Elementary modes of excitation in the nucleus*

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In the field of nuclear dynamics a central theme has been the struggle to find the proper place for the complementary concepts referring to the independent motion of the individual nucleons and the collective behavior of the nucleus as a whole. This development has been a continuing process involving the interplay of ideas and discoveries relating to all different aspects of nuclear phenomena. The multidimensionality of this development makes it tempting to go directly to a description of our present understanding and to the problems and perspectives as they appear today. However, an attempt to follow the evolution of some of the principal ideas may be instructive in illustrating the struggle for understanding many-body systems, which have continued to inspire the development of fundamental new concepts, even in cases where the basic equations of motion are well established. Concepts appropriate for describing the wealth of nuclear phenomena have been derived from a combination of many different approaches including the exploration of general relations following from considerations of symmetry, the study of model systems, sometimes of a grossly oversimplified nature, and, of course, the clues provided by the experimental discoveries which have again and again given the development entirely new directions.[1]

The situation in 1950, when I first came to Copenhagen, was characterized by the inescapable fact that the nucleus sometimes exhibited phenomena characteristic of independent-particle motion, while other phenomena, such as the fission process and the large quadrupole moments clearly involved a collective behavior of the whole nucleus.

It was also clear from the work of Rainwater that there was an important coupling between the motion of the individual particles and the collective deformation, and one was thus faced with the problem of exploring the properties of a dynamical system involving such coupled degrees of freedom (Foldy and Milford, 1950; Bohr, 1952; Hill and Wheeler, 1952; Bohr and Mottelson, 1953)

$$H = H_{\rm vib} + H_{\rm part} + H_{\rm coupl},$$

$$H_{\rm vib} = \tfrac{1}{2}C_\lambda \sum_\mu |\alpha_{\lambda\mu}|^2 + \tfrac{1}{2}D_\lambda \sum_\mu |\dot{\alpha}_{\lambda\mu}|^2, \qquad (1)$$

$$H_{\rm coupl} = \sum_p k(r_p) \sum_\mu \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta_p \varphi_p),$$

where $\alpha_{\lambda\mu}$ are the amplitudes of the nuclear deformation expanded in spherical harmonics and $(r_p, \theta_p, \varphi_p)$ are the coordinates of the particles considered. The coupling term represents the effect of the deformation on the one-particle potential.

I remember vividly the many lively discussions in these years reflecting the feeling of unease, not to say total disbelief, of many of our colleagues concerning the simultaneous use of both collective and single-particle coordinates to describe a system that we all agreed was ultimately built out of the neutrons and protons themselves. Niels Bohr participated very actively in these discussions. Something of the flavor of his contribution can perhaps be gathered from the exchange recorded in the Proceedings of the CERN International Physics Conference in Copenhagen from June 1952; I had given a report on our work and in the discussion Rosenfeld "asked how far this model is based on first principles." N. Bohr "answered that it appeared difficult to define what one should understand by first principles in a field of knowledge where our starting point is empirical evidence of different kinds, which is not directly combinable."

I would like to take this opportunity to acknowledge the tremendous inspiration it has been for me to have had the privilege of work for the entire period covered by this report within the unique scientific environment created by Niels Bohr.

INTERPRETATION OF LOW-ENERGY NUCLEAR EXCITATION SPECTRA

In the beginning of the 1950's, there existed very little evidence on nuclear spectra which could be used to test these ideas. In the following years, however, a dramatic development of nuclear spectroscopy took place. The new data made possible the identification of the characteristic patterns of rotational spectra (see Bohr, 1976) and shortly afterwards the recognition by Scharff-Goldhaber and Weneser (1955) that a significant class of spectra exhibit patterns corresponding to quadrupole vibrations about a spherical equilibrium.[2] The existence of the static deformations in certain classes of nuclei received further decisive confirmation in the successful classification of the intrinsic states of these spectra in terms of one-particle motion in an appropriately deformed potential (see Bohr, 1976).

A striking feature in the developing picture of nuclear excitation spectra was the distinction between a class of nuclei with spherical shape and others with large deformations. The clue to the origin of this distinction

*This lecture was delivered on the occasion of the presentation of the 1975 Nobel Prizes in Physics.

[1]We would like to take this opportunity to pay tribute to the ingenuity and resourcefulness of the generation of experimentalists whose untiring efforts have created the basis for the development sketched in our reports today.

[2]This step followed the recognition of striking regularities in the low-energy spectra of even–even nuclei, including the spins and parities (Goldhaber and Sunyar, 1951; Scharff-Goldhaber, 1952), energy systematics (Scharff-Goldhaber, 1952; Stähelin and Preiswerk, 1951; Rosenblum and Valdares, 1952; Asaro and Perlman, 1952), and selection rules (Kraushaar and Goldhaber, 1953).

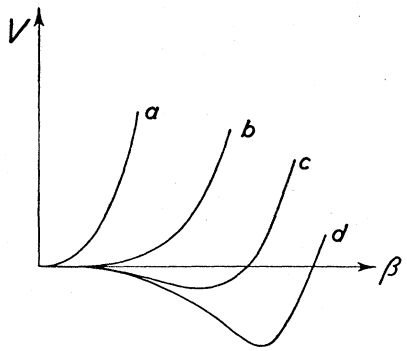

FIG. 1. Nuclear potential energy function. The figure, taken from Alder *et al.*, (1956), gives a schematic representation of the nuclear energy as function of the deformation β. The curve a represents a configuration with only relatively few particles outside of closed shells. As particles are added, the restoring force for the spherical shape ($\beta=0$) decreases (curve b). Still further from the closed shells, the spherical shape may become unstable (curves c and d) and the nucleus acquires a nonspherical equilibrium shape.

came, rather unexpectedly, from the analysis of the moments of inertia of the rotational spectra. The cranking model of Inglis (1954) had provided a starting point for a microscopic interpretation of the rotational motion, and the analysis showed that significant deviations from independent-particle motion were required to account for the observed magnitude of the moments of inertia. These correlations could be attributed to the residual interactions that tend to bind the nucleons into pairs with angular momentum zero. Such a pair is spherically symmetric, and this nucleonic correlation could therefore, at the same time, be seen to provide an effect tending to stabilize the spherical shape (Bohr and Mottelson, 1955).

