Double beta decay*

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The problem of double beta decay is reviewed with emphasis on its relevance to lepton number conservation. Recently, the ratio of the double beta-decay half-lives of ^{128}Te and ^{130}Te has been measured in a geological experiment and a limit for the ratio of the neutrinoless rate to the total rate for ^{82}Se decay has been obtained from a direct-detection experiment. For the first time, these results show conclusively that double beta decay is not primarily a lepton-number-violating neutrinoless process. However, they also do not agree with calculations which assume that only lepton-number-conserving two-neutrino double beta decay occurs. The conclusion that lepton number conservation is violated is suggested by limited experimental information. By considering contributions to the total rate from both the two-neutrino and the neutrinoless channels, we obtain data which are consistent with a lepton nonconservation parameter of order $\eta = 3.5\times10^{-5}$. Roughly the same value of η is obtained by assuming that the decay occurs either via lepton emission from two nucleons or via emission from a resonance in the nucleus.

CONTENTS

I. INTRODUCTION

The breaking of conservation laws has played a crucial role in the development of the theory of weak interactions. The discovery in 1957 that parity and charge conjugation are maximally violated in beta (β) decay (Wu *et al.*, 1957) has led to a careful study of many conservation laws. Small violations, such as the charge-parity nonconservation of order 10^{-3} discovered in K_L^0 decays (Christenson *et al.*, 1964) are extremely significant and further emphasize the importance of establishing conservation principles with high precision.

Lepton number conservation is another important law that has received much attention in the last few years. Table I gives some properties of the known leptons[1]: the electron (e^-), the electron neutrino (ν_e), the muon (μ^-), and the muon neutrino (ν_μ). The antileptons ($e^+, \overline{\nu}_e, \mu^+, \overline{\nu}_\mu$) have opposite lepton number and muon number. The electron and muon participate in both the weak and electromagnetic interactions, whereas the neutrinos are known to interact via the weak interaction alone. The standard V−A (vector − axialvector) theory of weak interactions allows only negative helicity neutrinos and positive helicity antineutrinos. However, the total polarization of the neutrinos has only been established to an accuracy of a few percent in β-decay and muon-decay experiments. The law of conservation of lepton number (L) states that the number of leptons less the number of antileptons is conserved in all reactions. For example, in the β decay of the neutron

$$n \to p + e^- + \overline{\nu}_e \tag{1}$$

both initial and final states have $L = 0$. A conserved muon number[2] accounts for the experimental absence of the decay $\mu \to e\gamma$ and for the observation that antineutrinos from pion decay $\pi^+ \to \mu^+ + \overline{\nu}_\mu$ do not produce posi-

TABLE I. Properties of known leptons.

	e^-	ν_e	μ^-	ν_μ
Mass (MeV)	0.511	<0.000 06 [a]	105.659	<0.65 [b]
Spin	1/2	1/2	1/2	1/2
Lepton number	1	1	1	1
Muon number	0	0	1	1

[a] Bergkvist (1972).
[b] Clark *et al.* (1974).

*Work supported by the Atomic Energy Control Board and the National Research Council of Canada.

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[1] Recent experiments suggest the existence of new heavy leptons (see Perl *et al.*, 1975 and Benvenuti *et al.*, 1977).

[2] For a discussion of muon number conservation, see Frankel (1975).

TABLE II. Isotopes for which $\beta\beta$ decay can occur. Single beta decay is energetically forbidden or strongly inhibited by angular momentum and parity considerations. Taken from Fiorini (1971).

Transition	A	Z	Isotopic abundance (%)	Transition energy (MeV)	Intermediate transition energy $(A,Z)-(A,Z+1)$
Ca–Ti	46	20	0.0033	0.985 ± 0.009	−1.382 ± 0.011
Ca–Ti	48	20	0.185	4.267 ± 0.009	+ 0.289 ± 0.012
Zn–Ge	70	30	0.62	1.008 ± 0.006	−0.653 ± 0.009
Ge–Se	76	32	7.67	2.045 ± 0.004	−0.923 ± 0.004
Se–Kr	80	34	49.82	0.138 ± 0.007	−1.871 ± 0.005
Se–Kr	82	34	9.19	3.003 ± 0.008	−0.089 ± 0.008
Kr–Sr	86	36	17.37	1.240 ± 0.006	−0.0537 ± 0.008
Zr–Mo	94	40	2.80	1.230 ± 0.005	−0.921 ± 0.014
Zr–Mo	96	40	17.40	3.364 ± 0.005	+ 0.215 ± 0.025
Mo–Ru	100	42	9.62	3.034 ± 0.006	−0.335 ± 0.060
Ru–Pd	104	44	18.5	1.321 ± 0.012	−1.145 ± 0.008
Pd–Cd	110	46	12.7	2.004 ± 0.013	−0.868 ± 0.015
Cd–Sn	114	48	28.86	0.547 ± 0.008	−1.439 ± 0.008
Cd–Sn	116	48	7.58	2.811 ± 0.006	−0.517 ± 0.024
Sn–Te	122	50	4.71	0.349 ± 0.008	−1.622 ± 0.008
Sn–Te	124	50	5.98	2.263 ± 0.007	−0.653 ± 0.008
Te–Xe	128	52	31.79	0.872 ± 0.007	−1.268 ± 0.010
Te–Xe	130	52	34.49	2.543 ± 0.008	−0.407 ± 0.030
Xe–Ba	134	54	10.44	0.731 ± 0.039	−1.328 ± 0.039
Xe–Ba	136	54	8.87	2.718 ± 0.080	−0.112 ± 0.080
Ce–Md	142	58	11.07	1.379 ± 0.049	−0.777 ± 0.050
Nd–Sm	148	60	5.71	1.936 ± 0.021	−0.514 ± 0.030
Nd–Sm	150	60	5.60	3.390 ± 0.020	−0.036 ± 0.062
Sm–Gd	154	62	22.61	1.260 ± 0.022	−0.718 ± 0.024
Gd–Dy	160	64	21.75	1.782 ± 0.030	−0.029 ± 0.030
U–Pu	238	92	99.275	1.173 ± 0.034	−0.117 ± 0.031

trons but only muons in reactions with protons

$$\bar{\nu}_\mu + p \to n + \mu^+ . \tag{2}$$

Is lepton number conserved absolutely, as charge and baryon number seem to be, or is there a small, but observable, violation? An attractive way of investigating this question centers on the process of double beta ($\beta\beta$) decay. There are two possible modes of $\beta\beta$ decay for a nucleus with Z protons and $A-Z$ neutrons:

$$(Z,A) \to (Z+2,A) + e_1^- + e_2^- + \bar{\nu}_1 + \bar{\nu}_2 \tag{3}$$

and

$$(Z,A) \to (Z+2,A) + e_1^- + -e_2^- . \tag{4}$$

Table II lists the isotopes for which $\beta\beta$ decay can occur. These are primarily transitions between nuclear ground states with spin zero and positive parity (0^+) for which single beta decays are energetically forbidden or strongly inhibited by angular momentum and parity considerations. The half-lives for these decays are expected to be $\gtrsim 10^{19}$ yr.

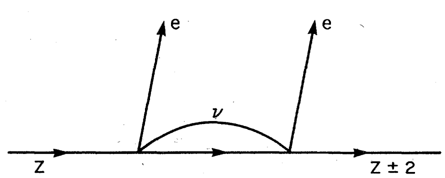

FIG. 1. Second-order neutrinoless double beta decay by emission and reabsorption of a neutrino.

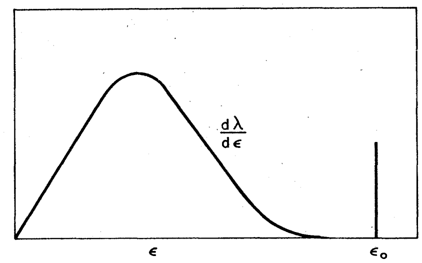

FIG. 2. Theoretical electron energy-sum spectrum from double beta decay in the two-neutrino mode. The fixed energy release in the neutrinoless mode is indicated by the vertical line at the maximum energy of the two-neutrino spectrum.

