Dynamo theory of the earth's varying magnetic field

David Rittenhouse Inglis

University of Massachusetts, Amherst, Massachusetts 01003

Various forms of the dynamo theory are presented in a graphic manner. Each of them depends on a flow pattern of presumably thermal convection in the earth's fluid core. The conducting fluid moves in magnetic fields generated by currents induced by the motion. Each of the flow patterns includes vortices, with helicity induced by the Coriolis force, that twist the magnetic fields in such a way as to regenerate an initial field. Some of them involve also differential rotation, with the parts of the core near the axis rotating more rapidly than the outer parts. Some forms of the theory are more successful than others in accounting qualitatively for the various observed aspects of the field, particularly the westward drifts and the occasional polarity reversals. The thermal energy source may be radioactivity of heat or crystallization at the inner-core surface.

CONTENTS

I. INTRODUCTION

A simple self-sustaining dynamo may consist of a loop of wire in which a current flows, inducing a magnetic field, and a conductor moving across the magnetic field to generate an electromotive force that drives the current in the loop. Under appropriate circumstances an initial stray field can be augmented by this feedback and would grow indefinitely if conditions were to remain the same. Such indefinite growth is of course not possible because power is needed to maintain the motion. In a conventional commercial direct-current dynamo the field is enhanced by the presence of iron, the residual magnetism of which also provides the initial stray field. Without the iron, such a dynamo would not function unless spun at impracticable speeds or scaled up to enormous size. If it were scaled up to the size of the earth, only very slow motion would be needed, of the order of the radius per century.

Any dynamo theory of the earth's magnetism postulates that the fluid core is churning with motions in magnetic fields that generate currents to support the magnetic fields. This can be described either in terms of the currents generated or, hydromagnetically, in terms of the tendency of a moving imperfect conductor to drag magnetic lines of force along with it.

It is enigmatic that we know so little about the motions within the core of our own planet when we know so much about distant parts of the universe, there being such a difference in observing power between the seismic waves and magnetic fields with which we observe the one and the electromagnetic radiation with which we observe the other. One may postulate various types of motion within the core, and there are accordingly several forms of the dynamo theory that have been rather extensively studied without, however, arriving at a basis for an unequivocal choice between the possibilities. Some experts favor one and some another. The purpose of this review is to provide a graphic description of the general nature of some of the most interesting possibilities, complete enough to satisfy the curiosity of the general reader about progress in this fascinating field without going into the detail that a specialist might prefer to do justice to the theoretical accomplishments.

The slow time variations of the field have some interesting aspects that require correspondingly complex fluid motions to explain them. We shall consider these aspects separately, first examining various possible types of flow that could give rise to the gross phenomenon, the existence of a fairly steady field of predominantely dipole character, and then go on to consider the observed westward drift and reversals and some complexities of the motion that could produce them.

The first proposal of a dynamo origin of the field was made by Elsasser (1946, 1947) who provided the first step of the regenerative process. Tidal deceleration of the mantle was taken as the energy source making the core fluid circulate eastward. This relative motion drags initially poloidal or north-south lines of force into the shape of a bent hairpin wound around the axis, generating a toroidal or east-west field that may be stronger than the initial poloidal field, a step of amplification. Figure 1 illustrates this bending by the relative rotation of two concentric spherical conductors (Elsasser, 1950).

The other step, a feedback mechanism, was provided by Bullard (1949). He postulated convective flow including radial currents presumably driven by a central radioactive heat source, for example two outward streams near the equator and two return streams at the poles. Coriolis force or conservation of angular momentum makes the inner part of the core rotate eastward relative to the outer part, again as in Fig. 1. He showed that the toroidal-poloidal feedback has the sign required for a self-sustaining dynamo. This was later discussed in further analytic detail by Bullard and collaborators (1950 and 1954) and in graphic terms by Inglis (1955 and 1964).

The next refinement was introduced by Parker (1955),

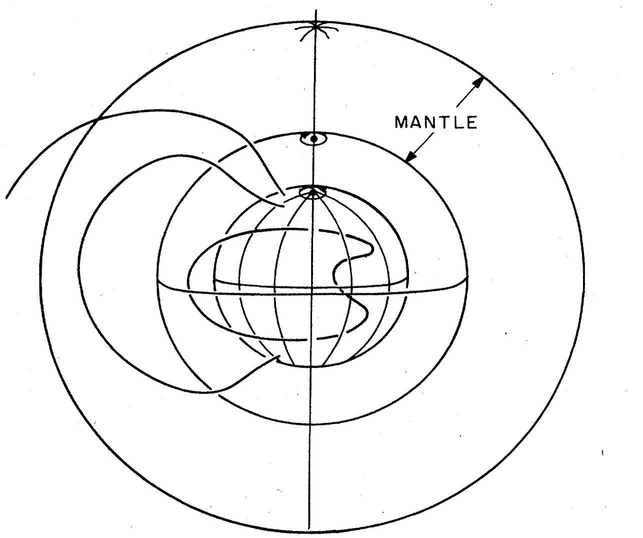

FIG. 1. Generation of a toroidal field from a poloidal field by the more rapid rotation of the inner part of a two-sphere model of the core.

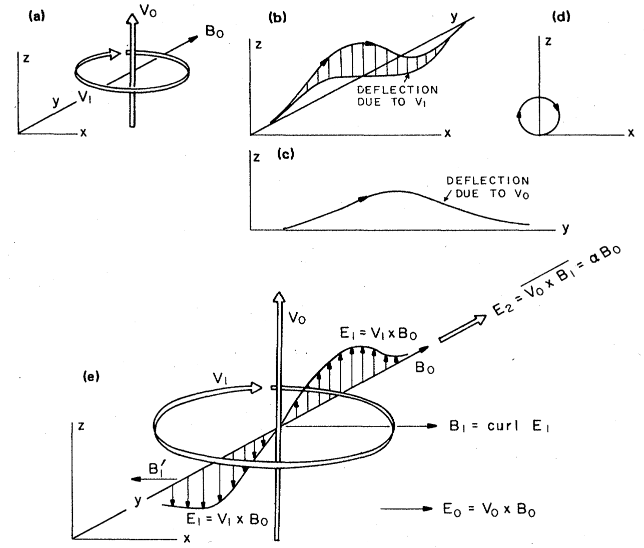

FIG. 2. Origin of the Alpha effect. A conducting stream with left-handed helicity twists a transverse line of force to give it right-handed helicity.

who considered a random distribution of a few large upward vortices, each perhaps associated with one of the magnetic anomaly regions observed on the surface, as well as the possibility of many smaller upward vortices. He showed that their cyclonic helicity, arising from the Coriolis force, twists the toroidal field in such a way as to regenerate an initial poloidal field. There have since been analyzed a number of interesting variations of this theme, each of them postulaing some either random or systematic distribution of vortices having preponderately left-handed helicity in the Northern Hemisphere and right-handed in the Southern as a result of Coriolis force.

Basic to all of this is the way a twisting stream of fluid crossing a magnetic field distorts that field to produce a circulation of the field about its original direction, the sense of the circulation of the field depending on the helicity of the stream, as shown in Fig. 2. The stream, having a mean velocity V_0 and a superimposed circulating motion V_1, crosses the magnetic field with a mean intensity B_0 in Fig. 2(a). In Figs. 2(b)–2(d) we consider the effect hydromagnetically, with the conducting flows dragging the lines of force with them. In the xy plane of Fig. 2(b), we see the distortion caused by the circulating component V_1. Superimposed on this is the deflection due to V_0, as shown also in Fig. 2(c). The net effect as viewed along the y axis is the circulation in the xz plane, shown in Fig. 2(d), superimposed on B_0. Here a left-handed helicity of the flow has produced a right-handed helicity in the magnetic field, as would be appropriate for a rising cyclone in the Southern Hemisphere. Reversing the former of course reverses the latter: In the Northern Hemisphere a rising cyclone is right-handed and produces left helicity in the field, which is regenerative. The basic equation

$$\mathrm{curl} B = 4\pi J \tag{1}$$

implies, from Fig. 2(d), that there is a current J in the y direction.

As is usual with hydromagnetic explanations, the same situation can be explained less simply in terms of the induced electric fields and currents which give rise to the modified magnetic fields. This is shown in Fig. 2(e). Here the electric field E_1 arises from V_1 and its curl is the superposed magnetic field B_1. E_1 has a shape similar to that of the deflection due to V_1 in Fig. 2(b), the x component of B being proportional to the y derivative of either curve in this simple geometry in which $\mathrm{curl} E_1$ consists of a y derivative. B_1 interacting with the main stream V_0 generates the component of electric field E_2 in the direction of the original magnetic flux B_0,

$$E_2 = \alpha B_0 , \tag{2}$$

with α positive arising from the left helicity of the stream V. For right-handed helicity α is instead negative. This equation is known as the "alpha effect" (Krause, 1977). Near the edges of the stream, B_1, shown there as B_1', is reversed but there V_0 is small so the contribution to E_2 is minor.

We shall later on describe various ways by which large parts of the conducting fluid in the earth's core can be filled with twisting streams of appropriate helicity to produce the alpha effect, with alpha predominantly positive in the Northern Hemisphere and negative in the Southern. Before going into these details of the source of the alpha effect, let us assume that the fluid core is endowed with the alpha effect and examine how this can lead to a self-sustaining global magnetic field of mainly dipole character. As a first example, the set of rings shown in Fig. 3 is to be imagined as imbedded in the fluid of the Northern Hemisphere of the core. Consider first the two rings that carry labels, B for magnetic field, E for electromotive force, and i for current, two rings that are looped through each other. (The ring carrying B_0 is of course representa-

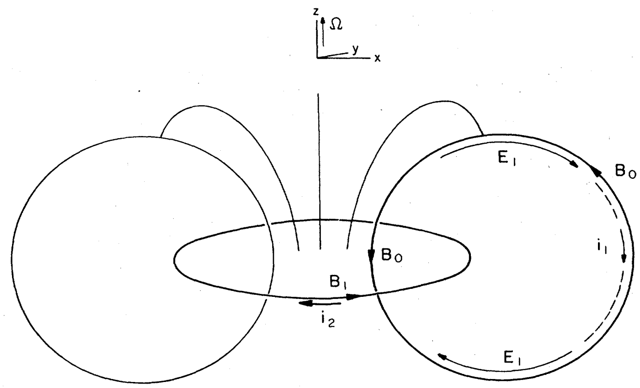

FIG. 3. Regeneration of a magnetic field by the alpha effect in linked rings.

