Pomeron decoupling theorems*

Richard C. Brower†

Physics Department, University of California-Santa Cruz, Santa-Cruz, California 95060

J. H. Weis‡

Physics Department, University of Washington, Seattle, Washington 98195

We investigate the hypothesis that diffractive scattering is dominated (as $s \to \infty$) by a Pomeron Regge pole $[\alpha_P(t)]$. This hypothesis is particularly attractive since a Regge pole and its attendant cuts satisfy t-channel unitarity. For a complete theory the constraints of s-channel unitarity must also be satisfied. In addition to Froissart's bound and its well-known consequence, $\alpha_P(0) \leq 1$, s-channel unitarity implies a large number of decoupling theorems for an isolated Pomeron pole with $\alpha_P(0) = 1$. Here we systematically review these decoupling theorems. The theorems are treated in order of increasing strength so as to clearly distinguish the "strong theorems" which can be used to prove the complete decoupling of the Pomeron (e.g., $\sigma_{\text{Tot}} \to 0$) from the "weak theorems" which cannot. This review is undertaken with two goals in view: (1) To focus attention on possible points of departure for more realistic treatments of diffractive scattering, and (2) to emphasize the importance of s-channel unitarity which is expected to strongly constrain diffractive production in certain regions of phase space regardless of the exact nature of the Pomeron singularity.

CONTENTS

I. INTRODUCTION

Diffractive high energy scattering for hadrons appears to offer rather simple phenomena, at least in contrast to the complex resonance and exchange effects at low energies. The "asymptotic" cross sections (Amaldi *et al.*, 1971)[1] are roughly constant (see Fig. 1), and the exchanges (in t-channel) carry only vacuum quantum numbers. It may not be a utopian dream that a simple and consistent theory of diffractive scattering can precede a detailed understanding of hadron dynamics.[2]

Indeed one is reminded of the simple geometrical description for classical diffractive scattering. The elastic amplitude for $a+b \to a+b$ (see Fig. 2) is expressed as absorption from a disk of radius $R = r_a + r_b$ and opacity C,

$$A^{ab}(s,t) \simeq is\pi CR^2 e^{tR^2/8} \tag{1.1}$$

at small t and large s. ($s = E^2$, $t \simeq -E^2 \sin^2\theta/2$ in terms of center of mass energy E and angle θ). The optical theorem then gives a constant cross section

$$\sigma_{\text{Tot}}{}^{ab} \simeq \pi CR^2. \tag{1.2}$$

However, the theory of relativistic diffractive scattering cannot be that simple.

A. Regge theory of diffractive scattering

Unitarity in the exchange (t) channel (see Fig. 2) is inconsistent at $t = 4m_\pi{}^2$ with this fixed power of energy (s^a, a independent of t). Indeed in models with t-channel unitarity [e.g., potential theory (Regge, 1960) or ϕ^3 theory (Amati *et al.*, 1962b; Lee and Sawyer, 1962)] these fixed powers are replaced by the behavior $s^{\alpha(t)}$ arising from a factorized Regge pole at $j = \alpha(t)$. Thus t-channel unitarity suggests that diffractive scattering may be given by a factorized Regge pole[2] called the Pomeron, which has trajectory $\alpha_P(t)$ and coupling $\beta_P(t)$. Its contribution is

$$A^{ab}(s,t) \sim -e^{-i\pi\alpha_P(t)/2} s^{\alpha_P(t)} \beta_P{}^{aa}(t)\beta_P{}^{bb}(t) + \text{cuts}, \tag{1.3}$$

where $\alpha_P(t) \simeq \alpha_P(0) + \alpha' t + 0(t^2)$ so that with $\alpha_P(0) = 1$ the cross section goes to a constant. The cuts give corrections

---

* Text based in part on lectures presented at the University of California Santa Cruz Summer School on Particle Physics, 25 June–6 July 1973, and talk at Argonne Symposium of the Pomeron, 1 March 1973.

† Supported in part by the National Science Foundation, under Grant GP-41111X.

‡ Supported in part by U. S. Atomic Energy Commission.

[1] Logarithmic departures from constancy are regarded as "fine structure" and are discussed in Sec. IV.

[2] For an excellent and up to date reference for virtually all the basic concepts used in this article see D. Horn and F. Zachariasen (1973).

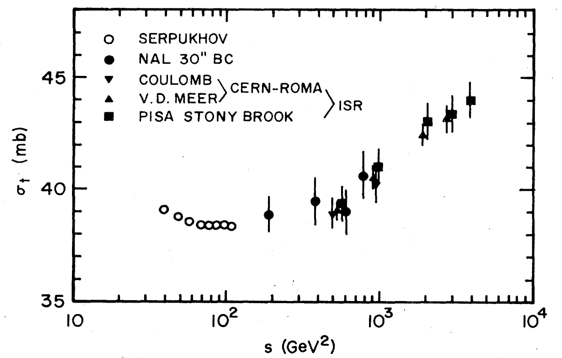


FIG. 1. The total pp cross section $\sigma_{\mathrm{Tot}}{}^{pp}(s)$ from Amaldi *et al.* (1973).

down by inverse powers of log s at $t = 0$ and are assumed to account for the observed departure from constant cross sections. The fact that Regge poles and their attendant cuts are the only structures[3] known to be consistent with t-channel unitarity is one of the most attractive features of the Pomeron Regge pole hypothesis.[4]

We note that this hypothesis has immediate and powerful consequences. For instance, the differential cross section must shrink at a universal rate (Giacomelli, 1972; Bartenev *et al.*, 1973) ($\alpha' \simeq 0.3$, see Fig. 3) and total cross sections factorize,[5]

$$\sigma_{\mathrm{Tot}}{}^{ab} \propto \beta_P{}^{aa}(0)\beta_P{}^{bb}(0). \tag{1.4}$$

Both these predictions are absent in the classical pictures [see Eq. (1.1)] and are supported to some extent by the present data (for other consequences see Sec. I.B).

Of course, a theory of diffractive scattering must also satisfy s-channel unitarity. Indeed it is well known that s-channel unitarity gives constraints as $s \to \infty$. The celebrated Froissart bound (Froissart, 1961)

$$\sigma_{\mathrm{Tot}}{}^{ab} \leq \frac{\pi}{m_\pi{}^2} \log^2 s, \tag{1.5}$$

follows from s-channel unitarity [Im $a_l(s) \geq \rho_l a_l(s) a_l{}^*(s)$] and t-channel analyticity in the neighborhood of $t = 0$. Since the Froissart bound uses only a small part of the full content of unitarity and analyticity, *stronger* bounds may well exist. For example, if we assume the Pomeron pole solution to t-channel unitarity (1.3), we obtain from Eq. (1.5) the stronger bound[6]

$$\sigma_{\mathrm{Tot}}{}^{ab} \sim s^{\alpha_P(0)-1} \leq \text{const.} \tag{1.6}$$

[3] We should emphasize that we do not regard Regge theory as fundamental. Indeed, it should be derived from unitarity and analyticity requirements. Roughly, it appears that (i) t-unitarity and (ii) s-analyticity give Regge poles plus cuts but with many undetermined parameters [$\alpha(t)$, $\beta(t)$, etc.]; and that (iii) s-unitarity and (iv) t-analyticity constrain these parameters (as illustrated by this article). The realistic hope for a theory of diffraction supposes that for $s \to \infty$, (i)–(iv) are soluble (by iteration) with few unknown constants.

[4] Other popular choices are: colliding $\frac{3}{2}$ root cuts, $a(j,t) = [(j-1)^2 - R_0{}^2 t]^{-\frac{3}{2}}$ to saturate the Froissart bound; a fixed pole $a(j,t) = (j-1)^{-1}$ and a masking cut; Schwartz square root branch points with $\alpha_c(t) = 1 + a(-t)^{\frac{1}{2}}$. None of these have been shown to be consistent with t-channel unitarity.

[5] Diffractive production cross sections also factorize. For evidence for this see Leith (1972).

[6] For logarithmic shrinkage one can obtain the intermediate bound $\sigma_{\mathrm{Tot}} \leq C \log s$. Applying $\sigma_{\mathrm{el}} \leq \sigma_{\mathrm{Tot}}$ to the amplitude $A_{\mathrm{el}}(s,t) \simeq$ if(s) $\exp[\frac{1}{2}b(s)t + i\frac{1}{2}d(s)t]$ you get $\sigma_{\mathrm{Tot}} \leq 16\,\pi b(s)$ or $\sigma_{\mathrm{Tot}} \leq 32\,\pi\alpha' \ln s$ for logarithmic shrinkage.

Alternatively, we can say that the s-channel unitarity requirement (1.5) restricts the Pomeron intercept to

$$\alpha_P(0) \leq 1. \tag{1.7}$$

Even further s-channel constraints on the Pomeron can be obtained for the critical value of the intercept $\alpha_P(0) = 1$. It is found that an isolated Pomeron pole with $\alpha_P(0) = 1$ must essentially completely decouple from all processes. This raises the possibility that an even stronger bound exists, namely $\alpha_P(0) < 1$ or

$$\sigma_{\mathrm{Tot}}{}^{ab} \sim 0 \tag{1.8}$$

as $s \to \infty$. However, we believe that it is much more likely that Regge cuts play a crucial, and, as yet, not fully understood role which will allow the construction of a theory with $\sigma_{\mathrm{Tot}}{}^{ab} \sim$ const (see Sec. IV). Regge cuts are, of course, required by t-channel unitarity through the iteration of the pole.

Here we wish to focus attention on these issues by studying in detail the idealized case of an *isolated* Pomeron pole with $\alpha_P(0) = 1$. For other discussions of Pomeron structure, see Gribov (1972) and White (1973). [This work generally follows the outline of Brower and Weis (1973).]

In Sec. II we give a systematic review of the Pomeron decoupling theorems (Gribov *et al.*, 1968a; Abarbanel *et al.*, 1971; DeTar *et al.*, 1971; DeTar and Weis, 1971; Abarbanel *et al.*, 1972a, b; Lee, 1973; Jones *et al.*, 1972) derived from s-channel unitarity. We present those theorems in order of increasing strength of assumptions necessary for their derivation and consequent increasing strength of their experimental implications. In particular, we separate those results (weak theorems) which are consistent with $\sigma_{\mathrm{Tot}} \to$ const. from those results (strong theorems) that conflict with $\sigma_{\mathrm{Tot}} \to$ const. It is conceivable that the weak theorems are satisfied in nature but the strong ones are not.

In Sec. III we combine the strongest decoupling results of Sec. II with t-channel analyticity to show the vanishing of the elastic couplings of the Pomeron (Brower and Weis, 1972) (e.g., vanishing of total cross sections). This shows the complete decoupling of an isolated Pomeron Regge pole at zero momentum transfer.

In Sec. IV we use the decoupling theorems as a focus for discussing some theoretical possibilities for complete theories of diffractive scattering. Multi-Pomeron cuts play a very weak role for $\alpha_P(0) < 1$, but an essential role for $\alpha_P{}^{\mathrm{input}}(0) > 1$ in eikonal models. The situation for the critical value of $\alpha_P(0) = 1$ is not, as yet, fully understood. However, considerable progress is being made by the application of renormalization group techniques to the Gribov Reggeon field theories [see discussion in Sec. IV.B and the review of White (1974)]. While it appears that absorptive effects can cause some of the theorems of Sec. II to break down, they may at the same time destroy some of the popular phenomenology based on a simple Pomeron pole.

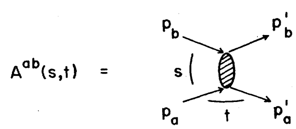


FIG. 2. We use the convention that the high energy limit $s = (p_a + p_b)^2 \to \infty$ is controlled by exchanges in the $t = (p_a - p_a')^2$ channel.

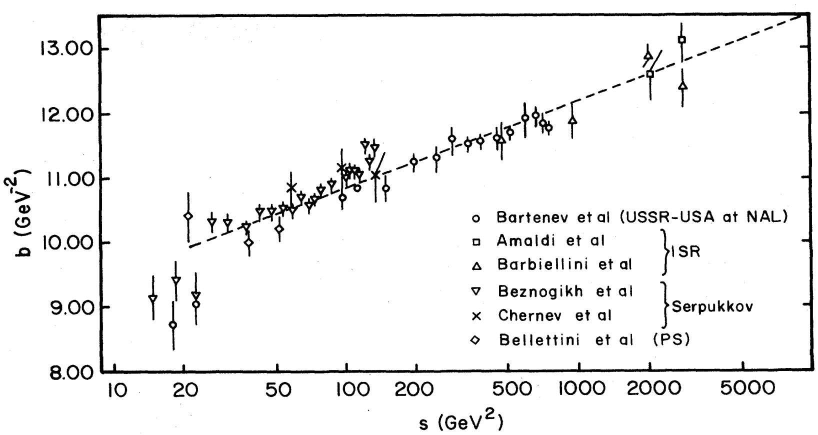


FIG. 3. The increasing value of $b(s) = (d/dt)$ log $(d\sigma^{pp}/dt)$ for $|t| < 0.1$ exhibits shrinkage corresponding to $\alpha' \simeq 0.3$ (Giacomelli, 1972; Bartenev *et al.*, 1972).

In conclusion, we note that regardless of the Pomeron fine structure (i.e., the detailed nature of the j-plane singularity or, alternatively, powers of log s), s-channel unitarity conditions of the type discussed here will place strong constraints on diffractive behavior in certain regions of phase space.

We conclude this section with a discussion of the Pomeron pole hypothesis in order to establish our notation and remind the reader of some of its attractive features.

B. Pomeron Regge pole dominance and applications

We have assumed that the Pomeron is a Regge pole [at $j = \alpha_P(t)$] passing exactly through $j = 1$ at $t = 0$. Moreover, the leading tips of the multi-Pomeron cuts (Fig. 4) have trajectories

$$\alpha_{\text{cut}}^{(n)}(t) = n\alpha_P(t/n^2) - n + 1 \tag{1.9}$$

and also pass through $j = 1$ at $t = 0$. Very little is rigorously known about these cuts, but if, as indicated in models (Goddard and White, 1972; Muzinich *et al.*, 1972) for the elastic amplitude, their discontinuity is nonsingular at the tip, and they give nonleading contributions[7] by factors of $1/\ln s$. Hence, for elastic amplitudes we expect the Pomeron Regge pole to dominate at $t = 0$.

However, much of the theoretical and phenomenological interest in the factorized Pomeron Regge pole consists in its application to production amplitudes. In multibody amplitudes almost nothing is known about the Regge cut contribution. So we make the conventional (and naive) hypothesis that the Pomeron pole also dominates at $t = 0$ in these multiparticle amplitudes. This assumption yields a unified theory of diverse phenomena. To emphasize this unification and to clarify the assumption we list the major consequences.

(1) Cross sections are asymptotically constant for large $\ln s$ and factorize

$$\sigma_{\text{Tot}}{}^{ab} \simeq \beta_P{}^{aa}(0)\beta_P{}^{bb}(0) + O(1/\ln s). \tag{1.10}$$

A negative cut as favored by Gribov, Pomeranchuk, and Ter-Martirosyan (1965) and White (1972) of course requires the cross section to rise to this constant.

(2) The elastic peak shrinks for small t at a universal rate,

$$b \equiv \frac{d}{dt} \ln\left(\frac{d\sigma^{ab}}{dt}\right) \approx a_a + a_b + 2\alpha' \ln s \tag{1.11}$$

$$\frac{d\sigma^{ab}}{dt} \simeq Ae^{tb(s)}.$$

Experimentally (Fig. 3) for the pp cross section b is independent of t for $-0.1 \leq t \leq 0$ and $\alpha' \approx 0.3$. Present estimates of the cut corrections give around 25% effects at Serpokhov energies, so that these features (1) and (2) are visible even with moderate $\ln s$. Note, however, that a cut leads the pole away from $t = 0$ ($\alpha_{\text{cut}} \simeq \frac{1}{2}\alpha' t + 1$) and that if it is negative in sign it can cause a gradual decrease in the rate of shrinkage. Also, if the cut has a broader t distribution ($\sim e^{2bt}$) as given in models, interference effects are expected (Chou and Yang, 1970; Sukhatme and Ng, 1973) and have been observed (Aachen Conf., 1972) for $t \approx -1.4$ (see Fig. 5).

(3) Feynman–Yang scaling is obtained for the inclusive reaction $a + b \to c +$ anything. This scaling limit is defined by taking the energy $s = (p_a + p_b)^2$ to infinity with $x = 2p_{\parallel c}{}^{\text{c.m.}}/s^{\frac{1}{2}}$ and $\mathbf{p}_{\perp c}$ fixed, where $p_\parallel{}^{\text{c.m.}}$ and $\mathbf{p}_\perp$ are the momenta along $\mathbf{p}_a$ and transverse to $\mathbf{p}_a$, respectively, in the center of mass of a and b. With $\alpha_P(0) = 1$, the limiting distribution for $x > 0$ is (see Sec. II.A.1)

$$E_c \frac{d\sigma^{ab}}{d^3p_c} \simeq \beta^{bb}(0)F_P{}^{a\to c}(x,p_{\perp c}) + O(1/\ln s), \tag{1.12}$$

where the fragmentation vertex $F_P{}^{a\to c}$ is independent of $p_\parallel$ and because of factorization of the Pomeron it is independent

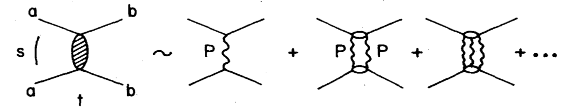


FIG. 4. In the Regge model, diffractive scattering is given by a Pomeron Regge pole (giving $\sigma_{\text{Tot}} \to s^{\alpha_P(0)-1}$) plus a series of multi-Pomeron cuts (fine structure). Presumably for $\alpha_P(0) = 1$, the cuts give small departures from constant cross sections, interference effects, long-range correlations, etc.

