Recent experimental advances in positronium research

Arthur Rich

Physics Department, University of Michigan, Ann Arbor, Michigan

Positronium is a two-body, leptonic, particle–antiparticle system which possesses self-annihilation channels not directly present in any system studied to date. These features make it ideal for the study of the relativistic, two-body problem in quantum electrodynamics. This review will focus on recent experimental advances in fundamental positronium research. In addition, a less detailed discussion of recent theoretical advances is outlined. The review also contains a fairly detailed historial introduction and a section discussing uses of positronium in research not related to tests of quantum electrodynamics.

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I. INTRODUCTION

A. Purpose of review and extent of coverage

The past five years have been witness to a number of major advances in understanding of the positronium (Ps) system and in the use of Ps to explore the basic structure of quantum electrodynamics (QED). These new and volatile advances have followed on a 20-year period of steady improvements and extensions of the pioneering theoretical and experimental research which occurred during the decade 1946–1956. In this review attention will be focused primarily on the recent experimental advances but with consideration being given to very recent theoretical work on hyperfine structure and decay-rate calculations. In order that the article be self-contained, an introduction to the theoretical background of the subject matter covered has been given in Secs. I.B and I.C. The experimental achievements that will be explored have, as is often the case in physics, gone hand in hand with innovations of either a methodological type (e.g., discovery of slow positron emission from various surfaces, discovery of Ps formation on metal surfaces and in fine powders) or of a technological nature (e.g., introduction of the channeltron electron multiplier).

The most important of these advances will be discussed insofar as they relate to fundamental Ps research. In order to place the current situation in perspective a history of positron and Ps research has been included at the beginning of the article, with particular emphasis on those aspects related to the subject matter of the review (Table I). Finally in the summary, an attempt will be made to pinpoint the most fruitful areas for research in this field in the near future.

The cgs system of units will be used throughout this review and the following symbols will be employed:

Electron (positron) mass: m;
Electron (positron) charge: $e^- = -e$, $e^+ = +e$;
Electron (positron) spin: $\uparrow$, $\downarrow$;
Electron (positron) magnetic moment: μ;

TABLE I. Historical Outline.

Date	Participants	Contributions
1930	Dirac (Di)	Prediction of antimatter (nature of antiparticles unclear).
1931	Weyl (We)	Antielectron = positive electron.
1933	Anderson (A)	Discovery of positron.
1934	Mohorovičić (Mo)	Postulated the possible existence of Ps.
1945	Ruark (R)	Qualitative discussion of the Bohr spectroscopic structure of Ps.
1946	Pirenne (P)	Calculated for arbitrary n: (i) Ps fine structure to order $\alpha^4 mc^2$, (ii) singlet decay rates to lowest order as $\lambda_0(n^1S_0)=\frac{\alpha^5}{2n^3}\left(\frac{mc^2}{\hbar}\right)$. Estimated triplet decay rates.
1946	Wheeler	Calculated singlet decay rate to lowest order and estimated triplet decay rate. Noted (i) $^3S_1 \rightarrow 2\gamma$ is forbidden by space symmetry, (ii) γ's in singlet decay have orthogonal planes of polarization. Showed that the complexes $2e^+ + e^-$, $2e^- + e^+$, and $2e^+ + 2e^-$ are bound.
1948, 50	Bleuler and Bradt (BB) Hanna (H), and Wu and Shaknov (WS)	Experimental verification that the γ's from $1^1S_0 \rightarrow \gamma + \gamma$ are plane polarized in perpendicular planes. This implies that e^-, e^+ have opposite intrinsic parity.
1948, 50	Landau (L), Yang (Y)	Discussion of selection rules based on invariance under rotation and inversion for the decay $1^1S_0 \rightarrow 2\gamma$.
1949	Berestetski and Landau (BL)	Calculation of Ps fine structure to order $\alpha^4 mc^2$ and calculation of Zeeman effect in $n=1$ Ps.
1949	Ore and Powell (OP)	Calculated zeroth order decay rate of 1^3S_1 Ps $[\lambda_0(1^3S_1)]$ not including radiative corrections. Result: $\lambda_0(1^3S_1)=\frac{2}{9\pi}(\pi^2-9)\alpha^6\frac{mc^2}{\hbar}=7.2\times10^6\ \text{sec}^{-1}$.
1951	Deutsch (D)	First production of Ps (in gases) and measurement of $\lambda(1^3S_1)$ to $\pm 10\%$.
1951	Deutsch and Dulit (DD)	Measurement of W(hfs) by magnetic quenching experiments ($\pm 15\%$) —verification of virtual annihilation term.
1951	Ferrell (F)	Calculation of Ps fine structure to order $\alpha^4 mc^2$ including corrections to the work of (P) and (BL).
1952	Deutsch and Brown	Measurement of W(hfs) to 1500 ppm by rf quenching technique.
1952	Karplus and Klein (KK)	Calculation of W(hfs) to order $\alpha^5 mc^2$. Result: $W(\text{hfs})=\left[\frac{7}{3}-\frac{\alpha}{\pi}\left(\frac{32}{9}+2\ln 2\right)+\text{order }(\alpha^2, \alpha^2\ln\alpha^{-1})\right]\frac{\alpha^4 mc^2}{4}=203\,381\ \text{MHz}$. Unevaluated terms of order α^2 and $\alpha^2\ln\alpha^{-1}$ will contribute 25 MHz to W(hfs) if each of their coefficients is unity.
1952, 53	Wolfenstein and Ravenhall (WR) and Michel (M)	Showed, using charge conjugation (C) selection rules, that $^1S_0 \rightarrow 3\gamma$ is forbidden and that the $^3S_1$ ($m=\pm 1$) states are unaffected by a magnetic field.
1954	Fulton and Martin (FM)	Calculation of $n=2$ energy levels to order $\alpha^5 mc^2$, i.e., including order α radiative correction.
1957	Harris and Brown (HB)	Calculation of order α radiative corrections to $\lambda(1^1S_0)$. Result: $\lambda(1^1S_0)=\lambda_0[1^1S_0]\left[1-\frac{\alpha}{\pi}\left(5-\frac{1}{4}\pi^2\right)\right]=0.798\times10^{10}\ \text{sec}^{-1}$.
1957	Hughes, Marder, and Wu (HMW)	Measurement of W(hfs) to 200 ppm by rf quenching.
1958	Cherry (C)	Preliminary evidence of slow positron reemission when high-energy positrons are incident on metal surfaces.
1967	Mills and Berko (MB)	Set the branching ratio limit $(^1S_0 \rightarrow 3\gamma)/(^1S_0 \rightarrow 2\gamma)=2.8\times10^{-6}$ as a test of C-invariance in Ps decay.
1967	Theriot *et al.* (T)	Measurement of W(hfs) to 60 ppm using rf techniques and determination that $\lambda(1^1S_0)=(0.799\pm0.10)\times10^{10}\ \text{sec}^{-1}$ by use of hfs linewidth. Results in agreement with theory (KK, 1952 and HB, 1957).

TABLE I. (*Continued*)

Date	Participants	Contributions
1968	Paulin and Ambrosino (PA)	Discovery of nearly free 1^3S_1 Ps in low-density (0.1 gm cm^{-3}), small-grained (~100 Å) powders of SiO_2, Al_2O_3, and MgO. Observed intensities are up to 20–30% of all incident e^+.
1968	Brandt and Paulin (BP)	Explanation of (PA) as Ps diffusion in the grains and calculation of Ps diffusion constants and annihilation lengths.
1968	Beers and Hughes (BH)	Measurement of $\lambda(1^3S_1)$ in gases. Result: $\lambda(1^3S_1) = (7.27 \pm 0.015) \times 10^6$ sec^{-1}.
1968–79	Groce, Costello McGowan, and Herring (GC), 1968; Canter, Coleman, Griffith, and Heyland (CC), 1972; Coleman, Griffith, and Heyland (CG), 1973; Mills (M) 1979	Discovery that when high-energy positrons are incident on gold-covered mica surface, 10^{-7} emerge at 1 eV. Major advances after this (using different moderators) were made by (CC) (10^{-6} slow e^+/incident fast e^+) (CG) (3×10^{-5} slow e^+/incident fast e^+), and (M) (10^{-3} slow e^+/incident fast e^+). Slow e^+ are crucial in recent Ps research.
1970	Fulton, Owen, and Repko (FOW)	First calculation of some of the order $\alpha^2 \ln \alpha^{-1}$ radiative corrections to W(hfs).
1973	Coleman and Griffith (CG)	Measurement of $\lambda(1^3S_1)$ as $\lambda(1^3S_1) = (7.26 \pm 0.015) \times 10^6$ sec^{-1} in agreement with (BH).
1974	Marko and Rich (MR)	Set the branching ratio limit $(1^3S_1 \to 4\gamma / 1^3S_1 \to 3\gamma) < 8 \times 10^{-6}$ as a test of C-invariance in Ps decay.
1974	Stroscio (S) and Stroscio and Holt (SH)	Calculation of order α radiative corrections to $\lambda(1^3S_1)$. Result: $\lambda(1^3S_1) = \lambda_0(1^3S_1)\left(1 + (1.86 \pm 0.5)\frac{\alpha}{\pi}\right) = (7.24 \pm 0.008) \times 10^6$ sec^{-1}.
1974	Canter, Mills, and Berko (CMB)	Discovery that Ps is formed with up to 80% efficiency from slow e^+ beams incident on various metal surfaces.
1975–77	Mills and Bearman (MBe) and Egan, Carlson, Hughes, and Yam (EC)	Determination of W(hfs). Results (expressed in frequency units) f(MBe) = (203,387 ± 1.6) MHz and f(EC) = (203 384 ± 1.2) MHz in agreement with theory (KK, 1952).
1975	Canter, Mills, and Berko (CMB) and Mills, Berko, and Canter (MBC)	Discovery of $n = 2$ Ps and measurement of the 2^3S_1-2^3P_2 fine-structure splitting as $f_0 = (8628 \pm 6)$ MHz. Results verify calculation of (FM, 1954), including 231 MHz order α radiative correction.
1976	Gidley, Marko, and Rich (GMR), and Gidley *et al.* (Gi)	Measurement of $\lambda(1^3S_1)$ in SiO_2 powders and vacuum. Results: $\lambda(SiO_2) = (7.10 \pm 0.006) \times 10^6$ sec^{-1}; λ (vac) = $(7.09 \pm 0.02) \times 10^6$ sec^{-1} in disagreement with previous theory and experiment.
1977	Lepage (L)	New approach to solution of the relativistic two-body problem with application to muonium and Ps.
1977	Caswell, Lepage, and Sapirstein (CLS)	Recalculation of $\lambda(1^3S_1)$. After correction of (SH, 1974) the result is $\lambda(1^3S_1) = \lambda_0(1^3S_1)[1 - (10.35 \pm 0.07)(\alpha/\pi)] = (7.038 \times 10^6)$ sec^{-1}.
1978	Gidley *et al.* (Gi) Gidley and Zitzewitz (GZ), Griffith *et al.* (G)	Reevaluation (Gi) of 1976 work of (GMR) and (G) to yield $\lambda(SiO_2) = (7.067 \pm 0.021) \times 10^6$ sec^{-1}, λ (vac) = $(7.050 \pm 0.013) \times 10^6$ sec^{-1}. New measurement of $\lambda(1^3S_1)$ in gases: λ(GZ) = $(7.056 \pm 0.007) \times 10^6$ and λ(G) = $(7.045 \pm 0.006) \times 10^6$ sec^{-1}.
1978	Caswell and Lepage (CL), Bodwin and Yennie (BY)	Completion of all order $\alpha^2 \ln \alpha^{-1}$ terms in W(hfs). Result: $W(\text{hfs}) = \left[\frac{7}{3} - \frac{\alpha}{\pi}\left(\frac{32}{9} + 2\ln 2\right) + \frac{5}{6}\alpha^2 \ln \alpha^{-1} + O(\alpha^2)\right]\frac{\alpha^4 mc^2}{4} = (203\,400 \text{ MHz})$. Uncalculated terms of order α^2 would contribute 7 MHz to W(hfs) if their coefficient is unity.
1979	Caswell and Lepage	Calculation of $O(\alpha^2 \ln \alpha^{-1})$ radiative corrections to $\lambda(1^1S_0)$ and $\lambda(1^3S_1)$: $\lambda(1^1S_0) = \lambda_0(1^1S_0)\left[1 - \frac{\alpha}{\pi}\left(5 - \frac{1}{4}\pi^2\right) + \frac{2}{3}\alpha^2 \ln \alpha^{-1} + O(\alpha^2)\right] = (0.7984 \pm 0.0001) \times 10^{10}$ sec^{-1}. $\lambda(1^3S_1) = \lambda_0(1^3S_1)\left(1 - (10.266 \pm 0.008)\frac{\alpha}{\pi} - \frac{1}{3}\alpha^2 \ln \alpha^{-1}\right) = (7.0386 \pm 0.0016) \times 10^6$ sec^{-1}.

TABLE I. (*Continued*)

Date	Participants	Contributions
1979	Mills and Pfeiffer (MP)	Observation of thermal Ps emission from a Cu (111) surface bombarded by 0–100 eV positrons.

Electron (positron) g factor: g;
Bohr magneton: $\mu_B = e\hbar/2mc$;
Fine structure constant: $\alpha = e^2/\hbar c \approx 1/137$;
Electron (positron) g factor anomaly: $a = (g-2)/2$.

Note that static electron-positron properties (mass, magnitude of charge, etc.) will be assumed to be identical (*TCP* theorem) unless it is specifically noted otherwise.

The literature search for this article was completed in June 1980, although several articles of particular interest which appeared through August have been included.

B. Qualitative introduction to the Ps system

Positronium (Ps) is the electron–positron bound state. The possibility of its existence was first suggested by Mohorovicic in 1934. Its gross (Bohr) spectroscopic structure is equivalent to that of hydrogen, but with each energy level being half that of hydrogen, since the reduced mass M is half of the electron mass $[M = (m_{e^-})(m_{e^+})/(m_{e^-}+m_{e^+}) = \frac{1}{2}m_{e^-}]$. Thus, for example, the ionization potential of Ps is 6.8 eV. The reduced mass effect also causes the Ps distance scale to be twice that in hydrogen. As in hydrogen, the ground state of Ps is composed of three triplet spin states of total spin one and a spin-zero singlet state, i.e.,

$$\psi_T(m=1) = \uparrow\uparrow, \quad \psi_T(m=-1) = \downarrow\downarrow,$$

and

$$\psi_T(m=0) = \psi_T = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow), \tag{1.1a}$$

$$\psi_S(m=0) = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow). \tag{1.1b}$$

Here $\uparrow$, $\downarrow$ refer to electron and positron spin, respectively, and m denotes the projection of spin onto an arbitrary quantization axis. These states are split by the usual spin-spin hyperfine (hfs) interaction, which in Ps is about 200 times larger than in hydrogen. The size of the Ps hfs splitting in comparison to the hydrogen hfs splitting is primarily due to the positron-to-proton magnetic moment ratio of 900. The hfs is then reduced because of the increased distance scale and, in addition, increased by a quantum electrodynamic effect, virtual Ps annihilation and re-creation, which accounts for almost half of the hyperfine splitting W(hfs). The singlet and triplet states annihilate, subject to various selection rules, into primarily two and three photons, respectively, with lifetimes of about 0.1 and 140 nsec. The $n=2$ level structure of Ps is similar to that of He with a singlet-triplet separation possible. The typical fine-structure splittings between various values of J (total angular momentum) are of order 10^4 MHz, as in hydrogen. The degeneracy with respect to J, which in hydrogen is lifted by the Lamb shift and by the hfs interaction, is lifted by $\mathbf{L}\cdot\mathbf{S}$ coupling in $n=2$ Ps.

Although the similarities between Ps and either hydrogen or helium are interesting, the primary reasons for studying Ps lie in its differences from these systems rather than in its similarities to them. The most important of these differences and their effects on the Ps system may be classified as follows.

(1) Ps is a purely leptonic system; consequently, high precision measurements of the $n=1$ hyperfine structure and decay rate and of the $n=2$ fine structure provide unambiguous and stringent tests of quantum electrodynamic predictions. The difficulties of hadronic structure calculations present in normal atoms need not be considered in Ps. In addition, the above measurements indirectly test *TCP*, since it is assumed in all calculations that $m_{e^+} = m_{e^-}$ and $|\mu_{e^+}| = |\mu_{e^-}|$. Deviations from exact equality would show up in the level separations. etc., however, these indirect tests are not as sensitive as various direct tests of particle-antiparticle equality which have been performed, such as the electron-positron mass and g-factor comparisons which show equality to 10^{-7} and 10^{-8}, respectively.

(2) The positron and electron are antiparticles; therefore Ps can be used to study various symmetry principles such as charge conjugation (C) invariance and intrinsic electron-positron parity. The particle-antiparticle nature of the system also makes possible the investigations of both real and virtual self-annihilation effects—processes which to date have been checked in detail only in Ps.

(3) The positron is a "light" nucleus; consequently, when calculations of an accuracy comparable to the current experimental precision of the $n=1$ hfs or triplet decay rate are needed, a number of interesting effects related to the two-body formulation of QED (via the Bethe-Salpeter approach—or more recently the Lepage approach) are tested.

II. HISTORICAL OVERVIEW OF FUNDAMENTAL POSITRONIUM RESEARCH (1946-1966)

There have been several comprehensive reviews which have concentrated on various aspects of fundamental Ps research. The most complete of these are by Deutsch (1953), De Bennedetti and Corben (1954), and Stroscio (1975), while a review stressing the solid-state aspects of positron and Ps research is currently being prepared (Berko, 1980). Those interested in a comprehensive and detailed historical review of the subject are invited to use Table I of this article in conjunction with this section, the above reviews, and, of course, the original articles. The work by Stroscio primarily stresses theory, while the Deutsch and De Bennedetti reviews include both theory and experiment. In addition to the above articles, excellent but

less comprehensive surveys of fundamental Ps research have been written by Hughes (1969), Mills, Berko, and Canter (1977), and Berko, Canter, and Mills (1979). Finally, a very extensive review related to recent work on the production and use of low-energy positron beams has recently appeared (Griffith and Heyland, 1978a). In the historical overview which we now present we will limit ourselves to mentioning only the highlights of the period 1946–1966. A detailed review of work since 1966 with particular emphasis on research performed in the past five years will take place within the context of the article itself. In addition, we will use the overview presented in this section as a vehicle to introduce, without derivation, the primary equations and concepts relating to the basic properties of Ps as known up to 1966.

A. The theoretical foundation prior to the discovery of positronium (1946-1951)

The earliest specific calculations involving Ps were by Pirenne, who calculated the hfs of $n=1$ Ps and the fine structure of Ps for arbitrary n. This rather comprehensive attack on the Ps level structure was completed in 1943, but not published until 1946 (Pirenne, 1946). Its publication preceeded by six years the actual discovery of Ps. Pirenne's major result was that the $n=1$ triplet and singlet state energies W_T(hfs), W_S(hfs) are given by the expressions

$$W_T(\text{hfs}) = [-\tfrac{1}{4} + \alpha^2(\tfrac{1}{12} - \tfrac{5}{64} + \tfrac{1}{4})]\alpha^2 mc^2 , \tag{2.1a}$$

$$W_S(\text{hfs}) = [-\tfrac{1}{4} + \alpha^2(-\tfrac{1}{4} - \tfrac{5}{64})]\alpha^2 mc^2 . \tag{2.1b}$$

Here the term $-\frac{1}{4}\alpha^2 mc^2$ in W_T and W_S is the Coulomb interaction (−6.8 eV), the terms $\frac{1}{12}\alpha^4$ and $-\frac{1}{4}\alpha^4$ represent the triplet and singlet spin-spin interaction with Hamiltonian

$$H_{ss} = \mu^2\left(\frac{\sigma_1\cdot\sigma_2}{r^3} - \frac{\sigma_1\cdot \mathbf{r}\sigma_2\cdot \mathbf{r}}{r^5}\right),$$

and the term $-\frac{5}{64}\alpha^4$ represents the usual hydrogenic type of relativistic effect due, for example, to the relativistic mass increase, etc. These terms are summarized in the Feynmann diagram of Fig. 1(a). The extra term $+\frac{1}{4}\alpha^4 mc^2$ in W_T is due to virtual annihilation of Ps [Fig. 1(b)]. Conservation of C forbids this process for singlet states since they are even under C, while the one-photon virtual state is odd (see the latter part of this section for a discussion of C selection rules in Ps decay). Pirenne also calculated the n^1S_0 Ps decay rate into two gammas (not including radiative corrections) as

$$\lambda_0(n^1S_0) = \frac{4\rho\sigma(2\gamma)v}{n^3} = \frac{8\times 10^9}{n^3}\ \text{sec}^{-1} . \tag{2.2a}$$

Here ρ is the electron density at the position of the positron

$$\rho = \frac{1}{\pi}(\alpha mc/2\hbar n)^3 ,$$

$\sigma(2\gamma)$ is the Dirac plane-wave cross section for two-photon pair annihilation, which in the limit $\gamma \to 1$ is given by $\sigma(2\gamma) = \pi r_0^2(c/v)$, where v is the relative electron-positron velocity, $r_0 = e^2/mc^2$ the classical electron radius, and the subscript on λ_0 refers to the zeroth order calculation, not including radiative and other types of corrections. The factor of 4 multiplying $\rho\sigma(2\gamma)v$ in Eq. (2.2a) occurs because $\sigma(2\gamma)$ represents an average over initial electron and positron spins, while in Ps only the singlet state contributes to two-photon annihilation. We note that $\lambda_0(n^1S_0)$ may be more suggestively written as

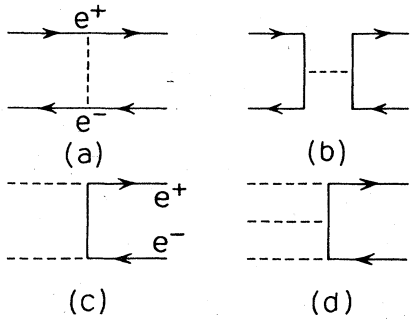


FIG. 1. (a) Feynman diagram representing the exchange of a covariant photon. This diagram includes all effects in expressions (2.1a) and (2.1b) with the exception of the term $\frac{1}{4}\alpha^4 mc^2$ in Eq. (2.1a). (b) Feynman diagram representing virtual annihilation of Ps. This effect adds $+\frac{1}{4}\alpha^4 mc^2$ to W_T and comprises 40% of the triplet-singlet splitting W(hfs). (c) Feynman diagram representing the lowest order decay of singlet ($n=1$) Ps [$\lambda_0(1^1S_0)$]. (d) Feynman diagram representing the lowest order decay of triplet ($n=1$) Ps [$\lambda_0(1^3S_1)$].

$$\lambda_0(n^1S_0) = \frac{\alpha^5}{2n^3}\left(\frac{mc^2}{\hbar}\right), \tag{2.2b}$$

where the nuclear unit of frequency $mc^2/\hbar$ is 2×10^{22} sec^{-1}. The Feynman diagram representing this calculation is shown in Fig. 1(c). In addition to the above work, Pirenne calculated the fine structure for $n>1$ and estimated the decay rates for three-photon emission as being down by a factor of α from the two-photon case.

Wheeler in 1946 made similar decay rate estimates and also predicted that the γ's from bound or free singlet annihilation would be plane polarized with orthogonal planes of polarization if electron and positron have opposite intrinsic parity. This prediction was verified by a number of workers during the period 1948–50 (see Table I). In addition, further calculations of the fine structure to order $\alpha^4 mc^2$ for arbitrary n were completed by Berestetski and Landau (1949) and Berestetski (1949) with final results including virtual annihilation terms and corrections to previous work being presented by Ferrell (1951).

The next major advance came in 1949 when Ore and Powell succeeded in calculating the decay rate (not including radiative corrections) for $1^3S_1 \to 3\gamma$ [Fig. 1(d)] as

$$\lambda_0(1^3S_1) = \frac{2}{9\pi}\alpha^6\frac{mc^2}{\hbar}(\pi^2 - 9) \simeq 7\times 10^7\ \text{sec}^{-1} . \tag{2.3}$$

Ore and Powell also showed that for this process the probability of emission of a γ ray of energy E is essentially linear in E. Most of the theoretical structure of Ps, including calculations of the hyperfine and fine structure splittings, decay rates, some selection rules affecting annihilation, etc., had thus been deter-

mined by 1949. There had not as yet been any calculations of radiative corrections; but, as mentioned in later references, work on radiative corrections to the fine and hyperfine structures was apparently already in progress. The extent of theoretical interest in Ps was quite striking, considering that the object of interest itself had not as yet been produced. There was, in fact, no way that the numerous theorists working on Ps calculations could easily have anticipated that, even if Ps were to be produced, the experimental precision in determining energy splittings, decay rates, etc., would ever be sufficient to compare quantitatively with theory.

B. The discovery of positronium and succeeding advances (1951-1968)

The experimental situation changed dramatically in 1951 when Deutsch reported that he had produced Ps in gases and in addition had made the first crude but important measurements of $W(\text{hfs})$ to ±15% and of $\lambda(1^3S_1)$ to ±10%. The first hyperfine structure determination was based on a magnetic quenching technique. In order to understand the technique, as well as much other Ps-related research, it is necessary to make a short digression at this point in order to outline the behavior of $n=1$ Ps in a magnetic field. Magnetic fields cannot split the $m=\pm1$ states, as can be shown by applying the combined operation of rotation about the magnetic field of 180° and C (or TCP) to the Hamiltonian operating on these states (Wolfenstein and Ravenhall, 1952). The magnetic part of the Hamiltonian H_M mixes $\psi_T(m=0)$ and ψ_S. It is given to first order in the field B by

$$H_M=\frac{g'}{2}\mu_B B[\sigma_z(e^-)-\sigma_z(e^+)]\,.$$

Here we take account of quantum electrodynamic effects (the g-factor anomaly—a), as well as relativistic binding and center of mass motion effects [see, e.g., Lewis and Hughes (1973) or Grotch and Kashuba (1973)] by writing

$$\frac{g'}{2}=(1+a)\left(1-\frac{5}{24}\alpha^2-\frac{T(\text{center-mass})}{2mc^2}\right).$$

Diagonalization of the energy matrix including H_M but not including effects due to the annihilation of the singlet and triplet states lead to the following well-known (Berestetski, 1949; Halpern, 1954; Bisi *et al.*, 1962) approximate expressions for the field perturbed eigenstates, energies, and decay rates:

$$\psi'_T=\frac{1}{(1+y^2)^{1/2}}(\psi_T+y\psi_S)\,, \tag{2.4a}$$

$$\psi'_S=\frac{1}{(1+y^2)^{1/2}}(\psi_S-y\psi_T)\,, \tag{2.4b}$$

$$W'_T=\tfrac{1}{2}W_T[1+(1+x^2)^{1/2}]\underset{x\ll1}{\simeq}W\{1+\tfrac{1}{4}x^2\}\,, \tag{2.4c}$$

$$W'_S=\tfrac{1}{2}W_T[1-(1+x^2)^{1/2}]\underset{x\ll1}{\simeq}-\tfrac{1}{4}x^2W\,, \tag{2.4d}$$

$$\lambda'_T=\frac{1}{1+y^2}(\lambda_T+y^2\lambda_S)$$

$$=\lambda'_T(3\gamma)+\lambda'_T(2\gamma)\underset{x\ll1}{\simeq}\lambda_T+\tfrac{1}{4}x^2\lambda_S\,, \tag{2.4e}$$

$$\lambda'_S=\frac{1}{1+y^2}(\lambda_S+y^2\lambda_T)=\lambda'_S(2\gamma)+\lambda'_S(3\gamma)\underset{x\ll1}{\simeq}\lambda_S\,. \tag{2.4f}$$

Here $x=2g'\mu_B B/W(\text{hfs})\simeq B/36.5$ kg; $y=x/[1+(1+x^2)^{1/2}]$; $\lambda'_T(2\gamma)$, $\lambda'_T(3\gamma)$, $\lambda'_S(2\gamma)$, and $\lambda'_S(3\gamma)$ refer to partial decay rates into the 2γ or 3γ channels from ψ'_T and ψ'_S, respectively; and we have taken the zero of energy to be W_S, i.e., $W_T=W(\text{hfs})=W$. The quantities W'_T, W'_S, λ'_T, and λ'_S are graphed, to scale, as functions of B in Fig. 2.

