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Addenda: Inelastic collisions of fast charged particles with atoms and molecules—The Bethe theory revisited [Rev. Mod. Phys. 43, 297 (1971)]*

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Since the appearance of the title paper, a number of new developments have occurred which need to be included in that body of material. We present additional remarks and clarifications which supplement and update numerous aspects of the Bethe theory discussed in the earlier paper. We also bring the bibliography up to date. Plasma stopping power, the z^3 effect, and stopping power for particles at extreme relativistic energies are among the new topics included. We make several comments on Fano's earlier review article, Ann. Rev. Nucl. Sci. **13**, 1 (1963).

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I. INTRODUCTION

This article brings up to date the earlier review by Inokuti (1971), "Inelastic Collisions of Fast Charged Particles with Atoms and Molecules—The Bethe Theory Revisited." Since its publication, further developments have occurred in a number of the areas covered, and, in addition, several new topics have emerged to join the body of literature on the Bethe theory. We summarize such material here and update the earlier list of references. We also offer several comments on other matters discussed by Inokuti (1971) and by Fano (1963). A list of errata to these two papers is included.

The more substantial additions to specific sections of Inokuti's review (1971) include the discussions below of electron impact with very large momentum transfer (Sec. II.D); sum rules pertaining to transfer of momentum, rather than energy (Sec. II.E); minima in the generalized oscillator strength (Sec. II.F); the doubly differential cross sections for electron ejection (Sec. II.K); and distorted-wave analyses (Sec. II.M). The new topics, not covered in the earlier review (Inokuti, 1971), are plasma stopping power (Sec. II.N); ultrarelativistic stopping power (Sec. II.O); neutron stopping power (Sec. II.P); and the dependence of stopping power on charge—the z^3 effect (Sec. II.Q).

Our discussions chiefly concern theoretical matters; documentation of numerical data, experimental or theoretical, is beyond the scope of this article.

II. MAJOR POINTS

A. Alternative definitions of the variable Q

In both Fano's (1963) article [Footnote 5, p. 5, and Eq. (18), p. 9] and Inokuti's (1971) review (p. 303), a stronger emphasis would have been appropriate on the difference between Fano's variable Q [which is called Q_r by Inokuti (1971)] and Bethe's (1933) variable Q. That difference persists even for zero scattering angle,

at which the variables become smallest for given v and E_n. More specifically, note that

$$Q_{\min}(\text{Fano}) = [(mc^2)^2 + E_n^2/\beta^2]^{1/2} - mc^2$$
$$\cong E_n^2/2mv^2, \quad \text{if } E_n \ll mc^2\beta,$$

while

$$Q_{\min}(\text{Bethe}) = E_n^2(1-\beta^2)/2mv^2.$$

The two definitions become indistinguishable for nonrelativistic speeds $v = \beta c \ll c$. Each of them has advantages and disadvantages. Bethe's Q certainly makes the differential cross-section formula [Eq. (2.27) of Inokuti (1971)] simple, but includes explicit dependence on E_n, as seen in Eq. (2.26) of Inokuti (1971). Therefore, in the stopping-power evaluation, the summation over E_n (including integration over continuous spectra) and the integration over Q become mutually dependent. Fano's Q is conceptually clear; it represents the kinetic energy of an ejected electron when its initial binding energy is ignored and has no explicit dependence on E_n. But the differential cross section becomes more complicated [Eq. (16) of Fano (1963)] when expressed in terms of Fano's Q.

B. The influence of a finite angular resolution on the forward energy-loss spectrum

To the discussion on p. 306 of Inokuti (1971) we may add that the precise shape of an energy-loss spectrum depends, in general, upon the geometry of the apparatus used in the measurement. Specifically, Eq. (3.5) applies to an apparatus in which the scattered electrons pass through a circular aperture. For an apparatus with a slit, the energy-loss spectrum has a different functional dependence on the variable ζ (Skerbele *et al.*, 1969).

The second term in the square brackets of Eq. (3.5) should read $(E_n^2/4RT)f_n^{(1)}$. The unnumbered equation below Eq. (3.5) should read $\zeta = 4(\hat{\theta}T/E_n)^2$.

C. Expression for the sum rule $S(-1,0)$

Equation (3.18) of Inokuti (1971) is correct for atomic and molecular states that have no permanent dipole moment. More generally, that equation should read

$$S(-1,0) = \sum_j \sum_k \langle x_j x_k \rangle / a_0^2 - \left(\sum_j \langle x_j \rangle \right)^2 / a_0^2 .$$

For a polar molecule, the second term is not, in general, vanishing, even after averaging over molecular orientations. The discussion by Hirschfelder, Brown, and Epstein (1964) in this respect correctly keeps the second term, and also takes into account the nuclear masses (which, for brevity, we do not treat here). No quantitative assessment of the role of the second term appears to have been made. A similar comment applies to Eq. (3.29).

Note added in proof. Chipman *et al.* (1977) have recently studied this matter for H_2O.

D. Collisions with large momentum transfer

Inelastic collisions of any charged particle resulting in extremely large momentum transfer are basically similar to collisions with a free electron. Then, the momentum transfer and the energy transfer are correlated with each other owing to kinematics. This fact is a key element of the Bethe theory, and manifests itself in the Bethe ridge, as discussed in Sec. III.2 of Inokuti's review (1971). Recent years have seen some progress in the understanding of electron impact with extremely large momentum transfer.

When the momentum transfer $\hbar K$ is so large that $(\hbar K)^2/2m$ greatly exceeds the binding energy of one of the electrons in an atom or molecule, most frequently that electron will be knocked out with very high speed. In the simplest treatment the wave function of the ejected electron may be approximated by a plane wave representing the eigenstate of momentum $\mathbf{p}$. Then the matrix element $\epsilon_n(K)$ in the cross-section expression [Eq. (2.6) of Inokuti (1971)] is written mainly in terms of

$$\Phi_s(\mathbf{p}_r) = \int d\mathbf{r}_Z \exp(i\hbar^{-1}\mathbf{p}_r \cdot \mathbf{r}_Z) \int d\mathbf{r}_1 \cdots d\mathbf{r}_{Z-1}$$
$$u^*_{\text{ion},s}(\mathbf{r}_1, \ldots, \mathbf{r}_{Z-1}) u_0(\mathbf{r}_1, \ldots, \mathbf{r}_{Z-1}, \mathbf{r}_Z),$$

where $\mathbf{p}_r = \hbar\mathbf{K} - \mathbf{p}$ is the recoil momentum received by the ion. Further, $u_{\text{ion},s}$ represents the sth eigenstate of the ion left behind and u_0 is the ground state of the target. If we assume further that $u_{\text{ion},s}$ and u_0 are products of one-electron orbitals, then $\Phi_s(\mathbf{p}_r)$ reduces to a product of two factors, i.e., the overlap integral of spectator electrons $1, \ldots, Z-1$, on the one hand, and, on the other hand, the Fourier transform (i.e., the momentum representation) of the orbital from which the electron is ejected. Clearly, only the second factor depends upon $\mathbf{p}_r$. Similar results are derived from less restrictive treatment, using various versions of impulse approximations. For electron collisions, one must account for electron-exchange effects, and the differential cross section at high incident energy and at high momentum transfer takes the form of the Mott cross section multiplied by a square of $\Phi_s(\mathbf{p}_r)$ (or its linear combination). Consequently, measurements of energy and angular distributions of two electrons, i.e., a scattered electron and an ejected electron, in coincidence may be interpreted largely in terms of the momentum distribution in an atom or molecule, although departures from the simple picture appear with decreasing incident energy and with decreasing momentum transfer. Recent years have seen a considerable growth of the literature on the topic exemplified by a number of studies (Glassgold and Ialongo, 1968; Neudatchin *et al.*, 1968; Amaldi and degli Atti, 1970; Camilloni *et al.*, 1972; Weigold *et al.*, 1975; Ugbabe *et al.*, 1975; Levin *et al.*, 1975; Guidoni *et al.*, 1976; and McCarthy and Weigold, 1976).

