Pion exchange at high energies

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The state of Regge pion exchange calculations for high-energy reactions is reviewed. Experimental evidence is summarized to show that (i) the pion trajectory has a slope similar to that of other trajectories; (ii) the pion exchange contribution can dominate contributions of higher trajectories up to quite a large energy; (iii) many two-body cross sections with large pion contributions can be fit only by models which allow for kinematical conspiracy at $t = 0$. The theory of kinematic conspiracy is reviewed for two-body amplitudes, and calculations of the conspiring pion–Pomeron cut discussed. The author then summarizes recent work on pion exchange in Reggeized Deck models for multiparticle final states, with emphasis on the predictions of various models (with and without resonances) for phases of the partial wave amplitudes.

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I. FOREWORD

In the last few years, the literature in elementary particle theory has been heavily dominated by studies of quantum chromodynamics. This spate of activity has tended to obscure important advances (both experimental and theoretical) in hadronic physics which are most easily interpreted in terms of the "old fashioned" ideas of Regge theory. This is unfortunate, because the differential cross sections for most exclusive and inclusive reactions cannot be estimated in a simple way from the fundamental QCD field theory, but can be estimated using Regge techniques. Furthermore, these techniques have a rather large degree of success. Hence, rather than neglecting Regge methods, we should be actively involved in improving and testing them so that agreement can be reached on those components (such as Regge trajectories and residues) which might some day conceivably be calculated from QCD.

Reiterating, even if QCD is the fundamental field theory of all hadronic interactions, it is not a good approximation to begin practical calculations of most experimental rates with the fundamental quarks and gluons. Each Regge exchange sums an infinite set of QCD graphs. Regge phenomenology seeks to parametrize these sums, and to discover the extent to which the data is characterized by particularly simple configurations of them (Regge pole exchange is probably one set of ladder graphs; Regge cut exchange, two or more ladders).

Within this framework, pion exchange has been maddeningly difficult. Evidence for pion exchange occurs in a very large fraction of all hadronic experiments (it occurs at some point in almost all multiparticle reactions) but it has been hard to obtain conclusive evidence that the exchange in fact has Regge characteristics. The size of the contribution is easy to estimate crudely (compared to other Regge exchanges) but among the most complicated to estimate exactly.

Recent developments in the experimental situation for two-body reactions and the theoretical situation for multibody reactions make it worthwhile to review the status of pion exchange. The data, discussed in Sec. II, make it appear highly likely that pion exchange has all the usual Regge properties in spite of the many complications associated with accurate estimate of its size and shape. These complications, which have been more or less understood for some time in two-body reactions, are summarized in Sec. III. This older material is included both for completeness and to provide a framework against which the efforts for multibody final states can be measured. Also, there are an increasing number of situations in nuclear physics in which one-pion exchange effects similar to those known in particle physics appear. It is hoped that our review of the role of the cut or absorption corrections will prove helpful in analyzing these lower-energy situations.

Calculations of pion exchange in multiparticle final states should then reflect the properties of the pion learned from two-body interactions. They must also properly include the constraints of unitarity and analyticity. These constraints are rather more complex for multibody reactions than for two-body reactions; as a result it is only within the past few years that theoretical work has really grappled with the phase of the multibody amplitude. This work is summarized in Sec. IV. Section V essays an overview of the situation.

II. IS THE PION A NORMAL REGGEON?

A. What do we expect?

One of the most successful hypotheses in hadron physics is the idea that the hadrons are not elementary particles, but rather composite objects made up of quarks or, equally well (in the bootstrap theory) of other hadrons. Forces which bind such constituents tend to produce poles (composite states) in more than one partial wave of the scattering amplitude; hence the hadrons should be classified into families (Regge trajectories) in which the different members of a family have different angular momenta, but have the same g parity, isospin, etc.

This classification into Regge trajectories has worked with spectacular success for the natural parity $[P=(-1)^J]$ mesons. The rho trajectory $\alpha_\rho(t)$ has been determined by Barnes *et al.* (1976) from its influence on charge exchange scattering ($\pi^- p \to \pi^0 n$) at high energy $[d\sigma/dt \sim f(t)s^{2\alpha_\rho(t)-2}$ for negative $t]$ and by the rho and g mesons ($\alpha_\rho = 1$ at $t=m_\rho^2$; $\alpha_\rho=3$ at $t=m_g^2$). Approximate exchange degeneracy of the forces makes this trajectory overlap substantially with the f trajectory (Mandula *et al.*, 1970) on which the f and h mesons have been found. Combined, this information yields the trajectory shown in Fig. 1 which is almost linear over the region $-1.2 \leq t \leq +4$ GeV^2. The ω trajectory is also determined both by scattering data at negative t (Michael, 1973a) and by two mesons (the 1^- at 784 MeV and the 3^- at 1675 MeV) for positive t. Although most other trajectories studied to date are determined by only one resonance and scattering data for the negative t region, they all seem similar to the results for the rho: straight line trajectories with slopes close to unity (Irving and Worden, 1977). There are many theoretical arguments for infinitely rising Regge trajectories (Mandelstam, 1974), and many simple dual models have used trajectories which rise linearly all the way to ∞ [Veneziano (1968) and refinements too numerous to mention]. Furthermore, implementation (Jones, 1972) of the concept of duality, as well as calculations in simple quark models (De Grand *et al.*, 1975), lead one to believe that the dynamics in unnatural parity systems $[P=-(-1)^J]$ must be similar to that in natural parity systems. In particular the trajectories should have similar properties, at least near the asymptotic limit $t\to\infty$.

With this background, one expects the pion trajectory to resemble the rho trajectory. It must go through the pion ($\alpha=0$ at $\sqrt{t}=0.138$ GeV), and should certainly be measurable in the scattering region (where we might expect it to be approximately linear near $t=0$). We would also expect to find a 2^- recurrence somewhere near $t=2$ GeV^2, i.e., at a mass near 1.4 GeV or somewhat higher (if the trajectory has a slope less than one). This behavior is sketched in Fig. 1.

For many years the experimental evidence for these properties of pion exchange was much less clear than the comparable situation for the rho. The "effective" trajectory calculated from the energy dependence of cross sections expected to be dominated by pion exchange (np backward scattering, photoproduction of charged pions and production of rhos by incident pions) tended to be approximately flat at $\alpha=0$ over a wide range of experimental t (Kreisler *et al.*, 1975; Diebold, 1969; Fox, 1969; Wolf, 1969; Bolotov, 1974). Lack

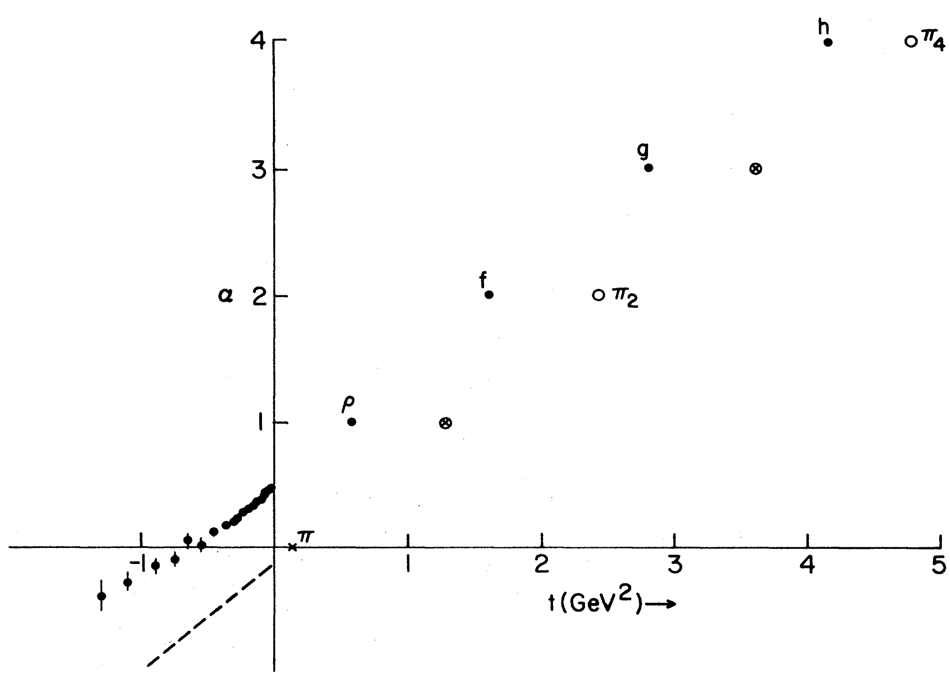

FIG. 1. Expectations for the π trajectory, using the ρ trajectory as a model. (⧫) ρ trajectory measured in $\pi^- p \to \pi^0 n$ by Barnes *et al.* (1976); (•) known resonances on the ρ trajectory and the exchange degenerate f trajectory; (----) predicted π trajectory; (○) predicted π recurrences; (⊗) predicted resonances on trajectory exchange degenerate with π.

of resonant phase variation in the A_3 bump cast doubt on the existence of a 2^- recurrence (Ascoli *et al.*, 1973; Antipov *et al.*, 1973) as did the similar situation for the A_1 bump (Ascoli, 1972; Antipov *et al.*, 1973; Ascoli and Wyld, 1975; Schult and Wyld, 1977) (the A_1 trajectory was expected in many models to be similar to the pi trajectory).[1] In fact the situation was so much less clear that many workers in the field continued to treat the energy dependence and phase of pion exchange as though the pion were elementary.

B. Extraction of trajectories for negative t

Recent high statistics experiments on photoproduction of pions by polarized photons and on the density matrices of rho mesons produced in $\pi^- p \to \rho^0 n$ have allowed effective trajectories to be computed for the exchange of states of definite quantum numbers. In Fig. 2 we show the results of Sherden *et al.* (1973) from photoproduction with polarized photons (see also Quinn, 1973); in Fig. 3 we show the trajectories extracted by Estabrooks, Martin, and Michael (1974) from data on rho production. In both cases there is clear evidence for a "normal" sort of pion trajectory. A similar pion trajectory is obtained (Bolotov *et al.*, 1976) from the reaction $\pi^- p \to \pi^0 \pi^0 n$ for low mass in the $(\pi^0\pi^0)$

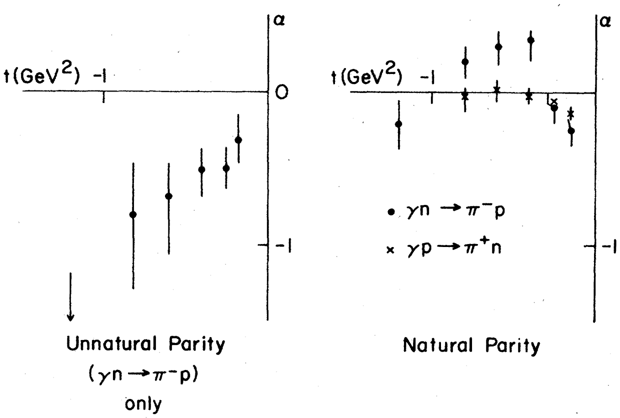

FIG. 2. Trajectories seen in pion photoproduction as shown by Quinn (1973). The unnatural parity trajectory from $\gamma p \to \pi^+ n$ (not shown) is similar to that found in $\gamma n \to \pi^- p$, but has larger uncertainties.

[1] The A_1 trajectory corresponds to the quantum numbers $P = -(-1)^J$, $C = -(-1)^J$, whereas the π and B trajectories correspond to $P = -(-1)^J$, $C = (-1)^J$. Arguments about exchange degeneracy therefore relate the π and B; additional assumptions must be made to include the A_1. These assumptions do not, however, seem very remarkable. As pointed out by Kane (1975), there are two possible sets of arguments: (i) one can observe that the mesons are $q\bar{q}$ trajectories in which the square of the mass is roughly proportional to L, the orbital angular momentum between quark and antiquark. The B and A_1 both have $L = 1$; they differ by having different total spin S for the $q\bar{q}$ pair. Hence their masses should be roughly equal, and their trajectories very close; (ii) duality arguments for $\pi^+\rho^+ \to \rho^+\pi^+$ with helicity 0 rhos require the π trajectory to be exchange degenerate with a $g = -$ trajectory. This can be either the A_1 or the H (central member of the A_1 octet); however, ordinary SU(3) symmetry implies that the A_1 and H trajectories are almost identical.

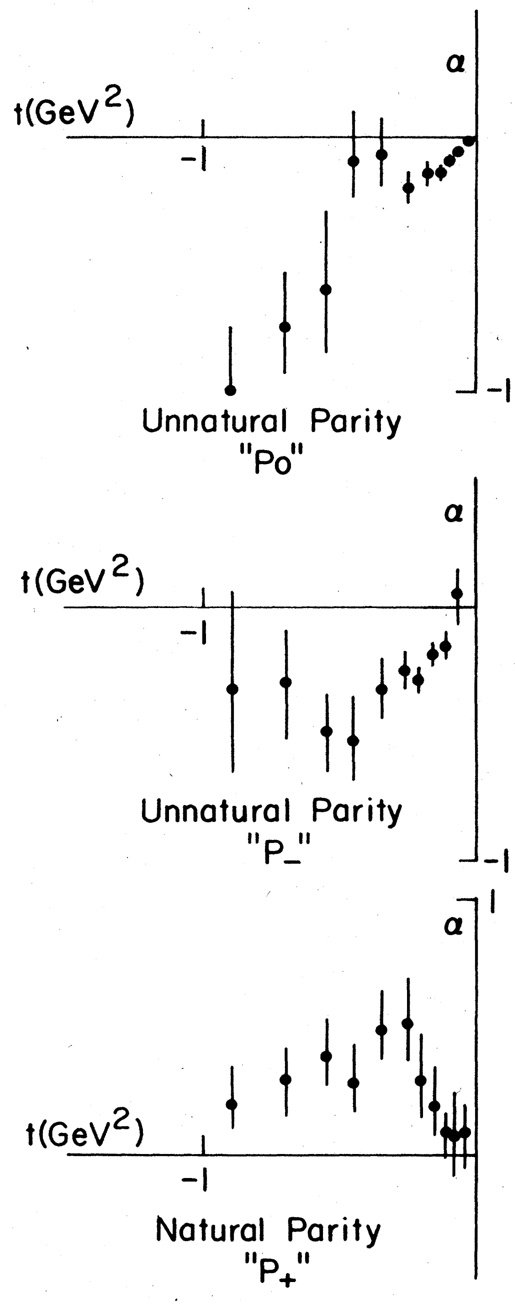

FIG. 3. Trajectories obtained by Estabrooks *et al.* (1974) from data for $\pi^- p \to \pi^- \pi^+ n$ (4–17 GeV/c). The amplitudes P_+ and P_- are for helicity 1 rhos; P_0 represents helicity 0 rhos.

system. The requirement of low mass puts the system into an s wave, and thus limits the production mechanism to pure unnatural parity exchange.

