofeev, 1977, Sov. Astron.—AJ **21**, 55.
Manchester, R. N., 1972, Astrophys. J. **172**, 43.
Manchester, R. N., 1974, Science **186**, 66.
Manchester, R. N., 1978, Proc. Astron. Soc. Aust. **3**, 200.
Manchester, R. N., and J. H. Taylor, 1977, *Pulsars* (Freeman, San Francisco).
Manchester, R. N., and E. Tademaru, 1971, Nature (London) Phys. Sci. **232**, 164.
Manchester, R. N., E. Tademaru, J. H. Taylor, and G. R. Huguenin, 1973, Astrophys. J. **185**, 951.
Manchester, R. N., J. H. Taylor, and Y. Y. Van, 1974, Astrophys. J. Lett. **189**, L119.
Manchester, R. N., L. M. Newton, W. M. Goss, and P. A. Hamilton, 1978, Mon. Not. R. Astron. soc. **184**, 35P.
Manchester, R. N., L. M. Newton, D. J. Cooke, and A. G. Lyne, 1980, Astrophys. J. Lett. **236**, L25.
Manchester, R. N., J. H. Taylor, and A. G. Lyne, 1981, preprint.
Mandrou, P., G. Vedrenne, and J. L. Masnou, 1980, Nature (Lond.) **287**, 124.
Maraschi, L., and A. Cavaliere, 1977, *Highlights in Astronomy 4*, 127.
Maraschi, L., and A. Treves, 1974 in *Proceedings of the International Conference Supernovae and Supernova Remnants* (Reidel, Dordrecht), p. 307.
Margolis, S. H., D. N. Schramm, and R. Silberberg, 1978, Astrophys. J. **221**, 990.
Massaro, E., and M. Salvati, 1979, Astron. Astrophys. **71**, 51.
Massaro, E., M. Salvati, and R. Buccheri, 1979, Mon. Not. R. Astron. Soc. **189**, 823.
Masnou, J., 1980, private communication.
Mast, T. S., J. E. Nelson, J. Saarloos, R. A. Muller, and B. A. Bolt, 1972, Nature (London) **240**, 140.
Mast, T. S., J. E. Nelson, and J. A. Saarloos, 1974, Astrophys. J. Lett. **187**, L49.
Matese, J. J., and D. P. Whitmire, 1980, Astrophys. J. **235**, 587.
Max, C., and F. Perkins, 1971, Phys. Rev. Lett. **27**, 1342.
Max, C., and F. Perkins, 1972, Phys. Rev. Lett. **29**, 1731.
Max, C., 1973a, Phys. Fluids **16**, 1277.
Max, C., 1973b, Phys. Fluids **16**, 1480.
Maxwell, O. V., 1979, Astrophys. J. **231**, 201.
McCrea, W. H., 1972, Mon. Not. R. Astron. Soc. **157**, 359.
McIlraith, A. H., 1968, Nature (London) **220**, 461.
Melosh, H. L., 1969, Nature (London) **224**, 781.
Melrose, D. B., 1978, Astrophys. J. **225**, 557.
Melrose, D. B., 1979, Austral. J. Phys. **32**, 61.
Melrose, D. B., and R. J. Stoneham, 1977, Proc. Astron. Soc. Aust. **3**, 120.
Mertz, L., 1974, Astrophys. Space Sci **30**, 43.
Mestel, L., 1966, Congré Colloq. Univ. Liège **41**, 351.
Mestel, L., 1968, Mon. Not. R. Astron. Soc. **140**, 177.
Mestel, L., 1971, Nature (London) Phys. Sci. **233**, 149.
Mestel, L., 1973, Astrophys. Space Sci. **24**, 289.
Mestel, L., and Y.-M. Wang. 1979, Mon. Not. R. Astron. Soc. **188**, 799.
Mestel, L., G. A. E. Wright, and K. C. Westfold, 1976, Mon. Not. R. Astron. Soc. **175**, 257.
Mestel, L., P. Phillips and Y.-M. Wang, 1979, Mon. Not. R. Astron. Soc. **188**, 385.
Michel, F. C., 1969a, Phys. Rev. Lett. **23**, 247.
Michel, F. C., 1969b, Astrophys. J. **158**, 727.
Michel, F. C., 1970a, Nature (London) **228**, 1072.
Michel, F. C., 1970b, Comments Astrophys. Space Phys. **2**, 227.
Michel, F. C., 1970c, Astrophys. J. Lett. **159**, L25.
Michel, F. C., 1971, Comments Astrophys. Space Phys. **3**, 80.
Michel, F. C., 1972, Astrophys. Space Sci. **15**, 153.
Michel, F. C., 1973a, Astrophys. J. Lett. **180**, L133.
Michel, F. C., 1973b, Astrophys. J. **180**, 207.
Michel, F. C., 1974a, Phys. Rev. Lett. **33**, 1521.
Michel, F. C., 1974b, Astrophys. J. **187**, 585.
Michel, F. C., 1974c, Astrophys. J. **192**, 713.
Michel, F. C., 1975a, Astrophys. J. Lett. **195**, L69.
Michel, F. C., 1975b, Astrophys. J. **196**, 579.
Michel, F. C., 1975c, Astrophys. J. **197**, 193.
Michel, F. C., 1975d, Astrophys. J. **198**, 683.
Michel, F. C., 1978a, Astrophys. J. **220**, 1101.
Michel, F. C. 1978b, Astrophys. J. **224**, 998.
Michel, F. C., 1979a, Space Sci. Rev. **24**, 381.
Michel, F. C., 1979b, Astrophys. J. **227**, 579.
Michel, F. C., 1980, Astrophys. Space Sci. **72**, 175.
Michel, F. C., 1982, Rice University Preprint: SPA 81-41.
Michel, F. C., and A. J. Dessler, 1981, Bul. Am. Astron. Soc. **13**, 557.
Michel, F. C., and A. J. Dessler, 1981, Rice University Preprint: SPA 81-01.
Michel, F. C., and H. C. Goldwire, Jr., 1970, Astrophys. Lett.