Deutsch's first measurement of W used a magnetic quenching technique. Magnetic quenching refers to the decrease of 3γ annihilation events from ψ'_T as a function of B. The ratio of 2γ to 3γ decays from ψ'_T is

$$r=\lambda'_S(2\gamma)/\lambda'_S(3\gamma)=(y^2)(\lambda_S/\lambda_T)\underset{x\ll1}{\simeq}\tfrac{1}{4}x^2(\lambda_S/\lambda_T)$$

as can be seen from Eq. (2.4a). Thus the fraction $F_{3\gamma}$ of 3γ decays from the triplet states is ($m=\pm1$ unaffected by B)

$$F_{3\gamma}=\frac{2}{3}+\frac{1}{3}\left(\frac{1}{1+r}\right). \tag{2.5}$$

The fraction of 3γ decays actually observed depends on detector configuration, the number of γ's (one, two, or three) required for an event, and detector energy settings; but if λ_S/λ_T is presumed to be known, it is clear that measurement of $F_{3\gamma}$ vs B yields x and therefore W. Deutsch and Dulit used this technique to determine W to ±15%, thereby confirming the virtual annihilation term (40%) of W).

Immediately thereafter, however, Deutsch improved the precision of the measurement enormously by introducing a radiofrequency (rf) quenching technique which has been the prototype for all the subsequent high-precision measurements of W. The technique consists of applying a static magnetic field B (typically $B\sim8$–10 kG) in the Ps-forming region and then applying an rf magnetic field $B_y\cos(2\pi ft)$ perpendicular to B. For technical convenience (Sec. IV.A2) B (i.e., x), rather than f, is then varied and when for a fixed f ($f=f_0$)

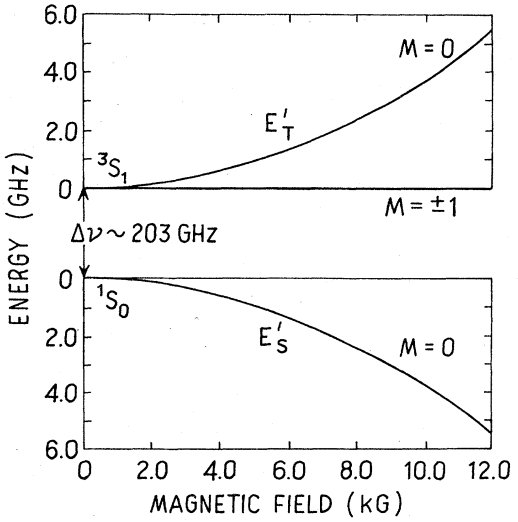


FIG. 2. The energy levels of Ps in a magnetic field (to scale).

$W'_T(x)$ satisfies the approximate condition

$$W'_T - W_T = hf_0 = \tfrac{1}{2}h\nu[(1+x^2)^{1/2} - 1], \quad h\nu = W \tag{2.6a}$$

transitions are induced from $\psi_T(\pm 1)$ to ψ'_T and vice versa with equal probability. Since λ'_T is $(10-20)\lambda_T$ due to the presence of the field B, the transition $\psi_T(\pm 1) \rightarrow \psi'_T$ predominates if B_y is chosen so that the transition rate (λ_{rf}) for the process is much less than λ'_T, but of order λ_T (typically $B_y \sim 10$ G), implying $\lambda_{rf} \sim \frac{1}{2}\lambda_T \sim (2-3) \times 10^{-2}\lambda'_T$. The $\psi_T(\pm 1) \rightarrow \psi'_T$ transition causes an increase (decrease) in the 2γ (3γ) decay fraction, since at the value of B used ψ'_T decays primarily into 2γ's. Detection of the change in one or the other of these fractions with B yields an approximately Lorentzian resonance line whose fractional natural linewidth (full width at half maximum intensity or FWHM) is given by the usual uncertainty principle relation as

$$\left(\frac{\delta B}{B}\right) = \frac{\lambda'_T + \lambda_T}{4\pi f_0} . \tag{2.6b}$$

If we use Eqs. (2.4c) and (2.4e), under the conditions $x \ll 1$, and $x^2\lambda_S \gg \lambda_T$, we may rewrite Eq. (2.6b) as

$$\frac{\delta B}{B} \simeq \frac{\lambda_S}{4\pi\nu} = 3.1 \times 10^{-3} . \tag{2.6c}$$

One obtains W from f_0 and the value of $x(B)$ at resonance using Eq. (2.6a). In principle one could also excite the direct singlet-triplet transition at $\nu = 203$ GHz; however, the microwave power necessary to do this is still extremely difficult if not impossible to obtain at the frequency in question (Sec. IV.A.2).

By 1954 Deutsch had succeeded in measuring W to 250 ppm, with his results being confirmed by a 200 ppm measurement shortly thereafter (Hughes, Marder, and Wu, 1957). Recently measurements at 6 and 8 ppm have been published (Mills and Bearman, 1975; Egan *et al.*, 1977). The fundamental problem in the recent work has been that of splitting the line to more than one part in one thousand in the face of a 5–10% signal amplitude, with the need for line-shape corrections due to the non-Lorentzian nature of the line. In addition, there is a need for systematic corrections for pressure shifts caused by collisions of the Ps with the background gas in which it is formed. These matters and the experiments themselves will be dealt with in detail in Sec. IV.A.2. We note here again, however, that these experiments, the most precise performed on Ps, are of the utmost importance, both as a test of quantum electrodynamics and as a test of the two-body formulation of relativistic quantum mechanics.

The next major steps taken to understand the Ps system were theoretical in nature. In 1952 Karplus and Klein were the first to calculate radiative corrections in Ps [see also Fulton and Karplus (1954) for further work on the two-body formalism and a check of the Karplus and Klein calculation]. Their calculation of radiative and other relativistic corrections to W to order $\alpha^5 mc^2$ was in fact necessary in order to obtain agreement with the measurements of Deutsch *et al.* The result of their calculation may be expressed as

$$W = W_0 + W_1 + W_2 + \cdots , \tag{2.7a}$$

$$W_0 = \frac{\alpha^4 mc^2}{4}\,\frac{7}{3} , \tag{2.7b}$$

$$W_1 = \frac{\alpha^4 mc^2}{4}\left(\frac{\alpha}{\pi}\right)\left(\frac{32}{9} + 2\ln 2\right) . \tag{2.7c}$$

Here W_0 is the zeroth-order hyperfine interaction (of order 203 000 MHz) as calculated by Pirenne (Feynman diagrams shown in Fig. 1) and W_1 is the order $\alpha^5 mc^2$ (1000 MHz) contribution due to first-order radiative corrections to the basic interaction as well as to other effects (Fig. 3). Only a partial calculation of the second-order (α^2 and $\alpha^2 \ln\alpha$) corrections to W, W_2 has as yet been completed. Since W_2 could well contribute a 100 ppm (10 MHz) correction to W, it must be calculated before a precision comparison with the 5 ppm experimental values can be carried out. We present a more detailed account of recent contributions towards evaluation of W_2 in Sec. IV.A.1.

An important clarification of the selection rules in Ps decay also occurred in 1952–3 when Wolfenstein and Ravenhall (and independently, Michel) showed explicitly that Ps in a state of definite relative angular momentum (l) and spin (total spin, s) is an eigenstate of the charge-conjugation operator, C, with eigenvalues $(-1)^{l+s}$, i.e.,

$$C\psi(n, l, s) = (-1)^{l+s}\psi(n, l, s) . \tag{2.8}$$

Since, as the above authors also showed, the N photon state is also an eigenstate of C with eigenvalues $(-1)^N$, conservation of C in the decay process leads to definite selection rules for the number (N) of gammas emitted from the decay of any specific state. Thus, for example, for the ground state $(n = 1, l = 0)\psi_S \rightarrow$ odd number of γ's or $\psi_T \rightarrow$ even number of γ's is greater than two, is strictly prohibited by C. Workers as far back as Pirenne had recognized that the decay $\psi_T \rightarrow 2\gamma$ was forbidden by considerations of space symmetry and spin but until the above research, the decay $\psi_S \rightarrow 3\gamma$, for example, was considered possible although the probability for this process had been shown to be

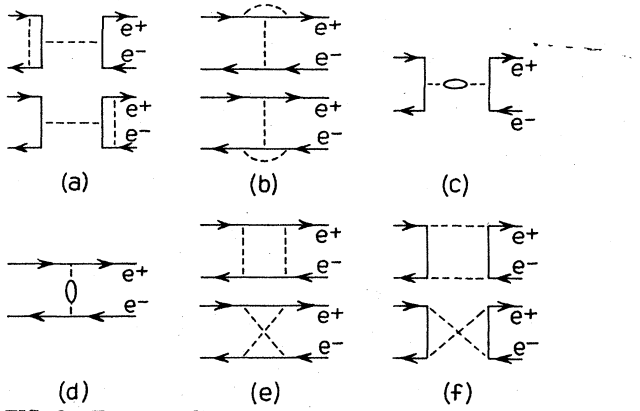


FIG. 3. Feynman diagrams representing all contributions to W_1 [Eq. (2.7c)] [$W_1 = W_{1a} + \cdots + W_{1f} = -(\alpha/4\pi)(32/9 + 2\ln 2) + i\alpha/4$ (units $\alpha^4 mc^2$)]. (a) Virtual annihilation plus photon exchange: $W_{1a} = -(\alpha/\pi)$. (b) Vertex correction diagrams $W_{1b} = (\alpha/3\pi)$. (c) Virtual annihilation plus vacuum polarization ($^3S_1$ only): $W_{1d} = (-2\alpha/9\pi)$. (d) Vacuum polarization: $W_{1c} = 0$. (e) Two-photon exchange: $W_{1e} = -\alpha/2\pi$. (f) Two-quantum annihilation ($^1S_0$ only): Re $W_{1f} = (\alpha/2\pi)(1 - \ln 2)$, Im$W_{1f} = (\hbar/2)\lambda_s = (\alpha/4)$.

zero to order α by explicit calculation (Drisko, 1952; Wolfenstein and Ravenhall, 1952). Additional calculations by Wolfenstein and Ravenhall relating to the lack of effect of magnetic fields on the energy levels of the $^3S_1$ ($m = \pm 1$) states have already been mentioned.

Two further significant theoretical contributions rounded out the first decade of Ps research. The first of these was the calculation of all corrections of order $\alpha^5 mc^2$ to the $n = 2$ energy levels by Fulton and Martin (1954). Their main results were that the magnitude of the radiative (Lamb shift), recoil (finite "nuclear" mass), and virtual annihilation contributions to order $\alpha^5 mc^2$ were most pronounced in the 2^3S_1 and 2^1S_0 states. These were typically 50–300 MHz for each type of contribution, as compared to fine-structure shifts of order 10^4 MHz as calculated by Pirenne. The order $\alpha^5 mc^2$ effects on the $2P$ states were shown to be only of order 1–20 MHz, though the fine-structure shifts in these states is similar to those in the $2S$ states. The reason for the smaller $2P$ shifts is that the contributions mentioned above are "short range" or contact, i.e., depend strongly on the size of the wave function near the origin. The $n = 2$ level structure is shown in Fig. 4.

Experimental verification of the work by Fulton and Martin did not occur until the recent breakthrough by the Brandeis group, who succeeded in producing the $n = 2$ state for the first time (Canter, Mills, and Berko, 1975). This group also measured the 2^3S_1-2^3P_2 fine-structure splitting to an accuracy of 0.1% (Mills, Berko, and Canter, 1975), thereby quantitatively confirming the 3% radiative correction to the 2^3S_1 level

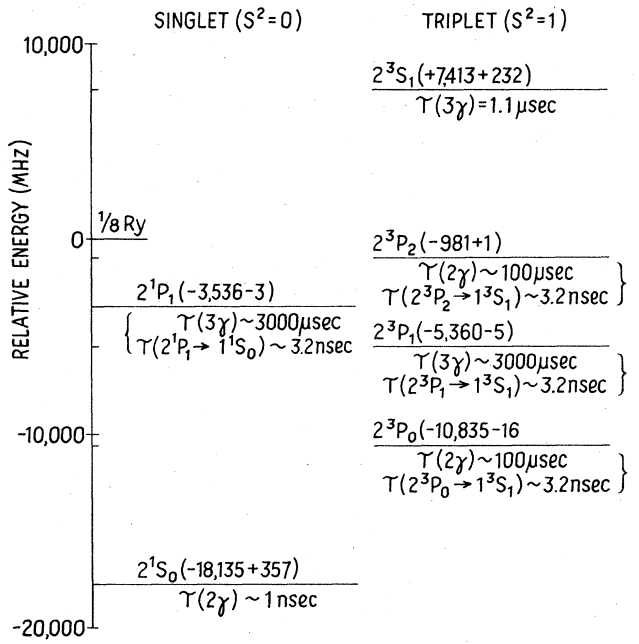


FIG. 4. The $n = 2$ level structure of Ps. The numbers in parenthesis represent the $O(\alpha^4)$ (fine structure) and $O(\alpha^5)$ (radiative corrections) to the Bohr energy level ($\frac{1}{8}$ Ry). Thus, for example, these corrections in 2^1P_1 are −3536 and −3 MHz, respectively. Estimates of the leading annihilation modes and optical decay rates are also included.

predicted by Fulton and Martin (see Sec. V.B. for a discussion of the experiment).

The next calculation of importance occurred in 1957 when Harris and Brown computed the order α radiative corrections to $\lambda(1^1S_0)$ (see Fig. 9 for the Feynman diagrams representing radiative corrections to singlet Ps decay). This was the first calculation of radiative corrections to any decay rate. Their result may be written as

$$\lambda(1^1S_0) = \lambda_0(1^1S_0) + \lambda_1(1^1S_0) + \lambda_2(1^1S_0) + \cdots . \tag{2.9}$$

Here

$$\lambda_0(1^1S_0) = \frac{\alpha^5}{2}\frac{mc^2}{\hbar} \simeq 8\times 10^9 \text{ sec}^{-1} \text{ [Eq. (2.2b)]},$$

$$\lambda_1(1^1S_0) = -\lambda_0\frac{\alpha}{\pi}(5 - \tfrac{1}{4}\pi^2) \simeq -5\times 10^{-3}\lambda_0(1^1S_0),$$

and

$$\lambda_2(1^1S_0) \sim O(\alpha^2, \alpha^2 \ln\alpha^{-1})\lambda_0,$$

where the process $1^1S_0 - 4\gamma$, as well as radiative corrections of order α^2, is included in λ_2.

A detailed analysis of this calculation may be found in Stroscio (1975). The results have not yet been verified experimentally although an indirect measurement of $\lambda(1^1S_0)$ at the 10^{-2} level of accuracy has been performed [Theriot *et al.* (1967)—Sec. IV.B.2).

In the decade following the calculation by Harris and Brown (1957–66) very little real progress was made in the theoretical understanding of Ps, in measurements of its properties, or in its use as a tool for elucidation of more basic physical laws. The recent progress in the above areas seems, interestingly enough, to date fairly explicitly from the years 1967–8, as can be seen from Table I. As stated at the beginning of this section, we will not be looking at the past decade's work in this overview since it constitutes the subject matter of this review, and as such will be presented in detail beginning in the next section.

III. POSITRONIUM FORMATION TECHNIQUES

A. The standard technique—Ps formation in gases

The object of most of the experiments to be discussed in the subsequent sections is to ascertain a particular property of Ps (fine- or hyperfine-structure interval, decay rate, etc.) and eventually to compare this property with the value predicted theoretically. The value calculated theoretically always assumes Ps to be located in vacuum, while, until very recently, the Ps formed experimentally was located in gas (Ar, N_2, etc.). Corrections necessitated by this fact will be mentioned below and discussed in detail in Sec. IV.

The process by which a positron of typically several hundred kilovolts energy emitted from a radioactive source slows down in a gas and eventually picks up an electron to form Ps has been thoroughly discussed in the literature (DeBenedetti and Corben, 1954; Massey, Burhop, and Gilbody, 1974). For our purposes, the principal features of the process are that under typical experimental conditions often including the presence of static and rf electric and magnetic fields, in chambers

of volumes of a few liters and at gas pressures of order one atmosphere, as many as 25–50% of the incoming positrons from a source can form Ps, typically within 10^{-10} sec of positron emission. The Ps so formed is perturbed due to collisions with the gas, but the effect of the perturbation on the quantities of interest mentioned above cannot be reliably calculated and corrected for. Consequently, the data necessary for an empirical plot of, e.g., W(hfs) or $\lambda(1^3S_1)$ vs gas density (ρ) are obtained, and with assumptions concerning the linearity of the extrapolation, the vacuum value of W or λ is estimated. The extrapolation from the measured points to the intercept or vacuum value is often over several standard deviations of the quoted result. Naturally, the extrapolation and its possible nonlinearity may be reduced by reducing ρ. This, however, results in lower Ps formation efficiency and, in most cases, an unsupportably long time for data collection. The parameters presented above represent an optimization with regard to data collection time, extrapolation, and other systematic effects. The need for Ps formation in a background gas presents a problem in high-precision Ps experiments which is either not present or present to a reduced degree in analogous experiments on, for example, hydrogen.

B. Ps formation in small grained powders

Paulin and Ambrosino (1968) discovered that if positrons from a source were incident on a variety of small-grained (70–90 Å size), low-density (0.5 gm cm^{-3}) powders of MgO, Al_2O_3, or SiO_2, then up to 30% of the positrons can form Ps. The triplet component of the Ps so formed was found to have its lifetime decreased by less than 1% from the vacuum value of 140 nsec. This was surprising in view of the fact that, up to that time, 1^3S_1 Ps formed in solids showed lifetimes of only one-two nanoseconds—as determined by annihilation with an electron of the solid in a relative spin singlet state (pick off). The powder phenomenon was explained by Brandt and Paulin (1968), who showed that 1^3S_1 Ps formed in a grain can diffuse out of the grain in a time well under the 1–2 nsec pick-off lifetime in the bulk solid. They estimated a Ps diffusion constant, $D=6\times10^{-5}$ cm^2 sec^{-1}. Once out of the grain, the Ps makes many thousands of collisions with grains in a lifetime (assuming thermal velocity); however, it is presumably energetically forbidden from reentering a grain. In addition, it is kept sufficiently far from the grain by short-range repulsive forces so that the annihilation probability per collision is small enough to give the minimal observed change in lifetime. These latter ideas have been explored quantitatively by Ford, Sander, and Witten (1976), who expect a short-range repulsive force due to interaction of Ps with the electrons of the grain with a potential of the form

$$V(x)=V_0e^{-x/a}. \tag{3.1}$$

Here, x is the Ps-grain distance, V_0 the work function of Ps (of order 1 eV), and a the penetration distance of the electron wave function into the vacuum (of order 0.5–1 Å). Such a potential, with parameters as chosen above and assuming a reasonable pick-off rate near the grain surface, gives a good estimate of decay rate and the correct dependence of decay rate on bulk density and grain size.

The importance of the discovery that copious amounts of nearly free Ps can be formed in powders lies in the facts (to be discussed in detail later) that the Ps can be formed in volumes of order (1–10) cm^3, rather than a few liters as with gases, and that at the densities in use the overall shift of W(hfs) and $\lambda(1^3S_1)$ from their presumed vacuum values may be smaller than in a gas under the conditions discussed in the preceding section. We note, however, that measurements of W(hfs) have not yet been made in low-density powders so that the above statement is, in fact, an extrapolation from measurements of W(hfs) in compressed ($\rho\sim1$ g cm^{-3}) pellets of Al_2O_3 and SiO_2, etc. (Judd *et al.*, 1973; Yam *et al.*, 1976). These workers found that W(hfs) was shifted down by 100 ppm in the pellets. Both the "point" geometry and the possible reduced extrapolation have been or can be of major importance in increasing the precision of measurements made on Ps. A negative factor in the use of powders is, however, the possibility that the extrapolation with respect to density in order to obtain a vacuum value may be nonlinear. This possibility, along with a similar possibility in the case of measurements performed in gases, must be considered and will be discussed at length in the context of specific experiments.

C. Production of slow positron beams and the formation of Ps on solid surfaces

Clearly, the confinement of Ps in vacuum would constitute a great improvement over Ps formation in gases or powders, at least insofar as the need for systematic corrections to measured values is necessary. A start towards the production and confinement of "vacuum" Ps has indeed been made; and although the amount of Ps so formed is sufficient to perform the experiments to be discussed, an increase in the amount of vacuum Ps available would constitute an important improvement in these experiments.

Vacuum Ps has been produced through a synthesis of two techniques, each of importance and interest in its own right. The first of these involved the discovery [Cherry (1958); Groce *et al.* (1968)—Table I] that when positrons from a radioactive source (mean energy typically several hundred keV) are incident on various types of surfaces, up to 1 in 10^7 emerge at an energy of about 1 eV. Subsequent research on this phenomenon has, through use of single-crystal Cu targets in 10^{-10} torr vacuum, improved the efficiency of low-energy emission to about $1:10^3$ slow e^+ per incident fast e^+ with eventual efficiencies as high as 6×10^{-3} considered as a possible limit (Mills, 1979). Typical efficiencies possible using various powder-coated surfaces in vacuums of 10^{-7}–10^{-8} torr are 10^{-5}–10^{-4} (Pendyala and McGowan, 1980; Zitzewitz, 1980). The width of the energy distribution of the slow positron component varies between roughly 0.2 and 0.5 eV, depending on the method used for slow positron production. Slow positron beams of typically 10^4–10^6 positrons/sec are now available in many laboratories, and beams of (10^6–10^7) positrons/

sec should be feasible using radioactive source strengths of only a few curies. Finally, it has recently been shown [Zitzewitz *et al.* (1979)—Sec. IV.D.2] that the slow positrons emitted from MgO-coated gold foil are polarized with degree of polarization $P=0.22^{+0.04}_{-0.02}$. This discovery should have ramifications of importance in solid-state physics and, if some increase in the accuracy of measurement of the polarization can be obtained, to an analysis of the slow-emission process itself.

Research concerning the production of slow positron beams and some of the hypotheses advanced to explain the phenomena of slow positron emission and its associated features of interest are discussed in detail in a number of recent articles presenting new work on the subject, particularly Lynn (1979) and Mills (1979). In addition, production of slow positron beams and their use in positron-atom scattering experiments are discussed in the review articles by Griffith and Heyland (1978) and Berko, Canter, and Mills (1979).

The technique leading to the production of vacuum Ps was the discovery by Canter, Mills, and Berko (1974) that when positrons of order several to several hundred eV of energy formed from initial beams of 1 eV positrons, created as described in the previous paragraph, are incident on various surfaces (Au, Ti, Cu, etc.), up to 80% of the incident positrons can form Ps, which subsequently leaves the target surface and enters the surrounding vacuum region. This technique led immediately (Canter, Mills, and Berko, 1975) to the discovery of $n=2$ Ps ($1:10^4$ of the Ps leaving the surface was in the $n=2$ state). Confinement of the $n=2$ Ps in a microwave cavity then led to the first measurement of the $n=2$ fine structure by Mills, Berko, and Canter (1975) (Sec. V). Finally, a similar technique has been used by Gidley *et al.* (1976) to measure accurately $\lambda(1^3S_1)$ in a vacuum (Sec. IV.B.4).

The process by which Ps is formed at or near the surface of the moderator in question is of interest for solid-state and surface physics as well as for Ps research, and it has been explored in detail by a number of experimenters. The most recent published research of interest (Mills and Pfeiffer, 1979) presents evidence for thermal as well as nonthermal Ps emission from a Cu (111) surface. The consequences of this discovery for fundamental Ps experiments could be far-reaching, since, for example, as discussed by the above authors, Ps at thermal energies would result in fewer Ps-wall collisions when Ps is confined in vacuum experiments which measure the $n=1$ triplet decay rate (Sec. IV.B.4) and the $n=2$ fine structure intervals (Sec. V.C). In addition, Ps moving at thermal velocities would decrease Doppler broadening in the fine-structure measurements and would necessitate reduced laser power (due to localization of the Ps) as well as reduced second-order Doppler shifts in possible $n=1$ to $n=2$ Ps excitation experiments, experiments which have been under consideration by various groups for a number of years.

The most recent theoretical attacks on the problems of Ps formation at surfaces and slow positron emission are presented by Oliva (1979, 1980), Nieminen and Oliva (1980), Chu, Mills, and Murray (1980), and Pendry (1980). These works contain references to prior theoretical and experimental research in this area.

