Rotational motion in nuclei*

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The exploration of nuclear structure over the last quarter century has been a rich experience for those who have had the privilege to participate. As the nucleus has been subjected to more and more penetrating probes, it has continued to reveal unexpected facets and to open new perspectives. The preparation of our talks today has been an occasion for Ben Mottelson and myself to relive the excitement of this period and to recall the interplay of so many ideas and discoveries coming from the worldwide community of nuclear physicists, as well as the warmth of the personal relations that have been involved.

In this development, the study of rotational motion has had a special role. Because of the simplicity of this mode of excitation and the many quantitative relations it implies, it has been an important testing ground for many of the general ideas on nuclear dynamics. Indeed, the response to rotational motion has played a prominent role in the development of dynamical concepts ranging from celestial mechanics to the spectra of elementary particles.

EARLY IDEAS ON NUCLEAR ROTATION

The question of whether nuclei can rotate became an issue already in the very early days of nuclear spectroscopy (Thibaud, 1930; Teller and Wheeler, 1938). Quantized rotational motion had been encountered in molecular spectra (Bjerrum, 1912), but atoms provide examples of quantal systems that do not rotate collectively. The available data on nuclear excitation spectra, as obtained for example from the fine structure of α decay, appeared to provide evidence against the occurrence of low-lying rotational excitations, but the discussion was hampered by the expectation that rotational motion would either be a property of all nuclei or be generally excluded, as in atoms, and by the assumption that the moment of inertia would have the rigid-body value, as in molecular rotations. The issue, however, took a totally new form with the establishment of the nuclear shell model (Mayer, 1949; Haxel *et al*., 1949).

Just at that time, in early 1949, I came to Columbia University as a research fellow and had the good fortune of working in the stimulating atmosphere of the Pupin Laboratory where so many great discoveries were being made under the inspiring leadership of I. I. Rabi. One of the areas of great activity was the study of nuclear moments, which was playing such a crucial role in the development of the new ideas on nuclear structure.

Today, it is difficult to fully imagine the great impact of the evidence for nuclear shell structure on the physicists brought up with the concepts of the liquid-drop and compound-nucleus models, which had provided the basis for interpreting nuclear phenomena over the previous decade (N. Bohr, 1936; N. Bohr and Kalckar, 1937; Weisskopf, 1937; Meitner and Frisch, 1939; N. Bohr and Wheeler, 1939; Frenkel, 1939).[1] I would like also to recall my father's reaction to the new evidence, which presented the sort of dilemma that he would respond to as a welcome opportunity for deeper understanding. In the summer of 1949, he was in contact with John Wheeler on the continuation of their work on the fission process, and in this connection, in order to "clear his thoughts," he wrote some tentative comments on the incorporation of the contrasting evidence into a more general picture of nuclear constitution and the implications for nuclear reactions (N. Bohr, 1949). These comments helped to stimulate my own thinking on the subject, which was primarily concerned with the interpretation of nuclear moments.[2]

The evidence on magnetic moments, which at the time constituted one of the most extensive quantitative bodies of data on nuclear properties, presented a special challenge. The moments showed a striking correlation with the predictions of the one-particle model (Schmidt, 1937; Mayer, 1949; Haxel *et al*., 1949), but at the same time exhibited major deviations indicative of an important missing element. The incomparable precision that had been achieved in the determination of the magnetic moments, as well as in the measurement of the hyperfine structure following the pioneering work of Rabi, Bloch, and Purcell, was even able to provide information on the distribution of magnetism inside the nucleus (Bitter, 1949; Bohr and Weisskopf, 1950).

A clue for understanding the deviations in the nuclear coupling scheme from that of the single-particle model was provided by the fact that many nuclei have quadrupole moments that are more than an order of magnitude larger than could be attributed to a single particle.[3]

*This lecture was delivered on the occasion of the presentation of the 1975 Nobel Prizes in Physics. © The Nobel Foundation 1975.

[1]The struggle involved in facing up to the new evidence is vividly described by Jensen (1964). Our discussions with Hans Jensen over the years concerning many of the crucial issues in the development provided for us a special challenge and inspiration.

[2]The interplay between individual particle and collective motion was also at that time taken up by John Wheeler. Together with David Hill, he later published the extensive article on "Nuclear Constitution and the Interpretation of Fission Phenomena" (1953), which has continued through the years to provide inspiration for the understanding of new features of nuclear phenomena.

[3]The first evidence for a nonspherical nuclear shape came from the observation of a quadrupole component in the hyperfine structure of optical spectra (Schüler and Schmidt, 1935). The analysis showed that the electric quadrupole moments of the nuclei concerned were more than an order of magnitude greater than the maximum value that could be attributed to a single proton and suggested a deformation of the nucleus as a whole (Casimir, 1936; see also Kopfermann, 1940). The problem of the large quadrupole moments came into focus with the rapid accumulation of evidence on nuclear quadrupole moments in the years after the war and the analysis of these moments on the basis of the shell model (Townes *et al*., 1949).

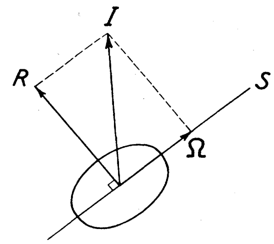

FIG. 1. Coupling scheme for particle in slowly rotating spheroidal nucleus. The intrinsic quantum number Ω represents the projection of the particle angular momentum along the nuclear symmetry axis S, while R is the collective angular momentum of the nuclear core and is directed perpendicular to the symmetry axis, since the component along S, which is a constant of the motion, vanishes in the nuclear ground state. The total angular momentum is denoted by I. The figure is from Bohr (1954).

