# A theoretical and experimental review of the weak neutral current: a determination of its structure and limits on deviations from the minimal $SU(2)_L \times U(1)$ electroweak theory

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A detailed analysis of existing neutral-current data has been performed in order (a) to determine as fully as possible the structure of the hadronic and leptonic neutral currents without recourse to a specific weak-interaction model; (b) to search for the effects of small deviations from the Weinberg-Salam (WS-GIM) model; and (c) to determine the value of $\sin^2\theta_w$ as accurately as possible. The authors attempt to incorporate the best possible theoretical expressions in the treatment of each of the reactions. For deep-inelastic scattering, for example, the effects of quantum chromodynamics, including the contributions of the $s$ and $c$ quarks, have been included. The sensitivity of the results both to systematic uncertainties in the data and to theoretical uncertainties in the treatment of deep-inelastic scattering, semi-inclusive pion production, $\nu$ elastic scattering from protons, and the asymmetry in polarized $eD$ scattering have been considered; the systematic errors are generally found to be smaller than the statistical uncertainties. In the model-independent analyses the authors find that the hadronic neutral-current parameters are uniquely determined to lie within a small domain consistent with the WS-GIM model. The leptonic couplings are determined to within a twofold ambiguity; one solution, the axial-vector-dominant, is consistent with the WS-GIM model. If factorization is assumed then the axial-dominant solution is uniquely determined and null atomic parity violation experiments are inconsistent with other neutral-current experiments. Within generalized $SU(2)\times U(1)$ models we find the following limits on mixing between right-handed singlets and doublets: $\sin^2\alpha_u \leq 0.103$, $\sin^2\alpha_d \leq 0.348$, and $\sin^2\alpha_e \leq 0.064$. Assuming these mixing angles to be zero, a fit to the most accurate data (deep-inelastic and the polarized $eD$ asymmetry) yields $\rho = 0.992\pm0.017$ ($\pm0.011$) and $\sin^2\theta_w = 0.224\pm0.015$ ($\pm0.012$), where $\rho = M_W^2/M_Z^2 \cos^2\theta_w$ and the numbers in parentheses are the theoretical uncertainties. The value of $\rho$ is remarkably close to 1.0 and strongly suggests that the Higgs mesons occur only as doublets and singlets. If one makes this assumption, then the limit on $\rho$ implies $m_L \leq 500$ GeV, where $m_L$ is the mass of any heavy lepton with a massless partner. In addition, for $\rho = 1.0$, the authors determine $\sin^2\theta_w = 0.229\pm0.009$ ($\pm0.005$). Fits which also include the semi-inclusive, elastic, and leptonic data yield very similar results. A two-parameter fit gives $\rho = 1.002\pm0.015$ ($\pm0.011$) and $\sin^2\theta_w = 0.234\pm0.013$ ($\pm0.009$), while a one-parameter fit to $\sin^2\theta_w$ gives $\sin^2\theta_w = 0.233\pm0.009$ ($\pm0.005$). Finally, the authors have found no evidence for a violation of factorization or for the existence of additional $Z$ bosons. Fits to two explicit two-boson models yield the lower limits $M_{Z_2}/M_{Z_1} > 1.61$ and 3.44 for the mass of the second $Z$ boson. The desirability of a complete analysis of radiative and higher-order weak corrections, which have not been included in the authors' theoretical uncertainties, is emphasized.

## CONTENTS



## I. INTRODUCTION

### A. Motivations

One of the most important advances in elementary particle physics in the last decade has been the development of gauge theories of the strong, weak, and electromagnetic interactions. In particular, the $SU(2)_L \times U(1)$ model due to Glashow, Weinberg, Salam, Ward, and others (Schwinger, 1957; Bludman, 1958; Glashow, 1961; Salam and Ward, 1964; Weinberg, 1967, 1971; Salam, 1969; Glashow, Iliopoulos, and Maiani, 1970)—the WS-GIM model—of the weak and electromagnetic interactions has been spectacularly successful in describing and predicting an enormous range of phenomena within a theoretically satisfactory framework. Perhaps the single most important prediction of the model was that neutral currents should exist. Neutral currents were subsequently discovered, and extensive experimental data on neutral currents now exist which are well known to be in good agreement with the predictions of the WS-GIM model.

In this paper we examine systematically all of the neutral-current experiments, attempting to extract the best possible values for the neutral-current parameters. Let us first describe the motivations for the analysis and explain how the present work differs from earlier studies.

The first motivation concerns the question of whether the WS-GIM model is indeed the correct theory of the weak and electromagnetic interactions at least at low energies. Many competing gauge theory models have been proposed which often differ from the WS-GIM model in their neutral-current structure. It is therefore important to extract the neutral-current parameters as fully as possible without reference to a specific weak-interaction model, and then compare the results with various gauge models. This we do in Secs. III and IV of this paper. We do, however, make the assumption that the neutral-current interactions are of a vector and axial-vector structure, as is implied by gauge theories and supported by experiment, and that they do not contain any admixture[1] of $S$, $P$, and $T$. It is of course logically possible that the effective neutral-current interactions at present energies are identical to those of the WS-GIM model but are not in fact based on a gauge theory (Bjorken, 1977, 1979; Hung and Sakurai, 1978; Bludman, 1979).[2] The resolution of this question must await a systematic search for gauge bosons and Higgs bosons.

Granted the approximate validity of the WS-GIM model, one must still ask whether it is the complete theory of weak and electromagnetic interactions, or whether it is just the low-energy limit of a more complex theory, which might involve additional heavy gauge bosons (charged and neutral), new types of Higgs particles, more complicated gauge couplings for right-handed fermions, etc. One sensitive probe for such effects, if they are not too small, is in the neutral current. Therefore, in Sec. VI we shall make various fits to the data that are sensitive to small deviations from the WS-GIM model, and shall attempt to place limits on the effects of additional $Z$ bosons, new representations of Higgs particles, the weak isospins of right-handed fermions, and the masses of additional heavy leptons.

Finally, if one assumes the correctness of the WS-GIM model, it is important to obtain the most accurate value possible for the single free parameter of the model, $\sin^2\theta_w$, which describes the amount of mixing between the SU(2) and U(1) gauge bosons. The precise value of $\sin^2\theta_w$ is especially interesting because of grand unified theories (Georgi and Glashow, 1974; Georgi, Quinn, and Weinberg, 1974; Buras *et al.*, 1978)[3] in which weak, strong, and electromagnetic interactions are unified in a simple group. The most popular model, based on the group SU(5), relates $\sin^2\theta_w$, the proton lifetime, and the strong coupling constant (Marciano, 1979; Goldman and Ross, 1979; Paschos, 1979a). In a careful analysis, Marciano (1979) argues that the present lower limit on the proton lifetime implies $\sin^2\theta_w(M_W) \leq 0.21$, where $\sin^2\theta_w(M_W)$ is the value of $\sin^2\theta_w$ evaluated at the $W$ boson mass scale. Marciano argues that the effective $\sin^2\theta_w$ measured by present experiments should agree with $\sin^2\theta_w(M_W)$ to within 0.01. Therefore it is very important to determine whether present neutral-current data are compatible with this limit on $\sin^2\theta_w$.

Essentially all experimental results are known to be in approximate agreement with the WS-GIM model. Furthermore, there have been many previous analyses of some or all of the data, many of which have focused on the determination of the neutral-current couplings relevant for the neutrino-hadron scattering without assuming a specific weak-interaction model. Such model-independent analyses were especially advocated by Bjorken (1976) and by Hung and Sakurai (1976, 1977a, b). Sehgal (1977a, 1978) combined deep-inelastic inclusive data with results on the semi-inclusive production of pions [$\nu(\bar{\nu})N \to \nu(\bar{\nu})\pi X$] to show that the hadronic neutral-current couplings must lie in one of four allowed regions in coupling parameter space. (Unfortunately, it was necessary to describe the semi-inclusive data using the parton model, the validity of which is questionable in the energy region of the experiments). Hung and Sakurai (1976, 1977a, b), Langacker and Sidhu (1978), and Ecker (1977, 1978, 1979) showed that only two of these regions were consistent with $\nu(\bar{\nu})p$ elastic scattering data. Subsequently, Abbott and Barnett (1978a, b) argued that the exclusive pion production reactions $\nu(\bar{\nu})N \to \nu(\bar{\nu})\pi N$, although difficult to treat theoretically, favored one solution, which corresponded to the neutral-current parameters of the WS-GIM model. More recent elastic scattering data confirmed this conclusion (Abbott and Barnett, 1979; Sidhu and Langacker, 1978; Paschos, 1979; Claudson *et al.*, 1979; Hendrick and Li,

[1] The difficulties of distinguishing between $S$, $T$, $P$ neutral current interactions and vector, axial-vector interactions has been extensively discussed by Kayser *et al.*, 1974, 1975; Fischbach *et al.*, 1977; Kayser, 1978; Cho and Gourdin, 1976; and Wu, 1979. The interference between weak and electromagnetic amplitudes observed in polarized electron-hadron scattering rules out a neutral current consisting entirely of $S$, $P$, and $T$. Small admixtures of $S$, $P$, and $T$ are much harder to rule out.

[2] A possibility of a nonvanishing charge radius of the neutrino was raised by Bernstein *et al.* (1963) and Bernstein and Lee (1963). Possible effects of the neutrino charge radius and the magnetic moment in neutral-current interactions were discussed by Kim *et al.* (1974) and Kim (1978).

[3] For reviews of this subject see Goldhaber, Langacker, and Slansky (1980) and Langacker (1980). Though not in a simple group, the first specific grand unified model which naturally has baryon nonconservation was proposed by Pati and Salam (1973, 1974).

1979; Williams, 1978): The hadronic neutral-current parameters are uniquely determined to lie in a small domain consistent with the WS-GIM model.[4] The parameters relevant to $\nu e$ and $eN$ scattering, though not uniquely determined, are consistent with the WS-GIM model.

The purpose of the present paper, in addition to incorporating the newest experimental results, is to perform a systematic analysis of all of the data with special emphasis placed on examining the reliability of the results.[5] Such an emphasis is needed if one is to place meaningful limits on small deviations from the WS-GIM model or determine an accurate value for $\sin^2\theta_w$. Therefore we shall examine the possible systematic errors in the various experiments and consider their effects on the fits. We shall use what we believe to be the best available theoretical expressions for each reaction involved; this includes using the quantum chromodynamics (QCD) parametrization of deep-inelastic scattering and the effects of heavy quarks wherever possible. We have tried to evaluate realistically the uncertainties in the theoretical expressions and input parameters and to estimate the resulting uncertainties in the fits. Finally we follow Roos and Liede (1979a, b) and Liede, Maalampi and Roos (1978) in performing a complete and correct statistical treatment of confidence levels. The minimizing program MINUIT (James and Roos, 1975) is used in all of the fits.

## B. Outline

The introduction will conclude with a summary of our notation for the neutral-current parameters. Section II is a summary of the experimental data we have used, as well as a critical discussion of the possible systematic uncertainties in the experiments. Section III is concerned with neutrino-hadron scattering. The theoretical expressions that we use for deep-inelastic reactions ($\nu N \to \nu + X$), semi-inclusive pion production ($\nu N \to \nu\pi X$), and elastic scattering ($\nu p \to \nu p$) are presented and their uncertainties are described. We concentrate especially on the theoretical interpretation of the deep-inelastic scattering, for which the most precise data are available. The neutral-current (NC) cross sections are parametrized using QCD expressions for the structure functions. The various parameters are chosen to fit the charged-current (CC) data and the sensitivity of the results to changes in the parameters is discussed. Simple parton model results are also given for comparison. Exclusive pion production is briefly discussed, although these reactions are not used in our fits because of the uncertainties in the theory. Parity violation in the nucleon-nucleon interaction is omitted because of the confusing experimental and theoretical situation. (See, for example, Fischbach and Tadic, 1973; Donnelly and Peccei, 1979; Korner, Kramer, and Willrodt, 1979; Desplanques, Donoghue, and Holstein, 1980; Fischbach, 1978; Ranft and Ranft, 1979; Cheng and Fischbach, 1979; Dubovik, Zamiralov and Zenkin, 1979.)

We then describe various model-independent fits to the hadronic data. Deep-inelastic scattering data from (approximately) isoscalar targets determines the magnitudes of the left- and right-helicity neutral-current couplings, but yields almost no information on the isospin structure of the neutral current. Inclusion of existing deep-inelastic data from proton and neutron targets determines the isospin of the left-handed current to within a twofold ambiguity, but does not significantly constrain the right-handed isospin. When the data on semi-inclusive pion production are also incorporated, the couplings are constrained to lie in one of four regions, the analog of Sehgal's (1977a) four solutions. Finally, when elastic scattering data are combined with the other constraints, the hadronic neutral-current parameters are determined to lie in one of two allowed domains. One of these solutions is predominantly isoscalar and is incompatible with exclusive pion production data. The remaining unique solution is in the region predicted by the WS-GIM model. We also discuss the sensitivity of these fits to theoretical and experimental uncertainties, display the results of fits to deep-inelastic and elastic data alone, and describe the implications of exclusive pion production and $\bar{\nu}_e D \to \bar{\nu}_e np$ data Pasierb *et al.*, 1979) (which are not directly included in the fits). Finally, we show that the results are not substantially changed even if one takes the matrix element of the axial-vector isoscalar current between proton states to be a completely free parameter, as suggested by Wolfenstein (1979).

In Sec. IV we consider neutrino-electron scattering. As is now well known, there are two possible solutions for the neutral-current parameters: a dominantly axial-vector solution, as in the WS model of leptons (Schwinger, 1957; Bludman, 1958; Glashow, 1961; Salam and Ward, 1964; Weinberg, 1967, 1971; Salam, 1969; Glashow, Iliopoulos, and Maiani, 1970) and a dominantly vector solution, such as would occur in an $SU(2)\times U(1)$ model with the right-handed electron in a doublet. Section V is devoted to the electron-hadron neutral current, which is relevant to atomic parity violation experiments and to the SLAC polarized electron-hadron scattering experiment. The constraints from the SLAC experiment and the theoretical uncertainty are discussed.

Section VI is devoted to simultaneous fits to all of the data within specific models. We first consider generalized $SU(2)\times U(1)$ models. We find the following upper limits for mixing betweeen right-handed singlets and doublets:

$$\sin^2\alpha_u < 0.103, \quad \sin^2\alpha_d < 0.348, \quad \sin^2\alpha_e < 0.064.$$

We then assume these mixing angles to be zero and perform two-parameter fits to the overall strength parameter $\rho = M^2_W/M^2_Z\cos^2\theta_w$ and $\sin^2\theta_w$. Fits to the most accurate experiments, namely deep-inelastic scattering and the SLAC polarized electron-deuteron experiment, yield $\rho = 0.992 \pm 0.017$ $(\pm 0.011)$ and $\sin^2\theta_w = 0.224 \pm 0.015$ $(\pm 0.012)$, where the numbers in parentheses are the the-

[4]See also Rizzo and Mathur, 1978a, b, 1979; Gourdin and Pham, 1979; Uchiyama, 1979; Iwao, 1979; Konuma and Oka, 1977b; Bailin and Dombey, 1978; Kim, 1978; Petcov, 1978; Hung and Sakurai, 1979.

[5]A preliminary version of this work is Langacker *et al.* (1979). Small changes in the results are due to the inclusion of more recent data in the present analysis.

oretical uncertainties. The value of $\rho$ is remarkably close to 1 and strongly suggests that the Higgs mesons occur only in SU(2) doublets and singlets. If one assumes that the unrenormalized value of $\rho$ is 1.0, then from the limit on the renormalized value of $\rho$ (Veltman, 1977; Chanowitz, Furman, and Hinchliffe, 1978) one can set a limit of $m_L < 500$ GeV on the mass of any charged heavy lepton with a massless partner. A two-parameter fit that also includes the semi-inclusive, elastic, and leptonic data yields the similar values $\rho = 1.002 \pm 0.015$ ($\pm 0.011$) and $\sin^2\theta_w = 0.234 \pm 0.013$ ($\pm 0.009$). Finally, we perform one-parameter fits to $\sin^2\theta_w$ (with $\rho$ constrained to be 1.0). We obtain $\sin^2\theta_w = 0.229 \pm 0.009$ ($\pm 0.005$) from the deep-inelastic and SLAC data alone, and $\sin^2\theta_w = 0.233 \pm 0.009$ ($\pm 0.005$) from a fit to all of the data.

We then proceed to consider factorization constraints and tests (factorization would hold in any gauge model involving a single $Z$ boson). In agreement with Hung and Sakurai (1977a, 1979) we find that if factorization holds then the axial-dominant solution for neutrino-electron scattering is uniquely determined and that the other neutral-current experiments are incompatible with null atomic parity violation experiments. These conclusions are true for any single $Z$ boson model and do not require any *a priori* assumption concerning the strength of the neutrino-$Z$ boson coupling. We also evaluate certain factorization tests proposed by Bernabeu and Jarlskog (1977) and others (Gupta and Paranjape, 1977; Liede, Maalampi, and Roos, 1978; Sidhu, 1979; Mohapatra and Sidhu, 1979; Maalampi, Roos and Liede, 1979). We find no evidence for a violation of factorization, but the constraints are not especially stringent.

In the last part of Sec. VI we investigate limits on the existence of a second $Z$ boson. Since there is *a priori* no information on the nature of the couplings of such a boson, it is necessary to use specific models in which the additional couplings are described by one or more free parameters. We consider two such models: a model based on $SU(2)_L \times (T_3)_R \times U_Y(1)$ owing to Deshpande and Iskandar (1979, 1980) and an $SU(2)_L \times SU(2)_R \times U(1)$ model.[6] We find lower limits for the mass of the second $Z$ boson of $M_{Z_2}/M_{Z_1} \geq 1.61$ and 3.44 for the two models, respectively. Section VII is a summary and discussion of our results and of outstanding problems.

## C. Notation

Let us define the effective Lagrangian for low-energy neutral-current interactions as[7]

$$L = L^{\nu H} + L^{\nu e} + L^{eH} + L^{e\mu} + \cdots, \tag{1.1}$$

where $L^{\nu H}$ contains the terms relevant for neutrino-hadron scattering, etc. For $L^{\nu H}$ we take[8]

$$L^{\nu H} = -\frac{G_F}{\sqrt{2}}\, \bar{\nu}\gamma^\mu(1+\gamma_5)\nu J^H_\mu , \tag{1.2}$$

where the hadronic neutral current is given by

$$\begin{aligned} J^H_\mu &= \sum_i [\varepsilon_L(i)\bar{q}_i\gamma_\mu(1+\gamma_5)q_i] \\ &\quad + [\varepsilon_R(i)\bar{q}_i\gamma_\mu(1-\gamma_5)q_i] \\ &= \sum_i \bar{q}_i\gamma_\mu(g^i_V + g^i_A\gamma_5)q_i . \end{aligned} \tag{1.3}$$

The sum extends over the quark flavors (i.e., $q_i = u, d, s, c, \ldots$). The vector and axial-vector couplings $g^i_{V,A}$ are related to the chiral couplings $\varepsilon^{(i)}_{L,R}$ by

$$\begin{aligned} g^i_V &= \varepsilon_L(i) + \varepsilon_R(i) \\ g^i_A &= \varepsilon_L(i) - \varepsilon_R(i) . \end{aligned} \tag{1.4}$$

Equation (1.2) actually implies a sum over neutrino types. In performing fits, we shall always assume $\varepsilon_{L,R}(s) = \varepsilon_{L,R}(d)$ and $\varepsilon_{L,R}(c) = \varepsilon_{L,R}(u)$, which is strongly suggested but not rigorously proven by the experimental absence of significant flavor-changing neutral-current effects.

We shall occasionally use the alternate notation

$$\begin{aligned} J^H_\mu &= \tfrac{1}{2}\alpha(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) + \tfrac{1}{2}\beta(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d) \\ &\quad + \tfrac{1}{2}\gamma(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) + \tfrac{1}{2}\delta(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d) + \cdots \\ &= \alpha V^3_\mu + \beta A^3_\mu + \gamma V^0_\mu + \delta A^0_\mu + \cdots , \end{aligned} \tag{1.5}$$

where $V^3_\mu (V^0_\mu)$ is the isovector (isoscalar) vector current, and similarly for $A^3_\mu(A^0_\mu)$. The dots refer to heavy quark terms. The two sets of couplings are related by

$$\begin{aligned} \varepsilon_L(u) &= \tfrac{1}{4}(\alpha + \beta + \gamma + \delta) , \\ \varepsilon_R(u) &= \tfrac{1}{4}(\alpha - \beta + \gamma - \delta) , \\ \varepsilon_L(d) &= \tfrac{1}{4}(-\alpha - \beta + \gamma + \delta) , \\ \varepsilon_R(d) &= \tfrac{1}{4}(-\alpha + \beta + \gamma - \delta) . \end{aligned} \tag{1.6}$$

Similarly, we write

$$L^{\nu e} = -\frac{G_F}{\sqrt{2}}\, \bar{\nu}\gamma_\mu(1+\gamma_5)\nu J^e_\mu , \tag{1.7}$$

where

$$\begin{aligned} J^e_\mu &= \varepsilon_L(e)\bar{e}\gamma_\mu(1+\gamma_5)e + \varepsilon_R(e)\bar{e}\gamma_\mu(1-\gamma_5)e \\ &= \bar{e}\gamma_\mu(g^e_V + g^e_A\gamma_5)e \end{aligned} \tag{1.8}$$

and where

$$g^e_{V,A} = \varepsilon_L(e) \pm \varepsilon_R(e) . \tag{1.9}$$

For electron-hadron interactions, the parity-violating interaction is given by[9]

$$\begin{aligned} L^{eH} &= -\frac{G_F}{\sqrt{2}} \sum_i (C_{1i}\bar{e}\gamma_\mu\gamma_5 e\bar{q}_i\gamma_\mu q_i + C_{2i}\bar{e}\gamma_\mu e\bar{q}_i\gamma_\mu\gamma_5 q_i) \\ &= -\frac{G_F}{\sqrt{2}} [\bar{e}\gamma_\mu\gamma_5 e(\tilde{\alpha}V^3_\mu + \tilde{\gamma}V^0_\mu) + \bar{e}\gamma_\mu e(\tilde{\beta}A^3_\mu + \tilde{\delta}A^0_\mu) + \cdots] , \end{aligned} \tag{1.10}$$

where again $q_i = u, d, s, c, \ldots$. The two sets of couplings are related by

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[6]We use the parametrization of Mohapatra (1978) and Sidhu (1979b). Other recent studies of the neutral current in $SU(2)_L \times SU(2)_R \times U(1)$ models include Liede, Maalampi, and Roos, 1978; Pati, Rajpoot, and Salam, 1978; Rizzo and Sidhu, 1980. These articles contain references to the original $SU(2)_L \times SU(2)_R \times U(1)$ papers.

[7]Our conventions for $\gamma$ matrices, etc. are the same as those of Bjorken and Drell (1965), except we normalize our spinors to $\bar{u}u = 2M$ and choose the opposite sign for $\gamma_5$.

[8]We use the value $G_F M_p^2 = 1.027 \times 10^{-5}$ given in the compilation of Nagels *et al.* (1979).

[9]Some authors define $C_{1i}$ and $C_{2i}$ with a sign that is opposite of ours.

$$C_{1u}=\tfrac{1}{2}(\tilde{\alpha}+\tilde{\gamma})\,,\quad C_{2u}=\tfrac{1}{2}(\tilde{\beta}+\tilde{\delta})\,,$$
$$C_{1d}=\tfrac{1}{2}(-\tilde{\alpha}+\tilde{\gamma})\,,\quad C_{2d}=\tfrac{1}{2}(-\tilde{\beta}+\tilde{\delta})\,. \tag{1.11}$$

Finally, the effective interaction for $e^+e^-\to\mu^+\mu^-$ is

$$\begin{aligned} L^{e\mu}=-\frac{G_F}{\sqrt{2}}\,[&h_{VV}(\bar{e}\gamma_\mu e+\bar{\mu}\gamma_\mu\mu)(\bar{e}\gamma^\mu e+\bar{\mu}\gamma^\mu\mu)\\ &+2h_{VA}(\bar{e}\gamma_\mu e+\bar{\mu}\gamma_\mu\mu)(\bar{e}\gamma^\mu\gamma_5 e+\bar{\mu}\gamma^\mu\gamma_5\mu)\\ &+h_{AA}(\bar{e}\gamma_\mu\gamma_5 e+\bar{\mu}\gamma_\mu\gamma_5\mu)(\bar{e}\gamma^\mu\gamma_5 e+\bar{\mu}\gamma^\mu\gamma_5\mu)]\,, \end{aligned} \tag{1.12}$$

where $\mu-e$ universality has been assumed. The interaction relevant for $e^+e^-\to\tau^+\tau^-$ is obtained from Eq. (1.12) by replacing $\mu$ by $\tau$ (again assuming $\mu-e-\tau$ universality).

It must be emphasized that the effective Lagrangians are only valid so long as the momentum transfers (or the center of mass energy for $e^+e^-\to\mu^+\mu^-$) are small compared to $Z$ boson mass(es). The parameters defined in this section are independent of specific weak-interaction models. Their expressions in terms of the parameters of specific models will be given in Sec. VI.

## II. NEUTRAL-CURRENT EXPERIMENTS

In this section we summarize the data[10] used to determine the structure of the neutral current and try also to include some discussion of the systematic uncertainty in each experiment. While there is clearly no substitute for reading the original papers, it was thought useful to have a summary of the systematic errors as well as the nominal results. For earlier reviews, see Amaldi, 1979; Baltay, 1979a, b; Dydak, 1979; Musset, 1979; Sakurai, 1978, 1979; and Winter, 1979.

### A. Deep-inelastic scattering from isoscalar targets

Since the initial discovery of neutral-current interactions, a large number of experiments, using both bubble chamber and counter techniques, have studied inclusive neutral-current interactions from isoscalar (or primarily isoscalar) targets. To date the primary neutral-current quantities that have been measured are the NC/CC ratios

$$R_\nu=\frac{\nu N\to\nu X}{\nu N\to\mu^- X}\,,\quad R_{\bar{\nu}}=\frac{\bar{\nu}N\to\bar{\nu}X}{\bar{\nu}N\to\mu^+X}\,, \tag{2.1}$$

and the differential cross sections with respect to the hadron energy $d\sigma/dy(\nu N\to\nu X)$ and $d\sigma/dy(\bar{\nu}N\to\bar{\nu}X)$, where $y=E_h/E_\nu$, $E_h$ is the hadron energy, and $E_\nu$ is the incident $\nu$ energy. [Recently, preliminary results (Mess, 1979) on the $x$ distribution in neutral-current interactions have been reported for the first time.] Currently there exist five high-energy experiments (i.e., $\langle E_\nu\rangle>50$ GeV), performed by the HPWF (Wanderer *et al.*, 1978), CITF (Merritt *et al.*, 1978), CDHS (Geweniger, 1979), BEBC (Deden *et al.*, 1979), and CHARM (Mess, 1979) collaborations, that have presented measurements of at least moderately high precision. We choose these as input to the analysis; the results are presented in Table I. While the measurements by GGM (Hasert *et al.*, 1979), BNL (Cazzoli *et al.*, 1977), and other groups provide valuable information on any energy dependence, the theoretical uncertainties inherent in the interpretation of those measurements are significantly larger than for the high-energy experiments.

The primary corrections to the neutral-current measurements consist of (1) charged-current (CC) events misidentified as neutral current (NC), (2) background from $\nu_e$ interactions, and (3) for the narrow-band beam experiments, background from $\nu$ and $\bar{\nu}$ produced prior to the momentum selection of the parent pions and kaons. In addition, for the bubble-chamber experiments the background from $n$ interactions must be dealt with, though generally kinematic cuts are made to reduce this background to a small level. These corrections are summarized for the CDHS and CHARM experiments in Table II, along with the experimenters' estimate of the statistical and total error in $R_\nu$ and $R_{\bar{\nu}}$.

For the determination of $R_\nu$ (the value of which largely determines $\sin^2\theta_w$), the dominant backgrounds of the CDHS and CHARM experiments (which quote the most precise results) are misidentified CC events and $\nu_e$ interactions. Background from the former is determined in the CDHS experiment by the measured rate for CC events and an extrapolation that is particularly model independent; the experimenters estimate an uncertainty in this background of $\approx 5\%$. Uncertainty in the number of $\nu_e$ interactions results primarily from the uncertainty in the $K/\pi$ ratio, estimated to be $\pm 10\%$. These systematic uncertainties (and other lesser ones) are combined in quadrature to yield for the CDHS experiment a total systematic error in $R_\nu$ of $\pm 0.005$ which is added linearly to the statistical error.

For the CHARM experiment, the uncertainty in the backgrounds from wide-band-beam (WBB) events, misidentified CC events, and $\nu_e$ interactions are small compared to the uncertainty produced by a class of events ambiguous between NC and CC; combining the systematic variation of $R_\nu$ that is obtained by alternately ascribing these ambiguous events to NC or CC with uncertainties in the other backgrounds yields a total systematic error in $R_\nu$ of $\pm 0.02$.

For the determination of $R_{\bar{\nu}}$, the largest correction for the CDHS experiment is the subtraction of wide-band-beam background (30.4% of the neutral-current sample) while for the CHARM experiment the dominant uncertainty is again produced by events ambiguous between NC and CC.

Both the CDHS and CHARM experiments (as well as HPWF, CITF, and BEBC) have made reasonably careful estimates of the systematic uncertainties, which in the case of the first two experiments dominate the statistical error for both $R_\nu$ and $R_{\bar{\nu}}$. That these estimates are reasonable is best justified by the excellent agreement between the five experiments.

To compare the measured values of $R_\nu$ and $R_{\bar{\nu}}$ with a particular theoretical model, or to abstract the coupling constants in a model-independent way, two addi-

[10] For additional reviews of the experimental results on neutral current interactions see Amaldi, 1979; Baltay, 1979a, b; Dydak, 1979; Musset, 1979; Sakurai, 1978, 1979; and Winter, 1979.

TABLE I. Measurements of $R_\nu$ and $R_{\bar\nu}$ from the five high-energy experiments ($\langle E_\nu\rangle \gtrsim 50$ GeV) studying deep-inelastic $\nu$ scattering from an (approximately) isoscalar target. Small differences in the target composition and in the hadron energy cut lead to slightly different expected results as discussed in Sec. III. For the CDHS and CHARM experiments, the number in parentheses indicates the size of the statistical error. For the HPWF experiment only, the numbers presented have been corrected for the energy cut (by the experimenters).

| Experiment | Target | Hadron energy cut (GeV) | $R_\nu$ | $R_{\bar\nu}$ |
|---|---|---|---|---|
| HPWF (Wanderer *et al.*, 1978) | $CH_2$ | 4 | $0.30 \pm 0.04$ | $0.33 \pm 0.09$ |
| CITF (Merritt *et al.*, 1978) | Iron | 12 | $0.28 \pm 0.03$ | $0.35 \pm 0.11$ |
| BEBC (Deden *et al.*, 1979) | $H_2$-Neon | 15 | $0.32 \pm 0.03$ | $0.39 \pm 0.07$ |
| CDHS (Geweniger *et al.*, 1979) | Iron | 10 | $0.307 \pm 0.008$ ($\pm 0.003$) | $0.373 \pm 0.025$ ($\pm 0.014$) |
| CHARM (Mess *et al.*, 1979) | Marble | 2, 10, 17 | $0.30 \pm 0.021$ ($\pm 0.006$) | $0.39 \pm 0.024$ ($\pm 0.014$) |

tional effects must be accounted for: (1) Each experiment imposes a minimum hadron energy cut, $E_h > E_0$ and (2) the targets used typically deviate slightly from pure isoscalar. Within a given theoretical framework, e.g., the quark-parton model or QCD, and with specific assumptions concerning $\bar q/q$, the fraction of $s$ quarks, etc., the correction for these effects is unambiguous. Thus any uncertainty relating to these is considered to be theoretical rather than experimental and is discussed in Sec. III.