Additional evidence against "flat" trajectories comes from inclusive cross sections. In Fig. 4 we show the trajectories extracted (Brasse, 1973) from inclusive pion photoproduction; in Fig. 5 we show a trajectory extracted from ISR (intersecting storage rings) inclusive neutron spectra (Engler *et al.*, 1975). Neither set of data allows a flat trajectory.

Some insight into the complications present in these

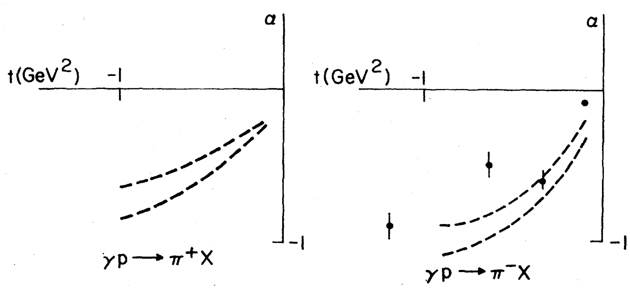

FIG. 4. Effective Regge trajectories obtained from π^+ and π^- inclusive photoproduction as shown by Brasse (1974). (♦) SLAC 9.3 GeV; the dashed curves bracket values allowed by DESY data at 6 GeV.

reactions may be gained from the effective trajectories for *natural* parity exchange obtained in the analyses presented above. The natural parity trajectory obtained from polarized photoproduction is shown in Fig. 2; that from rho production in Fig. 3. These trajectories tend to be somewhat high when compared with the results for $\pi^- p \to \pi^0 n$ shown in Fig. 1. Since the pion exchange dominates for small t (because of the pion pole at small positive t) and the natural exchanges for larger t, the effective trajectory for the net unpolarized cross section tends to remain near zero over a wide range of t.

Ultimately for large enough energy the higher trajectories must dominate at all t, but the relatively large size of the pion residues is such that this does not happen until very high energies. Figure 6 shows the energy dependence of $pp \to n\Delta^{++}$ as obtained by De Kerret *et al.* (1977). Note that pion exchange dominates the cross section up to an energy of $\sqrt{s} = 31$ GeV. Figure 7 shows the change in shape of the forward spike in np backward scattering as the energy increases. The prominent π exchange peak for $|t| < 0.02$ GeV^2 slowly disappears as the energy increases; however, even at the

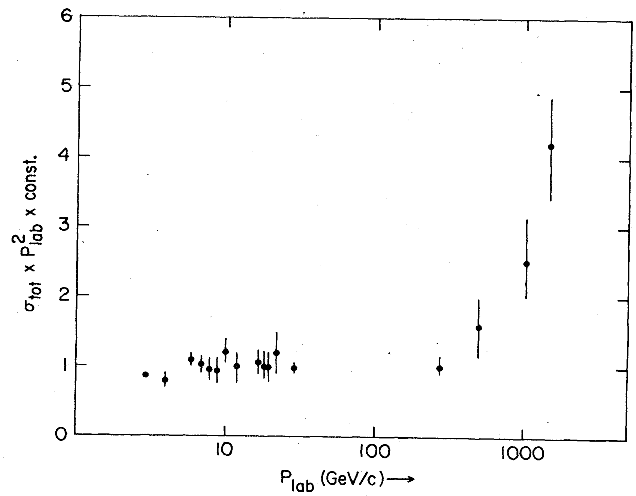

FIG. 6. Total cross section of $pp \to n\Delta^{++}(1232)$ multiplied by $P^2_{\rm LAB}$, as shown by De Kerret *et al.* (1977). The cross section behaves like $P^{-2}_{\rm LAB}$ up to $\sqrt{s} = 31$ GeV; above this point it behaves like $P^{-1}_{\rm LAB}$.

highest energies the forward turnover expected from simple models of ρ exchange is not present. The effective trajectory calculated from the cross section data at Fermilab energies is shown in Fig. 8 (Barton *et al.*, 1976). We see that this is similar to the natural parity

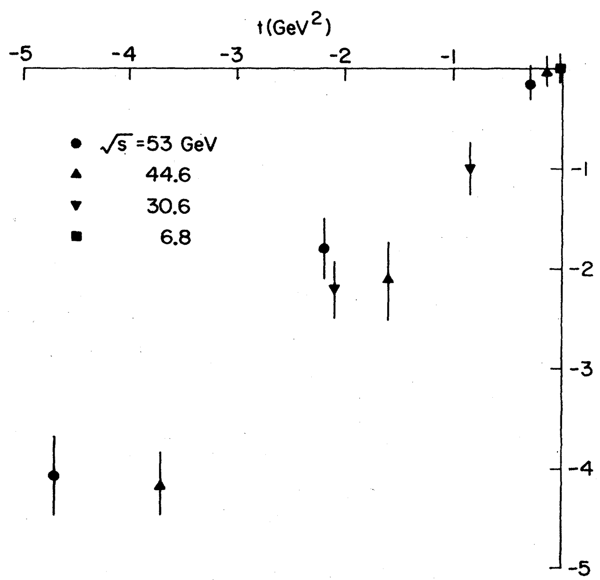

FIG. 5. Effective trajectories obtained from $pp \to nX$, as shown by Engler *et al.* (1975).

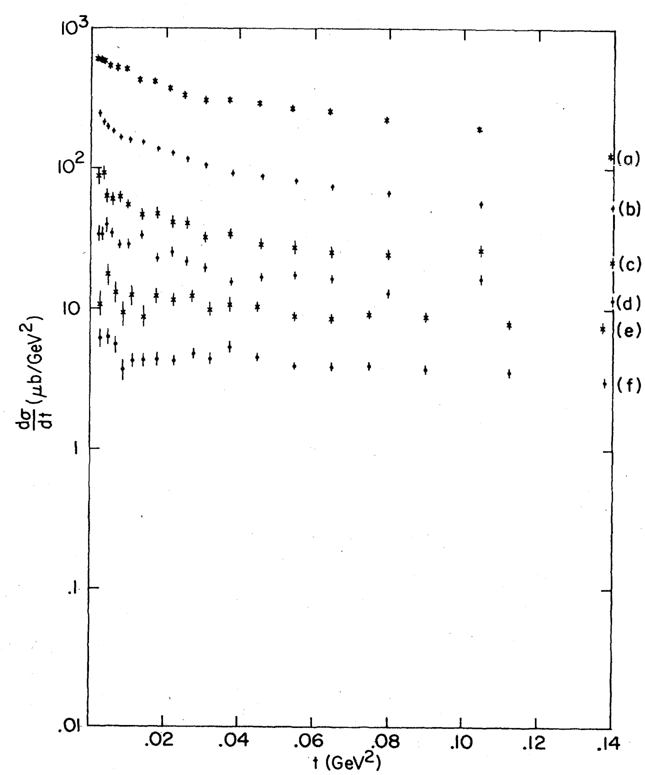

FIG. 7. The t dependence of $np \to pn$ near $t = 0$ flattens out as energy increases. (a) 9–12 GeV/c from Bohmer *et al.* (1976). (b) 19–21 GeV/c from Bohmer *et al.* (1976). (c) 37.5 GeV from Babaev *et al.* (1976). (d) 62.5 GeV from Babaev *et al.* (1976). (e) 90–120 GeV from Barton *et al.* (1976). (f) 240–300 GeV from Barton *et al.* (1976).

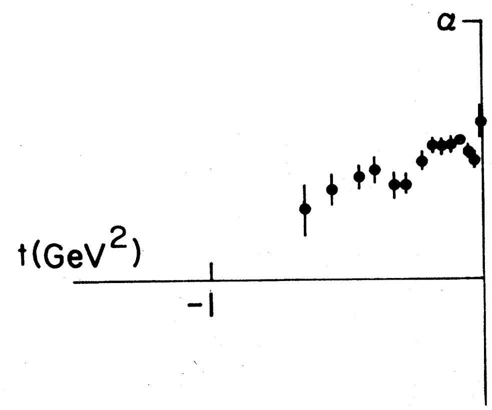

FIG. 8. Effective trajectory obtained from cross-section data on $np \to pn$ between 60 and 300 GeV/c, as obtained by Barton *et al.* (1976). The similarity of this to the natural parity trajectories shown in Figs. 2 and 3, plus the alteration in shape of the forward cross section shown in Fig. 7, indicate that natural parity exchanges are finally becoming important in this energy range. The order of magnitude of the energies involved agrees with the transition shown in Fig. 6.

trajectory calculated from the polarized photon studies at lower energies (Fig. 2).

Summing up the experimental situation at this time, we can say that there is no convincing evidence that the pion trajectory is flatter than the natural parity exchanges. The data obtained since the review of Fox and Quigg (1973) tends to add on the side of an exchange of normal slope. Because of the complexity of the production mechanisms, cross sections alone can lead to misleading effective trajectories; polarization data is very useful in sorting out the exchanges of different quantum numbers. The $np \to pn$ data at Fermilab energies, with a high effective trajectory and no turnover at small t, probably indicate that models for the *natural* parity exchanges need to be improved from simple use of factorization and data from pi-N charge exchange.[2]

C. Trajectory at positive t

The search for the 2^- pion recurrence has taken some time to yield a good candidate, probably because the most detailed analyses have focused on the diffractive production of three pions, $\pi^- p \to \pi^-\pi^-\pi^+ p$. In this channel there are large (Ascoli *et al.*, 1973; Antipov *et al.*, 1973) backgrounds from an f pi Deck effect which tend to obscure resonance phenomena. No doubt the best place to look for these particles is in charge exchange reactions, backward reactions, or decay of higher mass states. However, recent analysis of a diffractive production experiment with very high statistics (Daum *et al.*, 1980b) indicates a $J^{PC} = 2^{-+}$ resonance in the $f\pi$, $\rho\pi$, and $\epsilon\pi$ channels with a mass of 1670 MeV. This corresponds to $m^2 = 2.79$ GeV^2, almost exactly the position of the π_2 suggested by Fig. 1.

In addition, the recent observation of a particle which is almost certainly the A_1 in heavy lepton decays (Jaros *et al.*, 1978) and in backward production (Gavillet *et al.*, 1977; Ferrer *et al.*, 1977) lends strength to the hypothesis that the pion trajectory will continue to rise into the resonance region. While many details remain to be clarified [the diffractive production experiment of Daum *et al.* (1980a) prefers an A_1 resonance with mass 1280 MeV, different from the mass found in τ decay], there are now many indications of the presence of this exchange-degenerate partner of the pion.[1] See Secs. III.C.3 and IV.B for further discussions of A_1 exchange and production.

D. Couplings

Another set of measurements directly affected by the Reggeization of pion exchange is the density matrices of produced resonances. Elementary pion exchange has spin 0; hence it can couple only to spin-nonflip amplitudes in the t channel. The other contributions on the trajectory, from higher spin particles, can couple to other amplitudes. These contributions will show up on the density matrices of produced rho, f, or Δ in the reactions $\pi N \to \rho N$, $\pi N \to fN$, $\pi N \to \rho\Delta$, $\pi N \to f\Delta$. Determination of nonzero couplings of the trajectory in these amplitudes is, however, made difficult because the pion-Pomeron cut will also contribute to these spin-flip amplitudes (even if the pion exchange has only elementary couplings). Most cut calculations have adjustable parameters which can be varied almost at will in each helicity amplitude. Hence it is difficult to disentangle contributions of the "nonelementary" pion residue functions from those of the cut. To date, therefore, study of these additional pion trajectory couplings has principally focused on theoretical models (Jones and Wyld, 1969; Michael, 1973) with few applications (Irving and Michael, 1974).

III. CALCULATION OF PION EXCHANGE IN TWO-BODY REACTIONS

Calculation from field theory of the Born term for exchange of a single elementary pion is straightforward for all the processes of interest. Unfortunately the result of this calculation suffers from two difficulties insofar as shape in t is concerned: (i) In all reactions the predicted cross section is much too large at large t, compared to the high-energy data; (ii) in many reactions the predicted shape of the cross section has totally incorrect behavior at $t = 0$, as well. Let us discuss these problems in turn.

A. Behavior at large t

Early attempts to correct the behavior at large t applied a lesson learned from one-photon exchange—the concept that strongly interacting particles might have form factors rather than point couplings. Various studies of such form factors were made, and successful fits to the lower-energy data for some reactions achieved, using the energy dependence of elementary one-pion exchange (Dürr and Pilkuhn, 1965; Benecke and Dürr, 1968; Wolf, 1967 and 1969).

At this stage it was noted that the Regge approach had some advantages despite the small evidence for shrinkage in the cross sections, because the energy factor

[2] Some preliminary work in this direction has been done by Bouquet and Diu (1977).

$s^{\alpha(t)}$ tends to damp large t and produce peaked cross sections even before the t dependence of the Regge residues is inserted. Once the freedom in the Regge residues is exploited, it is not difficult to achieve a fit to the cross sections (Frautschi and Jones, 1967b). Thus from the point of view of t channel exchanges, the behavior at large t (for fixed energy) is just something to be fit by varying the parameterization of the vertices. Although many schemes were championed from the point of view of simplicity or ease of interpretation, this behavior was not really considered a crucial test of basic theoretical ideas.

Another school of thought analyzed the large t behavior from the s channel point of view. This group (Gottfried and Jackson, 1964; Durand and Chiu, 1965) pointed out that cross sections which are too flat in angle tend to contain too much s wave (or other low partial waves). They approached the problem by applying s channel absorptive corrections to the elementary one-pion exchange graph; removal of the low partial waves tended to make the cross sections more peaked as required by the data. This approach had the appeal that unitarity (an absolute constraint) puts an upper limit on the size allowed to individual partial waves, and any theory violating this limit must be wrong. The corrections to the Born diagram were believed to come from multiparticle intermediate states; these were generally parameterized by a phenomenological function which drastically reduced the size of the low partial waves.

In these early studies of pion exchange, absorption modelists eschewed Regge models because they felt they were not required by the energy dependence. Likewise, Regge modelists eschewed absorption because the ladder graphs which sum to Regge exchanges already contain multiparticle intermediate states and it was felt that it would be incorrect to include such effects twice. However, due to the freedom allowed Regge residues, there was no guarantee that the s channel unitarity constraint would be satisfied by an arbitrary Regge parameterization at arbitrary energy. Hence, Regge fitters had to admit that s channel constraints needed to be inserted into the calculations at some point; likewise absorption modelists had to admit that it would take more than absorption to correct the energy dependence of elementary high-spin exchanges to agree with data (in the case of rho exchange) or with the Froissart bound (in the case of f exchange).