Thus, quite suddenly the way was opened to a qualitative understanding of the whole pattern of the low-energy excitation spectra in terms of a competition between the pairing effect and the tendency toward deformation implied by the anisotropy of the single-particle orbits. The outcome of this competition depends on the number of particles in unfilled shells; for few particles, the deformation in the absence of interactions is relatively small and can easily be dominated by the tendency to form spherical pairs; but with increasing number of particles, the spherical equilibrium shape becomes less stable and eventually a transition takes place to a deformed equilibrium shape. These considerations are illustrated by the potential energy surfaces shown in Fig. 1.

MICROSCOPIC DESCRIPTION OF COLLECTIVE MOTION

This qualitative interpretation of the nuclear coupling schemes could soon be given a firmer basis in terms of many-body wave functions that describe the correlation effects governing the low-energy nuclear spectra.

A step towards a microscopic understanding of the deformation effect resulted from the discovery of rotational spectra in light nuclei.[3] For these nuclei, even a few particles represent a significant fraction of the total and can give rise to deformations that are among the largest observed. The spectra of some of these nuclei had previously been successfully analyzed in terms of shell-model configurations (see Bohr, 1976). Thus, for the first time one had a many-body wave function with rotational relationships and one could see explicitly that the main effect of the rather complicated finite range interactions employed in the shell model calculations had been to generate a deformed average potential.

The essence of this development was brought into focus by Elliott's discovery that the SU_3 classification scheme for particles in a harmonic oscillator potential leads to multiplets with rotational relations (Elliott, 1958). The effective two-body interaction that is invariant under SU_3 symmetry (when acting within the configurations of a major shell) and thus leads to the rotational coupling scheme, is given by the scalar product of the quadrupole moments of each pair of particles

$$V_{\text{eff}} = \tfrac{1}{2}\kappa \sum_{ij} (q(i)q(j))_0 ,$$
$$q_\mu(i) = r_i^2 Y_{2\mu}(\theta_i \varphi_i) . \tag{2}$$

Such a two-body force is equivalent to the interaction of each particle with the total quadrupole moment of the system and thus to the effect of an ellipsoidal deformation in the average potential.

In retrospect, the important lesson of this development was the recognition that the aligned wave function

$$\Psi = \mathcal{A}(\psi_{k_1}(x_1)\psi_{k_2}(x_2)\cdots\psi_{k_A}(x_A)) \tag{3}$$

obtained as a simple product of single-particle states in a self-consistent deformed potential provides a starting point for the full many-body wave function.[4] This viewpoint had indeed been implied by the establishment of the classification based on the Nilsson scheme, but the revelation of the exact SU_3 solution, even in such an oversimplified model, contributed greatly to the confidence in this approach.

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[3]Rotational band structure and the classification of the intrinsic states for (sd)-shell nuclei was first established in 1955 following the extensive series of experiments at Chalk River [see the survey by A. Litherland *et al.* (1958)]. For the classification of the p-shell nuclei in terms of the rotational coupling scheme, see Kurath and Pičman (1959). For our own understanding of the special flavor of these very light nuclei, the discussions over the years with Tom Lauritsen were a continuing challenge and source of inspiration.

[4]The wave function given by Eq. (3) represents the intrinsic state in the absence of rotation, and can be directly employed in obtaining the leading order intensity relations. The I-dependent terms, such as the rotational energy, are obtained by including the rotational perturbations in the intrinsic motion, as in the cranking model. The SU_3 coupling scheme represents a special case in which the total wave function with the inclusion of rotational effects can be expressed as a projection of the intrinsic wave function onto a state of specified angular momentum (Elliott, 1958). Such projected wave functions had been employed earlier (Hill and Wheeler, 1953; Peierls and Yoccoz, 1957; Yoccoz, 1957; Villars, 1957). See also the discussion in Bohr and Mottelson (1975).

The second major development involved the many-particle interpretation of the nuclear pairing effect. As we have seen, this problem had become a crucial one for the quantitative analysis of collective motion in the nucleus, but the story of the pairing effect goes back much further to the very earliest days of nuclear physics (Rutherford, Chadwick, and Ellis, 1930). The discovery of the neutron made it possible to interpret the accumulated systematics concerning the differences in stability of odd and even nuclei in terms of an additional binding associated with even numbers of protons or neutrons (Heisenberg, 1932). This effect later provided the basis for understanding the striking difference in the fission of the odd and even isotopes of uranium (N. Bohr and Wheeler, 1939). The pairing effect also played an important role in the development of the shell model since it provided the basis for the interpretation of many of the properties of odd-A nuclei in terms of the binding states of the last odd particle (Mayer, 1950; Racah, 1952; Racah and Talmi, 1953).