IV. RESEARCH ON GROUND-STATE POSITRONIUM

A. Hyperfine-structure measurements

1. Theoretical advances

A summary of $n=1$ hfs calculations up to 1966 has been given in Sec. II [Eqs. (2.1a and 2.1b) and (2.7a–2.7c)] and in Fig. 3. Rewriting Eqs. (2.7a)–(2.7c) for completeness, we have $W=W_0+W_1+W_2+\cdots$, where in units of $\frac{1}{4}\alpha^4mc^2\simeq 87\,000$ MHz, $W_0=\frac{7}{3}$ and

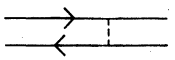
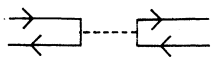
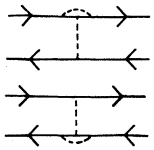
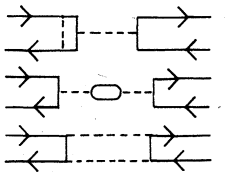
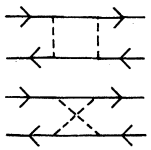
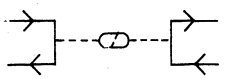
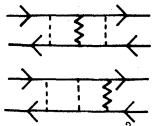
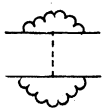
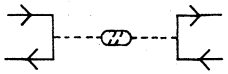
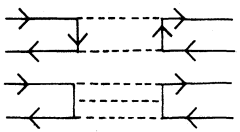
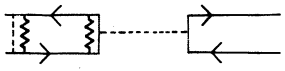
$$W_1=-\frac{\alpha}{\pi}\left(\frac{32}{9}+2\ln 2\right).$$

The term W_2 contains contributions of order α^2 and $\alpha^2\ln\alpha^{-1}$, i.e., contributions of order 5–20 MHz or (25–100) ppm of W_0 if the coefficients of α^2 and $\alpha^2\ln\alpha^{-1}$ are of order unity. The primary directions of theoretical research since 1967 have been (i) calculation of W_2 using essentially the techniques employed in the calculation of W_1 by Karplus and Klein and (ii) a new two-body formalism in which the Bethe-Salpeter (BS) equation is reduced to an equivalent Dirac equation (Lepage, 1977) or for essentially nonrelativistic problems (muonium or Ps) to an equivalent Schrödinger equation with reduced mass (Caswell and Lepage, 1978, 1979; Barbieri and Remiddi, 1978).

We first direct our attention to the current situation regarding calculation of W_2. This is summarized in Table II (Repko, 1978), in which we also include W_0 and W_1 for completeness. The first contributions to W_2 to be calculated were the recoil diagrams of order $\alpha^2\ln\alpha^{-1}$ (Fulton, Owen, and Repko, 1970, 1971). Next to be calculated were contributions, also of order $\alpha^2\ln\alpha^{-1}$, arising from the vacuum polarization correction to the annihilation diagram (Barbieri, Christillin, and Remiddi, 1973; Owen, 1973). We note that it was originally felt (Karplus and Klein, 1952) that three-photon diagrams of this type could only contribute terms of order α^2. Yet further contributions of a similar nature have recently been discovered (Barbieri and Remiddi, 1976; Lepage, 1977; Caswell and Lepage, 1978; Bodwin and Yennie, 1978). Since $\ln\alpha^{-1}=4.9$, terms of order $\alpha^2\ln\alpha^{-1}$ have a sizeable effect on W_2. The coefficient of $\alpha^2\ln\alpha^{-1}$ is currently $\frac{5}{6}$ (Table II), so that the total contribution of the order $\alpha^2\ln\alpha^{-1}$ diagrams to W is 19.1 MHz or 100 ppm! It is hoped that all corrections of this order have now been calculated. However, in view of the ever new contributions which keep coming to light, it is perhaps useful to quote the feelings of Barbieri and Remiddi, who state at the end of an article (Barbieri and Remiddi, 1976, p. 261) in which they discovered a new $\alpha^2\ln\alpha^{-1}$ term, "Even without a detailed scrutiny of the problem, recent history of the Ps ground state splitting calculation indicates that it is legitimate to ask the question whether equation 19 (their final result for the $\alpha^2\ln\alpha^{-1}$ terms) is the full story for the $\alpha^2\ln\alpha^{-1}$ terms." Since Lepage did indeed uncover new contributions within a year, the caution evidenced by Barbieri and Remiddi has already been borne out.

We now turn to the order α^2 corrections in W_2. The

TABLE II. Summary of PS hyperfine structure calculations (based on a similar table provided by Professor Repko).

Order	Static contribution Units of $\frac{1}{4}\alpha^4 m_0 c^2$	Annihilation contribution Units of $\frac{1}{4}\alpha^4 m_0 c^2$	Recoil contribution Units of $\frac{1}{4}\alpha^4 m_0 c^2$	Total contribution Units of $\frac{1}{4}\alpha^4 m_0 c^2$	Total contribution (MHz)
W_0	$\frac{4}{3}$	1	0	$\frac{7}{3}$	204 386.7
Typical Feynman diagram					
W_1	$\frac{4\alpha}{3\pi}$	$\frac{-2\alpha}{\pi}\left(\frac{13}{9}+\ln 2\right)$	$-\frac{2}{\pi}\alpha$	$\frac{-2\alpha}{\pi}\left(\frac{16}{9}+\ln 2\right)$	−1 005.5
Typical Feynman diagrams					
$W_2(\alpha^2 \ln \alpha^{-1})$	0	$\frac{1}{6}\alpha^2 \ln \alpha^{-1}$	$\frac{2}{3}\alpha^2 \ln \alpha^{-1}$	$\frac{5}{6}\alpha^2 \ln \alpha^{-1}$	19.1
Typical diagrams					
$W_2(\alpha^2)$	$-3.88\alpha^2$	$-0.6\alpha^2$ (Vac. Pol.)	$1.09\alpha^2$	$-0.64\alpha^2$	−4.5
Typical diagrams					
		$+2.62\alpha^2(2\gamma+3\gamma\,\text{ann})$ 			
		$0.09\alpha^2$ 			
$W = W_0 + W_1 + W_2$					203 395.8

first such correction to be calculated was the static term, $-3.88\alpha^2$ (Fulton, 1973). At about the same time α^2 contributions from the fourth-order vacuum polarization contribution to W_2 were also calculated (Barbieri *et al.* 1973; Samuel, 1974). A slight discrepancy (1 MHz) between the Barbieri and Samuel results still exists, and we have used the mean of the two calculations in quoting the coefficient of the vacuum polarization term and the final value of W.

A program to calculate the two-photon and the three-photon virtual annihilation contributions to order α^2 was begun in 1977 with the calculation of the three-photon terms (Cung *et al.*, 1977), with the result that such diagrams (Table II) contribute only -0.91 MHz to W_2. The two-photon calculation, completed in 1978 (Cung *et al.*, 1979), gave a much larger result, viz., 13.13 MHz! The total coefficient for the 2γ plus 3γ terms is $2.62\alpha^2$, as shown in Table II.

Finally, we turn to the recent work of Lepage (1977) and Caswell and Lepage (1978, 1979). Lepage has, in effect, introduced a new two-body bound state formalism into the theory. In this formalism the exact Bethe-Salpeter equation is reduced to either an equivalent Dirac equation (useful when the binding is relativistic as in high Z atoms) or to an equivalent Schrödinger equation useful for muonium or Ps [see also Barbieri and Remiddi (1978) for a similar formalism applied to Ps]. This is done by placing one of the two particles on the mass shell. The major advantages of this approach (paraphrasing the Lepage and Caswell-Lepage articles) are that the bound-state equation is essentially an equation for a single particle with reduced mass. In the nonrelativistic case the solutions of the zeroth-order problem are essentially the hydrogen atom wave functions—i.e., unlike the usual BS equation, an exact solution exists. The corrections to this first approximation are then readily given by a systematic perturbation theory approach. Other advantages to Lepage's formulation, of specific interest for Ps hfs calculations, are that the zeroth order wave functions are finite at the origin, so that the expectation value of the one-photon annihilation kernel is finite. In the BS formalism this kernel can be made finite only after a complex (infinite order in α) renormalization procedure. In fact, all renormalization related infinities can be removed order by order in Lepage's method in a manner identical to that used for treating on-shell amplitudes. This simplifies the analysis and numerical evaluation of high-order terms. Finally, Caswell and Lepage note that the free-particle spinor structure of the wave functions facilitates the use of computers for doing the large amount of algebra implicit in all of the order α^2 Ps hfs diagrams.

The first result of this new treatment (Lepage, 1977) was the reevaluation of recoil terms of order $\alpha^2 \ln\alpha^{-1}$ previously considered (Fulton, Owen, and Repko, 1971) but not fully evaluated because the above authors used one rather than two iterations of the BS equation. This new contribution is included in the $\frac{2}{3}\alpha^2 \ln\alpha^{-1}$ term in Table II. A further calculation using the new techniques yielded new annihilation terms of order $\alpha^2 \ln\alpha^{-1}$ as well as the first ($1.09\alpha^2$) recoil contribution to W_2 and also the $0.09\alpha^2$ addition to the vacuum polarization terms in the same order. Both of the latter terms involve infinite Coulomb exchange (Table II). In addition, a start has been made on order α^2 radiative corrections to one-photon annihilation (Buchmüller and Remiddi, 1980).

We conclude this brief summary of recent theoretical work on a hopeful note by paraphrasing Caswell and Lepage (1978, p. 816), who state in their conclusion, "... only terms in W_3 (order α^3) present a major conceptual problem. Evaluation of the remaining terms in W_2 is straightforward though perhaps tedious." It is now anticipated that calculation of W_2 may be completed in the near future [private communication, G. P. Lepage (1980)]. Since terms in W_3 (order α^3 and $\alpha^3 \ln^2(\alpha^{-1})$ are only of order 1 ppm of W (coefficient of $\alpha^3 \ln\alpha^{-1}$ taken as unity), a completed calculation of W_2 will allow an unambiguous test of Lepage's work at the level of the current experimental accuracy (6 ppm). More specifically, this would mean that typical coefficients of order $\alpha^2 \ln\alpha^{-1}$ in W_2 would be tested at the 5% level, while coefficients of order α^2 would be tested at the 25% level. This would certainly be an exciting and important test of the relativistic two-body formulation of quantum electrodynamics.

2. Experimental advances

a. Introductory remarks

Major improvements in the accuracy of measurements of W have recently been made by the Brandeis (Mills and Bearman, 1975) and Yale (Egan *et al.*, 1977) groups. Current accuracies in the 5 ppm regime have been reported. Both groups used essentially the same technique, observation of the rf perturbation of the 2γ and 3γ decay of $\psi_T(\pm 1)$ and ψ_T' as pioneered by Deutsch (see Sec. I.B for an introduction to this technique). Improvements over Deutsch's original measurements have come about largely as a result of greatly improved γ-detection efficiencies which permit large numbers (of order 10^{10}) of Ps decay events to be observed with reasonable run times. The major uncertainties in this measurement are associated with the extreme line splitting necessary (the natural line width is 6200 ppm) and with the effect of the background Ps-forming gas on the Ps itself. In this section we will outline the essential details of the experiment with major focus on the above uncertainties.

Our point of departure for discussion of the hfs measurement will be Eq. (2.4a)–(2.4f), Fig. 2, and Fig. 5 (which shows a schematic diagram of the experiment). As discussed in Sec. II.B, the essence of the experiment is to form Ps in a static magnetic field B and then to drive the transitions $\psi_T(\pm 1) \rightleftarrows \psi_T'$ by means of an rf magnetic field $B_y \cos 2\pi f t$ oriented perpendicular to B. The maximum in the resonance curve (not including decay effects) occurs, as expected, at the frequency [Eq. (2.6a)]

$$f_0 = \frac{(W_T' - W_T)}{h} = \frac{1}{2}\frac{W[(1+x^2)^{1/2} - 1]}{h} .$$

As previously discussed, W is then extracted from the above equation with μ and B assumed known ($x = 4\mu B/W$). The indirect method for determining W which we have just described is used because excitation of the direct triplet-singlet transition ($\nu = w/h$) is not

yet feasible, due to the need for a microwave field of about 10 G to drive the transition at $\nu = 203$ GHz ($\lambda = 1.5$ mm) directly and with about 50% probability. By contrast at fields typical of the Yale or Brandeis experiments ($B \sim 8-9$ kG, $f_0 \sim 2.5$ GHz, $\lambda_0 \sim 15$ cm), the 10 G microwave field necessary to drive the $\psi_T(\pm 1) \rightleftharpoons \psi_T'$ transition is easily obtainable.

A series of extremely complete and well organized descriptions of the Yale research have been published, the most recent being the Egan *et al.* (1977) reference. The Brandeis work has not been written up in as much detail in journal articles; however, it does parallel the Yale research in many respects. Since the documentation on the Yale work is both up-to-date and readily accessible, we will present only a brief description of the specific experimental technique used and then proceed to a discussion of systematic errors and a projection of the accuracy that may be obtained in the future.

b. Description of the rf quenching experiments

The experimental apparatus used in the Yale work is shown in Figs. 5 and 6 (Carlson *et al.*, 1977). Referring to Fig. 6, we see that positrons are emitted from a 2.5 mCi source of ^{22}Na (5–10 mCi were used by Egan *et al.*) placed in a microwave cavity which is itself located in the 7.9 kG magnetic field of a precision shimmed electromagnet. The cavity contains a Ps-forming gas such as N_2, He, etc. About 50% of the emitted positrons form Ps in either the triplet or singlet states. The quantity which is directly observed is the number of antiparallel two-photon events detected by the various oppositely situated RCA 4524 photo tube-NaI scintillator combinations shown in Fig. 5. The procedure for observing resonance is to keep the microwave frequency fixed and to vary the main field B. This method is used because it is more difficult technically to accurately measure and control the microwave power as frequency is varied than it is to measure the average magnetic field at each of a series of values spanning a few hundred gauss near 8 kG. The effect of passage through resonance is to increase the two-photon coincidence rate by about 10% over the background (Fig. 7). The fact that the 2γ rate should increase at resonance, the approximate linewidth, etc., has been discussed qualitatively in Sec. II.B, while a complete quantitative discussion may be found in the references. The remainder of this section will therefore be a discussion of the major errors present in the measurement.

c. Principle errors in the rf quenching experiments

i. Lineshape theory—statistical error. Under the data-taking conditions just described, the observed 2γ counting rate will be proportional to a quantity S which may be written (schematically) as

$$S = P_1 + P_{-1} + P_0 . \tag{4.1}$$

Here P_m is the probability of two-photon annihilation from o-Ps, which is initially in the mth magnetic substate. Thus P_1 and P_{-1} refer to the transitions $\psi_T(\pm 1) \rightarrow \psi_T' \rightarrow 2\gamma$, while P_0 refers to the change in the 2γ decay of ψ_T' due both to the slight change in λ_T' as B varies (magnetic quenching) and to the loss in 2γ decays from the rf induced process $\psi_T' \rightarrow \psi_T(\pm 1) \rightarrow 3\gamma$. As B is increased by approximately 200 G through the resonance line (Fig. 7), $P_1 \simeq P_{-1}$ varies from 0 to 0.25 for the typical experimental parameters discussed. On the other hand, P_0 remains relatively constant, since the transition $\psi_T' \rightarrow \psi_T(\pm 1) \rightarrow 3\gamma$ which reduces the 2γ signal has a probability $\lambda_T'/\lambda_T \sim 0.07$ of that for $\psi_T(\pm 1) \rightarrow \psi_T'$. At the same time, P_0 increases by only 0.1% due to magnetic quenching, i.e., the fraction $[\lambda_T'(2\gamma)/\lambda_T']$ of 2γ decays from ψ_T' changes by about this amount as B varies from 7700 to 7900 G (see the slope in the background in Fig. 7).

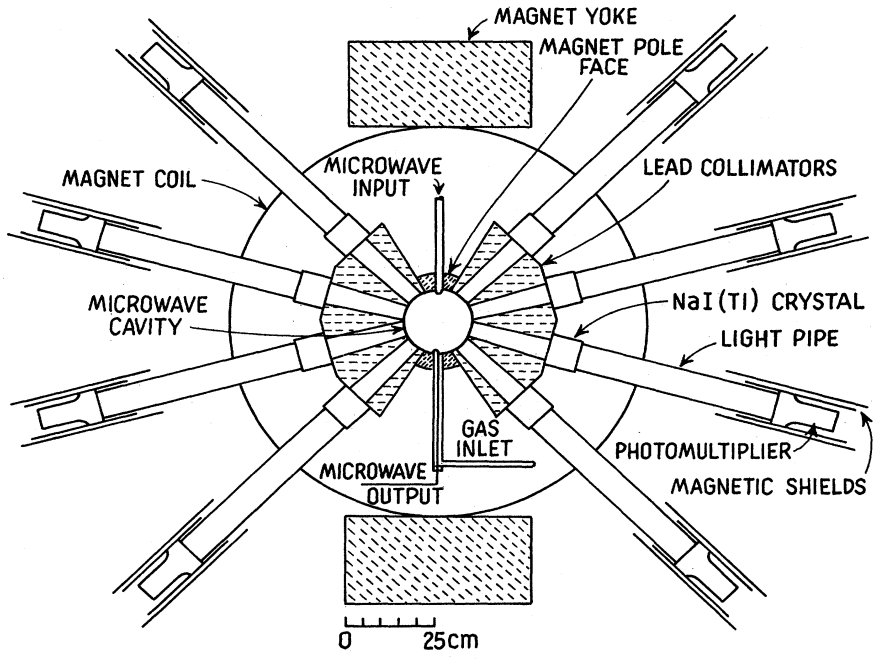


FIG. 5. The Yale Ps hyperfine structure apparatus—overall view.

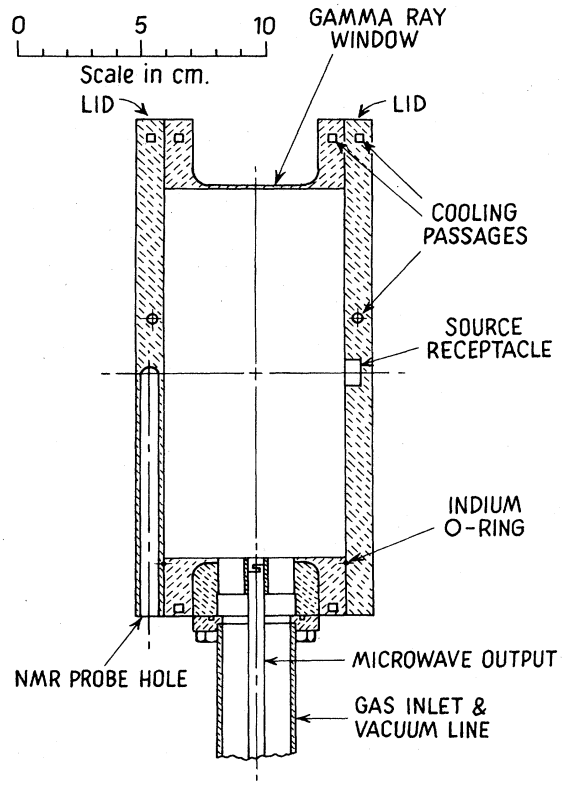


FIG. 6. The Yale Ps hyperfine structure experiment—microwave cavity—side view.

The quantities $P_m(x)$ are evaluated by solving the time-dependent equations for the amplitudes $a_K(t)$ of the o-Ps sublevels:

$$i\hbar\dot{a}_K(t) = \sum_{n=1}^{4} a_n H'_{kn}(t) e^{-i(\omega_n-\omega_k)t} - \left(\frac{i\hbar\lambda_k a_k}{2}\right). \tag{4.2}$$

Equation (4.2) is the usual time-dependent representation of the Schrödinger equation obtained from $i\hbar\dot{\chi} = H\chi$, with

$$\chi(t) = \sum_{k=1}^{4} a_k(t)\psi_k e^{-i\omega_k t} \quad (\psi_{1,2} = \psi_T(\pm 1),\ \psi_3 = \psi'_T,\ \psi_4 = \psi'_S)$$

and $H = H_0 + H'(t)$. Here H_0 is the full time-independent Hamiltonian for Ps in a magnetic field, which we have discussed up to this point, and

$$H'(t) = \frac{g'}{2}\mu_0 B_y[\sigma_y(e^-) - \sigma_y(e^+)]\cos\omega t\,. \tag{4.3}$$

$H'(t)$ represents the interaction Hamiltonian of the rf magnetic field with Ps [see Eq. (2.4) for the notation used here], and its matrix element $H'_{kn}(t)$ is $\langle\psi_k|H'(t)|\psi_n\rangle$. The term $i\hbar\lambda_k a_k/2$ accounts for the annihilation of the state ψ_k. The quantities P_m are given by

$$P_1 = \int_0^\infty |a_1(t)|^2 \lambda'_T(2\gamma)dt\,, \tag{4.4a}$$

$$P_{-1} = \int_0^\infty |a_2(t)|^2 \lambda'_T(2\gamma)dt\,, \tag{4.4b}$$

$$P_0 = \int_0^\infty |a_3(t)|^2 \lambda'_T(2\gamma)dt\,. \tag{4.4c}$$

These expressions follow from the fact that, e.g., $|a_k(t)|^2$ is the probability of finding Ps initially in state χ in ψ_k at time t where it has a 2γ decay rate $\psi'_T(2\gamma)$.

The solution of Eq. (4.2) for the $a_k(t)$ was discussed first by Halpern (1954) and, in more detail, by Hughes *et al.* (1957) and Theriot *et al.* (1970). The approximations made in solving the set of four coupled equations are (i) neglect of the singlet state ($k=4$), (ii) neglect of nonresonant terms, viz., $e^{-i\omega t}$ in the expansion $\cos\omega t = \frac{1}{2}(e^{i\omega t} + e^{-i\omega t})$, and (iii) neglect of positron polarization effects, since negligible shifts in the line center occur if one ignores the fact that the ratio of positrons which form the ψ'_T state to those which form the $\psi_T(+1)$ or $\psi_T(-1)$ states is actually $1 - [xP/(1+x^2)^{1/2}]$, where P is the residual polarization on Ps formation of the initially polarized positrons. This approximation is implicit in Eq. (4.1), in which the relative populations of the o-Ps substates are obviously taken as equal. The function S determined using the above approximations is

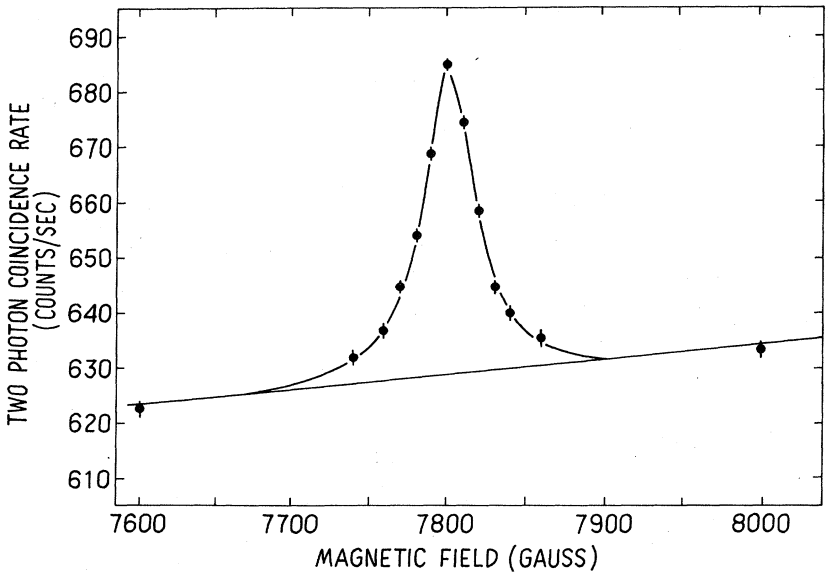


FIG. 7. Yale Ps hyperfine structure resonance data. The value of $(\omega_0/2\pi)$ extracted from this data is 2.323 364 GHz. The increase of the background with field is discussed in the text.

$$S=\frac{\lambda_T'(2\gamma)}{4\lambda_T'}+\left[\frac{\lambda_T'(2\gamma)}{\lambda_T'}(\lambda_T+\lambda_T')\frac{V^2}{\hbar}\left(\frac{1}{2\lambda_T}-\frac{1}{\lambda_T'}\right)\frac{1}{(\omega-\omega_0)^2+D^2}\right]. \tag{4.5}$$

Here V, the time-independent part of H_{kn}', is given by

$$V=\frac{H_{kn}'}{2\cos\omega t}\underset{x\ll1}{\simeq}\frac{xg'\mu_B B}{4\sqrt{2}}[1+O(x^2)],$$

and we note that $H_{13}'=H_{23}'$ is the only nonzero matrix element. D is given by

$$D^2=\tfrac{1}{4}(\lambda_T+\lambda_T')^2+[2(\lambda_T+\lambda_T')^2V^2/\lambda_T'\lambda_T\hbar^2].$$

The exact lineshape given by Eq. (4.5) is slightly non-Lorentzian, but it is symmetric in B, and the frequency at the line center has been identified as f_0. In the limit $x\ll1$ the quantity $S'\equiv S-[\lambda_T'(2\gamma)/\lambda_T']$ may be written explicitly as a power broadened Lorentzian in the static field B, i.e.,

$$S'=\frac{A\,B_y^2}{(B-B_0)^2+E^2}. \tag{4.6}$$

Here B_0 is the value of B on resonance, $A=\pi/8(\lambda_S/\lambda_T)\times(\omega_0/\omega)\simeq 4.5$ and $E^2=\frac{1}{4}(\delta B)^2+aB_y^2$ with $\delta B/B_0\simeq\lambda_S/4\pi\nu\approx 3\times10^{-3}$ [Eq. (2.6c)] and a a parameter due to power broadening which is determined from a fit to the linewidth. Thus at the peak of the line ($B=B_0$) and neglecting power broadening ($\delta B\simeq 3\times10^{-3}B_0$), $S'\simeq 2\times10^6(B_y/B_0)^2$. Since rf fields of order 10 G are readily obtained at $f_0\sim2$ GHz, one may cause $S'\sim1$. Thus the transition may, in fact, be saturated, and line broadening will occur. In practice, B_0 is obtained from a fit of expression (4.5) to data of the type shown in Fig. 7. The statistical error associated with the determination of B_0 was 2.5 ppm in the latest Yale measurement. This leads to an error of ±5 ppm in the result for W [Eq. (2.6a)], due to the fact that for $x\ll1$, W is approximately proportional to B_0^2.