The observation of neutrinoless $\beta\beta$ decay, Eq. (4), would directly indicate a violation of lepton number conservation. Let us assume that this could occur by the emission and subsequent reabsorption of a neutrino, as shown in Fig. 1. If the neutrinos are massless and completely polarized, as is normally assumed in the V–A theory, the neutrino emitted in Fig. 1 could not be reabsorbed because it would have the wrong polarization. However, since the neutrinos are not expected to be completely polarized if lepton conservation fails, the absence of neutrinoless $\beta\beta$ decay would be attributed to lepton number conservation.

$\beta\beta$ decay half-lives have been measured using geochemical techniques. These experiments use mass spectrometry to determine the relative abundance of $\beta\beta$ decay daughter nuclei in geologically old ores. Although leptons are not detected in these experiments, much insight regarding lepton conservation can be gained by comparing the results to the theoretical predictions for the half-lives of Processes (3) and (4).

The best way to determine whether $\beta\beta$ decay occurs with or without neutrinos would be to observe the energy spectrum of the emitted electrons. In neutrinoless decay the electron energy-sum spectrum has a sharp peak at the total available energy. If neutrino emission occurs, the electron energy-sum distribution is continuous with a broad maximum at half the energy release, as shown in Fig. 2. No direct observation of $\beta\beta$ decay with or without neutrinos has been reported, because of the difficulties involved in reducing background sufficiently. Nevertheless, lower limits on $\beta\beta$ decay half-lives obtained in several searches place constraints on calculations of Processes (3) and (4).

In this paper, Sec. II reviews the techniques used in the latest geological and direct-detection experiments and Sec. III reviews the theoretical efforts and calculations of lepton violation parameters. Section IV compiles the results of these studies and outlines prospects for the future. For references to earlier studies of $\beta\beta$ decay see Rosen and Primakoff (1965) and Fiorini (1971).

II. EXPERIMENTS

A. Geological methods

Geological methods have been used to measure the $\beta\beta$ half-lives of $^{130}Te \to {}^{130}Xe$, $^{128}Te \to {}^{128}Xe$, and $^{82}Se \to {}^{82}Kr$.

These experiments were made possible by the high sensitivity available with noble gas mass spectrometry.

Te is particularly interesting, because it is possible to look for the $\beta\beta$ decay of two isotopes. Assuming that the nuclear matrix elements for the two decays are approximately equal, the calculated ratio of the $\beta\beta$ decay half-lives for the two isotopes reduces to the ratio of the phase spaces (Pontecorvo, 1968; Primakoff and Rosen, 1969; Smith, Picciotto, and Bryman, 1973a, b, c). The neutrinoless phase space is roughly proportional to the fourth through fifth power of the energy release, which in ^{130}Te is three times that for ^{128} Te (see Table II), giving $S \equiv \tau_{1/2}(^{128}\mathrm{Te})/\tau_{1/2}(^{130}\mathrm{Te}) \sim 10^2$. The phase space for two-neutrino decay is roughly proportional to the eighth through eleventh power of the energy release, giving $S \sim 10^4$. Comparison with measured values of S provides a sensitive way of distinguishing between $\beta\beta$ decay channels.

The first experiments which provided definite observation of $\beta\beta$ decay and a determination of its half-life were those of ^{130}Te by Kirsten, Gentner, and Schaeffer (1967) and Kirsten, Schaeffer, Norton, and Stoner (1968). The reliability of these studies was based on an evaluation of the age of the mineral, $T = (1.31 \pm 0.14) \times 10^9$ yr, by K–Ar dating. The age measurement was consistent with the geological situation of the deposit. By locating an ore which had a large excess of ^{130}Xe above atmospheric abundance and which was not accompanied by other Xe isotopic anomalies, it was possible to show that the ^{130}Xe excess was due unambiguously to the $\beta\beta$ decay $^{130}\mathrm{Te} \to {}^{130}\mathrm{Xe}$. The agreement of the age determinations indicated that Ar in the ore was probably not lost by diffusion; it was then assumed that Xe, being a heavier noble gas, also did not escape. By doing a mass-spectroscopic analysis of the xenon, the ratio (^{130}Xe excess)/(^{130}Xe atmospheric) $\simeq 50$ was obtained. The half-life for $\beta\beta$ decay of ^{130}Te can be calculated from the following equation:

$$\tau_{1/2}(^{130}\mathrm{Te}) = T\,[N(^{130}\mathrm{Te})/N^{\mathrm{excess}}(^{130}\mathrm{Xe})]\ln 2\,, \tag{5}$$

where $N(^{130}\mathrm{Te})$ = number of atoms of ^{130}Te/g of sample, $N^{\mathrm{excess}}(^{130}\mathrm{Xe})$ = number of atoms of excess ^{130}Xe/g of sample. Equation (5) yields

$$\tau_{1/2}(^{130}\mathrm{Te}) = 10^{21.34 \pm 0.12}\ \mathrm{yr}\,. \tag{6}$$

A similar result was found by Srinivasan *et al.*, (1972):

$$\tau_{1/2}(^{130}\mathrm{Te}) = 10^{21.38 \pm 0.10}\ \mathrm{yr}\,. \tag{7}$$

In the latest studies of Te done by Hennecke, Manuel, and Sabu (1975) sufficient sensitivity was achieved to measure the ratio $S = \tau_{1/2}(^{128}\mathrm{Te})/\tau_{1/2}(^{130}\mathrm{Te})$. Xenon was extracted by stepwise heating of the sample, and its isotopic abundances were measured in the mass region $A = 122$ to $A = 136$ in a mass spectrometer. For ^{130}Xe the excess over atmospheric abundance was (^{130}Xe excess)/(^{130}Xe atmospheric) = 712 ± 2, which was more than an order of magnitude greater than in the previous experiments. For ^{128}Xe it was found that the ratio of excesses of ^{128}Xe to ^{130}Xe was (^{128}Xe excess)/(^{130}Xe excess) = $5.8 \pm 0.2 \times 10^{-4}$. Many sources of systematic errors in measuring the individual $\beta\beta$ decay half-lives, such as errors in ore age, in the tellurium determination, and in measurement of the xenon content, cancel out in the determination of the ratio of the half-lives. The experimental result found by this group is

$$S = 10^{3.20 \pm 0.01}\,. \tag{8}$$

Using the previously known ^{130}Te half-life, Eq. (6), the half-life for ^{128}Te is

$$\tau_{1/2}(^{128}\mathrm{Te}) = 10^{24.54 \pm 0.12}\ \mathrm{yr}\,. \tag{9}$$

In an earlier geological experiment, Takaoka and Ogata (1966) reported a value $S = 10^{1.2 \pm 0.6}$, making the neutrinoless interpretation plausible. However, the experimenters stressed that there were difficulties in the analysis of the background for the ^{128}Te case and they questioned the validity of the result. The latest result, Eq. (8), is in clear disagreement with that of Takaoka and Ogata and the extent to which it is consistent with present theories will be discussed in Sec. III.

Other similar geological experiments were performed for the decay $^{82}\mathrm{Se} \to {}^{82}\mathrm{Kr}$, with the result (Srinivasan, Alexander, Beaty, Sinclair, and Manuel, 1973)

$$\tau_{1/2}(^{82}\mathrm{Se}) = 10^{20.42 \pm 0.14}\ \mathrm{yr}\,. \tag{10}$$

In support of the experimental evidence for the existence of $\beta\beta$ decay in these geological experiments, other possible origins for the results have been considered and eliminated. These include $(\alpha, 2n)$, (α, n), and (α, γ) reactions generated by U and Th α decay in surrounding rocks, solar neutrino interactions, reactions induced by cosmic-ray primaries and secondaries, and neutron capture processes.

B. Direct detection

Because of the long lifetimes associated with $\beta\beta$ decay, the identification of events and the analysis of the background are particularly difficult problems. There have been many direct-detection experiments performed but only recently have their sensitivities matched that of geological methods. So far only lower limits have been obtained for $\beta\beta$ decay half-lives. In this section experiments for ^{76}Ge, ^{82}Se, and ^{48}Ca will be discussed.

