Solar neutrino experiments

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New results are presented for absorption cross sections of nine possible detectors of solar neutrinos (^{7}Li, ^{37}Cl, ^{51}V, ^{55}Mn, ^{71}Ga, ^{81}Br, ^{87}Rb, ^{115}In, and ^{205}Tl). Special attention is given to nuclear physics uncertainties. The calculated cross sections are used (with the aid of illustrative solar models and *ad hoc* assumptions about neutrino propagation) to discuss what can be learned about the sun or weak interactions from each of the nine suggested solar neutrino experiments. An experimental program for neutrino spectroscopy of the solar interior is outlined. It is shown in addition that stellar collapses can be detected to typical distances of several kpc (kiloparsecs) by the proposed ^{7}Li, ^{37}Cl, and ^{115}In solar neutrino detectors (provided that electron neutrinos do not decay or oscillate).

CONTENTS

I. INTRODUCTION

A. The problem

Solar neutrino experiments offer a unique opportunity for studying the interior of a star. Conventional information about stars is provided by photons that are emitted from stellar surfaces. The mean free path for photons in stellar interiors, where nuclear fusion occurs, is much less than a centimeter. Neutrinos, unlike photons, interact so weakly with matter that they can escape directly from a stellar interior. Thus neutrinos allow us to look inside a star and to test directly the theoretical predictions for the rates at which certain nuclear reactions occur.

The predicted solar neutrino fluxes make possible well-defined tests of the theory of stellar evolution. We know more about the sun than about any other star. We know its mass, luminosity, radius, surface temperature, surface composition, and age much more accurately than for any other star. The sun is also in what is believed to be the best-understood stage of stellar evolution, the quiescent main sequence phase. If we are to have confidence in the many astronomical and cosmological applications of the theory of stellar evolution, it ought at least to give the right answers for the sun.

The first solar neutrino experiment, which has been performed using ^{37}Cl by R. Davis, Jr. and his associates (Davis, Harmer, and Hoffman 1968; Davis, 1969; Davis and Evans, 1973; Rowley *et al.*, 1977) has revealed a serious discrepancy (see Bahcall and Davis, 1976) between observation and the stardard theory of stellar evolution (using the best estimates for all atomic and nuclear parameters). The origin of this disagreement is unknown. It is not even known for certain that the fault lies in the standard astronomical model of the sun rather than in the conventional physical theory for the propagation of neutrinos.

A number of exotic solutions, modifying either the physics or the astronomy (and in some cases both), have been proposed. Even if one grants that the source of the discrepancy is astronomical, there is no general agreement as to what aspect of the theory is most likely to be incorrect. Many of the proposed solutions of the solar neutrino problem have broad implications for conventional astronomy and cosmology. Some of them would change the theoretical ages of old stars or the inferred primordial element abundances. On the other hand, modified theories of the weak interactions have been proposed in which neutrinos may disappear by mixing or decay in transit from the sun to the earth, but for which there are no terrestrially measurable consequences. It is conceivable that one of these modified theories of the weak interactions is correct and the standard solar model is not in conflict with observations.

The present paper contains discussions of proposed solar neutrino experiments with attention to both the technical questions regarding their sensitivity (i.e., absorption cross sections) and the broader questions of what such experiments can teach us about neutrino physics and astrophysics.

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B. Contents of this paper

I discuss in detail the neutrino absorption cross sections of the nine detectors that have received the greatest attention from experimentalists over the past few years. These are: ^{7}Li, ^{37}Cl, ^{51}V, ^{55}Mn, ^{71}Ga, ^{81}Br, ^{87}Rb, ^{115}In, and ^{205}Tl. I discuss also the sensitivity of each of the targets to the different neutrino sources in the sun and compare the theoretical advantages and disadvantages of the different detectors. Almost all of the numerical results are new (although many of them have been circulated informally to interested experimentalists over the past few years).

The above-mentioned targets can also be used as efficient detectors of stellar collapse at the same time a solar neutrino experiment is being performed. I therefore tabulate some (unaveraged) cross sections that are useful for the application to stellar collapses.

This is *not* a review paper. Results are presented without detailed explanations or derivations. On the other hand, I have tried to be sufficiently complete and specific so that practioners of the various arts that are used will know exactly what was done in order to obtain the numbers that are given. The reader who is interested in obtaining more background information about various aspects of the solar neutrino problem can do so by consulting one or more of the following review articles (Bahcall and Davis, 1976; Bahcall, 1967; Reines, 1967; Salpeter, 1968; Davis, 1969; Davis *et al.*, 1972; Bahcall and Sears, 1972; Fowler, 1972; Ulrich, 1974; Kuchowicz, 1976).

The sections of this paper are arranged in a seemingly logical order. However, I suggest that the reader ignore the formal organization and skip immediately to those sections that are of interest to him or her. I summarize below the different parts of the paper so that a reader can choose easily what he wants to read.

The neutrino sources are described briefly in Sec. II. The formulae used to calculate the absorption cross sections are presented in Sec. III. Special attention is given to atomic physics effects. Overlap and exchange effects among electrons that are present in the initial and final atomic states of the neutrino capture reaction are included (for the first time). The (small) probability for the creation of a bound electron is also calculated.

The detailed results for each of the nine targets listed above are given in Sec. IV. Absorption cross sections for terrestrial ^{51}Cr and ^{65}Zn sources are presented here as well as cross sections for all of the solar neutrino sources. (It is presumed that future solar neutrino experiments will be tested with a terrestrial neutrino source.) The accuracy with which the most important nuclear matrix elements can be inferred from the available experimental data is investigated in detail. In several of the most interesting experiments, the absorption cross sections are found to be much less accurately known than was suggested in the original experimental proposals. Having in mind the application to stellar collapse, cross sections as a function of energy are given for ^{7}Li and ^{37}Cl (and are estimated at a typical energy for ^{71}Ga and ^{115}In).

The last section, Sec. V, contains a discussion of what can be learned from solar neutrino experiments. The counting rates are predicted for all the targets discussed in this paper using a standard solar model, some illustrative but concocted nonstandard models, and two *ad hoc* hypotheses about neutrino propagation (neutrino decay and neutrino oscillations). The detectors are calssified according to their relative sensitivity to different parts of the solar neutrino spectrum. The major nuclear physics uncertainties are then summarized. Three experiments (with ^{7}Li, ^{71}Ga, and ^{115}In targets) are preferred on the basis of the accuracy with which their absorption cross sections are known and their utility in helping to discriminate between theoretical alternatives. The part of the solar neutrino spectrum that is determined by each of the preferred experiments is established by analyzing the coupled equations that relate the measured capture rates (with their errors) to the individual fluxes. The great precision to which one can check the law of electrical charge conservation in nucleon decays as a byproduct of a ^{71}Ga solar neutrino experiment is also described. Finally, it is shown that stellar collapses can be detected to a typical distance of a few kpc with the ^{7}Li, ^{37}Cl, and ^{115}In solar neutrino detectors.

I suggest that the majority of readers may find it most useful to begin by glancing quickly through Sec. II and then jumping immediately to Sec. V. Those readers who have an interest in a particular experiment can then turn to the relevant subsection of Sec. IV. Section III is primarily for the specialist with a need to know.

II. NEUTRINO SOURCES

The most important reactions that are believed to produce solar neutrinos with continuous energy spectra are (see Bahcall and Sears, 1972; Parker, Bahcall, and Fowler, 1968; and references quoted therein):

$$p+p \rightarrow {}^2\mathrm{H}+e^+ +\nu \quad (0.420 \text{ MeV}), \tag{1}$$

$$^8\mathrm{B} \rightarrow {}^8\mathrm{Be}^* + e^+ + \nu \quad (14.02 \text{ MeV}), \tag{2}$$

$$^{13}\mathrm{N} \rightarrow {}^{13}\mathrm{C} + e^+ + \nu \quad (1.199 \text{ MeV}), \tag{3}$$

$$^{15}\mathrm{O} \rightarrow {}^{15}\mathrm{N} + e^+ + \nu \quad (1.732 \text{ MeV}). \tag{4}$$

The numbers given in parenthesis are, except for reaction (2), the maximum neutrino energies for the respective reactions (computed from the atomic mass differences given by Wapstra and Bos, 1977). The state in ^{8}Be that is populated by the beta decay of ^{8}B is approximately 2 MeV wide; the number given in parentheses beside reaction (2) is the maximum neutrino energy corresponding to the ^{8}B beta decay to the peak of the first 2^+ resonance in ^{8}Be. Because of the broad character of the ^{8}Be state, a special treatment is necessary in order to calculate accurately the average cross section for neutrinos from ^{8}B decay (see Sec. III.D).

The following reactions produce neutrinos with *discrete energies* except for a small thermal broadening due to the spread in initial electron energies:

$$p+e^- + p \rightarrow {}^2\mathrm{H} + \nu \quad (1.442 \text{ MeV}), \tag{5}$$

$$^7\mathrm{Be} + e^- \rightarrow {}^7\mathrm{Li} + \nu \quad (0.862 \text{ MeV}, \ 89.6\%), \tag{6a}$$

$$(0.384 \text{ MeV}, \ 10.4\%). \tag{6b}$$

Alvarez (1973) suggested that the ^{37}Cl solar neutrino experiment might be tested by using a terrestrial source of ^{65}Zn. Similar suggestions have been made by Reines (1978) and Raghavan (1978) for testing the proposed ^{71}Ga and ^{115}In experiments with a terrestrial source of ^{51}Cr. I present in Sec. IV the cross sections for absorption of ^{51}Cr and ^{65}Zn neutrinos by all the detectors that are discussed in this paper.

The discrete electron capture decays of ^{65}Zn are (Auble, 1975):

$$e^- + {}^{65}\mathrm{Zn} \rightarrow {}^{65}\mathrm{Cu} + \nu \quad (0.227\ \mathrm{MeV},\ 50.75\%), \tag{7a}$$

$$\rightarrow {}^{65}\mathrm{Cu} + \nu \quad (1.343\ \mathrm{MeV},\ 47.8\%). \tag{7b}$$

There is also a weak positron branch that produces a neutrino continuum

$${}^{65}\mathrm{Zn} \rightarrow {}^{65}\mathrm{Cu} + e^+ + \nu \quad (0.330\ \mathrm{MeV},\ 1.45\%). \tag{7c}$$

The decay branches of ^{51}Cr are (Rao and Rapaport 1970)

$$e^- + {}^{51}\mathrm{Cr} \rightarrow {}^{51}\mathrm{V} + \nu \quad (0.746\ \mathrm{MeV},\ 90.1\%), \tag{8a}$$

$${}^{51}\mathrm{V} + \nu \quad (0.426\ \mathrm{MeV},\ 9.9\%). \tag{8b}$$

In computing the neutrino energies shown in (7a)–(8b), I have subtracted (see Bahcall, 1963) from the atomic mass differences the average excitation energy of the final atom for positron decays and the binding energy of a K-shell electron in the final atom for electron captures.

In Sec. IV, I tabulate the weighted-average cross section for a given neutrino source (e.g., ^{7}Be or ^{65}Zn). In computing the averages, I include the fact that some decay branches are below threshold (i.e., contribute zero to the rate) for the particular target under discussion.

III. CALCULATIONS

The equations that are used to calculate neutrino absorption cross sections are given in this section (Sec. III.A). Some accurate numerical ft values are presented (in Sec. III.B and Table I) for cases in which the ground state to ground state transition can be observed in the laboratory as allowed electron capture. The computer code that has been used to calculate the weighted-average phase-space factors is described in Sec. III.C. In Sec. III.D, a discussion is given of the special treatment that is required for ^{8}B decays because of the broad final state in ^{8}Be through which the decay proceeds.

Special attention is given throughout this section to atomic physics effects. I calculate for the first time overlap and exchange effects among the initial and final electrons that occur in neutrino capture because the nuclear charge changes by one unit. I also evaluate numerically the (small) probability for the creation of a bound electron in the neutrino capture process. Ground-state ft values are presented in Table I for the cases where they could be calculated accurately from laboratory data. The (large) relativistic and (small) overlap and exchange effects are properly included and the method by which they have been determined are described. I also summarize briefly how the various physical effects (electron screening, finite nuclear size, and relativistic increase in electron density) are treated in the calculation of the weighted-average phase-space factors.

A. General formulae

The usual formula for allowed neutrino capture cross sections in the general reaction

$$\nu + {}^{Z-1}A \rightarrow e^- + {}^{Z}A\ . \tag{9}$$

may be written (Bahcall, 1964a)

$$\sigma \equiv \sigma_0 \langle w_e^2 G(Z, w_e) \rangle\ , \tag{10}$$

where

$$\sigma_0 \equiv \frac{1.206 \times 10^{-42}}{(ft_{1/2})_{I' \to I}} \left[\frac{(2I'+1)}{(2I+1)}\right] Z\,\mathrm{cm}^2\ , \tag{11}$$

and

$$G(Z, w_e) \equiv (p_e F(Z, W_e)/2\pi\alpha Z w_e)\ . \tag{12}$$

Here Z is the atomic number of the final nucleus (mass number A), $w_e \equiv W_e/m_e c^2$ is the energy (in units of the electron's mass) of the electron that is produced, p_e is the electron's momentum (in units of $m_e c$), and $F(Z, W_e)$ is the familiar Fermi function (Konopinski, 1966). The spin of the initial nuclear state is I and of the final state, I'; note for allowed decays $I - I' = 0, \pm 1$. The bracketed term in Eq. (11) is the same as the statistical factor S used by Sunyar and Goldhaber (1960).

1. Overlap and exchange effects

The usual theory (e.g., Konopinski, 1966) of neutrino capture by atoms neglects two physical effects: (i) the increase in nuclear charge by one unit, causing a change in the atomic Hamiltonian, from initial to final atomic state; and (ii) the indistinguishability of the electron that is created from the initially present atomic electrons. Similar atomic-physics considerations lead to calculable corrections, known respectively as overlap and exchange effects, to the usual theory of electron and positron emission and to electron capture. A theory for treating the weak interaction process with proper attention to the atomic variables has been described by Bahcall (1963); the predicted corrections are, in some cases, easily measurable and are then in agreement with laboratory experiments (e.g., Bahcall, 1965; Bambynek *et al.*, 1977). It is easy to apply the previously derived formalism to neutrino capture since electron beta decay and neutrino capture are formally very similar. One finds that atomic overlap effects are accounted for to high accuracy if the energy of the continuum electron that is produced in reaction (9) is calculated from an equation that includes the average excitation energy of the final atom $\bar{E}_{\mathrm{ex}}$, i.e.:

$$W_e = -\bar{E}_{\mathrm{ex}} + q + (M(A, Z-1) - M(A, Z)) + m_e c^2\ , \tag{13}$$

where q is the neutrino energy, and the term in parenthesis is the atomic mass difference. A satisfactory approximation for $\bar{E}_{\mathrm{ex}}$ is (see Eq. (30) of Bahcall, 1963): $\bar{E}_{\mathrm{ex}} \approx 24.5 Z^{1/3}$ eV for $Z < 10$ and $23 Z^{2/5}$ eV for $Z > 10$. Exchange effects between the final continuum electron and the electrons bound in the initial atom interfere in a way that reduces the capture rate. I find [cf. Eq. (57) of Bahcall, 1963] for the cross section at a specific neutrino energy q

$$\sigma(q)_{\mathrm{atomic}} \cong \sigma(q)\left[1 - 2Z^{-1}\theta(E_K - 2(W_e - m_e c^2))\right]\ , \tag{14}$$

where $\sigma(q)$ is the cross section calculated ignoring atomic effects, E_K is the binding energy of the K electron in the initial atom, and $\theta(x)=0$ if $x<0$, and $\theta(x)=1$ if $x>0$. Numerical evaluations of Eqs. (13) and (14) show that atomic overlap and exchange effects affect the calculated neutrino absorption cross sections by less than one percent for all the cases considered in this paper.

