Radiative decays of mesons

Patrick J. O'Donnell

Department of Physics and Scarborough College, University of Toronto, West Hill, Ontario M1C 1A4 Canada

New and improved data on the photonic decays of the old (noncharm) mesons has appeared within the past six years. It is now possible to put many of the ideas of the past twenty years to an exhaustive test. A review of these developments is given and the implications for our understanding of electromagnetic interactions of mesons are noted.

CONTENTS

I. INTRODUCTION

Over twenty years ago vector mesons were suggested (Nambu, 1957; Frazer and Fulco, 1960) as the means by which the isotopic scalar and vector electromagnetic form factors of the nucleon could be explained. Shortly after this many people used the idea of vector-meson dominance (VMD) in a number of problems in particle physics based on the pioneering work of Gell-Mann (1961) and Sakurai (1960). The study of the couplings of the ρ, ω, and ϕ mesons to the electromagnetic current (Gell-Mann and Zachariasen, 1961; Gell-Mann *et al.*, 1962; Nambu and Sakurai, 1962; Dashen and Sharp, 1964; Kroll *et al.*, 1967) proved to be a useful heuristic method for estimating decay rates and consequences of SU(3) symmetry.

By 1965 the quark model (Gell-Mann, 1964; Zweig, 1964) had been used to estimate some of the decays of the vector mesons (Anisovich *et al.*, 1965; Thirring, 1965; Soloviev, 1965; Becchi and Morpurgo, 1965, 1966) in a nonrelativistic SU(6) symmetry approach (Gursey and Radicati, 1964; Sakita, 1964). At this period also the width for the decay $\omega\rightarrow\pi\gamma$ had been measured and was given to be about 1.2 MeV. A full account of the use of the quark model in deriving the decay rates of hadrons was given by Van Royen and Weisskopf (1967a, 1967b).

Since then we have seen in particle physics many new theoretical developments such as quantum chromodynamics, (QCD) and the electroweak theory (Glashow, 1961; Weinberg, 1967; Salam, 1968) and many exciting new experimental discoveries (Particle Data Group, 1980) involving new flavors of quarks. Somewhat paradoxically much of the spectroscopy and many of the transition processes of the new charm- and bottom-quark constituent states can be obtained in relatively simple nonrelativistic models (Eichten *et al.*, 1975). New experimental data on the radiative decays of the "old" mesons appearing at this time seemed to remove the simple vector-meson dominance model or quark model understanding achieved by 1967. It is this development that we shall treat in this review, where we shall attempt to resolve the experimental and theoretical problems associated with the new and improved data on the old mesons.

II. THEORETICAL IDEAS AND MODELS

A. Some basic ideas

Since the basic ideas we shall deal with were formulated over 15 years ago, there are review articles and books which give comprehensive accounts of the theory (e.g., Morpurgo, 1969; Bernstein, 1968; Feld, 1969). In this section we shall introduce the concepts only in sufficient detail to establish notation and to indicate some subtleties which have a bearing on the subject matter of this review.

The original motivation of vector-meson dominance arose from attempts to interpret the electromagnetic form factors of the nucleon in dispersion relation calculations (Frazer and Fulco, 1960). The idea of approximating dispersion relations by a number of pole terms still forms the basis of the extended vector-meson dominance approach (Bramón and Greco, 1973; Cordes and O'Donnell, 1968, 1969).

To a large extent, however, the language of a local Lagrangian field theory is used to define the couplings of the ρ, ω, and ϕ mesons to the electromagnetic current (Gell-Mann and Zachariasen, 1961; Gell-Mann, 1962; Gell-Mann *et al.*, 1962; Nambu, 1957; Nambu and Sakurai, 1962; Dashen and Sharp, 1964; Kroll *et al.*, 1967. See also Feynman, 1972 for an excellent review).

We define the electromagnetic current $j_\mu(x)$ by

$$j_\mu(x)=\frac{em_\rho^2}{2\gamma_\rho}\rho_\mu(x)+\frac{em_\omega^2\sin\theta_Y}{2\sqrt{3}\,\gamma_Y}\omega_\mu(x)+\frac{em_\phi^2\cos\theta_Y}{2\sqrt{3}\,\gamma_Y}\phi_\mu(x)\,. \tag{2.1}$$

In this definition, $\rho_\mu(x)$, $\omega_\mu(x)$, and $\phi_\mu(x)$ are the field operators for the neutral ρ meson, ω meson, and ϕ mesons, respectively, e is the usual electric charge, and the masses of the ρ, ω, and ϕ mesons are denoted by m_ρ, m_ω, and m_ϕ. In the limit of SU(3) symmetry $\gamma_Y=\gamma_\rho$. The remaining terms define the coupling constant where θ_Y denotes the vector-meson mixing angle of the $\omega-\phi$ system. This angle enters when one describes the physical ω, ϕ states in terms of the singlet

(ω_1) and octet (ω_8) $I=Y=0$ members of the SU(3) representations (Sakurai, 1963; Glashow, 1963; Dashen and Sharp, 1964). That is, we write

$$\phi = \omega_8 \cos\theta_V - \omega_1 \sin\theta_V , \tag{2.2a}$$

$$\omega = \omega_8 \sin\theta_V + \omega_1 \cos\theta_V . \tag{2.2b}$$

It will prove to be convenient later when we describe the quark model to introduce a slightly different mixing angle notation, first introduced by Bramón and Greco (1973, 1974), which describes the deviation of mixing from the "ideal" mixing angle, $\hat{\theta} = \tan^{-1}(1/\sqrt{2})$. Let us define the mixing angle α_V for vector mesons by $\alpha_V \equiv \theta_V - \hat{\theta}$. In this basis we have

$$\phi = \frac{\cos\alpha_V}{\sqrt{3}}(\sqrt{2}\ \omega_8 - \omega_1) - \frac{\sin\alpha_V}{\sqrt{3}}(\sqrt{2}\,\omega_1 + \omega_8) , \tag{2.2c}$$

$$\omega = \frac{\sin\alpha_V}{\sqrt{3}}(\sqrt{2}\ \omega_8 - \omega_1) + \frac{\cos\alpha_V}{\sqrt{3}}(\sqrt{2}\ \omega_1 + \omega_8) . \tag{2.2d}$$

The significance of this choice becomes apparent when we consider the decay of the ρ, ω, and ϕ vector mesons into lepton pairs e^+e^- or $\mu^+\mu^-$. The width for such a decay is given by

$$\Gamma(V \to e^+e^-) = \frac{\pi\alpha^2}{3} m_V g_{V\gamma}^2 \left(1 + \frac{2m_l^2}{m_V^2}\right)\left(1 - \frac{4m_l^2}{m_V^2}\right)^{1/2} , \tag{2.3}$$

where V denotes any one of ρ, ω, or ϕ, and m_l is the mass of the electron. The coupling $g_{V\gamma}$ is related to those defined by Eq. (2.1) as follows:

$$g_{\rho\gamma} = 1/\gamma_\rho , \tag{2.4a}$$

$$g_{\omega\gamma} = \sin\theta_V/\sqrt{3}\ \gamma_Y , \tag{2.4b}$$

$$g_{\phi\gamma} = \cos\theta_V/\sqrt{3}\,\gamma_Y . \tag{2.4c}$$

In terms of the mixing angle α_V we have

$$g_{\omega\gamma} = (\sqrt{2}\sin\alpha_V + \cos\alpha_V)/3\gamma_Y , \tag{2.5a}$$

$$g_{\phi\gamma} = (\sqrt{2}\cos\alpha_V - \sin\alpha_V)/3\gamma_Y , \tag{2.5b}$$

and hence, as $\alpha_V \to 0$ we recover the SU(3) relation

$$g_{\rho\gamma}^2 : g_{\omega\gamma}^2 : g_{\phi\gamma}^2 = 9:1:2 . \tag{2.6}$$

We should emphasize here that although the literature is full of definitions of the coupling of the ρ, ω, and ϕ to the electromagnetic current differing from Eq. (2.1), use of Eq. (2.3) resolves any ambiguity by comparing directly with experiment. There is, however, some ambiguity in arriving at the values of the couplings $g_{V\gamma}$ from the experimental data, since we have treated the three vector mesons in a narrow width approximation (Cordes and O'Donnell, 1969a, 1969b; Gourdin, 1970; Feynman, 1972).

From the most recent collection of experimental values for the widths (Particle Data Group, 1980) we obtain the values $\Gamma(\rho^0 \to e^+e^-) = 6.79 \pm 0.82$ keV, $\Gamma(\omega \to e^+e^-) = 0.77 \pm 0.17$ keV, and $\Gamma(\phi \to e^+e^-) = 1.27 \pm 0.07$ keV. These are displayed in Fig. 1, where the ϕ width is arbitrarily set to have the value 2 and the other widths have been multiplied by the same factor. From this display it would appear that the data are not inconsistent with the SU(3) ratio, Eq. (2.6), following from setting $\alpha_V = 0$. Despite this, it is not clear how to extract a meaningful value for $\gamma_\rho^2/4\pi$, say, since the SU(3) ratio, Eq. (2.6), was derived for the dimensionless couplings and not the widths. Indeed, as shown also in Fig. 1, the data for $m_V\Gamma_V$ do even better in following the ratio 9:1:2 (Yennie, 1975; Jackson, 1976).