The situation at small t provided a great deal more information, and brought to the fore the importance of spin effects in the study of dynamical models. In order to provide a basis for this we need some background in kinematic constraints at small t. The following paragraphs are designed to provide this; we will then apply it to the analysis of data in the reactions of interest.

B. Kinematics of reactions with spin at small t: Conspiracy and evasion

In the reaction $A+B\to C+D$, the forward direction occurs at $\phi=0$, where ϕ is the function $\phi(s,t) = st(\Sigma m_i^2 - s - t) - t(m_b^2 - m_d^2)(m_a^2 - m_c^2) - s(m_a^2 - m_b^2)^* \times (m_c^2 - m_d^2) - (m_a^2 m_d^2 - m_c^2 m_b^2)(\Sigma m_i^2)$. If $m_A = m_C$ and $m_B = m_D$, ϕ vanishes at $t=0$; otherwise the constraint $\phi=0$ gives a curve $t(s)$, where t approaches zero as s approaches infinity. There are two approaches used in studying behavior near the forward direction. One can look at $t=0$, and observe that the forward direction will approach this as the energy gets high. Alternatively, one can focus on the $\phi=0$ position. The two approaches ultimately give the same information. We briefly review both approaches, since the arguments tend to be rather different and to complement each other in physical understanding.

The simplest situation is the case of equal mass scattering, $m_A = m_C, m_B = m_D$. In this case the point $t=0$ can be reached physically, and corresponds to the forward direction for the process. Because the s-channel helicity amplitudes behave like $f^s_{\lambda_1\lambda_2;\lambda_3\lambda_4} \sim (\phi)^{|\lambda_1-\lambda_2-\lambda_3+\lambda_4|/2} \sim (\sin\frac{1}{2}\theta s)^{|\lambda_1-\lambda_2-\lambda_3+\lambda_4|}$ we see that those s-channel amplitudes for which $\lambda_1-\lambda_2-\lambda_3+\lambda_4 \neq 0$ must vanish at $t=0$ to conserve angular momentum, whereas all other amplitudes may contribute with full strength. In $np\to pn$, for example, angular momentum conservation requires $f^s_{+-;-+}$ to vanish at $t=0$, while $f^s_{++;--}$ can contribute at that point. To see the constraints placed by this on exchanges, we must use the crossing relations to express these s channel amplitudes in terms of t channel ones:

$$f^{s(s,0)}_{+-;-+} = 0 = \tfrac{1}{2}(f^t_{++;++} - f^t_{++;--}) + \tfrac{1}{2}(f^t_{+-;-+} - f^t_{+-;+-}), \tag{3.1}$$

$$f^{s(s,0)}_{++;--} = \tfrac{1}{2}(f^t_{++;++} - f^t_{++;--}) - \tfrac{1}{2}(f^t_{+-;-+} - f^t_{+-;+-}). \tag{3.2}$$

Pion exchange populates only the amplitude combination $f^t_{++;++} - f^t_{++;--}$. Thus any theory that has only pion exchange in the t channel (and hence has $f^t_{+-;-+} - f^t_{+-;+-} = 0$) will require that the pion contribution vanish at $t=0$ in order that $f^s_{+-;-+}$ vanish there.[3] If, however, the theory also allows for population of $f^t_{+-;-+} - f^t_{+-;+-}$, the pi-like contribution need not vanish.

Equations like (3.1) are called conspiracy relations. These can be satisfied in two ways: either the contribution from each set of quantum numbers independently vanishes as $t\to 0$, or the contributions for each set of quantum numbers are individually nonzero, but collaborate in such a way as to make the constraint hold (Leader, 1968). The first possibility (where each contribution independently vanishes) is called evasion; the second is called conspiracy. Note that the term conspiracy (at least as used by this author) applies only to the means of satisfying the kinematical constraint; no judgment is applied about whether the amplitudes are populated by Regge poles, cuts, etc.

One important consequence of conspiracy is that the double flip amplitude $f^s_{++;--}$ can remain full strength in the forward direction (Frautschi and Jones, 1967a, 1968). We will see that this very strongly influences the shape of the $np\to pn$ and charged pion photoproduction reactions. Another important feature is the fact that conspiracy requires collaboration between amplitudes of different quantum numbers. If we decompose Eq. (3.1) into amplitudes dominated by exchanges of definite parity (Volkov and Gribov, 1963)

[3]Although we have not proved it here, the argument in fact requires that $f^t_{++;++} - f^t_{++;--}$ vanish like t or some higher power of t. See Fox and Leader (1967).

$$0 = f^t_{++;++} - f^t_{++;--} + \tfrac{1}{2}(\bar{f}^t_{+-;-+} - \bar{f}^t_{+-;+-})$$
$$- \tfrac{1}{2}(\cos\theta_t)(\bar{f}^t_{+-;-+} + \bar{f}^t_{+-;+-}) , \qquad (3.3)$$

we see that conspiracies involving the pion will involve either a natural parity exchange (pole or cut) of the same α, or an unnatural parity, unnatural charge parity exchange with $\alpha = \alpha_\pi + 1$. [$\bar{f}^t_{+-;-+} - \bar{f}^t_{+-;+-}$ is dominated by contributions from $P = -(-1)^J$, $C = -(-1)^J$; $\bar{f}^t_{+-;-+} + \bar{f}^t_{+-;+-}$ is dominated by contributions from $P = (-1)^J$, $C = (-1)^J$.][4] We emphasize at this point that there is nothing really remarkable about conspiracy; Feynman graphs frequently conspire with themselves. In particular, Born diagrams with s-channel poles, which have no reason to populate a particular set of t-channel quantum numbers, almost always satisfy the kinematic constraints by conspiracy. We will come back and discuss the phenomenological consequences of this shortly; for the moment let us move on to other ways of viewing the conspiracy relations.

A different approach to the conspiracy relations for the equal mass case is to observe that at $t = 0$ the entire four-momentum vector for the exchange is zero. Hence its Little group is $O(4)$ rather than $O(3)$. The exchanges should thus be classified by representations of $O(4)$ (Toller, 1965 and 1968; Sertorio and Toller, 1964; Freedman and Wang, 1967b,c; Domokos and Suranyi, 1964; Domokos, 1967). When this is carried out, the $O(4)$ representations can be broken down into their $O(3)$ content and one finds the classifications: (i) $M = 0, s = 0$ for single natural parity objects like rho, etc.; (ii) $M = 0, s = 1$ for pairs of unnatural parity poles separated by one unit of spin and having opposite charge conjugation naturality; (iii) $M = 1, s = 1$ for families with poles of $P = -(-1)^J$, $C = (-1)^J$ and $P = (-1)^J$, $C = +(-1)^J$ at some spin α and a pole of $P = -(-1)^J$, $C = -(-1)^J$ at $\alpha - 1$. $O(4)$ representations with M higher than 1 also have parity doubled content. These are exactly the families deduced from the analysis of the crossing relation as sketched above. One should again bear in mind that, although the papers on $O(4)$ representations tend to be written in terms of Regge poles (Lorentz poles (Lorentz poles, at $t = 0$), all the arguments except those dealing with factorization of Regge residues apply no matter what type of singularity in angular momentum is assumed.

The arguments for the unequal mass case are similar but have a slightly different emphasis (Domokos and Tindle, 1968; Stack, 1968) so we repeat them here. Since the point $t = 0$ cannot be reached at any finite energy, the angular momentum conservation arguments in the s channel are not a particularly good way of deriving the results, although we will see that consequences can be interpreted in the same way.

Once again the crossing matrix between s- and t-channel helicity amplitudes is used to derive the maximal singularities of regularized t-channel helicity amplitudes at $t = 0$ and constraints relating these amplitudes at that point (Hara, 1964; Wang, 1966 and 1967; Cohen-Tannoudji, Morel, and Navelet, 1968). Again we find that, if the amplitudes possess their maximally singular behavior at $t = 0$, they must conspire to satisfy the constraint equations. When the constraint equations are satisfied by conspiracy, only those s-channel helicity amplitudes required to vanish at $\theta = 0$ by angular momentum conservation will be suppressed near the forward direction: all other s-channel amplitudes will contribute with full strength in this region. This can be contrasted with the evasive situation, where s-channel double flip amplitudes are suppressed near the forward direction even in the cases where this is not required by angular momentum conservation. Study of the behavior near $t = 0$ for the unequal mass case requires inclusion of "daughter" trajectories (with singular residues) at $\alpha - 1, \alpha - 2, \ldots$ (Freedman and Wang, 1966 and 1967a; Jones and Shepard, 1968). These same daughters are predicted by $O(4)$ symmetry for the equal mass case.

In summary, then, for all possible mass configurations, we find that exchange of a unique set of quantum numbers (such as the elementary pion pole or a single pion Regge trajectory) will have to satisfy the kinematic constraints at $t = 0$ by evasion. This means that the contributions will be suppressed near the forward direction in some s-channel double flip amplitudes which are not required to vanish by angular momentum conservation. In order to restore those amplitudes to full strength, we must have exchange in the t channel of more than one quantum number set.

The conspiracy can be achieved by many different dynamical mechanisms.

(i) One can have a Regge trajectory with natural parity at the same α as the pion or (for the $M = 0$ type of conspiracy) an unnatural parity trajectory at $\alpha_\pi + 1$. Both of these are ruled out by experiment.

(ii) One can have an s-channel Born term pole or some other such s-channel structure. This possibility has been extensively discussed (Diu, 1975; Diebold, 1969; Gluck, 1974) with respect to pion exchange for two reasons: (a) The Born terms tend to have the same energy dependence as elementary one-pion exchange, so they fit with the measured behavior of the low-energy cross sections; (b) This approach works reasonably well numerically in a number of cases, especially charged pion photoproduction. The presence of such terms (which could produce a flat effective trajectory at large t) seems to have been ruled out by the newer experiments. Nonetheless, there may be important lessons to learn from these terms with respect to duality (the idea that the s-channel poles make up the t-channel exchanges and vice versa).

(iii) One can begin with a t-channel Regge pole exchange and absorb it [Kane and Seidl (1976) and references therein]. This imposition of s-channel requirements will affect the t-channel quantum number content of the amplitude and produce the desired conspiracy.

(iv) One can construct the pion-Pomeron cut by a Mandelstam-type diagram with both pion and Pomeron Regge exchanges (Chia, 1972 and 1973). This cut will have both natural and unnatural parity in the t channel and will satisfy the constraint equation by conspiracy.

[4]When $f^t_{+-;-+}$ behaves like s^α, $\bar{f}^t_{+-;-+} = f^t_{+-;-+}/(\sin^2 \tfrac{1}{2}\theta_t)$ behaves like $s^{\alpha-1}$. Similar considerations apply to $\bar{f}^t_{+-;+-} = f^t_{+-;+-}/(\cos^2 \tfrac{1}{2}\theta_t)$.

C. Behavior at low *t*

In the two-body reactions studied to date, conspiracy is necessary to produce many of the effects expected from pion exchange. In some reactions one cannot see the very sharp spike in $d\sigma/dt$ due to the nearby pion pole unless conspiracy is included. In the next subsections we review the most important reactions with special attention to their behavior near $t=0$ and to the various calculational techniques which have proven useful in studying this.

1. $np \to pn$

This reaction was one of the first to attract attention. The elementary one-pion exchange graph fails at both large t and small t as shown in Fig. 9. However, the energy dependence of the reaction looks remarkably like that of elementary one-pion exchange up to 25 GeV/c (Miller *et al.*, 1971; Engler *et al.*, 1971). Polarization data are available up to 12 GeV/c (Abolins *et al.*, 1973), but the measurements give values for only three points with $|t|<0.1$, so there is little information about the region where the pion unambiguously dominates. Hence most theoretical studies of pion exchange have been forced to fit the cross section. As we have pointed out above, this complicates matters considerably. However, both the energy dependence at low t and the presence of the sharp forward spike indicate that there is an important pion exchange contribution (the spike's width of approximately $m_\pi^2 = 0.02$ GeV^2 can be obtained from Fig. 9 even though the logarithmic scale used there obscures the peaking that is so marked on the normal linear scale). This appears to dominate at small t for the lower energy range. We are thus challenged to fit these data with conspiratorial models.

As is the case with all these reactions, there have been numerous contributions to the literature which produce the conspiracy by elementary pole exchange in another channel. As mentioned previously, this has the disadvantage that the contributions are fixed poles at $J=0$ in the complex angular momentum plane of the t channel, and there really is no good evidence for such poles when polarization data is available. We will therefore not discuss such fits here and will concentrate on the s-channel absorption or pion-Pomeron cut models to be described below. It is important to realize, however, that the constraints of duality on these reactions are not entirely clear and the cuts discussed below may be in some way related to the properly Reggeized pole contributions in these other chan-