The key to understanding the correlation effect underlying the odd-even differences came from the discovery by Bardeen, Cooper, and Schrieffer of the profound new concepts for treating the electronic correlations in superconductors (Bardeen, Cooper, and Schrieffer, 1957a and 1957b).[5] It was a marvelous thing that the correlations, which might appear to be associated with such complexity, could be simply expressed in terms of a generalized one-body problem in which the particles move in a potential which creates and annihilates pairs of particles giving rise to the quasiparticles that are superpositions of particles and holes (Bogoliubov, 1958; Valatin, 1958). It could also be seen that the many-body wave function represented a generalization of Racah's seniority coupling scheme (Racah, 1943) which had been exploited in the interpretation of the one-particle model in nuclei.

One thus had available the basic tools for a microscopic analysis of the coupling schemes encountered in the low-energy nuclear spectra. These tools were rapidly exploited to treat the moments of inertia of rotating nuclei (Belyaev, 1959; Migdal, 1959; Griffin and Rich; 1960; Nilsson and Prior, 1961), the potential energy surfaces and inertial parameters for the vibrations of spherical nuclei (Belyaev, 1959; Kisslinger and Sorensen, 1960), as well as the effects of pair correlations on a variety of nuclear processes (Soloviev, 1961 and 1962; Yoshida, 1961 and 1962; Mang and Rasmussen, 1962; Cohen and Price, 1961).

This was indeed a period of heady development in the understanding of many-body problems with a fruitful interplay of experience gained from the study of so many different systems that nature had provided, including the "elementary particles" that had stimulated the development of the powerful tools of relativistic field theory. An important clarification in the description of collective motion was the new way of viewing the normal modes of vibration as built out of correlated two-quasiparticle (or particle-hole) excitations. The significant part of the interactions creates and annihilates two such basic excitations and the vibrations can thus be obtained from the solution of a generalized two-body problem.[6] This approach not only complemented the previously applied adiabatic treatment of nuclear collective motion, but also gave a broader scope to the concept of vibration that was to be important for the subsequent development.

The whole picture of nuclear physics at this stage in the development is beautifully expressed by Weisskopf in his summary talk at the Kingston Conference in 1960, where the recurring theme is his comment again and again: "It works surprisingly well."[7]

THE GREAT VARIETY OF COLLECTIVE MODES

While the low-frequency spectra are dominated by transitions of particles within the partly filled shells, new aspects of nuclear dynamics are associated with the excitation of the closed shells. The classic example of a collective excitation of this type is the "giant dipole resonance" which was discovered in the study of the photo-processes soon after the war (Baldwin and

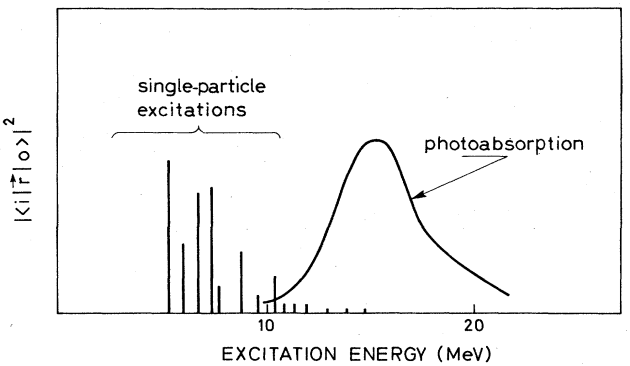

FIG. 2. Frequency distribution of nuclear electric dipole excitations. The figure is a schematic representation (for $A \approx 100$) of the dipole strength for single-particle excitations as compared with the observed frequency distribution of the photo-absorption cross section.

[5]It was a fortunate circumstance for us that David Pines spent a period of several months in Copenhagen in the summer of 1957, during which he introduced us to the exciting new developments in the theory of superconductivity. Through the discussions with him, the relevance of these concepts to the problem of pair correlations in nuclei became apparent (Bohr, Mottelson, and Pines, 1958). An important component in these discussions was the fact that the experimental evidence had been accumulating for the existence of an energy gap in the excitation spectra of nuclei reminiscent of that observed in superconductors (Bohr and Mottelson, 1955; Bohr, Mottelson, and Pines, 1958). For the recognition of the odd–even difference in nuclear excitation spectra, striking evidence had come from the high-resolution spectroscopic studies of ^{182}W and ^{183}W made possible by the bent crystal spectrometer (Murray *et al.*, 1955).

[6]See, for example, Glassgold, Heckrotte, and Watson, 1959; Ferrell and Fallieros, 1959; Goldstone and Gottfried, 1959; Takagi, 1959; Ikeda *et al.*, 1959; Arvieu and Véneroni, 1960 and 1961; Baranger, 1960; Kobayasi and Marumori, 1960; Marumori, 1960; Thouless, 1961.

[7]We would like to acknowledge the deep importance for us of the close personal contact with Viki Weisskopf who has provided inspiration for a whole generation of nuclear physicists.

Klaiber, 1947 and 1948) and which could be given an interpretation in terms of collective motion of the neutron and proton fluids with respect to each other (Goldhaber and Teller, 1948; Steinwedel and Jensen, 1950).

After the development of the shell model, attempts were made to describe the photo-absorption in terms of single-particle excitations (see especially Wilkinson, 1956) but one encountered the problem that the one-particle excitations that should carry the main part of the dipole strength appeared in a part of the spectrum quite distinct from that in which the strong dipole absorption was observed (see Fig. 2). This led to a period of lively discussions, and for a time it was felt that the single-particle and collective descriptions represented opposite and mutually exclusive interpretations.[8]

A step in the resolution of the problem resulted from a study of the interaction effects in the single-particle excitations of the closed-shell configuration of ^{16}O, which revealed a strong tendency towards the formation of linear combinations of different particle–hole configurations collecting the major part of the dipole strength and shifting it to higher energy (Elliot and Flowers, 1957). A highly simplified model based on degenerate single-particle excitations, as in the harmonic oscillator potential, again provided valuable insight by exhibiting exact solutions, in which the total dipole strength was collected into a single high-frequency excitation (Brink, 1957; Brown and Bolsterli, 1959). These schematic models could soon be seen in the more general framework of the normal modes treatment referred to above.