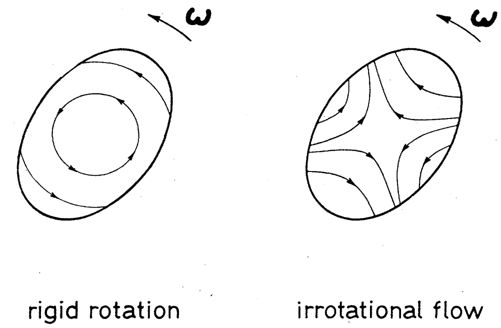

FIG. 2. Velocity fields for rotational motion. For the rotation generated by irrotational flow, the velocity is proportional to the nuclear deformation (amplitude of the traveling wave). Thus, for a spheroidal shape, the moment of inertia is $\mathcal{I}=\mathcal{I}_{\mathrm{rig}}(\Delta R/R)^2$, where $\mathcal{I}_{\mathrm{rig}}$ is the moment for rigid rotation, while R is the mean radius and ΔR (assumed small compared with R) is the difference between major and minor semi-axes. The figure is from Bohr (1954).

This finding directly implied a sharing of angular momentum with many particles, and might seem to imply a breakdown of the one-particle model. However, essential features of the single-particle model could be retained by assuming that the average nuclear field in which a nucleon moves deviates from spherical symmetry (Bohr, 1951a). This picture leads to a nuclear model resembling that of a molecule, in which the nuclear core possesses vibrational and rotational degrees of freedom. For the rotational motion there seemed no reason to expect the classical rigid-body value; however, the large number of nucleons participating in the deformation suggested that the rotational frequency would be small compared with those associated with the motion of the individual particles. In such a situation, one obtains definite limiting coupling schemes (see Fig. 1) which could be compared with the empirical magnetic moments and the evidence on the distribution of nuclear magnetism, with encouraging results (Bohr, 1951a, 1951b).[4]

In the meantime, and, in fact, at nearly the same point in space, James Rainwater had been thinking about the origin of the large nuclear quadrupole moments and conceived an idea that was to play a crucial role in the following development. He realized that a nonspherical equilibrium shape would arise as a direct consequence of single-particle motion in anisotropic orbits, when one takes into account the deformability of the nucleus as a whole, as in the liquid-drop model (Rainwater, 1950).

On my return to Copenhagen in the autumn of 1950, I took up the problem of incorporating the coupling suggested by Rainwater into a consistent dynamical system describing the motion of a particle in a deformable core. For this coupled system, the rotational motion emerges as a low-frequency component of the vibrational degrees of freedom, for sufficiently strong coupling. The rotational motion resembles a wave traveling across the nuclear surface, and the moment of inertia is much smaller than for rigid rotation (see Fig. 2).

[4]The effect on the magnetic moments of a sharing of angular momentum between the single particle and oscillations of the nuclear surface was considered at the same time by Foldy and Milford (1950).

Soon, I was joined by Ben Mottelson in pursuing the consequences of the interplay of individual-particle and collective motion for the great variety of nuclear phenomena that was then coming within the range of experimental studies (Bohr and Mottelson, 1953a). In addition to the nuclear moments, important new evidence had come from the classification of the nuclear isomers (Goldhaber and Sunyar, 1951) and beta decay (Mayer *et al.*, 1951; Nordheim, 1951), as well as from the discovery of single-particle motion in nuclear reactions (Barschall, 1952; Weisskopf, 1952). It appeared that one had a framework for bringing together most of the available evidence, but, in the quantitative confrontation with experiment, one faced the uncertainty in the parameters describing the collective properties of the nucleus. It was already clear that the liquid-drop description was inadequate, and one lacked a basis for evaluating the effect of the shell structure on the collective parameters.

THE DISCOVERY OF ROTATIONAL SPECTRA

At this point, one obtained a foothold through the discovery that the coupling scheme characteristic of strongly deformed nuclei with the striking rotational band structure was in fact realized for an extensive class of nuclei. The first indication had come from the realization by Goldhaber and Sunyar that the electric quadrupole transition rates for the decay of low-lying excited states in even–even nuclei were, in some cases, much greater than could be accounted for by a single-particle transition, and thus suggested a collective mode of excitation (Goldhaber and Sunyar, 1951). A rotational interpretation (Bohr and Mottelson, 1953b) yielded values for the nuclear eccentricity in promising agreement with those deduced from the spectroscopic quadrupole moments.

Soon after, the evidence began to accumulate that these excitations were part of a level sequence with angular momenta $I=0, 2, 4, \ldots$ and energies proportional to $I(I+1)$ (Bohr and Mottelson, 1953c; Asaro and Perlman, 1953); examples of the first such spectra are shown in Fig. 3. For ourselves, it was a thrilling experience to receive a prepublication copy of the 1953

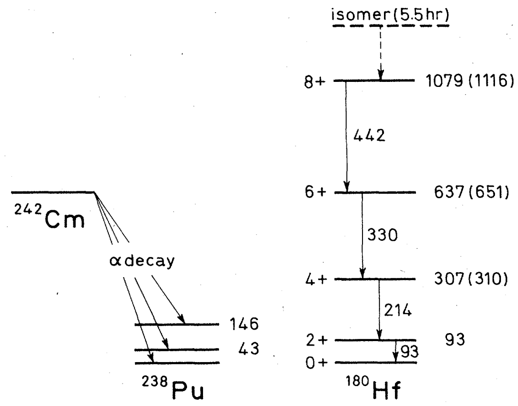

FIG. 3. Rotational spectra for ^{238}Pu and ^{180}Hf. The spectrum of ^{180}Hf (from A. Bohr and Mottelson, 1953c) was deduced from the observed γ lines associated with the decay of the isomeric state (Burson *et al.*, 1951). The energies are in keV, and the numbers in parenthesis are calculated from the energy of the first excited state, assuming the energies to be proportional to $I(I+1)$. The spectrum of ^{238}Pu was established by Asaro and Perlman (1953) from measurements of the fine structure in the α decay of ^{242}Cm. Subsequent evidence showed the spin-parity sequence to be 0+, 2+, 4+, and the energies are seen to be closely proportional to $I(I+1)$.

compilation by Hollander, Perlman, and Seaborg (Hollander *et al.*, 1953) with its wealth of information on radioactive transitions, which made it possible to identify so many rotational sequences.