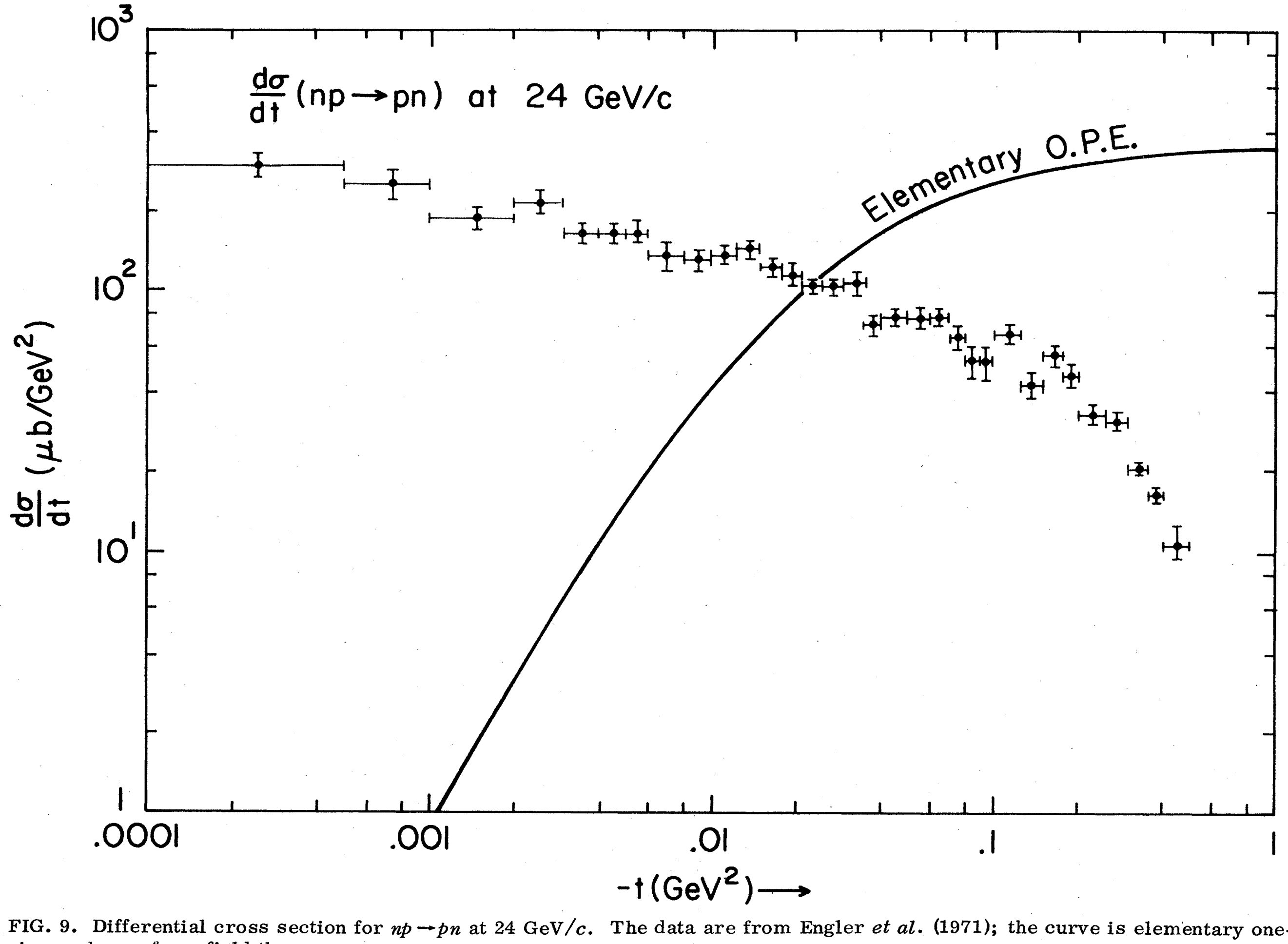

FIG. 9. Differential cross section for $np \to pn$ at 24 GeV/c. The data are from Engler *et al.* (1971); the curve is elementary one-pion exchange from field theory.

nels.

The forward peak of np backward scattering has been studied in the absorption model by Henyey *et al*, (1969) and Kane *et al.* (1970), and in the pion-Pomeron cut model by Chia (1972), and by Kaidalov and Karnakov (1968 and 1969). It happens that the two models give very similar results for this reaction. The cut contribution is almost constant near the forward direction, whereas the pole contribution is rapidly varying—it must vanish at $t=0$ (since the pole itself is evasive), and it contains the sharp pion pole. The sign of the cut relative to the pole is completely determined by the structure of the reaction (the two sides are the same). This requires that the pole and cut contributions interfere destructively; this produces a sharp dip near $t=-m_\pi^2$, which "creates" the forward spike seen in the data. As shown in a more phenomenological way by Engler *et al.* (1972), this interference is at the heart of any fit to the data.

We take the time here to sketch the basics of the absorption and cut calculations so we can refer back to them in later sections for the more complicated reactions.

The cut calculation involves evaluation of graphs of the sort shown in Fig. 10. These explicitly contain the third double spectral function necessary to produce Mandelstam-type Regge cuts. In his evaluation of the graphs, Chia (1972) made the σ particle a scalar, isoscalar meson to simplify the work. The loop calculations required cutoffs for some reactions; hence, this approach (although parameter free in principle) does contain some parameters in practice, as does the absorption model discussed below. One feature of the pion-Pomeron cut that should be kept in mind is that it does not populate (to highest order in s) the amplitudes with A_1-like quantum numbers. (See the further discussions of this in Sec. III.C.3.)

Two main approaches to the absorption model have dominated attempts to evaluate it. In the strong-cut Regge absorption model, one begins by writing the amplitudes for Regge pole exchange in the s channel; a function is then introduced to absorb the low partial waves. A previous article in this journal by Kane and Seidl (1976) describes this technique in detail. This approach has many features in common with the evaluation of the box graphs shown in Fig. 11. As such, the calculation does not really produce a Regge *cut*, since the box graph does not contain a third double spectral function (Mandelstam, 1963). Nevertheless, proponents of this model claim that the results effectively mimic the cut and this appears to be true for pion exchange [although not necessarily for other exchanges—see Chia (1977)].

The other approach to absorption, the Williams method or "poor man's" absorption model (Williams, 1970) is much simpler. One begins by writing down the elementary one-pion exchange graph for the s-channel helicity amplitudes. In the amplitudes where angular momentum conservation requires vanishing at $\theta=0°$, no alteration is made. In the others, the t dependence is "corrected" by evaluating everything but the pole at $t=m_\pi^2$. As explained above, this introduces a conspiracy because when these amplitudes are crossed back to the t channel both natural parity and unnatural parity contributions are present. The large t behavior and s dependence are then manipulated at will by introduction of form factors or Regge behavior. The "poor man's" absorption model has been used in many data analyses for the pion exchange reactions, with a substantial degree of success.

We emphasize that the important feature for fitting the shape of the cross sections at $t=0$ is the incorporation of an $M=1$ conspiracy into the models. The many approaches discussed here have all been reasonably successful because the region to be fitted is quite small in t (everyone is willing to cede the behavior at large t to the rho and A_2 exchanges) and because the energy dependence is close to that of the elementary pion pole in this range. Polarization data at larger s, which will be extremely useful in unravelling the explicit shape of

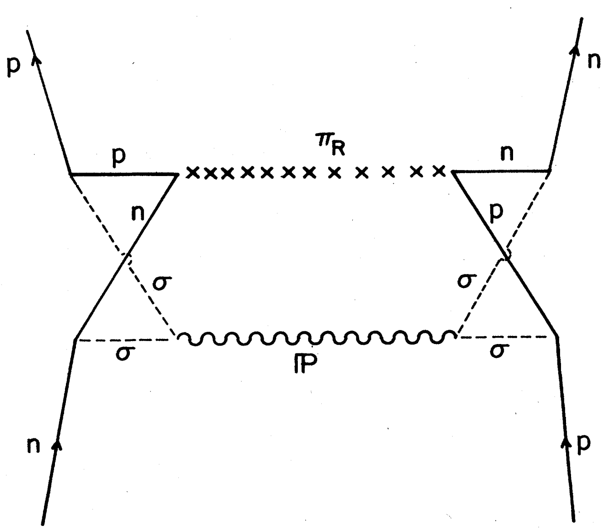

FIG. 10. Graph evaluated by Chia (1972) to compute pion-Pomeron cut.

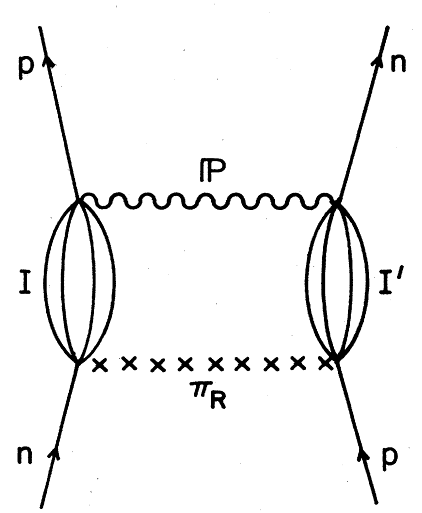

FIG. 11. Schematic graph evaluated by absorption model. Intermediate state I may include resonances.

the natural parity contributions [see, for example, Froyland and Winbow (1971)], will probably not tell us very much more about the pion and pion-cut contributions. However, it should confirm the pion trajectory extracted from other reactions (i.e., that the trajectory is in fact of normal slope and not flat).

2. $\gamma p \to \pi^+ n$

The photoproduction of charged pions is directly analogous to np backward scattering in that the elementary pion exchange graph goes to zero at $t=0$, whereas the cross section shows a sharp spike with width characteristic of the pion pole. Once again the conspiracy equation

$$\bar{f}^t_{01;++}(s,0) - \bar{f}^t_{01;--}(s,0) = -i[\bar{f}^t_{01;+-}(s,0) + \bar{f}^t_{01;-+}(s,0)] \tag{3.4}$$

shows that we must have a collaboration between exchanges with different quantum numbers in order to restore the double flip amplitude to its full value in the forward direction. If one begins with only a Reggeized pion exchange it is clear that such a conspiracy can be produced by absorption or the pion-Pomeron cut, and indeed very good fits to the data can be produced this way (Henyey *et al.*, 1969).

Spice is added to the interpretation of this reaction by the fact that gauge-invariant perturbation theory requires the pion pole to be accompanied by a direct channel nucleon pole (see Fig. 12). This nucleon Born term, when analyzed in the t channel, turns out to contain contributions from both parities which conspire with each other. Furthermore, calculation of this gauge invariant set yields a size for the cross section which agrees reasonably well with the data in the forward direction (Harari, 1967; Richter, 1967; Diebold, 1969). As long as the data was consistent with the presence of a fixed pole at $J=0$, therefore, it appeared that these gauge-invariant Born terms played a special role. Many papers appeared calculating the minimal gauge-invariant set of graphs for other photoproduction reactions, with the idea that a similar situation would obtain in, for instance, $\gamma p \to \pi^- \Delta^{++}$ (Gluck, 1974; Clark and Ugaz, 1975).

This information gained from perturbation theory cannot be totally disregarded when the process is Reggeized. Various subtleties needed for proper Reggeization of the pion exchange are best learned from study of the perturbation graphs, and I would like to review some of them here.

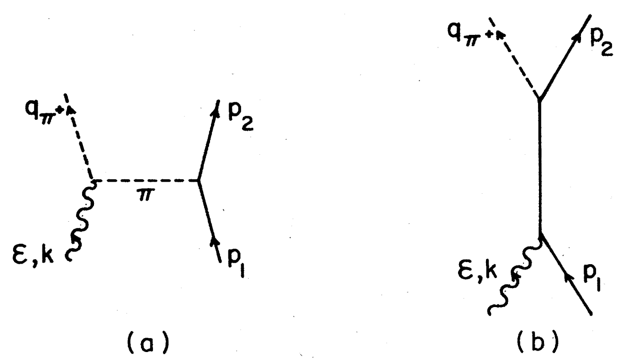

FIG. 12. Gauge-invariant set of Born graphs for the reaction $\gamma p \to \pi^+ n$.

If one inserts an ordinary "naive" Regge formula in the t-channel helicity amplitude for pion exchange $f^t_{10;++} - f^t_{10;--}$, one is led to the conclusion that there should be no pion pole at all since the elementary pion with $\alpha = 0$ cannot couple to the t-channel helicity one system (and all t-channel pion-photon states have helicity one). Indeed if one calculates the perturbation theory graphs shown in Fig. 12, the contribution from Fig. 12(a)

$$e 2\epsilon \cdot q [g/(t-\mu^2)] \bar{N}\gamma_5 N\,, \tag{3.5}$$

will vanish when calculated in the t channel in the gauge where the polarization vector for the photon is $\epsilon^t = 1/\sqrt{2}\,(0, \pm\cos\theta_t, i, \mp\sin\theta_t)$ [in the t channel $q = (q^0, q\sin\theta_t, 0, q\cos\theta_t)$]. This is the gauge normally used for Reggeization of photon processes because the polarization vectors are the natural extension from the polarization vectors for massive vector mesons. In this gauge, however, the contribution from the nucleon pole graph [Fig. 12(b)] produces a pion pole in the cross section! Where did this come from?

Within this framework, the pion pole shape of the nucleon exchange contribution came from *kinematic* factors in the t-channel decomposition. Normal (non-charge-coupling) behavior of these amplitudes at $t = \mu^2$ would be (Jones, 1969)

$$f^t_{10;++} - f^t_{10;--} \sim \sin\theta_t (t-\mu^2)^\alpha (\cos\theta_t)^{\alpha-1} \alpha_\pi \frac{e^{-i\pi\alpha_\pi}+1}{\sin\pi\alpha_\pi(t)}$$
$$\underset{t\to\mu^2}{\sim} \left(\frac{1}{t-\mu^2}\right)(t-\mu^2)^\alpha \left(\frac{1}{t-\mu^2}\right)^{\alpha-1} \sim 1\,, \tag{3.6}$$

using the fact that

$$\cos\theta_t = \frac{\sqrt{t}\,(2s - 2M^2 + t - \mu^2)}{(\mu^2 - t)(t - 4M^2)^{1/2}}\,, \tag{3.7}$$

where μ is the pion mass and M is the proton mass. However, the charge coupling contribution behaves like $(t-\mu^2)^{\alpha-1}$ near threshold rather than like $(t-\mu^2)^\alpha$ so the net result is a pole in the "pi-like" amplitude from the kinematic factor $\sin\theta_t$.

In Reggeizing pion exchange, one must therefore write the amplitude $f^t_{10;++} - f^t_{10;--}$ in the form

$$(t-\mu^2)^{\alpha-1}[A + B(t-\mu^2)]\, d^\alpha_{10}(\theta_t)\, \frac{e^{-i\pi\alpha_\pi(t)}+1}{\sin\pi\alpha_\pi(t)}\,,$$

where the constant A is determined by the charge coupling, whereas B represents all the other higher-order graphs. Near the pion pole, the combination $(t-\mu^2)^{\alpha-1} d^\alpha_{10}(\theta_t)$ becomes constant, so we pick up the behavior of the dynamical pole.

A similar result for the t dependence can easily be obtained if the charge coupling is associated with a fixed pole at $J=0$ in the partial wave amplitude. In this case the normal threshold behavior can be assigned to the whole amplitude, but it is multiplicative with a fixed pole

$$B_\pi(J,t) \sim (t-\mu^2)^J (x/J + y)\,. \tag{3.8}$$

The ratio $d^J_{10}(\theta_t)/J$ has no net pole at $J=0$, so this factor of $1/J$ will *not* affect the energy dependence. This is analogous to the removal of the kinematic zero in forward Compton scattering by pions by a fixed pole at $J=1$ (Abarbanel *et al.*, 1967)

In summary, then, proper Reggeization of photoproduction processes requires careful parametrization of the charge coupling contribution near the photon threshold. In perturbation theory this behavior is characteristic of all the contributions from the charge coupling in the gauge-invariant Born term, whether or not they explicitly have a pion propagator. If the threshold behavior is thus correctly incorporated into the Reggeized expression, the problem of providing a pion pole in the correct amplitude is solved. We note in passing that these apparent pathologies of the charge coupling are in fact derived in a straightforward way as the limit $m_v \to 0$ of the usual (Wang, 1966 and 1967) kinematic expressions for a vector meson of mass m_v. It is the noncharge couplings which are truly pathological—these have extra zeroes for photon processes because the formulae for energy and momentum become the same as $m_v \to 0$ (Henyey, 1968).