We note here that as was remarked in Sec. II.B, Eq. (2.6a) is correct only if decay is neglected in diagonalizing the energy matrix. This point (implicit in the 1954 calculation of Halpern) has been mentioned by Hughes *et al.* (1957), who observed that shifts in W_T' of order $(\lambda_S/2\pi\nu)^2\simeq 40$ ppm might occur. This is, in fact, correct; and the exact shift may be readily estimated by diagonalizing the usual energy submatrix, but including decay for Ps in a static field B:

$$H_d=\begin{pmatrix} w_S & p\\ p & w_T\end{pmatrix}. \tag{4.7}$$

Here $w_S=-i\lambda_S/2$, $w_T=W_T-i\lambda_T/2$, and $p=\langle\psi_S|H_M|\psi_T\rangle=\langle\psi_T|H_M|\psi_S\rangle=-2\mu B_0$. The (complex) eigenvalues of H_d are (Rich, 1965)

$$\mathcal{W}_T'(d)=\tfrac{1}{2}[(w_S+w_T)+\delta(1+Z^2)^{1/2}], \tag{4.8a}$$

$$\mathcal{W}_S'(d)=\tfrac{1}{2}[(w_S+w_T)-\delta(1+Z^2)^{1/2}], \tag{4.8b}$$

where $\delta\equiv w_T-w_S$ and $Z\equiv 2p/\delta$. The real and imaginary parts of $\mathcal{W}_T'(d)$, $\mathcal{W}_S'(d)$ yield exact expressions for W_T', W_S' and λ_T', λ_S', respectively. If we define the constant g as $g=(\lambda_S-\lambda_T)/4\pi\nu\simeq 3\times10^{-3}$, then

$$\mathrm{Re}\mathcal{W}_T'(d)=W_T'(d)=\frac{1}{2}\left[W+\left(\frac{\alpha+(\alpha^2+\beta^2)^{1/2}}{2}\right)^{1/2}\right], \tag{4.9}$$

where $\alpha=W^2[(1-g^2)+x^2]$ and $\beta=2gW^2$. Expanding $W_T'(d)$ to first order in x yields

$$W_T'(d)\underset{x\ll1}{\simeq}W\left(1+\frac{1}{4}\frac{x^2}{1+g^2}\right). \tag{4.10}$$

This expression differs from the expansion of Eq. (2.4c) in the factor $(1+g^2)^{-1}$, which should result in an increase in the value quoted for W by 9 ppm (2 MHz). This result for the change in the eigenvalues of the static problem does not necessarily mean that the true value of W should, in fact, be shifted by 9 ppm, since inclusion of decay in the static case may also cause a change in the lineshape (and a shift in the line center or peak) for the driven (rf-on) situation. A reexamination of the resonance lineshpae is clearly required at the current levels of experimental accuracy, particularly in view of the possibility that the order α^2 terms in W may be calculated in the near future so that comparison of theory and experiment at the few ppm level will be possible.

ii. Ps hfs pressure shifts. Collisions of o-Ps with the background gas in which it is formed are expected to produce both a broadening of the resonance line and a shift in the line center. The broadening occurs because for the finite x ($x\sim\frac{1}{4}$) used in the experiments the expression (2.6b) for the linewidth [$\delta B/B\simeq(\lambda_T'+\lambda_T)/2\omega_0$] will clearly increase if λ_T and λ_T' are increased by collisional pickoff. Since $\lambda_T'\sim60\lambda_T$ and λ_T are increased by less than five per cent because of collisions at the typical pressures of order one atmosphere of N_2 used in the hfs determinations, the increase in $\delta B/B$ is negligible.

The shift in the line center due to collisions (hyperfine pressure shift or HPS) is, however, not negligible. As in hydrogen and other gases, the shift in W due to collisions may be considered as due to exchange effects at short range (typical contribution about 10% of the shift) and the long-range van der Waals interaction, which contributes the remaining 90% of the HPS. Since the van der Waals force is attractive, it increases the electron-positron separation, thereby reducing W. The magnitude of the HPS, first observed by Theriot *et al.* (1970) may be expressed in terms of a fractional density shift

$$a=\frac{1}{\nu}\left(\frac{\partial\nu}{\partial D}\right),$$

where D is the pressure of the particular gas in which Ps is formed. The value of a for N_2 is $a(N_2)=(-3.2\pm0.5)\times10^{-5}$/atm at $T=0$ °C, while for noble gases it ranges from $+1.6\times10^{-5}$/atm for He to -7.3×10^{-5}/atm for Xe (Bearman, 1975; Bearman and Mills, 1976; Egan *et al.*, 1977). According to the argument presented above, a negative value of a is expected if the van der Waals interaction dominates. Only data acquired for Ps in N_2 were used in determining W and these data are shown in Fig. 8. As can be seen, the data points at lowest pressure (0.25 atm) are within just a few standard deviations of the intercept and thus constitute but a minimal extrapolation. There is no evidence for a quadratic pressure (density) dependence. A careful theoretical analysis of the HPS for Ps (Bear-

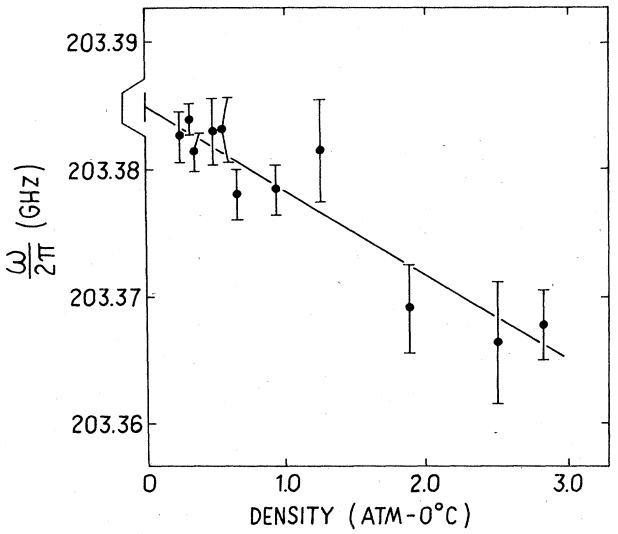


FIG. 8. Data for ω versus nitrogen density in the Yale hfs experiment. The solid line is a best fit to the data assuming a linear density dependence.

man, 1975; Bearman and Mills, 1976) yielded the results $a(N_2) = (-8.5 \pm 2.5) \times 10^{-5}$/atm in qualitative agreement with experiment. Roughly similar agreement (i.e., to within a factor of 2) was also obtained for the noble gases, with the exception of He, for which the theoretical result was $(-1.9 \pm 0.6) \times 10^{-5}$/atm.

iii. Magnetic field inhomogeneity and other magnetic field associated effects. Magnetic effects are not specifically associated with the properties of Ps; consequently, we will not discuss them in detail, but simply note that all magnetic uncertainties contribute an rms error of 3.3 ppm to W. The major part (2.4 ppm) of this error is associated with an rms uncertainty of 1.2 ppm in the value of B over the region from which the Ps annihilates. The error in W is twice the error in B, since $W \sim B^2$. The other field-associated uncertainties arise from NMR probe susceptibility, diamagnetic shielding corrections (1.6 ppm), and magnetic field offset and field reproducibility effects (1.4 ppm).

d. Conclusions and projections of future accuracy in the measurement of W

The final result of the most recent 6 ppm Yale experiment (Egan *et al.*, 1977) was $f = (203.384 \pm 0.0012)$ GHz, with component errors as discussed in the preceding section. This result is in excellent agreement with the work of the Brandeis group (Mills and Bearman, 1975), who obtained the result $f = (203.3870 \pm 0.0016)$ GHz. Both of these results should nominally be increased by 0.002 GHz, as discussed in Sec. IV.A.2.c. Although I have chosen to concentrate on the Yale experiment in our description, I should like to emphasize here that the Brandeis result is of approximately equal accuracy. Since the two experiments use essentially the same technique, they are not completely independent checks from the viewpoint of all systematic errors and assumptions. For example, both experiments apparently use the same form for the signal lineshape, a form which neglects the various effects mentioned in Sec. IV.A.2.c.

The major systematic difference between the experiments is that the Brandeis work uses SF_6 as the Ps-forming gas from which the majority of the data are derived, as opposed to N_2 in the Yale experiment. The agreement of the two results indicates that the gas extrapolation procedure is reliable. The other less important, though interesting, differences between the experiments were the magnetic fields used (7.9 kG—Yale, 9.25 kG—Brandeis) and the fact that in the Brandeis research the first difference signal (essentially the derivative of S with respect to B) was used in the analysis. This difference in the form of the signal used might reveal problems present in the line-fitting procedure. It would not be sensitive to differences in the form of S, since, as noted earlier, both groups use the same functional form for S. In any case, however, agreement of the two experiments at the 5–10 ppm level certainly constitutes corroboration of the validity of each.

We now turn to an analysis of the precision which may be possible in future measurements of W. From the discussion of this section we can readily identify counting statistics as the major source of error in the latest experiments. The Yale group has stated that counting rate increases of a factor four-five are feasible through increases in both source strength and detector solid angle; and, in addition, they feel that the already small magnetic field uncertainties can be further reduced. Thus they contemplate fairly straightforward improvements in the current technique leading to a precision of perhaps 3 ppm in the near future (Hughes, 1980, and private communication).

An alternative technique to the one that has been used in the past may also be feasible. Briefly stated, this would involve the use of a Ps formation $t = 0$ signal obtained as usual from either a nuclear γ-ray coincident with positron emission (e.g., the 1.3 MeV γ ray in ^{22}Na) or from the positron's energy loss in passing through a thin scintillator [a technique which the Michigan group has perfected, using ^{68}Ge as a source—see Sec. IV.B.2 or Gidley *et al.* (1978)] or by use of beam bunching (Mills, 1980). The signal in the proposed new experiment could be obtained in either of two ways. The first would be to observe the increase at resonance in the two-photon decays from ψ_T' which appear in a time window of width several $\tau_T'[\tau_T' \equiv (\lambda_T')^{-1} \sim 10$ nsec at $B \sim 8$ kG] and whose initial time is set to exclude most events within the prompt resolution curve. The signal-to-noise ratio (currently about 1:10—Fig. 7) would be greatly improved by such a scheme, since most 2γ events are prompt and are due to $1^1S_0 \rightarrow 2\gamma$ or direct annihilation. The signal would be only slightly decreased. Nanosecond coincidence timing under these conditions, using NaI detectors similar in size to those used in previous hfs experiments should be feasible, as demonstrated in our laboratory (Gidley, 1979). The second and more powerful method would involve observation of a signal produced by the decrease in delayed 3γ decays from $\psi_T(\pm 1)$ at resonance. The utility of this method lies in the fact that if one only accepts γ's with a time delay greater than several τ_T', the back-

ground should be negligible, and, in addition, only one of the three γ's from the ψ_T decay need be detected. Thus the overall signal may be significantly improved. In addition, line-narrowing techniques based on detection of γ's emitted at times delayed from the Ps-formation signal might be feasible. Finally, in any future hfs experiments the possibility of forming Ps in a vacuum (Sec. III.C) can be considered so that the extrapolation to zero gas pressure may be dispensed with.

B. Decay rate measurements

1. The 1^1S_0 decay rate—theoretical advances

Theoretical calculations of the singlet decay rate, including the first-order radiative corrections by Harris and Brown (HB) (1957), were discussed in Sec. II B. The (HB) results were

$$\lambda(1^1S_0)=\lambda_0(1^1S_0)+\lambda_1(1^1S_0)+\lambda_2(1^1S_0)+\cdots,$$

with $\lambda_0=(\alpha^5/2)\,(mc^2/\hbar)\simeq 8\times 10^9\ \text{sec}^{-1}$ and the radiative corrections λ_1 and λ_2 given as $\lambda_1=-\lambda_0(\alpha/\pi)(5-\frac{1}{4}\pi^2)$ $\simeq -5\times 10^{-3}\lambda_0$ and $\lambda_2\sim O(\alpha^2,\ \alpha\ln\alpha^{-1})$. The Feynman diagrams representing the processes responsible for λ_0 and λ_1 are shown in Fig. 9.

Recent confirmation of the 1957 calculations has been provided by Cung *et al.* (1978), who performed an analytical calculation of radiative corrections to the two-photon virtual annihilation diagrams for the $n=1$ hyperfine interval (Sec. IV.A.1). The imaginary part of their expression should yield $-\frac{1}{2}\lambda_1$ and, in fact, their result is in complete agreement with the direct decay rate calculation of (HB). Further checks of the 1957 result using somewhat different approaches have been performed by Freeling (1979) and by Tomozawa (1979). Both verify the (HB) result; and, in addition, Caswell and Lepage (1979) have calculated the coefficient of the $\alpha^2\ln\alpha^{-1}$ term to be $+\frac{2}{3}$ so that we may finally write

$$\lambda_2=[+\tfrac{2}{3}\alpha^2\ln\alpha^{-1}+O(\alpha^2)]\lambda_0\simeq 10^{-4}\lambda_0$$

and $\lambda(1^1S_0)=(0.798\,42\pm 0.0001)\times 10^{10}\ \text{sec}^{-1}$. The error represents the uncertainty in $\lambda(1^1S_0)$ caused by uncalculated terms of order α^2, assuming the coefficient of these terms to be of order unity.

2. The 1^1S_0 decay rate—experimental advances

The singlet lifetime $\tau(1^1S_0)=\lambda^{-1}(1^1S_0)$ is only 0.1 nsec; consequently, its direct measurement to the precision necessary in order to test the first-order radiative corrections $[\lambda_1(1^1S_0)]$ is precluded for technical reasons, reasons primarily associated with timing jitter in photomultipliers and with the one-to-two nanosecond decay times characteristic of plastic scintillator. An indirect method for measuring $\lambda(1^1S_0)$ has, however, been employed by the Yale group (Theirot *et al.*, 1970) and a completely different, though also indirect, experiment to measure $\lambda(1^1S_0)$ is now under way at Michigan.

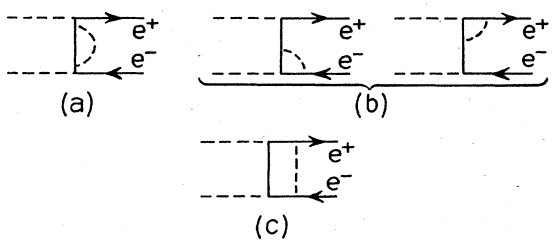


FIG. 9. Feynman diagrams representing order α radiative corrections to $\lambda(1^1S_0)$. Infrared divergent terms have been omitted, and the Feynman gauge has been used throughout in the expressions for $\lambda_{1a}-\lambda_{1c}$. (a) Self-energy correction $\lambda_{1a}=(\alpha/2\pi)(1+4\ln 2)\lambda_0(1^1S_0)$. (b) Vertex corrections λ_{1b} $=(\alpha/2\pi)(-4+\frac{1}{4}\pi^2-4\ln 2)\lambda_0(1^1S_0)$. (c) Binding diagram λ_{1c} $=(\alpha/2\pi)(-2)\lambda_0(1^1S_0)$.

The Yale measurement is an offshoot of the hfs experiment and may be understood by referring to Eq. (4.6) ($S'=AB_y^2/[(B-B_0)^2+E^2]$ with $E^2=\frac{1}{4}(\delta B)^2+aB_y^2$ and $\delta B/B_0\simeq(\lambda_T'+\lambda_T)/2\omega_0$). The FWHM ($\delta B$) of the natural (non-power-broadened) line shape may be obtained by measuring the peak in the resonance line $[S'(B=B_0)$ $=S_0'=AB_y^2/E^2]$ at various values of B_y and using the relation (obtained from substituting $B_y^2=S_0'E^2/A$ into the expression for E^2):

$$\frac{1}{E^2}=\left(\frac{-4a}{A(\delta B)^2}\right)S_0'+\frac{4}{(\delta B)^2}\ . \tag{4.11}$$

A plot of $1/E^2$ vs S_0' should then yield a straight line with intercept (at $S_0'=0$) of $4/(\delta B)^2$ so that δB may be determined. An implicit expression for λ_S then follows from the relation $\delta B/B_0=(\lambda_T'+\lambda_T)/2\omega_0$ if λ_T is known to reasonable accuracy, since $\lambda_T'=(1+y^2)^{-1}(\lambda_T+y^2\lambda_S)$. The result for λ_S from this procedure was $\lambda_S=(0.799\pm 0.01)$ $\times 10^{10}\ \text{sec}^{-1}$. This is in agreement with the theoretical value; however, the one-percent accuracy achieved is not sufficient to test the order α corrections.

An entirely new approach to the measurement of $\lambda(1^1S_0)$ was begun at Michigan by Gidley, Rich, and West in 1978. Briefly stated, we are now forming Ps in gases such as isobutane situated in a magnetic field B of 4.10 kG. The positron from a ^{68}Ge source, after appropriate collimation and energy selection, passes through a thin plastic scintillator to produce a $t=0$ (start) signal and form Ps in the gas. The annihilation γ rays which constitute the stop signal are also detected using a plastic scintillator. A time-to-amplitude-converter–multichannel-analyzer (TAC–MCA) timing system (discussed in Sec. IV.B.4) permits us to record the time spectrum and thereby extract directly the quantities λ_T and λ_T'. Referring to Eq. (2.4e) $[\lambda_T'$ $=(1+y^2)^{-1}(\lambda_T+y^2\lambda_S)]$, we see that knowledge of λ_T', λ_T and $B[y=y(B)]$ will permit a determination of λ_S. Analysis of the statistical and systematic errors associated with the proposed measurement, as well as preliminary data obtained in our initial runs, leads us to believe that a precision of order 0.2% in λ_S should be feasible. Such accuracy should be sufficient for a verification of the $\lambda_1(1^1S_0)$ term. Preliminary results of this work are now available and they show agreement with theory at the 0.5% level, thus verifying the approximate magnitude of $\lambda_1(1^1S_0)$.

3. The 1^3S_1 decay rate—theoretical advances

There has been significant theoretical progress in calculation of $\lambda(1^3S_1)$ since 1967 and particularly during the past few years. Using the notation adopted for the singlet state, we write

$$\lambda(1^3S_1)=\lambda_0(1^3S_1)+\lambda_1(1^3S_1)+\lambda_2(1^3S_1)+\cdots\cdot \tag{4.12}$$

The value of λ_0 (the 1^3S_1 notation will be dropped for the remainder of Secs. IV.B.3 and IV.B.4) as calculated by Ore and Powell in 1949 [$\lambda_0 = 2\alpha^6(mc^2/\hbar)(\pi^2 - 9)/9\pi = 7.211\times10^6$ sec^{-1} was discussed in Sec. II.A]. The first-order radiative corrections (Fig. 10) were, however, not completed until 1974 when Stroscio (1974) and Stroscio and Holt (1974) reported the result $\lambda_1 = (1.8 \pm 0.6)(\alpha/\pi)\lambda_0 \simeq 4\times10^{-3}\lambda_0$ [some preliminary results may also be found in Holt (1970)]. This result was further refined by Stroscio (1975), who reported $\lambda_1 = (1.86 \pm 0.45)(\alpha/\pi)\lambda_0$ or $\lambda = \lambda_0 + \lambda_1 = (7.242 \pm 0.008)$ μsec^{-1}. The error quoted above was primarily due to uncertainty in numerical evaluation of integrals. The importance of verifying this result may be understood when we note that it was and still is the only case for which the predicted radiative corrections to a decay rate can be verified experimentally.

The value of λ presented above was in good agreement with the (two) experiments performed up to that time which gave the results $\lambda = (7.262 \pm 0.015)$ μsec^{-1} (Coleman and Griffith, 1973) and $\lambda = (7.275 \pm 0.015)$ μsec^{-1} (Beers and Hughes, 1968; Hughes, 1973) for Ps decaying in background gases at pressures of 1–10 atm. It was, however, in marked disagreement with experiments at Michigan, which yielded the values $\lambda = 7.104 \pm 0.006$ μsec^{-1} for Ps decaying in SiO_2 powders (Gidley *et al.*, 1976) and $\lambda = (7.09 \pm 0.02)$ μsec^{-1} for Ps decaying essentially in a vacuum (Gidley *et al.*, 1977). These experiments will be discussed in the next subsection; however, we note here that they caused the SLAC group already mentioned in connection with recent advances in the hyperfine structure calculation (Sec. IV.A.1) to reexamine and extend the Yale work using their new techniques. The result of their initial attack on the problem was the revised value $\lambda_1 = -(10.35 \pm 0.07)(\alpha/\pi)\lambda_0$ or $\lambda = (7.0379 \pm 0.001)$ μsec^{-1} (Caswell *et al.*, 1977). The change in λ_1 from its previous value of ±1.86 was primarily due to a sign error in one of the integrals which comprise diagram (a) in Fig. 10. In addition to this specific error, Caswell *et al.*, (1977) pointed out that relativistic corrections from the wave function and propagators of diagram (d) (Fig. 1), the contribution which gives λ_0, can cause corrections of order α in λ. This contradicts statements made in previous work (Stroscio and Holt, 1974; Stroscio, 1974) to the effect that, to order α, only constant and linear terms in the momentum **p** need be retained in an expansion of the plane-wave matrix element in powers of **p**, since for Ps, $v/c \sim O(\alpha)$. In spite of this statement by the above authors, however, they did in fact correctly take account of the relativistic effects in their calculations (but not including the sign error).

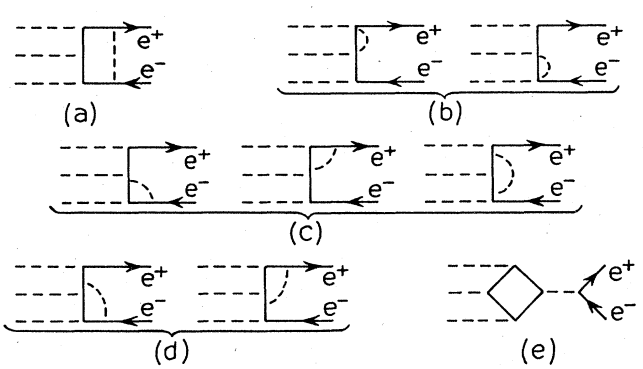


FIG. 10. Feynman diagrams representing order α radiative corrections to $\lambda(1^3S_1)$. Infrared divergent terms and the Coulomb correction term have been omitted and the Feynman gauge is used in the expressions for $\lambda_{1a}\cdots\lambda_{1e}$. (a) Photon (transverse plus Coulomb) exchange $\lambda_{1a} = \lambda_0 - (7.90 \pm 0.07)(\alpha/\pi)\lambda_0(1^3S_1)$. (b) Self-energy $\lambda_{1b} = (4.791 \pm 0.003)(\alpha/\pi)\lambda_0$. (c) Vertex corrections $\lambda_{1c} = (-2.868 \pm 0.003)(\alpha/\pi)\lambda_0$. (d) Double vertex correction $\lambda_{1d} = (-3.562 \pm 0.004)(\alpha/\pi)\lambda_0$. (e) Virtual annihilation $\lambda_{1e} = (-0.809 \pm 0.004)(\alpha/\pi)\lambda_0$.

The initial work by Caswell *et al.* was followed by a series of more accurate calculations of λ_1 and an extension of the research to a calculation of all $\alpha^2\ln\alpha^{-1}$ terms present in λ_2. The latest results (Caswell and Lepage, 1979) for λ_1 and λ_2 are $\lambda_1 = -(10.266 \pm 0.011)(\alpha/\pi)\lambda_0 = -(0.1720 \pm 0.000\,16)$ μsec^{-1} and $\lambda_2 = -(\frac{1}{3}\alpha^2\ln\alpha^{-1})\lambda_0 = (-0.000\,63$ μsec$^{-1})$, so that $\lambda = (7.0386 \pm 0.000\,16)$ μsec^{-1}. The uncertainty quoted above is due to the uncertainty in λ_1. The still-to-be-calculated $(\alpha/\pi)^2$ term in λ_2, i.e., $\lambda_2 = [-\frac{1}{3}\alpha^2\ln\alpha^{-1} + O(\alpha^2/\pi^2)]\lambda_0$ makes a contribution of about (0.000 04) μsec^{-1} if its coefficient is unity. Thus, for unity coefficient it contributes about one-fourth of the error in λ as is due to error in λ_1. It is therefore of interest to calculate the $O(\alpha^2/\pi^2)$ term as opposed to a further refinement of the λ_1 calculation. Although one cannot speculate quantitatively on the possibility of the coefficient exceeding four, we note that the coefficient of (α/π) for $\lambda(1^3S_1)$ is 10! Thus radiative corrections can lead to large coefficients, and it would be of interest to establish, even crudely, the size of the $(\alpha/\pi)^2$ term. This is particulary true in view of experiments now under way (see the following section) which may be able to detect the change in λ due to an $(\alpha/\pi)^2$ coefficient of order 20.

4. The 1^3S_1 decay rate—experimental advances

The first measurement of $\lambda(1^3S_1)$ (hereafter in this section λ) of accuracy sufficient to test the order α radiative corrections was performed by Beers and Hughes (Beers and Hughes, 1968; Hughes, 1973). The technique they used is still employed, though with numerous technical modifications, in some of the most recent experiments. Consequently, we will discuss the early work in some detail.

Essentially, high energy positrons emitted from a ^{22}Na source were slowed down in a gas (Freon-12 or a Freon 12-Argon mixture), where they formed Ps with about 40% probability (Beers, 1968), as described in Sec. III. The gas chamber used was surrounded with four Pilot B scintillator-photomultiplier detectors (Fig. 11) in which the decay of 1^3S_1 Ps was observed by detecting one of the three emitted annihilation photons. This constituted a stop signal for the timing system, the start signal being the detection of the 1.28 MeV gamma ray which accompanies each positron emitted by ^{22}Na with a time delay of about 10^{-4} nsec. The time interval between the start and stop signals was measured by using either the familiar time-to-amplitude converter–multichannel analyzer (TAC–MCA) system

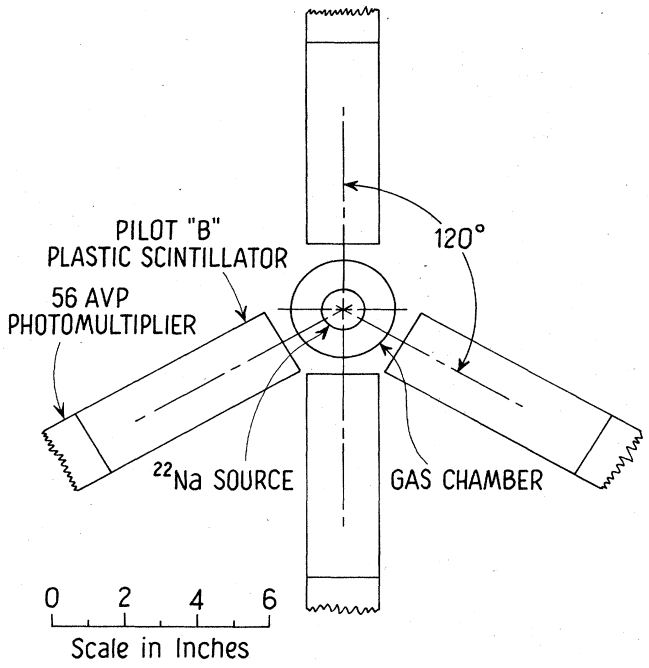


FIG. 11. Schematic of the Yale 1^3S_1 decay rate experiment. The height of the stainless-steel gas chamber is 2 inches and the wall thickness 0.1 inches.

or by use of a digitron, a device which counts external oscillator pulses during the start-stop interval. The digitron method was eventually adopted in this experiment because the TAC–MCA systems available at the time were susceptible to systematic timing drifts greater than the desired error in the final experimental result. Schematic diagrams of both types of timing systems as they are currently used are shown in Figs. 12(a) and 12(b).

The errors introduced by systematic effects involving drifts, nonlinearities and absolute calibration of the timing systems are somewhat smaller than other systematic effects present (particularly in the earlier experiments); consequently, we will not discuss the specifics of these systems in the article [see Griffith and Heyland (1977) and Gidley (1979) for detailed discussions of precision timing techniques].