In carrying through this program, one faced the uncertainty in the effective forces to be employed, but it was found possible to represent the interactions by an oscillating average potential acting with opposite sign on neutrons and protons, the strength of which could be related to the isovector component in the static central potential that is present in nuclei with a neutron excess (Bohr and Mottelson, 1969 and 1975).[9] Indeed, it appears that all the collective nuclear modes that have been identified can be traced back to average fields of specific symmetry generated by the effective interaction.

The new insight into the manner in which the vibrations are generated by the interactions in the various channels of particle excitations opened a whole new perspective since one became liberated from the classical picture of vibrations and could begin to imagine the enormously greater variety of vibrational phenomena that are characteristic of quantal systems. This perspective became apparent 10 to 15 years ago but there was at that time very little experimental evidence on

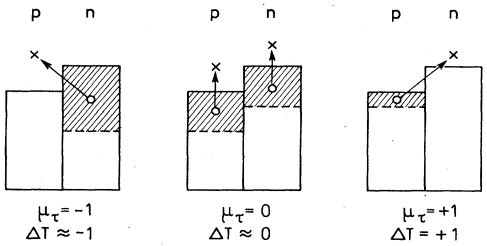

FIG. 3. Single-particle dipole excitations in a nucleus with neutron excess. The boxes represent the occupied proton (p) and neutron (n) orbits and the hatched domains correspond to the particle orbits that can be excited by the isovector dipole field with different components, μ_τ. For large values of the neutron excess, the excitations lead to a change ΔT in the total isospin quantum number equal to μ_τ. The figure is from Bohr and Mottelson (1975).

which to build. The understanding of some of the features in this rich fabric of possibilities has been the result of a gradual process [which added a decade to the gestation of Vol. II of our work on Nuclear Structure], and which is still continuing. A few examples may give an impression of the scope of the new phenomena.

The dipole mode is of isovector character and each quantum of excitation carries unit isospin. It is thus a component of a triplet, which also includes excitations that turn neutrons into protons and *vice versa*. In a nucleus with equal numbers of neutrons and protons, and total isospin $T_0 = 0$ in the ground state, the triplet of excitations represents an isobaric multiplet and are therefore directly related in terms of rotations in isospace. However, in a nucleus with neutron excess and isospin $T_0 \neq 0$ in the ground state, the dipole excitations with charge exchange may be very different from those with zero component of the isospin (see Fig. 3). The resulting dipole excitation spectrum is schematically illustrated in Fig. 4 and presents an example of symme-

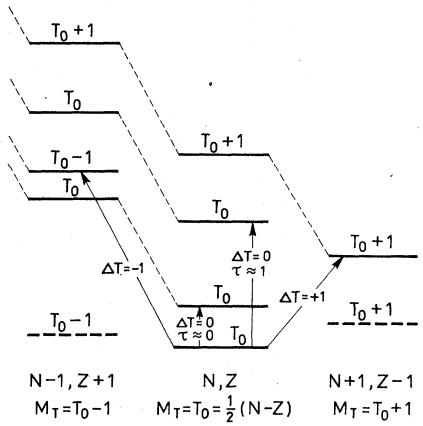

FIG. 4. Isospin of vibrational excitations in nucleus with neutron excess. The ground state of such a nucleus has a total isospin component $M_T = \frac{1}{2}(N-Z)$, and total isospin $T_0 = M_T$. The figure gives a schematic illustration of the pattern of states formed by adding vibrational quanta with isospin $\tau = 0$ and $\tau = 1$. Isobaric analog states are connected by thin broken lines. The ground states of the isobaric nuclei with $M_T = T_0 \pm 1$ are indicated by dashed lines. The figure is taken from Bohr and Mottelson (1975).

[8]See, for example, the discussion in *Proceedings of the Glasgow Conference on Nuclear and Meson Physics*, edited by E. H. Bellamy and R. G. Moorhouse (Pergamon, London, 1955).

[9]The close similarity of the results of the hydrodynamic and the microscopic treatments is a special feature of the dipole mode (Bohr and Mottelson, 1975), associated with the fact that the single-particle response function for this channel is concentrated in a single frequency region (see Fig. 2).

try breaking resulting from the lack of isobaric isotropy of the "vacuum" (the nuclear ground state). Some of the features in the pattern indicated in Fig. 4 have been experimentally confirmed, but the major part of this rich structure remains to be explored.[10]

Another dimension to the vibrational concept is associated with the possibility of collective fields that create or annihilate pairs of particles, in contrast to the field associated with the dipole mode that creates particle–hole pairs and therefore conserves particle number. The new fields are connected with the pairing component in the nuclear interactions which tend to bind pairs into a highly correlated state of angular momentum zero. The addition of such a pair to a closed shell constitutes an excitation that can be repeated and which can thus be viewed as a quantum of a vibrational mode. Figure 5 shows the pair vibrational spectrum with the two modes associated with addition and removal of pairs of neutrons from the closed-shell configuration of ^{208}Pb. One thus encounters a vibrational band in which the members belong to different nuclei. In systems with many particles outside closed shells the ground state can be viewed as a condensate of correlated pairs, as in the superconductor (Anderson, 1964). Such a condensate can be expressed as a static *deformation* in the magnitude of the pair field, and the addition and removal of pairs from the condensate constitute the associated rotational mode of excitation.