2. Bound-state capture

For completeness, we need to consider one more process: bound-state neutrino capture. In this process, no continuum electrons are produced; a bound electron is created in a previously unoccupied bound state of the final atom. The ratio of bound-state neutrino capture to the usually considered continuum capture process is (Bahcall, 1964a)

$$\frac{\sigma_{\text{bound state}}}{\sigma_{\text{continuum}}} \approx \frac{\Pi}{4}(\hbar/m_e c)^3 |g_{K,\mu}(0)|^2 \times \frac{m_e c^2 P(q_b)}{(\overline{W}_e/m_e c^2)(\overline{p}_e/m_e c)(\overline{F}(Z,W_e))}, \quad (15a)$$

where $g_{K,\mu}(0)$ is the radial wave function of the bound electron in the final atom, and $P(q_b)$ is defined by

$$P(q_b)=\phi(q_b)/\int dq\,\phi(q)\,. \quad (15b)$$

Here $\phi(q)$ is the incident neutrino flux with energy q; q_b is the value of the neutrino energy that just corresponds to producing the electron in a bound state; and the spectrum average of various quantities is denoted by a bar. Using the atomic wave functions cited in Sec. III.B, I find that bound-state neutrino capture amounts to less than one percent of the usual continuum process for all the cases considered in this paper.

B. $ft_{1/2}$ values

The $ft_{1/2}$ values to be used in Eq. (11) for the cross section constants σ_0 can be calculated directly from laboratory data if the inverse to reaction (9), the electron capture process $e^- + {}^ZA \rightarrow {}^{Z-1}A + \nu$, has been observed. The dimensionless phase-space factor for allowed electron capture is (Bahcall, 1966a)

$$f_{\text{ec}} = 2\pi^2(q_{1s}/m_e c^2)^2 \times [1+(q_{2s}^2 L/q_{1s}^2 K)]\,|\psi_{1s}(R)|^2(\hbar/m_e c)^3\,, \quad (16)$$

where q_{1s} is the neutrino energy (see below) when a $1s$ (i.e., K) electron is captured, L/K is the L to K capture ratio, and $|\psi_{1s}(R)|^2$ is the value of the square of the modulus of an $1s$ electron's wave function at the nucleus.

In writing Eq. (16), I have made use of the fact that atomic overlap and exchange effects are very small for *total* electron capture rates although these effects are easily measurable for capture ratios. There has occasionally been some confusion about this point in the literature because the cancellations that cause the overlap and exchange effects to be small for total decay rates are not obvious with the necessarily inaccurate theoretical results (often numerical extrapolations) that are used for small Z. One can show, using Eqs. (71), (80), and (87) of Bahcall (1963), that overlap and exchange effects amount to less than a percent correction to the total capture rates for all the cases discussed in this paper.

The quantity q_{1s} is to be interpreted [Bahcall, 1963, Eq. (70)] as the difference of atomic masses minus the (positive) binding energy of the $1s$ electron in the *final* atomic state that results from electron capture. The K-shell binding energy is important for large Z and small decay energies, q. In the extreme case shown in Table I, ^{87}Rb-^{87}Sr^m, the value of f_{ec} is increased by 31% (and the cross-section factor correspondingly decreased) when the K-shell binding is included compared to an estimate in which it is ignored. I have used values of electron binding energies taken from Bearden and Burr (1967).

Some of the important ground-state $ft_{1/2}$ values, and the cross-section factors, σ_0, are tabulated in Table I for cases in which the neutrino reaction is the inverse of an electron capture reaction that is observed in the laboratory. The half-lives, $t_{1/2}$, have been taken from recent laboratory measurements (see references in Table I). The phase-space factors f have been computed from Eq. (16) using self-consistent field Dirac–Hartree wave functions that include relativistic and nuclear size effects as well as electron exchange (Martin and Blic-

TABLE I. ft and σ_0 values for cases in which the electron capture is observed in the laboratory.

Reaction	f	$t_{1/2}$ (10^6 sec)	$ft_{1/2}$ (sec)	$I(I')$	σ_0 (10^{-46}cm^2)	References
$e^- + {}^7\text{Be} \rightarrow {}^7_3\text{Li} + \nu$	4.115×10^{-4}	5.15	2.12×10^3	3/2(3/2)	22.75	a, b
$e^- + {}^7_4\text{Be} \rightarrow {}^7_3\text{Li}^* + \nu$	8.165×10^{-5}	44.4	3.625×10^3	3/2(1/2)	6.65	a, b
$e^- + {}^{37}_{18}\text{A} \rightarrow {}^{37}_{17}\text{Cl} + \nu$	4.16×10^{-2}	3.03	1.26×10^5	3/2(3/2)	1.725	c
$e^- + {}^{51}_{24}\text{Cr} \rightarrow {}^{51}_{23}\text{V} + \nu$	9.14×10^{-2}	2.66	2.43×10^5	7/2(7/2)	1.19	d
$e^- + {}^{55}_{26}\text{Fe} \rightarrow {}^{55}_{25}\text{Mn} + \nu$	1.09×10^{-2}	85.2	9.28×10^5	5/2(3/2)	0.225	e
$e^- + {}^{71}_{32}\text{Ge} \rightarrow {}^{71}_{31}\text{Ga} + \nu$	2.26×10^{-2}	1.02	2.32×10^4	3/2(1/2)	8.33	f
$e^- + {}^{87}_{38}\text{Sr}^m \rightarrow {}^{87}_{37}\text{Rb} + \nu$	8.34×10^{-3}	3.37	2.81×10^4	3/2(1/2)	8.16	g

[a]Bahcall, 1966a.
[b]Ajzenberg-Selove and Lauritsen, 1974.
[c]Kishore *et al.*, 1975.
[d]Rao and Rappaport, 1970.
[e]Auble and Rappaport, 1970.
[f]Alvar, 1973.
[g]Verheul, 1971.

kart-Toft, 1970; Suslov, 1968; Behrens and Janecke, 1969). Values of $|\psi_{1s}|^2$ from the three sources listed above agree with each other to within an accuracy of about one percent.

The relativistic and nuclear size effects increase the calculated $f_{\rm ec}$ over their nonrelativistic values by, for example, a factor of 1.13 for the ^{37}Ar decay, and by a factor of 1.4 for the ^{71}Ge decay (the effects are only ~1% for ^{7}Be). The size of these corrections can be estimated by comparing the moduli obtained from nonrelativistic wave functions (e.g., Watson and Freeman, 1961; Clementi and Roetti, 1974; see also Mallow *et al.*, 1976) with the moduli found with relativistic wave functions (see references cited above). Equally well, one can estimate the corrections (to an accuracy ~one percent) by using an approximate formula derived by G. Racah (1932) (see also Shirley, 1964).

The ft value given in Table I for the ^{37}Ar decay is eleven percent larger than the value calculated by Bahcall (1964a), mainly because of relativistic and nuclear size effects on the electron wave functions. These effects cause a corresponding decrease in the calculated cross sections for neutrino captures from ^{37}Cl to the ground state of ^{37}Ar. The ft values given here for the ^{7}Be transitions are six percent larger than used by Bahcall (1969); more accurate wave functions (Behrens *et al.*, 1969) were used in the present calculation.

C. Computer code

1. Weighted-average phase space

A computer code has been developed to calculate the neutrino absorption cross sections for all the targets of interest in solar neutrino experiments using the neutrino energy spectra that are appropriate to each of the reactions (1)–(8). In particular, this code calculates the weighted-average dimensionless phase-space factor $\sigma_{\rm av}(Z)\equiv\langle w_e^2 G(Z,w_e)\rangle$ defined by Eq. (10) and more explicitly by

$$\sigma_{\rm av}(Z)=(2\pi\alpha Z)^{-1}\times\int_1^{w_e^{\rm max}} dw_e w_e p_e F(Z,w_e)\phi(q_\nu)\Big/\int_0^{q_\nu^{\rm max}} dq_\nu\phi(q_\nu)\,. \quad (17)$$

Here q_ν is the dimensionless neutrino energy, $q_\nu=q/m_ec^2$, which can be calculated from Eq. (13) for a specified electron energy. For continuum positron emitters

$$\phi(q_\nu)\propto q_\nu^2 w_e[w_e^2-1]^{1/2}F(-Z_{\rm emitter},w_e)\,. \quad (18)$$

Here $w_e\equiv[1+q_{\nu,\rm max}-q_\nu]$. For the special case of ^{8}B decay, in which the final nuclear state is broad, an additional average over $q_\nu^{\rm max}$ (corresponding to an average over the ^{8}Be* profile) is required. Neutrinos produced by electron capture [see reactions (5)–(8)] have spectra $\phi(q_\nu)$ that are effectively delta functions, except for the (usually) small effect of thermal broadening.

2. Fermi functions

Many of the complications that exist in computing accurately the dimensionless factors $\sigma_{\rm av}$ are associated with the Fermi functions $F(Z,w_e)$. Various corrections due to special relativity, finite nuclear size, and electron screening in terrestrial atoms must be included. The usual definition is Konopinski, 1966:

$$F(\pm Z,w_e)=2(1+\gamma_0)(2p_eR)^{-2(1-\gamma_0)}|\Gamma(\gamma_0+i\eta)|^2/(\Gamma(2\gamma_0+1))^2 \quad (19)$$

where $\gamma_0\equiv(1-(\alpha Z)^2)^{1/2}$, and $\eta\equiv\pm\alpha Zc/v$. The positive sign applies in electron and neutrino capture and the negative sign in positron and neutrino emission. The complex gamma function was calculated using a numerical approximation due to Lanczos (1964), which is much more accurate than is required by the present work. The nuclear radius R was taken to be (Elton, 1961)

$$R=[2.908A^{1/3}+6.091A^{-1/3}-5.361A^{-1}]\times10^{-3}(\hbar/m_ec)\,. \quad (20)$$

Actually, the $F(Z,w_e)$ of Eq. (19) was averaged over a uniform sphere of radius R, which results in a small correction,

$$F(Z,w_e)_{\rm av}=[1-(2/3)(1-\gamma_0)]^{-1}F(Z,w_e)\,, \quad (21)$$

caused by the fact that the electron capture can occur anywhere in the nuclear volume.

The prescription described above is a good analytic approximation. An even more accurate (but much more time-consuming) method is to solve numerically the Schrödinger equation with a Coulomb potential representing a finite-size nucleus. The appropriate numerical procedures for obtaining this solution have been described by Rose (1961) and by Bhalla and Rose (1962). Extensive tables of numerical solutions have been presented by Behrens and Janecke (1969). Comparing my results for Fermi functions to theirs, I have derived a correction factor, $(1+C)$, by which the Fermi functions for neutrino capture of Eq. (19) should be multiplied in order to bring them into exact agreement with those of Behrens and Janecke (1969). For energies less than or of order 2 MeV (i.e., for all sources except ^{8}B)

$$C\approx0.01\exp[2.054\ln Z-5.757]\,. \quad (22)$$

The above expression is accurate to a few tenths of a percent from $Z=4$ to $Z=80$. For $Z=18$ (^{37}Cl$\rightarrow{}^{37}$Ar), C is about 1% and for $Z=32$ (^{71}Ga$\rightarrow{}^{71}$GE) C is about 4%. The net correction for ^{8}B neutrinos is of the order of a percent for all the targets considered here and is typically small compared to the other uncertainties involved in calculating the ^{8}B cross sections. I have therefore taken C equal to unity for ^{8}B neutrinos.

The correction due to electron screening in terrestrial atoms was made in the same way as in Bahcall (1966a), using the expression for the screening-induced potential shift derived by Durand (1964). One can easily show, with the formalism developed by Rose (1936), that the effect of screening is small for the positron continuum wave functions in the solar reactions that produce neutrinos, Eq. (1)–(6). The correction to the continuum wave functions can be shown to be of order $V_0/(W-m_ec^2)_{\rm av}$, where V_0 is the value of the positron potential energy at the nucleus. Numerically,

$$V_0 \sim 90\text{eV} Z_{\text{nucleus}} \, [64 T_6/\rho(3+X)]^{-1/2} \tag{23}$$

is the value of the Debye–Huckel potential at the nucleus; T_6 is the temperature in units of $10^6\,°K$; X is the mass fraction of hydrogen; and ρ is the density (in gm/cm^3). In the solar emission processes (1)–(6), the screening correction $V_0/(W - m_e c^2)$ is negligible.

D. Average over ^{8}B endpoint energies

The final state of ^{8}Be that is populated by ^{8}B beta decay is very broad. The most probable maximum neutrino energy is $q_{\max} = 14.02$ MeV [Eq. (2)], but values of $q_{\max}$ as large as 16 MeV, or as small as 7 MeV, are possible. In the previous subsection, we have described the procedure for calculating cross-section factors, $\sigma_{av}(q_{\max})$, for specified values of $q_{\max}$ [cf. Eq. (17)]. In the special case of ^{8}B decay, $\sigma_{av}(q_{\max})$ must be averaged over the probability $P(q_{\max})$ that a given value of $q_{\max}$ occurs [Bahcall, 1964a, 1966b; Kopysov and Kuzmin, 1968]. Let β be the ratio of the weighted-average cross section to the cross section computed at the most probable $q_{\max}$, i.e.,

$$\beta \equiv \frac{\int \sigma_{av}(q_{\max}) P(q_{\max}) dq_{\max}}{\sigma(q_{\max} = 14.02\text{MeV})} . \tag{24}$$

The values of $P(q_{\max})$ that are to be used in Eq. (24) can be determined from a number of different experiments. The most directly relevant data are the energy distributions of α particles from the breakup of ^{8}Be formed by the beta decay of ^{8}B. I have evaluated β for different neutrino targets using four separate sets of numbers for $P(q_{\max})$: (1) values of $P(q_{\max})$ computed from the corrected α-particle spectrum in Fig. 2 of Clark, Treacy, and Tucker (1969) with a *smooth* extrapolation (cf. Alburger, Donovan, and Wilkinson, 1963) to zero intensity on the low-energy side of the observed profile; (2) values from Clark *et al.* (1969) obtained by assuming that the profile is exactly zero below the lowest measured point; (3) values from the experimental $2-\alpha$ spectrum of Farmer and Class (1960) for ^{8}B decay extrapolated smoothly to zero at both low and high energies; and (4) the same as (3) except that the profile was assumed zero outside the measure range. The computed values of β are the same within $\pm 1\%$ for all four assumptions in the cases we have considered. This excellent agreement amond the various ways of calculating β results from the fact that $\sigma_{av}(q_{\max})$ is a rather slowly varying function of $q_{\max}$, that all of the experiments are in reasonable agreement with each other, and that very few decays can occur on any hypothesis with $q_{\max}$'s that correspond to unobservably small or large α energies.

TABLE II. Some Li absorption cross sections as a function of neutrino energy E. Here $1.44E+2 \equiv 144$.

E (MeV)	σ (10^{-46}cm^2)	E (MeV)	σ (10^{-46}cm^2)
1.0	1.4 E+2	7.0	2.9 E+4
1.25	3.5 E+2	9.0	4.9 E+4
1.50	6.8 E+2	10.0	6.1 E+4
2.0	1.6 E+3	15.0	1.4 E+5
3.0	4.3 E+3	20.0	2.6 E+5
4.0	8.4 E+3	25.0	4.1 E+5
5.0	1.4 E+4	30.0	5.9 E+5

IV. RESULTS

A. ^{7}Li

The cross sections for ^{7}Li are given in the first row of Table III.

The thermal average over the energy distribution of the electrons captured in the sun is important for ^{7}Be neutrinos incident on ^{7}Li (Domogatsky, 1969). In this case, the solar neutrinos have exactly threshold energy if one neglects electron screening in the terrestrial target and thermal energy in the sun. In fact, if one includes electron screening in terrestrial atoms, but not thermal energy, then neutrinos from ^{7}Be in the sun are below threshold and the absorption cross section is zero. The appropriate average cross section factor for ^{7}Be neutrinos produced by the capture of electrons from the continuum is [cf. Eq. (17) and Domogatsky, 1969]

TABLE III. Neutrino absorption cross sections. All cross sections are given in units of 10^{-46}cm^2 and are averaged over the appropriate energy spectra for the solar neutrino sources, (1)–(6), and terrestrial ^{51}Cr and ^{65}Zn, (7)–(8). The neutrino threshold energies are given in MeV.