One must still ask whether there is any significance in the fact that the Born terms reproduce the correct size of the cross section near the forward direction. This is related to the question of whether the pion-Pomeron cut is somehow dual to the direct channel nucleon pole. Thus far very little definitive is known about this matter. (See, however, Jackson and Quigg, 1969 and 1970; Gluck, 1974; Argyres *et al.*, 1971; and Clark, 1978).

3. $\pi^- p \to \rho^0 n$

The rho production reaction is very similar to charged pion photoproduction, except that the produced rho also has a helicity zero state. In the t channel, the elementary pion exchange couples to this (helicity zero) state only, and the resulting cross section (after large t values have been damped by the usual methods) resembles the measured cross section quite closely. However, careful measurement of the density matrix of the produced rhos shows that the s-channel helicity one contributions have a sharp spike very similar to that seen in photoproduction (as one would expect from vector dominance arguments).

Once again, absorption corrections to the basic Regge pion exchange can be used to produce this spike. Again, calculations of the pion-Pomeron cut yield results similar to those of absorption (Henyey *et al.*, 1969) although there are some interesting differences in detail. In particular, Chia (1972) found that there is no reason for the cut contribution to the helicity zero amplitude to interfere destructively with the pole contributions. In fact, his fit to this reaction has a constructive interference in this amplitude.

Data from the reaction $\pi N \to \pi\pi N$ has been used extensively to extract pi-pi scattering, by extrapolation to the pion pole. Since only the t-channel helicity zero amplitudes couple to the elementary exchanged pions, it is important to first extract this amplitude from the data prior to the pole extrapolation. One must then correct somehow for absorption effects so that the true pole residue is extracted. Discussions of how to do this have been offered by Williams (1970), by Froggatt and Morgan (1972), and by Estabrooks and Martin (1975) (see also references therein).

One problem which affects such extrapolations to the pion pole has been the lack of experimental information about spin structure at the nucleon end. There are two contributions to the t-channel helicity zero rho-pion exchange and "A_1"-like exchange. These couple to different nucleon-antinucleon spin states: $|++\rangle - |--\rangle$ for the pion, and $|+-\rangle - |-+\rangle$ for the A_1. In the absence of information about the nucleon spin structure, it has been assumed in the past that there is no A_1-like amplitude so that the entire t-channel helicity zero contribution comes from pion exchange.

There are two reasons for this assumption—the shaky experimental status of the A_1 resonance, and the fact that the pion-Pomeron cut should not produce any A_1-like quantum numbers to first order. We repeat the argument for this last assertion. The A_1 contributes to the amplitude with $C=-(-1)^J$, $P=-(-1)^J$. The pion-Pomeron cut will have even signature, since the signature of a cut is just the product of the signatures of the poles (Branson, 1969). Hence, it will contribute only to amplitudes of even J. Furthermore, the cut has even charge conjugation parity because both the constituents have even charge conjugation parity. Hence, $C=+\neq-(-1)^J$ for even J.

New data on polarized targets (DeGroot, 1977) show that this assumption (the absence of contributions to the A_1 exchange amplitudes) is not valid.[5] Hence, the extrapolation to the pion pole is somewhat more complicated than had been anticipated previously. Preliminary results show, however, that the results obtained for the pion-pion scattering amplitudes will not be very much changed by the inclusion of the proper spin structure at the nucleon end.

4. Other reactions

There are many other two body reactions in which pion exchange plays an important role—$\pi N \to fN$, $\pi N \to \rho\Delta$, $pp \to \Delta\Delta$, etc. All analyses of these reactions to date show that the patterns discussed above for $np \to pn$, $\gamma p \to \pi n$, and $\pi^- p \to \rho^0 n$ recur again and again. In particular, (i) If the elementary pion exchange contributes only to direct channel helicity flip amplitudes (as was the case in np backward scattering and in charged pion photoproduction) then conspiracy (attributed to the pion-Pomeranchon cut or absorption) will be important and will dramatically change the shape of the cross section in the forward direction, making the spike of the pion pole evident. (ii) If the elementary pion pole has a large contribution to the direct channel nonflip amplitude (as was the case in $\pi^- p \to \rho^0 n$), then the cut effects will modify this amplitude only slightly. They will produce effects in the flip amplitudes similar to those discussed in (i); frequently these can only be examined by studying spin dependence (i.e., density matrices) in the data.

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[5] This supports an earlier analysis of Field (1972) showing that the strange member of the A_1 octet is exchanged in vector meson production.

A very interesting pion exchange situation in nuclear physics is discussed by Moake *et al.* (1979). They examine the reactions $p+{}^6\mathrm{Li}\to n+{}^6\mathrm{Be}$ and $p+{}^{12}\mathrm{C}\to n+{}^{12}\mathrm{N}$ at an incident proton energy of 144 MeV. Despite the very low energy, there is good reason to believe the reaction is dominated by pion exchange with cut-type corrections. These reactions have the same type of spin-parity structure as appears in $\pi^- p\to\rho^0 n$; however, the masses at each vertex in the nuclear reactions are essentially equal. This creates an important shift in the relative contributions of the pole term to various amplitudes: while $f^s_{0\,1/2;0\,-1/2}$ dominates the pole contributions for $\pi^- p\to\rho^0 n$, $f^s_{1\,1/2;0\,-1/2}$ will have the dominant pole contributions in the nuclear case. After cut effects are added, $d\sigma/dt$ for $\pi^- p\to\rho^0 n$ still turns over in the forward direction as it does for the case of elementary pion exchange. By contrast, the $p+{}^6\mathrm{Li}\to n+{}^6\mathrm{Be}$ and $p+{}^{12}\mathrm{C}\to n+{}^{12}\mathrm{N}$ cross sections measured by Moake *et al.* show the increase near $t=0$ seen in $np\to pn$, another reaction in which the helicity double flip amplitudes dominate (see Fig. 13).

These data on nuclear reactions, and the calculation presented in the same paper fitting it with absorbed pion exchange, demonstrate that cut corrections are important even at very low energies where the Regge energy dependence of pion exchange is irrelevant. One hopes that other pion exchange reactions will be studied using nuclear targets, since they offer a different set of mass and spin configurations than is available with more "elementary" particles.

Additional interaction between particle and nuclear physicists would be very desirable for two other reasons: (i) Both sets know a great deal about pion exchange, but they have used such different language in the past that communication of this knowledge has been rare. It seems clear, however, that the effects described by high-energy physicists as "pole + cut" are very similar to those described by nuclear physicists as "addition of a sigma particle"; (ii) wider appreciation of the effects by nuclear physicists might lead to more experimental work. The $p+{}^6\mathrm{Li}\to n+{}^6\mathrm{Be}$ study apparently was considered unfeasible by some nuclear experts; they claimed it was "well known" that the pion exchange vanished at $t=0$, making it very small in the region of interest!

D. Comparison with calculations of other Regge exchanges in two-body reactions

Regge cuts or absorption effects appear to play a larger role for pion exchange than for other exchanges, such as rho exchange. Two reasons for this have been advanced:

(i) Pion exchange couples largely to helicity flip at the nucleon end in the direct channel. Hence there is a substantial chance that the contribution will be to the sort of amplitude (double flip) which is suppressed in the forward direction unless conspiracy is present. This means that the presence or absence of conspiracy has a good chance of affecting the shape of the cross section.

(ii) The nearness of the pion pole to the physical scattering region tends to enhance the size of the cut contributions.

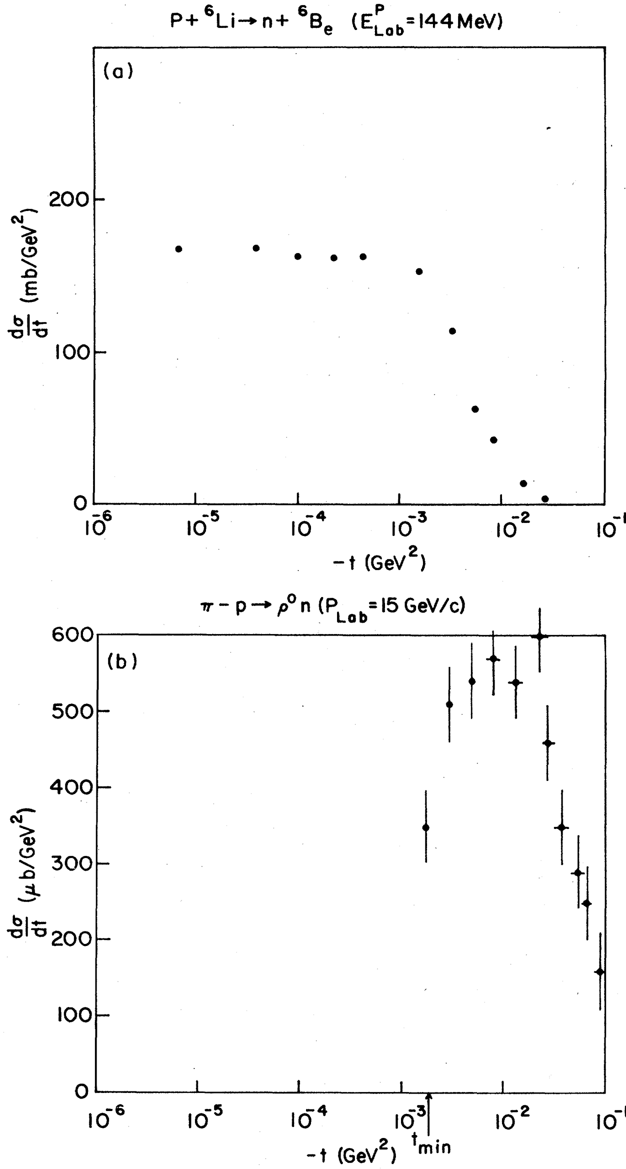

FIG. 13. The two reactions $p+{}^6\mathrm{Li}\to n+{}^6\mathrm{Be}$ and $\pi^- p\to\rho^0 n$ have similar spin-parity structure. However, because the mass difference $m_\rho^2-m_\pi^2$ is large compared to $m^2({}^6\mathrm{Li})-m^2({}^6\mathrm{Be})$, the pattern of amplitude population by the pion pole is quite different in the two cases. (a) $p+{}^6\mathrm{Li}\to n+{}^6\mathrm{Be}$ at $E_m=144$ MeV from Moake *et al.* (1979). As in the case of $np\to pn$ (Fig. 9), the pole term contributes mostly to direct channel helicity double flip. The conspiring cut is nonzero in the forward direction, producing forward peaking. (b) $\pi^- p\to\rho^0 n$ at $p_{\mathrm{lab}}=15$ GeV/c from Bulos *et al.* (1971). Here the pole term contributes mostly to the direct channel single flip. Even after the conspiring cut is added, the cross section turns over in the forward direction. The crucial difference between the reactions lies in the ratio of the mass difference to t, the momentum transfer. Because the ρ production data is at 15 GeV/c, effects due to the ratio of the mass difference to s are small (the only one of importance is the nonzero location of $t_{\min}$).

A detailed comparison of relative effects in the cut model can be found in Chia (1977).

In the next section we consider pion exchange in multiparticle reactions. Almost all the work to date on multiparticle final states ignores cut-type corrections, despite their importance in two-body reactions. From the point of view of (i) above, this is reasonable, since in the multi-Regge world helicity nonflip couplings are frequently available. However, most of the other reasons advanced by Chia for the importance of the pion-Pomeron cut are still present. We should therefore ask at each stage in the multiparticle studies whether cut effects are needed.

IV. CALCULATIONS FOR MULTIPARTICLE REACTIONS

As in the case of two-body and quasi-two-body reactions, pion exchange plays an important role in multibody final states because the pion couplings are so large, and the pole is so near the physical region, that the pion trajectory will dominate higher trajectories over quite a large energy range. When one realizes that the subchannel energies are smaller than the overall reaction energy by a substantial amount in most cases, and that the pion will dominate at these lower energies if it can contribute at all, it is clear that almost all multiparticle final states will contain an important pion exchange component.[6]

On the one hand, this means that a relatively large amount of phenomenological work has been done on pion exchange in multiparticle final states. On the other hand, the complexity of these reactions has meant that most of this work has been done using "naive" models, in order to simplify the calculations. Many of these naive models have failed to Reggeize the pion exchange; most of the others have inserted a Reggeized energy dependence but have not inserted a Reggeized phase for the pion exchange. Effects of the pion-Pomeron cut or absorption have likewise been treated by only a few authors.

Our study of pion exchange in two-body reactions has shown that careful attention to all the possible measurables (in that case spin) pays off in better understanding of the underlying dynamical mechanisms. One set of measurables available in all reactions is the shape of the cross section in various final-state angles; this allows one to determine interference phases between the partial waves of the system. Some relative phases which can be obtained from multiparticle final states cannot be found in any other way. Hence the interference phases can be measured, should be measured, and in some cases have been measured. They then need to be compared with the models at hand. Clearly such models should also be sophisticated enough to predict these phases.

[6] When I began writing this review paper, I had the ambition of including an Appendix with references to all experiments since 1970 showing important pion exchange characteristics. This proved to be very difficult; fortunately, I have been spared this task by the appearance of the "Indexed Compilation of Experimental High Energy Physics Literature" from the Particle Data Group (LBL-90, September 1978). This lists experimental papers by reaction, thus allowing one to quickly locate the charge exchange situations which pion exchange frequently dominates.

While all students of quantum mechanics are familiar with the phase-shift parametrization of two-body partial wave amplitudes, few have been similarly indoctrinated in the expected phase behavior of multibody reactions. In fact, there are constraints on the multibody amplitudes similar to those in the two-body case. These are very briefly reviewed in Sec. IV.A, to provide some background for the discussion of phases in recent calculations of pion exchange-dominated multibody final states.

In the subsequent sections we review the current status of these calculations, with special emphasis on the so-called "Deck-like" models for $\pi N \to \pi\rho N$. The reason for this emphasis is that this final state has received a great deal of experimental and theoretical attention in hopes of pinning down the parameters of the A_1 particle; hence much more detailed information on phases is available for this case than for most others. In Sec. IV.B we discuss treatments of $\pi N \to \rho\pi N$; in the following section we describe calculations for other reactions in which one or two pions are produced. Finally, in Sec. IV.D, we describe reactions initiated by leptons in which pion exchange forms an important part of the contribution at the hadronic end.