A search for the neutrinoless $\beta\beta$ decay of ^{76}Ge was carried out by Fiorini, Pullia, Bertollini, Capellani, and Restelli (1973). The decay scheme for ^{76}Ge is shown in Fig. 3. Single β decay to the $Z - 1$ nucleus ^{76}As is energetically forbidden and the $\beta\beta$ decay energy available for transitions to the ground state of ^{76}Se is 2.045 MeV. ^{76}Ge was chosen primarily because of the availability of

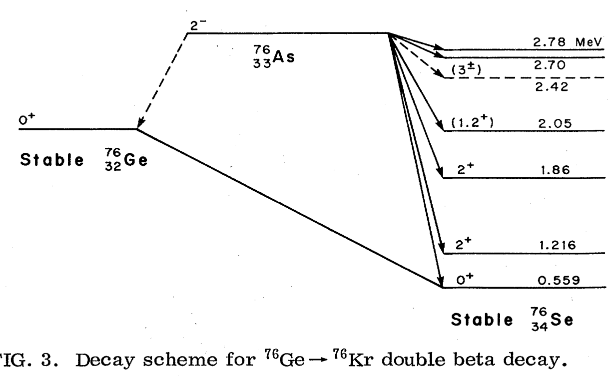

FIG. 3. Decay scheme for $^{76}\mathrm{Ge} \to {}^{76}\mathrm{Kr}$ double beta decay.

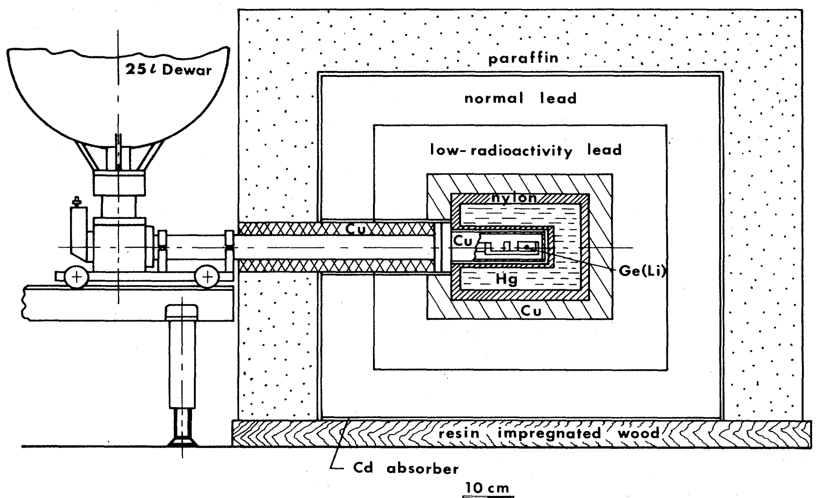

FIG. 4. The apparatus and local shielding used in the ^{76}Ge double beta-decay experiment of Fiorini *et al*. (1973).

large high-resolution Ge(Li) detectors. Ge(Li) crystals are normally fabricated with very high purity, free from radioactive contaminants. The natural abundance of ^{76}Ge, 7.67%, enables an ordinary Ge(Li) crystal to be both a clean source and a clean detector of $\beta\beta$ decay. A crystal enriched in ^{76}Ge would be highly desirable for this experiment, but the cost appears to be prohibitive.

The Ge(Li) detector used by Fiorini *et al*. (1973) had a 68.5 cm^3 active volume and an energy resolution of 6 keV at 2615 keV. Pulses from the detector were amplified and recorded in a 4096 channel pulse-height analyzer which employed spectrum stabilization for long runs. A diagram of the experimental apparatus is shown in Fig. 4. The experiment was located in the Mont Blanc tunnel connecting Italy to France, virtually eliminating cosmic-ray backgrounds, and the crystal was shielded from local radioactivity by layers of paraffin, cadmium, low-activity lead, bi-distilled mercury, nylon, and high purity electrolytic copper. After 4400 h of running no peak was found in the 2.045 MeV region of expected neutrinoless $\beta\beta$ decay. Tests indicated that the residual backgrounds observed in this energy region were likely due to ^{40}K, ^{235}U, ^{238}U, and ^{232}Th contaminants of less than 10^{-5} ppm which originated inside the local shield, probably in the crystal's cryostat structure. Possible ^{222}Rn contaminants in the liquid nitrogen coolant were also suspected. Figure 5 shows the spectrum obtained in the final run of the experiment lasting 2300 h. The background counting rate in the 2.045 MeV region was $(2\pm0.2)\times10^{-3}$ counts keV^{-1} h^{-1}, allowing a limit to be set on the half-life for the neutrincless $\beta\beta$ decay of ^{76}Ge, $\tau_{0\nu} > 5\times10^{21}$ yr, at a 68% confidence level. Presently this is the highest limit set on the lifetime of a neutrinoless $\beta\beta$ decay.

The study of ^{82}Se was carried out by Cleveland, Leo, Wu, Kasday, Rushton, Gollon, and Ullman (1975), using apparatus similar to the one used for the ^{48}Ca $\beta\beta$ measurement of Bardin *et al*. (1970). The arrangement consisted of a helium-filled double gap streamer chamber in a 370 G magnetic field with 16 unwrapped plastic scintillation counters on each side of the selenium source, as shown in Fig. 6. Two coincident pulses whose energies exceeded set limits fired the streamer

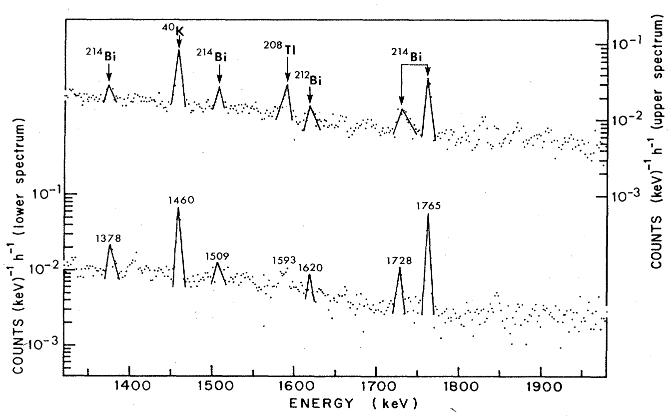

FIG. 5. The observed spectrum in the energy region of expected ^{76}Ge neutrinoless double beta decay, 2.045 MeV, from the experiment of Fiorini *et al*. (1973). (a) Upper spectrum: The data was taken in an initial run lasting 2100 h. (b) Lower spectrum: A lower background level was achieved in the final run of 2300 h by using higher purity materials for the crystal support structure and cryostat cap.

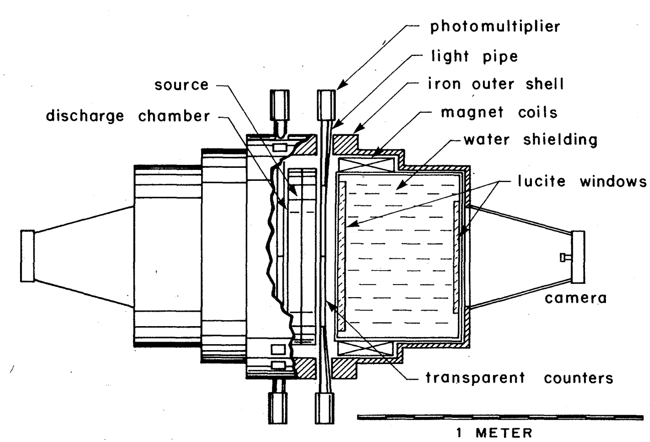

FIG. 6. The apparatus used by Cleveland *et al*. (1975) in the search for ^{82}Se neutrinoless double beta decay.