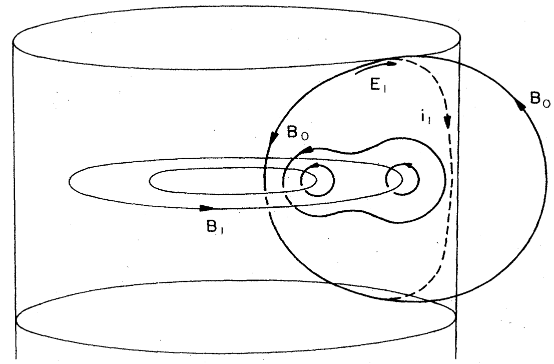

FIG. 4. Currents induced by the alpha effect in a finite fluid conductor regenerating a magnetic field that extends beyond the conductor, as in the earth's core.

tive of all the rings shown forming a torus that have a cumulative effect.) Start with B_0 as part of a stray field. The α effect with negative α produces the electric field E_1 which drives the current i_1. This current induces the magnetic field B_1 which, again with negative α, through E_1 drives i_2 which is of the correct sign to induce a field approximately following the path of the original B_0. If α is large enough and the resistance in the loops small enough, the regenerated B_0 can be stronger than the original B_0 and the field in both loops will grow. If α and the resistances were constant, it would grow indefinitely.

Such indefinite growth will of course not occur because energy is required to maintain the fluid currents responsible for the α effect. Magnetic field penetrating fluid flow impedes that flow, and as the flow slackens in the growing field, the numerical value of α decreases until either equilibrium is reached or, because of mechanical inertia and electric impedance, the strength overshoots equilibrium and oscillations set in around an average level, as is discussed below.

This regenerative mechanism is clearly quadratic in α, and this is known as an α^2 dynamo. It would regenerate as well with positive α, as presumably occurs in the Southern Hemisphere.

These ring currents are oversimplified. The α effect need not occur uniformly around the ring indicated and the currents need not follow the paths of the magnetic fields. This is indicated in Fig. 4, where B_0 penetrates outside the conducting medium (as the earth's field does through the mantle and beyond the surface of the earth), while i_1 does not. Also, α may be anisotropic and function, for example, only in the directions normal to the axis, such as where the symbol E_1 appears in Fig. 4. Even so, E_1 can drive a current i_1 approximately around the circuit indicated. In this figure, two horizontal rings are shown to suggest the cumulative effect of currents in a continuous medium.

A toroidal field similar to B_1 can instead be produced by differential rotation. In simple patterns of differential rotation, the inner parts of the fluid core, or the parts nearest the axis of rotation, have a higher angular velocity than the outer parts because of a tendency for conservation of angular momentum. The inner regions then rotate eastward relative to the outer parts. If a magnetic field loops deep into the core from north to south, it may be dragged eastward where it approaches nearest the center into the shape of a bent hairpin wrapped at least part way around the axis, as suggested in the nested-sphere model of Fig. 1. There the fluid core is represented by two solid parts, an inner sphere rotating more rapidly within a spherical shell. A more realistic model might have a continuous distribution of shear across cylindrical surfaces, but would lead to a similar effect, generating a toroidal field from a poloidal field. The toroidal field is eastward in the Northern Hemisphere, westward in the Southern. Through the α effect, with α again negative in the Northern Hemisphere and positive in the Southern, a westward toroidal current is generated in each, and this is in the correct sense to generate the original poloidal field, southward near the center and northward outside.

In a model introduced by Parker (1955) and further analyzed by Levy (1972) the α effect is attributed to a rather random distribution of upwelling streams of appropriate helicity, represented in Fig. 5 by short arrows with circles around them. The way these streams distort the hairpinlike toroidal field drawn out

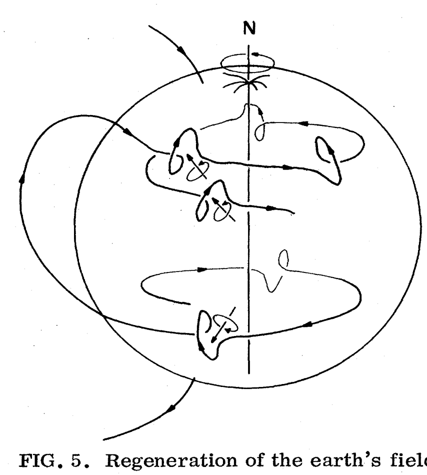

FIG. 5. Regeneration of the earth's field by differential rotation combined with the alpha effect induced by randomly distributed upwelling fluid streams, after Parker (1955).

from the poloidal field by differential rotation is indicated in the figure. It is seen that each of the local twists, similar to the one in Fig. 2, directs the field southward on the inside and northward on the outside to match the original poloidal field that it supports.

Here the toroidal field is generated from the poloidal field by differential rotation, conventionally denoted by ω, while the poloidal is generated from the toroidal by the α effect, and this is known as an $\alpha\omega$ dynamo.

II. FORCES GOVERNING FLUID MOTION

Let us now consider the kinds of motion that might exist in the earth's fluid core to give rise to the α effect there. The laws of motion in a rotating body are of course quite different from those in an inertial system, as is apparent in the familiar weather wisdom that winds blow not directly from a region of high pressure to one of low pressure but almost at right angles to the line between them in geostrophic flow, the pressure difference being supported by Coriolos force. The master equation governing the pattern of motion in the fluid core says that the density times the acceleration, $\rho\partial u/\partial t$, in the rotating coordinate system is equal to the sum of a number of forces, per unit volume. First there is the pressure-gradient force, $\nabla(P+\frac{1}{2}\rho u^2)$. Here P may be considered to include the gravitational force on the mean density that leads to hydrostatic equilibrium of a homogeneous fluid so that only small deviations $\rho\theta$ from the mean density ρ need to be further considered. Second, there is the important Coriolis term $2\rho\Omega\times\mathbf{u}$, containing the angular velocity Ω of the earth and local relative velocity $\mathbf{u}$, a force directed at right angles to each of them. (The angular velocity of local twisting, $\nabla\times\mathbf{u}$, is assumed to be so small compared to Ω that it is neglected.) Third, there is the gravitational buoyancy term, $\rho g\theta$. It is reasonable to assume that this arises from thermal expansion, the heat being supplied by radioactivity either within the solid inner core or distributed throughout the fluid. The resultant motion is a form of thermal convection. And fourth, there is a viscous term $\nu\nabla^2\mathbf{u}$ associated with the local rate of shear. The entire equation is thus

$$\rho\frac{\partial\mathbf{u}}{\partial t}=-\nabla P-2\rho\mathbf{\Omega}\times\mathbf{u}+\rho g\theta+\nu\nabla^2\mathbf{u}\,. \tag{3}$$

Here density variations are taken into account only in the buoyancy term, the fluid being almost homogeneous, and one may put $\nabla\cdot\mathbf{u}=0$.

If there is a magnetic field B present in a conducting fluid, there is an additional magnetic term representing the force on the current $\mathbf{j}=(1/4\pi)\nabla\times\mathbf{B}$,

$$\frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B}=-\frac{1}{8\pi}\nabla B^2+\frac{1}{4\pi}(\mathbf{B}\cdot\nabla)\mathbf{B}\,. \tag{4}$$

After decomposition into these two terms, B^2 enters the equation of motion in the same way as P and acts as a magnetic pressure, while the last term is interpreted as a tension along B, like that of a rubber band, since

$$(\mathbf{B}\cdot\nabla)\mathbf{B}=B\,d\mathbf{B}/ds=CB^2\hat{\mathbf{n}},$$

where s measures distance along the line of force B having curvature C and $\hat{\mathbf{n}}$ is the normal unit vector. Thus the tension on the curved line pulls sideways.

III. THE TAYLOR CONSTRAINT

The mechanics of the earth's fluid core, and particularly the magnetohydrodynamics, is a very complicated matter and can be approached as yet only through approximations and models. Of these there have been many interesting analyses (Gubbins, 1974; Levy, 1976; Moffatt, 1978). We can here describe only a few that may be among the most relevant.

One important guiding principle in such rotating systems is the Proudman-Taylor theorem stating that, in a nonviscous, homogeneous, rapidly rotating fluid with no opposing boundary constraints, steady motion is essentially two-dimensional, the same in all planes normal to the axis of rotation (Taylor, 1921). Under these circumstances the only remaining terms in the equation governing the motion are the first and third force terms, the pressure-gradient term, and the Coriolis term:

$$\nabla P+2\rho\mathbf{\Omega}\times\mathbf{u}=0\,. \tag{5}$$

Since the curl of a divergence is zero, it is useful to take the curl of this equation and make use of the vector identity $\nabla\times(\mathbf{A}\times\mathbf{B})=(\mathbf{B}\cdot\nabla)\mathbf{A}-(\mathbf{A}\cdot\nabla)\mathbf{B}$, obtaining

$$(\mathbf{u}\cdot\triangle)\mathbf{\Omega}-(\mathbf{\Omega}\cdot\nabla)\mathbf{u}=-\Omega\frac{\partial\mathbf{u}}{\partial z}=0\,. \tag{6}$$

This means that the velocity is independent of z and a function of only x and y in the plane normal to $\mathbf{\Omega}$.

This may be clearer if put in macroscopic terms in a simple case. Consider a laboratory demonstration of a fluid in a cylindrical container with flat bottom rotating about a vertical axis, such as that in which Taylor made a beautiful demonstration of his theorem by moving a short cylinder slowly across the bottom and observing that a column of fluid above it moved with it, apparently almost as if by magic. Imagine the forces around a rectangular path with two vertical sides and two horizontal sides. The total force integrated around the rectangle must be zero if the motion is steady. The gravitational pressure-gradient force is the same on the two vertical sides because the fluid is homogeneous. The Coriolis force on the two horizontal sides must thus also be equal. This means that the velocity of the fluid crossing these two sides must be equal, or that the velocity must be the same at the two levels (the choice of rectangle being arbitrary).

Whirling motions, or vorticity, can exist in such a Coriolis-dominated fluid only about axes parallel to the axis of rotation, and then only in the form of long columns of vorticity. There is an associated theorem of Taylor stating that the total amount of vorticity in the fluid remains constant. In ordinary turbulent vorticity, large-scale turbulence begets smaller-scale turbulence (and so on *ad infinitum*) but this is inhibited by the action of this theorem in rotating fluids (Hide, 1976).