Each of the experiments has studied the distribution of neutral-current events as a function of the hadronic energy $E_h$ or $y = E_h/E_\nu$; we show recent measurements of the CHARM experiment in Fig. 1. Information on the $y$ distribution yields additional constraints on the space-time structure.

In the event that $S$, $P$, *and* $T$ interactions are present, it is not possible to distinguish such contributions from those of vector, $V$, and axial-vector, $A$, currents. Hence it is customary to assume *no* tensor contributions, in which case the $y$ distribution may be parametrized by

$$\frac{d\sigma}{dy}(\nu N \to \nu X) = \frac{G_F^2 ME}{\pi}\,[h_1 + h_2(1-y)^2 + h_3 y^2]\,,$$
$$\frac{d\sigma}{dy}(\bar\nu N \to \bar\nu X) = \frac{G_F^2 ME}{\pi}\,[h_2 + h_1(1-y)^2 + h_3 y^2]\,, \qquad (2.2)$$

where $h_3$ measures the contribution from $S$ and $P$ interactions and $h_1$ and $h_2$ are determined by the $V$ and $A$ interactions. Given the existence of scaling violations, $h_1$ and $h_2$ are in fact average values of slowly varying functions of $y$. While each of the high-energy experiments excludes an interaction that is dominantly $S$ and $P$, the most restrictive limits come from the CDHS (Holder *et al.*, 1977a), and BEBC (Deden *et al.*, 1979) experiments: $h_3/(h_1+h_2) < 0.16$ and 0.12, respectively, at 95% confidence level. It should be noted, however, that the limits on the actual strength of an $S$ or $P$ interaction are much weaker than those implied by the

TABLE II. Summary of the primary backgrounds in the neutral-current sample for the CDHS and CHARM experiments. The systematic uncertainty for the CHARM experiment is dominated by events ambiguous between neutral current (NC) and charged current (CC).

| | $\nu$ | | $\bar\nu$ | |
|---|---|---|---|---|
| NC candidates | CHARM 9200 | CDHS 21 724 | CHARM 2700 | CDHS 4539 |
| WBB background | −0.8% | −3.9% | −4.4% | −30.4% |
| CC background | −6.7% | −17.8% | −2.4% | −7.1% |
| $\nu_e$ background | −7.0% | −11.6% | −2% | −5.7% |
| $R_\nu, R_{\bar\nu}$ | 0.30 | 0.307 | 0.39 | 0.373 |
| Statistical error | ± 0.006 | ± 0.003 | ± 0.014 | ± 0.014 |
| Systematic error | ± 0.02 | ± 0.005 | ± 0.02 | ± 0.011 |
| Total error [a] | ± 0.021 | ± 0.008 | ± 0.024 | ± 0.025 |

[a] The CHARM experiment does not present a total error; we have therefore combined their statistical and systematic errors in quadrature. The CDHS experiment, more conservatively, adds their statistical and systematic errors linearly to obtain the combined error.

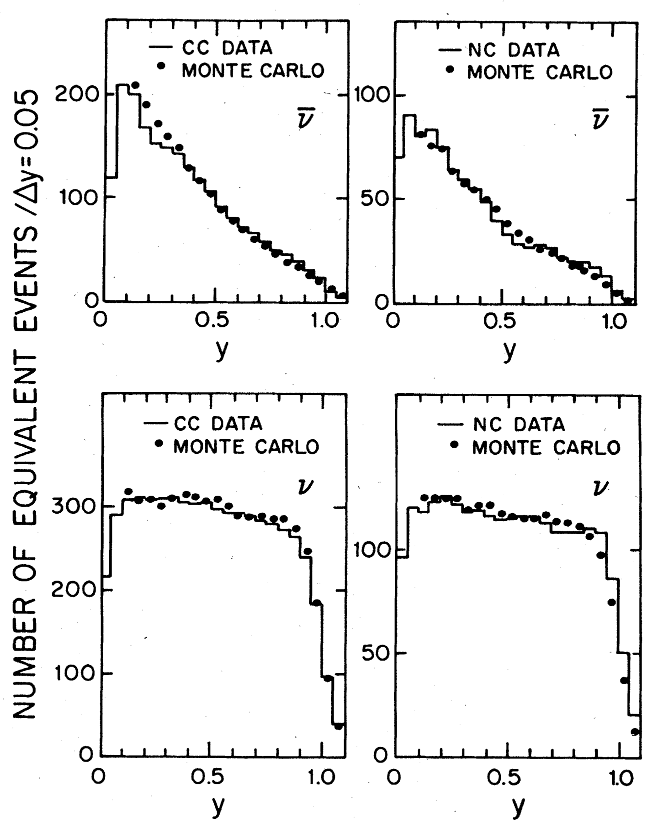


FIG. 1. Measurements of the $y$ distribution ($y = E_h/E_\nu$) for deep-inelastic $\nu(\bar\nu)$ scattering. Data are presented for both neutral-current (NC) and charged-current (CC) interactions. The Monte Carlo results assume the WS-GIM model with $\sin^2\theta_w = 0.25$.

limits on $h_3$. Defining $g_S(u)$, $g_S(d)$, $g_P(u)$, $g_P(d)$ analogously to $g_V(u)$, $g_V(d)$, etc., the BEBC group obtains

$$\frac{g_S(u)^2+g_S(d)^2+g_P(u)^2+g_P(d)^2}{g_V(u)^2+g_V(d)^2+g_A(u)^2+g_A(d)^2}<0.50 \tag{2.3}$$

at 90% confidence level.

To determine constraints on the left-handed coupling $g_L=[\varepsilon_L^2(u)+\varepsilon_L^2(d)]^{1/2}$ and right-handed coupling $g_R=[\varepsilon_R^2(u)+\varepsilon_R^2(d)]^{1/2}$ from information on the $y$ distribution $d\sigma/dy$ it is necessary to prescribe the composition of antiquarks in the nucleon as is discussed in Sec. III and to understand systematic effects as a function of $y$. The latter are more difficult to estimate than for the determination of $R_\nu$ and $R_{\bar\nu}$; hence, at this time we do not use information on the shape of the $y$ distribution to provide additional constraints on the coupling constants.[11] It should be noted that if a fit is performed simultaneously to the shape and magnitude of the $y$ distribution, the resulting information is *not* independent of the measurement of $R_\nu$ and $R_{\bar\nu}$.

### B. Deep-inelastic scattering from $n$ and $p$ targets

There now exist several measurements of deep-inelastic $\nu$ ($\bar\nu$) scattering from proton and neutron targets. The data are usually presented in terms of the ratio of neutral-current to charged-current cross sections,

$$R_\nu^p=\frac{\nu p\to\nu X}{\nu p\to\mu^- X}\,,\quad R_{\bar\nu}^p=\frac{\bar\nu p\to\bar\nu X}{\bar\nu p\to\mu^+ X}\,, \tag{2.4}$$

or in terms of the ratio of neutral-current cross sections for neutron and proton targets,

$$R_\nu^{n/p}=\frac{\nu n\to\nu X}{\nu p\to\nu X}\,,\quad R_{\bar\nu}^{n/p}=\frac{\bar\nu n\to\bar\nu X}{\bar\nu p\to\bar\nu X}\,. \tag{2.5}$$

Results of the first category include

$R_\nu^p=0.52\pm0.04$, BEBC (Blietschau *et al.*, 1979)[12]

$R_\nu^p=0.48\pm0.17$, FNAL 15′ BC (Harris *et al.*, 1977)

$R_{\bar\nu}^p=0.42\pm0.13$, FNAL 15′ BC (Derrick *et al.*, 1978).

All of the measurements have been obtained from bubble-chamber experiments and suffer from nontrivial problems of muon identification and neutron background. As the results of the BEBC group are the most recent and the most accurate, we limit a more detailed discussion to that experiment.

The initial sample of events, selected with the cut $E_H>5$ GeV, contains a contamination from misidentified charged-current events and neutron-induced events that is slightly larger than the number of genuine neutral-current events. This background may be greatly reduced, however, by selecting only events for which the total transverse momentum of the hadrons relative to the beam direction is large, e.g., $p_T^H>p_T^{\min}=1.5$ GeV/$c$. The experimenters estimate corrections to this sample to be small and quote $R_\nu^p=0.52\pm0.04$ where the statistical and systematic errors each contribute ±0.03. They also present $R_\nu^p$ as a function of $p_T^{\min}$; while the results for different values of $p_T^{\min}$ all agree within the errors, there is a variation of approximately 0.04 in the value of $R_\nu^p$ obtained (see Fig. 2). To be on the conservative side we therefore use the result $R_\nu^p=0.52\pm0.06$ in our analysis.

Measurements of $R_\nu^{n/p}$ and $R_{\bar\nu}^{n/p}$ include

$R_\nu^{n/p}=1.22\pm0.35$, FNAL 15′ BC (Marriner, 1977)

$R_{\bar\nu}^{n/p}=1.06\pm0.20$, FIIM 15′ BC (Bell *et al.*, 1979)[13]

where again both results derive from bubble-chamber experiments with the attendant problems of muon identification and neutron background. The latter experiment eliminates backgrounds from charged-current events by selecting only those events where all tracks interact. However, the experiment is performed using a neon target. The total observed hadronic charge is smeared considerably, due to nuclear effects, from the values 0 and 1 expected for interactions from $n$ and $p$ targets, respectively. A technique for unfolding the observed distribution is used to determine the number of interactions from $n$ and $p$ targets. The quoted error of 0.2 is dominated by systematic uncertainties.

### C. Semi-inclusive pion production

One of the earliest pieces of information on the isospin of the neutral current came from measurements of the $\pi^+/\pi^-$ ratio for inclusive pion production:

$$R_\nu^{+/-}=\frac{\nu N\to\nu\pi^+X}{\nu N\to\nu\pi^-X}\,,\quad R_{\bar\nu}^{+/-}=\frac{\bar\nu N\to\bar\nu\pi^+X}{\bar\nu N\to\bar\nu\pi^-X}\,. \tag{2.6}$$

Knowledge of the quark fragmentation function, i.e., the probability that a given type of quark, $u$ or $d$, produces a $\pi^+$ or $\pi^-$, allows one to extract from the above measurements information on the relative strength of the couplings to $u$ and $d$ quarks. The earliest results obtained in the Gargamelle experiment (Kluttig, Morfin,

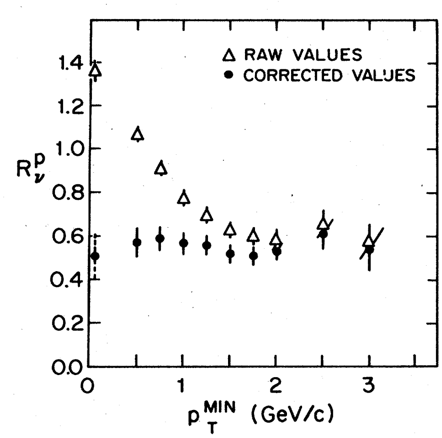


FIG. 2. Raw and corrected values of $R_\nu^p$ as a function of $p_t^{\min}$, the minimum transverse momentum of the hadrons with respect to the direction of the incident neutrinos.

[11]It is anticipated that both the CDHS and CHARM experiments will have completed soon their analysis of the $y$ distributions.

[12]Since our analysis was performed, the value reported by this group for $R_\nu^p$ has changed from 0.52 to 0.51. This change has no significant effect on the results we obtain.

[13]Note that the beam employed in this measurement has a mixture of $\bar\nu$: $\nu$ equal to 4.4:1.

and Van Doninck, 1977) at low energy ($E_\nu \approx 1$–5 GeV) have been supplemented by a recent measurement by the Fermilab-IHEP-ITEP-Michigan (FIIM) group (Roe, 1979; Bell *et al.*, 1979) using high-energy antineutrinos ($E_{\bar\nu} > 20$ GeV):

$$R_\nu^{+/-} = 0.77 \pm 0.14\,, \quad \text{GGM (Kluttig } et\ al.\text{, 1977)}$$

$$R_{\bar\nu}^{+/-} = 1.65 \pm 0.33\,, \quad \text{GGM (Kluttig } et\ al.\text{, 1977)}$$

$$= 1.27^{+0.36}_{-0.27}\,, \quad \text{FIIM (Roe, 1979)}$$

Both experiments were performed in heavy-liquid bubble-chamber experiments; in each case, the rather stringent kinematical cuts imposed to select pions in the "current fragmentation" region largely eliminate background from hadron-induced events, and the dominant uncertainties arise from (rather severe) ambiguities in the $\pi/K/p$ separation and from the effects of nuclear interactions. The fraction of the $\pi$ sample due to $p$ in the low-energy experiment and $K$ and $p$ in the high-energy experiment was evaluated by Monte Carlo calculations. The quoted errors on the Gargamelle result include a 11–14% uncertainty due to the nuclear corrections but no systematic error in the $p$ subtraction; the FIIM result explicitly includes a systematic uncertainty of approximately $\pm 0.04$ due to the $K/p$ subtraction. Perhaps the largest uncertainty in the use of these results to determine the isospin of the neutral current concerns the assumption that the pion production is dominated by quark fragmentation. This assumption is particularly dubious in the Gargamelle energy region, but it has also been pointed out recently (Musgrave, 1979) that even at high energies the data does not satisfy the factorization relations expected unless stringent cuts on $W$ and $Q^2$ are employed.

### D. Elastic scattering from protons

The elastic scattering of neutrinos and antineutrinos from protons, $\nu p \to \nu p$ and $\bar\nu p \to \bar\nu p$, are the simplest semileptonic neutral-current interactions to interpret theoretically, other than the deep-inelastic; it is therefore not surprising that data on these reactions have played an important role in determining the structure of the hadronic neutral current: When these reactions, which involve scattering from protons, are combined with the inelastic scattering from an isoscalar target, they yield important isospin information. In addition, they provide an independent measure of the strength of the interaction, with a different set of systematic errors than the inclusive measurements.

Since the initial measurements of the Columbia-Illinois-Rockefeller (Lee *et al.*, 1977a; Sokolsky, 1978) and Harvard-Pennsylvania-Wisconsin (Cline *et al.*, 1976) experiments, two other measurements of $\nu p \to \nu p$ have been reported, one by the Gargamelle group (Pohl *et al.*, 1978) in a bubble chamber, and the other by Aachen-Padova (AP) (Faissner *et al.*, 1980) in a counter experiment. The recent measurements by the Harvard-Pennsylvania-Brookhaven (HPB) collaboration (Entenberg *et al.*, 1979; Kozanecki, 1978; Strait, 1978) have the highest statistics and remain the most accurate, though the Columbia-Illinois-Brookhaven (CIB) group should have results of comparable accuracy available soon. The results for

$$R_\nu^{\rm el} = \frac{\nu p \to \nu p}{\nu n \to \mu^- p} \quad \text{and} \quad R_{\bar\nu}^{\rm el} = \frac{\bar\nu p \to \bar\nu p}{\bar\nu p \to \mu^+ n} \tag{2.7}$$

are summarized in Table III.

Potential systematic uncertainties arise from (1) $n$-induced background, (2) background from $\nu n \to \nu n\pi$ and $\nu n \to \nu n$, (3) normalization relative to the quasielastic reaction $\nu n \to \mu^- p(\bar\nu p \to \mu^+ n)$, and (4) nuclear effects. Both the HPB and CIB experiments have virtually eliminated $n$ background, while it remains a large correction for the GGM and AP experiments. Background from $\nu n \to \nu n$ is estimated to be small (10–15%) by all the groups except AP, which reports a large correction. The effect of interactions in the target nucleus is estimated to be small ($\approx 10\%$).

We include in the fits only the data from the HPB experiment which are virtually free of $n$ background, and include measurements of the differential cross section for both $\nu$ and $\bar\nu$ (Fig. 3). The systematic error is estimated to be approximately 25% and is dominated by the uncertainties in normalization, $\nu n \to \nu n\pi$ background subtraction, and detection efficiency. We combine this error in quadrature with the statistical error for each of the measured points.

### E. Single-pion production

Measurements of the cross sections for single-pion production

$$\begin{aligned} &\overset{(-)}{\nu} p \to \overset{(-)}{\nu} p\pi^0\,, \\ &\overset{(-)}{\nu} p \to \overset{(-)}{\nu} n\pi^+\,, \\ &\overset{(-)}{\nu} n \to \overset{(-)}{\nu} p\pi^-\,, \\ &\overset{(-)}{\nu} n \to \overset{(-)}{\nu} n\pi^0\,, \end{aligned} \tag{2.8}$$

TABLE III. Summary of results on the elastic scattering of neutrinos and antineutrinos from protons.

| Experiment | $\nu p \to \nu p$ Events observed | Background | $R_\nu^{\rm el}$ | $\bar\nu p \to \bar\nu p$ Events observed | Background | $R_{\bar\nu}^{\rm el}$ |
|---|---|---|---|---|---|---|
| HPB (Entenberg *et al.*, 1979; Kozanecki, 1978; Strait, 1978) | 217 | 82 | $0.11 \pm 0.02$ | 66 | 28 | $0.19 \pm 0.05$ |
| CIR (Lee *et al.*, 1977a; Sokolsky, 1978) | 71 | 30 | $0.20 \pm 0.06$ | | | |
| AP (Faissner *et al.*, 1980) | 217 | 155 | $0.10 \pm 0.03$ | | | |
| GGM (Pohl *et al.*, 1978) | 100 | 62 | $0.12 \pm 0.06$ | | | |

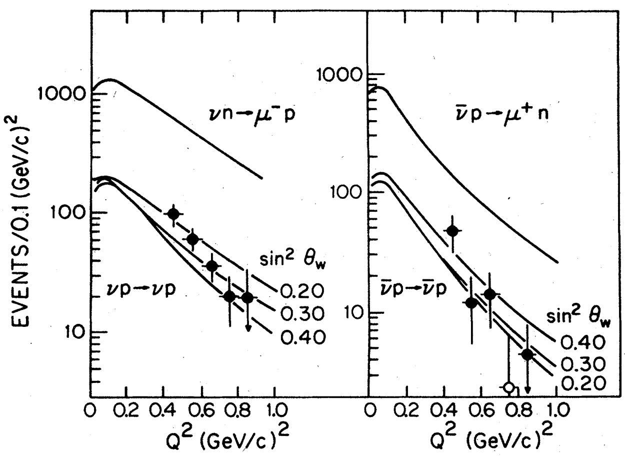


FIG. 3. Measurements by the Harvard-Pennsylvania-Brookhaven group of the differential cross section for $\nu p \to \nu p$ and $\bar\nu p \to \bar\nu p$. The curves show the prediction of the WS-GIM model.

can in principle yield strong constraints on the isospin of the weak neutral currents; however, the existence of significant nuclear corrections and theoretical uncertainties, even within the context of the gauge models, renders uncertain the extent to which one can rely quantitatively on theoretical predictions. For this reason we have not included data on these reactions in our fit to determine the coupling constants.

Measurements on reactions (2.8) do play an important role in excluding a dominantly isoscalar neutral current and require the existence of some isoscalar-isovector interference. Three particularly relevant experiments are the following

(1) A measurement of the $\pi^0/\pi^-$ ratio for inclusive pion production from a freon target (Bertrand-Coremans *et al.*, 1976) yields

$$\frac{\nu N \to \nu\pi^0 N'}{\nu N \to \nu\pi^- N'} = 1.4 \pm 0.2$$

and

$$\frac{\bar\nu N \to \bar\nu\pi^0 N'}{\bar\nu N \to \bar\nu\pi^- N'} = 2.1 \pm 0.4\,.$$

These results differ significantly from the ratio 0.9 which is expected for a purely isoscalar current and a freon target.

(2) Krenz *et al.* (1978) have measured the relative cross sections (in arbitrary units) for all four $\nu$-induced single-pion reactions. They obtain

$$\sigma(\nu p \to \nu p\pi^0) = 297 \pm 37\,,$$

$$\sigma(\nu p \to \nu n\pi^+) = 180 \pm 31\,,$$

$$\sigma(\nu n \to \nu n\pi^0) = 177 \pm 43\,,$$

$$\sigma(\nu n \to \nu p\pi^-) = 237 \pm 59\,.$$

The experimenters obtain a confidence level of $<10^{-4}$ when these measurements are fit to the prediction for a pure isoscalar current: $\sigma(p\pi^0):\sigma(n\pi^+):\sigma(n\pi^0):\sigma(p\pi^-) = 1:2:1:2$. Furthermore, the nonzero values of

$$N(p\pi^0) - N(n\pi^0) = 120 \pm 60\,,$$

$$N(p\pi^-) - N(n\pi^+) = 57 \pm 66\,,$$

imply that there must exist an isoscalar-isovector interference term. Finally, the observation of a clear $\Delta(1238)$ peak (Fig. 4) in the $N\pi$ effective-mass distributions also indicates that the neutral current must be dominantly isovector.

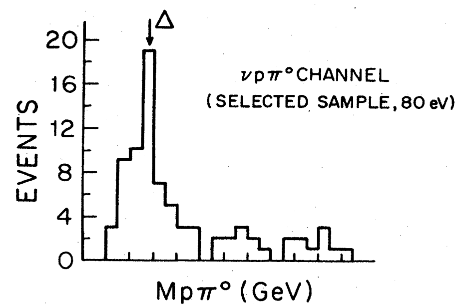


FIG. 4. Invariant mass distribution of the $N\pi$ system in the $\nu p\pi^0$ channel.

(3) Measurements of single-pion production (Erriques *et al.*, 1978) by $\bar\nu$ also yield evidence for $\Delta(1238)$ production in the $\bar\nu p \to \bar\nu p\pi^0$ and $\bar\nu n \to \bar\nu p\pi^-$ reactions; furthermore, the observed ratio (after nuclear corrections and for a target with $n/p = 1.22$)

$$\frac{\bar\nu N \to \bar\nu\pi^0 N'}{\bar\nu N \to \bar\nu\pi^- N'} = 2.4^{+0.8}_{-0.6}$$

is inconsistent (at the level of two standard deviations) with the values 1.12 and 4.4 expected for a pure isoscalar and pure isovector interaction, respectively. For completeness, we include also the recently reported cross-section ratios (Paty, 1979) for all four of the $\bar\nu$-induced single-pion reactions:

$$\frac{\sigma(\bar\nu p \to \bar\nu p\pi^0)}{\sigma(\bar\nu p \to \mu^+ n\pi^0)} = 0.30 \pm 0.14\,,$$

$$\frac{\sigma(\bar\nu n \to \bar\nu n\pi^0)}{\sigma(\bar\nu p \to \mu^+ n\pi^0)} = 0.49 \pm 0.21\,,$$

$$\frac{\sigma(\bar\nu p \to \bar\nu n\pi^+)}{\sigma(\bar\nu p \to \mu^+ n\pi^0)} = 0.54 \pm 0.22\,,$$

$$\frac{\sigma(\bar\nu n \to \bar\nu p\pi^-)}{\sigma(\bar\nu p \to \mu^+ n\pi^0)} = 0.41 \pm 0.20\,.$$

A quantitative analysis of the single-pion production data and a determination of the constraints imposed by it on the neutral-current coupling constants is presented by Abbott and Barnett (1978b) and by Monsay (1978). References to earlier measurements of $\nu n \to \nu n\pi$ are contained therein.

### F. Leptonic processes

A great deal of effort has gone into studying the pure leptonic processes since for these interactions theoretical predictions are completely unambiguous. The reactions that are experimentally accessible (all with difficulty) are

$$\begin{aligned}&\nu_\mu e \to \nu_\mu e\,,\\ &\bar{\nu}_\mu e \to \bar{\nu}_\mu e\,,\\ &\nu_e e \to \nu_e e\,,\\ &\bar{\nu}_e e \to \bar{\nu}_e e\,,\\ &e^+e^- \to \mu^+\mu^-\,.\end{aligned} \tag{2.9}$$

While experiments are underway[14] for the last two reactions, no results are currently available; we therefore focus on the first three reactions.

There exist four experiments (Blietschau *et al.*, 1976, 1978; Faissner *et al.*, 1978; Armenise *et al.*, 1979; Cnops *et al.*, 1978) that have published results on the cross section for $\nu_\mu e \to \nu_\mu e$. In addition, two recent experiments (Jonker *et al.*, 1979; Heisterberg *et al.*, 1980) have reported a signal for $\nu_\mu e \to \nu_\mu e$ and preliminary estimates of the cross section; the results are summarized in Table IV. The experiments may be roughly divided into two categories: bubble-chamber experiments which are very "clean" (background typically <10%) but have small numbers of events, and counter experiments which are able to obtain (in principle) moderately high data rates, but have significantly larger backgrounds. As always it is difficult to determine with certainty the systematic errors. However, uncertainty in the background for the three bubble-chamber experiments is small, and it is probably fair to say that in all four experiments systematic errors are dominated by the large statistical errors.

The results (Blietschau *et al.*, 1976; Faissner *et al.*, 1978; Bertrand *et al.*, 1979; Berge *et al.*, 1979; Armenise *et al.*, 1979) for $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$ are similarly summarized in Table V. In this case only two experiments have obtained a positive result, while three experiments have obtained upper limits near the expected level for the signal.

Finally, there exists one experiment (Reines, Gurr, and Sobel, 1976) which has observed the reaction $\bar{\nu}_e e \to \bar{\nu}_e e$ using reactor-produced $\bar{\nu}_e$. The experimenters divide their data to yield a cross section separately for "high-energy" recoil electrons ($E_e > 3$ MeV) and "low-energy" recoil electrons ($E_e < 3$ MeV). The results are

$$\sigma(\bar{\nu}_e e,\ \text{high}) = (1.86 \pm 0.48)\times 10^{-46}\ \text{cm}^2\,,$$
$$\sigma(\bar{\nu}_e e,\ \text{low}) = (7.6 \pm 2.2)\times 10^{-46}\ \text{cm}^2\,.$$

The $\bar{\nu}_e e \to \bar{\nu}_e e$ reaction (and $\nu_e e \to \nu_e e$) is particularly interesting since by observing the interference of the neutral- and charged-current interactions one may determine the sign of $g_A$. At present the data are not sufficiently accurate to allow this, however.

### G. Electron-hadron interactions

Reactions involving the interaction of electrons with hadrons play a unique role in the study of neutral currents: While the presence of the full-strength electromagnetic interaction dwarfs any weak effects, the interference of the weak and electromagnetic interactions allows one to test the parity-violating nature of neutral currents.

The cleanest and most precise result is obtained from a SLAC experiment (Prescott *et al.*, 1978, 1979) which measures the asymmetry in the scattering of polarized electrons from a deuteron target. Specifically, the asymmetry

$$A(Q^2, y) = \frac{d\sigma(+) - d\sigma(-)}{d\sigma(+) + d\sigma(-)}$$

is measured at 11 different values of $y$. The results are presented in Fig. 5. The experimenters have made a careful study of systematic effects; they include in the error bar for each data point the systematic error specific to that measurement, and in addition quote an overall systematic error due to the 5% uncertainty in the electron polarization. We have included each of these systematic errors in our fits to the measured asymmetries.

There also exist four experiments which have searched for the interference between the weak and electromagnetic interactions by attempting to observe parity violation in atomic transitions in bismuth and

TABLE IV. Summary of experiments on $\nu_\mu e$ scattering.

| Experiment | Sample of $\nu_\mu + N \to \mu^- + X$ | $\nu_\mu e$ candidates | Background | $\sigma/E$ [a] |
|---|---|---|---|---|
| GGM (Blietschau *et al.*, 1976, 1978) | | | | |
| CERN-PS | | 1 | $0.3 \pm 0.1$ | <3 (90% C.L.) |
| AP (Faissner *et al.*, 1978) | | | | |
| Counter expt. | | 32 | $20.5 \pm 2.0$ | $1.1 \pm 0.6$ |
| GGM (Armenise *et al.*, 1979) | | | | |
| CERN-SPS | 64000 | 9 | $0.5 \pm 0.2$ | $2.4^{+1.2}_{-0.9}$ |
| CB (Cnops *et al.*, 1978) | | | | |
| FNAL 15′ | 83700 | 11 | $0.5 \pm 0.5$ | $1.8 \pm 0.8$ |
| CHARM (Jonker *et al.*, 1979) | | | | |
| Counter expt. | 56000 | 11 | $4.5 \pm 1.4$ | $2.6 \pm 1.6$ [b] |
| VMWOF (Heisterberg *et al.*, 1979) | | | | |
| Counter expt. | | 46 | 12 | $1.4 \pm 0.3$ [b,c] |

[a] In units of $10^{-42}$ cm$^2$ GeV$^{-1}$.
[b] Preliminary.
[c] The error 0.3 is calculated with only the statistical fluctuation on the observed number of events (46). It assumes no contribution from the flux measurement, from the background calculation, and from the estimate of the efficiency (54%).

[14] An experiment to study $\nu_e e \to \nu_e e$ is being performed at LAMPF by H. Chen *et al.* For experiments on $e^+e^- \to \mu^+\mu^-$ see proposals submitted to PETRA and PEP.

TABLE V. Summary of experiments on $\bar{\nu}_\mu e$ scattering.

| Experiment | Sample of $\bar{\nu}_\mu + N \to \mu^+ + X$ | $\bar{\nu}_\mu e$ candidates | Background | $\sigma/E$ [a] |
|---|---|---|---|---|
| GGM (Blietschau *et al.*, 1976) CERN-PS | | 3 | $0.4 \pm 0.1$ | $1.0^{+2.1}_{-0.9}$ (90% C.L.) |
| AP (Faissner *et al.*, 1978) Counter expt. | | 17 | $7.4 \pm 1.0$ | $2.2 \pm 1.0$ |
| GGM (Bertrand *et al.*, 1979) CERN-SPS | 7400 | 0 | <0.03 | <2.7 (90% C.L.) |
| FMMS (Berge *et al.*, 1979) FNAL 15′ | 8400 | 0 | 0.2 | |
| BEBC TST (Armenise *et al.*, 1979) CERN-SPS | 7500 | 1 | $0.5 \pm 0.2$ | <3.4 (90% C.L.) |

[a] In units of $10^{-42}$ cm$^2$ GeV$^{-1}$.

thallium. The experimental situation has been confused since the two initial experiments [at Oxford (Baird *et al.*, 1977) and Seattle (Lewis *et al.*, 1977)] failed to observe any parity violation while two recent experiments [at Novosibirsk (Barkov and Zolotorev, 1978a, b) and Berkeley (Conti *et al.*, 1979; Commins, 1979)] claim to see an effect of about the expected magnitude. In addition, new experiments by the Seattle and Oxford groups yield a positive result though the results are still preliminary (Barkov, 1979). We do not include the atomic physics results in any of our fits save one: that is a combined fit to the results of the Novosibirsk and Berkeley experiments together with the measured asymmetries in the polarized electron scattering. There is no clear justification for selecting these two experiments; the results of the fit may simply be taken as indicative of the extent to which the coupling constants may be determined solely by electron-hadron interactions assuming the correctness of the experimental results.