[7] As the reader will see in the conclusion this assumption is equivalent to the "weak coupling" solution to the Gribov Reggeon field theory. For our present purpose any contribution less singular than a pole would suffice, as for example $(j - \alpha_{\text{cut}})^{1-\epsilon}$ gives $(\ln s)^{-\epsilon}$ suppression.

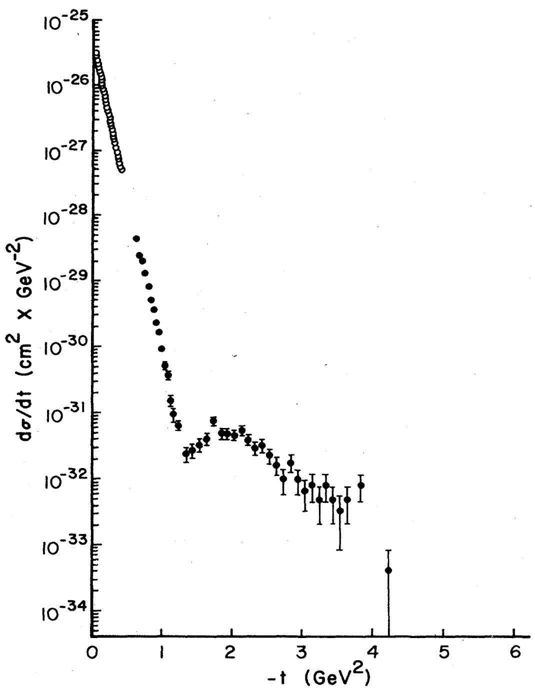


FIG. 5. The dip at $t \simeq -1.4$ in the elastic pp cross section (Aachen *et al.*, 1972) is predicted by a pole plus interfering cut model as for example in the prediction of Chou and Yang (1970).

of b. Here factorization has been checked to about 80% for several reactions (Chen *et al.*, 1971).

For $x < 0$, $p_{\parallel} \to \infty$ and the limit gives the fragmentation of b into c ($F^{b\to c}$). Often it is convenient to use the rapidity variable y which smoothly connects these two regions, $x > 0$ and $x < 0$. Rapidity y_i is defined by

$$p_{\parallel i}/E_i = \tanh y_i \tag{1.13}$$

and the fragmentation region of a ($x > 0$) or b ($x < 0$) is $(y_a - y_c)$ or $(y_c - y_b)$ finite as the total rapidity $Y = (y_a - y_b) \to \infty$.

We may visualize the longitudinal phase space of the final state $c + X$ (anything) in a rapidity (or longitudinal momentum) plot (see Fig. 6) extending from y_b to y_a of length $Y = y_a - y_b \approx \ln s +$ const. This is shown in Fig. 6 for the rest frame of b

$$y_b = 0, \quad y_a = Y \simeq \ln s.$$

For the process $a + b \to c + X$, in the fragmentation region of a particle a (or b), y_c is a finite distance from y_a (or y_b) as $y_a - y_b \simeq \ln s \to \infty$. The region between the two fragmentation regions contains the pionization or central region.

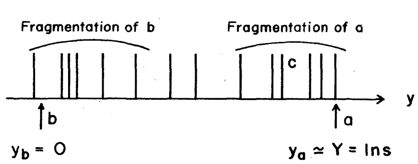


FIG. 6. Rapidity plot for final state in $a + b \to c + X$. Particle c is identified in the fragmentation region of a, a finite distance from y_a as $y_a - y_b \to \infty$. X contains all the other outgoing particles (lines).

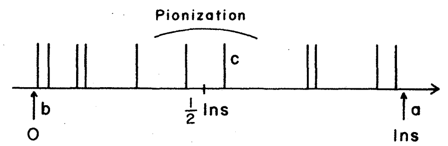


FIG. 7. The rapidity distribution for $a + b \to c + X$ with c in the pionization region. The rapidity of particle c is a fixed distance from $(y_a + y_b)/2 \simeq \frac{1}{2} \ln s$ as $y \to \infty$.

(4) Pionization (or AFS scaling) occurs for the inclusive reaction as $s \to \infty$ with $p_{\parallel c}{}^{\text{c.m.}}$ fixed in the center of mass of a and b (i.e., $x = 0$). That is

$$E_c \frac{d\sigma^{ab}}{d^3 p_c} \simeq \beta_P{}^{aa}(0) G_{PP}{}^c(p_{\perp c}) \beta_P{}^{bb}(0) + O(1/\ln s), \tag{1.14}$$

where $G^c(p_{\perp c})$ is independent of $p_{\parallel c}$ and, because of factorization of the Pomeron, independent of a and b. In rapidity space (Fig. 7) the pionization region is centered about $(y_a + y_b)/2$. Pionization has been observed at the ISR at CERN in the process $p + p \to \pi +$ anything, but until $\pi + p \to \pi +$ anything is measured at NAL factorization cannot be checked.

(5) Double fragmentation is obtained by measuring two particles in an inclusive reaction, $a + b \to c + d +$ anything, with c in the fragmentation region of a and d in the fragmentation of b. One has

$$E_a E_b \frac{d\sigma}{d^3 p_c d^3 p_d} \approx F_P{}^{a\to c}(x_c, p_{\perp c}) F_P{}^{b\to d}(x_d, p_{\perp d}) + O(1/\ln s), \tag{1.15}$$

where by factorization F_P is precisely given by the single fragmentation experiment (Brower *et al.*, 1973a). This may give a sensitive measure of cuts since the correlation,

$$C^{ab}(p_c, p_d) = \frac{E_c E_d}{\sigma^{ab}} \frac{d\sigma^{ab}}{d^3 p_c} \frac{d\sigma^{ab}}{d^3 p_d} - E_c E_d \frac{d\sigma}{d^3 p_c d^3 p_d} \tag{1.16}$$

is given entirely in terms of cuts to order $(s)^{\alpha_P(0)-1} \approx 1/s^{\frac{1}{2}} \sim \exp(-\frac{1}{2}|y_c - y_d|)$. The correlations due to this lower Regge term are clearly seen in the data giving the characteristic correlation length of two units of rapidity. With ultrahigh energies one can also look at the double pionization region (with $y_a - y_c$, $y_c - y_d$ and $y_d - y_b$ large)

$$\frac{E_c E_d}{\sigma_{ab}} \frac{d\sigma}{d^3 p_c d^3 p_d} \approx G_{PP}{}^c(p_{\perp c}) G_{PP}{}^d(p_{\perp d}) + O(1/\ln s). \tag{1.17}$$

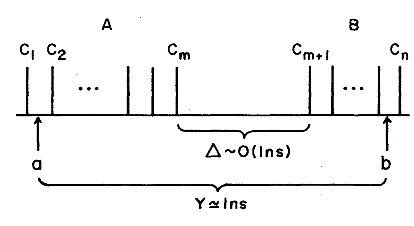


FIG. 8. The rapidity plot for the exclusive reaction $a + b \to c_1 + \cdots + c_n$ in the Regge limit with the rapidities for the left cluster $(y_1, \ldots, y_l)$ in a finite interval about y_a and the rapidities for the right cluster $(y_{l+1}, \ldots, y_n)$ about y_b.

There is an infinite set of these inclusive limits for $a+b \to c_1+c_2+\cdots+c_n+$ anything given in terms of the same functions F and G, although they rapidly become experimentally inaccessible.

(6) Double difffractive dissociation is given by factorization in the *exclusive* process $a+b\to c_1+\cdots+c_n$. We choose momenta p_i so that there is a left cluster ($p_1, p_2, \ldots, p_l$) and a right cluster ($p_{l+1},\cdots, p_n$) with rapidities at finite distance from the rapidity for p_a and p_b respectively, as $s\to\infty$ (see Fig. 8).

$$\frac{d\sigma}{dp_i\cdots dp_n} \simeq V_P{}^{Aa}(p_a\cdot p_b)^{\alpha_P(t)-1}V_P{}^{Bb}. \tag{1.18}$$

$V_P{}^{Aa}$ (or $V_P{}^{Bb}$) depends on the fixed cluster variables $p_i\cdot p_j$ for $i,\ j=a,\ 1,\ \ldots,\ l$ (or $i,\ j=l+1,\ \ldots,\ n,\ b$) and the fixed ratios $p_i\cdot p_j/p_a\cdot p_b$ with $i=a,\ 1,\ \ldots,\ l$, and $j=l+1,\ \ldots,\ n,\ b$. Factorization has been checked for a number of reactions (Leith, 1972).

We conclude this section by noting Fig. 9 which establishes our notation for the various Reggeon vertices discussed here and in the rest of the article.

II. DECOUPLING THEOREMS FROM UNITARITY

Here, we present a review of the constraints imposed by direct channel unitarity on diffractive production (Abarbanel *et al.*, 1971; DeTar *et al.*, 1971; DeTar and Weis, 1971; Abarbanel *et al.*, 1972a,b; Lee, 1973; Jones *et al.*, 1972). We have attempted to present a rather complete list in order of increasingly stronger *assumptions*. This has two ulterior purposes beyond pure pedagogy: First, although we shall only present the constraints for the leading Regge pole with $\alpha(0)=1$, we feel that the suppression of diffractive scattering (decoupling theorems) may be qualitatively valid in a wide class of models. Second, as we shall demonstrate in Sec. III, the strongest decoupling theorems are inconsistent with our hypothesis of $\sigma_T\sim$ const. Hence we would like to separate the "dangerous" results (leading to this inconsistency) from the "harmless" results. It is of great interest to test the experimental consequences of the decoupling theorems to clarify this separation.[8] Also, we hope to work backward from the "dangerous" results to expose the most natural further complication of Regge cuts that can save the Regge description of constant cross sections (see discussion in Sec. IV).

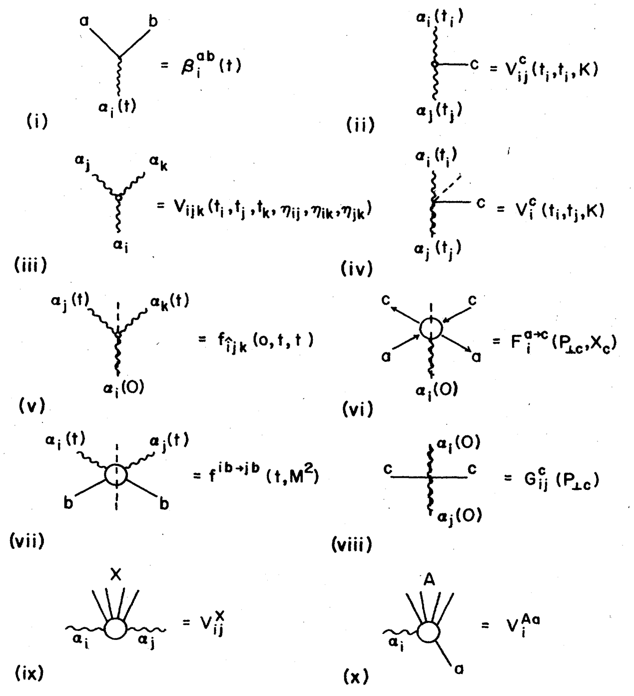


FIG. 9. The various Reggeon vertices encountered in this article are drawn here to establish our notation. Note that particles are indicated by superscripts ($a,b,c,\ldots$) and Reggeons by subscripts ($i,j,k,\ldots$). Dotted lines indicate a discontinuity of amplitude with respect to the energy variable for the cut Reggeon. Vertices (v)–(viii) are encountered in inclusive reactions $a+b\to c+X$ so the discontinuity is taken in $M^2=(p_a+p_b-p_c)^2$ and the kinematics is restricted to the forward direction for $a+b+c\to a+b+c$. Clusters of particles are designated by capital letters ($A,B,X,\ldots$). The reader is referred to the text for a more precise definition of the vertex functions.

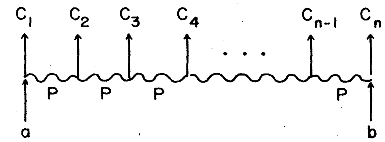
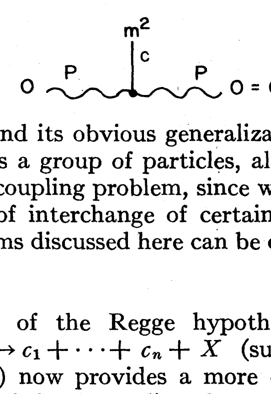


FIG. 10. The multiperiperal configuration used in proving the Finkelstein and Kajantie decoupling theorem.

Historically, the earliest decoupling theorem for diffractive production was obtained by Finkelstein and Kajantie (1968a and 1968b). They considered the multi-Regge (or multiperipheral limit) of an *exclusive* process $a+b\to c_1+\cdots+c_n$, and concluded that the Pomeron–Pomeron c_i vertex vanishes at zero momentum transfer for the Pomeron exchanges (Fig. 10).

$V_{PP}\ (t_1=0,\ t_2=0, K=m^2) =$  $= 0$ (2.1)

This decoupling theorem, and its obvious generalization to the case where c_i represents a group of particles, also goes right to the heart of the decoupling problem, since with the assumption of uniformity of interchange of certain limits all of the decoupling theorems discussed here can be derived from it.

However, the extension of the Regge hypothesis to *inclusive* processes $a+b\to c_1+\cdots+c_n+X$ (summing over X) by Mueller (1970) now provides a more elegant and economical derivation of the decoupling theorems. We shall therefore begin our discussion with the s-channel unitarity constraints on inclusive processes. But we shall return to the constraints on exclusive processes and the Finkelstein–Kajantie argument in Sec. II.B.2 since the full power of unitarity is manifested there. For example, the triple Pomeron zero removes $(\ln s)^n$ violations of exclusive sum rules instead of the $\ln\ln s$ violations of inclusive sum rules (Brower *et al.*, 1973c). Of course, to obtain the stronger constraints from exclusive processes, stronger assumptions about the existence of multi-Pomeron exchange are neces-

[8] This separation is also motivated by an observation we have made on couplings in the $\alpha(0)=1$ gauge invariant dual theory (see Appendix C).

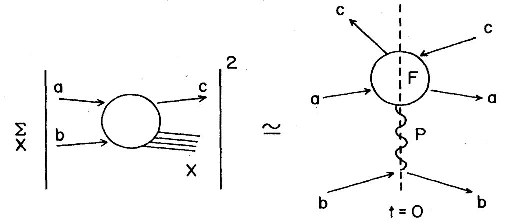


FIG. 11. Scaling limit for $a + b \to c + X$ expressed as a Mueller discontinuity in the mass of X (M^2).

sary so these results may also be more likely to fail in a theory with constant cross sections.

A. Weak decoupling theorems

In the inclusive approach to the decoupling theorems, all the results follow from the vanishing of the "triple Pomeron" vertex.[9] This condition, which was first discovered by Gribov and Migdal (1968a), is fundamental to an iterative approach to diffractive cuts in the Gribov calculus ("weak coupling") and the multiperipheral bootstrap (Abarbanel *et al.*, 1971).

1. Triple Pomeron zero

We consider the inclusive process $a + b \to c + X$, where X is any undetected state, and the center of mass momentum of c is $\mathbf{p}$ ($p_{\parallel} = xs^{\frac{1}{2}}/2$, $\mathbf{p}_{\perp}$ are components parallel and perpendicular to $\mathbf{p}_a$). The Feynman–Yang scaling hypothesis says

$$E_c \frac{d\sigma^{ab}}{d^3p_c} \sim \frac{1}{16\pi} s^{\alpha_P(0)-1}\beta_P{}^{bb}(0)F_P{}^{a\to c}(x,p_{\perp c}), \tag{2.2}$$

where $s = (p_a + p_b)^2 \to \infty$, and x, $p_\perp$ are fixed (see Fig. 11) and $\alpha_P(0) = 1$. The Pomeron exchange in the Mueller discontinuity accounts for scaling [with $\alpha_P(0) = 1$] in strict analogy to the manner in which the Pomeron via the optical theorem accounts for constant cross sections.

The total cross section is the totally inclusive process $a + b \to X$(anything), which the optical theorem relates to the elastic amplitude $A^{ab}(s,t)$

$$\begin{aligned}\sigma_{\mathrm{Tot}}{}^{ab}(s) &= \frac{1}{16\pi\lambda^{\frac{1}{2}}(s,m_a{}^2,m_b{}^2)}\,\mathrm{Im}A^{ab}(s,0)\\ &= \frac{1}{16\pi\lambda^{\frac{1}{2}}}\frac{1}{2i}\,\mathrm{Disc}_s A^{ab}(s,0),\end{aligned} \tag{2.3}$$

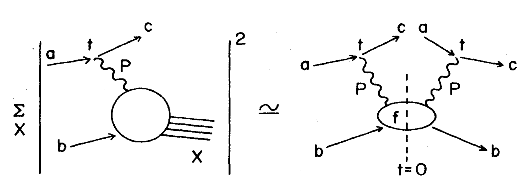


FIG. 12. Diffractive production for $a + b \to c + X$ expressed as a Mueller discontinuity in M^2.