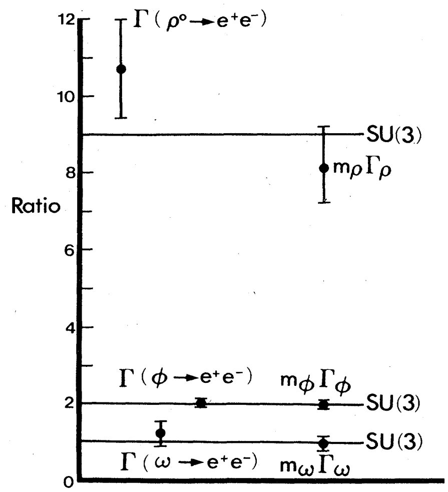

FIG. 1. Ratios of widths and mass x widths are shown for the leptonic decays of ρ, ω, and ϕ. Here Γ_ρ denotes the width for the decay $\rho \to e^+e^-$ and similarly Γ_ω, Γ_ϕ denote the widths for $\omega \to e^+e^-$ and $\phi \to e^+e^-$. Data are from the Particle Data Group (1980). The value two is arbitrarily given to Γ_ϕ or $m_\phi\Gamma_\phi$ and the other quantities are shown relative to this.

It is an interesting comment on the success of SU(3) (and on symmetry schemes in general) that in advance of the discovery of vector mesons, and of good data on them, the ratios of their leptonic decays was accurately predicted to be about 9:1:2, but with the better data now available it is apparent that SU(3) does not tell us what exactly should be in this ratio. Hence deviations from SU(3) are, to some extent, a matter of definition. Thus we shall adopt certain definitions and prescriptions in this article in order to bring some order to a review of the data on radiation decays, but remind the reader that other prescriptions can be (and have been) made. A comprehensive account of the leptonic decays of vector mesons (including the J/ψ and Υ mesons) has been given by Ong (1977, 1978).

For our present purposes we shall use the value $\Gamma(\rho^0 \to e^+e^-) = 6.79 \pm 0.82$ keV to estimate $\gamma_\rho^2/4\pi = 0.51 \pm 0.06$. The isoscalar pseudoscalar mesons η, X can be treated in a mixing scheme in a manner similar to the isoscalar vector mesons ω, ϕ by the definitions

$$\eta = \eta_8 \cos\theta_P - \eta_1 \sin\theta_P , \tag{2.7a}$$

$$X = \eta_8 \sin\theta_P + \eta_1 \cos\theta_P , \tag{2.7b}$$

where η_1, η_8 denote the singlet, octet $I=Y=0$ members of the SU(3) representations of pseudoscalar mesons and θ_P is the pseudoscalar mixing angle. [We denote the isoscalar state of mass 958 MeV by X rather than η'; this allows the prime to denote radial excitations (Graham and O'Donnell, 1979)]. As before we define also a pseudoscalar mixing angle $\alpha_P \equiv \theta_P - \hat{\theta}$, representing the shift from the "ideal" mixing angle $\hat{\theta} \equiv \tan^{-1}(1/\sqrt{2})$. Equations (2.7) are reexpressed then as

$$\eta=\frac{1}{\sqrt{3}}\cos\alpha_P(\sqrt{2}\ \eta_8-\eta_1)-\frac{1}{\sqrt{3}}\sin\alpha_P(\sqrt{2}\ \eta_1+\eta_8)\,, \qquad (2.7c)$$

$$X=\frac{1}{\sqrt{3}}\sin\alpha_P(\sqrt{2}\ \eta_8-\eta_1)+\frac{1}{\sqrt{3}}\cos\alpha_P(\sqrt{2}\ \eta_1+\eta_8)\,. \qquad (2.7d)$$

The quark model prediction of the ratio

$$\Gamma(\eta\to\gamma\gamma)/\Gamma(\pi^0\to\gamma\gamma) \qquad (2.8)$$

seems to favor the value $\theta_P\sim-10°$ or $\alpha_P\simeq-45°$. With such a value the combinations $(\sqrt{2}\eta_8-\eta_1)$ and $(\sqrt{2}\eta_1+\eta_8)$ are present in about equal amounts in the physical eigenvector. The values $\alpha_V\sim 0$ and $\alpha_P\sim-45°$ correspond roughly to the values obtained in quadratic mass formulas. Other types of mass formulas and mixing schemes are possible (further discussion and references may be obtained in Cordes and O'Donnell, 1969b; Boal *et al.*, 1976); a simple interpretation of these values can be given in the naive quark model, however, and we shall adopt them for the remainder of this article.

B. Vector dominance and SU(3) symmetry relationships

Although a gauge-invariant field theory has been obtained for the ρ^0, ω, and ϕ (Kroll *et al.*, 1967), it is now fairly well established that there exists at least one other particle, the $\rho'(1600)$, with the quantum numbers of the ρ meson (Atiya *et al.*, 1979; O'Donnell, 1980; Montanet, 1980). Such a particle couples to the electromagnetic current also (Fujikawa and O'Donnell, 1973; O'Donnell, 1980), so the gauge-invariant field theory must be only an approximate theory. In view of the emergence of quantum chromodynamics as the theory of strong interactions we shall take vector dominance to mean, as in its founding days, a phenomenological coupling of, for example, in the case of the ρ^0, a two-pion resonance in a narrow resonance approximation.

Thus it may be better to define the couplings of the vector mesons to the electromagnetic current by replacing Eq. (2.1) with

$$\sqrt{2q_0}\,\langle 0|j_\mu|\rho^0,q,\varepsilon\rangle=e\frac{m_\rho^2}{2\gamma_\rho}\varepsilon_\mu\,, \qquad (2.9a)$$

$$\sqrt{2q_0}\,\langle 0|j_\mu|\omega^0,q,\varepsilon\rangle=e\frac{m_\omega^2}{\sqrt{3}\,\gamma_Y}\sin\theta_V\,\varepsilon_\mu\,, \qquad (2.9b)$$

$$\sqrt{2q_0}\,\langle 0|j_\mu|\phi,q,\varepsilon\rangle=e\frac{m_\phi^2\cos\theta_V}{2\sqrt{3}\,\gamma_Y}\varepsilon_\mu\,, \qquad (2.9c)$$

where the vacuum is normalized by $\langle 0|0\rangle=1$ and a single-particle boson state has the covariant normalization

$$\langle p|q\rangle=(2\pi)^3\delta(\mathbf{p}-\mathbf{q})2p_0\,. \qquad (2.10)$$

For radiative decays such as $\omega\to\pi\gamma$, vector dominance relates the coupling constant $g_{\omega\pi\gamma}$ to the strong coupling $g_{\omega\rho\pi}$ by the relation

$$g_{\omega\pi\gamma}=\frac{e}{2\gamma_\rho}g_{\omega\rho\pi}\,. \qquad (2.11)$$

This would follow from the diagram in Fig. 2.

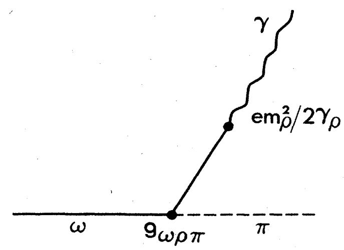

FIG. 2. This figure shows the application of vector dominance to the decay $\omega\to\pi\gamma$.

SU(3) symmetry relates the couplings $g_{V_iV_jP_k}(i,j,k=1,8)$ among vectors V_i, and pseudoscalars P_j of SU(3) octets. This allows us to relate processes such as $\omega\to3\pi$, $\phi\to3\pi$, etc. These have been considered in other papers (O'Donnell, 1976, 1977; Grunberg and Renard, 1976). Alternatively, we can relate the couplings for radiative decays directly, assuming that the photon has no SU(3) singlet component (Okubo, 1963a, 1963b; Glashow, 1963; Tanaka, 1964) by

$$(2\pi)^3(4p_0q_0)^{1/2}\langle V_ip,\varepsilon|j_j^\mu(0)|P_kq\rangle=\left(\frac{2eg}{m_\pi}\right)d_{ijk}\varepsilon^{\mu\nu\rho\sigma}\varepsilon_\nu p_\rho q_\sigma\,. \qquad (2.12)$$

Here p,ε are the four-momentum and polarization vector of the vector meson and q is the four-momentum of the pseudoscalar meson. The SU(3) invariant coupling is $2eg/m_\pi$, and d_{ijk} are the usual SU(3) coefficients (Gell-Mann, 1961). Notice that we have chosen to show the explicit mass dependence of the coupling by including m_π, the pion mass, in the definition (2.12). This is slightly unconventional since usually SU(3) is assumed for the mass-dependent coupling and SU(3) symmetry breaking is then introduced by using the real physical masses in the phase-space calculation of the widths. As we noted above in regard to the coupling of vector mesons to the electromagnetic current, SU(3) symmetry does not specify the kind of couplings for which the symmetry relationships will hold. The motivation behind the choice made in Eq. (2.12) is that when we come to compare with the quark model the coupling g will be a simple multiple (in a certain approximation) of the magnetic moments of the quarks.