A. Some features of phases in multibody final states

In the case of $2 \to 2$ reactions, the general arguments leading to the phase shift-elasticity parametrization are simple: (i) There is a cut in the energy plane for each partial wave, hence the partial wave cannot be purely real. The presence of this cut is related to causality requirements. (ii) Conservation of flux puts conditions on the size of the partial waves, thus leading in a straightforward way to the use of a phase shift.

Regge pole exchange in these cases leads naturally to a cut in energy and hence a well-defined phase

$$(-s)^{\alpha(t)} + \tau(s)^{\alpha(t)} = s^{\alpha(t)}(e^{-i\pi\alpha_\pi(t)} + \tau). \tag{4.1}$$

However, as we mentioned in our earlier discussion, the Regge parametrization does not automatically lead to unitary (flux conserving) partial wave amplitudes. This requirement must be added in "by hand"; in practice the pion-Pomeron cut corrections take care of it quite well.

For $2 \to n$ amplitudes, our first task is to determine the cut structure allowed by causality for this more complicated situation. This was done by Steinmann (1960). He found that causality allows discontinuities only in the places where tree graphs would have them. Hence, for instance, the reaction $\pi_1^- p_1 \to \rho^0 \pi_2^- p_2$ can have three types of double discontinuity: (i) the discontinuity in $s_{\pi_1^- p_1}$ can have a discontinuity in $s_{\rho^0 \pi_2^-}$; (ii) the discontinuity in $s_{\pi_1^- p_1}$ can have a discontinuity in $s_{\pi_2^- p_2}$; or (iii) the discontinuity in $s_{\pi_1^- p_1}$ can have a discontinuity in $s_{\rho^0 p_2}$. It is impossible to have a double discontinuity in $s_{\rho^0 \pi_2^-}$ and $s_{\pi_2^- p_2}$ (see Fig. 14).

Consequences for multi-Regge exchange were worked out much later [see Brower, deTar, and Weis (1974), and references therein]. The problem here is to reconcile the requirements that a double Regge exchange graph like that shown in Fig. 15 should (i) behave like $s_1^{\alpha_\pi(t_1)} s_2^{\alpha_P(t_2)}$ at large values of these subenergies, and

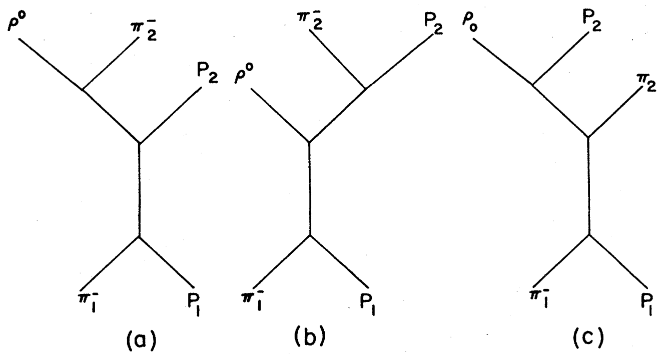

FIG. 14. Tree graphs illustrating double cut structures allowed for the reaction $\pi_1^- p \to \rho^0 \pi_2^- p_2$.

yet (ii) not violate Steinmann by having simultaneous discontinuities in s_1 and s_2. The solution achieved by Brower, deTar, and Weis is to parameterize the amplitude for Fig. 15 by a two-term expression

$$\beta_1(t_1)\beta_2(t_2)s_1^{\alpha_\pi(t_1)}s_2^{\alpha_{\mathbf{P}}(t_2)}(\xi_1\xi_{21}\eta_{12}^{\alpha_1}V_1+\xi_2\xi_{12}\eta_{12}^{\alpha_2}V_2)\,, \quad (4.2)$$

with

$$\xi_1=e^{-i\pi\alpha_\pi(t_1)}+1\,,\quad \xi_2=e^{-i\pi\alpha_{\mathbf{P}}(t_2)}+1\,,$$
$$\xi_{12}=e^{-i\pi[\alpha_\pi(t_1)-\alpha_{\mathbf{P}}(t_2)]}+1\,,$$
$$\xi_{21}=e^{-i\pi[\alpha_{\mathbf{P}}(t_2)-\alpha_\pi(t_1)]}+1\,,$$
$$\eta_{12}=s_{12}/s_1s_2\,, \quad (4.3)$$
$$V_1=\sum_{n=0}^{\infty}\frac{1}{n!}\Gamma(-\alpha_1+n)\Gamma(-\alpha_2+\alpha_1-n)\eta_{12}^{-n}\beta(\alpha_1-n,t_1,t_2)\,,$$
$$V_2=\sum_{n=0}^{\infty}\frac{1}{n!}\Gamma(-\alpha_2+n)\Gamma(-\alpha_1+\alpha_2-n)\eta_{12}^{-n}\beta(\alpha_2-n,t_1,t_2)\,.$$

The first term of Eq. (4.2) has the cut structure of Fig. 14(a) and the second the cut structure of Fig. 14(b).

As we will see in the sections below, phenomenological studies of the phases in multiparticle final states have gone through a considerable evolution. It is only quite recently that Steinmann-obeying parametrizations have been used in comparisons with data. One reason for this delay has been that some of the workers in the field feared that the "experimental" phases might be inaccurate, and hence detailed comparisons with them could be irrelevant. The chief criticism here revolved around implementation of unitarity (flux conservation) for a multiparticle amplitude. We now review the arguments involved.

A theorist with a model amplitude like Eq. (4.2) for the reaction $\pi^- p \to \rho^0\pi^- p$ shown in Fig. 15 will typically not compare it with the actual multipion angular distributions. Rather he will compare it with magnitudes and phases of partial wave amplitudes for $\pi^- p \to \rho^0\pi^- p$ extracted by fitting data of $\pi^- p \to \pi^-\pi^+\pi^- p$ to some preconceived form. These data contain reactions like $\pi^- p \to \epsilon^0\pi^- p$ and $\pi^- p \to f^0\pi^- p$ as well as the modeled $\pi p \to \rho^0 \pi p$; since all lead to the final state $\pi^+\pi^-\pi^- p$, the interference phases between the various final states can be extracted by examination of the angular shape of the cross section. However, some sort of parametrization of the individual partial waves as a function of the various dipion subenergies must be used. In principle, the interference phases depend crucially on this parametrization, since most of the information about them comes from places in the 3π Dalitz plot where two dipion resonances "overlap."

Early extractions of such phases from the data used an "isobar" model, which allowed for the two processes shown in Fig. 16. Resonance poles were inserted in the appropriate $\pi^-\pi^+$ subenergy, and the amplitudes added. While this prescription explicitly obeys the Steinmann requirements about cuts in the dipion and tripion subenergies, it can violate unitarity (Aaron and Amado, 1973 and 1976). Considerable effort has been expended in improving the isobar model fitting functions, and methods are now available for fitting with functions that have correct unitarity and analyticity properties *within* the $\pi\mathbf{P}\to 3\pi$ system (Schult and Wyld, 1977). Phases from such fits are available in the literature. This particular problem, therefore, has in some sense been solved.

One problem which remains, however, is the simultaneous implementation of proper cut structure and unitarity for the three body $\rho^0\pi p$, etc., states. This is a weak point in both the data analysis and the theory. While the best available form used for data fitting has correct analyticity and unitarity properties in the 3π subchannel, it makes no attempt to implement these constraints for the overall reaction. Likewise, the prescription of Brower, deTar, and Weis guarantees multi-Regge models with proper cut structure, but it does not guarantee that the resulting $2\to 3$ amplitude will satisfy flux conservation constraints. Some studies

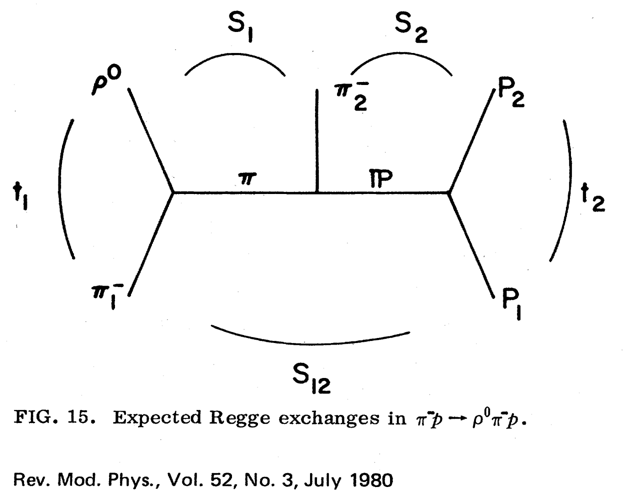

FIG. 15. Expected Regge exchanges in $\pi^- p \to \rho^0\pi^- p$.

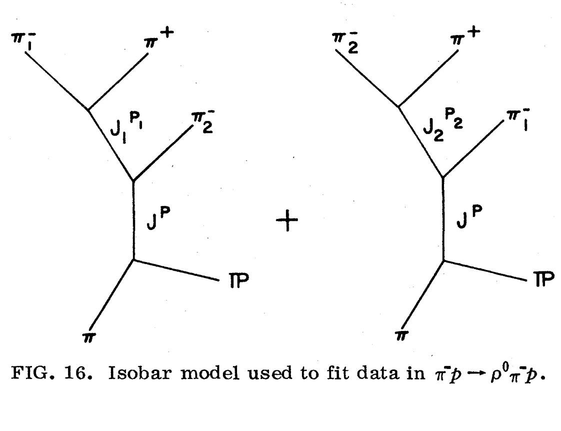

FIG. 16. Isobar model used to fit data in $\pi^- p \to \rho^0\pi^- p$.

have been made of cut corrections to multi-Regge calculations, but the unitarity requirements have not yet been as completely studied as in the case of two-particle final states.

On balance, therefore, there is potentially a lot of dynamical information in the phases of multiparticle amplitudes. As more and more data become available, this information can be determined more accurately and dynamical predictions such as those of Brower, deTar, and Weis can be checked. This has particularly interesting consequences for pion exchange, since various phases are determined by the pion trajectory and hence we have an additional check on its slope.

As we have indicated in this section, the study of these final-state interference phases has both theoretical and practical difficulties. However, it should ultimately provide us with substantial improvements in our models for pion exchange. The studies reviewed in the sections below show that while much progress has been made in this direction, there is room for further work.

B. Calculations for $\pi N \to \rho\pi N$ and related reactions

In two-body reactions, the most noticeable effect of pion exchange is the forward peaking that it produces in differential cross sections. While this *per se* is also important for the multiparticle final states, the chief feature normally discussed is not the peaking in momentum transfer itself but the peaking in invariant mass which results as a "kinematic reflection" of the momentum transfer behavior. For the reaction $\pi N \to \rho\pi N$, it was first pointed out by Deck (1964) that the process depicted in Fig. 17 would result in enhancement of the low $m_{\rho\pi}$ region because of the peaking produced by diffraction in t_2 and by pion exchange in t_1.

Berger (1968) then discovered that the shape of the $m_{\rho\pi}$ peak calculated in this way could be sharpened if the Regge energy dependence of the pion exchange was included. A very large amount of similar work on other reactions has followed the original Berger paper. While we cannot possibly do justice to all the papers, a fair sampling of the cross section calculations is displayed by the list given toward the end of Sec. IV.C below. The

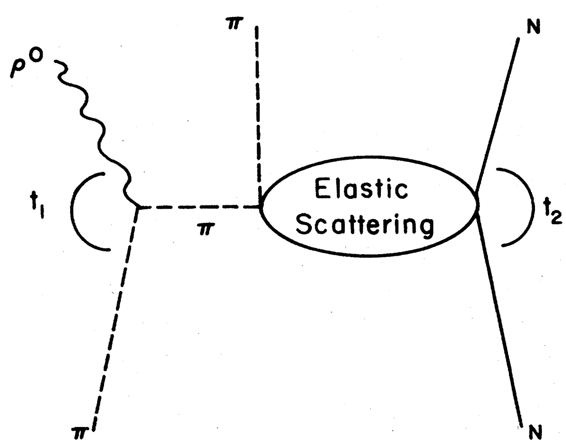

FIG. 17. Deck's model for $\pi N \to \rho\pi N$. In the original calculation, an elementary pion propagator is used. Berger (1968) Reggeized the $s_{\rho\pi}$ dependence.

partial wave analysis of the $\rho\pi$ system in $\pi N \to \rho\pi N$ by Froggatt and Ranft (1969) was one of the first attempts to look at the consequences of π exchange for amplitudes rather than cross sections.

The question of whether the bump seen in the data was really the A_1 resonance or "just" a kinematic reflection of the Deck-effect type then hinged essentially on the phase behavior in the rho-pi channel. Studies of isobar model fits to the data enabled people to extract relative phases for the partial waves in the $\epsilon\pi$, $\rho\pi$, and $f\pi$ channels (Brockway, 1970; Ascoli, 1974;[7] Antipov *et al.*, 1973; Otter *et al.*, 1974; Thompson *et al.*, 1974; Tabak *et al.*, 1974). The phase of the 1^+s wave $\rho\pi$ system, in which the mass enhancement occurs, was found to be relatively flat as a function of $m_{\rho\pi}$. This makes a resonance interpretation difficult. Improved fitting schemes have been studied and applied to the data (Ascoli and Wyld, 1975; Goradia *et al.*, 1974; Schult and Wyld, 1977); the resulting phases are not greatly different from those obtained using the isobar model despite the substantial theoretical improvements in the fitting functions.

One then must confront the essential question: are the phases in the data in agreement with (i) the phases of a properly Reggeized Deck model; (ii) the phases obtained by having a produced A_1 interfere with some sort of background (probably dominated by a Reggeized Deck model); or (iii) neither of the above. Thus far there have been fairly convincing arguments advanced for both (i) and (ii), so we will consider those in turn.

Although an early comparison of the phases in the $\epsilon\pi$ and $\rho\pi$ system was made by Ranft (1971), the first detailed calculation of phases in the reaction $\pi N \to \pi\pi\pi N$ was done by Ascoli *et al.* (1973, 1974) using the amplitude (see Fig. 18 for definition of kinematics)

$$M_{\pi\pi}(s_{\pi\pi}, t_{\pi\pi})\,\text{propagator}\,(t_R, s_{3\pi}, s_{\pi\pi})\,M_{\pi N}(s_{\pi N}, t_{NN})\,, \quad (4.4)$$

where $M_{\pi\pi}$ is the pi-pi scattering amplitude, $M_{\pi N}$ is the πN scattering amplitude, and the Reggeized pion propagator was taken to be

$$[s_{3\pi} + \tfrac{1}{2} t_R - \tfrac{1}{2}(s_{\pi\pi} + 2m_\pi^2 + t_{NN})]^{\alpha_\pi(t_R)} e^{-i\pi\alpha_\pi(t_R)/2}/(t_R - m_\pi^2)\,. \quad (4.5)$$

They were able to obtain substantial agreement with the phases extracted from the data—both with regard to the relative phases in the various channels at one $m_{3\pi}$, and with regard to the trends of these phases in $m_{3\pi}$ at low values of $m_{3\pi}$ (see Fig. 19).