chamber, and the tracks were photographed. The electrons' momenta are determined by their curvatures in the magnetic field. In order to reduce background by two orders of magnitude relative to "clean" surface conditions, the experiment was conducted in a salt mine situated 600 m below ground level where the primary residual radioactivity was due to ^{40}K. The energy losses of $\beta\beta$ decay electrons due to the thickness of the source was found by a Monte Carlo calculation, shifting the searched-for peak in the electron energy-sum spectrum for neutrinoless $\beta\beta$ decay from 3.0 MeV to 2.75 MeV with a width of ~0.3 MeV. The total number of events recorded was 65 500. The vast majority of these were background induced, primarily Compton scattering of γ rays in a scintillator, the recoil electron then passing through the chamber and hitting a second counter. The correct signature of $\beta\beta$ decay was two tracks leaving the source from a common vertex. Only 201 events of this type were seen. Many of these were caused by double scattering within the source and it was difficult to distinguish them from true $\beta\beta$ events. By restricting the energy range to between 2.4 and 3.2 MeV and imposing appropriate acceptance criteria on the track curvature, the overall selection efficiency became 19%. No events were found that could be attributed to $\beta\beta$ decay and, at a confidence level of 68%, this implied a lower limit for the half-life of neutrinoless $\beta\beta$ decay of $\tau_{1/2}^{0\nu}(^{82}\mathrm{Se}) > 3.1 \times 10^{21}$ yr. There is a great advantage in interpreting this result for ^{82}Se, since the $\beta\beta$ decay half-life has been definitely established with geological methods as $\tau_{1/2} = (2.76 \pm 0.88) \times 10^{20}$ yr (see Sec. II.A). A branching ratio limit could be determined for the first time:

$$R(^{82}\mathrm{Se}) = \frac{\text{no-neutrino rate}}{\text{total } \beta\beta \text{ rate}} \leqslant 9\% . \tag{11}$$

In the search for the $\beta\beta$ decay of ^{48}Ca, carried out by Bardin, Gollon, Ullman, and Wu (1970), the results were

$$\tau_{1/2}^{0\nu}(^{48}\mathrm{Ca}) > 10^{21.3}\ \mathrm{yr} \tag{12}$$

for the neutrinoless decay and

$$\tau_{1/2}^{2\nu}(^{48}\mathrm{Ca}) > 10^{19.56}\ \mathrm{yr} \tag{13}$$

for the two-neutrino decay at an 80% confidence level.

III. THEORY AND COMPARISON TO EXPERIMENT

A. Two-nucleon mechanism

1. Neutrinoless mode

The general derivation of the $\beta\beta$ decay rate with the two-nucleon mechanism

$$n + n \to p + p + e^- + e^- + \bar{\nu} + \bar{\nu} \tag{14}$$

has been given by Primakoff and Rosen (1969), Rosen and Primakoff (1965), and Konopinski (1966). The $\beta\beta$ decay matrix element can be written as

$$M_{\beta\beta} = \sum_n \frac{\langle \chi_f \psi_f | H_\beta | \psi_n \chi_n \rangle \langle \chi_n \psi_n | H_\beta | \psi_i \chi_i \rangle}{E_n - E_i} , \tag{15}$$

where $\psi_i, \psi_f, \psi_n, \chi_i, \chi_f, \chi_n$ are the initial, final, and intermediate nuclear and lepton wave functions, with no leptons in χ_i and two or four leptons in χ_f. H_β is the conventional weak Hamiltonian with lepton current L_λ. For a massless two-component neutrino, L_λ is given by

$$L_\lambda = \bar{\psi}_e \gamma_\lambda (1 + \gamma_5) \psi_\nu , \tag{16}$$

where ψ_e and ψ_ν are the lepton field operators and γ_λ and γ_5 are the Dirac matrices. The factor $(1+\gamma_5)$ is a helicity projection operator which ensures that a neutrino emitted together with an electron by one hadron cannot be reabsorbed with simultaneous emission of a second electron by another hadron. In that case, neutrinoless decay is forbidden and lepton number is conserved. Lepton nonconservation can be introduced by considering a current of the form

$$L_\lambda = \bar{\psi}_e \gamma_\lambda \frac{\{(1+\gamma_5) + \eta(1-\gamma_5)\}}{(1+\eta^2)^{1/2}} \left[\frac{\psi_\nu + \xi \psi_\nu^-}{(1+\xi^2)^{1/2}} \right] . \tag{17}$$

For simplicity, it will be assumed that the neutrino is a "Majorana" particle ($\xi = 1$) and the neutrinoless $\beta\beta$ decay amplitude is proportional to the "lepton-nonconservation" parameter η. L_λ of Eq. (17) predicts a longitudinal spin polarization for single β-decay electrons equal to $(v_{\mathrm{el}}/c)(1-\eta^2)/(1+\eta^2)$, where v_{el} is the electron velocity.

The process of neutrino emission and reabsorption is shown in Fig. 1. Although the second-order weak process contains a virtual-neutrino closed loop it is not divergent, since the emission and reabsorption involve different nucleons. The reciprocal of the two-nucleon separation serves as an effective cutoff for the virtual-neutrino energy. Since the nucleon charge makes it impossible for a single nucleon to emit and reabsorb the neutrino with the emission of two electrons, the matrix element is finite.

The current–current weak-interaction Hamiltonian density is

$$H_\beta(x) = (G/\sqrt{2}) [L_\lambda(x) J_\lambda^{(+)}(x) + J_\lambda^{(-)}(x) L_\lambda^\dagger(x)] , \tag{18}$$

where $J_\lambda^{(-)}(x) = [J_\lambda^{(+)}(x)]^\dagger$, $J_\lambda^{(\pm)}(x) = V_\lambda^{(\pm)}(x) + A_\lambda^{(\pm)}(x)$, $V_\lambda^{(\pm)}(x)$, and $A_\lambda^{(\pm)}(x)$ are polar and axial-vector hadron weak currents specified by the conserved vector current (CVC) and partially conserved axial-vector current (PCAC) hypotheses (see Marshak, Riazuddin, and Ryan, 1969), $L_\lambda(x)$ is given by Eq. (17), and G is the weak coupling constant.

The neutrinoless rate is (Primakoff and Rosen, 1969)

$$\lambda_{0\nu} = \frac{|\mathrm{ME}|^2}{f_2 \eta^2} \ln 2 , \tag{19}$$

where

$$f_2 = \frac{10^{19.9}}{f(\epsilon_0)} \left(\frac{1}{2\pi\alpha Z}\right)^2 [1 - \exp(-2\pi\alpha Z)]^2 \left(\frac{A}{130}\right)^{2/3} \mathrm{yr} , \tag{20}$$

ME is the nuclear matrix element, $f(\epsilon_0) = \epsilon_0^4(\epsilon_0^3 + 13\epsilon_0^2 + 77\epsilon_0 + 70)$, and ϵ_0 is the energy release in units of electron mass. This expression includes a Coulomb correction factor suitable for $\beta\beta$ decay (Konopinski, 1966),

$$f(v_1, v_2, Z) = \{2\pi Z\alpha / [1 - \exp(-2\pi Z\alpha)]\}^2 (v_1 v_2)^{-2} , \tag{21}$$

where the electron velocities, v_1 and v_2, were canceled out by other phase-space factors.

2. Two-neutrino mode

Similar techniques can be used to obtain the rate of two-neutrino $\beta\beta$ decay (Konopinski, 1966; Rosen and Primakoff, 1965):

$$\lambda_{2\nu} = \frac{|\mathrm{ME}|^2}{f_4} \ln 2 , \tag{22}$$

where

$$f_4 = \frac{3\times10^{20}}{f_{\beta\beta}} \left(\frac{1-\exp(-2\pi\alpha Z)}{2\pi\alpha Z}\right)^2 (\bar{\Delta} + \epsilon_0/2)^2 \text{ yr} , \tag{23}$$

$f_{\beta\beta} = (1+\epsilon_0/2 + \epsilon_0^2/9 + \epsilon_0^3/90 + \epsilon_0^4/1980)\,\epsilon_0^7/8!$, and $\bar{\Delta}$ is the average energy difference between initial and intermediate nuclear states. The electrons emerge predominantly antiparallel, with an angular correlation function

$$C(\mathbf{p}_1 \cdot \mathbf{p}_2) \simeq 1 - \frac{\mathbf{p}_1 \cdot \mathbf{p}_2}{(\epsilon_1+1)(\epsilon_2+1)} , \tag{24}$$

where $\mathbf{p}_i$ and ϵ_i are momentum and (kinetic) energy in units of the electron mass. Comparison of the two-neutrino and neutrinoless half-lives indicates that for $\eta \approx 1$ and comparable ϵ_0, Z, and nuclear matrix elements, the neutrinoless rate is greater by several orders of magnitude because of the difference between two-particle and four-particle phase spaces.