5, 21.

Michel, F. C., and R. Pellat, 1981, in *Pulsars* (IAU Symp. No. 95; Reidel, Dordrecht) p. 37.

Michel, F. C., and W. H. Tucker, 1969, Nature (London) **223**, 277.

Michel, F. C., and A. Yahil, 1973, Astrophys. J. **179**, 771.

Mikhailovskii, A. B., 1980, Plasma Phys., **22**, 133.

Mikhailovskii, A. B., 1981, Plasma Phys., **23**, 413.

Miller, R. H., B. M. Lasker, J. E. Hesser, and S. B. Bracker, 1975, Astrophys. J. **196**, 121.

Minkowski, R. 1942, Astrophys. J. **96**, 199.

Modisette, J. L., 1967, J. Geophys. Res. **72**, 1521.

Møller, C., 1952, *The Theory of Relativity* (Oxford University, New York/London), p. 330.

Morris, C. D., Jr., Ph.D. Thesis, Rice University, 1975.

Morris, D., V. Radhakrishnan, and C. Shukre, 1976, Nature (London) **260**, 124.

Morris, D., V. Radhakrishnan, and C. Shukre, 1978, Astron. Astrophys. **68**, 289.

Morrison, P., 1969, Astrophys. J. Lett. **157**, L73.

Mueller, R. O., A. R. P. Rau, and L. Spruch, 1971, Phys. Rev. Lett. **26**, 1136.

Mueller, R. O., A. R. P. Rau, and L. Spruch, 1975, Phys. Rev. A **11**, 789.

Mulholland, J. D., 1971, Astrophys. J. **165**, 105.

Nagel, W., 1981, Astrophys. J., in press.

Nakamura, Y., 1980, Sci. Rep. Tohoku Univ. I Ser. **62**, 121.