The exciting spring of 1953 culminated with the discovery of the Coulomb excitation process (Huus and Zupančič, 1953; McClelland and Goodman, 1953), which opened the possibility for a systematic study of rotational excitations (Heydenburg and Temmer, 1954, 1955). Already the very first experiments by Huus and Zupančič (see Fig. 4) provided a decisive quantitative test of the rotational coupling scheme in an odd nucleus, involving the strong coupling between intrinsic and rotational angular momenta.[5]

This was a period of almost explosive development in the power and versatility of nuclear spectroscopy, which rapidly led to a very extensive body of data on nuclear rotational spectra. The development went hand in hand with a clarification and expansion of the theoretical basis.

Figure 5 shows the region of nuclei in which rotational band structure has so far been identified. The vertical and horizontal lines indicate neutron and proton numbers that form closed shells, and the strongly deformed nuclei are seen to occur in regions where there are many particles in unfilled shells that can contribute to the deformation.

The rotational coupling scheme could be tested not only by the sequence of spin values and regularities in the energy separations, but also by the intensity relations that govern transitions leading to different members of a rotational band (Alaga *et al.*, 1955; Bohr

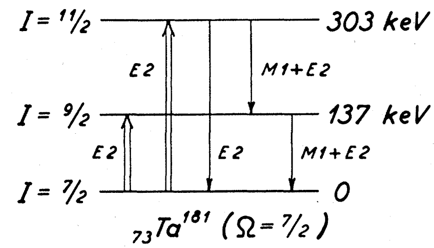

FIG. 4. Rotational excitations in ^{181}Ta observed by Coulomb excitation. In an odd-A nucleus with intrinsic angular momentum Ω (see Fig. 1), the rotational excitations involve the sequence $I = \Omega, \Omega+1, \Omega+2, \ldots$, all with the same parity. In the Coulomb excitation process, the action of the electric field of the projectile on the nuclear quadrupole moment induces $E2$ (electric quadrupole) transitions and can thus populate the first two rotational excitations. The observed energies (Huus and Zupančič, 1953) are seen to be approximately proportional to $I(I+1)$. The excited states decay by $E2$ and $M1$ (magnetic dipole) transitions, and the rotational interpretation implies simple intensity relations. For example, the reduced $E2$ matrix elements within the band are proportional to the Clebsch–Gordan coefficient $\langle I_i \Omega 2 0 | I_f \Omega \rangle$, where I_i and I_f are the angular momenta of initial and final states. The figure is from Bohr (1954).

et al., 1955; Satchler, 1955). The leading order intensity rules are of a purely geometrical character depending only on the rotational quantum numbers and the multipolarity of the transitions (see the examples in Fig. 4 and Fig. 10).

The basis for the rotational coupling scheme and its predictive power were greatly strengthened by the recognition that the low-lying bands in odd-A nuclei could be associated with one-particle orbits in the deformed potential (Nilsson, 1955; Mottelson and Nilsson, 1955; Gottfried, 1956). The example in Fig. 6 shows the spectrum of ^{235}U with its high level density and apparently great complexity. However, as indicated, the states can be grouped into rotational bands that correspond uniquely

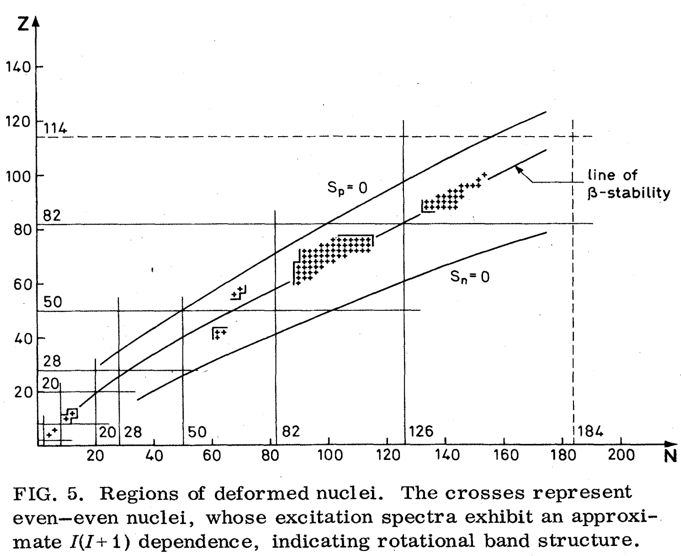

FIG. 5. Regions of deformed nuclei. The crosses represent even–even nuclei, whose excitation spectra exhibit an approximate $I(I+1)$ dependence, indicating rotational band structure. The figure is from Bohr and Mottelson (1975), and is based on the data in Sakai (1970, 1972). The curves labeled $S_n = 0$ and $S_p = 0$ are the estimated borders of instability with respect to neutron and proton emission.

[5]The quantitative interpretation of the cross sections could be based on the semiclassical theory of Coulomb excitation developed by Ter-Martirosyan (1952) and Alder and Winther (1953).