The raw data obtained in the Yale experiment, after background correction, is shown in Fig. 13. The signals-to-noise ratio at the data analysis start time (A/B), one of several figures of merit for the quality of data in a decay rate experiment, was about 1.3. The decay rate itself is obtained by fitting a function of the form

$$N=[N_0e^{-\lambda t}+B] \tag{4.13}$$

to the data under consideration (B is the background and is assumed constant). The value of χ^2 for the above hypothesis (single exponential decay) was reasonable.

We note here that several interesting corrections to the basic exponential decay spectrum must be considered in experiments of the type described above. These are due to the fact that stop signals occur for which there are no correlated starts (due to the cosmic ray and annihilation gamma, etc., background); start signals occur for which there are no correlated stops, etc. The perturbation that these effects cause in the decay spectrum was first analyzed by Lundy (1962) and has since been considered by Beers (1968), Coleman *et al.* (1974), and most recently by Gidley (1979). The corrections to Eq. (4.13) caused by these effects are small for the start and background rates which characterize the more recent of the above experiments (typically less than 10^{-3} effect on the measured decay rate); however, they must be considered at the level of accuracy of the current research.

The major systematic effect unrelated to specific timing and data analysis problems in the Yale experiment stemmed from the fact that, as in the hfs measurements, Ps was formed in a background gas. The Ps-gas collisions caused the Ps to decay more rapidly than it would in free space. There are a variety of possible effects which can increase the decay rate, the most important of which is that during the collision the positron has an enhanced probability of annihilation with a molecular electron which is in a spin-singlet state with respect to the positron (pick-off quenching). In order to obtain the true vacuum decay rate, $\lambda(1^3S_1)$, measurements are made at a number of different gas densities, and linear extrapolation to zero density is made, exactly as discussed in Sec. IV.A.2 for the hfs measurement. There are, however, physical effects such as three-body collisions, which could cause a weak quadratic dependence to the density extrapolation at high density. Finally, as the density is reduced, the lifetime of the free positron component increases to the point where it can cause an observable increase in the fitted decay rate. In addition, at reduced density, Ps can diffuse to the walls of the vacuum chamber, where it may have an increased probability of annihilation. Recent work at Michigan suggests that the former effect is of more importance than the latter in experiments performed to date. At any rate, an increase in decay rate was observed in the Yale work, so that data taken at pressures below 15 psi in Freon-12 were rejected. The final result obtained in this experiment was $\lambda(1^3S_1)=(7.275\pm0.015)$ sec^{-1}.

A somewhat different technique for measuring $\lambda(1^3S_1)$ based on a start signal obtained from direct detection of the emitted positron, rather than the prompt nuclear gamma ray, was reported in 1972–73 (Coleman *et al.*, 1972; Coleman and Griffith, (1973). The method is illustrated in Fig. 14. Essentially half of the positrons emitted by the ^{22}Na source pass through the thin scintillator shown in the inset, give a start pulse, and enter the gas chamber. The high voltage feedthrough shown permits an electric field to be set up in the gas so as to enhance Ps formation. The major advantage of this positron detection scheme is the improvement in the signal-to-noise ratio (A/B), since fewer start counts are lost than when the prompt gamma is detected. Typical signal-to-noise ratios in this work were 3–5, and the results of the experiment were $\lambda(1^3S_1)=(7.26\pm0.015)$ μsec^{-1}.

The next stage in the decay rate measurements was carried out at Michigan. The Michigan experiment (Gidley *et al.*, 1976) differed from preceding work primarily in the use of SiO_2 powder rather than gas as

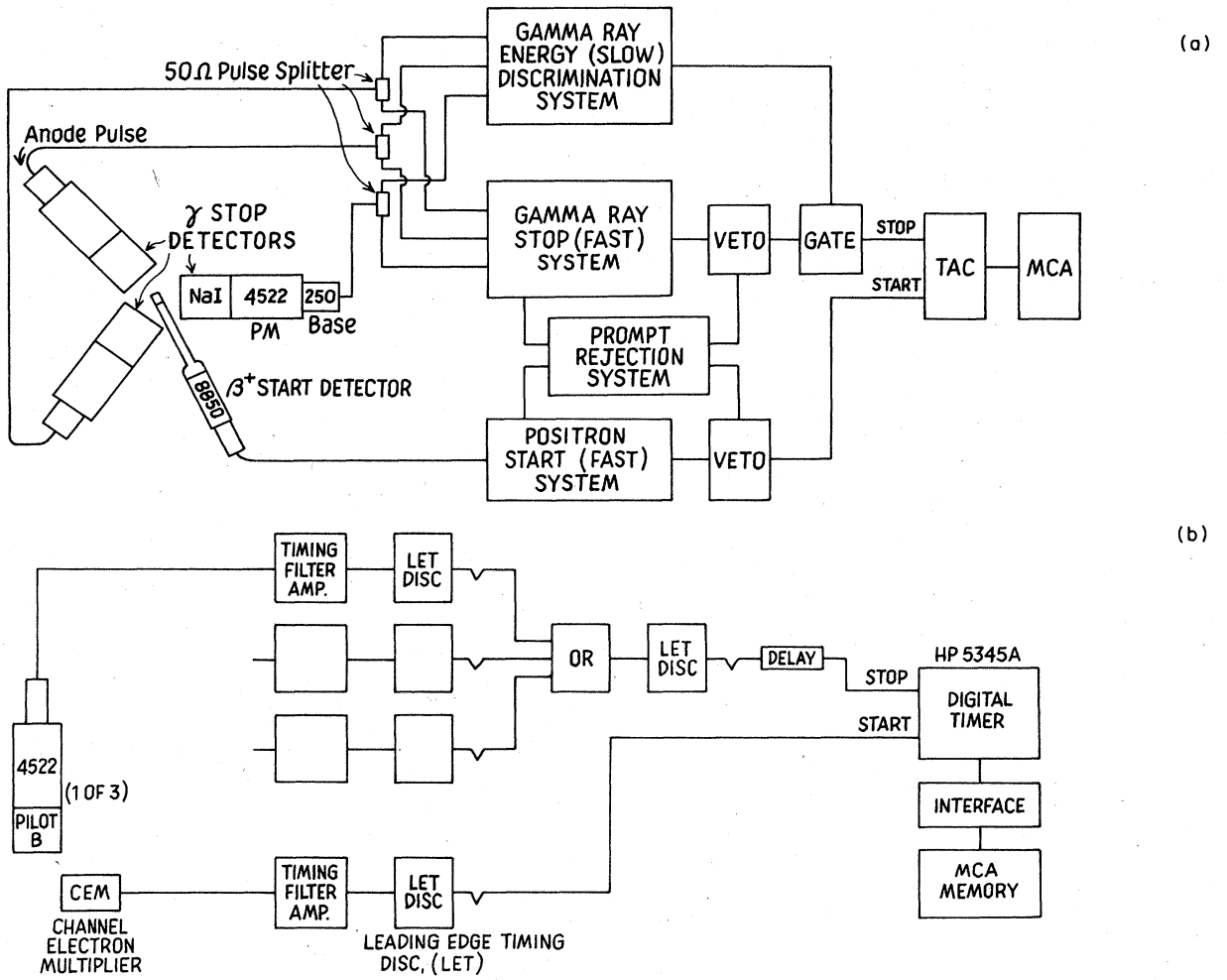


FIG. 12. (a) Abbreviated version of the timing system used in the first Michigan "powder" experiment. The Fast γ (Stop) – "Positron Start" systems reflect the usual start-stop timing arrangement. The prompt rejection unit (dotted lines) prevents start and stop events which occur within ±25 nsec of each other from reaching the time-to-amplitude converter. This reduces a major source of background due to positron starts and uncorrelated γ-stops where the γ's are in prompt coincidence with their associated positron. (b) Digital timing (digitron) type of system used in the Michigan "vacuum" o-Ps decay rate measurement. A continuous sequence of pulses at 500 MHz is put out by the digital timer. The number of these pulses between the start and stop inputs is counted and stored in the multichannel analyzer memory.

the medium in which Ps was formed (see Sec. III.B for a discussion of the powders used). A schematic diagram (to scale) of the experiment is shown in Fig. 15. As can be seen, the start signal is obtained from a positron passing through a thin piece of plastic scintillator, as in the London work, while one, two, or all three of the annihilation gammas were detected using NaI crystals. The major advantage of this technique is that the powder densities that can be used without significant perturbation of $\lambda(1^3S_1)$ are 10–100 times greater than the equivalent gas densities. This allows efficient Ps formation in regions of order 30 cm^3, as compared to the liter type volumes associated with gas measurements. This smaller volume allows for increased detector efficiency and a consequent reduction in the source strength necessary for a reasonable rate of data accumulation. The source reduction results in a very large improvement in the signal-to-noise ratio, which was typically 200–400 for single gamma detection and was as high as 2000–3000 if it was required that all three gammas be detected simultaneously. Samples of raw data associated with these runs are shown in Fig. 16.

The major uncertainty in this experiment was associated with the question of the linear extrapolation to zero density of data taken at the powder densities used. It can be shown that an extrapolation of λ with respect to the *free* volume density ρ^*, i.e., the mass per unit volume excluding the grain volume, should be linear if the Ps is essentially free between collisions, i.e., if it is not trapped at grain surfaces and is not acted on by atomic size electric fields in the intergrain region. Such an extrapolation for two different sizes of grain, along with the gas extrapolating carried out by the London and Yale groups, is shown in Fig. 17. As can be seen, the slope of the powder extrapolation was much smaller than was the slope of the gas extrapolation—an important point in terms of systematic relia-

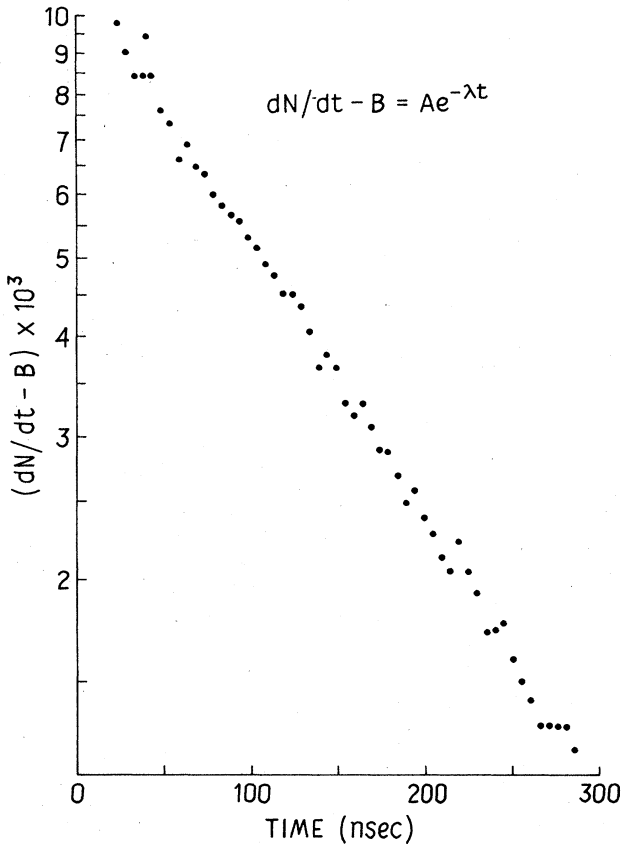


FIG. 13. Raw data after noise (B) subtraction from the Yale (1^3S_1) decay rate measurement.

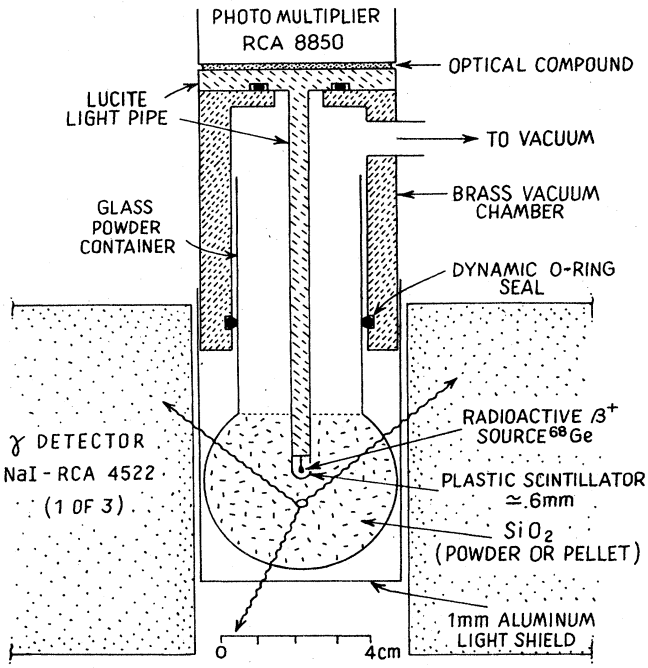


FIG. 15. Schematic diagram (to scale) of the Michigan measurement of the 1^3S_1 decay rate in SiO_2 powder.

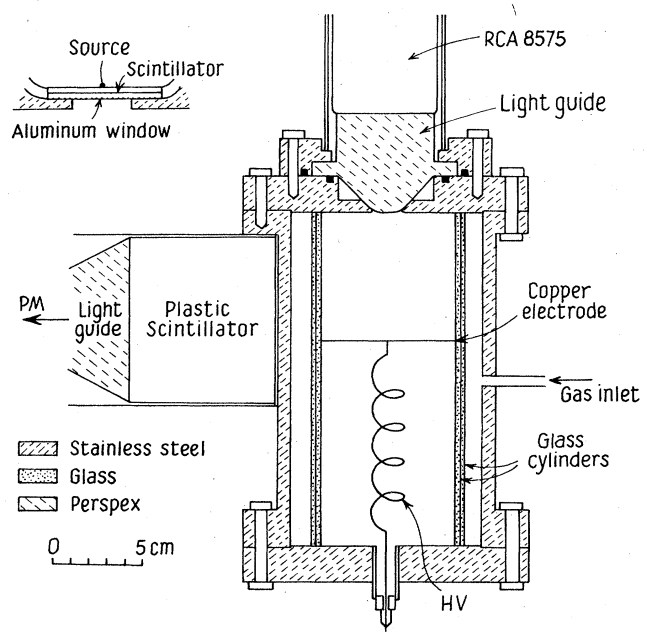


FIG. 14. Schematic (to scale) of the London measurement of the 1^3S_1 decay rate.

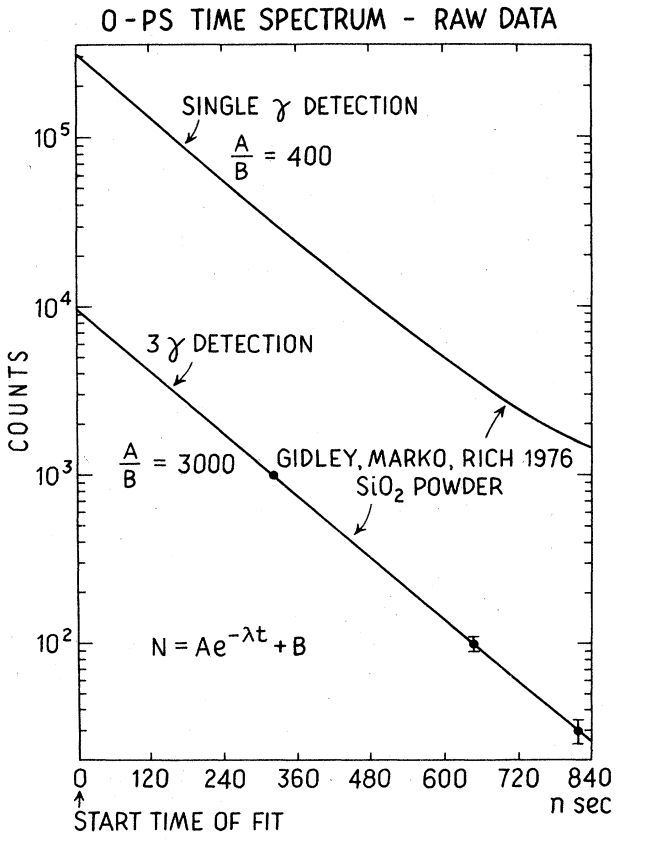


FIG. 16. Raw data and best-fit exponentials for the Michigan SiO_2 powder experiment. Data are shown in which it is required that stop pulses be recorded in either one or simultaneously in all three NaI detectors.

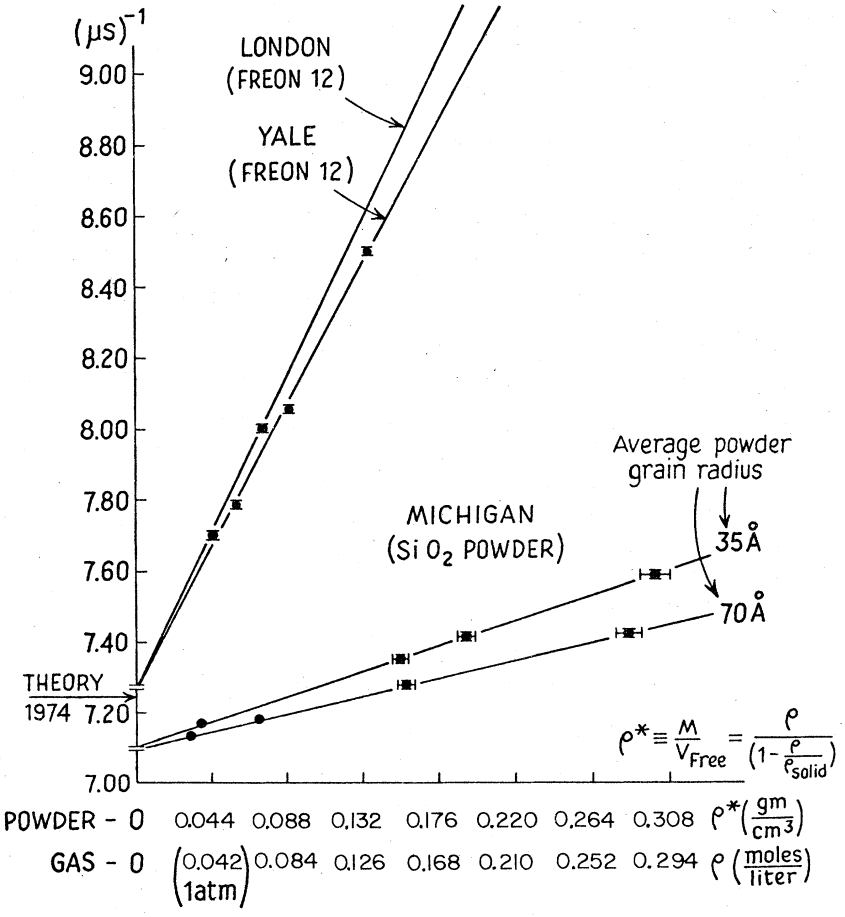


FIG. 17. Extrapolation with respect to density of the 1976 Michigan powder experiments and the pre-1976 London and Yale gas results. The intercepts were taken as the vacuum value for the 1^3S_1 decay rate.

bility if nonlinearities are present. In addition, the values of the intercepts obtained for the gas *vis-à-vis* the powder [$\lambda(\text{gas}) \simeq (7.27 \pm 0.015)\ \mu\text{sec}^{-1}$ and $\lambda(SiO_2) \simeq (7.10 \pm 0.006)\ \mu\text{sec}^{-1}$] were highly discrepant.

In order to clarify the situation an entirely new experiment (Fig. 18) was undertaken (Gidley *et al.*, 1976) in which Ps was formed from a beam of 400 eV positrons incident on the cone of a channel electron multiplier (see Sec. III.C for discussion of Ps formation by slow positrons incident on solid surfaces). The secondary electrons ejected by the incident positron were used to give a "start" signal (thus constituting the first

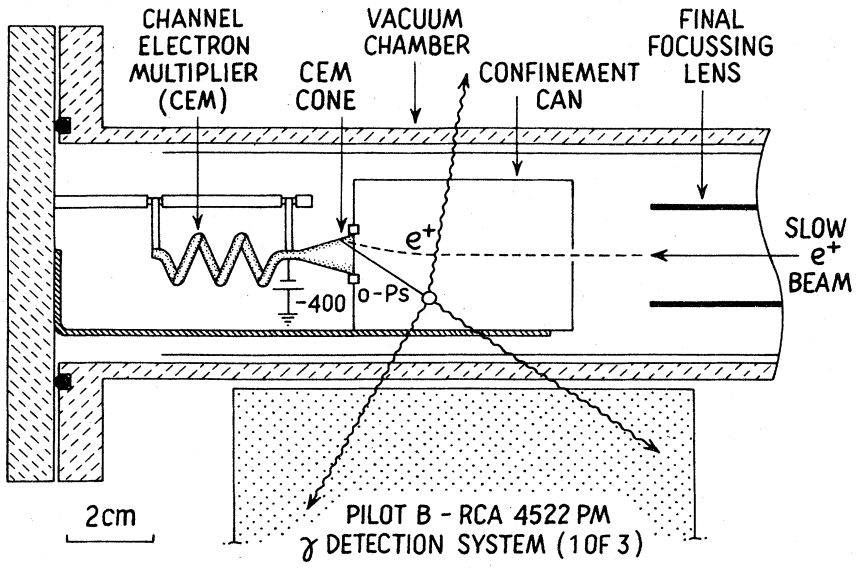


FIG. 18. Schematic diagram (to scale) of the Michigan vacuum experiment to measure the 1^3S_1 decay rate.

time that slow positrons were used for timing), while most of the Ps formed on the cone moved into the vacuum region of the confinement can. In this region the Ps makes just a few wall collisions (as opposed to the thousands of Ps-grain or Ps-molecule collisions in SiO_2 or gas) prior to annihilation.

Data obtained from this system had a signal-to-noise ratio (A/B as defined previously) of 1500 for single gamma detection and 9000 if it were required that two gammas be detected simultaneously! The quality of the data was thus extremely high, and a statistical error of $\pm 10^{-3}$ (± 0.006 μsec^{-1}) could be obtained with reasonable run times. The major error in this work was due to a systematic effect related to the passage of Ps through the beam entrance hole shown in Fig. 18. Since all of the decay rate experiments discussed so far actually measure the disappearance rate of Ps, passage through the hole and subsequent movement into a region from which the gamma rays are detected with lower probability is equivalent to an increased decay rate. The final result of the experiment, including this and other systematic errors, was $\lambda(1^3S_1) = (7.09 \pm 0.02)$ μsec^{-1}.

Following this demonstration that λ was definitely in sharp disagreement with theory, the SLAC group undertook the calculation described in the preceding section, obtaining the result $\lambda(\text{theory}) = (7.038 \pm 0.001)$ μsec^{-1}. Subsequent decay rate measurements in gases at Michigan (Gidley *et al.*, 1978) and London (Griffith *et al*, 1978) yielded the respective results $\lambda = (7.056 \pm 0.007)$ μsec^{-1} and $\lambda = (7.045 \pm 0.006)$ μsec^{-1}. These measurements were analogous in technique to the previous gas experiments, although a number of new features were incorporated in each and although in each the primary results were obtained in isobutane rather than freon or freon and argon. It should be noticed in particular that the signal-to-noise ratios were much improved, now lying in the range 10–75, depending on pressure (compared to the 2–5 signal-to-noise ratio in the 1968–73 Yale-London experiments), and that careful investigations of wall quenching effects were made, so that the error from this source was considered to be only 0.001 μsec^{-1} in both experiments. Unfortunately, the cause of the discrepancy between these later measurements and the Yale-London work of 1968–73 has never been determined. In addition to the gas measurement, the Michigan group also improved the precision of λ in both the powder and vacuum experiments. Small systematic effects not considered in the preliminary work were uncovered in each experiment, so that final results were

$\lambda(SiO_2) = (7.067 \pm 0.021)$ μsec^{-1} (Gidley *et al.*, 1978) and

$\lambda(\text{vacuum}) = (7.050 \pm 0.013)$ μsec^{-1} (Gidley and Zitzewitz, 1978).

I should like to summarize the current situation and indicate directions for future positronium decay rate measurements. The weighted average ($\bar{\lambda}$) and mean error [$\sigma(\bar{\lambda})$] of the latest experimental results [$\lambda(SiO_2) = 7.067 \pm 0.021$ μsec^{-1}, $\lambda(\text{vacuum}) = 7.050 \pm 0.013$ μsec^{-1}, $\lambda(\text{gas-Mich}) = 7.056 \pm 0.007$ μsec^{-1}, and $\lambda(\text{gas-London}) = 7.045 \pm 0.06$ μsec^{-1}], are $\bar{\lambda} = (7.050$ $\mu sec^{-1})$ and $\sigma(\bar{\lambda}) = 0.004$ μsec^{-1}. The weights used were the inverse squares of the respective assigned errors, and $\sigma(\bar{\lambda}) = \pm 0.004$ μsec^{-1} was calculated as the inverse root of the sum of the weights. The meaning of the above estimates of $\bar{\lambda}$ and $\sigma(\bar{\lambda})$ is, of course, open to question since the individual errors given represent a combination of statistical and systematic effects. In particular, the two gas experiments which have the major influence on $\bar{\lambda}$ and $\sigma(\bar{\lambda})$ have errors which are primarily statistical (Michigan) and primarily systematic (London). It is clear, however, from a perusal of the above separate results that $\bar{\lambda} = 7.050$ μsec^{-1} would represent a reasonable mean and $\sigma(\bar{\lambda}) = 0.006$ μsec^{-1} a conservative error of the mean in this case. We adopt $\sigma(\bar{\lambda}) = 0.006$ μsec^{-1}, rather than 0.004 μsec^{-1}, because the major error in the London work is systematic in origin.

The difference between this value and $\lambda(\text{theory}) = (7.0386 \pm 0.00016)$ μsec^{-1} is $\Delta\lambda = \bar{\lambda} - \lambda(\text{theory}) = (0.011 \pm 0.006)$ μsec^{-1}. Although $\Delta\lambda$ only differs from zero by two standard deviations we consider that it represents a real discrepancy, probably due to an experimental systematic effect, rather than a statistical fluctuation. We say this in light of the fact that all of the measurements made to date do lie above λ(theory) and most systematic effects present in these experiments (as discussed previously) tend to raise rather than lower the measured decay rate.