Using Eq. (2.12) we derive the SU(3) relations

$$\frac{g}{3}=g_{\rho^0\pi^0\gamma}=g_{\rho^\pm\pi^\pm\gamma} \qquad (2.13a)$$

$$=g_{K^{*\pm}K^\pm\gamma} \qquad (2.13b)$$

$$=-\tfrac{1}{2}g_{K^{*0}K^0\gamma} \qquad (2.13c)$$

$$=\frac{1}{\sqrt{3}}g_{\omega_8\pi^0\gamma} \qquad (2.13d)$$

$$=\frac{1}{\sqrt{3}}g_{\rho^0\eta_8\gamma} \qquad (2.13e)$$

$$=-g_{\omega_8\eta_8\gamma}\,, \qquad (2.13f)$$

where we have explicitly labeled the couplings with the particle name and removed the common factor $(2e/m_\pi)$. Further relationships can be obtained using the concept of U-spin conservation (Meshkov, *et al.*, 1963; Feld, 1969; Bernstein, 1968). The photon is a U-spin scalar, i.e., it conserves U spin. Conservation of U spin means conservation of charge within a SU(3) multiplet and, in particular, gives the following relationships:

$$g_{\rho\eta_1\gamma}=\sqrt{3}\,g_{\omega_8\eta_1\gamma}\equiv\sqrt{2/3}\,g_1' \,, \tag{2.14a}$$

$$g_{\omega_1\pi^0\gamma}=\sqrt{3}\,g_{\omega_1\eta_8\gamma}\equiv\sqrt{2/3}\,g_1 \,, \tag{2.14b}$$

$$g_{\omega_1\eta_1\gamma}=0 \,, \tag{2.14c}$$

where the last relation also assumes that the photon has no SU(3) singlet component. At this stage there are three independent coupling constants, one describing the coupling $V_8P_8\gamma(g)$, one describing the coupling $V_8P_1\gamma(g_1')$, and the last describing $V_1P_8\gamma$ coupling (g_1), where the $V(P)$ denotes vector (pseudoscalar) and the suffixes show the octet (8) or singlet (1) representation content.

Using Eqs. (2.2), (2.7), (2.13), and (2.14), we can summarize the above in the following set of relationships:

$$g_{\rho^0\pi^0\gamma}=g/3 \,, \tag{2.15a}$$

$$g_{\omega\pi\gamma}=\tfrac{1}{3}[\sqrt{2}\,(g-g_1)\sin\alpha_V+(2g_1+g)\cos\alpha_V] \,, \tag{2.15b}$$

$$g_{\phi\pi\gamma}=\tfrac{1}{3}[\sqrt{2}\,(g-g_1)\cos\alpha_V-(2g_1+g)\sin\alpha_V] \,, \tag{2.15c}$$

$$g_{K^{*0}K^0\gamma}=-\tfrac{2}{3}g \,, \tag{2.15d}$$

$$g_{K^{*\pm}K^{\pm}\gamma}=g/3 \,, \tag{2.15e}$$

$$g_{\rho\eta\gamma}=\tfrac{1}{3}[\sqrt{2}\,(g-g_1')\cos\alpha_P-(2g_1'+g)\sin\alpha_P] \,, \tag{2.15f}$$

$$g_{X\rho\gamma}=\tfrac{1}{3}[\sqrt{2}\,(g-g_1')\sin\alpha_P+(2g_1'+g)\cos\alpha_P] \,, \tag{2.15g}$$

$$\begin{aligned}g_{\omega\eta\gamma}=&\tfrac{1}{9}\cos\alpha_V\sin\alpha_P(g-2g_1'-2g_1)\\&-\tfrac{2}{9}\sin\alpha_V\cos\alpha_P(g+g_1'+g_1)+\frac{\sqrt{2}}{9}\sin\alpha_V\sin\alpha_P(g-2g_1'+g_1)+\frac{\sqrt{2}}{9}\cos\alpha_V\cos\alpha_P(-g-g_1'+2g_1) \,,\end{aligned} \tag{2.15h}$$

$$\begin{aligned}g_{\phi\eta\gamma}=&-\tfrac{1}{9}\sin\alpha_V\sin\alpha_P(g-2g_1'-2g_1)\\&-\tfrac{2}{9}\cos\alpha_V\cos\alpha_P(g+g_1'+g_1)+\frac{\sqrt{2}}{9}\cos\alpha_V\sin\alpha_P(g-2g_1'+g_1)-\frac{\sqrt{2}}{9}\cos\alpha_V\cos\alpha_P(-g-g_1'+2g_1) \,,\end{aligned} \tag{2.15i}$$

$$\begin{aligned}g_{X\omega\gamma}=&-\tfrac{1}{9}\cos\alpha_V\cos\alpha_P(g-2g_1'-2g_1)\\&-\tfrac{2}{9}\sin\alpha_V\sin\alpha_P(g+g_1'+g_1)-\frac{\sqrt{2}}{9}\sin\alpha_V\cos\alpha_P(g-2g_1'+g_1)+\frac{\sqrt{2}}{9}\cos\alpha_V\sin\alpha_P(-g-g_1'+2g_1) \,,\end{aligned} \tag{2.15j}$$

$$\begin{aligned}g_{\phi X\gamma}=&\tfrac{1}{9}\sin\alpha_V\cos\alpha_P(g-2g_1'-2g_1)\\&-\tfrac{2}{9}\cos\alpha_V\sin\alpha_P(g+g_1'+g_1)-\frac{\sqrt{2}}{9}\cos\alpha_V\cos\alpha_P(g-2g_1'+g_1)-\frac{\sqrt{2}}{9}\sin\alpha_V\sin\alpha_P(-g-g_1'+2g_1) \,.\end{aligned} \tag{2.15k}$$

The width formulas for decay of the type $V\to P\gamma$ are, in terms of these couplings, given by

$$\Gamma(V\to P\gamma)=\frac{4}{3}\alpha\frac{k^3}{m_\pi^2}\,|g_{VP\gamma}|^2 \,, \tag{2.16}$$

where $k=(m_V^2-m_P^2)/2m_V$. For decays of the form $P\to V\gamma$, we interchange P and V and multiply the right-hand side of Eq. (2.16) by three, since in Eq. (2.16) there was an average taken over the initial spin states.

Finally, in this section we note that the ideas of vector dominance and U spin enable us to relate other processes to those already considered. First of all we note that the coupling constants for the two-photon decays of π, η, and X are related by U spin to be

$$\begin{aligned}g_{\pi\gamma\gamma}&=\sqrt{3}\,g_{\eta_8\gamma\gamma}\\&=g_{\eta\gamma\gamma}(\sqrt{2}\cos\alpha_P-\sin\alpha_P)\\&\quad+g_{X\gamma\gamma}(\cos\alpha_P+\sqrt{2}\sin\alpha_P),\end{aligned} \tag{2.17}$$

where the width for $\pi^0\to\gamma\gamma$ is defined to be

$$\Gamma(\pi^0\to\gamma\gamma)=\frac{\alpha m_\pi}{4}\,|g_{\pi\gamma\gamma}|^2 \,. \tag{2.18}$$

Alternatively, we may use vector dominance to relate the $\pi\gamma\gamma$ coupling constant to the couplings of vector mesons. This can be done in a number of ways, of which we illustrate two possibilities in Fig. 3.

In Fig. 3 we have one photon coupling through vector dominance to either a ρ meson or an SU(3) octet part of the isoscalar vector meson. In each case a further use of vector dominance for the remaining photon leg is possible; isospin implies that if one photon is coupled to the isovector vector meson then the other is coupled to the isoscalar vector meson. That is, we have the relations

$$\begin{aligned}g_{\pi\gamma\gamma}&=\frac{e}{2\gamma_\rho}g_{\pi\rho\gamma}+\frac{e}{2\sqrt{3}\,\gamma_Y}(\sin\theta_V\,g_{\pi\omega\gamma}+\cos\theta_V\,g_{\pi\phi\gamma})\\&=\frac{e}{3\gamma_\rho}g \,.\end{aligned} \tag{2.19}$$

The last version of Eq. (2.19) follows when the defini-

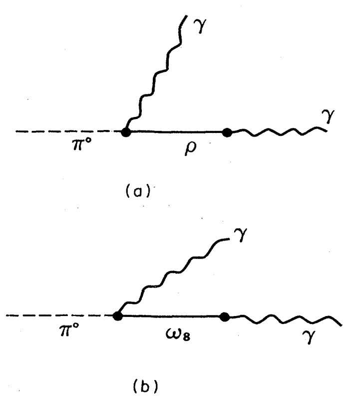

FIG. 3. This shows the application of vector dominance to the decay $\pi^0\to\gamma\gamma$.

tions (2.15) are used.

A simple consequence of these ideas is that the ratio of the decays $\pi^0 \to \gamma\gamma$ and $\rho^- \to \pi^-\gamma$ is

$$\frac{\Gamma(\rho^- \to \pi^-\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} \sim \frac{16}{3}\left(\frac{k}{m_\pi}\right)^3 \frac{(\gamma_\rho^2/4\pi)}{\alpha}, \tag{2.20}$$

where the momentum $k = (m_\rho^2 - m_\pi^2)/2m_\rho$. If we use the naive determination of $\gamma_\rho^2/4\pi$ from the decay $\rho \to e^+e^-$ as 0.51 then the ratio Eq. (2.20) is 7350. From the data tables (Particle Data Group, 1980) we find $\Gamma(\pi^0 \to \gamma\gamma) = 7.86 \pm 0.55$ eV. These numbers allow a prediction of 58 ± 8 keV for the decay $\Gamma(\rho^- \to \pi^-\gamma)$. The most recent measurement of this width, and one which is considered to be the most reliable, is that of Berg, *et al.*, 1980, who find $\Gamma(\rho^- \to \pi^-\gamma) = 67 \pm 7$ keV.

