On the origin of cosmic rays: Some problems in high-energy astrophysics*

V. L. Ginzburg

P. N. Lebedev Physical Institute, Acad. Sci. USSR, Moscow, USSR

V. S. Ptuskin

Institute of Terrestrial Magnetism, Ionosphere and Radiowave Propagation Acad. Sci. USSR, Moscow, USSR

This paper reviews the present state of the problem of the origin of cosmic rays . Primary attention is paid to galactic diffusion models with a halo, and questions of cosmic-ray chemical composition, electron component, and synchrotron galactic radioemission. The authors' conclusion is that models with a large halo with a characteristic cosmic-ray age $T_{cr} \sim 10^8$ years are confirmed by radio data, and at least do not contradict the information on cosmic-ray chemical composition. The paper also deals with the problems of anisotropy, plasma phenomena in cosmic rays, and the prospects of gamma-ray astronomy.

CONTENTS

I. INTRODUCTION

In spite of the fact that cosmic rays were discovered more than half a century ago, the question of their origin became a real astrophysical problem only much later—after 1948, when atomic nuclei were found in cosmic rays (although, to be sure, the dominating role of the proton component had been discovered before that) and in 1950–51 when the synchrotron nature of a considerable part of the cosmic radioemission was established. The latter fact made it possible to get information about the cosmic-ray electron component far from the Earth. The radio-astronomical method in combination with investigations of the primary cosmic rays near the Earth gave birth to cosmic-ray astrophysics or, as it is more often termed, high-energy astrophysics (the latter term is, however, wider for it includes also x-ray and gamma-ray astronomy, whereas cosmic rays are usually thought of as only charged cosmic particles of relatively high energy).

The outstanding achievements and discoveries connected directly or indirectly with the development of high-energy astrophysics are well known. It is sufficient to recall that relativistic or subrelativistic particles are for the most part or sometimes even completely responsible for the emission from radio galaxies, quasars, and pulsars. Against this bright background the problem of the origin of cosmic rays observed near the Earth is now only a particular case of a wider range of questions concerning acceleration, propagation, and various properties of relativistic particles in the cosmos. In some respects, the investigation of our Galaxy from the Earth is much more difficult than that of some other galaxies (e.g., when solving the question of the shape and other characteristics of the radio-halo). On the other hand, the requirements for the study of cosmic rays in our Galaxy are quite different from those in other galaxies and in quasars, where we must be satisfied with information about relativistic electrons, and where only additional, and for the most part, arbitrary hypotheses help us judge of the main (in all probability) proton–nuclear component.

In the present paper we shall discuss only the origin of cosmic rays observed near the Earth, although a number of the results presented here can of course be applied in some other cases. We shall not deal with cosmic rays of solar origin; and when speaking of the origin of cosmic rays we mean only those coming into the solar system from interstellar space and, in general, cosmic rays trapped in the Galaxy.

For the development of concepts concerning cosmic-ray origins, the reader is referred to the collection of original papers edited by Rosen (1969), to the monographs by Ginzburg and Syrovatskii (1964) and Hayakawa (1969), and, finally, to the materials of the discussion held in the Royal Society in 1974 (Rochester and Wolfendale, 1974).

Since the appearance, eleven years ago, of the book by Ginzburg and Syrovatskii (cited hereafter as GS), very much and at the same time rather little has been done

*This paper has been published in Russian in Uspekhi Fizicheskikh Nauk (Advances in Physical Sciences) a monthly publication of the USSR Academy of Sciences [Usp. Fiz. Nauk 117, 585 (1975)], and will appear in English in Soviet Physics-Uspekhi [Sov. Phys. Usp. 18, No. 12 (1975)].

toward solution of the problem of the origin of cosmic rays. In fact, there have appeared many new results on primary cosmic rays—protons, nuclei, electrons, and to a lesser extent positrons. X-ray and gamma-ray astronomy were born, giving us more and more data. But many questions, including the main ones concerning the problem under discussion, remain unsolved. These are the questions of the Galaxy radio-halo parameters, of the characteristic lifetime of galactic cosmic rays, and generally of the choice of the model describing observations. Suffice it to say that even metagalactic models of the origin of most cosmic rays observed near the Earth still have their supporters.

In this situation we do not think it would be timely to sum up or even to make a general review of the data available; nor would it be feasible, for it would require at least a vast monograph. The goal of the present paper is much narrower. After briefly recalling some facts and describing the general situation (Sec. II) we focus our attention on the methods and models which help to interpret the data on the chemical composition of cosmic rays (Sec. II) and on the composition of their electron–positron component (Sec. IV; we speak here also of the use of radio-astronomical information). Then we discuss gamma-ray-astronomy (Sec. V) and the character of cosmic-ray motion, as well as of plasma effects in cosmic-ray astrophysics (Sec. VI). We end the paper with some general remarks (Sec. VII).

In many papers published in the last ten years familiar calculations have appeared over and over and, more important, old errors have been repeated and some wrong conclusions have been drawn. For this reason we hope that the following comparison and a rather detailed presentation of part of the abovementioned material may prove useful for further investigations of the origin of cosmic rays.

II. GENERAL INFORMATION AND BASIC RELATIONS

A. Questions to be answered. The main models for the origin of cosmic rays.

To solve the problem of the origin of most cosmic rays means to answer the following questions.

(1) What is the region around the solar system in which the cosmic rays are trapped? In this region the parameters characterizing the cosmic rays are approximately the same as those near the Earth (the influence of the magnetic field of the Earth and the solar wind is not considered, or in other words we deal with cosmic rays at the periphery of the solar system). In terms of the most important parameter, cosmic-ray energy density w_{cr}, that means that we are considering the region where

$$w_{cr} \sim w_G \sim 10^{-12}\ \mathrm{erg/cm^3}, \tag{2.1}$$

where w_G is the cosmic ray energy density near the solar system.

In all probability the trapping region has no clear-cut boundary. For this reason, and also because of possible inhomogeneities in the cosmic-ray distribution, this question in a more complete form concerns the establishment of the dependence of w_{cr} and other parameters on the galactic coordinates.

(2) What are the main cosmic-ray sources in the trapping region? What are the characteristics of these sources?

(3) How do cosmic rays propagate in interstellar space and, also, in intergalactic space?

In particular, can the chemical composition of cosmic rays be explained as a function of their motion from the sources (and as a matter of fact within the sources)? It is also necessary to explain the high degree of isotropy in the observed primary cosmic rays. In this case a whole host of problems arise which are far from being solved, including the need to establish the limits of applicability of the diffusion approximation widely used in the description of cosmic-ray propagation in the interstellar and other magnetic fields of the cosmos. The role of plasma effects, particularly from the point of view of cosmic-ray isotropization, etc. must also be clarified.

(4) What are the acceleration mechanisms and other processes in the cosmic-ray sources?

This important question can in a certain sense (or it is better to say under certain assumptions) be rather clearly separated from the others. Specifically, if the main sources of cosmic rays—and thus their spatial distribution—are pointed out and the intensity and the spectrum of cosmic rays emitted by the sources are given, the problem of cosmic-ray origin can be divided into an external and an internal (source theory) problem.

(5) The cosmic-ray electron component (or more exactly, the electron-positron component) makes up in intensity and energy density only about one percent of the main, proton-nuclear component.[1] However it is just the electron component that is the source of synchrotron radiation. It can therefore be investigated far from the Earth mainly by means of radio astronomy. The trapping region and the sources of the electronic component need not, at least logically, coincide with those of the proton–nuclear component. The conditions of propagation of the electronic component are different from those for protons and nuclei since relativistic electrons (and positrons) undergo considerably more synchrotron and Compton energy losses. The electron acceleration mechanism may also differ from the acceleration mechanism for protons and nuclei. And finally we should mention that it is most probable that the positron component is completely secondary, i.e., it is generated by cosmic rays (protons and nuclei) in the interstellar medium and perhaps in the sources. In view of all this it seems reasonable to separate the question of the origin of the electron–positron component from questions (1)–(4) in application to this component.

All the abovementioned questions are interrelated to some extent and could, of course, be formulated differently, any division or classification being obviously subjective.

But clearly the answers to the whole group of questions depend critically upon our choice of cosmic-ray origin model ("theories" of cosmic-ray origin are more often spoken about, but the term theory is not very suitable

[1] When we speak simply of cosmic rays, without further clarification, we mean just their proton-nuclear component.

here). The main alternatives are the galactic and the metagalactic models (as their names imply, the first models assume that cosmic rays observed near the Earth are produced mainly in the Galaxy, while the second assume that they are produced outside it—in the Metagalaxy). Metagalactic models may be divided into universal (or quasihomogeneous) and local models. Galactic models, on the other hand, are primarily divided into halo and disk models. Some main features of these four types of models are compared in Table I. Of course, one can also think of many other intermediate types of models (e.g., a "disk" model with $h \sim 10^{22}$ cm, which is already similar to a model with a flattened halo), models with various trapping regions for protons and nuclei, etc. We should especially emphasize that all the models mentioned in Table I were considered to be quasistationary, i.e., their parameters do not change much during the characteristic galactic time $T_G \gtrsim 10^9$ years. There are reasons for this assumption, though it cannot be considered as strictly proved.

In our opinion it is most reasonable at this stage to restrict ourselves to consideration of the models listed in Table I. However, in different calculations presented hereafter some models will be defined more concretely and sometimes greatly simplified (see Secs. III and IV). Although the question of an adequate model has not yet been resolved this does not mean that we regard the abovementioned models as equal. On the contrary, as we have stated in (GS), we think that the most probable model is a galactic one with supernovae (and pulsars) as the main cosmic-ray sources. As such a model has repeatedly been discussed (see particularly GS and Ginzburg, 1975), some features of it, e.g., energy balance, will not be touched upon below. As concerns a more definite identification of the trapping region in galactic models, this question seems less definite and remains open. But, in any case, we do not believe that there are at present sufficient grounds for giving preference to the disk model, which is the most popular one in the literature of the last period. We consider the model with a pronounced halo to be more probable. However the very use of the term "probability" in the discussion of such questions is disputable. It is obvious that one should not estimate this probability, but rather, on the basis of calculations and observations, choose and specify the appropriate model. To do this is not easy, and it was in the hope of facilitating the choice of a model that the authors decided to write the present paper (for a short summary see Ginzburg and Ptuskin, 1975).

We shall not discuss metagalactic models, which though they have not been strictly disproved, face rather serious objections. These objections have recently been reviewed in a paper by Ginzburg (1974), while the arguments in favor of metagalactic models can be found in a paper by Burbidge (1974).

B. The quantities used and a brief summary of the data on cosmic rays near the Earth.

As has already been said, only charged particles (protons, nuclei, electrons, and positrons) of rather high energy are called cosmic rays. Specifically, only particles with a kinetic energy $E_k > 100$ MeV are treated as cosmic rays, whereas softer, but still fast, superthermal particles are sometimes called subcosmic rays.

The main quantity characterizing cosmic rays is their intensity I. By definition I is the number of incident particles per unit solid angle, per unit time on a unit area perpendicular to the direction of observation. The unit of measurement of I is the quantity (the number of particles)$\cdot$cm$^{-2}\cdot$sec$^{-1}\cdot$sr^{-1}.

The particle flux of the sort i for which the intensity

TABLE I. Models of cosmic-ray origin.

Models	The trapping region (in this region $w_{cr} \sim 10^{-12}$ erg/cm^3)	Basic sources [a]
A. Galactic Models		
Models with halo	Quasispherical halo with radius $R_h \sim (3-5)\times 10^{22}$ cm or flattened halo with $R \sim 5\times 10^{22}$ cm and $h \sim 5\times 10^{21}$ cm	Supernovae (including cosmic ray acceleration by pulsars). Galactic nucleus (explosive or continuous activity of the nucleus)
Disk models	Disk (of the radio-disk type) with $R \sim 5\times 10^{22}$ cm (Galaxy radius) and half-thickness $h \sim (1-2)\times 10^{21}$ cm	Stars of different types (for example, magnetic stars and, particularly, magnetic white dwarfs)
B. Metagalactic Models		
Universal (quasihomogeneous) model	The whole Metagalaxy (we mean, however, the region with redshift parameter $z \lesssim 5-100$)	Galaxies of different types (particularly, radiogalaxies) and quasars
Local models	A certain region of the Metagalaxy surrounding the Galaxy (local group of galaxies, local superclusters of galaxies, etc.)	

[a] While we cite some possible sources discussed in literature, we do not by any means regard them as equal (specifically, in our opinion, the basic cosmic-ray sources in galactic models are supernova explosions and particularly cosmic-ray acceleration by pulsars arising from these explosions).

is equal to I_i is $F_{i,\Omega}=\int I_i \cos\theta d\Omega$, where θ is the angle between the normal to the area and the particle velocity direction, and $d\Omega$ is the element of the solid angle. For isotropic radiation the particle flux F_i from a hemisphere of directions equals

$$F_i = 2\pi \int_0^{\pi/2} I_i \cos\theta \sin\theta d\theta = \pi I_i . \tag{2.2}$$

In the case of isotropic radiation concentration N_i of the particles of velocity v is given by

$$N_i = (4\pi/v_i)I_i . \tag{2.3}$$

Usually one has to deal not with monoenergetic particles but with energy distribution of particles. The main quantity here is the spectral (differential) intensity $I_i(E)$, so $I_i(E)dE$ is the intensity of particles of the sort i in the energy range from E to $E+dE$. The intensity of particles with the total energy exceeding E (integral intensity) is equal to

$$I_i(>E) = \int_E^\infty I_i(E)dE . \tag{2.4}$$

For an isotropic distribution of particles with the mass M_i

$$N_i(>E) = 4\pi \int_E^\infty \frac{I_i(E)}{v} dE , \quad E = \frac{M_i c^2}{\sqrt{1-v^2/c^2}} = M_i c^2 + E_k . \tag{2.5}$$

The kinetic energy density of isotropic cosmic rays is given by

$$w_i = \int E_k N_i(E) dE = \int \frac{4\pi}{v} E_k I_i(E) dE . \tag{2.6}$$

For nuclei it is convenient to use not only the total energy E or the kinetic energy E_k but also the total energy per nucleon $\epsilon = E/A$ or else the kinetic energy per nucleon $\epsilon_k = E_k/A$, where A is the nucleus mass number.

The expressions used above have been written down in the assumption that the direction distribution of particles is isotropic. These definitions are convenient because, the influence of the magnetic field of the Earth being excluded, the cosmic rays near the Earth are highly isotropic. However, in some cases when anisotropy is of interest it is convenient to introduce a quantity characterizing the degree of cosmic ray anisotropy. It is defined as

$$\delta = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} , \tag{2.7}$$

where I_{max} and I_{min} are, respectively, the maximal and minimal particle intensity depending on direction [the function $I(\theta)$ is supposed here to have only one peak, say in the $\theta = 0$ direction; in other words a relation of the type $I(\theta) = I_0 + I_1 \cos\theta$ is taken so that $\delta = I_1/I_0$]. What, then, do we know about cosmic rays (of galactic origin) near the Earth?

It should be noted here that at energies lower than dozens of GeV the cosmic-ray energy spectrum changes with changes in solar activity. Such a modulation effect increases as the energy decreases. Even during minimum solar activity the intensity of cosmic rays at nonrelativistic energies is much lower near the Earth than

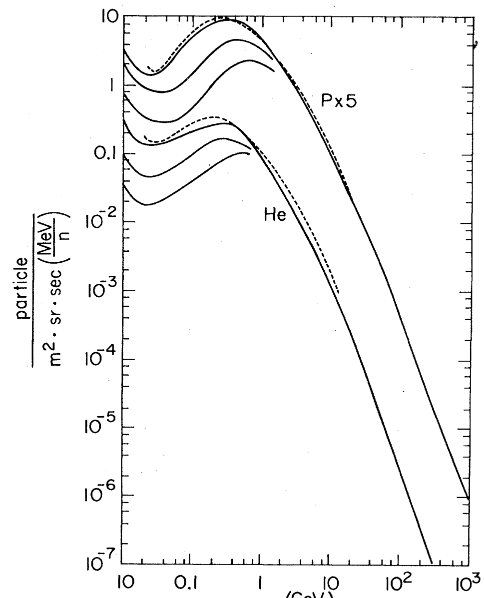

FIG. 1. Spectra of hydrogen (intensity p is multiplied by 5) and helium. Several curves at low energies correspond to the measurements in different periods of Solar activity.

outside the solar system. This circumstance is not important in what follows, since we shall for the most part be dealing with cosmic rays with energies $\epsilon_k \gtrsim 1-3$ GeV/nucleon. In this energy region the influence of solar modulation upon the particle spectra and, what is even more important, upon the relative composition of nuclei is comparatively small.

The most abundant elements in cosmic rays are hydrogen and helium. Their energy spectra are presented in Fig. 1 (Webber and Lezniak, 1974). Detailed information on the composition of heavier cosmic ray nuclei is obtained only for particle energies up to $\epsilon_k < 10$ GeV/nucleon. Figure 2 shows the relative compositions of

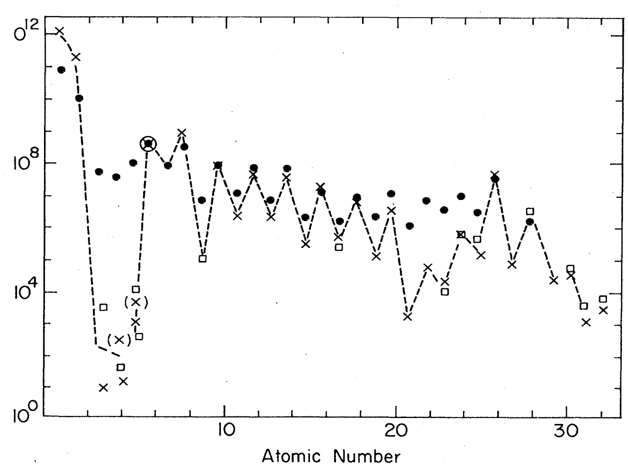

FIG. 2. Relative amount of elements in cosmic rays and in the Solar system (×, the Sun; □, meteorites; ●, cosmic rays). Content of hydrogen on the Sun is normalized to 10^{12}. It is assumed that the carbon content in cosmic rays and in the solar system coincides.

nuclei with energies 1–10 GeV/nucleon (Shapiro and Silberberg, 1974). The composition of different elements in the solar system is also presented for comparison. Both distributions are normalized so that the carbon nuclei content in cosmic rays coincides with that in the solar system.

As is clear from Fig. 2, the chemical composition of cosmic rays is characterized by the presence of a considerable flux of light nuclei (L-nuclei, i.e., Li, Be, B), in spite of their negligible average amount in the nature. This feature, which is also confirmed for other rare nuclei (for example those of ^{2}H, ^{3}He, F, K, Sc, V), points to an important transformation of the cosmic-ray chemical composition during propagation in interstellar space and perhaps even in the sources (i.e., in the acceleration region).

In recent years data have appeared on the energy spectra of different types of nuclei from hydrogen to iron for energies up to $\epsilon \sim 100$ GeV/nucleon (Juliusson, 1974; see also the review by Webber, 1974). Direct measurements of the intensities of protons and helium nuclei have been carried out up to energies $\epsilon \sim 10^3$ GeV/nucleon (see Fig. 1, where the data of Ryan *et al.*, 1972 are used). At energies greater than 10^4-10^5 GeV the spectrum is obtained almost exclusively from the data on extensive air showers. Because of differences in the methods of measuring the spectra in different energy regions, and also because of contradictions between experimental results obtained by different authors, it is difficult to establish a universal cosmic-ray spectrum for the complete energy range of particles coming to the Earth (the maximum energy registered is about 10^{20} eV; see the review by Hillas, 1974). For the energies from $E \sim 10$ GeV to $E \sim 10^6$ GeV the cosmic-ray proton spectrum has apparently a power-law form with a constant exponent, so that $I_p(E) \sim E^{-\gamma_p}$, where $\gamma_p \simeq 2.75$. At an energy $E \sim 3.10^6$ GeV a "break" is observed in the spectrum, i.e., the spectrum is steepened (for more details see Nesterova and Nikolskii, 1973; Hillas, 1974; Kristiansen, 1974).

The spectrum of cosmic rays of the highest energies, according to different groups of authors, is presented in Fig. 3 (Hillas, 1974).

The steepening of the spectrum at $E \sim 10^6$ GeV is usually associated with a change in the character of cosmic-ray propagation or, more precisely, with a rapid decrease in particle trapping efficiency in the Galaxy at very high energies. Cosmic-ray propagation in the interstellar medium in the energy range from several hundred MeV/nucleon to 10^6 GeV/nucleon is apparently regulated by a universal mechanism. Therefore in what follows we shall restrict ourselves to the consideration of particles with energies $E \lesssim 10^6$ GeV.

The integral cosmic-ray energy density w_{cr} is mainly determined by the contribution of nuclei with energies from dozens of MeV/nucleon to dozens of GeV/nucleon. Therefore, the estimation of w_{cr} from direct data on the particle intensity near the Earth requires that the influence of solar modulation be taken into account. The estimate obtained with the use of the spectrum of Fig. 1 gives $w_{cr} \simeq 0.5$ eV/cm$^3 \simeq 10^{-12}$ erg/cm^3.

The electron component of cosmic rays has not been investigated as thoroughly as the proton–nuclear component. The energy range up to 1 GeV is particularly sensible to the processes on the Sun and in the solar system; the spectrum is rather complex and it changes in time with the solar activity. At energies greater than about 5 GeV the electron spectrum is consistent with a power law $I_e(E) \propto E^{-\gamma_e}$, where the values of the exponent γ_e given by different experimenters range from 2.7 to 3.4 (Freier *et al.*, 1975). The data are consistent with a constant value for γ_e up to $E \sim 500-1000$ GeV (Müller and Meyer, 1973).

Figure 4 summarizes the data on the electron spectrum. A detailed discussion of the observational methods and results of different authors can be found in the review by Anand *et al.* (1975). At a given energy, say at $E = 1-3$ GeV, the electron intensity in cosmic rays is of the order of 1 percent of the proton intensity. The electron component energy density is thus $w_e \sim 10^{-2} w_{cr} \sim 10^{-14}$ (erg/cm^3).