The clarification of the dynamical role of pair fields in the nucleus has resulted from a close interplay of experimental and theoretical work.[11] From the experimental side, the decisive contribution came from the study of reactions in which a correlated pair of nucleons is added or removed from the nucleus as in the (tp) or (pt) reactions (see Broglia *et al.*, 1973).

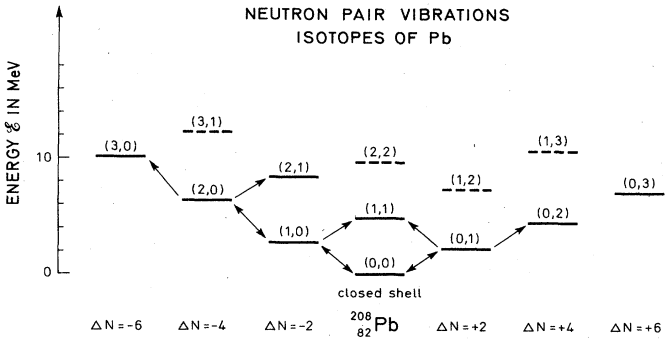

FIG. 5. Neutron monopole pair vibrations based on ^{208}Pb. The levels in the pair vibrational spectrum are labeled by the quantum numbers (n_-, n_+), where $n_\pm$ corresponds to the number of correlated $J=0$ pairs that have been added to or removed from the closed-shell configuration of ^{208}Pb. Thus, the levels $(n_-, 0)$ and $(0, n_+)$ correspond to the ground states of the even Pb isotopes. The observed levels are indicated by solid lines, while the dashed lines indicate the predicted positions of additional levels. The strong two-neutron transfer processes [(pt) and (tp)] that have so far been observed are indicated by arrows. The figure is from Bohr and Mottelson (1973).

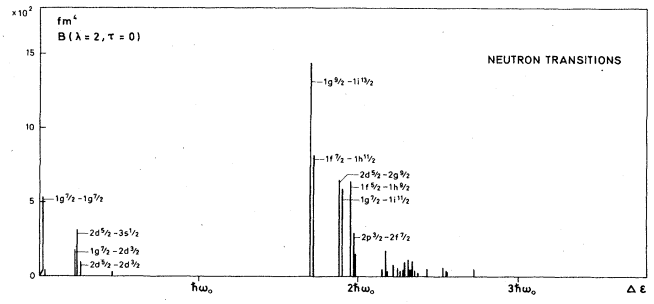

FIG. 6. Single-particle response function for quadrupole excitations. The figure gives the strength of the transitions produced by the quadrupole operator $r^2Y_{2\mu}$ acting on a nucleus with neutron number $N=60$. The single-particle spectrum has been obtained from a potential represented by a harmonic oscillator with the addition of spin-orbit coupling and anharmonic terms reflecting the flatter bottom and steeper sides of the nuclear potential. The excitation energies are plotted in terms of the oscillator frequency ω_0, and for the nucleus considered $\hbar\omega_0 \approx 8.7$ MeV. The figure is from Bohr and Mottelson (1975).

The new views of vibrations also lead to important insight concerning shape oscillations. While the early considerations were guided by the classical picture provided by the liquid-drop model (N. Bohr and Kalckar, 1937; Bohr and Mottelson, 1953) the lesson of the microscopic theory has been that one must begin the analysis of the collective modes by studying the single-particle excitations produced by fields of the appropriate symmetry.

For quadrupole excitations, an example of such a single-particle response function is shown in Fig. 6 and reveals that the quadrupole excitations involve two very different frequency regions. The first is associated with transitions within the partially filled shells and gives rise to the low-frequency quadrupole mode discussed above. The second frequency region in the quadrupole response function is associated with transitions between orbits separated by two major shells and contains most of the oscillator strength. This group of transitions generates a high-frequency collective

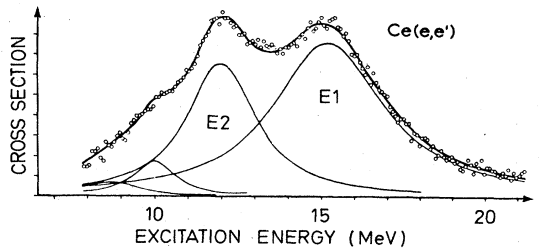

FIG. 7. Inelastic electron scattering on Ce. The figure is from Pitthan and Walcher (1971) and (1972). The highest energy resonance line corresponds with the well-known isovector dipole resonance observed in photo-absorption, while the resonance at an excitation energy of about 12 MeV is identified with the isoscalar quadrupole mode.

[10]For a summary of this development, see Fallieros (1973) and Bohr and Mottelson (1975).

[11]The concept of pair vibrations in nuclei evolved through the discussions of Högaasen-Feldman (1961), early versions of A. Bohr and Mottelson (1975) [see, for example, Bohr (1964) and (1968)] and Bès and Broglia (1966). Excited states of pair vibrational type were identified in the region of ^{208}Pb by Bjerregaard, Hansen, Nathan, and Hinds (1966).