C. Quark model

In the preceding section we have seen some indication of the efficacy of SU(3) symmetry ideas and simple vector dominance. It is fair to say, however, that the conventional wisdom is to apply the quark model to all processes involving elementary particles. In the context of this review we shall use a nonrelativistic quark model based on static SU(6) ideas. For reviews covering many aspects of such a model, see, for example, Morpurgo, 1969; Kokkedee, 1969; and Close, 1979.

The model is based on the pioneering work of Morpurgo, 1965 and Dalitz, 1966, and considers the u, d quarks with a mass of about 0.3 GeV and the strange s quark with a mass of about 0.5 GeV. This idea of having confined light quarks moving in a nonrelativistic potential has proved to be fruitful in applications to both the baryon spectra (Isgur and Karl, 1977) and the meson spectra (Graham and O'Donnell, 1979), even though the nonrelativistic nature of the model seems to make the model not self-consistent. We note that in applications to the new particles $J/\psi, \gamma$, etc., containing heavy quarks of masses $\gtrsim 2$ GeV, the nonrelativistic approach not only works well (Eichten *et al.*, 1975; Schnitzer, 1975, 1976) but is internally consistent with the nonrelativistic quark model assumptions. Furthermore, within the bag model (Chodos *et al.*, 1974a, 1974b; DeGrand *et al.*, 1975; Donoghue *et al.*, 1975), which is a model of relativistic, confined quarks, the boundary conditions force the "large" component of the Dirac wave function to dominate over the "small" components throughout the interior of the bag (Hackman *et al.*, 1978; Frank *et al.* 1981). Since this is the approximation which is used in taking the nonrelativistic limit of a relativistic theory (Bjorken and Drell, 1961) the success of the nonrelativistic approach may in fact be due to this "accident."

The early triumphs of the quark model were the magnetic dipole transitions typified by the decay $\omega \to \pi\gamma$ (Becchi and Morpurgo, 1965) and the static moment calculations of baryons, especially those of the neutron and of the proton (Beg *et al.*, 1964; Morpurgo, 1965). Since new data have appeared recently we shall review these topics in some detail.

In the SU(3) quark model the three quarks (u, d, s) shown in Fig. 4 are taken to form the fundamental three-dimensional representation ($\underline{3}$) of SU(3). If we make

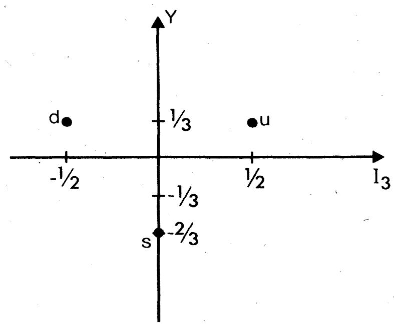

FIG. 4. $\underline{3}$ representation of the three quarks u, d, and s plotted against hypercharge (Y) and the third component of isotopic spin (I_3).

objects from two such sets of quarks, in all possible ways we find that they fall into two classes according to the symmetry behavior under interchange of one quark with another. These are grouped as follows:

Symmetric.

$$\begin{aligned} & uu \\ & \frac{1}{\sqrt{2}}(ud + du), \\ & dd, \\ & \frac{1}{\sqrt{2}}(us + su), \\ & \frac{1}{\sqrt{2}}(ds + sd), \\ & ss. \end{aligned} \tag{2.21a}$$

Antisymmetric.

$$\begin{aligned} & \frac{1}{\sqrt{2}}(ud - du), \\ & \frac{1}{\sqrt{2}}(us - su), \\ & \frac{1}{\sqrt{2}}(ds - sd). \end{aligned} \tag{2.21b}$$

A succinct notation for such a decomposition is

$$\underline{3} \times \underline{3} = \underline{6} + \underline{\bar{3}}$$

(Kokkedee, 1969; Lichtenberg, 1978; Close, 1979). The notation $\underline{\bar{3}}$ shows that there are two inequivalent representations of SU(3); if we reflect the triangle of Fig. 4 through the origin we obtain an inverted triangle for the antiparticles $(\bar{u}, \bar{d}, \bar{s})$ or $\underline{\bar{3}}$. If we now add a third quark from the set (u, d, s) then we obtain the decomposition

$$\underline{3} \times \underline{3} \times \underline{3} = \underline{1} + \underline{8} + \underline{8} + \underline{10}\,.$$

In this case, as one can readily check by explicit construction, the ten-dimensional representation is completely symmetric under the interchange of any of the quarks with another, and the singlet is completely antisymmetric under such an interchange. The two remaining eight-dimensional representations have a mixed behavior under quark interchanges. Since the eight-dimensional representation coincides with the correct number of low-mass baryons we display below in Table I forms of these representations in the two cases, with the appropriate particle label.

The extension to the SU(6) case involves taking the product of the SU(3) fundamental $\underline{3}$ representation with the fundamental $\underline{2}$ representation (spin $\frac{1}{2}$) of SU(2) (Gursey and Radicati, 1964). This is an exercise in Clebsch-Gordan coefficients and for baryons is summarized in the decomposition,

$$\underline{2}\times\underline{2}\times\underline{2}=\underline{4}+\underline{2}+\underline{2},$$

where the two $\underline{2}$ representations are also of mixed symmetry under interchange of the quark states. We can represent this pictorially by denoting spin up (down) by the symbol $\uparrow(\downarrow)$. The two-dimensional, mixed-symmetry representations are given in Table II. The nucleons appear in the $\underline{56}$ symmetric representation of SU(6) formed by multiplying the two representations of Tables I and II together, i. e., the SU(6) wave function, suitably normalized, is

$$\psi=\frac{1}{\sqrt{2}}(\psi_1\chi_1+\psi_2\chi_2). \qquad (2.22)$$

1. Static magnetic moments

The magnetic moment operator is

$$(\mu)_z=\sum_{i=1}^{3}\mu e_i\sigma_{iz}, \qquad (2.23)$$

where e_i is the charge operator for the ith quark and σ_{iz} is the appropriate Pauli matrix. Since the wave function Eq. (2.22) is fully symmetric under the interchange of all three quarks, including spin, and since the suffixes 1 or 2 denote symmetry or antisymmetry, respectively, under interchange of quarks 1 and 2, we

TABLE I. Two forms of the mixed-symmetry eight-dimensional representations of SU(3) in the quark model with identification with the eight baryons.

Particle	ψ_1	ψ_2
P	$\frac{1}{\sqrt{6}}\{(ud+du)u-2uud\}$	$\frac{1}{\sqrt{2}}(ud-du)u$
N	$-\frac{1}{\sqrt{6}}\{(ud+du)d-2ddu\}$	$\frac{1}{\sqrt{2}}(ud-du)d$
Ξ^-	$-\frac{1}{\sqrt{6}}\{(ds+sd)s-2ssd\}$	$\frac{1}{\sqrt{2}}(ds-sd)s$
Ξ^0	$-\frac{1}{\sqrt{6}}\{(us+su)s-2ssu\}$	$\frac{1}{\sqrt{2}}(us-su)s$
Σ^+	$\frac{1}{\sqrt{6}}\{(us+su)u-2uus\}$	$\frac{1}{\sqrt{2}}(us-su)u$
Σ^-	$\frac{1}{\sqrt{6}}\{(ds+sd)d-2dds\}$	$\frac{1}{\sqrt{2}}(ds-sd)d$
Σ^0	$\frac{1}{\sqrt{12}}\{(sd+ds)u+(su+us)d-2(du+ud)s\}$	$\frac{1}{2}\{(ds-sd)u+(us-su)d\}$
Λ^0	$\frac{1}{2}\{(ds+sd)u-(us+su)d\}$	$\frac{1}{\sqrt{12}}\{(sd-ds)u+(us-su)d-2(du-ud)s\}$

TABLE II. Two forms of the mixed-symmetry two-dimensional representations of SU(2). The labels 1 and 2 are similar to the previous table and denote symmetry (antisymmetry) under the interchange of the first two particles.