Nerney, S., 1980, Astrophys. J. **242**, 723.

Newton, R. G., 1971, Phys. Rev. D **3**, 626.

Nordtvedt, K., Jr., 1975, Astrophys. J. **202**, 248.

Novick, R., M. C. Weisskopf, J. R. P. Angel, and P. G. Sutherland, 1977, Astrophys. J. Lett. **215**, L117.

Ochelkov, Yu. P., and V. V. Usov, 1979, Sov. Astron. Lett. **5**, 180.

Ochelkov, Yu. P., and V. V. Usov, 1980, Sov. Astron. Lett. **6**, 414.

Ochelkov, Yu. P., I. L. Rozental', and I. B. Shukalov, 1972, Sov. Astron.—AJ **16**, 244.

O'Connell, R. F., 1975, Astrophys. J. **195**, 751.

O'Connell, R. F., and K. M. Roussel, 1972, Astron. Astrophys. **18**, 198.

Oide, K., H. Hirakawa, and M.-K. Fujimoto, 1979, Phys. Rev. D **20**, 2480.

Okamoto, I., 1974, Mon. Not. R., Astron. Soc. **167**, 457.

Okamoto, I., 1975, Mon. Not. R. Astron. Soc. **170**, 81.

Okamoto, I., 1978, Mon. Not. R. Astron. Soc. **185**, 69.

Onishchenko, G., 1975, Sov. Astron.—AJ **19**, 171.

Onishchenko, O. G., 1979, Astrophysics **15**, 169.

Oppenheimer, J. R., and G. M. Volkoff, 1939, Phys. Rev. **55**, 374.

Osborne, J. L., and A. W. Wolfendale, 1975, (Eds.) *Origin of Cosmic Rays* (Reidel, Dordrecht).

Oster, L., 1975, Astrophys. J. **196**, 571.

Oster, L., and W. Sieber, 1976a, Astrophys. J. **203**, 233,

Oster, L., and W. Sieber, 1976b, Astrophys. J. **210**, 220.

Oster, L., and W. Sieber, 1978, Astron. Astrophys. **65**, 179.

Oster, L., D. A. Hilton, and W. Sieber, 1976a, Astron. Astrophys. **57**, 1.

Oster, L., D. A. Hilton, and W. Sieber, 1976b, Astron. Astrophys. **57**, 323.

Ostriker, J., 1968, Nature (London) **217**, 1227.

Ostriker, J. P., and J. E. Gunn, 1969a, Astrophys. J. **157**, 1395.

Ostriker, J. P., and J. E. Gunn, 1969b, Nature (London) **223**, 813.

Ostriker, J. P., and J. E. Gunn, 1971, Astrophys. J. Lett. **164**, L95.

Ostriker, J. P., and J.-L. Tassoul, 1968, Nature (London) **219**, 577.

Ostriker, J. P., M. J. Rees, and J. Silk, 1970, Astrophys. Lett. **6**, 179.

Ozernoi, L. M., and V. V. Usov, 1972, Sov. Astron.—AJ **17**, 270.

Ozernoi, L.M., and V. V. Usov, 1973a, Astrophys. Space Sci. **25**, 149.

Ozernoi, L. M., and V. V. Usov, 1973b, Astrophys. Lett. **13**, 151.

Ozernoi, L. M., and V. V. Usov, 1977, Sov. Astron.—AJ **21**, 425.

Pacini, F., 1967, Nature (London) **216**, 567.

Pacini, F., 1968, Nature (London) **219**, 145.

Pacini, F., 1969, Nature (London) **224**, 160.

Pacini, F., and M. J. Rees, 1970, Nature (London) **226**, 622.

Pacini, F., and E. E. Salpeter, 1968, Nature (London) **218**, 733.

Papini, G., and S.-R. Valluri, 1975, Can. J. Phys. **53**, 2312.

Popoyan, V. V., D. M. Sedrakyan, and E. V. Chubaryan, 1973, Sov. Astron.—AJ **16**, 615.