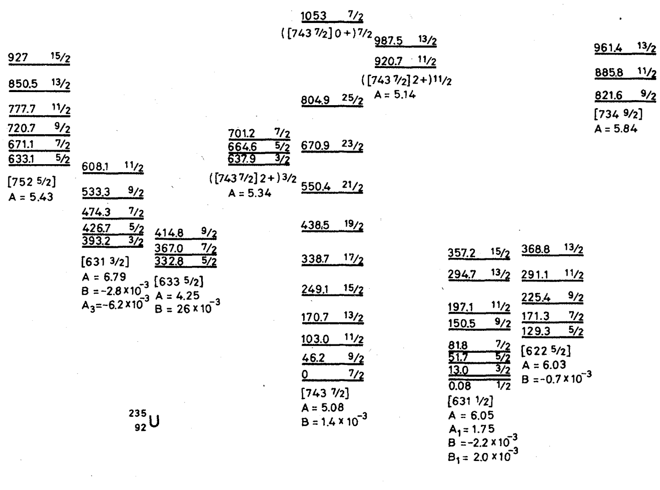

FIG. 6. Spectrum of ^{235}U. The figure is from Bohr and Mottelson (1975) and is based on the experimental data from Coulomb excitation (Stephens *et al.*, 1968), ^{239}Pu α-decay (Cline, 1968), one-particle transfer (Elze and Huizenga, 1969; Braid *et al.*, 1970), and the ^{234}U$(n\gamma)$ reaction (Jurney, 1969). All energies are in keV. The levels are grouped into rotational bands characterized by the spin sequence, energy dependence, and intensity rules. The energies within a band can be represented by a power series expansion of the form $E(I) = AI(I+1) + BI^2(I+1)^2 + \cdots (-1)^{I+\Omega}(I+\Omega)!((I-\Omega)!)^{-1}(A_{2\Omega} + B_{2\Omega}I(I+1) + \cdots)$, with the parameters given in the figure. The low-lying bands are labeled by the quantum numbers of the available single-particle orbits (see Fig. 7), with particle-like states drawn to the right of the ground-state band and hole-like states to the left. The bands beginning at 638, 921, and 1053 keV represent quadrupole vibrational excitations of the ground-state configuration.

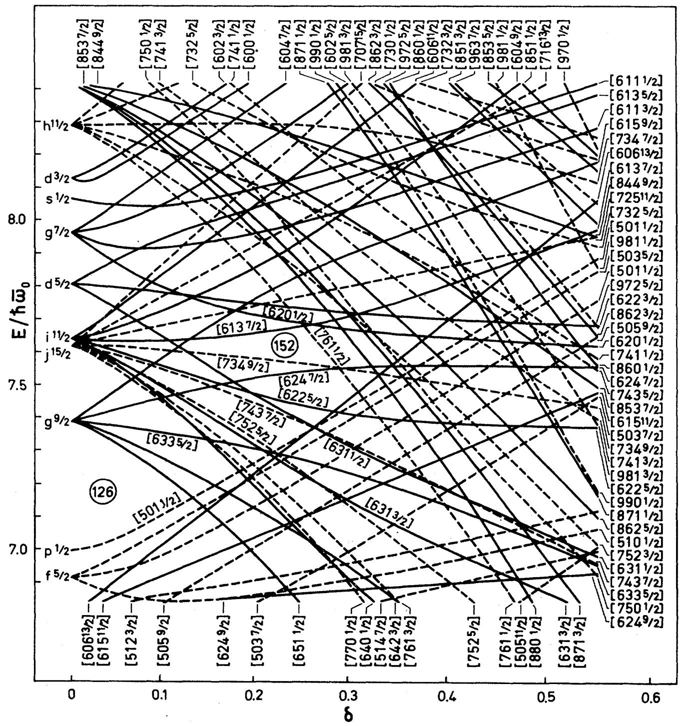

FIG. 7. Neutron orbits in prolate potential. The figure (from Bohr and Mottelson, 1975) shows the energies of single-particle orbits calculated in an appropriate nuclear potential by Gustafson, Lamm, Nilsson, and Nilsson (1967). The single-particle energies are given in units of $\hbar\bar{\omega}$, which represents the separation between major shells, and, for ^{235}U, has the approximate value 6.6 MeV. The deformation parameter δ is a measure of the nuclear eccentricity; the value determined for ^{235}U, from the observed $E2$ transition moments, is $\delta \approx 0.25$. The single-particle states are labeled by the "asymptotic" quantum numbers $[Nn_3\Lambda\Omega]$. The last quantum number Ω, which represents the component j_3 of the total angular momentum along the symmetry axis, is a constant of the motion for all values of δ. The additional quantum numbers refer to the structure of the orbits in the limit of large deformations, where they represent the total number of nodal surfaces (N), the number of nodal surfaces perpendicular to the symmetry axis (n_3), and the component of orbital angular momentum along the symmetry axis (Λ). Each orbit is doubly degenerate ($j_3 = \pm\Omega$), and a pairwise filling of orbits contributes no net angular momentum along the symmetry axis. For ^{235}U, with neutron number 143, it is seen that the lowest two configurations are expected to involve an odd neutron occupying the orbits [743 7/2] or [631 1/2], in agreement with the observed spectrum (see Fig. 6). It is also seen that the other observed low-lying bands in ^{235}U correspond to neighboring orbits in the present figure.

to those expected from the Nilsson diagram shown in Fig. 7.

The regions of deformation in Fig. 5 refer to the nuclear ground-state configurations; another dimension is associated with the possibility of excited states with equilibrium shapes quite different from those of the ground state. For example, some of the closed-shell nuclei are found to have strongly deformed excited configurations.[6] Another example of shape isomerism with associated rotational band structure is encountered in the metastable, very strongly deformed states that occur in heavy nuclei along the path to fission (Polikanov *et al.*, 1962; Specht *et al.*, 1972).

New possibilities for studying nuclear rotational motion were opened by the discovery of marked anisotropies in the angular distribution of fission fragments (Winhold *et al.*, 1952), which could be interpreted in terms of the rotational quantum numbers labeling the individual channels through which the fissioning nucleus passes the saddle-point shape (Bohr, 1956). Present developments in the experimental tools hold promise of providing detailed information about band structure in the fission channels and thereby on rotational motion under circumstances radically different from those studied previously.