The model of Ascoli *et al.* has the disadvantage that it contains a "double discontinuity" in $s_{\rho\pi}$ and $s_{\pi N}$, forbidden by the Steinmann prescription described in Sec. IV.A. Since the constraints on the discontinuities are intrinsically related to phases calculated from the amplitudes, this raised the possibility that the calculation of Ascoli *et al*, although interesting, was theoretically unsupported. However, it has been shown (Jones, 1976; Puhala, 1978) that for the $\rho\pi$ channel a Steinmann-obeying model can be constructed which approximately re-

[7]Although the main thrust of Ascoli's 1974 talk at the Conference on Experimental Meson Spectroscopy is the unitarized fit, he also presents here the results of the basic isobar model fit.

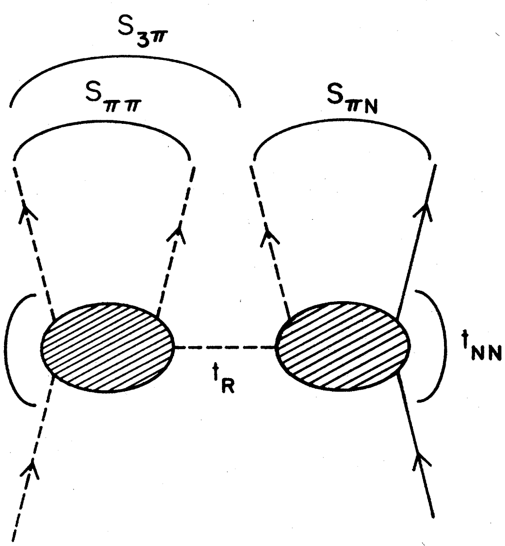

FIG. 18. Labeling of kinematics for the Deck model of $\pi N \to \pi\pi\pi N$.

produces the results of Ascoli *et al.* Although the extension of this model to the full 3π final state is still in progress at the time of writing, it appears likely that most of the previous results can be obtained with a theoretically sound model of double Regge exchange type. There is thus good evidence that the data in the region of the "A_1" peak can be approximated by a modern Deck type model. It should be noted, however, that perfect agreement has not been reached, nor has it been sought, in the following sense: the parameters in these calculations have been fixed to agree with other reactions, but have not been fit to the details of this particular reaction. Hence, it is not surprising that certain details such as the shape of the "A_1" peak are not exactly reproduced.

In an attempt to improve these various details, several groups have tried fitting the data with a superposition of a pion-exchange Deck-like background and a resonant A_1. Motivated by the idea that phase structure in the region of a resonance should be strongly influenced by unitarity, these groups have carefully included final-state interactions in the rho-pi system after the basic Deck mechanism. Three different sets of results have emerged:

(i) The work of Basdevant and Berger (1977, 1978) indicates that there is a resonant A_1 which is not directly produced by Pomeron exchange from the incident pion beam, but which is mainly present because of final-state interactions in the 1^+ wave of the rho-pi system after the initial production mechanism of the Deck effect. These authors have several solutions for the possible mass of their A_1, but prefer the solution with 1185 MeV because they can make this agree with the data on decay of the heavy lepton τ.

(ii) Another study, by Longacre and Aaron (1977), also improves the fit to the data by interfering a resonant A_1 with a Deck-type background and including final-state interactions. These workers believe that the mass of the resonant A_1 is 1450 MeV; they also find agreement with the mass distribution of the three pion states in heavy lepton decay [Aaron, Goldberg, and Longacre (1978); see also Aaron, Longacre, and Sacco (1979)].

(iii) Early studies by Bowler *et al.* (1975), with a similar Deck-resonance final-state interaction model, implied a resonant A_1 with mass of 1300 MeV. Recent fits of the Bowler parametrization to new high statistics data by Daum *et al.* (1980a) give the value 1280 MeV.

Typically these authors begin with an amplitude for Fig. 17 which has the phase of Pomeron exchange i and a real pion propagation. Near $t_2=(t_2)_{\min}$, this amplitude is largely a $1^+\rho\pi$ s wave, a feature first discovered by Stodolsky (1967). They then focus their attentions on the inclusion of 3π resonance parametrizations in this partial wave.

The Deck background in this partial wave contributes mostly to terms with the discontinuity structure of Fig. 14(b), regardless of the particular model used. A two-term model like Eq. (4.2) was studied by Puhala (1978); the Argand diagrams displayed in his paper show that the 1^+s $\rho\pi$ wave is dominated by discontinuities in $s_{\pi_2-p_2}$ rather than by the cut in $s_{\rho^0\pi_2^-}$. A different possibility was suggested by Jones (1977), who showed that if the Pomeron exchange in Fig. 15 preserves s-channel helicity in the subamplitude $\pi_R p \to \pi p$, *only* discontinuities of the sort symbolized by the tree graph of Fig. 20 [and hence Fig. 14(b)] contribute. In general, then, the Deck background supplies little or nothing to discontinuities of the sort shown in Fig. 14(a).

Resonances in the $\rho\pi$ system, however, should produce exactly the discontinuity structure of Fig. 14(a). The aim of the three groups mentioned above has therefore been to add these resonances to the Deck background in such a way that the overall size of the 1^+s $\rho\pi$ wave is kept within reasonable bounds.

This would seem to be a reasonable method of procedure. Why then have these three groups obtained such different values for the A_1 mass? Why do Schult and Wyld, using a less model-dependent (but still unitary) fitting routine find no need for an A_1 resonance when fitting the same data used by groups (i) and (ii)? Clearly more work needs to be done. This new work should incorporate the lessons learned from the various investigations we have mentioned. In particular:

(a) A definitive study would fit the new CERN data, discussed by Daum *et al.* This experiment on diffractive production of $\pi^+\pi^-\pi^-$ has much better statistics than previous experiments; if the effect is indeed a delicate one with a "shy" A_1 resonance hiding behind the Deck background, small error bars in the data may be crucial to the interpretation.

(b) Unless all parties can agree on a partial wave fitting routine, theorists should fit the actual $(3\pi)p$ data rather than magnitudes and phases obtained by some other group.

(c) In any case, it is necessary to simultaneously model the $\rho\pi$, $\epsilon\pi$, and $f\pi$ states. Most of the information in the data comes from the relative phases and magnitudes; restricting oneself to a model for only $\rho\pi p$ automatically reduces one to either ignoring much of this information or guessing at the contributions from the

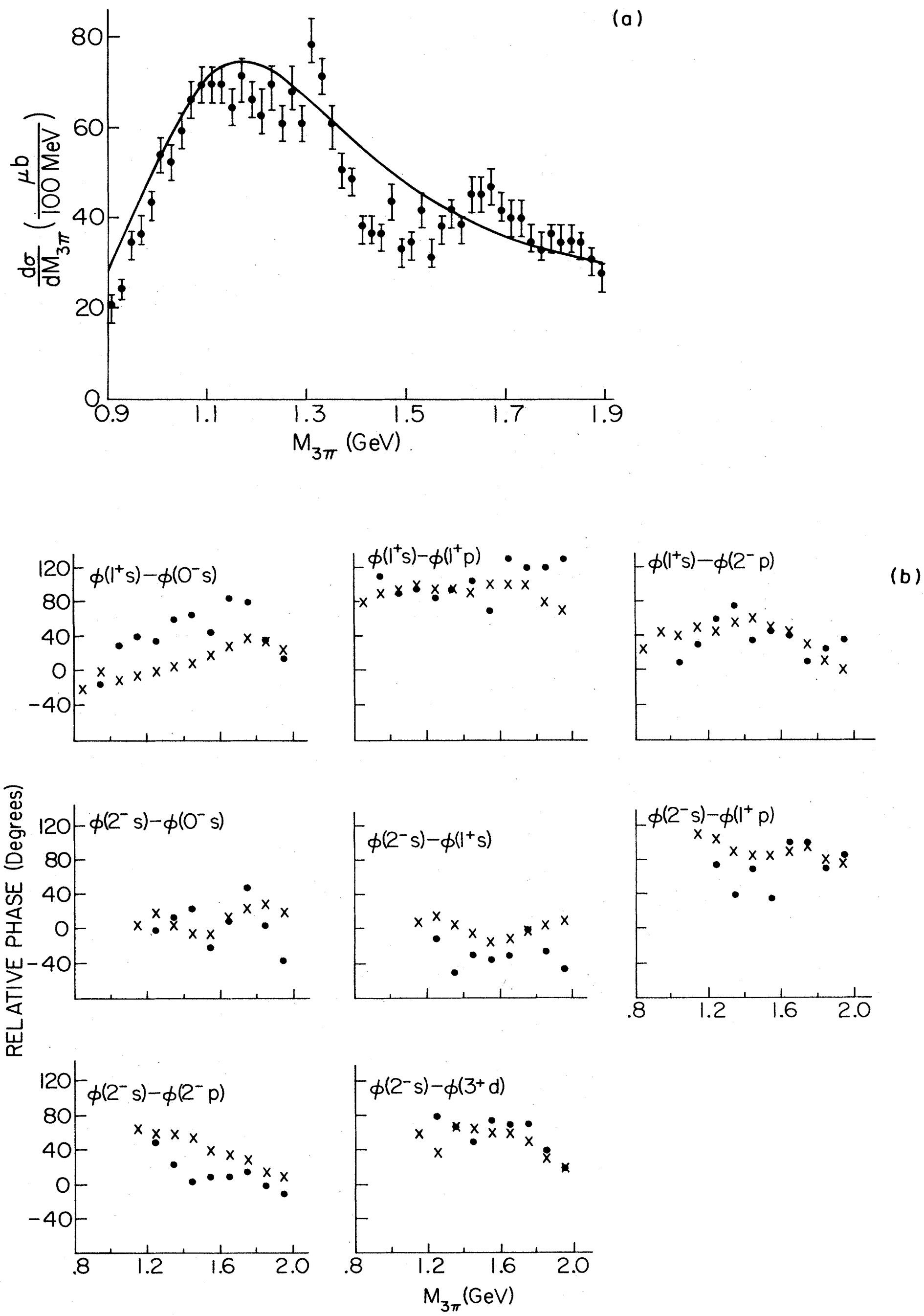

FIG. 19. Comparison of double Regge exchange model of Ascoli *et al.* (1973 and 1974) with combined data on $\pi^- p \to \pi^+\pi^-\pi^- p$ from 18–25 GeV. (a) Distribution in $M_{3\pi}$. Solid line is calculations; points with error bars are data. (b) Relative phases between 3π partial waves. (●) Data obtained from isobar model fit; (×) theory, calculated from the "two diagram" model of Ascoli, Jones, Weinstein, and Wyld (1973). The theoretical points were obtained by generating Monte Carlo events from the theory, then fitting these with the *same* fitting program applied to the data. Error bars have been omitted for clarity; they can be found in the original literature. The notation is as follows: 1^+s $J=1$ $\rho^0\pi^-$ in an s wave; 0^-s $J=0$ $\varepsilon^0\pi^-$ in an s wave; 1^+p $J=1$ $\varepsilon^0\pi^-$ in a p wave; 2^-p $J=2$ $\rho^0\pi^-$ in a p wave; 2^-s $J=2$ $f^0\pi^-$ in an s wave; 3^+d $J=3$ $\rho^0\pi^-$ in a d wave.

other states.

(d) To properly fit the interference phases between resonant and nonresonant partial waves (and between two nonresonant partial waves), one must have a theoretically acceptable model for the phase of the background Reggeized pion exchange. As we have pointed out above, the exact features of this phase depend on the coupling at the central vertex in Fig. 15. The prop-

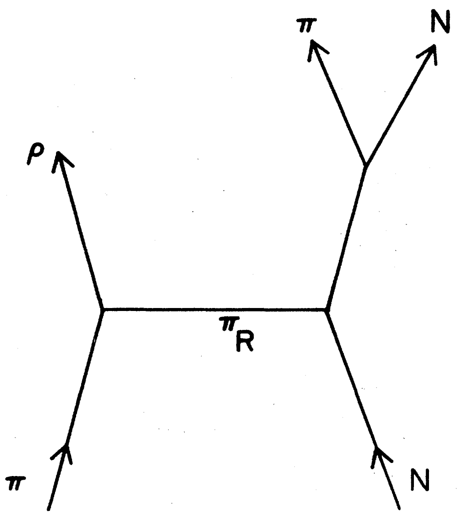

FIG. 20. Tree graphs included in a Steinmann-obeying model for $\pi N \to \rho\pi N$ with s-channel helicity conservation for the subprocess $\pi_R N \to \pi N$.

erties of this coupling are important theoretically, and efforts should be made to determine them from the data.

At this point, we do not have a program which simultaneously extracts from the data the phase properties of the background Regge pion exchange and the resonance parameters. (Various groups have done this for the $1^+\rho\pi$ s wave; but none have tried to fit all the waves simultaneously.) Until such a study is undertaken, resolution of the A_1 problem is perhaps best left to other reactions such as backward scattering, forward charge exchange processes (which have a totally different background shape from the diffractive processes), and leptonic decays. Once the parameters of the resonance are known, it will be necessary to add the resonance to the Reggeized pi exchange background *without double counting* to obtain a proper amplitude for the $\pi N \to (3\pi)N$ reaction. Because we currently do not know very much about the couplings of the Pomeron exchange in multibody final states, it is not even clear whether the double Regge (pion plus Pomeron) amplitude should be a sum of Steinmann-obeying pieces such as in the model of Puhala (this is appropriate for a Pomeron which does not obey s-channel helicity conservation in the subchannel) or a single term like the ones used by Basdevant and Berger or Longacre and Aaron (these authors use a double Regge amplitude which has the right Steinmann-obeying phase if the Pomeron is s-channel helicity conserving).