Parish, J. L., 1974, Astrophys. J. **193**, 225.

Parker, L., and J. Tiomno, 1972a, Astrophys. J. **178**, 809.

Parker, L., and J. Tiomno, 1972b, Nature (London) Phys. Sci **238**, 57.

Pavlov, G. G., and Yu. A. Shibanov, 1978, Sov. Astron.—AJ **22**, 214.

Pavlov, G. G., and Yu. A. Shibanov, 1979, Sov. Phys. JETP **49**, 741.

Pelizzari, M. A., 1975, M. S. Thesis, Rice University.

Pelizzari, M. A., 1976, Ph.D. Thesis, Rice University.

Petravic, M., 1976, Comput. Phys. Commun. **12**, 9.

Petschek, A. G., S. A. Colgate, and J. D. Colvin, 1976, Astrophys. J. **209**, 356.

Pfarr, J., 1972, Z. Phys. **251**, 152.

Pfarr, J., 1976, Gen. Rel. and Grav. **7**, 459.

Piddington, J. H., 1969, Nature (London) **222**, 965.

Pilipp, W. G., Astrophys. J. **190**, 391.

Pines, D., 1980a, J. d'Astrophys. **C2**, 11.

Pines, D., 1980b, Science **207**, 597.

Pines, D., and J. Shaham, 1974a, Comments Astrophys. Space Sci **6**, 37.

Pines D., and J. Shaham, 1974b, Nature **248**, 483.

Pines, D. J., Shaham, M. A. Alper, and P. W. Anderson, 1980 Prog. Teo Phys. Suppl. **69**, 376.

Pollack, J. B., P. D. Guthrie, and B. S. P. Shen, 1971, Astrophys. J. Lett. **169**, L113.

Proszynski, M., 1979, Astron. Astrophys. **79**, 8.

Pustil'nik, L. A., 1977, Sov. Astron-—AJ **21**, 432.

Radhakrishnan, V., 1071, in *The Crab Nebula* (IAU Symp. No. 46; Reidel, Dordrecht), p. 441.

Radhakrishnan, V., and D. J. Cooke, 1969, Astrophys. Lett. **3**, 225.

Rankin, J. M., and J. A. Roberts, 1971, in *The Crab Nebula* (IAU Symp. No. 46; Reidel, Doredrecht) p. 114.

Rappaport, S., P. C. Joss, J. E. McClintock, 1976, Astrophys. J. Lett. **206**, L103.

Rau, A. R. P., and L. Spruch, 1976, Astrophys. J. **207**, 671.

Rau, A. R. P., R. O. Mueller, and L. Spruch, 1975, Phys. Rev. A **11**, 1865.

Rawls, J. M., 1972, Phys. Rev. D. **5**, 487.

Ray, A., 1980, Phys. Fluids **23**, 898.