[9] The dynamical origin of $f_{\hat{P}PP} = 0$ from t-channel unitarity has recently been extensively studied in an S-matrix approach by Bronzan (1972, 1973).

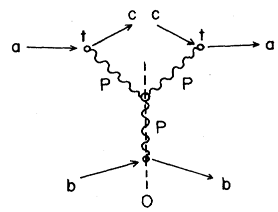


FIG. 13. The triple Pomeron contribution to $a + b \to c + X$ (anything) for $s = (p_a + p_b)^2 \gg M^2 = (p_a + p_b - p_c)^2 \gg 1$.

where

$$\lambda = s^2 + m_a{}^4 + m_b{}^4 - 2sm_a{}^2 - 2sm_b{}^2 - 2m_a{}^2m_b{}^2.$$

For the Pomeron exchange

$$A^{ab}(s,0) \sim is^{\alpha_P(0)}\beta_P{}^{aa}(0)\beta_P{}^{bb}(0),$$

we obtain constant cross sections. For the inclusive reaction $a + b \to c + X$, we use the generalized optical theorem of Mueller to do the sum over X (see Fig. 11)

$$E_c \frac{d\sigma^{ab}}{d^3p_c} = \frac{1}{16\pi}\frac{1}{\lambda^{\frac{1}{2}}(s,m_a{}^2,m_b{}^2)}\frac{1}{2i}\,\mathrm{Disc}_{M^2}A^{ab\to c}(t,s,M^2), \tag{2.4}$$

where $M^2 = (p_a + p_b - p_c)^2$, $s = (p_a + p_b)^2$, $t = (p_a - p_c)^2$. The Pomeron exchange in the forward six-particle amplitude for $a + b + c \to a + b + c$,

$$\frac{1}{2i}\,\mathrm{Disc}_{M^2}A^{ab\to c} \sim (M^2)^{\alpha_P(0)}\beta_P{}^{bb}(0)F_P{}^{a\to c}(x,p_\perp)$$

as $s \to \infty$ with x, $p_\perp$ fixed gives scaling (2.2).

Alternatively, we may consider the diffractive production of a fixed mass state M^2 plus c,

$$E_c \frac{d\sigma^{ab}}{d^3p_c} = \frac{[\beta_P{}^{ac}(t)]^2}{16\pi s}(s/M^2)^{2\alpha_P(t)}f^{Pb\to Pb}(M^2,t), \tag{2.5}$$

where f is the imaginary part of the "Pomeron" plus b elastic amplitude $A^{Pb}(M^2,t,t_{b\bar{b}})$ at $t_{b\bar{b}} = 0$ (see Fig. 12).

Now taking the high M^2 limit, we have

$$f^{Pb\to Pb}(M^2,t) \simeq (M^2)^{\alpha_P(0)}\beta_P{}^{bb}(0)f_{\hat{P}PP}(0,t,t). \tag{2.6}$$

The caret (ˆ) over the P indicates a discontinuity taken through that Reggeon (discontinuity in M^2). The leading (Pomeron) term gives that portion of diffractive production that contributes to scaling and is the so-called "triple Pomeron term."[10] Hence for $s \gg M^2 \gg 1$ and t fixed (see Fig. 13)

$$\begin{aligned}E_c(d\sigma/d^3p_c) &= G_P(t)s^{\alpha_P(0)-1}(s/M^2)^{2\alpha_P(t)-\alpha_P(0)},\\ G_P(t) &= (16\pi)^{-1}[\beta_P{}^{ac}(t)]^2 f_{\hat{P}PP}(0,t,t)\beta_P{}^{bb}(0).\end{aligned} \tag{2.7}$$

We now ask whether the factor,

$$(s/M^2)^{2\alpha_P(t)-\alpha_P(0)} \simeq \left(\frac{1}{1-x}\right)^{2\alpha' t+\alpha_P(0)},$$

[10] Actually, for the six particle amplitude, this is a helicity pole limit on the outside Pomerons [see Appendix A, η_{12}, $\eta_{31} \to \infty$ in (A.5)]. Furthermore only one term in the triple vertex contributes to the discontinuity in M^2. Therefore note that $f_{\hat{i}jk}(0,t,t)$ is symmetric in j and k, but not in i and j. The amplitude $f^{Pb\to Pb}$ is the maximum spin flip helicity for Pomerons with helicity $\lambda = \alpha(t)$. Theoretical evidence for this di-helicity Regge term comes from ϕ^3-theory (Chang *et al.*, 1971; Gordon, 1972; Mueller and Trueman, 1972; Neff, 1973) and dual theories (DeTar and Weis, 1971), while experimental evidence has come from the recent ISR and NAL data.

which is singular at the phase space boundary $x = 1$, is consistent with unitarity and constant cross sections.

The conservation of energy sum rule requires (DeTar *et al.*, 1971b)

$$(p_a + p_b)^\mu \sigma_{\text{Tot}}{}^{ab}(s) = \sum_c \int d^3p_c p_c{}^\mu \frac{d\sigma}{d^3p_c}, \tag{2.8}$$

where the sum is over all particle types c (e.g., π, K, N, etc.). Taking the energy component in the center of mass $p_a{}^0 + p_b{}^0 = s^{\frac{1}{2}}$, we get the *exact* unitarity sum rule

$$\sigma_{\text{Tot}}{}^{ab}(s) = \tfrac{1}{2} \sum_c \int d^2p_\perp \int dx \left(E_c \frac{d\sigma^{ab}}{d^3p_c} \right), \tag{2.9}$$

where $x = 2p_{\parallel}/s^{\frac{1}{2}}$.

Restricting ourselves to the channel ($c = \bar{a}$) and the phase space region dominated by the triple Pomeron, we have an inequality

$$\sigma_{\text{Tot}}{}^{ab}(s) \geq \pi s^{\alpha_P(0)-1} \int_0^c dp_\perp{}^2 \int_\delta^{1-M_0{}^2/s} \frac{dx}{1-x} \times G_P(t)(1-x)^{\epsilon + 2\alpha' t}, \tag{2.10}$$

where $\alpha_P(t) = 1 - \epsilon + \alpha' t + O(t^2)$ and $p_\perp{}^2 = -t + t_{\min}(s,x)$. For the present let us look at the leading term coming from the singularity at $x = 1$, for $\alpha_P(0) = 1$ ($\epsilon = 0$). For $s \gg M^2$, $t_{\min}$ goes to zero. Integrating by parts over $p_\perp{}^2$, we obtain

$$\frac{\pi G_P(0)}{2\alpha'} \int_\delta^{1-M_0{}^2/s} \frac{dx}{1-x} \left(\frac{-1}{\ln(1-x)} \right) + \text{const}$$

and performing the integral over $-d[\ln(1-x)]$, the integral diverges

$$\sigma_{\text{Tot}}{}^{ab} \geq \frac{\pi G_P(0)}{2\alpha'} \ln\ln(s/M_0{}^2).$$

Consequently a nonvanishing triple Pomeron at $t = 0$ gives a term in violation of $\sigma_{\text{Tot}} \to \text{const}$ as $s \to \infty$. Consistency with constant σ_{Tot} requires

$$f_{PPP}(0,t,t) = 0, \qquad \text{for} \qquad t = 0. \tag{2.11}$$

The source of the difficulty is the collision of the Pomeron pole $[\alpha_P(t)]$ and the $P - P$ cut $[\alpha_c(t) = 2\alpha_P(t/4) - 1]$. To see this we evaluate the triple Pomeron piece away from the collision, by going away from $t = 0$, or $\alpha_P(0) - 1 = 0$ or both.

The integral for Fig. 14

$$I = s^{\alpha_P(t)} \int \frac{dt' dt''}{[-\lambda(t,t',t'')]^{\frac{1}{2}}} \frac{dx}{1-x} e^{at'} e^{at''} \times (1-x)^{1+\alpha_P(t)-\alpha_P(t')-\alpha_P(t'')} \tag{2.12}$$

gives the pole-cut collision. Other contributions to the pole and the cut come from other parts of phase space, but they cannot cancel this piece since they are positive definite as well. We have approximated G_P by

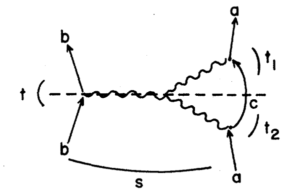


FIG. 14. The Reggeon diagram for the pole-cut collision in the triple Pomeron sum rule, or in Gribov's language the "enhanced graph."

$$G_P(t,t',t'') \sim e^{at'} e^{at''} \tag{2.13}$$

and neglected phase space terms of $O(M^2/s)$ which are less singular at $x = 1$, since they give lower cuts at $\alpha_c - 1$. Performing the t', t'' integrals and letting $y = -\ln(1-x)$ we have

$$I = \frac{\pi}{2} s^{\alpha_P(t)} e^{at/2} \int_{y_0}^{\ln s/M_0{}^2} dy \frac{e^{-\delta y}}{a + \alpha' y}, \tag{2.14}$$

where δ is the displacement of the pole above the cut

$$\delta = \alpha_P(t) - \alpha_{\text{cut}}(t) = 1 - \alpha_P(0) + \tfrac{1}{2}\alpha' t.$$

We make the change of variables,

$$z = \delta y + a\delta/\alpha' \qquad \bar{y}_0 = y_0 + a/\alpha'$$
$$\bar{M}_0{}^2 = M_0{}^2 e^{a/\alpha'} \qquad g(t) = \pi e^{at/2} e^{a/\alpha' \delta} \tag{2.15}$$

and separate the cut from the pole

$$I = \frac{g(t)}{\alpha'} \left[s^{\alpha_P(t)} \int_{\delta \bar{y}_0}^{\infty} \frac{e^{-z}}{z} dz - s^{\alpha_c(t)} \int_{\delta \ln(s/\bar{M}_0{}^2)}^{\infty} \frac{e^{-z} s^\delta}{z} dz \right]. \tag{2.16}$$

As $\delta \to 0$, the first integral is $-\ln|\delta|$ and the second may be approximated (for $\delta \geq 0$) by

$$\int_{\delta \ln(s/M_0{}^2)}^{\infty} \frac{dz}{z} e^{-(z-\delta \ln s)} \sim \int_{\delta \ln(s/M_0{}^2)}^{1+\delta \ln(s/M_0{}^2)} \frac{dz}{z} \tag{2.17}$$

for $\delta \ln s \to 0$ and $\delta \ln s \to \infty$.

The resulting expression

$$I \simeq g(t) \left[s^{\alpha_P(t)}(-\ln|\delta|) - s^{\alpha_c(t)} \ln\left(1 + \frac{1}{\delta \ln(s/M_0{}^2)}\right) \right] \tag{2.18}$$

can be studied for $\delta \approx 0$. For $s \to \infty$, but δ fixed, the triple Pomeron gives

$$I \sim s^{\alpha_P(t)} \ln \frac{1}{\delta} - \frac{s^{\alpha_c(t)}}{\delta \ln(s/M_0{}^2)} \tag{2.19}$$

which is the standard pole and negative cut ($\delta < 0$). The pole and cut have singular residues that do not cancel, but if we look at the limit $\delta \ln s \to 0$,

$$I \sim s^{\alpha_P(t)} \ln \frac{1}{\delta} + s^{\alpha_C(t)} \left(-\ln \frac{1}{\delta} + \ln \ln(s/M_0{}^2) \right). \tag{2.20}$$

The singular residues cancel as $\delta \to 0$, and at collision we get

$$I \simeq g(t) s^{\alpha_P}(\ln \ln(s/M_0{}^2) + \text{const}). \tag{2.21}$$

Notice that the cut is negative when it trails the pole but that it is positive when it leads the pole ($\delta < 0$).

To see further the j-plane structure we note that

$$A(j,t) = \frac{1}{j - \alpha_P(t)} \int_{\infty}^{\alpha_c(t)} \frac{dj' e^{(j'-j)y_0}}{j - j'} \tag{2.22}$$

is the partial wave amplitude with a pole-cut collision (i.e., multiplicative pole and cut) that gives exactly our asymptotic amplitude (2.16) after the Sommerfeld–Watson transform is performed.

Since $\ln\ln s$ comes from a pole-cut collision, one may ask whether the cut should not be inserted in the sum rule at the start. Obviously, the answer is yes. However, how cuts do or do not affect the sum rule arguments is not yet fully understood and we defer a discussion of this point to Sec. IV.

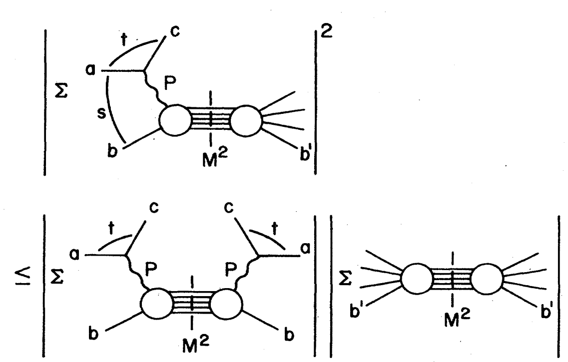
The $\ln\ln s$ term comes from the region (see Fig. 15) $\epsilon < 1 - x < M_0^2/s$ with ϵ arbitrarily small. One can avoid in a sense the divergence by reducing the range of integration (Neff, 1973) with the replacement $\epsilon \to s^{-\gamma}$ or $M_0^2/s \to \epsilon'$. However, such modifications are tantamount to eliminating what is usually called triple Pomeron behavior, i.e., a $1/(1-x)$ behavior in the scaling function. Such modifications could conceivably come about as a result of absorptive corrections (Neff, 1973; Ciafaloni and Marchesini, 1974), but this destroys conventional phenomenology unless the onset is delayed to ultrahigh energies.

2. Schwartz inequality theorems

Since the triple Pomeron vertex arose from a sum over diffractive dissociation processes, we might ask what its vanishing implies from these individual processes. A cute way to obtain constraints on these processes is to use the Schwartz inequality (Abarbanel *et al.*, 1972a,b; Lee, 1973)

$$|\langle a|b\rangle|^2 \le \langle a|a\rangle\langle b|b\rangle. \tag{2.23}$$

We choose $|a\rangle$ and $|b\rangle$ to be certain multiparticle states and the inner product to be a sum over a complete set of intermediate states



(2.24)

As long as these represent multiparticle processes inside the physical region, unitarity can be used to replace the sum over intermediate states by a discontinuity in M^2. Taking M^2 and s_{ab}/M^2 large, the leading term on the right-hand side

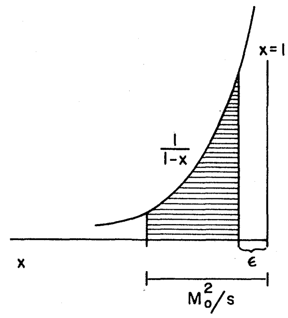
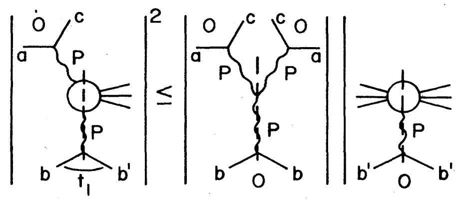


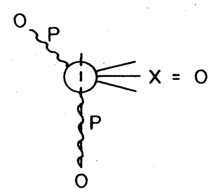
FIG. 15. The region of integration giving $\ln\ln s$ divergence in triple Pomeron integral.

is the triple Pomeron.



(2.25)

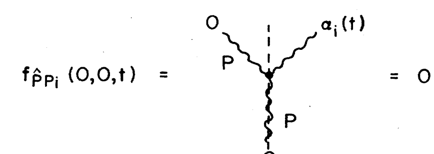
For $t_1 = 0$ we can compare the coefficient of $(s/M^2)^{2\alpha_P(0)}$ on both sides of (2.25) and use $f_{\hat{P}PP} = 0$ to obtain



(2.26)

for any state X. If Pomeron cuts are weak this can be obtained for arbitrary t_1 of the lower Pomeron. It should be carefully noted that one Pomeron in (2.26) has a discontinuity taken through it.

Clearly from (2.26) one can obtain an endless variety of decoupling theorems. For example, X may be a two-particle state ($p_{a'} \to p_{c'}$) in the inclusive process, $a'b' \to c' +$ anything. Then the triple Regge vertices[11] (Abarbanel *et al.*,



(2.27)

1971a, b; Lee, 1973) must all vanish (e.g., $\hat{P}PP$, $\hat{P}Pf$, $\hat{P}P\rho, \ldots$). It should be noted that the full triple Regge vertex occurs in (2.27) and not just the contribution of the maximum helicity for the cut Pomeron. A decoupling of the maximum helicity amplitude would be obtained if (2.25) held for a helicity limit (Goddard and White, 1971; DeTar

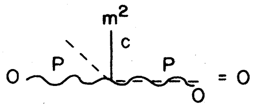
[11] Again the caret (ˆ) indicates the Reggeon with a discontinuity taken through it. Actually as we explain below and in Appendix A, the notation $f_{ijk}(0,0,t)$ is ambiguous because for $t \neq 0$ a series of helicity poles contribute and the vertex depends also on helicity angles.

et al., 1971a; Jones *et al.*, 1971). However (2.25) holds only in the physical region and a helicity limit is only inside the physical region if the three momentum transfers of the vertex satisfy (Misheloff, 1969; Abarbanel and Schwimmer, 1972)

$$\lambda(t_1,t_2,t_3) = t_1^2 + t_2^2 + t_3^2 - 2t_1t_2 - 2t_2t_3 - 2t_3t_1 \leq 0.$$

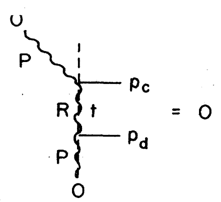
Therefore (2.27) can only be obtained for the full triple Regge vertex including the contributions of all helicities. (At $t_1 = t_2 = t_3 = 0$ the helicity limit can be taken, of course. At least for Toller quantum number $M = 0$ trajectories this will coincide with the $t \to 0$ limit of the full Regge vertex.) We refer the reader to Appendix A for a detailed discussion of the structure of triple Regge vertices and this point.