### H. $\bar{\nu}_e D \to \bar{\nu}_e pn$

Finally, we note for completeness a recent measurement (Pasierb *et al.*, 1979) of the reaction $\bar{\nu}_e D \to \bar{\nu}_e pn$

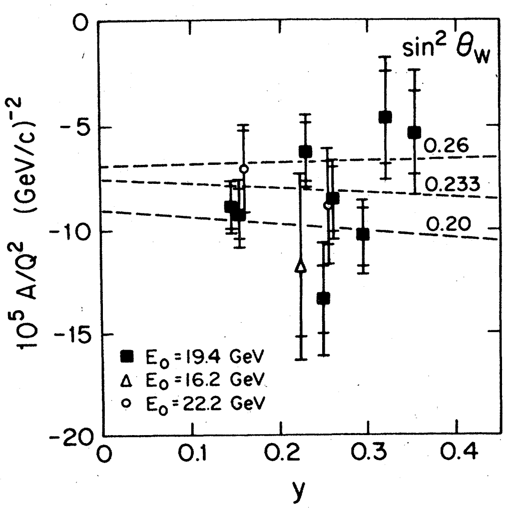


FIG. 5. Asymmetry in the scattering of polarized electrons from deuterium. The error bars illustrate only the statistical error and the total error. The curves are the predictions of the WS-GIM model.

by an Irvine group, using the $\bar{\nu}_e$ flux produced by the Savannah River fission reactor. An interesting feature of this reaction is that it proceeds only via the axial-vector interaction. The reported cross section,

$$(3.8 \pm 0.9) \times 10^{-45}\ \mathrm{cm}^2,$$

is in agreement with the SU(2)×U(1) prediction of $5.0 \times 10^{-45}$ cm$^2$. We do not include this result in our fit, but it does serve to verify the dominant axial-vector nature of the weak coupling.

### I. Summary

We summarize the experimental results included in our analysis in Table VI. In addition, we qualitatively anticipate the results of Sec. III-V by noting the following: The deep-inelastic experiments now have rather small statistical and systematic errors, and hence determine the strength of the left-handed and right-handed couplings with reasonable precision. However, the $\nu$ hadron experiments which determine the isospin of the neutral current—inelastic scattering from $p$ and $n$ targets, semi-inclusive pion production, and elastic scattering from protons—all have relatively large statistical or systematic errors. It is important that the accuracy and precision of these experiments be increased. Similarly, a precise determination of the leptonic currents will require high-statistics, low-background measurements of $\nu_\mu e \to \nu_\mu e$ and $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$.

## III. NEUTRINO-HADRON SCATTERING

In this section we concentrate on an analysis of neutrino-hadron scattering. We show that the parameters $\varepsilon_{L,R}(u)$ and $\varepsilon_{L,R}(d)$ are uniquely determined by the data to lie in a small allowed domain (which coincides with the WS-GIM model). The analysis is independent of any specific weak-interaction model, but various theoretical assumptions must be made concerning the necessary hadronic matrix elements. We have attempted to use the most accurate and reliable theoretical and experimental information available in our estimate of these matrix elements and to make realistic evaluations of the uncertainties involved.

We first describe the expressions we have adopted for each process and then describe the fit to $\varepsilon_{L,R}(u)$ and $\varepsilon_{L,R}(d)$. Fits to specific models are described in Sec. VI.

TABLE VI. Experimental data used in fits.

| | Reaction | Quantity measured | Data $\pm 1\sigma$ | Ref. | WS theory $\sin^2\theta_W = 0.233$ |
|---|---|---|---|---|---|
| (1) | $\nu N \to \nu X$ | $R_\nu$ | $0.307 \pm 0.008$ | Geweniger *et al*., 1979 | 0.305 |
| (2) | | | $0.30 \pm 0.021$ | Mess *et al*., 1979 | 0.312 |
| (3) | | | $0.30 \pm 0.04$ | Wanderer *et al*., 1978 | 0.326 |
| (4) | | | $0.28 \pm 0.03$ | Merritt *et al*., 1978 | 0.304 |
| (5) | $\bar\nu N \to \bar\nu X$ | $R_{\bar\nu}$ | $0.373 \pm 0.025$ | Geweniger *et al*., 1979 | 0.386 |
| (6) | | | $0.39 \pm 0.024$ | Mess *et al*., 1979 | 0.374 |
| (7) | | | $0.33 \pm 0.09$ | Wanderer *et al*., 1978 | 0.365 |
| (8) | | | $0.35 \pm 0.11$ | Merritt *et al*., 1978 | 0.399 |
| (9) | $\nu N \to \nu X$ | $g_L^2$ | $0.32 \pm 0.03$ | Deden *et al*., 1979 | 0.297 |
| (10) | $\bar\nu N \to \bar\nu X$ | $g_R^2$ | $0.04 \pm 0.03$ | Deden *et al*., 1979 | 0.030 |
| (11) | $\nu p \to \nu X$ | $R_\nu^p$ | $0.52 \pm 0.06$ | Blietschau *et al*., 1979 | 0.447 |
| (12) | | | $0.48 \pm 0.17$ | Harris *et al*., 1977 | 0.414 |
| (13) | $\bar\nu p \to \bar\nu X$ | $R_{\bar\nu}^p$ | $0.42 \pm 0.13$ | Derrick *et al*., 1978 | 0.383 |
| (14) | $\nu_p^n \to \nu X$ | $R_\nu^{n/p}$ | $1.22 \pm 0.35$ | Marriner, 1977 | 1.12 |
| (15) | $\bar\nu_p^n \to \nu X$ | $R_{\bar\nu}^{n/p}$ | $1.06 \pm 0.20$ | Bell *et al*., 1979 | 0.935 |
| (16) | $\nu N \to \nu\pi^\pm X$ | $R_\nu^{+/-}$ | $0.77 \pm 0.14$ | Kluttig *et al*., 1977 | 0.84 |
| (17) | $\bar\nu N \to \bar\nu\pi^\pm X$ | $R_{\bar\nu}^{+/-}$ | $1.65 \pm 0.33$ | Klutting *et al*., 1977 | 1.16 |
| (18) | | | $1.27^{+0.36}_{-0.27}$ | Roe, 1979 | 1.01 |
| (19)–(23) | $\nu p \to \nu p$ | $d\sigma/dq^2$ | See Ref.[c] | Entenberg *et al*., 1979; Kozanecki, 1978; Strait, 1978 | |
| (24)–(28) | $\bar\nu p \to \bar\nu p$ | $d\sigma/dq^2$ | See Ref.[c] | Entenberg *et al*., 1979; Kozanecki, 1978; Strait, 1978 | |
| (29) | $\nu_\mu e \to \nu_\mu e$ | $\sigma/E$ | $2.4^{+1.2}_{-0.9}$ [a] | Armenise *et al*., 1979 | 1.52 |
| (30) | | | $1.8 \pm 0.8$ [a] | Cnops *et al*., 1978 | 1.52 |
| (31) | | | $1.1 \pm 0.6$ [a] | Faissner *et al*., 1978 | 1.52 |
| (32) | $\bar\nu_\mu e \to \bar\nu_\mu e$ | $\sigma/E$ | $2.2 \pm 1.0$ [a] | Faissner *et al*., 1978 | 1.32 |
| (33) | | | $1.0^{+1.3}_{-0.6}$ [a] | Blietschau *et al*., 1976 | 1.32 |
| (34) | $\bar\nu_e e$ (low $E$) | $\sigma$ | $7.6 \pm 2.2$ [b] | Reines, Gurr, and Sobel, 1976 | 6.37 |
| (35) | $\bar\nu_e e$ (high $E$) | $\sigma$ | $1.86 \pm 0.48$ [b] | Reines, Gurr, and Sobel, 1976 | 1.21 |
| (36)–(46) | $eD \to eX$ | Asymm. | See Ref.[d] | Prescott *et al*., 1978, 1979 | |

[a] Units of $10^{-42} E_\nu$ (cm$^2$ GeV$^{-1}$).
[b] Units of $10^{-46}$ cm$^2$.
[c] The data include measurements at 5 values of $q^2$ for $\nu p$ and $\bar\nu p$.
[d] We fit directly to the 11 measurements at different values of $y$.

### A. Deep-inelastic scattering

In this subsection we describe our parametrization of the structure functions appearing in deep-inelastic inclusive scattering. As some of the experimental data are quite precise, we have taken great care to explore the sensitivity of our results to the various input parameters. We concentrate on a QCD parametrization of the structure functions, but simple parton model (SPM) results are also shown for contrast. The various parameters that enter the calculation, such as the amount of antiquarks, the magnitude of the strange sea, the $x$ and $Q^2$ dependence of the distribution functions, etc., are chosen to fit the neutrino charged-current data in a similar kinematic region.

#### 1. Theoretical framework

We describe lepton-nucleon scattering at high energy in the framework of the quark-parton model. The cross sections[15] for neutrino and antineutrino on a nuclear target $i$ $(=p,n)$ are

[15] We ignore the $Q^2$ dependence of the propagator, the effect of which is almost negligible (Hinchliffe and Llewellyn-Smith, 1977a). For the presently favored intermediate vector boson masses, inclusion of this effect would reduce the cross sections by $\sim 2\langle Q^2\rangle/M_w^2 \simeq 0.006$ for $\langle Q^2\rangle = 20$ GeV$^2$ and $M_w = 80$ GeV.

$$\frac{d^2\sigma_{cc}^{\nu i}}{dx\,dy}=\frac{2G_F^2M_pE}{\pi}[xd^i(x,Q^2)(\cos^2\theta_c+\sin^2\theta_c\xi_c)+xs^i(x,Q^2)(\sin^2\theta_c+\cos^2\theta_c\xi_c)+x\bar{u}^i(x,Q^2)(1-y)^2+x\bar{c}^i(x,Q^2)(1-y)^2], \quad (3.1)$$

$$\frac{d^2\sigma_{cc}^{\bar{\nu} i}}{dx\,dy}=\frac{2G_F^2M_pE}{\pi}[xu^i(x,Q^2)(1-y)^2+xc^i(x,Q^2)(1-y)^2+x\bar{d}^i(x,Q^2)(\cos^2\theta_c+\sin^2\theta_c\xi_c)+x\bar{s}^i(x,Q^2)(\sin^2\theta_c+\cos^2\theta_c\xi_c)], \quad (3.2)$$

$$\begin{aligned}\frac{d^2\sigma_{nc}^{\nu i}}{dx\,dy}=\frac{2G_F^2M_pE}{\pi}\{&[|\varepsilon_L(u)|^2+|\varepsilon_R(u)|^2(1-y)^2][xu^i(x,Q^2)+\xi_cxc^i(x,Q^2)]\\&+[|\varepsilon_L(d)|^2+|\varepsilon_R(d)|^2(1-y)^2][xd^i(x,Q^2)+xs^i(x,Q^2)]\\&+[|\varepsilon_R(u)|^2+|\varepsilon_L(u)|^2(1-y)^2][x\bar{u}^i(x,Q^2)+\xi_cx\bar{c}^i(x,Q^2)]\\&+[|\varepsilon_R(d)|^2+|\varepsilon_L(d)|^2(1-y)^2][x\bar{d}^i(x,Q^2)+x\bar{s}^i(x,Q^2)]\}, \quad (3.3)\end{aligned}$$

$$\begin{aligned}\frac{d^2\sigma_{nc}^{\bar{\nu} i}}{dx\,dy}=\frac{2G_F^2M_pE}{\pi}\{&[|\varepsilon_R(u)|^2+|\varepsilon_L(u)|^2(1-y)^2][xu^i(x,Q^2)+\xi_cxc^i(x,Q^2)]\\&+[|\varepsilon_R(d)|^2+|\varepsilon_L(d)|^2(1-y)^2][xd^i(x,Q^2)+xs^i(x,Q^2)]\\&+[|\varepsilon_L(u)|^2+|\varepsilon_R(u)|^2(1-y)^2][x\bar{u}^i(x,Q^2)+\xi_cx\bar{c}^i(x,Q^2)]\\&+[|\varepsilon_L(d)|^2+|\varepsilon_R(d)|^2(1-y)^2][x\bar{d}^i(x,Q^2)+x\bar{s}^i(x,Q^2)]\}, \quad (3.4)\end{aligned}$$

where $E$ is the incident $\nu$ or $\bar{\nu}$ energy, $M_p$ is the nucleon mass, and $cc$ and $nc$ refer to charged current and neutral-current, respectively. We have assumed generation symmetry for the neutral-current couplings, i.e., $\varepsilon_L(u)=\varepsilon_L(c)$, etc. The results are insensitive to small deviations from this assumption. Heavier quark contributions ($b$ or $t$) are ignored. We assume slow rescaling[16] as modified by Buras and Gaemers (1977; see also Hinchliffe and Llewellyn-Smith, 1977b) for charmed particle production; that is, we take

$$\xi_c xq(x,Q^2)=zq(z,Q^2)\theta(E_h-M_c) \quad (3.5)$$

where $z=(Q^2+M_c^2)/2M_p\nu$. As usual, the scaling variables are $x=Q^2/2M_p\nu$, $y=\nu/E$ with $\nu=E-E'$ ($E'$ is the final lepton energy). $E_h=\nu+M_p$ is the hadron energy and $M_c=1.5$ GeV. If we assume quark model expressions for $F_2$ and $F_3$ but allow a nonvanishing $R=(F_2-2xF_1)/2xF_1$, then Eqs. (3.1)–(3.4) are modified by

$$\begin{pmatrix}1\\(1-y)^2\end{pmatrix}\rightarrow\begin{pmatrix}1\\(1-y)^2\end{pmatrix}-\frac{1}{2}\frac{Ry^2}{1+R}. \quad (3.6a)$$

For the $(1-y)^2$ part of Eq. (3.6a), $y_{max}=[(1+R)/(1+0.5R)]\times\{1-[0.5R/(1+R)]^{1/2}\}$ from the positivity of cross sections. We shall, in various runs, take $R=0$, $R=$ constant $\neq 0$, and also utilize the approximate QCD formula (Calvo, 1977; Zee Wilczek, and Treiman, 1974; Nanopoulos and Ross, 1975; Hinchliffe and Llewellyn-Smith, 1977b; Gross, 1976), with $N_F=4$,

$$R=\frac{12}{(33-2N_f)\ln(Q^2/\Lambda^2)}\left(\tfrac{2}{3}-\tfrac{2}{3}x\right), \quad (3.6b)$$

where the bracket linearly extrapolates between $\frac{2}{3}$ for $x=0$ and 0 for $x=1$. It will turn out that $R_\nu$ and $R_{\bar{\nu}}$ are insensitive to $R$. For the Fermi constant and the Cabibbo angle (Nagels *et al.*, 1979; Shrock and Wang, 1978) we use

$$G_FM_p^2=1.027\times10^{-5}, \quad \theta_c=13.17°. \quad (3.7)$$

The simple parton model (SPM) approximation for the quark distribution functions, in which $q^i(x,Q^2)$ is independent of $Q^2$, is illustrated in Fig. 6. Corrections occurring in QCD are given in Fig. 7. The probe ($\gamma, W, Z$) of the hadron sees a smaller momentum in QCD than in the SPM, because the quark may emit a gluon before being scattered. Therefore the quark distribution is shifted to a smaller $x$ region. This behavior is manifested as the shrinkage (increase) of the quark distribution for $x>0.1\sim0.2$ ($x<0.1\sim0.2$) as $Q^2$ increases (for larger $Q^2$, there is more phase space for Fig. 7); that is, the slopes $\eta_1'$ and $\eta_3'$ ($\eta_2'$ and $\eta_4'$) in Eqs. (3.10) and (3.21) are negative (positive). This scaling violation is of order

$$\alpha_s(Q^2)=\frac{12\pi}{(33-2N_f)\ln Q^2/\Lambda^2} \quad (3.8)$$

where $N_F$ is the number of flavors below $Q^2$. By the above relation, we can express $\alpha_s$ in terms of a dimensional parameter $\Lambda$. The SPM is the limit $\alpha_s=0\,(\Lambda=0)$ of QCD. Larger values of $\Lambda$ imply larger $\alpha_s$ and therefore more scaling violation.

If the quark (and gluon) distribution functions are

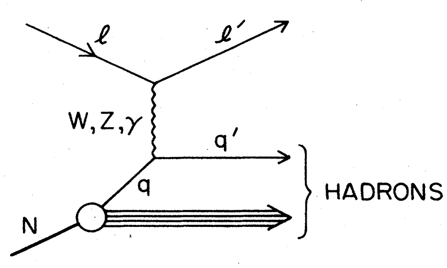


FIG. 6. Lepton-nucleon scattering in the SPM.

[16]Slow rescaling has been discussed by Georgi and Politzer (1976) and Barnett (1976a, b).

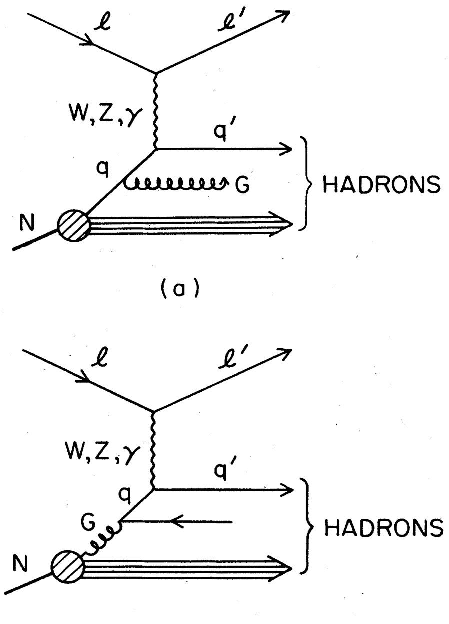


FIG. 7. QCD corrections to Fig. 6.

known for some momentum transfer $Q_0^2$ then QCD predicts their values for other values of $Q^2$. In the following we shall utilize approximate analytic expressions for the $Q^2$ evolution of the valence and sea distributions that are basically those given by Buras and Gaemers (1977, 1978), but generalized to include arbitrary fractions of strange and charmed sea and different initial $x$ distributions. These expressions correctly describe the low-order moments of the distributions and are adequate for phenomenological purposes.

We now define the quark distributions in the proton, omitting the superscript $p$ (i.e., $u=u^p$, etc.). The distributions in the neutron are related by isospin invariance (e.g., $u^n=d^p$, $s^n=s^p$). For the $u$ and $d$ quark distributions we have $u=u_v+\bar{u}$, $d=d_v+\bar{d}$, where the valence quark distributions are given by

$$\begin{aligned} xu_v(x,Q^2)&=\tfrac{1}{2}[xV_8(x,Q^2)+xV_3(x,Q^2)]\,,\\ xd_v(x,Q^2)&=\tfrac{1}{2}[xV_8(x,Q^2)-xV_3(x,Q^2)]\,,\end{aligned} \tag{3.9}$$

where

$$\begin{aligned} xV_8(x,Q^2)&=\frac{3}{B[\eta_1(S),1+\eta_2(S)]}\,x^{\eta_1(S)}(1-x)^{\eta_2(S)}\\ xV_3(x,Q^2)&=xV_8(x,Q^2)\\ &\quad-\frac{2}{B[\eta_3(S),1+\eta_4(S)]}\,x^{\eta_3(S)}(1-x)^{\eta_4(S)}\,. \end{aligned} \tag{3.10}$$

In Eq. (3.10), $S$ is defined as

$$S\equiv\ln\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\,, \tag{3.11}$$

so that $S=0$ at the initial momentum $Q_0^2$. The coefficients in Eq. (3.10) are determined from the valence number conditions

$$\int_0^1[u_v(x,Q^2)+d_v(x,Q^2)]\,dx=3\,,$$

$$\int_0^1[u_v(x,Q^2)-d_v(x,Q^2)]\,dx=1\,.$$

which hold for all $Q^2$. The functions $\eta_i(S)$ are approximated by the linear forms

$$\eta_i(S)\cong\eta_i(0)+\eta_i'GS\,,\quad i=1,2,3,4\,. \tag{3.12}$$

Here $G$ is the group-theoretic constant

$$G=\frac{4}{33-2N_f}\to\frac{4}{25}\quad\text{for } N_f=4\,. \tag{3.13}$$

The initial values $\eta_i(0)$ and the slope parameters $\eta_i'$ are chosen to fit the charged current and electroproduction structure functions and to approximately reproduce the predictions of QCD for the $Q^2$ dependence of the moments of $V_3$ and $V_8$.

We have modified Buras and Gaemers (1978) slightly to allow an initially SU(3)-asymmetric sea [the sea becomes SU(4) symmetric as $Q^2\to\infty$]. The quark sea and gluon distributions are (Buras and Gaemers, 1978; Buras, 1977)[17]

$$\begin{aligned} x\bar{u}(x,Q^2)&=x\bar{d}(x,Q^2)=C_{\bar{u}}(S)(1-x)^{\eta_{\bar{u}}(S)}\,,\\ xs(x,Q^2)&=x\bar{s}(x,Q^2)=C_{\bar{s}}(S)(1-x)^{\eta_{\bar{s}}(S)}\,,\\ xc(x,Q^2)&=x\bar{c}(x,Q^2)=C_{\bar{c}}(S)(1-x)^{\eta_{\bar{c}}(S)}\,,\\ xG(x,Q^2)&=C_G(S)(1-x)^{\eta_G(S)}\,, \end{aligned} \tag{3.14}$$

where $G(x,Q^2)$ is the gluon distribution. With an SU(3) symmetric sea one has $C_{\bar{u}}=C_{\bar{s}}$, $\eta_{\bar{u}}=\eta_{\bar{s}}$. QCD predicts the $S$ dependence of the $C$ and $\eta$ functions. The relevant formulas are given in Appendix A. At $Q_0^2$ we assume that the sea and gluon distributions have a universal $x$ dependence, so that $\eta_{\bar{u}}(0)=\eta_{\bar{s}}(0)=\eta_{\bar{c}}(0)=\eta_G(0)\equiv\eta_s(0)$, the value being determined from the charged-current data (de Groot, *et al.*, 1979). We use the same power for the sea distributions of quarks and gluons. Though there is some evidence of $\eta_{\bar{u}}\neq\eta_{\bar{s}}$ the neutral-current interaction is not sensitive to the variation of the power [see Eq. (3.29)].

We define $R_{\rm sea}$ and $A_{q_i}$ by

$$R_{\rm sea}(Q^2)\equiv\frac{\int_0^1dx\sum_i x\bar{q}_i(x,Q^2)}{\int_0^1dx\sum_i x[q_i(x,Q^2)+\bar{q}_i(x,Q^2)]}\,, \tag{3.15}$$

which is closely related to

$$R_{cc}\equiv\frac{\sigma(\bar{\nu}N\to\mu^+X)}{\sigma(\nu N\to\mu^-X)}\,, \tag{3.16}$$

and

$$A_{q_i}(Q^2)\equiv\frac{\int_0^1x\bar{q}_i(x,Q^2)\,dx}{\int_0^1x\bar{u}(x,Q^2)\,dx}\,. \tag{3.17}$$

The initial values for $C_{\bar{q}_i}(0)$ can be determined by specifying $R_{\rm sea}(Q^2)$, $A_s(Q^2)$, and $A_c(Q^2)$ at some value of $Q^2$ (such as $Q_0^2$). In fact, we shall always specify $A_c=0$ at $Q^2=1.8$ GeV$^2$ and specify

$$\begin{aligned} r_{\rm sea}&\equiv R_{\rm sea}(Q_0^2)\,,\\ a_s&\equiv A_s(Q_0^2)\,. \end{aligned} \tag{3.18}$$

[17]A similar form in the SPM is given by Barger and Phillips (1974). See also Close (1979), Fox (1977), and Field (1979).

$C_G(0)$ is determined by requiring that the momentum sum rule

$$\int_0^1 dx\left(\sum_i x[q_i(x,Q^2)+\bar{q}_i(x,Q^2)]+xG(x,Q^2)\right)=1 \tag{3.19}$$

be satisfied at $Q_0^2$. In fact, for the approximate linear form for the $\eta_i(S)$ in Eq. (3.12) the sum rule is satisfied to within 1% over the presently available $Q^2$ range.

Two groups (Buras and Gaemers, 1978; de Groot *et al.*, 1979) have determined the parameters $\eta_i(S)$ and $\eta_s(0)$.

*(a) Parametrization A; $Q_0^2=1.8$ GeV$^2$.* Buras and Gaemers (1978) determined parameters from $eN$ and $\mu N$ data. They obtained (in our notation)

$$R_{\rm sea}(1.8)=0.092\,,\quad A_s(1.8)=1\,,\quad A_c(1.8)=0 \tag{3.20}$$

and

$$\begin{aligned}\eta_1(S)&=0.70-1.1GS\,,\\ \eta_2(S)&=2.60+5.0GS\,,\\ \eta_3(S)&=0.85-1.5GS\,,\\ \eta_4(S)&=3.35+5.1GS\,,\\ \eta_s(0)&=10\,.\end{aligned} \tag{3.21}$$

The sea distribution at $Q^2=1.8$ is then

$$\bar{u}+\bar{d}=\frac{0.4}{x}(1-x)^{10}\,, \tag{3.22}$$

$$s+\bar{s}=\frac{f}{x}(1-x)^{10}\,.$$

Buras and Gaemers (1978) took $f=0.4$, but we shall allow $f\leq 0.4$. [Changing $f$ will also change the condition (3.20).] The conditions (3.20)–(3.22) will be referred to as parametrization $A$. This parametrization will be used in the analysis of the BEBC experiment.

*(b) Parametrization B; $Q_0^2=5$ GeV$^2$.* The CDHS group has recently determined the parameters

$$\begin{aligned}\eta_1(S)&=(0.56\pm0.2)-0.92GS\\ \eta_2(S)&=(2.71\pm0.11)+5.08GS\end{aligned} \tag{3.23}$$

from their charged-current neutrino data. Since the determination is based on (approximately) isoscalar target data, $\eta_3$ and $\eta_4$ are not given. However, we need these for calculations of cross sections on proton, neutron, and nuclear targets. We use the result of Field and Feynman (1977),

$$\int_0^1 xd(x)\,dx \Big/ \int_0^1 xu(x)\,dx=0.51\,,$$

which we interpret to hold at $Q^2=1.8$ GeV$^2$, to obtain $\int xd_v dx/\int xu_v dx=0.51$ at $Q_0^2=5$ GeV$^2$. Then, with the same slopes as in Eq. (3.21) we obtain

$$\begin{aligned}\eta_3(S)&=0.75-1.5GS\,,\\ \eta_4(S)&=3.94+5.1GS\,.\end{aligned} \tag{3.24}$$

The sea power is given by (de Groot *et al.*, 1979)

$$\eta_s(0)=8.1\pm0.7 \tag{3.25}$$

and

$$\Lambda=0.47\pm0.11 \text{ GeV}. \tag{3.26}$$

Further we have independently determined $r_{\rm sea}=0.12$ [defined in Eqs. (3.15) and (3.18)] for $Q_0^2=5$ GeV$^2$.

The CDHS group did not attempt to determine $a_s$ in their fit. However, from data on di-muon production by $\bar{\nu}$ (which is dominated by charm production from the $\bar{s}$ quarks in the sea), the CDHS (Holder *et al.*, 1977b) and HPWF (Mann, 1979) experiments obtain[18]

$$\frac{\int_0^1 x\bar{s}(x)\,dx}{\int_0^1 x[q(x)+\bar{q}(x)]\,dx}=0.025\pm0.01 \quad \text{(CDHS)}$$
$$=0.032\pm0.015 \quad \text{(HPWF)}.$$

A value of 0.024 for the above ratio corresponds to $a_s=0.5$.

## 2. The evolution of quark distributions

The most important consequence of QCD for the present analysis is that the magnitude and flavor composition of the sea change as a function of $Q^2$; the $q$ and $\bar{q}$ distributions evolve to SU(4) symmetric values as $Q^2$ increases (this follows from the preceding expressions and those in Appendix A). This approach is not rapid. In Fig. 8 (for parametrization $B$) and Fig. 9 (for parametrization $A$), we show $R_{\rm sea}(Q^2)$, $A_s(Q^2)$, and $A_c(Q^2)$. In Fig. 8(a), $R_{\rm sea}(Q^2)$ is shown for several values of $\Lambda$ with $a_s=0.5$, $r_{\rm sea}=0.12$. A change of $a_s$ to 1 does not lead to a noticeable change (i.e., they are almost identical to those given in Fig. 8). However, variation of $r_{\rm sea}$ shifts the curves. For $\Lambda=0.47$, $R_{\rm sea}$ increases

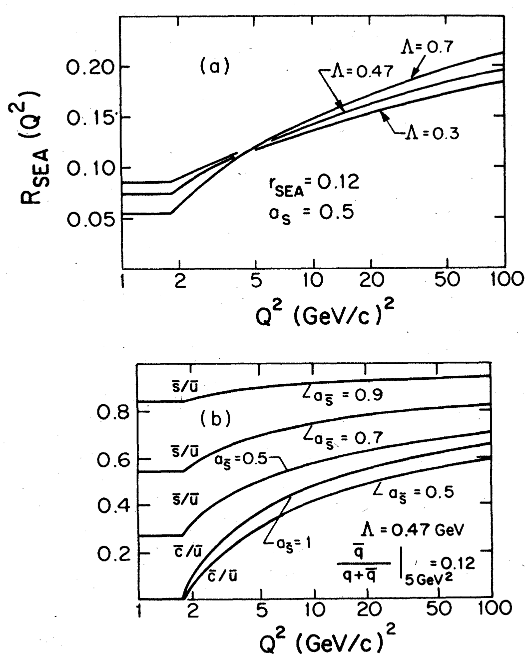


FIG. 8. $Q^2$ evolution of quark distributions for parametrization $B$. (a) $R_{\rm sea}(Q^2)$ for $r_{\rm sea}=0.12$, $a_s=0.5$, and $\Lambda=0.3$, 0.47, 0.7. (b) $A_s=\bar{s}/\bar{u}$ and $A_c=\bar{c}/\bar{u}$ for $r_{\rm sea}=0.12$, $\Lambda=0.47$, and varying values of $a_{\bar{s}}$.

[18]In these experiments the determination is an underestimate.

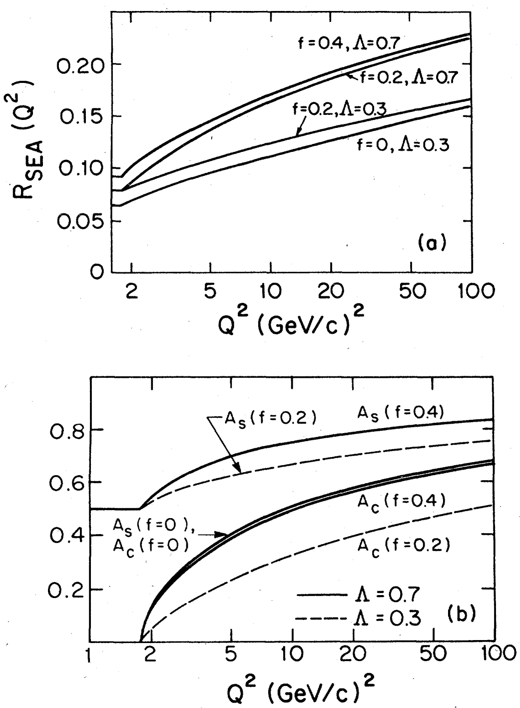


FIG. 9. $Q^2$ evolution of quark distributions for parametrization $A$. (a) $R_{\rm sea}(Q^2)$ for $\Lambda=0.3$ and 0.7. (b) $A_s=\bar{s}/\bar{u}$ and $A_c=\bar{c}/\bar{u}$ for $f=0.0$, 0.2, 0.4, which correspond, respectively, to $\bar{s}/\bar{u}=0$, 0.5, 1.0, at $Q_0^2=1.8$ (GeV/$c$)$^2$.