In most cases, and particularly in the papers cited above, electrons are not distinguished from positrons. At energies of order $E \sim 1$ GeV the positron content

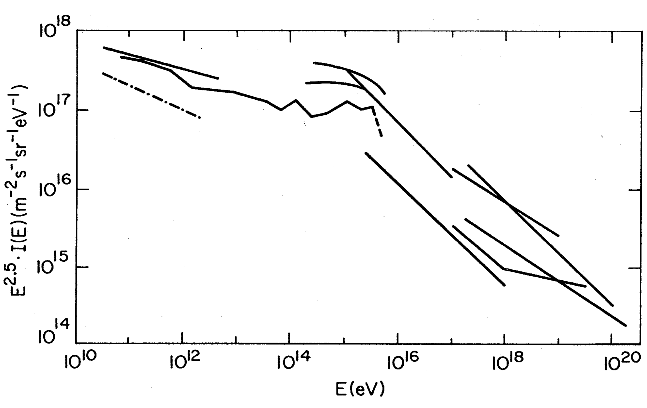

FIG. 3. General information on the spectrum of cosmic rays of the highest energies.

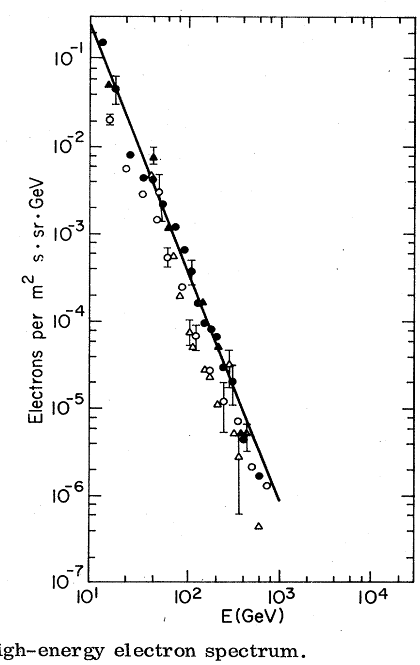

FIG. 4. High-energy electron spectrum.

amounts to about 10% of the electron concentration. Figure 5 presents the data on positrons obtained by different authors (see Buffington *et al.*, 1974). The part of the positron spectrum with energies $E > 1$ GeV is particularly important for a specification of the model of cosmic-ray propagation in the Galaxy. Unfortunately the positron spectrum has not yet been reliably determined in this region.

In concluding this section we present some data on cosmic-ray anistropy near the Earth. Reliable information on cosmic-ray anisotropy in the Galaxy, using the earth's measurements, can be obtained only for particles of energies not lower than 100–1000 GeV, since the motion of particles of lower energies is greatly distorted by the magnetic field of the solar system. But even at $E > 100$ GeV the anisotropy has not yet been reliably determined, and as a matter of fact only estimates of the upper anistropy limits are known. Figure 6 presents the results of different authors on the cosmic-ray anisotropy measurements (Hillas and Ouldrige, 1975). In 1974 some evidence of a strong anisotropy of cosmic rays with energies $E > 2.10^{19}$ eV was obtained (Krasilnikov *et al.*, 1974). These data, if correct, testify in favor of a galactic origin for even high-energy cosmic rays (Hillas and Ouldridge, 1975; see, however, Kiraly *et al.*, 1975).

C. Fundamental equations describing cosmic-ray propagation in the Galaxy

A high isotropy and a rather large content of secondary nuclei in cosmic rays indicate an effective "mixing" and a long wandering of high-energy particles in the Galaxy. Such a mixing and isotropization can be ascribed to several different causes: a stochastic structure of the galactic magnetic field and its large-scale inhomogeneities (Fermi, 1949; GS §10); an instability of anisotropic distributions of relativistic particles in the interstellar plasma (Ginzburg, 1965; Wentzel, 1974); macro-instability of the system formed by the relativistic gas of cosmic rays, the interstellar magnetic field, the interstellar plasma, and the gravitational field (Parker, 1969); or a strong particle reflection on the Galaxy boundaries. Unfortunately, complete analysis of all the known possibilities and a choice of a concrete physical mechanism responsible for cosmic-ray propagation in the Galaxy have not been worked out. This is mainly owing to the absence of complete enough information on the interstellar medium parameters and on the galactic magnetic field structure. It is natural that various approximate models are widely used in a situation where there is no consistent theory which could explain the character of cosmic-ray propagation proceeding from a strict picture of charged relativistic particle interaction with the interstellar plasma. These approximate methods make it possible to systematize and coordinate numerous experimental facts, and to explain characteristic features of the composition, spectra, and anisotropy of different cosmic-ray components. Since within each of the models cosmic-ray propagation has received in more detailed attention the corroboration of the models themselves, we shall first formulate a phenomenological theory of relativistic particle propagation in the Galaxy (Secs. III and IV) and only after that shall we discuss in more detail some possible ways to confirm it (Sec. VI).

Cosmic-ray propagation is most often considered within the diffusion approximation. Moreover (this assumption is in some sense independent), we shall think of cosmic rays as locally isotropic, which means that anisotropy may appear only when account is taken of the spatial inhomogeneity of particle concentration $N_i(\vec{r}, t, E)$.

The general transport equation for N_i in the approximation under discussion takes the form (for more details see GS, §14)

$$\frac{\partial N_i}{\partial t} - \text{div}(D_i \nabla N_i) + \frac{\partial}{\partial E}(b_i N_i) = Q_i - p_i N_i + P_i \,. \qquad (2.8)$$

If only the first two terms are kept in this equation (the rest will be dealt with below), we are led to the diffusion equation

$$\partial N_i/\partial t - \text{div}(D_i \nabla N_i) = 0\,, \qquad (2.9)$$

where $D_i(\vec{r}, E)$ is the diffusion coefficient; the equations are easily extended to the case when D_i is a tensor.

We should note here that the applicability of the diffusion approximations (2.8)–(2.9) to cosmic-ray propagation in the magnetic fields is not at all obvious. For this approximation to be valid it is not enough that the field have a strongly pronounced irregular random component since in this case there also exists a strong tendency for particle propagation along the lines of force of the magnetic field, even if they are rather tangled. But in the

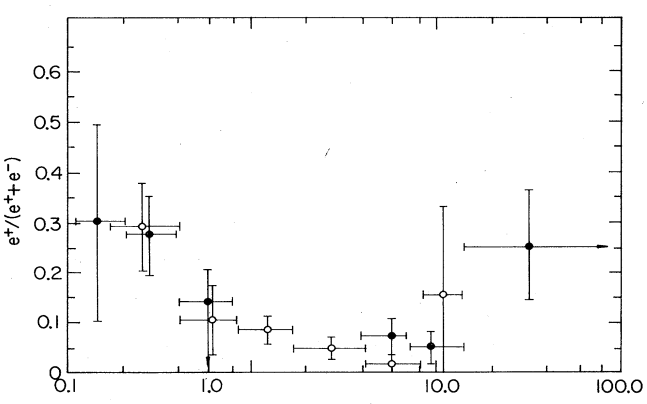

FIG. 5. Relative content of positrons in the cosmic-ray electron-positron component.

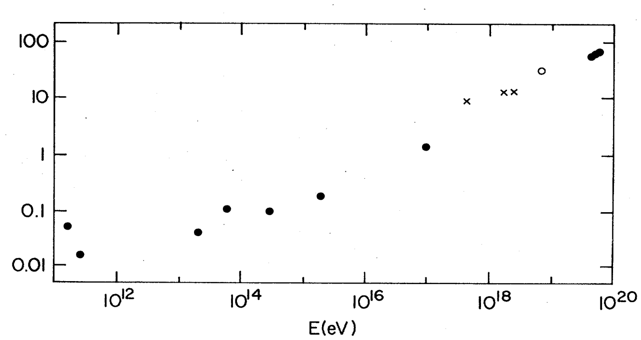

FIG. 6. Upper limits of cosmic-ray anisotropy (in %).

Galaxy, for example, differential rotation and the motion of gas clouds and spiral arms cause a constant mixing of the lines of force. At the same time we are usually interested not only in a picture averaged over rather large space regions (say, regions of tens and hundreds of parsecs) but also in a picture which is extended in time. To estimate average cosmic-ray gradients and their lifetime T_{cr} in the Galaxy it is in fact sufficient to know the concentration N_i averaged for the time $t \ll T_{cr} \sim 10^6 - 10^8$ yr, which means that the time of averaging may well be 10^5 yr.

In view of all these circumstances, the diffusion approximation seems already more suitable, particularly when the diffusion coefficient is chosen as a free parameter. Of course, this does not at all exclude the possibility of calculating D_i by means of a more detailed approximation (e.g., taking into account particle scattering by the magnetic field irregularities) and does not exclude verification of the very assumption of diffusion by comparing observations with the results of calculations in the diffusion approximation—on the basis of equations of the type (2.8), of the chemical composition, anisotropy, and other quantities characterizing all cosmic rays or their different components.

In the diffusion picture the resulting cosmic-ray flux is

$$F_i = 2\pi \int_0^{\pi} I_i(\theta) \cos\theta \sin\theta d\theta = D_i |\nabla N_i| . \tag{2.10}$$

Setting $I(\theta) = I_0 + I_1 \cos\theta$, we obtain for the degree of anisotropy the expression

$$\delta = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{I_1}{I_0} = \frac{3F}{4\pi I_0} = \frac{3D}{c} \frac{|\nabla N_{cr}|}{N_{cr}} , \tag{2.11}$$

where the relation $I \simeq I_0 = (v/4\pi) N_{cr} \simeq (c/4\pi) N_{cr}$ is used and ultrarelativistic particles are considered.

Note that it is easy to introduce in Eqs. (2.8)–(2.9) an additional term that takes into account a convective particle transport.

Let us now consider the remaining terms in the transfer equation (2.8). The quantity $(b_i N_i)$ is a particle flux in the "energy space," where b_i is the velocity in the energy space, i.e., the change of the particle energy per unit time

$$dE/dt = b_i(E) . \tag{2.12}$$

Within the accuracy of the approximation used, the change in the particle energy described by the term $b_i N_i$ should be smooth and continuous. As to the energy losses, $b_i < 0$. An example of such practically continuous losses might be ionization or cyclotron-radiation losses. Both in the case of losses and under particle acceleration, energy fluctuations may take place along with a regular mean energy change during some time interval. As a result of such fluctuations the energy distribution of the particles changes even if the mean particle energy remains constant. In the presence of energy fluctuations the term $-\frac{1}{2}(\partial^2/\partial E^2)(d_i N_i)$ [where $d_i(E) = (d/dt)\overline{(\Delta E)^2}$ and $\overline{(\Delta E)^2}$ is the mean square of the energy change under fluctuations] should be added under certain conditions to the left-hand side of Eq. (2.8).

The term $Q_i(\vec{r}, t, E)$ in (2.8) is the power of the "external" particle sources, i.e., the number of particles entering the system from the sources per unit time in the vicinity $d\vec{r}dE$ of the "point" $\vec{r}$, E is $Q_i d\vec{r}dE$. The term $-p_i N_i$ in (2.8) takes into account "catastrophic" processes of particle exit from the considered interval $d\vec{r}dE$, for example the transformation of nuclei when a nucleus of the sort i vanishes altogether and transforms itself into another nucleus. If the inelastic collision cross section is σ_i, the particle velocity is v, and the concentration of particles, say, of nuclei in the interstellar gas with which the particles collide is n, then

$$p_i = n v \sigma_i . \tag{2.13}$$

Another example of "catastrophic" losses is bremsstrahlung (radiative) losses in electron collisions with other particles with the emission of a rather hard photon.

The last term in (2.8) takes into account the particle coming into the interval $d\vec{r}dE$ also as a result of "catastrophic" collisions. One may, for example, write down

$$P_i = \sum_k \int p_i^k(E', E) N_k(\vec{r}, t, E') dE' , \tag{2.14}$$

where p_i^k is the probability of the process changing a particle of the sort k into a particle of the sort i (including the case of $k = i$) from the energy region E' into E.

In the general case the transport equation (2.8) is rather complicated. But in the analysis of the chemical composition of relativistic nuclei it may be considerably simplified. The point is that under nuclear transformations in the interstellar medium, and when some inelastic collisions (with the production of mesons etc.) are neglected, the energy per nucleon is conserved. Therefore it is reasonable to go over from the variable E to the variable ϵ; in this case $p_i^k(E', E) = p_i^k \cdot \delta(\epsilon - \epsilon')$ and $P_i = \sum_{k<i} p_i^k N_k(\vec{r}, t, \epsilon)$. This expression indicates that nuclei of type i may result only from the fragmentation of heavier nuclei for which the index k is assumed to be less than i. For relativistic nuclei the energy losses are mainly due to ionization, are relatively small, and may be neglected. As a result we are led to the equation widely used in the analysis of the chemical composition of cosmic-ray relativistic nuclei:

$$\partial N_i/\partial t - \operatorname{div}(D_i \nabla N_i) = Q_i(\vec{r}, t) - nc\sigma_i N_i + \sum_{k<i} nc\sigma_{ik} N_k . \tag{2.15}$$

We use here a relation of the type (2.13) between the quantities p_i, p_i^k and the cross sections σ_i, σ_{ik}, the particle velocities being set equal to the light velocity c. The variable ϵ is implicit in Eq. (2.15) and is a parameter.

Let us now consider a concrete application of the transfer equation (2.8) to electrons and positrons.

In this case we should note that $N_i = N_e(\vec{r}, t, E)$ or separately N_{e^-} (electrons) and N_{e^+} (positrons). Simplifications appear when "catastrophic" energy losses like nuclear reactions and bremsstrahlung can be neglected. Then we have

$$\frac{\partial N_e}{\partial t} - \operatorname{div}(D_e \nabla N_e) + \frac{\partial}{\partial E}(b_e N_e) = Q_e . \tag{2.16}$$

The sources Q_e should take into account the appearance of electrons and/or positrons due not only to their acceleration in the sources but also to different decays of unstable particles ($\mu^{\pm}$-mesons etc.) produced during nuclear collisions of cosmic rays with the interstellar gas (δ-electrons and electron–positron pairs produced by gamma-rays may be included). For positrons, the term which takes into account their annihilation should be introduced into an equation of the type (2.16).

The essential difference between the cosmic-ray electron component and the proton–nuclear component consists generally in the need to take into account for the electron component the energy losses [the third term in the left-hand side of Eq. (2.16)]. Therefore the energy variable E in Eq. (2.16) does not remain a parameter as it does in Eq. (2.15).

Equations (2.15) and (2.16) will serve us hereafter as a basis for the study of relativistic particle propagation in the Galaxy. However we shall systematically compare the results obtained within the diffusion model with those of the so-called homogeneous model. In a certain sense the homogeneous model may be thought of as an extremely simplified version of the diffusion model. The diffusion is considered here to proceed very rapidly and therefore the cosmic-ray concentration in the whole system (the Galaxy) is considered to be constant. One must, of course, for the cosmic rays in the system, some lifetime that depends on the rate of their escape from the system. In other words, the terms $(\partial N_i/\partial t) - \mathrm{div}(D_i \nabla N_i)$ in (2.15) are replaced by $N_i/T_{\rm cr}^{(\rm hom)}$, where $T_{\rm cr}^{(\rm hom)}$ is the parameter which has time dimensionality and characterizes the cosmic-ray leakage from the Galaxy. Then instead of (2.15) we obtain (the homogeneous or "leaky box" model)

$$N_i/T_{\rm cr}^{(\rm hom)} = \overline{Q}_i - \overline{n}c\sigma_i N_i + \sum_{k<i} \overline{n}c\sigma_{ik}N_k . \tag{2.17}$$

The definition of the homogeneous model requires that in (2.17) certain quantities be averaged over the galactic volume (the source power $\overline{Q}_i$, the interstellar gas density $\overline{n}$) and that the cosmic-ray density N_i be coordinate independent.

The homogeneous model can be obtained as a limiting case of the diffusion model if the particle leakage from the system is weak, i.e., if there is a strong particle reflection from the galactic boundaries. On the other hand when the reflection is strong at the boundaries and a particle passes through the Galaxy many times before going out of it, the approximation necessary for obtaining the homogeneous model is fulfilled automatically. The particle motion inside the propagation region may, in this case, be not diffusion but free. Therefore, to a certain extent, the diffusion and the homogeneous models may be treated as independent. In our analysis of the diffusion model we shall consider only the case of a free particle outflow at the galactic boundaries.

Note also that to describe the transformation of the cosmic-ray chemical composition one can use not only the equations for the concentration of different types of nuclei N_i but also the method of particle path length distribution functions (Davis, 1960). The function $G(\vec{r}, t, y)$ [or $G(\vec{r}, t, \tau)$] used here represents the probability that, after leaving the source, the particle found at moment t in point $\vec{r}$ has passed through the thickness of matter y (or has moved for the time τ) without taking into account fragmentation. The function G contains implicitly information concerning both the particle-propagation character and the distribution of cosmic-ray sources. The connection between these two methods is discussed in the next section.

III. PROPAGATION OF COSMIC-RAY NUCLEI

A. Transformation of nuclear chemical composition in the homogeneous model.

Equations for the nuclear concentration in the homogeneous model have the form (2.17). The values of the quantities $\overline{Q}_i$, i.e., the content of nuclei in the sources (more precisely, nuclei escaping them), are not known in advance.

As has been noted in Sec. II, there exists, however, a group of secondary nuclei which is practically absent from the sources (this conclusion is confirmed by spectroscopic data, analysis of meteorite composition, and calculations of nuclear reactions in the stars), and their content in cosmic rays is rather high. Secondary nuclei are thought to be the result of fragmentation of heavier nuclei during their propagation in the interstellar medium (we do not at present consider an alternative possibility, that secondary nuclei are produced by fragmentation near the cosmic-ray sources). Therefore for secondary nuclei one may assume that $\overline{Q}_i = 0$ which reduces Eq. (2.17) to

$$N_i\left(\frac{1}{x} + \sigma_i\right) = \sum_{j<i} \sigma_{ij} N_j , \tag{3.1}$$

where

$$x = \overline{n}c\, T_{\rm cr}^{(\rm hom)} . \tag{3.2}$$

The parameter x characterizes the mean path length of relativistic nuclei when they move from the sources to the observer (see also below). The value of the effective thickness x is evidently the only parameter characterizing the transformation of the chemical composition of the cosmic-ray nuclei in the homogeneous model. But to find the value of x one must, of course, know the relative contents of different types of nuclei in cosmic rays and the fragmentation cross section of these nuclei in the interstellar gas. In the simplest case, where secondary nuclei with concentration N_2 appear only from one group of primary nuclei with concentration N_1, we have [from Eq. (3.1)]

$$\frac{N_2}{N_1} = \frac{\sigma_{21}}{(1/x) + \sigma_2} . \tag{3.3}$$

Information on the cross sections necessary for the calculation of nuclear fragmentation can be found, for example, in papers by Shapiro and Silberberg (1970) and Silberberg and Tsao (1973). Table II presents some interaction cross sections of relativistic nuclei whose energy is $\epsilon_k \gtrsim 2$ GeV/nucleon with hydrogen nuclei. As is well known, the interstellar gas consists mainly of hydrogen nuclei with a small proportion (of about 10% of the number of nuclei) of helium nuclei, whereas the contribution of heavier elements to cosmic-ray fragmentation is negligibly small. The cross sections given by Table II in the relativistic region depend weakly on the

TABLE II. Cross sections of formation of light nuclei during interaction of relativistic protons with some target nuclei (in mb).

Nucleus product	Target nucleus					
	^{12}C	^{16}O	^{20}Ne	^{24}Mg	^{28}Si	^{56}Fe
^{6}Li	7	14	12	13	13	30
^{7}Li	6	14	11	11	11	20
^{7}Be	10	11	10	10	10	8.5
^{9}Be	6	3.7	3	3	3	5
^{10}Be	3.5	1.0	1.9	1.9	1.9	4
^{10}Be	14	12	9	8	7	7
^{11}B	51	25	18	15	12	9

energy and may be considered constant at $\epsilon_k > 2$ GeV/nucleon.

The cosmic-ray chemical composition and, specifically, the relative content of secondary nuclei are, on the contrary, energy dependent.

Let us first consider the energy range 1 GeV/nucleon $\lesssim \epsilon_k \lesssim 5$ GeV/nucleon. The data often used to find the effective thickness in this region are the light nucleus content (Li, Be, B). From the ratio of concentrations $N_L/N_M = 0.23 \pm 0.02$ of the groups of light nuclei L to medium nuclei M(C, N, O, F) observed near the Earth one can obtain the value of the thickness $x \simeq 3.10^{24}$ cm^{-2} in hydrogen with an error of about 50% (Shapiro and Silberberg, 1970; Meneguzzi, Audouze, Reeves, 1971; Ramaty and Lingenfelter, 1971; Ptuskin, 1972). The quantity x is usually expressed in other units: multiplying the value of x in cm^{-2} by the hydrogen nuclear mass we have $x \simeq 5$ g/cm^2—the matter thickness in g/cm^2 traversed by relativistic nuclei in hydrogen (if we take account of the contribution from the ten percent of helium nuclei, then $x \simeq 6.5$ g/cm^2). Sometimes the thickness is expressed in mb^{-1}; then $x \simeq (1/330)$ mb^{-1}. These units are particularly convenient in the analysis of Eq. (3.1) since the cross section is usually measured in mb. The above value of x found from the relative concentration of the whole group of light nuclei indicates clearly the content of both individual elements of this group and of secondary nuclei of the group Cl–Mn (Shapiro *et al.*, 1970), as well as the content of relativistic deuterium nuclei (Apparao, 1973).[2] Thus the homogeneous model of cosmic-ray propagation in the Galaxy gives a good description of the relative content of stable secondary nuclei.

Note that the quantity $1/x$, the inverse of the matter thickness traversed by particles in the Galaxy, which is determined by the cosmic-ray leakage from the Galaxy, appears to be approximately equal to the total cross section of inelastic interaction of nuclei from oxygen to silicon with the interstellar gas. This means that the loss of these relativistic nuclei because of their flow out of the Galaxy is about the same as their loss owing to fragmentation in the interstellar medium. For lighter nuclei

[2]The treatment of the data on the content of deuterons and ^{3}He in cosmic rays at nonrelativistic energies $\epsilon_k \lesssim 500$ MeV/nucleon does not contradict the homogeneous model with the mean thickness $x \simeq 5$ g/cm^2 (see, for example, Simpson, 1971, Ramadurai and Biswas, 1974).