Rees, M., 1971a, Nature (London) **229**, 312.
Rees, M. J., 1971b, Nature (London) PHys. Sci. **230**, 55.
Rees, M. J., and J. E. Gunn, 1974, Mon. Not. R. Astron. Soc. **167**, 1.
Rees, M. J., V. L. Trimble, and J. M. Cohen, 1971, Nature (London) **229**, 395.
Rickett, B. J., 1975, Astrophys. J. **197**, 185.
Rivlin, L. A., 1980, Sov. J. Quantum Electron. **10**, 612.
Roberts, D. H., 1976, Astrophys. J. **207**, 949.
Roberts, D. H., and P. A. Sturrock, 1972a, Astrophys. J. **172**, 435.
Roberts, D. H., and P. A. Sturrock, 1972b, Astrophys. J. Lett. **173**, L33.
Roberts, D. H., and P. A. Sturrock, 1973, Astrophys. J. **181**, 161.
Roberts, J. A., and G. G. Fahlman, 1969, Nature (London) **222**, 862.
Rosen, N., 1978, Astrophys. J. **221**, 284.
Rosi, L. A., and R. L. Zimmerman, 1976, Astrophys. Space Sci **45**, 447.
Rossi, B., and S. Olbert, 1970, *Introduction to the Physics of Space* (McGraw-Hill, New York).
Ruderman, M., 1969, Comments Nucl. Par. Phys. **3**, 37.
Ruderman, M., 1971, Phys. Rev. Lett. **27**, 1306.
Ruderman, M. A., 1976, Astrophys. J. **203**, 206.
Ruderman, M., 1979, J. Magn. and Magn. Mater. **11**, 269.
Ruderman, M. A., 1981, in *Pulsars* (IAU Symposium No. 95; Reidel, Dordrecht), p. 87.
Ruderman, M. A., and P. G. Sutherland, 1975, Astrophys. J. **196**, 51.
Ruffini, R., 1971, in *The Crab Nebula* (IAU Symp. No. 46; Reidel, Dordrecht), p. 382.
Rylov, Yu. A., 1976, Sov. Astron.—AJ **20**, 23.
Rylov, Yu. A., 1977, Astrophys. Space Sci. **51**, 59.
Rylov, Yu. A., 1978, Astrophys. Space Sci. **53**, 377.
Sadeh, D., 1972, Nature (London) **240**, 139.
Sadeh, D., J. P. Hollinger, S. H. Knowles, and A. B. Youmans, 1968, Science **162**, 897.
Saggion, 1975, Astron. Astrophys. **44**, 285.
Salvati, M., 1973, Astron. Astrophys. **27**, 413.
Salvati, M., and E. Massaro, 1978, Astron. Astrophys. **67** 55.
Saslaw, W. C., J. Faulkner, and P. A. Strittmatter, 1968, Nature (London) **217**, 1222.
Sato, H. 1977, Prog. Theor. Phys. **58**, 549.
Sazhin, M. V., 1978, Vestn. Mosk. Univ. Fiz. Asronomiya **19**, 118.
Sazonov, V. N., 1973, Sov. Astron.—AJ **16**, 971.
Scargle, J. D., 1969, Astrophys. J. **156**, 401.
Scargle, J. D., and E. A. Harlan, 1970, Astrophys. J. Lett. **159**, L143.
Scargle, J. D., and F. Pacini, 1971, Nature (London) Phys. Sci. **232**, 144.
Scharlemann, E. T., 1974, Astrophys. J. **193**, 217.
Scharlemann, E. T., and R. V. Wagoner, 1973, Astrophys. J. **182**, 951.
Scharlemann, E. T., J. Arons, and W. M. Fawley, 1978, Astrophys. J. **222**, 297.
Schlickeiser, R., 1980, Astrophys. J. **236**, 945.
Schmalz, R., H. Ruder, and H. Herold, 1979, Mon. Not. R. Astron. Soc. **189**, 709.
Schmalz, R., H. Ruder, H. Herold, and C. Rossmanith, 1980, Mon. Not. R., Astron. Soc. **192**, 409.
Schmid-Burgk, J., 1973, Astron. Astrophys. **26**, 335.
Schweizer, M., and N. Straumann, 1979, Phys. Lett. A **71**, 493.
Sedrakyan, D. M., 1970a, Nature (London) **228**, 1074.