The complete magnetohydrodynamic problem of the earth's core is too complicated to have been solved at all completely. A useful approach is to consider, even if one cannot uniquely solve, the dynamics in the absence of a magnetic field, then to consider whether the resulting kinematics will regenerate and amplify an original stray magnetic field as long as the field remains weak enough not to modify the kinematics, and then only after this to consider the extent to which the growing magnetic field may modify the kinematics. The parameters involved are sufficiently uncertain that one cannot decide uniquely on the nature of the kinematics, so we shall consider alternative possibilities and the consequences thereof, confining ourselves to graphic descriptions of motions that have been subjected to varying degrees of detailed analysis.

We first seek to understand what nearly steady fluid motions are apt to take place within the core, without the drag effect of a magnetic field, when the driving force is a slight thermal expansion due to an internal heat source. The core is so large, and thus the general circulation patterns within it so large, that in the bulk of the volume the rate of shear is slow and the viscous term may, as one possibility, be neglected in comparison with the pressure and Coriolis forces. Then the Taylor constraint is relevant. The part of the volume very near the boundaries is another matter. There the velocity variations may be on a scale much smaller than the whole volume and viscosity can change the nature of the motion.

IV. CONVECTION ROLLERS

We may assume that the heat driving the convection is being transported mainly by convection in the fluid from the solid inner core to the inner surface of the mantle. The inner radius of the fluid core is about a quarter of its outer radius. The dynamics of convection are quite different in the parts of the fluid near the axis and the poles from those near the equator. We may consider the fluid volume to be divided into two parts, the part near the poles (like the core of an apple) inside a cylinder just wide enough to include the inner core, and the part outside that cylinder. In the inner part the convection transporting the heat can travel almost parallel to the axis without being strongly affected by the Coriolis force, and so can resemble a simple Bénard convection cell, like that in a room with a warm floor. In the outer part the buoyant force tends to drive the convection in the normal direction and the motion is strongly affected by the Coriolis force.

Let us consider a cylindrical model of the convective motion in the outer part, thus ignoring at first the complications posed by the core's spherical outer boundary. We thus assume a thick slab of fluid with a warm inner cylindrical boundary and a cool outer cylindrical boundary, with plane boundaries normal to the axis and with gravity directed radially inward towards the axis of rotation. Here the buoyant force is radial, and without rotation a radial convection pattern would be established, a Bénard cell of toroidal shape. With slow rotation a weak Coriolis force would deflect the outward streams one way and the inward flow the other way, making a toroidally spiral flow pattern. This does not comply with Taylor's theorem but is possible because viscous and/or buoyancy forces are comparable in strength with the weak Coriolis force. With more rapid rotation the stronger Coriolis force makes this simple flow unstable and, it turns out, can curl the flow up into long rollers parallel to the axis, with adjacent rollers rolling on one another, as we shall describe in more detail.

We next narrow our attention to a smaller model, a sample of the normal-plane area of this model large enough to contain several rollers but small enough, compared with the radius, that the cylindrical coordinates, r, $r\theta$, and z can be replaced in sufficient approximation with Cartesian coordinates x, y, z, respectively. This model presents the possibility of motions that are dependent only weakly on z, and are periodic in x and y, so that they may be analyzed in terms of simple functions with this periodicity (Roberts, 1970, 1972). The consequent motions have been discussed by Stix (1977) following early ideas of Ekman and more recent ideas of Busse (1971, 1977). It is shown that, with rather strong Coriolis force, the preferred motion consists of an array of what are known as "Ekman spirals" in the bulk of the volume and a phenomenon known as "Ekman pumping" fostered by viscosity in the "Ekman layer" near the boundaries. The flow pattern is indicated in an axial cross section in Fig. 6, and in a normal cross section in Fig. 7. It consists of a number of rollers parallel to the axis, with adjacent rollers rotating in opposite directions so that they roll on one another. They also have a relatively slow axial flow in opposite directions in adjacent rollers. The cylindrical roller flow is the most prominent motion and is consistent with the Taylor constraint, but the axial flow is not. The transverse scale of length in such a pattern is much smaller than the size of the vessel, thus making viscosity more important than it would be in large-scale currents, so that the Taylor constraint is not too restrictive and a stable pattern is possible that transports heat in spite of the strong Coriolis deflections. Pressure differences are important in the establishment of this pattern, since the curvature of the rolling motion is in the direction of the Coriolis deflection in one roller and contrary to it in the next. The pressure is high inside the rollers rotating in the opposite way to the rotation of the system, where the Coriolis force makes the roller squeeze its core, so to speak, and low in the others. These are similar to the highs and lows in the atmosphere and for the same reason. Where the two rollers come into contact between a high and a low, their combined flow is like a geostrophic wind, and the Coriolis force on it maintains the pressure difference.

Near the boundaries in the Ekman layer, viscosity

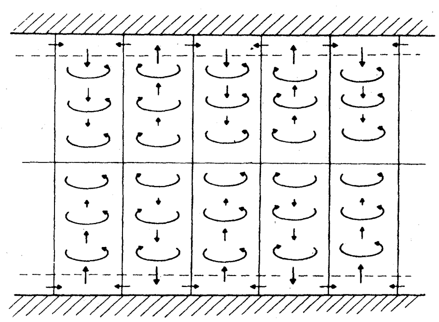

FIG. 6. "Ekman pumping" axial flow within convection rolls.

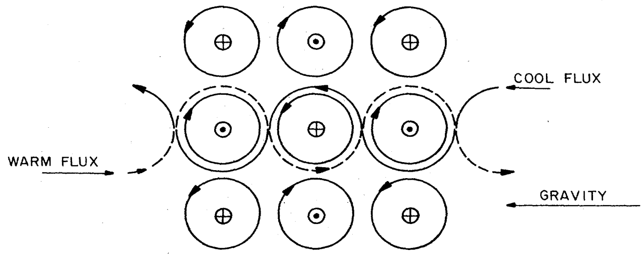

FIG. 7. "Grand-right-and-left" thermal convection flow that drives the convection rolls.

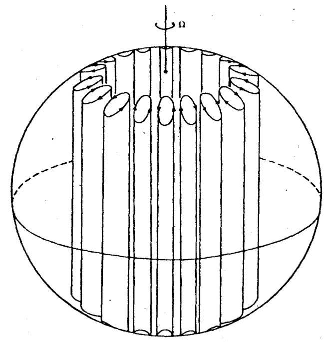

FIG. 9. Single layer of convection rolls within a sphere, after Busse.

slows down the rotation, breaking the Taylor constraint, reducing the Coriolis force, and permitting a flow from highs to lows. In a steady state, this is compensated by a contrary flow from the low to the high in the interior where the rollers are in contact, propelled by the Coriolis force in that direction as influenced by a bit of viscosity. This flow may be seen as a merging of the two rollers into a common stream between them and then a different partition as they separate, as dictated by local pressures, to maintain the pressure and the Coriolis forces nearly in balance in this almost homogeneous and slightly viscous fluid. Since this transfer takes place all along the length of the rollers, the axial current is stronger near the boundaries than near the median plane, as indicated by the long and short arrows in Fig. 6.

This Ekman pumping action mixes the axial flow in adjacent columns, as indicated in Fig. 8. This means that each column is exchanging warmer fluid from one side and cooler from the other, and this process, too, is part of the thermal convection.

If taking place in the earth's fluid core, this process is complicated by the sloping boundaries, part of the spherical surface, as indicated in Fig. 9. With this in mind, Busse has analyzed the process for the case of gently sloping boundaries and shown that the roller pattern occurs in this case too. The sloping boundary impedes the horizontal type of flow mandated by the Proudman-Taylor theorem, and thus serves, along with viscosity, to violate the Taylor constraint in the Ekman layer near the surface.

Busse and Carrigan (1976) nicely demonstrated this effect in a water-filled transparent rotating sphere simulating the earth's core, with the rather spectacular result shown in Fig. 10. The experiment is further described in a review article by Carrigan and Gubbins (1979), where the result is displayed still more clearly in color. As the rotational speed is increased the convection rolls occur first in a single layer in the form of a cylinder at a radius about half of the sphere radius, an optimum radius where the centrifugal force is great enough and the slope of the boundary not too great. With more rapid rotation convection rolls in several layers fill most of the volume.

Busse's theoretical treatment showed that this phenomenon could, with appropriate parameters, occur in the spherically symmetric gravitational field of the

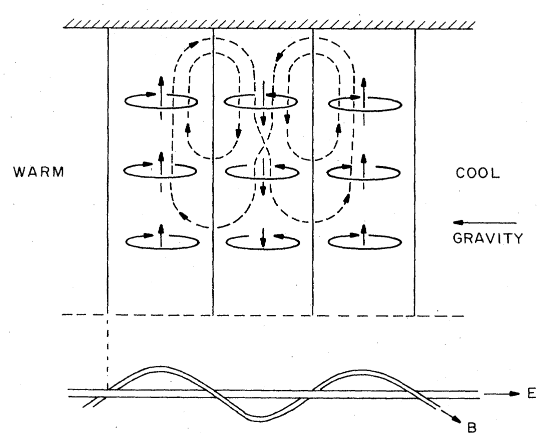

FIG. 8. Flow in the convection-roll pattern which, by dragging the magnetic field lines, twists them to produce the alpha effect as in Fig. 2.

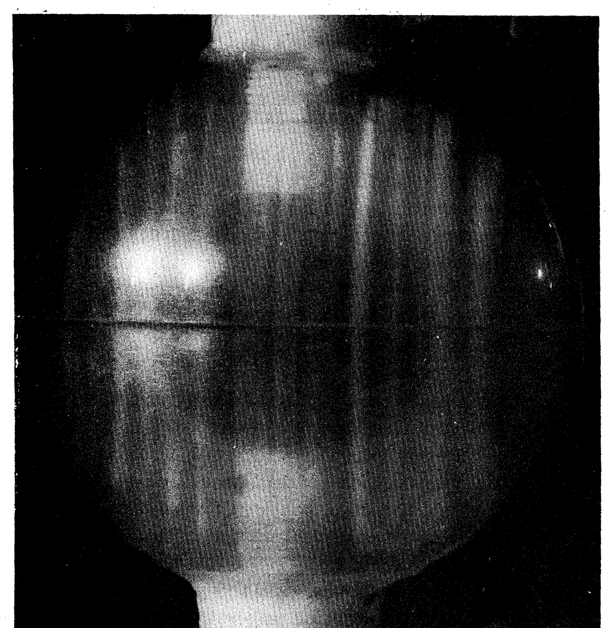

FIG. 10. Photograph of convection rolls in the demonstration of Busse and Carrigan. The fluid is water loaded with aluminum powder, and centrifugal force replaces the gravity in the earth's core.

earth or another planet. One persistent limitation of experimental geophysics is that it is not possible to simulate in the laboratory this spherically symmetric gravitational field. However, in this phenomenon the component of the gravitational field normal to the axis is the more important one, and this can be simulated by centrifugal force which has the same radial dependence, proportional to r. Because it has opposite sign, outward rather than inward, the temperature gradient is reversed in the Busse-Carrigan demonstration, the inner boundary being cool and the outer one warm. The warm fluid "rises" downward against centrifugal force!