TABLE III. Calculated amount of different cosmic-ray nuclei in the sources (normalized to 100 for carbon nuclei).

H	5×10^4	Na	0.8 ± 0.4	Ar	0.7 ± 0.5
He	2600	Mg	23 ± 2	Ca	2.2 ± 0.8
C	100	Al	2 ± 1	Cr	0.3 ± 0.3
N	11 ± 2	Si	20.5 ± 3	Mn	$0.2^{+0.4}_{-0.2}$
O	109 ± 2	P	$0.2^{+0.4}_{-0.2}$	Fe	22 ± 3
Ne	15 ± 2	S	3 ± 0.6	Ni	0.8 ± 0.2

the outflow dominates over spallation losses, while for heavier ones fragmentation dominates.

Having estimated the effective thickness x, one may proceed to establish the content of various primary nuclei in the sources. Table III presents the calculated content of various primary nuclei with charge $Z \leqslant 28$ in the sources, and Fig. 7 gives the ratios of the abundances of nuclei in the cosmic-ray sources to the corresponding abundance of the same nuclei in the solar system; the distribution is normalized to unity for iron nuclei (Shapiro and Silberberg, 1974; see also the review by Price, 1973). The composition of different elements in the sources is in good agreement with the assumption that the main cosmic-ray sources in the Galaxy are supernovae with the mass $M \gg M_\odot$ (Arnett and Schramm, 1973). Particularly important information on the cosmic-ray sources can be obtained from the content of heavy nuclei, i.e., those with the charge $Z > 30$ and up to transuranic elements (see the reviews by Price, 1971; Fowler, 1973; Zhdanov, 1974).

One should bear in mind, however, that the composition of nuclei accelerated in the sources, calculated with equations of the type (2.17), may not coincide with the composition of an exploding star substance. The difference may be due to the mechanism of particle injection into the accelerated region or to the acceleration mechanism itself. The dominating injection may be connected, for example, with a low value of the first ionization potential (Havnes, 1971) or with a small ionization cross section (Kristiansson, 1971) of the corresponding atoms (see also GS, § 9).

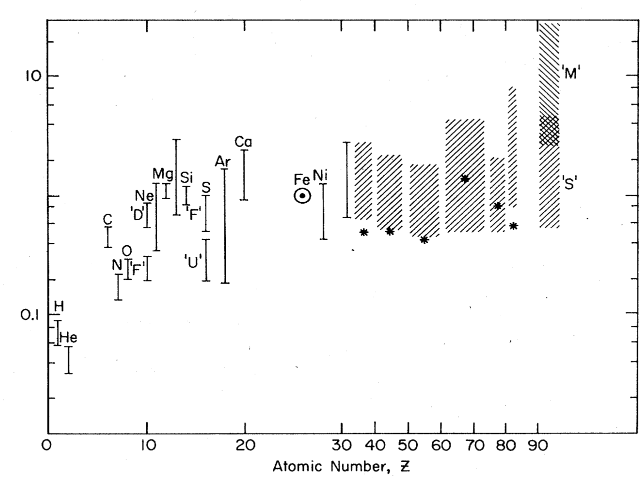

FIG. 7. Ratio of the element content in the cosmic-ray sources to their content in the solar system. Both sets of data are normalized to unity for the iron nuclei. The notation is that of Shapiro and Silberberg (1975).

After these brief remarks on cosmic-ray composition in the sources, let us return to the problem of relativistic particle propagation in the interstellar medium. As was noted at the end of Sec. II, the transformation in nuclear composition can be described with the aid of the path length distribution function $G(y)$. For the homogeneous model (2.17) the function $G(y)$ takes the exponential form $G(y)=\exp(-y/x)$ (Davis, 1960). For nuclei i Eq. (2.17) takes the form

$$N_i(1/x+\sigma_i)=q_i+\sum_{j<i}\sigma_{ij}N_j\,, \tag{3.4}$$

where

$$q_i=\overline{Q}_i/\overline{n}c\,.$$

By definition the function $G(y)$ gives the probability that the observed particle has passed through the amount of matter y if fragmentation is neglected. If fragmentation is taken into account, the number of nuclei detected by the observer is $G(y)e^{-\sigma_i y}$. (The intensity of the sources implied by all the terms of the right-hand side of (3.4) has been normalized to unity.) Integrating over all the y's we have

$$N_i=\int_0^\infty G(y)e^{-\sigma_i y}dy\,.$$

Comparing it with Eq. (3.4) we find

$$1/x+\sigma_i=\int_0^\infty G(y)e^{-\sigma_i y}dy\,,$$

and therefore

$$G(y)=\exp(-y/x)\,. \tag{3.5}$$

The function giving the distribution of propagation times τ from the sources to the observer can be found in a similar way

$$G(\tau)=\exp(-\tau/T_{\rm cr}^{\rm (hom)})\,. \tag{3.6}$$

Expressions (3.5) and (3.6) show that in the homogeneous model the thickness of matter traversed by nuclei in the Galaxy and the mean time of cosmic-ray wandering in the Galaxy neglecting fragmentation are equal to x and $T_{\rm cr}^{\rm (hom)}$, respectively. In fact,

$$\langle y\rangle=\int G(y)y\,dy\Big/\int G(y)dy=x\,, \tag{3.7}$$

$$\langle \tau\rangle=\int G(\tau)\tau\,d\tau\Big/\int G(\tau)d\tau=T_{\rm cr}^{\rm (hom)}\,. \tag{3.8}$$

However, the quantity $T_{\rm cr}^{\rm (hom)}$ is not measured directly (see above). The connection between the quantities x and $T_{\rm cr}^{\rm (hom)}$ is given by Eq. (3.2). As has been shown above, the value of the thickness x is determined from the relative composition of secondary nuclei; therefore, in order to calculate the age of cosmic rays $T_{\rm cr}^{\rm (hom)}$ it is necessary to know the mean gas density $\overline{n}$ in the cosmic-ray propagation region. The main galactic gas mass is concentrated in the galactic disk with a half-thickness b of the order of $\simeq 100$ pc, the mean gas density in the disk n_d being ~1 cm^{-3}. Above the disk there is apparently a region of an extensive gas halo with a density of $n\sim 10^{-2}$ cm^{-3} up to distances of the order of 1 kpc and a density of $n\sim 10^{-3}$ cm^{-3} at distances up to 10 kpc (Silk, 1974).

As has already been noted in Sec. I, the dimensions of the region of the Galaxy where cosmic rays are effectively trapped (the dimensions of the cosmic-ray halo) have not yet been established reliably. If cosmic rays are assumed to be concentrated in the gas disk, $\overline{n}=n_d\sim 1$ cm^{-3} and $T_{\rm cr}^{\rm (hom)}=x/\overline{n}c\sim 3.10^6$ years. In another limiting case, the model with a large halo 10–15 kpc, the mean gas density $\overline{n}$ is about 10^{-2} cm^{-3} and $T_{\rm cr}^{\rm (hom)}=(x/\overline{n}c)\sim 3.10^8$ years. The time $T_{\rm cr}^{\rm (hom)}$ has been estimated under the assumption that the thickness x through which nuclei propagate is in the interstellar medium. If a considerable part of the effective thickness x is traversed by the particle in the source region, the time that the cosmic ray spends wandering in the interstellar medium decreases correspondingly.

In calculating the mean path length x we have used data on the chemical composition of nuclei with energies of several GeV/nucleon. The change of composition which accompanies a change of the particle energy leads to the energy dependence of the thickness x. Figure 8 shows, for example, the observed energy dependence of the ratio $(B+N)/C$ (Juliusson, 1974; see also the review by Webber, 1974). The decrease in the amount of various secondary nuclei with the increase of the energy in the interval 1 GeV/nucleon $\lesssim \epsilon_k \lesssim$ 100 GeV/nucleon measured in the experiment is interpreted as a decrease of the thickness x with the energy. If x is of order $E^{-\mu}$, the exponent μ is $\mu\sim 0.3$, but a great uncertainty in the experimental data puts μ in the range $0\leqslant\mu\lesssim 0.6$. The power law in the energy dependence of the mean thickness and therefore of the time $T_{\rm cr}^{\rm (hom)}$ was chosen because, for the proton concentration in the homogeneous model, the relation

$$N_p(E)\simeq \overline{Q}_p(E)T_{\rm cr}^{\rm (hom)}(E) \tag{3.9}$$

is valid (the term $\overline{n}c\sigma_p$ is not important since $1/x\gg\sigma_p \simeq 30$ mb). Since the observed spectrum $N_p(E)$ is power-like, it is natural to consider the spectrum in the sources $\overline{Q}_p(E)$ and the age of the cosmic rays $T_{\rm cr}^{\rm (hom)}(E)$ to be also a power function of energy. The measured proton spectrum can be represented by a power function up to energies of 10^6 GeV and therefore the dependence $x(E)\sim E^{-\mu}$ with a constant index μ should be valid up to the same energies.

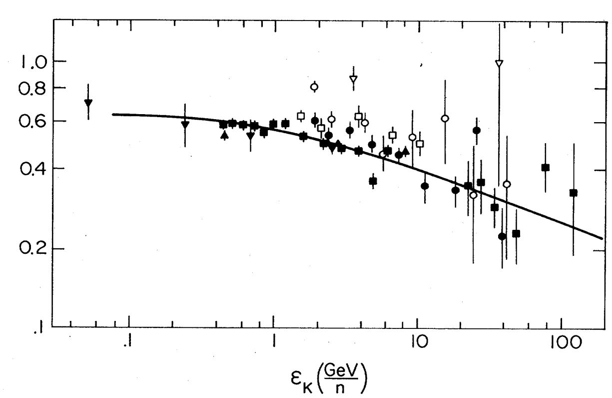

FIG. 8. Energy dependence of the ratio $(B+N)/C$ in cosmic rays.

When flowing out of the Galaxy cosmic rays cannot traverse an amount of matter less than $x_{min} \sim n_d b \sim 10^{-6}$ mb^{-1}. Taking into consideration that at energies of several GeV/nucleon the thickness $x \simeq 3.10^{-3}$ mb^{-1}, we obtain the maximum value of $\mu_{max} \sim 0.6$. Thus $\mu \lesssim 0.6$, which does not contradict observations. Note that the cosmic-ray source power increases as the quantity μ increases. In fact, if we assume that $\mu < \gamma_p - 2 \simeq 0.7$ the total integral power of relativistic proton sources is equal to

$$P_p(>E) = \int_E \overline{Q}_p(E) E\, dE = \int_E \frac{N_p(E)E}{T_{cr}^{(hom)}(E)} dE \sim E^{-\gamma_p+2+\mu} \simeq E^{-0.7+\mu}. \quad (3.10)$$

We assume here that all the thickness x is traversed when nuclei propagate in the interstellar gas. But if we believe that a considerable part of the thickness x is traversed in the cosmic-ray sources, the interpretation of the dependence $x(E)$ may change essentially. Specifically, that part of the thickness connected with the sources can change with energy. In this case the time of cosmic-ray wandering in interstellar space possibly does not at all depend on the energy. Besides, x will not necessarily vary like E raised to some power. For a more detailed discussion of energy dependence in the abundance of various nuclei (including primary ones), see the reviews by Webber (1974) and Meyer (1974), as well as the paper by Ptuskin (1974). We shall return to this question in Sec. III, where it turns out that the strongest limitations on a possible energy dependence of $x(E)$ can be obtained from the data on cosmic-ray anisotropy.

B. The diffusion model (stable nuclei)

The transformation of the chemical composition of cosmic-ray nuclei within the diffusion model is investigated in the same way as it was done in the homogeneous model approximation. Instead of the simple algebraic set of equations (2.17), a set of equations of the diffusion type with account taken of fragmentation [see (2.15)] should be analyzed here. However, in many important cases, to determine concentration of different nuclei N_i it is sufficient to solve one equation of the diffusion type, instead of a cumbersome set (2.15). For more details see the monograph GS (§ 14, 15). Presented here is its extension to the case when the gas density n in Eq. (2.15) is coordinate dependent.

We assume that the diffusion coefficient D is independent of the kind of nuclei, i.e., the operator

$$\hat{L} = (\partial/\partial t) - \text{div}(D_i \nabla)$$

on the left-hand side of (2.15) does not depend on the index i. We further assume that the spatial distribution of cosmic-ray sources and their evolution in time are also independent of the kind of nuclei, i.e., $Q_i(\vec{r}, t) = g_i X(\vec{r}, t)$, where the g_i are constants determining the relative content of different nuclei in the sources. Under these conditions the solution of the set of equations (2.15) can be represented in the form

$$N_i(\vec{r}, t) = \int_0^\infty N_i^{(\sigma)}(y) G(\vec{r}, t, y) dy, \quad (3.11)$$

where the function G satisfies the requirement ($G = 0$ at $y < 0$)

$$nc(\partial G/\partial y) + \hat{L}G = \delta(y)\chi, \quad (3.12)$$

and the functions $N_i^{(\sigma)}$ are determined from the set of equations[3]

$$dN_i^{(\sigma)}/dy = -\sigma_i N_i^{(\sigma)} + \sum_{k<i} \sigma_{ik} N_k^{(\sigma)} + g_i \delta(y) \quad (3.13)$$

($N_i^{(\sigma)} = 0$ at $y < 0$).

Thus when determining the nuclear concentration N_i, the diffusion of particles and their fragmentation may be investigated independently. The function G in the integral (3.11) is the particle distribution function with respect to the path length y without account taken of fragmentation.

The solution of the set of equations (3.13) has the form

$$N_i^{(\sigma)} = \sum_{k=1}^{i} a_{ik} e^{-\sigma_k y}, \quad (3.14)$$

where the coefficient a_{ik} is a linear combination of quantities $g_j (j \leqslant i)$; for example: $a_{11} = g_1$,

$$a_{21} = \frac{\sigma_{21}}{\sigma_2 - \sigma_1} g_1, \quad a_{22} = g_2 + \frac{\sigma_{21}}{\sigma_1 - \sigma_2} g_1 \quad \text{etc.}$$

(for more details see GS, § 14).

From Eqs. (3.11) and (3.14) we have

$$N_i(\vec{r}, t) = \sum_{k=1}^{i} a_{ik} F_k(\vec{r}, t), \quad (3.15)$$

where

$$F_k = \int_0^\infty G(\vec{r}, t, y) e^{-\sigma_k y} dy. \quad (3.16)$$

The problem of determining the concentrations N_i is reduced to obtaining the functions $F_k(\vec{r}, t)$. The equation for $F_k(\vec{r}, t)$ can be obtained by taking into account definition (3.16) and integrating Eq. (3.12) over the variable y

$$\hat{L}F_k + nc\sigma_k F_k = \chi. \quad (3.17)$$

The latter coincides with Eq. (2.15) for the heaviest nuclei $k = 1$ or, more precisely, for those nuclei for which one may neglect in the right-hand side of (2.15) the terms $\sum_{k<i} \sigma_{ik} N_k$ which take into account the fragmentation of nuclei heavier than i. In other words, Eq. (3.17) has the form

$$\frac{\partial N_i}{\partial t} - \text{div}(D \nabla N_i) + nc\sigma_i N_i = Q_i \quad (3.18)$$

since $N_1 = g_1 F_1$ and $Q_1 = g_1 \chi$.

Thus to find the concentrations N_i it is sufficient to solve Eq. (3.17) and use formula (3.15).

The methods described above can also be used for the other operators $\hat{L}$. Note that for the homogeneous model (2.17) the operator $\hat{L}$ has a simple form $\hat{L} = 1/T_{cr}^{(hom)}$.

The transformation of the chemical composition of

[3]The δ functions involved on the right-hand sides of Eqs. (3.12) and (3.13) define initial conditions for the functions G and N_i^{σ}. For a correct definition of the integral $\int_0^\infty$ in Eq. (3.11), it should be treated as the limit $\int_0^\infty = \lim_{\epsilon \to +0} \int_{-\epsilon}^\infty$. The boundary conditions for the function G must coincide with those for the functions $N_i(\vec{r}, t)$.

cosmic rays in the galaxy has been investigated by means of the diffusion approximation for different cases of spatial source distribution, and different dimensions and geometry of the cosmic-ray propagation region (see GS, §15; Shapiro and Silberberg, 1970; Ramaty and Lingenfelter, 1971; Pacheco de Freitas, 1970; Ginzburg and Syrovatskii, 1971; Ptuskin, 1972; 1974; Guet and Stanton, 1974).

The present paper considers only the models which take into account inhomogeneous gas distribution in the Galaxy. We assume that the cosmic-ray propagation region in the Galaxy has the form of a cylinder of radius R and height $2h$ (see Fig. 9) and that the cosmic-ray sources are homogeneously distributed in the internal disk of thickness $2b$. The gas density in the internal (gas) disk is $n=n_d$ and in the external disk (halo) $n=n_h \ll n_d$. The cosmic-ray diffusion is considered stationary, i.e., $\partial N_i/\partial t=0$. At the halo boundaries (the surface Σ) the particles go freely into intergalactic space, where the cosmic-ray concentration is negligible, i.e., $N_i/\Sigma=0$.

We assume first that the diffusion coefficient D is a constant in the entire propagation region. Then Eq. (2.15) takes the form

$$-D\Delta N_i + nc\sigma_i N_i = Q_i + \sum_{k<i} nc\sigma_{ik} N_k \,. \tag{3.19}$$

Equation (3.17) is written as follows

$$-D\Delta F_i + nc\sigma_i F_i = \theta(b-|z|)\theta(R-r)\,, \tag{3.20}$$

where $\theta(b-|z|)=1$ at $|z|\leq b$, and $\theta(b-|z|)=0$ at $|z|>b$. Before obtaining a complete solution of Eq. (3.20), let us investigate the simple one-dimensional case in which diffusion proceeds along one co-ordinate z only (Ptuskin, 1972, 1974). The solution of (3.20) will then take the form (the index i is omitted; $|z|\leq b$,

$$F_D(z)=\frac{1}{n_d c\sigma}\left\{1-\cosh\left(\frac{n_d c\sigma z^2}{D}\right)^{1/2}\times\left\{\cosh\left(\frac{n_d c\sigma b^2}{D}\right)^{1/2}+\left(\frac{n_d}{n_h}\right)^{1/2}\sinh\left(\frac{n_d c\sigma b^2}{D}\right)^{1/2}\times\tanh\left[\left(\frac{h}{b}-1\right)\left(\frac{n_h c\sigma b^2}{D}\right)^{1/2}\right]\right\}^{-1}\right\}\,. \tag{3.21}$$

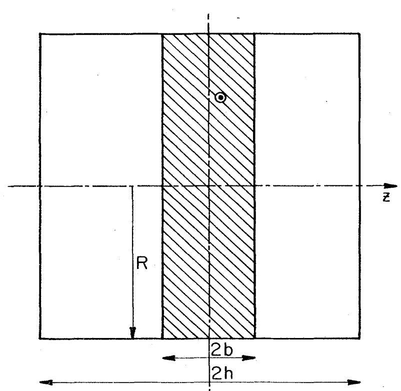

FIG. 9. Schematic presentation of the cosmic-ray propagation region in the Galaxy in the diffusion model. ⊙, the solar system's position.

The diffusion coefficient D and the halo half-thickness h should be considered unknown parameters in this model. Just as in the homogeneous model, these unknown parameters can be fixed from the requirement that cosmic-ray secondary nuclei be absent at the sources. A numerical calculation shows, however, that in practice a relative content of different secondary nuclei makes it possible to determine only the value of a certain combination of D and h. This fact is, generally speaking, connected with the inaccuracy of measurements of the nuclear chemical composition and the fragmentation cross sections, and therefore is valid for other versions of the diffusion model. That it is impossible to obtain separately the values of D and h follows automatically from the form of the function F_D (3.21). The point is that for the nuclei usually used in calculations (from hydrogen to iron) the fragmentation cross sections are comparatively small. Therefore when the hyperbolic functions in (3.21) can be expanded in a series only the first expansion terms can be taken. In this case, setting $z=0$, since the solar system is near the galactic symmetry plane ($b/h\ll 1$) we obtain:

$$F_D \simeq \frac{bh}{D}\,\frac{1}{1+(n_d cbh/D)\sigma}\,. \tag{3.22}$$

It was assumed in the derivation that $(b/h)\sigma x \ll 1$ (it is practically sufficient that $(b/h)\sigma x < \frac{1}{3}$ and

$$\frac{n_h}{n_d} \ll \frac{b}{h}\,\frac{1}{\sigma x}\,.$$

From Eq. (3.22) it follows that it is only in combination h/D that D and h can enter the function F_D. The parameter h/D can be found from the calculation of the chemical composition in the homogeneous model. In fact, from Eq. (2.16), with account taken of (3.2), one can obtain for the homogeneous model

$$F^{(\mathrm{hom})} = T_{\mathrm{cr}}^{(\mathrm{hom})}[1/(1+x\sigma)]\,. \tag{3.23}$$

The factors bh/D from (3.22) and $T_{\mathrm{cr}}^{(\mathrm{hom})}$ from (3.23) are not significant for the calculation of the chemical composition, since only relative concentrations are used in such calculations; it is only the dependence of F on the cross section σ that matters. Comparing Eqs. (3.22) and (3.23) we find that models (3.20) and (2.17) are equivalent in their description of the cosmic-ray chemical composition, the effective thickness x being related to the diffusion model parameters by

$$x \simeq n_d cbh/D \quad (b/h \ll 1)\,. \tag{3.24}$$

From relation (3.24) at $n_d = 1$ cm^{-3}, $b=100$ pc, $x=3.10^{24}$ cm^{-2} (the nuclear energy 1 GeV/neutron $\lesssim \epsilon_k \lesssim$ 5 GeV/neutron) we obtain $D\simeq 3.10^{27}$ cm^2/sec for $h=300$ pc, and $D\simeq 10^{29}$ cm^2/sec for $h=10$ kpc. Thus at the present stage it is still impossible to determine the halo dimensions from cosmic-ray composition, either in the diffusion or in the homogeneous model.

Formula (3.24) may be interpreted in terms of the theory of random particle wanderings during diffusion in the Galaxy. During such wandering, but before leaving the halo, a particle passes through the disk approximately h/b times, and in each passage through the disk it "gains" a path length of about $n_d cb^2/D$ (the thickness traversed by a particle in the halo turns out to be small if $n_h/n_d \ll b/h$, a condition which holds in the Galaxy).

