Isobaric quartets in nuclei

Walter Benenson and Edwin Kashy

Physics Department and Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824

The present experimental evidence on $T = 3/2$ states in nuclei is presented with particular attention to quartets in which the properties of all four members are known. A relation between the masses of the quartet, the isobaric multiplet mass equation, is shown to hold extremely well. The significance of the coefficients of the equation is discussed, and a brief review of the present status of quintets is also presented.

CONTENTS

I. INTRODUCTION

The concept of isobaric spin has now been in existence for more than forty years. Although first used to put charge independence explicitly into descriptions of the nucleon–nucleon force, it has grown to be a quantum number of prime importance in nuclear and particle physics. Although technically not a good quantum number, it is conserved sufficiently to permit first-order perturbation theory calculations to work well. In this article we are mainly restricting our attention to the shifting and mixing of nuclear levels due to the breaking of isobaric spin conservation by charge-dependent forces for the particular case of isobaric spin $T = 3/2$ levels.

The splitting of otherwise degenerate levels by a force which destroys that degeneracy has been an important source of information on quantum levels in all fields of physics. The Zeeman effect and prediction of the Ω particle are two very important examples. The first case just involves the projection of J, the angular momentum, on the magnetic field axis, whereas the second involves the projections of strangeness and isobaric spin. For cases like this it is only necessary to know the form of the perturbing force (or in fact the rank of its representation) to predict the energy or mass dependence of the splitting, provided the splitting is small compared to the total energy. A beautiful example occurs in nuclear physics for $T = 3/2$ levels. These levels have four possible electric charges which correspond to the $2T+1$ projections on the z axis. These four levels would have identical energies were it not for isobaric spin violation. They are easily distinguished from each other because they occur in four different nuclei, and in fact the most difficult experimental problem is often distinguishing them from levels of lower isobaric spin in the same nucleus. In this review article we will attempt to cover all of the available experimental information on isobaric quartets in nuclei and the relevant properties of the individual levels with $T = 3/2$. The last reviews of this subject were published ten years ago (Cerny, 1968, Jänecke, 1969). Particular attention will be paid to the mass of the levels and a relation between the masses, known as the isobaric multiplet mass equation (IMME). The astonishing success of the IMME will be shown in 22 cases, and some discussion will be given on possible reasons for this success. No details of the method of measurement of the quantities presented in the tables will be given; these can be found in the references.

II. ISOBARIC QUARTETS

In principle every nuclear state with isobaric spin T is a member of a $2T+1$ multiplet of levels with very similar wave functions but different charge, as measured by the z component of the isobaric spin T_z. Levels with $T = 1/2$ form doublets which are usually referred to as mirror levels. A state with T greater than the T_z of the nucleus is usually called an analog state. The reason for this nomenclature is that the state in question has an almost identical structure to that of a state in the nearby $|T_z| = T$ nucleus, and therefore is analogous to it. Examination of the schematic representation of the $A = 9$ system shown in Fig. 1 reveals that the $T_z = \pm 1/2$ nuclei ^{9}Be and ^{9}B both have well defined $T = 3/2$ levels which bear a striking similarity to the level structure of the $T_z = \pm 3/2$ nuclei, ^{9}C and ^{9}Li. For the purposes of showing the correspondence between the nuclei, the level schemes for each nucleus have been shifted by the Coulomb energy and neutron–proton mass difference to line up the $T = 1/2$ and $T = 3/2$ levels.

In this article we are mainly studying how much shift is actually caused by Coulomb and other charge-dependent forces, and as a consequence we shall discuss the extent to which $T = 3/2$ levels contain $T = 1/2$ components. The total energy of a level, as measured by its mass, will actually be represented for convenience by its mass excess, and this will mask the fact that the shifts between levels are really quite small compared to the total mass or the total binding energy. Also, there is no requirement that the levels in question be bound. In fact, most $T = 3/2$ levels in $T_z = \pm 1/2$ nuclei

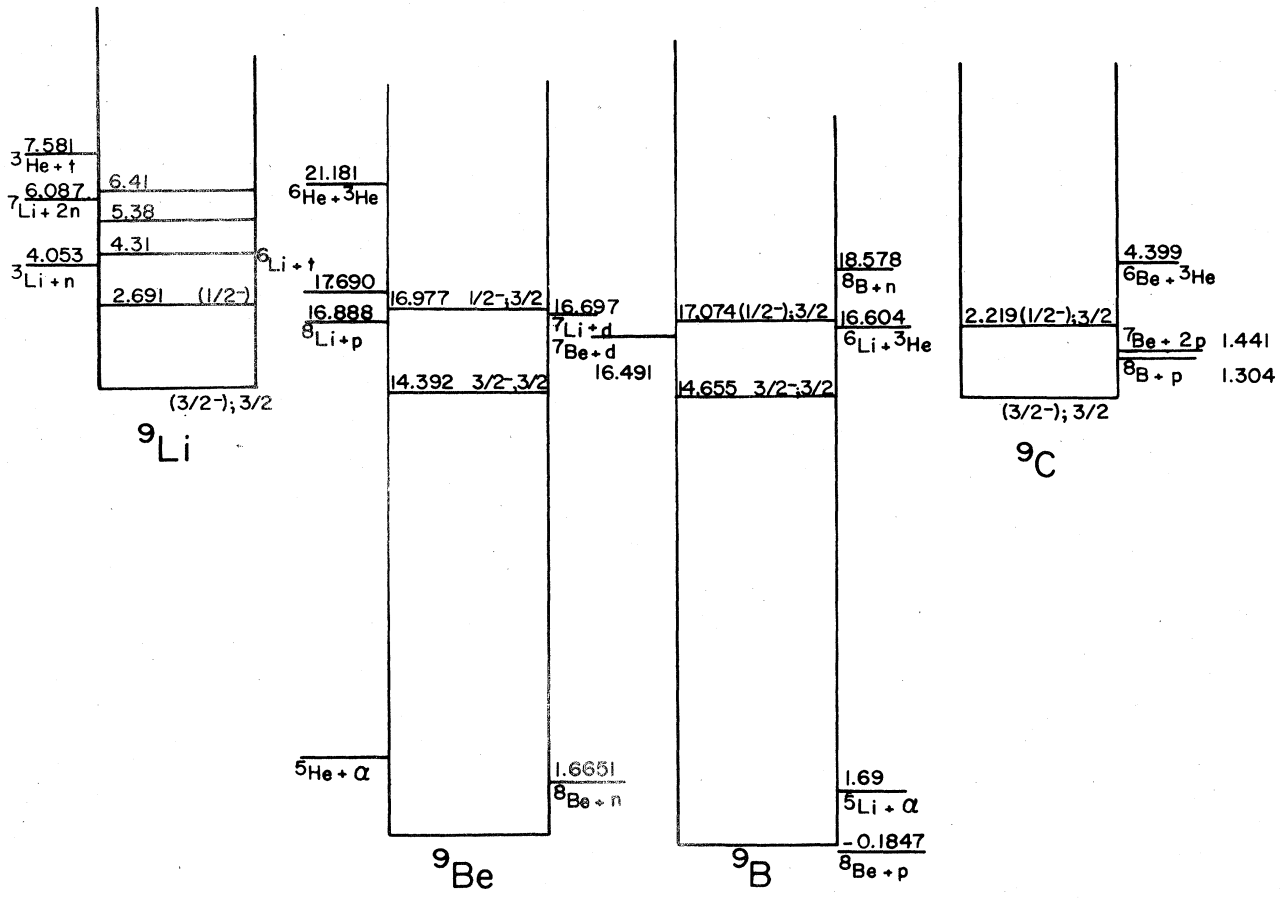

FIG. 1. Energy levels of the $A = 9$ nuclei.

are unbound to isospin forbidden particle decay, and several are unbound to allowed particle decay. The first excited state in ^{9}Li shown in Fig. 1 is bound, but its analogs in ^{9}Be, ^{9}B, and ^{9}C are particle unstable to the extent that their widths are easily observable as a peak broadening in high-resolution experiments. These same high-resolution experiments also show that the shifts between the four levels of the quartet are state dependent to a small extent, and this can yield very detailed information about the shell structure of the levels.

III. THE ISOBARIC MULTIPLET MASS EQUATION

We will begin our treatment of isobaric mass quartets with a discussion of the isobaric multiplet mass equation, which was first proposed by Wigner in 1957 (Wigner, 1957). This occurs in an address presented to a "Robert A. Welch Conference on Chemical Research" and is very simplified and clearly presented. Later derivations and discussions (Weinberg, 1959; Garvey, 1969; Wilkinson, 1966) are more mathematical. The derivation is essentially a classical use of the Wigner–Eckhart theorem.