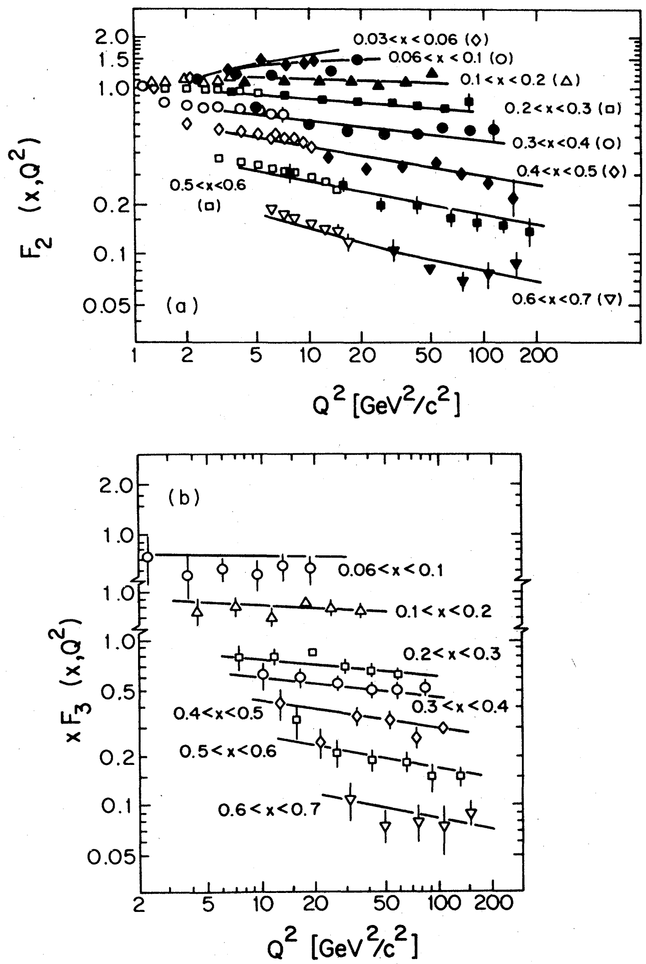


FIG. 10. Curve presents our fit to the CDHS charged current data in parametrization $B$ with $\Lambda=0.47$ and $r_{\rm sea}=0.12$. (a) $F_2(x,Q^2)$. (b) $xF_3(x,Q^2)$.

from 0.076 at low $Q^2$ to 0.195 at $Q^2=100$ GeV$^2$. For $Q^2=20$ GeV$^2$ (the CDHS average value), this ratio is 0.161. In Fig. 8(b), $A_s$ and $A_c$ are shown for several values of $a_s$ assuming $A_c$ ($Q^2=1.8$)$=0$. As $Q^2\to\infty$, $A_s$ and $A_c\to 1$, and this trend is shown in the figure. For parametrization $B$, $a_s\geq 0.3$ so that the strange quark distribution is not negative at $Q^2=1.8$. The input $a_s=0.5$ corresponds to $A_s$ ($Q^2=20$)$=0.61$. In Fig. 9(a), $R_{\rm sea}(Q^2)$ in parametrization $A$ is shown for $f=0, 0.2, 0.4$ [see Eq. (3.22)] and $\Lambda=0.3, 0.7$. In Fig. 9(b), $A_s$ and $A_c$ are shown. The trends are similar to those in Fig. 8.

### 3. CDHS experiment

For each deep-inelastic experiment we integrate over the appropriate $\nu, \bar{\nu}$ spectrum and include hadron energy cuts and the $n/p$ ratio of the target in our theoretical expressions for the charged- and neutral-current cross sections. The determination of parameters in $B$ [Eqs. (3.23)–(3.26)] is from the CDHS charged-current reactions. We obtain a reasonably good fit to $F_2$, $xF_3$, and the charged-current ratio $R_{cc}$ for

$$r_{\rm sea}=0.12\ ,\quad \Lambda=0.47\ \text{GeV}\ .$$

The variation of the power of the sea distribution does not change $R_{cc}$ or $xF_3$, but slightly changes $xF_2$ for $x<0.1$. We shall use the power determined by the CDHS group. The variation of $a_s$ ($\bar{s}/\bar{u}$ at $Q^2=5$ GeV$^2$) does not change $F_2$ and $xF_3$ so long as $r_{\rm sea}$ is fixed, but slightly changes $R_{cc}$ (less than 0.5%). Hence, fixing $r_{\rm sea}=0.12$ almost guarantees a good fit to $F_2$, $xF_3$, and $R_{cc}$ (also the individual $\sigma_\nu^{cc}$ and $\sigma_{\bar{\nu}}^{cc}$). These fits to the CDHS data are given in Fig. 10(a), 10(b), and in Fig. 11.

We now consider the CDHS neutral-current data. The ratios of NC to CC cross sections on iron with a hadron

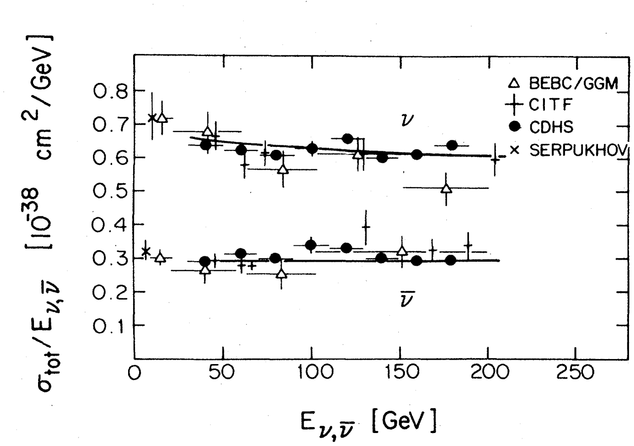


FIG. 11. Charged-current cross sections measured by the CDHS, BEBC, GGM, CITF, and Serpukhov experiments (see de Groot *et al.*, 1979). The curve represents our fit to the CDHS charged-current data with $\Lambda=0.47$ and $r_{\rm sea}=0.12$.

energy cut of 10 GeV are

$$R_\nu^{\mathrm{Fe}}=0.307\pm0.008\,,\qquad R_{\bar\nu}^{\mathrm{Fe}}=0.373\pm0.025\,. \tag{3.27}$$

In the SPM, the expressions for $R_\nu$ and $R_{\bar\nu}$ in terms of the neutral-current parameters are given in Table VII. For example, $R_\nu$ and $R_{\bar\nu}$, flux averaged on an iron target are, for $R_{cc}=0.48$ and $\bar s=\frac{1}{2}\bar u$,

$$\begin{aligned}R_\nu^{\mathrm{Fe}}&=0.899\,|\varepsilon_L(u)|^2+1.016\,|\varepsilon_L(d)|^2+0.349\,|\varepsilon_R(u)|^2+0.435\,|\varepsilon_R(d)|^2\,,\\ R_{\bar\nu}^{\mathrm{Fe}}&=0.849\,|\varepsilon_L(u)|^2+1.064\,|\varepsilon_L(d)|^2+2.290\,|\varepsilon_R(u)|^2+2.585\,|\varepsilon_R(d)|^2\quad\text{(CDHS, SPM)}\,.\end{aligned} \tag{3.28}$$

With these expressions one can compare the data with any gauge theory model. The comparison with the WS-GIM model is given in Figs. 12(a) and 12(b). Figure 12(a) shows the effect of varying the amount of $s$ quark for fixed $R_{cc}=0.48$, while Fig. 12(b) shows the effect of varying $R_{cc}$ for a fixed $s$-quark ratio $\bar s=\frac{1}{2}\bar u$. These variations change $R_{\bar\nu}$ up to 4%, and $R_\nu$ by less than 3.0%; hence the value of $\sin^2\theta_w$ determined from $R_\nu$ changes by less than 0.009.

In QCD (parametrization $B$), the results for $R_\nu$ and $R_{\bar\nu}$ are given in Table VIII.[19] These expressions can be succinctly written as

$$\begin{aligned}R_\nu^{\mathrm{Fe}}=\;&[\varepsilon_L(u)]^2(0.941-0.448\Delta_r-0.054\Delta_{a_s}-0.242\Delta_{n/p}+0.037\Delta_\Lambda-0.0001\Delta_{hc}+0.000R-0.0004\Delta_{\eta_s})\\ &+[\varepsilon_L(d)]^2(1.018+0.338\Delta_r+0.034\Delta_{a_s}-0.002\Delta_{n/p}-0.018\Delta_\Lambda-0.0003\Delta_{hc}-0.001R+0.0006\Delta_{\eta_s})\\ &+[\varepsilon_R(u)]^2(0.372+0.838\Delta_r-0.048\Delta_{a_s}-0.079\Delta_{n/p}+0.033\Delta_\Lambda-0.0043\Delta_{hc}-0.049R-0.0019\Delta_{\eta_s})\\ &+[\varepsilon_R(d)]^2(0.416+1.695\Delta_r+0.040\Delta_{a_s}-0.018\Delta_{n/p}-0.023\Delta_\Lambda-0.0046\Delta_{hc}-0.052R-0.0013\Delta_{\eta_s})\,,\\ R_{\bar\nu}^{\mathrm{Fe}}=\;&[\varepsilon_L(u)]^2(0.960-1.310\Delta_r-0.147\Delta_{a_s}-0.003\Delta_{n/p}+0.105\Delta_\Lambda-0.0009\Delta_{hc}-0.005R+0.0000\Delta_{\eta_s})\\ &+[\varepsilon_L(d)]^2(1.079+0.608\Delta_r+0.096\Delta_{a_s}+0.174\Delta_{n/p}-0.046\Delta_\Lambda-0.0004\Delta_{hc}+0.003R+0.0022\Delta_{\eta_s})\\ &+[\varepsilon_R(u)]^2(2.582-11.980\Delta_r-0.189\Delta_{a_s}-0.121\Delta_{n/p}+0.202\Delta_\Lambda+0.0376\Delta_{hc}+0.167R+0.0151\Delta_{\eta_s})\\ &+[\varepsilon_R(d)]^2(2.792-10.657\Delta_r+0.051\Delta_{a_s}+0.584\Delta_{n/p}+0.058\Delta_\Lambda+0.0403\Delta_{hc}+0.450R+0.0178\Delta_{\eta_s})\,,\end{aligned} \tag{3.29}$$

where $\Delta_r=r_{\mathrm{sea}}-0.12$, $\Delta_{a_s}=a_s-0.5$, $\Delta n/p=(n/p)-1.18$, $\Delta_\Lambda=\Lambda-0.47$, $\Lambda_{hc}=E_h^{\mathrm{cut}}-10$, and $\Delta_{\eta_s}=\eta_s(0)-8.1$; the terms involving $R$ indicate the effect of a constant (independent of $x$ and $Q^2$), nonzero $R$.

The $\Delta_{n/p}$ and $\Delta_{hc}$ terms in (3.29) are useful for adopting our expressions to other experimental conditions when the $\nu$, $\bar\nu$ fluxes of the other experiments are approximately the same as those of the CDHS group. The other terms are used to estimate the sensitivity of the expressions to reasonable variations in the input parameters. One sees that variations in $\Lambda$, $\eta_s(0)$, and $R$ produce negligible changes in the coefficients of the chiral couplings. From Table VIII one can also see that incorporating the QCD expression for $R$ has no significant effect. The results are also insensitive to reasonable variations in $\eta_i(0)$, $\eta_i'$, and to small violations of the generation symmetry assumption $\varepsilon_{L,R}(c)=\varepsilon_{L,R}(u)$, $\varepsilon_{L,R}(s)=\varepsilon_{L,R}(d)$.

TABLE VII. Coefficients of chiral couplings for iron target averaged over CERN SPS neutrino spectrum in the SPM satisfying the constraint $E_h^{\mathrm{cut}}=10$ GeV.

| Constraint | $R_{\nu,\bar\nu}$ | $[\varepsilon_L(u)]^2$ | $[\varepsilon_L(d)]^2$ | $[\varepsilon_R(u)]^2$ | $[\varepsilon_R(d)]^2$ |
|---|---|---|---|---|---|
| $R_{cc}=0.48$ | $R_\nu$ | 0.958 | 1.000 | 0.406 | 0.417 |
| $A_s=0$ | $R_{\bar\nu}$ | 1.000 | 1.027 | 2.455 | 2.564 |
| $R_{cc}=0.48$ | $R_\nu$ | 0.899 | 1.016 | 0.349 | 0.435 |
| $A_s=0.5$ | $R_{\bar\nu}$ | 0.849 | 1.064 | 2.290 | 2.585 |
| $R_{cc}=0.48$ | $R_\nu$ | 0.865 | 1.025 | 0.316 | 0.445 |
| $A_s=1.0$ | $R_{\bar\nu}$ | 0.764 | 1.085 | 2.197 | 2.598 |
| $R_{cc}=0.46$ | $R_\nu$ | 0.905 | 1.014 | 0.335 | 0.411 |
| $A_s=0.5$ | $R_{\bar\nu}$ | 0.862 | 1.063 | 2.444 | 2.733 |
| $R_{cc}=0.50$ | $R_\nu$ | 0.892 | 1.018 | 0.362 | 0.458 |
| $A_s=0.5$ | $R_{\bar\nu}$ | 0.837 | 1.064 | 2.152 | 2.453 |

Changes in $r_{\mathrm{sea}}(R_{cc})$ and $a_{\bar s}$ are considerably more important, as is the difference between the QCD and SPM expression for the structure functions. The most important consequence of the QCD parametrization is that the charmed sea is predicted to rise from an initial value of zero to substantially larger values at large $Q^2$, as can be seen in Figs. 8 and 9. These variations in the amount and composition of the sea lead to significant ($\lesssim 15\%$) variations in $R_\nu$ and $R_{\bar\nu}$ for some values of the neutral-current parameters but are considerably less important for neutral-current parameters in the vicinity of the WS-GIM model. The QCD predictions of WS-GIM for the CDHS experiment are shown in Fig. 12(c) and 12(d). We see that $R_{\bar\nu}$, which is most sensitive to sea effects, is changed substantially as $R_{cc}$ and $a_s$ are varied and as one goes from the SPM to QCD (which increases $\bar c/\bar u$). $R_\nu$ and hence the best value for $\sin^2\theta_w$ are changed very little. For example, for $\sin^2\theta_w=0.23$, $R_\nu$ changes from 0.3063 to 0.3059 as $R_{cc}$ is varied from 0.48 to 0.50 [equivalent to changing the average value of $\bar q/(q+\bar q)$ from 0.16 to 0.18]; hence the value of $\sin^2\theta_w$ changes by less than 0.0006.

### 4. BEBC experiment

The BEBC group (Deden *et al.*, 1979) have performed their own QCD analysis of their total cross sections

[19]Except for the BEBC experiment, the QCD parametrization $B$ will be used.

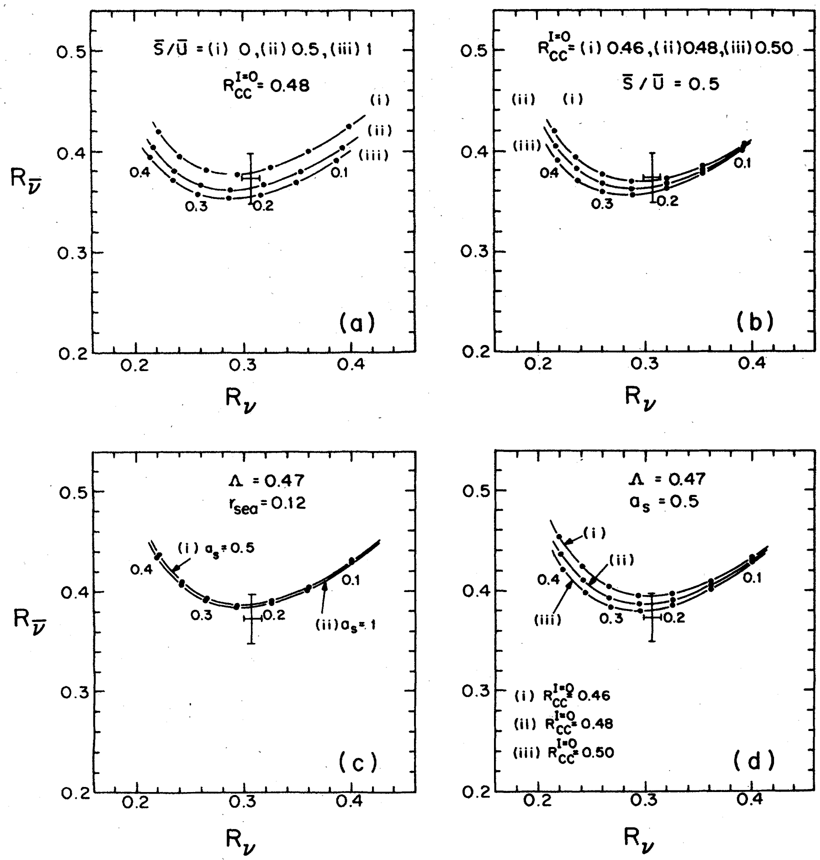


FIG. 12. Comparison of the WS-GIM prediction for $R_\nu$ and $R_{\bar\nu}$, assuming an iron target, $E_H^{\rm cut}=10$ GeV, and the CDHS $\nu$ spectrum, with the measurements of the CDHS experiment. (a) Simple parton model with $\bar{s}/\bar{u}=0$, 0.5, 1.0, and $R_{cc}^{I=0}=\sigma_{\bar\nu}^{cc}/\sigma_\nu^{cc}=0.48$ (for an isoscalar target). (b) Simple parton model with $R_{cc}^{I=0}=0.46$, 0.48, 0.50, and $\bar{s}/\bar{u}=0.5$. (c) QCD with $\Lambda=0.47$, $r_{\rm sea}=0.12$, and $a_s=0.5$ or 1.0. (d) QCD with $\Lambda=0.47$, $a_s=0.5$, and $R_{cc}^{I=0}=0.46$, 0.48, 0.50.

TABLE VIII. Coefficients of chiral couplings in QCD (parametrization $B$). These can be compared to the CDHS experiment in iron with the hadron energy cut of 10 GeV. $R=0$, except in the last row, in which the QCD expression [Eq. (3.6b)] is used. $R_{cc}^{I=0}$ is the charged-current ratio at $E=150$ GeV.

| Constraints | | $R_{cc}^{I=0}$ | $R_{\nu,\bar\nu}$ | $\varepsilon_L(u)^2$ | $\varepsilon_L(d)^2$ | $\varepsilon_R(u)^2$ | $\varepsilon_R(d)^2$ |
|---|---|---|---|---|---|---|---|
| $\Lambda=0.47$ | $r_{\rm sea}=0.12$ | 0.479 | $R_\nu$ | 0.941 | 1.018 | 0.372 | 0.416 |
| | $a_s=0.5$ | | $R_{\bar\nu}$ | 0.960 | 1.079 | 2.582 | 2.792 |
| | $r_{\rm sea}=0.12$ | 0.481 | $R_\nu$ | 0.914 | 1.035 | 0.348 | 0.436 |
| | $a_s=1.0$ | | $R_{\bar\nu}$ | 0.886 | 1.127 | 2.488 | 2.818 |
| | $r_{\rm sea}=0.104$ | 0.460 | $R_\nu$ | 0.948 | 1.013 | 0.359 | 0.389 |
| | $a_s=0.5$ | | $R_{\bar\nu}$ | 0.981 | 1.069 | 2.784 | 2.971 |
| | $r_{\rm sea}=0.137$ | 0.500 | $R_\nu$ | 0.934 | 1.024 | 0.386 | 0.445 |
| | $a_s=0.5$ | | $R_{\bar\nu}$ | 0.939 | 1.089 | 2.391 | 2.622 |
| $\Lambda=0.36$ | $r_{\rm sea}=0.12$ | 0.478 | $R_\nu$ | 0.938 | 1.020 | 0.369 | 0.418 |
| | $a_s=0.5$ | | $R_{\bar\nu}$ | 0.949 | 1.084 | 2.563 | 2.787 |
| $\Lambda=0.58$ | $r_{\rm sea}=0.12$ | 0.481 | $R_\nu$ | 0.947 | 1.016 | 0.376 | 0.413 |
| | $a_s=0.5$ | | $R_{\bar\nu}$ | 0.972 | 1.074 | 2.608 | 2.800 |
| $\Lambda=0.47$ | $r_{\rm sea}=0.12$ | 0.481 | $R_\nu$ | 0.941 | 1.018 | 0.367 | 0.410 |
| | $a_s=0.5$ | | $R_{\bar\nu}$ | 0.959 | 1.079 | 2.625 | 2.837 |
| | $R$ =QCD | | | | | | |

and $y$ distributions, assuming an SU(3)-asymmetric sea, to obtain $g_L^2 = [\varepsilon_L(u)]^2 + [\varepsilon_L(d)]^2 = 0.32 \pm 0.03$ and $g_R^2 = [\varepsilon_R(u)]^2 + [\varepsilon_R(d)]^2 = 0.04 \pm 0.03$. Although we shall use these values in our fits, we first study the sensitivity of the results to the parametrization.

The BEBC group has given absolute values of the total cross sections for CC and NC events. Their fit to the NC data is done with parametrization $A$ with an SU(3) asymmetric sea ($f = 0.2$); they determine $\Lambda = 0.7$ from a fit to the CC data. Our calculation for the CC cross section, which is shown in Fig. 13 (for various values of $f$), agrees well with their fit and with their measured cross sections.

The NC cross section in the WS-GIM model is shown in Fig. 14 using (a) parametrization $A$ with an *ad hoc* sea distribution obtained by calculating the SU(3) symmetric sea distribution and artifically suppressing one-half of the strange quark contribution and all of the charm quark contribution to the cross section, (b) parametrization $A$ with an SU(3) asymmetric sea with $f = 0.2$.

The ratio of the NC and CC cross sections measured by the BEBC group on an isoscalar target with $E_h^{\rm cut} = 15$ GeV are

$$R_\nu^{I=0} = 0.32 \pm 0.03\,, \quad R_{\bar\nu}^{I=0} = 0.39 \pm 0.07\,. \tag{3.30}$$

For the SPM, the results for $R_\nu$ and $R_{\bar\nu}$ are given in Table IX. $R_\nu$ and $R_{\bar\nu}$ flux-averaged on an isoscalar target for $R_{cc} = 0.47$, and $\bar s = \frac{1}{2}\bar u$ are

$$\begin{aligned} R_\nu^{I=0} &= 0.9435\,[\varepsilon_L(u)]^2 + 1.014\,[\varepsilon_L(d)]^2 \\ &\quad + 0.3319\,[\varepsilon_R(u)]^2 + 0.4019\,[\varepsilon_R(d)]^2\,, \\ R_{\bar\nu}^{I=0} &= 0.8454\,[\varepsilon_L(u)]^2 + 1.033\,[\varepsilon_L(d)]^2 \\ &\quad + 2.574\,[\varepsilon_R(u)]^2 + 2.762\,[\varepsilon_R(d)]^2\,. \end{aligned} \tag{3.31}$$

In Fig. 15, the data are compared with the WS-GIM model for $\bar s/\bar u = 0, \frac{1}{2}, 1$ with $R_{cc} = 0.47$. The comparison with Buras and Gaemers with $\Lambda = 0$ is also shown but in this case $R_{cc} = 0.43$, which is a bit lower than the charged-current data.

For QCD (parametrization $A$), the results for $R_\nu$ and

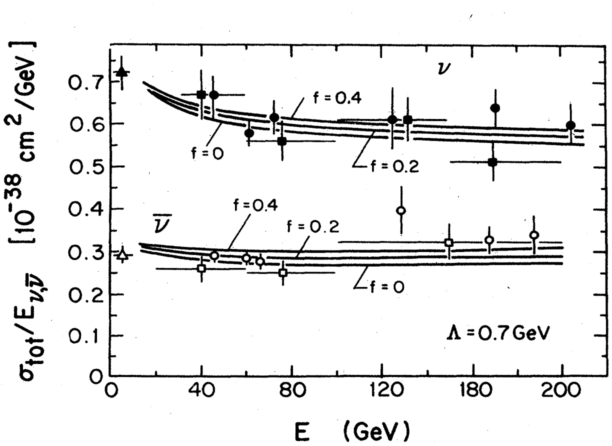


FIG. 13. Comparison of our fit, assuming parametrization $A$, $\Lambda = 0.7$, and $f = 0.0$, 0.2, and 0.4, to the charged-current cross sections measured by BEBC (squares), CITF (circles), and GGM (triangles). Solid points ($\nu$); open points ($\bar\nu$).

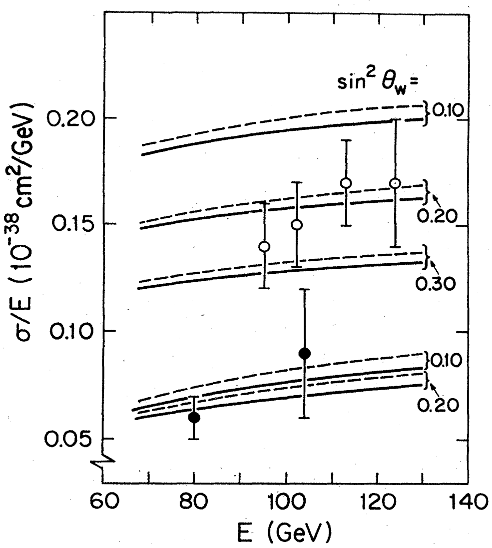


FIG. 14. Neutral-current cross sections in the WS-GIM model. The solid curves assume parametrization $A$ with $f = 0.4$, but include only half of the nominal $s$ and $\bar s$ contribution and no $c$ and $\bar c$ contribution to the cross section. The dashed curves are for $f = 0.2$ and include the full strange and charm sea contribution. The data are from the BEBC experiment. The open points and the associated curves are for $\nu$, the solid points for $\bar\nu$.

$R_{\bar\nu}$ are given in Table X. In the vicinity of $A_s(Q^2 = 1.8) = 0.5$, $R = 0$, $\Lambda = 0.7$ GeV, and $E_h^{\rm cut} = 15$ GeV, $R_\nu$ and $R_{\bar\nu}$ are

$$\begin{aligned} R_\nu^{I=0} &= (0.968 - 0.059\Delta_{f_r})\,[\varepsilon_L(u)]^2 + (1.030 + 0.033\Delta_{f_r}[\varepsilon_L(d)]^2 \\ &\quad + (0.358 - 0.024\Delta_{f_r})[\varepsilon_R(u)]^2 + (0.420 + 0.067\Delta_{f_r}[\varepsilon_R(d)]^2\,, \\ R_{\bar\nu}^{I=0} &= (0.913 - 0.152\Delta_{f_r})[\varepsilon_L(u)]^2 + (1.081 + 0.085\Delta_{f_r})[\varepsilon_L(d)]^2 \\ &\quad + (2.683 - 0.430\Delta_{f_r})[\varepsilon_R(u)]^2 + (2.850 - 0.194\Delta_{f_r})[\varepsilon_R(d)]^2\,, \end{aligned} \tag{3.32}$$

where $\Delta_{f_r} = (f - 0.2)/0.4$. For parametrization $B$, if one assumes that fluxes of the CDHS and BEBC groups are the same, the results can be obtained from Eqs. (3.29) by using $E_h^{\rm cut} = 15$ GeV and $(n/p) = 1$. The effect of varying $f$ is shown in Fig. 15 together with the predictions of parametrization $B$ for the WS-GIM model. It is seen that for neutral current parameters in the vicinity of

TABLE IX. Same as in Table VII, but in deuteron with $E_h^{\rm cut} = 15$ GeV.

| Constraint | $R_{\nu,\bar\nu}$ | $[\varepsilon_L(u)]^2$ | $[\varepsilon_L(d)]^2$ | $[\varepsilon_R(u)]^2$ | $[\varepsilon_R(d)]^2$ |
|---|---|---|---|---|---|
| Buras–Gaemers sea at $Q^2 = 1.8$ | $R_\nu$ | 0.936 | 1.015 | 0.284 | 0.364 |
| ($R_{cc} = 0.43$) | $R_{\bar\nu}$ | 0.802 | 1.042 | 2.869 | 3.109 |
| $R_{cc} = 0.47$ | $R_\nu$ | 1 | 1 | 0.386 | 0.386 |
| $A_s = 0$ | $R_{\bar\nu}$ | 1 | 1 | 2.753 | 2.753 |
| $R_{cc} = 0.47$ | $R_\nu$ | 0.944 | 1.014 | 0.332 | 0.402 |
| $A_s = 0.5$ | $R_{\bar\nu}$ | 0.845 | 1.033 | 2.574 | 2.762 |
| $R_{cc} = 0.47$ | $R_\nu$ | 0.911 | 1.021 | 0.301 | 0.411 |
| $A_s = 1.0$ | $R_{\bar\nu}$ | 0.759 | 1.052 | 2.474 | 2.766 |

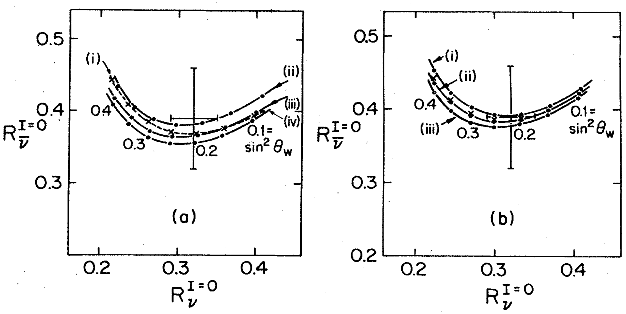


FIG. 15. Comparison of the WS-GIM prediction for $R_\nu$ and $R_{\bar\nu}$, assuming an isoscalar target and $E_H^{\rm cut}=15$ GeV, with the BEBC data. (a) Simple parton model with (i) Buras-Gaemers parametrization with $\Lambda=0$, (ii) $R_{cc}^{I=0}=0.47$, and $\bar s/\bar u=0$, (iii) $R_{cc}^{I=0}=0.47$, and $\bar s/\bar u=0.5$, (iv) $R_{cc}^{I=0}=0.47$. (b) QCD assuming parametrization $A$, $\Lambda=0.7$, and (i) $f=0.0$, (ii) $f=0.2$, (iii) $f=0.4$. The predictions of parametrization $B$, indicated by cross marks (×), are almost indistinguishable from parametrization $A$ with $f=0.2$.

the WS-GIM model the difference between parametrizations $A$ and $B$ is negligible.

### 5. HPWF experiment

The HPWF experiment (Wanderer *et al.*, 1978) on a liquid scintillator target ($CH_2$) obtains[20] for $E_h^{\rm cut}=0$

$$R_\nu^{CH_2}=0.30\pm0.04\,,\qquad R_{\bar\nu}^{CH_2}=0.33\pm0.09\,. \tag{3.33}$$

$R_\nu$ and $R_{\bar\nu}$ are obtained by averaging three experiments performed with different $\nu$, $\bar\nu$ beams (Horn, QT 300, and QT 380). Therefore, we average the fluxes of the three experiments by weighting each one according to the number of CC events obtained. The target deviates slightly from pure isoscalar ($n/p=0.75$). We obtain (again for $r_{\rm sea}=0.12$ and $a_s=0.5$)

$$\begin{aligned} R_\nu^{CH_2}&=1.069[\varepsilon_L(u)]^2+1.030[\varepsilon_L(d)]^2\\ &\quad+0.482[\varepsilon_R(u)]^2+0.494[\varepsilon_R(d)]^2\,,\\ R_{\bar\nu}^{CH_2}&=0.988[\varepsilon_L(u)]^2+1.004[\varepsilon_L(d)]^2\\ &\quad+2.293[\varepsilon_R(u)]^2+2.197[\varepsilon_R(d)]^2\,. \end{aligned} \tag{3.34}$$

Comparison to the WS-GIM model is given in Fig. 16 (solid lines).

