The deuteron in high-energy physics*

Lars Bergström and Sverker Fredriksson†

Department of Theoretical Physics, Royal Institute of Technology, S-100 44 Stockholm, Sweden

The deuterium nucleus plays an important role in several branches of high-energy physics. We review its present status as a neutron source, a relativistic bound state, a collective six-quark state and a double scatterer.

CONTENTS

I. INTRODUCTION

The importance of the deuterium nucleus has been tremendous for our understanding of nuclear forces, and hence of strong interactions. The successful theory, due to Hulthén and Sugawara (1957) and others, for the bound proton–neutron state is, however, not the last word about the usefulness of deuterium nuclei for basic research. In recent years there has been a growing consensus that the deuteron has some fundamental properties that make it a particularly promising tool for high-energy physics research. The hope is that it will help to unmask secrets about elementary collisions that would be invisible in collisions with single protons in, for instance, a hydrogen target. Here we do not think of the purely technical advantage of having deuterium in a heavy-liquid bubble chamber in order to improve the rate of inelastic collisions. The real challenge with a deuteron target is that it is more complicated than a single proton, but still much easier to handle theoretically than heavier nuclei.

This sounds cryptic enough, regarding that the main trend in today's high-energy experimentation is to simplify the initial state as much as possible, for instance by turning to electron–positron and neutrino-proton collisions. Some physicists regard even a single proton as such a mishmash of quarks and gluons that a deuteron would be far too complicated to be of any use in high-energy physics. That might be true for deuteron production in high-energy collisions, but it is precisely the *complications* with a deuteron target or projectile that make it so attractive, irrespective of (and even because of!) the fact that we do not clearly understand the structure of a single nucleon.

Regarding the deuteron as a set of two quasifree nucleons, the most self-evident use comes from the fact that it provides the simplest possible *neutron* target. Heavier nuclei contain up to 60% neutrons, but are nevertheless not as appealing since they bias the elementary interactions with other, less understood, nuclear effects. Neutron beams are also in use, but experimental data here suffer from considerable uncertainties due to technical problems. Deuteron targets have indeed given us unique information about the difference in internal structure between a proton and a neutron. By applying symmetry laws to a neutron target one can also draw conclusions about the proton itself. These aspects of using deuteron targets will be discussed in Sec. II.A.

However, the proton and the neutron in a deuteron are not free. In fact, this is probably the theoretically most studied bound state in physics. These studies have led to an understanding of, for instance, the relativistic effects on the wave functions, which has been useful in other branches of physics. In particular, positronium and, more recently, quarkonium physics has benefitted from these developments, as will be discussed in Sec. II.B.

The most interesting complication in a high-energy hadron-deuteron collision is that only 80–85% of the collisions seem to be with one of the nucleons with the other as a passive "spectator." In the remaining 15%–20% of the events both nucleons take active part, one way or another, in the interaction with the projectile. Such a double scattering would be rather uninteresting were it not for the fact that it occurs *within an extremely short period of time*. In the rest system of an energetic projectile (with $p_{\rm lab} > 5$ GeV/c, say) the whole process is so fast that the uncertainty principle simply forbids it to look like two successive collisions with normal properties. There is, for instance, not time enough for the final state from the first collision to develop fully before the second collision. The deuteron therefore serves as a tiny "bubble chamber" that records the space-time development of a hadronic production process at its very earliest stage. Heavier nuclei are naturally even more interesting in this respect, because with them one can "follow" the developing process in the nucleus for a longer time. It is, on the other hand, impossible experimentally to keep track of double, triple, and more complicated scatterings. With deuterons this is, however, a rather straightforward procedure. The interesting double scatterings must occur in so-called even-topology events ("hp"), where the number of outgoing charged tracks is even. These events also include all collisions on the proton alone, as well as the (few) hits on the neutron where the spectator proton happens to have a

*Dedicated to Lamek Hulthén on his 70th birthday.

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Fermi momentum large enough to make a visible track. Odd-topology events ("hn") come from the remaining collisions with a neutron alone. Since hp and hn collisions are almost equally abundant, the double scatterings make even-topology events dominate over odd-topology events. The probability of having a visible proton spectator can be computed from the deuteron wave function, so by a simple subtraction procedure one can estimate all relevant observables for double scattering events alone, once they are known for odd- and even-topology events separately.

The properties and understanding of these double collisions will be the topic of the remaining Secs. III and IV of this review. There are in fact two schools of opinions on how to look upon an extremely fast double collision. They are built on orthogonal views on the time development of a hadronic collision.

Section III presents the idea that some or all events where both nucleons interact with the projectile are nothing but immediate collisions, with the whole deuteron behaving as one single particle. This is the extreme view of *collective* models. It is based on the conjecture that the projectile, under certain conditions, does not have time to experience separate collisions, and therefore feels only one encounter with the whole deuteron. Different models prescribe different conditions for such a collective interaction.

The *fluctuon* model and related pictures ("bag," "cluster," etc.) postulate that a deuteron reacts like one single particle whenever the two nucleons are closer together than a typical nucleon radius of around 1 fm. The probability for this is 8%–12%, so these models allow also ordinary double collisions on the 10% level for a hadronic projectile.

The *collective tube model* is more radical. At high energies it does not allow any conventional double collisions. The proton and the neutron are assumed to react collectively whenever they overlap in the transverse direction relative to the path of the projectile, irrespective of their mutual distance in the longitudinal direction. All "double scattering" events are hence of a collective nature.

Collective models have exciting theoretical consequences. They claim that the deuteron behaves now and then like a heavy *six-quark* particle. If this is true we have here a unique possibility to "manipulate" the number of quarks in a particle and hence to give a new dimension to the tests of quark–parton models. It is also interesting to study the trivial change in the kinematics of a production process due to the doubled target mass of a collective interaction. It leads to production in regions of phase space that are forbidden in collisions on a stationary nucleon, and the effects are much stronger than what is expected from ordinary Fermi motion.

Section IV is devoted to more conventional models for the double scattering in deuterium. Also these contain interesting assumptions about the space-time development of particle interactions that can be tested only with nuclear targets.

According to one such picture, with *fan diagram dominance*, all particles coming from the first collision cannot rescatter on the other nucleon for the simple reason that they do not "exist" until after a certain time has passed. This formation time is determined from the uncertainty relation and the velocity of the particle in question. Rescattering can take place only if the newborn hadron has had time to develop its final state before passing the next nucleon.

The orthogonal *eikonal model* prescribes that only the projectile itself can rescatter. The newborn hadrons from either collision fly directly out without further interaction.

A problem here is that one cannot be sure that the projectile rescatters in the shape of the original particle. Perhaps it is sometimes excited to a resonant state before interacting again. This question of "inelastic screening" is best studied in elastic proton–deuteron collisions, where the proton in such a situation would be excited to a baryon resonance and then de-excited to a proton by the second collision.

The *additive quark model* is a sort of compromise between those two extremes. Here the projectile rescatters via its own "spectator" quarks, i.e., the valence quarks that do not take part in the first collision. Since these already "exist," they do not need any formation time to rescatter. The newborn quarks from the first collision, however, have to wait for a while before being able to interact with the second nucleon.

It goes without saying that the confrontation of these ideas with experimental data is of utmost importance for our understanding of the details of particle interactions. Although the deuterium nucleus plays a special role here, there is naturally also a wealth of data from heavier nuclei that has been, and will be, used to test these theoretical ideas. For further details we direct the reader to an upcoming review article on high-energy collisions with nuclear targets by Bergström, Berlad, Eilam and Fredriksson (1980).

Finally a few words about experimental techniques. Deuterium is mostly used in heavy-liquid bubble chambers, where it serves both as target and detector, but also in internal gas-jet targets. Several important experiments are performed with the deuteron as a projectile for collisions with protons and heavier nuclei. This makes it easier to study particle production in the deuteron fragmentation region. An extreme example is the use of deuterons in one or both of the rings at the CERN ISR.

II. THE DEUTERON AS A NEUTRON-PROTON BOUND STATE

A. The relevance of the neutron

One of the most apparent advantages of using a deuterium target is the possibility to extract information on the neutron. It is true that recently high-energy neutron beams with reasonably well-defined properties have been prepared. However, one disadvantage with these is that only *np* and *nA* reactions can be studied. For meson–neutron and lepton–neutron interactions one is presently in need of using deuterium and heavier nuclei as target, and to extract from those experiments the properties of the neutron. As we will see in Sec. II.B, the extraction procedure is not without complications, especially if one is interested in, say, differential cross sections near the kinematical boundaries.

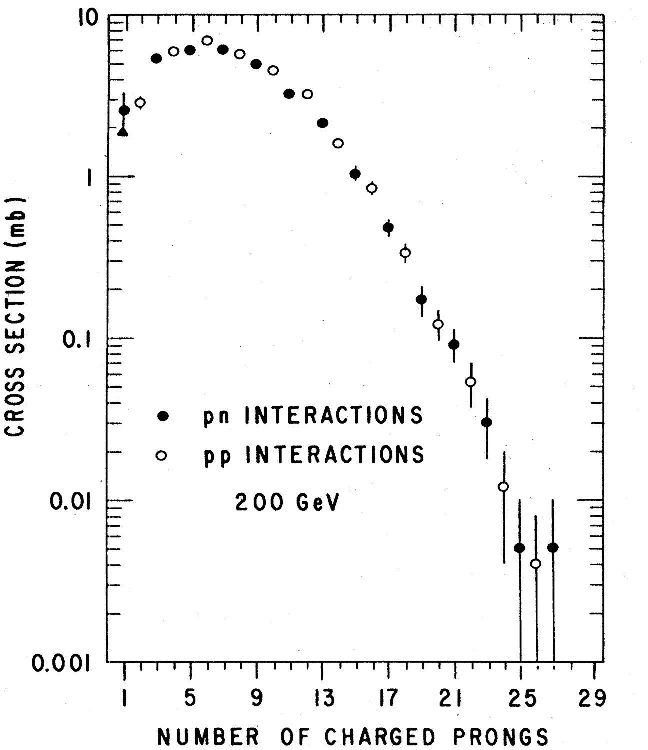

FIG. 1. The charged multiplicity distribution for 200 GeV/c pn and 205 GeV/c pp interactions (Dombeck *et al.*, 1978).

For more general properties, such as total and topological cross sections, the situation is more favorable and simple subtraction recipes usually suffice.

The notion of isospin symmetry is a cornerstone in particle physics, and a good way to test it in inelastic scattering is to use a deuterium target. When the rescattering events are subtracted and the effect of Fermi motion corrected for, the spectra of hn collisions can be deduced. In Fig. 1 the topological cross section σ_N is shown as a function of N, the number of charged prongs, for pp and pn interactions at 200 GeV/c (Dombeck *et al.*, 1978). In Fig. 2 the corresponding quantity is shown for π^+n and π^-p interactions at 100 GeV/c (Lys *et al.*, 1977b).

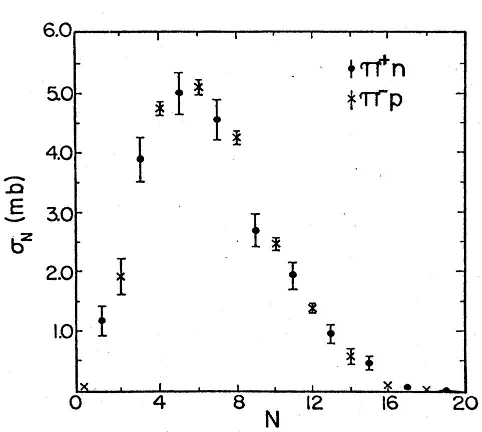

FIG. 2. The charged multiplicity distribution for 100 GeV/c π^+n and π^-p interactions (Lys *et al.*, 1977b).

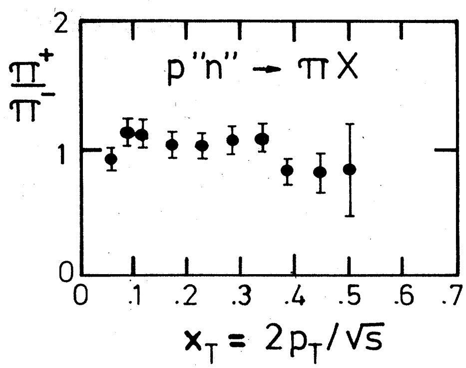

FIG. 3. The ratio of π^+ to π^- production yields versus $x_T = 2p_T/\sqrt{s}$ for 400 GeV/c $p''n''$ collisions (Antreasyan, 1979). See text for definitions.

The pioneering experiment on large-p_T production off nuclear targets [by the Chicago-Princeton group, Cronin *et al.* (1973); Antreasyan *et al.* (1979)] included a measurement of the ratio of produced π^+ to π^- for $p''n''$ collisions, where $p''n''$ is simply the difference between pd and pp (see Fig. 3).

As can be seen, all these experiments nicely confirm the isospin invariance (in the last experiment, the drop of π^+/π^- at large x_T has possibly the explanation that the spectrometer is no longer at 90° in the cms in that kinematical region).

An interesting experiment with deuterons has been performed at the intersecting storage rings (ISR) at CERN (Clark *et al.*, 1978). The reactions $dd \to \pi^0 X$ and $dp \to \pi^0 X$ have been measured at $\sqrt{s} = 53$ and 63 GeV. The pion spectrum resembles very much that of pp collisions at the same cms energy, apart from an overall factor of 4 and 2, respectively (from charge symmetry). This would favor an independent collision model, were it not for the low statistics.

A very interesting thing to measure, in view of the current interest in quark-parton models, is the "deep-inelastic" structure of the deuteron. The idea is to use a simple probe, i.e., one with no internal structure, e.g., an electron, to measure the (interior) charge distribution in a deuteron. Since the electron has a pointlike coupling to the photon, that part of the scattering process is calculable in quantum electrodynamics

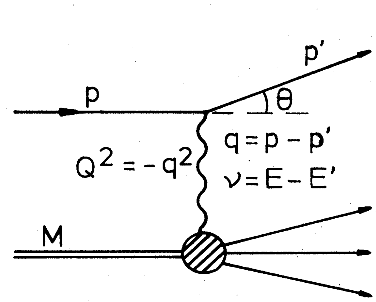

FIG. 4. One-photon exchange diagram for deep-inelastic lepton–hadron scattering. The four-momentum of the ingoing and outgoing electron is p and p', respectively. E, E' denote the corresponding energies, θ is the laboratory scattering angle, and M is the target mass.