S_z	χ_1	χ_2
$\frac{1}{2}$	$\frac{1}{\sqrt{6}}\{(\uparrow\downarrow+\downarrow\uparrow)\uparrow-2\uparrow\uparrow\downarrow\}$	$\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\uparrow$
$-\frac{1}{2}$	$-\frac{1}{\sqrt{6}}\{(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow\}$	$\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\downarrow$

rewrite Eq. (2.23) as

$$(\mu)_z=3\mu e_3\sigma_{3z}. \qquad (2.24)$$

This simplifies the calculation of static properties immensely. For example, take the proton wave function corresponding to $S_z=+\frac{1}{2}$. This is

$$|P\rangle=\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{6}}[(ud+du)u-2uud]\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\uparrow-2\uparrow\uparrow\downarrow]+\frac{1}{\sqrt{2}}(ud-du)u\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\uparrow\right). \qquad (2.25)$$

Thus

$$\mu_P=\langle P|(\mu)_z|P\rangle=\tfrac{3}{2}\mu\{\tfrac{1}{6}[2\times(\tfrac{2}{3})+4(-\tfrac{1}{3})]\tfrac{1}{6}[2\times 1+4\times(-1)]+\tfrac{1}{2}(2\times(\tfrac{2}{3}))\tfrac{1}{2}(2\times 1)\}$$
$$=\mu. \qquad (2.26)$$

Similarly, for the neutron

$$|N\rangle=\frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{6}}[(ud+du)d-2ddu]\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\uparrow-2\uparrow\uparrow\downarrow]+\frac{1}{\sqrt{2}}(ud-du)d\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\uparrow\right) \qquad (2.27)$$

and hence

$$\mu_N=\tfrac{3}{2}\mu\{\tfrac{1}{6}[2\times(-\tfrac{1}{3})+4\times\tfrac{2}{3}]\tfrac{1}{6}[2\times 1+4\times(-1)]+\tfrac{1}{2}(2\times-\tfrac{1}{3})\tfrac{1}{2}(2\times 1)\}$$
$$=-\tfrac{2}{3}\mu. \qquad (2.28)$$

This gives the famous nonrelativistic SU(6) ratio $\mu_P/\mu_N=-\frac{3}{2}$ and furthermore predicts that the magnetic moment of the quark is the same as that of the proton. In units of the proton charge and with $\hbar=c=1$ we have

$$\mu_q\equiv\mu=\frac{2.79}{2m_P} \qquad (2.29a)$$

$$=\frac{g_q}{2m_q}, \qquad (2.29b)$$

where the first equation uses the value of the proton magnetic moment and the second introduces the gyromagnetic ratio of the quark. If the quark is a pointlike Dirac particle (i.e., $g_q=1$), then we obtain

$$m_q=\frac{m_p}{2.79}$$
$$\sim 340\text{ MeV}. \qquad (2.30)$$

In Table III we give the expectation values of e_3 for the

TABLE III. The expectation values of the charge for the third quark for the wave functions of Tables I and II. For $S_z = +\frac{1}{2}$ we have $\langle\chi_1|\sigma_z^3|\chi_1\rangle = -\frac{1}{3}$ and $\langle\chi_2|\sigma_z^3|\chi_2\rangle = 1$.

	$\langle\psi_1\|e_3\|\psi_1\rangle$	$\langle\psi_2\|e_3\|\psi_2\rangle$
P	0	$\frac{2}{3}$
N	$\frac{1}{3}$	$-\frac{1}{3}$
Ξ^-	$-\frac{1}{3}$	$-\frac{1}{3}$
Ξ^0	$\frac{1}{3}$	$-\frac{1}{3}$
Σ^+	0	$\frac{2}{3}$
Σ^-	$-\frac{1}{3}$	$-\frac{1}{3}$
Σ^0	$-\frac{1}{6}$	$\frac{1}{6}$
Λ^0	$\frac{1}{6}$	$-\frac{1}{6}$

wave functions ψ_1 and ψ_2. Also given are the values of σ_z^3 for the spin $S_z = \frac{1}{2}$ parts of χ_1 and χ_2. To get the baryon moments listed in the first column of Table IV, we form the quantity

$$\tfrac{3}{2}\mu[\langle\psi_1|e_3|\psi_1\rangle\langle\chi_1|\sigma_z^3|\chi_1\rangle + \langle\psi_2|e_3|\psi_2\rangle\langle\chi_2|\sigma_z^3|\chi_2\rangle].$$

When we compare this with the latest experimental values we see that all of the signs and magnitudes are given correctly, but that there are some discrepancies with the more exact values. To do better we are forced to break the simple symmetry. As we discussed in the previous sections this is a matter of definition, for we could ascribe the symmetry breaking wholly to quark masses or to differing gyromagnetic ratios or some combination of both.

One simple prescription is to take all of the quark masses as being determined by the magnetic moments of the proton, neutron, and lambda. That is, write the magnetic moment operator for quarks as

$$\mathbf{M} = \tfrac{2}{3}\mu_u\sigma_u - \tfrac{1}{3}\mu_d\sigma_d - \tfrac{1}{3}\mu_s\sigma_s\,, \tag{2.31}$$

and assume that $\mu_i = e/2m_i$ for the ith quark. We may rewrite Eq. (2.31) as follows:

$$\mathbf{M} = \mu_u(\tfrac{2}{3}\sigma_u - \tfrac{1}{3}\sigma_d - \tfrac{1}{3}\sigma_s) + \tfrac{1}{3}(\mu_u - \mu_d)\sigma_d + \tfrac{1}{3}(\mu_u - \mu_s)\sigma_s\,. \tag{2.32}$$

This shows the explicit breaking of SU(3), which demands that $M_i \;\alpha\; e$ (Close, 1979). One might worry here about the self-consistency of such an approach, since an intrinsic mass difference between quarks presumably suggests a complex structure for the quarks instead of the pointlike behavior assumed in all of the above. However, at the level of the existing data and with the relativistic problem not really having been solved, it is presumably all right to adopt this simple prescription, which has been shown to account successfully for the baryon mass splitting (De Rújula *et al.*, 1975). The quark masses which result from this procedure are $m_u = 338$ MeV, $m_d = 322$ MeV, and $m_s = 606$ MeV, and the overall comparison with experiment as shown in Table IV seems fair. Possible improvements to these results arising from configuration mixing of non-$\underline{56}$ SU(6) wave functions have been given by Geffen and Wilson (1980).

TABLE IV. The values of the baryon magnetic moments are given in the simplest version of the static quark model (second column) and with a simple mass correction as discussed in the text (column 3). The last column shows recent experimental values. The numbers in column 3 are the result of fitting μ_P, μ_N, and μ_Λ.

Baryon	μ	(With mass corrections)	Experiment
P	1	$\frac{8}{9}\mu_u + \frac{1}{9}\mu_d$	2.792 846
N	$-\frac{2}{3}$	$-\frac{2}{9}\mu_u - \frac{4}{9}\mu_d$	−1.913 042
Ξ^-	$-\frac{1}{3}$	$\frac{1}{9}\mu_d - \frac{4}{9}\mu_s = -0.50$	−0.75 ± 0.06
Ξ^0	$-\frac{2}{3}$	$-\frac{2}{9}\mu_u - \frac{4}{9}\mu_s = -1.44$	−1.237 ± 0.16
Σ^+	1	$\frac{8}{9}\mu_u + \frac{1}{9}\mu_s = 2.26$	2.33 ± 0.13
Σ^-	$-\frac{1}{3}$	$-\frac{4}{9}\mu_d + \frac{1}{9}\mu_s = -1.09$	−1.48 ± 0.37
Σ^0	$\frac{1}{3}$	$\frac{4}{9}\mu_u - \frac{2}{9}\mu_d + \frac{1}{9}\mu_s = 0.88$	-
Λ^0	$-\frac{1}{3}$	$-\frac{1}{3}\mu_s$	−0.613 8 ± 0.0047

It should be noted that m_u, m_d as calculated here differ somewhat from the results one would obtain from naively forming the baryon masses. In particular, m_d is about 5% smaller than m_u. This might be due to the crudity of the calculation or it may be an indication of a deeper result. If the quark masses are dynamical in origin in the manner we expect from QCD, then the values of such quantities may differ among hadrons and between magnetic moment calculations. It is interesting to note here that these estimates of quark masses from baryon magnetic moments are not too different from what may be deduced from the Geffen and Wilson (1980) anomalous moment results.

2. Magnetic dipole ($M1$) transitions

The magnetic dipole transition has had a long history of application in atomic and molecular physics and naturally has been applied to the decay of a vector meson into a pseudoscalar meson and a photon at an early stage of the development of the quark model (Becchi and Morpurgo, 1965). A good review of $M1$ transitions in atomic and particle physics, in particular with regard to relativistic $M1$ transitions and the J/ψ spectra, has recently been provided by Sucher (1978). In the quark model the $M1$ transitions take place in analogy with ordinary $M1$ transitions in hydrogenlike atoms for decays ${}^3S_1 \to {}^1S_0 + \gamma$. Since some subtleties arise in the application of $M1$ transitions in the quark model, we shall derive the decay formula in some detail.