Turning to the possibility of increased precision in these experiments, we note that in their latest paper the London group (Griffith *et al.*, 1978, p. L748) states that "further refinements of this method should enable an accuracy of 1 or 2 parts in 10^4 to be achieved." This seems reasonable for gas measurements, since in both the London and Michigan work the current accuracy of about 10 parts in 10^4 is comprised of systematic effects at roughly the 5 in 10^4 level which are themselves primarily related to determining the pressure in the gas and then converting from pressure to density (i.e., knowledge of the virial coefficients at the 1% level). These are certainly nontrivial improvements, but ones which we feel are possible. The problem of wall quenching has been investigated experimentally at Michigan with the result that wall effects are absent at the 10^{-4} level. This conclusion is in agreement with theoretical estimates by the London group. Other systematic effects, such as timing calibration, spectrum distortion due to photomultiplier afterpulsing, etc., are already at the 10^{-4} level and can probably be reduced even further. Finally, appropriate modifications of the apparatus—such as the use of a magnetic field to confine the positrons to a small cylindrical region at the center of the chamber (a technique already proven for our λ_s determination, as described in Sec. IV.B.2) should make it possible to increase the data rate with no loss in signal to noise. This is because Ps has a diffusion length of less than 1 mm at the gas densities to be used and is therefore also confined to the center of the chamber. Statistical uncertainty—now at the level of $(5-10):10^4$—may therefore be reduced to less than $1:10^4$ with reasonable run times; and, of course, wall effects will not be present. Thus, as long as the linearity of the extrapolation of λ versus gas density holds, a determination of λ to a few parts in 10^4 does appear possible.

The Michigan group has also investigated the feasi-

bility of performing the vacuum experiment to the same level of precision, and we feel that here also such precision can be reached. Systematic effects in such an experiment would, of course, be completely different than in a gas experiment, so that the pair of experiments would constitute an excellent check on the number obtained. Finally, consideration of systematic effects in a powder decay rate experiment, effects primarily related to measurement of the powder density and determination of density uniformity as well as to possible effects which are nonlinear in density, such as surface trapping of Ps, make higher precision experiments in this medium more difficult than in the vacuum or gas. It would seem that a measurement of λ at the 10^{-4} level is feasible at present and that particularly in view of the current difference between theory and experiment, as discussed above, the successful performance of such an experiment would be of importance.

C. Search for forbidden decay modes in positronium annihilation

The search for the decay of 1^1S_0 or 1^3S_1 Ps into γ-ray decay schemes forbidden by various symmetry principles can be used to set upper limits on the validity of the symmetries; or, alternatively, such limits can be used to search for types of decay not included in the usual QED framework. In this section I will discuss two experiments which set limits on C violation in the Ps system, as well as a suggested new experiment to search for axion emission in 1^3S_1 annihilation.

1. Limits on the $1^1S_0 \to 3\gamma$ decay mode

As discussed in Sec. II.B, it is a rigorous consequence of conservation of charge conjugation (C) that the decays $n^1S_0 \to 3\gamma, 5\gamma, \ldots$ and $n^3S_1 \to 4\gamma, 6\gamma, \ldots$ are forbidden. The normal electromagnetic interaction does, of course, conserve C; however, it is possible to construct C and T or C and P violating interactions for which the decay $1^1S_0 \to 3\gamma$ is allowed, and a search for this Ps decay mode has been undertaken (Mills and Berko, 1967; Mills, 1967.) A discussion of proposed C-violating Hamiltonians is also contained in the above references. The initial discussion of possible C violation in the electromagnetic interactions was the observation (Bernstein, Feinberg, and Lee, 1965) that there was no experimental evidence for C and T invariance in the electromagnetic interactions of the strongly interacting particles. Bernstein *et al.* calculated that such violations of C, consistent with the available experimental limits, could through second-order (virtual electromagnetic) processes, manifest themselves in the CP violating decay $K_2^0 \to \pi^+ + \pi^-$ with an amplitude of order (α/π) times that for $K_1^0 \to \pi^+ + \pi^-$. Based on the suggestions in this article a number of experiments to search for C-forbidden decays or decay distributions of the π^0, η^0, etc. were undertaken. To date no such C-violating effects in the strongly interacting particles have been observed.

The Bernstein *et al.* analysis assumed that the usual C-conserving electromagnetic interaction ($e\bar{\psi}\gamma_\mu\psi A_\mu$) was correct for leptons since predictions based on this interaction (Lamb shift, electron g-2, etc.) had been observed experimentally to high accuracy and agreed with theory. It is, however, possible to construct C-violating interaction Hamiltonians for the electron-positron system which, with suitably chosen coupling constants, would not cause observable changes in the Lamb shift, g-2 value, etc., for the limits on the coupling constants which can be derived from direct searches for C-violating effects in Ps.

The only definitive search for the decay $1^1S_0 \to 3\gamma$ was conducted by Mills and Berko as noted. In this experiment Ps was formed in SF_6 gas, and the ratio of the decay rate into three counters in the symmetric configuration (120°–120°–120°) to the decay rate into three counters in the asymmetric configurations (60°–150°–150°) or (90°–120°–150°) is formed. The decay rate into the symmetric configuration vanishes even if C is violated because of a symmetry argument based only on conservation of angular momentum and Bose statistics of the gammas. Thus only 3γ emission from a $^3S_1$ state can be present in this configuration. If a Ps-quenching gas such as NO is then added and the decay rates into the symmetric and the asymmetric configuration remeasured, any change observed in the ratio must be due to the presence of decays from $^1S_0 \to 3\gamma$, since the fraction of singlet to triplet annihilations is changed by quenching. Numerous systematic effects exist which could also cause the ratio to change when Ps is quenched. After correction for these effects and calculation or measurement of a number of factors, such as the quenching ratio, effect of finite solid angle on the prohibition of $^1S_0 \to 3\gamma$ in the symmetric configuration, etc., the branching ratio defined as $b = \lambda_S^{3\gamma}/\lambda_S^{2\gamma}$ ($\lambda_S^{2\gamma}$, $\lambda_S^{3\gamma}$ = decay rate for $^1S_0 \to 2\gamma$, $^1S_0 \to 3\gamma$, respectively) was determined to be $b \leq 2.8 \times 10^{-6}$. The interpretation of this result in terms of the coupling constants on the C-violating portions of the various interactions proposed (for example, the C-violating Lagrangian

$$\mathcal{L} = g\left(\frac{1}{2m}\right)^8 \bar{\psi}\gamma_5\gamma_\mu\psi F_{\alpha\beta}F_{\alpha\beta,\gamma\delta}F_{\mu\gamma,\delta},$$

where $F_{\mu\nu}$ is the electromagnetic field operator and ψ the electron field operator) give limits $g < 1$ for m taken as the electron mass.

2. Limits on the C-violating $1^3S_1 \to 4\gamma$ decay mode

An experiment analogous to the Brandeis work was later performed (Marko and Rich, 1974; Marko, 1974) at the University of Michigan. In this work the C-violating decay mode $1^3S_1 \to 4\gamma$ was searched for. The technique used to detect $^3S_1 \to 4\gamma$ was to form Ps from positrons emitted from a ^{22}Na source embedded in a spherical MgO powder sample and then to search for $^3S_1 \to 4\gamma$ events using four NaI detectors placed at the vertices of a tetrahedron. This configuration was chosen because the C-allowed process $^1S_0 \to 4\gamma$ is forbidden to decay into a tetrahedral geometry by considerations of rotational invariance (Mani and Rich, 1971) and because under a specific C-violating Hamiltonian (Mani and Rich, 1971) the matrix element for $^3S_1 \to 4\gamma$ is maximum in the tetrahedral mode. A set of strict conditions was imposed on the accepted events, so that instead of searching for a small change in a large background count rate as in

the $^1S_0 \to 3\gamma$ experiment, limits on the $^3S_1 \to 4\gamma$ rate could be set directly. The principle conditions required were that (i) the four signals be simultaneous in time to within the 16 nsec coincidence resolving time of the detection system, (ii) the signals occur within a 20–120 nsec interval after a $t=0$ signal taken from the 1.3 MeV gamma ray coincident with positron emission from ^{22}Na, (iii) each signal be within an energy window determined by the detector solid angle and decay kinematics, and (iv) the sum of the four energies equal $m_{Ps}c^2 = 1.02$ MeV to within the system energy resolution.

The result of the experiment was that four candidate events were found during a 24-day run. These events were consistent with the expected background rate; consequently, the data set an upper limit on the branching ratio $b = \lambda_T^{4\gamma}/\lambda_T^{3\gamma}$ ($\lambda_T^{3\gamma}$, $\lambda_T^{4\gamma}$ = decay rate for $^3S_1 \to 3\gamma$, $^3S_1 \to 4\gamma$, respectively) of $b < 8\times10^{-6}$. This was equivalent to determining the coupling constant λ in the Mani-Rich Hamiltonian,

$$H_i = \frac{\lambda}{m^8} e^4 \partial_\alpha \rho_\beta F^{\alpha\delta} F_\delta^\beta F^{\mu\nu} F_{\mu\nu} \quad (\rho_\beta = \bar{\psi}\gamma_\beta\psi)$$

to be $\lambda < 0.8$ for m taken equal to the electron mass.

Finally, I should remark that if a C-violating interaction was detected at the level of sensitivity of the searches conducted to date, the Hamiltonians referred to above would lead to anomalous results in $e^- e^+$ interactions at high energy if their high energy behavior were not appropriately modified (Christ, 1974; Kane, 1974). These Hamiltonians are, of course, only effective in the low-energy region. At higher energy, the more fundamental structure should be taken into account, and this will modify the energy dependence. It is an open question as to whether a reasonable theory can be invented that agrees with experiments in the low-energy regime and that is consistent with an effective coupling λ that can be detected in experiments of the type described above.

3. Search for the allowed decay $1\,^1S_0 \to 4\gamma$ or $e^- + e^+ \to 4\gamma$

The decay $1^1S_0 \to 4\gamma$ or the direct annihilation $e^- + e^+ \to 4\gamma$ is allowed and would be interesting to detect and study, in that it would be the first direct four-gamma decay process observed at any energy. A calculation of the branching ratio for the process $1^1S_0 \to 4\gamma$ gives the result $b = \lambda_s^{4\gamma}/\lambda_s^{2\gamma} = 3\times10^{-7}$ (McCoyd, 1965), while an estimate based on phase-space considerations yields approximately 2×10^{-8} for the process (Mani and Rich, 1971). An analysis of a possible experiment to search for the decay $e^- + e^+ \to 4\gamma$ has been carried out by the author for electron-positron annihilation in a metal (no Ps formation). The results are that it might be possible to detect a 4γ to 2γ branching ratio of 10^{-7} in runs of approximately one-day duration.

4. Search for axion emission in $1\,^3S_1$ decay

The possibility of adding a chiral U(1) symmetry in the unified gauge theories leads to the possibility of a neutral pseudoscalar particle—the axion or higglet (Weinberg, 1978; Wilczek, 1978). The mass and lifetime of the axion, if it exists, may be estimated from an analysis of existing experiments and from current algebra considerations to be 50 KeV $< m_a c^2 <$ 200 KeV and $\tau_a(a \to 2\gamma) \sim (G_F \alpha^2 m_a^3)^{-1} \sim (100\ \text{keV}/m_a c^2)^3 \sim 1$ sec, although the above ranges are model dependent. Values for m_a somewhat greater than 1 MeV with lifetimes much shorter than τ_a, as given above, because the channel $a \to e^+e^-$ would be opened, are unlikely, but cannot be precluded. Analysis of the data from various beam dump and reactor experiments (Donnelly *et al.*, 1978) shows that under reasonable theoretical assumptions [use of the SU(2) × U(1) weak interaction model with an additional U(1) chiral symmetry, etc.] axions would have been detected already. However, according to Donnelly *et al.*, these experiments would not have detected axions if (i) they are predominantly isoscalar, (ii) $m_a c^2 \sim 100$ keV, (iii) their high-energy coupling to hadrons is weak, or possibly if (iv) a different weak-interaction model is used.

An alternative to the various high-energy and nuclear experiments which have been suggested to search for axions has been proposed (Mikaelian, 1978) in which triplet Ps decay into an axion and a gamma ray ($1^3S_1 \to \gamma + a$) would be detected. This process is allowed if $m_a < 2m$, although we note that $^1S_0 \to \gamma + a$ if forbidden by angular momentum conservation, since the axion has spin zero. If the coupling strength of the axion-electron vertex is $2^{1/4} G_F^{1/2} m c_e$ (G_F = the Fermi coupling constant, and c_e is a model-dependent parameter of order unity), then the partial decay rate into the gamma-axion channel is

$$\lambda(1^3S_1 \to \gamma + a) = \frac{\alpha^4 G_F m^3 \left[1 - \left(\frac{m_a}{2m}\right)^2\right]}{12\sqrt{2}\,\pi} c_e^2 \ \text{sec}^{-1}$$

$$\simeq \frac{1}{8}\left[1 - \left(\frac{m_a}{2m}\right)^2\right] c_e^2 \ \text{sec}^{-1} \qquad (4.14)$$

Thus the branching ratio for $1^3S_1 \to \gamma + a$ vs $1^3S_1 \to 3\gamma$ is about $10^{-1}/7\times10^6 \sim 10^{-8}$ for $m_a < m$ and $c_e \simeq 1$. Mikaelian has suggested that one might search for the single quantum decay of 1^3S_1 Ps as an indirect signal of axion production. The energy of the γ emitted (E_γ) is related to m_a (by conservation of energy and momentum) through the expression $m_a^2 = 4m[m - (E_\gamma/c^2)]$ so that a measurement of E_γ would permit a determination of m_a which, when combined with the branching ratio, would also yield c_e.

Experimentally the detection of a branching ratio of 10^{-8} is reasonable from considerations of the signal alone, since 10^9 triplet events can be accumulated in a reasonable time with singles rates (10^5 sec^{-1}) which preserve the possibility of doing delayed coincidence timing from a $t=0$ signal to isolate the triplet decay. The major problem in such an experiment would be the suppression of non-axion-related single-pulse events of the proper energy in delayed coincidence with the start signal. Such events could arise at the 10^{-8} level through a varity of processes—for example, $1^3S_1 \to 3\gamma$ with one of the gammas being of low enough energy so that it had a reasonable probability of remaining undetected, even for photomultiplier discriminators set at the single photon level. The other gamma would simply pass through a detector without interaction. Consideration of such processes readily leads to the conclusion

that if, for example, NaI detectors were to be used, they would have to subtend nearly 4π solid angle around the Ps formation region and in addition would have to be of order 30 cm in length. A preliminary investigation by the author of processes such as these leads to the conclusion that a dedicated experiment to search for single-quantum annihilation of triplet Ps may be feasible at the 10^{-8} level.

D. Miscellaneous positronium related experiments

In this section we will discuss several applications of fundamental interest in which the use, or proposed use, of Ps as a tool will be of central importance in the research. Many other investigations could have been included in this subsection. I have, however, simply tried to illustrate situations in which the unique properties of Ps have mandated its use in a particular investigation, even though the object of the work is essentially unrelated to Ps *per se*.

1. Proposed measurement of the electron compton wavelength ($\lambda_c = h/mc$)

Measurement of λ_c is of interest for its own sake, since λ_c is a fundamental constant of physics and also because one can write

$$\alpha = (2\lambda_c R_\infty)^{1/2}, \tag{4.15}$$

where R_∞ is the Rydberg constant for infinite mass ($R_\infty = 4\pi\hbar^3 c/me^4$). Since R_∞ is currently known to 0.003 ppm, a determination of λ_c at the one ppm level would allow a very direct non-QED determination of α at the half ppm level. Such a measurement would initially be less accurate than the current non-QED determination of α (Williams and Olsen, 1979) which is obtained from the relation

$$\alpha = [(4R_\infty/c)(\Omega_{\rm NBS}/\Omega)(\mu_P/\mu_B)^{-1}\gamma_P/(2e/h)]^{1/2},$$

where Ω_{NBS}/Ω represents the ratio of the National Bureau of Standards ohm to the absolute ohm, μ_P/μ_B is the proton magnetic moment in Bohr magnetons, γ_P is the proton gyromagnetic ratio, and the ratio $(2e/h)$ is obtained from the a.c. Josephson effect. The uncertainty in α from the above combination of separate measurements is 0.11 ppm and is due primarily to an error of 0.21 ppm in γ_P. Clearly, a measurement of α from the relation $\alpha = (2\lambda_c R_\infty)^{1/2}$ would constitute an important systematic check on the current work and, in addition, might eventually prove to be more accurate.

The technique which as been proposed to measure λ_c (Sauder and Deslattes, 1967; Sauder, 1970) is essentially to form Ps in helium gas cooled to its critical point and placed in a magnetic field (B). The value of the field would be chosen high enough that the perturbed triplet state (ψ_T') would decay primarily by two-photon annihilation. The field would at the same time be low enough that the lifetime of the perturbed state (τ_T') would be sufficiently in excess of the Ps thermalization time to assure thermalization of the state prior to decay. We note that if Ps at rest in the state ψ_T' decays into two gamma rays, conservation of momentum and energy $[2(hc/\lambda) = 2mc^2 - W_T']$ leads to the relation for the wavelength of each photon:

$$\lambda = \frac{\lambda_c}{1 - \frac{1}{2}(W_T'/mc^2)}, \tag{4.16}$$

where W_T' is the energy of the state ψ_T' [Eq. (2.4c)]. Since $W_T'/2mc^2 \simeq 6.6\times10^{-6}$ (accurate to parts per million), we see that a measurement of λ to any given accuracy yields λ_c to the same accuracy.

The inherent problem in any measurement of λ_c using positron-electron or Ps annihilation is, however, precisely the fact that the system annihilates not from rest but with a finite center of mass momentum. Following the notation of Sauder and Deslattes, let E_0 be the total energy of the positron-electron (or Ps) complex, i.e., E_0 will be the sum of the center of mass kinetic energy, the binding energy if Ps is formed, and the rest energy $(2mc^2)$. Then the energy of the emitted gammas may be written as

$$h\nu_1 = \tfrac{1}{2}E_0 + \delta, \tag{4.17a}$$

$$h\nu_2 = \tfrac{1}{2}E_0 - \delta. \tag{4.17b}$$

Here δ represents the Doppler broadening, which, from conservation of momentum, is

$$\delta = \frac{-pc}{2}\left[\frac{pc}{E_0} - \left(1+\frac{2\delta}{E_0}\right)\cos\theta\right], \tag{4.18}$$

where θ is the angle between photon 1 ($h\nu_1$) and the center of mass momentum p. Since $E_0 \simeq 2mc^2$ and p is characteristic of an e^+-e^- system in thermal equilibrium, i.e., $pc/E_0 \ll 1$, $2\delta/E_0 \ll 1$, Eq. (4.18) reduces to the approximate expression $\delta \simeq (pc/2)\cos\theta$. Thus to a good approximation the measured fractional energy spread of the annihilation gammas will be approximately $pc/mc^2 \simeq v/c$ or 1400 ppm for an e^--e^+ system with center of mass energy of 1 eV. Such an energy is characteristic of free e^--e^+ annihilation in matter. Extraction of a one-ppm measurement of λ from an initial 1000-pm linewidth is essentially impossible, due to lack of a sufficiently well-known theoretical line shape and to the enormous statistical precision that would be needed for the part-per-thousand line splitting necessary.

This problem may be resolved if Ps is formed in a low-temperature gas (a gas should be used, since pick-off must be avoided) and the decay of the singlet state observed, since considerably narrower lines would be achieved if the singlet Ps were thermalized. Unfortunately, typical thermalization times in gases are far in excess of the 0.1 nsec lifetime of ψ_s. As mentioned above, Deslattes and Sauder propose to overcome the thermalization problem by utilizing the 2γ decay of ψ_T', which they note occurs in about half of the ψ_T' decays at a field of 2 kG. The value of τ_T' in such a field is 70 nsec, which is well in excess of the calculated 1 to 10 nsec thermalization time of Ps in He gas at the critical point ($T_c = 5.2$ K, pressure = 2.3 atom). The value of (v/c) for Ps at 5.2 K is only 30×10^{-6}. Detailed consideration of the lineshape for the 2γ decay of thermalized Ps leads to the result that the shape will be Gaussian with fractional full width at half maximum in-

tensity of 1.4 (v/c). This is sufficiently narrow that splitting to 1 ppm becomes feasible.

As a final remark, I note that the actual measurement of λ is made using a crystal spectrometer. The wavelength of the annihilation photon (λ) is determined from accurate measurements of the crystal spacing d and the Bragg angle θ at which the γ ray is diffracted, according to the usual relation $\lambda = 2d\sin\theta$. Since $\lambda \sim \lambda_c = 0.024$ Å and $d \sim 1$ Å, $\theta \sim 10^{-2}$ rad, so that measurement of λ to 1 ppm requires measurement of θ to 10^{-3} arcsec (both in absolute accuracy and measurement precision) and concomittantly knowledge of d to 10^{-6}. Both of these requirements are feasible, as has been demonstrated in recent work (Kessler *et al.*, 1978, 1979), in which a number of γ-ray lines between 63 and 675 keV were measured to accuracies of better than 1 ppm, using Ge and Si crystals. In particular, ^{192}Ir lines at 468 and 484 keV were measured to accuracies of 0.57 and 0.85 ppm, respectively. Thus if the work on a low-temperature Ps annihilation source currently in progress at the NBS (Deslattes, private communication) is successful, a determination of λ_c at the ppm level appears feasible.

2. Use of positronium in precision positron polarimetry

a. General methods for determining positron polarization

Measurement of positron polarization ($P = \langle\boldsymbol{\sigma}\rangle$) and helicity ($h = \langle\boldsymbol{\sigma}\cdot\mathbf{p}\rangle/|\mathbf{p}|$) with both high statistical precision and high absolute accuracy is of importance in a number of current experiments, as well as in a number of proposed and ongoing tests of the weak interactions. In this subsection we will discuss the design and operation of a positron polarimeter (based on Ps formation in a magnetic field) which is now in everyday use in our laboratory. The instrument combines the qualities of high statistical efficiency, reasonable absolute accuracy, and potentially extremely high precision when used to compare the polarization of positrons from different beta-decay sources. I will also outline the numerous experiments in which the polarimeter can be employed.

Measurement of electron polarization at characteristic beta-decay energies (100–1000 keV) has, in the past, been accomplished using either Mott (electron-nucleus) or Moller (electron-electron) scattering with efficiencies (fraction of scattered to incident electrons) of order 10^{-5}–10^{-4}, asymmetries [fractional change in counting rate when a completely polarized beam ($|\mathbf{P}| = 1$) has its polarization flipped, i.e., $\mathbf{P} \to -\mathbf{P}$] of less than 25%, and, finally, maximum overall accuracies in the measurement of P quoted as no better than $\pm 1\%$ and more typically $\pm 10\%$. Similar methods, i.e., Mott or Bhabba scattering could in principle be used to determine P for positrons. Unfortunately, the asymmetry in Mott scattering of positrons is typically only 10% of that for electrons, since positrons are repelled from the nucleus, so that the motional magnetic fields [$\mathbf{v}$(projectile) $\times$ $\mathbf{E}$(nucleus)] responsible for Mott scattering asymmetries are reduced. Bhabba asymmetries are also much reduced over Moller asymmetries at energies below 1 MeV, due essentially to the Pauli principle's not being operative for e^--e^+ collisions. A general discussion of e^- and e^+ polarization and helicity may be found in van Klinken and Koks (1978), who emphasize the theoretical implications of beta-decay polarization measurements.

A different method of polarization analysis, applicable only to positrons and not based on scattering was used by Hanna and Preston (1957) to demonstrate that positrons from ^{64}Cu polarized as predicted by the then recently enunciated V-A theory of weak interactions. These workers stopped positrons in magnetized iron and showed that the differential angular correlation curve shifted when the magnetizing field was reversed with respect to the direction of the incoming positron beam, i.e., the presumed direction of positron polarization. These and subsequent angular correlation experiments of a similar nature have been reviewed and analyzed by Berko (1967). Essentially, the effect seen is due to the fact that for e^+-e^- annihilation at rest parallel spins (triplet configuration) result in 3γ-decay, while antiparallel spins (singlet-triplet mixture) decay into 2γ's or 3γ's with equal probability. Thus the normalized 2γ angular correlation rate will clearly be affected. The asymmetry actually observed is less than 0.05, with the maximum values occurring at large (10^{-2} rad) angular correlations. The low value for the asymmetry is due primarily to the fact that the positrons annihilate with greatest probability with the unpolarized rather than polarized electrons in iron and secondarily to possible spin depolarization of the positrons as they slow down from their initial MeV beta emission energies to the thermal or near thermal energies which characterize their annihilation in the iron.

An alternative though similar method for detecting positron polarization was employed by Page and Heinberg (1957) and reviewed by Page (1959). In this technique the positrons were stopped in a gas placed in a magnetic field oriented parallel or antiparallel to the initial presumed polarization direction, and Ps was formed. As we will discuss in more detail below, the amount of singlet versus perturbed triplet [$\psi'_T(m=0)$] Ps formed depends on the relative orientation of **P** and **B**. At the fields used, the decay of ψ'_T is predominantly into 2γ's; consequently, if the 2γ decay of ψ'_T could be distinguished from the 2γ decay of ψ_S so that only the process $\psi'_T \to 2\gamma$ were detected, reversal of **B** or **P** should cause a change in the number of γ's detected per incident positron. The 2γ decay of ψ'_T can, in fact, be distinguished from the 2γ decay of ψ_S, because the much shorter-lived ψ_S will not thermalize at the typical gas densities necessary for efficient Ps formation in a reasonable volume, while the perturbed state will. The perturbed state will therefore have a narrow angular correlation (low center-of-mass momentum) as compared to the singlet decay. The number of 2γ events in the narrow component of the angular correlation curve should therefore change on field reversal, and, in fact, Page and Heinberg observed a field-up–to–field-down counting rate asymmetry of up to $(5\pm 1)\%$ on performing the experiment using a number of different combinations of gases and with magnetic fields in the 10–15 kG range. The error in the asymmetry quoted above is statistical and would allow a measurement of P to $\pm 20\%$ if no systematic errors were present. The inherent

limitation in the statistical precision of the experiment is the fact that only a small fraction of the Ps formed decays into γ's which enter the angular correlation slits. For example typical 2γ coincidence counting rates for a 15 mCi ^{22}Na source (6×10^8 disintegrations sec^{-1}) were only 1 count sec^{-1}.

b. The Ps time-resolved positron polarimeter

The shortcoming of the Page technique from the viewpoint of statistical efficiency was effectively overcome by an idea suggested by Telegdi (1959) and independently by Lundby (1960). The essence of the idea was that rather than using angular correlation techniques to isolate ψ'_T, one could use timing coincidence techniques to accomplish the same goal. The advantage here lies in the fact that the probability of detecting a gamma from a time-delayed ψ'_T decay is of order unity. Thus the statistical efficeincy is improved by orders of magnitude without any loss in the asymmetry produced. The method was first implemented at CERN using plastic scintillator as the Ps-forming material (Dick *et al.*, 1961, 1963; Bisi *et al.*, 1962). An improved version of the polarimeter as regards both the magnitude of the asymmetry obtained and the absolute precision of the polarization measurements possible was described by a Michigan group, Gerber *et al.* (1977), who used MgO powder as the Ps-forming material. It is this current form of the instrument which we now discuss.