The HPWFRO experiment does not directly measure $R_{cc}$. However, it has recently presented (Benvenuti *et al.*, 1979) values of

$$B^{\bar\nu}=-\int_0^1 xF_3^{\bar\nu}(x)dx\Big/\int_0^1 F_2^{\bar\nu}(x)dx$$

and $B^\nu-B^{\bar\nu}$, assuming $\int xc(x)dx\ll\int xs(x)dx$ for the charged current, which are consistent with $R_{cc}=0.48\sim0.53$. They find $B^\nu-B^{\bar\nu}=0.13\pm0.06$ and $B^{\bar\nu}=0.63\pm0.03$ (for $E_\nu>80$ GeV). But[21]

$$\begin{aligned} B^\nu-B^{\bar\nu}&=\frac{2R_{\rm sea}(A_s-A_c)}{1+(A_s/2)+(A_c/2)}\\ &\xrightarrow[A_c=0]{}\frac{4R_{\rm sea}A_s}{2+A_s}=4\frac{\bar s}{q+\bar q}\,, \end{aligned} \tag{3.35}$$

so that $\bar s/(q+\bar q)=0.032\pm0.015$. Also, for $A_c=0$ we have

$$\begin{aligned} R_{\rm sea}&=\tfrac{1}{2}(1-B^{\bar\nu})-\frac{\bar s}{q+\bar q}\\ &=0.15\pm0.02\,, \end{aligned}$$

consistent with the value of 0.12 used previously.

### 6. CITF experiment

The neutral-current experiment of the CITF group (Merritt *et al.*, 1978) on an iron target obtains

$$R_\nu^{\rm Fe}=0.28\pm0.03\,,\qquad R_{\bar\nu}^{\rm Fe}=0.35\pm0.11\,, \tag{3.36}$$

with $E_h^{\rm cut}=12$ GeV from the Fermilab narrow-band beams. Again using the values $r_{\rm sea}=0.12$, $a_s=0.5$, $\Lambda=0.47$, and $E_h^{\rm cut}=12$ GeV, the theoretical ratios in QCD folded with the Fermilab spectrum are

$$\begin{aligned} R_\nu^{\rm Fe}&=0.942[\epsilon_L(u)]^2+1.018[\epsilon_L(d)]^2\\ &\quad+0.331[\epsilon_R(u)]^2+0.373[\epsilon_R(d)]^2\,,\\ R_{\bar\nu}^{\rm Fe}&=0.956[\epsilon_L(u)]^2+1.086[\epsilon_L(d)]^2\\ &\quad+2.990[\epsilon_R(u)]^2+3.233[\epsilon_R(d)]^2\,. \end{aligned} \tag{3.37}$$

The comparison of the CITF data with the WS-GIM model is given in Fig. 16 as dashed lines. This experiment

TABLE X. Same as in Table VIII, but in QCD parametrization $A$ in deuteron with $E_h^{\rm cut}=15$ GeV.

| Constraints | | $R_{cc}$ | $R_{\nu,\bar\nu}$ | $[\varepsilon_L(u)]^2$ | $[\varepsilon_L(d)]^2$ | $[\varepsilon_R(u)]^2$ | $[\varepsilon_R(d)]^2$ |
|---|---|---|---|---|---|---|---|
| $\Lambda=0.7$ | $f=0$ | 0.481 | $R_\nu$ | 0.999 | 1.023 | 0.371 | 0.384 |
| | | | $R_{\bar\nu}$ | 0.997 | 1.034 | 2.919 | 2.957 |
| | $f=0.2$ | 0.495 | $R_\nu$ | 0.968 | 1.030 | 0.358 | 0.420 |
| | | | $R_{\bar\nu}$ | 0.913 | 1.081 | 2.683 | 2.850 |
| | $f=0.4$ | 0.508 | $R_\nu$ | 0.940 | 1.045 | 0.347 | 0.451 |
| | $(\bar s=\bar u)$ | | $R_{\bar\nu}$ | 0.845 | 1.119 | 2.490 | 2.763 |
| $\Lambda=0.47$ | Parametrization $B$ | 0.479 | $R_\nu$ | 0.985 | 1.017 | 0.364 | 0.396 |
| | $a_s=0.5$ | | | | | | |
| | $r_{\rm sea}=0.12$ | | $R_{\bar\nu}$ | 0.956 | 1.047 | 2.795 | 2.886 |

[20]The experiment imposes a cut $E_h>4$ GeV, but the results presented in Eq. (3.33) have been corrected for this cut by the experimenters.

[21]The quantities $B^\nu$, $B^{\bar\nu}$, $R_{\rm sea}$, $A_s$, and $A_c$ in Eq. (3.35) and the following paragraph are taken to be the average values integrated over $Q^2$.

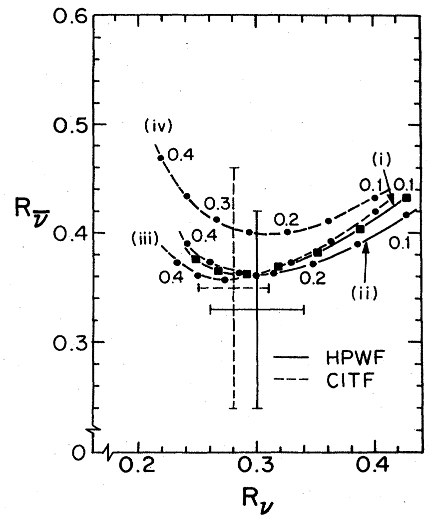


FIG. 16. Comparison of the WS-GIM predictions for $R_\nu$ and $R_{\bar{\nu}}$ with the measurements of the HPWF (solid line) and CITF (dashed line) experiments. Parametrization $B$ of QCD is assumed with $\Lambda = 0.47$ and $a_s = 0.5$. For HPWF the conditions are $E_h^{\text{cut}} = 0$, a $p/n$ target ratio of 1.3, and (i) $R_{cc} = 0.55$ at 100 GeV, (ii) $R_{cc} = 0.477$ at 100 GeV. For CITF the conditions are $E_h^{\text{cut}} = 12$ GeV, an Fe target, (iii) $R_{cc} = 0.6$ at 100 GeV, and (iv) $r_{\text{sea}} = 0.12$.

gives a larger value of $\sin^2\theta_w$ than those of the CDHS and BEBC groups, though it is completely consistent within the error.

### 7. CHARM experiment

The neutral current experiment of the CHARM group (Mess, 1979) on an isoscalar target obtains (see Table II)

$$R_\nu^{I=0} = 0.30 \pm 0.021\,, \qquad R_{\bar{\nu}}^{I=0} = 0.39 \pm 0.024\,, \tag{3.38}$$

with $E_h = 2$, 10, and 17 GeV for the events occurring in the radial intervals $0 < r < 50$ cm, $50\text{ cm} < r < 90$ cm, and $90\text{ cm} < r < 120$ cm, respectively. The theoretical ratios in QCD, integrated over the $\nu$, $\bar{\nu}$ spectra obtained by the CHARM group, are

$$\begin{aligned} R_\nu^{I=0} &= 0.988[\epsilon_L(u)]^2 + 1.022[\epsilon_L(d)]^2 + 0.416[\epsilon_R(u)]^2 + 0.450[\epsilon_R(d)]^2\,, \\ R_{\bar{\nu}}^{I=0} &= 0.969[\epsilon_L(u)]^2 + 1.047[\epsilon_L(d)]^2 + 2.316[\epsilon_R(u)]^2 + 2.394[\epsilon_R(d)]^2\,. \end{aligned} \tag{3.39}$$

### 8. Deep-inelastic scattering from $n$ and $p$ targets

The theoretical formalism described above is valid for $\nu$, $\bar{\nu}$ scattering from $n$ and $p$ targets. However, the accuracy of the data for those targets is such that we neglect the effects of QCD and use the SPM expressions. We also neglect the effects of $s$ and $c$ quarks, but include the SU(2) symmetric contributions of $u$, $\bar{u}$, $d$, and $\bar{d}$ in the sea.

For the experimentally measured quantities

$$\begin{aligned} R_\nu^{n/p} &= \frac{\nu n \to \nu x}{\nu p \to \nu x}\,, \quad R_{\bar{\nu}}^{n/p} = \frac{\bar{\nu} n \to \bar{\nu} x}{\bar{\nu} p \to \bar{\nu} x}\,, \\ R_\nu^{p} &= \frac{\nu p \to \nu x}{\nu p \to \mu^- x}\,, \quad R_{\bar{\nu}}^{p} = \frac{\bar{\nu} p \to \bar{\nu} x}{\bar{\nu} p \to \mu^+ x}\,, \end{aligned} \tag{3.40}$$

we have the following expressions:

$$\begin{aligned} R_\nu^{n/p}(\xi,\alpha) &= \frac{\frac{u}{d}[\epsilon_L(d)^2 + \xi\epsilon_R(d)^2] + [\epsilon_L(u)^2 + \xi\epsilon_R(u)^2] + \frac{\bar{d}}{d}[\xi\epsilon_L(u)^2 + \epsilon_R(u)^2] + \frac{\bar{u}}{d}[\xi\epsilon_L(d)^2 + \epsilon_R(d)^2]}{\frac{u}{d}[\epsilon_L(u)^2 + \xi\epsilon_R(u)^2] + [\epsilon_L(d)^2 + \xi\epsilon_R(d)^2] + \frac{\bar{u}}{d}[\xi\epsilon_L(u)^2 + \epsilon_R(u)^2] + \frac{\bar{d}}{d}[\xi\epsilon_L(d)^2 + \epsilon_R(d)^2]}\,, \\ R_{\bar{\nu}}^{n/p}(\bar{\xi},\alpha) &= \frac{\frac{u}{d}[\epsilon_R(d)^2 + \bar{\xi}\epsilon_L(d)^2] + [\epsilon_R(u)^2 + \bar{\xi}\epsilon_L(u)^2] + \frac{\bar{d}}{d}[\bar{\xi}\epsilon_R(u)^2 + \epsilon_L(u)^2] + \frac{\bar{u}}{d}[\bar{\xi}\epsilon_R(d)^2 + \epsilon_L(d)^2]}{\frac{u}{d}[\epsilon_R(u)^2 + \bar{\xi}\epsilon_L(u)^2] + [\epsilon_R(d)^2 + \bar{\xi}\epsilon_L(d)^2] + \frac{\bar{u}}{d}[\bar{\xi}\epsilon_R(u)^2 + \epsilon_L(u)^2] + \frac{\bar{d}}{d}[\bar{\xi}\epsilon_R(d)^2 + \epsilon_L(d)^2]}\,, \\ R_\nu^{p}(\xi,\alpha) &= \frac{\frac{u}{d}[\epsilon_L(u)^2 + \xi\epsilon_R(u)^2] + [\epsilon_L(d)^2 + \xi\epsilon_R(d)^2] + \frac{\bar{u}}{d}[\xi\epsilon_L(u)^2 + \epsilon_R(u)^2] + \frac{\bar{d}}{d}[\xi\epsilon_L(d)^2 + \epsilon_R(d)^2]}{1 + \xi\frac{\bar{u}}{d}}\,, \\ R_{\bar{\nu}}^{p}(\bar{\xi},\alpha) &= \frac{[\epsilon_R(u)^2 + \bar{\xi}\epsilon_L(u)^2] + \frac{d}{u}[\epsilon_R(d)^2 + \bar{\xi}\epsilon_L(d)^2] + \frac{\bar{u}}{u}[\bar{\xi}\epsilon_R(u)^2 + \epsilon_L(u)^2 + \frac{\bar{d}}{u}[\bar{\xi}\epsilon_R(d)^2 + \epsilon_L(d)^2]}{\bar{\xi} + \frac{\bar{d}}{u}}\,. \end{aligned} \tag{3.41}$$

Assuming $u_v/d_v = 2$, we have

$$\frac{u}{d} = 2 - \frac{\bar{u}}{d} = 2 - \alpha'\,,$$

where

$$\alpha' = \frac{\bar{u}}{d} = \frac{\bar{d}}{d} = \frac{3\alpha}{2+\alpha}\,, \qquad \alpha = \frac{\sum_i \bar{q}_i}{\sum_i q_i}\,, \tag{3.42}$$

and where

$$q_i = \int_0^1 x q_i(x) dx \, .$$

As with the experiments discussed in Sec. III. A, each of the experiments measuring the above quantities employs, in its selection of neutral-current events, a cut requiring the hadron energy $E_h$ to be greater than some minimum value $E_h^{\text{cut}}$; the effect of this cut is realized through the parameters

$$\xi = \frac{\int dE_\nu E_\nu \phi_\nu(E_\nu) \int_{a(E_\nu)} dy (1-y)^2}{\int dE_\nu E_\nu \phi_\nu(E_\nu) \int_{a(E_\nu)}^1 dy} \, ,$$

$$\bar{\xi} = \frac{\int dE_{\bar\nu} E_{\bar\nu} \phi_{\bar\nu}(E_{\bar\nu}) \int_{a(E_{\bar\nu})}^1 dy (1-y)^2}{\int dE_{\bar\nu} E_{\bar\nu} \phi_{\bar\nu}(E_{\bar\nu}) \int_{a(E_{\bar\nu})}^1 dy} \, , \tag{3.43}$$

where $\phi_\nu(E_\nu)$, $\phi_{\bar\nu}(E_{\bar\nu})$ are the neutrino and antineutrino flux distributions and $a(E_\nu) = E_h^{\text{cut}}/E_\nu$.

For the measurements of $R_\nu^p$, $R_{\bar\nu}^p$, and $R_\nu^{n/p}$ by Harris *et al.* (1977), Derrick *et al.* (1978), and Marriner (1977), respectively, we have used the values of $\xi$, $\bar\xi$, and $\alpha$ quoted by Abbott and Barnett (1979). For the value of $R_{\bar\nu}^{n/p}$ reported recently by Bell *et al.* (1979), we also use the same values. The values of $\xi$ and $\bar\xi$ are summarized in Sec. III. E. Finally, for the measurement of $R_\nu^p$ reported by Blietschau *et al.* (1979), we use their calculated expression

$$R_\nu^p = 2.1[\epsilon_L(u)]^2 + 1.0[\epsilon_L(d)]^2 + 0.70[\epsilon_R(u)]^2 + 0.36[\epsilon_R(d)]^2 \, . \tag{3.44}$$

### B. Semi-inclusive pion production

We adopt the simple parton model (SPM) description for the semi-inclusive reactions, $\nu(\bar\nu) + N \to \nu(\bar\nu) + \pi^\pm + X$, because the complicated nature of the reactions, the uncertainties in the fragmentation functions, and the experimental uncertainties do not warrant a full QCD treatment. We basically follow the approach of Sehgal (1975) and Abbott and Barnett (1978b), and[22] include the (small) corrections due to antiquarks.

For the ratios of $\pi^+$ and $\pi^-$ produced in $\nu$ and $\bar\nu$ reactions we obtain

$$R_\nu^{+/-} \equiv \left(\frac{\pi^+}{\pi^-}\right)_{\nu\to\nu} = \frac{\eta + r_\nu}{1 + \eta r_\nu} \, , \tag{3.45a}$$

$$R_{\bar\nu}^{+/-} \equiv \left(\frac{\pi^+}{\pi^-}\right)_{\bar\nu\to\bar\nu} = \frac{\eta + r_{\bar\nu}}{1 + \eta r_{\bar\nu}} \, , \tag{3.45b}$$

$$R_\nu^{+/+-} \equiv \left(\frac{\pi^+}{\pi^+ + \pi^-}\right)_{\nu\to\nu} = \frac{R_\nu^{+/-}}{1 + R_\nu^{+/-}} \, , \tag{3.45c}$$

$$R_{\bar\nu}^{+/+-} \equiv \left(\frac{\pi^+}{\pi^+ + \pi^-}\right)_{\bar\nu\to\bar\nu} = \frac{R_{\bar\nu}^{+/-}}{1 + R_{\bar\nu}^{+/-}} \, . \tag{3.45d}$$

The ratio of the probability for a $u$ quark to fragment into a $\pi^+$ to the probability for a $u$ quark to fragment into a $\pi^-$ is

$$\eta = \frac{\int_{z_1}^{z_2} D_u^{\pi^+}(z) dz}{\int_{z_1}^{z_2} D_u^{\pi^-}(z) dz} \, . \tag{3.46}$$

The integral extends over the range of $z$ values allowed in a particular experiment ($z = E_\pi/E_h$). For the GGM experiment (Kluttig, Morfin, and Van Doninck, 1977), $\eta$ is $2.8 \pm 0.7$. (This number is obtained from electroproduction and charged-current neutrino reactions.) Also

$$r_\nu = \frac{b}{a} \, , \quad r_{\bar\nu} = \frac{\bar b}{\bar a} \tag{3.47}$$

where

$$\begin{aligned} a &\equiv |\epsilon_L(u)|^2 + \xi |\epsilon_R(u)|^2 + \alpha\xi |\epsilon_L(d)|^2 + \alpha |\epsilon_R(d)|^2 \, , \\ b &\equiv |\epsilon_L(d)|^2 + \xi |\epsilon_R(d)|^2 + \alpha\xi |\epsilon_L(u)|^2 + \alpha |\epsilon_R(u)|^2 \, , \\ \bar a &\equiv \bar\xi |\epsilon_L(u)|^2 + |\epsilon_R(u)|^2 + \alpha |\epsilon_L(d)|^2 + \alpha\bar\xi |\epsilon_R(d)|^2 \, , \\ \bar b &\equiv \bar\xi |\epsilon_L(d)|^2 + |\epsilon_R(d)|^2 + \alpha |\epsilon_L(u)|^2 + \alpha\bar\xi |\epsilon_R(u)|^2 \, , \end{aligned} \tag{3.48}$$

and

$$\alpha = \int_0^1 x\bar q(x) dx \Big/ \int_0^1 x q(x) dx \, , \tag{3.49}$$

and

$$\xi = \frac{\int dE_\nu E_\nu \phi(E_\nu) \int_{a(E_\nu)}^1 (1-y)^2 dy}{\int dE_\nu E_\nu \phi_\nu(E_\nu) \int_{a(E_\nu)}^1 dy} \, . \tag{3.50}$$

$\phi_\nu(E_\nu)$ is the neutrino spectrum and $a(E_\nu) = E_h^{\text{cut}}/E_\nu$ incorporates the effects of the experimental hadron energy cut ($\xi = \frac{1}{3}$ for $E_h^{\text{cut}} = 0$). $\bar\xi$ is the same as Eq. (3.50) with the replacement $\nu \to \bar\nu$. For the GGM experiment $\xi_\nu = 0.15$, $\xi_{\bar\nu} = 0.13$, and $\alpha = 0.05$. (Kluttig, Morfin, and Van Doninck, 1977; Sehgal, 1977b).

The Fermilab experiment (FIIM) (Roe, 1979) uses a mixture of $\nu$ and $\bar\nu$ beams. If $l$ is the ratio of the $\bar\nu$ flux to the $\nu$ flux, one obtains

$$R_{\nu+l\bar\nu}^{+/+-} = \frac{\eta + r'_{\nu+l\bar\nu}}{(1+\eta)(1 + r'_{\nu+l\bar\nu})} \, , \tag{3.51}$$

with

$$r'_{\nu+l\bar\nu} \equiv \frac{b + l\bar b}{a + l\bar a} \, . \tag{3.52}$$

For the FIIM experiment, $l = 4.5 \pm 0.4$, and, integrating over the $\nu$ and $\bar\nu$ spectra, we obtain $\xi = 0.194$ and $\bar\xi = 0.157$. We take $\alpha = 0.12$ and use the same value for $\eta$ ($2.8 \pm 0.7$) as for the GGM experiment.

It is useful to invert Eqs. (3.45) and (3.51) so that only the measured quantities $R_\nu^{+/-}$, $R_{\bar\nu}^{+/-}$, $R_{\nu+l\bar\nu}^{+/+-}$, and $\eta$ appear on the left-hand side, and only the "theoretical" quantities $r_\nu$, $r_{\bar\nu}$, and $r'_{\nu+l\bar\nu}$ appear on the right-hand side. We then obtain

$$\frac{R_\nu^{+/-} - \eta}{1 - \eta R_\nu^{+/-}} = r_\nu \, , \quad \frac{R_{\bar\nu}^{+/-} - \eta}{1 - \eta R_{\bar\nu}^{+/-}} = r_{\bar\nu} \, , \tag{3.53}$$

and

$$\frac{\eta - (1+\eta) R_{\nu+l\bar\nu}^{+/+-}}{(1+\eta) R_{\nu+l\bar\nu}^{+/+-} - 1} = r'_{\nu+l\bar\nu} \, . \tag{3.54}$$

We may now include the uncertainty in $\eta$ by combining its error in quadrature with the experimental error in $R_\nu^{+/-}$, $R_{\bar\nu}^{+/-}$, and $R_{\nu+l\bar\nu}^{+/-}$. From the measured values $R_\nu^{+/-}$

[22]See also Cleymans and Sehgal (1974) and Hung (1977). QCD and mass corrections were estimated to be small by Sarkar (1979).

$=0.77\pm0.14$, $R_{\bar\nu}^{+/-}=1.65\pm0.33$, and $R_{\nu+t\bar\nu}^{+/-}=0.56\pm0.06$, we obtain

$$\begin{aligned} 1.76\pm0.75 &= r_\nu \quad (\text{GGM},\ \nu)\,,\\ 0.32\pm0.21 &= r_{\bar\nu} \quad (\text{GGM},\ \bar\nu)\,,\\ 0.59\pm0.33 &= r'_{\nu+t\bar\nu} \quad (\text{FIIM},\ \nu+4.5\bar\nu)\,. \end{aligned} \tag{3.55}$$

For reasonable values of $\alpha$, and for the neutral-current parameters in the vicinity of the WS-GIM model, the antiquark contributions to $r_\nu$ and $r_{\bar\nu}$ are small. We have neglected $s$ quarks because their effects are expected to be small compared to the uncertainties in the data and because their fragmentation functions into $\pi^\pm$ are not well known. It should be remarked that the SPM is reasonably successful in predicting the ratios of charged to neutral pions for both $\nu$ and $\bar\nu$ scattering.

### C. Elastic scattering[23]

The hadronic matrix element relevant to elastic $\nu(\bar\nu)p \to \nu(\bar\nu)p$ scattering is

$$\langle f|J_\mu^H|i\rangle = \bar u_f\left(\gamma_\mu F_1 + \frac{i\sigma_{\mu\nu}q^\nu}{2M}F_2 + \gamma_\mu\gamma_5 G_A\right)u_i\,, \tag{3.56}$$

where $q=p_f-p_i$. The expression for $d\sigma/dQ^2$ for $\nu p\to\nu p$ and $\bar\nu p\to\bar\nu p$ in terms of the form factors are given in Kim, Langacker, and Sarkar (1978).

The vector form factors $F_1(Q^2)$ and $F_2(Q^2)$ are easily computed from known electromagnetic form factors, with the result

$$\begin{aligned} G_M &\equiv F_1+F_2 = \alpha G_V^3 + \gamma G_V^0\,,\\ F_2 &= \alpha F_V^3 + \gamma F_V^0\,, \end{aligned} \tag{3.57}$$

where, in the dipole representation,

$$\begin{aligned} G_V^3 &= \frac{1}{2}\,\frac{1+\kappa_p-\kappa_n}{\left(1+\dfrac{Q^2}{M_V^2}\right)^2}\,,\\ G_V^0 &= \frac{3}{2}\,\frac{1+\kappa_p+\kappa_n}{\left(1+\dfrac{Q^2}{M_V^2}\right)^2}\,,\\ F_V^3 &= \frac{1}{2}\,\frac{\kappa_p-\kappa_n}{(1+\tau)\left(1+\dfrac{Q^2}{M_V^2}\right)^2}\,,\\ F_V^0 &= \frac{3}{2}\,\frac{\kappa_p+\kappa_n}{(1+\tau)\left(1+\dfrac{Q^2}{M_V^2}\right)^2}\,. \end{aligned} \tag{3.58}$$

Here $Q^2=-q^2>0$ is the squared momentum transfer, $\tau=Q^2/4M_p^2$, $M_V^2=0.71\ \text{GeV}^2$ is the dipole mass parameter, and $\kappa_p=1.793$ and $\kappa_n=-1.913$ are the anomalous magnetic moments of the proton and neutron. Expressions (3.57) and (3.58) assume that the neutron electric form factor $G_E^n=F_1^n-\tau F_2^n$ is zero for all $Q^2$. Gourdin (1974) states that the data are compatible with $G_E^n=-\tau G_M^n$. In this case, the formulas for $G_V^3$ and $G_V^0$ are unchanged, while $\kappa_n$ is replaced by $(1+\tau)\kappa_n$ in the expressions for $F_V^3$ and $F_V^0$. The expressions are strictly valid only if the proton matrix elements of the vector currents $\bar s\gamma_\mu s$, $\bar c\gamma_\mu c$, etc., can be ignored. We believe this to be a very good approximation for the vector form factors because in perturbation theory the lowest-order graph contributing to $\langle p_f|\bar s\gamma_\mu s|p_i\rangle$ involves the exchange of three virtual gluons between a heavy quark loop and the valence quarks. Hence the corrections to $F_1$ and $F_2$ should be of $O(\alpha_s^3)\lesssim 0.01$, where $\alpha_s$ is the strong fine-structure constant.

The axial-vector form factor $G_A(Q^2)$ is more difficult. If one ignores the contributions of the $s$ and heavy quarks, one has

$$G_A(Q^2) = \beta G_A^3 + \delta G_A^0\,. \tag{3.59}$$

The axial-isovector form factor $G_A^3$ is parametrized by the dipole form

$$\begin{aligned} G_A^3(Q^2) &= \tfrac{1}{2}g_A(Q^2)\,,\\ g_A(Q^2) &= \frac{g_A(0)}{\left(1+\dfrac{Q^2}{M_A^2}\right)^2}\,, \end{aligned} \tag{3.60}$$

where $g_A(0)=1.260\pm0.012$ from $\beta$ decay (Nagels *et al.*, 1979). For the dipole mass parameter $M_A$, we take $(1.00\pm0.05)$ GeV.

More generally, we take

$$G_A^3(Q^2) = \tfrac{1}{2}g_A(Q^2)(1+\varepsilon) \tag{3.61}$$

where $\varepsilon$ is a correction for heavy quark currents such as $\bar c\gamma_\mu\gamma_5 c - \bar s\gamma_\mu\gamma_5 s$ (this correction would contribute to $\nu n\to\nu n$ with the same sign as $\nu p\to\nu p$). Collins, Wilczek, and Zee (1978; see also Mohapatra and Senjanovic, 1979; Oneda, Tanuma, and Slaughter, 1979) have estimated $\varepsilon=0.04$ from the diagram in which two gluons are exchanged between a heavy quark loop and the valence quarks $[O(\alpha_s^2)]$. Wolfenstein (1979) has estimated $\varepsilon=0.10\pm0.15$. We shall take $\varepsilon=0.0^{+0.2}_{-0.0}$.

The axial-isoscalar form factor $G_A^0$, for which there is no independent experimental measurement, is trickier. We shall assume a dipole form

$$G_A^0(Q^2) = \frac{G_A^0(0)}{\left(1+\dfrac{Q^2}{(M_A^0)^2}\right)^2} \tag{3.62}$$

and take $M_A^0=M_A$. $G_A^0(0)$ can be estimated from SU(3) symmetry if one also assumes $\langle p_f|\bar s\gamma_\mu\gamma_5 s|p_i\rangle=0$. Then

$$G_A^0(0) = \left(3-4\frac{D}{F+D}\right)\frac{g_A(0)}{2} \equiv \frac{\lambda g_A(0)}{2}\,. \tag{3.63}$$

The $D/(D+F)$ ratio determined from semileptonic hyperon decays is (M. Roos, quoted in Nagels *et al.*, 1979) $D/(D+F)=0.65\pm0.01$, which gives $\lambda=0.40\pm0.04$. On the other hand, SU(6) gives $D/(D+F)=0.60$, which implies $\lambda=0.60$. For further discussion, see the paper of Hung (1978).

Wolfenstein (1979) has recently argued that the theoretical predictions for $G_A^0(0)$ are unreliable because of anomalies, and he has suggested that $G_A^0(0)$ should be kept as a free parameter in the fits. In most of our fits we shall take $D/(D+F)=0.65\pm0.05$, so that $\lambda=0.4\pm0.2$. The large uncertainty represents not only the error in

[23]Previous analyses of elastic $\nu(\bar\nu)p$ scattering include Barnett, 1975; Sidhu, 1976; Albright *et al.*, 1976; Barger and Nanopoulos, 1976; Abbott and Barnett, 1979; Sidhu and Langacker, 1978; Paschos, 1979; Claudson *et al.*, 1979; Hendrick and Li, 1979; and Williams, 1978.

$D/(D+F)$ but also the uncertainties in the use of SU(3), the treatment of the strange and heavy quarks, and possible difficulties with anomalies. We shall also perform fits in which $\lambda$ is treated as a free parameter, in accordance with Wolfenstein's suggestion. Fortunately, it will turn out that $\delta$ is very small (it is zero in the WS-GIM model), so the uncertainties due to $G_A^0(Q^2)$ are minimized.

In summary, then, we take

$$G_A(Q^2)=\frac{1}{2}\frac{1.26}{\left(1+\frac{Q^2}{M_A^2}\right)^2}[\beta(1+\varepsilon)+\delta\lambda]\,, \tag{3.64}$$

where $M_A=(1.00\pm0.05)$ GeV, $\varepsilon=0.0^{+0.2}_{-0.0}$, and either $\lambda=0.4\pm0.2$ or $\lambda=$ (free parameter), depending on the run. We fit the parameters directly to the differential cross-section data points.

### D. Other processes

As noted in Sec. II, experiments on exclusive pion production reactions $\nu(\bar{\nu})N\to\nu(\bar{\nu})+\pi+N$ have now clearly observed the $\Delta$ resonance (Krenz *et al.*, 1978; Erriques *et al.*, 1978; Barish *et al.*, 1974; Lee *et al.*, 1977b); this fact together with the observed ratios of the cross sections for different charge states ($p\pi^-$, $n\pi^+$, $n\pi^0$, $p\pi^0$) indicates that the hadronic neutral current $J_\mu^H$ is not purely isoscalar. Beyond this observation we shall not use information from these reactions in our fits. The reason is that the hadronic matrix elements $\langle \pi N|J_\mu^H|N\rangle$ are not known in any reliable way. Abbott and Barnett (1978b) and Monsay (1978) have analyzed the reactions using the Adler model (Adler, 1968, 1975; Adler *et al.*, 1976)[24] for the matrix elements. They have found that the data are only compatible with the WS-GIM model if a large (30%) theoretical uncertainty is assumed for the matrix elements. Since these theoretical uncertainties are large and difficult to estimate reliably, the exclusive data would not usefully constrain our fits.

A number of authors have suggested other types of experiments which would be sensitive to particular aspects of the hadronic neutral current. These include measurements of final-state polarizations (Kim, Langacker, and Sarkar, 1978; Le Yaouanc *et al.*, 1977; Doncel, Michel, and Minnaert, 1978; Donoghue, 1978; Cho and Gourdin, 1976), specific nuclear transitions (Gounaris and Vergados, 1977, 1978; Walecka, 1977a; Konuma and Oka, 1978a; Avilex, Kobayashi, and Paez, 1979; Bernabeu and Pascual, 1979a), and other exclusive reactions (Mintz, 1978; Biswas *et al.*, 1978).