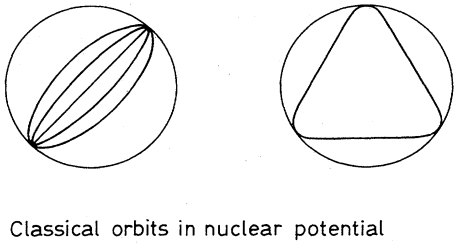

FIG. 8. Periodic orbits in nuclear potential. For small values of angular momentum the motion resembles the elliptical orbits in the oscillator potential. For larger values of angular momentum the effects of the rather sharp nuclear surface can give rise to approximately triangular orbits.

mode which has been eagerly expected for many years (Mottelson, 1960); a few years ago, the study of inelastic electron scattering led to the identification of this mode (Pitthan and Walcher, 1971 and 1972) (see Fig. 7), which has since been found as a systematic feature in a wide variety of inelastic scattering experiments (see for example the review by Satchler, 1974). This discovery opens the possibility for a deeper probing of one of the fundamental degrees of freedom in the nucleus.

Returning to the quadrupole response function, the low-frequency excitations reflect a degeneracy in the single-particle spectrum, which is responsible for the tendency to break away from spherical symmetry and to form a spheroidal equilibrium shape (Jahn-Teller effect). One may ask: What underlies this degeneracy in the single-particle spectrum? Apparently this question was never seriously asked until the discovery of the fission isomers (Polikanov *et al.*, 1962) revealed the occurrence of important shell-structure effects in potentials that deviate in a major way from spherical symmetry (Strutinsky, 1967) (saddle-point shape). These developments posed in an acute way the question of the general conditions for the occurrence of significant degeneracies in the eigenvalue spectrum for the wave equation. It has been possible to relate this question to the occurrence of degenerate families of periodic orbits in the corresponding classical problem (Balian and Bloch, 1971; Wheeler, 1971; Swiatecki, 1971; Bohr and Mottelson, 1975), and the instabilities that arise for partially filled shells directly reflect the geometry of these classical orbits. Thus, the observed quadrupole deformations in nuclei can be associated with the elliptical orbits for particle motion in an harmonic oscillator potential (see Fig. 8). The nuclear potential in heavy nuclei also supports orbits of triangular symmetry and indeed there is evidence for an incipient octupole instability in heavy nuclei.

MODERN VIEW OF PARTICLE-VIBRATION COUPLING

The picture of nuclear dynamics that has emerged from these developments thus involves a great variety of different collective excitations that are as elementary as the single-particle excitations themselves, in the sense that they remain as approximately independent entities in the construction of the nuclear excitation spectrum. Examples of the superposition of elementary

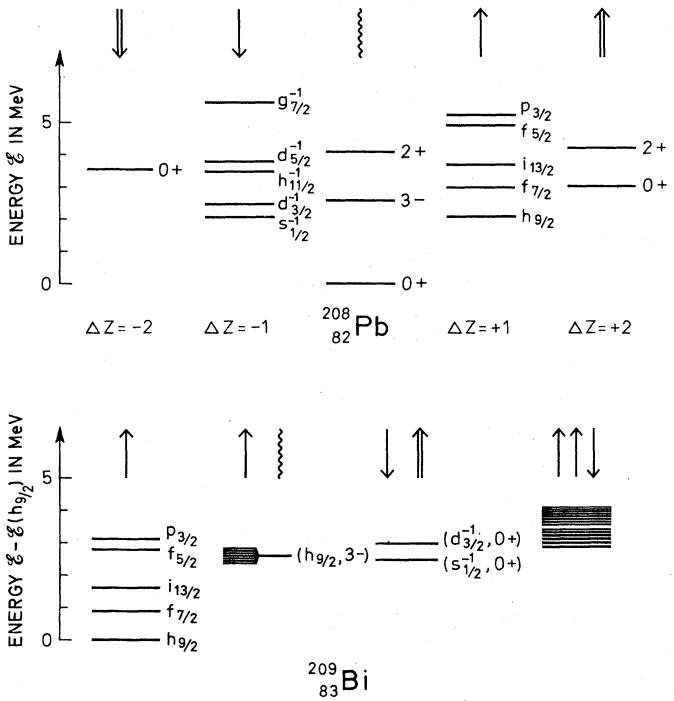

FIG. 9. Elementary excitations based on the closed shell of ^{208}Pb. The upper part of the figure shows fermion excitations involving the addition or removal of a single proton ($\Delta Z=+1$ or $\Delta Z=-1$), and boson excitations involving correlated pairs of protons ($\Delta Z=\pm 2$), as well as collective shape oscillations (particle–hole excitations) in ^{208}Pb itself. The lower part of the figure gives the observed spectrum of $^{209}_{83}$Bi which comprises partly the single-proton states, and partly states involving the combinations of a single particle or a single hole with a collective boson. The configuration $(h_{9/2},3-)$ gives rise to a septuplet of states with $I=\frac{3}{2},\ \frac{5}{2}\ldots\frac{15}{2}$ which have all been identified within an energy region of a few hundred keV (see Fig. 12). At an excitation energy of about 3 MeV a rather dense spectrum of two-particle one-hole states sets in, as indicated to the right in the figure. The figure is taken from Bohr and Mottelson (1973).

modes of excitation are given in Fig. 9 (see also Fig. 5).

In the analysis of the elementary modes and their interactions, a central element is the particle–vibration coupling which expresses the variations in the average potential associated with the collective vibrational amplitude. This coupling is the organizing element that generates the self-consistent collective modes out of the particle excitations. At the same time it gives rise to interactions that provide the natural limitation to the analysis in terms of elementary modes.