(QED), and from measurements on the final state one can extract information on the "blob" illustrated in Fig. 4. In fact, the electron merely serves as a source of virtual photons, which can then probe the target. Since the photons are virtual, both their energy $\nu = E - E'$, and momentum transfer squared, $q^2 \equiv -Q^2$, can be varied independently by specifying the scattering angle θ and the energy of the scattered electron in the laboratory system. For deep-inelastic lepton–nucleon scattering, current conservation (gauge invariance) and parity conservation can be used to show that the information is contained in only two structure functions, W_1 and W_2. Hence, the differential cross section can be written

$$\frac{d\sigma}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\left[W_2(\nu, Q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu, Q^2)\sin^2\frac{\theta}{2}\right]. \tag{2.1}$$

In models for the hadrons, these structure functions can be calculated and compared with the data. In parton models, where the charge is carried by spin-1/2 constituents within the nucleons, the main prediction is that (Bjorken, 1969; Feynman, 1972)

$$2MW_1(\nu, Q^2) \to F_1(x) \text{ and } \nu W_2(\nu, Q^2) \to F_2(x) \tag{2.2}$$

in the deep-inelastic limit, $Q^2, \nu \to \infty$, where the scaling variable $x \equiv Q^2/(2M\nu)$ is kept fixed. If the charged constituents have spin 1/2 (which is the case if one identifies partons with quarks), then

$$F_2(x) = 2xF_1(x). \tag{2.3}$$

Here x can be interpreted as the momentum fraction of the nucleon which is carried by the constituent coupling to the photon.

The quark content of the proton is expected to be uud + quark–antiquark ($q\bar{q}$) pairs, that of the neutron ddu + $q\bar{q}$ pairs. The virtual quark–antiquark pairs ("sea quarks") are concentrated in the low-x region, which has been confirmed by neutrino experiments. It is generally believed that the admixture of charm–anticharm ($c\bar{c}$) and heavier quark pairs is very small in a nucleon, and is therefore often neglected. In some special processes (e.g., diffractive production of charmed particles) they may be of importance, as has been noted by Brodsky *et al.* (1980).

Now the structure function of the proton can be written as

$$F_1^p(x) = \tfrac{4}{9}(u+\bar{u}) + \tfrac{1}{9}(d+\bar{d}) + \tfrac{1}{9}(s+\bar{s}) + \text{heavier } q\bar{q} \text{ pairs} \tag{2.4}$$

and for the neutron

$$F_1^n(x) = \tfrac{4}{9}(d+\bar{d}) + \tfrac{1}{9}(u+\bar{u}) + \tfrac{1}{9}(s+\bar{s}) + \text{heavier } q\bar{q} \text{ pairs}, \tag{2.5}$$

where $u = u(x)$ etc. are the x distributions of the respective quarks. These relations come about because the isospin hypothesis implies that the distribution of u quarks in a proton equals that of d quarks in a neutron, and so on. One can then see that the ratio of F_1^n to F_1^p has the bound

$$1/4 \le F_1^n(x)/F_1^p(x) \le 4. \tag{2.6}$$

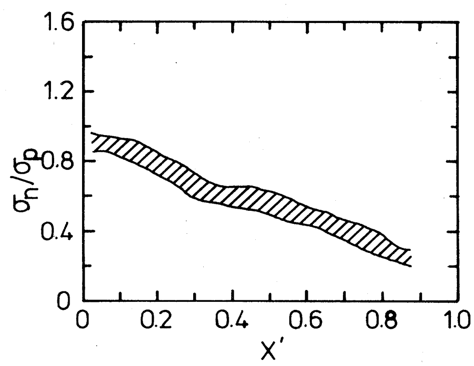

FIG. 5. The ratio of the deep-inelastic neutron cross section, extracted from deuteron data, to the proton cross section [Stein *et al.* (1975); for a recent data summary see Bodek *et al.* (1979)]. The variable x' equals $1/\omega'$, where $\omega' = (2M + M^2)/Q^2$. The band contains the bulk of data.

A famous SLAC experiment with a deuterium target (Bodek *et al.*, 1973, 1979; Stein *et al.*, 1975; Miller *et al.*, 1972) nicely confirmed this, as shown in Fig. 5. The ratio is indeed within the limits, and seems to approach 1/4 for large x. This indicates that the d quarks in the proton have a faster falloff with x than the u quarks (and *vice versa* for the neutron). For large x, the ratio then simply measures the ratio of (quark charge)^2 for d and u. In the quark model this ratio is 1/4.

It is, however, difficult to unfold the deuterium data for large x to extract the neutron part. Close to the kinematic limit there are considerable "smearing corrections" which should be applied due to the internal (Fermi) motion of the nucleons in the deuteron. To solve this problem one is of course in need of detailed information on the deuteron wave function.

B. Aspects of the bound state

A fundamental problem in the relativistic field theories of particles is that of the bound states. Whereas the problem is easily formulated and solved for the two-body bound states in the nonrelativistic theory, the complications are enormous in the relativistic case. Here the deuteron has a very important role as a model system which can be used when comparing theory with experiment. Of course the deuteron bound state is interesting in its own right, but the knowledge gained when studying it can be applied to other interesting two-body systems, such as positronium and muonium, not to mention the mesons, which are believed to be bound quark–antiquark states.

The appropriate bound-state equation in relativistic field theory is the Bethe–Salpeter equation (Salpeter and Bethe, 1951; Gell-Mann and Low, 1951). For a two-fermion state (with momenta p_1 and p_2, respectively) the amplitude $\chi(p_1, p_2)$ fulfills the equation

$$\chi(p) = S_F'^{(1)} S_F'^{(2)} \int \frac{d^4p'}{(2\pi)^4} K(p'-p)\chi(p'), \tag{2.7}$$

where the fermion propagators are $S_F'^{(i)}(p_i)$, and the "kernel" responsible for the interaction between the particles is denoted by $K(p'-p)$. The equation is stated in the center of mass, where $P=p_1+p_2=(M,0)$; the relative momentum is $p=\frac{1}{2}(p_1-p_2)$. This equation is exceedingly difficult to solve in realistic cases, and therefore some approximation has to be made to be able to proceed. One way is to get away from the four-dimensional complications by considering only instantaneous kernels, i.e., independent of relative time. The integration over p^0 can then be carried out in Eq. (2.7). A further approximation is to retain only the positive energy projections of the fermion propagators, which essentially amounts to neglecting virtual pair creation in the bound state. The main advantage with this procedure is that the resulting equation is very similar to a Schrödinger equation, but with the nonrelativistic potential replaced by a "quasipotential" containing the kernel and the positive energy projectors.

Methods like these were successfully applied to the deuterium system by Gross (1966). His treatment has since then paved the way for highly accurate calculations of properties of positronium by similar methods (Caswell and Lepage, 1979; Lepage 1978). Related pseudopotential approaches were developed by Logunov and Tavkhelidze (1963) and Blankenbecler and Sugar (1966). Modern work on heavy quark–antiquark systems in quantum chromodynamics (QCD) has benefitted from these ideas (e.g., Bergström, Snellman, and Tengstrand, 1980; Bergström and Snellman, 1980). It still seems to be the case that the methods of dealing with the deuteron are more matured and are likely to be subsequently taken over to the other fields of relevance.

Of course, there are still aspects of the fully relativistic treatment of the deuteron which are not entirely understood. The approximations introduced at various stages of the treatment may be justified for some applications but dangerous for others. In particular, there still seems to be a controversy concerning West's (1972) so-called β-correction. The cross section on deuterium can generally be written

$$\sigma_d=\sigma_n+\sigma_p-\sigma_\beta-\sigma_G\,, \tag{2.8}$$

where $\sigma_{n,p}$ are the cross sections on free nucleons, σ_G is the Glauber correction for shadowing (usually important in hadron–deuteron reactions only), and σ_β is the "smearing" correction caused by the Fermi motion. By essentially using nonrelativistic wave functions, West (1972) drew the conclusion that σ_β could be quite sizable. Other authors (Frankfurt and Strikman, 1979; Landshoff and Polkinghorne, 1978) claimed that a relativistic treatment showed the correction to be negligible. The controversy has recently been reviewed by Kusno and Moravcsik (1979), who conclude that West's correction indeed vanishes for hadronic scattering, but is still important in the leptonic case.

Recently, relativistic calculations of the deuteron wave functions have been done by Buck and Gross (1979), and Arnold, Carlson, and Gross (1980). As one of the consequences can be mentioned the fact that the wave function (or the Bethe–Salpeter amplitude in this case) contains not only the usual S and D parts but also an admixture of P states. This is possible due to the extra degrees of freedom present when one nucleon is virtual. Of course this does not violate parity conservation, because for these parts of the wave function the overall parity is opposite to the spatial parity (as for the small components in the Dirac equation).

The most ambitious relativistic treatment has been applied to elastic lepton–deuteron scattering (Arnold, Carlson, and Gross, 1980), to be briefly touched upon in Sec. III.

Another type of experiment where one is really looking for parity violating effects, is the scattering of polarized electrons off a deuteron target. A recent such experiment at SLAC by Prescott *et al.* (1979) gave a very important verification of the unified gauge theory for weak and electromagnetic interactions by Glashow, Salam, and Weinberg.

The nonconservation of parity for this process is due to interference between the weak neutral current and the usual (parity conserving) electromagnetic current. The helicity dependence of the cross section can be defined by the asymmetry parameter

$$A=(d\sigma_R-d\sigma_L)/(d\sigma_R+d\sigma_L)\,, \tag{2.9}$$

which can be calculated in the parton model [see, e.g., Cahn and Gilman (1978)]. The photon couples to the fermions through the electromagnetic charge, which is the same for the right-handed as for the left-handed projections. On the other hand, the Z^0 boson mediating the weak neutral current couples to the weak charges, which are generally different for the left- and right-handed pieces. If one introduces the dimensionless variable $y=\nu/E$, it can be shown (analogously as in neutrino physics) that a left-handed electron scatters off a left-handed quark with a flat distribution in y, whereas a left-handed electron scatters off a right-handed quark with a distribution proportional to $(1-y)^2$. By summing all possible helicity amplitudes, one gets for the asymmetry

$$A(x,y)/Q^2=-M_Z^{-2}\sum_i f_i(x)[a_i+b_i(1-y)^2]\Big/\left\{\sum_i f_i(x)[1+(1-y)^2]\right\}, \tag{2.10}$$

where M_Z is the mass of the weak neutral vector boson, x is the scaling variable defined before, and a_i and b_i are proportional to the difference between the left- and right-handed weak charges (so that they would vanish if those charges were coinciding). The $f_i(x)$ are essentially the probabilities to find a quark of type i with momentum fraction x within the target. The usefulness of the deuterium target comes about because there $f_u(x)=f_d(x)$, and the dependence on x disappears if one neglects the antiquark contribution (which is a good approximation in the SLAC experimental configuration). The result for the asymmetry is then simply

$$a=a_1+a_2[1-(1-y)^2]/[1+(1-y)^2]\,. \tag{2.11}$$

Now a measurement of A for different y values can be used to determine a_1 and a_2. Due to the ingenious experimental setup of Prescott *et al.* (1979), these small parameters [of the magnitude of 10^{-5} $(\mathrm{GeV}/c)^{-2}$] could indeed be measured and were found to be in agreement with the Glashow–Salam–Weinberg model, whereas some other models could be ruled out.

III. THE DEUTERON AS A SIX-QUARK STATE

A. General considerations

The main trend in high-energy physics today is without doubt towards an understanding of most phenomena in terms of interacting constituents. The quark–gluon theory of hadrons, QCD, thus opens exciting perspectives also for our knowledge of nuclear matter. New phenomena are indeed to be expected for a larger collection of quarks than is present within single hadrons.

The key to looking for finer structure within hadrons and nuclei is to increase the momentum transfer. This is clearly seen in the case of elastic scattering, since the elastic form factor is connected to the probability for the target to remain intact after the collision. To achieve this for a target with n constituents, the "potential" keeping the constituents together must act $n-1$ times, so that each constituent gets its share of the total momentum transfer. Otherwise the target would break. Here one sees the possibility to measure the number of active constituents for different values of q^2, the squared four-momentum transfer. In many-body Schrödinger theory it is known that, asymptotically, for an n-constituent bound state (Stack, 1967; Brodsky and Chertok, 1976)

$$F_n(\bar{q}^2) \sim [2m\bar{q}_1^{-2}V(\bar{q}_1^2)]^{n-1}\,|\psi_n(0)|^2\,, \tag{3.1}$$

where $\bar{q}_1 = \bar{q}/n$ is the average momentum transferred to each constituent, and

$$\psi_n(0) = \int \prod_{i=1}^{n-1} d^3k_i\,\psi(k_i) \tag{3.2}$$

(provided of course that the wave function is finite at the origin). For a nonrelativistic Coulomb or Yukawa potential $V(\bar{q}^2) \sim 1/\bar{q}^2$, which gives

$$F_n(\bar{q}^2) \sim (\bar{q}^2)^{-2n+2}\,. \tag{3.3}$$

A relativistic treatment with the Bethe–Salpeter equation has been performed by Brodsky and Chertok (1976). In this case it is convenient to replace each hadron of four-momentum p with a collection of constituents with four-momenta $p_i = x_i p + \kappa_i$, where the momentum fractions x_i fulfill $\Sigma x_i = 1$, and for the momenta κ_i relative to the rest frame of the hadron, $\Sigma\kappa_i = 0$. The elastic transition amplitude for $A+B \to C+D$ is

$$M_{\mathrm{el}} = \int \prod_i d^4k_i \psi_C^{+}\psi_D^{+} M_n \psi_A \psi_B\,, \tag{3.4}$$

where M_n is the connected multiparticle scattering amplitude and ψ is the n-particle Bethe-Salpeter wave function. For weak binding $x_i \sim m_i/M$, and M_n is well approximated by the free scattering amplitude with the momenta partitioned according to the x_i. For a scale-invariant theory (in particular QCD) the high-momentum behavior of the Bethe–Salpeter amplitude is effectively controlled by the behavior of the kernel (as can be seen by iterating the amplitude), and one readily derives the "dimensional counting rules" (Brodsky and Farrar, 1973; Matveev, Muradyan, and Tavkhelidze, 1973)

$$F_n(q^2) \sim (1/q^2)^{n-1}\,. \tag{3.5}$$

This formula is expected to be valid modulo (sometimes quite large) powers of $\ln(q^2/m^2)$ for large q^2. In QCD, for instance, the one-gluon exchange kernel is $\sim \alpha_s(q^2)/q^2$, where $\alpha_s(q^2)$ is believed to fall off as $1/\ln(q^2)$ (asymptotic freedom).