Target	Threshold	$p-p$	$p+e^-+p$	$e^- + {}^8$Be	^{8}B	^{13}N	^{15}O	^{51}Cr	^{65}Zn
^{7}Li	0.862	0	600	9.5 [a]	3.1×10^4	41.7	230	0	225
^{37}Cl	0.814	0	15.6	2.38	1.08×10^4	1.66	6.61	0	6.1
^{51}V	0.751	0	11.0	2.23	$\sim 1 \times 10^4$	1.52	4.94	1.3	4.3
^{55}Mn	0.231	0.282	4.3 [b]	1.72	$\sim 6 \times 10^3$	1.44	2.5 [b]	1.4	1.8 [b]
^{71}Ga	0.236	10.7	157 [b]	64 [b]	$\sim 3 \times 10^3$ [b]	53 [b]	92 [b]	52 [b]	67 [b]
^{81}Br [c]	0.459	0	31.9	10.6	$\sim 1 \times 10^3$	8.6	17.3	8.1	13
^{87}Rb	0.115	22.9	182 [b]	81 [b]	$\sim 3 \times 10^3$ [b]	70 [b]	112 [b]	68 [b]	89 [b]
^{115}In [d]	0.120	8.76E+1	6.4E+2	2.9E+2	9E+3	2.5E+2	4.0E+2	2.5E+2	3.2E+2
^{205}Tl [e]	0.062	7.2E+1	3.4E+2	1.8E+2	~3E+3	1.6E+2	2.3E+2	1.6E+2	1.85E+2

[a] Evaluated for $1.5 \times 10^7\,°K$ [see Eq. (27)].
[b] Assumes $Q = 1.0$ [cf. Eqs. (41)–(44)].
[c] Assumes a nominal $\sigma_0 = 2.18 \times 10^{-46}$cm^2 (see text) and includes only transitions to the $1/2^-$ isomeric state.
[d] Assumes $\sigma_0 = 24.12 \times 10^{-46}$cm^2 and includes only the $9/2^+$ to $7/2^+$ transition discussed in the text [$Q = 1.0$, Eq. (45)].
[e] Assumes a nominal $\sigma_0 = 4.95 \times 10^{-46}$cm^2 and $Q = 1.0$ [see Eq. (46)].

$$\sigma_{\rm av;cont} = \frac{\int_0^\infty\int_0^\infty dE_{\rm Be}\, dE_e E_{\rm Be}^{1/2} E_e^{1/2} e^{\frac{-(E_e+E_{Be})}{kT}} \int_{q_{\rm min}}^{q_{\rm max}} dq\, v_{\rm rel} \frac{d\sigma_{ec}}{dq}\sigma_{\rm abs}(q)}{\int_0^\infty\int_0^\infty dE_{\rm Be}\, dE_e E_{\rm Be}^{1/2} E_e^{1/2} e^{\frac{-(E_e+E_{be})}{kT}} v_{\rm relative}\,\sigma_{\rm ec}}, \qquad (25)$$

where the absorption cross section $\sigma_{\rm abs}$ contains all the corrections discussed in Sec. III.C. The differential cross section for the electron capture process with the production of a neutrino of energy q, $d\sigma/dq_{\rm ec}$, must be computed by taking account of the thermal motion of both the ^{7}Be ions and the electrons. Making a nonrelativistic approximation for the thermal motion and neglecting terms of the order of the square root of the ratio of the masses, $(m_e/M_{\rm Be})^{1/2} \ll 1$, I obtain $\sigma_{\rm av;cont} = 0.517 \pm 0.015$, for all temperatures in the range 10×10^6 °K to 20×10^6 °K.

If the neutrinos are produced by the capture of electrons originally *bound* to Be ions, (see Iben, Kalata, and Schwartz, 1967; Bahcall and Moeller, 1969) the appropriate cross section should be averaged over the initial thermal motion of the bound system (see Domogatsky, 1969). In this case, the thermal motion must compensate for the binding energy of the electron as well as for the kinematic effects that also enter the continuum calculation described above. One finds

$$\sigma_{\rm av;bound} \approx \pi^{-1/2}\sigma(q_{\substack{\rm threshold\\ \rm absorption}}) \times \int_0^\infty dx\, x^{1/2} e^{-x} \int_{-1}^{+1} \theta(q(\mu) - q_{\rm threshold})\, dx, \qquad (26a)$$

where $\theta(y) = 1$ if $y \geq 0$ and $\theta(y) = 0$ if $y < 0$, μ is the cosine of the angle between the initial ion's direction of motion and the momentum of the neutrino, and x is the initial kinetic energy divided by kT. The cross section factor for the capture of neutrinos produced by Be IV ions can be found by performing the indicated integrations in Eq. (26a). I find

$$\sigma_{\rm av;bound} \cong 0.52[1 - (0.84\, T_7^{-1/2})(1 - 0.19 T_7^{-1})], \quad \text{where } T_7 \equiv (T/10^7\,°K). \qquad (26b)$$

The weighted-average cross section can be calculated using the average ratio (0.2) of bound-to-continuum electron capture estimated by Bahcall and Moeller (1969). The appropriate average over all quantities, including the mode of capture (bound or continuum) and the branching ratios in Eq. (6), is

$$\sigma(^7\text{Be}) = 8.8 \times 10^{-46}\text{cm}^2[1 + 0.2\{1 - (0.84 T_7^{-1/2}) \times (1 - 0.19 T_7^{-1})\}]. \qquad (27)$$

Equation (27) should be used with an expression for the total electron capture rate, such as Eq. (12) of Bahcall and Moeller (1969), that takes account of all neutrinos that are produced both by bound and continuum capture in the sun.

All of the cross sections given in Table III take account of transitions, when energetically possible, to the first excited state ($J = \frac{1}{2}^-$) of ^{7}Be at an excitation energy of 0.43 MeV. Transitions from the ground state of ^{7}Li to either the ground or first excited state of ^{7}Be are superallowed (cf. Table I). The relative contributions of transitions to the first excited state of ^{7}Be are about five percent for the ^{15}O and ^{65}Zn spectra, seven percent for the pep neutrinos, and about twenty-one percent for the average ^{8}B spectrum (^{13}N neutrinos do not have sufficient energy to populate the excited state). An average over possible endpoint energies of the ^{8}B spectrum yields a correction factor of $\beta = 0.86 \pm 0.01$ ($1-\sigma$ spread; see Eq. 24 and the following discussion for a definition of β and the cases considered) for transitions to both the ground and first excited states. The lowest state in ^{7}Be, beside the ground and first excited state, to which an allowed transition is possible is the $J = \frac{5}{2}^-$ state at 6.7 MeV; only ^{8}B neutrinos (among the sources listed in Table III) have enough energy to excite this state. The fractional contribution of this state to the total capture rate due to ^{8}B is only

$$\frac{\sigma(6.7\text{ MeV})}{\sigma(\text{ground state}) + \sigma(0.43\text{ MeV})} = 0.002[10^5\text{sec}/(ft_{1/2})_{6.7\,\text{MeV}}]. \qquad (28)$$

I conclude from Eq. (28) that transitions to excited states at 6.7 MeV and above are negligible. In radiochemical experiments, they would be unobservable anyway since the highly excited levels of ^{7}Be are particle unstable and would not lead to detections in an experiment designed to observe ^{7}Be electron captures.

It is useful to have available cross sections as a function of energy for other applications, for example, for determining the sensitivity of a ^{7}Li detector to neutrinos from a supernova explosion. Table II gives cross sections for neutrinos with energies between 1 MeV and 30 MeV. For neutrino energies E between 5 and 30 MeV,

$$\sigma(E) \approx 5.0 \times 10^{-44}\text{cm}^2(E/1\text{MeV})^{2.09}. \qquad (29)$$

Neutrino–antineutrino oscillations can be studied with terrestrial experiments that use ^{7}Li as a detector and reactor antineutrinos as a source [Domogatsky, 1976; Bahcall and Primakoff, 1978]. The relevant cross section for interpreting the reactor experiments can be found by averaging the cross sections $\sigma_\nu(E)$ from Table II and Eq. (29) over the spectrum of antineutrino energies from a reactor (Avignone, 1970). I find

$$\langle\sigma(\bar\nu = \nu)\rangle_{\rm reactor} = 2.9 \times 10^{-43}\text{cm}^2. \qquad (30a)$$

The appropriate average interaction energy that enters discussions of neutrino oscillations is

$$\langle 1/E^2\rangle \equiv \int dE\,\phi_B\sigma(E)E^{-2} \Big/ \int dE\,\phi_B\sigma(E) \approx (2.9\text{ MeV})^{-2}. \qquad (30b)$$

The result given in Eq. (30a) for the average cross section is a factor of 2.4 times larger than the value quoted by Domogatsky (1976).

^{7}Li was first proposed as a solar neutrino detector by

Bahcall (1964c; see also Reines and Woods, 1965; Kuzmin and Zatsepin, 1966; Bahcall, 1969). Various authors have evaluated the experimental possibilities (see especially Pomanskii, 1966; Davis, 1969; Rowley, 1974; and references in the review articles listed in the introduction). The ^{65}Zn cross sections are calculated here for the first time. The cross section for ^{7}Be neutrinos is a factor of two lower than previously given by Domogatsky (1969) due to an error in his derivation of the explicit form of Eq. (25). The factor of two corresponds to the fact that only the neutrinos, approximately one half of the total, that are emitted in the forward direction with respect to the continuum electron's momentum are above threshold. The other cross sections given in Table III are in generally good agreement (typically 10%) with the earlier results of Bahcall (1964c, 1969). The present values are all slightly smaller than previous values because of the six percent larger *ft* value used here (cf. Sec. III. B), the inclusion of the bracketed statistical factor of Eq. (11) for the relatively infrequent transitions to the excited states at 0.43 MeV, and the average over the ^{8}B endpoint energies [Eq. (24)]. These refinements are numerically important only for the ^{8}B cross section, which is thirty percent smaller than the previously used value.

B. ^{37}Cl

The cross sections for solar neutrino absorption by ^{37}Cl are given in the second row of Table III. They are, with only minor changes (~1%), identical with those given recently by Bahcall (1977).

1. Ground-state transitions

Only ground state to ground state transitions are energetically possible for all of the important solar neutrino sources with the exception of ^{8}B. The first excited state in ^{37}Ar is at 1.41 MeV. Since the threshold for ground-state neutrino absorptions is 0.814 MeV, only neutrinos with energies greater than 2.12 MeV can cause transitions to excited states in ^{37}Ar. Thus, of the eight neutrino sources listed in Sec. II, only the ^{8}B neutrinos are energetic enough to produce excited-state transitions.

The only previously published calculation of the ground state to ground state solar neutrino cross sections is that of Bahcall (1964a, b). The present results are in good agreement with the previous values when account is taken of the eleven percent larger *ft* values found here (see Sec. III. B) for the laboratory electron capture using more accurate relativistic atomic wave functions. This larger ground-state *ft* value reduces the expected cross sections by a corresponding factor. The *pep*, ^{7}Be, and ^{15}O cross sections given in Table III for ^{37}Cl are approximately ten to fifteen percent smaller than the previously quoted values. The relatively unimportant ^{13}N cross section is decreased, in the present calculation, by a greater amount, twenty-one percent. This larger decrease in the ^{13}N cross section is the result of a combination of small effects, including the improved *ft* value, a round-off error, and a smaller estimated end point neutrino energy, all of which affected the cross section in the same direction.

Alvarez (1973) estimated the ^{65}Zn cross section in his original discussion of the possibilities for a laboratory calibration of the ^{37}Cl experiment. The present detailed calculation is in good agreement with (i.e., 15 percent lower than) Alvarez's estimate.

2. The ^{8}B cross section

Table IV summarizes the results of seven different phenomelogical calculations of the ^{8}B cross section. All of the different methods of calculating the cross section lead to values between 1.05×10^{-42}cm^2 and 1.11×10^{-42}cm^2.

Most of the cross section for ^{8}B neutrinos on ^{37}Cl is due to transitions to excited states in ^{37}Ar, especially the analog state at 5 MeV (Bahcall 1964a, b). The calculation of the cross sections to the individual excited states can be made in an essentially empirical fashion (see Bahcall, 1966b; Bahcall and Barnes, 1964) using the branching ratios measured in the beta decay of the isotopic analog to neutrino absorption by $^{37}_{17}$Cl, i.e., in the decay of ^{37}Ca:

$$^{37}_{20}\mathrm{Ca} \rightarrow {^{37}_{19}K} + e^{+} + \nu . \qquad (31)$$

Sextro, Gough, and Cerny (1974) have recently published a high-resolution study of the ^{37}Ca beta decay, reaction (31), using delayed protons from the populated excited levels in ^{37}K. Their results for the various *ft* values can be translated to the case of neutrino absorption by ^{37}Cl and are given in Table V (which is adapted from Table VI of Sextro *et al.*, 1974). The actual values of the energies of the lowest excited states in ^{37}Ar are taken from Endt and van der Leun (1973) in the region in which the density of levels is sufficiently low ($E_x \lesssim 3.5$ MeV) that an accurate correspondence can be es-

TABLE IV. Various estimates of the ^{8}B solar neutrino cross section on ^{37}Cl.

Model	^{37}Cl data	Assumption	σ (10^{-42}cm^2)
A	Sextro *et al.* (1974)	$\log ft = 3.30$ analog	1.05
B	Sextro *et al.* (1974)	$\log ft = 3.256$ analog (a minimum)	1.11
C	Sextro *et al.* (1974)	2.1% decay to $E_x = 6.5$ MeV	1.10
D	Poskanzer *et al.* (1966)	18% decay to the 1.41 MeV state	1.05
E	Poskanzer *et al.* (1966)	18% decay to the 2.80 MeV state	1.09
F	Poskanzer *et al.* (1966)	λ(1.4 MeV)/λ(2.8 MeV) = 2.4 [a]	1.06
G	Poskanzer *et al.* (1966)	λ(1.4 MeV)/λ(2.8 MeV) = 0.18 [b]	1.08

[a] Haxton and Donnelly (1977).
[b] Lanford and Wildenthal (1972).

TABLE V. Individual ft values and cross sections for ^{8}B neutrinos incident on ^{37}Cl. The ft values are from measurements by Sextro, Gough, and Cerny (1974) of ^{37}Ca decay ratios.

Excitation energy (MeV)	$(10^5 s/ft)$ [a]	σ_{av} [Eq. (17)]	β [b]	σ $(10^{-46} cm^2)$
0.0	0.794	397	0.872	596
1.41	0.826	272	0.857	418
2.80	1.351	176.5	0.833	429
3.53	1.471	135	0.820	354
3.84	1.919	120	0.809	405
4.40	0.752	96	0.803	126
4.50	1.449	92	0.800	231
4.66	0.909	86	0.798	135
4.95	1.923	75.5	0.790	249
4.98	50.0	74.5	0.790	6,375
5.12	6.711	70	0.788	799
5.32	1.408	64	0.785	153
5.45	0.769	60	0.782	78
6.02	1.667	45	0.773	126

[a] Taken from Sextro, Gough, and Cerny (1974).
[b] See Eqs. (24) and (34).

tablished between the levels in ^{37}Ar and the levels in ^{37}K that are populated by ^{37}Ca decay. The energies of the isotopic spin $\frac{3}{2}$ states, at 4.98 MeV and 6.65 MeV, have been taken from the accurate measurements by Parker, Howard, and Goosman (1975). The remaining excitation energies are those measured for the ^{37}Ca decays by Sextro *et al.* (1975). The energy sensitivity of the calculated cross sections is sufficiently small [see, e.g., Eq. (33) below] that the remaining uncertainties in the excitation energies do not contribute significantly to estimates of the total uncertainty in the ^{8}B cross section.

The measurements of Sextro *et al.* (1974) permit a basically empirical specification of the matrix elements in the neutrino capture reaction, $\nu_e + {}^{37}_{17}\mathrm{Cl} \to e^- + {}^{37}_{18}\mathrm{A}$, which is the inverse of the ^{37}Ca decay. Their results represent an important improvement over the previously standard measurements of Poskanzer, McPherson, Esterlund, and Reeder (1966) in three respects: (1) the detection of proton decay from the lowest unbound level capable of being fed by allowed beta decay; (2) the resolution of the decay peak near the 5 MeV analog level into three parts; and (3) the use of a more accurate determination of the ^{37}Ca mass excess (−13.161 MeV, Butler *et al.*, 1968; Benenson *et al.*, 1973).