Collisions with large momentum transfer $\hbar K$ seldom lead to lower excitations, which imply modest energy transfer. This observation is seen in the Bethe theory as the rapid drop of the generalized oscillator strength for any low excitation with $\hbar K$ (cf. Sec. III.2 of Inokuti, 1971). Kelsey (1976) points out, however, that the scattering amplitude for this class of collisions *evalu-*

ated in the second Born approximation contains a component behaving as $(\hbar K)^{-2}$ due to the Rutherford scattering by nuclei. This component declines with $\hbar K$ much less rapidly than the first Born component and may often govern the behavior of measured differential cross sections at large scattering angles.

E. Various integrals of the generalized oscillator strength with respect to the momentum transfer

Section III.3 of Inokuti's review (1971) treats integrals (including sums over discrete spectra) of the generalized oscillator strength with respect to the *energy transfer*, i.e., the usual sum rules. Here we point out another family of relations involving integrals with respect to the *momentum transfer* and discuss applications of some of these relations.

We begin with the definition

$$G_n(\lambda) = \int (Ka_0)^\lambda f_n(K) d(Ka_0) , \tag{1}$$

where $f_n(K)$ is the generalized oscillator strength for excitation to state n (either discrete or continuum) at momentum transfer $K\hbar$. The integration domain is taken over all positive values of Ka_0 for $0 \le \lambda < 9 + 2(l + l')$, where l and l' are the orbital angular momentum quantum numbers of the active electron in the initial and final atomic states, respectively. For larger values of λ, the integral diverges because of the asymptotic behavior $f_n(K) = O[K^{-10-2(l+l')}]$ for large K (Rau and Fano, 1967). For $\lambda < 0$, the integrand becomes singular in general at $K = 0$; then we consider the integral with a small but positive lower limit $K_{\min}$, in accordance with the physical fact that the momentum transfer for any inelastic collision is never zero (p. 301 of Inokuti, 1971). [A comprehensive treatment of $G_n(\lambda)$ will be published by M. Matsuzawa, S. Mitsuoka, and M. Inokuti; what follows is an excerpt from this work.]

Using Eqs. (2.5) and (2.9) of Inokuti (1971) and formally carrying out the K integration first, we can write

$$G_n(\lambda) = \frac{E_n}{R} \int d\mathbf{r} \rho_{0n}(\mathbf{r}) \int d\mathbf{r}' \rho_{n0}(\mathbf{r}') g(\lambda, \mathbf{r} - \mathbf{r}') , \tag{2}$$

where we define

$$g(\lambda, \mathbf{r} - \mathbf{r}') = a_0^{\lambda-1} (4\pi)^{-1} \int d\mathbf{K} K^{\lambda-4} \exp[i\mathbf{K} \cdot (\mathbf{r} - \mathbf{r}')] , \tag{3}$$

and

$$\rho_{0n}(\mathbf{r}) = (0 | \sum_{j=1}^{Z} \delta(\mathbf{r}_j - \mathbf{r}) | n) \tag{4}$$

is the one-electron density for the transition between the ground state 0 and the state n. In the derivation of Eqs. (2) and (3) and throughout the present discussion, we presume that the atoms or molecules are either spherical or randomly oriented and imply that all matrix elements have been averaged over magnetic sublevels of the ground state and summed over magnetic sublevels of the state n; this stipulation permits one to replace $\int dK$ by $(4\pi)^{-1} \int d\mathbf{K} K^{-2}$.

Thus we see that $g(\lambda, \mathbf{r} - \mathbf{r}')$ is the object of study here and is independent of the target species. By virtue of the orthogonality of the states 0 and n, we may add to $g(\lambda, \mathbf{r} - \mathbf{r}')$ any constant or any function of $\mathbf{r}$ or $\mathbf{r}'$ only without affecting the value of $G_n(\lambda)$. The integral of Eq. (3) is improper in general but may be readily interpreted in the sense of generalized functions (Lighthill, 1958). For example, some specific forms of $g(\lambda, \mathbf{r} - \mathbf{r}')$ are as follows:

$$g(4, \mathbf{r} - \mathbf{r}') = 2\pi^2 a_0^3 \delta(\mathbf{r} - \mathbf{r}') , \tag{5}$$

$$g(3, \mathbf{r} - \mathbf{r}') = (a_0 / |\mathbf{r} - \mathbf{r}'|)^2 , \tag{6}$$

$$g(2, \mathbf{r} - \mathbf{r}') = (\pi/2)(a_0 / |\mathbf{r} - \mathbf{r}'|) , \tag{7}$$

$$g(1, \mathbf{r} - \mathbf{r}') = -\ln(|\mathbf{r} - \mathbf{r}'| / a_0) , \tag{8}$$

$$g(0, \mathbf{r} - \mathbf{r}') = -(\pi/4) |\mathbf{r} - \mathbf{r}'| / a_0 . \tag{9}$$

Insertion of each of these explicit forms into Eq. (2), together with Eq. (1), gives a nontrivial relation between the generalized oscillator strength and the transition density. Some of those relations and their variants, e.g., for elastic scattering form factors, have been discussed in the literature (Silverman and Obata, 1963; Tavard and Roux, 1965; Tavard *et al.*, 1965; Kohl and Bonham, 1967; Bonham, 1967; Sahni and Krieger, 1972; Lassettre and Dillon, 1973; and O'Connell and Lightbody, 1975). A remarkable application has been found by Lassettre and Dillon (1973) for $\lambda = 2$. According to Eqs. (2) and (7), the essential factor in $G_n(2)$ is the integral

$$I = \int d\mathbf{r} \rho_{0n}(\mathbf{r}) \int d\mathbf{r}' \rho_{n0}(\mathbf{r}') |\mathbf{r} - \mathbf{r}'|^{-1} . \tag{10}$$

If one considers a single-electron excitation from a closed-shell ground state, then one can equate, within a certain approximation, I with the exchange integral well known in spectroscopy and then relate I with the singlet–triplet splitting in the excited state n.

For $\lambda < 0$, we set the lower limit in Eq. (1) or Eq. (3) at $K_{\min} > 0$, as we stated before. Let us consider $\lambda = -1$ as an example. Through repeated partial integrations, it is possible to show (Matsuzawa and Inokuti, 1976) that

$$g(-1, \mathbf{r} - \mathbf{r}') = a_0^{-2} \int_{K_{\min}}^{\infty} dK K^{-3} \frac{\sin(K|\mathbf{r} - \mathbf{r}'|)}{K|\mathbf{r} - \mathbf{r}'|}$$

$$= \tfrac{1}{2}(K_{\min} a_0)^{-2} + [-\tfrac{11}{36} + \tfrac{1}{6}\gamma + \tfrac{1}{6}\ln(K_{\min} a_0)](|\mathbf{r} - \mathbf{r}'|/a_0)^2 + \tfrac{1}{6}(|\mathbf{r} - \mathbf{r}'|/a_0)^2 \ln(|\mathbf{r} - \mathbf{r}'|/a_0) + O(K_{\min}^2 a_0^2) , \tag{11}$$