Sedrakyan, D. M., 1970b, Astrophysics **6**, 339.
Sedrakyan, D. M., and K. M. Shakhabasyan, 1972, Astrophysics **8**, 326.
Sedrakyan, D. M., K. M. Shakhabasyan, and Y. A. Mücket, 1975, Astrophysics **10**, 154.
Sedrakyan, D. M., K. M. Shakhabasyan, and R. Rudolph, 1977, Astrophysics **13**, 78.
Setti, G., and L. Woltjer, 1970, Astrophys. J. **159**, L87.
Shang-Hui, G., G. Yun-Zao, Y. C. Leung, L. Zong-Wei, and L. Shao-Rang, 1981, Astrophys. J. **245**, 1110.
Shapiro, M. M., and R. Silberberg, 1979, Rep. N. R. L. Prog. (July), 1.
Shaposhnikov, V. E., 1976, Astrophysics **12**, 43.
Shklovsky, I. S., 1968, *Supernovae* (Wiley-Interscience, New York), p. 318.
Shklovsky, I. S., 1970a, Astrophys. J. Lett. **159**, L77.
Shklovsky, I. S., 1970b, Nature (London) **225**, 251.
Shklovsky, I. S., 1977, Sov. Astron.—AJ **21**, 371.
Shvartsman, V. F., 1970, Radiophys. Quantum Electron. **13**, 1428.
Shvartsman, V. F., 1971, Sov. Astron.—AJ **15**, 342.
Sieber, W., and L. Oster, 1977, Astron. Astrophys. **61**, 445.
Silk, J., 1971, Astrophys. J. Lett. **166**, L39.
Silverstein, S. D., 1969, Phys. Rev. Lett. **23**, 139.
Simard-Normandin, M., and P. P. Kronberg, 1980, Astrophys. J. **242**, 74.
Simon, M., and D. L. P. Strange, 1969, Nature (London) **224**, 49.
Simon, N. R., and V. K. Sastri, 1971, Bul. AAS **3**, 479.
Skilling, J., 1968, Nature (London) **218**, 923.
Skobelev, V. V., 1976, Sov. Phys. JETP **44**, 660.
Smith, F. G., 1969, Nature (London) **223**, 934.
Smith, F. G., 1970, Mon. Not. R. Astron. Soc. **149**, 1.
Smith, F. G., 1971a, in *The Crab Nebula* (IAU Symp. No. 46; Reidel. Dordrecht), p. 431.
Smith, F. G., 1971b, Mon. Not. R. Astron. Soc. **154**, 5P.
Smith, F. G., 1971c, Nature (London) Phys. Sci. **231**, 191.
Smith, F. G., 1971d, Nature (London) Phys. Sci. **232**, 164.
Smith, F. G., 1973a, Nature (London) **243**, 207.
Smith, F. G., 1973b, Mon. Not. R. Astron. Soc. **161**, 9p.
Smith, F. G., 1974, Mon. Not. R. Astron. Soc. **167**, 43P.
Smith, F. G., 1976, Q. J. R. Astron. Soc. **17**, 383.
Smith, F. G., 1977, *Pulsars* (Cambridge University, Cambridge) p. 221.
Smoluchowski, R., 1972, Nature (London) Phys. Sci. **240**, 54.
Soper, S. R. K., 1972, Astrophys. Space Sci. **19**, 249.
Stecker, F. W., 1971, Nature (London) **229**, 105.
Stenflo, L., 1980, Phys. Scr. **21**, 831.
Stewart, P., 1974, Astron. Astrophys. **32**, 13.
Stewart, P., 1975, Astron. Astrophys. **41**, 169.
Stewart, P. 1977, Astron. Astrophys. **55**, 387.
Stothers, R., 1969, Nature (London) **223**, 279.
Sturrock, P. A., 1970, Nature (London) **227**, 465.
Sturrock, P. A., 1971a, Astrophys. J. **164**, 529.
Sturrock, P. A., 1971b, Astrophys. J. Lett. **169**, L7.
Sturrock, P. A., 1971c, Astrophys. J. **170**, 85.
Sturrock, P. A., and K. B. Baker, 1979, Astrophys. J. **234**, 612.
Sturrock, P. A., R. N. Bracewell, and P. Switzer, 1971, Nature (London) **229**, 186.
Sturrock, P. A., K. Baker, and J. S. Turk, 1976, Astrophys. J. **206**, 273.