Only one assumption must be made in order to use the Sextro *et al.* (1974) measurements to calculate neutrino capture on ^{37}Cl: the specification of the ft value for the superallowed transition to the analog 5 MeV level. Various estimates, both theoretical and empirical (see Bahcall, 1964a, 1966b; Hardy and Margolis, 1965; Engelbertink and Brussard, 1966; Lanford and Wildenthal, 1972; Haxton and Donnelly, 1977), have shown that the ft value for the analog transition is restricted to a relatively narrow range

$$3.23 \leq (\log ft)_{\rm analog} \leq 3.30\,. \qquad (32)$$

The reason the uncertainty is so small in Eq. (32) is that the Fermi matrix element can be calculated accurately from isotopic spin considerations and is large, $\langle 1\rangle^2 = 3$, while the Gamow–Teller matrix element, estimated by any method, is always relatively small, $\langle\sigma\rangle^2/3 \leq 0.1$. We shall see below (Eq. 36) that the value of $(\log ft)_{\rm analog}$ can be restricted even further by the measurements of Sextro *et al.* (1974).

I follow Sextro *et al.* (1974) in adopting $\log ft_{\rm analog} = 3.3$ for the first model listed in Table IV. In what follows, I estimate the sensitivity of the calculated total cross section to this assumption by varying $\log(ft)_{\rm analog}$. The weighted-average phase-space factors, σ_{av}, have been calculated by averaging over the ^{8}B neutrino spectrum with a nominal end point energy of 14.02 MeV, as described in Eq. (17). The results are given in column three of Table V. The results are not sensitive to the precise values of the excitation energies of the final nuclear states. For example, in the vicinity of the 5 MeV superallowed transition

$$\sigma^{-1}\frac{d\sigma}{dE_x}\bigg|_{E_x\approx 5\ \rm MeV} \cong -0.0045/(10\ \rm keV)\,. \qquad (33)$$

The average over the shape of the state of ^{8}Be that is populated by the ^{8}B decay has been carried out for the four separate probability distributions described in Sec. III. D; the different distributions lead to reduction factors, β (see Eq. 24), that are the same within a $1-\sigma$ dispersion of approximately one percent. The appropriate values of β are given in column 4 of Table V. The following interpolation formula can be used to estimate β to an accuracy of the order of one percent:

$$\beta = 0.872 - 0.0165\ (\text{excitation energy}/1\ \text{MeV})\,. \qquad (34)$$

Equation (34) is useful for calculations of individual cross sections that are made (see below) with a variety of different assumptions.

The individual cross sections are listed in the last column of Table V. They have been computed from the relation [cf. Eqs. (10), (11), (17) and (24)].

$$\sigma(^{37}\mathrm{Cl}) = 2.1708\times 10^{-46}\mathrm{cm}^2(10^5 s/ft_{^{37}\mathrm{Cl}})[\beta\sigma_{av}]\,. \qquad (35)$$

The total cross section computed from the sum of the individual cross sections in Table V is $\sigma = 1.05\times 10^{-42}$ cm^2; this result is listed in the first row of Table IV under the label model A.

Models B and C of Table IV are also based on the experimental results of Sextro *et al.* (1974), but involve different assumptions. In model B, the $\log ft$ value for the analog transition is allowed to be as small as possible without conflicting with the decay *ratios* measured by Sextro *et al.* (1974) and the known ground state to ground state ft value. One finds

$$(\log ft)_{\rm analog}\ (\text{Sextro } et\ al.\ 1974) \geq 3.256\,. \qquad (36)$$

Equation (36), when combined with the previous limits given in (32), yield a rather accurate determination of the superallowed ft value, i.e., 3.28 ± 0.02. The ^{8}B cross section that is computed by normalizing the measured decay ratios to the minimum allowed analog ft value [Eq. (36)] is 1.11×10^{-42} cm^2 and is listed under Model B in Table IV. The two models, A and B, are opposite extremes since the normalizing ft value is assumed to be equal to its maximum or minimum value, respectively, in the two cases. No decays were observed by Sextro *et al.* (1974) to excited state above 6.02 MeV. Model

TABLE VI. Individual *ft* values and ^{8}B cross sections obtained using the measured decay ratios of Poskanzer, McPherson, Esterlund, and Reeder (1966) for the ^{37}Ca decay.

Excitation energy (MeV)	$(10^5 s/ft)$ [a]	σ_{av} [Eq. (17)]	β [b]	σ $(10^{-42} cm^2)$
0.0	0.79	397	0.872	596
1.41	···	272	0.857	
2.80	···	176.5	0.833	
3.53	1.53	135	0.820	369
3.84	1.34	120	0.809	282
4.12	0.815	107.5	0.804	153
4.40	2.00	96	0.803	334
4.66	2.85	86	0.798	424
4.83	<2.8	80	0.793	<384
5.02	54.5	73	0.7895	6,828
6.03	2.73	45	0.773	204
6.32	1.845	38	0.768	117
6.54	2.35	34	0.764	131
6.65	3.08	31.5	0.750	159

[a] Decay ratios taken from Poskanzer *et al.*, (1966). The *f* values were computed using the ^{37}Ca mass excess quoted by Sextro *et al.*, (1974).

[b] See Eqs. (24) and (34).

C was computed by assuming that there were actually some very weak decays in this region. To be specific, I assumed a total decay rate above 6.02 MeV that was in the same proportion to the superallowed transition as was found by Poskanzer *et al.* (1966). This results in an estimated decay rate of 2.1% to states at about 6.5 MeV, with a corresponding reduction in the fractional decay rate to the 1.4 MeV excited state. The cross section computed on the basis of this assumption is 1.10×10^{-42} cm^2.

Models D, E, F, and G were computed using the decay ratios measured by Poskanzer, McPherson, Esterlund, and Reeder (1966) and the more accurate mass estimate given by Sextro *et al.* (1974). Table VI contains the results for the corrected individual *ft* values and cross sections that were obtained with the aid of Poskanzer *et al.* (1966) measured decay ratios.

The decay rates to the two lowest excited states, at 1.4 and 2.8 MeV, are not specified by the data of Poskanzer *et al.* (1966). Models D and E represent the two extreme assumptions about the *ft* values of these states. In model D, all of the otherwise unaccounted for decay rate, 18%, is assigned to the 1.4 MeV excited state. In model E, all of the unaccounted for decay rate is assigned to the 2.8 MeV state. The ^{8}B cross sections computed in these two different ways are, respectively, $\sigma = 1.05 \times 10^{-42}$ cm^2 and $\sigma = 1.09 \times 10^{-42}$ cm^2. In model F, I have assumed that the decay to the 1.4 MeV state is 2.4 times more probable than the decay to the 2.8 MeV state; the factor of 2.4 is the ratio obtained by Haxton and Donnelly (1977) from extensive calculations with nuclear models of the mass 37 system. The cross section calculated with the aid of the Haxton–Donnelly ratio of decay rates is 1.06×10^{-42} cm^2 and is listed as model F in Table IV. Lanford and Wildenthal (1972) calculated *ft* values for the transitions from the ^{37}Cl ground state to the lowest two excited states in ^{37}K using detailed shell-model calculations; their results imply that decay rate to the 1.4 MeV state is only 0.18 of the rate to the 2.8 MeV state. The cross section computed with the Lanford–Wildenthal model is 1.08×10^{-42}cm^2 and is listed as model G in Table IV. The ratio of decays to the 1.4 MeV state versus those to the 2.8 MeV state was found to be ≈ 1.3 in the ^{37}Ca experiment of Sextro *et al.* (1974), compared to the theoretical values of 2.4 (Haxton et al., 1977) and 0.18 (Lanford and Wildenthal, 1972.)

All of the nuclear models and both sets of experimental data on the ^{37}Ca yield essentially the same ^{8}B cross section. I adopt

$$\sigma(^8B) = (1.08 \pm 0.1) \times 10^{-42} \text{ cm}^2 ; \quad (37)$$

the estimate of the uncertainty may be conservative (cf. Table IV).

There have been three previous calculations of the ^{8}B solar neutrino cross section on ^{37}Cl that have made extensive use of detailed experimental information on the matrix elements in the ^{37}Ca decay. These three calculations all yielded cross sections that were about twenty-five percent higher than the value given in Eq. (37), with a remarkably small dispersion among the different estimates. The separate calculations yielded the following values: 1.35×10^{-42}cm^2 (Bahcall, 1966b); 1.31×10^{-42}cm^2 (Sextro, Gough, and Cerny, 1974); and 1.27×10^{-42} cm^2 (Haxton and Donnelly, 1977). (There have also been several other estimates that have relied more heavily on specific theoretical nuclear models; these include calculations by Bahcall, 1964a; Domogatsky, Garvin, and Eramajn, 1965; Engelbertink and Brussard, 1966; and Lanford and Wildenthal, 1972. These more theoretical calculations were less precise, but were in rough agreement with the empirical estimates.)

The difference between the present and previous calculation of Bahcall (1966b) is due to two factors that contributed about equally and in the same direction: (1) more accurate *f* values were used in the present work, largely due to an improved ^{37}Ca mass excess (see discussion above of the Poskanzer *et al.*, 1966, results) and (2) a normalizing error in the earlier calculation of β has been corrected. There is not sufficient detail given in the paper of Sextro *et al.* (1974) to allow one to trace the discrepancy between their quoted cross section and the value derived here. Haxton and Donnelly (1977) estimated $\sigma(^8B) = (1.27 \pm 0.01) \times 10^{-42}$ cm^2 for the conditions listed as model F in Table IV. The main reason Haxton *et al.* (1977) obtained a larger cross section is that they used the *ft* values directly from the paper of Poskanzer (1966) which has a less precise estimate for the ^{37}Ca mass excess; this results in an approximately fifteen percent overestimate of $\sigma(^8B)$. There is also a numerical approximation in the work of Haxton *et al.* (1977) which further increased the estimated cross section. They used a nonrelativistic approximation to the Fermi function (see Eq. 2.50 of Haxton, 1976), which in addition neglects nuclear size and electron screening effects. The nonrelativistic approximation used by Haxton *et al.* (1977) causes a six percent error in $\sigma(^8B)$.

3. Cross sections as a function of energy

Absorption cross sections as a function of incident neutrino energy are useful in interpreting reactor searches for $\bar{\nu}_e + {}^{37}\mathrm{Cl} \to {}^{37}\mathrm{Ar} + e^-$ (Davis, 1955, 1958; Domogatsky, 1976; Bahcall and Primakoff, 1978), in setting upper limits on the local neutrino energy density as a function of energy (Bahcall, 1977), and in estimating the expected counting rate from supernova explosions (see Bahcall, 1977). The absorption cross sections can be calculated from the data of Sextro *et al.* (1974) (using columns 1 and 2 of Table V) and the computer code for neutrino lines described in Sec. III. C. The results are summarized in Table VII. The following interpolation formulae are accurate to about fifteen percent for the indicated energy range.

$$\sigma(E) \approx 5.6(E/1\ \mathrm{MeV})^{2.85} \times 10^{-46}\mathrm{cm}^2,\ 1 \leq (E/1\ \mathrm{MeV}) \leq 5, \tag{38a}$$

$$\sigma(E) \approx 4.6(E/1\ \mathrm{MeV})^{3.7} \times 10^{-46}\mathrm{cm}^2,\ 8 \leq (E/1\ \mathrm{MeV}) \leq 15. \tag{38b}$$

The large increase in the cross section between 5 MeV and 8 MeV is caused by the analog state.

The relevant cross section for interpreting reactor experiments can be found by averaging $\sigma_\nu(E)$ [cf. Eqs. (38)] over the anti-neutrino spectrum from a reactor (Avignone, 1970). The average cross section calculated in this way is

$$\langle\sigma(\bar{\nu} = \nu)\rangle_{\mathrm{reactor}} = 1.3 \times 10^{-44}\ \mathrm{cm}^2. \tag{39a}$$

The appropriate average interaction energy that enters discussions of neutrino oscillations is

$$\bar{E} \equiv \left[\int dE \phi_E \sigma_E E^{-2} / \int dE \phi_E \sigma_E\right]^{-1/2} = 3.5\ \mathrm{MeV}. \tag{39b}$$

The result given in Eq. (39a) is about a factor of three larger than the value quoted by Domogatsky, (1976).

C. ^{51}V

Total absorption cross sections for neutrinos incident on ^{51}V are given in the third row of Table III.

Allowed transitions to excited states in ^{51}Cr are energetically possible only for ^{8}B neutrinos, for which they are dominant. The ground state to ground state capture rate is $\sigma_{gs} = 3.7 \times 10^{-44}$ cm^2 for ^{8}B neutrinos. The analog state in ^{51}Cr corresponding to the ground state of ^{51}V lies at an excitation energy of 6.6 MeV (Courtney and Fox, 1975). The superallowed capture rate to this state is (cf. Bahcall, 1964a):

$$\sigma_{sa} = 5.4 \times 10^{-43} M\ \mathrm{cm}^2, \tag{40a}$$

where

$$M \equiv [(\langle 1\rangle^2 + (C_A/C_V)^2 \langle\sigma\rangle^2)/5], \tag{40b}$$

and $\langle 1\rangle$, $\langle\sigma\rangle$ are the usual reduced beta-decay matrix elements (Konopinskii, 1966). The quantity M is equal to one in the limit, suggested by a single-particle calculation, that $\langle\sigma\rangle^2$ is small compared to $\langle 1\rangle^2 (=5)$. The total ^{8}B cross section may be estimated crudely by the following argument. About three-quarters of the total cross section for ^{8}B neutrinos on ^{37}Cl arises from transitions to the ground state (6 percent) and the analog state (68 percent). If one assumes that the same ratio exists between total versus ground-plus-superallowed transitions for $^{51}\mathrm{V} \to {}^{51}\mathrm{Cr}$ as for $^{37}\mathrm{Cl} \to {}^{37}\mathrm{Ar}$, then one obtains $\sigma_{\mathrm{total}} \approx 0.75 \times 10^{-42} M$ cm^2. [This result, as well as Eq. (40a), was obtained using approximate values from Eq. (34) for the correction due to the average over the ^{8}B endpoint energy.] The cross section given in Table III was obtained by rounding-off the above expression for σ_{total}.

The ^{51}V cross sections presented here are in good agreement with the less precise values given by Bahcall (1967).

TABLE VII. Some ^{37}Cl absorption cross sections as a function of neutrino energy E.

E (MeV)	σ (10^{-46}cm^2)	E (MeV)	σ (10^{-46}cm^2)
0.85	2.4	4.0	2.5E+2
0.9	3.25	5.0	5.4E+2
1.0	5.0	6.0	1.5E+3
1.1	6.95	7.0	4.6E+3
1.3	11.6	8.0	1.0E+4
1.5	17.1	9.0	1.7E+4
2.0	35.4	10.0	2.7E+4
2.5	67	15.0	1.1E+5
3.0	110	25.0	4.4E+5

D. ^{55}Mn

The cross sections for neutrino capture by ^{55}Mn are given in the fourth row of Table III. The total ^{8}B cross section, including the superallowed transition to the 7.6 MeV ($T = \frac{5}{2}$) analog state in ^{55}Fe, has been estimated in the same manner as described previously (Sec. IV. C) for the $^{51}\mathrm{V} \to {}^{51}\mathrm{Cr}$ reaction.

The $J = \frac{5}{2}^-$ excited state of ^{55}Fe, which lies 0.94 MeV above the ground state, may contribute a significant amount to the *pep*, ^{15}O, and ^{65}Zn cross sections. Letting

$$\sigma_{\mathrm{total}} \equiv \sigma_{gs}\, Q, \tag{41}$$

where σ_{gs} is the cross section to the ground state of ^{55}Fe, one finds

$$Q(pep) = 1 + 2.66(10^5\mathrm{s}/ft)_{5/2^- \to 5/2^-}, \tag{42a}$$

$$Q(^{15}\mathrm{O}) = 1 + 1.38(10^5\mathrm{s}/ft)_{5/2^- \to 5/2^-}, \tag{42b}$$

and

$$Q(^{65}\mathrm{Zn}) = 1 + 2.18(10^5\mathrm{s}/ft)_{5/2^- \to 5/2^-}. \tag{42c}$$

It would be difficult to make an accurate calculation of the relevant ft value for the transition from the first $J = \frac{5}{2}^-$ excited state of ^{55}Fe to the ground state of ^{55}Mn.