Since the charge-independent Hamiltonian, H_{CI}, conserves T, its eigenvalues are independent of T_z but, of course, depend on the other quantum numbers labeled α below

$$H_{CI}|\alpha T T_z\rangle = E_{\alpha,T}|T T_z\rangle . \tag{1}$$

Now any charge-dependent *two-body* force can be expressed in terms of isospin by

$$V_{ij} = V_1(t_{zi} + t_{zj}) + V_2(t_{zi} t_{zj}) , \tag{2}$$

where i and j label the particles, and t_{zi} and t_{zj} their isobaric spin projection. V_{ij} can be written as a linear combination of scalar, vector, and tensor terms, and therefore the perturbing Hamiltonian H', if it comes from two-body forces, can be written

$$H' = H^{(0)} + H^{(1)} + H^{(2)} , \tag{3}$$

where $H^{(n)}$ is the nth order isotensor with zero projection on the z axis. The Wigner–Eckart theorem applied to the states which are eigenvalues of H_{CI} yields

$$\begin{aligned}\langle \alpha T T_z|H'|\alpha T T_z\rangle = {} & \langle \alpha T|\,||H^{(0)}||\,|\alpha T\rangle \\ & + \frac{T_z}{T(T+1)}\langle \alpha T|\,||H^{(1)}||\,|\alpha T\rangle \\ & + \frac{3T_z^2 - T(T+1)}{[(2T-1)T(T+1)(2T+3)]^{1/2}} \\ & \times \langle \alpha T|\,||H^{(2)}||\,|\alpha T\rangle \end{aligned} \tag{4}$$

All of the above terms are energies expressed in terms of reduced matrix elements and coefficients involving only T and T_z. Therefore

$$\langle \alpha T T_z | H_{\rm CI} + H' | \alpha T T_z \rangle = a + bT_z + cT_z^2 \tag{5}$$

since no other T_z dependences are present.

In Wigner's original presentation a and c are interchanged. When this permutation occurred is unknown, but we will follow the convention which has an a and not c as the constant term, and also the reader should note the isobaric spin in nuclear physics has the opposite projection on the charge axis as is the convention in particle physics. That is:

$$T_z = (N - Z)/2 \tag{6}$$

or the neutron is considered to be the $T_z = +\frac{1}{2}$ member of the nucleon doublet. That Wigner himself has switched to the opposite convention will not dissuade us from the normal nuclear physics (and original) convention.

A more graphic understanding of the IMME can be obtained by looking at one charge-dependent force by itself, the Coulomb force between nucleons, which can be expressed in isobaric spin notation as

$$H_C = e^2 \sum_{ij} \frac{(1/2 - t_{zi})(1/2 - t_{zj})}{r_{ij}} \tag{7}$$

This was shown by MacDonald (1955) to be expressable as

$$H_C = e^2 \sum_{ij} \frac{1}{r_{ij}} \left(\left[\frac{1}{4} + \frac{\mathbf{t}_i \cdot \mathbf{t}_j}{3} \right] - \left[\frac{t_{zi} + t_{zj}}{3} \right] + \left[t_{zi} t_{zj} - \frac{\mathbf{t}_i \cdot \mathbf{t}_j}{3} \right] \right) \tag{8}$$

which is a sum of tensors of rank 0, 1, and 2. Likewise Garvey (1969) has derived the form of the spin-orbit or magnetic perturbing Hamiltonian in terms of tensors of rank 0, 1, and 2.

So we see that in order for the form of the IMME to be violated, for example with a dT_z^3 term, one needs to look either to the next order in the perturbation theory or to many-body forces. The most convenient access to violations of the perturbation theory approximation is via the wavefunctions. Consequently we will spend some time examining them.

It is enlightening to calculate the coefficients b, c, and d for various simple models; a triangular three-nucleon system, a uniformly charged sphere, and a uniformly charged spheroid give

$$\begin{aligned} b &= -0.6[(A-1)e^2/r_c] + (M_n - M_{1H}), \\ c &= 0.6e^2/r_c, \\ d &= 0. \end{aligned} \tag{9}$$

If we assume

$$r_c = r_{c0} A^{1/3} \tag{10}$$

and define a new quantity

$$b^* = b - (M_n - M_{1H}) \tag{11}$$

then we get the simple prediction that

$$b^*/c = 1 - A \tag{12}$$

which holds fairly well, as will be seen later.

One can also make a simple calculation in which there is a core containing $Z_{\rm core}$ protons which is unaffected either in radius or charge by T_z, and an outer shell of valence particles which changes charge with T_z. In this case one gets

$$b^*/c = 2Z_{\rm core} - A + 1. \tag{13}$$

Examination of the data shows that $Z_{\rm core}$ is essentially zero, as one would expect, since the valence particles are not localized on a shell on the nuclear surface. However, this formula is useful in understanding in simple terms the small changes in the coefficients which do occur as the valence nucleons fill first one subshell and then another.

A. Validity of the isobaric multiplet mass equation

Since any set of three or less numbers can be fit exactly with a quadratic equation, a multiplet with $T \geq 3/2$ is required to verify the predictions of the IMME. Thus experimentally what one needs is accurate mass measurements of four particular nuclear states. Historically the experimental difficulty lay in the $T_z = -3/2$ nucleus, which is in all cases classified as "far from stability" by nuclear physicists. These nuclei are accessible only by low-cross-section, highly negative-Q-value, multinucleon transfer reactions. Consequently the first quartet was not completed until 1964 by Cerny and co-workers (Cerny *et al.*, 1964). This was achieved by measuring the mass of ^{9}C with the ^{12}C(^{3}He, ^{6}He) reaction. As will be seen below, the missing member of a mass quartet is now much more likely to be located in a $T_z = \pm 1/2$ nucleus. Determining accurately the mass excess of these levels is three times as important as the $T_z = \pm 3/2$ levels in verifying the validity of the IMME. This follows from evaluating d, the coefficient of a T_z^3 term in the equation

$$d = \tfrac{1}{6}\left[M(-3/2) - M(3/2) - 3(M(-1/2) - M(1/2))\right].$$

In other words the IMME prediction of $d = 0$ amounts to stating that the energy difference between $T_z = \pm 1/2$ members is one-third that between $T_z = \pm 3/2$ members. The above result also shows that the errors in the mass of the $T_z = \pm 1/2$ levels are three times as important as those in the $T_z = \pm 3/2$ levels for an evaluation of d and therefore in checking the validity of the IMME.

Table I gives a list of all the experimentally determined mass excesses and excitation energies relating to mass quartets which are presently known (March, 1978). These are the data that were used to evaluate the coefficients of the IMME presented and discussed below. Fortunately in almost every case there is a recent measurement considerably more accurate than the preceding ones, and it was not necessary to make an arbitrary choice between differing results, nor were attempts to average such numbers required. However, the data that come from compilations and are referred to as such may have been manipulated in this manner. In this case we defer to the good judgment of the compilers.

The degree to which the four masses in a quartet agree with a quadratic form for the IMME can be mea-

TABLE I. A list of properties of $T=\frac{3}{2}$ levels which are members of isospin quartets with three or four known members. The errors in parentheses are in units of the last significant digit of the value itself. All ground-state masses are from the 1977 mass table (Wapstra and Bos, 1977) except for the most recent references.