Sturrock, P. A., V. Petrosian, and J. S. Turk, 1975, Astrophys. J. **196**, 73.
Sunyach, C., 1980, private communication.
Sutherland, P. G., 1979, Fundam. Cosmic Phys. **4**, 95.
Suvorov, E. V., and Yu. V. Chugnov, 1973, Astrophys. Space Sci. **23**, 189.
Swaneburg, B. N., K. Bennett, G. F. Bignami, R. Buccheri, P. Caraveo, W. Hermsen, G. Kanbach, G. G. Lichti, J. L. Masnou, H. A. Mayer-Hasselwander, J. A. Paul, B. Sacco, L. Scarsi, and R. D. Wills, 1981, Astrophys. J. Lett. **243**, L69.
Swank, J. H., R. M. Becker, E. A. Boldt, S. S. Holt, S. H. Pravdo, and P. J. Serlemitsos, 1977, Astrophys. J. **212**, L73.
Sweeney, G. S. S., and P. Stewart, 1974, Astron. Astrophys. **37**, 201.
Symbalisty, E. M. D., and D. N. Schramm, 1981, Enrico Fermi Institute preprint 81-43.
Synge, J. L., 1969, Nature (London) **223**, 161.
Taam, R. E., and R. E. Picklum, 1978, Astrophys. J. **224**, 210.
Tademaru, E., 1971, Astrophys. Space Sci. **12**, 193.
Tademaru, E., 1973, Astrophys. J. **183**, 625.
Tademaru, E., 1974, Astrophys. Space Sci. **30**, 179.
Tademaru, E., 1976, Astrophys. J. **209**, 245.
Tademaru, E., 1977, Astrophys. J. **214**, 885.
Tademaru, E., and E. R. Harrison, 1975, Nature (London) **254**, 676.
Takakura, T., 1969, Nature (London) **224**, 252.
Taylor, J. H., and G. R. Huguenin, 1971, Astrophys. J. **167**, 273.
Taylor, J. H., R. A. Hulse, L. A. Fowler, G. E. Gullahorn, and J. M. Rankin, 1976, Astrophys. J. Lett. **206**, L53.
Taylor, J. H., L. A. Fowler, and P. M. McCulloch, 1979, Nature (London) **277**, 437.
Tennakone, K., 1972, Lett. Nuovo Cimento **3**, 583.
Thorne, K. S., and J. R. Ipser, 1968, Astrophys. J. Lett. **152**, L71.
Treves, A., 1971a, Astron. Astrphys. **15**, 471.
Treves, A., 1971b, Nuovo Cimento B **4**, 88.
Trimble, V., 1968, Astron. J. **73**, 535.
Trimble, V., and M. Rees, 1970, Astrophys. Lett. **5**, 93.
Trimble, V., and M. J. Rees, 1971a, *The Crab Nebula* (IAU Symp. No. 46; Reidel, Dordrecht), p. 273.
Trimble, V., and M. J. Rees, 1971b, Astrophys. J. Lett. **166**, L85.
Trümper, J., W. Pietsch, C. Reppin, W. Voges, R. Staubert, and E. Kendziorra, 1978, Astrophys. J. Lett. **219**, L105.
Tsai, W.-Y., and T. Erber, 1974, Phys. Rev. D **10**, 492.
Tsuruta, S., 1974, in *Physics of Dense Matter,* IAU Symp. No. 53 (Reidel, Dordrecht), p. 209.
Tsuruta, S., 1975, Astrophys. Space Sci. **34**, 199.
Tsuruta, S., V. Canuto, J. Lodenquai, and M. Ruderman, 1972, Astrophys. J. **176**, 739.
Tsygan, A. I., 1977, Sov. Astron. Lett. [Pis'ma V. Astron. Zh. 3, 531].
Tsytovich, V. N., and A. S. Chikhachev, 1969, Sov. Astron.—AJ **13**, 385.
Tsytovich, V. N., and S. A. Kaplan, 1972, Astrophysics **8**, 260.
Tsytovich, V. N., J. W. Buckee, and D. Ter Haar, 1970, Phys. Lett. A **32**, 471.