3. Evaluation of lepton-violation parameter by comparison to experiment

As mentioned earlier, by assuming that the nuclear matrix elements are equal for the isotopes ^{128}Te and ^{130}Te, it is possible to compare the theoretical predictions to the experiment without explicit evaluation of the nuclear matrix elements. The prediction for neutrinoless decay from Eq. (19) is $S_{0\nu} = [\tau_{1/2}(^{128}\mathrm{Te})/\tau_{1/2}(^{130}\mathrm{Te})]_{0\nu} = 10^{2.4}$ and that for two-neutrino decay from Eq. (22) is $S_{2\nu} = [\tau_{1/2}(^{128}\mathrm{Te})/\tau_{1/2}(^{130}\mathrm{Te})]_{2\nu} = 10^{3.8}$. The experimental ratio $S_{\mathrm{exptl}} = 10^{3.20\pm0.01}$ (Hennecke *et al.*, 1975) cannot be explained quantitatively by either the neutrinoless or the two-neutrino modes alone. Assuming that both types of decay occur, the total rate can be calculated (Vergados, 1976):

$$\frac{1}{\tau_{1/2}} = \left(\frac{\eta^2}{f_2} + \frac{1}{f_4}\right) |\mathrm{ME}|^2 . \tag{25}$$

The lepton-violation parameter can be obtained by calculating $S_{\mathrm{tot}} = [\tau_{1/2}(^{128}\mathrm{Te})/\tau_{1/2}(^{130}\mathrm{Te})]_{0\nu+2\nu}$ using Eq. (25) and comparing this with S_{exptl}. Assuming that the matrix elements for the two isotopes cancel each other, the result is

$$\eta = (4.3\pm0.1)\,10^{-5} , \tag{26}$$

where the uncertainty in η comes from the experimental uncertainty in S_{exptl}. To the extent that the two-nucleon models offer a correct description of $\beta\beta$ decay, this analysis of the geological data indicates a small violation of lepton number conservation. For this result to be accepted it must pass the more stringent test of direct observation of neutrinoless decay.

Equation (26) can be used to calculate the ratio of neutrinoless to total rates for ^{82}Se,

$$R(^{82}\mathrm{Se}) = \frac{\text{neutrinoless rate}}{\text{total } \beta\beta \text{ rate}} = \frac{\eta^2 f_4}{\eta^2 f_4 + f_2} = 0.09 , \tag{27}$$

which is comparable to the experimental result $R(^{82}\mathrm{Se}) \leqslant 0.09$ found by Cleveland *et al.* (1975) (Sec. II.B.1).

For Te, the only other element whose total $\beta\beta$ rates are known, the calculated ratios are $R(^{130}\mathrm{Te}) = 0.13$ and $R(^{128}\mathrm{Te}) = 0.77$.

4. Evaluation of nuclear matrix elements and comparison to experimental half-lives

Earlier efforts to calculate the nuclear matrix elements (Rosen and Primakoff, 1965) assumed that the $\beta\beta$ transition is of the type with ground state $0^+ \to 0^+$ and estimated that the only contribution was

$$|\mathrm{ME}|^2 \sim |\langle \psi_f^* | \sum_{mn} \tau_n^{(\mp)} \tau_m^{(\mp)} / r_{mn} | \psi_i \rangle|^2 \sim 0.01 R^{-2} , \tag{28}$$

where $R = 1.2\times10^{-13} A^{1/3}$ cm is the nuclear radius, r_{mn} is the separation between the nth and mth nucleons, and $\tau^{(\mp)}$ are isospin raising and lowering operators. Assuming that only neutrinoless decay occurs, these calculations yielded limits on the lepton-violation parameter of order $\eta \leqslant 10^{-3}$ (Primakoff and Rosen, 1969).

Recently it has become possible to obtain reliable nuclear wave functions by diagonalizing realistic Hamiltonians. Using these techniques Vergados (1976) calculated $\beta\beta$ decay matrix elements considering the process as a second-order Gamow–Teller β-decay transition. The first case treated was $^{48}\mathrm{Ca} \to {}^{48}\mathrm{Ti}$, which is considered a good candidate for being a closed-shell nucleus, making an exact shell-model calculation possible. It was found that transitions to the ground state of ^{48}Ti are rather weak, the stronger transitions to excited states being energetically forbidden. Due to large cancellations among the various components of the ^{48}Ti wave functions, the $\beta\beta$ decay of ^{48}Ca has a particularly low rate. This result was obtained earlier by Khodel (1970), using a theory of finite nuclear systems. Since the matrix element is small due to large cancellations, its precise value is difficult to calculate reliably. Because of this, ^{48}Ca may be a poor choice for the study of lepton nonconservation in $\beta\beta$ decay.

The other cases Vergados studied were $^{130}\mathrm{Te} \to {}^{130}\mathrm{Xe}$ and $^{128}\mathrm{Te} \to {}^{128}\mathrm{Xe}$. These nuclei are far from being closed shell, so an approximation scheme was employed. The calculations yielded values for the matrix element in Eqs. (19) and (22) of $\mathrm{ME} = -0.496$ for ^{130}Te and $\mathrm{ME} = -0.568$ for ^{128}Te. The assumption that the two matrix elements are equal in making the analysis of Te $\beta\beta$ decay independent of nuclear structure is thus a reasonable one. Note also that the above values obtained for Te are considerably greater than that obtained for ^{48}Ca, for which $\mathrm{ME} = 1.7\times10^{-2}$.

Following the procedure leading to Eq. (26) and using the above matrix elements, we find

$$\eta = (3.5\pm 0.1)\times10^{-5} . \tag{29}$$

The results from Eq. (25) are then $\tau_{1/2}(^{48}\mathrm{Ca}) = 10^{19.9}$ yr compared to $\tau_{\mathrm{exptl}}(^{48}\mathrm{Ca}) \geqslant 10^{19.56}$ yr (Bardin *et al.*, 1970), $\tau_{1/2}(^{130}\mathrm{Te}) = 10^{21.7}$ yr compared to $\tau_{\mathrm{exptl}}(^{130}\mathrm{Te}) = 10^{21.34\pm0.12}$ yr (Kirsten *et al.*, 1968), and $\tau_{1/2}(^{128}\mathrm{Te}) = 10^{24.9}$ yr compared to $\tau_{\rm exptl}(^{128}{\rm Te}) = 10^{24.54\pm0.12}$ yr (Hennecke *et al.*, 1975).

One should bear in mind that the uncertainty quoted in Eq. (29) results only from experimental error, and may be optimistic in terms of the reliability of the matrix elements for ^{130}Te and ^{128}Te.