A/J^π	T_z	Nucl.	E_x (MeV)	Mass excess (MeV)	Γ (keV)	Ref.
7	$-\frac{3}{2}$	^{7}B	g.s.	27.94(10)	1400(200)	Ajzenberg-Selove and Lauritsen, 1974
	$-\frac{1}{2}$	^{7}Be	11.010(30)	26.780(30)	320(30)	Ajzenberg-Selove and Lauritsen, 1974
$\frac{3}{2}^-$	$\frac{1}{2}$	^{7}Li	11.245(31)	26.153(31)	258(33)	Ajzenberg-Selove and Lauritsen, 1974
	$\frac{3}{2}$	^{7}He	g.s.	26.111(30)	200	Ajzenberg-Selove and Lauritsen, 1974
	$-\frac{3}{2}$	^{9}C	g.s.	28.9121(39)	Bound	
9	$-\frac{1}{2}$	^{9}B	14.6554(25)	27.0711(23)	2.750(43)	Kashy *et al.*, 1974
	$\frac{1}{2}$	^{9}Be	14.3922(18)	25.7406(17)	3.290(67)	Kashy *et al.*, 1974; McDonald *et al.*, 1977
$\frac{3}{2}^-$	$\frac{3}{2}$	^{9}Li	g.s.	24.9554(20)	Bound	
9	$-\frac{3}{2}$	^{9}C	2.219(10)	31.131(11)	100(20)	Benenson and Kashy, 1974
	$-\frac{1}{2}$	^{9}Be	17.046(4)	29.492(4)	22(5)	Benenson and Kashy, 1974
$\frac{1}{2}^-$	$\frac{1}{2}$	^{9}Be	16.9773(15)	28.3257(14)	25	Ajzenberg-Selove and Lauritsen, 1974
	$\frac{3}{2}$	^{9}Li	2.691(5)	27.646(5)	25	Ajzenberg-Selove and Lauritsen, 19
	$-\frac{3}{2}$	^{11}N	g.s. mass unknown	25.23(10)	740(100)	Ajzenberg-Selove and Lauritsen, 1974
11	$-\frac{1}{2}$	^{11}C	12.50(3)	23.150(30)	490(40)	Ajzenberg-Selove and Lauritsen, 1974
	$\frac{1}{2}$	^{11}B	12.91(2)	21.580(20)	240(30)	Ajzenberg-Selove and Lauritsen, 1974
$\frac{1}{2}^-$	$\frac{3}{2}$	^{11}Be	0.3198(2)	20.496(6)	Bound	Ajzenberg-Selove and Lauritsen, 1974
	$-\frac{3}{2}$	^{13}O	g.s.	23.105(10)	Bound	Ajzenberg-Selove, 1976
13	$-\frac{1}{2}$	^{13}N	15.0651(9)	20.4107(13)	0.77(15)	Ajzenberg-Selove, 1976; McDonald *et al.*, 1977
	$\frac{1}{2}$	^{13}C	15.106(2)	18.231(2)	5.8(7)	Ajzenberg-Selove, 1976; McDonald *et al.*, 1977
$\frac{3}{2}^-$	$\frac{3}{2}$	^{13}B	g.s.	16.562(4)	Bound	Ajzenberg-Selove, 1976
	$-\frac{3}{2}$	^{15}F	g.s. mass unknown	16.90(20)	900	Benenson *et al.*, 1978
15	$-\frac{1}{2}$	^{15}O	Unknown			
	$\frac{1}{2}$	^{15}N	11.615(4)	11.717(4)	405(6)	Ajzenberg-Selove, 1976
$\frac{1}{2}^+$	$\frac{3}{2}$	^{15}C	0.7400(15)	10.6132(15)	Bound	Ajzenberg-Selove, 1976
	$-\frac{3}{2}$	^{17}Ne	g.s.	16.478(20)	Bound	Robertson *et al.*, 1978
17	$-\frac{1}{2}$	^{17}F	11.1921(23)	13.1448(23)	$0.43(^{+13}_{-22})$	Ajzenberg-Selove, 1977; McDonald *et al.*, 1977
	$\frac{1}{2}$	^{17}O	11.077(3)	10.266(4)	5(1)	Ajzenberg-Selove, 1977; McDonald *et al.*, 1977
$\frac{1}{2}^-$	$\frac{3}{2}$	^{17}N	g.s.	7.870(5)	Bound	
	$-\frac{3}{2}$	^{17}Ne	1.330(15)	17.805(35)	Bound	Robinson *et al.*, 1978
17	$-\frac{1}{2}$	^{17}F	12.5500(14)	14.5020(14)	2.87(12)	Ajzenberg-Selove, 1977
	$\frac{1}{2}$	^{17}O	12.466(4)	11.654(5)	8(2)	Ajzenberg-Selove, 1977; McDonald *et al.*, 1977
$\frac{3}{2}^-$	$\frac{3}{2}$	^{17}N	1.3739(3)	9.244(15)	Bound	Ajzenberg-Selove, 1977
	$-\frac{3}{2}$	^{19}Na	g.s.	12.930(13)	Bound	
19	$-\frac{1}{2}$	^{19}Ne	7.495(22)	9.246(22)	Bound	Nann, 1978
	$\frac{1}{2}$	^{19}F	7.538(2)	6.0506(2)	Bound	Ajzenberg-Selove, 1978
$\frac{5}{2}^+$	$\frac{3}{2}$	^{19}O	g.s.	3.3314(27)	Bound	
	$-\frac{3}{2}$	^{19}Na	0.120(10)	13.048(15)	Bound	Ajzenberg-Selove, 1978
19	$-\frac{1}{2}$	^{19}Ne	7.616(16)	9.367(16)	Bound	Ajzenberg-Selove, 1978

TABLE I. (*Continued*)

A/J^π	T_z	Nucl.	E_x (MeV)	Mass excess (MeV)	Γ (keV)	Ref.
	$\frac{1}{2}$	^{19}F	7.660(2)	6.173(2)	Bound	Ajzenberg-Selove, 1978
$\frac{3^+}{2}$	$\frac{3}{2}$	^{19}O	0.0960(5)	3.4274(27)	Bound	Ajzenberg-Selove, 1978
	$-\frac{3}{2}$	^{21}Mg	g.s.	10.916(16)	Bound	
21	$-\frac{1}{2}$	^{21}Na	8.970(5)	6.784(5)	$0.75(^{+5}_{-2.5})$	Endt and Van der Leun, 1973; McDonald *et al.*, 1977
	$\frac{1}{2}$	^{21}Ne	8.856(6)	3.123(6)	2.8(0.5)	Endt and Van der Leun, 1973; McDonald *et al.*, 1977
$\frac{5^+}{2}$	$\frac{3}{2}$	^{21}F	g.s.	−0.047( )	Bound	
	$-\frac{3}{2}$	^{21}Mg	0.210(10)	11.126(19)		Endt and Van der Leun, 1973
21	$-\frac{1}{2}$	^{21}Na	9.219(5)	7.033(5)	1.2	Endt and Van der Leun, 1973
	$\frac{1}{2}$	^{21}Ne	9.139(6)	3.406(6)		Endt and Van der Leun, 1973
$\frac{3^+}{2}$	$\frac{3}{2}$	^{21}F	0.2792(6)	0.233(7)		Endt and Van der Leun, 1973
	$-\frac{3}{2}$	^{21}Mg	1.080(20)	11.996(20)		Endt and Van der Leun, 1973
21	$-\frac{1}{2}$	^{21}Na	Unknown	Unknown		Endt and Van der Leun, 1973
	$\frac{1}{2}$	^{21}Ne	9.963(6)	4.230(6)		Endt and Van der Leun, 1973
$(\frac{1^-}{2}, \frac{3^-}{2})$	$\frac{3}{2}$	^{21}F	1.1009(20)	1.054(7)		Endt and Van der Leun, 1973
	$-\frac{3}{2}$	^{23}Al	g.s.	6.768(25)		
23	$-\frac{1}{2}$	^{23}Mg	7.788(25)	2.317(25)		Endt and Van der Leun, 1973
	$\frac{1}{2}$	^{23}Na	7.888(3)	−1.642(3)		Endt and Van der Leun, 1973
$\frac{5^+}{2}$	$\frac{3}{2}$	^{23}Ne	g.s.	−5.1551(21)		
	$-\frac{3}{2}$	^{25}Si	g.s.	3.824(10)		
25	$-\frac{1}{2}$	^{25}Al	7.902(4)	−1.010(4)	0.1551(50)	Benenson *et al.*, 1973; McDonald *et al.*, 1977
	$\frac{1}{2}$	^{25}Mg	7.7879(9)	−5.4029(14)		Weigmann *et al.*, 1976
$\frac{5^+}{2}$	$\frac{3}{2}$	^{25}Na	g.s.	−9.357(7)		
	$-\frac{3}{2}$	^{25}Si	0.040(5)	3.864(11)		Endt and Van der Leun, 1973
25	$-\frac{1}{2}$	^{25}Al	7.969(2)	−0.939(2)		Rogers *et al.*, 1977
	$\frac{1}{2}$	^{25}Mg	7.8647(9)	−5.3261(9)		Weigmann *et al.*, 1976
$\frac{3^+}{2}$	$\frac{3}{2}$	^{25}Na	0.089 53(10)	−9.268(7)		Endt and Van der Leun, 1973
	$-\frac{3}{2}$	^{25}Si	0.815(15)	4.639(18)		Endt and Van der Leun, 1973
25	$-\frac{1}{2}$	^{25}Al	Unknown			
	$\frac{1}{2}$	^{25}Mg	8.8355(14)	−4.3553(18)		Weigmann *et al.*, 1976
$\frac{1^+}{2}$	$\frac{3}{2}$	^{25}Na	1.069 32(19)	−8.288(7)		Endt and Van der Leun, 1973
	$-\frac{3}{2}$	^{27}P	g.s.	−0.753(35)		Benenson *et al.*, 1977
27	$-\frac{1}{2}$	^{27}Si	6.628(5)	−5.757(5)		Benenson *et al.*, 1977
	$\frac{1}{2}$	^{27}Al	6.815(2)	−10.379(2)		Benenson *et al.*, 1977
$\frac{1^+}{2}$	$\frac{3}{2}$	^{27}Mg	g.s.	−14.5850(14)		
	$-\frac{3}{2}$	^{29}S	g.s.	−3.160(50)		
29	$-\frac{1}{2}$	^{29}P	8.382(5)	−8.567(4)	0.360(50)	Endt and Van der Leun, 1973
	$\frac{1}{2}$	^{29}Si	8.291(7)	−13.603(7)		Endt and Van der Leun, 1973
$\frac{5^+}{2}$	$\frac{3}{2}$	^{29}Al	g.s.	−18.212(5)		
	$-\frac{3}{2}$	^{31}Cl	g.s.	−7.070(50)		Benenson *et al.*, 1977
31	$-\frac{1}{2}$	^{31}S	Unknown	Unknown		
	$\frac{1}{2}$	^{31}P	6.381(3)	−18.059(3)		Endt and Van der Leun, 1973
$\frac{3^+}{2}$	$\frac{3}{2}$	^{31}Si	g.s.	−22.9487(10)		