A reasonable estimate for the maximum likely effect of the $\frac{5}{2}^-$ excited state can be obtained by setting $(ft)_{5/2^- \to 5/2^-}$ equal to 10^5s in Eq. (42). A somewhat similar transition occurs between the $\frac{5}{2}^-$ ground state of ^{57}Mn and the $\frac{5}{2}^-$ excited state of ^{57}Fe (excitation energy = 0.14 MeV). In this case the measured ft value is 4×10^5s (Auble, 1977). We denote by Q_0 the values of Q calculated from Eqs. (42) with $ft = 10^5$s. For the solar models considered in Sec. V (see Table X), the differences between the capture rates predicted with $Q = Q_0$ and with $Q = 1.0$ are less than or equal to ten percent except for the CNO model (in which case the rate is increased by a factor of 1.9).

The ^{8}B capture cross section is, with the available experimental information, rather uncertain (perhaps by 50 percent). In the standard model, about half the predicted capture rate comes from ^{8}B neutrinos although for all the other models ^{8}B contributes less than or of order of twenty-five percent to the total capture rate.

The results given here are in good agreement with the estimates first made by Bahcall (1967) when the statistical factor of equation (11) $[(2I'+1)/(2I+1)=0.67]$ is included. The present results (for $Q=1.0$) are also in excellent agreement with the calculations of Domogatsky (1977) (who, however, used somewhat less precise Fermi functions and *ft* values). Domogatsky did not discuss the effect of the 0.9 MeV excited state of ^{55}Fe.

E. ^{71}Ge

The ground state to ground state cross sections for neutrino absorption by ^{71}Ge are given in the fifth column of Table III. The most important neutrino source for this detector is expected to be the $p-p$ reaction. [The fractional change of the $p-p$ cross section with threshold energy is approximately $\Delta\sigma/\sigma\Delta E_{\rm th}\approx -0.0078$ per keV .]

There are two excited states of ^{71}Ge that must be considered in calculating the total cross sections for all the neutrino sources we discuss. These states are the $J=\frac{5}{2}^-$ level at an excitation energy of 0.175 MeV in ^{71}Ge and the $J=\frac{3}{2}^-$ level at 0.50 MeV. In addition, there is another $J=\frac{3}{2}^-$ level at 0.71 MeV that might be significant for some energetic sources.

It is useful to define, via Eq. (41), a correction factor, Q, by which the ground state to ground state cross sections must be multiplied to obtain the total neutrino absorption cross sections. With no loss of generality, one can write

$$Q=\left[1.+0.069(10^6{\rm s}/ft)_{\rm ex=0.17}\left(\frac{\sigma_{\rm av}({\rm ex}=0.17)}{\sigma_{\rm av}({\rm ex}=0.0)}\right)+0.46(10^5{\rm s}/ft)_{\rm ex=0.5}\left(\frac{\sigma_{\rm av}({\rm ex}=0.5)}{\sigma_{\rm av}({\rm ex}=0.0)}\right)+0.46(10^5{\rm s}/ft)_{\rm ex=0.7}\left(\frac{\sigma_{\rm av}({\rm ex}=0.7)}{\sigma_{\rm av}({\rm ex}=0.0)}\right)\right], \quad (43)$$

where ex is the excitation energy in MeV of the final state in ^{71}Ge, and the values of $\sigma_{\rm av}$ are the appropriate kinematic cross sections, averaged over neutrino spectra, that are defined by Eq. (17). The required numerical values for the cross section factors, $\sigma_{\rm av}$, are listed in Table VIII for all the neutrino sources of interest here.

Fortunately, a number of measured *ft* values are available for decays that are related to the excited-state transition rates that enter Eq. (43). These experimentally studied transitions have the same crude shell-model description as the transitions of interest and occur among neighboring nuclei in the periodic table. The data on these transitions are summarized in various issues of the Nuclear Data Tables (Auble, 1975, 1976, 1977).

There are seven well-studied ground state to ground state transitions that are analogous to the $\frac{3}{2}^-\to\frac{5}{2}^-$ transition (crudely: $2p_{3/2}\to 1f_{5/2}$) that populates the 0.17 MeV excited state of ^{71} Ge. These are (with their measured log *ft* values in parenthesis): $^{69}_{32}$Ge$_{37}\to^{69}_{31}$Ga$_{38}$(6.3); $^{67}_{31}$Ga$_{36}\to^{67}_{30}$Zn$_{37}$($\geq$ 6.4); $^{67}_{29}$Cu$_{38}\to^{67}_{30}$Zn$_{37}$(6.3); $^{65}_{28}$Ni$_{37}\to^{65}_{29}$Cu$_{36}$(6.6); $^{65}_{30}$Zn$_{35}\to^{65}_{29}$Cu$_{36}$(7.5); $^{65}_{31}$Ga$_{34}\to^{65}_{30}$Zn$_{35}$($\geq$ 5.9); and $^{65}_{32}$Ge$_{33}\to^{65}_{31}$Ga$_{34}$($\geq$ 6.0). In addition, there is a very similar transition to the ^{71}Ga$\to^{71}$Ge transition in which we are interested that occurs between the $J=\frac{3}{2}^-$ ground state of $^{69}_{29}$Cu$_{40}$ and the first $\frac{5}{2}^-$ excited state of $^{69}_{30}$Zn$_{39}$ at ex = 531 keV. This transition has a log *ft* value of 6.4. The transition between the $\frac{3}{2}^-$ ground state of $^{61}_{29}$Cu$_{32}$ and the first $\frac{5}{2}^-$ excited state of $^{61}_{28}$Ni$_{33}$ at ex = 0.067 MeV has a log *ft* value of 6.2.

The nine transitions mentioned above are all consistent with the factor $(10^6{\rm s}/ft)_{\rm ex=0.17MeV}$ that occurs in Eq. (43) being less than unity.

There are eight well-studied $\frac{3}{2}^-\to\frac{3}{2}^-$ transitions that are at least somewhat similar to the $\frac{3}{2}^-\to\frac{3}{2}^-$ transitions from ^{71}Ga to ^{71}Ge. They are: $^{69}_{29}$Cu$_{40}\to^{69}_{30}$Zn$_{39}$(5.6); $^{67}_{31}$Ga$_{36}\to^{67}_{30}$Zn$_{37}$(5.5); $^{67}_{29}$Cu$_{38}\to^{67}_{30}$Zn$_{37}$(5.8); $^{65}_{31}$Ga$_{34}\to^{65}_{30}$Zn$_{35}$(5.0); $^{63}_{30}$Zn$_{33}\to^{63}_{29}$Cu$_{34}$(5.4); $^{61}_{29}$Cu$_{32}\to^{61}_{28}$Ni$_{33}$(5.1); $^{61}_{29}$Cu$_{32}\to^{61}_{28}$Ni$_{33}$(5.0); and $^{61}_{30}$Zn$_{31}\to^{61}_{29}$Cu$_{32}$(5.4). These data suggest that $(10^5{\rm s}/ft)_{\rm ex=0.5MeV}$ and $(10^5{\rm s}/ft)_{\rm ex=0.7MeV}$ are both likely to be less than, or of the order of, unity.

A conservative estimate of the maximum likely contribution of excited states may be obtained by setting $(10^6{\rm s}/ft)_{\rm ex=0.17}=(10^5{\rm s}/ft)_{\rm ex=0.5}=(10^5{\rm s}/ft)_{\rm ex=0.7}=1.0$ in Eq. (43). The resulting values of Q are denoted by Q_0 and

TABLE VIII. Kinematic cross sections for neutrino absorption by ^{71}Ga. See Eq. (17) of the text for a definition of $\sigma_{\rm av}$. The excitation energies of final states in ^{71}Ge are denoted by E_x. The correction factor Q_0 is defined by Eq. (43) with $(ft)_{\rm ex=0.17\,MeV}=10^6$s and $(ft)_{\rm ex=0.5\,MeV}=(ft)_{\rm ex=0.7\,MeV}=10^5$s.

Source	Maximum neutrino energy (MeV)	$\sigma_{\rm av}(E_x=0.0)$	$\sigma_{\rm av}(E_x=0.17$ MeV)	$\sigma_{\rm av}(E_x=0.5$ MeV)	$\sigma_{\rm av}(E_x=0.70$ MeV)	Q_0
$p-p$	0.420	1.28	$4.8E-3$	0.0	0.0	1.0003
^{8}B	14.02	$3.8E+2$	$3.65E+2$	$3.38E+2$	$3.21E+2$	1.86
^{13}N	1.198	6.41	4.3	1.42	$4.5E-1$	1.18
^{15}O	1.737	$1.11E+1$	8.4	4.4	2.6	1.34
pep	1.442	$1.88E+1$	$1.52E+1$	9.44	6.54	1.45
^{7}Be	0.862	8.22	5.83	2.38	0.0	1.18
^{7}Be	0.384	2.57	0.0	0.0	0.0	1.00
^{51}Cr	0.426	2.95	1.56	0.0	0.0	1.04
^{51}Cr	0.746	6.59	4.50	1.51	0.0	1.15
^{65}Zn	1.342	$1.67E+1$	$1.33E+1$	7.93	5.27	1.42
^{65}Zn	0.330	$3.5E-1$	0.0	0.0	0.0	1.00

are listed in the last column of Table VIII. For $Q = Q_0$ the counting rates are increased by less than or of order seven percent over the values computed for $Q = 1.0$ for all the solar models considered in Sec. V (see Table X) except for the CNO model (where the increase is 28 percent).

The ^8B-induced transitions to the analog state in ^{71}Ge (of the ground state of ^{71}Ga) lead to proton or neutron emission. The average cross section for the reaction $\nu_e + {}^{71}Ga \to {}^{70}Ge + n + e^-$ (or ${}^{70}Ga + p + e^-$), summed over both nuclear final states, is 4×10^{-43} cm^2 for ^8B neutrinos.

The results given here are in agreement with the less precise estimates of Bahcall (1967) when (Raghaven, 1976) allowance is made for the statistical factor of one-half $[(2I' + 1/(2I + 1)]$ that occurs in Eq. (11). The ground-state rates are also in good agreement with the estimates of Domogatsky (1977). The experimental possibilities for using ^{71}Ga as a detector of solar neutrinos were apparently first discussed by Kuzmin (1965), Kuzmin and Zatsepin (1966), and Pomanski (1965). The cross sections given here were used as a basis for the recent proposal for a large scale ^{71}Ga experiment by Bahcall *et al.* (1978).

F. ^{81}Br

The cross sections for neutrino absorption by ^{81}Br leading to the $\frac{1}{2}^-$ isomeric state of ^{81}Kr (at 190 keV excitation energy) are given in the sixth row of Table III for a nominal cross section factor $\sigma_0 = 2.18 \times 10^{-46}$ cm^2 [cf. Eq. (11)]. The above value of σ_0 corresponds to an assumed [Scott, 1976] log *ft* value of 5.3 for the transition from the $\frac{3}{2}^-$ ground state of ^{81}Br to the $\frac{1}{2}^-$ isomeric state of ^{81}Kr. Scott (1976) justified this choice of the log *ft* value in his original proposal for a ^{81}Br experiment by noting that there are, in the relevant region of the periodic table, four transitions from $\frac{3}{2}^-$ to $\frac{1}{2}^-$ states (all states with small or zero excitation energies) that have log $ft \approx 5.2 \pm 0.1$. There are also a number of other similar transitions in which the order of initial and final spins is reversed ($\frac{1}{2}^-$ to $\frac{3}{2}^-$). These transitions should be equally good measures of the expected log *ft* value. I mention specifically the following transitions (Lemming, 1975; Urone and Kocher, 1975) with the measured log *ft* values given in the parentheses: ${}^{81}_{34}Se_{47} \to {}^{81}_{35}Br_{46}(5.0)$; ${}^{81}_{38}Sr_{43} \to {}^{81}_{37}Rb_{44}(6.5)$; ${}^{79}_{32}Ge_{47} \to {}^{79}_{33}As_{46}(5.5)$; and ${}^{79}_{36}Kr_{43} \to {}^{79}_{35}Br_{44}(5.6)$. All of the above transitions are between ground states and may be crudely characterized as $p_{3/2} \to p_{1/2}$. Two related transitions involving metastable initial states are ${}^{87}_{38}Sr^m_{49} \to {}^{85}_{37}Rb_{48}(4.6)$ and ${}^{77}_{32}Ge^m_{45} \to {}^{77}_{33}As_{34}(5.2)$. These values suggest a log *ft* value anywhere between 4.9 and 6.8 for the $\frac{3}{2}^-$ to $\frac{1}{2}^-$ transition when corrected (by +0.3 in the log) for the reversed order of the spins. The neutrino absorption cross section under discussion is therefore uncertain by at least an order of magnitude.

The transition from the ground state of ^{81}Br to the 636 keV, $J = \frac{3}{2}^-$ excited state of ^{81}Kr is also allowed and could be induced by *pep*, ^8B, ^{13}N, and ^{15}O neutrinos. Moreover, the sources mentioned above could all induce transitions to other low-lying states in ^{81}Kr. Not enough is known at present about the character of any of the states above 190 keV to make possible a useful calculation of the relevant cross sections.

Hampel and Kristen (1978) have described an elegant experiment that is designed to measure the *ft* value between the ground state of ^{81}Br and the lowest $\frac{1}{2}^-$ excited state of ^{81}Kr. In order to supply the complete information necessary to interpret a solar neutrino experiment, it may be necessary to determine the nuclear properties of other low-lying states of ^{81}Kr and to perform a calibration experiment with ^{51}Cr or ^{65}Zn.

G. ^{87}Rb

Neutrino absorptions by ^{87}Rb populate excited states of ^{87}Sr since transitions to the ground state are highly forbidden. The first excited state of ^{87}Sr has an excitation energy of 0.39 MeV. Sunyar and Goldhaber (1960) pointed out that solar neutrinos could reverse the decay process that is studied in the laboratory; they drew attention to the allowed capture reaction $\nu_{solar} + {}^{87}Rb_{gs} \to {}^{87}Sr(ex = 0.39\ MeV) + e^-$. The calculated cross sections for this transition are given in row seven of Table III.

There is another excited state at ex = 0.87 MeV ($I' = \frac{1}{2}^-, \frac{3}{2}^-$) that could be significant for solar neutrino experiments. The next candidate state for an allowed transition lies at ex ≈ 1.23 MeV and is not expected to be important for solar neutrino experiments. Defining, as before, $\sigma_{total} = \sigma_{ex=0.39 MeV} \times Q$, we may write

$$Q \cong \left[1 + 2.81(I' + 1/2)(10^4 s/ft)_{0.87MeV} \left\{\frac{\sigma_{av}(ex = 0.87\ MeV)}{\sigma_{av}(ex = 0.39\ MeV)}\right\}\right] . \quad (44)$$

The calculated values of $R \equiv [\sigma_{av}(ex = 0.87\ MeV)/\sigma_{av}(ex = 0.39\ MeV)]$ for the neutrino branches for which this transition is energetically possible are $R = 0.55(pep)$; 0.37(^7Be); 0.89(^8B); 0.31(^{13}N); 0.45(^{15}O); and 0.53(^{65}Zn).

For most models of the solar interior, the transition to the excited state at 0.87 MeV is not very important. A $\log(ft)_{0.87MeV}$ as low as $4.0(I' = \frac{1}{2})$ would only increase the capture rate predicted from the standard solar model by 23 percent. For the low-Z, homogenized, and $p-p$ + *pep* models considered in Sec. V, the increase in the predicted capture rate with the same assumptions would be less than or equal to eleven percent. On the other hand, if the energy source of the sun were CNO neutrinos then the rate to the 0.87 MeV excited state would be $1.11(I' + \frac{1}{2})(10^4 s/ft)$ times the rate to the 0.39 MeV state.

Sunyar and Goldhaber (1960) first estimated the capture rate of solar neutrinos by ^{87}Rb assuming that two-thirds of the incident neutrinos were $p-p$ neutrinos and one-third were ^7Be neutrinos. Our average cross section for this assumed spectrum, with $Q = 1.0$, is about a factor of three larger than estimated by Sunyar and Goldhaber. The results given in Table III are in rough agreement with the less precise estimates calculated by Bahcall (1967). Our results for $Q = 1.0$ are typically twelve percent larger than the values quoted by Domogatsky (1977), except for ^8B where the difference is about twenty percent.