Consider Ps formed from a beam of 100% polarized positrons ($\mathbf{p}=\hat{z}$) incident on unpolarized electrons situated in a magnetic field $\mathbf{B}=\hat{z}B$ which defines the spin-quantization axis. Assuming, as is reasonable, that there is no spin flip on Ps formation (negligible spin-dependent forces exist during the e^+-e^- capture process), the spin characterization of the Ps states formed may be written as

$$\psi_T(m=1)=\uparrow\uparrow,\quad \psi_T(m=-1)=\downarrow\downarrow,\quad \psi_1=\uparrow\downarrow,\quad \psi_2=\downarrow\uparrow. \tag{4.19}$$

Here the latter two states are not eigenstates of the Hamiltonian if the spin-spin and Zeeman terms discussed in Sec. II are included and up ($\uparrow$) or down ($\downarrow$) refers to the direction defined by **B**. The existence of an asymmetry on reversal of **P** may easily be seen qualitatively if we consider the expansion of $\psi_1(t)$ and $\psi_2(t)$ in terms of the Ps eigenstates ψ'_S and ψ'_T (see Sec. II.B for notation):

$$\psi_1(t)=\left(\frac{1+\varepsilon}{2}\right)^{1/2}\psi'_T\exp\left[\left(\frac{-iW'_T}{\hbar}-\frac{\lambda'_T}{2}\right)t\right] +\left(\frac{1-\varepsilon}{2}\right)^{1/2}\psi'_S\exp\left[\left(\frac{-iW'_S}{\hbar}-\frac{\lambda'_S}{2}\right)t\right], \tag{4.20a}$$

$$\psi_2(t)=\left(\frac{1-\varepsilon}{2}\right)^{1/2}\psi'_T\exp\left[\left(\frac{-iW'_T}{\hbar}-\frac{\lambda'_T}{2}\right)t\right] +\left(\frac{1+\varepsilon}{2}\right)^{1/2}\psi'_S\exp\left[\left(\frac{-iW'_S}{\hbar}-\frac{\lambda'_S}{2}\right)t\right]. \tag{4.20b}$$

Here, $\varepsilon\equiv x/(1+x^2)^{1/2}$ and at high field ($x\gg1$, $\varepsilon\to1$),

$$\psi_1(t)\to\psi'_T\exp\left[\left(\frac{-iW'_T}{\hbar}-\frac{\lambda'_T}{2}\right)t\right]$$

and

$$\psi_2(t)\to\psi'_S\exp\left[\left(\frac{-iW'_S}{\hbar}-\frac{\lambda'_S}{2}\right)t\right].$$

Since a $\mathbf{P}=+\hat{z}$ positron beam forms the states $\psi_T(+1)$ and ψ_2 while a $\mathbf{P}=-\hat{z}$ beam forms the states $\psi_T(-1)$ and ψ_1 we see that at high field, to the extent that $\lambda'_T<\lambda'_S$, the state ψ_1 is longer lived than ψ_2, so that the time spectrum of an ensemble of decaying Ps atoms formed from a beam with $\mathbf{P}\cdot\mathbf{B}<0$ will have a larger fraction of counts in the late portion of the spectrum than the corresponding fraction for a beam of reversed polarization ($\mathbf{P}\cdot\mathbf{B}>0$). The asymmetry goes to zero at infinite field, since, from Eqs. (2.4e) and (2.4f), as $x\to\infty$, $\lambda'_T\to\lambda'_5\to\frac{1}{2}(\lambda_T+\lambda_S)$; however, for values of B in the range 5–20 kG the asymmetry is sizeable and may easily be observed.

An exact expression for A may be obtained by noting that the time spectrum for an ensemble of decaying Ps atoms initially populating the states $\psi_T(+1)$ and $\psi_2(\mathbf{P}\cdot\mathbf{B}>0)$—or, alternatively, $\psi_T(-1)$ and $\psi_1(\mathbf{P}\cdot\mathbf{B}<0)$—is

$$r_+(t)=\frac{d}{dt}[\,|\psi_T(+1)|^2+|\psi_2|^2\,], \tag{4.21a}$$

$$r_-(t)=\frac{d}{dt}[\,|\psi_T(-1)|^2+|\psi_1|^2\,], \tag{4.21b}$$

where $r_+(t)$ and $r_-(t)$ represent the rate of annihilation at time t for $\mathbf{P}\cdot\mathbf{B}>0$ and $\mathbf{P}\cdot\mathbf{B}<0$, respectively. The asymmetry at a given point in the time spectrum is defined as

$$A(t)\equiv 2[r_- - r_+]/[r_- + r_+]. \tag{4.22}$$

An approximate result for $A(t)$ obtained by direct evaluation of (4.21a) and (4.21b) using the expansions (4.20a) and (4.20b) for $\psi_1(t)$ and $\psi_2(t)$ and assuming t large enough that the term $\lambda'_S\exp(-\lambda'_S t)$ may be neglected when compared to $\lambda'_T\exp(-\lambda'_T t)$ is

$$A(t)=\frac{2\varepsilon}{1+2(\lambda_T/\lambda'_T)\exp[(\lambda'_T-\lambda_T)t]}. \tag{4.23}$$

This expression may easily be generalized to situations in which **P** makes an arbitrary angle θ with respect to **B** (spin or field reversal implies $\theta\to\theta+\pi$) and in which $|\mathbf{P}|<1$ (Rich, 1965; Rich and Crane, 1967) with the result

$$A(t)=\frac{2P\varepsilon\cos\theta}{1+2(\lambda_T/\lambda'_T)\exp[(\lambda'_T-\lambda_T)t]}. \tag{4.24}$$

In actual experiments the effects of prompt decays from both ψ'_S and direct annihilation events, which together comprise typically about 75% of all annihilations, must be considered. This is because, due to the finite time resolution (typically 1 nsec) of the equipment, such events can cause small but observable perturbations in the delayed portion of the time spectrum. Such effects are not, however, of sufficient significance to be considered in the general analysis of the polarimeter which we present below.

A schematic diagram of a preliminary version of the polarimeter (Gerber *et al.*, 1977) which used MgO powder as the Ps-forming substance is shown in Fig. 19. This instrument was used to measure the polarization of positrons from ^{68}Ga decay to an accuracy of 11%, a slight improvement over a previous 12% measurement, however, its primary purpose was to test the feasibility of the polarimeter for comparative polarization mea-

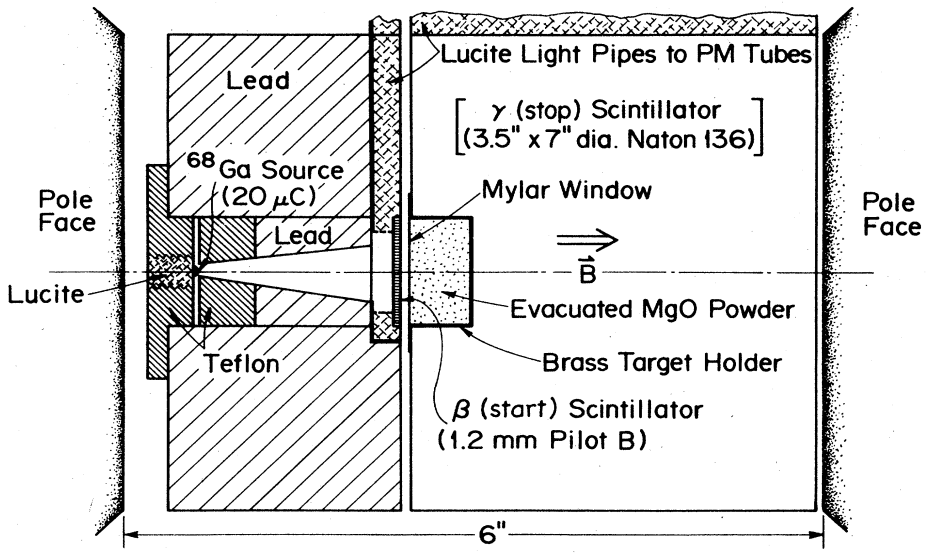


FIG. 19. Schematic diagram of a positron polarimeter based on Ps formation in evacuated MgO powder placed in a magnetic field of 2.9 kG. The polarimeter apparatus has cylindrical symmetry.

surements of precision exceeding 10^{-3}. If such accuracy is indeed possible, a number of novel tests of the weak interactions would become feasible. We will now discuss the preliminary results obtained with this instrument, primarily by way of illustrating the technique used to measure positron polarization, and then turn to an outline of the various experiments which can be undertaken with a second generation instrument.

As can be seen from Fig. 19 positrons emitted into a forward cone of half angle 7° pass through a 2 mm piece of plastic scintillator which gives the required start signal. They then stop and either annihilate directly or form Ps in the evacuated MgO powder. The annihilation radiation then gives a stop signal in the Naton 136 plastic scintillator. The entire source start-stop system lies within the 2.9 kG field of a Varian electromagnet, although in principle of course, only the Ps formation region need be in the field. The raw data and the associated computer fit lifetime spectra are shown in Fig. 20. The change in intensity on field reversal can clearly be seen in this figure. Finally, Fig. 21 shows the measured asymmetry (individual points) for this experiment, while the solid line represents the best fit to the data and implies that the polarization on Ps formation is $P = 0.58 \pm 0.03$ with 60% of the error being statitical in nature. This value is in agreement with the value of P calculated from the initial beam helicity and the estimated depolarization prior to Ps formation.

Second-generation versions of the above described polarimeter have already proven to be, and should in the future prove to be, of great importance in numerous types of investigations in which the physics of interest depends on measurement of absolute positron polarization or on comparison of the polarization from a number

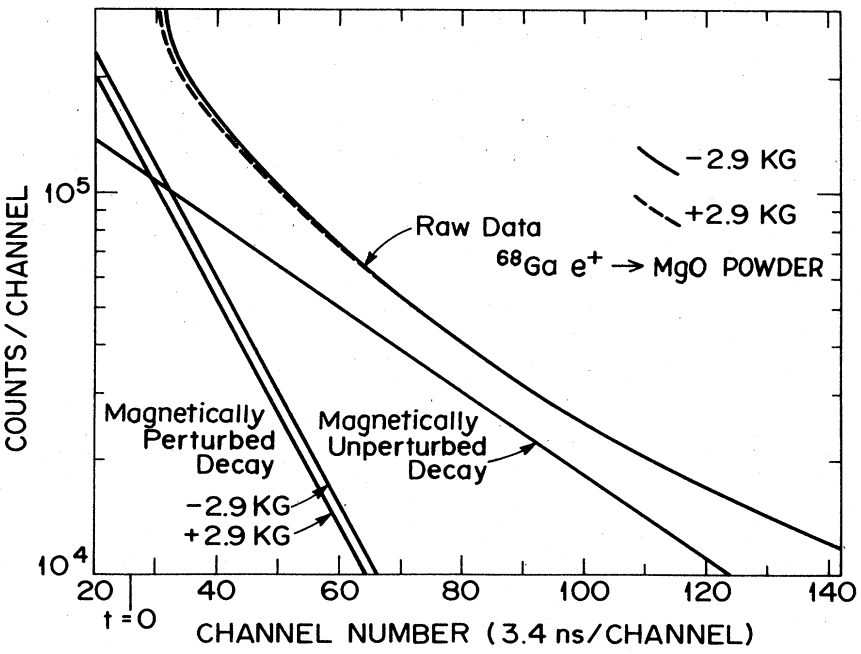


FIG. 20. Raw Ps-decay data obtained with the apparatus of Fig. 19. The various lifetime components were found by computer fit. The polarization-dependent asymmetry is given by the difference in count rates in the early channels, labeled perturbed decay, when B is reversed.

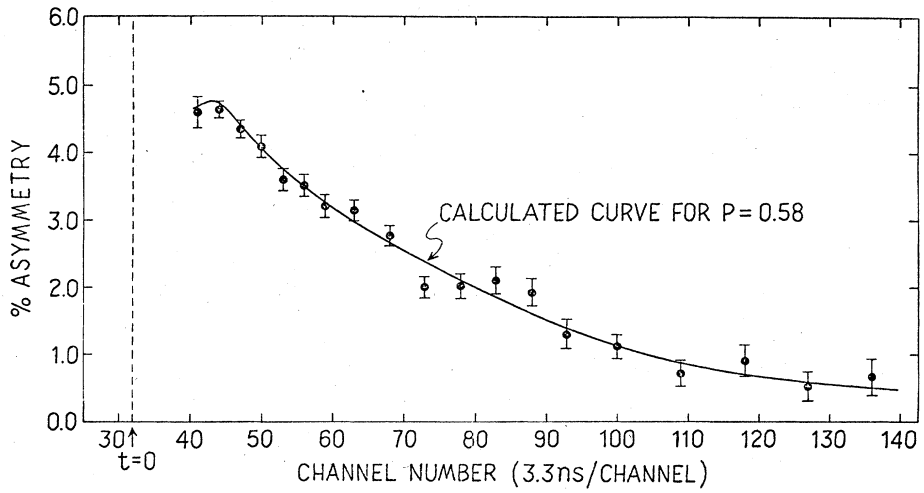


FIG. 21. Measured polarimeter asymmetry at $B = 2.9$ kG. The curve is best fit of expression (4.24) to the data and yields $P = 0.58$.

of different sources. Before discussing these applications we note that the overall efficiency of the instrument was about 10%, i.e., 10% of the incident positrons formed $\psi_T(\pm 1)$ and ψ'_T, decayed in the appropriate time window, and were detected. This clearly represents a major improvement when compared to the 10^{-3}–10^{-5} efficiencies characteristic of the polarization detection methods described previously. Finally, the peak asymmetry achieved ($A_{max} = 5\%$, Fig. 21) could be increased to 15–20% by use of higher magnetic fields.

c. *Applications of the time-resolved positron polarimeter*

We now list the most important applications in which the polarimeter has been used, and then discuss in some detail several possible applications which are of general interest and which have only been mentioned in the recent literature. The polarimeter in its initial form was first used to measure the polarization of positrons in the decay $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ (Dick *et al.*, 1963). A short time later a similar polarimeter was used to observe the g-2 rotation of positron spins in an experiment which measured the positron g-2 to about 1% (Rich and Crane, 1966) and eventually using the same type of polarimeter to 0.1% (Gilleland and Rich, 1969).

Most recently (Zitzewitz *et al.*, 1979) the instrument has been used to measure the polarization of the 1-eV positron beam (Sec. III.C) emerging from an MgO-coated gold moderator. A schematic diagram of this experiment is shown in Fig. 22. Slow positrons emitted into a 2π solid angle at the moderator were electrostatically accelerated to 500 eV, focused into a beam which

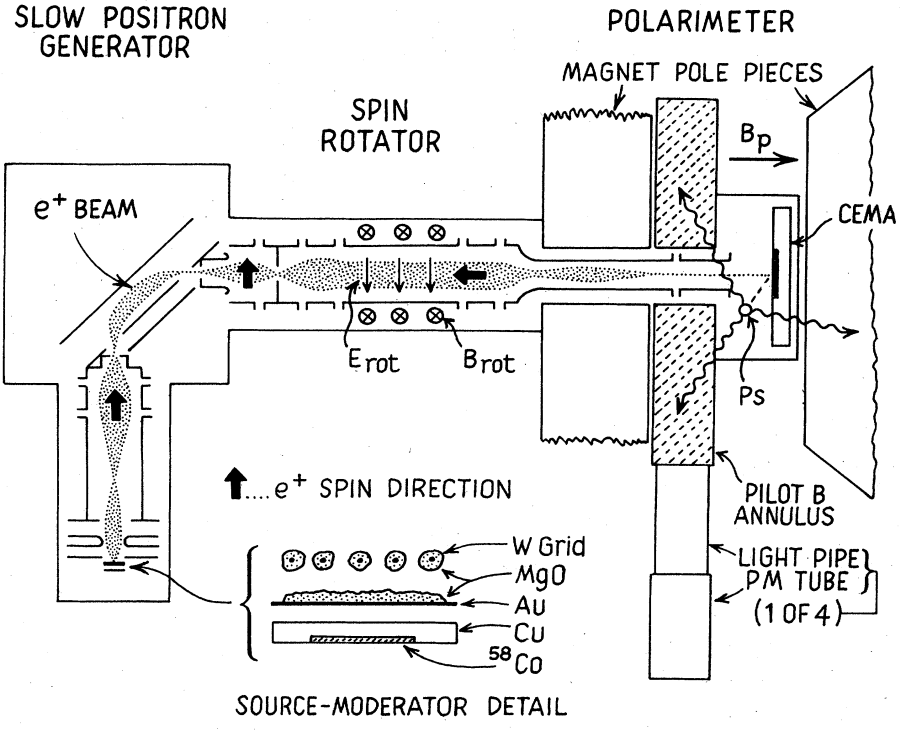


FIG. 22. Schematic diagram of the slow-positron source – spin rotator and polarimeter used in the recent discovery that the slow positrons emitted from an MgO-coated gold surface are polarized.

was bent through 90° in a cylindrical mirror analyzer, sent through a Wien filter spin rotator, and then by way of a hole in the yoke of a magnet sent into a polarimeter whose working field B_p was 6.3 kG. The polarimeter was of the type described previously, but with the essential difference that the positrons were allowed to impinge on a chevron electron multiplier array (CEMA). They formed Ps with about 10% efficiency on the CEMA and simultaneously caused a start signal by ejection of secondary electrons. The stop signal was, as usual, detection of the decay gammas in the plastic scintillator shown in Fig. 22. The reason for this modification is that it is impossible for the low-energy positrons used in this work to pass through a thin scintillator in order to give a start signal. The result of the measurement was that the polarization of the slow beam had a most probable value of $0.22^{+0.04}_{-0.02}$. The polarization of the positrons emerging from the source was calculated (after appropriate angle and energy averages) to be 0.34 ± 0.07 and with an estimated $(15 \pm 5)\%$ depolarization, an estimate based on an extrapolation of the calculations of Bouchiat and Lévi-Leblond (1964), the polarization of the emerging slow positron beam being thought to be 0.29 ± 0.06. This calculated result when compared with the measured value of $0.22^{+0.04}_{-0.02}$ shows that some additional depolarization (possibly spin relaxation) may be present in the MgO.

Several immediate applications of polarized slow positrons were discussed in the Zitzewitz *et al.* paper. We note here in particular the positron analogy to recent work on polarized low-energy electron diffraction (PLEED). This work demonstrates that PLEED complements low-energy electron diffraction (LEED) in investigations of crystal structure with PLEED being more sensitive to certain surface parameters [see Zitzewitz *et al.* (1979) for recent references to PLEED]. The substitution of polarized low-energy positron diffraction, PLEPD, for PLEED or low-energy positron diffraction (LEPD) for LEED should provide a consistency check on crystal structure models. Both LEPD and PLEPD should be feasible from the viewpoint of the slow positron beam intensities available; and, in fact, the first such work (LEPD—experimental detection: Rosenberg and co-workers 1980a, 1980b; Platzman and Mills, 1980; LEPD and PLEPD theoretical analysis: Feder, 1980) has recently been published.

The final subject which we address in this section is the proposed use of the positron polarimeter in a number of tests related to the weak interactions. A number of applications in this area were noted by Gerber *et al.* (1977) with the one of most general interest being the suggestion (Michel, 1976) that one could search for the effects of interesting forbidden or recoil order corrections, such as the weak magnetism and induced tensor terms on positron polarization. A detailed calculation of these effects was undertaken by Holstein (1977a, 1977b). The result of the calculation was that the helicity of the positrons emitted in nuclear beta decay could be written to first order in recoil as

$$h \simeq \beta\left[1+\frac{2}{3}\left(\frac{c^2}{a^2+c^2}\right)\left(\frac{m}{M}\right)\left(\frac{mc^2}{E}\right)\left(1-\frac{b}{c}-\frac{d}{2c}\right)\right] = \beta[1+\varepsilon]. \tag{4.25}$$

Here

m, M is the positron and parent-daughter average mass, respectively

$E = \gamma mc^2$ is the positron energy

a is the Fermi (vector) form factor

c is the Gamow-Teller (axial vector) form factor. Typically c and a are of similar magnitude for mixed (super-allowed) transitions.

b is the weak magnetism form factor. This form factor is of interest, apart from specific nuclear physics consideration, because its value may be directly related, through the conserved vector current (CVC) hypothesis (Feynman and Gell-Mann, 1958) to the often well measured isovector magnetic moment (e.g., the $M1$ gamma width or the magnetic-moment difference between initial and final states if these states are members of a common isomultiplet). Typically, for allowed transitions, $b \simeq 4Ac$ where A is the atomic weight of the decaying state. Measurement of b in situations where its value as predicted by CVC can be independently determined experimentally thus constitutes a test of CVC. We note that the CVC hypothesis is a requirement of the electro-weak gauge theories (Weinberg, 1977).

d is the induced tensor form factor. We assume in what follows that d arises from first-class currents only.

ε is the deviation of h from β due to the above terms.

We see from Eq. (4.25) and the above definitions that $\varepsilon \simeq 10^{-3}(mc^2/E) \simeq 10^{-3}$, where we have taken $b/c \sim d/c \sim 4A \gg 1$, $c^2/(a^2+c^2) \sim 1$, $M = 1800AM$, and $E \sim mc^2$. In light of previous remarks related to the uncertainty in depolarization estimates, a deviation of such small magnitude would not be directly accessible by an absolute polarization determination. It may, however, be possible to compare the polarizations of positrons of the same energy from two different radioactive sources to the requisite precision, and such an attempt is now being made at Michigan.

The essence of the technique is to *compare* the polarization of the positrons from the source of interest with the polarization of positrons of equal energy (as selected by a beta spectrometer) emitted from a comparison or normalization source such as a Fermi transition decay for which $b = c = d = 0$, so that ε (Fermi) $\simeq 0$. In general, the value of ε for the normalization source $[\varepsilon(N)]$ should be small enough that it may be neglected with respect to the value of ε for the source of interest. The comparison technique has the advantage that a number of systematic effects, such as positron depolarization on stopping in the Ps-forming substance, should be equal for positrons of equal energy from either source. These effects should therefore cancel to high order when the polarizations from the sources are compared. Since such systematic uncertainties are the primary source of error in the polarization measurements contemplated, a comparison of the polarization from the two sources can be made to an accuracy far greater than the absolute accuracy which can be attained in measurements on a single source. In effect, one could take the ratio of the experimental asymmetries measured for the two sources without even attempting to extract an actual polarization for either, since the ratio

yields the result $(1+\varepsilon)/[1+\varepsilon(N)]$, from which ε can be obtained if $\varepsilon(N) \ll \varepsilon$. We see from Eq. (4.25) that measured values of ε and E, along with the known values of m, M, a, and c, permit a direct determination of $b + \frac{1}{2}d$. The quantities of interest, b and d, may then be separately obtained by either (i) using a transition within an isomultiplet (e.g., ^{19}Ne) for which $d=0$, or (ii) by using a transition in a nucleus such as ^{12}N, where the combination $b+\frac{1}{2}d$ has been measured directly in positron-nuclear spin-alignment experiments [e.g., Calaprice (1978), which contains a review of various measurements of b and d].

The precision polarization comparison technique can also be used in a number of other weak interaction investigations including, in particular (i) improved limits (Holstein, 1977a) on the size of Fierz interference terms (mixture of S- and T-type weak interactions into the canonical V-A form) in nuclear beta decay, and (ii) tests of spontaneous left-right symmetry breaking (Beg *et al.*, 1977; Holstein and Treiman, 1977). In both of the above cases improved upper limits on the size of the effects would follow from polarization comparisons at the 10^{-3} level, i.e., the criterion for useful precision here is somewhat less stringent than that in the weak magnetism situation.

3. Ps in astrophysical environments

An active phase of current research in γ-ray astronomy consists of the search for γ-ray lines in the energy range below a few MeV. Such work is generally undertaken using balloon borne NaI or Ge (Li) detectors with either active (NaI) or passive (e.g., Pb) collimation to give angular resolution. Several gamma-ray lines in the MeV energy range have in fact been reported, with the first and still the strongest being at about 0.5 MeV (Johnson *et al.*, 1972; Chupp *et al.*, 1973, 1975; Leventhal *et al.*, 1978, 1980). The lines detected by Johnson and Leventhal originated from the direction of the galactic center, while the line observed by Chupp was associated with a solar flare. Explanations for the source of the galactic center radiation have been summarized by Leventhal *et al.* (1978), with one of the possibilities being direct e^+-e^- and Ps annihilation. The 0.5 MeV line, observed in solar flares, is also thought to have its origin in e^+-e^- and Ps annihilation. A detailed consideration of such processes in the medium of a solar flare was carried out by Crannel *et al.* (1976), while Bussard *et al.* (1979) worked on the analogous problem for the intergalactic medium. The research presented in these articles showed that the fraction of Ps (3γ) annihilation relative to free (2γ) annihilation, the width of the 511 keV line and the intensity of the line relative to other observed gamma lines can give quantitative information concerning the state of the medium (temperature, degree of ionization, etc.) from which the radiation is emitted. It should be emphasized, however, that while evidence for a 511 keV line from both solar flares and the galactic center is strong, identification of the 3γ background due to Ps annihilation is still marginal. It is hoped that further observations from satellite mounted gamma-ray detectors will yield the data necessary to observe Ps formation at the galactic center unambiguously, if such formation is indeed occurring.

4. Ps formation in optically active substances

Most naturally occurring sugars and amino acids are composed of only one of the two possible isomers (L or D) of the substance. The cause of this phenomenon, first noted by Pasteur in 1861, is still a subject of speculation and experimentation. The discovery of parity violation in weak interactions immediately led to the suggestion (Krauch and Vestor, 1957; Vester *et al.*, 1959; Ulbricht, 1959) that the molecular violation of parity noted by Pasteur might somehow be linked to nuclear parity violation and in particular to the fact that nuclear parity violation implies that the emitted beta particles have a net helicity. One of the initial suggestions was that the electrons from naturally occurring radionucleides might have an interaction different for the L isomer than the D isomer of the same molecule. The L and D isomers were presumably present in equal numbers (racemic mixture) when they were initially created chemically; however, even a slight preferential effect in one versus the other might, through evolutionary amplification, lead to the complete asymmetry now observed.

Numerous experiments were attempted following this suggestion. The experiments generally involved searches for preferential destruction of one of the isomers of an initially racemic mixture after exposure, either to beams of polarized electrons, or to electrons with an initial helicity emitted from radioactive sources. Positive effects were reported for some of this work [see, for example, Bonner *et al.* (1975)], but these have not proved to be reproducible [see, for example, Hodge *et al.* (1979)].