For a deuterium target, the elastic electron scattering cross section can be written (Rosenbluth's formula):

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}\bigg|_{\mathrm{Mott}}\left[A(q^2) + B(q^2)\tan^2\frac{\theta}{2}\right]\,. \tag{3.6}$$

An important experiment on elastic ed scattering by Arnold *et al.* (1975) covered the range $0.8 \lesssim q^2 \lesssim 6$ (GeV/c)^2, and the angle of the outgoing electron was as small as 8°. The last term in (3.6) can then be neglected, and the prediction (3.5) applies to $\sqrt{A}(q^2)$.

For very small momentum transfers the virtual photon couples to the whole deuteron as if it were elementary, which is easily understood. For somewhat larger $|q^2|$ the neutron and the proton can be "resolved" by the virtual photon. But what happens at very large $|q^2|$? The interesting thing is that for the deuteron, the form factor shown in Fig. 6 seems to have the behavior (Arnold *et al.*, 1975)

$$F(q^2)_{\mathrm{deuteron}} \sim 1/q^{10}\,, \tag{3.7}$$

which in terms of the counting rules (3.5) corresponds to $n=6$, i.e., six active constituents! The deuteron thus seems to behave as a "bag" of six quarks (for large momentum transfers). The corresponding behavior for the proton form factor is $F(q^2)_{\mathrm{proton}} \sim 1/q^4$, which indicates three active constituents, in full agreement with what one expects in QCD. In fact, the counting rule scaling behavior is almost too good. From a theoretical viewpoint, one would believe that the approximation of neglecting the "soft" hadronic structure (i.e.,

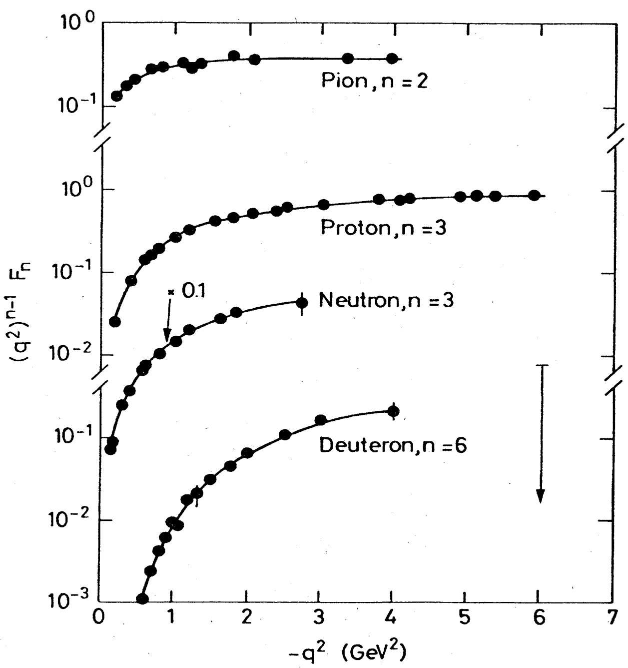

FIG. 6. Elastic electromagnetic form factors of hadrons for large spacelike q^2 in terms of the dimensional scaling quark model (Brodsky and Chertok, 1976).

the bound state wave function) should be valid only for very high momentum transfers not yet attained by the experiments. Another problem is that the most significant feature of QCD, the asymptotic freedom as reflected in the logarithmic decrease of the coupling constant α_s with q^2, is not seen in this experiment. An ambitious effort to calculate the proton form factor in QCD (Brodsky and Lepage, 1979a) indicated a significant logarithmic reduction of the form factor multiplied by q^4 over the measured q^2 range, whereas that quantity was found to be roughly constant in the experiment. It is conceivable that the hadronic wave functions must be taken better into account in the theoretical calculations, and the q^2 be increased in the experiments before such detailed comparisons between QCD and data are meaningful.

What about the $n=6$ component of the deuteron? In QCD, this is not unexpected, since besides the usual state of one color singlet neutron and one proton, there should exist "mixed color" states. This has been most thoroughly discussed in the framework of "bag" models for hadrons [see, e.g., Matveev and Sorba (1977) and references therein].

These models were devised to take care of the hitherto unsolved problem of confinement in QCD. It turns out that in this framework the deuteron should be a linear combination of the usual pn state, a $\Delta\Delta$ state and a baglike six-quark state. Høgaasen, Sorba, and Viollier (1980) estimate the corresponding fractions to be 93.8%, 0.6%, and 5.6%, respectively. The $6q$ baglike state lies 270 MeV higher than the pn state. The "core" of many potentials [e.g., the soft core potential of Reid (1968)] can then be interpreted as the energy barrier which must be penetrated to enter the $6q$ state. At short distances, i.e., large momentum transfers, the core suppresses the usual pn wave function, and the admixture of $6q$ becomes increasingly important.

Elastic scattering is the prototype of an exclusive process, i.e., the final state is completely measured. Of course one is also interested in inclusive cross sections, where only part of the final state is specified (e.g., measurement of one-particle spectra in inelastic collisions). There are theoretical arguments suggesting that these types of processes are connected (Brodsky and Lepage, 1979b). In particular, for deep-inelastic scattering the structure functions have the counting rule behavior

$$F_2(x, Q^2) \sim (1-x)^{2n_s-1}\,, \tag{3.8}$$

where n_s is the number of passive spectator fields in the hadron. This is in accordance with the Drell-Yan (1971) and West (1970) relation, which states that for large x

$$\nu W_2 \sim x(1-x)^a \tag{3.9}$$

if

$$F(q^2) \sim (q^2)^{-1/2(a+1)}\,. \tag{3.10}$$

For deep-inelastic scattering on a deuteron the possible existence of a collective $6q$ state would be reflected in processes not kinematically allowed on a single nucleon. The so-called cumulative processes will be discussed in more detail later in this section. A clearly "forbidden" process is when $x > 1/2$ (if x is the fraction of the whole deuteron's momentum carried by the constituent). Part of that type of cross section may be explained by the high momentum tail of the two-nucleon wave function [see, e.g., Frankfurt and Strikman, 1979). However, Kobushkin (1976) has analyzed the process in terms of just the "baglike" six-quark admixture described above. Using the composite quark model of Altarelli *et al.* (1974), he could make a good fit to the deep-inelastic scattering experiment of Schütz *et al.* (1977), as shown in Fig. 7. The estimated amount of $6q$-admixture in the wave function was 2%–3%. For an elaboration of this method, see Kobushkin (1979).

A more general analysis, which can also be applied to heavier nuclei, has been carried out by Schmidt and Blankenbecler (1977a, b). The nonrelativistic wave function in momentum space for a Hulthén potential can be parametrized as

$$\psi_{\mathrm{NR}} = (a\epsilon + \bar{k}^2)^{-1}(a\epsilon_1 + \bar{k}^2)^{(1-g)/2}\,. \tag{3.11}$$

A relativistic generalization which reduces to this form in the nonrelativistic limit is

$$\psi = N(x)(k^2 - a^2)^{-1}(k_1^2 - a_1^2)^{(1-g)/2}\,. \tag{3.12}$$

The square of this wave function gives the structure function $G_{a/A}(x)$, i.e., the probability of extracting a subsystem a with momentum fraction x from the nucleus A. This ansatz for the wave function is probably an oversimplification, but nevertheless some characteristic features of the nuclear wave functions seem to be contained in it. If one rewrites the Bethe–Salpeter equation in terms of the variables x and k_T (the transverse momentum of the constituent), the structure function can be computed from the formula

$$G_{a/A}(x, k_T) = \frac{1}{2(2\pi)^3}\,\frac{x}{1-x}\,|\psi(x, \bar{k}_T)|^2\,; \tag{3.13}$$

and with the generalized Hulthén wave function, this becomes

$$G(x, k_T) = \frac{N^2}{2(2\pi)^3}\,x(1-x)^g (M^2 + k_T^2)^{-2}(M_1^2 + k_T^2)^{1-g}\,. \tag{3.14}$$

Here $N(x)$, $M(x)$, and $M_1(x)$ are slowly varying functions of

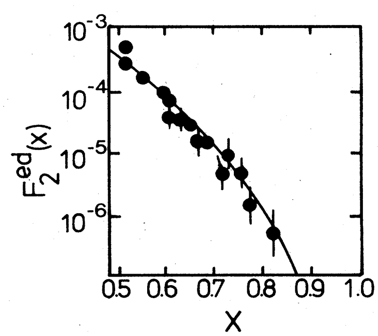

FIG. 7. Prediction of the deep-inelastic structure function $F_2^{ed}(x)$ from a model with a 6% six-quark admixture in the deuteron wave function (Kobushkin, 1976). Data are from Schütz *et al.* (1977).

x for x near 1 [see Schmidt and Blankenbecler (1977a) for explicit expressions]. Then one can see that the numerator in (3.14) controls the behavior for large x

$$G \sim (1-x)^g. \tag{3.15}$$

By choosing a proper generalized wave function (3.12), g, and consequently G, is determined. If a is a bound state of n_a nucleons, a similar analysis can be performed to yield

$$g = 2T(A-n_a)-1. \tag{3.16}$$

Here $T=3$ in most models (e.g., quark counting). It is assumed that the target nucleus breaks up fully after the reaction. To use Eq. (3.15) for pion production, say, one needs in addition $G_{\pi/N}$. This can be deduced from, for instance, the reaction $pC \to \pi^- X$, where experimentally one finds (Papp *et al.*, 1975) a pion spectrum varying like $(1-x)^3$. This value of 3 for the exponent adds to $g=5$ from Eq. (3.16), and one extra unit in the exponent from the kinematics finally gives the total behavior $(1-x)^9$ for the combined process deuteron → nucleon → pion. The exponent 9 is again in agreement with the dimensional counting rule (3.8) and is tested against data on $dC \to \pi^- X$ at 2.1 and 1.05 GeV kinetic energy per nucleon in Fig. 8.

To produce a pion in the forward direction a valence quark from the projectile deuteron may be used, with the necessary antiquark picked up from the low momentum sea. If the other five valence quarks in the deuteron remain spectators, then $2n_s - 1 = 9$. The corresponding prediction for $\alpha C \to \pi^- X$ in the projectile fragmentation region is $(1-x)^{21}$, since removing one of the twelve quarks in the alpha particle leaves eleven spectators, so that $2n_s - 1 = 21$. This prediction is also in remarkable agreement with the experimental data of Papp *et al.* (1975). Successful fits apply to some extent also for $^{12}C + {}^{12}C \to pX$ and αX, which should behave as $(1-x)^{65}$ and $(1-x)^{47}$, respectively, although data are more scarce.

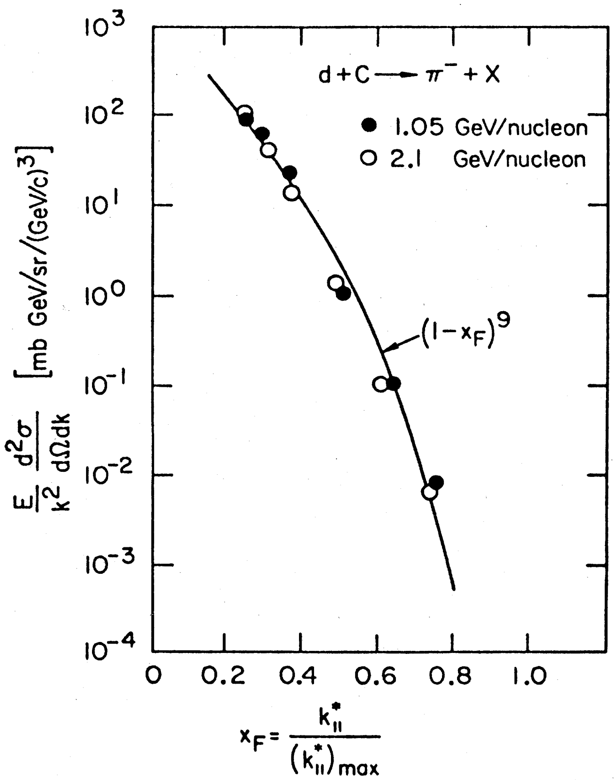

FIG. 8. Prediction of π^- x_F spectrum in dC collisions for $T=3$ in Eq. (3.16) (Schmidt and Blankenbecler, 1977) compared to the data of Papp *et al*. (1975) from two different energies.

The analysis of $dA \to \pi^- X$ has also been made by Lehman (1976), who found behavior consistent with $(1-x)^9$ and an admixture of 8% of the $6q$ component in the deuterium wave function. Here the data of Baldin *et al.* (1973) were used.

A very recent experiment (Schroeder *et al.*, 1979) on $pA \to \pi^\pm X$ below 5 GeV incoming kinetic energy shows deviations from these scaling laws in the fragmentation region of the nucleus, where the $1-x$ powers are expected to be quite high for heavy nuclei. The requirement that each quark should have relativistic momentum is, however, barely fulfilled below 5 GeV with so many spectators. The relevance of these data for the fluctuon and tube models discussed below has recently been analyzed by Fredriksson (1980).

The idea that hadron production in the fragmentation region more or less directly reflects the valence quark content of the interacting hadrons has been quite successful also for hadron–hadron collisions, in the guise of the so-called parton recombination models [Das and Hwa (1977); Duke and Taylor (1978); DeGrand and Miettinen (1978); for a recent review of parton recombination models, see also Hwa (1979)].

The kind of "collectivity" shown by the $6q$ component of the deuteron in, for instance, deep-inelastic scattering has its analogue in other types of reactions. One has very early seen "deuteron peaks" in quasielastic proton scattering on nuclei (Azhgirei *et al.*, 1957). It is difficult to understand the appearance of such fast, loosely bound subsystems unless the neutron and the proton were strongly correlated in the nucleus at the time of reaction. This inspired the construct of "fluctuon" models, of which the one of Blokhintsev (1957) was the first.

The idea is that occasionally strongly correlated clusters of nucleons should appear in nuclei, and that these clusters have their origin in (and can be calculated from) the overlap of single-particle wave functions in the nucleus. These early ideas remained virtually forgotten until some experiments of Baldin and collaborators (1971, 1973) revealed that occasionally in hA collisions particles can be produced in parts of phase space forbidden with single nucleon targets. This group named such events "cumulative" since a cumulation of single nucleons seemed to be needed. Recent theoretical developments, including quark effects, have in fact made the fluctuon models very similar to the $6q$ bag models discussed above, but in addition they generalize to heavier nuclei and other processes.

In its simplest version (Baldin 1974; Burov, Lukyanov, and Titov, 1977; Lukyanov and Titov, 1979) the fluctuon model treats the nucleons as moving freely within the nuclear volume, and the probability for a cluster of n_c nucleons to be found within the fluctuon volume V_f in the nucleus with A nucleons is simply

$$P(n_c, A) = \binom{A}{n_c} q^{n_c}(1-q)^{A-n_c}, \tag{3.17}$$

where $q = V_f/V_A$ (V_A is the nuclear volume). Here the only free parameter is the fluctuon radius r_f, which is estimated from data fits to be 0.5–0.7 fm.