where $\gamma = 0.5772$ is the Euler constant. When we insert Eq. (11) into Eq. (2), the first term of Eq. (11) gives zero contribution because of the orthogonality of the states 0 and n. Further we note that

$$\int d\mathbf{r} \rho_{0n}(\mathbf{r}) \int d\mathbf{r}' \rho_{n0}(\mathbf{r}') (|\mathbf{r} - \mathbf{r}'|/a_0)^2 = -2 |(n| \sum_{j=1}^{Z} \mathbf{r}_j |0)|^2 / a_0^2 = -6M_n^2 , \tag{12}$$

where M_n^2 is the squared dipole matrix element in atomic units, as defined by Eq. (2.11) of Inokuti (1971). Thus we may write

$$G_n(-1)=(E_n/R)\left\{[-\ln(K_{\min}a_0)+\tfrac{11}{6}-\gamma]M_n^2+\frac{1}{6}\int d\mathbf{r}\rho_{0n}(\mathbf{r})\int d\mathbf{r}'\rho_{n0}(\mathbf{r}')\left(\frac{|\mathbf{r}-\mathbf{r}'|}{a_0}\right)^2\ln\left(\frac{|\mathbf{r}-\mathbf{r}'|}{a_0}\right)+O(K_{\min}^2a_0^2)\right\}. \tag{13}$$

Actually, $G_n(-1)$ is the essential factor in the integrated cross section as defined by Eq. (4.1) of Inokuti (1971), and Eq. (13) is an alternative form of the Bethe asymptotic cross section, i.e., Eq. (4.15) of Inokuti (1971). The major distinction is that Eq. (13) expresses the cross section in terms of the transition density directly and thus bypasses the generalized oscillator strength. Equation (13) not only gives a general insight into the cross-section formula, but also may prove useful in numerical work, especially because some theories are now beginning to give as an output the transition density (as opposed to wave functions) of atoms and molecules. [See, for example, McCurdy and McKoy (1974), Yeager *et al.* (1975), and Chang and Fano (1976)].

The quantity $G_n(\lambda)$ for $\lambda<0$ has other applications, as we illustrate below.

The angular distribution and the polarization of light emitted by targets excited by a fast charged particle are governed by the angle χ defined by

$$\cos\chi=\mathbf{k}\cdot(\mathbf{k}-\mathbf{k}')/[k|\mathbf{k}-\mathbf{k}'|]=(\mathbf{k}\cdot\mathbf{K})/kK, \tag{14}$$

where $\hbar\mathbf{k}$ and $\hbar\mathbf{k}'$ are the momenta of the particle before and after the collision [p. 512 of Bethe (1933); Vriens and Carrière (1970); Fano and Macek (1973); and McFarlane (1974)]. The importance of this angle χ is readily apparent if one recalls that the target response is specified within the first Born approximation by the momentum transfer $\hbar\mathbf{K}=\hbar(\mathbf{k}-\mathbf{k}')$ and the measurement always uses the incident momentum $\hbar\mathbf{k}$ as a reference direction. By simple algebra, one can express $\cos^2\chi$ as

$$\cos^2\chi=\frac{1}{4}\left(\frac{m}{M}\right)^2\frac{R}{T}\,(Ka_0)^{-2}\left((Ka_0)^2+\frac{ME_n}{mR}\right)^2$$
$$=\left(\frac{E_n^2}{4RT}\right)\left[(Ka_0)^{-2}+\left(\frac{m}{M}\right)^2\left(\frac{R}{E_n}\right)^2(Ka_0)^2\right]+\frac{mE_n}{2MT}, \tag{15}$$

where E_n is the excitation energy of state n and M is the mass of the charged particle. What counts in practice is the average of $\cos^2\chi$ over different momentum transfers, i.e., the integral

$$\int d\sigma_n\cos^2\chi,$$

$d\sigma_n$ being the differential cross section given by Eq. (2.14) of Inokuti (1971). This integral thus reduces to $G_n(-3)$, $G_n(1)$, and $G_n(-1)$, the last of which is related to the integrated cross section we have already discussed. The quantity $G_n(1)$ may be treated by use of Eq. (8). Consequently, only $G_n(-3)$ calls for a new analysis (Matsuzawa, Mitsuoka, and Inokuti, to be published).

Finally, the sum

$$\sum_n\int d\sigma_n\cos^2\chi$$

(including the integration over continuous spectra as well as contributions from elastic collisions, i.e., $n=0$) gives an average deflection of a beam of charged particles transversing matter [Fox (1970) and p. 169 of Dettmann (1971)].

F. Minima of the generalized oscillator strength

The general significance of the minima was discussed in Sec. III.4 of Inokuti's review (1971). Since then, many examples of the minima have been studied both experimentally and theoretically, e.g., in the papers by Hanne and Kessler (1972), Miller (1973), Kim and Bagus (1973), Robb (1974), Wong *et al.* (1975), and Matsuzawa *et al.* (1976). A new topic concerns the consequence of the minima on the *integrated* cross section for inner-shell ionization by protons and other heavy particles (Nikolaev and Kruglova, 1971; Kruglova *et al.*, 1973; Kruglova and Nikolaev, 1974; Merzbacher, 1973; Choi, 1973; Datz *et al.*, 1974; and Manson and Msezane, 1975). The integrated cross section is given by an integral of the generalized oscillator strength with respect to $\ln(Ka_0)^2$ [Eq. (4.1) of Inokuti (1971)], the lower and upper limits of the integral being

$$(K_{\min}a_0)^2\cong E_n^2/4RT$$

and

$$(K_{\max}a_0)^2\cong(4T/R)(M/m)^2.$$

For inner-shell ionization, the excitation energy E_n is very great, and therefore $K_{\min}$ often becomes so great that $(K_{\min}a_0)^2>E_n/R$; then minima of the generalized oscillator strength at extremely large K (which are difficult to observe in the angular distribution of scattered particles) manifest themselves in the dependence of the integrated cross section upon the incident particle velocity v, or equivalently upon $T=mv^2/2$. With increasing T, $K_{\min}$ decreases steadily and may pass any one of the minima of the generalized oscillator strength; then the integrated cross section fails to increase (as it would do in the absence of such a minimum). Therefore, one expects to see a shoulder in the T dependence of the inner-shell ionization cross section if the corresponding generalized oscillator strength has minima. This expectation has been borne out for the ionization from the $2s$ inner subshell, where the generalized oscillator strength usually has a minimum attributable to the node of the $2s$ orbital (Datz *et al.*, 1974; Manson and Msezane, 1975). In contrast, the ionization from the $2p$ inner subshell has no minimum in the generalized oscillator strength and no shoulder occurs in the T dependence of the integrated cross section.

G. Recent calculations of the generalized oscillator strength

To supplement the discussion near the end of Sec. III.4 of Inokuti's review (1971), we point out here recent calculations that specifically include electron correlation effects and appear to be accurate. Our listing of references is representative rather than exhaustive.

The first class of calculations uses the traditional method of the superposition of configurations to obtain accurate eigenfunctions and thence the generalized oscillator strength. Typical studies of this class include those of Kim and Bagus (1973), Robb (1974), Banyard and Taylor (1974), and Wells and Miller (1975).

The second class is exemplified by the work of Davis and Sinanoğlu (1975), who apply a many-electron theory emphasizing intrashell pair correlations.

The third class is the application of another technique of many-electron problems, i.e., the random-phase approximation. A review of the current status has recently been published by Amusia and Cherepkov (1975). Together with the work of McKoy and co-workers [as seen in the paper by McCurdy and McKoy (1974)], this approach differs from the traditional calculations in directly obtaining the generalized oscillator strength without constructing a pair of eigenfunctions first.