In Figs. 6 and 8 the motion in each roller is indicated simply as a circular motion surrounding an axial motion, but these are actually compounded into a spiral motion varying with radius. The axial motion is caused by the pumping action at the boundaries and has mirror symmetry about the mid-plane between the boundaries in Fig. 6, or about the equatorial plane in the earth. Two adjacent rolls in one hemisphere have opposite rotation and opposite axial flow, thus the same helicity. The helicity is left-handed in the Northern Hemisphere and right-handed in the Southern.

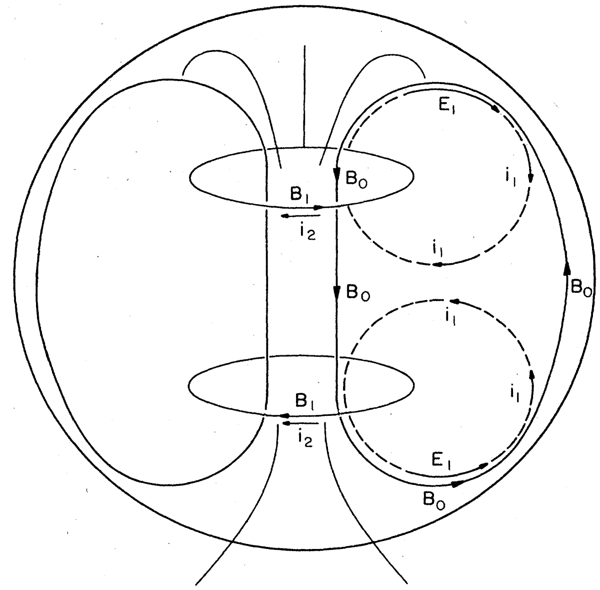

FIG. 11. Regenerative process of Fig. 4 extended to the two hemispheres.

V. THE ALPHA EFFECT OF THE ROLLER PATTERN

This, as we have seen, is just the combination of helicities that give rise to regeneration of the terrestrial dynamo, with a positive α in the Northern Hemisphere and negative in the Southern. The mechanism by which the roller pattern having left-handed helicity gives rise to right-handed helicity of the magnetic field and positive α is made clear by the lower diagram in Fig. 8. There one sees that the field line B is dragged hydromagnetically into the paper at the left side of the pattern by the rotation of the roller, then is deflected upward by the axial current in the first roller, then out of the paper at the junction of the first two rollers, then downward at the middle of the second roller, etc., to twist the line B with right-handed helicity giving E $=$ curlB to the right and positive α. This is of course the same reasoning as for Fig. 2, but it is somewhat more striking in this periodic pattern.

The α effect thus generated provides the basis for the large-scale field regeneration shown schematically in Figs. 3 and 4 for the Northern Hemisphere. The way this scheme fits into the two hemispheres is shown in Fig. 11. The toroidal field B_1 has opposite signs in the two hemispheres but, because of their different signs of α, the toroidal current i_2 circulates in the same sense in both, so that they reinforce one another in inducing the global poloidal field B_0, the earth's dipole field. Similarly, B_0 induces E_1 driving i_1 around circuits in opposite senses in the two hemispheres, inducing B_1 circulating in opposite senses. This provides a simplified basis for understanding a possible regeneration process in the earth's core.

While the convection-roll mechanism, with its impressive experimental demonstration, is an elegant and appealing possibility, it may be that conditions in the earth's core are not favorable for it. Another possibility is that convection is carried largely by randomly distributed turbulence. This may take various forms, either large-scale or small-scale turbulence. In large-scale turbulence, such as originally envisaged by Parker and suggested in Fig. 5, the Taylor constraint may be broken more by time variation than by viscosity, large upward bursts occurring rather suddenly and spontaneously at all latitudes. Each such burst might correspond to a region of magnetic anomaly observed at the earth's surface. Here a preponderance of the appropriate helicity is obtained by assuming that only upward bursts occur. This favoring of upward streams arises from the fact that the surface of the solid core, where the heat source presumably resides, has only about eight percent of the area of the inner surface of the mantle, to which the heat is being conducted (the two radii being almost 1300 and 4500 km, respectively). There is thus a greater tendency for the warm fluid to concentrate into upward streams, while cooler fluid at the large outer surface can return downward in a general subsidence over a large area. The upward stream is cyclonic, its helicity being determined by the Coriolis force on the converging fluid as it is drawn into the base of the rising column, as in a "low" in the atmosphere. Viscosity being relatively unimportant in large-scale motions, this helicity persists in the rising column, producing the magnetic regeneration indicated in Fig. 5. Being rather short-lived, the upward bursts may penetrate the shearing layers of differential rotation only temporarily and are thus not necessarily incompatible with the shear that tends to cut them off.

VI. THE DYNAMO WITH RANDOM SMALL-SCALE TURBULENCE

It is, however, not clear that they are compatible with the differential rotation. It is perhaps more plausible that there could be smaller-scale randomly distributed turbulence within the general differential rotation, the natural time scale being shorter and the shearing difficulty less severe. This turbulence would include many short upward and downward currents, each dissipating or turning horizontal after traveling only a fraction of the radial thickness of the fluid core involved.

Turbulence may ordinarily be pictured as a rather random collection of eddy-current loops. With Coriolis force present, where two adjacent eddy streams are parallel, they tend to twist around one another and lose their identification with a particular eddy. This turbulence may be pictured as a continual confluence and subsequent dissipation of short segments of twisted currents. On this small scale we may assume that there is so much damping of the motion, probably more from magnetic drag than from viscosity, that the angular momentum of the twist in the stream is not conserved along the length of the stream, but rather that the helicity is determined by the local convergence at the beginning of the stream and the divergence near its end. Thus in the Northern Hemisphere an upward stream has right-handed helicity near its beginning and left-handed near its end. These are reversed in a downward stream. Incidentally, this concept of the helicity being determined by the local Coriolis influence is similar to the normal situation in sunspots, where the helicity of a stream of compressible fluid is determined by its radial expansion as it rises and its pressure decreases. In this behavior, turbulence in a gas is very different from that in a liquid.

Turbulence can occur on various scales, smaller-scale turbulence within large-scale turbulence. Turbulence on a given scale S may be expected to be of about uniform intensity in parts of the fluid much further than S from a boundary but of decreasing intensity nearer to a boundary where the formation of eddies is impeded by the proximity of the boundary.

Let us consider the matter in a spherical shell between r and $r+dr$ in a region of varying turbulent intensity, influenced by proximity to a spherical boundary. Suppose, for example, that this is in a region near the upper boundary where turbulence is decreasing with increasing r. Consider first the helicity involved in the exchange between the shell and the region below it through upward streams ending in the shell and downward streams forming in it. In the Northern Hemisphere both have left-handed helicity. There is a similar exchange with the region above the shell, involving right-hand helicity. With a uniform density of turbulence the two would cancel, but because the turbulence density is greater beneath, there is more exchange with the lower region and a preponderance of left-handed helicity, as indicated in Fig. 12. For the Northern Hemisphere of the fluid core, the inner surface of the mantle forms the upper or northern boundary and the inner solid core forms a lower boundary of the part of it near the axis of rotation.

In the alpha effect, the average α is then negative near the upper boundary and similarly positive near the southern boundary in the Northern Hemisphere, and the other way around in the Southern Hemisphere (Steenbeck and Krause, 1969). This distribution of α can lead to regeneration of the field in an α^2 dynamo that does not depend on differential rotation. The essential dynamo mechanism is shown in Fig. 13. At the northernmost boundary, for example, the toroidal field B_1 circulates westward, as does i_2 with positive α, and this i_2 spread over the polar region generates B_0 southward most strongly near the axis where a southward E_1 drives i_1 around the circuit that regenerates B_1. Though based on a different type of turbulence, this regeneration scheme is similar to that operating in Fig. 4. At the smaller southern boundary on the north side of the solid core where α is negative, B_1 again circulates westward but i_2 eastward so that it annuls the B_0 that would otherwise pass inside this loop of i_2 and deflects B_0 outside the loop, where it generates a northward E_1. This being now on the outside drives i_1 around in the same sense as that near the northern boundary, so as likewise to regenerate the westward B_1. In the Southern Hemisphere the effect is similar with B_1 eastward. This is the essence of the α^2 dynamo analyzed by Steenbeck and Krause (1969).

The regeneration scheme in the north polar region at the top of Fig. 13, based on the westward circumpolar current i_2, is a self-contained local dynamo, not dependent on the fact that B_0 extends far out into the mantle and beyond. The regeneration is a near-surface

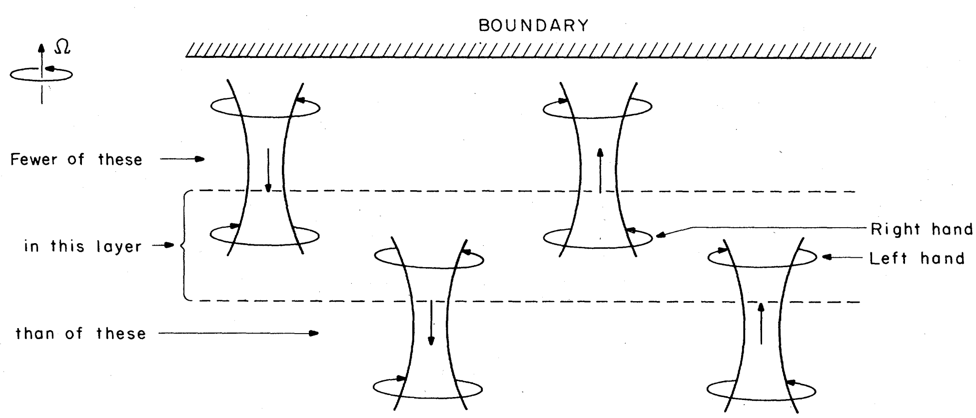

FIG. 12. Preponderant helicity near a boundary. Near a northern boundary of the fluid in the Northern Hemisphere, e.g., there is a gradient in the source density of short, randomly distributed convective streams causing a preponderance of left-handed helicity and hence negative alpha.