For the deuteron the probability $P(2,2)$ is simply reflecting the six-quark admixture in the wave function (Burov *et al.*, 1978). In the simple probabilistic model described by Eq. (3.16) this turns out to be somewhat larger than the 6%–8% discussed above, but as pointed out by Burov *et al.* (1978), the extraction of the $6q$ contribution to the elastic form factor is very sensitive to parametrization.

For other processes than deep-inelastic scattering and pion production in the beam fragmentation region the quark counting rules applied to fluctuons work less successfully. For cumulative production of pions in the backward regions the exponent $2n_s - 1$ in Eq. (3.8) must, for instance, be divided by 2 to get good agreement with data. This may be a consequence of the low incident energies in the experiment (Baldin *et al.*, 1974), since such a correction must be applied also to the counting rules for pp collisions at lower energies. In addition, it seems, however, as if the fluctuon model predicts a far too slow dependence on the nuclear radius in cumulative pion spectra (Fredriksson, 1980). Since such detailed questions can anyhow not be investigated with the deuteron, where only small clusters are excited, we drop this subject here and direct the interested reader to the review articles by Lukyanov and Titov (1979) and Bergström *et al.* (1980), which cover the whole subject.

B. The collective tube model

The CTM is the most extreme collective model in this field. For a high-energy collision with a nucleus it postulates that the projectile collides at once with all nuclear matter within a tube with cross section σ_{tot}^{pp} along its path, as illustrated by Fig. 9. There is at present no clear theoretical basis for this assumption. The model is hence purely phenomenological, and its present popularity comes from the fact that it reproduces a lot of experimental results. An intuitive picture is that the projectile "sees" such a strongly Lorentz contracted nucleus that it cannot discriminate a tube from an ordinary particle. Such an argument at least sets a lower limit on the projectile momentum for the applicability of the CTM: $p_{\text{lab}} \gtrsim 0.5 l_{\text{tube}}$, where p_{lab} is in GeV/c and the tube length l_{tube} is in fm.

The CTM has been invented and reinvented several times during the last 25 years. Review articles about its ability to reproduce experimental data from various nuclear targets have been presented by Afek *et al.* (1976), Fredriksson (1977), Takagi (1979a) and Bergström *et al.* (1980).

The most apparent nontrivial prediction of the CTM is that a collision on a heavy nucleus can involve very high target masses, which in turn leads to enhanced maximal energy available for particle production. The identification of the whole tube with one single particle also leads to the same kind of "abnormal" quark rules as described in Sec. III.A. This means that predictions from the CTM for a fixed tube with i nucleons (or $3i$ quarks) are identical to the corresponding ones from the baglike models in Sec. III.A. The difference lies mainly in the probabilities for meeting such targets in the nucleus. The CTM is "maximally" collective, in contrast to the fluctuon model, where the collectivity is rather small. While the models in Sec. III.A. do not exclude ordinary multiple collisions and intranuclear cascades in the bulk of events, the CTM claims that *all* collisions are collective and that cascades do not occur, since the whole tube is recoiling out of the nucleus before it fragments and before any new hadrons are created. So while the bag and fluctuon models can make predictions only for some rare phenomena, the CTM should be able to reproduce the bulk of data from multiparticle reactions on nuclear targets.

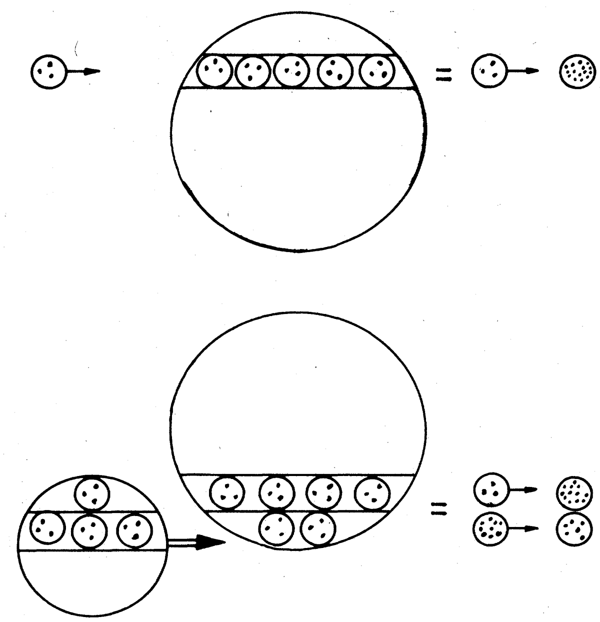

FIG. 9. The collective tube model for hadron–nucleus (upper figure) and heavy-ion (lower figure) collisions at high energies.

For a deuteron target the CTM claims that all "rescattering" events are nothing but single collisions with a six-quark hadron.

A first, and obvious, prediction is that the probability for hitting a two-nucleon tube must not depend on the final multiplicity, which it does in models with intranuclear cascades (see Sec. IV for details). As will be argued in Sec. IV.A the experimental situation here is unclear, with some data in support of both alternatives.

Let us start by studying quantities that do not depend on the number of quarks in the tube but only on its mass as it appears in the kinematics of a multiparticle production process. The predictions for the deuteron case have been worked out by Dar and Tran Thanh Van (1976). One assumes that the production cross sections depend only on the cms energy $\sqrt{s}$ of a collision. But since

$$s \approx 2m_{\text{target}} p_{\text{lab}} \quad \text{for } \sqrt{s} \gg m_{\text{target}}, \tag{3.18}$$

one gets the result that a collision with a two-nucleon tube at momentum $p_{\rm lab}$ resembles a collision with one single nucleon at a momentum equal to $2p_{\rm lab}$

$$\sigma^{h(2N)}(p_{\rm lab})=P_2\sigma^{hN}(2p_{\rm lab})\,, \tag{3.19}$$

with P_2 being the "rescattering" probability. Summing all hd collisions gives for the cross section σ_n for producing n charged particles

$$\sigma_n^{hd}(p_{\rm lab})=(1-P_2)[\sigma_n^{hp}(p_{\rm lab})+\sigma_n^{hn}(p_{\rm lab})]+P_2\sigma_n^{hN}(2p_{\rm lab})\,, \tag{3.20}$$

where p, n, and N are a proton, a neutron, and an average nucleon, respectively. The important even-topology events "hp", discussed in the Introduction, include all "rescatterings" and fulfill

$$\sigma_n^{``hp"}(p_{\rm lab})=\sigma_n^{hp}(p_{\rm lab})+P_2[\sigma_n^{hN}(2p_{\rm lab})-\sigma_n^{hp}(p_{\rm lab})]\,. \tag{3.21}$$

If differences between p, n, and N can be neglected in σ_n, one observes that the second term in Eq. (3.21), the "shadowing" correction, is positive if and only if σ_n^{hN} increases with $p_{\rm lab}$. The odd-topology events in "hn" always have a negative "shadowing" correction to the approximation $\sigma^{``hn"}\approx\sigma^{hn}$.

Equation (3.21) has been confronted with data in several publications succeeding the original one by Dar and Tran Thanh Van, which was built on data with rather low statistics. Figure 10 (Porter *et al.*, 1980) shows the result for 15 GeV/c π^-d collisions (the lower points are from the eikonal model discussed in Sec. IV.B.) The success of the CTM here is somewhat obscured by the fact that the same experiment shows a systematic rise in P_2 with n when calculated from sets of events

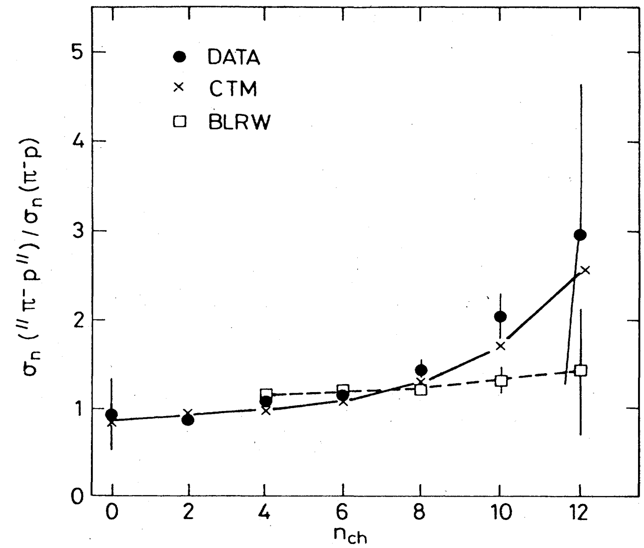

FIG. 10. Multiplicity distributions of charged particles in even-topology events of 15 GeV/c π^-d collisions according to the CTM, Eq. (3.21), and to the eikonal model, Eq. (4.17). Data are from Porter *et al.* (1980).

with a fixed number n of charged particles. Figure 11 (Lys *et al.*, 1977a) shows that Eq. (3.19) is in good agreement with data; "rescatterings" in πd and pd collisions at 100 GeV/c result in practically the same multiplicity distributions as πp and pp collisions at 200 GeV/c. A compilation by Moriyasu *et al.* (1978) is reproduced in Fig. 12. It shows the result of summing

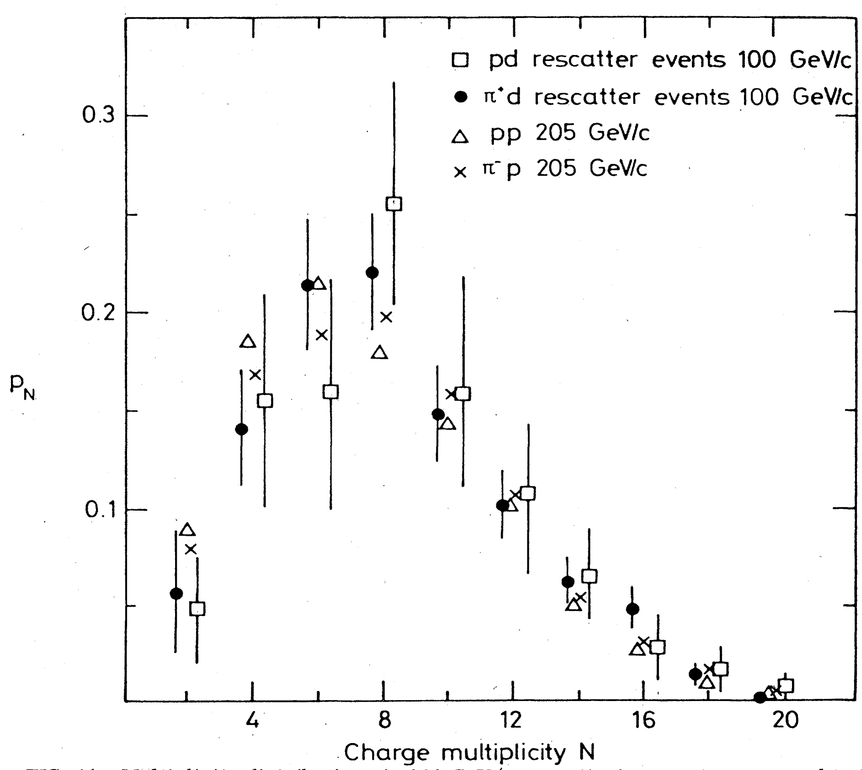

FIG. 11. Multiplicity distributions in 100 GeV/c rescattering events compared to the corresponding distributions in 205 GeV/c π^+p and pp collisions. The compilation is from Lys *et al.* (1977a). According to the CTM, Eq. (3.19), the data sets should be pairwise equal.

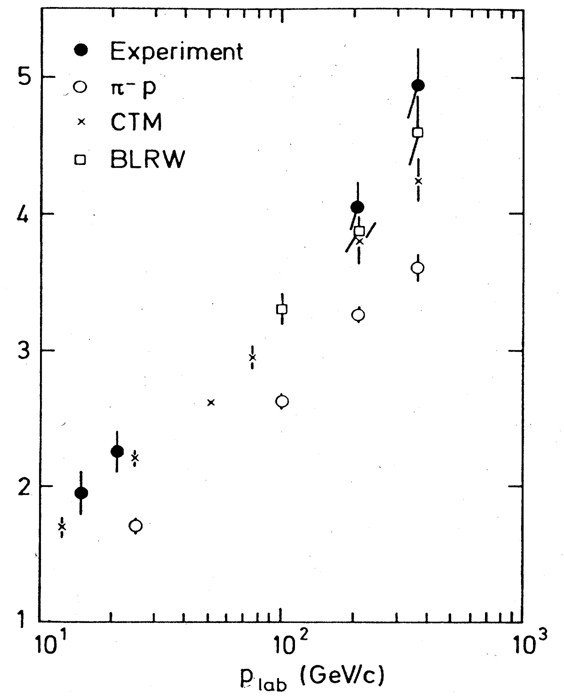

FIG. 12. The average negative multiplicity at various energies in $\pi^- d$ collisions. The figure is from Moriyasu *et al.* (1978) and shows data as well as the CTM and eikonal model predictions of Eqs. (3.22) and (4.17), respectively.

Eq. (3.20) over n in order to get the average multiplicity

$$\langle n_{hd}(p_{\rm lab})\rangle = 2(1-P_2)\langle n_{hp}(p_{\rm lab})\rangle + P_2\langle n_{hp}(2p_{\rm lab})\rangle \qquad (3.22)$$

at various projectile momenta $p_{\rm lab}$. Here the CTM result is indistinguishable from data and from that of the eikonal model. The same curve, but compiled for the excess multiplicity $\langle n_{re}\rangle - \langle n_{hp}\rangle$ in the "rescattering" events only, is shown in Fig. 13 together with other model estimates (see Sec. IV) and experimental data (Zieminski, 1977). A qualitative test of the ability to explain data down to the "theoretical" low-energy limit $p_{\rm lab}\approx 0.5 l_{\rm tube}$ of the CTM is provided by data from $\bar{p}d$ collisions at 3–15 GeV/c (Braun *et al.*, 1978). Equation (3.19) is indeed consistent with those results all the way down to 3 GeV/c, although the authors draw the opposite conclusion after having mixed up Eq. (3.19) with Eq. (3.20).