Finally, the energy and angular distribution of electrons ejected in an ionizing collision has been studied in great detail for He (Jacobs, 1974; Robb *et al.*, 1975; Burnett *et al.*, 1976), other rare gases (Amusia and Cherepkov, 1975), and atomic oxygen (Burnett, 1976). The studies on He (Jacobs, 1974; Robb *et al.*, 1975; Burnett *et al.*, 1976) use accurate eigenfunctions for both the initial and final states and may be regarded as being as trustworthy as the earlier work on discrete excitations (Kim and Inokuti, 1968 and 1969).

H. The Renner-Teller effect and the Jahn-Teller effect in the generalized oscillator strength

Section III.5 of Inokuti's review (1971) fails to point out some manifest departures from the Born–Oppenheimer separation of electronic and nuclear motions in molecules. In a non-Σ electronic state of a linear polyatomic molecule, vibronic couplings may cause the Renner–Teller effect (Herzberg, 1966). Some consequences of this effect in inelastic collision cross sections have been discussed (Kiselev *et al.*, 1971). A study of the generalized oscillator strength, especially its dependence on the momentum transfer, is likely to bring forth deeper understanding of the Renner–Teller effect than has been obtained so far solely by optical means. The same remark applies to the Jahn–Teller effect (Herzberg, 1966) in a nonlinear polyatomic molecule. No extensive work seems to have been reported yet.

I. Generalized oscillator strength for a purely vibrational excitation of a molecule

Expression (3.47) of Inokuti (1971) is correct only when electronic and vibrational transitions occur simultaneously (i.e., $n \neq n_0, v \neq v_0$). For a purely vibrational excitation ($n = n_0, v \neq v_0$), the nuclear contribution must be retained in the integrand of Eq. (3.47). That is, the expression for the electronic part of the form factor should read

$$\epsilon_n(\mathbf{K}; \mathbf{R}_1, \ldots, \mathbf{R}_n) = \int w^*_{n_0}(\mathbf{r}_1, \ldots, \mathbf{r}_Z; \mathbf{R}_1, \ldots, \mathbf{R}_N) \times \left[\sum_{j=1}^{Z} \exp(i\mathbf{K}\cdot\mathbf{r}_j) - \sum_{l=1}^{N} z_l \exp(i\mathbf{K}\cdot\mathbf{R}_l)\right] \times w_{n_0}(\mathbf{r}_1, \ldots, \mathbf{r}_Z; \mathbf{R}_1, \ldots, \mathbf{R}_N) d\mathbf{r}_1 \cdots d\mathbf{r}_Z ,$$

where z_l and $\mathbf{R}_l$ are, respectively, the charge and the position vector of the lth nucleus. For a diatomic molecule, $\mathbf{R}_l (l = 1, 2)$ can be reduced to the internuclear separation ρ. More details on this topic are given by Itikawa (1973).

Bonham and Geiger (1969) expanded the form factor for vibrational excitation in power series in K and showed that the expansion coefficient can be expressed in terms of the moments of the molecular charge distribution. [Some errors in their paper were corrected by Bonham (1975).]

J. The continuity of the generalized oscillator strength across an ionization threshold

According to the quantum-defect theory, atomic eigenfunctions of successively higher discrete states of a given symmetry connect smoothly as a function of excitation energy E with continuum eigenfunctions belonging to the same symmetry at the beginning of the continuum, i.e., at the ionization threshold. Therefore, the matrix element of any operator (presumed to be independent of E) for successively higher discrete excitations from a fixed initial state (e.g., the ground state) must connect smoothly with the corresponding matrix element for continuum excitation. The generalized oscillator strength whose essential part is the matrix element $\epsilon_n(K)$ as seen in Eq. (2.8) of Inokuti (1971) has the continuity property, and so does the dipole oscillator strength. Further, all quantities that depend linearly upon the generalized oscillator strength, e.g., the integrated cross section σ_n in the first Born approximation [Eq. (4.1) of Inokuti (1971)] must have the continuity property.

The above point was made by Inokuti (1971) [see Footnote 4 on p. 310 and the commentary on Eqs. (4.23) and (4.24) on p. 327], but a stronger emphasis would have been in order. Indeed, it has been demonstrated (Kim and Inokuti, 1973) that the continuity property of the generalized oscillator strength provides a powerful test of consistency of data on ionization with those on discrete excitation.

Actually, analyses of the dipole oscillator strength according to the quantum-defect theory have proven powerful and successful, especially in bringing out hidden systematics for cases when multichannel effects are important (Lu, 1971; Fano, 1970, 1975). Extension of those studies to the generalized oscillator strength is highly desirable.

K. Some properties of the cross sections for secondary-electron ejection

An analysis (Kim, 1972) of the angular distribution of secondary electrons resulting from ionizing collisions supplements the discussion of Sec. IV.1, IV.2, and V.3 of Inokuti's article (1971). Let $d^2\sigma/dE\,d\Omega$ be the (doubly) differential cross section for the ejection of electrons into the infinitesimal ring $d\Omega = 2\pi \sin\Theta d\Theta$ at angle Θ measured from the direction of incidence of a fast charged particle. Then a plot of $(T/R)(4\pi a_0^2 z^2)^{-1} d^2\sigma/dE\,d\Omega$ vs $\ln T$ (at fixed E and Θ) should approach a straight line at sufficiently great T. Further, the asymptotic slope on that plot at fixed E depends upon Θ in a simple analytic form, which is identical to the angular distribution of electrons emitted after absorption of unpolarized photons with energy E. This relation enables one to test consistency of secondary-electron data with photoelectron data (Kim, 1972; Hamnett *et al.*, 1976).

The cross section $d\sigma/dE$ per unit range of excitation energy E in a continuum is written as Eq. (4.22) of Inokuti (1971). If the (quantum) yield of ionization $\eta(E)$ is known, $\eta(E)d\sigma/dE$ gives the energy distribution of secondary electrons (integrated over the angle Θ of ejection). To study this distribution, it is useful to consider its ratio to the Rutherford cross section

$$d\sigma_{\text{Ruth}}/dE = 4\pi a_0^2 z^2 T^{-1}(R/E)^2$$

for energy transfer E to a single target electron. Several general properties of the ratio, which may be written as

$$Y(E, T) = T(E/R)^2 \eta(E)(d\sigma/dE)/(4\pi a_0^2 z^2),$$

are noteworthy. For E greatly exceeding the relevant ionization threshold I, $Y(E, T)$ should approach a constant, which represents an effective number of target electrons. For E up to a few multiples of I, the E dependence of $Y(E, T)$ should resemble that of $E\eta(E)df/dE$, i.e., the photoionization cross section (at photon energy E) multiplied by E, according to Eq. (4.22) of Inokuti (1971). Further, if one plots $Y(E, T)$ against R/E, the area under the curve represents contributions to the total ionization cross section σ_i [Eq. (4.29) of Inokuti (1971)] from each interval of R/E. Kim and coworkers (Kim 1975a, b, c; Kim and Noguchi, 1975; Tuckwell and Kim, 1976) have successfully exploited these and other properties of $d\sigma/dE$, first for testing the consistency of various experimental data, and eventually for constructing comprehensive and trustworthy sets of $d\sigma/dE$ to be used in many applications.

The validity of the first Born approximation has been recently examined in a comprehensive study of $d^2\sigma/dE\,d\Omega$ for proton impact on helium (Manson *et al.*, 1975). Similarly extensive studies on other targets are highly desirable.