Another way of obtaining Eq. (3.24) consists in the analysis of the function of particle path length distribution $G(\vec{r}, t, y)$. The G function is determined from Eq. (3.12), and when (3.22) holds, it has an exponential form $G \sim \exp(-y/x)$, where x is defined by Eq. (3.24).

In the diffusion model under consideration the same interpretation holds as in the homogeneous model for the observable decrease with energy of the fraction of secondary nuclei. It is most natural to believe that the corresponding change in the thickness $x(E) \sim E^{-\mu}$ is connected with the dependence of the diffusion coefficient on the energy $D \sim E^{\mu}$. It is of course quite possible logically that the variation $x(E)$ is due to the energy dependence of the thickness h of the region of particle storage (or, more generally, to the energy dependence of the boundary conditions for particles in the halo), or even to the $n_d(E)$ dependence, since more energetic particles may pass through the dense interstellar gas clouds more quickly, for which reason the average gas density in the disk is smaller for them.

In the previous section it was shown that in the homogeneous model the function giving the particle distribution of path lengths $G(y)$ and the function giving propagation times $G(\tau)$ coincide when $y = \bar{n}c\tau$ is taken into account. In the diffusion model under consideration the function $G(\vec{r}, t, y)$ satisfies Eq. (3.12) and for the function $G(\vec{r}, t, \tau)$ the equation ($G = 0$ at $\tau < 0$):

$$\partial G/\partial \tau + \hat{L}G = \delta(\tau)\chi \tag{3.25}$$

is valid. Due to inhomogeneous gas distribution in the Galaxy there exists in this case no simple connection between the distribution functions $G(\vec{r}, t, y)$ and $G(\vec{r}, t, \tau)$. With the aid of the function $G(\vec{r}, t, \tau)$ one can calculate different mean quantities characterizing cosmic-ray motion (see Ptuskin, 1974). For example, the mean age of cosmic rays in the Galaxy, neglecting fragmentation, turns out to be (for $b/h \ll 1$)

$$T = \frac{\int G(\vec{r}, \tau)\tau\, d\vec{r}\, d\tau}{\int G(\vec{r}, \tau)\, d\vec{r}\, d\tau} \simeq \frac{h^2}{2.4D}. \tag{3.26}$$

The mean age of the particles observed at the point $z = 0$ is equal to (again neglecting fragmentation and for $b/h \ll 1$)

$$T_0 = \frac{\int G(z=0, \tau)\tau\, d\tau}{\int G(z=0, \tau)\, d\tau} \simeq \frac{h^2}{3D}. \tag{3.27}$$

To estimate the power of the cosmic-ray source, it is convenient to define the time of particle exit out of the Galaxy (neglecting fragmentation and for $b/h \ll 1$) as:

$$T_s = \frac{\int N(\vec{r})\, d\vec{r}}{\int Q(\vec{r})\, d\vec{r}} \simeq \frac{h^2}{2D}. \tag{3.28}$$

The above calculations refer to a one-dimensional diffusion model (i.e., at $h/R \ll 1$) with the diffusion coefficient constant in the whole Galaxy. These simplifying assumptions make it possible to investigate the problem analytically and without particular difficulties. When we take into account the role of the side boundaries, i.e., when the halo half-thickness h and the disk radius R are comparable, the general expression for the nuclear concentration (for the function F) becomes more cumbersome. Besides, in a model with a large halo, even a weak coordinate dependence of the diffusion coefficient D may appear to be essential.

We shall consider the diffusion coefficient to be constant in the entire disk (the thickness $2b$) and equal to D_d, and the diffusion coefficient in the halo to be equal to D_h and also constant. The diffusion coefficient should be expected to decrease with increasing distance from the galactic plane, i.e., $D_h \geq D_d$ (though in principle this inequality may not always hold, e.g., at the halo boundaries there may exist an active turbulent region in which a strong particle scattering takes place). In this case the solution of Eq. (3.20) leads to the following expression (Ptuskin and Khazan, 1975):

$$\begin{aligned} N(r,z) &= QF(r,z) \\ &= \frac{2Q}{D_d}\sum_{k=1}^{\infty}\frac{J_0[\nu_k(r/R)]}{J_1(\nu_k)\nu_k\lambda_{kd}^2} \\ &\quad\times\left\{1-\cosh(\lambda_{kd}z)\left[\cosh(\lambda_{kd}b)+\frac{\lambda_{kd}}{\lambda_{kh}}\frac{D_d}{D_h}\sinh(\lambda_{kd}b)\right.\right. \\ &\qquad\left.\left.\times\tanh[\lambda_{kh}(h-b)]\right]^{-1}\right\}, \\ &\qquad (|z| \leq b), \end{aligned} \tag{3.29}$$

where r is the radial coordinate; $\lambda_{kd}^2 = (\nu_k^2/R^2) + (n_d c\sigma/D_d)$; $\lambda_{kh}^2 = (\nu_k^2/R^2) + (n_h c\sigma/D_h)$; J_0 and J_1 are the zero and the first Bessel functions; ν_k are the roots of the Bessel function $J_0(\nu_k) = 0$; and the cosmic-ray sources are concentrated in the disk so that

$$Q(\vec{r}) = Q \cdot \theta(b - |z|)\theta(R - r),$$

where θ is a step function [see Eq. (3.20)].

In a one-dimensional model with the diffusion coefficient D constant in space, the cosmic-ray chemical composition (more precisely, the relative abundance of secondary nuclei) determines the value of the mean path length x of the particles. When the halo dimension h is fixed and the rest of the parameters (b, n_d, n_h) are specified, we can determine the diffusion coefficient D. In a model with a variable diffusion coefficient, the diffusion coefficients in the disk D_d and halo D_h cannot be uniquely determined even for a definite choice of the quantities h, R, b, n_d, n_h, and with the use of the known data on the chemical composition. One can find only a certain combination of these quantities that defines the value of the effective thickness x. Figure 10 (curve A) presents possible values of the diffusion coefficients D_d and D_h established for a model with a large halo $h = R$ from the relative abundance of L nuclei at energies of 1 GeV/neutron $\lesssim \epsilon_k \lesssim 5$ GeV/neutron. In this case it was assumed that $h = R = 15$ kpc, $b = 150$ pc, $n_d = 0.5$ cm^{-3}, and the distance of the solar system from the galactic center $r = 10$ kpc. For comparison, the curve C in Fig. 10 gives the values of D_d and D_h calculated for a one-dimensional model with two diffusion coefficients. In such a model Eq. (3.24) is replaced by

$$x \simeq \frac{n_d c b^2}{D}\left(\frac{1}{2} + \frac{h}{b}\frac{D_d}{D_h}\right). \tag{3.30}$$

Curve B in Fig. 10 illustrates the effect of the spatial source distribution upon the quantities D_d and D_h—it is calculated on the assumption that the cosmic-ray sources are concentrated in the central region of the Galaxy with the radius equal approximately to 150 pc. On the whole Fig. 10 shows that the one-dimensional model approximation describes qualitatively the main features of a three-dimensional model even in the case of a very large halo $h \sim R$, particularly if in Eq. (3.30) some $h_{\rm eff} < h$ is substituted for h in order to take account of particle leakage through the side boundaries. Moreover, the spatial source distribution in the Galaxy turns out to have a weak effect on the content of stable secondary nuclei (the secondary and the primary nuclei are assumed to have sufficiently small cross sections for an interaction with the interstellar gas to satisfy the inequality $\sigma \ll (h/bx)$; specifically, no nuclei heavier than iron are considered).

As a result we may emphasize that a diffusion model with a free particle exit at the halo boundaries and with an inhomogeneous interstellar gas distribution gives a good description of the cosmic-ray chemical composition (the content of stable secondary nuclei) in the Galaxy. The same is true for the homogeneous model. From this point of view, neither of these two models can be preferred. Moreover the very expressions for relative concentrations of different nuclei in the homogeneous and the one-dimensional diffusion models coincide for $(b/h)\sigma x \ll 1$ and $(n_h/n_d) \ll (b/h)(1/\sigma x)$. From these inequalities it follows that with the increase of the halo dimensions the difference between the homogeneous and the one-dimensional diffusion models will grow smaller and smaller.

Related to what has been stated above is a general question: what limitations on a possible model of relativistic particle propagation in the Galaxy can be obtained from the analysis of the cosmic-ray nuclear chemical composition? It turns out that within the accuracy of present day measurements (of the order of 10–20% or even worse) of the relative element content in cosmic rays and of the nuclear fragmentation cross sections during interaction with the interstellar gas,

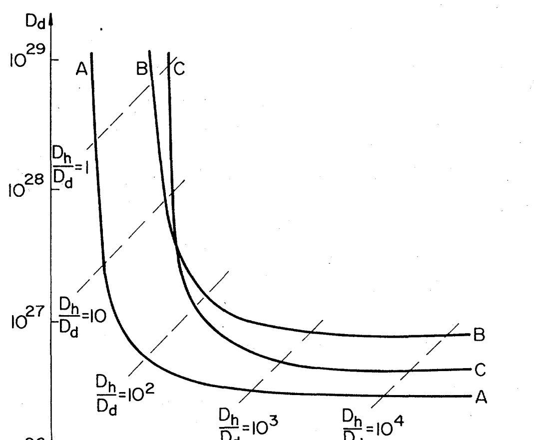

FIG. 10. Results of numerical calculations of the diffusion coefficients in halo D_h and in disk D_d (in cm^2/sec). Curve A, the sources are distributed uniformly in the galactic disk; B, the sources occupy the central galactic region; C, the one-dimensional approximation.

our choice of model is very wide. This problem is easiest to investigate with the aid of the functions of particle path length distribution $G(y)$ using information on the nuclear content in cosmic rays along with an extensive set of fragmentation cross sections (Syrovatskii and Kuzhevskii, 1969; Shapiro and Silberberg, 1970). In practice, only nuclei from hydrogen to iron are dealt with. The cosmic-ray chemical composition is well described by models with a large dispersion in path length, so that only one model $G(y) = \delta(y - x_0)$ is henceforth excluded, in which all the nuclei traverse the matter thickness $y = x_0$ (this is the so-called regular model or "slab" approximation). Physical realization of the regular model could be as follows: all the cosmic rays observed near the Earth emanate from one compact source and come to the observer on one and the same path, e.g., along one tube of the lines of force (although strictly speaking even in this case some spread in y would exist due to particle spread in pitch angle with respect to the magnetic field direction).

Recent data on the abundance of very heavy nuclei (up to uranium) make it possible, in principle, to obtain additional information on the distribution function $G(y)$. In particular, a high concentration of nuclei with large fragmentation cross sections indicates that the function $G(y)$ is not cut off at low values of path length y, of the order of several tenths g/cm^2. Unfortunately, very important information gained from relativistic superheavy nuclei (see, for example, Blanford *et al.*, 1973) cannot be used as yet because of low observational statistics. Measurement of a secondary antiproton flux (see Gaisser and Levy, 1974) is another source of information which can be used in the future.

In concluding this section we shall mention one of the possible diffusion model modifications—a "compound diffusion" model (Getmantsev, 1962; Lingenfelter *et al.*, 1971; Ramaty and Lingenfelter, 1971) in which a one-dimensional particle diffusion proceeds strictly along the magnetic field tubes, and in which there exists a three-dimensional diffusion of the tubes in the interstellar medium. As a result, the equations for nuclear concentration take the form

$$\frac{\partial N_i(\vec{r},t,s)}{\partial t} - D\frac{\partial^2 N_i(\vec{r},t,s)}{\partial s^2} + nc\sigma_i N_i(\vec{r},t,s) - \sum_{j<i} nc\sigma_{ij} N_j(\vec{r},t,s) = \int d\vec{r}_0 Q_i(\vec{r}_0,t) f(\vec{r},\vec{r}_0,s); \quad (3.31)$$

$$\frac{\partial f(\vec{r},\vec{r}_0,s)}{\partial s} - \frac{1}{3} l\Delta f(\vec{r},\vec{r}_0,s) = \delta(\vec{r}-\vec{r}_0)\delta(s), \quad (3.32)$$

where s is the path along the lines of force, and l the scattering length characterizing the diffusion of the magnetic field lines.

For the same scattering lengths the particle propagation in interstellar space is much slower in the compound diffusion model than in the normal diffusion model. For example, the time dependence of particle spatial displacement in the compound diffusion model is given by $L \sim t^{1/4}$, whereas in the usual diffusion model it is $L \sim t^{1/2}$. The compound diffusion model gives a satisfactory description of the experimental data available on the chemical composition of cosmic-ray nuclei and leads naturally to typical effective diffusion coefficients exceeding those of the usual diffusion model. However, this model faces some apparent difficulties in the cosmic-ray anisotropy analysis (Allen, 1972). There are no particular grounds for using this model, at least at the present stage.

C. Radioactive nuclei and the age of cosmic rays in the Galaxy

The age of cosmic rays in the Galaxy can, in principle, be determined by the relative abundance of radioactive nuclei. To this end one should use nuclei whose mean lifetime under decay is of the order of the time of cosmic-ray leakage from the Galaxy. The nucleus usually considered is ^{10}Be whose decay is ($^{10}\mathrm{Be} \xrightarrow{\beta^-} {}^{10}\mathrm{B}$) with an average time $\tau \simeq 2, 2.10^6 \times E/Mc^2$ yr. (Yiou and Raisbeck, 1972).

Let us consider the homogeneous model first. In Eq. (2.17) for the concentration of radioactive nuclei N_i an additional term N_i/τ_i appears now on the left-hand side, where (τ_i is the mean lifetime of the nuclei, under decay). For secondary radioactive nuclei we have [cf. Eq. (3.1)]:

$$N_i\left(\frac{1}{x} + \frac{1}{\bar{n}c\tau_i} + \sigma_i\right) = \sum_{j<i} \sigma_{ij} N_j \,, \tag{3.33}$$

where as before

$$x = \bar{n}cT_{\mathrm{cr}}^{(\mathrm{hom})} \,.$$

The thickness x is determined by the relative nuclear composition, so that when measuring the amount of radioactive nuclei from formulae of the type (3.33) one can find the average gas density $\bar{n}$ and, consequently, the cosmic-ray age (without account taken of fragmentation)

$$T_{\mathrm{cr}}^{(\mathrm{hom})} = x/\bar{n}c \,.$$

For example, in the simplest case, when secondary radioactive nuclei with concentration N_2 are produced by one group of primary nuclei with concentration N_1 (the index 2 at σ and τ is omitted) we obtain from Eq. (3.31)

$$\frac{N_2}{N_1} = \frac{\sigma_{21}}{1/x + (\bar{n}c\tau)^{-1} + \sigma} \,. \tag{3.34}$$

Note that from the general formula (3.33) one can also obtain a simple equation for the cosmic-ray age

$$T_{\mathrm{cr}}^{(\mathrm{hom})} = \frac{x}{\bar{n}c} = \frac{\eta_i}{1-\eta_i}(1 + x\sigma_i)\tau_i \,, \tag{3.35}$$

where η_i is the fraction of the isotope i which has decayed (i.e., $\eta_i = 1 - (N_i(\tau_i)/N_i(\tau_i \to \infty))$.

A practical determination of the time of cosmic-ray wandering in the Galaxy faces the difficulty that the content of the isotope ^{10}Be should only amount to about 15 percent of the total amount of beryllium produced in cosmic rays (if ^{10}Be does not decay), and this quantity is difficult to measure with sufficient accuracy. Because in most observations particular beryllium isotopes could not be separated at all, cosmic-ray age was determined by using the ratio Be/B. On the basis of observations of the ratio Be/B in cosmic rays and the use of the one-dimensional model, the estimate $T_{\mathrm{cr}}^{(\mathrm{hom})} \sim 3.10^6$ yr and the upper limit $T_{\mathrm{cr}}^{(\mathrm{hom})} \lesssim 10^7$ yr have been obtained (O'Dell *et al.*, 1973). It is not clear, however, whether or not these conclusions are reliable, since the fragmentation cross sections of beryllium isotope formation for various nuclei are only known with a relatively large error, of about 25 percent (Raisbeck and Yiou, 1973). A successful separation of beryllium isotopes in cosmic rays has been performed at the energy $\epsilon_k \sim 200$ MeV/nucleon (Webber *et al.*, 1973). The nuclei lifetime, taking into account fragmentation, was estimated as $T_{\mathrm{cr}}^{(\mathrm{hom})} + 1/\bar{n}v\sigma = (3.4^{+3.4}_{-1.4})$, 10^6 years, whereas when fragmentation was disregarded, the cosmic-ray age was estimated as $T_{\mathrm{cr}}^{(\mathrm{hom})} = 3.10^6 - 10^7$ years. This figure includes the possibility of a statistical error in measuring the amount of ^{10}Be. (When the uncertainty in the fragmentation cross section is taken into account, the spread of possible values of $T_{\mathrm{cr}}^{(\mathrm{hom})}$ is wider.)

Thus, because of the possibility of large statistical errors in measuring the content of certain isotopes and elements, and because of uncertainty in the magnitude of the cross-sections, even a conditional age for cosmic rays in the Galaxy $T_{\mathrm{cr}}^{(\mathrm{hom})}$, that is age in homogeneous model, has not yet been reliably established. At the same time the treatment of the available observations by the homogeneous model formulae gives a probable estimate of the age as $T_{\mathrm{cr}}^{(\mathrm{hom})} \sim 3.10^6 - 10^7$ years.[4] If the homogeneous model is applicable, the estimate by Webber *et al.* implies the absence of a considerable halo in our Galaxy (in the model with halo dimensions 10–15 *kpc* $T_{\mathrm{cr}}^{(\mathrm{hom})} \sim 1 - 3.10^8$ years).

Let us now consider the diffusion model (Prischep and Ptuskin, 1975). The radioactive nucleus concentration is now described in Eq. (2.15) by adding on the left-hand side the term N_i/τ_i to take account of the decay. For radioactive nuclei, contrary to the case of stable ones, the methods of solving sets of diffusion equations described at the beginning of Sec. III.B are not applicable, since the operator $\hat{L}$ now formally depends on the type of nucleus (through the term $1/\tau_i$). The only exception is the case of a constant gas density throughout the Galaxy. In this case the term N_i/τ_i may be taken to the right-hand side of the equation and grouped with the quantity $\sigma_i N_i : (\sigma_i + 1/\bar{n}c\tau_i)N_i \equiv \sigma_i^* N_i$, where σ_i^* is some effective cross section that takes into account the decay. Actually, that the case $n = \mathrm{const}$ is singled out is due to the fact that only here does the function of particle path length distribution $G(y)$ describing the nuclear fragmentation coincide with the function of age distribution $G(\tau)$ that describes the evolution of decaying nuclei disregarding fragmentation. In our diffusion model the gas is not distributed uniformly, and in the general case equations of the type (2.15) should be solved with the additional term N_i/τ_i in consecutive order for all the groups of nuclei. Practically, however, we only take into account the decay of ^{10}Be nuclei; therefore for the rest of the nuclei the solution procedure is similar to that described in Sec. III.B. For the ^{10}Be nuclei we have the equation (the index i is omitted)

[4] According to the recent data (Garcia-Munoz *et al.*, 1975), the most probable age is $T_{\mathrm{cr}}^{(\mathrm{hom})} \sim 2.10^7$ years.

$$-\mathrm{div}(D\nabla N)+nc\sigma N+(N/\tau)=Q(\vec{r}). \tag{3.36}$$

The ^{10}Be nuclei are secondary, and therefore the quantity $Q(\vec{r})$ describes also the fragmentation of nuclei heavier than ^{10}Be. We assume that the term $Q(\vec{r})$ is coordinate-independent inside the gas disk; this approximation is quite valid if the diffusion equations for stable nuclei are reduced to the homogeneous equations. Besides, we shall consider the one-dimensional model and disregard the gas density in the halo. Then Eq. (3.36) has the solution ($|z|\leq b$):

$$N=\frac{Q}{D\kappa_d^2}\left\{1-\cosh(\kappa_d z)\left[\cosh(\kappa_d b)+\frac{\kappa_d}{\kappa_h}\sinh(\kappa_d b)\tanh(\kappa_h(h-b))\right]^{-1}\right\} \tag{3.37}$$

where

$$Q(\vec{r})=Q\circ\theta(b-|z|);\quad \kappa_d^2=\frac{h_d c\sigma}{D}+\frac{1}{D\tau};$$

and

$$\kappa_h^2=\frac{1}{D\tau}.$$

It is convenient to consider separately three limiting cases (assume $z=0$ and $b/h\ll 1$):

(a) $\tau\gg h^2/D$, i.e., nuclei decay weakly for the time of diffision wandering in the Galaxy.

Then

$$N\simeq\frac{Q}{n_d c}\,\frac{1}{1/x+[3(b/h)n_d c\tau]^{-1}+\sigma}, \tag{3.38}$$

where x is defined by Eq. (3.24).

Expression (3.38) coincides with Eq. (3.32) for the homogeneous model ($Q_i/n_d c=\sum\sigma_{ik}N_k$), if the quantity $3b/hn_d$ is taken for the mean gas density. If η is the fraction of ^{10}Be nuclei which has decayed, by analogy with (3.35) we obtain

$$\frac{\eta}{1-\eta}\simeq\frac{T_0}{(1+x\sigma)\tau} \tag{3.39}$$

where $T_0=h^2/3D$ is the mean age of the particles detected by an observer at the point $z=0$, ignoring fragmentation and decay [see Eq. (3.27)].