Dar and Tran Thanh Van (1976) also extend Eq. (3.20) to rapidity spectra $f = dn/dy$. Due to the enhanced target mass in an h-tube collision, the cms will be shifted in comparison to an hp collision at the same projectile momentum. The result for an average hd collision reads

$$f^{hd}(p_{\rm lab}, y_{\rm lab}) = 2(1-P_2) f^{hp}(p_{\rm lab}, y_{\rm lab}) + P_2 f^{hp}(2p_{\rm lab}, y_{\rm lab} + \ln 2) \qquad (3.23)$$

(the original work has incorrectly $\ln\sqrt{2}$ instead of ln2). One assumes again that dn/dy depends on $\sqrt{s}$ and y only, and not on the exact quark content of the target. Figure 14 shows a comparison of Eq. (3.23) with 21 GeV/c $\pi^- d$ data, using 21 and 42 GeV/c $\pi^- p$ results, respectively, as input. It is taken from a compilation by Pawlik

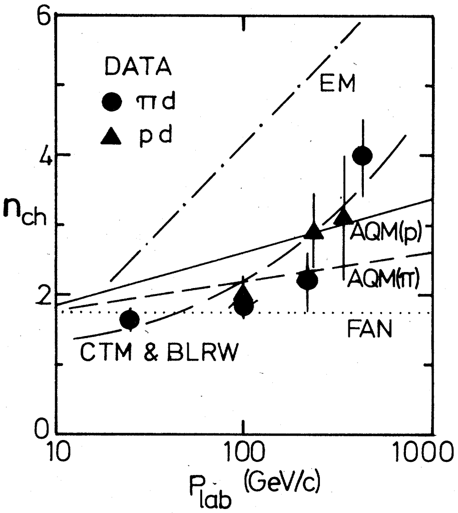

FIG. 13. The charge excess in hd double collisions over hp collisions. Data are from Zieminski (1977), and the lines show predictions from the CTM, Eq. (3.19), the eikonal (BLRW) model, Eq. (4.17), the eikonal model (EM) according to Nikolaev *et al.*, Eq. (4.28), the fan diagram model (FAN), Eq. (4.9), and the additive quark model (AQM), Eq. (4.32). The CTM and BLRW are similar (by coincidence here).

(1978) and corrected by Dar (private communication) for the erroneous $\sqrt{2}$ mentioned above.

Concluding the discussion on average multiplicities, the CTM gives a good description of the bulk of "rescattering" events in the deuteron. Concerning rarer phenomena, it was mentioned in Sec. III.A that at least the cumulative production is better explored with heav-

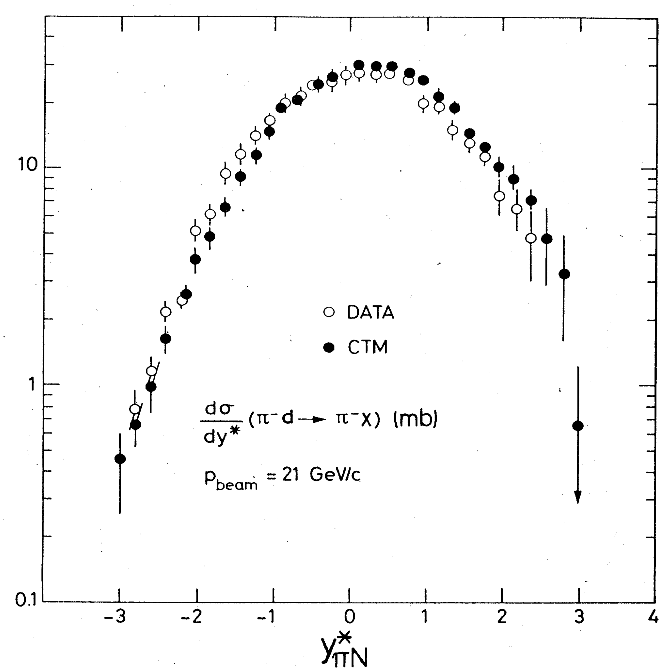

FIG. 14. A test of the CTM equation (3.23) for the rapidity spectrum in the process $\pi^- d \to \pi^- X$ at 21 GeV/c. The analysis is from Pawlik (1978) and Dar (private communication).

ier nuclear targets. It has been shown by Takagi (1979b), and later by Berlad, Dar, and Eilam (1979) that the CTM fits the data of Baldin *et al.* (1974) on backward π^- production in 8.4 GeV/c proton–nucleus collisions. Both groups use the quark counting rules discussed in Sec. III.A; but due to the large collectivity of the CTM, they do not have to "correct" these rules to be able to fit the data, as was done by Burov *et al.* (1978) in the less collective fluctuon model.

The only case where the deuteron plays a special role for cumulative production is when used as a projectile. It is technically easier to measure particle momenta in the projectile fragmentation region, and deuterons are simpler to handle as projectiles than are heavier nuclei. Here a cumulative phenomenon reveals itself by producing pions with momenta higher than the momentum of one single nucleon in the incoming deuteron. This signal has been observed in deuteron–nucleus collisions at 1.05 and 2.10 GeV/nucleon kinetic energy (Papp *et al.*, 1975), and at 8.4 GeV/c total deuteron momentum (Baldin *et al.*, 1973). The effect seems to be larger in the latter data, and there has been a dispute about the possibility of understanding the cumulative π^- yields in terms of a conventional Fermi motion in the deuteron. Baldin *et al.* favor a collective effect and claim that their pion spectrum at $p_\pi > 4.2$ GeV/c is orders of magnitude above any Fermi "tail," while Papp *et al.* succeed in fitting their data with a Hulthén wave function for the nucleonic motion and without collective effects. The two groups do not even agree on whether the two data sets are consistent with each other. Perhaps the difference is a low-energy effect, developing in the range below $p_{\mathrm{lab}} \approx 2$ GeV/nucleon, above which collective effects start to appear. Against this interpretation stands the fact that Takagi (1979b) and Mathis and Meng (1978) have presented detailed fits to the data of Papp *et al.* Figure 15 shows the one of Takagi, where the same quark counting rules are used as when explaining the target cumulative production on heavier nuclei. The agreement is good, but one must keep in mind that predictions close to the absolute kinematic boundaries are very sensitive to the exact choice of parameters.

A final area where deuterons are particularly suitable for testing the CTM is lepton–deuteron collisions. The main problem there is that the probability P_2 for a collision with a two-nucleon tube might differ drastically from the P_2 of an hd collision. Dar and Tran Thanh Van (1976) suggest that $P_2^{ld} = \frac{1}{2}P_2^{hd}$. They think of the tube effect as appearing when one nucleon is hit by the projectile and then pushes the other nucleon while recoiling. A hadronic projectile has almost no chance of penetrating a nucleon without interacting, so the $P_2^{hd} \approx 0.20$ is simply the geometric probability that the nucleons are piled up in front of each other at the impact. With a leptonic projectile both nucleons are practically transparent, so when a collision takes place, it could equally well be with either of the nucleons, even if one of them is geometrically shadowing the other. The authors now postulate that the "pushing" effect exists only when the first nucleon is hit inelastically. This argument leads to $P_2^{ld} \approx \frac{1}{2}P_2^{hd} \approx 0.10$. In events where the other nucleon is hit, it does not "drag" its colleague

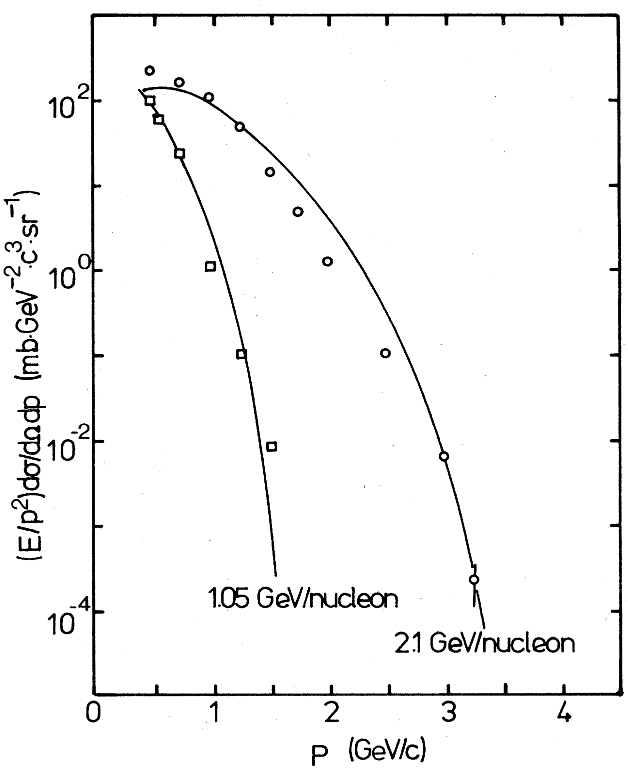

FIG. 15. CTM results (Takagi, 1979b) for cumulative production of forward π^- in the process $dC \to \pi^- X$ at 1.05 and 2.10 GeV/nucleon kinetic energy. Data are the same as in Fig. 8.

with it, and no collective effect appears. Although the physical picture is very different from the one behind the baglike models, it leads to similar results, since $P_2 \approx 0.10$ is close to the value chosen in those models.

In more recent CTM works it seems, however, as if most groups choose an approach with exactly the same tubes for all projectile types. This is partly motivated (Afek *et al.*, 1977) by the finding that neutrino–nucleus collisions are surprisingly similar to pion–nucleus collisions (Burnett *et al.*, 1978). The physical picture is that any target quark that recoils after being hit by something will instantaneously feel confined to the whole tube and drag it out of the nucleus. The best way to discriminate between these two completely different views of the mechanism behind a collective tube behavior is naturally to measure P_2 accurately in both ld and hd collisions. Dar and Tran Thanh Van argue that most ld data support $P_2 \approx 0.10$. The statistics of available ld data are not too good, however; and a value of 0.15–0.20 is not excluded.

Quite generally, one can conclude that a high-statistics lepton-deuteron experiment would be most welcome both for discriminating between collective and noncollective models and for solving those internal CTM problems.

IV. THE DEUTERON AS A DOUBLE SCATTERER

The common properties of models under this title are that there are indeed two consecutive collisions in a so-called rescattering event in the deuteron, and that the rescattering on the second nucleon in some respect is abnormal. As we will see in Sec. IV.B, the only picture with two fully normal scatterings is in clear disagreement with experimental data. The main problems are to clear out first what kind of particles are responsible for the rescatterings and then to determine what kind of "anomalies" are realistic to investigate. One can think of three types of double collisions, namely, by the whole projectile, by some fraction of the projectile (quarks), and by the hadrons (quarks) created in the first collision. The anomalies tested so far in the literature are that the projectile shares its momentum in a nontrivial way in between the two collisions and that the secondaries are unable to reinteract immediately after creation. Both conjectures serve to dampen the rescattering rates in comparison to the most naive pictures.

A. Fan diagram dominance

The name of this model refers to its close connection to so-called fan diagrams in reggeon field theories. The process and its fan diagram are illustrated in Fig. 16. The left-hand side shows the cross section for a collision with one of the nucleons, followed by a rescattering by one of the produced particles on the second nucleon. The right-hand side is the corresponding process if "ladders" are assumed to be equivalent to reggeons. For the actual computations we will, however, not refer to concepts from reggeon field theories, but instead try to stay on a physically intuitive basis (some notions from Regge theory will be used in Sec. IV.B). We thereby follow the presentation by Nikolaev and collaborators (Nikolaev and Zoller, 1979; Nikolaev, 1976b; Davidenko and Nikolaev, 1976).

The central concept in this model is the *formation length* l_f. A certain hadron is postulated to exist in the final state only from a distance l_f behind the point of collision, where l_f is given by the uncertainty relation (with equality) as

$$l_f = k/m^2 = de^y . \tag{4.1}$$

Here k means the momentum of the particle and m the characteristic mass in the Lorentz factor of the hadronic matter from which the particle is created. One defines the rapidity as $y = \ln k/m_\pi$, so that $d = m_\pi/m^2$. The mass m is a free parameter in the theory and is usually set equal to 1.4 GeV.

In order to find the probability $w^b_{\rm re}(y)$ for a produced hadron with rapidity y to rescatter on the second nucleon, one makes a few simplifying approximations. First of all, the whole process can be regarded as one-dimensional, since most produced particles in a high-energy collision proceed within a narrow cone in the forward direction. The probability of the second nucleon's lying on the path of a produced hadron b is $\sigma^{bN}_{\rm inel}/(4\pi r^2)$, where r is the distance between the two nucleons. An interaction is assumed to take place if and only if the particle b "exists" before passing the second nucleon N. The probability of this is given by the step function $\theta(r - l_f)$ times the geometric factor above. Integrating over the spherically symmetric deuteron wave function $\psi_d(r)$ finally gives

$$w^b_{re}(y) = \sigma^{bN}_{\rm inel} \int_0^\infty dr\, |\psi_d(r)|^2 \theta(r - de^y) . \tag{4.2}$$

This can be evaluated with, for instance, the Hulthén wave function

$$\psi_d(r) = A(1 - e^{-\alpha r}) e^{-\beta r}/r . \tag{4.3}$$

To compute various quantities for the rescattering events one needs to know the distribution in y from the first collision. Calling it $dn_b(y)/dy$ one gets at first the total probability for a rescattering as

$$P_{\rm re} = \sum_b \int_0^\infty dy\, w^b_{\rm re}(y) \frac{dn_b(y)}{dy} , \tag{4.4}$$

where one sums over all relevant species of produced particles. The total average multiplicity from double scattering events is

$$\langle n_{\rm re}\rangle = \langle n_{hN}\rangle - 1 + P^{-1}_{\rm re} \sum_b \int_0^\infty dy w^b_{\rm re}(y) \frac{dn_b(y)}{dy} \langle n_{bN}(y)\rangle . \tag{4.5}$$

The contribution $\langle n_{hN}\rangle - 1$ takes into account all particles from the first collision that do not collide with the second nucleon. Here $\langle n_{bN}(y)\rangle$ signifies the average total multiplicity when the produced hadron b with rapidity y collides with the second nucleon N.

The next step is to compute the rapidity spectrum of particle type i from the double collision events in the process $hd \to i + X$ compared to the process $hN \to i + X$. One defines the ratio $R^i_{\rm re}(y) = (dn^{\rm re}_i/dy)/(dn^{hN}_i/dy)$ and gets

$$R^i_{\rm re}(y) = 1 - \frac{w^i_{\rm re}(y)}{P_{\rm re}} + \left(P_{\rm re} \frac{dn^{hN}_i(y)}{dy}\right)^{-1} \times \sum_b \int_y^Y dy_1 w^b_{\rm re}(y_1) \frac{dn^{hN}_b(y_1)}{dy_1} \frac{dn^{bN}_i(y)}{dy} . \tag{4.6}$$

The various spectra dn/dy naturally depend implicitly on the maximally allowed value of the argument, which is y_1 for y, and Y for y_1.