CONNECTION BETWEEN ROTATIONAL AND SINGLE-PARTICLE MOTION

The detailed testing of the rotational coupling scheme and the successful classification of intrinsic spectra

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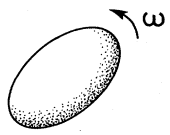
[6]The fact that the first excited states in ^{16}O and ^{40}Ca have positive parity, while the low-lying single-particle excitations are restricted to negative parity, implies that these states involve the excitation of a larger number of particles. It was suggested (Morinaga, 1956) that the excited positive parity states might be associated with collective quadrupole deformations. The existence of a rotational band structure in ^{16}O was convincingly established as a result of the ^{12}C$(\alpha\alpha)$ studies (Carter *et al.*, 1964), and the observation of strongly enhanced $E2$-transition matrix elements (Gorodetzky *et al.*, 1963).



$$H = H_0 - \hbar\omega J_x$$

$$\mathcal{J} = 2\hbar^2 \sum_i \frac{\langle i|J_x|o\rangle^2}{E_i - E_0}$$

cranking model

FIG. 8. Nuclear moment of inertia from cranking model. The Hamiltonian H describing particle motion in a potential rotating with frequency ω about the x axis is obtained from the Hamiltonian H_0 for motion in a fixed potential by the addition of the term proportional to the component J_x of the total angular momentum, which represents the Coriolis and centrifugal forces acting in the rotating coordinate frame. The moment of inertia is obtained from a second order perturbation treatment of this term and involves a sum over the excited states i. For independent-particle motion, the moment of inertia can be expressed as a sum of the contributions from the individual particles.

provided a firm starting point for the next step in the development, which concerned the dynamics underlying the rotational motion.

The basis for this development was the bold idea of Inglis (1954) to derive the moment of inertia by simply summing the inertial effect of each particle as it is dragged around by a uniformly rotating potential (see Fig. 8). In this approach, the potential appears to be externally "cranked," and the problems concerning the self-consistent origin for the rotating potential and the limitations of such a semiclassical description have continued over the years to be hotly debated issues. The discussion has clarified many points concerning the connection between collective and single-particle motion, but the basic idea of the cranking model has stood its tests to a remarkable extent (Thouless and Valatin, 1962; Bohr and Mottelson, 1975).

The evaluation of the moments of inertia on the basis of the cranking model gave the unexpected result that, for independent-particle motion, the moment would have a value approximately corresponding to rigid rotation (Bohr and Mottelson, 1955). The fact that the observed moments were appreciably smaller than the rigid-body values could be qualitatively understood from the effect of the residual interactions that tend to bind the particles into pairs with angular momentum zero. A few years later, a basis for a systematic treatment of the moment of inertia with the inclusion of the many-body correlations associated with the pairing effect was given by Migdal (1959) and Belyaev (1959), exploiting the new concepts that had, in the meantime, been developed for the treatment of electronic correlations in a superconductor (Bardeen, Cooper, and Schrieffer, 1957a, 1957b; see also Mottelson, 1975).

The nuclear moment of inertia is thus intermediate between the limiting values corresponding to rigid rotation and to the hydrodynamical picture of irrotational flow that was assumed in the early models of nuclear rotation. Indeed, the classical pictures involving a local flow provide too limited a framework for the description of nuclear rotation, since, in nuclear matter, the size of the pairs (the coherence length) is greater than the diameter of the largest existing nuclei. Macroscopic superflow of nuclear matter and quantized vortex lines may occur, however, in the interior of rotating neutron stars (Ruderman, 1972).

While these developments illuminated the many-body aspects of nuclear rotation, appropriate to systems with a very large number of nucleons, a parallel development took its starting point from the opposite side. Shell-model calculations exploiting the power of group-theoretical classification schemes and high-speed electronic computers could be extended to configurations with several particles outside of closed shells. It was quite a dramatic moment, when it was realized that some of the spectra in the light nuclei that had been successfully analyzed by the shell-model approach could be given a very simple interpretation in terms of the rotational coupling scheme.[7]

The recognition that rotational features can manifest themselves already in configurations with very few particles provided the background for Elliott's discovery that the rotational coupling scheme can be given a precise significance in terms of the SU_3 unitary symmetry classification, for particles moving in a harmonic oscillator potential (Elliott, 1958). This elegant model had a great impact at the time and has continued to provide an invaluable testing ground for many ideas concerning nuclear rotation. Indeed, it has been a major inspiration to be able to see through, even in this limiting case, the entire correlation structure in the many-body wave function associated with the collective motion. Thus, for example, the model explicitly exhibits the separation between intrinsic and collective motion and implies an intrinsic excitation spectrum that differs from that of independent-particle motion in a deformed field by the removal of the "spurious" degrees of freedom that have gone into the collective spectrum.

This development also brought into focus the limitation to the concept of rotation arising from the finite number of particles in the nucleus. The rotational spectrum in the SU_3 model is of finite dimension (compact symmetry group) corresponding to the existence of a maximum angular momentum that can be obtained from a specified shell-model configuration. For low-lying bands, this maximum angular momentum is of the order of magnitude of the number of nucleons A and, in some of the light nuclei, one has, in fact, obtained evidence for such a limitation in the ground-state rotation-

[7]In this connection, a special role was played by the spectrum of ^{19}F. The shell-model analysis of this three-particle configuration had been given by Elliott and Flowers (1955) and the rotational interpretation was recognized by Paul (1957); the approximate identity of the wave functions derived by the two approaches was established by Redlich (1958).

al bands.[8] However, the proper place of this effect in nuclear rotations is still an open issue due to the major deviations from the schematized SU_3 picture.

GENERAL THEORY OF ROTATION

The increasing precision and richness of the spectroscopic data kept posing problems that called for a framework, in which one could clearly distinguish between the general relations characteristic of the rotational coupling scheme and the features that depend more specifically on the internal structure and the dynamics of the rotational motion.[9] For ourselves, an added incentive was provided by the challenge of presenting the theory of rotation as part of a broad view of nuclear structure. The viewpoints that I shall try to summarize gradually emerged in this prolonged labor (Bohr and Mottelson, 1963; Bohr, 1974; Bohr and Mottelson, 1975).