(b) $h^2/D\gg\tau\gg b^2/D$, i.e., before decay the nuclei flow out of the galactic disk but do not reach the halo boundaries. In this case

$$N\simeq\frac{Q}{n_d c}\,\frac{1}{\sigma+(1/n_d c)\sqrt{(D/b^2\tau)}}=\frac{Q}{n_d c}\,\frac{1}{\sigma+(1/\bar{n}c)\sqrt{D/h^2\tau}}, \tag{3.40}$$

if the mean value

$$\bar{n}=n_d\frac{b}{h}$$

is introduced. Expression (3.40) can not be reduced to the homogeneous model. In particular, the concentration N has a different τ dependence. Instead of Eq. (3.39) we now have

$$\frac{\eta}{1-\eta}\simeq\frac{\sqrt{(h^2/D)\tau}}{(1+x\sigma)\tau}. \tag{3.41}$$

(c) $b^2/D\gg\tau$, when the nuclei do not flow out of the disk before decay; then the radioactive nuclei content does not correspond to that of the homogeneous model and is given by

$$N\simeq\frac{Q}{n_d c}\,\frac{1}{\sigma+(n_d c\tau)^{-1}}. \tag{3.42}$$

In this case

$$\frac{\eta}{1-\eta}\simeq\frac{(x/n_d c)-\tau}{(1+x\sigma)\tau}. \tag{3.43}$$

Thus the diffusion model for the case of radioactive nuclei does not, generally speaking, coincide with the homogeneous one. Only the concentration of slowly decaying nuclei (at $\tau\gg h^2/D$) is described by an expression corresponding to the homogeneous model. In this case the parameter $T_{\rm cr}^{(\rm hom)}$, determined by the homogeneous model formulae, turns out to be in fact the mean age of the particles seen by an observer $T_0=h^2/3D$. If the particle motion is of a diffusive character but in the treatment of experimental material the relations used are formally valid for the homogeneous model [see Eqs. (3.3) and (3.34)], then for $b^2/D\ll\tau\ll h^2/D$ (case b) the effective age $T_{\rm cr}^{(\rm hom)}$ turns out to be approximately equal to the geometric mean of the diffusion time for particle leakage from the Galaxy and the mean decay lifetime of the nucleus (i.e., $T_{\rm cr}^{(\rm hom)}\sim((h^2/D)\tau)^{1/2}$). When rapidly decaying nuclei do not flow out of the disk at all ($\tau\ll b^2/D$), the time $T_{\rm cr}^{(\rm hom)}$, determined formally by the homogeneous model formulae, bears no relation to the time of the particle leakage from the Galaxy, and according to (3.43) is approximately equal to $[(x/n_d c)-\tau]$.

The great difference between the diffusion and the homogeneous models in the description of motion of rapidly decaying nuclei is that in the homogeneous model the concentrations of all sorts of nuclei are coordinate-independent, whereas in the diffusion model the concentrations are not only coordinate-dependent but this dependence is different for different nuclei. For example, stable nuclei occupy both the disk and the halo, whereas nuclei with a small lifetime may not even leave the disk. It is clear that in the latter case the radioactive nucleus concentration measured near the Earth exceeds considerably the average concentration for the entire Galaxy and the apparent cosmic-ray age is less than the actual one.

For the ^{10}Be nuclei the case b approximation is practically always valid at $b/h\ll 1$. That is why if the cosmic-ray particle propagation is of a diffusive character with a free particle exit at the halo boundaries, the age $T_{\rm cr}^{(\rm hom)}$ obtained formally by the homogeneous model formulae [see Eq. (3.33)] is practically equal to $\sqrt{(h^2/D)\tau}$.[5] Therefore, for example, for $T_{\rm cr}^{(\rm hom)}\sim\sqrt{(h^2/D)\tau}$

[5]The homogeneous model may be considered to be applicable also in this case, but the region of averaging of $\bar{n}$ and Q in Eq. (2.17) is approximately equal to $\sqrt{D\tau}$—the distance traversed by radioactive nuclei. In fact, in this case (for $T_{\rm cr}^{(\rm hom)}\gg\tau$)

$$\frac{N_2}{\tau}\simeq\bar{n}c\sigma_{21}N_1\sim\frac{n_d Bc\sigma_{21}N_1}{\sqrt{D\tau}}\simeq(x\sigma_{21}N_1)/\sqrt{h^2\tau/D}$$

where the relation $x=n_d cBh/D$ has been used. With a typical (for homogeneous model) averaging over the whole galactic volume we obtain $N_2/\tau\simeq\bar{n}c\sigma_{21}N_1\sim(x\sigma_{21}N_1)T_{\rm cr}^{(\rm hom)}$. Thus we are led to the relation $T_{\rm cr}^{(\rm hom)}\sim\sqrt{(h^2/D)\tau}$.

$\sim 2\times10^7$ years (Garcia-Munoz *et al.*, 1975) and $\tau = 2.2\times10^6$ years the characteristic diffusion time for particle leakage from the halo is $T_{cr,h} \simeq h^2/2D \sim 10^8$ years. In a three-dimensional model for a given "age" $T_{cr}^{(hom)}$) the effective value of h will be greater than in a one-dimensional model. This is also the case in the diffusion model with a variable diffusion coefficient, since a rather small diffusion coefficient D_d in the disk leads to a still higher concentration of rapidly decaying nuclei near the Earth; there the value $T_{cr}^{(hom)} \sim 3\times10^6$ years is apparently compatible even with the presence of a large galactic halo with $h \sim R$ (Ptuskin and Khazan, 1975).

To sum up, we may say that the even disregarding a large uncertainty in the fragmentation cross sections, the conclusion based on the homogeneous model of cosmic-ray propagation that a sizeable halo is not present in our Galaxy, is not valid for the diffusion model del with a free particle exit at the halo boundaries and with the main gas mass concentrated in the galactic disk. Although such a diffusion model has not been proved valid, it is evidently more realistic than the homogeneous model. As will be seen below, the radio-astronomical method of determining cosmic-ray age does not give sufficient grounds for the estimate $T_{cr} \lesssim 3.10^6$ yr either, but testifies rather in favor of the age $T_{cr} \sim 10^8$ yr. So the assertion of the validity of the galactic disk model, ofter heard of late, and the use, in accordance with this, of the age $T_{cr} \sim 1-3.10^6$ years may be characterized as adopted by repetition.

D. On the anisotropy of cosmic rays

Measurements of cosmic-ray anisotropy are very important for clarifying the character of the motion of relativistic charged particles and spatial source distribution. It should be mentioned that the application of diffusion models to anisotropy calculation is less well grounded than their use in the analysis of cosmic-ray chemical composition (however, strict justification is absent even in this latter case). The point is that while mixing and wandering through the whole Galaxy, the cosmic-ray nuclei are transformed by interaction with the interstellar gas, so that the chemical composition of nuclei near the Earth is some quantity averaged over all possible propagation trajectories and large time intervals. By contrast, anisotropy can be determined for the most part by the local structure of the magnetic field near the solar system. Even in the simplest case, when a local magnetic field has no complicated features, like traps etc., but is homogeneous within, say, several parsecs (with weak perturbations necessary for the scattering of particles and their diffusion along the field), the simple connection (2.11) between the anisotropy δ and the cosmic-ray concentration gradient ∇N breaks down because of the tensor character of the diffusion coefficient. In fact, the effective "collision" frequency necessary to provide the characteristic diffusion coefficient $D \sim 10^{28}\text{cm}^2/\text{sec}$ for relativistic particles is $\nu \sim c^2/D \sim 10^{-7}\ \text{sec}^{-1}$. This estimate for ν is much less than the gyrofrequency

$$\Omega = \frac{ZeH}{Mc}\frac{Mc^2}{E} \sim 3.10^{-2}\frac{Mc^2}{E}\ \text{sec}^{-1}$$

(the estimate is for protons, Ze is the particle charge, $H \sim 3.10^{-6}G$ is a magnetic field in the interstellar medium) for particle energies $\epsilon \lesssim 3.10^5$ GeV/nucleon. Thus, the cosmic-ray relativistic gas is "magnetized" and its properties are essentially anisotropic.[6] In particular, if in this case the cosmic-ray concentration gradient is perpendicular to the main homogeneous magnetic field, the observed particle flux and anisotropy are perpendicular to the concentration gradient and to the magnetic field (Davis, 1954). From the viewpoint of plasma physics this is simply one form of drift.

Even in the absence of concentration gradients, magnetized particles in a homogeneous magnetic field can give anisotropy with two peaks due to anisotropic angular distribution of particle velocities with respect to the magnetic field direction.

Note that irrespective of the specific character of the motion of cosmic rays with energies $\epsilon_k \sim 1-10^3$ GeV/nucleon, this motion cannot be directed along the galactic disk. The latter could be expected because the mean magnetic field is apparently directed parallel to the galactic plane and it seems that particle propagation across the disk has to be inhibited. However, the particles in the gas disk are to pass through an amount of matter x_d no larger than the total thickness $x \simeq 3.10^{24}$ cm^{-2}. To this end the time $T_{cr,d} = x_d/n_d c < x/n_d c \sim 10^{14}$sec is necessary at a gas density in the disk of $n_d = 1\ \text{cm}^{-3}$. On the other hand, the upper limit on the anisotropy $\delta \lesssim 10^{-3}-10^{-4}$ leads to the limit on the bulk velocity of the cosmic-ray outflow from the disk $u_d \lesssim [\delta/(\gamma_p + 2)]c \sim 1-10\times10^6$ cm/sec (the so called Compton–Getting effect; for more details see Glesson and Axford, 1968). Therefore, at a directed motion from the disk, the average path of the cosmic rays in the disk (particularly along the disk) is $L = u_d T_{cr,d} < 1-10\times10^{20}$ cm, while the half-width of the disk itself is $b \simeq 3\times10^{20}$ cm. These rough estimates, which disregard the characteristic features of relativistic particle propagation, spatial distribution of sources, and the observer's position, show that cosmic rays cannot move freely in the disk and must be effectively displaced across the disk (Parker, 1969). Such a motion is evidently connected with mixing and tangling of the magnetic field lines of force which carry the imbedded cosmic-ray relativistic gas to the disk boundaries. So something like diffusion does in fact take place. In the framework of the diffusion model these considerations favor an isotropic character for the diffusion, at least over the Galaxy as a whole. Therefore, in what follows we shall use the simplest version of the diffusion approximation with an isotropic diffusion coefficient for lack of sufficiently detailed information on the local field structure. Since the diffusion character of the particle motion must be violated at very high energies, we shall restrict ourselves to particle energies $\epsilon \lesssim 10^6$ GeV/nucleon.

Roughly speaking, we may distinguish three effects

[6]If particle diffusion is due to the scattering on static or low-frequency magnetic inhomogeneities, the opposite case (i.e., when the cosmic-ray relativistic gas is not magnetized) cannot be realized at all since in the quasistatic case $\nu_{max} \sim \Omega$ (in other words $D_{min} \sim cr_H = c^2/\Omega$).

leading to cosmic-ray anisotropy in the Galaxy. The first is connected with the motion of the solar system at a velocity of the order of 30 km/sec relative to the overall stellar population, interstellar gas, and large-scale galactic magnetic field. Cosmic rays must be involved in the general Galaxy rotation, and even in the case of an isotropic particle distribution in the moving system the individual motion of the Sun leads to an observable anisotropy due to the Compton-Getting effect. This phenomenon was possibly discovered experimentally (see Speller *et al.*, 1972).

The second effect is due to close but separate cosmic-ray sources (e.g., pulsars). Its estimate is quite vague, since it requires a knowledge of the sources, spatial distribution, power, age, and evolutionary law. Lingenfelter (1969) estimated the anisotropy due to some close pulsars and obtained the value $\delta \sim 10^{-4}$. Cosmic rays were considered to have been generated by supernova explosions. The diffusion coefficient was assumed to be $D = 6\times10^{28}$ cm^2/sec. The value of δ can be lowered by assuming that cosmic rays are generated continuously or by choosing another diffusion coefficent.

Finally, the third effect is a general cosmic-ray leakage from the Galaxy. In this case the calculations of anisotropy with the aid of the discrete source model (see Jones, 1970; Ramaty and Lingenfelter, 1971; Dickinson and Osborn, 1974) coincide in general with the results obtained by the assumption of a spatially continuous source distribution (Ptuskin, 1974; Le Guet and Stanton, 1974; Ptuskin and Khazan, 1975). For the diffusion model, the anisotropy in the disk is given by Eq. (2.11)

$$\delta = \frac{3D_d}{c}\frac{|\nabla N_{cr}|}{N_{cr}}, \tag{2.11}$$

where N_{cr} is determined from Eq. (3.29). Figure 11 shows two components of the anisotropy near the Sun along (δ_r) and across the disk radius δ_z, as a function of the ratio of the diffusion coefficients in the disk and in the halo, the halo dimension being $h = 15$ kpc (Ptuskin and Khazan, 1975). The radial component of the anisotropy is inversely proportional to the ratio D_h/D_d and decreases from $\delta_r \sim 8\times10^{-5}$ at $D_h/D_d \sim 1$ to $\delta_r \sim 10^{-5}$ at $D_h/D_d \sim 8$ (for energies $\epsilon_k \sim 1$–5 GeV/nucleon). The value of δ_r decreases also with the halo dimensional h for a

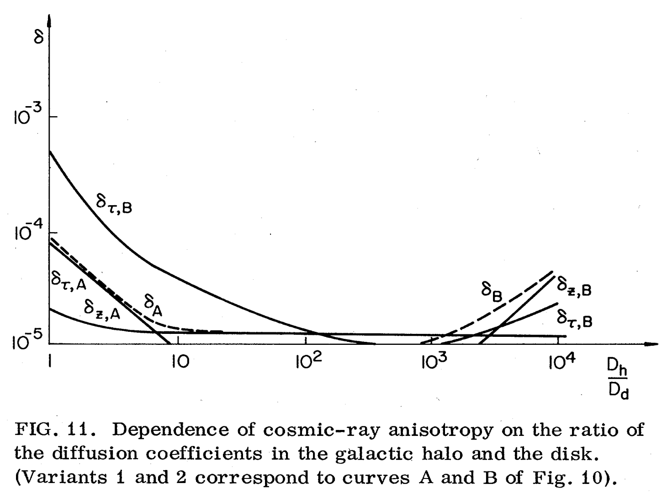

FIG. 11. Dependence of cosmic-ray anisotropy on the ratio of the diffusion coefficients in the galactic halo and the disk. (Variants 1 and 2 correspond to curves A and B of Fig. 10).

constant D_h/D_d. As to the value of the anisotropy component δ_z, its dependence on the model parameters is very weak (Ptuskin, 1974). For a fixed chemical composition of cosmic rays (more precisely, for a fixed value of the effective thickness x) the value of δ_z in the Galactic disk is determined only by the observer's position relative to the galactic symmetry plane $z = 0$ and by the gas density in the disk n_d (it is important that cosmic-ray sources be uniformly distributed in the disk):

$$\delta_z \simeq 3zn_d/x. \tag{3.44}$$

This formula is obtained analytically in the one-dimensional diffusion model approximation, but it is also confirmed for the three-dimensional case by numerical calculations. Thus the anisotropy component across the galactic disk is practically independent of the halo dimensions and of the ratio of the diffusion coefficients in the disk and in the halo (for a fixed thickness x traversed by nuclei). For the solar system $z \sim 10$ pc, therefore $\delta_z \sim 3\times10^{-5}$ (for particle energies $\epsilon_k \sim 1$–5 GeV/nucleon). Note that a possible asymmetry of the northern and southern parts of the halo was disregarded in the calculations. Anisotropy due to such an asymmetry does not exceed the value $\delta_z \sim 3\times10^{-5}$ at $\Delta h/h \lesssim 20$ percent where Δh is the difference in halo half-thickness in the north and in the south.

Summarizing, we should say that the minimum expected value of the cosmic-ray anisotropy in the Galaxy near the Sun is $\delta \sim 3\times10^{-5}$, at $\epsilon_k \sim 1$–5 GeV/nucleon, which is the only energy interval for which we can obtain reliable data at present on the diffusion coefficient from the analysis of the relative abundance of secondary nuclei. The quantity $\delta \sim 3\times10^{-5}$ is approximately one order of magnitude less than the observed upper limit on the anisotropy for particles of energy $E \sim 10^2$–10^3 GeV. However, the anisotropy apperars to grow with the energy. An indirect evidence of this is given, for example, by the energy dependence of the abundance of secondary nuclei which is associated with the increase of the diffusion coefficient with energy (see Sec. III.B). If we take the diffusion coefficient variation law $D \sim E^{\mu}$, then the data on the composition of secondary nuclei of energy $\epsilon_k \sim 1$–5 GeV/nucleon along with the data on cosmic-ray anisotropy up to energies 10^6 GeV/nucleon lead to the estimate $\mu < 0.35$ (Ptuskin, 1974). This does not contradict the data on the chemical composition of nuclei of energy $\epsilon \sim 50$–100 GeV/nucleon. Some additional remarks on cosmic-ray anisotropy at higher energies will be made in Sec. VI.

IV. THE COSMIC-RAY ELECTRON COMPONENT AND GALACTIC RADIOEMISSION [7]

A. Propagation of cosmic-ray electrons in the Galaxy

The energy losses of relativistic electrons in the galaxy play a very important role in the determination of their energy spectrum. For electron energies $E > 100$ MeV we may restrict ourselves to two main forms of losses, Compton and synchrotron (for more details see

[7]The authors would like to thank Dr. V. A. Dogel for the discussion of the material used in this section.

GS § 8), occurring due to relativistic electron scattering on photons and *tp* electron synchrotron radiation in the interstellar fields, respectively. If photons are distributed isotropically and the magnetic field is on the average also isotropic, the losses are given by the formula

$$-\frac{dE}{dt}=\frac{32\pi}{9}c\left(\frac{e^2}{mc}\right)^2\left(w_{\rm ph}+\frac{H^2}{8\pi}\right)\left(\frac{E}{mc^2}\right)^2 \qquad (4.1)$$

assuming that $mc^2 \ll E \ll mc^2(mc^2/\epsilon_{\rm ph})$, where m is the electron mass and $\epsilon_{\rm ph}$ is the mean energy of photons with energy density $w_{\rm ph}$. For an arbitrary quasihomogeneous magnetic field, the quantity $\frac{3}{2}H_\perp^2$ instead of the mean field squared H^2 enters in Eq. (4.1), where $H_\perp$ is the magnetic field component perpendicular to the electron velocity.

In more convenient units, Eq. (4.1) may be rewritten in the form

$$-\frac{dE}{dt}\equiv\beta E^2=8\times10^{-17}\left(w_{\rm ph}+6\times10^{11}\frac{H^2}{8\pi}\right)E^2({\rm GeV\ sec})^{-1} \qquad (4.2)$$

where $w_{\rm ph}$ is the energy density of electromagnetic radiation in eV/cm^3, H is the field intensity in gauss, and E is the electron energy in GeV.

The radiation field (photon background) in the Galaxy is due mainly to relict radiation with a temperature 2.7 °K($w_{\rm ph}$ = 0.25 eV/cm^3) and to star light (near the galactic plane $w_{\rm ph} \simeq 0.5$ eV/cm^3). The background infrared radiation, not yet reliably estimated, may also play some role. When all these sources are taken into account, the quantity $w_{\rm ph}$ can, in general, be estimated as $w_{\rm ph} \sim 1$ eV/cm^3. The value of the mean magnetic field in the interstellar medium is evidently $H \simeq 1-10.10^{-6}$ G. Thus the Compton and synchrotron losses are approximately the same, though if we accept the most probable values $w_{\rm ph} \sim 1$ eV/cm^3 and $H \sim 3\times10^{-6}$ G, the Compton losses dominate.

Equations (4.1) and (4.2) show that the energy losses of relativistic electrons increase with energy. The characteristic time of losses $T = E/(-dE/dt)$ is comparable to the characteristic time cosmic rays remain in the galactic disk, $T_{{\rm cr},d} \sim 3.10^6$ years, for a particle energy $E \sim 10^2$ GeV, and is comparable to the characteristic time $T_{{\rm cr},h} \sim 10^8$ years for halo models for $E \sim 3$ GeV. Therefore it is clear that the energy losses are really quite important for cosmic-ray electrons observed near the Earth with energies higher than several GeV. This means that the above discussed homogeneous and diffusion models of cosmic-ray propagation in the Galaxy will lead in many cases to different interpretations of the electron spectrum characteristics. The situation is similar to that described for radioactive nuclei, i.e., electrons of very high energies lose their energy so fast that they have no time to diffuse from the sources within the disk to the halo boundaries. As a result, particles of different energies occupy different energies occupy different volumes of the galactic region and therefore relativistic electron distribution in the Galaxy in the diffusion model with free particle exit at the boundaries is essentially inhomogeneous.

Analysis of relativistic electron propagation in the Galaxy and comparison of the results with the conclusions of the homogeneous model have been carried out by Syrovatskii, 1959; GS, § 14, 17; Jokipii and Meyer, 1968; Berkey and Shen, 1969; Jones, 1970; Webster, 1970; Bulanov *et al.*, 1972; Bulanov and Dogel, 1974.

The equation for the cosmic-ray electron concentration in the Galaxy within the homogeneous model has the form

$$\frac{N_e(E)}{T_{\rm cr}^{\rm (hom)}}+\frac{\partial}{\partial E}(-\beta E^2 N_e(E))=\overline{Q}_e(E) \qquad (4.3)$$

hence

$$N_e(E)=\int_E^\infty dE_2\overline{Q}_e(E_2)\exp\left(-\int_E^{E_2}\frac{dE_1}{\beta E_1^2 T_{\rm cr}^{\rm (hom)}(E_1)}\right). \qquad (4.4)$$

Here $T_{\rm cr}^{\rm (hom)}(E)$ is the characteristic time of electron leakage from the system in the homogeneous model. If the source spectrum and the leakage time are power-like, then $\overline{Q}_e \sim E^{-\gamma_0}$ and $T_{\rm cr}^{\rm (hom)}(E) \sim E^{-\mu}$; in the two limiting cases the spectrum $N_e(E)$ is

$$N_e(E) \sim E^{-\gamma_0-\mu}, \quad E \ll E_1, \qquad (4.5)$$

and

$$N_e(E) \propto E^{-\gamma_0-1}, \quad E \gg E_1 \qquad (4.6)$$

where the critical value of the energy E_1 is determined from the condition

$$\beta E_1 T_{\rm cr}^{\rm (hom)}(E_1)=1\,. \qquad (4.7)$$

For particles of relatively low energy $E \ll E_1$ the spectrum is due to the balance of particle generation in the sources and particle leakage from the Galaxy [see Eq. (4.5)]; the energy losses are not essential in this case. On the contrary, at energies $E \gg E_1$ losses dominate over leakage and the spectrum has the form (4.6).

In the energy range $E \sim E_1$ there is a "break" in the spectrum index, i.e., the index increases by $\Delta\gamma = 1-\mu$.