B. Double beta decay via resonances in the nucleus

1. Neutrinoless mode

It has been conjectured that nuclear states contain probability admixtures of baryon resonances of a few percent for the least massive resonances (Kerman and Kisslinger, 1969; Arenhövel, Danos, and Williams, 1970). This view is particularly relevant to the study of $\beta\beta$ decay, since an isospin 3/2 resonance could in principle emit and reabsorb a virtual neutrino with simultaneous emission of two electrons, as in the reactions

$$\Delta^- \to p + e^- + e^-, n \to \Delta^{++} + e^- + e^- . \tag{30}$$

The $\Delta Q = 2$ baryon current is possible with the $\Delta(1232)$ (the spin 3/2, isospin 3/2 pion–nucleon resonance), because of its four charge states (+2, +1, 0, −1). This mechanism was exploited by Primakoff and Rosen (1969), who assumed a 1% admixture of $\Delta(1232)$ in the nuclear ground state. The process now involves one particle inside the nucleus, instead of two, and the overlap between the wave functions of the parent and daughter nuclei is correspondingly larger than in the conventional two-nucleon mechanism. Their calculation is similar to the one presented in Sec. III.A.1, assuming that the nucleon resonances are composites of three quarks. The mean separation between the quarks serves to keep the matrix elements finite. It was taken to be equal to the charge radius of the proton, which is about an order of magnitude smaller than the mean separation between any two nucleons in medium-to-heavy nuclei. This compensates for the small probability of finding resonances inside the nucleus. The matrix element for the first process in Eq. (30) can be calculated by the same method used for the two-nucleon mechanism. The rate for no-neutrino double beta decay is given by

$$\lambda_{0\nu} = \ln 2 \left[\left(\frac{10^{17.5}}{\eta^2 g(\epsilon_0)}\right)\left(\frac{1-\exp(-2\pi\alpha Z)}{2\pi\alpha Z}\right)^2 \times \left(\frac{\langle r\rangle}{0.7\times10^{-13}\ \mathrm{cm}}\right)^2 \left(\frac{1}{P(\Delta)|\langle\Phi_f|\Phi_i\rangle|^2}\right)\right]^{-1} \mathrm{yr}^{-1}, \tag{31}$$

where $g(\epsilon_0) = \epsilon_0^2(\epsilon_0^5 + 14\epsilon_0^4 + 81\epsilon_0^3 + 221\epsilon_0^2 + 228\epsilon_0 + 140)$, $P(\Delta)$ is the probability of finding a $\Delta(1232)$ in the nucleus, and $|\langle\Phi_f|\Phi_i\rangle|^2$ is the overlap factor between final and initial nuclear states. This factor is normalized to unity for identical momentum distributions for the Δ-nucleon pair of Eq. (30) and of the other $A-1$ nucleons. The electron–electron angular correlation was found to be

$$C(\mathbf{p}_1\cdot\mathbf{p}_2) \sim (1-\hat{p}_1\cdot\hat{p}_2)(1-\tfrac{1}{3}\hat{p}_1\cdot\hat{p}_2); \quad \hat{p}_i \equiv \mathbf{p}_i/|\mathbf{p}_i| . \tag{32}$$

In comparisons with the experimental data, Primakoff and Rosen (1969) used $P(\Delta)\approx 0.01$ and $|\langle\Phi_f|\Phi_i\rangle|^2 \approx 0.1$. The empirical half-life $\tau_{1/2}(^{130}{\rm Te}) \approx 10^{21.34\pm0.12}$ yr (Kirsten *et al.*, 1969) yielded a violation coefficient $\eta \lesssim 10^{-4}$, assuming only neutrinoless decay occurs. Once the parameters η, $P(\Delta)$, and $|\langle\Phi_f|\Phi_i\rangle|^2$ have been fixed, the predictions of this model for decays of other nuclei were essentially the same as for the neutrinoless two-nucleon model described in Sec. III.A.1.

The work described above followed a suggestion by Pontecorvo (1968) that the $\beta\beta$ decays of Te are predominantly a first-order effect of a hypothetical superweak $\Delta Q = 2, \Delta S = 0$ lepton-violating interaction. Neutrinoless $\beta\beta$ decay would occur with the simultaneous emission of two electrons by a resonance in the nucleus rather than with a second-order process of emission and reabsorption of a neutrino as calculated by Primakoff and Rosen (1969). Since this is a first-order process, a very small coupling constant could yield rates as large as those for two-neutrino $\beta\beta$ decay.

This superweak process was calculated by Smith, Picciotto, and Bryman (1973a). It was assumed that there is an admixture of the $\Delta(1232)$ in the nucleus of a few percent. A π-core model of the Δ (see Feld, 1969) was used in which the pions are just enough off energy shell to allow the emission of the electrons. The process is illustrated in Fig. 7. A Lagrangian similar to that for pion beta decay $\pi^- \to \pi^0 + e^- + \bar{\nu}$ was used, to obtain an amplitude

$$A = G'(P_1+P_2)_\mu[\bar{u}(p_2)\gamma_\mu(1+\gamma_5)\nu(p_1) - (1\leftrightarrow 2)] , \tag{33}$$

where P_1, P_2, p_1, p_2 are the momenta of the pions and electrons, respectively, and G' is the superweak coupling constant. The following rate was obtained:

$$\lambda = 7.2\times10^{-6}\left(\frac{G'}{G}\right)^2\left(\frac{2\pi Z\alpha}{1-\exp(-2\pi Z\alpha)}\right)^2 P(\Delta)|\langle\Phi_f|\Phi_i\rangle|^2 \times \epsilon_0(\epsilon_0^4 + 6\epsilon_0^3 + 25\epsilon_0^2 + 40\epsilon_0 + 48)\ \mathrm{yr}^{-1} , \tag{34}$$

where G is the weak coupling constant.

Using the values for $P(\Delta)$ and $|\langle\Phi_f|\Phi_i\rangle|^2$ given above, the half-life $\tau_{1/2} = \ln 2/\lambda$ for ^{130}Te was found to be $\tau_{1/2}(^{130}{\rm Te}) = 3\times10^{-5}(G/G')^2$ yr. Assuming that only this process is responsible for the total $\beta\beta$ decay rate, a comparison with the experimental result yields a value for the superweak coupling constant $G' \approx 10^{-13}G$.

It should be noted that μ-capture (Bryman *et al.*, 1972) and K-decay (Beier *et al.*, 1972) experiments have shown that $\Delta Q = 2$ currents may be suppressed by a factor larger than 10^3 (Primakoff and Rosen, 1972).

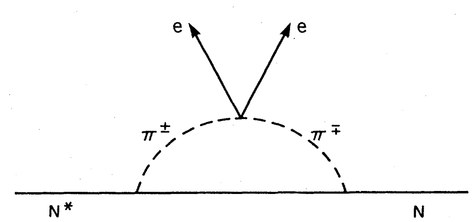

FIG. 7. First-order neutrinoless double beta decay occurring via a $\Delta(1232)$ in the nucleus.

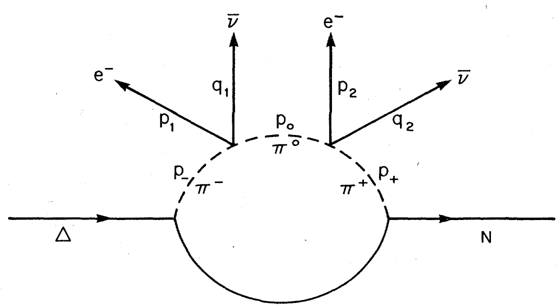

FIG. 8. Second-order two-neutrino double beta decay through the Δ(1232) mechanism.

2. Two-neutrino mode

The previous section demonstrates that resonances in the nucleus play an important role in determining limits on lepton-violating processes. Smith, Picciotto, and Bryman (1973b) have shown that a small probability admixture of resonances in the nucleus may also play an important role in lepton-conserving $\beta\beta$ decay.