In a general theory of rotation, symmetry plays a central role. Indeed, the very occurrence of collective rotational degrees of freedom may be said to originate in a breaking of rotational invariance, which introduces a "deformation" that makes it possible to specify an orientation of the system. Rotation represents the collective mode associated with such a spontaneous symmetry breaking (Goldstone boson).

The full degrees of freedom associated with rotations in three-dimensional space come into play if the deformation completely breaks the rotational symmetry, thus permitting a unique specification of the orientation. If the deformation is invariant with respect to a subgroup of rotations, the corresponding elements are part of the intrinsic degrees of freedom, and the collective rotational modes of excitation are correspondingly reduced, disappearing entirely in the limit of spherical symmetry.

The symmetry of the deformation is thus reflected in the multitude of states that belong together in rotational families and the sequence of rotational quantum numbers labeling these states, in a similar manner as in the symmetry classification of molecular rotational spectra. The nuclear rotational spectra shown in Figs. 3, 4, and 6 imply a deformation with axial symmetry and invariance with respect to a rotation of 180° about an axis perpendicular to the symmetry axis (D_∞ symmetry group). It can also be inferred from the observed spectra that the deformation is invariant with respect to space and time reflection.

The recognition of the deformation and its degree of symmetry breaking as the central element in defining rotational degrees of freedom opens new perspectives for generalized rotational spectra associated with deformations in many different dimensions including spin, isospin, and gauge spaces, in addition to the geometrical space of our classical world. The resulting rotational band structure may involve comprehensive families of states labeled by the different quantum numbers of the internally broken symmetries. Relations between quantum numbers belonging to different spaces may arise from invariance of the deformation with respect to a combination of operations in the different spaces.[10]

The Regge trajectories that have played a prominent role in the study of hadronic properties have features reminiscent of rotational spectra, but the symmetry and nature of possible internal deformations of hadrons remain to be established. Such deformations might be associated with boundaries for the regions of quark confinement.

The condensates in superfluid systems involve a deformation of the field that creates the condensed bosons or fermion pairs. Thus, the process of addition or removal of a correlated pair of electrons from a superconductor (as in a Josephson junction) or of a nucleon pair from a superfluid nucleus constitutes a rotational mode in the gauge space in which particle number plays the role of angular momentum (Anderson, 1966). Such pair rotational spectra, involving families of states in different nuclei, appear as a prominent feature in the study of two-particle transfer processes (Middleton and Pullen, 1964; see also Broglia *et al.*, 1973). The gauge space is often felt as a rather abstract construction but, in the particle-transfer processes, it is experienced in a very real manner.

The relationship between the members of a rotational band manifests itself in the simple dependence of matrix elements on the rotational quantum numbers, as first encountered in the $I(I+1)$ dependence of the energy spectra and in the leading order intensity rules that govern transitions leading to different members of a band. The underlying deformation is expressed by the occurrence of collective transitions within the band.

For sufficiently small values of the rotational quantum numbers, the analysis of matrix elements can be based on an expansion in powers of the angular momentum. The general structure of such an expansion depends on the symmetry of the deformation and takes an especially simple form for axially symmetric systems. As an example, Fig. 9 shows the two lowest bands observed in ^{166}Er. The energies within each band have been measured with enormous precision and can be expressed as a power series that converges rather rapidly for the range of angular momentum values included in the figure. Similar expansions can be given for matrix elements of tensor operators representing electromagnetic transitions, β decay, particle transfer, etc. Thus, ex-

[8]The evidence (Jackson *et al.*, 1969; Alexander *et al.*, 1952) concerns the behavior of the quadrupole transition rates, which are expected to vanish with the approach to the band termination (Elliott, 1958). This behavior reflects the gradual alignment of the angular momenta of the particles and the associated changes in the nuclear shape that lead eventually to a state with axial symmetry with respect to the angular momentum and hence no collective radiation (Bohr, 1967; Bohr and Mottelson, 1975).

[9]In this development, a significant role was played by the high-resolution spectroscopic studies (Hansen *et al.*, 1959) which led to the establishment of a generalized intensity relation in the $E2$ decay of the γ-vibrational band in ^{156}Gd.

[10]A well-known example is provided by the strong-coupling fixed-source model of the pion-nucleon system, in which the intrinsic deformation is invariant with respect to simultaneous rotations in geometrical and isospin spaces resulting in a band structure with $I=T$ (Henley and Thirring, 1962; Bohr and Mottelson, 1975).

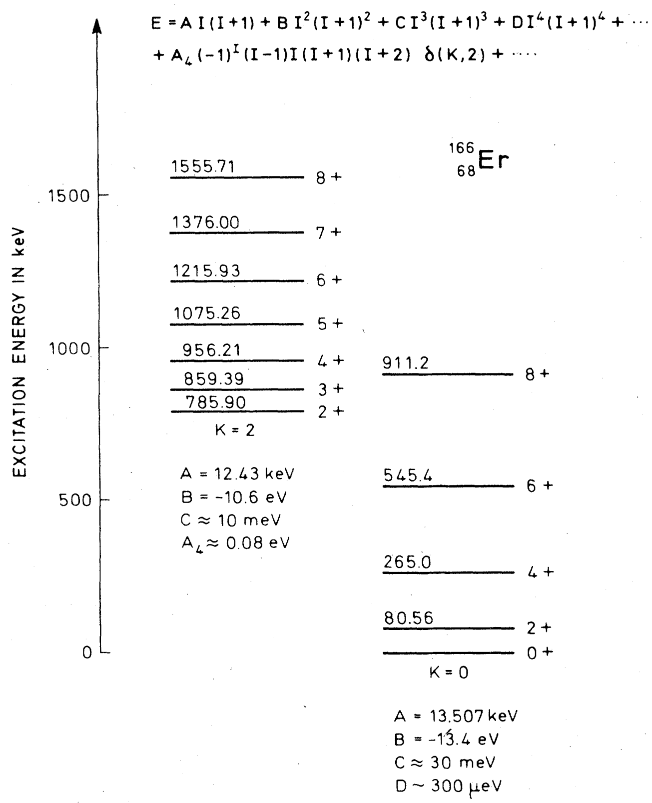


FIG. 9. Rotational bands in ^{166}Er. The figure is from Bohr and Mottelson (1975) and is based on the experimental data by Reich and Cline (1970). The bands are labeled by the component K of the total angular momentum with respect to the symmetry axis. The $K=2$ band appears to represent the excitation of a mode of quadrupole vibrations involving deviations from axial symmetry in the nuclear shape.