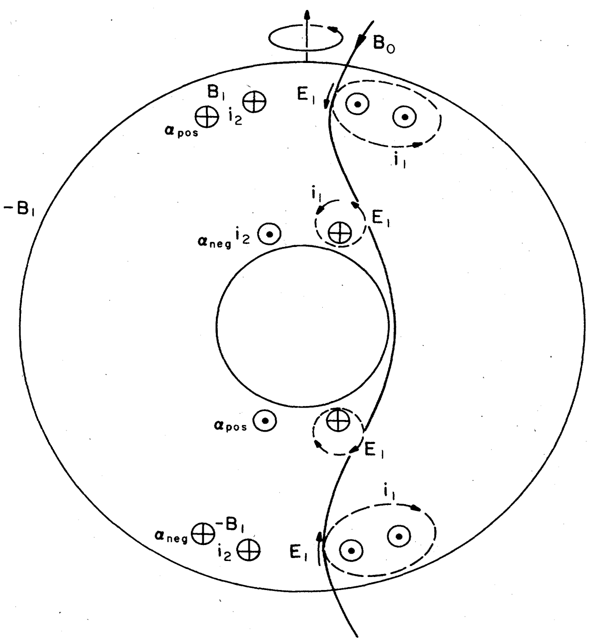

FIG. 13. Regeneration of the global field resulting from localized regeneration by randomly distributed convection near the boundaries.

effect, not a volume effect. Both i_1 and i_2 are driven by electric fields limited to the near-surface layer, and are opposed by the electric resistance of the fluid. This factor favors short loops of i, though they must be large enough to surround some magnetic flux. Presumably largely for this reason, the analysis of Steenbeck and Krause finds that this local dynamo tends to be concentrated close to the pole, mainly within the inner cylindrical region directly to the north and south of the solid core. The solid core presumably rotates relatively eastward under the influence of circulation near the equator. The field penetrating from the north polar regions down through the solid core is thus twisted towards the east. This produces an eastward toroidal component that is relatively weak because it is merely part of a thick twisted bundle of lines of force clustered about the axis, the pitch being steep so near the axis. This eastward component opposes the westward B_0 of Fig. 13 there and tends to turn the generator off, an example of the need already noted for a negative α in the Northern Hemisphere to support an $\alpha\omega$ dynamo. This situation with its negative feedback contains the possibility of an unsteady dynamo coupled to the inertia of the differential rotation. However, since the ω-induced toroidal field is relatively weak so near the axis, this may be merely a weak $\alpha\omega$ mechanism slightly reducing the regeneration in a strong α^2 dynamo.

VII. WESTWARD DRIFT

The earth's dipole field is inclined at about 11° to the axis of rotation, and over the past few centuries the magnetic poles have drifted westward at a rate of about 0.18° per year. The line of zero declination was in Europe five centuries ago and is in America now. There are also nondipole elements of the field, in particular the widely scattered, almost continent-sized, regions of anomalous field. While varying irregularly in ways that defy complete analysis, these show a general westward drift at a more rapid rate. We may first examine how a theory can account grossly for a westward drift without distinguishing between these two aspects of it. Since most of the irregularities of the field are observed at fairly low latitudes, a westward drift of the fluid in the outer part of the core, relatively far from the axis, should suffice. This outer part can contain irregularities moving with it that are observable at the surface to give evidence of its general drift.

For theories involving differential rotation, the differential rotation itself gives rise to a westward drift of the outer part in a simple way. Bullard's original explanation of the drift was of this type. Conservation of angular momentum of the exchanged matter makes the inner part rotate in space faster than the outer part and thus eastward relative to the outer part. Both are coupled to the mantle mainly through magnetic drag. In a steady state there must be no total torque between the mantle and the core so the magnetic drag on the inner part and outer part must have opposite signs. This requires that the inner part must rotate eastward and the outer part westward relative to the mantle, as illustrated in Fig. 14.

There is another, very different, theory of the regenerating process that leads to a very different theory of the westward drift. It involves hydromagnetic waves near the core-mantle interface that exhibit a dynamo action, a shorter wave moving eastward that is too rapidly varying for its field to penetrate the mantle to the earth's surface, and a slower westward-moving

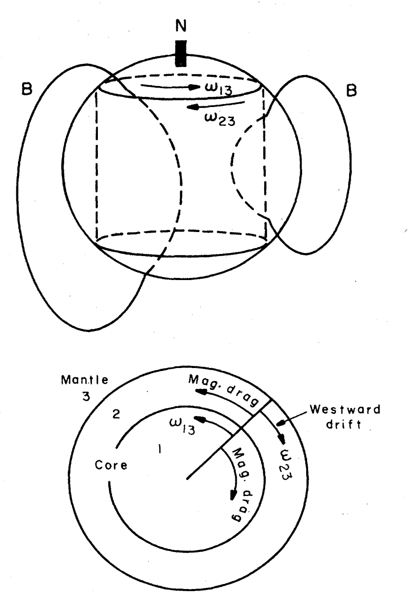

FIG. 14. Westward drift of the outer part of the core resulting from both parts being coupled to the mantle, with the inner part rotating eastward relative to the outer part due to conservation of angular momentum when matter is exchanged between them.

wave that could be observed as the westward drift (Acheson and Hide, 1973). This theory of the westward drift differs from others in that no massive relative rotation of the bulk of the fluid is involved, but merely a wave motion within the fluid.

The Busse convection-roller theory provides an attractive explanation of the magnetic dynamo regeneration, but in its simplest form does not account for the westward drift, a shortcoming that seems not to have been addressed. One mechanism we may propose even leads to an eastward drift of the convection-roll pattern. In the upper half of Fig. 6, for example, one observes that the helical flow within the rollers carries to the surface a flux of westward (on the outer side) rotation, or of angular momentum in the negative Ω direction. This flux consists of upward flow of negative angular momentum and downward flow of positive angular momentum on adjacent rollers, the transfer from one to the other being induced at least in part by frictional interaction with the surface. Through friction the fluid is thus imparting negative angular momentum to the surface and the surface is imparting positive angular momentum, or eastward rotation, to the fluid. This positive angular momentum is at first manifest in the small-scale rotation within the rollers, but as the fluid leaks through from the low-pressure to the oppositely rotating high-pressure roller in the body of the fluid, this positive angular momentum must be passed on to the fluid as a whole with its pattern of pressure differences, giving it an eastward rotation relative to the mantle.

However, with additional complexity and thus some loss of elegance one can contrive configurations involving convection rolls that might display a westward drift of the outer parts of the pattern despite this eastward tendency. An example would be two concentric cylindrical blocks of convection rolls with a slip plane between them where rigidity is broken because the periodicities on the inside and outside of it are incommensurate, as suggested in Fig. 15. Conservation of angular momentum of matter crossing it could then give rise to differential rotation and a westward motion of the outer block through magnetic couplings with the mantle as before.

VIII. α^2 DYNAMOS PLUS DIFFERENTIAL ROTATION

We have seen that there are several mechanisms that could support the earth's main magnetic field and even account for the existence of a westward drift. Differential rotation is a useful ingredient for the westward drift and also to support the regenerative process in $\alpha\omega$ dynamos, while α^2 dynamos can function either as global dynamos or, as seen near the pole in Fig. 13, as rather localized dynamos, but without differential rotation they do not display westward drifts. One can account for the additional complexity of the observations by making a more complex theory, in particular by superposing some of these ingredients in a single theory.

A combination of differential rotation with α^2 regeneration has been analyzed in a very interesting way by Rädler (1975). He postulates a meridional flow with axial symmetry and yet twisting in such a way as to exhibit differential rotation, and superposed on this a small-scale turbulence supporting the α effect. The meridional flow is similar to that used by Bullard, but with axial symmetry. It may be regarded as consisting of a sheet flowing outward in the equatorial plane in a flat spiral, then a flow along the spherical boundary in the general direction of the meridians towards the poles, also twisting to conserve angular momentum, and a downward flow at the poles. The random turbulence throughout the core gives rise to the same distribution of α as treated earlier by Steenbeck and Krause (1969) and employed in Fig. 13. Their formulation of mean-field electrodynamics (Krause, 1977) is used, in which turbulence increases large-scale electric resistance and there is also a transverse α effect generating an electromotive force normal to the mean magnetic field, but the essence of the theory is the α effect already described. Solutions of the magnetic field equations are sought of the form $e^{i(\omega t+m\phi)}F(r, \theta)$. The first factor leaves them free to display a rotation of the magnetic pattern. The factor F may be either symmetric (S) or antisymmetric (A) in the replacement of θ by $\pi-\theta$. The simplest modes, shown in Fig. 16, are specifically considered. The notation $A0$ means antisymmetric in θ, $m=0$, for example, and this represents the axial dipole, the α^2 regeneration of which is demonstrated in Fig. 13. The analysis shows that the other modes are

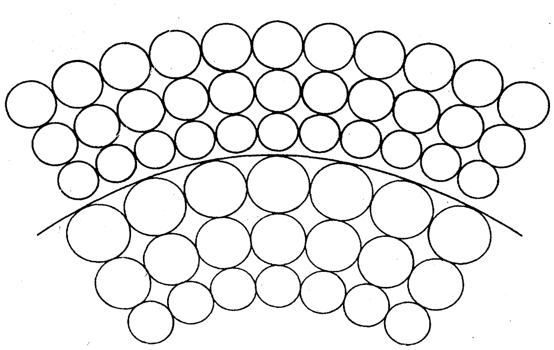

FIG. 15. Possible division of a convection-roll pattern into an inner and an outer part, as postulated in Fig. 14.