A pion–core model of the Δ(1232) was used with the $\beta\beta$-decay process $\Delta^- \to p + e^- + e^- + \bar{\nu} + \bar{\nu}$ as shown in Fig. 8. It involves two consecutive transitions of the type $\pi \to \pi + e^- + \bar{\nu}_e$, which are understood in terms of the CVC hypothesis. The differential decay rate is

$$d\lambda = \sum_{\text{spins}} |B|^2 |\langle \Phi_f | \Phi_i \rangle|^2 P(\Delta) \{2\pi\alpha Z / [1 - \exp(-2\pi\alpha Z)]\}^2 \, d\rho \,, \tag{35}$$

with

$$B = G^2 \{ [\bar{u}(p_1)\gamma_\alpha(1-\gamma_5)\nu(q_1)\bar{u}(p_2)\gamma_\beta(1-\gamma_5)\nu(q_2)] \times (p_+ + p_0)_\beta [(p_+ + p_2 + q_2)^2 - m_\pi^2]^{-1} (p_0 + p_-)_\alpha - (p_1 \leftrightarrow p_2) - (q_1 \leftrightarrow q_2) + (p_1 \leftrightarrow p_2, q_1 \leftrightarrow q_2) \} \tag{36}$$

and $d\rho$ is the Lorentz-invariant phase-space element. p_1, p_2, q_1, q_2 are the four-momenta of the electrons and antineutrinos, respectively, and p_+, p_0, p_- are the four-momenta of the pions, as labeled in Fig. 8. The rate was calculated to be

$$\lambda = \ln 2 \left[(2.22 \times 10^{25}) \left(\frac{1 - \exp(-2\pi\alpha Z)}{2\pi\alpha Z} \right)^2 \times \frac{1}{|\langle \Phi_f | \Phi_i \rangle|^2 P(\Delta) \bar{K} f_0(\epsilon_0)} \right]^{-1} \text{yr}^{-1} , \tag{37}$$

where

$$f_0(\epsilon_0) = \epsilon_0^7 (1 + \epsilon_0/2 + \epsilon_0^2/9 + \epsilon_0^3/90 + \epsilon_0^4/1980) . \tag{38}$$

$\bar{K}$ is an average of the function

$$K = [(E_1+\epsilon_1)^{-1} + (E_2+\epsilon_2)^{-1}] \, \{2[(E_1+\epsilon_1)^{-1} + (E_2+\epsilon_2)^{-1}] - [(E_1+\epsilon_2)^{-1} + (E_2+\epsilon_1)^{-1}]\} + (E_1 \leftrightarrow E_2) \tag{39}$$

with E_1, E_2, ϵ_1, and ϵ_2 the energies of electrons and neutrinos, respectively.

3. Comparison to experiment and evaluation of lepton-violation parameter

First we will compare the calculated two-neutrino rates to the experimental values. Then we will include the neutrinoless contribution to calculate η, as was done in Sec. III.A for the two-nucleon model.

Using $|\langle \Phi_f | \Phi_i \rangle|^2 = 0.1$, it is found that a probability $P(\Delta) = 1.4\%$ in Eq. (37) yields a half-life for ^{130}Te equal to the experimental value $\tau_{1/2}(^{130}\text{Te}) = 10^{21.34 \pm 0.12}$ yr of Kirsten *et al.* (1968). Using the same $|\langle \Phi_f | \Phi_i \rangle|^2$ and $P(\Delta)$ for other elements, one finds $\tau_{1/2}(^{82}\text{Se}) = 10^{21.0}$ yr compared to $\tau_{1/2}^{\text{exptl}}(^{128}\text{Se}) = 10^{20.42 \pm 0.14}$ yr (Srinivasan *et al.*, 1973), $\tau_{1/2}(^{48}\text{Ca}) = 10^{19.9}$ yr compared to $\tau_{1/2}^{\text{exptl}}(^{48}\text{Ca}) > 10^{19.6}$ yr (Bardin *et al.*, 1970), and $\tau_{1/2}(^{128}\text{Te}) = 10^{24.9}$ yr compared with $\tau_{1/2}^{\text{exptl}}(^{128}\text{Te}) = 10^{24.54 \pm 0.12}$ yr (Hennecke *et al.*, 1975). These results indicate that resonances in the nucleus with a probability of the order of 1% may contribute significantly to $\beta\beta$ decay.

It should also be noted that this model predicts an electron angular correlation $1 - \mathbf{v}_1 \cdot \mathbf{v}_2 / c^2$, which is the same as the one obtained by conventional two-nucleon calculations. Therefore it may be difficult to separate the two processes in future direct-detection experiments.

Following the procedure used for the two-nucleon mechanism (Sec. III.A.3), a lepton-violation parameter based on the Δ mechanism can be obtained using the neutrinoless rate given in Eq. (31) and the two-neutrino rate given in Eq. (37). The total rate becomes

$$\lambda_{\text{tot}} = \frac{\ln 2}{\tau_{1/2}} = \lambda_{2\nu} + \lambda_{0\nu} = \ln 2 \left(\frac{f_0(\epsilon_0)\bar{K}}{10^{25.3}} + \eta^2 \frac{g(\epsilon_0)}{10^{17.5}} \right) \times \left(\frac{2\pi\alpha Z}{1 - \exp(-2\pi\alpha Z)} \right)^2 P(\Delta) |\langle \Phi_f | \Phi_i \rangle|^2 . \tag{40}$$

Then, using Eq. (8), we obtain

$$\eta = (3.4 \pm 0.1) \times 10^{-5} . \tag{41}$$

This value is independent of $P(\Delta)$ and $|\langle \Phi_f | \Phi_i \rangle|$ and is similar to the value found earlier using the two-nucleon mechanism, Eq. (29).

It is also possible to calculate the ratio

$$R = \frac{\text{neutrinoless rate}}{\text{total rate}} = \frac{\eta^2 g}{\eta^2 g + 10^{-7.8} f_0 \bar{K}} \tag{42}$$

with the results $R(^{82}\text{Se}) = 0.03$, $R(^{130}\text{Te}) = 0.06$, and $R(^{128}\text{Te}) = 0.75$. The value for ^{82}Se is consistent with the experimental limit $R(^{82}\text{Se}) \lesssim 0.09$ and is somewhat smaller than that predicted by the two-nucleon mechanism.

4. Double beta decay and massive Majorana neutrino

Double beta-decay rates have been used ingeniously by Halperin, Minkowski, Primakoff, and Rosen (1976) to put a lower limit on the mass of heavy Majorana neutrinos. The possible existence of neutrinos such as these has been discussed in the context of vectorlike gauge theories (Fritzch, Gell-Mann, and Minkowski, 1975). They are too heavy (a few GeV) to be produced in low-energy processes, but they can serve as intermediaries in the neutrinoless $\beta\beta$-decay model described in Sec. III.B.1. The process has been calculated in a way similar to that for ν_e exchange with a weak current,

$$L_\lambda = \bar{e}\gamma_\lambda(1+\gamma_5)\nu_e + \bar{e}\gamma_\lambda(1+\gamma_5)N_e\,, \tag{43}$$

where N_e is the heavy Majorana neutrino. A suitable value for the mass of the N_e could explain the observed $\beta\beta$-decay rates with exact γ_5 invariance (i.e., $\eta=0$). The amplitude is given by noncovariant perturbation theory as

$$M_{\beta\beta} = \int d^3x\, d^3y \left[\sum_{p(N_e),k} \frac{\langle\psi_f e_2 e_1|H_w(\mathbf{y})|\psi_k e_1 N_e\rangle\langle\psi_k e_1 N_e|H_w(\mathbf{x})|\psi_i\rangle}{E(N_e)+E_1+E_k-E_i} + \sum_{p(N_e),l} \frac{\langle\psi_f e_1 e_2|H_w(\mathbf{x})|\psi_l e_2 N_e\rangle\langle\psi_l e_2 N_e|H_w(\mathbf{y})|\psi_i\rangle}{E(N_e)+E_2+E_l-E_i} - (e_1 \leftrightarrow e_2)\right], \tag{44}$$

where $H_w(\mathbf{z})$ is the weak interaction Hamiltonian density, ψ_i and ψ_f are the initial and final hadronic states, and ψ_k and ψ_l are the intermediate ones. The neutrino and electron energies are $E(N_e)$ and E_1, E_2. The energy of the intermediate neutrino is much larger than E_1, E_2, and the hadronic energy differences $(E_k - E_i), (E_l - E_i)$. Therefore, to a good approximation the energy denominators are set to $E(N_e)$ and the sum over the intermediate states is carried out by closure. Assumptions were made that the fundamental process occurs through a $\Delta(1232)$ resonance in the nucleus and that an effective quark–quark potential exists inside the nucleon and $\Delta(1232)$, which confines the quarks to distances $\lesssim(2m_\pi)^{-1}$ without any hard-core repulsion. To say that this model (which assumes $\eta=0$) would account for the observed $\beta\beta$-decay lifetimes is to say that it would predict the same results as the previous model of Sec. III.B.1 with $\eta \approx 10^{-4}$. After a lengthy calculation, the following comparison was obtained:

$$P(\Delta)^{1/2} M \frac{1}{a(\frac{1}{2}Ma+1)^2} = \eta\left(\frac{m_\pi}{70}\right)\left(\frac{m_\pi}{A^{1/3}}\right), \tag{45}$$

where M is the N_e mass. With $\eta \leqslant 5\times10^{-4}$, $A=100$, $a=(2m_\pi)^{-1}$, and $P(\Delta)=0.01$, the limits obtained for M were

$$M \lesssim 1\ \text{keV}\,, \qquad M \gtrsim 3\times10^5\ \text{GeV}\,. \tag{46}$$

The lower limit is much too large to be considered a reasonable physical possibility. Setting $M=2$ GeV would yield $\eta=(8\times10^4)\times(5\times10^{-4})$, leading to a neutrinoless $\beta\beta$-decay rate 10 orders of magnitude larger than the experimental limit. This physical result would be basically unchanged even if the probability of finding a Δ resonance in the nucleus were drastically reduced.