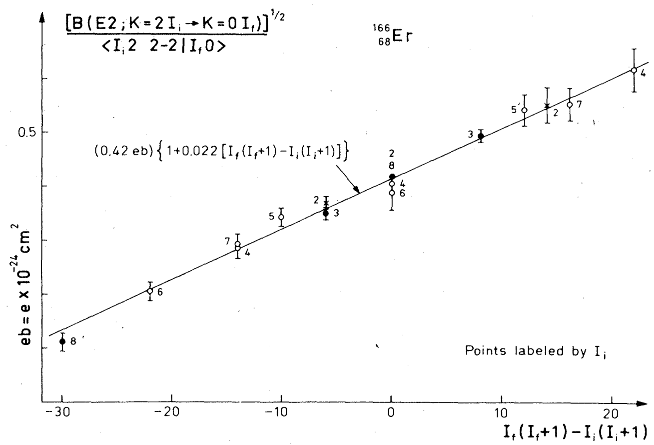


FIG. 10. Intensity relation for $E2$ transitions between rotational bands. The figure, which is from Bohr and Mottelson (1975) and is based upon experimental data in Gallagher *et al.* (1965), Günther and Parsignault (1967), and Domingos *et al.* (1972), shows the measured reduced electric quadrupole transition probabilities $B(E2)$ for transitions between members of the $K=2$ and $K=0$ bands in ^{166}Er (see Fig. 9). An expansion similar to that of the energies in Fig. 9, but taking into account the tensor properties of the $E2$ operator, leads to an expression for $(B(E2))^{1/2}$, which involves a Clebsch-Gordan coefficient $\langle I_i K_i | 2-2 I_f K_f \rangle$ (geometrical factor) multiplied by a power series in the angular momenta of I_i and I_f of the initial and final states. The leading term in this expansion is a constant and the next term is linear in $I_f(I_f+1)-I_i(I_i+1)$; the experimental data are seen to be rather well represented by these two terms.

tensive measurements have been made of the $E2$ transitions between the two bands in ^{166}Er, and Fig. 10 shows the analysis of the empirical transition matrix elements in terms of the expansion in the angular momentum quantum numbers of initial and final states.

Such an analysis of the experimental data provides a phenomenological description of the rotational spectra in terms of a set of physically significant parameters. These parameters characterize the internal structure of the system with inclusion of the renormalization effects arising from the coupling to the rotational motion.

A systematic analysis of these parameters may be based on the ideas of the cranking model, and this approach has yielded important qualitative insight into the variety of effects associated with the rotational motion. However, in this program, one faces significant unsolved problems. The basic coupling involved in the cranking model can be studied directly in the Coriolis coupling between rotational bands in odd-A nuclei associated with different orbits of the unpaired particle (Kerman, 1956). The experiments have revealed, somewhat shockingly, that this coupling is, in many cases, considerably smaller than the one directly experienced by the particles as a result of the nuclear rotation with respect to the distant galaxies (Stephens, 1960; Hjorth *et al.*, 1970; see also the discussion in Bohr and Mottelson, 1975). It is possible that this result may reflect an effect of the rotation on the nuclear potential itself (Migdal, 1959; Belyaev, 1961; Hamamoto, 1974; Bohr and Mottelson, 1975), but the problem stands as an open issue.

CURRENT PERSPECTIVES

In the years ahead, the study of nuclear rotation holds promising new perspectives. Not only are we faced with the problem already mentioned of a more deep-going probing of the rotational motion, which has become possible with the powerful modern tools of nuclear spectroscopy, but new frontiers are opening up through the possibility of studying nuclear states with very large values of the angular momentum. In reactions induced by heavy ions, it is in fact now possible to produce nuclei with as much as a hundred units of angular momentum. We thus encounter nuclear matter under quite novel conditions, where centrifugal stresses may profoundly affect the structure of the nucleus. The challenge of this new frontier has strongly excited the imagination of the nuclear physics community.

A schematic phase diagram showing energy versus angular momentum for a nucleus with mass number $A \approx 160$ is shown in Fig. 11. The lower curve representing the smallest energy, for given angular momentum, is referred to as the yrast line. The upper curve gives the fission barrier, as a function of angular momentum, estimated on the basis of the liquid-drop model (Cohen *et al.*, 1974). For $I \approx 100$, the nucleus is expected to

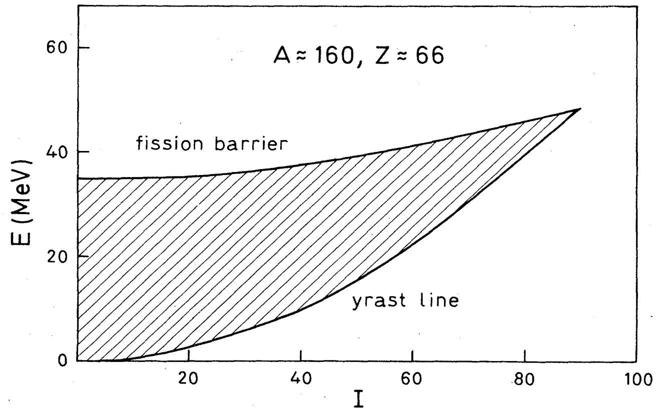


FIG. 11. Nuclear phase diagram for excitation energy versus angular momentum. The yrast line and the fission barrier represent estimates, due to Cohen, Plasil, and Swiatecki (1974), based on the liquid-drop model, with the assumption of the rigid-body value for the moment of inertia.