Measurements on the electron spectrum, according to different authors, give different electron spectrum indices from $\gamma_e \simeq 2.7$ to $\gamma_e \simeq 3.4$. However, most of the measurements are consistent with the assumption that the spectrum index does not change (within the accuracy $\Delta\gamma < 0.3$) in the energy range 5–500 GeV and perhaps even up to 10^3 GeV (Webber, 1973; Meyer, 1974[8]). Bearing in mind that the value of the parameter μ is within the limits $0 \leqslant \mu \lesssim 0.35$ (see Sec. III), we conclude that the electron spectrum has no break that corresponds to the requirement (4.7). This means that the time of cosmic-ray leakage from the Galaxy is either very small ($T_{\rm cr}^{\rm (hom)} < 10^6$ years, if $E_1 \gtrsim 500$ GeV and $\mu = 0$) or, just the opposite, comparatively large ($T_{\rm cr}^{\rm (hom)} \gtrsim 10^8$ years, if $E_1 \lesssim 5$ GeV). Since the measurements of the relative content of radioactive ^{10}Be, interpreted in the framework of the homogeneous model, permit the exclusion of the value $T_{\rm cr}^{\rm (hom)} \gtrsim 10^8$ years, we conclude that, for the homogeneous model, $T_{\rm cr}^{\rm (hom)} \lesssim 10^6$ years. If this time actually corresponded to the time of electron leakage from the system, obviously we could speak of the disk model

[8]See, however, Anand *et al.*, 1975, where this statement is disputed.

only. But as has been said, there are no grounds, generally speaking, for applying the homogeneous model to electrons.

Let us turn now to the diffusion model. The electron concentration is described here by Eq. (2.16), which in the stationary case and with account taken of Eq. (4.1) takes the form

$$-\operatorname{div}(D_e \nabla N_e) + \frac{\partial}{\partial E}(-\beta E^2 N_e) = Q_e(\vec{r}, E). \tag{4.8}$$

We shall consider the relativistic electron and proton diffusion coefficients to be the same at a given energy E and, therefore, we shall omit the index e in the diffusion coefficient.

If the electron sources are distributed uniformly in the disk and have a powerlike energy dependence, one can set

$$Q_e(\vec{r}, E) = \frac{K E^{-\gamma_0} \theta(b - |z|)\theta(R - r)}{2\pi R^2 b}, \tag{4.9}$$

where θ is a step function. If the diffusion coefficient has the form $D = D_0(E/E_0)^{\mu}$ and at the boundaries of the propagation region $N_e|_{\Sigma} = 0$, then (Bulanov and Dogel, 1974):

$$N_e = \frac{2KE^{-(\gamma_0+1)}}{\pi R^2 b(\gamma_0 - 1)\beta} \sum_{n=0}^{\infty} \frac{\sin[\pi(b/h)(n+\frac{1}{2})]}{n+\frac{1}{2}} \cos\pi\left[\frac{z}{h}\ (n+\tfrac{1}{2})\right] \times \sum_{m=1}^{\infty} \frac{J_0(\nu_m(r/R))}{\nu_m J_1(\nu_m)} \times {}_1F_1\left(1; \frac{\gamma_0 - \mu}{1-\mu}; -\left[\pi^2(n+\tfrac{1}{2})^2 + \frac{h^2}{R^2}\nu_m^2\right]\frac{D_0 E^{\mu-1}}{h^2(1-\mu)E_0^{\mu}\beta}\right) \tag{4.10}$$

where ${}_1F_1(\alpha;\beta;x)$ is the confluent hypergeometric function and it is assumed that $\mu < 1$.

By analogy with the case of propagation of radioactive nuclei one can obtain simple asymptotic expressions for N_e corresponding to three cases of relativistic electron spatial distribution in the Galaxy: (1) particles occupy all the halo; (2) particles flow out of the disk, but due to great energy losses they do not reach the halo boundaries; (3) particles lose their energy so fast that they practically do not leave the source region.

In what follows it is convenient to introduce the quantity

$$\lambda^2(E) = \int_E \frac{D(E^1)dE^1}{\beta E^2} = \frac{D_0 E^{\mu-1}}{|1-\mu| E_0^{\mu}\beta}, \tag{4.11}$$

$\lambda(E)$ being the mean distance traveled by the electron of energy E before it loses a considerable part (half) of its energy. From Eq. (4.10) it follows that the electron spectrum near the Earth ($z = 0, r = 10$ kpc) can be approximated by the power law $N \sim E^{-\gamma}$ and

$$\gamma = \gamma_0 + \mu \text{ at } \lambda(E) \gg h \quad (E \ll E_1), \tag{4.12}$$

$$\gamma = \gamma_0 + \frac{\mu+1}{2} \text{ at } b \ll \lambda(E) \ll h \quad (E_1 \ll E \ll E_2), \tag{4.13}$$

$$\gamma = \gamma_0 + 1 \text{ at } \lambda(E) \ll b \quad (E \gg E_2), \tag{4.14}$$

where the values of the energies E_1 and E_2 are determined from the conditions $\lambda(E_1) = h$ and $\lambda(E_2) = b$. Thus the electron spectrum near the Earth has two breaks, at energies E_1 and E_2, the index γ changing by the quantity $\Delta\gamma = (1-\mu)/2$ in each case (remember that all the above formulae are based on the assumption that $\mu < 1$).

The total steepening of the electron spectrum by $\Delta\gamma = 1 - \mu$ in the diffusion model occurs at two rather weakly pronounced breaks, whose absence or presence is difficult to establish accurately by means of present-day methods of observation. In this sense the diffusion model offers a wider choice of halo dimensions and, correspondingly, a wider choice of estimates as to the time of cosmic-ray leakage from the Galaxy, than the homogeneous model. Besides, even in the absence of breaks in the energy band 5 GeV $\lesssim E \lesssim$ 500 GeV in the diffusion model, the possibility of $E_1 < 5$ GeV, $E_2 > 500$ GeV is not excluded. These breaks correspond to the time $T_{cr} \sim h^2/2D \sim 10^8$ years and to large halo dimensions. This is a quite real possibility, since in the diffusion model the measurements of ^{10}Be content have not yet practically imposed limitations of T_{cr} (see Sec. III). We shall see that it is just this version that agrees best of all with the radio-astronomical data.

Note that actually the powerlike electron spectrum must have a cutoff at very high energies ($E > 10^3$ GeV) due to discreteness in the spatial source distribution (Shen, 1970). Discreteness has to manifest itself when particles turn out to be localized near the sources because of large energy losses, i.e., at $\lambda(E) \lesssim l$, where l is the mean distance between the sources (say, pulsars of supernova remnants). In particular, for the electrons observed near the Earth the spectrum should be cut off at an energy determined from the condition $\lambda(E) \sim L$, where L is the distance to the nearest source.

Part of the information about the model parameters, which can, in principle, be obtained from analysis of the electron spectrum observed near the Earth, is lost not only for lack of experimental accuracy but also because the spectrum index γ_0 in the sources is not known. From this point of view measurements of the intensity of the cosmic-ray positron component are very important.

The spectrum of particles produced is known in this case, since relativistic positrons appear during interaction of the cosmic-ray proton-nuclear component with the interstellar gas mainly owing to the decays $\pi^+ \to \mu^+ \to e^+$ and to other reactions. At energies higher than several GeV $\gamma_0 \simeq 2.7$. The available data on the abundance of positrons with energies higher than several GeV are insufficient to determine their spectrum. One can only estimate roughly the ratio of concentrations (Buffington *et al.*, 1974) $N_{e^+}/(N_{e^+} + N_{e^-}) \simeq 0.8 \pm 0.02$. Using the corresponding calculations of the secondary positron intensity and proceeding from the proton-component spectrum observed near the Earth (Ramaty and Lingenfelter, 1966; Perola *et al.*, 1967), it is possible to conclude that in the interstellar gas the cosmic-ray path length is $x = 3.5 \pm 1.5$ g/cm^2, which is in good agreement with the value of the mean thickness obtained from the measurements of the abundance of secondary nuclei.

B. Nonthermal galactic radioemission

Energy lost by relativistic electrons in the Galaxy due to synchrotron radiation is observable in the form of a general nonthermal radioemission of the Galaxy. The synchrotron radiation intensity I_ν at a frequency ν in the direction $\vec{l}$ from electrons of concentration $N_e(\vec{r}, E)$ located in a magnetic field H which is random in direction and constant in magnitude, is given by (GS, § 4; Ginzburg and Syrovatskii, 1969)

$$I_\nu = \int_{\vec{l}} \int_E d\vec{l}\, dE N_e(\vec{r}, E) p_\nu(E). \quad (4.15)$$

Here p_ν is the radiation intensity at the frequency ν of electrons with energy E. For ultra-relativistic electrons the function $p_\nu(E)$ has a maximum at the frequency

$$\nu \simeq 0.29 \frac{3eH}{4\pi mc}\left(\frac{E}{mc^2}\right)^2 = 1.2\times 10^6 H\left(\frac{E}{mc^2}\right)^2 \text{Hz}. \quad (4.16)$$

In Eq. (4.15) one integrates over $d\vec{l}$ along the line of sight.

In the investigation of nonthermal radioemission one should take into account that the contribution of the relict radiation (temperature 2.7°K) becomes considerable in the frequency range $\nu > 1400$ MHz; on the other hand, at $\nu < 50$ MHz it is necessary to take into account the absorption of radioemission in the ionized interstellar gas. Besides, in the general radioemission from the galactic disk at low galactic latitudes ($b < 5°$) the share of thermal radioemission is large. For this reason it is convenient to use measurements at higher latitudes for nonthermal radioemission. Even with all these circumstances in mind the interpretation of the galactic radio-map is rather difficult. This is connected first of all with the presence of inhomogeneities such as "spurs," "loops," "arcs" etc., whose nature is not yet clear enough (in this connection see, for example, Sofue *et al.* 1970), and secondly, with the somewhat indefinite contribution to the total radioemission from isotropic metagalactic components and individual discrete galactic radio sources (like old supernova remnants). For this reason, in the analysis of different models it is reasonable to use data on radioemission from those directions in which the distortion of the galactic radio background is minimal. Such characteristic directions are those toward "anticenter", toward "halo-minimum" and toward the "pole" (see Fig. 12). Calculations of radioemission must give absolut magnitudes of intensity in these directions and account for their basic features which are as follows:

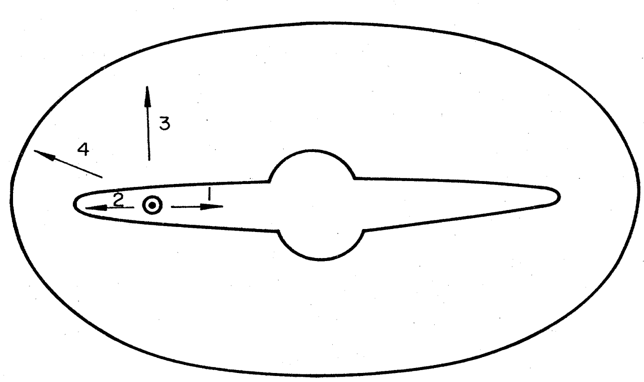

FIG. 12. Characteristic directions toward which galactic radioemission is observed: 1, center; 2, anticenter; 3, pole; 4, halo-minimum.

(1) The intensity of the galactic radio background decreases rather slowly with the increase of the galactic latitude b, i.e., of the angle between the galactic plane and the direction of observation.

2. The radioemission spectrum has a break in the frequency range of tens of MHz. The spectral index $\alpha = -d\lg I_\nu / d\lg_\nu$ changes from 0.3–0.6 in the region of lower frequencies (before the break) to 0.8–0.9 in the region of higher frequencies.

(3) The exponent α increases as the coordinate b increases. The difference in the values of α in the directions of small and large b is 0.1–0.2 (in the frequency region of tens of MHz).

For more detailed information on galactic radioemission see, for example, Baldwin (1967); Yates (1968); Landecker and Wielebinski (1970); Bridle (1967); Tokarev (1968); Krymkin (1971); Berkhuijsen (1971); Milogradov-Turin and Smith (1973); Webster (1974; 1975).

A consistent explanation of galactic radioemission has been possible up to now only in the framework of the diffusion model of relativistic electron propagation (Bulanov *et al.*, 1972, 1975; Bulanov and Dogel, 1974; the problem was formulated in GS § 17). To calculate the radiation intensity one should use Eq. (4.15) where the quantity $N_e(\vec{r}, E)$ is determined by Eq. (4.10). The corresponding cumbersome formulae are simplified considerably if the electron spectrum can be approximated by the power function [Eqs. (4.12)–(4.14)]. In this case in the anticenter direction one can obtain the following values of the radiation spectrum index α (assuming $\mu < 1$):

$$\alpha = \frac{\gamma_0 + \mu - 1}{1} \text{ at } \nu < \nu_1^A = \frac{3eH_A}{4\pi mc}\left(\frac{E_1}{mc^2}\right)^2, \quad (4.17)$$

$$\alpha = \frac{\gamma_0 + \mu - 1/2}{2} \text{ at } \nu_1^A \ll \nu \ll \nu_2^A = \frac{3eH_A}{4\pi mc}\left(\frac{E_2}{mc^2}\right)^2, \quad (4.18)$$

$$\alpha = \frac{\gamma}{2} \text{ at } \nu \gg \nu_2^A. \quad (4.19)$$

Here H_A is the mean value of the magnetic field in the galactic disk in the anticenter direction.

The frequency-dependent variations of the spectral index α in the pole direction (at $\mu < 1$) are

$$\alpha = \frac{\gamma_0 + \mu - 1}{2} \text{ at } \nu \ll \nu_1^p = \frac{3eH_p}{4\pi mc}\left(\frac{E_1}{mc^2}\right)^2, \quad (4.20)$$

$$\alpha = \frac{\gamma_0}{2} \text{ at } \nu \gg \nu_1^p. \quad (4.21)$$

Here H_p is the mean value of the magnetic field in the galactic pole direction; we recall that γ_0 is the exponent in the electron source differential spectrum [see Eq. (4.9)].

The change of the spectral index with frequency is different for different directions, owing to a nonuniform high-energy electron distribution in the Galaxy, for which reasons the integral $\int N(\vec{r}, E) d\vec{l}$ depends essentially on the choice of the direction $\vec{l}$ and the radiation spectrum has different shapes in different directions, even if the value of the magnetic field H is the same throughout the Galaxy. Note, for example, that it follows from

Eqs. (4.17), (4.19), and (4.20)–(4.21), that in the anticenter direction the radioemission spectrum has two breaks equal to $\Delta\alpha = (\mu - 1)/4$ each, and in the pole direction one break: $\Delta\alpha = (\mu - 1)/2$.

It should be observed, also, that when registering radiation in the anticenter direction one can obtain different forms of the spectra subject to the characteristics of the receiving antenna. If in the anticenter direction the angular width ϕ of the polar diagram of the antenna does not exceed tha angle $\phi = 2\arctan(b/(R-r))$, Eqs. (4.17)–(4.19) are valid, since only the electrons from the source region contribute to the registered radiation. In this same direction at $\phi > \arctan(h/(R-r))$ the radio spectrum will be analogous to the radiation spectrum in the halo direction, since in this case the radiation from the halo electrons dominates and the form of the spectrum is determined by relations (4.20) and (4.21). In the intermediate case the form of the spectrum depends on ϕ.

Without going into details of calculations and interpretation of the characteristic features of galactic radioemission, we present here only the main conclusions drawn in Bulanov *et al.* (1975) (for $\mu = 0$):

(1) The spectrum index of the electron sources in the Galaxy has the value $\gamma_0 = 2.2$.

(2) The value of the index $\gamma_e = 2.7$ observed near Earth at energies 5–50 GeV may be explained by the influence of synchrotron and Compton losses; these losses are responsible for the fact that with a disk distribution of sources and in the presence of a halo the spectrum has two breaks, at each of which the index changes by 1/2.

(3) The first break, at which the spectrum index changes from $\gamma_0 = 2.2$ to $\gamma = \gamma_0 + 1/2 = 2.7$ is located in the energy range 0.8–2.5 GeV.

(4) The second break, at which the index changes to $\gamma = \gamma_0 + 1 = 3.2$ is located in the region of several thousand GeV.

(5) The existing Galaxy radioemission can be explained within the considered model by the synchrotron radiation of relativistic electrons, models with a large halo ($h \sim 5-12$ kpc) giving the results closest to observations. The average field in the halo (subject to the contribution from galactic discrete sources and the role of the Metagalaxy) ranges in the interval $H = H_b = (4-6) \times 10^{-6}$ G and in the disk $H = H_A = (5-9) \times 10^{-6}$ G.

(6) For the models under consideration the cosmic-ray life-time turns out to be $T_{cr,D} \sim 10^8$ yr.

The parameters of the model can change somewhat if the energy dependence and coordinate dependence of the diffusion coefficient are introduced, but the main result, the presence of a clearly pronounced galactic radio and cosmic-ray halo, turns out to be rather firm. In this connection it may be instructive to consider the main arguments expressed lately against the existence of an extensive halo [in particular, in Razin (1971) and Legueux (1972), where the halo half-thickness was estimated as $h \sim 0.3 - 1$ kpc].

The first argument is connected with the presence of the abovementioned "spurs" and "arcs" etc. occupying a rather large part of the sky and making a considerable contribution to the observed radioemission. Most investigators regard these inhomogeneities as local. In this case the question arises whether or not it is possible to distinguish rather weak radioemission coming from the halo against the background of these bright inhomogeneities? As has been mentioned above, information about the halo still can be obtained by using observations in the directions where inhomogeneities are absent or rather small (anticenter, pole, etc). The main evidence for an extensive radio halo is a smooth variation of the radioemission intensity from the anticenter to the polar direction the minimum radiation is observed in the intermediate direction (halo-minimum). Alternative explanations of this fact can be connected with the presence of a very extensive metagalactic background or with an anomalously large radiation coming from a local quasispherical region with a very intense radioluminosity in the local arm; these explanations seem, however, improbable.[9]

The second argument against the existence of a considerable halo is derived from the small time of the cosmic-ray leakage out of the Galaxy ($T_{cr}^{(hom)} < 10^7$ yr) determined within the homogeneous model by the Galaxy's content of radioactive ^{10}Be. However, as has been discussed in detail in Sec. III, in a consistent calculation of the diffusion model one obtains an estimate for the diffusion time for cosmic-ray leakage of $T_{cr,D} \lesssim 3 \times 10^8$ years, which does not contradict the presence of an extensive halo.

The third argument is connected with the absence, in many cases, of a considerable radio halo in other galaxies. It should be taken into account here that different galaxies may have cosmic-ray sources of different intensity, and for the objects with weak sources the halo will be weak or absent altogether. Besides, to detect a radio halo one should observe at comparatively low frequencies (sse, for example, Ginzburg, 1967). This circumstance is connected, naturally, with a nonuniform distribution of electrons of different energies and may be connected, as well, with the decline of the magnetic field strength far from the galactic plane. For example, the whole halo of our Galaxy must radiate only at frequencies $\nu < 15$ MHz. Note in this connection that even the use of the wavelength 50 cm has led to the discovery of enormous halos in radio galaxies (Willis *et al.*, 1974) and a halo in galaxy NGC 4631 (private communication, J. H. Oort, 1974; R. D. Ekers and R. Soncisi, 1975).

V. COSMIC RAYS AND GAMMA-RAY ASTRONOMY

At present there exists only indirect information about cosmic rays (protons and nuclei) far from the Earth. This information is obtained for the most part from radio-astronomical data. These data permit us to find the form of a relativistic electron spectrum (i.e., the energy dependence of its intensity), but the concentration and the electron energy density can be determined only by making an additional assumption about the magnetic field

[9]The most recent estimation of the metagalactic radio background gives the effective temperature $T_{M_g} \sim 17°$ at $\nu = 178$ MHz (Dogel, 1975), whereas the general radiation from the halo-minimum direction corresponds to the temperature $T \simeq 80°$.

intensity in the radiating region. To find the concentration of all the cosmic rays N_{cr} it is necessary to assume additionally the form of the connection between N_{cr} and N_e (for more details see GS and Ginzburg, 1972; 1975). The only direct way of determining the concentration of the cosmic-ray proton–nuclear component far from the Earth consists in the use of gamma-ray astronomy. This is connected with the fact that when relativistic protons and nuclei collide with the nuclei of the interstellar gas, they produce various secondary particles which radiate gamma rays in the process of decay. The main role in this process is played by the π^0 mesons produced directly, but a contribution to gamma radiation is also made by the decay of Σ^0 hyperons and secondary π^0 mesons produced in the channels $K^\pm \to \pi^\pm + \pi^0$, $\Lambda \to n + \pi^0$ etc. As a result the intensity of gamma radiation $I_\gamma(E_\gamma)$ is determined by the product of the cosmic-ray intensity $I_{cr}(E)$ [or cosmic-ray concentration $N_{cr}(E)$] and the gas concentration n along the line of sight.

The radiating capacity of the unit volume q_γ may be represented in the form

$$q_\gamma(>E_\gamma) = n(\sigma I_{cr}), \qquad (5.1)$$

$$(\sigma I_{cr}) = \int_{E_\gamma}^{\infty}\int_{E}^{\infty} \sigma(E, E_\gamma) I_{cr}(E)\, dE\, dE_\gamma, \qquad (5.2)$$

where σ is the corresponding cross section of production of gamma quanta under the action of cosmic rays with intensity $I_{cr}(E)$. Calculations of the function (σI_{cr}) can be found in Stecker (1971) (see Fig. 13). For example, for galactic cosmic rays with an intensity equal to that observed near the Earth, $(\sigma I_{cr})_{E_\gamma > 100\,\mathrm{MeV}} \simeq 10^{-26}$ photons/sec. sr. at the energy $E_\gamma \geqslant 100$ MeV. The maximum of the differential spectrum of gamma radiation due to π^0-meson decay is observed at the energy $E_\gamma \simeq 67.5$ MeV. This feature leads to a flattening of the integral spectrum of gamma rays in the energy range $E_\gamma \simeq 10$–100 MeV and makes it possible to distinguish from among the general cosmic gamma radiation the component connected with the production and decay of π^0-mesons. Other possible mechanisms of high-energy gamma radiation (for example, the scattering of low-energy photons by ultrarelativistic cosmic-ray electrons) lead usually to steeper spectra.