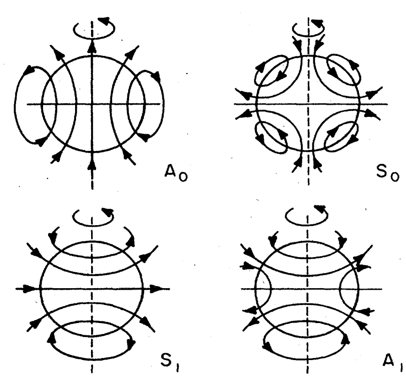

FIG. 16. Simplest modes in the expansion of the regenerated field in the theory of Steenbeck and Krause (1969).

similarly regenerated and that those with $m=1$ display a westward drift. While this drift is presented merely as emerging from the equations of the analysis, it appears to arise from essentially the same mechanism as already described, a balance of magnetic drag on the low-latitude and high-latitude parts of the poleward-flowing surface layer of the core, in which differential rotation is assumed. The two dipole fields, $A0$ parallel to the axis and $S1$ normal to it, are the two components of the earth's main dipole that is at present inclined 11° to the rotation axis. Its westward drift is that of $S1$. The westward drift of the magnetic anomalies may reasonably be associated with that of $A1$, which is found to be faster than that of $S1$ in nice qualitative agreement with this aspect of the observed field. The solution for each mode is in the form of an eigenvalue of the quantity $C=\mu\sigma\alpha R$, essentially a lower limit of the α to provide the regeneration, in terms of susceptibility μ and conductivity σ. The eigenvalue of $S1$ is higher than that of $A0$, suggesting that $S1$ should be excited less strongly than $A0$ by the actual turbulence, again in qualitative agreement with the fact that the dipole is oriented roughly but not exactly along the polar axis.

IX. GEOMAGNETIC REVERSALS

Perhaps the most remarkable feature of the earth's magnetic field is the fact that it reverses its direction at apparently random intervals, as revealed by paleomagnetic observations. The reversals have taken place irregularly on the scale of several hundred thousand or a million years, but within this pattern there are events of a shorter duration, the reversed polarity lasting for a time of the order of a hundred thousand years or less. The present or "normal" polarity has lasted for about eight hundred thousand years, and the period of reversed polarity preceding it was interrupted by an event of normal polarity lasting about 56 thousand years a million years ago (Cox, 1969). On the longer scale of almost a billion years there seem to have been periods, or at least one long period, without any reversals.

One widely discussed treatment of the reversal problem, by Rikitake (1958, 1966) and Allan (1958, 1962) makes no attempt to simulate the geometry of the earth's core, but shows that it is in principle possible for a self-sustaining dynamo consisting of coupled rotating discs to reverse its polarity spontaneously. A single disk with a coil producing a magnetic field that penetrates it, as shown at the top of Fig. 17, is a classic example of a simple self-sustaining dynamo. The coupled system shown below it is not a coupled pair of such single dynamos, but rather one two-stage dynamo, with two rotors, in which the magnetic field at each rotor depends on the rotation of the other. The identical rotors are assumed to be driven by equal and constant torques. Energy is dissipated through electric resistance losses. While a steady state is possible, most initial conditions lead to anharmonic oscillations. The unsteadiness depends not so much on exchange of kinetic energy between the two discs as on exchange of energy between magnetic energy and kinetic energy of rotation. Analysis by Allan (1962) yields the results shown in

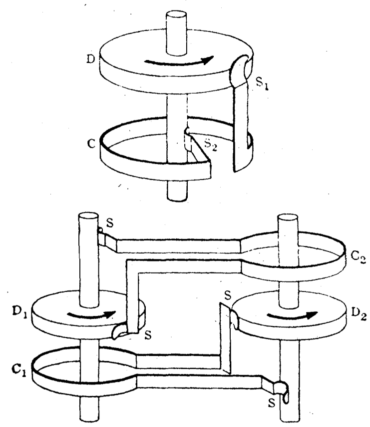

FIG. 17. One-stage and two-stage rotating-disc dynamos.

Fig. 18 for typical initial conditions. The two broken lines show currents in the two coils, which are seen to vary roughly in phase with one another and out of phase with the angular velocity of the rotors indicated by the solid line. A growing phase mismatch of the currents appears to trigger the reversal. The reversed spike at $\tau=103$ seems to simulate the event about a million years ago, for example. For such a crude model, this is tantalizing agreement with the observed pattern.

This magnetomechanical demonstration of a reversal could be a closer analog to a dynamo in the earth's fluid core than was at first envisaged, as has been pointed out by Moffatt (1978). The four equations governing the motion and currents in the two-rotor system in Fig. 17 take the form

$$\begin{aligned} I d\omega_i/dt &= T - M i_i i_j\,, \\ L di_i/dt + R i_i &= M\omega_i i_j\,, \end{aligned} \tag{7}$$

with $i,j=1,2$ or $2,1$, respectively. Here I is the moment of inertia of each rotor, L the self-inductance of each coil, T the constant torque applied to each rotor, and M the mutual inductance between either coil and the moving parts of the rotor to which it is magnetically coupled. The identity of the two I's, and likewise R's, L's, and T's, is a simplification introduced into the illustrative calculations. More generally, they could

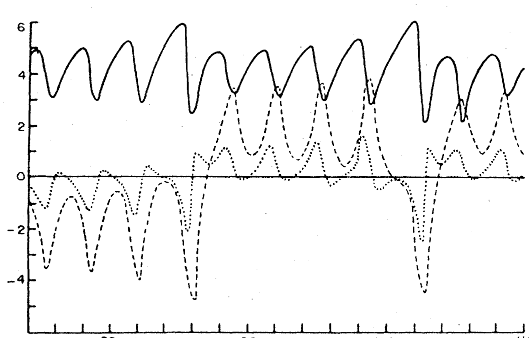

FIG. 18. Changing phase relations of magnetic and mechanical amplitudes in a two-stage disc dynamo, resulting in occasional magnetic reversals, according to calculations of Allan (1962).

be I_i, etc., presumably with qualitatively similar results (Cook and Roberts, 1970). The closeness of the analogy arises from the fact that the α effect is driven by a mechanical rotation that has a moment of inertia, and this, along with a linear motion also having inertia, in a magnetic field generates an electromotive force as illustrated in Fig. 2. This emf may then drive a current around a circuit, having resistance and inductance, to produce the next magnetic field. The combined rotation of all the vortices participating in this stage of the α effect may be represented by ω_i in Eq. (7).

In the α^2 dynamo of Fig. 11, for example, the initial field B_0 interacting with the rotation ω_i through the α effect produces E_1 that drives i_1 around a circuit to induce B_1. This in turn interacts with ω_2 to produce i_2 regenerating B_0, all as in the two-disk dynamo. The magnetic drag on the rotation producing the α effect tends to decelerate the rotation and would stop it and turn off the effect if a torque, represented by T, supporting the motion were not supplied by thermal convection. Aside from the unmanageable complexity of making the appropriate averages, Eq. (2) can thus be seen to be of an appropriate form to represent the inertial and inductive effects inherent in an α^2 dynamo in the earth, and the calculation in that simplified model may thus be relevant to the reversal of the earth's field.

A similar analogy may be made to an $\alpha\omega$ dynamo, the case treated by Moffatt. There ω_1 represents the differential rotation and ω_2 the rotations involved in the α effect.

A very different theory of the origin of the reversals is that proposed by Parker (1969) and treated in more detail by Levy (1972, 1976) on the basis of Parker's theory of regeneration that involves randomly distributed large-scale turbulence, occasional large upwellings distributed haphazardly over the core. It is not, like Rikitake's, a mechanism dependent on an exchange of energy between magnetic and mechanical forms. Indeed, it has no explicit inertial component. It depends instead on the interaction of two almost independent self-sustaining dynamos magnetically coupled. They are assumed to vary individually in regenerating strength, either statistically or with secular change of physical parameters. One is idealized in the calculations as a pair of rings of upward streams, or cyclones, at high latitude near the pole in each hemisphere. The other is such a pair of rings at low latitude near the equator. Normally when they are both operating, or when the distribution of upwellings is random, they are parts of the same global dynamo. However, if one of them ceases to function, the direction of the magnetic field at its position is determined by the toroidal field generated by the other. This can locally have reversed direction and, when the dormant ring starts generating again, it will be excited in the reverse direction and may take over if the other ring weakens. This is the general idea. Let us consider some of the details.

In the usual global view of a poloidal field being distorted into a toroidal field by differential rotation, the lines of force are drawn into hairpins in such a way that the generally southward poloidal field becomes an eastward toroidal field in the Northern Hemisphere and westward in the Southern. This is indicated in Fig. 19(a) by the feathered tails of the departing arrows on the lines of force in the Northern Hemisphere and the points in the circles in the Southern. The magnetic lines are deflected so as to point into the paper in the Northern Hemisphere because they are headed downward across the shear into more rapidly rotating parts of the fluid core and the rapidly rotating solid core; conversely in the Southern Hemisphere, they proceed up into slower layers. From symmetry, the turnaround point, the head of the bent hairpin, is in the equatorial plane.

In the form of the theory presented by Levy, the differential rotation is represented by a spherical shear pattern, rather than by the cylindrical shear pattern that we have discussed as perhaps more plausible. The general effect is similar whether the shear pattern is spherical or cylindrical.

Regeneration throughout most of the core would be required to support this simple poloidal field. When the regenerative mechanism that generates the poloidal field is more localized, the poloidal field has a more complicated shape. Here again, the local reversal of field direction comes about as a result of the way the poloidal field lines cross the shear planes locally, upward rather than downward, and the consequent direction in which the shear distorts this field to form a toroidal field.

The simple pattern of Fig. 19(a) we consider the

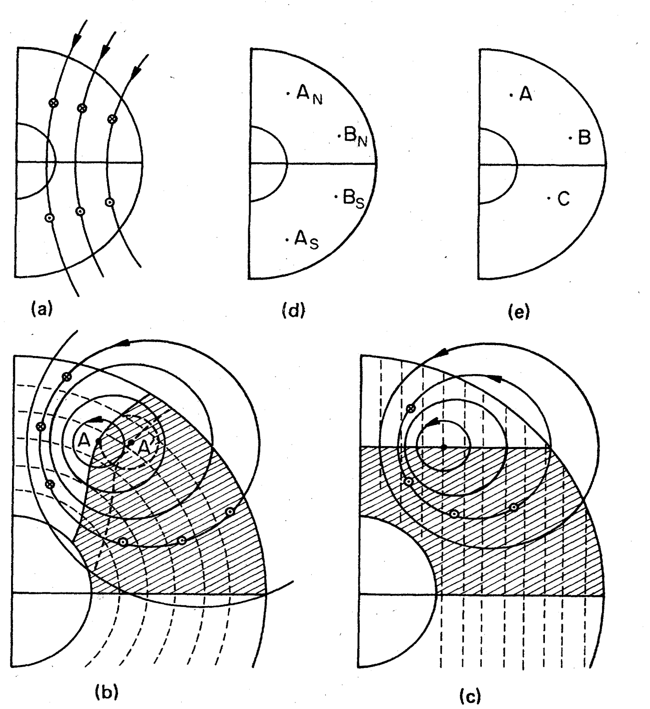

FIG. 19. Reversal mechanism in the earth's core. The poloidal field of a ring current at A, for example, is distorted by differential rotation into an eastward, or normal, toroidal field where the field penetrates downward across shear lines [unshaded area in (b) and (c)] and to a reverse field in the shaded areas where the penetration is upward. Variations in the strength of the alpha effect coupling various rings can then cause a global reversal.