If X is taken to be a single-particle state (with vacuum quantum numbers), we have



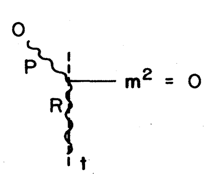
This appears to be the Finkelstein–Kajantie decoupling of the Pomeron–Pomeron–particle coupling (Finkelstein and Kajantie, 1968a and 1968b). However, here a discontinuity of the vertex is taken and one can easily show that this result is not strong enough to imply the vanishing of the full vertex.

Another example of particular interest is the case when X is an outgoing two-particle state (p_c,p_d) with large invariant mass $(p_c + p_d)^2$. Using the Regge expansion, we have



(2.28)

From (2.28) it might appear that the Pomeron–Reggeon particle coupling must vanish and thus, by continuation to $t = m^2$, the Pomeron–particle–particle coupling and total cross sections must vanish. However, this is not true because of the discontinuity taken (Moen and White, 1972). To see this we note that it is expected that discontinuities of multi-Regge amplitudes factorize just as do the full amplitudes themselves (Weis, 1973, 1974). Thus (2.28) can be written as the product of the appropriate discontinuities of the upper and lower vertices. The vanishing of the upper vertex does not imply the vanishing of the elastic coupling whereas the vanishing of the lower vertex part does. Thus the weakest assumption is



(2.29)

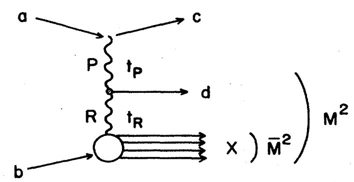


FIG. 16. The kinematic variables for the Jones *et al.* (1972) sum rule.

The part of the vertex in (2.29) is, by factorization of discontinuities, the same as that in the discontinuity in s_R of Fig. 24. From Eq. (3.15) we see that this is V_P, which by (3.19) does not contribute to the poles at $\alpha_R = 0, 1, 2, \ldots$.

Thus far all the theorems we have derived have been weak in the sense that they do not require the vanishing of the Pomeron's elastic coupling and thus $\sigma_{\mathrm{Tot}}{}^{ab} \sim 0$. It is amusing to note that all these theorems can be satisfied by requiring that the Pomeron couple like a conserved vector current at $t = 0$. Of course, since the Pomeron is a Reggeon at a *wrong* signature integer, it is not precisely like a vector particle. In particular, helicities $\lambda \neq \pm 1, 0$ are present. It is only the $\lambda = \pm 1, 0$ helicities of the Pomeron which have couplings like a conserved current. The Pomeron-conserved vector current analogy is developed in detail in Appendix B and we refer the reader to it for a precise definition.

To demonstrate conclusively that the decoupling theorems up to this point are consistent with nonvanishing elastic coupling, it is useful to construct an explicit example. We have investigated the usual planar dual model and found that all these theorems are realized, because with $\alpha(0) = 1$ it has a special gauge property. The dual model is discussed in detail in Appendix C.

B. Strong decoupling theorems

1. Theorems from inclusive cross sections

We now derive essentially the same decoupling theorems that one can obtain from (2.23) but in a strong form without the discontinuity taken. To obtain the strong decoupling theorems we again consider inclusive cross sections. We can use a relationship like (2.8) between the two-particle inclusive cross section and the single-particle inclusive cross section [or, equivalently, imagine using (2.8) when a is an external Pomeron].

The energy-momentum sum rule relating to the double inclusive $(a + b \to c + d + X)$ to the single inclusive cross section $(a + b \to c + X)$ is

$$(p_a + p_b - p_c)^\mu \frac{d\sigma^{ab}}{dp_c} = \sum_d \int dp_d p_d{}^\mu \left(\frac{d\sigma}{dp_c dp_d}\right), \qquad (2.30)$$

where $dp \equiv d^3p/(2E)$. Introducing the Feynman variables $x_c = 2p_{\parallel c}/s^{\frac{1}{2}}$, $x_d = 2p_{\parallel d}/s^{\frac{1}{2}}$ we obtain for the sum of the energy and $p_\parallel$ component of (2.30),

$$\frac{d\sigma^{ab}}{dp_c} \geq \frac{1}{2} \int d^2p_{\perp d} \int \frac{dx_d}{1 - x_c} \left(\frac{d\sigma^{ab}}{dp_{cd}p_d}\right). \qquad (2.31)$$

In the limit $x_c \to 1$, this becomes an inequality for the triple Pomeron vertex.

$$f\hat{p}_{PP}(0,t,t) \geq \tfrac{1}{2}\int d^2p_{\perp d}\int dy(1-y)^{\alpha_P(0)}B(t_P,t_R,y,\kappa), \qquad (2.32)$$

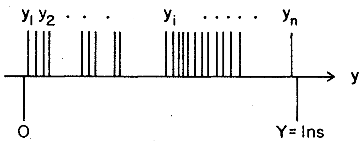
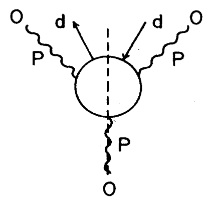


FIG. 17. Rapidities of produced particles in typical multiparticle event.

where y is the Feynman parameter for $P(t)+b\longrightarrow d+X$ with Mueller discontinuity B. In terms of $M^2=(p_a+p_d-p_c)^2$ and $\bar{M}^2=(p_a+p_d-p_c-p_d)^2$ (see Fig. 16), we have

$$1-y\simeq \bar{M}^2/M^2 \quad \text{and} \quad \kappa=M^2(p_c+p_d)^2/s. \tag{2.33}$$

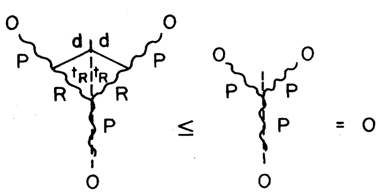
For $t_P=0$, the integral gives a lower bound on $f_{\hat{P}PP}(0,0,0)$ and must therefore vanish. The integral is a positive definite phase space integral so that

$B(0,t_R,y,\kappa)$ =  = 0 (2.34)

identically for p_d in the physical region (Jones *et al.*, 1972). This is an extremely strong result with many consequences. In particular if we consider the limit $y\longrightarrow 1$ (e.g., $s\gg M^2 \gg^2\bar{M}\gg 1$), we may pick out the Regge pole in the Pomeron d-channel, which gives[12]

$$B\sim (1-y)^{-2\alpha_R(t_R)}f_{\hat{P}RR}(0,t_R,t_R)\,|V_{PR}(0,t_R,\kappa)|^2=0 \tag{2.35}$$

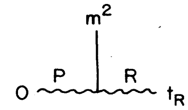
or representing this diagramatically we have

 (2.36)

Therefore either $f_{\hat{P}RR}$ or $V_{PR}{}^d$ must vanish. The vanishing of $f_{\hat{P}RR}$ is easily seen to lead to the vanishing of the Pomeron elastic coupling (and thus σ_{Tot}) since $f_{\hat{P}RR}$ has particle poles at $t_R=m^2$ [see Eq. (4.5)],

$$f_{\hat{P}RR}\approx \frac{1}{(t_R-m^2)^2}\beta_P{}^{dd}.$$

In order to avoid this result at this stage, we therefore require

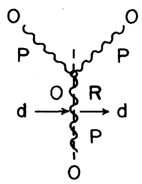
$V_{PR}(0,t_R,\kappa=m^2-t_R)$ =  = 0 (2.37)

for all $t_R\leq 0$. The restriction to $\kappa=m^2-t_R$ follows from the fact that we must approach the limit $y\longrightarrow 1$ and $t_P\longrightarrow 0$ from inside the physical region, thus real ω in

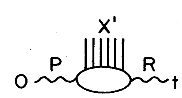
$$\kappa^{-1}=\frac{2(t_Pt_R)^{\frac{1}{2}}\cos\omega-t_P-t_R+m^2}{\lambda(t_P,t_R,m^2)}.$$

There can be no explicit ω dependence because the Pomeron is a nondegenerate (Toller $M=0$) trajectory, and therefore V_{PR} must be analytic in t_R and t_P for fixed κ.

In Jones *et al.* (1972) other regions of phase space for B are also considered. For example, with p_d in the central region, we get

 = 0 (2.38)

Further, by considering inclusive processes with more particles one can obtain an infinite number of decoupling theorems, now in the strong form. For example,

 = 0 (2.39)

For an application to couplings of Regge cuts, see Iwasaki and Yazaki (1973).

2. Decoupling theorems from exclusive cross sections

In this subsection we study the constraints on Pomeron couplings that can be obtained by studying the cross sections for multiparticle production that can occur through multiple Pomeron exchange. Thus we shall need to make stronger assumptions about Pomeron exchange than we made in the previous subsection—i.e., that factorized multiple-Pomeron exchanges exist. However, although we obtain no new decoupling theorems, we will obtain stronger forms of the old ones (Brower *et al.*, 1973). We find that a nonvanishing triple Pomeron vertex $f_{\hat{P}PP}(0,0,0)$ leads to contributions to the total cross section growing like $(\ln s)^n$ with n an arbitrary integer. These contributions are much larger than the $\ln\ln s$ contribution obtained from the inclusive sum rule above and indeed violate the Froissart bound by an arbitrary power of $\ln s$. In addition we can exclude counter examples to the sum rule decoupling proof which have trajectories with infinite slope at $t=0$ [note the factor $1/\alpha'$ in Eq. (2.10)] (Henyey and Zakrzewski, 1973; Oehme, 1973). Indeed any trajectory

[12] This limit is actually a rather unusual one. In fact, it is not a conventional Regge or helicity limit on the eight-particle amplitude (A. R. White, unpublished). When (2.34) or (2.36) is viewed as a sum over intermediate states and Regge pole factorization is assumed for each of them, (2.35) will follow if the sum over states commutes with the limit $M^2\longrightarrow\infty$. It is conceivable that this is not the case and that the vertices in (2.35) are not the usual Regge vertices although no completely satisfactory example of this has as yet been constructed. For a discussion of this possibility, see I. O. Moen and W. J. Zakrzewski (1973, 1974).

which gives a diffraction peak shrinking less or as rapidly as $[\ln s]^{-2}$ is inconsistent with constant total cross sections.[13] Finally, at the end of this subsection, we reproduce the Finkelstein–Kajantie decoupling theorem (Finkelstein and Kajantie, 1968a and 1968b).

Since we will be dealing with multiparticle final states at high energy, it is convenient to use the rapidity and transverse momentum variables. Thus for the process $a+b\rightarrow 1+2+\cdots+n$ we write

$$\begin{aligned} p_a &= (m_a \cosh y_a,\ \mathbf{0},\ m_a \sinh y_a),\\ p_b &= (m_b \cosh y_b,\ \mathbf{0},\ m_b \sinh y_b),\\ p_i &= (m_{\perp i} \cosh y_i,\ \mathbf{p}_{\perp i},\ m_{\perp i} \sinh y_i),\end{aligned} \tag{2.40}$$

where $m_{\perp i}^2 = p_{\perp i}^2 + m_i^2$. For large energies $s = (p_a+p_b)^2$ in the laboratory frame

$$\begin{aligned} y_a &= 0\\ y_b &\simeq \ln s = Y.\end{aligned}$$

Our attention will be focused on the rapidities, y_i, of the produced particles, since empirically the transverse momenta remain quite small for large s. Thus we characterize multiparticle events by the y_i as shown in Fig. 17.

In order to introduce the type arguments to be used in this subsection, we reproduce the triple Pomeron decoupling result in a slightly different way. Consider the rapidity configurations of all the final states produced by the collision of a and b at a given energy. Take that subset of events in which y_n differs from Y by a finite amount and all the other rapidities are less than $Y-\Delta$ (Δ large); i.e., take all events with a large gap in rapidity next to the leading particle n (the configuration in Fig. 17 satisfies this condition). Since the subenergies $s_{in} = (p_i+p_n)^2$ are large, we assume Pomeron exchange between n and the remaining cluster. If we call M^2 the invariant mass of the cluster, then summing over all events of a given M^2, we obtain the (Pomeron-particle a) total cross section which again is dominated by Pomeron exchange. Since the events satisfying our condition are a subset of all events, their partial cross section $\sigma^{(1)}$ satisfies

$$\sigma_{\mathrm{Tot}}^{ab} \geq \sigma^{(1)}. \tag{2.41}$$

On the other hand,

$$\begin{aligned}\sigma^{(1)} &\sim \frac{1}{16\pi s^2}\int_{M_0^2}^{e^{Y-\Delta}} dM^2 \int_{-\infty}^{0} dt \beta_P{}^{aa}(0) M^{2\alpha(0)} f_{\hat{P}PP}(0,t,t)\\ &\quad\times\left(\frac{s}{M^2}\right)^{2\alpha(t)}[\beta_P{}^{bn}(t)]^2\\ &= \frac{1}{16\pi}\int_{\mu_0}^{Y-\Delta} d\mu \int_{-\infty}^{0} dt \beta_P{}^{aa}(0) f_{\hat{P}PP}(0,t,t) e^{2\alpha' t(Y-\mu)}\\ &\quad\times[\beta_P{}^{bn}(t)]^2,\end{aligned} \tag{2.42}$$

where $t = (p_b - p_n)^2$ and $\mu = \ln M^2$. Since the main contribution comes for $t\approx 0$,

$$\begin{aligned}\sigma^{(1)} &\sim \frac{1}{16\pi}\beta_P{}^{aa}(0)\frac{f_{\hat{P}PP}(0,0,0)[\beta_P{}^{bn}(0)]^2}{2\alpha'}\int_{\mu_0}^{Y-\Delta} d\mu \frac{1}{Y-\mu}\\ &\sim \frac{1}{16\pi}\beta_P{}^{aa}(0)\frac{f_{\hat{P}PP}(0,0,0)[\beta_P{}^{bn}(0)]^2}{2\alpha'}\ln\ln s\end{aligned} \tag{2.43}$$

and Eq. (2.41) requires $f_{\hat{P}PP}(0,0,0) = 0$.

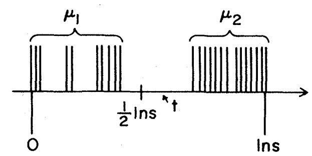


FIG. 18. Typical event contributing to $\sigma^{(2)}$.

We now consider (Brower *et al.*, 1973c) the partial cross section for "two cluster production," $\sigma^{(2)}$, defined as the cross section for all events with no particles in a large rapidity gap, fixed at a given point, say $y = \frac{1}{2}\ln s$.[14] Of course,

$$\sigma_{\mathrm{Tot}}^{ab} \geq \sigma^2. \tag{2.44}$$

A typical event is shown in Fig. 18 where the masses, M_i^2, of the clusters are

$$\ln M_i^2 \simeq \mu_i, \quad \mu_0 \leq \mu_i \leq \lambda_i \ln s < \tfrac{1}{2}\ln s. \tag{2.45}$$

Since the energy across the gap is large $[e^{(1-\lambda_1-\lambda_2)\ln s}]$, the behavior will be controlled by Pomeron exchange. Similarly, for large μ_1 and μ_2 we have Pomeron exchange in the two Pomeron-particle total cross sections. Thus we have

$$\begin{aligned}\sigma^{(2)} &\sim \frac{1}{16\pi s^2}\int_{M_{01}^2}^{e^{\lambda_1 \ln s}} dM_1^2 \int_{M_{02}^2}^{e^{\lambda_2 \ln s}} dM_2^2 \int_{-\infty}^{0} dt\\ &\quad \beta_P{}^{aa}(0) M_1^{2\alpha_0} f_{\hat{P}PP}(0,t,t)\left(\frac{s}{M_1^2 M_2^2}\right)^{2\alpha(t)}\\ &\quad\times f_{\hat{P}PP}(0,t,t) M_2^{2\alpha_0}\beta_P{}^{bb}(0)\\ &= \frac{\beta_P{}^{aa}(0)\beta_P{}^{bb}(0)}{16\pi}\int_{\mu_{01}}^{\lambda_1 \ln s} d\mu_1 \int_{\mu_{02}}^{\lambda_2 \ln s} d\mu_1 \int_{-\infty}^{0} dt\\ &\quad\times e^{2\alpha'(Y-\mu_1-\mu_2)}[f_{\hat{P}PP}{}^2(0,t,t)]^2\\ &\simeq \frac{\beta_P{}^{aa}(0)\beta_P{}^{bb}(0)}{16\pi}\frac{[f_{\hat{P}PP}(0,0,0)]^2}{2\alpha'}\int_{\mu_{01}}^{\lambda_1 \ln s} d\mu_1\\ &\quad\times\int_{\mu_{02}}^{\lambda_2 \ln s} d\mu_2 \frac{1}{Y-\mu_1-\mu_2}.\end{aligned} \tag{2.46}$$

Scaling μ_i by $\ln s$ ($\mu_i = u_i \ln s$), we find

$$\sigma^{(2)} \sim \beta_P{}^{aa}(0)\beta_P{}^{bb}(0)\frac{[f_{\hat{P}PP}(0,0,0)]^2}{(16\pi)(2\alpha')}\int_0^{\lambda_1} du_1$$

[13] Shrinkage greater than $(\ln s)^{-2}$ for elastic cross sections is excluded by unitarity—see Roy (1972). Thus no escapes of this type are possible.