An alternative type of experiment involving Ps formation in L and separately D isomers of amino acids was carried out by Garay *et al.* (1973, 1974), while searches for preferential polarized muon interactions were undertaken by Lemon *et al.* (1974). No muon or muonium asymmetries were found; however, Garay *et al.* reported preferential formation of triplet Ps, at levels ranging from 6% to 30%, for D vs L isomers of several amino acids. The cause of the asymmetric Ps formation was ascribed to a possible helicity of the electrons in optically active substances, the sign of the helicity depending on the L or D character of the isomer considered. If such a helicity is in fact present in optically active molecules, it was suggested (Hrasko, 1973; Garay *et al.*, 1973; Garay and Hrasko, 1975) that it might also cause a preferential interaction of electrons with net helicity from naturally occurring radionucleides with the L vs D isomers of the substance in question. Such an asymmetric interaction, amplified by evolution, might then constitute the reason for the existence of naturally occurring optical activity. The detection and quantitative determination of a net electron helicity as a general property of optically active substances would also naturally be of interest for its own sake. Preliminary estimates (Ford, 1979; Hegstrom, 1980) indicate that the magnitude of the helicity in chiral molecules is zero to first order in the spin-orbit coupling interaction but does appear in second

order.

Unfortunately, Garay's experiment, which indicated a large helicity for the outer electrons in the amino acids tested, was found to be nonreproducible by Dezsi *et al.* (1974), Brandt and Chiba (1976), and others, who obtained null results for the asymmetry at the few per cent level for many of the amino acids used by Garay. In addition, Rich (1976) pointed out that the residual helicity at Ps-formation energies of the beta-decay positrons which were used by the above authors (initial energy typically several hundred keV) should be negligible as a result of velocity randomization in the slowing down process. Since any asymmetry present that is related to a helicity-selective Ps-forming interaction will be proportional to the product of positron and electron helicity, both of which would almost certainly be smaller than 10^{-2} in the experimental configurations used, asymmetries in the part-per-million rather than part-per-hundred range might be expected.

An experiment to search for asymmetries at the 100 ppm level is now underway at Michigan (Gidley, Rich, Van House, and Zitzewitz, 1980). The experiment is being carried out with a low-energy positron beam of controlled helicity (spin rotation provided by a Wien filter), as discussed in Sec. IV.D.2. The low energy of the incident positrons should minimize their velocity randomization at Ps-formation energies; and the spin rotation (helicity reversal) which can be provided will make it possible to look for asymmetries in one isomer, thereby elininating major systematic effects which can occur in switching from L to D isomers. These and other features of the experiment are discussed in some detail in the above article by Gidley *et al.* Finally, we should note that a different and possibly more powerful experimental technique for detecting electron helicity in chiral molecules would be to carry out a scattering experiment with a polarized beam whose helicity can be varied, incident on a thin target isomer. The variation in scattered intensity with incident electron-beam helicity (due to the electron exchange interaction) would be the observed signal, in direct analogy with the recent SLAC neutral current parity violation work (Prescott *et al.*, 1978). The intense polarized electron beams (based on photoemission from GaAs) used in that work would in fact be ideal for such an experiment.

V. RESEARCH ON $n = 2$ POSITRONIUM

A. Theory

As mentioned in Sec. I (see also Table I), calculations of excited state fine structure and estimates of excited-state optical and annihilation decay rates commenced with the first detailed Ps calculations by Pirenne, Berestetski, Landau, and Ferrell, who by 1951 had completed fine-structure calculations to order $\alpha^4 mc^2$. The major result of interest to this order is that the degeneracy with respect to j in hydrogen ($2^2S_{1/2} - 2^2P_{1/2}$) is lifted, so that, for example, $E(2^3S_1) - E(2^3P_1) = 12\,773$ MHz, a typical fine-structure splitting.

These calculations were extended by Fulton and Martin (1954) to include all terms of order $\alpha^5 mc^2$ in any two-fermion system. In particular, the results were evaluated for $n = 2$ Ps and are shown in Fig. 4. Fulton and Martin found that contributions to order $\alpha^5 mc^2$ arose from (1) recoil (Coulomb field pair production) and retardation (exchange of 1 and 2 transverse photons) terms (ΔE_B), (2) radiative corrections due to self energy (L) and vacuum polarization (V) effects ($\Delta E_{L,V}$), and (3) virtual annihilation (ΔE_A). These processes make contributions of from 15 to 300 MHz in the 2^1S_0 and 2^3S_1 states, while only (ΔE_B) and ($\Delta E_{L,V}$) contribute from 1–15 MHz in the various P states. The sum of the various contributions is shown in Fig. 4 with a more detailed breakdown being given as (all energies in MHz)

$$2^1S_0 - \Delta E_B = 112, \quad \Delta E_{L,V} = 260, \quad \Delta E_A = 15.6, \tag{5.1a}$$

$$2^3S_1 - \Delta E_B = 61, \quad \Delta E_{L,V} = 294, \quad \Delta E_A = -124.3, \tag{5.1b}$$

$$2P - \Delta E_B = -2.91 \quad (\text{all } P \text{ states}), \tag{5.1c}$$

$$\Delta E_A = 0 \quad (\text{all } P \text{ states}), \tag{5.1d}$$

$$2^1P_1 - \Delta E_{L,V} = 0, \quad 2^3P_2 - \Delta E_{L,V} = 3.81,$$

$$2^3P_1 - \Delta E_{L,V} = -2.12, \quad 2^3P_0 - \Delta E_{L,V} = -12.72 \tag{5.1e}$$

The experimental verification of the results expressed in Eqs. (5.1a)–(5.1e) is of great interest, since, as with the $n = 1$ hfs and decay rate calculations, the work provides a general test of quantum electrodynamics in a purely leptonic two-body system. In particular, terms which can be identified as analogous to the Lamb shift in hydrogen ($\Delta E_{L,V}$) make a contribution of 291 MHz to the $2^3S_1 - 2^3P_2$ splitting, so that the experiment to be described constitutes verification of radiative corrections in Ps at the level of 2%. We also note here, however, that $\Delta E(2^3S_1 - 2^3P_2)$; and, in fact, any of the $n = 2$ fine structure levels has an additional source of theoretical uncertainty at the several MHz level, due to uncalculated terms of order $\alpha^6 mc^2$ (Fulton and Martin, 1954).

B. Measurement of $n = 2$ fine-structure intervals

The problem of producing Ps in the $n = 2$ state proved to be extremely intractable, with a long history of usually well conceived but unsuccessful experiments, beginning with the work of Kendall in 1954. The first definitive observation of Ps Lyman-α radiation was reported by Canter, Mills, and Berko (1975). These authors present a summary of previous attempts to observe the $n = 2$ state, one of which (Varghese *et al.*, 1974), may in fact have detected $n = 2$ Ps. Details of this first observation of Ps Ly-α have been reported in a number of review articles and conference proceedings (for example, Mills *et al.*, 1977; Berko *et al.*, 1979), as well as in the original discovery paper. Since observation of Ps Ly-α has not in itself yet led to any new results of interest, we will instead concentrate our attention on the second phase of the work of the Brandeis group, namely, measurement of the $2^3S_1 - 2^3P_2$ fine-structure splitting.

Details of the Brandeis apparatus are shown in Fig. 23 (taken from Mills *et al.*, 1975). Slow positrons, produced when the positrons from a ^{58}Co source are incident on an MgO-covered gold foil converter, are magnetically guided by a 75-G, 150 cm long, curved

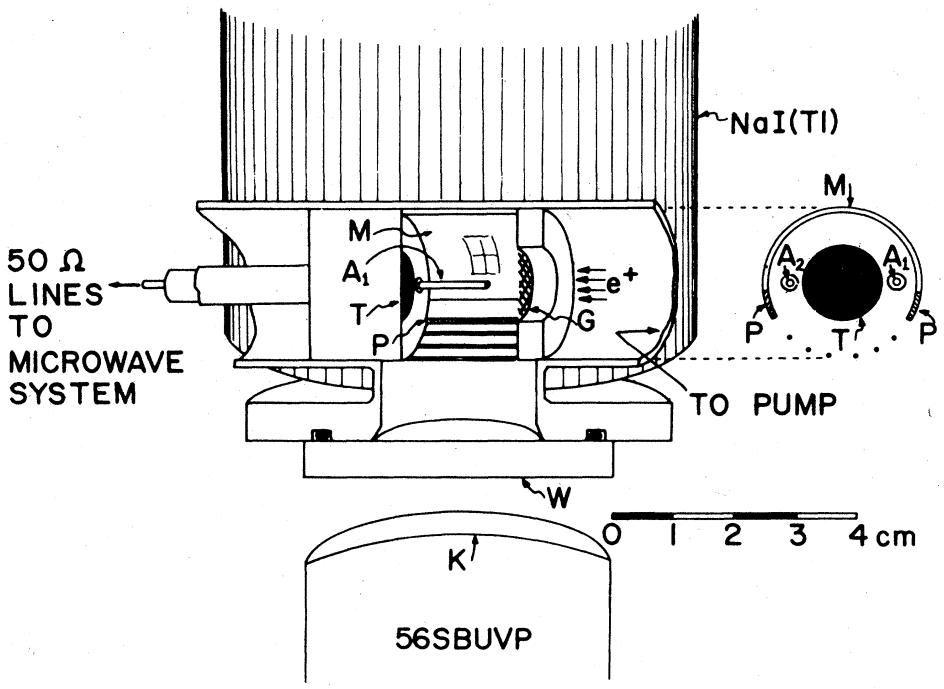


FIG. 23. Schematic diagram of the apparatus used to determine the $n=2$ fine-structure splitting in Ps. The various components are G, 95%-transmission tungsten grid; T, copper target; M, aluminized Suprasil quartz mirror; W, Suprasil quartz window; K, CsTe photocathode; P, support posts; A_1, input antenna; A_2 output antenna; NaI (Tl) annihilation detector coupled to an RCA 8575 phototube. (From Mills, Berko, and Canter, 1975).

solenoid to a copper surface (T) on which Ps in the $n=1$ and $n=2$ states is formed. The average residual field of the guiding solenoid in the cavity is about 54 G. Its effect on the data will be discussed later. The fraction of $n=2$ Ps formed is 10^{-3}–10^{-4}, with presumably roughly equal population of the 16 fine-structure states (six nondegenerate energy levels). The copper surface forms the end plate of a cylindrical TM_{010} microwave cavity with a resonant frequency of 8860 MHz and $Q \simeq 70$. Provision is made to observe the Lyman α (2430 Å) photons emitted from the $n=2 \rightarrow n=1$ transition (using the Amperex 56 SBUVP photomultiplier), as well as the annihilation radiation, using the NaI (Tl) detector. The Lyman-α photons can escape through a side of the cavity which has been replaced with parallel wires for this purpose.

In order to discuss the technique used to observe resonance, we should first note (see Fig. 4) that if there is no rf field present in the cavity, the 2^3S_1 state decays into 3γ's with a lifetime of 1.1 μsec, since the lifetime against an optical transition is essentially infinite. In addition, the various 2^3P states decay to the 1^3S_1 state with lifetimes of a few nanoseconds. The Lyman-α photons emitted can serve as the start signal in a delayed coincidence spectrum in which one of the 1^3S_1 annihilation gammas serves as a stop signal. This coincidence technique (Lyman-α photon start–1^3S_1 gamma stop) was, in fact, the method used to first identify $n=2$ Ps decay (Canter *et al.*, 1975). The presence of an rf electric field at the predicted $2^3S_1 - 2^3P_2$ transition frequency $[f(2^3S_1 - 2^3P_2)] = 8625$ MHz will drive this transition and therefore cause an increase in the gamma radiation emitted within a few 1^3S_1 lifetimes (several hundred nanoseconds) of a Lyman-α start. In other words, we have the time sequence

$$e^+ - 2^3S_1 \xrightarrow[\tau\sim 1\ \mu\text{sec}]{} 2^3P_2 \xrightarrow[\tau\sim 3\ \text{nsec}]{} 1^3S_1 \xrightarrow[\tau=140\ \text{nsec}]{} 3\gamma .$$

The microwave power necessary to saturate the $2^3P_1 - 2^3P_2$ transition is readily attainable, and at the power used in the experiment (0.41 mW) the resonance line width was approximately doubled due to power broadening.

The delayed coincidence time spectrum for rf on and rf off (Fig. 24) clearly shows the enhancement in counting rate due to the rf induced 2^3S_1-2^3P_2 transition. Thus $\Delta \equiv$ (rf on counting rate–rf off counting rate) should be zero in the absence of the transition. The shortening of the measured lifetime (123 ± 8) nsec for this signal from the 1^3S_1 vacuum lifetime (140 nsec) is attributed to wall collisions by the Brandeis group. The large $t=0$ intensity (about ten times the delayed intensity) is thought to be due primarily to signals in the 56 SBUVP tube caused by decay γ's. The 10 nsec component shown in Fig. 24 has not yet been completely accounted for, but could result from a high velocity $n=2$ Ps component (Mills *et al.*, 1977). In any case, it is only the long delayed signal which is used to determine the resonant frequency.

The resonance curve itself is obtained from the experimental signal defined as $S(f) = [N_{on}(f) - N_{off}(f)]/N_{off}$, where $N_{on}(f)$, $N_{off}(f)$ refer to the delayed coincidence counting rates, with rf on and off at frequency f, as discussed above. The theory of the resonance process shows that the signal should have a Lorentzian line shape, i.e., that $S(f)$ should be given by the equation

$$S(f) = \tfrac{1}{4} A \delta^2 [(f - f_0)^2 + \tfrac{1}{4}\delta^2]^{-1} .$$

Here A is proportional to the input power density and the time during which the signal is applied, while δ is the full width at half maximum signal strength. In the

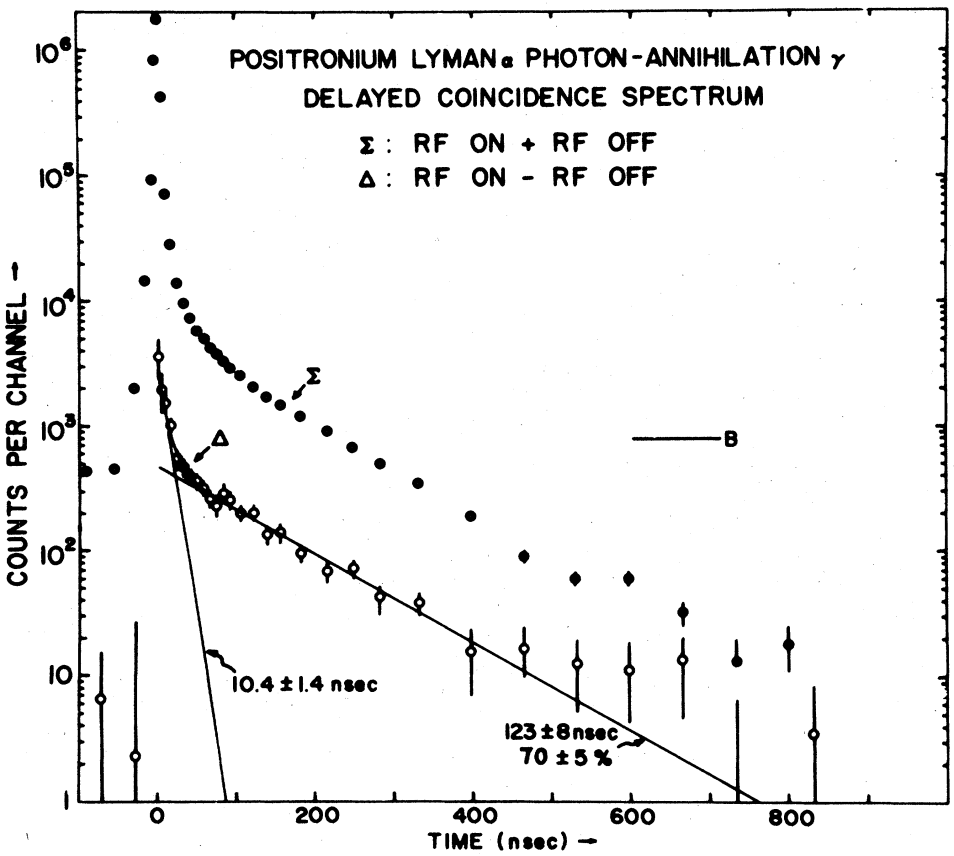


FIG. 24. The lifetime spectrum observed in the Ps $n=2$ fine-structure determination. The quantity B, the uncorrelated background, has been subtracted from the sum spectrum Σ.

absence of power broadening, δ is the natural linewidth, i.e.,

$$\delta_{nat} = \lambda(2^3P_2 - 1^3S_1)/2\pi ,$$

where λ is the optical decay rate (300 MHz) of the 2^3P_2 state, so that $\delta_{nat} \simeq 50$ MHz. As mentioned earlier, a power density of 0.4 mW was used, giving $A \sim 0.4$ and $\delta \sim 100$ MHz, i.e., the linewidth was doubled by power broadening and also, to a lesser extent, by Ps collisions with the cavity wall. Figure 25 shows the resonance curve $S(f)$ obtained by the Brandeis group. Also shown is a logarithmic first difference signal

$$S'(f) \equiv [N(f+\Delta) - N(f-\Delta)]/[N(f+\Delta) + N(f-\Delta)]$$

$$= [S(f+\Delta) - S(f-\Delta)]/[2 + S(f+\Delta) + S(f-\Delta)]$$

obtained in separate runs with $\Delta = 30$ MHz. The solid line is a best fit of S (or S') to the data and yields $A = (11.4 \pm 0.6)\%$, $f_0 = (8628.4 \pm 2.8)$ MHz, $\delta = (102 \pm 12)$ MHz, and $\chi^2 = 12.1$ for 10 degrees of freedom. These fitted values of A and δ are in rough agreement with values which can be calculated under a number of reasonable assumptions from knowledge of the experimental parameters (input power level, accidental background, etc.). Numerous systematic tests to investigate any possible dependence of f_0 on cavity geometry, cavity wall condition, rf power, etc., were conducted. The results were always consistent with the above value of f_0.

The experimental value of f_0 should only be compared to the theoretical value after corrections for Zeeman and motional Stark shifts due to the residual 54-G average field from the slow positron guiding solenoid which is present in the cavity. The effect on Ps of

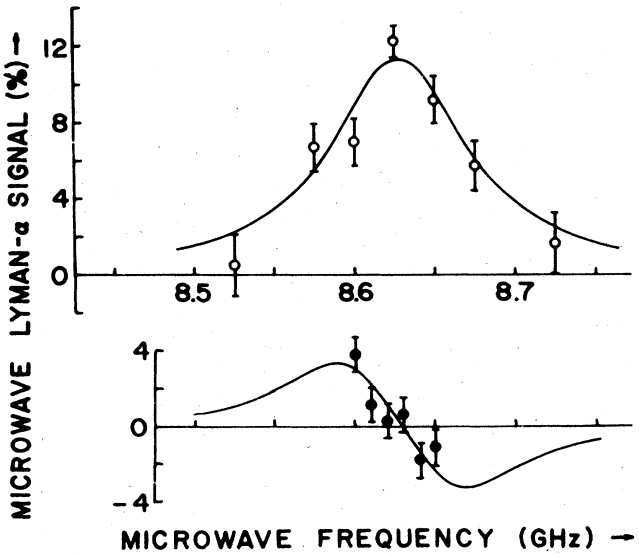


FIG. 25. Resonance curves obtained in the Ps $n=2$ fine-structure determination. The upper curve represents the observed signal (S), while the lower curve represents the logarithmic first difference signal (S') discussed in the text.

motion through a magnetic field has been calculated by Curry (1973) and Lewis and Hughes (1973). Using their results and assuming the Ps kinetic energy (T) to be in the range $0 < T < 1$ eV, the Zeeman and motional Stark shifts give corrections to $f(2^3S_1 - 2^3P_2)$ which range from -4.65 MHz ($T = 0$ eV) to -1.03 MHz ($T = 1$ eV and Ps moving perpendicular to B). Thus $f(2^3S_1 - 2^3P_2)$ should be between 8620 and 8624 MHz as compared to the measured value $f_0 = (8628.4 \pm 2.8)$ MHz, where the experimental error quoted is entirely statistical in nature. This result, although slightly above the range of theoretical values, is not considered inconsistent with theory. It is taken to confirm the 231 MHz $\alpha^5 mc^2$ correction to ΔE $(2^3S_1 - 2^3P_2)$ with an overall error of ±6 MHz, of which ±2.8 MHz is statistical, as mentioned above, the remainder being due to both uncertainty in the size of the motional Stark shift and to shifts which can originate from rf power drifts.

In considering prospects for improved accuracy in this work, we should note that replacing the slow-positron magnetic guiding field with an electrostatic lens system (for example, an electrostatic lens system with losses of less than 20% has been developed for the various Michigan slow-positron experiments) could, in conjunction with reduced power input, produce an unshifted and almost unbroadened line whose width would be the natural linewidth of 50 MHz, i.e., 6000 ppm of f. Zeeman and motional Stark shifts would be eliminated by the magnetic field reduction, but Doppler broadening would still contribute about 10 MHz to this linewidth if the mean kinetic energy of the Ps were about 1 eV $[(v/c) \times f \simeq 8$ MHz]. The possibility of producing thermal $n = 2$ Ps emitted from room temperature (or colder) surfaces (Sec. III.C) would, however, lead to a factor of at least 5–10 reduction in the Doppler broadening.

If one assumes that all systematic effects which could contribute to a shift in f are eliminated, it is interesting to speculate on the accuracy that could be obtained in Ps fine-structure measurements, statistics being the only limiting factor. In view of somewhat analogous experiments in hydrogen and more specifically, muonium, in which fine or hyperfine resonance lines are split to 10^{-3} with some confidence, it might be expected that similar splitting would be feasible in Ps, thus leading to a several-part-per-million range of accuracy for the Ps fine-structure energy level determination. Such accuracy represents roughly a one-hundred-fold improvement over the statistical precision of the current measurement and would therefore necessitate up to 10^4 times the amount of data as were obtained in that experiment. Since typical runs in the Brandeis work were of one-day duration, it is clear that major improvements in the slow-positron beam rate, or in the efficiency of $n = 2$ Ps production will be necessary before measurements at the level of accuracy now obtained for the hydrogen or the muonium fine structure can be contemplated. In this connection, we note that a strong start towards a beam rate improvement was discussed in Sec. III.C (Mills, 1979). In addition, an improved version of the experiment described in this section is currently under way at Brandeis.

VI. PROSPECTS FOR FUTURE PROGRESS

A. Summary

It is clear from the content of this review that during the past five years there has been outstanding progress, both experimental and theoretical, in the use of Ps to test quantum electrodynamics. As has been emphasized, Ps represents a particularly important theoretical challenge, because, unlike other simple systems which have been used to test various aspects of QED (electron and muon g-2, muonium, hydrogen, etc.), Ps combines the need for an exact two-body bound-state relativistic formulation in combination with the added feature of virtual annihilation channels ($e^+ + e^- \to \gamma,\ 2\gamma, \ldots \to e^+ + e^-$). The latter cause observable effects in the $n = 1$ hyperfine splitting (W) and triplet decay rate $[\lambda(1^3S_1)]$, as well as in the $n = 2$ fine structure interval $[E(2^3S_1 - 2^3P_2)]$.

In addition to its use as a rigorous testing ground for studying QED, I have shown that the Ps atom can be used for the investigation of problems in such diverse areas as astrophysics, biophysics, precision metrological measurements, surface physics, and in tests of symmetry principles. There seems to be every reason to believe that further progress will be made in extending and deepening these studies during the next five years.

As regards the major QED tests, it seems likely that improvements by a factor of two in the measurement of W (W determined to perhaps 3 ppm) will be forthcoming in the near future. Of more significance, however, is the possibility of completion of the calculation of all diagrams of order α^2 in W, so that a test of W at the 3 ppm level, rather than at the current level of 100 ppm, may then be achieved. Since radiative corrections constitute about 1% of W, such a comparison would directly test these corrections to a few hundred ppm, thus providing one of the most accurate tests to date of radiative corrections in a bound system.

New measurements of $\lambda(1^1S_0)$ and $\lambda(1^3S_1)$ are also under way, and improvements in the precision of these quantities in the neighborhood of a factor of three to ten [$\lambda(1^1S_0)$ at the 1000 ppm level and $\lambda(1^3S_1)$ at the 100 ppm level] may be envisioned during the next several years. The measurements would, however, still be somewhat behind the theoretical calculations, currently accurate to 10^{-4} and 10^{-5}, respectively. We note again, however, that these decay-rate measurements, and in particular the $\lambda(1^3S_1)$ determination, allow for the most accurate comparison between theory and experiment for any atomic, nuclear, or elementary particle decay rate measured to date. They thus have a significance beyond the order α QED test that they currently provide.

Turning last to the $n = 2$ measurements, I should emphasize that the preliminary fine-structure splitting obtained in 1975 led to a test of radiative corrections accurate to 2%, including all experimental and theoretical uncertainties. Thus this first measurement in the $n = 2$ level had approximately the same sensitivity, as a test of QED, as had been obtained from the more developed Ps hfs research, primarily because the order α^2 hfs terms are still uncalculated. It is emphasized

that an order of magnitude improvement in this measurement, coupled with a calculation of the order $\alpha^6 mc^2$ terms, mentioned in Sec. V.A, would lead to a test of radiative corrections in Ps of accuracy (10^{-3}), commensurate with that obtained from the current Ps hfs work *including* a calculation of order α^2 radiative corrections (W_2). It seems clear that in view of the importance of testing radiative corrections, as well as the relativistic two-body formulation of quantum electrodynamics in a bound purely leptonic system, the $n=2$ Ps fine-structure measurements should be given the highest priority.

Another technique of future interest as regards excited-state Ps is the possibility of two-photon Doppler-free pumping of states such as the $n=2$ state. As has been mentioned (Sec. III.C), such experiments have been under consideration for a number of years with the eventual goals being either a more efficient method of populating the excited levels and/or a precision measurement of the transition frequency, so that, as in recent work on hydrogen (Wieman and Hänsch, 1980), a determination of ground-state recoil and radiative corrections can be made.

B. Conclusions

As discussed throughout this review and summarized in the last section, there has been strong recent progress in the use of Ps as a tool for studying QED. This progress will in all likelihood continue and even accelerate in the near future. The question presents itself, as it has during the past decade, whether verification of QED calculations in various bound and free systems to higher and higher order improves our understanding of leptonic structure and interactions in a way commensurate with the level of effort involved. This question is of course subjective, so that a quantitative answer to it cannot readily be formulated. However, it can be noted that although the theory of quantum electrodynamics has numerous well-known logical inconsistencies, there is at present no alternative to or extension of QED that makes either new predictions or solves the logical problems referred to above. However, since major new conceptual breakthroughs often arise from small deviations in predictions of existing theories, in the absence of definitive new theoretical suggestions, the improvement of the classical low-energy tests of QED, with the Ps work an outstanding example, appears to be as fruitful a method as any for uncovering the next level of the theoretical structure. In addition, it can be noted that an extremely important and useful result of the research effort, motivated initially to study the fundamental questions referred to above, has been a host of applications which is particularly marked in the case of Ps (Sec. IV.D). Such "spin-off" has provided an exciting and important aspect to the basic QED-oriented research from which it was derived.

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