TABLE I. (*Continued*)

A/J^π	T_z	Nucl.	E_x (MeV)	Mass excess (MeV)	Γ (keV)	Ref.
	$-\frac{3}{2}$	^{33}Ar	g.s.	−9.385(30)	0.110(15)	Nann *et al.*, 1974 McDonald *et al.*, 1977
33	$-\frac{1}{2}$	^{33}Cl	5.546(3)	−15.457(3)		Endt and Van der Leun, 1973
	$\frac{1}{2}$	^{33}S	5.4754(14)	−21.1105(16)		Endt and Van der Leun, 1973
$\frac{1}{2}^+$	$\frac{3}{2}$	^{33}P	g.s.	−26.3369(21)		
	$-\frac{3}{2}$	^{33}Ar	1.340(20)	−8.040(40)		Nann *et al.*, 1974
33	$-\frac{1}{2}$	^{33}Cl	6.95(5)	−14.05(5)		Endt and Van der Leun, 1973
	$\frac{1}{2}$	^{33}S	6.905(4)	−19.6809(21)		Moalem and Wildenthal, 1975
$\frac{3}{2}^+$	$\frac{3}{2}$	^{33}P	1.4314(2)	−24.9055(21)		Endt and Van der Leun, 1973
	$-\frac{3}{2}$	^{33}Ar	1.790(20)	−7.598(40)		Nann *et al.*, 1974
33	$-\frac{1}{2}$	^{33}Cl	7.399(7)	−13.604(7)	<2	Endt and Van der Leun, 1973
	$\frac{1}{2}$	^{33}S	7.339(5)	−19.247(5)		Moalem and Wildenthal, 1975
$\frac{5}{2}^+$	$\frac{3}{2}$	^{33}P	1.847 60(15)	−24.4893(21)		Endt and Van der Leun, 1973
	$-\frac{3}{2}$	^{35}K	g.s.	−11.169(20)		
35	$-\frac{1}{2}$	^{35}Ar	Unknown	Unknown		
	$\frac{1}{2}$	^{35}Cl	5.651(3)	−23.363(3)		Endt and Van der Leun, 1973
$\frac{3}{2}^+$	$\frac{3}{2}$	^{35}S	g.s.	−28.846 27(21)		
	$-\frac{3}{2}$	^{37}Ca	g.s.	−13.144(25)		Benenson *et al.*, 1973*
37	$-\frac{1}{2}$	^{37}K	5.0469(24)	−19.752 5(28)	0.04(2)	Benenson *et al.*, 1973; Endt and Van der Leun, 1973, Goosman and Kavanagh, 1967
	$\frac{1}{2}$	^{37}Ar	4.993(6)	−25.955(6)		Benenson *et al.*, 1973; Endt and Van der Leun, 1973
$\frac{3}{2}^+$	$\frac{3}{2}$	^{37}Cl	g.s.	−31.761 76(13)		
	$-\frac{3}{2}$	^{37}Ca	1.613(17)	−11.531(35)		Benenson *et al.*, 1973; Butler *et al.*, 1968
37	$-\frac{1}{2}$	^{37}K	6.670(20)	−18.1294(20)		Benenson *et al.*, 1973
	$\frac{1}{2}$	^{37}Ar	6.660(15)	−24.2879(15)		Benenson *et al.*, 1973 Parker *et al.*, 1975
$(\frac{1}{2})$	$\frac{3}{2}$	^{37}Cl	1.7266(1)	−30.0352(2)		Endt and Van der Leun, 1973

[a] The g.s. mass is taken to be the more precise (Benenson, 1973) of two existing measurements, not the value given in the tables of Wapstra and Bos, 1977, which is an average.

sured by the coefficient of a cubic term, d, or by the X^2 of the fit without a cubic term. These are both given for the complete mass quartets in Table II, and a graph of d versus A is given in Fig. 2. Although a few cases could be fortuitous, the agreement of 21 of the 22 complete quartets represents a striking result. The one significant deviation, $A=9$ ground state, is also the most accurately measured case. One can draw several possible conclusions from this. A likely one is that the deviation is a real effect and that some of the other quartets would show deviations if measured as accurately. A second conclusion could be that the accuracy of the $A=9$ measurements was not as good as implied by the quoted errors. Some light can be shed on this question by examining the possible causes for a deviation.

One can identify readily two causes for a significant d coefficient which are calculable using simple models. These are: expansion of the wave function due to Coulomb effects and Coulomb mixing of $T=1/2$ states. A third effect, that due to the next-order term in the perturbation theory approximation, has not been calculated but to some extent is absorbed in the wave function changes discussed below.

As the neutrons in the neutron-rich members of the quartet are converted to protons, the Coulomb repulsion energy increases, and the nucleus must expand to some degree. This expansion will be accentuated for particles which are barely bound and therefore can be considered to be near the top of the potential well which binds them. In addition, as $s_{1/2}$ particle lies in a well without a centrifugal barrier and therefore would be expected to show a more pronounced expansion. Thus the most favorable case for looking at this effect ought to be the $A=27$, $1/2^+$ ground-state quartet. This quartet was recently completed (Benenson *et al.*, 1977) and although the accuracy of the ^{27}P mass determination is relatively poor due to target problems, there is no evidence of a

TABLE II. Coefficients of the isobaric multiplet mass equation for the energy levels in Table I. The errors are in the units of the last significant figure.

A	J^π	a (MeV)	b (MeV)	c (MeV)	d (keV)	χ^2
7	$\frac{3}{2}^-$	26.396(25)	−0.63(5)	0.280(28)	9(27)	
		26.396(25)	−0.617(27)	0.284(25)		0.1
9	$\frac{3}{2}^-$	26.3398(16)	−1.331(3)	0.2640(13)	5.8(16)	
		26.3381(15)	−1.3213(13)	0.2656(12)		14
9	$\frac{1}{2}^-$	28.8489(25)	−1.167(5)	0.2401(3)	2.3(29)	
		22.305(19)	−1.574(25)	0.246(18)		0.03
13	$\frac{3}{2}^-$	19.2567(15)	−2.1795(27)	0.2563(27)	−0.6(21)	
		19.2567(15)	−2.1801(20)	0.2559(23)		0.1
15	$\frac{1}{2}^+$		Incomplete			
		12.826(25)	−2.34(7)	0.25(3)		
15	$\frac{1}{2}^-$	13.811(11)	−2.494(21)	0.240(8)	1(10)	
		13.810(8)	−2.492(8)	0.240(6)		0.1
17	$\frac{1}{2}^-$	11.647(3)	−2.880(5)	0.234(5)	5(4)	
		11.647(3)	−2.876(4)	0.238(4)		1.3
17	$\frac{3}{2}^-$	13.022(4)	−2.847(6)	0.223(10)	−3(7)	
		13.023(4)	−2.849(5)	0.221(8)		0.2
19	$\frac{5}{2}^+$	7.589(12)	−3.194(25)	0.237(6)	−5(11)	
		7.594(3)	−3.205(4)	0.2343(24)		0.2
19	$(\frac{3}{2})^+$	7.589(12)	−3.194(25)	0.237(6)	−5(11)	
		7.7181(3)	−3.206(5)	0.230(3)		0.6
21	$\frac{5}{2}^+$	4.893(5)	−3.662(9)	0.241(5)	3(5)	
		4.894(5)	−3.657(5)	0.242(5)		0.5
21	$\frac{3}{2}^+$	5.162(5)	−3.627(9)	0.230(5)	−2(5)	
		5.162(5)	−3.629(5)	0.229(5)		0.2
21	$(\frac{1}{2}, \frac{3}{2})^-$		Incomplete			
		5.991(7)	−3.637(7)	0.231(5)		
23	$\frac{5}{2}^+$	0.279(14)	−3.96(3)	0.234(9)	−8(13)	
		0.287(4)	−3.973(8)	0.230(4)		0.3
25	$\frac{5}{2}^+$	−3.2614(25)	−4.393(5)	0.220(3)	−0.4(30)	
		−3.2652(19)	−4.396(3)	0.222(3)		0.02
25	$(\frac{3}{2}^+)$	−3.189(2)	−4.383(3)	0.217(3)	2(3)	
		−3.1898(16)	−4.381(2)	0.218(3)		0.1
25	$\frac{1}{2}^+$		Incomplete			
		−2.248(4)	−4.309(6)	0.188(4)		
27	$\frac{1}{2}^+$	−8.118(4)	−4.623(6)	0.200(9)	6(6)	
		−8.120(2)	−4.629(5)	0.207(3)		0.9
29	$\frac{5}{2}^+$	−11.135(6)	−5.038(9)	0.200(13)	9(9)	
		−11.137(5)	−5.032(7)	0.210(7)		1.0
31	$\frac{3}{2}^+$	−15.469(4)	−5.282(4)	0.204(13)	−5(8)	
		−15.467(2)	−5.282(3)	0.197(3)		0.3
33	$\frac{1}{2}^+$	−18.3366(27)	−5.654(4)	0.211(8)	1(5)	
		−18.3371(17)	−5.653(3)	0.213(2)		0.07
33	$\frac{3}{2}^+$	−16.916(28)	−5.63(6)	0.197(16)	3(26)	
		−16.919(7)	−5.622(13)	0.199(7)		0.01
33	$\frac{5}{2}^+$	−16.473(4)	−5.644(7)	0.191(10)	6(7)	
		−16.475(4)	−5.640(5)	0.198(5)		0.7
35	$\frac{3}{2}^+$		Incomplete			
		−20.468(4)	−5.892(7)	0.205(4)		
37	$\frac{3}{2}^+$	−22.904(4)	−6.202(8)	0.200(6)	−2(5)	
		−22.904(4)	−6.204(5)	0.199(5)		0.6
37	$(\frac{1}{2})$	−21.261(14)	−6.157(28)	0.213(9)	−5(13)	
		−21.262(14)	−6.168(8)	0.211(8)		1.5