[14] We should emphasize that our definition of a cluster or fireball differs from that often used. In our case, there can be large rapidity gaps (i.e., Pomeron exchanges) among the particles contained in the cluster, whereas many authors define a fireball as having no large internal rapidity gaps. It is important in our case to include the events with large internal gaps so that the full Pomeron exchange, as opposed to a "bare" Pomeron, will be built by the sum.

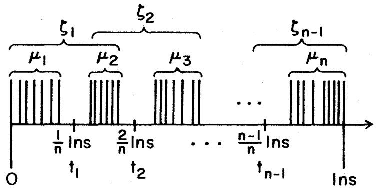


FIG. 19. Typical event contribution to $\sigma^{(n)}$.

$$\times \int_0^{\lambda_2} du_2 \frac{1}{1-u_1-u_2} \ln s = \sigma_{\text{Tot}}{}^{ab}\eta_P I^{(2)} \ln s, \tag{2.47}$$

where $\sigma_{\text{Tot}}{}^{ab} \sim \beta_P{}^{aa}(0)\beta_P{}^{bb}(0)$, $I^{(2)}$ is the double integral, and

$$\eta_P = \frac{f\hat{p}_{PP}{}^2(0,0,0)}{(16\pi)(2\alpha')}. \tag{2.48}$$

Equation (2.47) clearly is in contradiction with (2.44) by a factor of $\ln s$ unless the triple Pomeron vertex vanishes. Thus the inconsistency of the simple Pomeron pole model with unitarity for nonvanishing triple Pomeron vertex is greater than $\ln\ln s$ obtained in Eqs. (2.10) and (2.43).

The origin of the $\ln s$ in Eq. (2.47) can be traced to a factor of $(\ln s)^2$ from the phase space in $M_i{}^2$ and a factor $(\ln s)^{-1}$ from the shrinkage of the diffraction peak in t [e.g., $(Y-\mu_1-\mu_2)^{-1}$]. If the diffraction peak shrinks more rapidly, then the growth of (2.47) will be reduced. Any shrinkage of the diffraction peak less rapid than $(\ln s)^{-2}$ is inconsistent with constant total cross sections. Shrinkage of $(\ln s)^{-2}$ can also be excluded by letting λ_1 and λ_2 approach $\frac{1}{2}$ in which case $I^{(2)}$ diverges as $\ln\ln s$ analogously to (2.43).[15]

We can obtain partial cross sections which grow as an arbitrary power of $\ln s$ by considering cross sections for production of n clusters (Brower *et al.*, 1973c). Thus consider all those events with large gaps in rapidity centered at $(k/n)\ln s, k = 1, 2, \ldots, n-1$ (see Fig. 19). Again the partial cross section $\sigma^{(n)}$ satisfies

$$\sigma_{\text{Tot}}{}^{ab} \geq \sigma^{(n)}. \tag{2.49}$$

Again we define $\mu_i = \ln M_i^2$, $\mu_0 \leq \mu_i \leq \lambda_i \ln s \leq n^{-1} \ln s$, and also $\zeta_i = \ln s_{i,i+1}$, where $s_{i,i+1}$ is the total invariant mass in the clusters i and $i+1$.

Since the $s_{i,i+1}$ are large, we can use the multi-Regge form for the phase space integral (Finkelstein and Kajantie, 1968a and 1968b) where the $M_i{}^2$ are treated as fixed masses

$$d\phi_n = \frac{1}{2^{n+1}(2\pi)^{3n-5}s^2}\left[\prod_{i=1}^{n-1}\int d\zeta_i \delta\left(\sum_{i=2}^{n-1}\zeta_i - \sum_{i=2}^{n-2}\mu_i - Y\right)\right]$$
$$\times\left[\prod_{i=1}^{n-1}\int dt_i\right]\left[\prod_{i=2}^{n-1}\int \frac{dp_{\perp i}{}^2}{[-\lambda(t_{i-1}, t_i, -p_{\perp i}{}^2)]^{\frac{1}{2}}}\right] \tag{2.50}$$

and introduce a further set of integrals over the $M_i{}^2$. In Eq. (2.50) the t_i are the momentum transfers between clusters and the $p_{\perp i}$ are the transverse momenta of the clusters. Instead of the integrals over $p_{\perp i}{}^2$ one could use integrals over the Toller angles ω_i

$$p_{\perp i}{}^2 + M_i{}^2 = \frac{\lambda(t_{i-1}, t_i, M_i{}^2)}{2(t_{i-1}t_i)^{\frac{1}{2}}\cos\omega_i + M_i{}^2 - t_{i-1} - t_i}, \tag{2.51}$$

then

$$\int \frac{dp_{\perp i}{}^2}{[-\lambda(t_{i-1}, t_i, -p_{\perp i}{}^2)]^{\frac{1}{2}}} = \int \frac{d\omega_i(p_{\perp i}{}^2 + M_i{}^2)}{[\lambda(t_{i-1}, t_i, M_i{}^2)]^{\frac{1}{2}}}. \tag{2.52}$$

Since we will always be interested in the contribution for $t_i = 0$ where $p_{\perp i}{}^2 = 0$, we have

$$\int \frac{dp_{\perp i}{}^2}{[-\lambda(t_{i-1}, t_i, -p_i{}^2)]^{\frac{1}{2}}} = \int d\omega_i = 2\pi. \tag{2.53}$$

The integral over ω_i just gives 2π in this case, because the amplitude does not depend on ω_i as the momentum transfers vanish.[16] Therefore we drop the transverse momentum integral in the following.

As before, since the gaps are large we assume Pomeron exchange giving factors

$$\left(\frac{s_{i,i+1}}{M_i{}^2M_{i+1}{}^2}\right)^{2\alpha(t_i)} = \left(\frac{s_{i,i+1}}{M_i{}^2M_{i+1}{}^2}\right)^2 e^{2\alpha' t_i(\zeta_i-\mu_i-\mu_{i+1})}. \tag{2.54}$$

Using the relationship

$$1 = \frac{s_{12}s_{23}\cdots s_{n-1,n}}{s(p_{2\perp}{}^2+M_2{}^2)(p_{3\perp}{}^2+M_3{}^2)\cdots(p_{n-1\perp}{}^2+M_{n-1}{}^2)}$$
$$\approx \frac{[s_{12}s_{22}\cdots s_{n-1,n}]}{s[M_2{}^2M_3{}^2\cdots M_{n-1}{}^2]}$$

we obtain

$$\sigma^{(n)} = \frac{1}{2^{n+1}(2\pi)^{2n-3}}\left[\prod_{i=1}^{n-1}\int d\zeta_i\delta\left(\sum_{i=1}^{n-1}\zeta_i - \sum_{i=2}^{n-2}\mu_i - Y\right)\right]$$
$$\times\left[\prod_{i=1}^{n}\int d\mu_i\right]\left[\prod_{i=1}^{n-1}\int dt_i e^{2\alpha' t_i(\zeta_i-\mu_i-\mu_{i+1})}\right]$$
$$\times[\sigma^{aP}(\mu_1,t_1)\prod_{i=2}^{n-1}\sigma^{PP}(\mu_i, t_{i-1}, t_i)\sigma^{bP}(\mu_n, t_{n-1})], \tag{2.55}$$

where $\sigma^{xP}(\mu,t)$ is the total cross section for scattering a Pomeron of mass t from particle x at energy e^μ, and $\sigma^{PP}(\mu,t,t')$ is the total Pomeron (t)–Pomeron (t') cross section at energy e^μ.

We first consider the case where μ_i are large. Pomeron exchange gives constant cross sections

$$\sigma^{aP} \sim \beta_P{}^{aa}(0)f\hat{p}_{PP}(0,t,t)$$
$$\sigma^{PP} \sim f\hat{p}_{PP}(0,t,t)f\hat{p}_{PP}(0,t',t'). \tag{2.56}$$

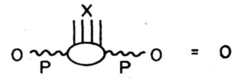


FIG. 20. The Finkelstein–Kajantie decoupling theorem.

[15] When shrinkage is not $(\ln s)^{-1}$ and the Regge trajectory is not linear, α' is the coefficient of dominant t behavior in $\alpha(t)-1$, i.e., $\alpha(t)-1 \cong \alpha'(-t)^\gamma$.

[16] This is true only for Toller quantum number $M = 0$ Regge trajectories. The Pomeron is $M = 0$ of course, since it contributes to helicity flip zero (elastic couplings) by assumption.

We therefore obtain

$$\sigma^{(n)} \approx \frac{\beta_P{}^{aa}(0)\beta_P{}^{bb}(0)}{2^{n+1}(2\pi)^{2n-3}}\left[\frac{(f\hat{P}_{PP}(0,0,0))^2}{2\alpha'}\right]^{(n-1)} \times\left[\prod_{i=1}^{n}\int_{\mu_{0i}}^{\lambda_i \ln s} d\mu_i\right]\left[\prod_{i=1}^{n-1}\int_{\mu_i+\mu_{i+1}+\nu_i\ln s+\delta_i}^{\infty} d\zeta_i \times\frac{1}{\zeta_i-\mu_i-\mu_{i+1}}\right]\delta\Big(\sum_{i=1}^{n-1}\zeta_i-\sum_{i=2}^{n-2}\mu_i-Y\Big).$$

The ζ_i integral extends over a gap of minimum $+\delta i$ length $\nu_i \ln s$. Scaling the integration variables $\mu_i = u_i \ln s$, $\zeta_i = z_i \ln s$ gives trivially a factor $(\ln s)^{n-1}$,

$$\sigma^{(n)} \simeq \sigma_{\text{Tot}}{}^{ab}(2/\pi)^{n-2}\eta_P{}^{n-1}I^{(n)}(\ln s)^{n-1}, \tag{2.57}$$

where

$$I^{(n)} = \left(\prod_{i=1}^{n}\int_0^{\lambda_i} du_i\right)\left(\prod_{i=1}^{n-1}\int_{u_i+u_{i+1}+\nu_i}^{\infty} dz_i \frac{1}{z_i-u_i-u_{i+1}}\right) \times\delta\Big(\sum_{i=1}^{n-1} z_i-\sum_{i=2}^{n-2}u_i-1\Big). \tag{2.58}$$

We see that multiple Pomeron exchange with nonvanishing triple Pomeron vertices produces a violation of the Froissart bound by an arbitrary power of $\ln s$.

As a second application of (2.55) we obtain generalized Finkelstein–Kajantie decoupling theorems. In this case we choose only a subset of the intermediate states in σ^{aP} and σ^{PP}, and fix the $\mu_i{}^2$, for example,

$$\begin{aligned}\sigma^{aP} &\approx [\beta_P{}^{aa}(0)]^2\delta(\mu_1{}^2-m_a{}^2),\\ \sigma^{Pb} &\approx [\beta_P{}^{bb}(0)]^2\delta(\mu_n{}^2-m_b{}^2),\\ \sigma^{PP}(\mu_i,0,0) &\simeq [V_{PP}{}^X(\mu_i)]^2,\end{aligned} \tag{2.59}$$

where $V_{PP}{}^X$ is the coupling of two Pomerons to a fixed state X (or fixed group of states). We drop the integrals over $\mu_i{}^2$ since we consider a fixed state, and again scale $\zeta_i = z_i \ln s$,

$$\sigma^{(n)} = \frac{\beta_P{}^{aa}(0)\beta_P{}^{bb}(0)^2}{2^{n+1}(2\pi)^{2n-3}}\left(\frac{[V_{PP}{}^X(\mu_i)]^2}{2\alpha'}\right)^{n-1}\frac{1}{\ln s} \times\sum_{i=1}^{n-1}\int_{\nu_i+\epsilon_i/\ln s}^{1/n}\frac{dz_i}{z_i}\,\delta\Big(\sum_{i=1}^{n-1}z_i-1\Big), \tag{2.60}$$

where $\epsilon_i = \mu_i+\mu_{i+1}+\delta_i$ and δ_i is the lower bound on the energy across the ith gap for which the Pomeron dominates. We can take $\nu_i \to 0$ and still have multiple Pomeron exchange. In this limit the z_i integrals diverge as

$$\prod_{i=1}^{n-2}\ln\frac{\ln s}{\delta_i} \approx (\ln\ln s)^{n-2},$$

so

$$\sigma^{(n)} = [V_{PP}{}^X(\mu_i)]^{2n-2}\frac{(\ln\ln s)^{n-2}}{\ln s}c^n. \tag{2.61}$$

In this case we can let n increase as s increases while still keeping the energy across each Pomeron above the minimum rapidities $\delta_i \simeq \delta$. Thus Eq. (2.61) holds for $n \simeq \ln s/\delta$. However, if the minimum rapidity gap across each Pomeron does not increase with s, the minimum momentum transfer does not vanish as $s \approx \infty$,[17]

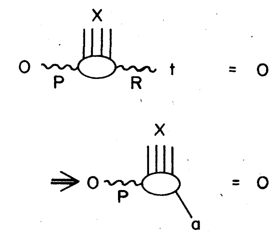


FIG. 21. Decoupling of the Pomeron–Reggeon-X vertex and continuation to the particle pole.

$$(t_i)_{\min} \approx \frac{s_{1,i-1}s_{i+1,n}}{s} \approx \frac{\exp(\sum_{j=1}^{i-1}\rho_j)\exp(\sum_{j=i+1}^{n}\rho_j)}{s} \approx \frac{1}{\delta}, \tag{2.62}$$

where $s_{i,i-1}$ and $s_{i+1,n}$ are the masses of the blobs produced on either side of the momentum transfer t_i. Thus we need a weak energy dependence in δ. Suppose $\delta \simeq \ln\ln s$, then $n \approx \ln s/\ln\ln s$ and

$$\sigma_{\text{Tot}}{}^{ab} > \sigma^{(n)} \sim (V_{PP}{}^{x2})^{\ln s/\ln\ln s-1}\frac{\exp(\ln s\ln\ln\ln s/\ln\ln s)}{\ln s}. \tag{2.63}$$

The inequality in (2.63) is violated unless (see Fig. 20)

$$V_{PP}{}^X = 0. \tag{2.64}$$

Finkelstein and Kajantie (1968a and 1968b) originally derived Eq. (2.64) for the case of X, a single particle, but the generalization we have presented here is clearly trivial. This result is clearly very powerful. All the decoupling theorems discussed in the previous subsection can be obtained from Eq. (2.64). For example, the vanishing of the triple Pomeron vertex is obtained as follows. Let X be a sum over all states of a given mass e^μ, then

$$V_{PP}{}^X(\mu)^2 = \sigma^{PP}(\mu) = 0$$

for all μ. Taking μ large[18]

$$\sigma^{PP}(\mu) \sim f\hat{P}_{PP}{}^2(0,0,0) = 0.$$

Thus the Finkelstein–Kajantie decoupling theorem is at once the most general and the strongest.

We close this section with a remark which will lead us into the discussion in the following section. We can imagine deriving further decoupling theorems by going to particle poles in the momentum transfers. Suppose for example we start with $V_{PP}{}^X$ and let X be a multiparticle state with one large subenergy, so

$$0 = V_{PP}{}^X = V_{PR}{}^{X_1}V_{RP}{}^{X_2}. \tag{2.65}$$

If we continue to the particle pole on the Reggeon[19] we have (Fig. 21)

$$V_P{}^{X_a} = 0. \tag{2.66}$$

[17] See also footnote in Neff (1973).

[18] This could have been done from the start in obtaining (2.63) from (2.55) as long as $\mu/\ln s \to 0$ as $s \to \infty$.

[19] This line of reasoning has been pursued by R. Rajaraman (1972).

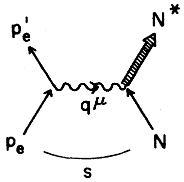


FIG. 22. Amplitude for $e + N \rightarrow e + N^*$ in the limit $s \rightarrow \infty$ and $q^2 \rightarrow 0$.

This procedure involves an analytic continuation from $t_R \leq 0$ to $t_R = m_a^2$ and thus clearly is outside the scope of the type of argument considered in this section. We discuss the validity of such analytic continuations in the following section and conclude that they are generally valid, although there are some potential hazards, particularly in the case $X = a$. We remark that in the multiperipheral model (Amati *et al.*, 1962; Lee and Sawyer, 1962; Chew and Pignotti, 1968) the validity of such continuations has always been assumed since the production amplitudes are amplitudes like Fig. 10 with alternating Pomeron and pion exchanges. Thus the inability to obtain $\alpha_P(0) = 1$ without a consequent vanishing of the elastic coupling is quite clear within the context of these models.

Concluding Sec. II, we note that Le Ballac (1971) has given an argument for the elastic decoupling directly from inclusive cross sections which avoids the analytic continuation. He notes that a *pure pole* Pomeron exchange in the central region implies that the average n-particle correlation function $\langle C_n \rangle$ increases like $\ln s$ and as a consequence $\sigma_{\rm el} \rightarrow 0$ faster than $(\ln s)^{-n+1}$. However, as Ellis, Finkelstein, and Peccei (1972) emphasize, $\langle C_n \rangle \sim (\ln s)^{n-1}$ behavior is expected as soon as Pomeron–Pomeron cuts are allowed, and then $\sigma_{\rm el} \sim (\ln s)^{-1}$ with no decoupling of the elastic Pomeron vertex. Therefore, unlike our argument from analyticity, the standard *soft PP* cut circumvents LeBallac's conflict with $\sigma_{\rm Tot} \rightarrow$ constant.