Finally, the general topic of secondary-electron spectra resulting from ionic collisions has seen considerable progress, as reviewed by Ogurtsov (1972) and by Rudd and Macek (1972). As already noted in Sec. V.2 of Inokuti's review (1971), a notable departure from the first Born approximation is seen in the forward peaking of secondary electrons that have velocities comparable to the velocity of an incident (positive) ion. This peaking is often referred to as charge transfer to the continuum. Section 6 of Rudd and Macek (1972) excellently summarizes the findings about this phenomenon. As for the importance of this phenomenon, Kim (1975b) has pointed out that the charge transfer to the continuum accounts for some 25% of the total ionization cross section for incident protons of 100–500 keV, in excess of the same cross section for incident electrons of the same speed. (It is suspected that, for negative massive particles such as π^- and μ^-, the secondary-electron spectra may show a depression in the forward direction for electrons having velocities similar to the velocity of the incident particles.) Parts of the z^3 effect, discussed extensively in Sec. II.Q, should be related to the forward peaking effect. However, the relation between the two effects has not been elucidated yet.

L. Condition for the Morse formula for the differential cross section for total inelastic scattering

The condition "$R/T \ll \theta \ll 1$" given in the line immediately above Eq. (4.69) of Inokuti (1971) should be replaced by "$\frac{1}{2}(m/M)(E_n/T) \ll \theta \ll 1$ for the majority of states n to be considered." To verify the condition, we may inspect Eq. (3.7) of Inokuti (1971); if $\frac{1}{2}\xi \ll \theta \ll 1$, then $\chi \cong \theta^2/2$ and hence we obtain Eq. (4.69) from Eq. (2.16).

M. The significance of the form-factor concept in distorted-wave approximations

In an extension of the discussion about the Coulomb–Born approximation on p. 341 of Inokuti (1971), let us consider the matrix element

$$(\mathbf{k}'n|V'|\mathbf{k}0) = \int [\psi^{(+)}_{\mathbf{k}'}(\mathbf{r})]^* u_n^*(\mathbf{r}_1, \ldots, \mathbf{r}_Z) V' \psi^{(-)}_{\mathbf{k}}(\mathbf{r}) \times u_0(\mathbf{r}_1, \ldots, \mathbf{r}_Z) d\mathbf{r}_1 \ldots d\mathbf{r}_Z d\mathbf{r}\,.$$

Here u_0 and u_n represent the initial state and the final state of the target, respectively, and $\psi^{(-)}_{\mathbf{k}}(\mathbf{r})$ and $\psi^{(+)}_{\mathbf{k}'}(\mathbf{r})$ may be Coulomb wave functions for the incident particle before and after the collision. But $\psi^{(-)}_{\mathbf{k}}(\mathbf{r})$ and $\psi^{(+)}_{\mathbf{k}'}(\mathbf{r})$ may be more generally distorted waves constructed as appropriate. The plus sign refers to the outgoing-wave boundary condition, and the minus sign to the incoming-wave boundary condition. The symbol V' represents the potential responsible for the inelasticity and its relevant part is the Coulomb interaction between the incident particle at position $\mathbf{r}$ on the one hand and the target electrons at position $\mathbf{r}_j$ on the other hand. Following the treatment of Sec. 2.3 of Fano (1963), we express the Coulomb interaction as a Fourier integral

$$\sum_{j=1}^{Z} ze^2/|\mathbf{r}_j - \mathbf{r}| = (2\pi^2)^{-1} ze^2 \sum_{j=1}^{Z} \int d\mathbf{q}\, q^{-2} \exp[-i\mathbf{q}\cdot(\mathbf{r} - \mathbf{r}_j)]\,.$$

By the use of this expression, we can write (Fano and Inokuti, 1976)

$$(\mathbf{k}'n|V'|\mathbf{k}0) = (2\pi^2)^{-1} z e^2 \int d\mathbf{q}\, q^{-2} (\mathbf{k}'|\exp(-i\mathbf{q}\cdot\mathbf{r})|\mathbf{k}) \times (n|\sum_{j=1}^{z} \exp(i\mathbf{q}\cdot\mathbf{r}_j)|0) .$$

The second factor

$$(n|\sum_{j=1}^{z} \exp(i\mathbf{q}\cdot\mathbf{r}_j)|0)$$

in the integrand above is the usual form factor $\epsilon_n(q)$, which is a target property. The first factor $(\mathbf{k}'|\exp(-i\mathbf{q}\cdot\mathbf{r})|\mathbf{k})$ is a matrix element between the distorted-wave states of the particle. In the (plane-wave) Born approximation, it reduces to $\delta(-\mathbf{k}'-\mathbf{q}+\mathbf{k})$ apart from a normalization factor, and specifies uniquely the momentum transfer $(\mathbf{k}-\mathbf{k}')\hbar$ to the target. In any distorted-wave approximation, $(\mathbf{k}'|\exp(-i\mathbf{q}\cdot\mathbf{r})|\mathbf{k})$ expresses a *distribution* of momentum transfer $\mathbf{q}$ to the target; physically, the distribution arises because the use of the distorted waves amounts to letting the incident particle change its momentum in the field of the target. Yet, the response of the target to each value of $\mathbf{q}$ is still described by the same form factor as in the Bethe theory. Additional remarks are given by Fano and Inokuti (1976).

The factorization of $(\mathbf{k}'n|V'|\mathbf{k}0)$ enables one to separate the issue of target wave functions (generally unknown) from the validity of any version of distorted-wave approximations. A suggested application is to evaluate $(\mathbf{k}'|\exp(-i\mathbf{q}\cdot\mathbf{r})|\mathbf{k})$ as soon as the distorted waves are determined, and then to use trustworthy data on form factors $\epsilon_n(q)$, either theoretical or experimental, in the factorized formula. An analysis of distorted-wave approximations along this line and others seems to be amply warranted in view of their recent successful applications (Madison and Shelton, 1973; Calhoun *et al.*, 1976; Thomas *et al.*, 1974; Chutjian and Thomas, 1975; Baluja and Taylor, 1976).

N. The stopping power of a plasma

The method of Secs. 2.1–2.11 of Fano's article (1963) can be adapted to the calculation of the energy loss of a heavy charged particle moving through a plasma. A difference appears in the low-Q range, where the nature of a plasma as a continuous medium should be fully taken into account. Replacing Formula (35) of Fano (1963) by the contributions from the collective excitation [Eqs. (45) and (47)] and using the dielectric constant of a plasma

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - i\omega\nu} \quad (\nu \to +0) ,$$

one obtains the stopping power of a plasma in the form

$$-\frac{dE}{ds} = \frac{4\pi z^2 e^2}{mv^2} N_e \left[\ln\frac{2mv^2}{\hbar\omega_p} + \frac{1}{2}\ln\frac{1}{1-\beta^2} - \frac{1}{2}\beta^2\right] .$$

Here $\omega_p = (4\pi N_e e^2/m)^{1/2}$ is the plasma frequency and N_e is the number density of the plasma electrons. With the use of different methods this formula has been derived by Tsytovich (1962), Gould (1972a, 1972b, 1972c), and Vriens (1973). A corresponding formula for the incident particle with a nonrelativistic velocity has been obtained also by Larkin (1959) and by Honda (1964). It should be noted that the above formula is valid only for $v \gg e^2/\hbar$. Otherwise, the close collision needs to be considered more carefully [see, for example, Honda (1964) and Gould (1972b)].