H. ^{115}In

Neutrino capture by $^{115}_{49}$In leaves $^{115}_{50}$Sn in an excited state since the transition to the ground state of ^{115}Sn is highly forbidden. Raghavan (1976) estimated the absorption cross sections for the $p-p$, pep, and ^{7}Be neutrinos leading to the $\frac{7}{2}^+$, 614 keV excited state of ^{115}Sn in his original proposal to use ^{115}In as a solar neutrino detector. All of the relevant cross sections for this transition are shown in the eighth row of Table III. Following Raghavan (1976), I have adopted (but see below) for the purpose of this table, an ft value of 2.5×10^4s for the ^{115}In $-$ ^{115}Sn ($\frac{9}{2}^+$ to $\frac{7}{2}^+$) transition, corresponding to $\sigma_0 = 2.412\times10^{-45}$ cm^2 [cf. Eq. (11)].

There are two complications regarding the ^{115}In $-$ ^{115}Sn cross sections that must be discussed. They are: (1) the uncertainty in the estimate of the ft value for the transition to the 614 keV excited state of ^{115}Sn; and (2) the contributions of other excited states.

The ft value for the transition to the 614 keV excited state of ^{115}Sn was estimated by Raghavan (1976) using beta-decay data on related In-Sn transitions for mass numbers 117, 119, and 121, some stripping measurements (Schneid, Prakash, and Cohen, 1967), and theoretical calculations based on model Hamiltonians for the pairing correlation (Kisslinger and Sorenson, 1963; Silverberg and Winther, 1963; Fujita, Futami, and Ikeda, 1967). Raghavan estimated that the ft value was uncertain by about ten percent.

It is difficult to justify a conventional error estimate for the ft value from the considerations mentioned above. The theoretical model involves simplifying assumptions regarding the form of the Hamiltonian and the complete basis set of state vectors. The experiments all refer to final-state nuclei with neutron numbers at least as large as 67 while ^{115}Sn has only 65 neutrons (the same problem exists in the comparison of the initial state nuclei). Recall that, in the simplest shell-model picture, the $2d_{3/2}$- neutron states are not occupied until neutron number 67. Thus there is a difference between the states involved in the beta-decay experiments and the states involved in the related ^{115}In$-$^{115}Sn transition. There is no principle of physics that allows one to calculate accurately the effect on the ft value of the above-mentioned differences in state vectors. I conclude that the Raghavan (1976) estimate of the uncertainty cannot be justified rigorously.

Raghavan (1978) has proposed calibrating the ^{115}In detector with a terrestrial source of ^{51}Cr. The calculated cross sections for this source are $\sigma(746\text{keV}) = 2.6\times10^{-44}$ cm^2 and $\sigma(426\text{ keV}) = 1.4\times10^{-44}$ cm^2. An accurate calibration with ^{51}Cr would eliminate the uncertainty, discussed above, in our knowledge of the ft value for the transition to the 614 keV excited state.

There are other states of ^{115}Sn at excitation energies of 1.25 MeV, 1.62 MeV, 1.65 MeV, and 1.84 MeV (see Raman and Kim, 1975) that might contribute to a solar neutrino experiment or a ^{65}Zn calibration experiment. Very little is known about the properties of these states except that the 1.84 MeV level may be a $\frac{7}{2}^+$ state.

It is useful to define, as before, a correction factor, Q, by which the cross sections listed in Table III must be multiplied in order to take account of transitions to levels other than the 0.614 MeV state. One can write

$$Q \equiv \left[1. + \sum_{i=1}^{4} \alpha_i R_i\right], \tag{45}$$

where

$$\alpha_i = \frac{\sigma_0(\text{ex}=E_i)}{\sigma_0(\text{ex}=0.614\text{ MeV})}, \quad R_i \equiv \frac{\sigma_{\text{av}}(\text{ex}=E_i)}{\sigma_{\text{av}}(\text{ex}=0.614\text{ MeV})},$$

and the sum over i includes the four excited states mentioned in the preceding paragraph. The values of R_i are straightforward to calculate. If we list the results in order of increasing excitation energy, then the nonvanishing values of R_i are: $R(pep) = 0.45$, 0.22, 0.21, and 0.12; $R(^7\text{Be}) = 0.26$ (for the 862 keV line); $R(^8\text{B}) = 0.865$, 0.79, 0.79 and 0.75; $R(^{13}\text{N}) = 0.18$, 0.005, and 0.002; $R(^{15}\text{O}) = 0.34$, 0.12, 0.10, and 0.04; $R(^{65}\text{Zn}) = 0.42$, 0.19, and 0.18. The ratios of the cross section factors α_i are [see Eq. (11)]

$$\alpha_i \equiv \left[\left(\frac{(ft)_{\text{ex}=0.614\text{ MeV}}}{(ft)_{\text{ex}=E_i}}\right)\left(\frac{(2I_i'+1)}{8}\right)\right].$$

The properties of the relevant four nuclear states must be determined before one can make a plausible estimate of the likely sizes of the α_i. It may be that the spins or parities of some of these states do not permit an allowed neutrino capture reaction. In this case, the appropriate value for the relevant α would be approximately zero.

One can estimate a plausible upper limit to the likely effect of the four excited states mentioned above by setting all of the $\alpha_i \equiv 1.0$. Denote the values of Q calculated in this fashion by $Q = Q_0$. Then the predicted increase in capture rate for $Q = Q_0$ (over what is calculated for $Q = 1.0$) is less than or equal to seven percent for the standard, low-Z, homogenized, and $p-p+pep$ models considered in Sec. V. Only for the pure CNO model is the expected increase large (44 percent in this case).

In the ^{115}In experiment proposed by Raghavan (1976), individual electrons are counted in real time and their energies are measured. The experiment differs from all the other experiments discussed in this paper by providing important information about the energy spectrum of the electrons that are produced (and hence, the incident neutrino energy spectrum). Table IX contains the calculated absorption cross sections for $p-p$ neutrinos to produce electrons with kinetic energies, $\bar{E}$, in the listed 10 keV intervals.

The production by ^{13}N and ^{15}O neutrinos of electrons with energies in the range that could be confused with events due to $p-p$ neutrinos ($\bar{E} \leq 0.3$ MeV) is negligible. I find that $\sigma(\bar{E} \leq 0.3\text{ MeV})/\sigma(\text{all energies})$ is 0.145 for ^{13}N neutrinos and 0.04 for ^{15}O neutrinos. Since the *total* contribution of ^{13}N and ^{15}O neutrinos is expected to be only 1 percent (cf. Table XI), their contribution in the $p-p$ energy range should be completely negligible.

The cross section for $p-p$ neutrinos obtained here is twenty-five percent smaller than estimated by Raghavan (1976). [The above difference would have been thirty percent had I used the same threshold (128 keV) for neutrino capture as did Raghavan ($\Delta\sigma/\sigma\Delta E_{\text{th}} \approx -0.0038$ per keV for $p-p$ neutrinos).] The pep cross section given in Table III is about seven percent larger than the value quoted by Raghavan. Because of the energy resolution of the ^{115}In detector, it is worthwhile separating

TABLE IX. Differential cross sections for $p-p$ neutrinos incident on ^{115}In. Here $\bar{E}$ is the mean kinetic energy of an electron that is produced by neutrino capture and $d\sigma(\bar{E})$ is the cross section for production of an electron in the interval $\bar{E} \pm dE$, where $dE = 10$ keV.

$\bar{E}$ (keV)	$d\sigma(\bar{E})$ (10^{-46}cm^2)	$\bar{E}$ (keV)	$d\sigma(\bar{E})$ (10^{-46}cm^2)
10	0.98	160	3.8
20	1.1	170	3.9
30	1.3	180	4.1
40	1.45	190	4.2
50	1.6	200	4.3
60	1.8	210	4.35
70	2.0	220	4.45
80	2.2	230	4.3
90	2.4	240	4.2
100	2.6	250	4.1
110	2.8	260	3.8
120	3.0	270	3.5
130	3.2	280	2.95
140	3.4	290	2.1
150	3.6	300	0.6

the cross sections for the two ^{7}Be branches. I find: $\sigma(862\text{ keV}) = 3.1 \times 10^{-44}$ cm^2, and $\sigma(384\text{ keV}) = 1.2 \times 10^{-44}$ cm^2. The cross section for the more energetic branch is about ten percent larger than the value quoted by Raghavan.

I. ^{205}Tl

The cross sections for neutrino capture by $^{205}_{81}\text{Tl}_{124}$ are uncertain because the inverse laboratory transitions are not observable (only excited states of $^{205}_{82}\text{Pb}_{123}$ are populated by solar neutrino capture) and because no precisely analogous transitions have been studied experimentally. Freedman *et al.* (1976), in their important paper suggesting ^{205}Tl as a detector of solar neutrinos that were emitted over the past 10^7 years, proposed that the *ft* value for the $1/2^+ \to 1/2^-$ transition to the lowest excited state of ^{205}Pb be estimated by analogy to similar transitions in three neighboring nuclei. These authors pointed out that the $1/2^+$ to $1/2^-$ transitions in three cases considered all have similar log *ft* values

$$^{205}_{80}\text{Hg}_{125} \to {}^{205}_{81}\text{Tl}_{124}\ (\log ft = 5.3);\ {}^{207}_{81}\text{Tl}_{126} \to {}^{207}_{82}\text{Pb}_{125} \quad (5.1);$$

$$\text{and } {}^{209}_{81}\text{Tl}_{128} \to {}^{209}_{82}\text{Pb}_{127} \quad (5.2).$$

However, none of the transitions listed above are exactly analogous to the $^{205}\text{Tl} \to {}^{205}\text{Pb}$ transition of interest. The $^{205}\text{Hg} \to {}^{205}\text{Tl}$ and $^{207}\text{Tl} \to {}^{207}\text{Pb}$ transitions are good single-particle transitions ($p_{1/2} \to s_{1/2}$) from the neutron $p_{1/2}$ shell to the proton $s_{1/2}$ shell with almost closed neutron and proton shells. The $^{209}\text{Tl} \to {}^{209}\text{Pb}$ transition contains extra neutrons outside the closed shell at neutron number 126. The ground state of ^{205}Tl is different from any of the states mentioned above since it is predominantly $(f_{5/2})^6$ and contains only a smaller, experimentally unknown amount of $(f_{5/2})^4(p_{1/2})^2$. It will be difficult to obtain an accurate value, with a well-determined estimate of the uncertainty, for the *ft* value of the Tl-Pb transition of interest. At present, it is probably not overly pessimistic to estimate the uncertainty as a factor of two in the rate (most likely, the *ft* value is larger than for the three transitions discussed above). In order to estimate the capture cross sections, I have adopted a nominal value of $\log ft = 5.3$ for the transition to the $1/2^-$ excited state of ^{205}Pb at ex = 0.0023 MeV (following Freedman *et al.*, 1976).

I have calculated also cross sections to the $3/2^-$ state of ^{205}Pb at an excitation energy of 0.26 MeV. The results may be summarized in terms of the nominal cross sections that are given in Table III and a correction factor Q. Here

$$\sigma_{\text{total}} = \sigma_{\text{nominal}} Q\,, \quad (46a)$$

where

$$Q = (2\times 10^5 s/ft)_{\text{ex}=0.002\text{ MeV}}\left[1 + 2(2\times 10^5 s/ft)_{\text{ex}=0.26\text{ MeV}}\left(\frac{\sigma_{\text{av}}(\text{ex}=0.26\text{ MeV})}{\sigma_{\text{av}}(\text{ex}=0.002\text{ MeV})}\right)\right]. \quad (46b)$$

Note that in order to calculate σ_{total}, or Q, two unknown *ft* values must be determined. The ratios $\sigma_{\text{av}}(\text{ex}=0.26\text{ MeV})/\sigma_{\text{av}}(\text{ex}=0.0\text{ MeV})$ can be calculated accurately and are, for neutrino continuum sources: 0.20($p-p$); 0.95(^{8}B); 0.66(^{13}N); 0.74(^{15}O); and 6×10^{-5}(^{65}Zn). For neutrino line sources, one obtains $[\sigma_{\text{av}}(\text{ex}=0.26\text{ MeV})/\sigma_{\text{av}}(\text{ex}=0.0\text{ MeV})] = 0.79(pep)$; 0.70(^{7}Be, $q_{\max} = 0.86$ MeV); 0.56(^{7}Be, $q_{\max} = 0.38$ MeV); and 0.79(^{65}Zn). The uncertainty in the total capture rate due to possible transitions to the $3/2^-$ state is appreciable, but difficult to estimate quantitatively without detailed knowledge of this state.

V. DISCUSSION

A. Neutrino fluxes

Table X contains a list of solar neutrino fluxes that were computed for some illustrative solar models and special physical hypotheses. All of the solar models and physical hypotheses in this table were concocted to explain the difference between theory and observation in the ^{37}Cl experiment. The fluxes will be used to explore the ability of the proposed experiments to discriminate between alternative explanations.

Of the various entries in Table X, only the standard solar model represents the results of a calculation based, without modification, on the conventional ideas of the theory of stellar evolution (see, e.g., Schwarzschild, 1958, or Clayton, 1968). The low-Z model was computed by assuming that the heavy element abundance, Z, in the solar interior is so low ($Z \lesssim 10^{-3}$) that only hydrogen and helium contribute significantly to the opacity. The homogenized model was computed by assuming that the inner 95% of the mass of the sun was continuously mixed for the entire lifetime of the sun. A *lower limit* (assuming that neutrinos do not change their

TABLE X. Neutrino fluxes calculated using some illustrative solar models or physical hypotheses. All fluxes are given in units of $10^{10} cm^{-2} sec^{-1}$ at the earth's surface. The ^7Be fluxes include neutrinos from both branches 6(a) and 6(b).

Model or hypothesis / Source	$p-p$	pep	^7Be	^8B	^{13}N	^{15}O	References
Standard	6.1	$1.5E-2$	$3.4E-1$	$3.2E-4$	$2.6E-2$	$1.8E-2$	a
Low Z	6.3	$1.6E-2$	$1.4E-1$	$5.4E-5$	$1.2E-3$	$4.3E-4$	a
Homogenized	6.45	$1.6E-2$	$1.44E-1$	$8E-5$	$7.2E-3$	$7.2E-3$	b
Only $p-p$ and pep	6.45	$1.6E-2$	0	0	0	0	-
CNO	0	0	0	0	3.38	3.38	c
Neutrino oscillations ($R_{osc} \leq 1/3$)	≤ 2	$\leq 5E-3$	$\leq 1.1E-1$	$\leq 1.1E-4$	$\leq 9E-3$	$6E-3$	d, e, f
Neutrino decay	0	0	0	0	0	0	g

[a] Bahcall *et al.*, 1973.
[b] Bahcall, Bahcall, and Ulrich, 1968.
[c] Bahcall, 1969.
[d] Gribov and Pontecorvo, 1969.
[e] Bahcall and Frautschi, 1969.
[f] Fritzsch and Minkowski, 1976.
[g] Bahcall, Cabbibo, and Yahil, 1972.

form on the way to the earth) to the expected counting rates in solar neutrino experiments is provided by the fluxes listed for the p-p and pep model. These fluxes were computed by supposing that all the nuclear fusion reactions in the sun are terminated via the $^3He-^3He$ reaction. An *upper limit* is provided by the CNO model, which was computed by assuming that the sun derives all of its luminosity from the CNO cycle. The last two rows of Table X describe situations in which the standard theory of weak interactions has been modified to include either neutrino oscillations or neutrino decay as the neutrinos travel from the center of the sun to the earth. These special physical hypotheses give much lower predicted capture rates (see Sec. V.B) than any of the astronomical hypotheses discussed above. The effect of neutrino oscillations, when averaged over the neutrino spectrum, yields a reduction factor, R_{osc}, that is the same for all experiments (see Bahcall and Frautschi, 1969). Comparison of the standard predictions for the ^{37}Cl experiment with the recent observations of Cleveland and Davis (1978) yields the value of $R_{osc} \lesssim 1/3$ that is used in Table X. This value, which does not include the errors in the theory or the observations, implies that the number of neutrinos which would have to be connected by oscillations in order to explain the ^{37}Cl observations is rather large ($\gtrsim 4$; see Nussinov, 1976). The series of zeros in the last row of Table X, corresponding to the hypothesis of neutrino decay, follows from the assumption that the ^8B neutrinos (which are most important in the ^{37}Cl experiment) decay on their way to the earth. If this assumption is correct, then certainly the lower energy p-p (and pep) neutrinos decay.