### E. Determination of hadronic weak neutral current

We now use the theoretical expressions derived in Secs. III A–III D in conjunction with the experimental results summarized in Sec. II to determine the hadronic neutral-current coupling constants. In the simplest parton model formalism, in which the effects of QCD, hadron energy cuts, and the neutral-current coupling to $s$ and $c$ quarks are ignored, measurements of deep-inelastic scattering from an isoscalar target simply determine the strength of the left- and right-handed couplings:

$$g_L=[\varepsilon_L(u)^2+\varepsilon_L(d)^2]^{1/2}$$

and

$$g_R=[\varepsilon_R(u)^2+\varepsilon_R(d)^2]^{1/2};$$

this is true both for measurements of

$$R_\nu=\frac{\sigma(\nu N\to\nu X)}{\sigma(\nu N\to\mu^- X)}$$

and

$$R_{\bar{\nu}}=\frac{\sigma(\bar{\nu}N\to\bar{\nu}X)}{\sigma(\bar{\nu}N\to\mu^+ X)}$$

and for measurements of $d\sigma/dy(\nu N\to\nu X)$ and $d\sigma/dy(\bar{\nu}N\to\bar{\nu}X)$. The allowed regions of coupling constants may be represented by an annulus in $\varepsilon_L(u)-\varepsilon_L(d)$ space and a similar annulus in $\varepsilon_R(u)-\varepsilon_R(d)$ space. The absence of any information on the angles $\theta_L=\arctan[\varepsilon_L(u)/\varepsilon_L(d)]$, $\theta_R=\arctan[\varepsilon_R(u)/\varepsilon_R(d)]$ reflects the absence of any information on the isospin structure of the neutral current, since the scattering is from an isoscalar target. The effect of QCD corrections, the neutron excess in the target, and the coupling to $s$ and $c$ quarks, which we include in our analysis, is to modify slightly these annuli.

To eliminate any ambiguity, we summarize here the specific constraints that follow from the data on deep-inelastic scattering from an (approximately) isoscalar target.[25]

*Constraints (1)–(10).*

(1) $0.307\pm0.008=R_\nu^{\rm Fe}=0.941[\varepsilon_L(u)]^2+1.018[\varepsilon_L(d)]^2+0.372[\varepsilon_R(u)]^2+0.416[\varepsilon_R(d)]^2\,,$

(2) $0.373\pm0.025=R_{\bar{\nu}}^{\rm Fe}=0.960[\varepsilon_L(u)]^2+1.079[\varepsilon_L(d)]^2+2.582[\varepsilon_R(u)]^2+2.792[\varepsilon_R(d)]^2\,,$

(3) $0.28\pm0.03=R_\nu^{\rm Fe}=0.9420|\varepsilon_L(u)|^2+1.0183|\varepsilon_L(d)|^2+0.3305|\varepsilon_R(u)|^2+0.3727|\varepsilon_R(d)|^2\,,$

(4) $0.35\pm0.11=R_{\bar{\nu}}^{\rm Fe}=0.9562|\varepsilon_L(u)|^2+1.0858|\varepsilon_L(d)|^2+2.9904|\varepsilon_R(u)|^2+3.2331|\varepsilon_R(d)|^2\,,$

(5) $0.30\pm0.04=R_\nu^{\rm CH_2}=1.0691|\varepsilon_L(u)|^2+1.0302|\varepsilon_L(d)|^2+0.4822|\varepsilon_R(u)|^2+0.4938|\varepsilon_R(d)|^2\,,$

---

[24] Quark model calculations have been performed by Korner, Kobayashi, and Avilez (1978).

[25] For the CHARM experiment, constraints (7) and (8) used in the fits were obtained assuming an average $E_h^{\rm cut}=6$ GeV; subsequently we were able to calculate expressions (3.39) incorporating the variable cut $E_h^{\rm cut}=2$, 10, 17 GeV depending on the radial position of the event (see Sec. III.A.7). The coefficients in constraints (7) and (8) and in Eqs. (3.39) agree within 1% except for the coefficients of $\varepsilon_R(u)^2$ and $\varepsilon_R(d)^2$, which differ by 3.9%. This difference has negligible effect on the determination of the coupling constants.

(6) $0.33\pm0.09=R_{\bar\nu}^{\mathrm{CH}_2}=0.9882\,|\varepsilon_L(u)|^2+1.0042\,|\varepsilon_L(d)|^2+2.2925\,|\varepsilon_R(u)|^2+2.1974\,|\varepsilon_R(d)|^2$,

(7) $0.30\pm0.021=R_{\nu}^{I=0}=0.9868\,|\varepsilon_L(u)|^2+1.0201\,|\varepsilon_L(d)|^2+0.4127\,|\varepsilon_R(u)|^2+0.4463\,|\varepsilon_R(d)|^2$,

(8) $0.39\pm0.024=R_{\bar\nu}^{I=0}=0.9648\,|\varepsilon_L(u)|^2+1.0461\,|\varepsilon_L(d)|^2+2.4060\,|\varepsilon_R(u)|^2+2.4868\,|\varepsilon_R(d)|^2$,

(9) $g_L^2=0.32\pm0.03=\varepsilon_L(u)^2+\varepsilon_L(d)^2$,

(10) $g_R^2=0.04\pm0.03=\varepsilon_R(u)^2+\varepsilon_R(d)^2$.

A fit of the parameters $g_L$, $g_R$, $\theta_L$, and $\theta_R$ to constraints (1)–(10) yields the allowed regions (at 90% confidence level) in the $\varepsilon_L(u)-\varepsilon_L(d)$ and $\varepsilon_R(u)-\varepsilon_R(d)$ planes shown in Fig. 17; the $\chi^2$ is 3.1 for six degrees of freedom. The values of $g_L$ and $g_R$ determined are

$$g_L=0.543\pm0.015\,,\quad g_R=0.172\pm0.027\,.$$

No significant information on $\theta_L$ and $\theta_R$ is obtained.

Information on the isospin of the neutral current comes from several $\nu$ reactions including (1) deep-inelastic scattering from neutrons and protons (Blietschau *et al.*, 1979; Harris *et al.*, 1977; Derrick *et al.*, 1978; Marriner, 1977; Bell *et al.*, 1979), (2) $\pi^+/\pi^-$ inclusive charge ratios (Kluttig *et al.*, 1977; Roe, 1979), (3) neutrino and antineutrino elastic scattering from protons (Lee *et al.*, 1977a; Sokolsky, 1978; Entenberg *et al.*, 1979; Kozanecki, 1978; Strait, 1978; Faissner *et al.*, 1980; Pohl *et al.*, 1978), and (4) single-pion production (Bertrand-Coremans *et al.*, 1976; Krenz *et al.*, 1978; Erriques *et al.*, 1978; Paty, 1979) and the reaction $\bar\nu_e D\to\bar\nu_e np$ (Pasierb *et al.*, 1979) as discussed in Sec. III. A–III.D, respectively. The additional information provided is best illustrated by presenting the region of coupling constants allowed when the data from each type of reaction are taken together with the deep-inelastic (isoscalar) data.

The data which currently exist on inelastic scattering from neutrons and protons yield the following constraints.

*Constraints (11)–(15).*

(11) $0.52\pm0.06=R_\nu^p=2.1[\varepsilon_L(u)]^2+1.0[\varepsilon_L(d)]^2+0.70[\varepsilon_R(u)]^2+0.36[\varepsilon_R(d)]^2$,

(12) $0.48\pm0.17=R_\nu^p(\xi,\alpha)$, $\xi=0.21$, $\alpha=0.08$

(13) $0.42\pm0.13=R_{\bar\nu}^p(\bar\xi,\alpha)$, $\bar\xi=0.13$, $\alpha=0.08$

(14) $1.22\pm0.35=R_\nu^{n/p}(\xi,\alpha)$, $\xi=0.21$, $\alpha=0.08$

(15) $1.06\pm0.20=R_{\bar\nu}^{n/p}(\bar\xi,\alpha)$, $\bar\xi=0.13$, $\alpha=0.08$.

$R_\nu^p(\xi,\alpha)$, $R_{\bar\nu}^p(\bar\xi,\alpha)$, $R_\nu^{n/p}(\xi,\alpha)$, and $R_{\bar\nu}^{n/p}(\bar\xi,\alpha)$ are functions of $|\varepsilon_L(u)|^2$, $|\varepsilon_L(d)|^2$, $|\varepsilon_R(u)|^2$, and $|\varepsilon_R(d)|^2$, and are presented in subsection III.A.8. A fit to constraints (11)–(15) in conjunction with the data from isoscalar targets [constraints (1)–(10)] yields the allowed regions shown in Fig. 18. While significant information is provided on $\theta_L$ which measures the isospin of the left-handed coupling, $\theta_R$, which determines the isospin nature of the right-handed coupling, is essentially undetermined.

Measurements of the charge ratios in semi-inclusive pion production $R_\nu^{+/-}=\sigma(\nu N\to\nu\pi^+X)/\sigma(\nu N\to\nu\pi^-X)$ and $R_{\bar\nu}^{+/-}=\sigma(\bar\nu N\to\bar\nu\pi^+X)/\sigma(\bar\nu N\to\bar\nu\pi^-X)$ yield the following.

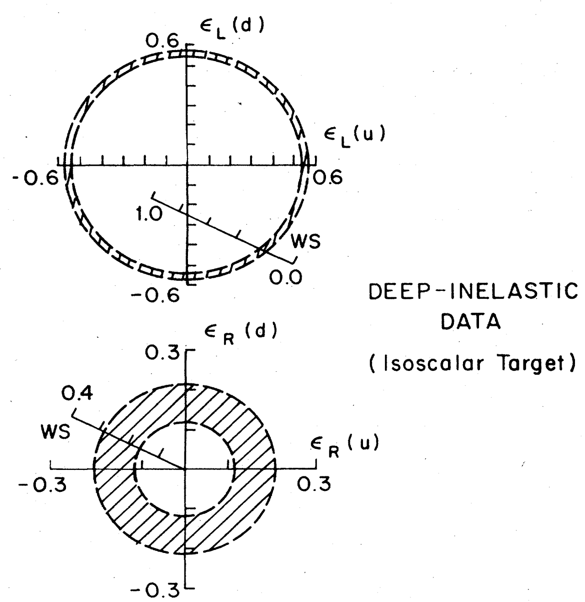


FIG. 17. Region of hadronic coupling constants allowed (at 90% confidence level) by the data on deep-inelastic $\nu$ and $\bar\nu$ scattering from an isoscalar target. The prediction of the WS-GIM model is shown as a function of $\sin^2\theta_w$.

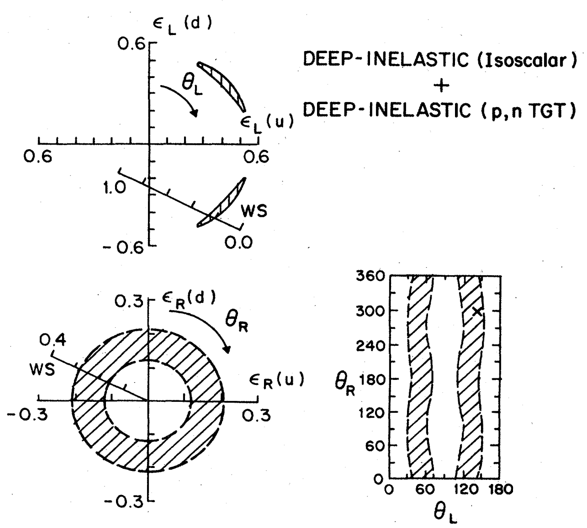


FIG. 18. Region of hadronic coupling constants allowed (at 90% confidence level) by the data on deep-inelastic $\nu,\bar\nu$ scattering from both isoscalar and $p,n$ targets. The prediction of the WS-GIM model is shown as a function of $\sin^2\theta_w$. The cross on the $\theta_R-\theta_L$ plot corresponds to $\sin^2\theta_w=0.233$.

*Constraints (16)–(18).*

(16) $1.76 \pm 0.75 = r_\nu$,

(17) $0.32 \pm 0.21 = r_{\bar\nu}$,

(18) $0.59 \pm 0.33 = r'_{\nu+l\bar\nu}$,

where $r_\nu$, $r_{\bar\nu}$, and $r'_{\nu+l\bar\nu}$ are functions of $|\varepsilon_L(u)|^2$, $|\varepsilon_L(d)|^2$, $|\varepsilon_R(u)|^2$, $|\varepsilon_R(d)|^2$ that are presented in Eqs. (3.47) and (3.52). These data when combined with constraints (1)–(10) yield the allowed regions presented in Fig. 19.

Finally, the data on elastic scattering of $\nu$ and $\bar\nu$ from protons provide information on both the strength and isospin of the neutral current. Comparison of the measured differential cross sections (Entenberg *et al.*, 1979; Kozanecki, 1978; Strait, 1978) with those calculated according to Sec. III.C yields constraints (19)–(28). The allowed regions, when these constraints are combined with the deep-inelastic isoscalar data, are presented in Fig. 20.

It is apparent from Figs. 18–20 that the data on inelastic scattering from protons and neutrons, charge ratios in semi-inclusive pion reactions, and the elastic scattering from protons restrict the isospin of the neutral current, and hence the values of $\varepsilon_L(u)$, $\varepsilon_L(d)$, $\varepsilon_R(u)$, $\varepsilon_R(d)$ to well defined regions. This is best shown in Fig. 21, which presents the results of a simultaneous fit to all the $\nu$-hadron data [constraints (1)–(28)]. The data allow (at 90% C.L.) two separate regions which have previously been referred to as solutions $A$ (dominantly isovector) and $B$ (dominantly isoscalar) (Hung and Sakurai, 1977b). The latter is excluded by the data on single-pion production as emphasized in Secs. II and III.D. [We also note for completeness that recent results (Pasierb *et al.*, 1979)[26] determining $\sigma(\bar\nu_e D \to \bar\nu_e np) = (3.8 \pm 0.9) \times 10^{-45}$ cm$^2$ are consistent with solution $A$ and exclude solution $B$ at the level of approximately two standard deviations. While solution $B$ is not allowed

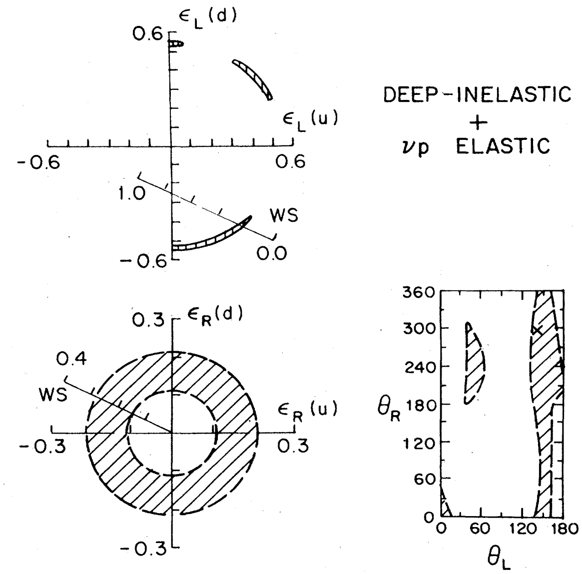


FIG. 20. Region of hadronic coupling constants allowed (at 90% confidence level) by the data on deep-inelastic $\nu,\bar\nu$ scattering from an isoscalar target and measurements of the elastic scattering of $\nu,\bar\nu$ from protons. The prediction of the WS-GIM model is shown as a function of $\sin^2\theta_w$. The cross on the $\theta_R-\theta_L$ plot corresponds to $\sin^2\theta_w = 0.233$.

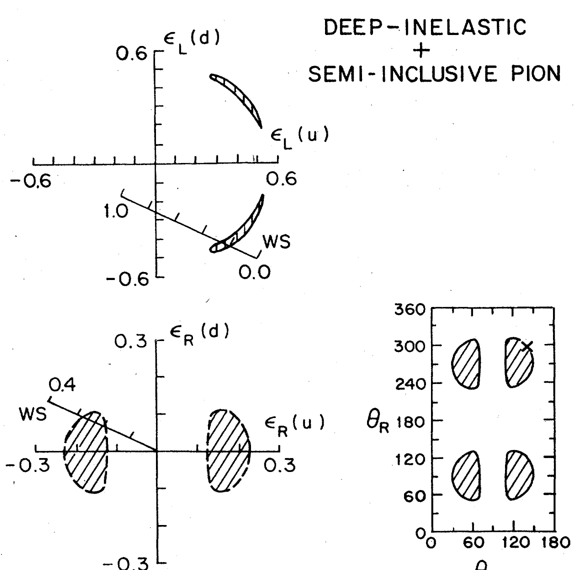


FIG. 19. Region of hadronic coupling constants allowed (at 90% confidence level) by the data on deep-inelastic $\nu,\bar\nu$ scattering from an isoscalar target and measurements of the $\pi^+/\pi^-$ ratio in semi-inclusive pion production. The prediction of the WS-GIM model is shown as a function of $\sin^2\theta_w$. The cross on the $\theta_R-\theta_L$ plot corresponds to $\sin^2\theta_w = 0.233$.

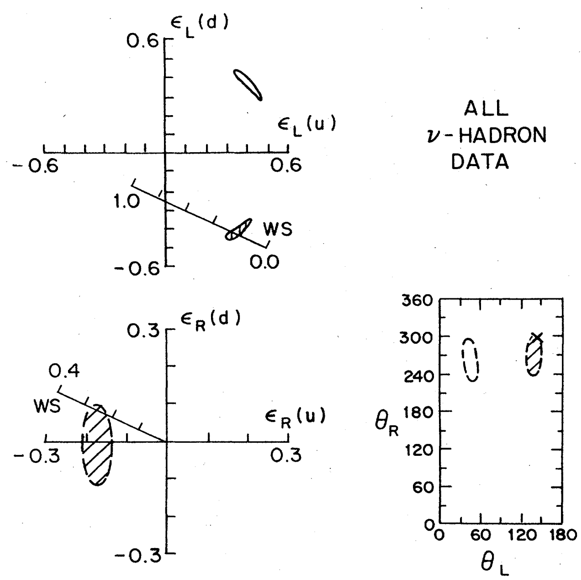


FIG. 21. Region of hadronic coupling constants allowed (at 90% confidence level) by a simultaneous fit to all the $\nu$-hadron data in Table VI. Only the shaded region is allowed by the measurements on exclusive pion production. The prediction of the WS-GIM model is shown as a function of $\sin^2\theta_w$. The cross on the $\theta_R-\theta_L$ plot corresponds to $\sin^2\theta_w = 0.233$.

[26] For the theory see the references therein and Ahrens and Gallaher (1979).

TABLE XI. Values of the hadronic neutral-current coupling constants determined by a fit to the $\nu$-hadron data [constraints (1)–(28)]. $(\chi^2/\mathrm{DOF})=13.9/24$. The results are presented for three equivalent parametrizations. Correlations between the fitted parameters are presented in Appendix C.

| | | | | | |
|---|---|---|---|---|---|
| $\varepsilon_L(u)$ | $0.340 \pm 0.033$ | $\alpha$ | $0.589 \pm 0.067$ | $g_L$ | $0.544 \pm 0.007$ |
| $\varepsilon_L(d)$ | $-0.424 \pm 0.026$ | $\beta$ | $0.937 \pm 0.062$ | $g_R$ | $0.180 \pm 0.019$ |
| $\varepsilon_R(u)$ | $-0.179 \pm 0.019$ | $\gamma$ | $-0.273 \pm 0.081$ | $\theta_L$ | $142 \pm 4°$ |
| $\varepsilon_R(d)$ | $-0.017 \pm 0.058$ | $\delta$ | $0.101 \pm 0.093$ | $\theta_R$ | $265 \pm 18°$ |

at 90% C.L. by constraints (1)–(28), the additional evidence against it is welcome.] The best fit values and the errors, for the equivalent parametrizations $\varepsilon_L(u)$, $\varepsilon_L(d)$, $\varepsilon_R(u)$, $\varepsilon_R(d)$; $\alpha$, $\beta$, $\gamma$, $\delta$; and $g_L$, $g_R$, $\theta_L$, $\theta_R$ are presented in Table XI. The fit is obtained with a $\chi^2$ of 13.9 for 24 degrees of freedom.

We now make an attempt, albeit crude, to estimate the systematic uncertainties in the determination of the coupling constants. It is most convenient to work with the variables $g_L$, $g_R$, $\theta_L$, $\theta_R$ since $g_L$,$g_R$ are predominantly determined by the deep-inelastic (on isoscalar target) experiments, while the determination of $\theta_L$ and $\theta_R$ follows exclusively from the other experimental input. With regards to experimental systematic errors, which are discussed in some detail in Sec. II, we note that most experiments have included reasonably generous estimates of systematic effects in their quoted errors. We have explicitly increased the errors on the BEBC measurement of $R_\nu^p$ and on the elastic scattering results to allow for systematic effects (see Sec. II). The theoretical uncertainties in the treatment of the deep-inelastic experiments are estimated assuming the basic validity of QCD; then, variations in the amount of sea and its strangeness content lead to variations in $g_L$ and $g_R$ of approximately $\pm 0.002$ and $\pm 0.015$, respectively. Similarly, with regard to deep-inelastic scattering from neutrons and protons, varying the ratio of $\bar{q}/q$ by $\pm 0.04$ changes $\theta_L$ and $\theta_R$ by $<1°$. Uncertainties in the theoretical treatment of the elastic scattering are simulated by varying, in Eqs. (3.64), $M_A=0.95$, 1.0, and 1.05, $\varepsilon=0.0$, 0.1, and 0.2, and $D/(D+F)=0.60$, 0.65, and 0.70. Combining each of the resulting variations in quadrature yields for $\theta_L$ and $\theta_R$ an uncertainty of $\pm 3°$ and $\pm 6°$, respectively. Indeed, even if the axial-isoscalar form factor in Eq. (3.64) is taken to be a free parameter $z$, the best fit values of the coupling constants are substantially unchanged. Finally, the uncertainty resulting from the treatment of the charge ratios in semi-inclusive pion production is perhaps the most difficult to estimate. The effect of neglecting strange quarks and the corrections of QCD are almost certainly small compared to the uncertainty in the basic assumption that the reactions are dominated by current fragmentation. While numerous authors have questioned the validity of such assumptions for the low-energy Gargamelle data, it has recently been emphasized that, even for the high-energy $\nu$ data, factorization is not satisfied to better than 20–30% unless cuts on $W$ and $Q^2$ are employed (Musgrave, 1979). We have crudely simulated these effects by including (in the fit) a 25% uncertainty in the fragmentation functions, and note that, at the present time, the experimental errors still dominate such an effect. However, the results are best treated with caution until it is more firmly established that current fragmentation is valid for the kinematic region of the data being used.

## IV. LEPTONIC INTERACTIONS

### A. Theoretical framework

The cross section for $\nu(\bar{\nu})e \to \nu(\bar{\nu})e$ is given ('t Hooft, 1971; Chen and Lee, 1972; Kim, 1977) by

$$\frac{d\sigma}{dE_e}=\frac{G_F^2 M_e}{2\pi}\left[A+B\left(1-\frac{E_e}{E_\nu}\right)^2-C\frac{M_e E_e}{E_\nu^2}\right], \tag{4.1}$$

where $E_e$ is the energy of the final electron. The coefficients $A$, $B$, and $C$ depend on the incident neutrino type, and are listed in Table XII. The cross section integrated over the electron energy is therefore (for $E_\nu \gg M_e$)

$$\sigma=\frac{G_F^2 M_e E_\nu}{2\pi}\left(A+\tfrac{1}{3}B\right). \tag{4.2}$$

The U.C.-Irvine (Savannah River) experiment (Reines, Gurr, and Sobel, 1976) on $\bar{\nu}_e e$ scattering presents separate cross sections for low-energy (1.5 MeV $<E_e<3.0$ MeV) and high-energy (3.0 MeV $<E_e<4.5$ MeV) electrons. Avignone and Greenwood (1977) have given expressions for $d\sigma/dE_e$ with the appropriate weighting with the $\bar{\nu}_e$ spectrum. From the tables in their paper,[27]

$$\sigma_{\bar{\nu}_e}(1.5<E_e<3.0)=D(0.4008A+0.0404B-0.0373C), \tag{4.3}$$
$$\sigma_{\bar{\nu}_e}(3.0<E_e<4.5)=D(0.1014A+0.0053B-0.0077C),$$

where $D=4.305\times 10^{-45}$ cm$^2$. The uncertainty in each coefficient is approximately 4% (which is negligible compared to the experimental uncertainties).

Kayser *et al.* (1979) have recently discussed the importance of observing interference in $\bar{\nu}_e e$ scattering, and several authors (Green and Veltman, 1980; Salomonson and Ueda, 1975; Byers, Rucke, and Yano, 1979; Marciano and Sirlin, 1980b) have estimated radiative corrections.

Many authors have discussed weak-electromagnetic interference in $e^+e^-$ reactions. As no data currently exist, we refer the reader to the literature.[28]

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[27]We follow the results of Liede, Maalampi, and Roos (1978).

[28]$e^+e^-\to\mu^+\mu^-$ has been considered by Godine and Hankey, 1972; Cung, Mann, and Paschos, 1972; Love, 1972; Budny, 1973; McDonald, 1974; Kayser, Rosen, and Fischbach, 1975; Ward, 1978; Dass and Ram Babu, 1979. $e^+e^-\to\tau^+\tau^-$, for which polarization effects may be observable, is considered by Koniuk, Leroux, and Isgur, 1978; Dahmen, Schulke, and Zech, 1979. $e^+e^-$ on resonance is considered by Isgur, 1975; Bigi, Kuhn, and Schneider, 1978; Hirata, 1979; Bernabeu and Pascual, 1979b. $e^+e^-\to q\bar{q}$ is studied by Nieves, 1979; Goggi and Penso, 1979. $e^+e^-\to e^+e^-$ near the $Z$ pole is considered by Field and Lautrup, 1979.

TABLE XII. The coefficients $A$, $B$, and $C$ for $\nu_\mu e$, $\bar\nu_\mu e$, $\nu_e e$, and $\bar\nu_e e$ elastic scattering. The expressions for $\nu_e$ and $\bar\nu_e$ include the charged-current contributions.

| | $\nu_\mu e \to \nu_\mu e$ | $\bar\nu_\mu e \to \bar\nu_\mu e$ | $\nu_e e \to \nu_e e$ | $\bar\nu_e e \to \bar\nu_e e$ |
|---|---|---|---|---|
| $A$ | $(g_V^e + g_A^e)^2$ | $(g_V^e - g_A^e)^2$ | $(2 + g_V^e + g_A^e)^2$ | $(g_V^e - g_A^e)^2$ |
| $B$ | $(g_V^e - g_A^e)^2$ | $(g_V^e + g_A^e)^2$ | $(g_V^e - g_A^e)^2$ | $(2 + g_V^e + g_A^e)^2$ |
| $C$ | $g_V^{e2} - g_A^{e2}$ | $g_V^{e2} - g_A^{e2}$ | $(g_V^e - g_A^e)(2 + g_V^e + g_A^e)$ | $(g_V^e - g_A^e)(2 + g_V^e + g_A^e)$ |

### B. Determination of leptonic weak neutral current

The determination of the leptonic neutral current is particularly straightforward due to the simplicity of the reactions. From the measurements of $\nu_\mu e \to \nu_\mu e$, $\bar\nu_\mu e \to \bar\nu_\mu e$, and $\bar\nu_e e \to \bar\nu_e e$, we have the following constraints.

Constraints (29)–(35).

$\nu_\mu e \to \nu_\mu e$.

$$\begin{aligned}
&(29)\quad 2.4^{+1.2}_{-0.9} = F(g_V^{e2} + g_V^e g_A^e + g_A^{e2}),\\
&(30)\quad 1.8 \pm 0.8 = F(g_V^{e2} + g_V^e g_A^e + g_A^{e2}),\\
&(31)\quad 1.1 \pm 0.6 = F(g_V^{e2} + g_V^e g_A^e + g_A^{e2}).
\end{aligned} \tag{4.4}$$

$\bar\nu_\mu e \to \bar\nu_\mu e$.

$$\begin{aligned}
&(32)\quad 2.2 \pm 1.0 = F(g_V^{e2} - g_V^e g_A^e + g_A^{e2}),\\
&(33)\quad 1.0^{+1.3}_{-0.6} = F(g_V^{e2} - g_V^e g_A^e + g_A^{e2}).
\end{aligned} \tag{4.5}$$

$\bar\nu_e e \to \bar\nu_e e$.

$$\begin{aligned}
(34)\quad 7.6 \pm 2.2 = D\{&0.4008[(1 + g_V^e) - (1 + g_A^e)]^2\\
&+ 0.0404[(1 + g_V^e) + (1 + g_A^e)]^2\\
&- 0.0373[(1 + g_V^e)^2 - (1 + g_A^e)^2\},\\
(35)\quad 1.86 \pm 0.48 = D\{&0.1014[(1 + g_V^e) - (1 + g_A^e)]^2\\
&+ 0.0053[(1 + g_V^e) + (1 + g_A^e)]^2\\
&- 0.0077[(1 + g_V^e)^2 - (1 + g_A^e)^2]\},
\end{aligned} \tag{4.6}$$

where $F = (2G_F^2 M_e/3\pi) = 5.75 \times 10^{-42}$ cm$^2$ GeV$^{-1}$, the left-hand side of Eqs. (4.4) and (4.5) are in units of $10^{-42}$ cm$^2$ GeV$^{-1}$, $D$ is given above, and the left-hand side of Eq. (4.6) is in units of $10^{-46}$ cm$^2$ GeV$^{-1}$.

The regions in the $g_V^e - g_A^e$ plane that are allowed by an overall fit to the above constraints are presented in Fig. 22. There are two allowed regions, one of which, the dominantly axial-vector, is in good agreement with the WS-GIM model. The other is dominantly vector and corresponds to an SU(2)×U(1) model with $e_R^-$ in a doublet. It will be seen below that inclusion of the data on the asymmetry in polarized electron scattering uniquely selects the dominantly axial-vector solution if factorization is assumed.

## V. THE ELECTRON-HADRON INTERACTION

### A. Deep-inelastic scattering

The SLAC experiment (Prescott *et al.*, 1978, 1979) on polarized-electron deep-inelastic scattering $eD \to e + X$ measures the parity-violating asymmetry

$$A = \frac{d\sigma(+) - d\sigma(-)}{d\sigma(+) + d\sigma(-)}, \tag{5.1}$$

where $\sigma(+)$ and $\sigma(-)$ are the cross sections for an electron polarized parallel or antiparallel to the beam, respectively. In the parton model, the interference between $Z$ boson exchange and electromagnetism leads to an asymmetry (Cahn and Gilman, 1978)

$$\frac{A(x,y)}{Q^2} = \frac{G_F}{2\sqrt{2}\pi\alpha} \frac{\sum_i x q_i(x) Q_i (C_{1i} \pm F(y) C_{2i})}{\sum_i x q_i(x) Q_i^2}, \tag{5.2}$$

where $Q^2 = -q^2 > 0$ is the momentum transfer, $xq_i(x)$ is the distribution function for parton $i$ of momentum fraction $x$, $eQ_i$ is the charge of parton $i$, and

$$F(y) = \frac{1 - (1-y)^2}{1 + (1-y)^2}. \tag{5.3}$$

The upper and lower signs refer to quarks and antiquarks, respectively ($Q_i$ is always the quark charge, i.e., the second term in the numerator changes sign when going from quark to antiquark).