significant d term. A calculation of the effect of the wave function expansion is discussed in the paper on ^{27}P. It is shown that although the expansion is big (causing a $\simeq 200$ keV shift in the energy of ^{27}P), when properly accounted for in the $T = 3/2$ levels of the $T_z = \pm 1/2$ nuclei, ^{27}Si and ^{27}Al, the effect on the IMME is virtually entirely in the b and c coefficients. A 200 keV change in the Coulomb energy represents approximately a 2% change in Coulomb radius and yet gives a d coefficient of a fraction of a keV. Thus we are led to the conclusion that the quadratic form of the IMME does not follow from the identical wave functions of the levels since in this case we have a 2% radius difference causing no d coefficient either experimentally or in the calculations. Similarly Bertsch and Kahana (Bertsch and Kahana, 1970) have calculated the effect of the wave function expansion in $A = 9$ and showed it to be a 2-keV effect and positive. This calculation was carried out

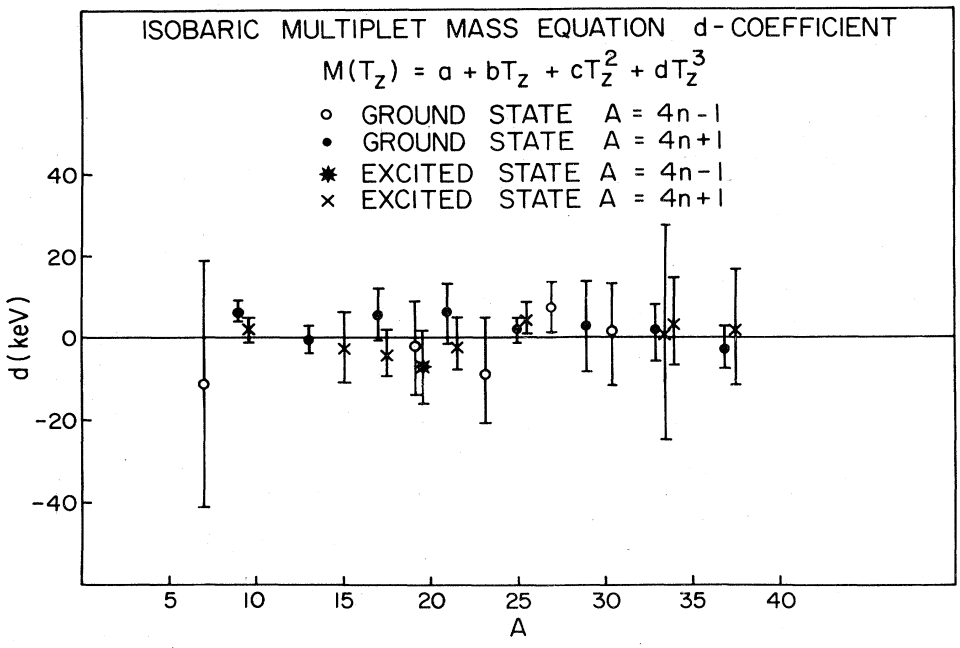

FIG. 2. The d coefficient of the IMME plotted vs A of the quartet. Excited states are slightly displaced from the integral A values.

before the $A=9$ first excited quartet was completed. The experimental d coefficient for this quartet was found to be consistent both with the ground-state quartet and with zero. Three of the states of the quartet have a finite particle width, and this prevents a very accurate experimental determination of their mass excess. There is also the question of how the energy of a broad peak should be related to its eigenvalue theoretically, but these levels are not sufficiently wide to raise this problem. If the d coefficients of the $A=9$ ground-state quartet were due to wave function expansion effects, then such effects would be expected to be enhanced by the $\simeq 2.3$ MeV less binding in the first excited-state quartet. Thus the data seem to be saying that wave function expansions do not affect the d coefficient of $A=9$, probably for the same reasons as in $A=27$, namely, that small expansions in the $T_z=\pm 1/2$ members cancel the effect in the $T_z=-3/2$ member.

A second effect, isobaric spin mixing, definitely occurs in the $T_z=\pm 1/2$ nuclei of every quartet. The particle decays of these levels in almost every case occur because of this mixing since they are isobaric spin forbidden. As is shown in Fig. 3, isospin mixing affects both $T_z=+1/2$ and $T_z=-1/2$, $T=3/2$ levels. Since the d coefficient depends only on the mass difference between the $T=3/2$ levels in the $T_z=\pm 1/2$ nuclei, an effect which displaces both of these levels equally does not produce a d coefficient because its contribution to the d coefficient is equal to one half the difference in the shift in the two nuclei. Unless the shifts are quite large, the effect of isospin mixing is expected to show up in the other coefficients.

An estimate of the shifts due to isospin mixing can be made from the data on widths of $T=3/2$ state compiled by MacDonald *et al.* (1976) for the s,d shell, and by Ajzenberg-Selove (1976, 1977, 1978; Ajzenberg-Selove and Lauritsen, 1974) for the p shell. As can be seen in Table I the most striking feature of these widths is their smallness (2 keV at the most), and since the particle decay width comes completely from this mixing, one can show that the shift must be smaller than the width of the level. Thus we are considering unmeasurably small effects for all the present quartets. In particular, the $A=9$ widths are so small that they rule out isospin mixing as a means for producing the significant d coefficient and rule out also the interesting nuclear structure effects pointed out by Hardy *et al.* (1971).

In summary, the IMME works very well because almost all the physical effects one can consider contribute to the b and c coefficients and tend not to produce a d coefficient or any other violation of the relation.

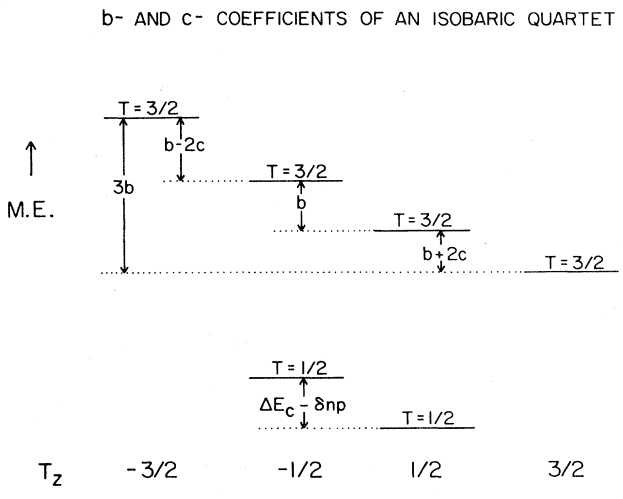

FIG. 3. The effect of isospin mixing on mass quartets and quintets.