IV. CONCLUSIONS

The results from the theoretical models described in the previous sections and the latest corresponding experimental values for the $\beta\beta$-decay half-lives have been summarized in Table III.

These results allow us to make more concrete statements about the nature of $\beta\beta$ decay than has been possible in the past. The measurement of the upper limit for neutrinoless $\beta\beta$ decay of $^{82}Se \rightarrow {}^{82}Kr$ decay leads to the experimental result $R_{\text{exptl}}(^{82}Se) = \lambda_{0\nu}/\lambda_{\text{tot}} \lesssim 9\%$. The conclusion that ^{82}Se decay is not primarily neutrinoless is independent of any theoretical analysis.

The extent to which the neutrinoless mode contributes

TABLE III. Summary of theoretical predictions and experimental values for the half-lives of double beta-decay nuclei discussed in the text. All theoretical half-lives are normalized to the ^{130}Te experimental result (Kirsten *et al.*, 1968), except for $\tau_{1/2}(^{48}Ca)$ from the two-nucleon models, which are reduced because of the smaller calculated matrix element (Vergados, 1976).

Transition	Energy (m_ec^2)	$\tau_{1/2}$ (years) theory: Two-neutrino two-nucleon [g]	Two-neutrino Δ mechanism [h]	Neutrinoless two-nucleon [i]	Neutrinoless two-quark [i]	Neutrinoless $\Delta Q=2$ [j]	$\tau_{1/2}$ (years) experiments
$^{48}Ca \rightarrow {}^{48}Ti$	8.4	$10^{20.3}$	$10^{19.9}$	$10^{20.6}$	$10^{20.5}$	$10^{20.9}$	$>10^{21.3}$ [a] neutrinoless $>10^{19.56}$ [a] two-neutrino
$^{76}Ge \rightarrow {}^{76}Se$	4.0	$10^{22.3}$	$10^{22.4}$	$10^{22.0}$	$10^{22.1}$	$10^{21.9}$	$>10^{21.7}$ [b] neutrinoless
$^{82}Se \rightarrow {}^{82}Kr$	5.9	$10^{20.9}$	$10^{21.0}$	10^{21}	$10^{21.2}$	$10^{21.3}$	$>10^{21.49}$ [c] neutrinoless $10^{20.42\pm0.14}$ [d]
$^{128}Te \rightarrow {}^{128}Xe$	1.7	$10^{25.1}$	$10^{24.9}$	$10^{23.7}$	$10^{23.5}$	$10^{22.7}$	$10^{24.54\pm0.12}$ [e]
$^{130}Te \rightarrow {}^{130}Xe$	5.0	$10^{21.3}$	$10^{21.3}$	$10^{21.3}$	$10^{21.3}$	$10^{21.3}$	$10^{21.34\pm0.12}$ [f]

[a] Bardin *et al.* (1970).
[b] Fiorini *et al.* (1973).
[c] Cleveland *et al.* (1975).
[d] Srinivasan *et al.* (1972).
[e] Hennecke *et al.* (1975).
[f] Kirsten *et al.* (1968).
[g] Konopinski (1966).
[h] Smith, Picciotto, and Bryman (1973b).
[i] Primakoff and Rosen (1969).
[j] Smith, Picciotto, and Bryman (1973a).

to $\beta\beta$ decay is more difficult to deduce. The relatively model-independent analysis of the calculated ratio $S=\tau_{1/2}(^{128}\mathrm{Te})/\tau_{1/2}(^{130}\mathrm{Te})$ (Sec. III.A.3) indicates that the experimental value for S cannot be quantitatively explained by either the two-neutrino mode alone or the neutrinoless mode alone. Assuming both modes of decay occur, it is possible to compare experimental and theoretical values for S to find $\eta \simeq (3.5\pm0.1)\,10^{-5}$. Although this result depends on the particular theoretical analysis used, basically the same value is obtained for two-nucleon and resonance models. The conclusion that lepton-number-violating neutrinoless $\beta\beta$ decay occurs is a difficult one to accept, especially when based on a single indirect experiment. Perhaps this interesting possibility will encourage further experimental work.

Using the above value of η, a theoretical $R(^{82}\mathrm{Se}) \approx 3-9\%$ is found which is consistent with the experimental $R_{\mathrm{exptl}}(^{82}\mathrm{Se}) \lesssim 9\%$. An improvement of the experimental sensitivity for neutrinoless $\beta\beta$ decay of ^{82}Se would be extremely useful. In addition, a reliable calculation of the $\beta\beta$-decay matrix element for $^{82}\mathrm{Se}\to{}^{82}\mathrm{Kr}$ transition would be important.

Possibly a search for neutrinoless $\beta\beta$ decay of ^{130}Te could be made using a technique similar to that of Fiorini *et al.* (1973), for ^{76}Ge, but with the Ge(Li) detector source replaced by a detector source made of cadmium telluride.[3] Small cadmium telluride detectors which have a 17% natural abundance of ^{130}Te have been fabricated giving energy resolutions ~0.5% in the few MeV range (Zanio *et al.*, 1974). Unlike Ge(Li) detectors, CdTe detectors operate at 300 °K, obviating the need for a cryostat and liquid nitrogen, which were the major sources of background suspected in the experiment of Fiorini *et al.* (1973). Thus there is a freer choice of local shielding, although the natural uranium content of the telluride may still be a major difficulty.

Experimental advances which would enable the direct observation of two-neutrino $\beta\beta$ decay would also be extremely important. The main obstacle is further reducing the already minute concentrations of extraneous radioactive nuclei found in the $\beta\beta$-decay source and the detection apparatus. The problem is illustrated in the analysis of the decay of ^{48}Ca (Bardin *et al.*, 1970), which was limited to an electron energy-sum greater than 2.2 MeV, because it was not possible to completely eliminate background from a small contamination of radium in the samples.

Even if the $\beta\beta$-decay spectrum is observed to proceed mainly through the lepton-conserving mode, there is the additional question: what internal mechanisms contribute? Of particular interest is the role of resonances in the nucleus. The $\Delta(1232)$ mechanism, in which two lepton pairs are emitted by the resonance, seems to explain the data as well as the two-nucleon model and may, therefore, be a significant contributor to $\beta\beta$ decay. Distinguishing between mechanisms is complicated by the fact that the angular correlations are similar to those predicted by the conventional two-nucleon mode.

Immediate future prospects for geological experiments on nuclei other than Se and Te appear to be limited. The techniques used rely on unambiguous mass-spectroscopic measurements of heavy noble gases, Kr and Xe. For other $\beta\beta$-decay candidates to be observed, there are the extremely difficult tasks of matching the mass-spectroscopic sensitivity attainable for noble gases (of order 10^{-14} cm^3 STP for Xe per gram of ore), finding a deposit dated by independent means of age ~10^9 yr, and eliminating competing reactions as alternate sources of the searched-for decay product.

[3]We are grateful to Dr. J. A. Macdonald for bringing cadmium telluride detectors to our attention.

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