If $\hbar\omega_p$ is regarded as a mean excitation energy of the plasma, the above formula for the stopping power can be understood easily. As is stated in Sec. 2.11 of Fano's (1963) article [specifically in the passage immediately after Eq. (47) on pp. 20 and 21], the low-Q transverse excitations in a plasma are restricted to the spectral range $\mathrm{Re}\,\epsilon(\omega) > 1/\beta^2$. This condition is not satisfied by the dielectric constant used here; as a consequence, the collective transverse excitations cannot be excited in such a plasma. For this reason, the stopping power reduces by the amount $(4\pi z^2 e^2 N_e/mv^2)[-\frac{1}{2}\ln(1-\beta^2) - \frac{1}{2}\beta^2]$, compared with the usual formula [Eq. (38) of Fano (1963) or Eq. (4.64) of Inokuti (1971)].

Finally, it should be noted that the above formula for the dielectric constant of a plasma has been derived on the assumption of a "cold" plasma. The thermal motion of plasma particles has little effect on the stopping power, unless the velocity of the incident particle is comparable to or smaller than the mean thermal velocity of electrons in the plasma.

O. The stopping power at extreme relativistic energies

An ion colliding with an atomic electron may be treated as a point particle when the impact parameter is large compared with atomic dimensions. Bethe's relativistic stopping-power formula for the distant ("low-Q") collisions, Eq. (35) of Fano (1963), thus applies to all types of charged particles and is valid at very high energies. In contrast, the formula for high-Q collisions, Eq. (37) of Fano (1963), was derived by making use of his relation (7), which is equivalent to writing $\gamma m/M \ll 1$, where $\gamma = (1-\beta^2)^{-1/2}$. The Bethe theory is thus restricted to heavy charged particles below certain energies, e.g., $\lesssim$10 GeV for muons and $\lesssim$100 GeV for protons, and the usual relativistic stopping-power formula does not depend on the spin or structure of the incident particle. Under the condition $\gamma m/M \ll 1$, the incident particle can lose only a small fraction of its energy in a collision with an atomic electron. Use of the small-recoil approximation [p. 6 of Fano's article (1963)] and of the small-angle scattering formulas simplifies the theory considerably.

At extremely high energies, a particle can lose a large fraction of its energy in a single collision and be scattered through a large angle. The high-Q contribution to stopping power at such energies can be calculated by utilizing the appropriate formula for electron scattering from a given particle. This procedure was carried out for protons and muons (Turner *et al.*, 1969). The characteristics of the incident particle itself enter the theory in a straightforward manner. The Rosenbluth formula (1950) was used to calculate the high-Q contribution to stopping power for ultrarelativistic protons. The additional terms, not present in the Bethe theory, depend on the spin, anomalous magnetic moment, and distributions of charge and magnetic moment. To calculate the effects of the proton's structure, the form factors of Hand *et al.*

(1963) were used by Turner *et al.* (1969). The new terms, which are all relatively small in magnitude when $\gamma \leq 10^4$, tend to offset one another; the force due to the spin of the proton increases the stopping power, but the finite extensions of the particle's charge and magnetic moment decrease it. The net effect is a decrease in stopping power of about 10% when the proton energy is 10^4 GeV.

The stopping power for muons at ultrarelativistic energies was obtained by adapting the proton formula to a point Dirac particle with no anomalous magnetic moment. The calculated ultrarelativistic decrease is about 10% for muons of energy 10^3 GeV (Turner *et al.*, 1969).

The differential electron scattering cross section of Gourdin (1963) was used to calculate ultrarelativistic stopping power for the deuteron (Vora and Turner, 1970). Form factors associated with the distributions of the deuteron's charge, electric quadrupole moment, and magnetic dipole moment were employed. At an energy 2×10^3 GeV, the ultrarelativistic effects decrease the deuteron stopping power at about 8%.

Following this work, a general formulation of the stopping-power problem for a nucleon was carried out without the restriction $\gamma m/M \ll 1$. Equation (32) of Turner *et al.* (1973a) gives the ultrarelativistic generalization of Eq. (16) of Fano (1963).

In practice, these ultrarelativistic effects would appear to be outweighed by other factors, such as the density effect [as shown by Turner *et al.* (1969) and by Vora and Turner (1970)] and radiative corrections. Furthermore, stopping power itself may no longer be a relevant quantity when significant amounts of energy are lost by means other than collisions with atomic electrons [cf. Sec. 2.12 of Fano's review (1963)]. A brief review of calculations of charged particles through matter at very high energies is given by Turner (1970).

P. Neutron stopping power

Although uncharged, a neutron can lose energy to atomic electrons by virtue of its magnetic moment and internal structure. A semiclassical calculation (Vora *et al.*, 1971) indicated that the stopping power for a point neutron is some five orders of magnitude less than that for the proton when $\gamma = (1-\beta^2)^{-1/2} = 10$. A quantum-mechanical calculation of neutron stopping power has subsequently been carried out (Turner *et al.*, 1973b). The high-Q contribution can be obtained readily from the corresponding formula for the proton, i.e., Eq. (19) of Turner *et al.* (1969), or computed from the ultrarelativistic generalization of the theory applied to nucleons (Turner *et al.*, 1973a). Analysis of the low-Q contribution shows that it is negligible at neutron energies $>4Z^{0.35}$ GeV, where Z is the atomic number of the medium traversed. The quantum-mechanical result for neutron stopping power shows a weaker dependence on energy than the semiclassical formula, although the two formulations give comparable results in the energy range 5–10 GeV. Numerical calculations were performed up to 7000 GeV by Turner *et al.* (1973b), without the inclusion of the density effect or radiative corrections.

Similarly, a moving magnetic monopole would lose energy to atomic electrons, and an extension of the Bethe treatment to the magnetic monopole is possible. Evidence for the detection of a magnetic monopole has been presented (Price *et al.*, 1975), but has met with mixed reception by other workers.[1]

Q. Dependence of stopping power on the charge of an incident particle[2]

The Bethe theory of stopping power assumes that the speed of the ion is much greater than that of the atomic electrons and uses the first Born approximation, according to which the cross section for inelastic collisions is proportional to $(ze)^2$, the square of the charge of the incident particle. (Incident electrons and positrons are excluded from this discussion.) Because successive Born approximations yield an expansion of scattering amplitudes in powers of the coupling constant for the interaction between the incident particle and the atomic electrons, inclusion of the second Born approximation in the calculation of stopping power gives an additional term proportional to z^3. The presence of the z^3 contribution would manifest itself experimentally in at least two ways. First, the stopping powers and ranges of particles of the same mass but opposite charge (e.g., pions, muons, proton–antiprotons, etc.) would be different. Second, the ratio of stopping powers for particles of the same velocity would not be exactly equal to the ratio of the squares of their charges. These points were mentioned on p. 23 of Fano's article (1963) and on p. 340 of Inokuti's review (1971). Considerable developments made after 1971 call for an amplified discussion here.

During the 1960s rather clear experimental evidence for the existence of departures from the z^2 dependence predicted by the Bethe theory was accumulated (Barkas *et al.*, 1963; Heckman and Lindstrom, 1969; Heckman 1970). Ashley, Ritchie, and Brandt (Ashley *et al.*, 1970, 1972, 1973) carried out a nonrelativistic calculation of the additional z^3 contribution to stopping power. They assumed that the incident particle moves in a straight-line trajectory and treated the electrons of the target medium classically as isotropic harmonic oscillators. As in Bohr's semiclassical treatment, one can identify z^3 contributions to stopping power that result from both "distant" and "close" collisions, depending upon whether the atomic binding of the target electrons is important. The classical terminology of "distant" and "close" corresponds in Bethe's quantum-mechanical theory to "low" and "high" momentum-transfer collisions. The Ashley, Ritchie, and Brandt analysis shows that the z^3 contribution arises principally from distant collisions, while close collisions contribute mainly as described nonrelativistically by the Rutherford scattering formula, which is proportional to z^2. Using the Lenz–Jensen statistical model of the atom to determine the distribution of oscillator strengths for the target atoms, they obtained ex-

[1] See, for example, Phys. Today **28**, No. 10, 17 (1975).