"normal" direction of the poloidal field. The situation is complicated by the way the lines of force, wrapped perhaps several times around the axis to make the toroidal field stronger than the poloidal, tend to move sideways due to ohmic resistance and thus to shrink towards the axis while being regenerated further out. We shall consider the effect of the local shear on the transition from poloidal to toroidal without this complication.

Consider that the distribution of strength of the regenerating mechanism, such as the magnitude of α, is uniform in azimuth, a function of r and θ but not of ϕ. Consider then the contribution from a single ring, A, a single element of area $r\,d\theta\,dr$, in Fig. 19(b), in which a spherical shear pattern is assumed. The poloidal field lines generated by A come inside the ring in the normal direction from the north and are deflected to form toroidal field in the normal way until they reach the point where they are tangent to the shear lines. From that point on a line of force starts rising into more slowly rotating layers and the shear twists the line out of the paper, westward, which is reversed for the Northern Hemisphere but normal where the line may reach into the Southern Hemisphere. The shaded area in the figure shows where the toroidal field generated by A is reversed. Thus the shaded area implies this reversal relative to the field at A that is regenerated by A, whereas in the unshaded area the field is of the same sign as that of the original field. Figure 19(c) shows the same thing for the case of a cylindrical shear pattern.

The discussion and analysis of the reversal problem are somewhat simplified if one adopts symmetry in the two hemispheres. If we simultaneously consider two rings behaving similarly, each a mirror image of the other in the equatorial plane, then each is in the unshaded or same-sign region of the other, and they reinforce one another in regenerating the field. A dynamo could thus consist of a ring and its mirror image in the other hemisphere, a dynamo A consisting of the pair of rings of $A_{\rm north}$ and $A_{\rm south}$, as in Fig. 19(d), which shows also a similar pair B. $B_{\rm north}$ is in the shaded region of $A_{\rm north}$ and in the unshaded region of $A_{\rm south}$, but since $A_{\rm north}$ is closer and more influential, the influence of A on $B_{\rm north}$, and by extension of A on B, is predominantly reversed. B can coexist with A as a self-sustaining dynamo if the mutual stimulation within the pair B is stronger than this reversed influence of A on B. This can happen, for example, if B is sufficiently far from A.

A reversal can be brought about by random or uncorrelated variations in the relative strength of the regenerating process at A and B, variations in the value of α, for example. Starting with A and B both strong and acting together as a dynamo, let B weaken. Then the reverse field impressed by A on B will predominate and B will generate weakly in reverse. Let B then strengthen or A weaken or both until the field of B at A is stronger than the field that A exerts on itself. A is in the unshaded or same-sign region of B, and this field at A is therefore reversed, so A must now generate in reverse. A and B are now operating together again, but with the entire field reversed from the original situation. This is one type of mechanism that seems to be capable of explaining the observed geomagnetic reversals.

Since there are several possible regeneration mechanisms, as we have seen, it seems plausible that there should be two separate dynamos in different azimuthal regions, each with north-south symmetry, interacting in this way. With rather different regeneration mechanisms, the two dynamos can depend on the varying physical conditions of the earth in different ways and thus have time variations independent of one another, so as to make the reversal occur at apparently random intervals.

An appropriate combination of flow patterns may consist of random turbulence at high latitudes, near the poles, and a convection-roll flow pattern in the large volume farther from the axis. Dynamo A could then be the α^2 dynamo of Fig. 13 and dynamo B the $\alpha\omega$ dynamo of Fig. 5.

Viewed in more detail, the success of the reversal process depends on how fast the fields grow or decay in the conducting medium as the strength of the sources changes. In his calculations, Levy models the decay by the formality of turning on the regeneration at A, then turning it off and waiting a while, then turning on the spherical shear pattern of Fig. 19(b), then turning that off, etc. During the first hiatus, the ring-shaped field lines shrink so as to be disposed about A' about as they were at first about A. When the shear is turned on, the shaded region is bounded by the broken line through A' rather than by the solid line through A. In Fig. 19(d), B_N is still the reverse region relative to A_N.

Parker and Levy based their discussions on turbulence such as indicated in Fig. 5, with the north-south symmetry of the two dynamos occurring only occasionally by chance, this being considered compatible with the random nature of the reversals. In their analysis, dynamo A consists of two rings of upward bursts of cyclones near the two poles, happening to occur simultaneously, and dynamo B similarly of two chance rings near the equator.

The north-south symmetry is not a requirement for reversal, as may be shown by consideration of Fig. 19(e). When rings A, B, and C are operating together normally to form a dynamo, each ring is in the normal region of the other except that B is in the reversed region of A. However, A is small, so the normal field of C at B is dominant there. In the reversal process, C weakens so A reverses B. Then the reversed B strengthens and reverses A and C. In the theory based on the random occurrence of ring-shaped bursts of cyclones, this would seem to be more probable than the reversal involving four rings as proposed by Parker and Levy.

There are thus several flow patterns that might reasonably give rise to the magnetic regeneration of a dynamo, and at least two mechanisms that could account for the field reversals. It would be premature to claim that any particular combinations of these interesting possibilities is operative within the earth or even that any of them is. One especially attractive candidate con-consists of the Steenbeck-Krause-Rädler modes of an α^2 dynamo with the Rikitake-Allan-Moffatt reversal mechanism, based on oscillations exchanging magnetic and mechanical energy, operative within the $A0$ mode.

While it taxes the limit of paleomagnetic observations to determine the course of events in the relatively short time interval of a reversal, there is some indication that the earth's magnetic moment does not disappear during the reversal, but rather becomes weak while retaining a component normal to the axis that seems to vary considerably (Cox, 1972). This is one more observation that favors the combination just described; it seems to imply that $A0$ reverses while $S1$ retains its strength. Thus one has a mechanism for this detail of the reversal while retaining the mechanisms for the general dynamo regeneration and for two separate aspects of the westward drift.

X. ENERGY CONSIDERATIONS

While our emphasis here has been on the mechanism whereby postulated motions generate magnetic fields, it is of interest to inquire whether there may be adequate energy sources to drive the postulated motions. The brief answer is that there are possible sources consistent with present data. However, evaluation of the performance of postulated energy sources, even in order of magnitude, involves estimates of material properties such as conductivity, melting point, and viscosity, at the enormous pressures in the earth's core, some thirty times the maximum pressure attained in some of the relevant laboratory measurements from which they are extrapolated. Shock-wave measurements extend to high enough pressures but do not accurately determine the needed quantities. There is thus so much uncertainty in the parameters used that the limits placed on energy requirements are not at all stringent.

There are uncertainties of a factor of 2 or so arising from spread in the laboratory measurements themselves (Verhoogan, 1961) but there could be greater uncertainties in some of the very long extrapolations made by use of questionable semiempirical formulas. It should be appreciated that, at the pressure of the inner-core surface (3 Mbars) the pressure energy PV is 15 eV per atomic volume, to be compared with a normal binding energy of iron of about 4 eV/atom, or an energy of fusion, more relevant to the melting point of 0.1 eV/atom. The electronic energy band structure, which is responsible for several important physical parameters, must be drastically altered by such high pressure. Unknown phase transitions are possible, as well as effects of impurities.

By way of example, static measurements of one important quantity, electric conductivity σ, have been made up to high enough pressures, though with some uncertainty of the pressure calibration (Kawai and Mochizuki, 1971). The measurements on FeO_3 follow a simple curve to pressure much higher than usual laboratory pressures, then suddenly jump down by three orders of magnitude before reaching pressures typical of the earth's core.

A rough idea of the energy requirement to support the field may be obtained by estimating the ohmic dissipation by the currents in the core required to support the field. With a conductivity σ obtained by simple extrapolation, if the dipole moment were simply decaying in a static core it would dissipate about 10^8 W. Estimates of the dissipation based on multipole expansions of the fields and motions in various models of the dynamo run from 10^9 to over 10^{12} W, depending partly on how much the toroidal field strength exceeds the poloidal (Stix, 1977).

Radioactivity within the solid core or the entire core has long been considered a likely source of the heat to drive thermal convection, though there is some reason for doubt. It may be that the iron meteorites originated in a planetary core similar to the earth's in chemical composition. The radioactive content of most, but not all, iron meteorites indicates an energy production in the core of 3×10^{12} W, apparently sufficient if the toroidal field is not too large. A similar indication for potassium is too small, but isosopic abundances in the sun suggest that there may be enough ^{40}**K** to provide an ample 10^{13} W in the earth's core.

Another possible source of thermal energy to drive the convection is freezing at the surface of the solid core in a cooling earth. Whatever the early thermal history of the earth may have been, whether the initial heating arose from accretion or from short-lived radioactivity in an earth almost as old as the universe, it is plausible to assume that internal heat sources are now insufficient to maintain its temperature and it is now in a cooling phase. In considering latent heat of crystallization as the source, two functions of pressure are relevant, one representing the melting point and the other the adiabatic curve in the liquid phase at a given stage of global cooling. Both curves rise with increasing pressure and decreasing radius. The melting-point curve rises because the crystalline solid phase is more compact than the fluid, and the higher the pressure, the higher the temperature required to provide energy for expansion on melting. The rising adiabat is the familiar heating on compression. Extrapolation by one method from data available at lower pressures indicates that, in a predominantly iron or iron-nickel core, the melting point curve is steeper than the adiabat. If we think of starting with a very hot earth, when the adiabat is everywhere higher than the melting point and the whole core is fluid, reduction of the global temperature would bring the adiabat down to meet the melting curve first at zero radius. On further cooling the curves meet at larger radii and the solid core grows. In the current situation, then, crystallization would be proceeding at the surface of the growing solid core. Heat of fusion is thus being released on solidification, and this can serve as the heat source driving the convection and thereby supporting the earth's magnetic field (Jacobs, 1961). A quantitative discussion indicates that, within the range of uncertainty of the parameters involved, ample energy is available (Verhoogen, 1961) with the core cooling at about 20 degrees per 10^9 years, within perhaps a factor of 2, and the radius of the inner core growing at about 30 km per 10^9 years. The corresponding total heat flow through the mantle is about 5×10^{11} cal/sec.