In a relativistic, covariant treatment of the decay $V \to P\gamma$ where $V(P)$ denotes a single-particle vector (pseudoscalar) boson, the S matrix may be written

$$S_{fi} = -i(2\pi)^4\delta(k_P + k - k_V)T_{fi}\,, \tag{2.33}$$

where k_P, k_V, and k are the four-momenta of the pseudoscalar boson, vector boson, and photon, respectively. In turn T_{fi} may be related to the invariant Feynman amplitude m_{fi} by

$$T_{fi} = \left(\frac{1}{2\omega 2E_V 2E_P}\right)^{1/2} m_{fi}\,, \tag{2.34}$$

which leads, in the rest frame of the initial particle, to the decay rate

$$\Gamma=(2\pi)^4\int\frac{d^3k}{(2\pi)^3}\frac{d^3k_P}{(2\pi)^3}\delta(\mathbf{k}_P+\mathbf{k})\delta(m_V-E_P-\omega)\times\sum\frac{1}{2m_V}\frac{1}{2\omega}\frac{1}{2E_P}|m_{fi}|^2 . \qquad (2.35)$$

Here, $\sum$ denotes the sum over final-state polarizations and spins. In the nonrelativistic treatment the bound-state wave functions are normalized in a noncovariant way by $\langle\psi_f|\psi_i\rangle=\delta_{fi}$, and the amplitude for a transition $|i\rangle\rightarrow|f\rangle+\gamma$ is usually denoted by $eM/\sqrt{2\omega}$. As Sucher (1978) has emphasized, this amplitude is regarded as an approximation for the matrix element T_{fi}, evaluated in the center-of-mass frame. Thus

$$m_{fi}\sim(2m_V 2E_P)^{1/2}eM , \qquad (2.36)$$

and from Eq. (2.35)

$$\Gamma(V\rightarrow P\gamma)=\frac{e^2}{2\pi}\frac{\omega E_P}{m_V}\int\frac{d\Omega}{4\pi}\sum|M|^2 , \qquad (2.37)$$

where $E_P=(\omega^2+m_P^2)^{1/2}=(m_V^2+m_P^2)/2m_V$ and where $\omega=|k|=(m_V^2-m_P^2)/2m_V$. Now the magnetic dipole interaction in the long-wavelength limit ($\exp i\mathbf{k}\cdot\mathbf{r}\approx1$) is given by

$$eM=\mu_q e_q\boldsymbol{\sigma}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})+\cdots , \qquad (2.38)$$

where the remainder denotes terms which are zero if the parity of the initial and final wave functions is unchanged. From Eqs. (2.37) and (2.38) we find, averaging over the initial spin states,

$$\Gamma(V\rightarrow P\gamma)=\tfrac{2}{3}\alpha\,\omega^3\left(\frac{E_P}{m_V}\right)\sum\left|\langle V|\frac{\mu_q e_q\sigma_q}{e}|P\rangle\right|^2 . \qquad (2.39)$$

For our present purposes we shall take $\mu_q=\mu$ for all u, d, and s quarks despite the analysis given above in the case of the static properties of the baryons. Furthermore, we list in Table V the SU(3) representations appropriate to quark–antiquark combinations forming S-state mesons. The spin-one and spin-zero mesons are then distinguished by their spin wave functions. For the specific decay $\omega\rightarrow\pi\gamma$, which was an early triumph for the quark model, the result of calculating $\sum|\langle V|e_q\mu\sigma_q/e|P\rangle|^2$ was $2\mu^2/e^2$, using the wave functions of Table V (Anisovitch *et al.*, 1965; Becchi and Morpurgo, 1965; Soloviev, 1965; Thirring, 1965). Hence

$$\Gamma(\omega\rightarrow\pi\gamma)=\tfrac{4}{3}\alpha\mu^2\omega^3\left(\frac{E_\pi}{m_\omega}\right). \qquad (2.40)$$

Equation (2.40) differs from the above references because of the last factor (E_π/m_ω), which arises from the nonrelativistic approximation. In the original calculations, made when the reported width was given to be about 1.2 MeV, it was noted that such a factor would be replaced by unity if the relativistically covariant form [Eq. (2.12)] for the $\omega\pi\gamma$ vertex was used. Setting $\mu=\mu_P$ and $(E_\pi/m_\omega)=1$ gives a value of 1.17 MeV. Actually the calculation neglects any recoil, $k\sim0$, and because of its nonrelativistic nature it should only hold if $m_\pi\approx m_\omega$, or, equivalently, $E_\pi/m_\omega\approx1$. Barnes (1976) has shown that a correction for recoil [using wavefunctions from the relativistic model of Feynman *et al.* (1971)] can in some cases give an enhancement by as much as a factor of 8. In the decays of heavy vector mesons, such as the J/ψ, corrections due to recoil are expected to be of the order of $k^2/m_P^2\sim7\%$. For the decay we are considering here k is so large that typically $E_P/m_V\sim\frac{1}{2}$. To further confuse the issue we note that recent estimates of the width give $\Gamma(\omega\rightarrow\pi\gamma)\sim789\pm92$ keV (Ohshima, 1980), which is much nearer what would be obtained from Eq. (2.40) with $E_\pi/m_\omega=0.52$ as calculated from the physical masses. The effect of the nonrelativistic factor has been calculated by Moorhouse (1975) and O'Donnell (1977), who show that inclusion of the factor E_P/m_V generally reduces the predicted decay widths by about a factor of 2. Such a factor swamps all of the other attempts to improve on the naive SU(6) quark model, such as introducing overlap integrals (Ono, 1973, 1975; Isgur, 1976) or using effective magnetic moments calculated as above or as free parameters (Geffen and Wilson, 1980). Thus although the matrix elements for magnetic transitions in quark models are related to the magnetic moments of the baryons in a way not obtainable in any other symmetry scheme, it seems that for the lighter mesons an absolute rate prediction cannot be made without additional assumptions. Furthermore, although the factor E_P/m_V affects the predicted widths by a factor of approximately 0.5, there is substantial variation from process to process. For example, in the decay $K^{*0}\rightarrow K^0\gamma$ the reduction from such a factor is 0.65, which introduces an approximately 20% ambiguity between calculations which include or omit such an effect. If we wish to turn the argument around in order to learn about the dynamics of light quark bound states from $M1$ transitions, this could be an important problem since it would tend to obscure wave-function overlaps and other effects.

TABLE V. The representations of quark-antiquark combinations for u, d, and s quarks.

π^+,ρ^+	$-u\bar{d}$
π^0,ρ^0	$\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$
π^-,ρ^-	$d\bar{u}$
K^+,K^{*+}	$u\bar{s}$
K^0,K^{*0}	$d\bar{s}$
$\bar{K}^0,\bar{K}^{*0}$	$-s\bar{d}$
K^-,K^{*-}	$s\bar{u}$
η_8,ω_8	$\frac{1}{\sqrt{6}}(u\bar{u}+d\bar{d}-2s\bar{s})$
η_1,ω_1	$\frac{1}{\sqrt{3}}(u\bar{u}+d\bar{d}+s\bar{s})$
X,ω	$\sin\alpha(-s\bar{s})+\cos\alpha\left(\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\right)$
η,ϕ	$\cos\alpha(-s\bar{s})-\sin\alpha\left(\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\right)$

D. Comparison of vector dominance and the quark model

The preceding two sections have briefly summarized many years of work on understanding symmetries and their consequences. We have also seen some of the limitations of these models. Here we shall use these models in a simple form to obtain approximate relationships expected among the various decays.

First we consider the mixing angle problem. Although many possibilities could exist for the values of α_V and α_P, depending on forms of symmetry breakings and other criteria (Cordes and O'Donnell, 1969), it is difficult to find any one today who would not take ideal mixing for vector mesons, that is $\alpha_V=0$ in our definition, and $\alpha_P \approx -45°$. From Table V we see that these possibilities translate in the quark model to statements about the wave-function structures. Thus the ϕ is pure $s\bar{s}$ and the ω has no $s\bar{s}$ content. This "explains" via the OZI rule (Okubo, 1963a, 1963b; Zweig, 1965; Iizuka *et al.*, 1966) the smallness of the decay $\phi \to \pi\gamma$, which would be strictly forbidden in the $M1$ case if α_V were not exactly zero. Even with $\alpha_V=0$, vector dominance does not prohibit this decay unless there is a nonet SU(3) symmetry, i.e., in Eq, (2.15c), $g=g_1$. For the pseudoscalars the choice $\alpha_P \sim -45°$ means that the η is composed of equal parts $s\bar{s}$ and $(u\bar{u}+d\bar{d})$. This is discussed by Feynman (1972), but the physical interpretation is unclear. A contrary view, that the η and X mesons behave like pure SU(3) octet and singlet states, has recently been advanced (Kenny and Taylor, 1980; Fukugita and Pham, 1980). Furthermore the U(1) problem (Crewther, 1978) seems to be closely related to the structure of the pseudoscalars. In addition there have been suggestions (Harari, 1976) that there may be some mixing of other quarks, such as $c\bar{c}$, in small amounts. For simplicity, then, we shall use nonet symmetry [$g=g_1=g_1'$ of Eqs, (2.15)] in the vector-dominance approach and in both cases take the mixing angles $\alpha_V=0$ and $\alpha_P=-45°$. Then both models are identical insofar as the relative strengths of the $V \to P\gamma$ matrix elements are concerned, and the ratios of a large number of processes are shown in Table VI. In this table we use the relativistic phase space as is appropriate to the vector-dominance approach to define a method of treating the quark model phase space, and we follow the timeworn prescription of using the physical masses in calculating phase space as a means of introducing SU(3) breaking.

The last elements for two-photon decays are calculated using Eq. (2.19) for the decay $\pi^0 \to \gamma\gamma$ and analogous equations for $\eta \to \gamma\gamma$ and $X \to \gamma\gamma$. In the quark model we use the $M1$ transition method and vector dominance as illustrated above. Table VII lists the appropriate matrix elements in this case; we have allowed for the possibility of isospin splitting in the magnetic moments of the u, d quarks.