To get transparent results Nikolaev *et al.* make a few approximations that are not very well motivated physically. All secondary particles are assumed to be pions with spectra dn/dy = constant at $0 < y < y_{\rm max}$. The expression (4.2) is approximated with a linear form

$$w_{\rm re}(y) = \frac{2P_{\rm re}}{y_c} \left(1 - \frac{y}{y_c}\right) \theta(y_c - y) , \tag{4.7}$$

with $y_c \approx 5.5$. Neither of these conjectures is particularly correct, but the authors claim that more realistic expressions for dn/dy and $w_{\rm re}$ give similar results, probably because smearing effects are considerable in the integrals.

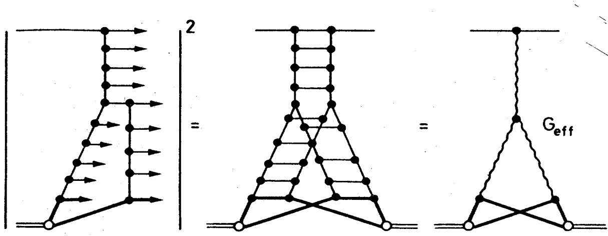

FIG. 16. A fan diagram in reggeon field theory.

The final results are

$$P_{\mathrm{re}} \approx 0.24 \tag{4.8}$$

$$\langle n_{\mathrm{re}}\rangle \approx \langle n_{hN}\rangle + 1.75 \tag{4.9}$$

and

$$R_{\mathrm{re}}(y) \approx 1 - \frac{2}{y_c}\left(1-\frac{y}{y_c}\right)\theta(y_c-y) + \left(1-\frac{y}{y_c}\right)^2\theta(y_c-y)\,. \tag{4.10}$$

The value of P_{re} is most likely too high, because all secondary hadrons are not prompt pions. Some of the pions one observes in the true final state might have rescattered several at a time in the form of, for instance, ρ and ω mesons. Trying to take this into account Nikolaev *et al.* get $P_{\mathrm{re}} \approx 0.15$, which is more in line with the data. Note that Eq. (4.9) is independent of the energy and quantum numbers of the projectile. The ratio $R_{\mathrm{re}}(y)$ has some features that are easy to understand from a physical point of view. For slow hadrons one finds $R \approx 2$, because the low y spectrum gets equal contributions from the two collisions due to the limiting fragmentation property in the target area. For very fast hadrons with $y > y_c$ one gets $R = 1$, which means that those secondary particles are too fast to reinteract. Consequently, all fast hadrons come from the first collision. A dip develops around $y = y_c - 1$ with a width of around two units in y and a depth of y_c^{-2} below the "plateau" at $R = 1$. The particles "lacking" here are the ones that rescatter.

Let us now take a look at experimental data. The probability P_{re} for a rescattering is around 0.15 for πd and around 0.18 for pd and $\bar{p}d$ collisions (Dziunikowska *et al.*, 1976; Lys *et al.*, 1977a, b; Porter *et al.*, 1980; Moriyasu *et al.*, 1978; Zieminski, 1977; Braun *et al.*, 1978; Sheng *et al.*, 1975; Dado *et al.*, 1976, 1979; Eisenberg *et al.*, 1976; Dombeck *et al.*, 1978; Csorna *et al.*, 1977a, b; Hanlon *et al.*, 1979; Bergier *et al.*, 1980) in fair agreement with the theoretical value $P_{\mathrm{re}} \approx 0.15$ given above. No relevant energy dependence is observed in P_{re}, which might seem a bit surprising in light of Eq. (4.4), where the spectrum dn_b/dy in the relevant region $y \lesssim 2$ is known to depend on the incoming momentum as long as $p_{\mathrm{lab}} \lesssim 30$ GeV/c. Some data (Braun *et al.*, 1978) show energy independence in P_{re} even down to $p_{\mathrm{lab}} \approx 3$ GeV/c. As mentioned in Sec. III.B above, a question of importance for the models discussed in this review is whether P_{re} depends on the final multiplicity when computed from a sample of events with fixed multiplicity. In any cascadelike model one expects a strongly growing P_{re} when the multiplicity from the first collision increases. This prediction is in a qualitative agreement with the data of Porter *et al.* from 15 GeV/c $\pi^- d$ collisions, but in disagreement with the conclusions of Zieminski (1977), Dombeck *et al.* (1978), Dziunikowska *et al.* (1976), and Bergier *et al.* from 200 GeV/c πd and pd collisions and 100 GeV/c $\bar{p}d$ collisions. It is unlikely that P_{re}, taken at a fixed multiplicity, depends so strongly on the incoming energy between 15 and 200 GeV/c, while the total P_{re} for all events is practically energy independent over the same energy interval. It is clear that more accurate experimental data and theoretical investigations of this seemingly trivial, but basic, point are needed.

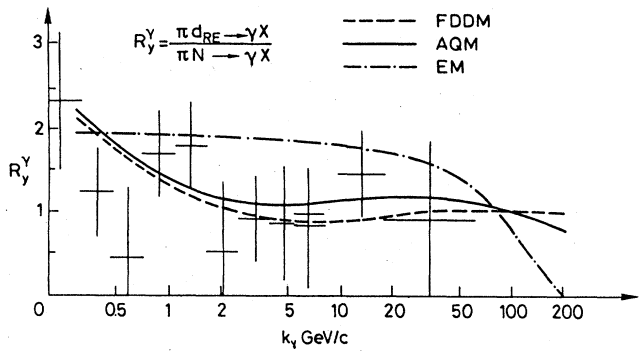

FIG. 17. The laboratory momentum (k_γ) spectrum of rescattering events in the process $\pi^- d \to \gamma X$ at 200 GeV/c, compared to the one in $\pi^- p$ collisions. The fan diagram model, Eq. (4.6), eikonal model, Eq. (4.27), and additive quark model predictions are all compiled by Nikolaev and Zoller (1979). Data are from Csorna *et al.* (1977a, b).

The experimental charge excess in Fig. 13 is consistent with $\langle n_{\mathrm{re}}\rangle - \langle n_{hN}\rangle = 1.75$ in Eq. (4.9) up to $p_{\mathrm{lab}} \approx 200$ GeV/c (other theoretical estimates, discussed elsewhere, are also shown).

The rapidity distribution in Eq. (4.10) has not been as conclusively compared with data. Figure 17 (Nikolaev and Zoller, 1979) shows $R(y)$ for rescatterings in the process $\pi^- d \to \gamma X$ at 200 GeV/c (most photons come from π^0 decays). Experimental results are from Csorna *et al.* (1977a, b). Data from $\pi^- d \to \pi^- X$ at 21 GeV/c (Zieminski, 1977) are in qualitative agreement with Eq. (4.10).

B. The eikonal model

Within the reggeon field theories mentioned in the beginning of Sec. IV.A there are reasons to believe that the effective coupling in the fan diagram is the same as the one at work in diffractive (inelastic) particle–particle collisions at high energies. It is well known from experiments that the latter interaction is rather weak, so with this line of thought the process in Fig. 16 would be of minor importance. In simpler words, one neglects the possibility that a created hadron will rescatter on the second nucleon. This leaves us with one single source for the rescatter events, the projectile itself. Two different physical situations are of particular interest, namely, elastic and inelastic double collisions.

For the *elastic* scattering with a multinucleon target there is an almost classical theory due to Glauber (1959). The situation for hd scattering is illustrated by Fig. 18. The Glauber multiple-scattering theory relates the amplitude for such a process to the amplitude for one single elastic scattering. The prediction for

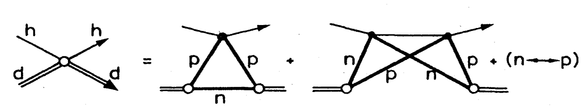

FIG. 18. An elastic hd scattering in the Glauber theory.

pd scattering was worked out by Glauber and Franco (1966) and further refined by, for instance, Michael and Wilkin (1969), Franco and Glauber (1969), Alberi and Bertocchi (1969), and Joachain and Quigg (1974). The theoretical development up to 1975 has been reviewed by Fridman (1975). Many experiments on elastic πd and pd scattering (Fellinger *et al.*, 1964; Hsiung *et al.*, 1968; Bradamante *et al.*, 1970b, 1971; Coleman *et al.*, 1967; Bennet *et al.*, 1967; Allaby *et al.*, 1969) have confirmed the main trends of these predictions, especially the one that single scattering events populate the small-angle region, while double scattering events dominate at large momentum transfer.

At least one work (Bradamante *et al.*, 1970a) has, however, demonstrated a clear systematic failure of the Glauber theory at large momentum transfer. Many possible explanations for this effect have been presented (Fäldt, 1971; Gunion and Blankenbecler, 1971; Namysłowski, 1972; Zovko, 1975; Harter and Julius, 1976), one being nontrivial corrections due to the deuteron recoil. Subsequent experiments at 24 GeV/c (Amaldi *et al.*, 1972), at 50–400 GeV/c (Akimov *et al.*, 1975), and at the CERN ISR (Armitage *et al.*, 1978) have not been able to discriminate between these suggested corrections to the Glauber theory. Experiments on elastic deuteron–deuteron collisions at 0.68–2.12 and 7.9 GeV/c (Goshaw *et al.*, 1969, 1970) have also been performed, because it is believed (Franco, 1968; Alberi *et al.*, 1970) that dd scattering should better clarify deviations from the basic theory. The statistics at large momentum transfer were, however, not good enough here to give any firm answers.

Quite recently a new and very interesting high-statistics experiment has been concluded at the CERN ISR by Goggi *et al.* It has resulted in several publications about both elastic and coherent inelastic pd and dd collisions. The works of greatest interest for this review are the ones (Goggi *et al.*, 1978a, b, 1979) that pinpoint the most probable mechanism behind the observed deviations from the original Glauber theory—the phenomenon of *inelastic screening.* This mechanism was first suggested by Abers *et al.* (1966), who pointed out that the double scattering process might contain inelastic intermediate states like the one shown in Fig. 19(a). Here the projectile has "surrounded" itself with a number of newborn particles, which are then reabsorbed in the second collision. Amplitudes for such processes can be estimated within Regge theory (Gribov, 1969; Kancheli and Matinyan, 1970; Anisovich, Volkovitsky and Dakhno, 1972; Quigg and Wang, 1973; Kwieciński, Leśniak and Zalewski, 1974; Baig and Pajares, 1979). The intermediate state has a rather low mass, so that it can be approximated by a baryon resonance (whereby the contribution from the excited proton dominates, of course). In the corrections to the purely elastic case one simply replaces the elastic amplitudes with resonance production amplitudes. The resulting formulas from Regge theory have been worked out by Alberi and Baldracchini (1978). One finds that the contribution f_{IIS} to the double scattering amplitude reads (neglecting the difference between a proton and a neutron)

$$f_{\mathrm{IIS}}=\frac{i}{2\pi p_{\mathrm{lab}}}\int d^2q_T'\sum_k f_k(\tfrac{1}{2}\bar{q}_T-\bar{q}_T')f_k(\tfrac{1}{2}\bar{q}_T+\bar{q}_T')S(\bar{q}_T',q_L)$$
$$+\frac{ip_{\mathrm{lab}}}{2\pi^2}\sum_{ijk}\eta_k e^{-i\pi\beta_k(q^2/4)}\int\frac{d\sigma^{ijk}}{dt\,dM^2}S(\bar{q}_T',q_L)d^2q_T'dM^2. \tag{4.11}$$

Here p_{lab} is the incoming proton momentum, $\bar{q}$ the total momentum transfer, $\frac{1}{2}\bar{q}_T\pm\bar{q}_T'$ the internal transverse momentum transfers, f_k the amplitude for a scattering between the proton and the reggeon k, η_k and β_k the signature and trajectory of the reggeon k, and $d\sigma^{ijk}/dt\,dM^2$ the triple-reggeon cross section. The first sum represents the diagram in Fig. 19(b), with two consecutive reggeon–proton interactions, while the second sum takes into account all possible triple-reggeon configurations, as illustrated by Fig. 19(c). The point is that the "unknown" $d\sigma^{ijk}/dt\,dM^2$ has already been derived (Field and Fox, 1974) from fits to the diffractive part of the cross section for $pp\to pX$. For dd scattering the situation is naturally much more complicated (Alberi and Baldracchini, 1978; Kancheli and Matinyan, 1971), with, for instance, triple collisions. Figure 20 (Goggi *et al.*, 1979) demonstrates the excellent agreement between the experimental data and the Glauber theory corrected for inelastic screening. Both pd and dd data at $\sqrt{s}=53$ GeV are shown. The lower figures show the failure of the original Glauber theory and give an impression of the size of the corrections. In the dd case one observes inelastic screening both in the double and the triple scatterings.

We can hence conclude that the intermediate resonance formation, defining the concept of inelastic screening, seems to play a crucial role in high-energy multiple scattering in nuclei. This important new experiment at the CERN ISR will perhaps settle the hot debate about the existence of such corrections to other quantities, like the total and elastic cross sections of heavier nuclei.

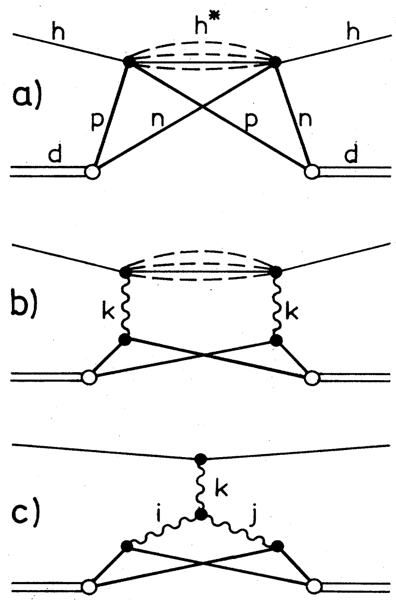

FIG. 19. (a) An elastic double scattering with inelastic screening. (b) A double reggeon exchange. (c) A triple-Pomeron interaction.