B. Predicted capture rates

Table XI contains the capture rates due to different neutrino sources that are predicted on the basis of a standard solar model [cf. Table X]. Table XII contains the *total* capture rates that are predicted by some nonstandard solar models and special physical hypotheses [cf. Table X]. I have given in Table XII only the total expected capture rate, summed over all neutrino branches, since radiochemical experiments have been proposed for all the targets listed in Table XII, with the exception of ^{115}In (see Raghavan, 1976, and Sec. IV.H). The results presented in Tables XI and XII are useful in determining which experiments can best decide between various theoretical alternatives.

TABLE XI. The expected capture rates for a standard solar model. All capture rates are given in units of 1 SNU $=10^{-36}$ captures per target atom per second.

Target \ Source	$p-p$	pep	^7Be	^8B	^{13}N	^{15}O	$\sum(\phi\sigma)$ (Total)
^7Li	0	9.0	3.2	9.9	1.1	4.1	27.3
^{37}Cl	0	0.23	0.81	3.46	0.04	0.12	4.66
^{51}V	0	0.17	0.76	~3	0.04	0.09	4
^{55}Mn	1.7	0.06	0.59	~2	0.04	0.05	4
^{71}Ga	65	2.4	21.8	~1	1.4	1.7	93
^{81}Br	0	0.48	3.6	~0.3	0.22	0.3	4.9
^{87}Rb	140	2.7	27.5	~1	1.8	2.0	$1.75E+2$
^{115}In	534	9.6	99	~3	6.5	7.2	$6.6E+2$
^{205}Tl	439	5.1	61	~1	4.2	4.1	$5.1E+2$

TABLE XII. Capture rates implied by some illustrative solar models and physical hypotheses. All capture rates are given in units of 1 SNU. The last two columns list the tons of each target that are required to achieve 1 CAP (one capture per day) assuming either the CNO or the $p-p$ and pep models.

Target \ Source	Standard	Low Z	Maximal mixing	Only $p-p$ and pep	CNO	Neutrino oscillations	Neutrino decay	Tons for 1 CAP (CNO)	Tons for 1 CAP ($p-p$ and pep)
^{7}Li	27	12.8	15.4	9.6	918	$\lesssim 9$	0	0.2	15
^{37}Cl	4.7	1.2	1.5	0.25	28	$\lesssim 1.6$	0	99	1.1×10^4
^{51}V	~4	~1	~1.3	0.18	22	$\lesssim 1.3$	0	45	5×10^3
^{55}Mn	~4	2.4	2.6	1.9	13	$\lesssim 1.3$	0	81	555
^{71}Ga	93	79	82	71.5	490	$\lesssim 30$	0	6.9	47
^{81}Br	4.9	2.1	2.3	0.5	875	$\lesssim 1.6$	0	36	6×10^3
^{87}Rb	175	159	164	151	615	$\lesssim 60$	0	9.6	39
^{115}In	659	604	622	575	2,200	$\lesssim 220$	0	1.0	4.0
^{205}Tl	514	485	499	470	1,318	$\lesssim 170$	0	4.2	12

C. Classification of the different experiments

The detectors for solar neutrinos can be classified according to their relative sensitivity to different parts of the solar neutrino spectrum (see Tables XI and XII). Three of the nine targets are primarily sensitive to ^{8}B neutrinos; these are ^{37}Cl, ^{51}V, and ^{55}Mn. A deuterium detector, which has most recently been discussed by Fainberg (1978), would be sensitive almost entirely to ^{8}B neutrinos (I have not discussed the ^{2}H cross sections in this paper since they have been accurately calculated by Ellis and Bahcall, 1968, and Kelly and Uberall, 1966).

Four of the nine detectors are primarily sensitive to neutrinos from the proton–proton reactions. These p-p detectors are: ^{71}Ga, ^{87}Rb, ^{115}In, and ^{205}Tl. *The expected capture rates for these detectors are practically independent of the astronomical assumptions that are made (see Table XII) provided only that the sun produces, in a steady/state fashion and via the proton-proton chain, the energy that it radiates from its surface.*

The *pep* neutrinos are expected to make the largest single contribution to the capture rate of a ^{7}Li detector, even for the standard solar model. The observational results from the ^{37}Cl experiment (see Cleveland and Davis, 1978) show, moreover, that the higher-energy ^{8}B neutrinos should contribute, for a ^{7}Li target, at most one-half the capture rate due to *pep* neutrinos. Since the *pep* neutrinos are as good a measure of the proton–proton reaction rate as are the $p-p$ neutrinos (see Bahcall and May, 1969), one can also classify the ^{7}Li detector as a $p-p$ sensitive target. The ^{7}Li and ^{115}In targets share the property of being reasonably sensitive to more than one neutrino branch (the *pep*, ^{7}Be, ^{8}B, and ^{15}O branches for the ^{7}Li detector; the $p-p$ and ^{7}Be branches for the ^{115}In target).

The ^{81}Br detector is primarily sensitive to ^{7}Be neutrinos.

The ^{115}In experiment (Raghavan, 1976, 1978) is the only one discussed in this paper in which the energies of the individual electrons that are produced by neutrino capture could be measured. The $p-p$ and ^{7}Be capture rates could be determined separately. Note also that in the ^{115}In experiment one could, in principle, measure the angular correlation between the direction of the sun and the direction of the electron that is produced, thus establishing directly that the neutrinos come from the sun. The correlation parameter α (where $d\sigma/d\Omega \propto (1+\alpha\cos\theta)$) is $-1/3$ for the transition from the ground state of ^{115}In to the $7/2^+$ excited state of ^{115}Sn (Bahcall, 1964c).

D. Nuclear physics uncertainties

The absorption cross sections for the ^{7}Li, ^{37}Cl, ^{71}Ga, and ^{87}Rb targets are known to an accuracy of the order of five percent for those transitions in which the nuclear matrix elements have been measured in laboratory experiments on the inverse decay [i.e., all transitions of interest with ^{7}Li and ^{37}Cl (except those induced by ^{8}B neutrinos on ^{37}Cl) and the most important, $p-p$, transitions with ^{71}Ga, and ^{87}Rb]. The cross sections for the ^{115}In detector are reasonably well known, although it is difficult to make a meaningful quantitative estimate of the uncertainty (see Sec. IV.H). The estimates could be refined further if the appropriate (p,n) or stripping experiments were done for ^{71}Ga, ^{87}Rb, and ^{115}In. The reactions of interest lead from the ground states of the target nuclei to the excited states of the daughter nuclei that were discussed in Secs. IV.E, IV.G, and IV.H.

All of the absorption cross sections for ^{81}Br and ^{205}Tl are poorly known at present (typical uncertainties may be a factor of two or more; see Sec. IV.F and Sec. Sec. IV.I). The cross sections for ^{51}V and ^{55}Mn to absorb ^{8}B neutrinos are poorly known because of the uncertain contribution (that I have guessed to be of order twenty-five percent) of the allowed transitions to various excited states (see Secs. IV.C and IV.D).

E. Recommended new experiments

In order for a solar neutrino experiment to be most useful, the absorption cross sections must be accurately known. Of the new targets discussed in this paper, only ^{7}Li, ^{71}Ga, ^{87}Rb, and ^{115}In (and ^{2}H; see Ellis and Bahcall, 1968) satisfy this requirement. A new detector should also help discriminate between the possible explanations of the discrepancy between theory and observation in the ^{37}Cl experiment. A ^{2}H experiment might not provide much new information since it is sensitive almost entirely to ^{8}B neutrinos that have already been shown to be in short supply by the ^{37}Cl experiment. There has not been a recent and detailed experimental feasibility study for the proposed ^{87}Rb experiment, perhaps because of the uncomfortably short lifetime (2.8 hrs) of the daughter nucleus, ^{87}Sr. If we set aside ^{87}Rb because of the absence of a feasibility study, then only three preferred targets remain: ^{7}Li, ^{71}Ga, and ^{115}In.

There are four major neutrino branches that must be measured in order to carry out a program of neutrino spectroscopy of the solar interior (cf. Bahcall, 1964c). These branches are the $p-p$, ^{7}Be, ^{8}B, and ^{13}N+^{15}O neutrinos. The future experimental solar neutrino program should include all three of the preferred new detectors (^{7}Li, ^{71}Ga, and ^{115}In). The ^{71}Ga experiment is primarily sensitive to $p-p$ neutrinos and the ^{37}Cl experiment to ^{8}B neutrinos. The ^{7}Li and ^{115}In experiments provide additional information about the ^{7}Be and ^{13}N+^{15}O fluxes. Taken together, the results of the four experiments (^{7}Li, ^{37}Cl, ^{71}Ga, and ^{115}In) may allow us to solve for the parameters of the solar interior (temperature range, density, and composition). If neutrinos decay or oscillate, these experiments will provide otherwise unobtainable information about the weak interactions (cf., Pontecorvo, 1968). Solar neutrino experiments have two advantages over accelerator experiments on neutrino decay or oscillation: a much larger distance (10^{11} cm) between the beam source (the sun) and the detector (on earth) and a much smaller time-dilation factor (MeV neutrinos versus multi-GeV neutrinos).

F. Neutrino spectroscopy

The solar neutrino spectrum can be determined by analyzing, for a given set of experiments, the coupled equations that relate the individual fluxes to the measured neutrino capture rates (and their errors). I assume in this section (except where explicitly stated otherwise) that the neutrino fluxes produced in the sun are not modified (by oscillations or decay) on their way to the earth. In the analysis given below, I show which part of the spectrum is determined by each of the recommended experiments.

It is convenient to define dimensionless ratios, ϵ, which are the actual neutrino fluxes, ϕ, received at earth divided by the fluxes calculated in a standard solar model, ϕ_{sm} (see Table X). Then

$$\epsilon(p-p)\equiv\phi(p-p)/\phi_{sm}(p-p)\,, \tag{47a}$$

$$\epsilon(^{7}\mathrm{Be})\equiv\phi(^{7}\mathrm{Be})/\phi_{sm}(^{7}\mathrm{Be})\,, \tag{47b}$$

$$\epsilon(^{8}\mathrm{B})\equiv\phi(^{8}\mathrm{B})/\phi_{sm}(^{8}\mathrm{B})\,, \tag{47c}$$

and

$$\epsilon(\mathrm{CNO})=\frac{\phi(^{13}\mathrm{N})\sigma(^{13}\mathrm{N})+\phi(^{15}\mathrm{O})\sigma(^{15}\mathrm{O})}{\phi_{sm}(^{13}\mathrm{N})\sigma(^{13}\mathrm{N})+\phi_{sm}(^{15}\mathrm{O})\sigma(^{15}\mathrm{O})}\,. \tag{47d}$$

On the assumption that the fluxes from the standard solar model provide an upper limit to the actual fluxes, each ϵ lies between zero and one. The ϵ(CNO) defined above is independent of the experiment to which the absorption cross sections σ refer if and only if $\phi(^{13}\mathrm{N})/\phi(^{15}\mathrm{O})=\phi_{sm}(^{13}\mathrm{N})/\phi_{sm}(^{15}\mathrm{O})$. In practice, one may suppose, for the four recommended experiments, that the absorption cross sections that appear in (47d) are those for the ^{7}Li experiment since ϵ(CNO) can best be determined from this experiment (see below).

The equations for the ϵ's which refer to the four recommended experiments are:

$$9.0\epsilon(p-p)+3.2\epsilon(^{7}\mathrm{Be})+9.9\epsilon(^{8}\mathrm{B})+5.2\epsilon(\mathrm{CNO})=E(\mathrm{Li})\pm\Delta E(\mathrm{Li})\,, \tag{48a}$$

$$0.23\epsilon(p-p)+0.81\epsilon(^{7}\mathrm{Be})+3.5\epsilon(^{8}\mathrm{B})+0.16\epsilon(\mathrm{CNO})=E(\mathrm{Cl})\pm\Delta E(\mathrm{Cl})\,, \tag{48b}$$

$$67.4\epsilon(p-p)+22\epsilon(^{7}\mathrm{Be})+1\epsilon(^{8}\mathrm{B})+3.1\epsilon(\mathrm{CNO})=E(\mathrm{Ga})\pm\Delta E(\mathrm{Ga})\,, \tag{48c}$$

and

$$544\epsilon(p-p)+99\epsilon(^{7}\mathrm{Be})+3\epsilon(^{8}\mathrm{B})+14\epsilon(\mathrm{CNO})=E(\mathrm{In})\pm\Delta E(\mathrm{In})\,. \tag{48d}$$

The measured capture rates are denoted by E and the experimental uncertainties by ΔE, both in units of 1 SNU. The coefficients of the epsilons that appear in Eqs. (48) are calculated using a standard solar model and each column may be somewhat uncertain. However, these relatively small uncertainties will not affect the general conclusions reached below. In writing Eqs. (48) I have assumed that $\phi(pep)/\phi(p-p)=[\phi(pep)/\phi(p-p)]_{sm}$. This relation is exact for the average result of neutrino oscillations and is also satisfied rather accurately for any of the so-far concocted nonstandard solar models since the ratio of pep to $p-p$ fluxes is insensitive to solar model parameters (see Bahcall and May, 1969).

The ^{71}Ga and ^{115}In experiments are sensitive only to the $p-p$ and ^{7}Be fluxes [cf. Eqs. (48c) and (48d)]. The quantities $\phi(p-p)$ and $\epsilon(^{7}\mathrm{Be})$ can be determined separately from the proposed ^{115}In counting experiment. If both the ^{71}Ga and ^{115}In experiments are performed, one can obtain a reasonably accurate measurement of $\epsilon(p-p)$ and of $\epsilon(^{7}\mathrm{Be})$. Neither experiment will provide significant information about $\epsilon(^{8}\mathrm{B})$ or ϵ(CNO).

The ^{37}Cl experiment is primarily sensitive to ^{8}B neutrinos. Equation (48b) may be rewritten in a form that makes this fact especially clear:

$$\epsilon(^{8}\mathrm{B})=0.3[E(\mathrm{Cl})-e\pm\Delta E(\mathrm{Cl})]\,. \tag{49a}$$

Here

$$e\equiv0.23\epsilon(p-p)+0.81\epsilon(^{7}\mathrm{Be})+0.16\epsilon(\mathrm{CNO})\,. \tag{49b}$$

The quantity e is expected to be rather small and is reasonably independent of which *ad hoc* assumption is used to "explain" the present results of the ^{37}Cl experiment. If one adopts the current values (Rowley *et al.*, 1977; Bahcall and Davis, 1976; Cleveland and Davis, 1978) for the capture rate ($E(\mathrm{Cl})=1.6$ SNU; $\Delta E(\mathrm{Cl})\sim0.4$ SNU) and assumes that this signal is all due to solar neutrinos, then there are only three models or hypotheses whose fluxes are given in Table X that are consistent with the observations. They are: the low-Z model, the homogenized model, and the hypothesis of neutrino oscillations. For all three of these cases e lies between 0.3 to 0.4. Using this value of e with the above described assumptions, the value of $\epsilon(^{8}\mathrm{B})$ is nominally well determined:

$$\epsilon(^8B) \simeq 0.35 \pm 0.1 . \qquad (50)$$

Experimental errors (that are less than or of the order of 25%) in the determination of the other fluxes are not likely to affect greatly the final determination of $\epsilon(^8B)$ if $E(Cl) = 1.6$.