Information about the particle-vibration coupling comes from a variety of sources. For some modes, such as the shape oscillations, the coupling can be related to observed static potentials. More generally, the couplings directly manifest themselves in inelastic scattering processes and indirectly in the properties of the modes and their interactions.

The average one-particle potentials appearing in the particle-vibration coupling are of course ultimately related to the underlying nucleonic interactions. Indeed many of our colleagues would stress the incompleteness

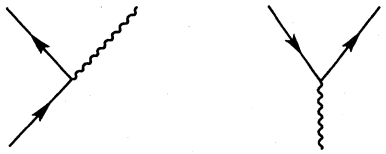

FIG. 10. Basic diagrams for particle-vibration coupling. The solid lines represent particles, and the wavy line a phonon of a collective excitation. The particle-vibration coupling creates or annihilates vibrational quanta and at the same time either scatters a particle (or hole) or creates a particle–hole pair.

in a description that is not explicitly based on these interactions. However, we would emphasize that the potentials are physically significant quantities in terms of which one can establish relationships between a great variety of nuclear phenomena.[12]

It is of course a great challenge to exploit the extensive and precise information available on the two-body forces and the structure of hadrons in order to shed light on the average nuclear potentials. The problem is a classical one in nuclear physics and has continued to reveal new facets, not only because of the complexity of the nuclear forces, but also due to the many subtle correlations that may contribute to the effective interactions in the nuclear medium.

The basic matrix elements of the particle-vibration coupling can be represented by the diagrams in Fig. 10, which form the basis for a nuclear field theory based on the elementary modes of fermion and boson type. In lowest order, the coupling gives rise to a renormalization of the effective moments of a particle illustrated by the diagrams in Fig. 11. This renormalization is a major effect in the transitions between low-lying single-particle states and provides the answer to the old dilemma concerning the distribution of the strength between the particle excitations and the collective modes. Thus, for example, for the dipole mode,

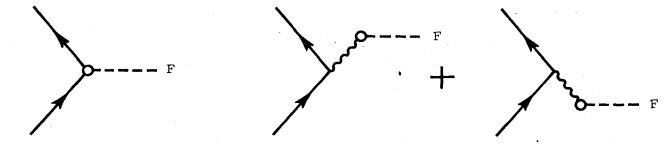

FIG. 11. Renormalization of the matrix elements of a single-particle moment resulting from particle vibration coupling. The moment F may be any operator that acts on the degrees of freedom of a single particle, such as an electric or magnetic moment, β-decay transition moment, etc.

[12]This issue appears to be endemic to all strongly interacting many-body systems ranging from condensed matter to elementary particles. The approach described here is closely related to that of the Fermi-liquid theory as developed by Landau (1956) and (1957). This formulation operates with a phenomenological effective interaction between the quasiparticles from which the coupling between the particles and the collective modes can be derived. The description of nuclear dynamics in terms of the concepts employed in the theory of Fermi liquids has been developed by Migdal (1967).

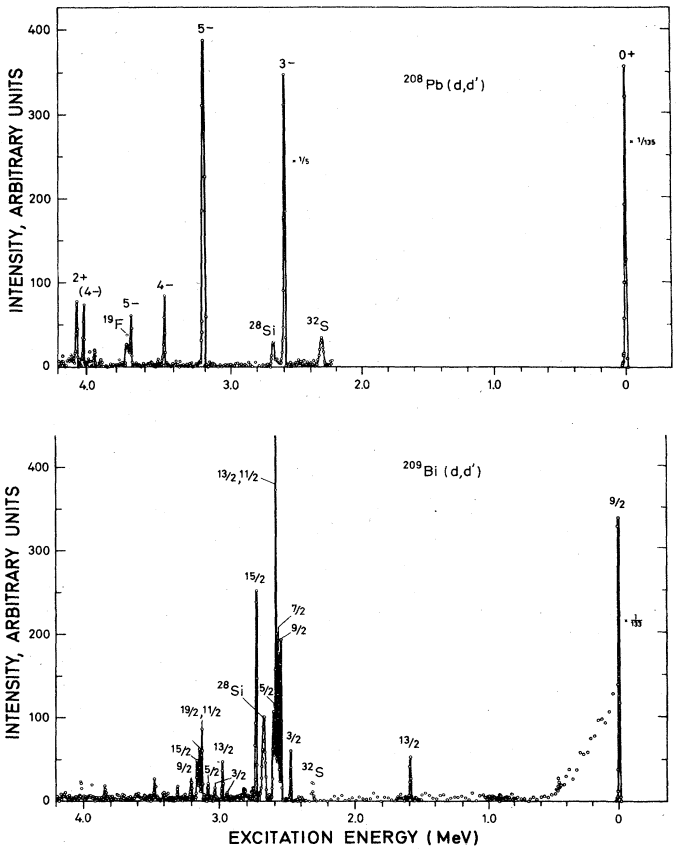

FIG. 12. Energy spectra of deuterons scattered from ^{208}Pb and ^{209}Bi. The figure is taken from Bohr and Mottelson (1975) and is based on data from Ungrin *et al.* (1971). The prominent inelastic group in ^{208}Pb corresponds to the excitation of an octupole vibrational phonon ($I\pi = 3-$; $\hbar\omega_3 = 2.6$ MeV). In ^{209}Bi the ground state has $I = \frac{9}{2}$, corresponding to a single $h_{9/2}$ proton outside the closed-shell configurations. The excitation of the octupole quantum leads to a septuplet of states in the neighborhood of 2.6 MeV with $I = \frac{3}{2}, \frac{5}{2} \ldots \frac{15}{2}$.