Indeed we should emphasize that all the results of Sec. II are *valid* in the presence of a Pomeron pole that is only separated by a factor of $(\ln s)^{-1}$ from its cuts at $t = 0$. That is why no *naive* cut mechanism avoids their conflict with constant cross sections.

III. DECOUPLING THEOREMS FROM ANALYTICITY

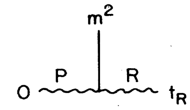
In this section we discuss further decoupling theorems that can be obtained by analytically continuing the results of Sec. II from the physical regions where they were originally obtained. Before discussing these results in detail, in Part A we give a simple example (Gribov, 1972; Brower and Weis, 1972) which illustrates some of the technical difficulties in such analytic continuations.

In part B we investigate the Pomeron particle–particle elastic coupling with a more detailed study of the analytic structure in the case of continuation of the Pomeron–Reggeon–particle vertex to the particle pole. We conclude that the coupling does vanish (Brower and Weis, 1972). In part C we consider other couplings obtained by continuation. Of particular interest are the "elastic" couplings occurring in the Mueller analysis of inclusive cross sections: the Pomeron four-particle fragmentation function and the two Pomeron two-particle pionization function. These have not been studied in detail but we believe they also must vanish.

A. Heuristic argument

There might be optimism that the decoupling of the Pomeron–Reggeon vertex for $t_R < 0$

$V_{PR}(0, t_R, K = m^2 - t_R) =$



$= 0$ (3.1)

might be consistent with the elastic pole at $t_R = m^2$. This optimism is based on the analogy with the longitudinal photon entering into electroproduction of a resonance (e.g., $e + N \rightarrow e + N^*$). Here the leading behavior at high energy has a form factor that vanishes at $q^2 = (p_e - p_e')^2 = 0$ (see Fig. 22)

$$F_{NN^*}(q^2) = 0 \qquad \text{at} \qquad q^2 = 0,$$

but the elastic form factor is nonzero and equal to the charge

$$F_{NN}(q^2) = e \qquad \text{at} \qquad q^2 = 0.$$

Might not the Pomeron decouple in the (inelastic) Pomeron–Reggeon–particle vertex, but *not* in the elastic vertex ($t_R = m^2$)? Our example clearly identifies the false element in this analogy and thus destroys the optimism based on it.

Suppose we can analytically continue the Pomeron–Reggeon two-particle vertex (or the two Pomeron–four-particle amplitude) to the particle pole to obtain the decoupling of the Pomeron from three particles (see Fig. 23). We would like to know whether it is possible to make a *further* continuation to an internal particle pole to obtain the vanishing of the Pomeron particle–particle coupling (and thus total cross sections); i.e., a continuation to, say, $s_{13} = m_2^2$.

The Pomeron three-particle coupling generally has the form

$$g(t, s_2/s_1, s_3/s_1, s_{12}, s_{23}, s_{31}),$$

where $s_{12} + s_{23} + s_{31} = m_1^2 + m_2^2 + m_3^2 + t$. However, the vanishing of (3.1) is only known in the physical region for $t = 0$. In this case there are further constraints among the variables (corresponding to $\kappa = m_2^2 - s_{31}$ in the double Regge limit)

$$\frac{s_1}{s_{23} - m_1^2} = \frac{s_2}{s_{31} - m_2^2} = \frac{s_3}{s_{12} - m_3^2}. \tag{3.2}$$

Equation (3.2) is derived in Appendix B—see Eq. (B5).

Since inelastic resonances decouple at this kinematical point for a gauge invariant Pomeron or photon (see Appendix B), it is plausible to consider just the sum of the

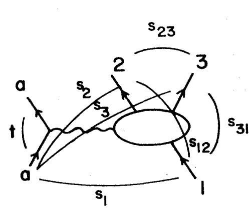


FIG. 23. Pomeron three-particle coupling.

three poles in the s_{ij} channels, as a simple model for g. Then

$$A_5 \simeq \beta_P{}^{aa}(t)[\xi_P\Gamma(-\alpha_P(t))s_1{}^{\alpha(t)}]g$$
$$\simeq i\beta_P{}^{aa}\left[\frac{s_1\beta_1}{s_{23}-M_1{}^2}+\frac{s_2\beta_2}{s_{31}-M_2{}^2}+\frac{s_3\beta_3}{s_{12}-M_3{}^3}\right], \quad (3.3)$$

where β_i is the coupling of the Pomeron to particle i at $t = 0$. Using Eq. (3.2) and $s_2 \simeq -s_1$, $s_3 \simeq -s_1$, the vanishing of Eq. (3.3) implies only

$$\beta_1 - \beta_2 - \beta_3 = 0 \quad (3.4a)$$

and not the individual elastic couplings β_i.

We can also consider the reactions obtained by crossing pairs of particles and obtain

$$\beta_{\bar{2}} - \beta_{\bar{1}} - \beta_3 = 0, \quad (3.4b)$$

$$\beta_{\bar{3}} - \beta_2 - \beta_{\bar{1}} = 0. \quad (3.4c)$$

Since the couplings of the Pomeron to particle and antiparticle are equal,

$$\beta_{\bar{i}} = \beta_i. \quad (3.5)$$

Equations (3.4) clearly give

$$\beta_1 = \beta_2 = \beta_3 = 0. \quad (3.6)$$

Therefore in this simple model the vanishing of elastic coupling is obtained, but it requires a knowledge of the crossing (signature or charge conjugation) properties of the Pomeron (3.5). This input is nontrivial since for exchanges of opposite signature like the photon we do not obtain decoupling. Indeed for the photon, charge conjugation is minus one,

$$\beta_i \to e_i, \quad \text{where} \quad e_{\bar{i}} = -e_i \quad (3.7)$$

and all three equations (3.4) reduce to the same charge conservation equation

$$e_1 - e_2 - e_3 = 0. \quad (3.8)$$

The analogy between the Pomeron and the photon (at $t = 0$, $j = 1$) breaks down for those properties related to signature or charge conjugation.

B. Vanishing of total cross sections

In order to find the implications of the vanishing of the Pomeron–Reggeon–particle vertex discussed in Sec. II, Eq. (2.37),

$$V_{PR}(t_P = 0, t_R; \kappa = m^2 - t_R) = 0 \quad (3.9)$$

for the Pomeron particle–particle coupling, we need to analytically continue in t_R from $t_R \leq 0$ where Eq. (3.9) was originally obtained to $t_R = m^2$. The correct performance of this continuation requires a detailed knowledge of the structure of $V(t_P,t_R;\kappa)$. Thus let us discuss this structure and the arguments with which it has been derived (DeTar and Weis, 1971; Drumond *et al.*, 1969; Goddard and White, 1971; Halliday, 1971; Weis, 1972). In doing this it is convenient to consider the vertex as it occurs in the $2 \to 3$ amplitude. By factorization this will of course be the same vertex which occurs in the doubly inclusive cross section considered above (Weis, 1973, 1974). Therefore, we consider the $2 \to 3$ process shown in Fig. 24.

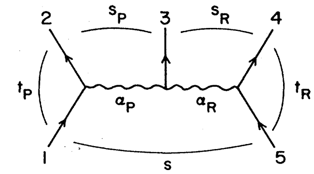


FIG. 24. Variables for five-particle amplitude.

The double Regge behavior is

$$A_5 \simeq \beta(t_P)[\xi_P\Gamma(-\alpha_P)s_P{}^{\alpha_P}]V_{PR}(t_P,t_R,\kappa)$$
$$\times[\xi_R\Gamma(-\alpha_P)s_R{}^{\alpha_R}]\beta(t_R), \quad (3.10)$$

where

$$\kappa = s_Ps_R/s. \quad (3.11)$$

From now on we shall drop the single Regge vertices $\beta(t_P)\beta(t_R)$ since they play no essential role.

The first step is to decompose the amplitude into signatured amplitudes which have only right-hand cuts in s_P, s_R, and s. Since in general those three invariants have both right- and left-hand cuts with no particular relations between them, we need $2^3 = 8$ different signatured amplitudes in order to have enough freedom to describe a general amplitude. These eight amplitudes arise from positive and negative signatures τ_P and τ_R for the two angular momenta (j_P in t_P-channel and j_R in t_R-channel) and positive and negative signature associated with the helicity m at the central vertex.[20] Thus we have

$$A_5(s_P,s_R,\kappa,t_P,t_R) = \sum_{\tau_P,\tau_R,\tau_V=\pm1}\{[A^{\tau_P\tau_R\tau_V}(s_P,s_R,\kappa,t_P,t_R)$$
$$+\tau_PA^{\tau_P\tau_R\tau_V}(-s_P,s_R,\kappa,t_P,t_R)$$
$$+\tau_RA^{\tau_P\tau_R\tau_V}(s_P,-s_R,\kappa,t_P,t_R)+$$
$$\tau_P\tau_RA^{\tau_P\tau_R\tau_V}(-s_P,-s_R,\kappa,t_P,t_R)+\tau_V[\kappa\to-\kappa]\}. \quad (3.12)$$

The assumption that signatured amplitudes $A^{\tau_P\tau_R\tau_V}$ have partial-wave projections with good (Carlsonian) continuations to complex angular momentum and helicity is the weakest link in our discussion. However, while their existence has never been rigorously proven,[21] the structure of the amplitudes which result has been found in all models of Regge poles studied (Drumond *et al.*, 1969).

The next step is to discuss the Regge behavior of the signature amplitude,

$$A^{\tau_P\tau_R\tau_V} = \Gamma(-\alpha_P)(-s_P)^{\alpha_P}V(t_P,t_R,\kappa)$$
$$\times\Gamma(-\alpha_R)(-s_R)^{\alpha_R}. \quad (3.13)$$

The double Regge residue of the signatured amplitude V has the behavior

$$V = (-\kappa)^{-\alpha_P}V_P(t_P,t_R,\kappa) + (-\kappa)^{-\alpha_R}V_R(t_P,t_R,\kappa), \quad (3.14)$$

where V_P and V_R have no cuts in κ (DeTar and Weis, 1971; Drumond *et al.*, 1969; Halliday, 1971). This is necessary to assure that A has no simultaneous discontinuities in overlapping invariants (i.e., s_P and s_R) in the physical region.[22]

[20] This signature, which is a new feature of multiparticle amplitudes, has been introduced by P. Goddard and A. R. White (1971).

[21] See A. R. White (1972) for a thorough discussion of this problem.

[22] For a recent discussion of this Steinmann property, see H. P. Stapp (1971).

Combining Eqs. (3.13) and (3.14) one easily verifies this property:

$$A^{\tau_P\tau_R\tau_V} = (-s)^{\alpha_P}(-s_R)^{\alpha_R-\alpha_P}V_P + (-s)^{\alpha_R}(-s_P)^{\alpha_P-\alpha_R}V_R. \tag{3.15}$$

Combining Eqs. (3.12), (3.13), and (3.14) we obtain the contribution of a double Regge pole with definite signatures (Brower and Weis, 1972; Goddard and White, 1971; Weis, 1972; Roth, 1972)

$$\begin{aligned} A_5 &= s^{\alpha_P}s_R^{\alpha_R-\alpha_P}\xi_P\xi_{RP}\Gamma(-\alpha_P)\Gamma(-\alpha_R) \\ &\times[V_P(t_P,t_R,\kappa) + \tau_P\tau_V V_P(t_P, t_R, -\kappa)] \\ &+ s^{\alpha_R}s_P^{\alpha_P-\alpha_R}\xi_R\xi_{PR}\Gamma(-\alpha_P)\Gamma(-\alpha_R) \\ &\times[V_R(t_P,t_R,\kappa) + \tau_R\tau_V V_R(t_P, t_R, -\kappa)], \end{aligned} \tag{3.16}$$

where

$$\xi_i = e^{-i\pi\alpha_i} + \tau_i,$$
$$\xi_{ij} = e^{-i\pi(\alpha_i-\alpha_j)} + \tau_i\tau_j, \quad (i, j = P \text{ or } R). \tag{3.17}$$

The signature factors in (3.16) give the full complex phase of the amplitude since the V_i are presumed real below the thresholds in t_P and t_R. Comparing Eqs. (3.10) and (3.11) we see that V_{PR} is not real

$$\begin{aligned} V_{PR} &= \xi_R^{-1}\xi_{RP}\kappa^{-\alpha_P}[V_P(\kappa) + \tau_P\tau_V V_P(-\kappa)] \\ &+ \xi_P^{-1}\xi_{PR}\kappa^{-\alpha_R}[V_R(\kappa) + \tau_R\tau_V V_R(-\kappa)]. \end{aligned} \tag{3.18}$$

The phase of V_{PR} results from the necessary cuts in κ (3.14) and, roughly speaking, is associated with cuts in s.

The final step is to obtain the structure of the V_i. This is done by arguments of the following type (DeTar and Weis, 1971). The poles at $\alpha_P = J_P$ must have polynomial residues of order J_P in the overlapping invariants s_P and s. For α_R nonintegral one sees from Eq. (3.15) or (3.16) that they can occur only in V_P. The double poles at $\alpha_P = J_P$ and $\alpha_R = J_R$ are accommodated by poles in V_P (and V_R) at $\alpha_P - \alpha_R$ integral. We finally have (DeTar and Weis, 1971)

$$\Gamma(-\alpha_P)\Gamma(-\alpha_R)V_P(t_P,t_R;\kappa) = \sum_{i=0}^{\infty}\Gamma(-\alpha_P+i)\Gamma(-\alpha_R+\alpha_P+i)U(\alpha_P-i, t_P, t_R)\frac{\kappa^i}{i!} \tag{3.19}$$

and similarly with $P \leftrightarrow R$. The function U has no singularities in the t_i below threshold.[23]

We remark that our basic formulas (3.13), (3.14), and (3.19) can be quite elegantly obtained using complex angular momentum and complex helicity analysis. However, we have preferred to give a rather direct derivation and refer the reader to Weis (1972) for discussion of complex helicity approaches. We only remark that the terms $\kappa^{-\alpha_P+i}$ and $\kappa^{-\alpha_R+i}$ in V arise from singularities in complex helicity at $\lambda = \alpha_P - i$ and $\lambda = \alpha_R - i$. For nonnegative i these are "sense" helicities, i.e., helicities less than the (complex) angular momentum. The absence of terms with negative i in Eq. (3.19) corresponds to the absence of "nonsense" helicity couplings (Brower *et al.*, 1973b).

[23] $U(\alpha_P - i, t_P, t_R)$ can have simple poles if nonsense wrong-signature multiplicative fixed poles are present (see Weis, 1972). These are located at $\alpha_P + n_P = 0\,[\tau_P = (-1)^{n_P+1}]$, $\alpha_R + n_R = 0\,[\tau_R = (-1)^{n_R+1}]$, and $\alpha_R - \alpha_P + n_{RP} = 0\,[\tau_P\tau_R = (-1)^{n_{RP}+1}]$, where the n are negative integers. These can be shown not to affect our conclusions.

Armed with Eqs. (3.18) and (3.19), which exhibit explicitly the behavior of the double Regge vertex in t_R, we can now examine the consequences of (3.9). We are interested in the behavior for $\alpha_P \simeq 1$, $\tau_P = +1$, and $\alpha_R \simeq 0$, $\tau_R = +1$ (since we are considering for simplicity a spin-zero particle). Keeping only the dominant behavior in Eq. (3.18), we may parametrize the vertex as

$$\Gamma(-\alpha_P)\Gamma(-\alpha_R)V_P \simeq \frac{U_0^{\tau_V}}{\alpha_P - 1} + \frac{U_1^{\tau_V}\kappa}{(\alpha_P - 1)(\alpha_R - \alpha_P + 1)}, \tag{3.20}$$

$$\Gamma(-\alpha_P)\Gamma(-\alpha_R)V_R \simeq \frac{U_1^{\tau_V}}{\alpha_R(\alpha_R - \alpha_P + 1)}. \tag{3.21}$$

We have exhibited the vertex signature explicitly in Eqs. (3.20) and (3.21) since in general both signatures can be present.[24] Inserting these expressions in (3.18) we have

$$0 \simeq \frac{e^{i\pi\alpha_R} - 1}{i\pi} \times \sum_{\tau_V=\pm1}\left\{\frac{e^{-i\pi\alpha_R} - 1}{2}\left[\frac{U_0^{\tau_V}(1+\tau_V)}{\kappa} + \frac{U_1^{\tau_V}(1-\tau_V)}{\alpha_R}\right] + \left[U_1^{\tau_V}(1+\tau_V)\frac{1}{\alpha_R}\right]\right\} \simeq 2U_1^{+1}. \tag{3.22}$$

In general the contributions of V_P and V_R occur in V_{PR} with different phases so they must individually vanish. For $\alpha_R = 0$, the contribution of V_P is purely imaginary and vanishes, so it gives no constraint on U_0 and U_1, but the contribution of V_R is real and only vanishes if

$$U_1^{+1} = 0. \tag{3.23}$$

However, returning to Eq. (3.18) we see that the vertex at $\alpha_R = 0$ (that is, the Pomeron–particle–particle coupling) is

$$\beta_P(0) = \lim_{t_P\to0} V_{PR}(t_P, t_R = m^2, \kappa) = 2U_1^{+1}. \tag{3.24}$$

Therefore the vanishing of the Pomeron–Reggeon–particle vertex implies[25] the vanishing of the Pomeron–particle–particle elastic coupling (3.24).