But this does not detract from the interest of the equation of nuclear physics; rather it transfers the focus to the b and c coefficients and what they can tell us about nuclei. Also, one can make practical use of the successes of the IMME to predict accurately levels and ground-state masses which are difficult or impossible to measure themselves, but which have three known analogs. In the next section we will discuss what is known about the b and c coefficients in quartets with $A \leq 37$. In some cases the quartets are not complete, and in those cases we will simply assume with a good assurance of correctness that $d=0$ and evaluate b and c from the three known members.

B. The b and c coefficients

Coulomb displacement energies in nuclei have been an important method for studying nuclei throughout the whole range of nuclear species. The physics of these energies has been well reviewed in many places, particularly by Nolen and Schiffer (Nolen and Schiffer, 1969). Normally a Coulomb displacement energy is derived from the shift between two levels, a ground state and its analog or a level and the corresponding level in a mirror nucleus. Interesting anomalies and also corroboration of features of the nucleus found from other methods have arisen from these studies. In the case of mass quartets we are in fact dealing with three different displacement energies, but the IMME shows that these are in fact only two meaningful displacements which can be discussed. These can be expressed in terms of the b and c coefficients, and therefore for convenience we will choose this representation. These two parameters can in fact be used for any multiplet with $T \geq 1$.

In Fig. 4 is shown the significance of the b and c coefficients in terms of the splitting of a mass quartet. If c were equal to zero, then the displacements between adjacent levels would be equal, and in fact the c coefficient does not enter in the displacement between $T_z = \pm 1/2$ levels. The quantity b^* is exactly the same as

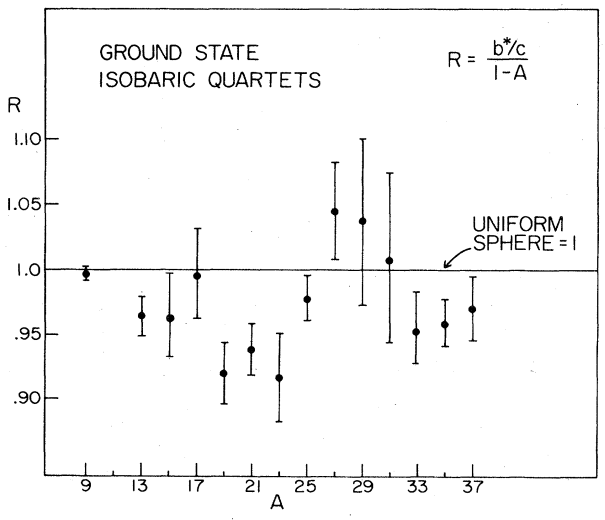

FIG. 4. Significance of the b and c coefficients of the IMME under the assumption that $d=0$.

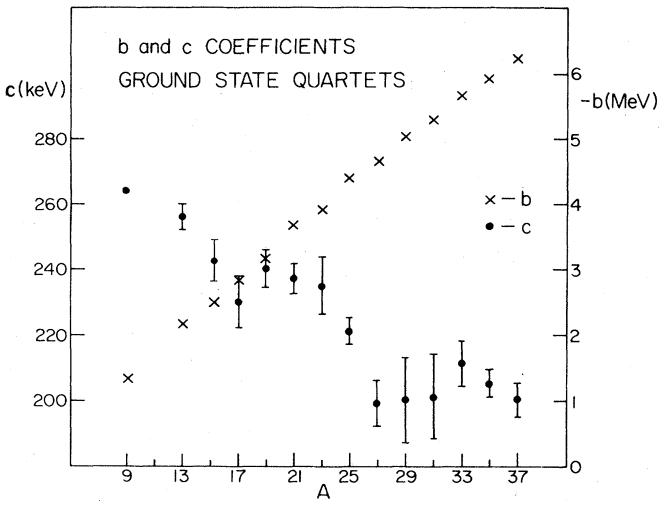

FIG. 5. The quantity R plotted vs A for ground-state quartets. For a uniform sphere, $R=1$.

the Coulomb displacement energy ΔE_c between $T=1/2$ levels in the $T_z = \pm 1/2$ nuclei except that it refers to the $T=3/2$ levels in these nuclei. The splitting between $T_z = \pm 3/2$ levels is given by the IMME to be just $3b$. The b coefficient is in every case negative, and the c coefficient is positive, so the spacing between $T_z=+3/2$ and $T_z=+1/2$ is actually less than that between $T_z=-3/2$ and $T_z=-1/2$. The b and c coefficients that are plotted in Figs. 5, 6, and 7 are only for one quartet in each mass system. The small differences within a given mass system would not be discernible on the figure.

In Fig. 5 the quantity R, which would be unity if the nucleus were a uniformly charged object, is plotted versus A. One can see definite deviations, but the agreement is amazingly good for such a simple model. In Fig. 6 are plotted the b and c coefficients versus atomic number, A, for the lowest-energy quartet

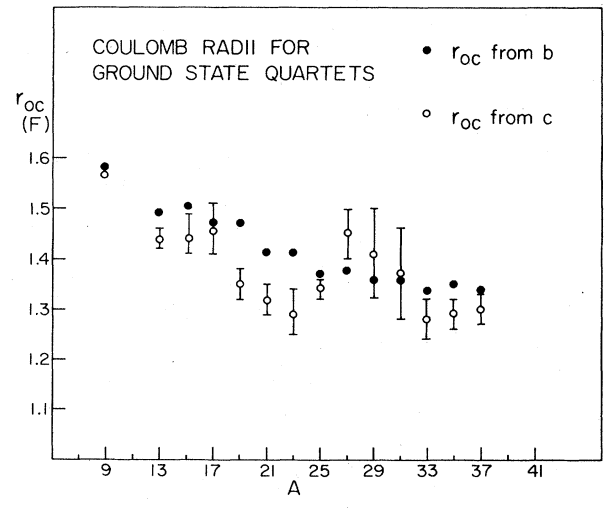

FIG. 6. The b and c coefficients of the IMME for ground-state quartets plotted vs A.

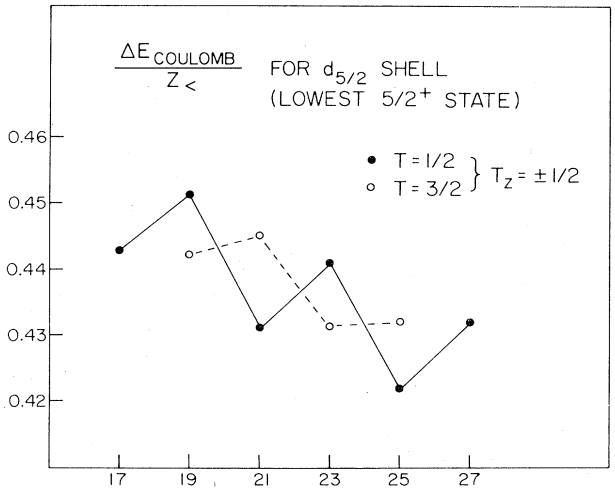

FIG. 7. Coulomb radii determined from the b and c coefficients of the IMME vs A.

known for each A. Two things are evident from this plot. The first is that b and c are quite different quantities as is evidenced by their A dependence and their opposite signs. The magnitude of b goes steadily up with a barely distinguishable pairing oscillation, whereas c falls and seems to fluctuate. A more detailed picture can be obtained by removing from b and c the known, gross A dependence. This is done by inverting the previously given equations for b and c in terms of the Coulomb radius and defining two new quantities, the Coulomb radius parameter determined from b

$$r_{0b} = 0.6(A-1)e^2/b^* A^{1/3}, \tag{14}$$

and from c

$$r_{0c} = 0.6e^2/cA^{1/3}. \tag{15}$$

A plot of the two radius parameters is given in Fig. 7. For the case of b one can now see very clearly the Coulomb pairing alternation which we will use later to measure the Coulomb pairing energy, but otherwise the data is fairly smooth. In the case of r_{0c} there seem to be definite shell effects and fluctuations. As discussed previously a reduced r_{0c} as compared to r_{0b} implies that the valence particles have a greater rms radius than the core. That b and c are very different quantites can be understood by examining the origin of these two terms. As the isobaric spin lowering operator converts neutrons to protons and raises the charge of the nucleus, there is an energy increase from the Coulomb interaction between the newly charged proton and the core. This energy is a constant displacement between members of the quartet and is therefore in the b coefficient. On the other hand, as neutrons are converted to protons, there is an energy associated with the electrostatic repulsion between these newly created valence protons. This term is not a constant but increases with each application of the lowering operator. Thus the c coefficient is sensitive to the wave functions of the valence particles, whereas the b coefficient is an average over all the core. Therefore for particular nuclear structure information one should look at the c coefficient, but for an average property like the Coulomb radius or pairing interaction one should use the b coefficient.