The ^{7}Li experiment can be used to determine the weighted-average CNO fluxes [cf. Eq. (47d)] once the $p-p$, ^{7}Be, and ^{8}B fluxes have been measured by the other recommended experiments. The accuracy with which one can determine ϵ(CNO) is limited by the fact that there are sizeable contributions [see Eq. (48a)] to the expected ^{7}Li capture rate from all four of the major solar neutrino sources. One may write schematically

$$\Delta\epsilon(\mathrm{CNO}) \approx 0.2[(\Delta E(\mathrm{Li}))^2 + (3\Delta E(\mathrm{Cl}))^2 + (9\Delta\epsilon(p-p))^2 + (3\Delta\epsilon(^7\mathrm{Be}))^2]^{1/2}, \qquad (51)$$

where $\Delta\epsilon$ is the uncertainty in the relevant ϵ. Note that $\Delta E(^7\mathrm{Li})$ must be less than or of the order of three and $\Delta\epsilon(p-p)$ must be less than or of the order of 0.3 to make possible even a crude determination of ϵ(CNO).

If the results from only one additional experiment (complementary to the ^{37}Cl experiment) are available, then the analysis must be different from that given above. I discuss this single-experiment case in some detail since the considerations given below may provide partial guidance as to the relative priorities among the recommended experiments. Either a ^{71}Ga or an ^{115}In experiment can distinguish between explanations that are based on presumed inadequacies in, respectively, the astronomical theory or the weak interaction theory, provided only that the sun produces in a steady-state fashion the energy it radiates from its surface. A low counting rate in either of these experiments could also arise, in principle, if the sun is now in an abnormal phase in which its nuclear energy generation is much less than its surface luminosity. However, for most of the models of this kind that have appeared in the literature (Rood, 1972; Ezer and Cameron, 1972; Ulrich and Rood, 1972), the reduction in the counting rate of a ^{71}Ga or an ^{115}In experiment would not be nearly as great as is expected on either the oscillation or the decay hypotheses. In fact, if the ^{37}Cl capture rate is really 1.6 SNU, then one can estimate crudely for one illustrative model of this kind [Rood's 1972 model 3] a capture rate of order 6×10^1 SNU for ^{71}Ga, and 5×10^2 SNU for ^{115}In. Moreover, these latter two hypotheses lead to specific predictions for the ^{71}Ga and ^{115}In experiments when combined with the results of the ^{37}Cl experiment.

A ^{7}Li experiment can also distinguish between the same large classes of explanations discussed in the previous paragraph provided only that the observed counting rate is distinguishable, within the errors, from $27\,R_{\mathrm{osc}}$ (here R_{osc} is the energy-independent reduction factor due to neutrino oscillations, cf. Sec. V.A and Table XII). Assume, for example, that R_{osc} is of the order of one-third and that the observed counting rate is of the order of $9 \pm \Delta E(^7\mathrm{Li})$. Under these circumstances, one will be unable to conclude either that the only allowed explanations are modifications of the standard astronomical theory or, alternatively, that only modifications of the conventional theory of weak interactions are acceptable solutions. Counting rates outside the range from $9-\Delta E(^7\mathrm{Li})$ to $9+\Delta E(^7\mathrm{Li})$ *would* permit one to rule out either the astronomical or the weak interaction explanations.

Note, moreover, that even an experiment in the potentially unclear range would permit some specific inferences. For example, a counting rate of $9\pm\Delta E(^7\mathrm{Li})$ would be inconsistent with an explanation of the solar neutrino problem that is based on neutrino decay or on the claim that only the rare ^{8}B flux is predicted incorrectly.

The financial expense and difficulty of performing each of the recommended experiments are important additional factors in establishing the chronological priorities of the various observations. These additional factors are beyond the scope of the present article; they must be determined by detailed analysis of the experimental proposals and by scaled-down pilot experiments.

G. Tests of the conservation of electric charge

The conservation of electric charge is widely believed to be an *absolute* conservation law, valid to an arbitrary accuracy. The validity of this law, like all the laws of physics, ultimately rests upon experiment. Feinberg and Goldhaber (1959) and Goldhaber (1975) have discussed a variety of possible experimental tests of charge conservation. The experimental situation has been summarized recently by Goldhaber (1977). The most stringent lower limits on the lifetime of the electron (conceivable decays are, e.g., $e\to\gamma+\nu_e, e\to 2\nu_e+\bar{\nu}_e$, etc.) are of order 10^{22} years for electrons in iodine and germanium atoms (Steinberg *et al.*, 1975; Moe and Reines, 1965). The most stringent limit on a nuclear decay (involving, e.g., $n\to p+\gamma$ or $n\to p+\bar{\nu}_e+\nu_e$) with which I am familiar is the lower limit of the order of 10^{16} yr on the spontaneous decay of ^{87}Rb to ^{87}Sr^m (Sunyar and Goldhaber 1959).

The validity of charge conservation in reactions involving nucleons is not guaranteed by experiments that demonstrate the stability of the electron. In principle, there could be a reaction which permitted, at a certain level, $n\to p+\gamma$ or $n\to p+\nu_e+\bar{\nu}_e$ while forbidding, to the same order of small quantities, all decays of the electron. Moreover, the lifetimes for nucleon decays ($n\to p+$ anything) that are accessible to solar neutrino experiments are much longer than the limits obtained so far on the lifetime of the electron, primarily because of the much larger amounts of material and the longer counting times that are involved in solar neutrino experiments. The nuclear experiments on charge conservation can provide stringent, independent tests of charge conservation.

Solar neutrino radiochemical experiments that involve targets for which the neutrino capture threshold is less than the mass of the electron provide sensitive tests of charge conservation with no extra effort. Some examples of the kinds of processes that are forbidden by charge conservation but are allowed by energy conservation and all the other known laws of physics are: $^{71}\mathrm{Ga}\to{}^{71}\mathrm{Ge}+\nu_e+\bar{\nu}_e$ and $^{71}\mathrm{Ga}\to{}^{71}\mathrm{Ge}+\gamma(E_\gamma\approx275$ keV). Targets composed of ^{55}Mn, ^{81}Br, ^{87}Rb, and ^{205}Tl are also suitable, in principle, for tests of charge conservation. ^{115}In cannot be used for experiments to test charge nonconservation since there is no practical way of detecting sufficiently small quantities of the stable product ^{115}Sn. Solar neutrino experiments with ^{115}In are feasible only because electrons produced by neutrino capture (but not by a hypothetical charge nonconserving reaction) are counted.

The great sensitivity of solar neutrino experiments to charge nonconservation can be seen immediately from the definition (Bahcall 1969) of a SNU, i.e., 10^{-36} daughter atoms produced per target atom per second. Translated into lifetimes measurable in a radiochemical experiment designed to detect $\nu_e + A_Z \to e^- + A_{(Z+1)}$, this gives:

$$\tau(A_Z \to A_{(Z+1)} + \text{anything}) \geq \frac{2.2\times 10^{28}\,\text{year}}{P(\text{SNU})}. \quad (52)$$

Here P(SNU) is the measured production rate in SNU's of daughter atoms $A_{(Z+1)}$ which, in principle, could be due either to solar neutrino captures, to background processes, or to charge nonconservation. Note that radiochemical experiments are sensitive to any mode by which the charge nonconservation is effected; this is indicated by the appearance of the word "anything" in Eq. (52). If the p-p solar neutrinos reach the earth, then the ultimate sensitivity to charge nonconservation obtainable with a ^{71}Ga experiment alone, $P \sim 10^2$, will be a lifetime $\sim 10^{26.5}$ years (see Steinberg, 1976). This is eleven orders of magnitude longer than the upper limit obtained by Sunyar and Goldhaber (1959) in the nuclear decay of ^{87}Rb and ^{87}Sr and four orders of magnitude longer than the most stringent limits on the lifetime of the electron (to specific decay modes).

The interpretation of charge nonconservation experiments requires some theoretical assumptions. A reasonably general, and perhaps plausible, assumption is that the interaction matrix element factors into a nuclear part times something else. The nuclear part itself can then be factored (for the relevant low momentum transfers) into an ordinary nuclear beta-decay matrix element (obtainable, e.g., from the electron capture rate of ^{71}Ge for a ^{71}Ga target) times a charge nonconserving nuclear matrix element, $\langle n|H_0|p\rangle$. Using Fermi's golden rule with the above assumption one can write

$$|\langle n|H_Q|p\rangle|^2\rho = (\ln 2\ \hbar/2\pi)(\tau_{1/2}(Q))^{-1}[ft_{1/2}/6\times 10^3\text{s}], \quad (53)$$

where ρ is the phase space for the decay (which depends on the decay mode and theory assumed), and $\tau_{1/2}(Q)$ is the lifetime, or upper limit to the lifetime, for charge nonconserving decays (cf. Eq. 52). The nuclear $ft_{1/2}$ factor that appears in Eq. (53) must be corrected for the statistical factor, $[(2I+1)/(2I'+1)]$, that takes account of the difference in spin between initial and final nuclear states [cf. Eq. (11)]. For $^{71}\text{Ga} \to {}^{71}\text{Ge}, ft_{1/2} \approx 4.6 \times 10^4$s (see Table I). In the form given in Eq. (53), the results of a suitable solar neutrino experiment can be interpreted in terms of a limit on a nucleon non-charge-conserving matrix element in a manner that is entirely analogous to, *but independent of*, the matrix element $\langle e^-|H_Q|0\rangle$ that is determined by electron decay experiments.

If one assumes that the weak interactions include a small charge nonconserving part that has the usual form except for a neutrino replacing the electron in the lepton current, $H_Q \equiv \epsilon H_{\text{usual form}}$, then one can obtain an interesting limit on ϵ. The result may be expressed in terms of the ratio of branching ratios for the elementary neutron decays: $\epsilon^2 = \Gamma(n\to p+\nu_e+\bar\nu_e)/\Gamma(n\to p+e^-+\bar\nu_e)$. I find:

$$\frac{\Gamma(n\to p+\nu_e+\bar\nu_e)}{\Gamma(n\to p+e^-+\bar\nu_e)} = \left[\frac{P(\text{SNU})t_{1/2}(n)}{2.2\times 10^{+28}\text{yr}}\right]\left[\left(\frac{W(n)}{W(A_Z)}\right)^5\frac{(ft)_{A_Z}}{(ft)_n}\right] \quad (54\text{a})$$

Here $W(n)$ is the mass difference (1.29 MeV) between a neutron and a proton; $W(A_Z)$ is the *nuclear* mass difference between the isotopes A_z and $A_{(Z+1)}$; $(ft)_{A_Z}$ includes the statistical spin factor; and $(ft)_n = 1.1\times 10^3$s, $t_{1/2}(n) = 6.5\times 10^2$s. For ^{71}Ga one finds

$$\frac{\Gamma(n\to p+\nu_e+\bar\nu_e)}{\Gamma(n\to p+e^-+\bar\nu_e)} = 6\times 10^{-29}P(^{71}\text{Ga};\text{SNU}). \quad (54\text{b})$$

Thus a ^{71}Ga solar neutrino experiment will be sensitive to a non-charge-conserving part of the weak interactions that is more than twenty-six orders of magnitude smaller than the main part of the weak interactions.

There is in principle some ambiguity in interpreting radiochemical solar neutrino experiments in which the neutrino capture threshold is less than the mass of the electron. How can one be sure that a measured counting rate in, for example, the ^{71}Ga experiment is due to solar neutrinos instead of charge nonconservation? There are two answers to this question: one theoretical, one experimental. First, all current theoretical considerations suggest that it is much more likely that the p-p solar neutrino flux is of the predicted order of magnitude than that charge is not conserved at a level that *accidentally* is of the right order of magnitude to be detectable in a solar neutrino experiment. Second, the capture rates in the ^{71}Ga and ^{115}In experiments can be compared. The experiment with ^{115}In is sensitive to $p-p$ and ^{7}Be neutrinos but in the proposed electron counting mode cannot detect charge nonconserving transitions of ^{115}In to ^{115}Sn. If the counting rates in the ^{71}Ga and ^{115}In experiments are consistent with the same solar neutrino fluxes, then the ^{71}Ga experiment may be interpreted additionally in terms of an upper limit on charge nonconserving reactions. Combining the ^{71}Ga and ^{115}In experiments may permit an increase by one order of magnitude in the sensitivity estimated above of the ^{71}Ga experiment to charge nonconservation.

H. Detection of stellar collapses

It is now generally believed by workers in the field that much of the potential energy which is released when stars collapse is emitted in the form of neutrinos (see, e.g., Colgate and White, 1966; Arnett, 1967; Domogatsky and Zatsepin, 1968; Wilson, 1974; Brown, 1977; Schramm, 1978 and references quoted therein). The rate at which optically undetected stellar collapses occur in the galaxy is not known, but plausible estimates might vary from once a year to once in a hundred years. The solar neutrino experiments that have been discussed in this paper can all be used to detect occasional stellar collapses in the galaxy. In fact, it has been suggested (Bahcall, 1977) that the one high result in the ^{37}Cl experiment might be interpreted in terms of a stellar collapse.

The effective flux associated with a stellar collapse at a distance R from the sun can be written

$$\phi_{\mathrm{eff}} \equiv \epsilon Mc^2/4\pi R^2 t\bar{E}\,, \tag{55}$$

where ϵ is the fraction of the rest mass energy Mc^2 that is released in the form of neutrinos of average energy $\bar{E}$, and t is the time between successive collections of the products of the neutrino capture. The distance R out to which a given experiment can detect stellar collapses can be calculated in terms of the above parameters and the design sensitivity, in SNU, of the apparatus. A reasonable criterion for R is that

$$\phi_{\mathrm{eff}}(\bar{E})\sigma(\bar{E}) \geq 3(\phi\sigma)_{\mathrm{av;design}}\,, \tag{56}$$

where $(\phi\sigma)_{\mathrm{av;design}}$ is the average capture rate for solar neutrinos that the experiment is designed to detect. Combining Eqs. (55) and (56) one finds that supernovae can be detected out to a distance

$$R = [\epsilon Mc^2\sigma(\bar{E})/12\pi\bar{E}t(\phi\sigma)_{\mathrm{av;design}}]^{1/2}\,. \tag{57}$$

An appropriate value for $\bar{E}$ is of order 10 MeV with $\epsilon \sim 0.1$ (see Brown 1977).

For the proposed ^{7}Li and ^{71}Ga experiments, it is reasonable to take $(\phi\sigma)_{\mathrm{design}}$ as the rate expected from *pep* and *p-p* neutrinos alone (9.6 SNU and 71.5 SNU, respectively). For the ^{37}Cl experiment it is reasonable to use $(\phi\sigma)_{\mathrm{av;design}} \approx 1.6$ SNU, since this is the rate actually measured by Cleveland and Davis (1978). A reasonable time between successive radiochemical purges is $t \sim 1$ month. The ^{115}In experiment is different from the others mentioned above since it is a counting experiment. At present, it seems likely that a complete ^{115}In experiment would be designed (see Raghavan, 1978) to detect the flux of *p–p* and *pep* solar neutrinos at a rate of approximately one capture per day. This corresponds to substituting t equal to 1 day into Eq. (57) with $(\phi\sigma)_{\mathrm{av;design}} = 5\times10^2$ SNU.

The absorption cross sections at $\bar{E}=10$ MeV have been given for ^{7}Li and ^{37}Cl in Tables II and VII. The ground state to ground state cross sections for ^{71}Ga and ^{115}In at 10 MeV are 0.5×10^{-42} cm^2 and 1.5×10^{-42} cm^2, respectively. It is not possible to estimate accurately their ground state to excited state transitions (see Secs. IV.E and IV.H). The sum of the excited-state cross sections could be a few times larger than the quoted ^{71}Ga and ^{115}In ground-state cross sections (and hence the sensitivity estimates quoted below could be somewhat pessimistic).

I conclude that (with the parameters given above) stellar collapses can be detected to a typical distance of a few kpc with the proposed solar neutrino detectors. The specific values are; ^{7}Li (3kpc), ^{37}Cl (4.5 kpc), ^{71}Ga ($\geq$0.3 kpc), and ^{115}In ($\gtrsim$1 kpc). It is interesting that the detector with the largest guaranteed sensitivity is the ^{37}Cl experiment. (The ^{71}Ga and ^{115}In targets are relatively insensitive to the higher energy neutrinos with $\bar{E}=10$ MeV since the analog state in ^{71}Ge is particle unstable and, in ^{115}Sn, is at too high an excitation energy to be important. For ^{37}Cl, the analog state at 5 MeV dominates the absorption cross sections.) The sensitivity of the ^{37}Cl experiment to stellar collapses is one reason for continuing to process samples from this experiment over an extended period.

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