We would now like to make a number of comments on this result. As we noted in the introduction to this section, the signature of the Pomeron plays an essential role. In Eq. (3.22) there is a term $1/\kappa$ which, due to the kinematic constraint (3.10), is $1/(m^2 - t_R)$ and looks like the particle pole. However, it is multiplied by the factor ξ_{RP}, which is a wrong signature factor for $\alpha_P = 1$ and therefore cannot compensate the particle pole in V_R. The harmlessness of this potentially dangerous kinematic singularity is thus a result of the positive signature for the Pomeron. For the photon,[25] on the other hand, only V_P contributes, and cur-

[24] Planar models have no left-hand κ cut (equal $\tau_V = +1$ and $\tau_V = -1$ contributions), but nonplanar models have left-hand κ cuts (see Ross, 1972).

[25] Equations (3.18), (3.19), and (3.20) can be applied equally well to the photon, since they just give the required polynomial dependence of the residue. The Regge argument can be compared step by step with the simple pole argument given in Sec. III.A for both Pomerons and photons.

rent conservation requires that the kinematic $1/\kappa$ singularity exactly cancel the particle pole.

Signature plays a crucial role in another amusing way. Consider a Regge trajectory of negative signature so $\alpha_R = 0$ is a wrong signature point. Such a trajectory, if it has the same $\alpha_R(t_R)$ as the positive signature trajectory, surprisingly has the same behavior as (3.22) near $\alpha_R = 0$. In this case from Eqs. (3.18), (3.20), and (3.21) we have

$$V_{PR}(0, t_R, \kappa = m^2 - t_R) \simeq \frac{2}{i\pi} \sum_{\tau V} \left\{ -\frac{U_0^{\tau V}(1+\tau_V)}{\kappa} - \frac{U_1^{\tau V}(1-\tau_V)}{\alpha_R} + \frac{U_1^{\tau V}(1-\tau_V)}{\alpha_R} \right\}$$

$$\simeq -\frac{4}{i\pi}\frac{U_0^{+1}}{\kappa} \simeq \frac{4}{i\pi}\frac{U_0^{+1}}{\alpha_R}. \qquad (3.25)$$

The signature factor ξ_{RP} is now right signature so the $1/\kappa$ kinematic singularity contributes.

Equation (3.22) immediately suggests a mechanism for avoiding the vanishing of the elastic coupling. If the negative signature trajectory is exactly degenerate with the positive signature trajectory for $t_R \leq 0$ (exchange degeneracy), then the decoupling theorems will apply only to the sum of both their contributions since they cannot be separated by their asymptotic behavior. If the negative signature trajectory also has $\alpha_R(m^2) = 0$, then its contribution will cancel the particle pole if $U_0^{+1}(\tau_R = -1) = U_1^{+1}(\tau_R = +1)$.[26] In this case there is a nonuniformity of interchange of the limits $t_P \to 0$ and $t_R \to m^2$. The particle pole at $t_R = m^2$ occurs in V_R which is multiplied by $\xi_R\xi_{PR}(\tau_R = +1) + \xi_R\xi_{PR}(\tau_R = -1) = 2\xi_P$. Combining this with Eq. (3.21) we see the term with the particle pole is

$$\frac{\xi_P}{\alpha_R - \alpha_P + 1}\cdot\frac{1}{\alpha_R} \simeq -i\pi\frac{\alpha_P - 1}{\alpha_P - 1 - \alpha_R}\cdot\frac{1}{\alpha_R}. \qquad (3.26)$$

The coefficient of $1/\alpha_R$ is finite for the limit $t_R \to m^2 (\alpha_R \to 0)$ followed by $t_P \to 0$ $(\alpha_P \to 1)$ but zero for the limit $t_P \to 0$ followed by $t_R \to m^2$. If one traces the origin of the factor ξ_P in (3.26), one sees that it arises because the amplitude now has no left-hand cut in s_R.

Unfortunately this exchange degeneracy mechanism for circumventing the vanishing of total cross sections is not physically reasonable. The pion's exchange degenerate partner would have the quantum number of the A_1 meson. Such a trajectory could very well exist. However, there is no evidence for a trajectory with $I^G = 1^+$ and $J^P = 0^+$ approximately exchange degenerate with the ρ. While decoupling of the ρ might be tolerated by some people, this absence of such a trajectory also requires the pion to decouple. This can be shown by considering the Pomeron-π-π-ρ vertex which can be shown to vanish using the inclusive sum rules or Finkelstein–Kajantie argument (see Fig. 25).

While the exchange degenerate πA_1 eliminate the left-hand cuts in $s_{\pi\pi}$ this is not sufficient to cause the coefficient

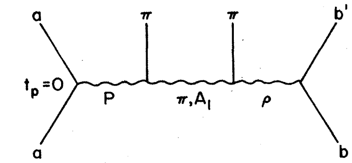


FIG. 25. A six-particle amplitude with P, πA_1, and ρ exchanges.

of α_π^{-1} to vanish at $t_P = 0$. The reason is basically that, while a single Regge trajectory of definite signature gives a factorizable contribution to the amplitude, the contribution of a Regge exchange without definite signature (e.g., an exchange degenerate pair) does not factorize. Thus the $P - \pi - \pi A_1$ "vertex" occurring in the $2 \to 3$ amplitude and vanishing is not the same $P - \pi - \pi A_1$ "vertex" occurring in the $2 \to 4$ amplitude, so the vanishing does not progagate to more complicated amplitudes.[27]

If the analytic structure of the Pomeron–Reggeon–particle vertex is different from that discussed here, the elastic decoupling theorem could be vitiated. We mention two possibilities. First, a singularity of the form

$$\frac{1}{\alpha_R - \alpha_P + 1}\cdot\frac{1}{\kappa} \qquad (3.27)$$

in V_R [see Eq. (3.21)] could cancel the particle pole in (3.22). Such inverse powers of κ correspond to "nonsense" singularities in complex helicity, and we have argued, though not completely rigorously for the case of the five-particle amplitude, that these are not allowed (Brower *et al.*, 1973b). Second, if $1/\kappa$ is replaced by

$$\frac{1}{\kappa - \alpha_R} \quad \text{or} \quad \frac{\kappa + \alpha_R}{\kappa - \alpha_R}\cdot\frac{1}{\alpha_R} \qquad (3.28)$$

the particle pole can still be cancelled. Again these behaviors are not expected in Reggeon couplings (Brower *et al.*, 1973). No known model for Regge poles produces the behaviors (3.27) or (3.28), so before one doubts the general arguments and takes these behaviors seriously he should find a mechanism which produces them.

There is one source of singularities in the Pomeron–Reggeon–particle vertex which must be taken seriously. This is the collision of the Reggeon pole with the Pomeron–Reggeon cut at $t_R = 0$. Although it is commonly assumed that this collision is weak {this idea is further supported by the vanishing of the Pomeron–Reggeon–Reggeon vertex obtained in Sec. II [Eq. (2.27)]}, if it were strong it would change the behavior of the Regge pole for $t_R \leq 0$ drastically. In Sec. IV we mention two proposals which vitiate the elastic decoupling theorem. We emphasize here that any such proposal means a nonuniformity of the interchange of limits $t_P \to 0$ and $t_R \to m^2$. This should be a quite drastic effect for the pion. Specifically, it means that if the pion–proton or pion–pion cross section is determined from the Chew–Low extrapolation, no Pomeron contribution will be found. This type of nonuniformity should be testable.

C. Other decoupling theorems

The crucial difficulty with continuation to particle poles in the elastic case is the presence of kinematic singularities

[26] Recall that Eq. (3.22) is multiplied by $e^{-i\pi\alpha_R} + 1$ and Eq. (3.25) by $e^{-i\pi\alpha_R} - 1$.

[27] These statements can be readily verified using the analysis developed in J. W. Weis (1972).

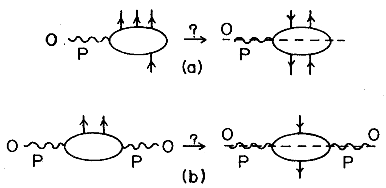


FIG. 26. Examples of Mueller vertices (right) and possible connections to other vertices.

at the same point. In other cases the kinematic singularities do not coincide with the particle pole so we expect that the continuation can be carried out without difficulty, although we have not investigated this point in detail.

There are other couplings which are rather like the elastic particle–particle coupling, however. These are the couplings occurring in the Mueller analysis of inclusive cross sections (see Fig. 26). These are similar to the elastic vertices in that the momenta of the particles on opposite sides of the discontinuity are equal. One might try to obtain decoupling of the Mueller vertices by analytically continuing the vertices on the left of Fig. 25 which vanish by the arguments of Sec. II. The analytic structure of such vertices is complicated, however, and to our knowledge no one has analyzed this problem completely. It is our feeling that the continuations can be performed to give vanishing of the Mueller vertices. We have investigated this question in the dual resonance model where the Pomeron is treated as an ordinary trajectory with intercept one. It is found that neither the vertices on the left nor those on the right in Fig. 26 vanish, and to obtain vanishing of those on the left we have to multiply in factors of t_P which cause those on the right to vanish also.[28] (See Appendix C.)

IV. THEORETICAL ALTERNATIVES AND IMPLICATIONS

A. Theoretical alternatives

Here we briefly discuss what role the decoupling theorems for a simple isolated Pomeron pole with $\alpha_P(0) = 1$ might be expected to play in a full theory of diffractive scattering. We conduct this discussion within the context of three popular models for diffractive scattering:

(i) $\sigma \to 0$ like $s^{-\epsilon}$; ϵ very small and the theorems are approximately true [e.g., MPM (Amati *et al.*, 1962a, Lee and Sawyer, 1962; Bertocchi *et al.*, 1962) or MRM (Chew and Pignotti, 1968) model with $\alpha_P(0) < 1$];

(ii) $\sigma \to$ const; some mechanism circumvents the decoupling theorems [e.g., Gribov Reggeon calculus (Gribov, 1968) with $\alpha_P(0) = 1$];

(iii) $\sigma \to \infty$ [e.g., $\ln^2 s$ in certain eikonal models (Hasslacher *et al.*, 1970; Cheng and Wu, 1970) where, effectively, $\alpha_P(0) > 1$].

(i). $\alpha_P(0) < 1$

The multiperipheral model (MPM) is perhaps the most highly developed model of diffractive scattering (Chew and Pignotti, 1968). The parameter ϵ is related to the triple Pomeron coupling G_P (Abarbanel *et al.*, 1971), and $\sigma_{\text{Tot}} \sim \beta(\epsilon)s^{-\epsilon}(\epsilon > 0)$. Generally then the size of G_P or β controls the point at which the cross sections are no longer nearly constant and $\beta(\epsilon) \to 0$ as $\epsilon \to 0$. The cuts are strictly nonleading by a power $[O(s^{-\epsilon})]$, and thus the fine structure in the j-plane is inessential, as far as the Pomeron couplings are concerned.

The difficulty in the MPM with taking $\alpha_P(0) \to 1$ may well be a consequence of the positive sign of the two Pomeron cut. Roughly, since the t-channel iteration adds to the cross section, unitarity bounds are difficult to satisfy. In any event, there are several "advantages" to a negative cut. The interface of a negative cut "explains" both a (temporarily) rising cross section [$-(a + \alpha' \ln s)^{-1}$ is an increasing function] and the dip at $t \simeq -1.4$ by destructive interference (Fig. 5).

However, it is interesting to note that Chew has shown that further fine structure can give a temporary increase in the cross section due to double diffractive production. In this model, fireballs (see Fig. 19) are defined as a cluster of particles with no rapidity gaps larger than $\Delta \sim 2$, and the sum over these particles is assumed to give a bare Pomeron. [This sum is not equivalent to our sum in Sec. II, since we made no cutoff at a maximal gap size. The unrestricted sum must give the full Pomeron, whereas the restricted sum leads to an unphysical signularity which may or may not be a Regge pole (bare Pomeron) as in the two-component model (Bishari *et al.*, 1974; Bishari and Koplik, 1974)]. Chew has shown that this model gives a leading pole, a positive cut, and complex conjugate poles. The complex poles cause oscillations that can explain a temporary rise in a cross section. Also for moderate $\ln s$ the j-plane is *approximated* by a pole and a negative cut.

Hence the crucial feature of the MPM model for $\alpha_P(0) \to 1$ is whether a positive cut is theoretically tenable as an *exact* principle. Gribov and White have advanced arguments that the negative sign is required by t-channel unitarity (Gribov *et al.*, 1965; White, 1972). We are impressed by both the phenomenological and theoretical arguments for a negative cut.

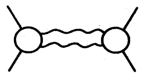
(ii). $\alpha_P(0) = 1$

Unitarity in the t-channel requires that with each Regge pole there are associated a series of cuts at $\alpha_{\text{cut}}^{(n)} = n\alpha(t/n^2) - n + 1$. If $\alpha_P(0) = 1$, all of the cuts will coincide with the pole at $t = 0$ and actually lead the pole in the scattering region $t < 0$. Therefore in this case the j-plane fine structure is potentially a crucial feature. The difficult problem of the interaction between the colliding pole and cuts has not been completely studied as yet so our discussion here will be only qualitative. The most sophisticated approach has been the application of the Renormalization Group approach to the Gribov local field theory approximation of the Reggeon calculus. We discuss the relationship of this work to decoupling problems in Sec. IV.B.

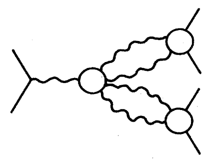
It is fairly clear that naive cuts do not affect the decoupling theorems. By naive cuts we mean contributions which look just like the pole contributions except divided by powers

[28] For some study of this question, see also C. H. Mehta and D. Silverman (1973).

of $\ln s$. For example,

$$\frac{s^{\alpha_c(t)}}{\ln s}\,\beta(t) \sim$$  (4.1)

where $\beta(t)$ has no singularities not present in the full amplitude. For example, in the inclusive cross section we might have

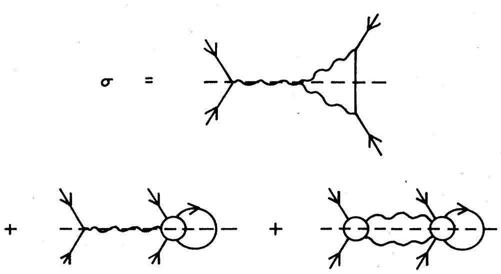
$$\frac{(s/M^2)^{2\alpha_c(t)}}{(\ln s/M^2)^2}\, M^{2\alpha_P(0)} \sim$$  (4.2)

Even though (4.2) dominates the triple Regge term for all $t \neq 0$, it is clear that its integral is $\sim (\ln s)^{-2} \ll \ln\ln s$ and so it does not affect the decoupling proof.

To see this another way, consider again the energy momentum conservation sum rule (DeTar, 1974 and Veneziano, 1974)[29]

$$\sigma_{\mathrm{Tot}}{}^{ab} = \pi \int_{-\infty}^{0} dt \int_{-1}^{1} dx\, \frac{d\sigma^{ab}}{dtdx}. \tag{4.3}$$

Separating the contribution of the triple Regge region (say, $M^2/s < \delta$) from the rest of the fragmentation region, we have for $\epsilon = 1 - \alpha_P(0) > 0$

$\sigma =$  (4.4)

$$= As^{-\epsilon} - R\frac{s^{-2\epsilon}}{\ln s} + Bs^{-\epsilon} + \frac{Cs^{-2\epsilon}}{\ln s},$$

where R is the fixed pole residue in Pomeron particle scattering and arises from the low M^2 part of the first term. The cut contribution from the remainder of the fragmentation region C can be such as to give the usual cut residue, i.e., $C = -R^2 + R$. However, as $\epsilon \to 0$ with $f_{\hat{P}PP}(0,0,0) \neq 0$, the first two terms become singular and give $\ln\ln s$ [see Eqs. (2.18)–(2.21)]. A compensating singularity is not expected in B or C since a negative $\ln\ln s$ behavior would give a negative inclusive cross section in some region of phase space in the absence of an even greater singularity.

The obvious place to look for a nontrivial role played by cuts is in contributions where one Reggeon spans two or more other Reggeons—see, for example, Figs. 27 and 28. Such contributions have a structure quite different from the pole or naive cuts and, as we shall see in an example below, can be quite singular. It is possible that such contributions spoil all the arguments presented in Secs. II and III.

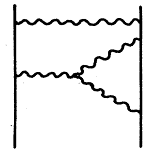


FIG. 27. An absorptive correction to the usual triple Pomeron term.

The most drastic effect of such contributions could be to allow a nonvanishing triple Pomeron coupling at $t = 0$ and thus violate the basic decoupling theorem. A popular plausibility argument for such an effect has been the analogy between the triple Pomeron diagram and the usual simple Regge cut diagram [Fig. 28(a)]. The usual heuristic argument for a negative sign of the two Reggeon cut says that the discontinuities through the Reggeons C_2 in Fig. 28(a) reverse the positive sign of the AFS term obtained by discontinuity C_1. Similarly, one might expect that the discontinuities C_2 of the triple Pomeron diagram [Fig. 28(b)] reverse the sign of the usual term C_1. In other words, there are other contributions to the cross section asymptotically as big as the triple Pomeron contribution. Such effects in planar Feynman diagrams (which in fact lead to a cancellation of the triple Pomeron term since a *Feynman* diagram like Fig. 28(b) actually has no cut) have been explicitly studied by Halliday and Sachrajda (1973).