Tucker, W. H., 1969, Nature (London) **223**, 1250.
Unwin, S. C., A. C. S. Readhead, P. N. Wilkinson, and M. S. Ewing, 1978, Mon. Not. R. Astron. Soc. **182**, 711.
Usov, V. V., 1975, Astrophys. Space Sci **32**, 375.
van Horn, H. M., 1968, Nature (London) **220**, 762.
van Horn, H. M., 1980, Astrophys. J. **236**, 899.
van Paradijs, J., 1978, Nature (London) **274**, 650.
Vandakurov, Yu. V., 1972, Sov. Astron.—AJ **16**, 265.
Vasyliunas, V. M., 1981, Space Science Rev. (in press).
Ventura, J., 1979, Phys. Rev. D **19**, 1684.
Vila, S. C., 1969. Nature (London) **224**, 157.
Virtamo, J., and P. Jauho, 1973, Astrophys. J. **182**, 935.
Virtamo, J., and P. Jauho, 1975, Nuovo Cimento B **26**, 537.
Vladimirskii, V. V., 1969, Zh. Eks. Teor. Fiz. Pis'ma Red. **9**, 116.
Wagoner, R. V., 1975, Astrophys. J. Lett. **196**, L63.
Wang, Y.-M., 1978, Mon. Not. R. Astron. Soc. **182**, 157.
Warner, B., and R. E. Nather, 1969, Nature (London) **222**, 157.
Warwick, J. W., 1969, Science **163**, 959.
Weber, E. J., and L. Davis, Jr., 1967, Astrophys. J. **148**, 217.
Wheaton, W. A., J. P. Doty, F. A. Primini, B. A. Cooke, C. A. Dobson, A. Goldman, M. Hecht, J. A. Hoffman, S. K. Howe, A. Scheepmaker, E. Y. Tsiang, W. H. G. Lewin, J. L. Matteson, D. E. Gruber, W. A. Baity, R. Rothschild, F. K. Knight, P. Nolan, and L. E. Peterson, 1979, Nature (London) **282**, 240.
Wheeler, J. A., 1966, Annu. Rev. Astron. Astrophys. **4**, 393.
Wheeler, J. C., 1975, Astrophys. J. Lett. **196**, L67.
Wiggins, R. A., and F. Press, 1969, J. Geophys. Res. **74**, 5351.
Will, C. M., 1975, Astrophys. J. Lett. **196**, L3.
Will, C. M., 1976, Astrophys. J. **205**, 861.
Will, C. M., 1977, Astrophys. J. **214**, 826.
Will, C. M., and D. M. Eardley, 1977, Astrophys. J. Lett. **212**, L91.
Wilson, L. W., 1974, Astrophys. J. **188**, 349.
Witten, T. A., Jr., 1974, Astrophys. J. **188**, 615.
Woodward, J. F., 1978, Astrophys. J. **225**, 574.
Woosley, S. E., and R. E. Taam, 1976, Nature (London) **263**, 101.
Wright, G. A. E., 1979, Nature (London) **277**, 363.
Yukhimuk, A. K., 1971, Astrophysics **7**, 366.
Van Riper, K. A., and D. Q. Lamb, 1981, Astrophys. J. Lett., **244**, L13.
Zheleznyakov, V. V., 1971, Astrophys. Space Sci. **13**, 87.
Zheleznyakov, V. V., and E. V. Suvorov, 1972, Astrophys. Space Sci. **15**, 24.
Zheleznyakov, V. V., and V. E. Shaposhnikov, 1972, Astrophys. Space Sci. **18**, 166.
Zheleznyakov, V. V., and V. E. Shaposhnikov, 1975, Astrophys. Space Sci. **33**, 141.
Zheleznyakov, V. V., and V. E. Shaposhnikov, 1979, Aust. J. Phys. **32**, 49.
Zimmerman, M., 1978, Nature (London) **271**, 524.
Zlobin, V. N., and V. A. Udal'tsov, 1975, Sov. Astron.—AJ **19**, 683.
Zwicky, F., 1938, Astrophys. J. **88**, 522.