We emphasize that the Steinmann relations are important because the amplitude must obey them *after* all interactions (including final-state interactions) have taken place. If proper unitarity and analyticity *in all channels* were enforced at all times in the calculations, the results would automatically be Steinmann obeying. However, it appears to be impractical to deal with the pi-nucleon channel at the same time as the rho-pi channel in inserting final-state interactions, because the interesting region of phase space is that where the pi-nucleon subenergy is quite large. Thus we feel that the most profitable course of action is to use parametrizations which are automatically Steinmann obeying, and to study the inclusion of resonances in a way which avoids double counting. Hence we feel that despite the great deal of work done on this subject, the definitive paper has yet to be written.

C. Other calculations involving production of one or two pions

Information about the relative phases between partial waves in multiparticle final states is available for only a few reactions other than $\pi N \to \rho\pi N$. Relative phases of $\pi p \to \pi\pi p$ have been extracted from the data by an isobar analysis (Longacre *et al.*, 1975), and a fit using three-body unitary states is under way (Arndt *et al.*, 1979). Phases in $Kp \to K\pi\pi p$ have been extracted using routines similar to those for $\pi p \to \pi\pi\pi p$ (Hansen, 1974; Carnegie *et al.*, 1977; Brandenburg *et al.*, 1976; Otter *et al.*, 1976). In both these reactions it is expected that important effects due to background pion exchange are present, in addition to resonances in the $\pi\pi p$ and $K\pi\pi$ systems.

A substantial amount of work has been done in both cases. Basdevant and Berger (1976) and Bowler (1977) have investigated the $K\pi\pi$ data, and find that it can be fit by one or two resonances in the $K\pi\pi$ system interfering with a Deck background due to pi and K exchange. The pi-pi-nucleon system is under study; pion exchange effects for this system have been estimated (Aaron *et al.*, 1977) and a fit to the overall reaction is under way.

Outside these two systems, no systematic study has been done of the phases in multiparticle final states produced by pion exchange. Hence less effort has been expended in the construction of sophisticated models for the other reactions. Instead, attention has focused on the calculation of mass spectra analogous to those seen in the Deck effect. Since phases are not observed, the pion signature factor is normally not included. Frequently the Regge energy dependence for the crossed channel subenergy has also been omitted. In spite of these approximations, qualitative agreement with mass spectra has been achieved in many cases, indicating that the main features of the phenomenology are probably quite simple.

Before we catalog these various calculations, we address the following question: Why should very simple treatment of the pion exchange in multiparticle reactions yield reasonably good results when the more careful tests in two-body reactions show the need for cut-type corrections, conspiracies, etc.? Cuts are certainly present in the multiparticle final states; their relative unimportance for the data fitting that has been done to date is to some extent a function of lack of detailed data. However, there are some features of the multiparticle final state which tend to obscure or dilute the more dramatic effects of the cuts in two-body reactions.

We saw in Sec. III that the pion-Pomeron cut was necessary in $np \to pn$ and in charged pion photoproduction in order to reproduce the sharp forward spike seen near $t=0$. This spike was produced by a conspiracy—i.e., exchange of both natural and unnatural parity quantum numbers. The conspiracy had the effect of restoring double helicity flip amplitudes to their full strength in the forward direction; the effect on the cross section was profound in these cases because the "normal" mechanisms did not produce a substantial nonflip amplitude.

In the multiparticle final state, the point $t=0$ is farther from the physical region (for a given overall energy) than in the two-particle final state, because the produced "masses" flanking the pion exchange piece can be much larger. The minimum value of t grows with these masses; hence observation of spikes in differential cross sections will be very hard (only a very small region of the multiparticle phase space is close to $t=0$). More important, perhaps, is the fact that the subreaction involving pion exchange is of the type "Reggeon" + particle $\rightarrow$ two particles, or Reggeon + Reggeon $\rightarrow$ two particles. The Reggeons normally carry all helicities. Hence one would not expect to have a situation analogous to charged pion photoproduction in which the importance of the spike rests on the limitation of the incident photon to helicity one (the spike is much less important for interpretation of the cross section in the related reaction $\pi+N\rightarrow\rho+N$ in which the rho has the full range of helicities available to it). From both of these considerations, we see that the $t=0$ spikes will probably not play an important role in multiparticle final states.

The other feature of the cuts, the fact that they bear both naturalities, becomes important only if the naturality of the exchanges is measured. For the three-body final state this means that density matrices (or polarizations) of one of the final-state particles must be measured; for the multibody final state some sort of partial wave analysis is necessary to pick out these features. To date, only a small amount of data of this sort has been collected.

It is likely, therefore, that any effects of absorption on the multiparticle final state will be observed in places that are unique to the multiparticle calculations. These include the mass spectra and various double differential cross sections. A study of Berger and Pirila (1975) using the absorption model indicates that the cuts affect the double differential cross section $d\sigma/dMdt$, and tend to improve the dependence of the slope in t on the produced mass (this is one place where Deck-type calculations tend to fail). They find very little change in the calculated mass spectra. Similar results were obtained by Berger and Irving (1975). Since most of the papers discussed below only calculate mass spectra, we will ignore absorptive corrections in what follows. The method used by Berger, Irving, and Pirila to include absorption is rather intuitive; no simple formula like Eq. (4.2) for a multiparticle final-state process with cuts is available, to our knowledge.

Other Deck-type calculations have been done for $pn\rightarrow N\pi\Delta$ (Berger and Morrow, 1970), for the $\pi\pi N$ system in $\pi N\rightarrow(3\pi)N$ (S. T. Jones, 1970) for the $\bar{p}\pi\pi$ system in $\bar{p}p\rightarrow\bar{p}\pi\pi p$ (Cutler and Wyld, 1975), for the $\bar{K}K\pi$ system in $\pi N\rightarrow K\bar{K}\pi N$ (Pietilainen and Lassila, 1977), for the 3π system in $\pi N\rightarrow(3\pi)X$ (Puhala, 1979), for the πN system in $aN\rightarrow a\pi N$ (Cutler and Berger, 1977), for the 3π system in $\pi C\rightarrow(3\pi)C$ (Cutler, 1974), and many others. Generally speaking, good agreement with the overall magnitude and shape of the mass spectrum and many angular distributions can be achieved; we refer the reader to the references listed for details of the individual calculations. Also, generally speaking, the mass-slope correlations cannot be properly reproduced in these models.

Some use has been made of Deck-type models plus the optical theorem to calculate distributions in inclusive reactions (Cutler and Wyld, 1975; Puhala, 1979). Results have some qualitative similarities to the data, but fewer details are reproduced than in the exclusive cases.

In summary, now that this substantial body of work has established that pion exchange does play an important role in multibody reactions, and that many of the main features can be reproduced by rather crude models, it seems time to call for experimental and theoretical studies of details. In particular, (a) Studies of polarization and density matrices of the produced final-state particles will give additional indications about reaction mechanisms; (b) studies of Steinmann-obeying models should be pursued to determine whether details like the mass-slope correlations and predicted phases are improved. At the same time, phases should be extracted from the data in as many reactions as possible. The effort already expended in constructing improved data analysis programs should be continued; (c) the theoretical study of inclusion of duality in multibody final states, which more or less foundered after the deficiencies of the Veneziano model were discovered, needs to be begun again. All results (such as those for the A_1) obtained by interfering contributions from resonances with those from pion exchange backgrounds must be regarded as tentative until we are sure that the formula used to obtain them does not violate some fundamental principle.

D. Pion exchange in lepton-induced reactions

Exchange of the pion trajectory plays an important role in inelastic electron scattering, $e^-p\rightarrow e^-X$, in at least two different places. Let us consider these in turn.

Exclusive electroproduction of charged pions $e^-p\rightarrow e\pi^{\pm}N$ is interesting because it allows one to study the photoproduction process $\gamma p\rightarrow\pi^{\pm}N$ at variable q^2. By vector dominance arguments, the reaction $\pi p\rightarrow\rho N$ can be considered to be related to photoproduction at positive q^2; the electroproduction data, by contrast, are basically photoproduction at negative q^2. Numerous theoretical papers have focused on the problem of continuation in q^2; the data are thus interesting.

The feature of most note in photoproduction of charged pions at $q^2=0$ is the pion-Pomeron cut. A series of phenomenological analyses of reactions dominated by pion exchange tends to show the following remarkable feature: The strength of the cut gets monotonically smaller as q^2 of the "photon" becomes large and positive (Irving, 1975; Hyams *et al.*, 1974; Ochs and Wagner, 1973), and mononically larger as q^2 becomes large and negative (Irving, 1975; Van Ryckegham, 1974). That is, production of large mass dipion resonances is described very well by just Reggeized one-pion exchange, in spite of the complexities of rho production; and the cut strength seems to be larger in electroproduction than in photoproduction. This feature of the data has not yet been completely explained theoretically, but it should be borne in mind when applying Regge concepts to multiparticle final states.

Inclusive electroproduction of charged pions has also been studied. A very detailed comparison of inclusive electroproduction to inclusive photoproduction is given by Brasse (1974). For the purposes of this review, we only mention a few salient features of the data. In $\gamma p \to \pi^{\pm} X$ there is a marked peaking as $k_{\perp}^2 \to 0$, similar to the peak in the exclusive cross section. Studies of this reaction by Craigie, Kramer, and Korner (1974) in the triple Regge model indicate that a model with only poles cannot fit this peak, as might be expected from our study of the exclusive case. They investigate also models containing absorption and, not surprisingly, find that this makes it possible to fit the peak. However, in contrast to the results found in the exclusive case by Irving and Van Ryckegham, the size of the cut needed for agreement with the data seems to decrease as q^2 increases. Also in contrast to the relatively slow variation observed in the exclusive case as a function of q^2, these authors state that the inclusive electroproduction changes very rapidly near $q^2 \approx 0$. Later work by Pumplin (1976) provides an absorbative term different in size by a factor of 2π from that of Craigie and Kramer, with somewhat different phenomenological consequences.

Further careful study of these reactions seems advisable, as one might expect to be able to form a unified viewpoint about the electroproduction case. Since this is one of the few cases available in nature where the dependence of strong interaction effects (i.e., cuts, etc.) can be measured as a continuous function of the external mass, and since data is relatively straightforward to obtain, we might hope that a thorough theoretical understanding of the phenomenological features could be obtained.

For large $-q^2$, of course, alternative interpretations of the $ep \to e\pi X$ data may be obtained from the parton model. The transition from $q^2 = 0$ to large negative q^2 thus provides another arena for study of the transition from Regge (summed ladders of fundamental particles) to parton (low order in terms of fundamental particles) ideas. To date, this particular aspect of these reactions has not been fully explored.

A different view of pion exchange in $ep \to eX$ was given by Sullivan (1972) who showed that the pion structure functions could be isolated by appropriate treatment of the data.

V. SUMMARY AND CONCLUSIONS

The available evidence currently indicates that the pion exchange is a normal Regge pole with a trajectory of slope somewhere near 0.8 or 1 per GeV^2. This pion exchange is accompanied by a pion-Pomeron cut which can be very important in explaining the shape near the forward direction of helicity double flip amplitudes; if the cross section is dominated by such amplitudes, the cut will be essential to obtain the correct shape of the cross section.

Born term models frequently approximate the right size for the cross section and have the correct conspiracy structure for reproduction of forward direction spikes. The Born terms must be rejected as fundamental mechanisms because they do not produce a t-dependent energy dependence as seen in the data. However, it is quite likely that the Born terms are approximately dual to the Regge cut structure which has the trajectory seen. The exact equivalence needs to be worked out in more detail.

In this regard, we note that study of duality for pion exchanges has been slighted because the small size of the pion trajectory near $t = 0$ means that the Reggeized propagator has only a small imaginary part. Most studies of duality look only at the imaginary part; thus it has been regarded as difficult to study the pion exchanges. However, we note here that the phases of the *partial wave amplitudes* need not be small; thus there may be important duality effects. To date the only place where the phase of the Reggeized pion propagator has been studied is the multibody reaction $\pi N \to (3\pi)N$.

The size of the cut appears to vary with the mass of the produced system. Studies in exclusive and inclusive pion electroproduction appear to give different dependencies of the size of the cut on q^2; clearly more work needs to be done. Cuts are apparently not very important in reproducing gross features of multiparticle final states produced by pion exchange; however, they can influence details such as doubly differential cross sections.

In view of the pervasive nature of pion exchange in strong interaction processes, we should be thankful that simple calculations can reproduce much of the data. However, we should equally well be interested in refining our understanding of various details, especially those linked to the presence of the pion on a Regge trajectory.

In particular, nuclear experiments (such as those of Moake *et al.*) which appear to be dominated by pion exchange should be performed at a series of energies, from the lowest available to nuclear physicists to the highest available to particle physicists. This will check our hypothesis that the same parametrization for (absorbed) Regge pion exchange should hold over an enormous range of energies.

The recently obtained high statistics data on $\pi N \to (3\pi)N$ also deserve detailed attention. Theorists should develop a general model including all the quasi-three-body final states ($\rho^0\pi$, $\epsilon^0\pi$, and $f^0\pi$) which obeys both the Steinmann relations for $2 \to 3$ processes and unitarity in the $\pi \mathrm{P} \to 3\pi$ subchannel. In order to address our major theoretical questions, the parameters in the model should be directly related to (a) the 3π resonances (A_1, A_2, A_3, etc.) and (b) the π_R-Pomeron-pion vertex at the center of Fig. 15. The development of such a model is a nontrivial task, since it may require decisions about the role of duality in multiparticle final states. Fitting such a general model to the high statistics data is also nontrivial. However, it appears that a unified approach in which detailed models confront the full $(3\pi)p$ final state is necessary if we are to unambiguously answer the following questions:

(i) Are the recurrences of the pion diffractively produced, as one might naively expect?

(ii) How does the Reggeized pion couple to the Pomeron?

Needless to say, similar analyses should be carried out on other reactions which also contain the π_R-Pomeron-pion vertex. For instance, $\gamma p \to \pi^+\pi^- p$ should be restudied over a broad range of mass for the $\pi^+\pi^-$ pairs. This reaction is simpler than $\pi N \to (3\pi)N$ in some regards, but it is complicated by the requirement of gauge invariance. As we have seen in the two-body case, this should not be an insurmountable problem.

Determination of the π_R-P-π coupling in these "elementary" reactions will allow better calculations of multiperipheral chains, where an iterative string of Regge pion and Pomeron exchanges produces n pions in the final state. These not only have phenomenological importance; they are also fundamental building blocks in many purely theoretical calculations.

Historically, a great deal of theoretical work has been devoted to the possible forms of the Regge-Regge-particle vertex, but phenomenologists have tended to use simplified models or multiparticle Veneziano models rather than exploring the full range allowed by theory. With the presently available high statistics data, we have every reason to expect that the Reggeized pion-Pomeron-pion vertex will be the first such vertex to receive thorough study.

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