The gamma-radiation flux from a discrete source located at some distance R is

$$F_\gamma(>E_\gamma) = \int_{\Omega_s}\int_{\vec{l}_s} q_\gamma\, d\vec{l}\, d\Omega \simeq (\sigma I_{cr}) \frac{N(V)}{R^2}$$

$$\simeq 5 \times 10^{23} (\sigma I_{cr}) \frac{\mathfrak{M}}{R^2}\ (\text{photons/cm}^2\,\text{sec}) \qquad (5.3)$$

where the integration is performed along the line of sight over the source region $\vec{l}_s$ and over the solid angle Ω_s at which the source is observed; $N(V) = nV$ is the total number of nuclei in the source of volume V and mean gas concentration n, $\mathfrak{M} \simeq 2.10^{-24} N(V)$ is the gas mass in the source (the chemical composition of the gas is considered to correspond to the universal abundances of the elements).

The intensity of the diffuse gamma radiation appearing in a continuous space distribution of radiation sources can be found by the formula

$$I_\gamma(>E_\gamma) = \int_{\vec{l}} q_\gamma\, d\vec{l} \simeq n(\sigma I_{cr}) L\ (\text{photons/cm}^2\,\text{sec sr}). \qquad (5.4)$$

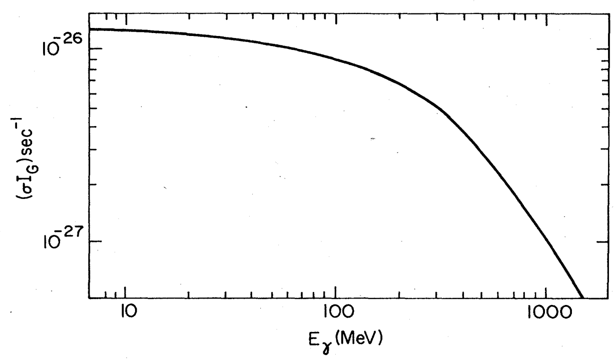

FIG. 13. Spectrum of generation of gamma quanta appearing resulting from interaction of cosmic rays with the interstellar gas (due to π^0-meson decay etc.).

The integration is performed here along the line of sight $\vec{l}$ throughout the radiating region, n is the mean gas concentration, and L is the characteristic dimension of the radiating region.

For a detailed account of experimental results of gamma-ray astronomy and their interpretation see Stecker (1971); Kraushaar *et al.* (1972); Ginzburg (1972); Galper *et al.* (1974); Fichtel *et al.* (1975); Stecker (1975); etc. Here we shall give only some examples.

In the general cosmic gamma radiation there exists an isotropic extragalactic component whose intensity is of the order of $I_\gamma(>100\ \mathrm{MeV}) = 2$–$3 \times 10^{-5}$ photons/cm² sec sr. This value makes it possible to obtain a limit for the cosmic-ray intensity and its spatial distribution in metagalactic space I_{Mg}. In particular, if $I_{Mg} = I_{cr}$, i.e., if the intensities in the Galaxy and in metagalactic space are equal, cosmic rays must be trapped in a region with the dimension $L \lesssim 50(10^{-5}/n_{Mg})$ Mpc, where n_{Mg} is the metagalactic gas concentration. This estimate argues against the metagalactic model of cosmic-ray origin (at $n_{Mg} \gtrsim 10^{-6}\ \mathrm{cm}^{-3}$), in the framework of which cosmic rays occupy the whole Metagalaxy with a constant intensity $I_{Mg} = I_{cr}$ (for values of cosmological distance $r = (\lambda - \lambda_0)/\lambda_0 \lesssim 1$).

For the determination of cosmic-ray intensity near the Galaxy another method, the measurement of a gamma flux from the Magellanic clouds, seems much more effective (Ginzburg, 1972). In this case Eq. (5.3) should be used. The overall gas mass in each of the Magellanic clouds can be determined with the aid of radio-astronomical methods (by observing the spectral line of neutral hydrogen $\lambda = 21$ cm). If the cosmic-ray intensities are equal in the Galaxy and in the Magellanic clouds, the fluxes of gamma radiation turn out to be $F_\gamma(>100\ \mathrm{MeV}) \simeq 2 \times 10^{-7}$ photons/cm² sec for the large Magellanic cloud, and $F_\gamma(>100\ \mathrm{MeV}) \simeq 10^{-7}$ photons/cm² sec for the Small Magellanic Cloud. Unfortunately, at the present level of sensibility of gamma-ray telescopes one can establish only upper boundaries for the flux from the Magellanic clouds $F_\gamma(>100\ \mathrm{MeV}) < 10^{-6}$ photons/cm² sec. If further measurements show that radiation fluxes from the Magellanic clouds are less than this value, it will mean that the cosmic-ray intensity outside the Galaxy is less than inside it.

Approximately the same consideration can be used in determining the cosmic-ray intensity outside the Galaxy by measuring gamma radiation from the galactic anticenter region (Dodds *et al.*, 1975).

For a consideration of the problem of cosmic-ray origin, measurements of gamma radiation from supernova remnants are very important. The determination of a gamma quantum flux from the extended remnant of the supernova Vela has made it possible to find the cosmic-ray energy stored in this remnant (Thompson *et al.*, 1974). The estimate $W_{cr} \sim 3.10^{50}$erg agrees well with the Galactic theory of cosmic-ray origin, in which the main relativistic particle sources are supernovae (for more detail see GS§11 and Ginzburg, 1975).

In conclusion we should like to mention some interesting results obtained from measurements of galactic gamma radiation with $E_\gamma \simeq 100$ MeV in the direction of galactic center (these measurements are discussed in detail in Fichtel *et al.*, 1975). The radiation turns out to originate in the region of the galactic disk. For the most part it is concentrated in latitude within the limits $-10° < b < 10°$ and in longitude from $l = 335°$ to $l = 40°$. The distribution in longitude is shown in Fig. 14. A heightened radiation intensity is observed from the region of the galactic center and from along the galactic arms. The latter can be explained by the fact that the product of the gas concentration and the cosmic-ray intensity inside the arms exceeds by an order of magnitude or even more the corresponding product in the space between the arms. However it is still not clear enough how the cosmic-ray intensity I_{cr} is distributed. For example, it is possible that in the galactic disk $I_{cr} \simeq$const and the gas concentration in the arms is much higher than between the arms (chiefly, owing to molecular hydrogen). From the analysis carried out in Stecker *et al.* (1975) for the region $0° \leq l \leq 180°$ it follows that the cosmic-ray intensity in the Galaxy increases comparatively little in the central region direction (in general, it is approximately double that observed near the Earth), and the enhanced gamma radiation is explained by an enormous ring of molecular hydrogen clouds at the distance of about 5 kpc from the galactic center.

Gamma-ray astronomy is undoubtedly a most promising field and its development brings cosmic ray astrophysics to a new stage. Specifically, the possibility is now before us to obtain more or less direct information about the cosmic-ray proton–nuclear component in various distant regions. Note in this connection that the diffusion approximation used above and its concrete application to the Galaxy describes a certain picture "in large," for large regions of cosmic space, or in other words, gives averaged characteristics. This is done on the assumption that cosmic-ray mixing in the Galaxy is effective for a period of the order of their lifetime T_{cr}. But of course there is no reason to consider the cosmic-ray distribution in the Galaxy to be actually strictly uniform. Moreover, in the framework of galactic models, in which cosmic-ray sources are within the Galaxy, it is quite natural to expect a somewhat larger cosmic-ray concentration near these sources (e.g., near supernova remnants). However the question of the time and the character of dissipation of cosmic-ray density inhomogeneities is a separate and quite interesting problem. We may hope that gamma-ray astronomy (along with radio astronomy in application to the electron component) will make it possible to investigate this problem. Now we would like to emphasize particularly that a certain inhomogeneity in the cosmic-ray density distribution in the Galaxy, with a possibly enhanced cosmic-ray concentration in the spiral arms, near supernova remnants, etc. does not in the least contradict the above galactic models with a large halo and an effective cosmic-ray mixing in the Galaxy as a whole.

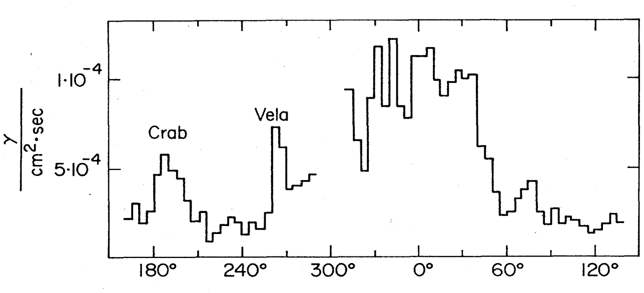

FIG. 14. Longitude intensity distribution for gamma radiation from the galactic plane ($-10° \leq b \leq 10°$) at $E_\gamma \geq 100$ MeV.

VI. ON THE CHARACTER OF COSMIC-RAY PROPAGATION IN THE GALAXY.

A. Cosmic-ray propagation in the interstellar magnetic fields

At the beginning of Sec. II.C we mentioned briefly some possible physical mechanisms for producing isotropy and cosmic-ray effective trapping in the Galaxy. Below we shall discuss this problem in somewhat more detail.

When studying the propagation of relativistic particles in the interstellar medium it is important to know the structure of the magnetic field which governs the cosmic-ray motion. The general large-scale field of the galactic disk has apparently an ordered spiral form and is extended along the galactic plane. A random component is superimposed upon this ordered field. The largest irregularities are about 100 pc. According to Wilkinson and Smith (1974) the random magnetic field in the vicinity of the solar system (at distances of several hundred parsecs) is 0.5–1 of the mean ordered field, the characteristic dimension of irregularities amounting to 10–50 pc. Such a structure is possibly connected with turbulent motions of the interstellar gas or dense clouds *HI*. The strength of the field is $1-10\times10^{-6}$ G.

It is quite natural to suppose that the magnetic field irregularities have an extensive spectrum in scale-length $l = 2\pi/k$. The main part of the turbulence energy is concentrated in the irregularity scale of dozens of parsecs and the spectrum decreases for smaller irregularities or, correspondingly, for larger values of the wave number k. For the scale of dozens of parsecs $H_\sim \sim H$ where H is the mean field and $H_\sim$ is the random component. Indirect data on the spectrum in the region of comparatively small irregularity scales ($l \sim 10^{11}$ cm) can be obtained from pulsar scintillation data (see, for example Rickett, 1970), which enable us to establish directly the scale $l = 10^{11.5 \pm 1.5}$ cm and the electron density fluctuations $\langle 4n_e^2\rangle^{1/2} = 10^{-4\pm1}$ cm^{-3}; the latter, in their turn, are clearly connected with the magnetic field fluctuations (Sheuer and Tsytovich, 1970). For example, for magnetosonic waves $\langle\Delta n_e^2\rangle/n_e^2 \simeq \sin^2\alpha(H_\sim^2/H^2)$ if the waves propagate at an angle α to the field $\vec{H}$. There are indications that fluctuations with characteristic dimensions $l \sim 10^{14}-10^{15}$ cm also exist in the interstellar medium (Shishov, 1973). The question of describing the whole irregularity spectrum from $l\sim 10^{10}$ cm to $l \sim 10^{20}$ cm by some universal law remains open and practically not investigated. On the one hand, it is hard to imagine that in such a large interval of the wave numbers k the fluctuations may have a common spectrum law, since the characteristic nonlinear processes and the wave damping mechanisms forming the spectrum are different for different k. On the other hand, the experimental data can be described approximately by one simple spectrum $W_k \sim 1/k^{1.5-2}$ where W_k is a spectral fluctuation density such that $\int W_k\, dk$ gives the total energy density of the fluctuating magnetic field. Such a spectrum would explain well the cosmic-ray diffusion in the Galaxy up to the energies $E\sim 10^6$ GeV (for more details see below).

Propagation of charged relativistic particles in interstellar magnetic fields is of a rather complicated character. It may be considered as a superposition of adiabatic motion along a large-scale field (where the dimension of field inhomogeneities is much larger than the particle gyroradius) and pitch-angle scattering in interactions with small-scale magnetic and electric fields. The stochastic structure ("tangling") of a large-scale field can, in principle, lead to an effective cosmic-ray diffusion (GS, §10). To provide a characteristic relativistic particle diffusion coefficient $D\sim 10^{28}-10^{29}$ cm^2/sec, inhomogeneities (or the distance between the "magnetic clouds") with a characteristic dimension of the order of a parsec must exist. However, as has been mentioned above, the energy of a random magnetic field in the Galaxy is stored mainly on a larger scale by about an order of magnitude. In general, the theory of cosmic-ray "mixing" due to a large-scale (in comparison with the particle gyroradius) turbulence of plasma with a frozen-in magnetic field remains at a qualitative level. Exceptions are the works investigating random wandering of individual lines of force in the galactic disk (Jokipii and Parker, 1969; Jones 1971; Jokippi, 1973). In these studies it was found that when moving along the magnetic field's lines of force the particles can be displaced considerably relative to the mean ordered field directed along the disk, and thus an effective cosmic-ray transfer takes place across the galactic disk. This result is in agreement with what has been said in Sec. IV.D.

Let us now consider the influence of a small-scale turbulent field. The theory of relativistic particle interactions with such a turbulent magnetic field has been developed rather thoroughly (mainly in application to the problems of cosmic-ray propagation in the solar wind; see, for example, Jokipii, 1971; Toptygin 1973). The energy of the fluctuating field is usually considered to be much lower than the energy of the mean field $H_\sim^2/8\pi \ll H^2/8\pi$. This condition permits the use of perturbation theory. Numerical calculations of particle interaction with individual strong inhomogeneities ($H_\sim \sim H$) of a given shape have also been carried out (Dorman and Sergeev, 1975).

It is convenient to consider particle scattering on small-scale turbulent pulsations of small-amplitude magnetic and electric fields from a general point of view as an interaction between particles and waves with random phases in the framework of a weakly turbulent plasma theory (see Ginsburg *et al.*, 1973, and the literature cited there). In this case it is easy to take into account interaction with all types of waves which can propagate in the magnetoactive interstellar plasma. Quasilinear approximation is used here, i.e., when calculating effective frequencies of particle collisions with turbulent pulsations, only the first term linear in the wave energy density is taken into account (see, for example, the book by Tsytovich, 1971). The interaction of particles with waves is of a resonant character. So, magnetohydrodynamic (mhd) waves scatter particles particularly effectively if the wavelength coincides with the particle Larmor radius. More precisely, the resonance condition for particles with the velocity $\vec{v}$ has the form

$$\omega(\vec{k}) - kv\cos\alpha\cos\theta = \pm\Omega \qquad (6.1)$$

or approximately

$$k\cos\alpha = ZeH/|\vec{p}|c\cos\theta\ , \qquad (6.2)$$

where $\omega(\vec{k})$ is the wave frequency, $\vec{p}$ is the momentum of the particle, $\Omega = (eH/Mc)(Mc^2/E)$ is the relativistic particle gyrofrequency, θ is the angle between the particle velocity vector $\vec{v}$ and the magnetic field, and α is the angle between the wave vector and the magnetic field; besides, the characteristic phase velocity of mhd waves $\omega(\vec{k})/k \sim v_a = H/\sqrt{4\pi\rho}$ (where ρ is the density of ionized gas) is assumed to be much smaller than the particle velocity (practically, under typical conditions of the interstellar medium, $v_a/c\sim 10^{-3}$).

The frequency of relativistic particle collisions ν_m with the mhd pulsations can be estimated by the formula

$$\nu_m \sim \Omega_H \frac{Mc}{p}\,\frac{k_r W_{k_r}}{H^2} \qquad (6.3)$$

where the resonance value of the wave number is

$$k_r = ZeH/pc$$

The quantity ν_m gives the inverse time for which a particle scatters at an angle of the order of unity under interaction with mhd waves.

Particles are scattered by waves also in the case when the Larmor radius is much larger than the wavelength, i.e., when conditions (6.1) and (6.2) are violated. The resonance is realized on high harmonics; condition (6.1) is replaced by Čerencov's condition

$$\omega(\vec{k}) = \vec{k}\vec{v}\ . \qquad (6.4)$$

The effective scattering frequency takes the form

$$\nu_m^* \sim \Omega_H \frac{Mc}{p} k_r \int_{k_r} \frac{W_k}{k}\,dk\ ,\ k_r = \frac{zeH}{pc}\ . \qquad (6.5)$$

For high-frequency whistler waves and Langmuir waves the interaction is also due to the Čerenkov resonance (5.4). The effective collision frequencies are, respectively, equal to

$$\nu_w \sim \Omega_H \left(\frac{Mc^2}{E}\right)^2 \frac{\Omega_H}{w(k)} \frac{W^w}{H^2} \tag{6.6}$$

and

$$\nu_l \sim \Omega_H \left(\frac{Mc}{p}\right)^2 \frac{v_a}{c} \left(\frac{m}{M}\right)^{1/2} \frac{W^l}{H^2}, \tag{6.7}$$

where W^w and W^l are the energy densities of whistlers and Langmuir waves.

The relativistic particle scattering in angles leads to a spatial diffusion with the coefficient $D \sim c^2/\nu$. From estimates (6.5), (6.6), and (6.7) it follows that under interaction of relativistic particles with waves the Čerencov resonance gives rise to a strong energy dependence of the diffusion coefficient $D \sim E^2 (E = pc)$, which does not correspond to $D \sim E^\mu$, obtained for cosmic rays, where $\mu < 0.4$. In the case of the cyclotron resonance (6.2) a weak E-dependence of D may be established by an appropriate choice of the wave spectrum. In fact, from Eq. (6.3) it follows that at $W_k \sim 1/k^{2-\mu}$, $D \sim E^\mu (E = pc)$.

If we assume that $D \sim 10^{28}$ cm²/sec for particle energies $E_k \sim 1$ GeV, and D depends on the energy as $D \sim E^{0.2}$ up to 3.10^6 GeV, then the irregularity spectrum must have the form $W_k \sim 1/k^{1.8}$. The irregularities interacting resonantly with particles of energy $E_k \sim 1$ GeV have the dimension $l \sim 5.10^{12}$ cm, and the turbulent field energy density in this region is characterized by the quantity $H_\sim^2/H^2 \sim 10^{-6}$. Particles of energy $E \sim 10^6$ GeV interact with the waves $l \sim 5.10^{18}$ cm and $H_\sim^2/H^2 \sim 0.1$. These values agree satisfactorily with the observations of galactic magnetic field irregularities.

Thus, by choosing a definite spectrum of magnetic irregularities, one can provide the necessary cosmic-ray diffusion in the Galaxy. However, the question of whether or not the necessary spectrum of inhomogeneities actually exists in the interstellar medium is far from being clear. Observational data are still very poor. On the otherhand, the simplest assumption—that the spectrum of irregularities is a superposition of linear mhd waves of small amplitude—leads to too great a power demand from the wave source if mhd wave damping on neutral atoms is taken into account. Therefore it is possible, though not yet proved, that the necessary spectrum of inhomogeneities consists of nonlinear waves for which the damping is weaker. For example, the system of magnetic field discontinuities gives the necessary spectrum of the type $W_k \sim 1/k^2$. If the condition $H_\sim^2/H^2 \ll 1$ is valid, Eq. (6.2) holds as before.

Additional important information on the character of cosmic-ray propagation can be obtained from measurements of the spectrum and anisotropy of particles of the highest energies from 10^5 GeV to 10^{11} GeV. The observed spectrum steepening at the energy $E \sim 3 \times 10^6$ GeV is usually associated with the fact that effectiveness of the trapping of particles of higher energy in the Galaxy falls sharply. This may be due to the change from the diffusive character of cosmic-ray propagation at energies $E < 3 \times 10^6$ GeV to a drift motion across the mean ordered magnetic field at $E > 3 \times 10^6$ GeV (Syrovatskii, 1971), or due to the flattening of the spectrum of irregularities which scatter particles in a resonant way [in Bell *et al.* (1973) see calculations of the high-energy cosmic-ray scattering on irregularities of the "magnetic cloud" type]. Recently measurements of the anisotropy of cosmic rays with energies $E > 2 \times 10^{10}$ GeV have appeared (Krasilnikov *et al.*, 1974). Interpretation of all the data on anisotropy and spectra of cosmic rays of superhigh energies leads, according to Hillas and Ouldridge (1975), to the conclusion that our Galaxy has a large halo, whose magnetic field is capable of trapping particles up to 10^{11} GeV (see, however, Kiraly *et al.*, 1975).

B. On collective (plasma) effects

Cosmic rays may be considered as a relativistic gas of charged particles. Therefore, apart from the effects of one-particle scattering on a given wave spectrum or wandering in a given magnetic field, there appear various collective effects attributable to them.

These effects and the corresponding problems are discussed in papers containing also review material (Parker, 1969, 1971; Kaplan and Tsytovich, 1972; Ginzburg *et al.*, 1973; Wentzel, 1974). A complete presentation of these interesting questions of cosmic-ray astrophysics would require an additional large review; therefore we shall restrict ourselves only to listing some of the considered problems.

A directed cosmic-ray gas bulk motion in the interstellar plasma induces a stream (beam) instability, oscillation excitation, particle scattering on these oscillations, and relaxation of the relativistic particle distribution function. Thus plasma effects could, in principle, account for cosmic-ray mixing and isotropization in the Galaxy. At present growth rates of different types of waves in cosmic plasma due to cosmic-ray flux instabilities have been found. Mhd waves have proved to be the most effective. However wave damping in the interstellar medium prevents cosmic rays with energies higher than tens of GeV from exciting oscillations. For particles with energies of several GeV the effects of mhd waves are rather important. It is more difficult to solve a self-consistent problem in which cosmic rays generate waves and are scattered by them. In this case it is necessary to take into account nonlinear effects of interaction between waves. In the investigated cases an anisotropy in a wide energy range has not yet been explained.

Interesting phenomena arise in the study of the evolution of spatially inhomogeneous low-energy cosmic-ray distributions, where particles can effectively generate mhd waves and be scattered by them. In this case the front of relativistic particles moves along the magnetic field at the Alfvén velocity. The cosmic-ray energy decreases adiabatically, is transferred to the mhd waves, and when the latter damp, it is removed by the interstellar gas. Large-scale effects of the relativistic gas upon the cosmic plasma can be described by hydrodynamic equations taking into account the momentum and energy transfer from cosmic rays to the background plasma. As a result, cosmic rays can regulate the interstellar gas motion and even induce such phenomena as galactic wind (Ipavich, 1975).