the one-particle excitations carry a very small admixture of the collective mode, which is sufficient to almost cancel the dipole moment of the bare particle.[13]

Acting in higher order, the particle-vibration coup-

[13]While the renormalization of the electric quadrupole operator followed directly from the coupling to the deformation of the nuclear surface (Alder *et al.*, 1956; Bohr and Mottelson, 1975) the occurrence of large deviations in the magnetic moments for configurations with a single particle outside of closed shells was felt as an especially severe challenge to the description in terms of particles coupled to surface oscillations (see, for example, Bohr and Mottelson, 1953). The clue to the understanding of this effect came from the recognition that special kinds of configuration mixings could give rise to large first-order effects in the magnetic moments (Arima and Horie, 1954; Blin-Stoyle and Perks, 1954). Later, it was recognized that this was a manifestation of the particle-vibration coupling involving collective modes of spin-flip type ($\lambda\pi = 1+$) (Mottelson, 1960; Bohr and Mottelson 1975). Experimental evidence for the occurrence of such modes in heavy nuclei came only at a much later time (Berman *et al.*, 1969; Pitthan and Walcher, 1972). The interpretation of the strong $M1$ transitions in light nuclei was discussed by Kurath (1963) in terms of an intermediate situation between (LS) and (jj) coupling.

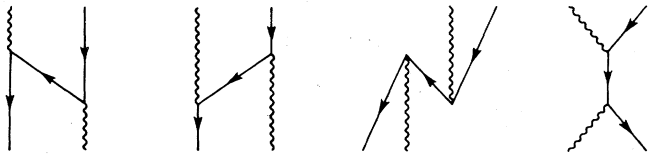

FIG. 13. Second-order diagrams contributing to the energy of a particle–phonon multiplet.

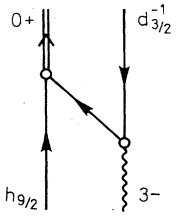

FIG. 15. Coupling between the configurations $(h_{9/2}, 3-)\frac{3}{2}+$ and $(d^{-1}_{3/2}, 0+)\frac{3}{2}+$, based on particle–vibration vertices.

ling gives rise to a wealth of different effects, including interactions between the different elementary modes, anharmonicities in the vibrational motion, self-energy effects, etc. An example is provided by the interaction between a single particle and a phonon in ^{209}Bi (see Fig. 12).[14] The lowest single-proton state $h_{9/2}$ can be superposed on the octupole excitation observed in ^{208}Pb and gives rise to a septuplet with $I = \frac{3}{2}, \ldots \frac{15}{2}$. The splitting of the septuplet receives contributions from the octupole coupling, which can be estimated from the second-order diagrams shown in Fig. 13 (and which are seen to correspond to those of the Compton effect in electrodynamics).

It is an important feature of this calculation that the interactions contain the effect of the antisymmetry between the single particle considered and the particles out of which the vibration is built. This effect is contained in the last diagram in Fig. 13, as schematically indicated in Fig. 14. In a similar manner, the third diagram in Fig. 13 contains the effect of the Bose symmetry of the two identical octupole quanta.

The particle vibration coupling also leads to the interaction between "crossed" channels, such as illustrated in Fig. 15, which exhibits the coupling of the $I = \frac{3}{2}$ member of the septuplet in ^{209}Bi to the state obtained by superposing a quantum consisting of a pair of protons coupled to angular momentum zero (as in the ground state of ^{210}Po) and a single-proton hole in the configuration $d^{-1}_{3/2}$ (as observed in the spectrum of ^{207}Tl). The treatment of this diagram takes proper account of the fact that the two configurations considered are not mutually orthogonal, as must be expected quite generally in a description that exploits simultaneously the quanta of particle–hole type as well as those involving two particles or holes.

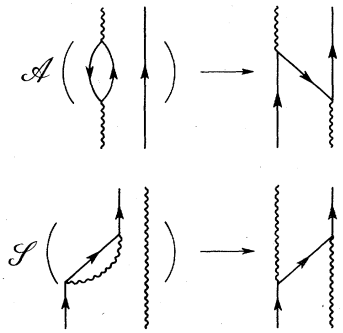

FIG. 14. Linked diagrams associated with symmetrization of particle plus phonon states.

As illustrated by these examples, it appears that the nuclear field theory based upon the particle-vibration coupling provides a systematic method for treating the old problems of the overcompleteness of the degrees of freedom, as well as those arising from the identity of the particles appearing explicitly and the particles participating in the collective motion (Bès *et al.*, 1974; Bohr and Mottelson, 1975). This development is one of the active frontiers in the current exploration of nuclear dynamics.

Looking back over this whole development one cannot help but be impressed by the enormous richness and variety of correlation effects exhibited by the nucleus. This lesson coincides with that learned in so many other domains of quantal physics and reflects the almost inexhaustible possibilities in the quantal many-body systems. The connections between the problems encountered in the different domains of quantal physics dealing with systems with many degrees of freedom have become increasingly apparent, and have been of inspiration, not least to the nuclear physicists who find themselves at an intermediate position on the quantum ladder. Looking forward, we feel that the efforts to view the various branches of quantal physics as a whole may to an even greater extent become a stimulus to a deeper understanding of the scope of this broad development.

[14]The discovery of the weak-coupling multiplet in ^{209}Bi (Alster, 1966; Hafele and Woods, 1966) was a major incentive to the exploration of the scope of the particle-vibration coupling (Mottelson, 1968; Hamamoto, 1969 and 1970; Bès and Broglia, 1971; Broglia, Paar, and Bès, 1971; Bohr and Mottelson, 1975).

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