If one neglects antiquarks as well as heavy quarks, then for a deuteron target Eq. (5.2) reduces to

$$\frac{A_D}{Q^2} = \frac{3G_F}{5\sqrt{2}\pi\alpha}[(C_{1u} - \tfrac{1}{2}C_{1d}) + F(y)(C_{2u} - \tfrac{1}{2}C_{2d})], \tag{5.4}$$

where $3G_F/5\sqrt{2}\pi\alpha = 2.16 \times 10^{-4}$ GeV$^{-2}$. In the WS-GIM model,

$$C_{1u} - \tfrac{1}{2}C_{1d} = -\tfrac{3}{4} + \tfrac{5}{3}\sin^2\theta_w$$

and

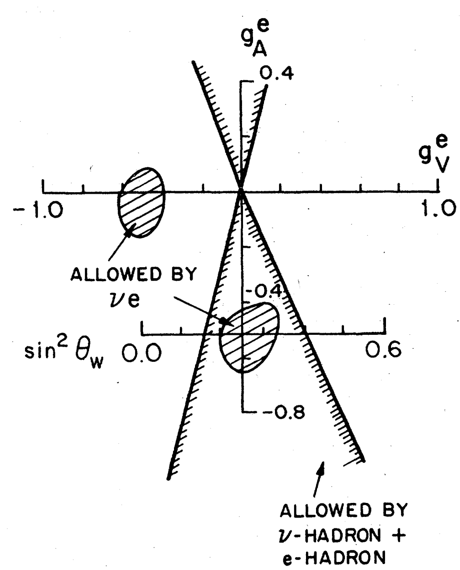


FIG. 22. Region of leptonic coupling constants allowed (at 90% confidence level) by a fit to the measurements of $\nu_\mu e \to \nu_\mu e$, $\bar\nu_\mu e \to \bar\nu_\mu e$, and $\bar\nu_e e \to \bar\nu_e e$. Also shown is the region allowed by the data on $\nu$-hadron scattering and the asymmetry in polarized electron scattering, assuming factorization.

$$C_{2u} - \tfrac{1}{2}C_{2d} = 3(\sin^2\theta_w - \tfrac{1}{4}) .$$

It has been emphasized by several authors (Wolfenstein, 1978; Bjorken, 1978; Derman, 1979; Fritzsch, 1979; Oka, 1979; Marciano and Sanda, 1978a; Bardin, Fedorenko, and Shomeiko, 1979; Miller and Ng, 1979) that Eq. (5.4) is very reliable in the relevant kinematic range, despite the relatively small $Q^2$ involved. Corrections to Eq. (5.4) involve such effects as $\bar{u}$ and $\bar{d}$ antiquarks, $R = \sigma_L/\sigma_T \neq 0$, logarithmic scaling violation, strange quarks, coherence between diagrams in which the $Z$ and $\gamma$ scatter from different quarks, and higher-order weak effects (such as the exchange of two $W$ bosons). The first three effects only modify the second ($y$-independent) term and are therefore suppressed, since $F(y)$ is small for $y \leq 0.3$. Moreover, these terms are multiplied by $C_{2u} - \frac{1}{2}C_{2d}$, which is small for neutral currents in the vicinity of the WS-GIM model (these corrections vanish for $\sin^2\theta_w = \frac{1}{4}$). For $\sin^2\theta_w < \frac{1}{4}$ they give (Fritzsch, 1979; Oka, 1979) a positive contribution of $\lesssim 5\%$ to the asymmetry; if included in the fits they would lead to a slightly smaller value of $\sin^2\theta_w$. The constant term is affected (Wolfenstein, 1978; Bjorken, 1978; Derman, 1979) by strange quarks, higher-order weak effects (Marciano and Sanda, 1978a), and coherence effects (Bjorken, 1978). The first two give negative contributions to the asymmetry and would lead to larger values for $\sin^2\theta_w$. The coherence effect has been estimated to be less than 6%.

As these various corrections appear to be small and enter with opposite signs, we shall not incorporate them explicitly into our expressions. Instead, we shall assume a 7% theoretical uncertainty in the coefficients of the two terms in Eq. (5.4).

For the asymmetry on a proton target one must make an assumption concerning the ratio $xu(x)/xd(x)$. If one assumes this ratio to be two, which is reasonable in the valence quark region ($x \gtrsim 0.2$), then the proton asymmetry is

$$\frac{A_p}{Q^2} \approx \frac{2G_F}{3\sqrt{2}\pi\alpha}[(C_{1u} - \tfrac{1}{4}C_{1d}) + F(y)(C_{2u} - \tfrac{1}{4}C_{2d})] . \qquad (5.5)$$

This expression is considerably less reliable than the expression for $A_D/Q^2$; we shall therefore not use the measurement (Prescott *et al.*, 1978) of $A_p$ in our fits.

Several authors have considered polarized lepton-nucleon elastic scattering (Feinberg, 1975; Cuthiell and Ng, 1977; Gilman and Tsao, 1979; Hoffman and Reya, 1978; Christova and Petcov, 1979; Klein, Reimann, and Savin, 1979; Rizzo, 1979) and other electron-hadron reactions (Walecka, 1977b; Bernabeu and Eramzhyan, 1979; Bernabeu and Pascual, 1979a; Yookoo and Usttio, 1979).

### B. Atomic parity violation

As noted in Sec. II, the existing data on atomic parity violation is confusing, with different groups reporting conflicting experimental results (Baird *et al.*, 1977; Lewis *et al.*, 1977; Barkov and Zolotorev, 1978a, b; Conti *et al.*, 1979; Commins, 1979; Barkov, 1979). In addition the reported positive results have large experimental errors, and the atomic physics calculations are difficult and uncertain. We therefore do not use these results in our analysis with the exception of the one fit described below.

It is now conventional to compare theoretical and experimental results by means of the weak charge[29] $Q_w$, which for heavy atoms is given by

$$Q_W(N,Z) = -2[C_{1u}(2Z+N) + C_{1d}(Z+2N)] . \qquad (5.6)$$

Hence

$$Q_W^{Bi} = Q_W(126,83) = -584C_{1u} - 670C_{1d} , \qquad (5.7)$$

$$Q_W^{Tl} = Q_W(123,81) = -570C_{1u} - 654C_{1d} . \qquad (5.8)$$

The Novosibirsk (Barkov and Zolotorev, 1978a, b) and Berkeley (Conti *et al.*, 1979; Commins, 1979) experiments determine

$$Q_W^{Bi} = -140 \pm 40 \text{ (Novosibirsk)} , \qquad (5.9)$$

$$Q_W^{Th} = -280 \pm 140 \text{ (Berkeley)} , \qquad (5.10)$$

which together with Eqs. (5.7) and (5.8) yield

$$C_{1u} + 1.15C_{1d} = 0.24 \pm 0.068 , \qquad (5.11)$$

$$C_{1u} + 1.15C_{1d} = 0.49 \pm 0.41 . \qquad (5.12)$$

According to Eq. (5.4), accurate measurements of $A_D/Q^2$ as a function of $y$ allow the determination of the linear combinations $C_{1u} - \frac{1}{2}C_{1d}$ and $C_{2u} - \frac{1}{2}C_{2d}$. Comparing the results from the SLAC polarized-electron experiment,

$$\frac{A_D}{Q^2} = [(-9.7 \pm 2.6) + (4.9 \pm 8.1)F(y)] \times 10^{-5} ,$$

with (5.4) yields

$$C_{1u} - \tfrac{1}{2}C_{1d} = -0.45 \pm 0.12 , \qquad (5.13)$$

$$C_{2u} - \tfrac{1}{2}C_{2d} = 0.23 \pm 0.38 . \qquad (5.14)$$

The regions in the $C_{1u} - C_{1d}$ plane allowed by Eqs. (5.11), (5.12), and (5.13) are nearly orthogonal and therefore determine $C_{1u}$ and $C_{1d}$ reasonably well, as shown in Fig. 23(a). In plotting the figure we have increased the errors in Eqs. (5.11)–(5.13) by 1.64 to obtain the 90% confidence intervals. As noted in Sec. II, there is no clear justification for selecting the results of the Novosibirsk and Berkeley experiments in favor of those from the Oxford and Seattle groups. Figure 23(a) is simply illustrative of the allowed regions, assuming the correctness of Eqs. (5.9) and (5.10).

Anticipating the results of Sec. VI, we note that if a single vector boson mediates the $e$-hadron, $\nu$-hadron, and $\nu$-$e$ interactions, then factorization relations enable one to place constraints on $C_{1u}$-$C_{1d}$ given measurements of the $\nu$-hadron coupling constants; the allowed region is shown in Fig. 22(a). One may then determine the allowed $C_{1u}$-$C_{1d}$ region in two different ways: (1) fit to the $eD$ and atomic physics results or (2) fit simultaneously to the $eD$, $\nu$-hadron, and $\nu$-$e$ data. The regions allowed by these two procedures are presented in Fig. 23(b), together with the prediction of the Weinberg-

[29]Some useful articles which contain references to the original papers are Wilets, 1978; Fortson, 1978; Feinberg, 1977, 1979; Sandars, 1977; Commins, 1979; Henley, 1977; Harris, Loving, and Sandars, 1978; Bernabeu and Pascual, 1980; Bouchiat, 1979.

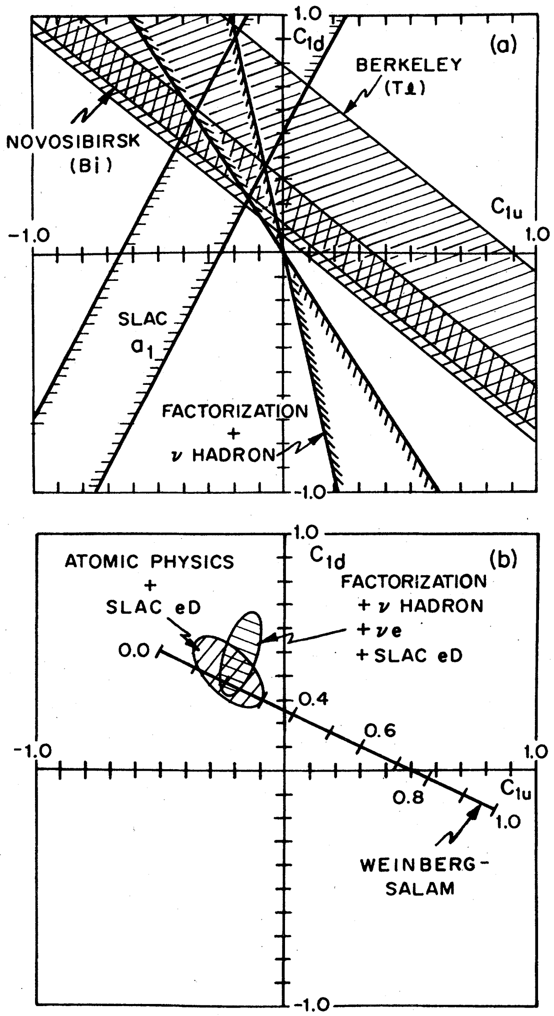


FIG. 23. (a) Region of $C_{1u}$ and $C_{1d}$ allowed (at 90% confidence level) by (i) the SLAC *eD* experiment, (ii) the Novosibirsk and Berkeley atomic physics experiments, and (iii) $\nu$-hadron scattering (assuming factorization). (b) Allowed regions for a simultaneous fit to (i) the *eD* and atomic physics results, or (ii) the $\nu$-hadron, $\nu e$, and *eD* results (assuming factorization).

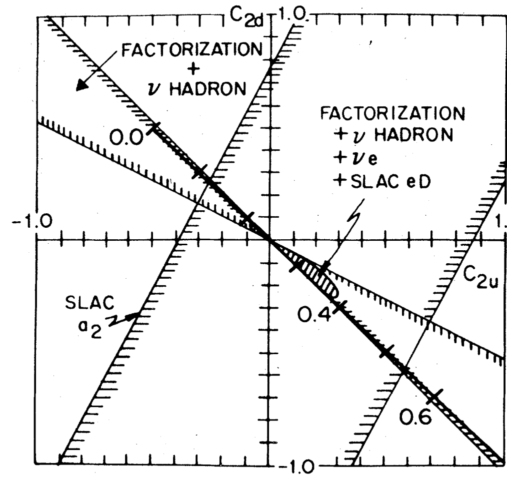


FIG. 24. Region of $C_{2u}$ and $C_{2d}$ allowed (at 90% confidence level) by (i) the SLAC *eD* experiment, (ii) $\nu$-hadron scattering (assuming factorization), and (iii) a simultaneous fit to the $\nu$-hadron, $\nu e$, and SLAC *eD* results (assuming factorization).

Salam model. We note that the null atomic physics results initially reported are not consistent with the *eD* and $\nu$-hadron results if a single intermediate boson is assumed.

We present in Fig. 24 the regions in the $C_{2u}$-$C_{2d}$ plane allowed separately by Eq. (5.14) (with the errors increased by 1.64 to obtain the 90% confidence interval) and by the $\nu$-hadron data, assuming factorization (see Sec. VI). Also shown is the region allowed by a simultaneous fit to the *eD*, $\nu$-*e*, and $\nu$-hadron data, assuming factorization as discussed in Sec. VI. Finally, we note that the *eD* measurements taken together with the $\nu$-hadron data and the assumption of a single intermediate boson restrict the possible domain of $g_V^e$ - $g_A^e$ as shown in Fig. 22.

## VI. MODELS AND FACTORIZATION TESTS

In the previous sections we have determined the constraints placed on the neutral-current parameters by the neutrino-hadron, neutrino-electron, and electron-hadron data individually. We have found that the neutrino-hadron neutral-current parameters $\varepsilon_{L,R}(u)$ and $\varepsilon_{L,R}(d)$ are uniquely determined and that the neutrino-electron parameters $g_V^e$ and $g_A^e$ must be in one of two allowed regions. All of the data (with the exception of some of the experiments on atomic parity violation in bismuth) are compatible with the WS-GIM model.

In this section we describe various simultaneous fits to all of the data. For this purpose it is necessary to assume specific models in which the $\nu$-*H*, $\nu$-*e*, and *e*-*H* couplings are related. We have concentrated on models based on the groups SU(2) × U(1), SU(2) × U(1) × U(1), and $SU(2)_L \times SU(2)_R \times U(1)$. We have two principal objectives in this section. Within the WS-GIM model we shall use all of the data simultaneously to obtain a best fit to the Weinberg angle, including a realistic estimate of the uncertainty due to systematic, statistical, and theoretical errors. Secondly, we are concerned with placing limits on (or seeking evidence for) small deviations from the WS-GIM model.

One class of deviations involves more complicated SU(2) × U(1) models in which the right-handed fermions are not all in pure singlet states or in which the Higgs structure is more complicated than in the WS-GIM model. Another involves models based on more complicated groups for which there is more than one *Z* boson. It is difficult to find a very general parametrization of such models, so we have concentrated on specific two-boson models, each of which is similar to the WS-GIM model for $\nu$-hadron scattering.[30] Various factorization tests are also considered.

### A. Factorization tests

In any model involving a single *Z* boson [including SU(2) × U(1)], one has the factorization relations (Hung

[30]There is a class of models with neutrino neutral currents which are identical to those of WS-GIM: Georgi and Weinberg (1978). See also Zee and Kim (1980).

and Sakurai, 1977a, 1979; Ecker, 1979)

$$\begin{aligned} C_{1i} &= 2g_A^e g_V^i/\rho\,, \\ C_{2i} &= 2g_V^e g_A^i/\rho\,, \end{aligned} \tag{6.1}$$

as well as

$$\begin{aligned} h_{VV} &= g_V^{e\,2}/\rho\,, \\ h_{AA} &= g_A^{e\,2}/\rho\,, \\ h_{VA} &= g_V^e g_A^e/\rho\,. \end{aligned} \tag{6.2}$$

One can interpret $\rho$ as the square of the coupling of the $Z$ to the neutrino. One can write various factorization tests from Eq. (6.1).

As one test of the single $Z$ boson hypothesis, we shall perform fits in which $g_{V,A}^e$ and $g_{V,A}^i$ are adjustable parameters but with $C_{1i}$ and $C_{2i}$ given by Eq. (6.1) (both with $\rho=1$ and with $\rho$ adjustable). For these runs we shall also check the relations (Bernabeu and Jarlskog, 1977; see also Gupta and Paranjape, 1977)

$$\begin{aligned} &\alpha+\beta+3(\gamma+\delta)+2(g_V^e+g_A^e)=0\,, \\ &\alpha+\beta-3(\gamma+\delta)-2\rho=0\,, \end{aligned} \tag{6.3}$$

which hold in any $SU(2)\times U(1)$ model with canonical values for the weak isospin of the left-handed fermions [these relations also hold in most theories based on the groups $SU(2)_L\times U(1)\times G_R$ (Maalampi, Liede, and Roos, 1979) and $G_{1L}\times G_{2R}$ (Mohapatra and Sidhu, 1979)].

A more specific test of the WS-GIM model is (Sidhu, 1979a)

$$\alpha-\beta+3(\gamma-\delta)+2(g_V^e-g_A^e)=0\,. \tag{6.4}$$

For more general $SU(2)\times U(1)$ models, the right-hand side of Eq. (6.4) is replaced by

$$4\rho[2T_{3R}(u)+T_{3R}(d)+T_{3R}(e)]\,. \tag{6.5}$$

## B. SU(2) × U(1) models

We first consider a general class of $SU(2)\times U(1)$ models,[31] of which the WS-GIM model is a special case. We define the $SU(2)$ and $U(1)$ coupling constants as $g$ and $g'$; the Weinberg angle $\theta_W$ is given by $\tan\theta_W=g'/g$. These parameters are related to the Fermi constant, the electron charge, and the $W$ boson mass by[32]

$$\begin{aligned} &G_F/\sqrt{2}=g^2/8M_W^2\,, \\ &e=g\sin\theta_W=\frac{gg'}{(g^2+g'^2)^{1/2}}\,, \\ &M_W=\left(\frac{\pi\alpha_{em}}{\sqrt{2}\,G_F}\right)^{1/2}\operatorname{cosec}\theta_W=\frac{38.5\text{ GeV}}{\sin\theta_W}\,. \end{aligned} \tag{6.6}$$

The $Z$ boson mass is

$$M_Z=\frac{M_W}{\sqrt{\rho}\cos\theta_W}=\frac{77.0\text{ GeV}}{\sqrt{\rho}\sin 2\theta_W}\,, \tag{6.7}$$

where $\rho=1$ if all Higgs bosons are in doublets (and singlets).[33] More generally (Lee, 1972)

$$\rho=\frac{\Sigma(I^2-I_3^2+I)\lambda_{I,I_3}^2}{2\Sigma I_3^2\lambda_{I,I_3}^2}\,, \tag{6.8}$$

where

$$\lambda_{I,I_3}=\langle\Phi_{I,I_3}\rangle_0$$

is the vacuum expectation value of a Higgs field with weak isospin $I$ and $z$ component $I_3$.

Let $T_{3L}(i)$ and $T_{3R}(i)$ represent the third component of weak isospin for the left- and right-handed components of fermion $i$. We always assume the canonical assignment $T_{3L}(\nu)=\frac{1}{2}$, $T_{3R}(\nu)=0$ for neutrinos. Then

$$\varepsilon_{L,R}(i)=\rho[T_{3L,R}(i)-\sin^2\theta_W Q_i]\,, \tag{6.9}$$

where $Q_i$ is the electric charge of fermion $i$.

We also assume the canonical values

$$T_{3L}(u)=\tfrac{1}{2}\,,\quad T_{3L}(d)=-\tfrac{1}{2}\,,\quad T_{3L}(e)=-\tfrac{1}{2} \tag{6.10}$$

for left-handed fields. The resulting neutral-current parameters are given in Table XIII.

The WS-GIM model corresponds to $T_{3R}(u)=T_{3R}(d)=T_{3R}(e)=0$ (all right-handed fermions in singlets) and $\rho=1$ (all Higgs fields in doublets). The minimal SU(5)

TABLE XIII. Values of the neutral-current parameters in several models, where $x=\sin^2\theta_W$ and $y=\sin^2\theta$. The WS-GIM model is the special case $\rho=1$, $T_{3R}(e)=T_{3R}(u)=T_{3R}(d)=0$ of the $SU(2)\times U(1)$ model. In each case, $g_{V,A}^i=\varepsilon_L(i)\pm\varepsilon_R(i)$. The parameter $\delta^2$ is positive.

| | $SU(2)\times U(1)$ | $SU(2)\times U(1)\times U(1)$ | $SU(2)\times SU(2)\times U(1)$ |
|---|---|---|---|
| $\varepsilon_L(e)$ | $\rho(-\frac{1}{2}+x)$ | $(-\frac{1}{2}+x)$ | $\frac{1}{2}c(-1+2y)-\frac{1}{2}d$ |
| $\varepsilon_R(e)$ | $\rho[T_{3R}(e)+x]$ | $x$ | $\frac{1}{2}c(-1+2y)+\frac{1}{2}d$ |
| $\varepsilon_L(u)$ | $\rho(\frac{1}{2}-\frac{2}{3}x)$ | $(\frac{1}{2}-\frac{2}{3}x)$ | $\frac{1}{2}c(1-\frac{4}{3}y)+\frac{1}{2}d$ |
| $\varepsilon_L(d)$ | $\rho(-\frac{1}{2}+\frac{1}{3}x)$ | $(-\frac{1}{2}+\frac{1}{3}x)$ | $\frac{1}{2}c(-1+\frac{2}{3}y)-\frac{1}{2}d$ |
| $\varepsilon_R(u)$ | $\rho[T_{3R}(u)-\frac{2}{3}x]$ | $-\frac{2}{3}x$ | $\frac{1}{2}c(1-\frac{4}{3}y)-\frac{1}{2}d$ |
| $\varepsilon_R(d)$ | $\rho[T_{3R}(d)+\frac{1}{3}x]$ | $\frac{1}{3}x$ | $\frac{1}{2}c(-1+\frac{2}{3}y)+\frac{1}{2}d$ |
| $C_{1u}$ | $2g_A^e g_V^u/\rho$ | $-(\frac{1}{2}-\frac{4}{3}x)+\frac{1}{2}\delta^2[1-\frac{8}{5}(1-x)]$ | $-a(1-\frac{4}{3}y)$ |
| $C_{1d}$ | $2g_A^e g_V^d/\rho$ | $\frac{1}{2}-\frac{2}{3}x-\frac{1}{2}\delta^2[1-\frac{4}{5}(1-x)]$ | $a(1-\frac{2}{3}y)$ |
| $C_{2u}$ | $2g_V^e g_A^u/\rho$ | $-(\frac{1}{2}-2x)+\frac{1}{10}\delta^2(12x-7)$ | $-a(1-2y)$ |
| $C_{2d}$ | $2g_V^e g_A^d/\rho$ | $-C_{2u}$ | $a(1-2y)$ |

[31]References to various specific models may be found in Langacker and Sidhu (1978).

[32]The numerical values for $M_w$ and $M_z$ include radiative corrections: Marciano (1979); Veltman (1980).

[33]Certain other Higgs representations with $I\geq 3$ can also give $\rho=1$.

grand unified model (Georgi and Glashow, 1974; see Footnote 3) predicts these values and also $\sin^2\theta_W \approx 0.20-0.21$. More generally, if $u_R$ has a component $u_R \cos\alpha_u$ in a singlet but $u_R \sin\alpha_u$ is in a doublet, then

$$
\begin{aligned}
T_{3R}(u) &= \tfrac{1}{2}\sin^2\alpha_u ,\\
T_{3R}(d) &= -\tfrac{1}{2}\sin^2\alpha_d ,\\
T_{3R}(e) &= -\tfrac{1}{2}\sin^2\alpha_e ,
\end{aligned} \tag{6.11}
$$

where $d_R \sin\alpha_d$ and $e_R \sin\alpha_e$ are in doublets.

We always take $\varepsilon_{L,R}(d)=\varepsilon_{L,R}(s)$ and $\varepsilon_{L,R}(u)=\varepsilon_{L,R}(c)$. If $T_{3R}(u)$ or $T_{3R}(d)$ is not zero, this need not be true. However, $\varepsilon_{L,R}(d)\approx\varepsilon_{L,R}(s)$ is strongly suggested by the absence of strangeness-changing neutral currents. Furthermore, it will turn out that $T_{3R}(d)$ and $T_{3R}(u)$ are close to zero. Since the effects of $c$ and $s$ quarks are relatively small for current experiments, we can safely neglect the possible differences of the $c$ and $s$ couplings from the $u$ and $d$ couplings.

### C. Other models

Another way to search for the effects of additional $Z$ bosons is to fit the data to specific models. One model by Deshpande and Iskandar (1979, 1980) is based on the group $SU(2)\times U(1)\times U(1)$, where the first U(1) is the $(T_3)_R$ of $SU(2)_L\times SU(2)_R\times U(1)$ models and the second is pure vector. It agrees with WS-GIM for neutrino scattering but allows more general electron-hadron couplings. The parameters are given in Table XIII.

We also consider a two-boson model (see footnote 6) based on the group $SU(2)_L\times SU(2)_R\times U(1)$. This model also reproduces the WS-GIM neutral-current structure in an appropriate limit and is sensitive to small deviations.

The neutral-current couplings, given in Table XIII, depend on four parameters, $a$, $c$, $d$, and $\sin^2\theta$. The WS-GIM limit is $a=c=d=\frac{1}{2}$, $y=\sin^2\theta=2\sin^2\theta_W$. The limits on these parameters and the $Z$ boson mass formulas are given in Appendix B.

### D. Results

With the assumption that a single intermediate boson mediates all neutral-current interactions, the cross section for any neutral-current process may be written in terms of seven independent coupling constants, i.e., $\varepsilon_L(u)$, $\varepsilon_L(d)$, $\varepsilon_R(u)$, $\varepsilon_R(d)$, $g_V^e$, $g_A^e$, and $\rho$. This then implies that there exist three independent relations between the ten parameters $\varepsilon_L(u)$, $\varepsilon_L(d)$, $\varepsilon_R(u)$, $\varepsilon_R(d)$, $g_V^e$, $g_A^e$, $C_{1u}$, $C_{1d}$, $C_{2u}$, $C_{2d}$. Two of these may be succinctly stated as

$$
\frac{C_{1d}}{C_{1u}}=\frac{g_V^d}{g_V^u}, \qquad \frac{C_{2d}}{C_{2u}}=\frac{g_A^d}{g_A^u}. \tag{6.12}
$$

That is, the isospin structure of the hadronic current is the same in $\nu$-$H$ and $e$-$H$ interactions. We have fit $g_V^d/g_V^u$ and $g_A^d/g_A^u$ directly to the $\nu$-hadron data and obtain

$$\frac{g_V^d}{g_V^u}=-2.76^{+1.12}_{-2.08}\quad (90\%\ \text{C.L.}),$$

$$\frac{g_A^d}{g_A^u}=-0.78\pm0.26\quad (90\%\ \text{C.L.}),$$

where the errors represent the 90% confidence interval. The allowed regions in the $C_{1u}-C_{1d}$ and $C_{2u}-C_{2d}$ plane are shown in Figs. 23 and 24; the extent to which they are in agreement with the regions allowed by the measurements of $e$-$H$ interactions provides a test of factorization. As noted in Sec. V, the null results initially reported by atomic physics experiments are inconsistent with the assumption of factorization if the $\nu$-hadron and $eD$ experiments are correct.

The third factorization relation allows the measurements of $eD$ scattering and the $\nu$-hadron data to put constraints on $g_V^e/g_A^e$. We obtain the result implicitly by fitting to the $eD$ and $\nu$-$H$ data directly in terms of $g_V^e/g_A^e$. The allowed region in the $g_V^e$-$g_A^e$ plane is shown in Fig. 22; note that it selects the axial-vector-dominant solution as the correct one. We emphasize that it has not been assumed that $\rho=1.0$. Again, comparison of the $g_V^e$-$g_A^e$ region allowed by the $\nu$-hadron and $eD$ data with that determined from $\nu$-$e$ scattering provides a test of factorization.

A simultaneous fit to all the data in Table VI with $\varepsilon_L(u)$, $\varepsilon_L(d)$, $\varepsilon_R(u)$, $\varepsilon_R(d)$, $g_V^e$, $g_A^e$, and $\rho$ as free parameters yields the best fit values of the parameters, both for $\rho$ a free parameter and for $\rho=1.0$, presented in Table XIV. The correlations between the fitted parameters are presented in Appendix C.

We also check the Bernabeu-Jarlskog relations [Eqs. (6.3)] and obtain

$$\text{(BJ1)}\quad \alpha+\beta+3(\gamma+\delta)+2(g_V^e+g_A^e)=0.01\pm0.41,$$

$$\text{(BJ2)}\quad \alpha+\beta-3(\gamma+\delta)-2\rho=0.03\pm0.62.$$

Similarly, for Eq. (6.4) we obtain

$$\text{(SD1)}\quad \alpha-\beta+3(\gamma-\delta)+2(g_V^e-g_A^e)=-0.32\pm0.31.$$

Within the context of generalized $SU(2)\times U(1)$, the weak neutral currents, leptonic and hadronic, are described by five parameters: $\rho=M_W^2/M_Z^2\cos^2\theta_W$, $\sin^2\theta_W$, $T_{3R}(u)$, $T_{3R}(d)$, and $T_{3R}(e)$. A simultaneous fit to all

TABLE XIV. Determination of $\varepsilon_L(u)$, $\varepsilon_L(d)$, $\varepsilon_R(u)$, $\varepsilon_R(d)$, $g_V^e$, $g_A^e$, and $\rho$ for (1) factorization assumed, $\rho$ a free parameter, (2) factorization assumed and $\rho=1.0$. The predictions of the WS-GIM model for $\sin^2\theta_W=0.233$ are presented in column 3.

| | Factorization assumed | Factorization plus $\rho=1.0$ | WS-GIM $\sin^2\theta_W=0.233$ |
|---|---|---|---|
| $\chi^2/DOF$ | 26.2/39 | 26.2/40 | |
| $\varepsilon_L(u)$ | $0.339\pm0.033$ | $0.339\pm0.030$ | 0.345 |
| $\varepsilon_L(d)$ | $-0.424\pm0.026$ | $-0.425\pm0.025$ | $-0.423$ |
| $\varepsilon_R(u)$ | $-0.179\pm0.019$ | $-0.179\pm0.018$ | $-0.155$ |
| $\varepsilon_R(d)$ | $-0.016\pm0.058$ | $-0.016\pm0.052$ | +0.077 |
| $g_V^e$ | $0.043\pm0.063$ | $0.043\pm0.056$ | $-0.036$ |
| $g_A^e$ | $-0.545\pm0.056$ | $-0.545\pm0.044$ | $-0.50$ |
| $\rho$ | $1.001\pm0.21$ | | 1.0 |

the data in Table VI yields

$$\rho = 1.018 \pm 0.045, \quad \sin^2\theta_W = 0.249 \pm 0.031,$$
$$T_{3R}(u) = -0.010 \pm 0.040, \quad T_{3R}(d) = -0.101 \pm 0.058,$$
$$T_{3R}(e) = 0.039 \pm 0.047.$$

The values of $T_{3R}(u)$, $T_{3R}(d)$, $T_{3R}(e)$ are all close to zero, as is expected in a model with no right-handed doublets. However, the limits are not very restrictive. Using the relations (6.11) we obtain the upper limits (90% C.L.)

$$\sin^2\alpha_u \leq 0.103, \quad \sin^2\alpha_d \leq 0.348, \quad \sin^2\alpha_e \leq 0.064.$$

While the value of $\rho$ is remarkably close to 1.0, its uncertainty, and that of $\sin^2\theta_W$ are not particularly restrictive.