Another type of instability connected with cosmic rays manifests itself when the galactic gravitational field is taken into account. We assume that in the Galaxy as a whole there exists a state of equilibrium between the relativistic cosmic-ray gas, the interstellar gas, the magnetic field, and the stellar gravitational field. This equilibrium, however, turns out to be unstable[10] and its violation causes the effective flow of cosmic rays to the galactic disk's boundaries. The magnetic field at disk surface forms typical loops which are then broken by relativistic particles. This mechanism acts like a valve regulating the cosmic-ray pressure in the galactic disk when the magnetic field of the disk has a closed structure. At present only the instability increase rate in such a process has been calculated (it is about 10^{-7} years^{-1}; a characteristic scale of the resulting magnetic field perturbations is nearly 100 pc (Parker, 1969).

We think that the examples presented are sufficient to justify the statement that cosmic rays are an important dynamical factor in the interstellar medium. The wide range of problems they pose must be the object of further investigations.

VII. CONCLUDING REMARKS

The galactic model for the origin of most cosmic rays observed near the Earth, the main sources of which are supernovae and the trapping region of which is a large halo, was developed as far back as 1953. The state of the problem of cosmic-ray origin as it was in 1963–1964 was discussed in detail in the book by Ginzburg and Syrovatskii (1964), where preference was also given to the abovementioned galactic model. Since that time many new data have been obtained, naturally, and high-energy astrophysics as a whole has made great progress. Nevertheless, as has been emphasized in the introduction to this paper, in 1975 discussions are still going on concerning such fundamental problems as the role of metagalactic cosmic rays, the shape and even the very existence of a radio-halo, the characteristic cosmic-ray age T_{cr} in the Galaxy, etc. In particular, the opinion is widespread now that $T_{cr} \sim T_{cr,d} \sim 3\times10^6$ years, an age which corresponds to the disk model.

The aim that the authors have pursued in their recent work was to reevaluate the situation. Of course, we have tried to present a balanced picture, but it cannot be denied that the old affection for the halo model has played its role here. Warning the reader about this, we hope at the same time that we have presented the material clearly enough and in enough detail for the reader to form his own judgment on that score.

Meanwhile our opinion is that the estimate $T_{cr} \lesssim 3\times10^6 y$ is not supported by experimental data and, on the contrary, a "large" $T_{cr} \sim 10^8$ yr is more probable. Specifically, it is just the latter estimate that is in agreement with radio observations favoring the fact that the Galaxy has a large and rather luminous radio halo. The corresponding data on the electron component and galactic radioemission need, however, specification and confirmation. As far as the information on the amount of ^{10}Be nuclei is concerned, it does not yet contradict the "large" age and, according to the latest data (Garcia-Munoz *et al.*, 1975) probably even confirms this as a choice. Because the question of establishing the age T_{cr} on the basis of investigation of the cosmic-ray chemical composition has been repeatedly, and often incorrectly, discussed in the literature, we have considered this problem especially thoroughly (Sec. III). As to a number of other aspects, important in the framework of any galactic model of cosmic-ray origin (such as the basic sources and energy balance, the diffusion mechanism and cosmic-ray isotropization, the role of gamma-ray astronomical observations), they were treated more briefly or in a few cases were hardly even considered. The latter also refers to the analysis of metagalactic models of cosmic-ray origin. We should note that apart from the obvious limitation of space, there was no particular reason to present certain material since we could not say anything new (this concerns, for instance, the problem of supernovae as the basic sources of galactic cosmic rays and the criticism of metagalactic models; see Ginzburg, 1975).

There cannot be any doubt that comparatively slow progress in the investigation of a number of problems of high-energy astrophysics has been caused by difficulties in the experiments. An isotropic analysis of the primary cosmic-ray composition and, specifically, measurement of the amount of ^{10}Be nuclei at different energies may serve as a striking example. But now, after numerous attempts, the first and apparently decisive step in this direction has been taken. Obvious progress in the study of the electron component has been made of late and gamma-ray astronomical measurements have become a reality. The determination of the positron component spectrum is not far off.

In conclusion it seems to us that the hope is justified that the abovementioned uncertainties as to the choice of a sufficiently firm model of cosmic-ray origin will be eliminated in the very near future. It should be noted, however, that similar optimistic predictions were made earlier too, and they did not always fully come true. But if it is really difficult to predict reliably the character and the rate of further development, it is quite possible to ascertain the progress already made and the existence of quite definite and clearly formulated problems; and it is also not hard to believe that these problems will be solved in the foreseeable future.

AUTHOR'S NOTE (NOVEMBER 26, 1975)

In August 1975 the 14th International Cosmic-Ray Conference took place in Munich. The paper by the present authors (Ginzburg and Ptuskin, 1975) was also submitted to this conference. Its Proceedings contain 9 volumes (besides 2 volumes for rapporteur papers and index) including 2 volumes devoted to the problem of the origin of cosmic rays. These volumes are rich in material which obviously could not be dealt with in the present paper. This material, however, does not seem to the authors to call for any basic changes in the arguments presented here. Moreover we did see some papers submitted to the Conference as preprints and have taken them into account.

[10]Instability is due to the rising of the magnetic field and the cosmic rays and the sinking of the interstellar gas in the galactic disk.

ACKNOWLEDGMENT

The authors would like to thank cordially, unknown referees and Mrs. K. Friedman for their remarks which helped us to improve the text.

REFERENCES

Allan, H. R., 1972, Astrophys. Lett. **12**, 237.
Anand, K. C., R. P. Daniel, and A. A. Stephens, 1975, Astrophys. Space Sci. **13**, 169.
Apparao, K. M. V., 1973, in *Proceedings of the XIII International Conference on Cosmic Rays, Denver* (University of Denver, Denver), **1**, 126.
Arnett, W. D., and D. N. Schramm, 1973, Astrophys. J. **184**, L47.
Baldwin, I. E., 1967, in IAU Symposium No. 31, edited by H. van Woerden (Academic, London).
Bell, M. C., J. Kota, and A. W. Wolfendale, 1973, in *Proceedings of the XIII International Conference on Cosmic Rays, Denver* (University of Denver, Denver), Vol. 1, p. 390.
Berkey, G. B., and C. S. Shen, 1969, Phys. Rev. **188**, 1994.
Berkhuijsen, E. M., 1971, Astron. Astrophys. **14**, 359.
Blanford, G. E., M. W. Freidlander, J. Klarmann, R. M. Walker, and J. P. Wefel, 1973, Phys. Rev. D **8**, 1722.
Bridle, A. H., 1967, Mon. Not. R. Astron. Soc. **136**, 219.
Buffington, A., C. D. Orth, and G. F. Smoot, 1974, Phys. Rev. Lett. **33**, 34.
Bulanov, S. V., and V. A. Dogel, 1974, Astrophys. Space Sci. **28**, 305.
Bulanov, S. V., V. A. Dogel, and S. I. Syrovatskii, 1972a, Kosm. Issled. **10**, 532.
Bulanov, S. V., V. A. Dogel, and S. I. Syrovatskii, 1972b, Kosm. Issled. **10**, 721.
Bulanov, S. V., V. A. Dogel, and S. I. Syrovatskii, 1975a, Kosm, Issled. (in press).
Bulanov, S. V., V. A. Dogel, and S. I. Syrovatskii, 1975b, in *Proceedings of the XIV International Conference on Cosmic Rays, Munich*, Vol. 2, p. 100.
Burbidge, G., 1974, Philos, Trans. R. Soc. Lond. **A277**, 481.
Daniel, R. R., and A. A. Stephens, 1970, Space Sci. Rev. **10**, 599.
Davis, L., 1954, Phys. Rev. **96**, 743.
Davis, L., 1960, in *Proceedings of the X International Conference on Cosmic Rays, Moscow*, Vol. 3, p. 220.
Dickinson, G. J., and J. L. Osborne, 1974, J. Phys. A **7**, 728.
Dodds, D., A. W. Strong, and A. V. Wolfendale, 1975, Mon. Not. R. Astron. Soc. **171**, 569.
Dogel, V. A., 1975, Astron. Zh. (in press).
Dorman, L. I., and A. V. Sergeev, 1975, in *Proceedings of the XIV International Conference on Cosmic Rays, Munich*, Vol. 3, p. 928.
Fermi, E., 1949, Phys. Rev. **75**, 1169.
Fichtel, C. E., R. C. Hartman, D. A. Kniffen, D. J. Thompson, G. F. Bignami, H. Ogelman, M. E. Özel, and T. Tümer, 1975, Astrophys. J. **198**, 163.
Fowler, P. H., 1973, in *Proceedings of the XIII International Conference on Cosmic Rays, Denver* (University of Denver, Denver), Vol. 5, p. 3627.
Freier, X., *et al.*, in *Proceedings of the XIV International Conference on Cosmic Rays, Munich*, Vol. 1, p. 425.
de Freitas, Pacheco J. A., 1971, Astron. Astrophys. **13**, 58.
Gaisser, T. K., and E. H. Levy, 1974, Phys. Rev. D **10**, 1731.
Galper, A. M., V. G. Kirrilov-Ugrumov, and B. I. Luchkov, 1974, Usp. Fiz. Nauk **112**, 491 [Sov. Phys.—Usp. **17**, 186].
Garcia-Munoz, M., G. M. Mason, and J. A. Simpson, 1975, in *Proceedings of the XIV International Conference on Cosmic Rays, Munich*, Vol. 1, p. 331; see also, 1975, Astrophys. J. Lett. **201**, L141, L145.
Getmantzev, G. G., 1968, Astron. Zh. **39**, 607 [Sov. Astron. **6**, 477].
Ginzburg, V. L., 1965, Astron. Zh. **42**, 1129 [Sov. Astron. **9**, 877].
Ginzburg, V. L., 1967, in IAU Symposium No. 31, edited by H. van Woerden (Academic, London).
Ginzburg, V. L., 1972, Usp. Fiz. Nauk **108**, 273 [Sov. Phys. Usp. **15**, 626 (1973)].
Ginzburg, V. L., 1972, Nat. Phys. Sci. **239**, 8.
Ginzburg, V. L., 1974, Philos. Trans. R. Soc. Lond. **A277**, 463.
Ginzburg, V. L., and V. S. Ptuskin, 1975, in *Proceedings of the XIV International Conference on Cosmic Rays, Munich*, Vol. 2, p. 265.
Ginzburg, V. L., V. S. Ptuskin, and V. N. Tsytovich, 1973, Astrophys. Space Sci. **21**, 13.
Ginzburg, V. L., and S. I. Syrovatskii, 1964, *The Origin of Cosmic Rays* (Pergamon, London and New York).
Ginzburg, V. L., and S. I. Syrovatskii, 1965, Annu. Rev. Astron. Astrophys. **3**, 297.
Ginzburg, V. L., and S. I. Syrovatskii, 1969, Annu. Rev. Astron. Astrophys. **7**, 375.
Ginzburg, V. L., and S. I. Syrovatskii, 1971, in *Proceedings of the XII International Conference on Cosmic Rays, Hobart* (University of Tasmania, Hobart), Vol. 1, p. 53.
Gleeson, L. J., and W. I. Axford, 1968, Astrophys. Space Sci. **2**, 431.
Le Guet, F., and M. Stanton, 1974, Astron. Astrophys. **35**, 165.
Havnes, O., 1971, Nature **229**, 548.
Hayakawa, S., 1969, *Cosmic Ray Physics* (Wiley—Interscience, New York).
Higdon, J. C., and R. E. Lingenfelter, 1975, Astrophys. J. **198**, L17.
Hillas, A. M., 1974, Philos. Trans. R. Soc. Lond. **A277**, 413.
Hillas, A. M., and M. Ouldridge, 1975, Nature **253**, 609.
Hristiansen, G. B., 1974, *Cosmic Rays of Superhigh Energy* (in Russian) (Moscow State University, Moscow).
Ipavich, F. M., 1975, Astrophys. J. **196**, 107.
Jokippi, J. R., 1971, Rev. Geophys. Space Phys. **9**, 27.
Jokipii, J. R., 1973, Astrophys. J. **183**, 1029.
Jokippi, J. R., and P. Meyer, 1968, Phys. Rev. Lett. **20**, 752.
Jokippi, J. R., and E. N. Parker, 1969, Astrophys. J. **155**, 799.
Jones, F. C., 1970, Phys. Rev. Lett. **25**, 1534.
Jones, F. C., 1971, Astrophys. J. **169**, 477.
Juilusson, E., 1974, Astrophys. J. **191**, 331.
Kaplan, S. A., and V. N. Tsytovich, 1972, *Plasma Astrophysics* (Nauka, Moscow) [Pergamon, Oxford and New York (1973)].
Kiraly, P., J. L. Osborne, M. White, and A. W. Wolfendale, 1975, Nature **255**, 619.
Krasilnikov, D. D., A. I. Kuzmin, J. Linsley, V. A. Orlov, R. J. O. Reid, A. A. Watson, and J. C. Wilson, 1974, J. Phys. A **7**, L176.
Kraushaar, W. L., G. W. Clark, G. P. Garmire, R. Borken, P. Higbie, C. Leong, and T. Thorsos, 1972, Astrophys. J. 177, 341; Erratum **186**, 401.
Kristiansson, K., 1971, Astrophys. Space Sci. **16**, 405.
Krymkin, V. V., 1971, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **14**, 199.
Landecker, T. L., and R. Weilebinski, 1970, Austral. J. Phys. Astrophys. Suppl. N 16.
Lequeux, J., 1972, Astron. Astrophys. **24**, 395.
Lingenfelter, R. E., 1969, Nature **224**, 1182.
Meneguzzi, M., J. Audouze, and H. Reeves, 1971, Astron. Astrophys. **15**, 337.
Meyer, P., 1974, Philos. Trans. R. Soc. Lond. **A277**, 349.
Milogradov-Turin, J., and F. C. Smith, 1973, Mon. Not. R. Astron. Soc. **161**, 269.
Müller, D., and P. Meyer, 1973, Astrophys. J. **186**, 841.

Nesterova, N. M., and S. I. Nikolskii, 1973, Proc. P. N. Lebedev Physical Inst. Acad. Sci. USSR **64**, 49.
O'Dell, F. W., M. M. Shapiro, R. Silberberg, and C. H. Tsao, 1973, in *Proceedings of the XIII International Conference on Cosmic Rays, Denver* (University of Denver, Denver), Vol. 1, p. 490.
Parker, E. N., 1969, Space Sci. Rev. **9**, 651.
Perola, G. C., L. Scarsi, and G. Sironi, 1967, Nuovo Cim. **52B**, 455.
Price, P. B., 1971, in *Proceedings of the XII International Conference on Cosmic Rays, Hobart* (University of Tasmania, Hobart), **1**, 453.
Price, P. B., 1973, Space Sci. Rev. **15**, 69.
Prischep, V. L., and V. S. Ptuskin, 1975, Astrophys. Space Sci. **32**, 265.
Ptuskin, V. S., 1972, Kosm. Issled. **10**, 351.
Ptuskin, V. S., 1974, Astrophys. Space Sci. **28**, 17.
Ptuskin, V. S., and Ya. M. Khazan, 1975, in *Kosmicheskie Luchi* (Cosmic Rays), Collection No. 17 (Nauka, Moscow).
Raisbeck, G., and F. Yiou, 1973, in *Proceedings of the XIII International Conference on Cosmic Rays, Denver* (University of Denver, Denver), **1**, 494.
Ramadurai, S., and S. Biswas, 1974, Astrophys. Space Sci. **30**, 187.
Ramaty, R., and R. E. Lingenfelter, 1966, J. Geophys. Res. **71**, 3687.
Ramaty, R., and R. E. Lingenfelter, 1971, *Isotopic Composition of the Primary Cosmic Radiation* (Danish Space Research Institute), p. 203.
Razin, V. A., 1971, Astron. Zh. **48**, 46 [Sov. Astron. **15**, 35 (1971)].
Rickett, B. J., 1970, Mon. Not. R. Astron. Soc. **150**, 67.
Rochester, G. D., and A. W. Wolfendale, 1974, Philos. Trans. R. Soc. Lond. **A277**, 318.
Rosen, S., 1969, editor, *Selected Papers on Cosmic Rays Origin Theories* (Dover, New York).
Ryan, M. J., J. F. Ormes, and V. K. Balasubrahmanyan, 1970, Phys. Rev. Lett. **24**, 913.
Shapiro, M. M., and R. Silberberg, 1974, Philos. Trans. R. Soc. Lond. **A277**, 319.
Shen, C. S., 1970, Astrophys. J. **162**, L. 181.
Scheuer, P. A. G., and V. N. Tsytovich, 1970, Astrophys. Lett. **7**, 185.
Shishov, V. I., 1973, Astron. Zh. **50**, 941 [Sov. Astron. **17**, 598].
Silberberg, R., and C. H. Tsao, 1973, Astrophys. J. Suppl. **25**, 315; 335.
Silk, J., 1974, Comments Astrophys. Space Phys. **6**, 1.
Simpson, J. A., 1971, in *Proceedings of the XII International Conference on Cosmic Rays, Hobart* (University of Tasmania, Hobart), Vol. 1, p. 324.
Sofue, Y., K. Hamajima, and M. Fujimoto, 1974, Publ. Astron. Soc. Jpn. **26**, 399.
Speller, R., T. Thambyaphillai, and H. Elliot, 1972, Nature **235**, 25.
Stecker, F. W., 1971, *Cosmic Gamma Rays* (NASA Office of Scientific and Technical Information, Washington, D. C.).
Stecker, F. W., P. M. Solomon, N. Z. Scoville, and C. E. Ryter, 1975, Astrophys. J. **201**, 90.
Syrovatskii, S. I., 1959, Astron. Zh. **36**, 17 [Sov. Astron. **3**, 22].
Syrovatskii, S. I., 1971, Comments Astrophys. Space Phys. **3**, 155.
Syrovatskii, S. I., and B. M. Kuzhevskii, 1969, in *Proceedings of the XI International Conference on Cosmic Rays, Budapest*, edited by A. Somogyi, E. Nagy, M. Pósch, F. Telbisz, and G. Vesztergombi (Akadémiai Kiadó, Budapest), published as a Supplement to Vol. 29 of Acta Physica.
Thompson, D. S., G. F. Bignami, C. E. Fichtel, and D. A. Kniffen, 1974, Astrophys. J. **190**, L51.
Tokarev, J. V., 1969, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **12**, 161.
Toptygin, I. N., 1973, Astrophys. Space Sci. **20**, 329.
Tsytovich, V. N., 1971, *Theory of Plasma Turbulence* (Atomizdat, Moscow) [Plenum, New York (1972)].
Webber, W. R., 1974, in *Proceedings of the XIII International Conference on Cosmic Rays, Denver* (University of Denver, Denver), Vol. 5, p. 3568.
Webber, W. R., and J. A. Lezniak, 1974, Astrophys. Space Sci. **30**, 361.
Webber, W. R., J. A. Lezniak, J. Kish, and S. V. Damle, 1973, Astrophys. Space Sci. **24**, 17.
Webster, A. S., 1969, Astrophys. Lett. **5**, 189.
Webster, A. S., 1974, Mon. Not. R. Astron. Soc. **166**, 355.
Webster, A. S., 1975, Mon. Not. R. Astron. Soc. **171**, 143.
Wentzel, D., 1974, Annu. Rev. Astron. Astrophys. **12**, 71.
Wilkinson, A., and F. G. Smith, 1974, Mon. Not. R. Astron. Soc. **167**, 593.
Willis, A. G., R. G. Stro, and A. S. Wilson, 1974, Nature **250**, 625.
Yiou, F., and G. M. Raisbeck, 1972, Phys. Rev. Lett. **29**, 372.
Yates, K. W., 1968, Austral. J. Phys. **21**, 167.
Zdanov, G. B., Usp. Fiz. Nauk **111**, 109 [Sov. Phys. Usp. **16**, 642 (1973)].

Note added in proof: We would like to mention some additional references which were published recently:

Badhwar, G. D., R. L. Golden, M. L. Brown, and J. L. Lacy, 1975, "Cosmic ray antiproton production and positron production in the interstelar medium," Astrophys. Space Sci. 37, 283.
Daniel, R. R. and S. A. Stephens, 1975, "Propagation of cosmic rays in the galaxy," Space Sci. Rev. 17, 45.
Garazzi, G. and G. Sironi, 1975, "Cosmic-ray electrons and galactic radio noise: some problems," Riv. Nuovo Cim. 5, 155.
Hainebach, K. L., E. B. Norman, and D. N. Sieber, 1976, "Consistency of cosmic-ray composition, acceleration mechanism, and supernova models," Astrophys. J. 203, 245.
Haymes, R. C. *et al.*, 1975, "Detection of nuclear gamma rays from the galactic center region," Astrophys. J. 201, 593.
Hillas, A. M., 1975, "Some recent developments in cosmic rays," Phys. Rep. Letters 20 N2, 59.
Mael, R. C., A. J. Fisher, F. A. Hagen, and J. F. Ormes, 1975, "Neutron-rich isotopes of cosmic rays with $g \leq Z \leq 16$," Astrophys. J. (Letters) 202, L119.
Rasmussen, I. L., and B. Peters, 1975, "The nuclear composition of cosmic rays at their source in closed galaxy," Nature 258, 412.
Schmidt, W. K. H., K. Alallah, T. F. Cleghorn, W. V. Jones, A Modlinger and M. Simon, 1976, "Results of cosmic ray chemical composition at energies up to 100 GeV/nucl.," Astron. Astrophys. 46, 49.
Skilling, J. "Cosmic ray streaming," 1975, Nature 248, 687, Mon. Not. R. Astron. Soc. 172, 566; 173, 245, 255.
Tkaczyk, W., J. Wdowczyk, and A. W. Wolfendale, 1975, "Extragalactic heavy nuclei in cosmic rays?" J. Phys. A.: Math. Nucl. Gen. 8, 1518.
Wdowczyk, J. and A. W. Wolfendale, 1975, "Energy spectra of ultra high energy cosmic rays," Nature 285, 217.
S. E. Woosley, 1976, "Importance of isotopic composition of iron in cosmic rays," Astrophys. Space Sci. 39, 103.