The effect of Coulomb pairing is very evident in the plot of r_{0b} versus A, Fig. 7. Although all of the mass numbers, A, are odd, the number of protons, Z, alternates from even to odd from one case to the next. Because of the strong pairing correlation between nucleons, more energy is released when a paired proton is converted to a neutron than when an unpaired one is. This effect is discussed in Talmi and deShalit (Talmi and deShalit, 1963) and is very evident for Coulomb displacement energies ΔE_c in the s, d, and $f_{7/2}$ shells. For example, in Fig. 8, ΔE_c is given for the lowest, $T = 1/2$ and $3/2$, $5/2^+$ states in the $d_{5/2}$ shell. For $T = 1/2$ levels the average displacement between odd- and even-Z Coulomb energies is 150 keV. The upper half of Fig. 9 illustrates the origin of this shift. For the ^{19}Ne–^{19}F mirror pair a paired proton is converted to a neutron, whereas for ^{21}Na–^{21}Ne an unpaired proton is converted. In the first case we have an increase in the Coulomb energy of Δp (one broken pair). In the second case we have no broken pairs. For b^*, the Coulomb energy between $T = 3/2$ levels in the mirror pairs in the same masses, it is convenient to find the Coulomb energy difference between $T_z = \pm 3/2$ levels and divide by three as prescribed by the IMME. That is,

$$b^* = \tfrac{1}{3}[M(-3/2) - M(3/2)] - (M_n - M_{1_H}). \tag{16}$$

One can now see in Fig. 9 that the ^{19}Na–^{19}O Coulomb energy involves one broken pair, but ^{21}Mg–^{21}F involves two. Therefore b^* will increase in magnitude by $\Delta p/3$ in going from $A = 19$ to 21, whereas ΔE_c will decrease by Δp. Thus one can determine the Coulomb pairing energy from either ΔE_c or b^*, and although the former is much more accurate (since it involves masses of low-lying states of nuclei near stability or actually stable) a value of $\Delta p = 150$ keV fits both sets of data well. (This quantity Δp is unfortunately called b in the treatment in Talmi and deShalit.)

In the preceding discussion of the b coefficient, a small correction for the magnetic spin-orbit term

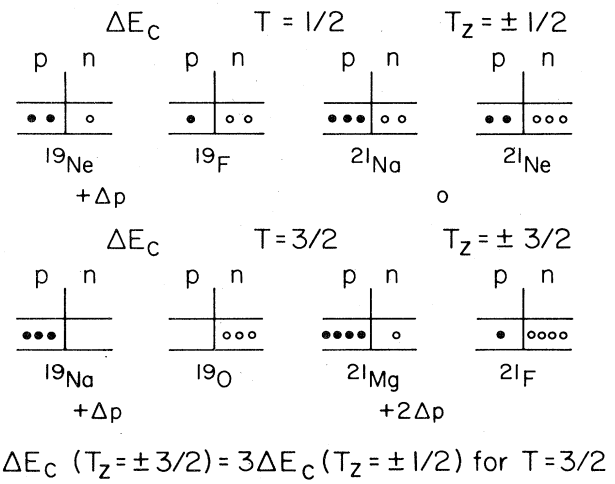

FIG. 8. Pairing effects in $T = 1/2$ and $T = 3/2$ Coulomb energies.

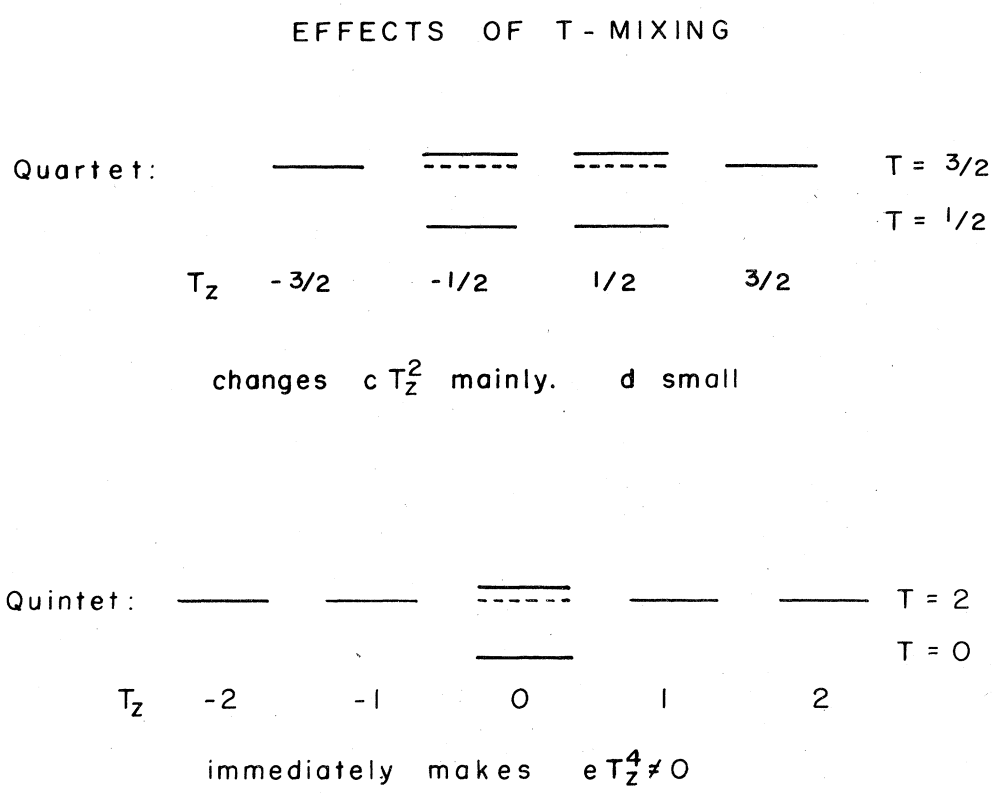

FIG. 9. Shell-model diagram useful in explaining the pairing alternation of Coulomb energies. See text for explanation.

should be included. This term arises from the different magnetic moments of the neutron and proton and was evaluated by Garvey (1969) for the case of three particles with the same angular momentum, j, coupled to total angular momentum $J=j$. This condition applies approximately to most of the quartets discussed above and can be considered to be an upper limit in the other cases. The contribution to b from this effect is between −11 and +35 keV and hence is only of interest for theoretical calculations of Coulomb effects with an accuracy of this order. The shell model calculations described later in the paper do take account of this effect in various manners. The general trend of b with A is hardly affected by this charge-dependent effect and, once the pairing effect is accounted for, shows that the Coulomb radius calculated from b does vary smoothly but not as $A^{1/3}$. This is undoubtedly due to the simplicity in the assumption of uniformly charged sphere, and a more detailed model such as the shell model should account for the variation.

IV. THEORETICAL CALCULATIONS

There have been only four attempts to calculate the shifts between members of isobaric spin quartets. All four use the shell model as a basis for the calculations with various degrees of sophistication. The shell model has been used with great success in the region of light nuclei discussed in the present article, but in virtually every case isobaric spin is considered to be a good quantum number, and no account is taken of the z projection of the isobaric spin. In fact, in the first step of the shell model calculation, the single-particle energies for the various orbitals are taken to be an average between the two mirror nuclei. There is a very good reason for doing this—neutrons and protons do not then have to be considered separately, and consequently the size of the basis required to carry out the calculations is reduced by a factor of two. As was pointed out by McGrory *et al.* (1978), single-particle energies can be taken separately for neutrons and protons from experiment and the calculation then carried out with a bigger basis, one in which isobaric spin is not a good quantum number. The advantage of this method is that almost all charge-dependent effects are automatically taken into account either via the single-particle energies, the Coulomb two-body matrix elements, or the diagonalization of the matrix. Using the new techniques developed by Whitehead (1972) for diagonalizing the large matrices required, it is now possible to include all of the active s,d shell neutrons and protons separately in most cases. The $T=3/2$ states in the $T_z=\pm 1/2$ nuclei are mixed with $T=1/2$ states in these calculations, but unfortunately the isospin forbidden particle decays have not yet been calculated. With the single-particle energies taken from ^{17}F and ^{17}O, $5/2^+$, $3/2^+$, and $1/2^+$ states, they automatically take into account the magnetic shift in energy due to the opposite sign of neutron and proton magnetic moment and the expansion of the wave function for the proton. However, since the expansion depends on the binding energy, the single-particle energies taken from ^{17}F–^{17}O should really be adjusted for the correct binding energy in each nucleus, but this has not been done. In the upper end of the s,d