III. THE EXPERIMENTAL SITUATION

If this review had been written about one year earlier, then this section would have reviewed some seemingly controversial measurements. In particular, the measurements of the widths for $\rho^- \to \pi^-\gamma$ (Gobbi *et al.*, 1974, 1976) and $K^{*0} \to K^0\gamma$ (Carithers *et al.*, 1975) by the Primakoff effect (Primakoff, 1951; Good and Walker, 1960; Berman and Drell, 1964; Halprin *et al.*, 1966) gave values much smaller than the naive quark model prediction. Since then the decay $\rho^- \to \pi^-\gamma$ has been remeasured (Berg *et al.*, 1980) and a reanalysis of the Primakoff experiments has suggested that A_2 exchange effects could alter the previous conclusions (Kamal and Kane, 1979). Furthermore, Ohshima (1980) has made a critical reevaluation of many of the partial widths tabulated in the tables published by the Particle Data group

TABLE VI. The ratio of radiative decays calculated under the simplifying conditions described in the text.

Decay	Relative weight	Phase space	Relative ratio
$\rho \to \pi\gamma$	$\frac{1}{9}$	0.0527	5.8
$\omega \to \pi\gamma$	1	0.0549	54.9
$\phi \to \pi\gamma$	0	0.1258	0
$K^{*0} \to K^0\gamma$	$\frac{4}{9}$	0.0289	12.8
$K^{*+} \to K^+\gamma$	$\frac{1}{9}$	0.0296	3.3
$\rho \to \eta\gamma$	$\frac{1}{2}$	0.0073	3.65
$X \to \rho\gamma$	$\frac{1}{2}$	0.0132	6.6
$\omega \to \eta\gamma$	$\frac{1}{18}$	0.0079	0.4
$\phi \to \eta\gamma$	$\frac{4}{18}$	0.0474	10.54
$X \to \omega\gamma$	$\frac{1}{18}$	0.0120	0.7
$\phi \to X\gamma$	$\frac{4}{18}$	2.2×10^{-4}	4.8×10^{-2}
$\pi \to \gamma\gamma$	$4\pi\alpha/9\gamma_\rho^2$	4.61×10^{-4}	$\frac{3.74\times10^{-4}}{\gamma_\rho^2/4\pi}$
$\eta \to \gamma\gamma$	$\frac{4\pi\alpha}{27\gamma_\rho^2}\cos^2\theta_P(1-2\sqrt{2}\tan\theta_P)^2$	0.0310	$\frac{1.825\times10^{-2}}{\gamma_\rho^2/4\pi}$
$X \to \gamma\gamma$	$\frac{4\pi\alpha}{27\gamma_\rho^2}\sin^2\theta_P(1+2\sqrt{2}\cot\theta_P)^2$	0.1648	$\frac{0.304}{\gamma_\rho^2/4\pi}$

TABLE VII. Relative strengths of matrix elements, allowing for mixing angles and distinct quark magnetic moments.

Decay	Matrix element
$\rho\to\pi\gamma$	$(2\mu_u-\mu_d)/3$
$\omega\to\pi\gamma$	$\frac{1}{3}(2\mu_u+\mu_d)\cos\alpha_V$
$\phi\to\pi\gamma$	$-\frac{1}{3}(2\mu_u+\mu_d)\sin\alpha_V$
$K^{*0}\to K^0\gamma$	$-\frac{1}{3}(\mu_s+\mu_d)$
$K^{*+}\to K^+\gamma$	$-\frac{1}{3}(\mu_s-2\mu_u)$
$\rho\to\eta\gamma$	$-\frac{1}{3}(2\mu_u+\mu_d)\sin\alpha_P$
$X\to\rho\gamma$	$\frac{1}{3}(2\mu_u+\mu_d)\cos\alpha_P$
$\omega\to\eta\gamma$	$-\frac{1}{3}(2\mu_u-\mu_d)\cos\alpha_V\sin\alpha_P-\frac{2}{3}\mu_s\sin\alpha_V\cos\alpha_P$
$\phi\to\eta\gamma$	$-\frac{2}{3}\mu_s\cos\alpha_V\cos\alpha_P+\frac{1}{3}(2\mu_u-\mu_d)\sin\alpha_V\sin\alpha_P$
$X\to\omega\gamma$	$-\frac{2}{3}\mu_s\sin\alpha_V\sin\alpha_P+\frac{1}{3}(2\mu_u-\mu_d)\cos\alpha_V\cos\alpha_P$
$\phi\to X\gamma$	$-\frac{2}{3}\mu_s\cos\alpha_V\sin\alpha_P-\frac{1}{3}(2\mu_u-\mu_d)\sin\alpha_V\cos\alpha_P$
$\pi^0\to\gamma\gamma$	$\frac{1}{9\gamma_\rho}[4\mu_u-\mu_d]$
$\eta\to\gamma\gamma$	$-\frac{1}{9\gamma_\rho}\{(4\mu_u+\mu_d)\sin\alpha_P+\sqrt{2}\,\mu_s\cos\alpha_P\}$
$X\to\gamma\gamma$	$\frac{1}{9\gamma_\rho}\{(4\mu_u+\mu_d)\cos\alpha_P-\sqrt{2}\,\mu_s\sin\alpha_P\}$

(1980). We show in Table VIII the values quoted in the Particle Data Tables and those given by Ohshima.

If we compare these data to those given in my earlier review (O'Donnell, 1977) we can see that a number of new measurements have appeared ($\rho\to\eta\gamma$, $\omega\to\eta\gamma$, $\phi\to\eta\gamma$, $X\to\gamma\gamma$, $X\to\rho\gamma$, $X\to\omega\gamma$, and $K^{*-}\to K^-\gamma$) and that some substantial changes have taken place; in particular, the width reported for $\rho^-\to\pi^-\gamma$ has doubled. It has been pointed out by the authors of the new $\rho^-\to\pi^-\gamma$ measurement (Berg *et al.*, 1980) that there is less than 1% likelihood that their experiment and the earlier, somewhat controversial result (Gobbi *et al.*, 1974, 1976) are both correct with their quoted errors. There are good reasons for favoring the later result (Berg *et al.*, 1980) due to the higher beam energies used (156 and 260 GeV/c in place of 23 GeV/c) and a technically better apparatus.

In addition we have mentioned the critical analysis of the data (Particle Data Group, 1980) done by Ohshima (1980). We show, in Fig. 5, the results of this analysis compared with the expectations from Table VI. Since publication of Ohshima's paper a final value for $K^{*-}\to K^-\gamma$ has been given (Berg *et al.*, 1981). This is shown in the Table and has been used in Fig. 5. We have normalized the ratios to the new measurement of the $\rho\to\pi\gamma$ width. If we bear in mind that Table VI is the result of a number of simplifying assumptions, especially $\alpha_V=0$ which forbids $\phi\to\pi\gamma$, the agreement is good in most cases, notable exceptions being $\phi\to\eta\gamma$ and $K^{*0}\to K^0\gamma$.

The reader should be aware that in the data analyses done by Ohshima, and especially in his reevaluation of the width for $\omega\to\pi\gamma$, a number of experiments have been rejected. In particular, the three most recent measurements of the branching ratio $\omega\to\pi\gamma/\omega\to\pi^+\pi^-\pi^0$ were the only ones kept. It is not clear that there is sufficient justification for this choice. In addition the usual assumption is made that $\omega\to\pi\gamma$ is the same as $\omega\to$ neutrals. A reduction of the most likely value for $\Gamma(\omega\to\pi\gamma)$ would be possible if another neutral decay mode at the 1% level were observed. New measurements of the decay modes of the ω could be of great help in settling this issue.

TABLE VIII. The second column shows data as given in the tables of the Particle Data Group. The third column is a selection from these using additional criteria. In the multiple entries for $\rho\to\eta\gamma$ and $\omega\to\eta\gamma$ all upper values or all lower values must be used since the experimental results are correlated. The fourth column is the prediction of a three-parameter fit to the latter data (Ohshima, 1980). For comparison an earlier, slightly different, prediction is shown in parenthesis (O'Donnell, 1977). The latter prediction also has results for the two-photon decays.

Decay	Experiment: Particle data group	Experiment: Selected (by Ohshima)	SU(3) symmetry	
$\rho\to\pi\gamma$	38 ± 11 keV	67 ± 7 keV	67 ± 7	(65)
$\omega\to\pi\gamma$	889 ± 57 keV	789 ± 92 keV	789 ± 120	(723)
$\phi\to\pi\gamma$	5.7 ± 2.1 keV	6.5 ± 1.9 keV	11.1 ± 14.5	(4.9)
$K^{*0}\to K^0\gamma$	75 ± 35 keV	75 ± 35 keV	147 ± 16	(144)
$K^{*-}\to K^-\gamma$		62 ± 14 keV	37.5 ± 4	(36)
$\rho\to\eta\gamma$	50 ± 13 keV	52.5 ± 13.7 keV	45.6 ± 21.7	(40)
	76 ± 15 keV	79.8 ± 15.9 keV		
$X\to\rho\gamma$	83 ± 30 keV	93.1 ± 25.1 keV	93.1 ± 24.1	(77)
$\omega\to\eta\gamma$	3.0 ± 2.5 keV	3.2 ± 2.6 keV		
		1.9	9 ± 2.5	(6)
	29.0 ± 7.0	30.5 ± 7.4 keV		
$\phi\to\eta\gamma$	62 ± 9 keV	67.7 ± 9.5 keV	137 ± 18	(117)
$X\to\omega\gamma$	7.6 ± 3.1 keV	8.4 ± 2.7 keV	8.4 ± 2.4	(7.5)
$\phi\to X\gamma$			0.7 ± 0.1	(0.5)
$\pi^0\to\gamma\gamma$	7.86 ± 0.54 eV	7.86 ± 0.54 eV		(7.92 eV)
$\eta\to\gamma\gamma$	323 ± 46.4 eV	323 ± 46.4 eV		(380 eV)
$X\to\gamma\gamma$	5.32 ± 1.98 keV	5.66 ± 1.45 keV		(6.5 keV)