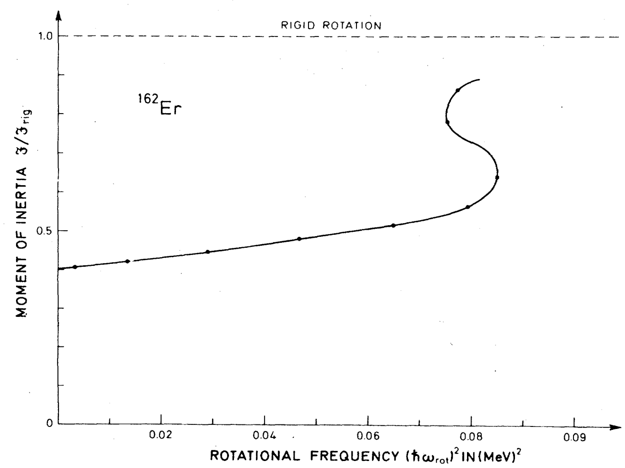


FIG. 12. Moment of inertia as function of rotational frequency. The figure is from Bohr and Mottelson (1973) and is based on the experimental data of Johnson, Ryde, and Hjorth (1972). The rotational frequency is defined as the derivative of the rotational energy with respect to the angular momentum and is obtained by a linear interpolation in the variable $I(I+1)$ between the quantal states. The moment of inertia is defined in the usual manner as the ratio between the angular momentum and the rotational frequency.

become unstable with respect to fission, and the available data on cross sections for compound-nucleus formation in heavy ion collisions seem to confirm the approximate validity of this estimate of the limiting angular momentum (Britt *et al.*, 1975; Gauvin *et al.*, 1975).

Present information on nuclear spectra is confined almost exclusively to a small region in the left-hand corner of the phase diagram, and a vast extension of the field is therefore coming within range of exploration. Special interest attaches to the region just above the yrast line, where the nucleus, though highly excited, remains cold, since almost the entire excitation energy is concentrated in a single degree of freedom. One thus expects an excitation spectrum with a level density and a degree of order similar to that near the ground state. The extension of nuclear spectroscopy into this region may therefore offer the opportunity for a penetrating exploration of how the nuclear structure responds to the increasing angular momentum.

In recent years, it has been possible to identify quantal states in the yrast region up to $I \approx 20-25$, and striking new phenomena have been observed. An example is shown in Fig. 12, in which the moment of inertia is plotted against the rotational frequency. This "back-bending" effect was discovered here in Stockholm at the Research Institute for Atomic Physics, and has been found to be a rather general phenomenon.

In the region of angular momenta concerned, one is approaching the phase transition from superfluid to normal nuclear matter, which is expected to occur when the increase in rotational energy implied by the smaller moment of inertia of the superfluid phase upsets the gain in correlation energy (Mottelson and Valatin, 1960). The transition is quite analogous to the destruction of superconductivity by a magnetic field and is expected to be associated with an approach of the moment of inertia to the rigid-body value characteristic of the normal phase.

The back-bending effect appears to be a manifestation of a band crossing, by which a new band with a larger moment of inertia and correspondingly smaller rotational frequency for given angular momentum, moves onto the yrast line. Such a band crossing may arise in connection with the phase transition, since the excitation energy for a quasiparticle in the rotating potential may vanish, even though the order parameter (the binding energy of the correlated pairs) remains finite, in rather close analogy to the situation in gapless superconductors (Goswami *et al.*, 1967). In fact, in the rotating potential, the angular momentum carried by the quasiparticle tends to become aligned in the direction of the axis of rotation. The excitation of the quasiparticle is thus associated with a reduction in the angular momentum and, hence, of the energy that is carried by the collective rotation (Stephens and Simon, 1972).

It must be emphasized that, as yet, there is no quantitative interpretation of the striking new phenomena, as exemplified by Fig. 11. One is facing the challenge of analyzing a phase transition in terms of the individual quantal states.

For still larger values of the angular momentum, the centrifugal stresses are expected to produce major changes in the nuclear shape, until finally the system becomes unstable with respect to fission. The path that a given nucleus follows in deformation space will depend on the interplay of quantal effects associated with the shell structure and classical centrifugal effects similar to those in a rotating liquid drop. A richness of phenomena can be envisaged, but I shall mention only one of the intriguing possibilities.

The classical centrifugal effects tend to drive the rotating system into a shape that is oblate with respect to the axis of rotation, as is the case for the rotating earth. An oblate nucleus, with its angular momentum along the symmetry axis, will represent a form for rotation that is entirely different from that encountered in the low-energy spectrum, where the axis of rotation is

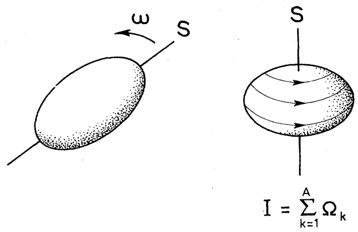


FIG. 13. Collective rotation contrasted with alignment of particle angular momenta along a symmetry axis.

perpendicular to the symmetry axis (see Fig. 13). For a nucleus spinning about its symmetry axis, the average density and potential are static, and the total angular momentum is the sum of the quantized contributions from the individual particles. In this special situation, we are therefore no longer dealing with a collective rotational motion characterized by enhanced radiative transitions, and the possibility arises of yrast states with relatively long lifetimes (Bohr and Mottelson, 1974). If such high-spin metastable states (super-dizzy nuclei) do in fact occur, the study of their decay will provide quite new opportunities for exploring rotational motion in the nucleus at very high angular momenta.

Thus, the study of nuclear rotation has continued over the years to be alive and to reveal new, challenging dimensions. Yet, this is only a very special aspect of the broader field of nuclear dynamics that will be the subject of the following talk.

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