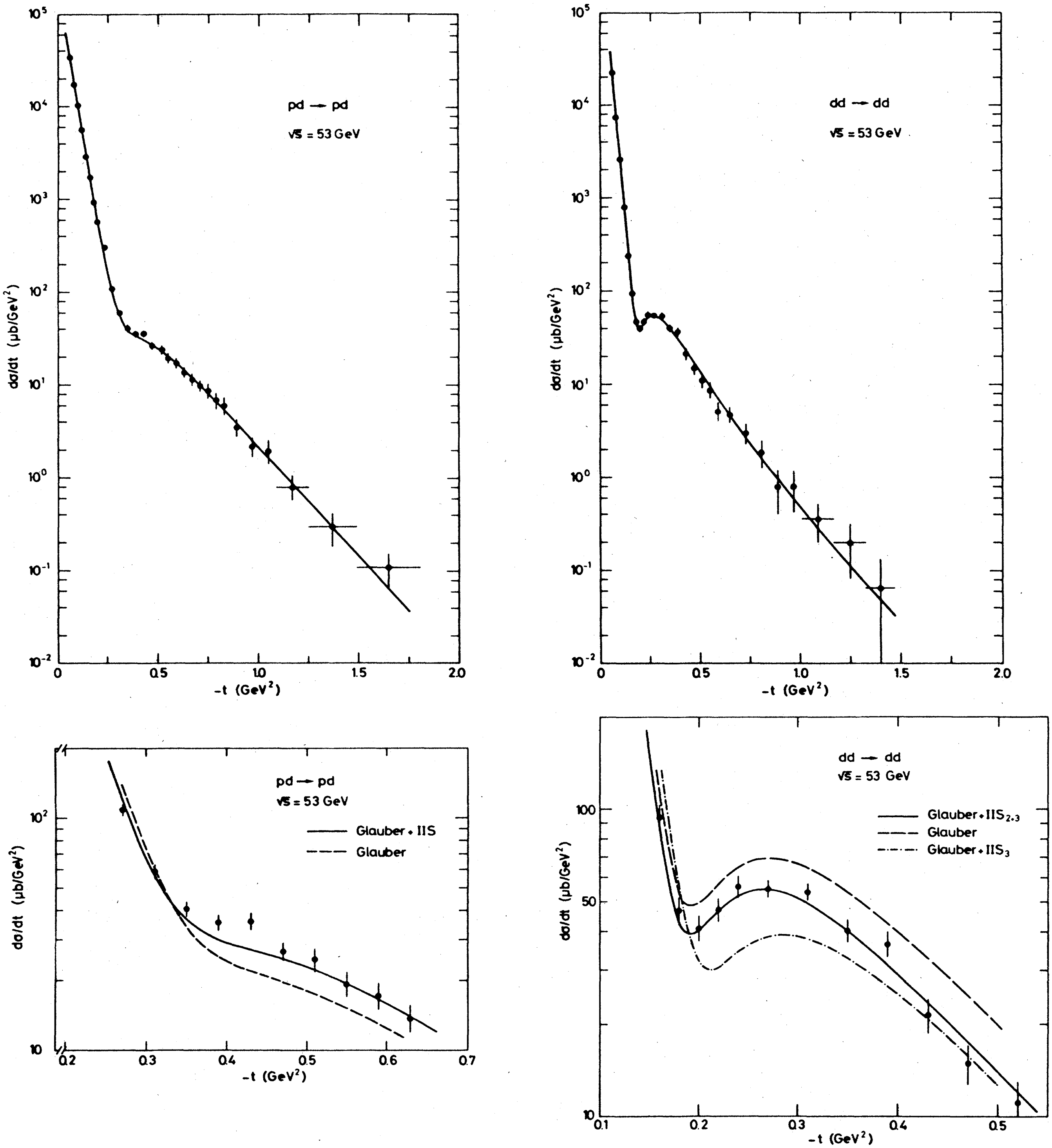

FIG. 20. Data from elastic *pd* and *dd* collisions in the CERN ISR according to Goggi *et al.* (1979). The lines in the upper figures show a fit with inelastic screening corrections added to the Glauber theory. The lower figures illustrate the relative contributions of this screening in the most sensitive kinematic region.

One might ask if the nucleons in the deuteron are not excited analogously to baryon resonances. Most probably they are, but the effect is weaker, since either the excitation or reabsorption (or both) must be caused by the other nucleon. There the crucial momentum transfer is small, as it is due to "nuclear physics," and an excitation is accordingly very rare. The problem of such "isobaric configurations" in nuclei is nevertheless of great importance for nuclear structure theory (Arenhövel and Weber, 1972; Brown and Weise, 1975; Green, 1976; Weber and Arenhövel, 1978), since a measurement of, for instance, the $\Delta\Delta$ component in the deuteron wave function would be most useful for learning about details of two-nucleon forces. As mentioned earlier the $\Delta\Delta$ configuration is expected in bag models, so experimental evidence here would also shed light on baryon and six-quark physics. In addition, a baryon resonance is expected to have a higher Fermi momentum than a proton or a neutron (perhaps 3–6 times as high on the average), which should result in a large correction to the high momentum tail of the ordinary Fermi distribution.

A process on the borderline between elastic and inelastic collisions is *deuteron break-up*, $hd \to hpn$. It is of considerable interest for testing details of Regge theory. More specific reactions, like $hd \to hN^*N$, are naturally of value for investigating the idea of isobars in nuclei. We will not penetrate the delicate problems here, but instead direct the interested reader to the review article by Baur and Trautmann (1976) and to the works by Bertocchi and collaborators (Alberi and Bertocchi, 1969; Bertocchi, Craigie and Weis, 1979).

It is not obvious that *inelastic* collisions in general can be taken into account by any extension of Glauber's theory. The most straightforward generalization seems to be the one of Abramovskii, Gribov, and Kancheli (1973), where the Regge technique is used. The procedure is to let the Glauber diagrams in Fig. 18 represent *inelastic* cross sections with the help of Regge theory and the generalized optical theorem. The necessary steps are illustrated in Fig. 21, where the single scattering amplitude is taken as an example. The elastic Glauber diagram is assumed to contain a Pomeron exchange at the vertex. A Pomeron is in turn thought of as a so-called multiperipheral ladder of particles. Using the generalized optical theorem backwards, one can cut this diagram in the middle and arrive at the cross section for multiparticle production shown at the far right. Hence the cross section for one inelastic collision can be derived from the Glauber amplitude for single scattering. Similarly, the cross section for one elastic plus one inelastic collision corresponds to the double scattering diagram of Fig. 18. The situation with two "elastic" collisions with fragmentation of the projectile is represented by the inelastic screening contribution (Fig. 19) to the Glauber theory. However, the very important physical case that the projectile undergoes two inelastic collisions has no counterpart in the Glauber formalism. The extended Glauber theory was therefore supplemented (Abramovskii, Gribov and Kancheli, 1973) with so-called nonplanar Mandelstam diagrams (Mandelstam, 1963a, b) to take this into account. An example is shown

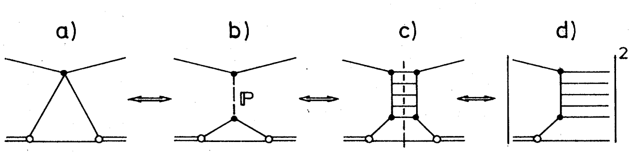

FIG. 21. An example of how the Glauber technique for elastic scattering can be extended to incorporate also particle production.

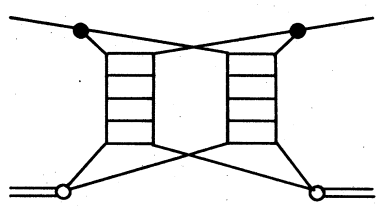

FIG. 22. A nonplanar Mandelstam diagram, for which the procedure of Fig. 21 does not work.

in Fig. 22. It is clear that such a diagram cannot be reduced to any of those in Fig. 18. The article by Abramovskii, Gribov, and Kancheli, in addition, contains the proof of the celebrated AGK cutting rules, which are very useful observations that many Mandelstam diagrams cancel in the actual cross sections. Baker *et al.* (1977, 1978) have applied these rules for the three diagrams at work in the hd double collisions and arrive at a remarkably simple relation among the production cross sections in hd collisions.

Defining the amount Δ of double scattering in the total inclusive cross section as

$$\Delta = (\sigma^{\text{“}hp\text{”}} - \sigma^{\text{“}hn\text{”}}) - (\sigma^{hp} - \sigma^{hn}) , \tag{4.12}$$

where "hp" and "hn" are the even- and odd-topology events discussed in the Introduction, and the cross section "defect" $\delta\sigma$ as

$$\delta\sigma = \sigma^{hp}_{\text{tot}} + \sigma^{hn}_{\text{tot}} - {}^{hd}_{\text{tot}} , \tag{4.13}$$

one gets

$$\Delta = 2\delta\sigma . \tag{4.14}$$

This comes about because the contributions to the double collision cross section are related through the simple AGK equations

$$\sigma(0) = \delta\sigma , \tag{4.15a}$$

$$\sigma(1) = -4\delta\sigma , \tag{4.15b}$$

$$\sigma(2) = 2\delta\sigma . \tag{4.15c}$$

Here $\sigma(i)$ comes from the process where the projectile undergoes exactly i inelastic collisions. Then $i = 0$ corresponds to the diagram of Fig. 18, and $i = 2$ to the nonplanar diagram of Fig. 22. One neglects $\sigma(0)$ for nondiffractive events (high multiplicities) and observes that $\sigma(1)$ gives equal contributions to "hn" and "hp" events, while $\sigma(2)$ contributes only to "hp". Hence

$$\sigma(\text{“}hn\text{”}) \approx \sigma(hn) + \tfrac{1}{2}\sigma(1) , \tag{4.16a}$$

$$\sigma(\text{“}hp\text{”}) \approx \sigma(hp) + \tfrac{1}{2}\sigma(1) + \sigma(2) , \tag{4.16b}$$

from which Eq. 4.14 follows trivially.

The data of Sheng *et al.* (1975), Dombeck *et al.* (1978) and Dziunikowska *et al.* (1976) are in fair agreement with this prediction from the AGK rules, which Baker *et al.* interpret as a support of the space-time structure implicit in the diagramatic technique of reggeon field theories.

The authors also use the AGK rules to derive an equivalent relation between the corresponding cross sections σ_N for producing N charged particles. It reads

$$\alpha_N \equiv \sigma_N^{\text{“}hp\text{”}} / \sigma_N^{hp} = C + \sigma_N(2)/\sigma_N^{hp} , \tag{4.17}$$

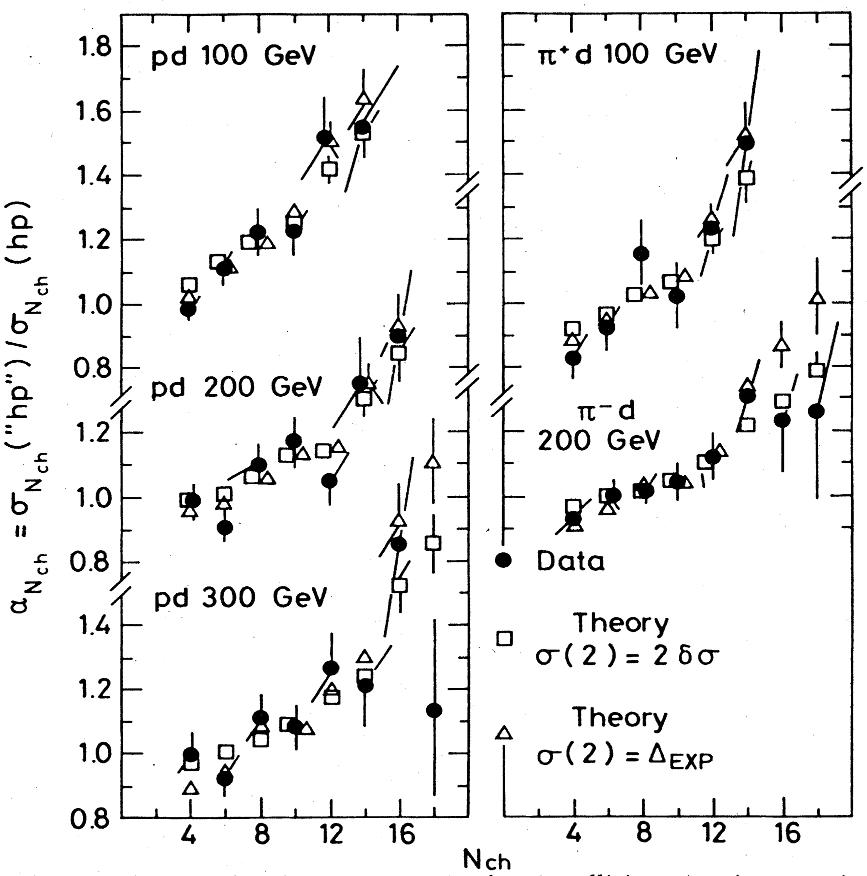

FIG. 23. Multiplicity distributions in even-topology *hd* collisions at various energies. According to the eikonal model, Eq. (4.17), the open "data" points should lie on top of the closed data points. The fits are from Baker *et al.* (1977, 1978).

where C is a constant given by

$$C=(\sigma^{\text{"}hp\text{"}}-2\delta\sigma)/\sigma^{hp}\,. \tag{4.18}$$

For simplicity it is assumed that these two collisions "share" the projectile momentum equally, so that $\sigma_N(2)$ can be evaluated by folding the multiplicity distributions from an hp and an hn collision at half the incoming momentum:

$$\sigma_N(2)=\sigma(2)\rho_N(2)\,, \tag{4.19}$$

where

$$\rho_N(2)=\sum_{i=0}^{N}\rho_i^a(p_{1\text{ab}}/2)\rho_{N-i}^b(p_{1\text{ab}}/2)\,. \tag{4.20}$$

Here ρ_i^a is the probability for having i charged prongs in an ha collision.

Equations (4.17–4.20) are confronted with 100–300 GeV/c pd and $\pi^{\pm}d$ data in Fig. 23. The phenomenological success of the theory is obvious, especially as α_N varies strongly with N in a nontrivial fashion. By summing over σ_N one gets a prediction for $\langle N\rangle$. This has been done for N_-, the number of negative particles, in π^+d collisions at several energies (Moriyasu *et al.*, 1978). The result is shown in Fig. 12.

A more recent π^-d experiment at 15 GeV/c shows, on the other hand, a strong disagreement between the measured α_N and the prediction from Eq. (4.18), as illustrated by Fig. 10 (Porter *et al.*, 1980). This deviation can be a "harmless" energy effect, but the most tempting guess, that the incoming momentum is not shared equally in double collisions at lower energies, is working in the wrong direction. Any asymmetric sharing of the incoming momentum over the two targets would give even lower total multiplicities. Perhaps the whole idea of an "energy sharing" is wrong at lower energies, when the time between the two collisions exceeds some critical value (more on that below).

It should be kept in mind that the application by Baker *et al.* of reggeon field theory to a deuteron target (the BLRW model) is identical, except for details, to the works by Capella and Krzywicki (1978) on collisions on heavier nuclei. In the latter case it is rather clear that the success of the formalism in explaining some of the data on rapidity distributions is more a test of the additional energy sharing hypothesis than of the reggeon field theory itself.