[2] The authors are grateful to R. H. Ritchie, W. Brandt, and J. C. Ashley for substantial contributions to this section.

cellent agreement with the data of Andersen, Simonsen, and Sørensen (1969). Measurements made by Sellers, Hanser, and Kelley (Sellers *et al.*, 1973; Kelley *et al.*, 1973) show further evidence of the deviation of stopping power from a strict z^2 dependence in the energy range of a few MeV/amu. Using the z^3 correction of Ashley, Ritchie, and Brandt (1972), these investigators obtained agreement between theory and their measured values, which have only small experimental uncertainties.

Hill and Merzbacher (1974) reported the results of another nonrelativistic calculation of the z^3 effect for distant collisions. They treated the incident particle classically and the atomic electrons quantum mechanically as harmonic oscillators. Their results confirm the predictions of Ashley, Ritchie, and Brandt (1972) and demonstrate the exact equivalence of the classical and quantum-mechanical treatments of the z^3 effect in the harmonic-oscillator approximation.

Ashley, Ritchie, and Brandt (1972) employed an adjustable parameter to fit the data of Andersen, Simonsen, and Sørensen (1969). Later, they (Ashley *et al.*, 1973) eliminated the adjustable parameter from their theory by including the deviation of actual oscillator-strength distributions for atoms from those predicted by the statistical model. They give simple formulas and tables for evaluating the z^3 correction to stopping powers and ranges in elemental and compound target materials. The predicted range differences are in good agreement with the measurements of Barkas, Birnbaum, and Smith (1956) and Tovee *et al.* (1971). Like some of the data of Tovee *et al.* (1971), however, their values are about 40% smaller than the range differences reported by Barkas, Dyer, and Heckman (1963) for stopping sigma hyperons. These discrepancies have not been fully analyzed. Ashley (1974) has also studied the effects of the z^3 dependence on the evaluation of mean excitation energies and shell corrections.

Jackson and McCarthy (1972) chose a somewhat different minimum impact parameter in the formulas of Ashley, Ritchie, and Brandt (1972) and considered the effect of close collisions in the z^3 correction. As already pointed out by Ashley, Ritchie, and Brandt (1972), the Rutherford formula describes the collision of two unbound charged particles at low velocities, both classically and quantum mechanically, the exact scattering cross section being proportional to z^2. However, at high velocities, the Mott cross section, which is not strictly proportional to z^2, applies. Eby and Morgan (1972) also used the Mott cross section to study close collisions. As Jackson and McCarthy relate, Fermi, in a letter to Barkas in 1953 (Barkas *et al.*, 1956), pointed out this fact and calculated the differences in the average momentum lost by electrons in collisions with positive and negative mesons. The Mott term taken from the literature by Fermi, was, however, in error. The correct formula was used by Jackson and McCarthy, who carried through the calculation of the close-collision contribution to the z^3 effect. This contribution approaches a constant value as $\beta \to 1$, implying that the relativistic z^3 term introduces a constant fractional difference in the ranges of positive and negative particles of the same mass and velocity at high energies. This fractional difference, which does not depend strongly on the atomic number of the stopping material, is of the order of 2×10^{-3}.

Jackson and McCarthy compare the results of their calculations with available experimental data. Theoretical predictions of the z^3 effect are in good agreement for energy-loss measurements (Andersen *et al.*, 1969) of slow helium ions and protons at the same speed. The predictions are also in general agreement with other measurements in which there are large uncertainties (Jackson and McCarthy, 1972). Less definitive is the general agreement with the data of Heckman and Lindstrom (1969) for the difference in energy loss between slow negative and positive pions in emulsions. Jackson and McCarthy also calculate range differences for positive and negative particles in a number of absorbing materials. They report that the calculated differences are in rough agreement with the emulsion data for stopping sigma hyperons (Barkas *et al.*, 1963) and pions (Heckman and Lindstrom, 1969). They obtain good agreement with data for fast muons (Clark *et al.*, 1972).

Recently, Lindhard (1976) has considered this problem anew. Hill and Merzbacher (1974) had already noticed that the Bloch (1933) stopping-power formula, valid at large velocities, is to be replaced by the semiclassical Bohr formula at low velocities. Lindhard (1976) has shown that this correction to the Bethe formula may be represented in lowest order in z by a term proportional to z^4. Further, he has given qualitative arguments to the effect that a contribution proportional to z^3 originates from the region of small impact parameters and that this contribution is roughly equal to that calculated by Ashley, Ritchie, and Brandt (1972, 1973) from a model which emphasizes the importance of contributions from large impact parameters. Recent experimental work by Andersen, Bak, Knudsen, and Nielsen (1977), which compares the stopping power of several materials for protons, alpha particles, and Li^{3+} ions at the same velocity, tends to support Lindhard's analysis, if one includes the Bloch z^4 term. This is equivalent to choosing the minimum impact parameter to be somewhat smaller than the first trial value used by Ashley, Ritchie, and Brandt to fit these data satisfactorily (Brandt and Ritchie, to be published). Thus the question of the contribution of small impact parameters to the z^3 effect is still not completely resolved. It is possible that experimental data on the charge dependence of the stopping power of matter from channeled particles may be helpful in settling these questions.

The underlying physics responsible for the z^3 effects in stopping power manifests itself in other phenomena. Deviations from Born-approximation predictions have been observed for K x-ray emission from materials excited by alpha particles and deuterons of the same velocity (Lewis *et al.*, 1971). Basbas, Brandt, and coworkers (Basbas *et al.*, 1971a, 1971b, 1973) have used the theory of Ashley, Ritchie, and Brandt in a modified form to account for the variation of K-shell ionization data with the charge of fast projectiles. Additional K-shell ionization data of Cue and co-workers (Cue *et al.*, 1974) with heavy projectiles also show a dependence on z^3. Still other departures from z^2 dependence must apparently be attributed to different physical processes. For example, some inner-shell ionization effects have been ascribed to perturbations of the target atomic states by

the passage of a slow charged particle. Some discussion of these and other problems is given elsewhere (Brandt, 1973; Madison and Merzbacher, 1975). New effects, which augment stopping power through the proximity of projectiles moving in clusters, have recently been observed (Brandt *et al.*, 1974) and reviewed (Brandt and Ritchie, 1976).

R. K-shell ionization by relativistic particles

Several recent papers on K-shell ionization add important information on relativistic effects discussed in Sec. II.3 of Inokuti's review (1971). This topic was touched upon also by Fano (1963), who cited the earlier work of Perlman (1960) in connection with the stopping power of K electrons in heavy elements. Theories of K-shell ionization by high-energy electrons in medium- and high-Z materials have been reviewed by Madison and Merzbacher (1975). The subject of inner-shell ionization cross sections has also been surveyed for incident electrons by Powell (1976), and more generally by Tawara (1977).