The validity of all this has been challenged (Higgins and Kennedy, 1971) on the basis of a second method of extrapolation which indicates that the slope of the adiabat is greater than the slope of the melting-point curve, so the two would not meet at the surface of a solid core. In this extrapolation the melting temperature is taken to be linear in relative volume expansion, $\Delta V/V$, in agreement with the data for most metals at laboratory pressures. The first method of extrapolation agrees instead with the laboratory data for van der Waals solids such as argon and helium. Since the core pressures are great enough to distort electronic energy bands radically, probably erasing the importance of valence-type binding and enhancing interaction of closed shells such as dominant in van der Waals-type solids, the first method of extrapolation seems at least as reasonable as the second, and the challenge need not be considered conclusive.

If the convection in the core is indeed thermally driven, no matter whether by radioactivity or by latent heat or both, the slope of the adiabat is relevant to the possible efficiency of the dynamo. If the actual temperature within the core were to follow the adiabat, transport of a sample of fluid along an adiabat would yield no external power to drive convection. The fluid would remain static but the heat of the source would be dissipated by conduction. It is only to the extent that convection maintains a temperature gradient steeper than this that energy is available from the energy transport to drive convection. This part of the energy, which may be less than half and perhaps only a tenth (Braginskii, 1964) of the total, can through appropriate dynamo action support the magnetic field while being dissipated in ohmic resistance.

An upper limit to the total heat flow in the core is imposed by observations of the outward heat flux at the surface of the earth, 3×10^{13} W, if we reasonably assume that the heat is being transported steadily through the mantle (which requires convection therein). Estimates of the radioactive content of the earth's crust indicate that much or even most of the observed flow arises in the crust, leaving only a part to be attributed to the core. Even if only a tenth of the total arose in the core, and even if the dynamo efficiency in producing electric currents were only ten percent, this limitation would exclude only a small upper part of the range of estimates of the dynamo's electric dissipation. It thus permits thermally driven dynamo action but casts in doubt models with the largest toroidal fields.

It has been proposed that turbulent motions in the core need not be thermally driven but could arise from the 26 000-year precession of the earth's axis, that is, essentially from torques applied by the sun and moon, proportionately greater to the mantle than those to the core.

The velocity of a parcel of fluid in the core may be written

$$\mathbf{v}=\boldsymbol{\omega}\times\mathbf{r}+\mathbf{v}' ,$$

where the angular velocity $\boldsymbol{\omega}$ of the mantle in space precesses with the very slow angular velocity $\boldsymbol{\Omega}$ and $\mathbf{v}'$ is velocity relative to the mantle. The acceleration $\dot{\mathbf{v}}$ then contains the familiar centrifugal term $\boldsymbol{\omega}\times[\boldsymbol{\omega}\times\mathbf{r}]$ and Coriolis term $\boldsymbol{\omega}\times\mathbf{v}'$ and an additional term due to the precession $\dot{\boldsymbol{\omega}}\times\mathbf{r}=(\boldsymbol{\Omega}\times\boldsymbol{\omega})\times\mathbf{r}$. The direction of this latter term, being normal to a vector almost fixed in space, rotates daily in the rotating coordinate system; Malkus (1968) proposes that, since it can cause turbulence, it might be the source of the energy driving a geodynamo. He estimates that the energy available from the relative motion of core and mantle is sufficient, but Loper (1975) finds that almost all of this is dissipated in the boundary layers and not nearly enough is left to support the dynamo.

It has also been proposed that flow patterns, perhaps similar to those we have postualted as thermally driven, could instead be driven by the selective settling of a heavy component of a mixed fluid on the surface of the solid core. There being no geologic evidence for such a mixture, it seems more plausible to assume that the drive is thermal.

XI. CONCLUDING REMARKS

The motions within the earth's core present a dynamical problem of surprising complexity that is far from being resolved. Some aspects of the fluid motions within a steadily rotating earth have been analyzed in simplified models to justify some of the motions postulated in our discussion of regeneration of a magnetic field. The reaction of the magnetic field on the motions further complicates the dynamics, most simply slowing down the motions to match the field strength to the power input, but also making possible, for example, hydromagnetic waves of predominantly horizontal motion under the surface (Acheson and Hide, 1973). But the earth's rotation is not steady. Lunar and solar torques on the equatorial bulge and tides cause both a precession and a deceleration of the mantle's rotation, behind which the core fluid tends to lag by different amounts at different depths (Inglis, 1941; Roberts, 1972), superposing shear and perhaps additional turbulence, at least on a small scale near the surface, on the motions assumed for the dynamo models. It is clear from the mobility of both the dipole and the multipole components of the observed magnetic field that it arises from motion within the fluid core. Despite the unresolved complexity it is gratifying to understand how several reasonable assumed patterns of flow can exhibit dynamo action, each of them involving streams to which the Coriolis force imparts helicity that tends to regenerate at least the main dipole field.

REFERENCES

Acheson, D. H., and R. Hide, 1973, Rep. Prog. Phys. **36**, 159.
Allan, D. W., 1958, Nature **182**, 469.
Allan, D. W., 1962, Proc. Cambridge Philos. Soc. **58**, 671.
Braginskii, S. I., 1964, Geomagn. Aeron. **4**, 698.
Bullard, E. C., 1949, Proc. R. Soc. London Ser. A **197**, 433.
Bullard, E. C., C. Freedman, H. Gellman, and J. Nixon, 1950, Philos. Trans. R. Soc. London Ser. A **243**, 67.
Bullard, E. C., and H. Gellman, 1954, Philos. Trans. R. London Ser. A **247**, 213.
Busse, F. H., 1971, Z. Geophys. **37**, 153.
Busse, F. H., 1977, J. Geophys. **43**, 441.
Busse, F. H., and C. R. Carrigan, 1976, Science **191**, 81.
Carrigan, C. R., and D. Gubbins, 1979, Sci. Am. **240** (Feb.), 118.
Cook, A. H., and P. H. Roberts, 1970, Proc. Cambridge Philos. Soc. **68**, 547.
Cox, A., 1969, Science **173**, 237.

Cox, A., 1972, Trans. Am. Geophys. Union **53**, 613.
Elsasser, W. M., 1946, Phys. Rev. **69**, 106; **70**, 202.
Elsasser, W. M., 1947, Phys. Rev. **72**, 821.
Elsasser, W. M., 1950, Rev. Mod. Phys. **22**, 1.
Gubbins, D., 1974, Rev. Geophys. Space Phys. **12**, 137.
Hide, R., 1976, Geophys. Fluid Dyn. **7**, 157.
Higgins, G., and G. C. Kennedy, 1971, J. Geophys. Res. **76**, 1870.
Inglis, D. R., 1941, Phys. Rev. **59**, 178.
Inglis, D. R., 1955, Rev. Mod. Phys. **27**, 212.
Inglis, D. R., 1964, J. Geomagn. Geoelectr. **17**, 517.
Jacobs, J. A., 1961, in *The Earth Today*, edited by A. H. Cook and T. F. Gaskel (Interscience, New York), p. 267; also Geophys. J. R. Astrom. Soc. **4**, 267.
Kawai, N., and S. Mochizuki, 1971, Phys. Lett. A **36**, 54.
Krause, F., 1977, J. Geophys. **43**, 421.
Levy, E. H., 1972, Astrophys. J. **171**, 635.
Levy, E. H., 1976, Annu. Rev. Earth Planet. Sci. **4**, 159.
Loper, D. E., 1975, Phys. Earth Planet. Inter. **11**, 43.
Malkus, W. V. R., 1968, Science **160**, 259.
Moffatt, H. K., 1978, *Magnetic field generation in electrically conducting fluids* (Cambridge University Press, Cambridge, England), especially pp. 318 ff.
Parker, E. N., 1955, Astrophys. J. **122**, 293.
Parker, E. N., 1969, Astrophys. J. **158**, 815.
Rädler, K.-H., 1975, Mem. Soc. R. Sci. Liege Collect. 6° Serie, **VIII**, 109.
Rikitake, T., 1958, Proc. Cambridge Philos. Soc. **54**, 89.
Rikitake, T., 1966, *Electromagnetism and the Earth's Interior* (Elsevier, Amsterdam).
Roberts, G. O., 1970, Philos. Trans. R. Soc. London Ser. A **266**, 535.
Roberts, G. O., 1972, Philos. Trans. R. Soc. London Ser. A **271**, 411.
Roberts, P. H., 1972, J. Geomagn. Geoelectr. **24**, 321.
Steenbeck, M., and F. Krause, 1969, Astron. Nachr. **291**, 49 and 271.
Stix, M., 1977, J. Geophys. **43**, 695.
Taylor, G. I., 1921, Proc. R. Soc. London Ser. A **100**, 114.
Verhoogen, J., 1961, *The Earth Today*, edited by A. H. Cook and T. F. Gaskel (Interscience, New York), p. 276.

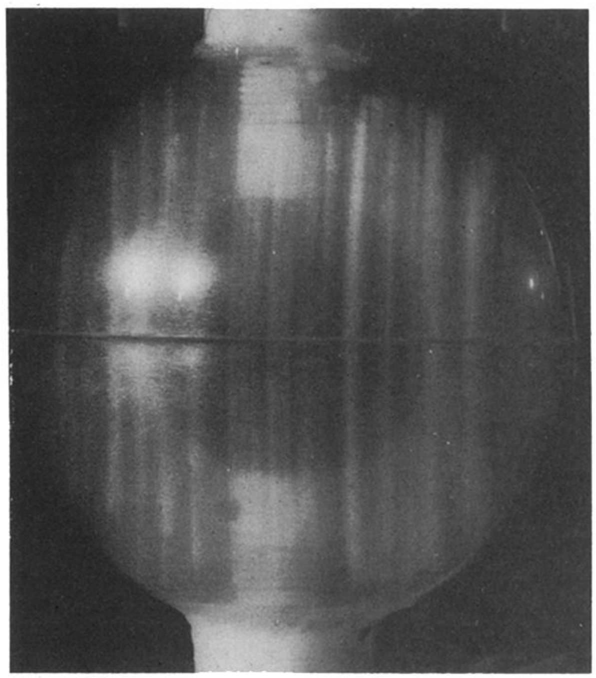

FIG. 10. Photograph of convection rolls in the demonstration of Busse and Carrigan. The fluid is water loaded with aluminum powder, and centrifugal force replaces the gravity in the earth's core.