The eikonal model as applied to nuclear collisions by Baker *et al.*, Capella and Krzywicki, and others (Shabelski, 1978; Capella and Kaidalov, 1976; Lehman, 1977) is criticized by Nikolaev and collaborators (Nikolaev and Zoller, 1979; Nikolaev, 1976a, b; Davidenko and Nikolaev, 1976; Nikolaev and Ostapchuk, 1978). They claim that the partial success in reproducing data trends is accidental and caused by an incorrect treatment of the leading particle spectrum after the first collision. This is naturally the same as questioning the energy sharing principle. According to Nikoalev *et al.*, a more natural way to compute multiplicities and y spectra is to use the formalism from the fan diagram model in Sec. IV.A, but with

$$w_{re}(y)=0 \tag{4.21}$$

for the hadrons created in the first collision. The only double collider, the projectile itself, has

$$w_{re}(y)=\sigma^{hN}\langle(4\pi r_d^2)^{-1}\rangle \tag{4.22}$$

(from a simple geometric consideration). Assuming for simplicity that the projectile (leading particle) after the collision has the y spectrum

$$L(y)=\exp(y-Y) \tag{4.23}$$

and that the created (secondary) particles have

$$S(y)=1-\exp(y-Y)\,, \tag{4.24}$$

one gets from Eq. (4.6) that

$$R_{re}^L(y)=Y-y\,, \tag{4.25}$$

$$R_{re}^S(y)=2-(Y-y)(e^{Y-y}-1)^{-1}\,, \tag{4.26}$$

and for the total yield of outgoing particles

$$R_{re}^{S+L}(y)=2(1-e^{Y-y})\,. \tag{4.27}$$

More realistic input spectra give similar results, which in turn are quite inconsistent with the earlier results that the authors criticize. Nikolaev *et al.* take into account also the inelastic intermediate states of the leading particle in a phenomenological way, but the effect on y spectra in general and on ratios $R(y)$ in particular turns out to be small. Figure 17 shows that the resulting eikonal model prediction is in disagreement with the (low-statistics) rescattering data from $\pi^- d \to \gamma X$ at 200 GeV/c (Csorna *et al.*, 1977a, b).

The prediction for the excess multiplicity is

$$\langle n_{re}\rangle-\langle n_{hN}\rangle\approx\langle n_{hN}(p_{eff})\rangle-1\,. \tag{4.28}$$

Here p_{eff} is the average momentum of the projectile after the first collision:

$$p_{eff}=p_{lab}(1-K)\approx 0.5\,p_{lab}\,, \tag{4.29}$$

because K is the mean inelasticity in an hN collision. Equation (4.28) results in the upper line in Fig. 13. The apparent failure to reproduce data here in comparison to the success of the BLRW curve is not as mysterious as claimed by Nikolaev and Zoller (1979). The reason is simply that Eq. (4.17) is achieved for two collisions with momenta $p_{lab}/2$, while Eq. (4.28) contains one collision with momentum p_{lab} followed by another with momentum $p_{lab}/2$. The dispute between these two schools of eikonal models is therefore a question of how a very fast projectile disposes its original momentum in an almost immediate double collision. The most intuitive picture is of course the one by Nikolaev *et al.*, that the projectile first collides and slows down before the second collision. This idea can, on the other hand, not reproduce the experimental data, in contrast to the conjecture that the collision is so fast that it forces the projectile to share its incoming momentum between the two collisions. Observe that we do not mean the distribution of the momentum *loss* experienced by the projectile; this is assumed to be normal for each separate collision, taking into account the reduced incoming momentum.

How can one understand such a splitup of the momentum of an elementary particle? The Mandelstam diagrams formally look as if it were the projectile itself that split up in a double collision. This observation leads the mind to the additive quark model for collisions with nuclei, but as we will see in the next subsection, one does not arrive at the eikonal model results simply by assuming a hadron substructure. The physical interpretation of the energy sharing property of Mandelstam diagrams is still obscure.

C. The additive quark model

No model in today's high-energy physics can survive if it is not compatible with the quark–parton picture. The particular model which is simplest to apply to collisions with nuclear targets is the additive quark model. It prescribes that a hadron is nothing but a collection of free and independent quarks. A proton (pion)–nucleus collision therefore resembles a sum of three (two) consecutive and independent quark–nucleus encounters, except for the requirement that free quarks in the final state must somehow be glued together to real hadrons after the interaction. The details for nuclear targets of this model have been worked out by Anisovich *et al.* (Anisovich and Shekhter, 1973; Anisovich, 1975; Anisovich, Shabelski, and Shekhter, 1978), Nikolaev *et al.* (Nikolaev and Zoller, 1979; Nikolaev, 1976a, b, 1977; Davidenko and Nikolaev, 1976), and Białas *et al.* (Białas, Czyż, and Furmanski, 1977; Białas, 1979). It turns out that the model becomes a sort of hybrid between the fan diagram model and the eikonal model.

The main assumption is that each quark has a rather small probability to collide inelastically with a nucleon:

$$\sigma_{inel}^{qN}\approx\tfrac{1}{2}\sigma_{inel}^{\pi N}\approx\tfrac{1}{3}\sigma_{inel}^{pN}\,. \tag{4.30}$$

Double collisions in the deuteron can occur in two different ways. First there may be rescatterings of the quarks created in the first collision. Here one recovers Eq. (4.2) for the rescattering probability $w_{re}(y)$, but with two important modifications: σ^{hN} is substituted with σ^{qN} according to Eq. (4.30), and the best-fit value for the mass m appearing in the formation length l_f in Eq. (4.1) is changed to 0.85 GeV according to Nikolaev *et al.* It is quite natural that the mass scale is smaller for quark than for pion production. The other origin of rescatterings is the collisions between the second nucleon and the projectile "spectator" quarks that do not take part in the first collision. As these spectators "exist" all the time, no formation time is needed. The $w_{re}(y)$ is hence given by Eq. (4.22) with σ^{hN} again changed to σ^{qN}.

The incoming quark responsible for the first collision is assumed to escape rescattering completely, or, alternatively, to hide among the secondary quarks, being able to rescatter only after a certain time has passed. In any case, its chance to rescatter on the second nucleon is much smaller than for the spectators that follow it in the projectile. As this is absolutely essential for the many successful reproductions of experimental data presented within this model, it is interesting to speculate about its physical meaning. Does it indicate that a colliding projectile quark drastically stops when hitting a target quark, like in the model of Andersson *et al.* (1977)? Or is a colliding quark just "wounded" a certain time after the collision and thereby hindered from reinteracting as suggested by Bialas *et al.* (1977)? Another way of expressing this is to assume that two quarks interact only via clouds of gluons, or virtual $q\bar{q}$ pairs, surrounding each of them. Gluons have a strong self-interaction, so the two clouds arrest each other in the collision and the quarks have to continue "naked." It takes some (formation) time for them to "redress" by polarizing the vacuum, and during that time they are unable to interact with other nearby quarks. If the redressing time is long enough, one can neglect the possibility of multiple collisions by any of the incoming quarks. Here one might object that the redressing does not take place in vacuum, but in closely packed nuclear matter, which should help make the formation time shorter.

Although these arguments are nothing but wild speculations, they illustrate the great potential of using nuclear targets for exploring the inner structure of hadrons and the properties and interactions of quarks. There is simply no other way to probe experimentally the predictions of field theories for the short-time and small-distance properties of strong interactions than to let a newborn hadronic system reinteract as early as possible.

Before presenting the details of the additive quark model, let us discuss whether it becomes identical to the phenomenologically successful version of the eikonal model (see Sec. IV.B) in the limiting case of a large l_f, i.e., when the secondary quarks do not rescatter. Then an average πd double collision event at momentum $p_{\rm lab}$ will obviously look like two qN collisions with momentum $p_{\rm lab}/2$ (or, rather, one qN and one $\bar{q}N$). This is, however, *not* equivalent to two πN collisions at momentum $p_{\rm lab}/2$, like in the eikonal model of Baker *et al.* (1977, 1978), but instead similar to two πN collisions at momentum $p_{\rm lab}$, since an average πN collision resembles a qN collision at half the momentum. The difference becomes even more transparent on heavier nuclei. According to the eikonal model, an average pion–nucleus collision at momentum $p_{\rm lab}$ resembles n πN collisions at momentum $p_{\rm lab}/n$, where n grows in proportion to the nuclear radius. In the cascade-free quark model it looks more like two πN collisions at momentum $p_{\rm lab}$, since both pion quarks have a high chance to collide in a heavy nucleus. Unfortunately, data from such nuclei cannot yet discriminate between these two options. Thanks to the possibility of identifying double collisions in the deuteron, one can, however, conclude that data favor the eikonal model in case of the excess multiplicity. In the quark model without cascades one would get $\langle n_{\rm re}\rangle - \langle n_{hN}\rangle \approx \langle n_{hN}\rangle - 1$ due to the argument given above. Since Eq. (4.28) already gives an overestimate of data, this would be far too high.

For a deuteron target we can hence conclude that rescattering must occur among the secondaries, in case we want to take the additive quark model seriously. The spectators in fact cause fewer rescatterings than do the secondary quarks, but the latter give much smaller multiplicites when colliding with the second nucleon, and the hadrons produced in such cascades tend to be slow and hence limited to rather low y values. One of the most discussed phenomenological topics in the field of collisions with nuclei is the existence or nonexistence of experimental evidence for such intranuclear cascades. The situation is still unclear; and we can only say that some theoretical models need them to reproduce data, while others do not.

For describing double collisions in the additive quark models we again follow the review article by Nikolaev and Zoller (1978). One recovers approximately the linear expression (4.7) for the $w_{\rm re}(y)$ of secondaries, but with $y_c \approx 4.5$ due to the smaller value on m. Let n_q^c mean the number of secondary quarks from the first collision with $y < y_c$, i.e., with a theoretical chance to rescatter, and assume that the distribution of n_q^c is the same in πd and pd collisions. One sums rescatterings of secondaries and spectators and gets

$$P_{\rm re} = \sigma^{qN}\langle(4\pi r_d^2)^{-1}\rangle(\langle n_q^c\rangle + \alpha)\,, \tag{4.31}$$

where $\alpha = 1$ for πd, and 2 for pd collisions. Now one must relate quark distributions to measurable pion distributions. First

$$\sigma^{qN}\langle n_q^c\rangle = \sigma^{\pi N}\langle n_\pi^c\rangle\,, \tag{4.32}$$

where n_π^c is the number of secondary (prompt) pions that can be built of the quarks with $y < y_c$. The quantity $\langle n_\pi^c\rangle$ can be measured (with proper corrections for pions that are not promptly produced). The result is $P_{\rm re} \approx 0.15$ for πd and $P_{\rm re} \approx 0.175$ for pd collisions. Note here the obvious advantage with a quark model that one gets straightforward predictions for the differences between using a pion and a proton projectile. This is naturally not possible in the other models described in this section, where the only differences come from using different πp and pp data as input.

The excess multiplicity of *quarks* becomes

$$\langle n_{\rm re}^q\rangle - \langle n_{hN}^q\rangle = (1-\eta)\langle n_q^c\rangle + \eta\langle n_q\rangle - 1\,, \tag{4.33}$$

where n_q is the multiplicity of outgoing quarks from a spectator-nucleon collision, considering the fact that a proton (pion) spectator carries one third (half) of the projectile momentum on average. The first term in Eq. (4.33) is obviously the contribution from rescattering secondaries, the second comes from spectator collisions, and the third is the "missing" quark that rescatters. The relative number η of spectator rescatterings is given by

$$\eta = \alpha(\langle n_q^c\rangle + \alpha)^{-1}\,. \tag{4.34}$$

The prediction for the excess multiplicity is compared with data and with other theoretical results in Fig. 13. It can be seen that the result is better than for the fan diagram result and for the eikonal model as interpreted by Nikolaev and Zoller, but still not as good as the predictions from the CTM and from the eikonal model by Baker *et al.*

The rapidity distribution dn_q/dy of produced quarks is assumed to be twice the spectrum of produced pions at a rescaled outgoing momentum given by $p_\pi = 2p_q$. The y spectrum of outgoing hadrons in rescattering events can now be derived. For the ratio $R(y)$ to hN spectra one gets easily the approximate value

$$R \approx 1 + \eta \quad (4.35)$$

in the central ("plateau") y region, and

$$R \approx 1 - \eta \quad (4.36)$$

in the high y region. The full result with realistic input distributions is shown as the middle curve in Fig. 17. Data cannot help to discriminate the quark model from the fan diagram model discussed in Sec. IV.A.

We conclude that the additive quark model for high-energy collisions with nuclei is intuitively very appealing and that it is in a fair agreement with experimental data from double collisions in the deuteron. When going to heavier nuclei the cascade computations become very complex in contrast to the situation in the eikonal model, where such cascades are postulated not to exist. It therefore seems most rewarding in the future to try to confront the quark model with accurate data taken in the extreme forward region of high rapidities. Here cascades have no influence and the rescatterings of spectators will be more clearly seen.

V. CONCLUSIONS

The deuteron has been used most fruitfully in high-energy physics. It forms a natural bridge between nuclear and particle physics. As we have seen, some unique information on the strong interaction and its dependence on different hadronic matter has been obtained by using deuterium targets and projectiles. The new experimental techniques have successively enabled a deeper study of the deuteron's interior. The most interesting results (both theoretical and experimental) are perhaps those indicating a six-quark component in the deuteron wave function. This opens exciting possibilities to explore quark matter in a way complementary to, e.g., lepton–hadron scattering. It also implies interesting isobaric excitations within the deuteron and heavier nuclei.

Another important recent result was the verification of the unification of weak and electromagnetic interactions, using a deuterium target.

On the phenomenological side the most severe problem is also the most fascinating one: We still do not know if a "double collision" in the deuteron is of a collective nature or if it is just "abnormal in a conventional way."

It is possible that one can find the answer to this question without forcing the various models to make detailed predictions about complicated quantities. We suggest that one first of all makes a *high-statistics* determination of the probability P_2 for a double scattering, using various projectiles and incoming momenta.

A strong variation of P_2 with projectile type would rule out the fan diagram model and one of the CTM versions discussed in Sec. III.B, but support the eikonal model. Even a moderate variation of P_2 with p_{1ab} would contradict the fluctuon model, the CTM, and the eikonal model, but support the fan diagram and additive quark models. At $p_{1ab} \lesssim 2$ GeV/c, the CTM also expects a change in P_2, while the fluctuon effect should remain at all energies. Finally, a strong rise in P_2 with increasing multiplicity would be in disagreement with the CTM and the eikonal model, but in line with the fan diagram and additive quark models.

It might be that there already exist enough experimental data on double collisions to solve these problems, although they have not been thoroughly analyzed, as the main interest has been centered so far on the neutronlike events alone. We would therefore like to end this review by urging the experimental groups that look upon the deuteron primarily as a neutron source to analyze also the double collision events. The chance is high that they contain some really exciting physics!

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