Davidović and Moiseiwitsch (1975) calculated K-shell ionization cross sections for incident relativistic electrons as functions of the electron energy and the atomic number of the target atom. They employed the Darwin approximation for the K electron. Anholt and coworkers (1976) measured cross sections for producing K-shell vacancies with 4.88-GeV protons incident on a number of elements between Ni and U. They also calculated Born-approximation cross sections with the relativistic proton–electron interaction, but used nonrelativistic, single-electron $1s$ and continuum wave functions for the electron. The measured cross sections were significantly larger than their calculated values. Moiseiwitsch, Norrington, and Davidović (1977) extended the work of Davidović and Moiseiwitsch (1975) to relativistic protons, allowing also for a change in spin direction of the atomic electron. They calculated the K-shell vacancy cross section as a function of atomic number for 4.88-GeV protons. The new computed cross sections appear to be in satisfactory agreement with the measurements of Anholt *et al.* (1976). Moiseiwitsch, Norrington, and Davidović attribute the difference between their calculations and those of Anholt *et al.* to the latters' only partial inclusion of relativistic effects.

Despite all the new developments, the subject remains open for further study. It is desirable to formulate calculations in such a way that one clearly distinguishes the effects of the relativistic motion of target electrons from the effects of incident particles' relativistic kinematics and interactions. Presumably the treatment by Bethe (1933) and by Fano (1963), as summarized in Sec. II.3 of Inokuti (1971), should fully incorporate the latter effects. In that treatment an important assumption is the validity of Eqs. (2.29) and (2.30), which concern the dynamics of target electrons. It remains unclear how well these equations are satisfied, because none of the theoretical results have been analyzed or even presented from our point of view.

III. ERRATA

A. Corrections to Inokuti's review (1971)

Page 298, left column. Line 10 should read "...the Franck-Hertz ex-".

Page 300, Fig. 1. Plot (a) and (b) should be interchanged.

Page 301, left column. The second line below Eq. (2.16) should read "occurs at $\theta=0$, and...".

Page 301, left column. The second term in the brace in Eq. (2.17) should read

$$+\frac{1}{2}\left(\frac{mE_n}{MT}\right).$$

Page 302, left column. The equation in line 5 should read

$$Q=(\hbar K)^2/2m.$$

Page 302, left column, footnote 2. The content within the parentheses in the second line should read "at a fixed T."

Page 304, right column. The argument of the exponential function of Eq. (2.34) should be $-iEt/\hbar$.

Page 308, right column. The content in the first square brackets of Eq. (3.9) should read $\frac{1}{3}(n^2-1)+(nKa_0)^2$.

Page 309, left column. The argument of the exponential function in the third line of Eq. (3.10) should read

$$\left\{-\frac{2}{\kappa a_0}\arctan\left[\cdots\cdots\right]\right\}.$$

Page 317, left column. Line 17 should read "Massey-Mohr result was..." .

Page 318, right column. Add a closing parenthesis ")" after "...a summary" on line 13.

Page 325, left column. The first term on the right-hand side of Eq. (4.12) should read

$$\int_0^\infty \frac{f_n(K)}{f_n}\,d[\ln(Ka_0)^2].$$

Page 326, right column, line 14 from bottom. Change "causal" into "casual."

Page 327, right column. The right-hand side of Eq. (4.23) should read

$$[(R^2/E)(df/dE)]_{E=I}.$$

Page 335, right column. The second to the last line of Eq. (4.78) should read as follows: $\exp(i\mathbf{K}\cdot\mathbf{r}_k)\exp\{(it/\hbar)[H+(\hbar K)^2/2m$.

Page 336, left column. On the right-hand side of Eq. (4.83) change $\mathbf{p}_i$ to $\mathbf{p}_j$.

Page 336, right column. In the right-hand side of the unnumbered equation above Eq. (4.87), the second term within the bracket should read

$$+\frac{3}{E^{5/2}}\,\frac{p_{zj}}{(2m)^{1/2}}.$$

Page 340, right column. The reference on line 19 from bottom should read ARB72.

Page 343–347. Bibliographic data on Inokuti's (1971) references A71, ARB72, CK72, H71, HBE64, M71,

MGV63, OM71, OPB71, PD71, VW71, and YS71 were given incompletely or incorrectly (ARB72 and CK72 appear as ARB71 and CK71). For correct citations, see the following in the list of references of the present article: Amusia, 1971; Ashley *et al.*, 1972; Cooper and Kolbenstvedt, 1972; Hudson, 1971; Hirschfelder *et al.*, 1964; Miller, 1971; Meyerson *et al.*, 1963; Oldham and Miller, 1971; Opal *et al.*, 1971; Peart and Dolder, 1971; Van der Wiel and Wiebes, 1971; Yamamoto and Suzuki, 1971. The work cited as RP65–70 has been published by Rieke and Prepejchal (1972). The work cited as OPB71 has been extended to many molecules (Peterson *et al.*, 1971 and 1972; Opal *et al.*, 1972)

Page 347, right column. The reference code CDJ70 should be changed to VDJ70.

B. Corrections to Fano's article (1963)

Page 7. The first term within the braces of Eq. (12) should carry a minus sign. [The same error first occurred in Eq. (2) of an earlier paper (Fano, 1956).] Because the two terms within the braces differ in the parity of atomic states involved, the present correction is of no consequence to the treatment of particle penetration, to which the Fano article is exclusively devoted. That correction, however, is significant when one uses Eq. (12) to evaluate the angular distribution of electrons ejected by charged particles. The difference in the signs in the two terms may be remembered in terms of the elementary fact that electric charges of the same sign repel each other, while electric currents in the same direction attract each other.

Page 8. The exponents in Eq. (17) should read $2\pi i\mathbf{q}\cdot\mathbf{r}_j/\hbar$

Page 9. Equation (19), as it stands, has no clearcut mathematical meaning. Probably the bracket on the left-hand side was intended to denote an average over collisions with different Q but with a fixed E_n. With this interpretation, one finds that

$$\left(\frac{E_n - Q}{E_n + Q}\right)^2 \sim \frac{\langle K\rangle_0}{E_n}$$

from inspection of hydrogenic matrix elements $F_n(q)$. The above estimation also results from a general argument based upon the binary-encounter theory, e.g., by use of Eq. (4.83) of Inokuti (1971).

Page 12. Change $+\beta^4$ to $-\beta^4$ in Eq. (25).

Page 12. In the line above Eq. (26), "order Q/mc^2" should read "order $(Q/mc^2)^2$." Terms of order Q/mc^2 actually cancel out, making Eq. (26) more nearly exact.

Page 13. The second term within the braces of Eq. (32) should read $\ln 1/(1-\beta^2)$.

Page 24. Footnote 20 may be brought up to date by noting the work of Ball, Wheeler, and Fireman (1973), who solved the Bloch equation for the dipole oscillation of the Thomas–Fermi model and thereby determined I/Z to be 4.95 eV. This value, however, is much too low compared to experimental data of Table I of Fano (1963), and exemplifies unrealistic aspects of that particular model. Indeed, the dipole spectrum of the Thomas–Fermi model is nonvanishing at arbitrarily low frequencies, whereas the spectrum of any real atom begins at its first excitation threshold.

Atomic shell structure gives rise to a periodic variation of I/Z as a function of atomic number Z. The variation is largest for low Z and damps for high Z [see Sec. III C of the paper by Dehmer, Inokuti and Saxon (1975) for more details].

Page 41. The last term in the parentheses of Eq. (65) should have $2mc^2$ instead of mc^2 in the denominator.

Page 43. The last term within the braces of Eq. (77) should have a factor 2/3 instead of 1/3.

Page 44. Equation (80) should start with $f(E, s)$.

Page 52. In the caption to Fig. 7, the equation for "Theoretical asymptotic law" should begin with $(137\beta^2\sigma_i/4\pi a_0^2$.

ACKNOWLEDGMENTS

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