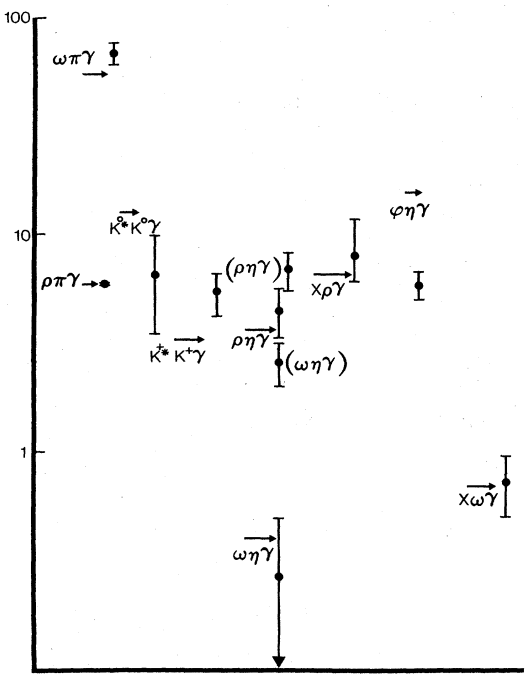

FIG. 5. Arrows show the predictions of the radiative decays when a number of simplifying assumptions are made (as described in the text). The ambiguous experimental determinations of $\rho\to\eta\gamma$ and $\omega\to\eta\gamma$ are shown with the parenthesis denoting one of the two solution fits (Andrews, 1975).

IV. WHAT REMAINS TO BE DONE?

A. Quantitative results

Figure 5 shows that vector dominance and SU(3) symmetry or equivalently the simple quark model account pretty well for the ratios of the radiative decays of the mesons. What are the expected results if we try to do somewhat better? Reference to Eqs. (2.15) shows that to obtain a nonzero width for the decay $\phi\to\pi\gamma$ in the vector-dominance model we must have $\alpha_V\neq 0$ or $g\neq g_1$ or both. In the vector-dominance approach it is simplest to keep $\alpha_V=0$ and to forsake nonet symmetry. The results of this procedure are given in Table VIII.

For comparison we show in Table IX the predictions of a quark model calculation (Geffen and Wilson, 1980)

TABLE IX. Two recent model predictions.

Decay	Effective quark model Geffen and Wilson (1980) (keV)	Nonet symmetry Ohshima (1980) (keV)
$\rho\to\pi\gamma$	67	73.8 ± 5.5
$\omega\to\pi\gamma$	861	684 ± 51
$\phi\to\pi\gamma$	5.9	6.5 ± 2
$K^{*0}\to K^0\gamma$	139	162 ± 12
$K^{*-}\to K^-\gamma$	96	41.3 ± 3.1
$\rho\to\eta\gamma$	57	47.8 ± 3.5
$X\to\rho\gamma$	108	79.9 ± 5.9
$\omega\to\eta\gamma$	4.4	4.4 ± 0.4
$\phi\to\eta\gamma$	57	135 ± 10
$X\to\omega\gamma$	8.7	10.3 ± 0.8
$\phi\to X\gamma$	0.23	0.6 ± 0.04

which includes effective magnetic moments for the u, d, and s quarks. The structure of the matrix elements is calculated from $\mathrm{tr}\,P\{V,A\}$ where P, V are the standard 3×3 matrix representations of SU(3) for pseudoscalar and vector nonets, respectively, and the photon matrix

$$A=\mathrm{diag}\{\tfrac{2}{3}\lambda_u, -\tfrac{1}{3}\lambda_d, -\tfrac{1}{3}\lambda_s\}\,.$$

This leads to the entries shown in Table VII with μ replaced by λ. The reason given for replacing the magnetic moment by an effective magnetic moment is that the photon can interact with three gluons, and such interactions may be important but not easily calculated. In practice this calculation is of the same form as that which one would obtain from Eqs. (2.15) with $\alpha_V\neq 0$ and assuming nonet symmetry to be good. However, since $\lambda_s\neq\lambda_d\neq\lambda_u$ the magnetic moment operator explicitly breaks SU(3), as discussed after Eq. (2.32), whereas Eq. (2.15) was derived by taking the photon to be a U-spin scalar and a member of an SU(3) octet. In the calculation of Geffen and Wilson (1980) the fit was made to the world averages as given by the Particle Data Group (1980). Most of the predictions are in reasonable agreement with the data, although the widths for $K^{*0}\to K^0\gamma$ and $K^{*+}\to K^+\gamma$ are too large. Since Geffen and Wilson interpret the large observed value for the ratio $\Gamma(\omega\to\pi\gamma)/\Gamma(\rho\to\pi\gamma)$ as evidence for a charge-independent quark anomalous moment, and since the branching ratio should be independent of the ambiguities of overlap integrals and phase-space considerations, a lower value of $\Gamma(\omega\to\pi\gamma)$ would reduce the magnitude of this anomalous moment. Ohshima (1980) has done essentially the same calculation as part of his paper, but has used his new analysis of the data to compare with his predictions. His prediction for $K^{*-}\to K^-\gamma$ is close to the new experimental value but he is in disagreement with the results for $K^{*0}\to K^0\gamma$ and $\phi\to\eta\gamma$.

These two calculations show the dependence of the parameters on the method of using the data for normalization and on the way in which a nonzero width for the decay $\phi\to\pi\gamma$ is obtained. There seems to be no fundamental reason in the quark model, if we allow for multigluon couplings, for the interaction to be of the form shown in Table VII and not of the form of Eqs. (2.15) with an appropriate identification of g, g_1, and g_1'. On the other hand, there is no fundamental reason for choosing in Eqs. (2.15) the value $\alpha_V\simeq 0$, whereas in the quark model such a choice is associated with the Zweig rule.

A number of attempts have been made to achieve a more exact fit to the data than we have presented here. These can be grouped into two types.

(a) Broken SU(3) symmetry models in which SU(3) breaking in couplings and in phase space is assumed. For example, the papers of Edwards and Kamal (1976, 1977), Thews (1976), and Verma (1980) fall into this class. Of course more parameters are introduced and there are many possible ways of doing this. The earlier papers attempted to account for the small value for the $\rho^-\to\pi^-\gamma$ width. Since this has now been superceded by the reported width of 67 ± 7 keV these calculations no longer obtain. The calculation of Verma gives results similar to Geffen and Wilson in that the K^* decays are not accounted for.

(b) There have been a number of extended vector dominance models proposed to deal with the radiative decay problem, for example by Bramon and Greco (1973), Fujikawa and Kuroda (1976), Gounaris (1976), Etim and Greco (1977), and Grunberg and Renard (1976). Of particular interest are the results of Etim and Greco (1977), who find a good overall fit to the data but obtain too high a value for the width for $\rho \to \pi\gamma$ and too low for $K^{*+} \to K^+\gamma$. To what extent this set of results would remain when compared to the more recent data is not known.

Finally we note that an attempt has been made (Hackman *et al.*, 1978) to consider $M1$ transitions in a bag model. This calculation was performed in the absence of the more recent data and only modest success is claimed. Since the static cavity approximation seems to be necessary for phenomenological calculations, this does not seem to be a reliable way to calculate radiative decays into pions.

B. Summary

We see from the above that at last the old ideas on SU(3) symmetry, vector dominance, and the quark model are now in a position to be tested quantitatively. Depending on the choice of accounting for the nonzero width $\phi \to \pi\gamma$ it would appear possible to relate, with few parameters, many decays involving radiative transitions among vector and pseudoscalar mesons. At the present time the agreement is good except for the two decays $K^{*0} \to K^0\gamma$ and $K^{*-} \to K^-\gamma$ (in the naive quark model approach) or, $K^{*0} \to K^0\gamma$ and $\phi \to \eta\gamma$ (in the vector-dominance approach). A repeat of the measurements of these decays should settle completely the efficacy and correctness of the ideas described in the early part of this review. Also such measurements should provide information on the proper way to handle the breaking of the Zweig rule for $\phi \to \pi\gamma$ and, from the K^* decays, information on the photon coupling to multigluon states. Since there has been only one measurement of the decay $K^{*0} \to K^0\gamma$, it would seem to be an important experiment remaining to be checked with greater accuracy.

It should be emphasized that a new measurement of the decay modes of the ω takes on an added importance now. Not only do most fits to the radiative decay processes depend crucially on the value of the width for $\omega \to \pi\gamma$ but the deduction of the dynamics of light quark bound states may depend in an essential way on the ratio $\Gamma(\omega \to \pi\gamma)/\Gamma(\rho \to \pi\gamma)$.

ACKNOWLEDGMENTS

I am grateful to David Toms and Christos Papavasiliou for reading the manuscript and for useful discussions. Research was supported by the Natural Science and Engineering Council of Canada.

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