Much smaller errors are of course obtained from fits to a smaller set of parameters. In particular, if one assumes that there are no right-handed doublets, i.e., $T_{3R}(u) = T_{3R}(d) = T_{3R}(e) = 0$, a fit to $\rho$ and $\sin^2\theta_W$ yields ($\chi^2/DOF = 33.1/44$)

$$\rho = 1.002 \pm 0.015, \quad \sin^2\theta_W = 0.234 \pm 0.013 \quad \text{(All data)},$$

in excellent agreement with the hypothesis $\rho = 1.0$. Following the prejudice of no right-handed doublets *and* $\rho = 1.0$, a fit solely to $\sin^2\theta_W$ ($\chi^2/DOF = 33.1/45$) yields

$$\sin^2\theta_W = 0.233 \pm 0.009 \quad \text{(All data)}.$$

As a test of the sensitivity of these results to particular experiments or reactions, we have repeated the fits omitting, one at a time, each type of experiment. The results, presented in Table XV, show very little sensitivity to which data is included.

We have also attempted to estimate the uncertainty in the obtained values of $\rho$ and $\sin^2\theta_W$ due to theoretical uncertainties. For the deep-inelastic $\nu$ scattering we assume the correctness of the quark parton model with QCD corrections, but vary the total antiquark content of the nucleon (parametrized by $R_{cc}$) and the strangeness composition of the sea (parametrized by $a_s$) over the range allowed by the data on charged-current and dimuon production. As noted above, reasonable variations in $\Lambda$, $\eta_{\bar{q}}(0)$, and $R = \sigma_L/\sigma_T$ produce much smaller effects which are therefore negligible. Uncertainties in the predictions for the $\nu p\,(\bar{\nu}p)$ elastic scattering are simulated by varying the contributions from the axial-isovector form factor, as represented by $\varepsilon$ in Eq. (3.61), and by varying the axial-vector mass $M_A$ [see Eq. (3.60)]. For $eD$ data, we estimate an overall uncertainty in the predicted asymmetry of $\pm 7\%$ as discussed in Sec. V.

The changes in the values obtained for $\rho$ and $\sin^2\theta_W$ as one changes the theoretical assumptions are shown in Table XVI. Combining the variations in Table XVI in quadrature, one obtains an estimate of $\pm 0.005$ for the theoretical uncertainty in the value of $\sin^2\theta_W$ obtained from a fit to all the data; similarly, for the combined fit to $\rho$ and $\sin^2\theta_W$ one obtains an uncertainty of $\pm 0.009$ for $\sin^2\theta_W$ and $\pm 0.011$ for $\rho$ due to the theoretical uncertainties.

Needless to say, one of the reasons for the insensitivity of the results to which reactions are included (Table XV) and to theoretical assumptions concerning the elastic scattering (Table XVI) is that the determination of $\rho$ and $\sin^2\theta_W$ is currently dominated by the experiments on $\nu$ deep-inelastic scattering and the asymmetry in $eD$ scattering; hence we focus on these experiments which in general have the smallest statistical errors and a fairly careful analysis of experimental systematic effects. The values obtained for fits to $\sin^2\theta_W$ alone, and to $\rho$ and $\sin^2\theta_W$ are shown in Table XVII.

Again it is important to consider systematic uncertainties. As noted in Sec. II, most of the deep-inelastic experiments have included reasonable estimates of the experimental systematic effects in their quoted errors. Furthermore, the good agreement between the various results on deep-inelastic scattering excludes large systematic errors particular to individual experiments (though not effects common to all experiments). The $eD$ experiment quotes errors on the measured asymmetry at each value of $y$, which include statistical and point-to-point systematic errors, and in addition, an overall error of $\pm 5\%$ resulting from the uncertainty in the electron polarization. These two effects (which produce uncertainties in $\sin^2\theta_W$ of $\pm 0.012$ and $\pm 0.008$, respectively, for a fit to the $eD$ data alone) have been combined in quadrature in the above numbers.[34]

With regard to theoretical uncertainties, we repeat the procedure discussed for Table XVI, and present in Table XVIII the variations in the values of $\rho$ and $\sin^2\theta_W$

TABLE XV. Summary of fits to $\sin^2\theta_W$, and $\rho$ and $\sin^2\theta_W$, dropping, one at a time, each type of experimental input.

| Data omitted | $\sin^2\theta_W$ | $\sin^2\theta_W$ | $\rho$ |
|---|---|---|---|
| None | $0.233 \pm 0.009$ | $0.234 \pm 0.013$ | $1.00 \pm 0.015$ |
| Deep-inelastic (isoscalar target) | $0.232 \pm 0.014$ | $0.239 \pm 0.015$ | $1.04 \pm 0.046$ |
| Deep-inelastic ($n, p$ target) | $0.234 \pm 0.009$ | $0.234 \pm 0.013$ | $1.00 \pm 0.015$ |
| Semi-inclusive pion | $0.231 \pm 0.009$ | $0.231 \pm 0.013$ | $0.999 \pm 0.017$ |
| $\nu p$ elastic scattering | $0.231 \pm 0.009$ | $0.233 \pm 0.013$ | $1.00 \pm 0.016$ |
| $\nu e$ | $0.230 \pm 0.009$ | $0.228 \pm 0.014$ | $0.996 \pm 0.016$ |
| $eD$ asymmetry | $0.239 \pm 0.010$ | $0.253 \pm 0.017$ | $1.017 \pm 0.017$ |

[34] This is somewhat less conservative than the approach used by the experimenters, who add the two errors linearly. The latter approach may be overly conservative, particularly if one considers 2–3 standard deviations, in which case the systematic error would be multiplied by 2–3. In any event, the difference is not large. We would obtain an error of $\pm 0.010$ instead of $\pm 0.009$ for the combined fit to $\sin^2\theta_W$ if the errors were added linearly.

TABLE XVI. Variations in the value obtained for $\sin^2\theta_W$ ($\rho=1$) and for $\rho$ and $\sin^2\theta_W$ due to uncertainties in the theoretical predictions for $\overset{(-)}{\nu}N \to \overset{(-)}{\nu}X$, $\overset{(-)}{\nu}p \to \overset{(-)}{\nu}p$, and $eD \to eX$. All of the data in Table VI is included in the fits. The first row of the table presents the results obtained for the "normal" theoretical assumptions (see Sec. III) while the succeeding rows present the changes in $\sin^2\theta_W$ and $\rho$ as the theoretical assumptions ("conditions") are changed. For the $eD$ experiment, the overall theoretical uncertainty is assumed to be ±7%.

| Reactions to which conditions refer | Conditions | $\sin^2\theta$ | $\sin^2\theta$ | $\rho$ |
|---|---|---|---|---|
| All | Normal ($R_{cc}=0.48$, $a_s=0.5$, $\varepsilon=0.0$, $M_A=1.0$ GeV) | $0.233 \pm 0.009$ | $0.234 \pm 0.013$ | $1.002 \pm 0.015$ |
| $\overset{(-)}{\nu}N \to \overset{(-)}{\nu}X$ | $R_{cc}=0.53$, $a_s=0.5$ | 0.001 | 0.004 | 0.005 |
| $\overset{(-)}{\nu}N \to \overset{(-)}{\nu}X$ | $R_{cc}=0.45$, $a_s=0.5$ | 0.000 | −0.003 | −0.006 |
| $\overset{(-)}{\nu}N \to \overset{(-)}{\nu}X$ | $R_{cc}=0.48$, $a_s=1.0$ | 0.000 | 0.000 | −0.000 |
| $\overset{(-)}{\nu}p \to \overset{(-)}{\nu}p$ | $\varepsilon=0.1$, $M_A=1.0$ GeV | 0.001 | −0.001 | −0.003 |
| $\overset{(-)}{\nu}p \to \overset{(-)}{\nu}p$ | $\varepsilon=0.2$, $M_A=1.0$ GeV | 0.002 | −0.002 | −0.007 |
| $\overset{(-)}{\nu}p \to \overset{(-)}{\nu}p$ | $\varepsilon=0.0$, $M_A=1.05$ GeV | 0.000 | −0.001 | −0.001 |
| $\overset{(-)}{\nu}p \to \overset{(-)}{\nu}p$ | $\varepsilon=0.0$, $M_A=0.95$ GeV | −0.000 | 0.001 | 0.001 |
| $eD \to eX$ | Theoretical prediction ×0.93 | −0.004 | −0.008 | −0.007 |
| $eD \to eX$ | Theoretical prediction ×1.07 | +0.004 | 0.007 | +0.006 |

obtained (from a simultaneous fit to the deep-inelastic $\nu$ data and the $eD$ data) as the theoretical assumptions are modified.

While it is not possible to exclude either experimental systematic effects that have not been anticipated or the possibility that there are significant inadequacies of QCD, it nonetheless appears that one can assume with a considerable degree of confidence that

$$\sin^2\theta_W = 0.224 \pm 0.015\ (\pm 0.012)\,,$$

$$\rho = 0.992 \pm 0.017\ (\pm 0.011)\,,$$

and for $\rho = 1$,

$$\sin^2\theta_W = 0.229 \pm 0.009\ (\pm 0.005)$$

within the context of SU(2)× U(1) with no right-handed doublets. The numbers in parentheses are an estimate of the theoretical uncertainty, obtained by combining the contributions from Table XVIII in quadrature.

For completeness we mention the Paschos-Wolfenstein (1973) formula for $\sin^2\theta_w$ in the WS-GIM model:

$$\frac{\sigma_\nu^{NC} - \sigma_{\bar\nu}^{NC}}{\sigma_\nu^{CC} - \sigma_{\bar\nu}^{CC}} = \tfrac{1}{2} - \sin^2\theta_w\,. \tag{6.13}$$

This formula is attractive theoretically in that it is almost independent of any details of the structure functions; it is valid in the presence of arbitrary amounts of scaling violation, for example. The only corrections to the formula involve the kinematic suppression of $c$ (or $s$) quarks in charged-current interactions. The formula as written is strictly valid only to the extent that $m_c$ is negligible. The experimental systematic effects are somewhat more difficult than in the measurement of $R_\nu$ since Eq. (6.13) involves a difference of cross sections that are measured with different spectra ($\nu$ vs $\bar\nu$); we have therefore not attempted to use this expression. The CDHS group has made the appropriate corrections and obtains (Geweniger, 1980) $\sin^2\theta_w = 0.228 \pm 0.018$; they expect to reduce the error further.

We fit the $SU(2)_L \times U(1) \times U(1)$ model of Deshpande and Iskandar (1979, 1980) to the data in terms of $x = \sin^2\theta_w$ and $\delta^2$, where $\delta^2$ is a function of coupling constants and vacuum expectation values of the Higgs doublets and singlets and obeys $\delta^2 \geq 0$. The WS-GIM limit is $\delta^2 = 0$. We obtain $\sin^2\theta_w = 0.238 \pm 0.010$ and $\delta^2 = 0.15 \pm 0.22$ which is consistent with 0. The 90% confidence level lower limit on $M_{Z_2}$, the mass of the heavier neutral $Z$ boson, is 142 GeV; the corresponding mass of the lighter neutral boson is $M_{Z_1} = 88$ GeV. For $\delta^2 = 0$, $M_{Z_1} = M_Z$ as given in Eq. (6.7) and $M_{Z_2} = \infty$.

We have also fit an $SU(2)_L \times SU(2)_R \times U(1)$ model (see footnote 6) to the data, parametrizing it in terms of $y$

TABLE XVII. Values of $\rho$ and $\sin^2\theta_W$ determined from fits to the data on neutrino deep-inelastic scattering and the asymmetry in $eD$ scattering. The correlation between $\rho$ and $\sin^2\theta_W$ for the combined fit is 0.60, and the $\chi^2/DOF$ is 13.8/44.

| $\sin^2\theta_W$ | $\rho$ | $\sin^2\theta_W$ | |
|---|---|---|---|
| $0.234 \pm 0.011$ | $0.999 \pm 0.025$ | $0.232 \pm 0.027$ | Deep-inelastic $\nu$ |
| $0.223 \pm 0.015$ | $1.74 \pm 0.36$ | $0.293\ ^{+0.033}_{-0.100}$ | $eD$ asymmetry |
| $0.229 \pm 0.009$ | $0.992 \pm 0.017$ | $0.224 \pm 0.015$ | Deep-inelastic $\nu$ and $eD$ asymmetry combined |

TABLE XVIII. Variations in the values obtained for $\sin^2\theta_W$ alone, and $\rho$ and $\sin^2\theta_W$ arising from (1) variations in the total antiquark content and strangeness composition of the sea assumed for deep-inelastic $\nu$ scattering and (2) a 7% theoretical uncertainty in the predicted asymmetry for $eD$ scattering.

| | $\sin^2\theta_W$ | $\sin^2\theta_W$ | $\rho$ |
|---|---|---|---|
| Normal | 0.229 ± 0.009 | 0.224 ± 0.015 | 0.992 ± 0.017 |
| $eD$ theory × 0.93 | −0.005 | −0.011 | −0.009 |
| $eD$ theory × 1.07 | +0.005 | +0.010 | +0.007 |
| $R_{cc}=0.45$, $a_{\bar{s}}=0.5$ | 0.000 | −0.003 | −0.006 |
| $R_{cc}=0.53$, $a_{\bar{s}}=0.5$ | 0.001 | +0.004 | +0.004 |
| $R_{cc}=0.48$, $a_{\bar{s}}=1.0$ | 0.000 | 0.000 | +0.000 |

$=\sin^2\theta$, $\varepsilon$, and $\sigma$ where $\varepsilon$ and $\sigma$ are functions of the Higgs fields, and obey the conditions $0\leq\varepsilon\leq 1$ and $-1\leq\sigma\leq 1$. The WS-GIM limit is $\varepsilon=0$, $\sigma=1$, and $y=2\sin^2\theta_w$. The best fit, subject to these constraints, is in fact $\varepsilon=0$, $\sigma=1$, and $y=0.467$, corresponding to $\sin^2\theta_w=0.234$. The 90% confidence level lower limit on $M_{Z_2}$, the mass of the heavier neutral boson, is 310 GeV; the corresponding mass of the lighter neutral boson is $M_{Z_1}=90$ GeV. These numbers do not include the theoretical uncertainties or the 5% systematic effect in the $eD$ measurements due to uncertainty in the electron polarization.

## VII. SUMMARY AND CONCLUSIONS

We have performed a model-independent analysis of all existing neutral-current data incorporating the best theoretical expressions currently available. Analysis of the $\nu$-hadron experiments leads to a unique and rather accurate determination of the hadronic coupling constants: $\varepsilon_L(u)=0.340\pm0.033$, $\varepsilon_L(d)=-0.424\pm0.026$, $\varepsilon_R(u)=-0.179\pm0.019$, $\varepsilon_R(d)=-0.017\pm0.058$ (Table XI). Theoretical uncertainties, assuming the basic validity of QCD, lead to variations in the coupling constants that are in all cases smaller than the statistical errors. Allowance for experimental systematic effects, to the extent that such effects can be determined, has been included in the error bars. We emphasize, however, that of all the $\nu$-hadron experiments, only the deep-inelastic scattering results are quite precise; information on the isospin structure results largely from several experiments with relatively large errors and systematic uncertainties. More accurate results on semi-inclusive pion production by high-energy neutrinos, deep-inelastic $\nu$ scattering from proton and neutron targets, and the elastic scattering of $\nu$ and $\bar{\nu}$ from protons is important, particularly with regard to the determination of the isospin structure of the right-handed coupling. The $\nu$-hadron results together determine the left- and right-handed couplings to be $g_L=0.544\pm0.007$ ($\pm0.002$) and $g_R=0.180\pm0.019$ ($\pm0.015$), where the errors in parentheses result from theoretical uncertainties.

Analysis of the leptonic reactions $\nu_\mu e\to\nu_\mu e$, $\bar{\nu}_\mu e\to\bar{\nu}_\mu e$, and $\bar{\nu}_e e\to\bar{\nu}_e e$ leads to two solutions, one of which is dominantly vector, the other axial-vector. Inclusion of the results on the asymmetry in polarized $eD$ scattering (together with the assumption of factorization) uniquely selects the axial-vector-dominant solution; the results are $g_V^e=0.043\pm0.063$ and $g_A^e=-0.545\pm0.056$. These results follow from several experiments, each of which possesses large statistical or systematic errors. A precise determination of the leptonic current will require several high-statistics measurements of these reactions.

Because of the inconsistencies in the results reported on parity violation in atomic physics, we did not include these data in any of our fits save one: in that one, we combined the data from the Novosibirsk (Bi) and Berkeley (Tl) experiments with the $eD$ asymmetry to determine the electron-hadron couplings ($C_{1u},C_{2u},C_{1d},C_{2d}$) in a model-independent way. The results (Fig. 23) are in good agreement with the WS-GIM model and with the region allowed by all other neutral-current data assuming factorization.

Assuming the existence of a single, neutral intermediate vector boson, all the data may be described by seven parameters, $\varepsilon_L(u)$, $\varepsilon_L(d)$, $\varepsilon_R(u)$, $\varepsilon_R(d)$, $g_V^e$, $g_A^e$, and $\rho$; the values obtained (Table XIV) are very similar to the results of the model-independent fits. For the Bernabeu-Jarlskog (1977) relations, which must equal zero in any SU(2) × U(1) model with the canonical values of weak isospin for the left-handed fermions, we obtain

$$\alpha+\beta+3(\gamma+\delta)+2(g_V^e+g_A^e)=0.01\pm0.41\,,$$

$$\alpha+\beta-3(\gamma+\delta)-2\rho=0.03\pm0.62\,.$$

A similar relation by Sidhu (1979a), which must equal zero if the WS-GIM model is correct, yields

$$\alpha-\beta+3(\gamma-\delta)+2(g_V^e-g_A^e)=-0.32\pm0.31\,.$$

Within the context of generalized SU(2) × U(1) models, which are specified by $\rho$, $\sin^2\theta_w$, and the third component of isospin for the right-handed fermions, we obtain $\rho=1.018\pm0.045$, $\sin^2\theta_w=0.249\pm0.031$, $T_{3R}(u)=-0.010\pm0.040$, $T_{3R}(d)=-0.101\pm0.058$, and $T_{3R}(e)=0.039\pm0.047$. These may be interpreted to yield upper limits on the mixing angles of the light fermions with heavier fermions: $\sin^2\alpha_u<0.103$, $\sin^2\alpha_d<0.348$, $\sin^2\alpha_e<0.064$ (90% confidence level).

Assuming the validity of $SU(2)_L\times U(1)$, we determine the best estimates of $\rho$ and $\sin^2\theta_w$. Using only the deep-inelastic $\nu$ scattering and $eD$ results, these being the most precise and the best understood systematically, we obtain $\rho=0.992\pm0.017$ ($\pm0.011$) and $\sin^2\theta_w=0.224\pm0.015$ ($\pm0.012$), where the errors in parentheses result from theoretical uncertainties. For $\rho=1$, $\sin^2\theta_w=0.229\pm0.009$ ($\pm0.005$). The theoretical uncertainty is dominated by the 7% uncertainty assumed for the theoretical prediction for $eD$ scattering. For deep-inelastic $\nu$ scattering, changing $\bar{q}/(q+\bar{q})$ from 0.16 to 0.18 changes $R_\nu$ by only 0.0004 and hence $\sin^2\theta_w$ by 0.0006 (for $\sin^2\theta_w$ in the vicinity of 0.23). Inclusion of the results from all the experiments (Table VI) yields very similar results: $\rho = 1.002 \pm 0.015$ ($\pm 0.011$) and $\sin^2\theta_w = 0.234 \pm 0.013$ ($\pm 0.009$). For $\rho = 1$, $\sin^2\theta_w = 0.233 \pm 0.009$ ($\pm 0.005$).

With the assumption that $\rho = 1.0$ before renormalization effects, then the 90% confidence level upper limit on $\rho$ implies $M_L < 500$ GeV, where $M_L$ is the mass of any heavy lepton with a massless partner (Veltman, 1977; Chanowitz, Furman, and Hinchliffe, 1978).

Finally, we have analyzed two specific two-boson models and determine $M_{Z_2} > 310$ and 142 GeV for an $SU(2)_L \times SU(2)_R \times U(1)$ and $SU(2)_L \times U(1) \times U(1)$ model, respectively. Barr and Zee (1979) have analyzed, within the context of grand unified models, possible deviations from $SU(2)_L \times U(1)$.

Our results show that a value of $\sin^2\theta_w$ larger than the SU(5) prediction is significantly favored if $\rho$ is identically 1.0. It has been emphasized by Marciano (1979) that more precise determinations of $\rho$ and $\sin^2\theta_w$ will require a careful analysis of higher-order effects, both in the charged- and neutral-current cross sections. Not only electromagnetic radiative corrections[35] but also higher-order charged- and neutral-current weak effects[36] may modify cross sections and renormalize the effective values of $\rho$ and $\sin^2\theta_w$. These effects are expected to depend not only on the reaction but also on the energy and momentum transfer involved. A complete analysis would be very desirable. Veltman (1980) has also emphasized that precise measurements of $\sin^2\theta_w$ at low and high energies will yield quantitative information on higher-order effects.

We close on an experimental note: While we have emphasized above the importance of improving existing measurements, experiments which directly probe new sectors of the neutral-current interaction, such as the muon or heavy quark couplings, would be extremely useful. Who knows where the next surprise may lie?

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## APPENDIX A: THE EVOLUTION OF DISTRIBUTION FUNCTIONS

The $Q^2$ dependence of the functions $C_{\bar{q}_i}(S)$, $C_G(S)$, $\eta_{\bar{q}_i}(S)$, and $\eta_G(S)$ of Eq. (3.14) are as follows. Let

$$C_{\bar{q}_i}(S) = P_{\bar{q}_i}\left(\frac{1}{\langle x\rangle_{\bar{q}_i}} - 1\right) \tag{A1}$$

and

$$\eta_{\bar{q}_i}(S) = \frac{1}{\langle x\rangle_{\bar{q}_i}} - 2\,, \tag{A2}$$

where

$$P_{\bar{q}_i} \equiv \langle \bar{q}_i(Q^2)\rangle_2\,, \tag{A3}$$

$$\langle x\rangle_{\bar{q}_i} \equiv \frac{\langle \bar{q}_i(Q^2)\rangle_3}{P_{\bar{q}_i}}\,, \tag{A4}$$

and where

$$\langle \bar{q}_i(Q^2)\rangle_n \equiv \int_0^1 dx\, x^{n-1} q(x, Q^2)\,. \tag{A5}$$

Analogous expressions hold for $C_G(S)$ and $\eta_G(S)$. Then,

$$\langle \bar{u}(Q^2) + \bar{d}(Q^2)\rangle_n = \tfrac{1}{4}D_2^{(n)} + \tfrac{1}{12}D_1^{(n)} + \tfrac{1}{6}D_0^{(n)}\,, \tag{A6}$$

$$\langle \bar{s}(Q^2)\rangle_n = \tfrac{1}{8}D_2^{(n)} + \tfrac{1}{24}D_1^{(n)} - \tfrac{1}{6}D_0^{(n)}\,, \tag{A7}$$

$$\langle \bar{c}(Q^2)\rangle_n = \tfrac{1}{8}D_2^{(n)} - \tfrac{1}{8}D_1^{(n)}\,, \tag{A8}$$

where

$$D_0^{(n)}(Q^2) = 2\langle \bar{u}(Q_0^2) + \bar{d}(Q_0^2) - 2\,\bar{s}(Q_0^2)\rangle_n e^{-\gamma_n S}\,, \tag{A9}$$

$$D_1^{(n)}(Q^2) = 2\langle \bar{u}(Q_0^2) + \bar{d}(Q_0^2) + \bar{s}(Q_0^2) - 3\bar{c}(Q_0^2)\rangle_n e^{-\gamma_n S}\,, \tag{A10}$$

$$\begin{aligned} D_2^{(n)}(Q^2) = {} & [(1-\alpha_n)\langle q^S(Q_0^2)\rangle_n - \beta_n\langle G(Q_0^2)\rangle_n]e^{-\gamma_n^+ S} \\ & + [\alpha_n\langle q^S(Q_0^2)\rangle_n + \beta_n\langle G(Q_0^2)\rangle_n]e^{-\gamma_n^- S} - \langle V_8(Q_0^2)\rangle_n e^{-\gamma_n S}\,, \end{aligned} \tag{A11}$$

with

$$\langle q^S(Q_0^2)\rangle_n = \langle 2\bar{u}(Q_0^2) + 2\bar{d}(Q_0^2) + 2\,\bar{s}(Q_0^2) + 2\,\bar{c}(Q_0^2) + V_8(Q_0^2)\rangle_n \tag{A12}$$

and

$$\begin{aligned} \langle G(Q^2)\rangle_n = {} & \left(\alpha_n\langle G(Q_0^2)\rangle_n - \frac{\alpha_n(1-\alpha_n)}{\beta_n}\langle q^S(Q_0^2)\rangle_n\right)e^{-\gamma_n^+ S} \\ & + \left((1-\alpha_n)\langle G(Q_0^2)\rangle_n + \frac{\alpha_n(1-\alpha_n)}{\beta_n}\langle q^S(Q_0^2)\rangle_n\right)e^{-\gamma_n^- S}\,. \end{aligned} \tag{A13}$$

The $\alpha$, $\beta$, and $\gamma$ are

$$\begin{aligned} &\gamma_2 = 0.427\,, \quad \gamma_3 = 0.667\,, \\ &\gamma_2^+ = 0.747\,, \quad \gamma_3^+ = 1.386\,, \\ &\gamma_2^- = 0\,, \quad \gamma_3^- = 0.609\,, \\ &\alpha_2 = 0.429\,, \quad \alpha_3 = 0.925\,, \\ &\beta_2 = 0.429\,, \quad \beta_3 = 0.288\,. \end{aligned} \tag{A14}$$

[35]Radiative corrections to deep-inelastic charged-current scattering are considered by Kiskis, 1973; De Rujula, Petronzio, and Savoy-Navarro, 1979; Barlow and Wolfram, 1979; Marciano and Sirlin, 1980a, b; Dawson, Hagelin, and Hall, 1980; Sakakibara, 1980. For the $eD$ asymmetry, see Bardin, Fedovenko, and Shomeiko, 1979; Dawson, Hagelin, and Hall, 1980.

[36]Higher-order contributions to atomic parity violation and the polarized $eD$ asymmetry have been considered by Marciano and Sanda (1978a, b).

## APPENDIX B: DETAILS OF THE $SU(2)_L \times SU(2)_R \times U(1)$ MODEL

The neutral-current couplings in Table XIII depend on constants $a$, $c$, $d$, and a mixing angle $\sin^2\theta$. These constants are expressed (see footnote 6) in terms of two ratios of Higgs vacuum expectation values $\varepsilon$ and $\sigma$, where $0 \leq \varepsilon \leq 1$ and $-1 \leq \sigma \leq 1$. The WS-GIM limit is

$$\varepsilon = 0\,, \quad \sigma = 1\,, \quad \sin^2\theta = 2\sin^2\theta_w\,.$$

We have

$$a = \frac{1}{2}\,\frac{\sin 2\varphi}{\cos\theta}\left(\frac{1}{R_1} - \frac{1}{R_2}\right),$$

$$c = \frac{\sec^2\theta}{2}\left(\frac{\sin^2\varphi}{R_1} + \frac{\cos^2\varphi}{R_2}\right) + \frac{\sec\theta}{2}\,\sin\varphi\cos\varphi\left(\frac{1}{R_1} - \frac{1}{R_2}\right),$$

$$d = \frac{1}{2}\left(\frac{\cos^2\varphi}{R_1} - \frac{\sin^2\varphi}{R_2}\right) + \frac{\sec\theta}{2}\,\sin\varphi\cos\varphi\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \tag{B1}$$

In Eq. (B1), $\varphi$ is defined by

$$\begin{pmatrix}\sin\varphi \\ \cos\varphi\end{pmatrix} = \begin{pmatrix}(\varepsilon - 1)\sigma \\ r\end{pmatrix} \Big/ [(1-\varepsilon)^2\sigma^2 + r^2]^{1/2}\,, \tag{B2}$$

where

$$r = -\tfrac{1}{2}(1-\varepsilon)\sin^2\theta\sec\theta + \varepsilon\cos\theta - \{[\tfrac{1}{2}(1-\varepsilon)\sin^2\theta\sec\theta - \varepsilon\cos\theta]^2 + (1-\varepsilon)^2\sigma^2\}^{1/2}\,. \tag{B3}$$

$R_1$ and $R_2$ are defined by

$$R_{1,2} = \frac{M_{Z_{1,2}}^2}{M_{W_L}^2} = \frac{1}{2[1-(1-\varepsilon)\sigma]}\Big((1-\varepsilon)\sec^2\theta + (1+\varepsilon) \mp \{[(1+\varepsilon) - (1-\varepsilon)\sec^2\theta]^2 + 4(1-\varepsilon)^2\sigma^2\sec^2\theta\}^{1/2}\Big)\,. \tag{B4}$$

The $W_L^{\pm}$ masses are

$$M_{W_L}^{\pm} = \left[\frac{\pi\alpha}{\sqrt{2}\,G_F}\right]^{1/2}\frac{\sqrt{2}}{\sin\theta}\,. \tag{B5}$$

## APPENDIX C: CORRELATION MATRICES

TABLE XIX. Correlation coefficients for fits to $\nu$-hadron data. $\theta_L$ and $\theta_R$ are expressed in radians.

| | $\varepsilon_L(u)$ | $\varepsilon_L(d)$ | $\varepsilon_R(u)$ | $\varepsilon_R(d)$ |
|---|---|---|---|---|
| $\varepsilon_L(u)$ | 1.0 | | | |
| $\varepsilon_L(d)$ | 0.94 | 1.0 | | |
| $\varepsilon_R(u)$ | 0.21 | 0.01 | 1.0 | |
| $\varepsilon_R(d)$ | −0.21 | −0.25 | −0.011 | 1.0 |
| | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ |
| $\alpha$ | 1.0 | | | |
| $\beta$ | 0.02 | 1.0 | | |
| $\gamma$ | −0.06 | 0.12 | 1.0 | |
| $\delta$ | 0.15 | −0.38 | 0.12 | 1.0 |
| | $g_L$ | $g_R$ | $\theta_L$ | $\theta_R$ |
| $g_L$ | 1.0 | | | |
| $g_R$ | −0.59 | 1.0 | | |
| $\theta_L$ | −0.11 | 0.07 | 1.0 | |
| $\theta_R$ | 0.12 | −0.20 | 0.08 | 1.0 |

TABLE XX. Correlation coefficients for fit of $\varepsilon_L(u)$, $\varepsilon_L(d)$, $\varepsilon_R(u)$, $\varepsilon_R(d)$, $g_V$, $g_A$, and $\rho$ to all data.

| | $\varepsilon_L(u)$ | $\varepsilon_L(d)$ | $\varepsilon_R(u)$ | $\varepsilon_R(d)$ | $g_V$ | $g_A$ | $\rho$ |
|---|---|---|---|---|---|---|---|
| $\varepsilon_L(u)$ | 1.0 | | | | | | |
| $\varepsilon_L(d)$ | 0.95 | 1.0 | | | | | |
| $\varepsilon_R(u)$ | 0.22 | 0.06 | 1.0 | | | | |
| $\varepsilon_R(d)$ | −0.24 | −0.21 | −0.41 | 1.0 | | | |
| $g_V$ | 0.00 | 0.00 | 0.00 | 0.03 | 1.0 | | |
| $g_A$ | 0.00 | −0.02 | 0.09 | 0.15 | 0.34 | 1.0 | |
| $\rho$ | 0.42 | 0.34 | 0.44 | −0.62 | −0.45 | −0.62 | 1.0 |

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