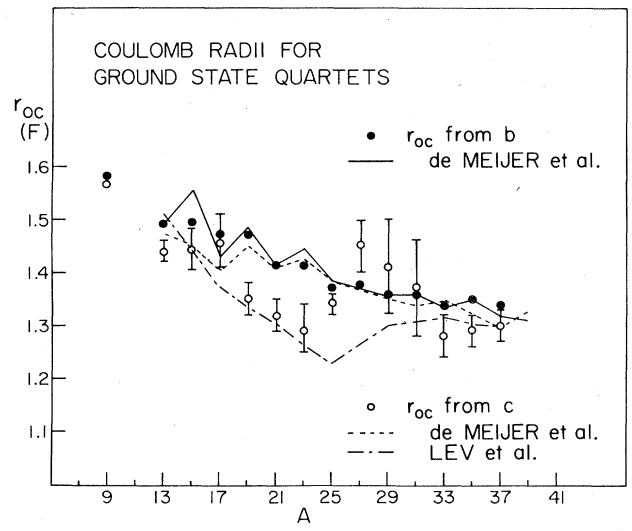

FIG. 10. The Coulomb radii from b and c coefficients of the IMME compared to theoretical calculations.

shell the single-particle energies are taken from ^{39}K-^{39}Ca hole states. Once again the Coulomb radius is intended only as a parameter which removes from b and c the gross A dependence. The agreement with b is excellent, but c is only qualitatively accounted for. It would be very interesting to know if the problem with the c coefficient originates from the fact that the binding-energy effects were not included or from the Coulomb two-body matrix elements, which were calculated with harmonic oscillator wave functions.

Another shell model approach was made by DeMeijer, Van Royen, and Brussaard (DeMeijer *et al.*, 1971). Whereas the calculations described above involve redoing the shell model calculation with neutrons and protons treated separately, DeMiejer *et al.* worked in perturbation theory using previously calculated wave functions with good isobaric spin. They attempted to calculate the effect of each charge-dependent effect separately and add them. Several approximations were made to carry out this ambitious project, which covered all states of $T=0$, 1/2, 1, 3/2, and 2 in the s,d shell except $A=29$. Coulomb two-body matrix elements are taken to be independent of J which is an approximation good to about 20%, failing worst for $J=0$. All off-diagonal Coulomb matrix elements are taken to be zero. Unlike the calculations in which neutrons and protons are treated separately, a small d term often appears, but this is assumed to be due to the calculational errors and not a real physical effect. The calculations which are presented in Fig. 10 give a good agreement with b and fail to represent the variations in c in a way very similar to the results of McGrory *et al.* (1978).

The shell model calculations of Auerbach, Lev, and Kashy (Auerbach *et al.*, 1971) are the simplest of the three since they assume the lowest-order shell model configuration without any mixing. In addition, the Coulomb radius was adjusted to give agreement with b and then c was calculated. The results are given in Fig. 10, therefore, for the c coefficient only.

V. ISOBARIC QUINTETS

In the sections above we have discussed only mass quartets, but experimental evidence on quintets (of $T=2$ levels) is beginning to become available. Besides providing a more stringent test of the IMME, Coulomb energies in mass quintets are of high interest in nuclear structure studies. We will briefly review the existing experimental data and give the coefficients of quintets with four or more known members, and then examine some of the special interests in complete quintets.

Although there are six accurate mass measurements for $T_z=-2$ nuclei, there is only one complete quintet, the $A=8$ (Robertson *et al.*, 1975). This situation arises because no good general method has been found for identifying $T=2$ states in $T_z=-1$ nuclei. The $A=8$ case was very favorable because the $T=2$ state in ^{8}B lies in a continuum region of high-level density of $T=1$ states. It thus appears as a narrow well defined peak in the $^{11}B(^{3}He,^{6}He)$ reaction. Attempts to use the same method in other nuclei have been thwarted by the high-level density of observed $T=1$ levels and the lack of selective population of $T=2$ states. The case of ^{20}Na is being pursued because there appears to be a strong narrow peak at the IMME prediction, but $T=2$ states in ^{10}C, ^{16}F, and ^{24}Al have been searched for unsuccessfully.

A summary of the properties of the members of the $A=8$ quintet is given in Table III, and the coefficients of the IMME, including now a possible eT_z^4 term, are given in Table IV. The results do not favor an e term over a d term, but theoretical expectations do. The expected origin of an e term is diagrammed in Fig. 3. In this figure the situation for quintets is contrasted with that for quartets. Isospin mixing produces level shifts, but only in quintets does this produce a violation of the IMME directly. As described previously only the difference in shifts violates the IMME for quartets. The $T=2$ state in the $T_z=0$ nucleus is usually a very well studied level which is known to be mixed with lower T states and thereby shifted. This evidence comes from the forbidden particle decays of the state. Any shift of the $T=2$ state in a $T_z=0$ nucleus due to mixing will produce an e coefficient and no d coefficient. In the case of ^{8}Be, the sign and magnitude of the e coefficient can be used to predict the missing 0^+, $T=0$ state expected theoretically to lie nearby. The importance of locating this state is high because it may permit a clear-cut determination of the $\Delta T=2$, charge-dependent matrix

TABLE III. Summary of properties of the $A=8$ isobaric quintet.[a]

Nucl.	T_z	Mass excess (MeV)	Γ (keV)
^{8}C	−2	35.097(24)	245(40)
^{8}B	−1	23.542(9)	32(25)
^{8}Be	0	32.4358(18)	5.5(20)
^{8}Li	1	31.7694(54)	12
^{8}He	2	31.595(7)	Bound

[a] From Robertson *et al.* (1978).

TABLE IV. Coefficients of the IMME for quintets with three or more known members.

A/J^π	a (MeV)	b (MeV)	c (MeV)	d (keV)	e	χ^2	Reference
8	32.4346(17)	−0.8823(40)	0.2294(24)	···	···	7.7	Robertson *et al.*, 1978
0^+	32.4353(18)	−0.8944(63)	0.2258(28)	5.6(22)	···	1.5	
	32.4358(18)	−0.8819(40)	0.2137(69)	···	4.2(17)	1.8	
	32.4358(18)	−0.8899(73)	0.2173(74)	3.6(27)	2.6(21)	···	
12	27.610(18)	−1.769(26)	0.239(15)	···	···	0.0	Kekelis *et al.*, 1978
0^+	27.611(20)	−1.770(40)	0.239(15)	0.(11)	···	···	
16	17.983(3)	−2.584(12)	0.216(8)	···	···	2.8	Kekelis *et al.*, 1978
0^+	17.984(3)	−2.587(13)	0.206(10)	8.(5)	···	···	
16	19.772(10)	−2.591(17)	0.209(10)	···	···	6.3	Kekelis *et al.*, 1978
2^+	19.785(1)	−2.604(17)	0.187(13)	15(6)	···	···	
20	9.6911(23)	−3.4361(52)	0.2462(31)	···	···	1.35	Tribble, 1976
0^+	9.6903(24)	−3.4336(57)	0.2486(39)	2.3(20)	···	···	
24	1.502(3)	−4.174(7)	0.224(5)	···	···	···	
0^+							
28	−6.267(3)	−3.805(5)	0.215(2)	···	···	···	
0^+							
32	−13.966(2)	−5.468(3)	0.202(2)	···	···	0.04	Hagberg *et al.*, 1977
0^+	−13.965(4)	−5.469(3)	0.202(5)	0.5(25)	···	···	Tribble *et al.*, 1977
36	−19.3760(16)	−6.0481(32)	0.2016(15)	···	···	0.78	
0^+	−19.3763(16)	−6.0503(40)	0.2058(50)	−1.6(18)	···	···	

element in a case for which the wave functions are known. For more details on this, see Robertson *et al.* (1975).

The b and c coefficients for the remaining partially complete quintets in Table IV are not plotted in Figs. 4–7, but they fall right on the systematics for each parameter. As expected, the b coefficient lies essentially directly on a line between the next lowest and next highest odd-A b coefficient from the quartets. The only theoretical treatment of the b and c coefficients for quintets is found in the work of De Meijer *et al.* (1971). The b-coefficient agreement is much better than the c-coefficient agreement just as is the case for quartets. The field of mass quintets is really just getting started and one can hope for a summary review article on them in 5–10 yr.

VI. CONCLUSIONS

We have shown that the IMME works extremely well for quartets and quintets, and therefore we have studied the significance of its coefficients. In gross the coefficients show that the nucleus is not so different from a uniformly charged sphere, but the differences, particularly in the quadratic term, have not been fully explained by the shell model either in an exact or perturbation theory treatment.

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