Let us examine this effect in more detail. Whereas the discontinuity C_1 in Fig. 28(b) corresponds to single diffractive dissociation with a dominant contribution to the single-particle inclusive cross section (2.2) near $x = 1$, the discontinuity C_2 corresponds to an absorptive correction to the usual multi-peripheral process and is expected to contribute almost equally for all x—see Fig. 28(c). Therefore it corresponds to a *negative* contribution to the inclusive cross section in certain regions of the phase space. Unlike the simple cut diagram Fig. 28(a) where such contributions are $(1/\ln s)$ and nonleading, this contribution is $O(\ln\ln s)$ and dominates. Positivity can only be insured by the presence of other still larger [e.g., $(\ln\ln s)^2$, $(\ln\ln s)^3, \ldots$] contributions to the inclusive cross section such as the contribution corresponding to a discontinuity through both right-hand

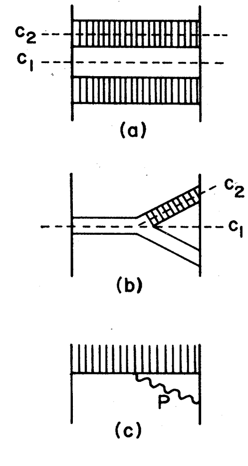


FIG. 28. Analogy between (a) Regge cut diagram and (b) triple Pomeron contribution. (c) Exclusive process contributing to discontinuity C_2 of (b).

[29] We are indebted to C. E. DeTar and A. Patrascioiu for discussions on this point.

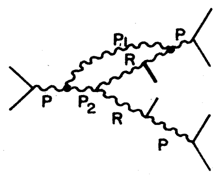


FIG. 29. Term leading to violation of argument for $V_{PR} = 0$ proposed by Cardy and White (1973a,b).

Pomerons in Fig. 28(b). Such contributions probably mean that we really do not have a theory with $\alpha_P(0) = 1$.[30]

Because of the likelihood of arbitrary powers of lnlns, we feel that the possibility of a nonvanishing triple Pomeron can only be treated in a framework where all orders are summed, like the eikonal model. Blankenbecler, Fulco, and Sugar (Blankenbecler *et al.*, 1974; Blankenbecler, 1973) have studied this problem. They find that to lowest order the sign reversal of the triple Pomeron takes place as discussed above and

$$\sigma_{\mathrm{Tot}} \simeq C - G_P \ln\ln s \tag{4.5}$$

and full eikonalization gives

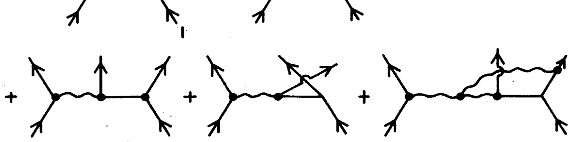
$$\sigma_{\mathrm{Tot}} \simeq \frac{C^2}{C + G_P \ln\ln s}. \tag{4.6}$$

Thus constant cross sections can only be obtained if the triple Pomeron coupling vanishes at $t = 0$.

Of course it is obvious that what is usually called triple Pomeron behavior, namely $F(x, t = 0) \simeq (1 - x)^{-1}$ for $x_0 \leq x \leq 1 - M_0^2/s$, is probably inconsistent with constant cross sections, since the inclusive cross section is positive definite. Any scheme which gives constant cross sections must change this behavior. The change could be mild [for example, $F(x, t = 0) \simeq (1 - x)^{-1}[\ln(1 - x)]^{-1}$] or more drastic [for example, restricting the range of validity to $1 - M_1^2/s^\gamma < x < 1 - M_0^2/s$ as suggested by Neff (1973), which means the triple Pomeron region shrinks to zero as $s \to \infty$]. Strictly speaking then, the triple Pomeron behavior is destroyed, and this may also destroy the attractive phenomenology based on a simple pole.

For some recent work on these possibilities, see also Ciafaloni and Marchesini (1974).

We feel it is much more likely that absorptive effects play an essential role in the strong decoupling theorems of Sec. II.B. As a related example, let us first discuss a mechanism proposed by Gribov (1972). Let us suppose the diffractive dissociation of one particle into two is required to vanish at $t_P = 0$ (this might be obtained by continuing the Pomeron–two-particle–Reggeon vertex to the particle pole). Gribov writes this amplitude as



(4.7)

[30] This is essentially the same point made after Eq. (4.4).

Thus in addition to the usual pole terms there is an absorptive correction to one of the poles. Let us discuss the structure of this term in more detail (Baker and Weis, unpublished). In the Pomeron channel it has also a two-Pomeron cut and in the particle channel it has a Pomeron–particle cut. Roughly speaking, at $t_P = 0$ it has a behavior like the function

$$\frac{\beta s}{(s_{31} - m_2^2) + (\ln s)^{-1}}. \tag{4.8}$$

In the scattering region $s_{31} < 0$, it behaves like the three-pole terms as $s \to \infty$ and will allow (4.7) to vanish if

$$\beta_1 = \beta_2 = \beta_3 = \beta. \tag{4.9}$$

However, it has no elastic pole at $s_{31} = m_2^2$, so the elastic couplings are unchanged.

Unfortunately this specific scheme has several undesirable features. It requires all elastic couplings (and consequently asymptotic total cross sections) to be equal. This means the theory is not relevant at present energies. Perhaps more importantly, in order to obtain this singular behavior a nonvanishing triple Pomeron coupling at $t = 0$ is required. This seems in conflct with the requirement of a vanishing triple Pomeron coupling from inclusive sum rules or the Reggeon calculus. However, Gribov has suggested a modification of the vertex which allows the contribution to (4.7) to be nonzero but the contribution to the inclusive sum rule (2.6) to vanish. However, if we look at the inclusive cross section, we find

$$F(x,t) \propto f_{\hat{P}PP}(0)(1 - x)^{-1-2\alpha' t}[1 - 2\alpha' t \ln(1 - x)]. \tag{4.10}$$

The usual triple Pomeron coupling now does not vanish at $t = 0$ but there is an additional contribution to the cross section which is not positive definite.

Quite generally one can imagine that the Pomeron–Reggeon cut can mask the contribution of the Reggeon [e.g., in Eqs. (2.36) or (4.7)]. Thus the full contribution can vanish for $t_R < 0$ where the Pomeron–Reggeon cut leads the pole, but have nonvanishing couplings for $t_R > 0$ and, in particular, $t_R = m^2$. Quite general classes of functions with such behavior can be written down (Brower and Zachariasen, unpublished), but consistent models which produce them have not been constructed. We note that such strong collisions between the Regge pole and the Pomeron–Reggeon cut will also mean that our assumptions on the analytic structure of the Pomeron–Reggeon–particle vertex in Sec. III will undoubtedly be violated.

Cardy and White (1973a,b) have recently made an extensive study of cuts in the Reggeon calculus. They argue that contributions of the form of Fig. 29 will cause the sum rule arguments of Sec. II.B for the vanishing of Pomeron couplings to break down. Contributions like these are very similar to those proposed by Gribov [Eq. (4.7)] and have corresponding singular behaviors. In the proposal of Cardy and White the bare triple Pomeron coupling is nonzero and the zero of $f_{\hat{P}PP}(0)$ arises only after a sum over all two-Pomeron iterations (Bronzan, 1972, 1973). Thus if the bare coupling is used in Fig. 29, the diagram will have the required singular behavior. However, it is not at all clear why exchanges of further Pomerons between P_1 and P_2 should not generate the full vertex $f_{\hat{P}PP}(0)$ and thus remove the desired singular behavior.

The detailed form of the inclusive cross sections has not yet been studied in the scheme proposed by Cardy and White. However, any mechanism like this and the others discussed above which violates the proof of the vanishing of the Pomeron–Reggeon–particle coupling is expected to violate some of our cherished phenomenological notions. In particular the Chew–Low extrapolation would be expected to fail for the Pomeron part of $\pi\pi$ scattering since the Pomeron–Reggeon cut masks the Reggeon (pion) in the scattering region. This phenomenon is explicitly illustrated by Eq. (4.8).

We remark that it is possible that all the weak decoupling theorems hold and only the strong theorems are violated, although this is not what Cardy and White envisage. In Sec. II.A we have noted that the weak decoupling theorems are all satisfied by a pure pole if it has a specific gauge property. The hypothesis of such a "gauge invariant Pomeron" is attractive because it is possible that such a pole may approximately satisfy unitarity with cuts relegated to a minimal, albeit vital, role. The analogy with Q.E.D. where gauge invariance gives stability to a pole at $j = 1$ at $t = 0$ is also inviting. We refer the reader to Appendixes B and C for a brief discussion of the properties of such a Pomeron (Ravndal, 1971).

Finally, it is possible that all the unitarity decoupling theorems of Sec. II hold and that only the analytic continuation argument of Sec. III breaks down. We have discussed and dismissed several possible mechanisms for this there. We believe this is a rather unlikely possibility.

(iii). $\alpha_P(0) > 1$

Case (iii) was suggested by Froissart (1961) and has been developed in the eikonal model (Cheng and Wu, 1970; Hasslacher, 1970). Here one begins with a pole with $\alpha_P(0) > 1$ (or any "input" singularity above one)

$$i\lambda e^{at} s^{\alpha_P(t)}, \qquad \alpha_P(t) = 1 - \epsilon + \alpha' t \tag{4.11}$$

and exponentiates the Fourier transform to insure elastic unitarity

$$A_{\rm el} = is \int \frac{d^2 x_\perp}{2\pi} \exp(i x_\perp \cdot q_\perp)(1 - e^{-D}), \tag{4.12}$$

where

$$D = \frac{1}{2} \frac{\lambda s^{-\epsilon}}{a + \alpha' \ln s} \exp\left(-\frac{x_\perp^2}{a + \alpha' \ln s}\right).$$

By replacing $a \to a - \frac{1}{2}(i\pi)\alpha'$ and $\lambda \to \lambda \exp\{-\frac{1}{2}(i\pi)[\alpha_P(0) - 1]\}$, the correct Regge phase can be introduced.

If we expand e^{-D} we get a series

$$\sigma_{\rm Tot}(s) = \sum_{n-1}^{\infty} s^{-\epsilon} \lambda \left(\frac{\lambda s^{-\epsilon}}{\ln s/s_0}\right)^{n-1} g_n \tag{4.13}$$

which gives the Regge pole plus multiple cuts. For $\alpha(0) > 1$, the limit $\ln s \to \infty$ cannot be interchanged with the sum, and the sum actually goes like $\log^2 s$. There is a singularity at $\ln s = \infty$ in $\ln s$. Here cuts play a dominant role.

It would clearly be very interesting to extend this approach to production processes and see how the unitarity constraints are satisfied in detail. So far little is known about this in general. Caneschi and Schwimmer (1972a,b) have studied the behavior of the model of Finkelstein and Zachariasen (1971) for a self-consistent Pomeron with $\sigma_{\rm Tot} \sim \ln^2 s$. They find, for example, that

$$\frac{1}{\sigma_{\rm Tot}} \frac{d\sigma}{d^3p/E}$$

scales and has the behavior $(1 - x)^{-1}[\ln(1 - x)]^{-1}$ for $x \simeq 1$ and $t \simeq 0$. Recently the absorptive mechanism for satisfying the Froissart bound has been employed to construct models with a bare Pomeron with $\alpha_P(0) = 1$ and nonvanishing triple Pomeron coupling (Amati *et al.*, 1973; Finkelstein, 1973). We shall discuss the philosophy of these approaches further below.

We remind the reader that whereas s-channel unitarity is built into such models and thus constraints like those of Sec. II will certainly be satisfied, t-channel unitarity will now be the crux in constructing a complete model. It has not been shown that j-plane singularities of the type occurring in the eikonal model are consistent with t-channel unitarity. Thus what is explicit in the Regge model becomes nontrivial in the eikonal model, and vice versa.

B. Renormalization group approach to Gribov Reggeon field theory

In the entire discussion so far we have avoided, for the most part, any effects due to the *infinite* accumulation of multi-Pomeron cuts at $t = 0$ for $\alpha_P(0) = 1$. Recently, significant progress has been made in summing the infinite series of cuts near $t \simeq 0$ and $j \simeq 1$ as they appear in the Gribov Reggeon calculus (A. A. Midgal *et al.*, 1974a,b; Abarbanel and Bronzan, 1974a,b). While the precise relationship between these results and the s-channel decoupling problems is still a little obscure, considerable progress has been made. Here we briefly review the results and comment on their possible implications. (For another review, see White, 1974).

Some years ago Gribov and co-workers (Gribov, 1968, and subsequent papers) showed that the unitarity conditions on Regge poles and cuts at low momentum transfer were similar to the unitarity conditions on nonrelativistic field theories. Hence the Pomeron and its cuts near $t \simeq 0$ could be studied by an analogue method, treating the Pomeron as a nonrelativistic quasiparticle, where rapidity (y) and impact parameter (**b**) play the role of one time and two space dimensions. The conjugate variables are "energy" $E = 1 - j$ and the transverse momentum $\mathbf{k} = (k_x, k_y)$ given by Mellin and Fourier transforms of y and **b**, respectively. Gribov and others used perturbation theory and other standard field theory techniques, and proposed two types of solutions—a "strong coupling" solution (Gribov and Migdal, 1968b) in which $\sigma_{\rm Tot}$ rises as $(\log s)^\eta$, and "weak coupling" solution (Gribov and Migdal, 1968a) in which $\sigma_{\rm Tot}(s)$ goes to a constant with $(\log)^{-1}$ corrections. The

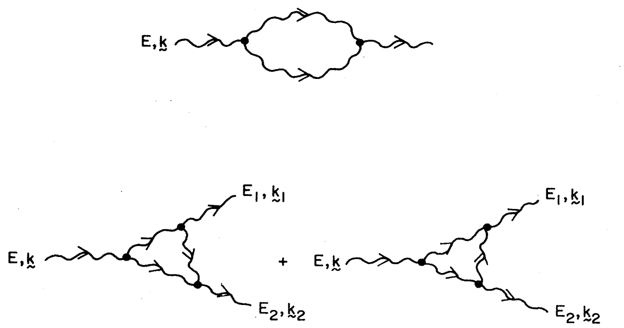


FIG. 30. Lowest order $(\Psi)^3$ corrections to the propagator $\Gamma^{(1,1)}$ and the triple Pomeron vertex $\Gamma^{(1,2)}$.

"weak coupling" solution only occurred if the triple Pomeron vanished in accordance with the decoupling theorem. Recently, renormalization group techniques have been applied to Reggeon field theories to obtain both the "strong coupling" (Migdal *et al.*, 1974a,b; Abarbanel and Bronzan, 1974a,b) and the "weak coupling" (Brower and Ellis, 1974; R. Jengo, 1974).

For the Pomeron trajectory near $t = -\mathbf{k}^2 \simeq 0$, we have the energy-momentum relation

$$E = 1 - j = \Delta_0 + \alpha_0' k^2 + O(k^2) \tag{4.14}$$

with $\Delta_0 = 1 - \alpha(0) \simeq 0$, and we recognize the standard Regge pole as the Green's function,

$$G_0^{(1,1)}(E,\mathbf{k}) = \frac{i}{E - \alpha' k^2 - \Delta_0} \simeq -\frac{i}{j - \alpha_P(t)}, \tag{4.15}$$

for the (free) Schrödinger equation.

So we may introduce a nonrelativistic Pomeron field $\Psi(y,\mathbf{b})$ with a free Lagrangian

$$\mathcal{L}_0(y,\mathbf{b}) = \frac{i}{2}\Psi^+ \frac{\partial}{\partial y}\Psi - \alpha_0' \boldsymbol{\partial}\Psi' \boldsymbol{\partial}\Psi + \Delta_0 \Psi^+\Psi, \tag{4.16}$$

where we calculate the high $y = \ln s$ contribution to the elastic amplitude by the Mellin transform

$$A_{\mathrm{el}}(s,t) \simeq s \int \frac{dE}{2\pi i} e^{-Ey} G_0{}^{(1,1)}(E,\mathbf{k}). \tag{4.17}$$

For $\alpha(0) = 1$, the most singular multi-Pomeron corrections to the pole arise from the triple Pomeron region. Hence we introduce $(\Psi)^3$ interactions[31] (Fig. 30)

$$\mathcal{L}_I = -\tfrac{1}{2}(ig_0)\Psi^+\Psi(\Psi + \Psi^+) - \tfrac{1}{2}(i\lambda_0)(\Psi^+\boldsymbol{\partial}\Psi\cdot\boldsymbol{\partial}\Psi + \Psi\boldsymbol{\partial}\Psi^+\cdot\boldsymbol{\partial}\Psi^+) + \cdots \tag{4.18}$$

and replace G_0 by the full renormalized Green's function $G(E,k)$ for $\mathbf{k} \simeq 0$, $E \simeq 0$ in Eq. (4.17). Of course calculating $G(E,k)$ is difficult, but in principle the high energy contribution of multi-Pomeron exchange consistent with all j-plane unitarity discontinuities (near $E \simeq \mathbf{k} = 0$) is reduced to a field theoretical infrared problem.

[31] The anti-Hermitian form for the trilinear contributions to $\mathcal{L}_{\mathrm{int}}$ is required by the same argument that Gribov (1965) gives for the negative sign for the Pomeron–Pomeron cut.

We should emphasize the limited domain of the Gribov field theory formulation. The discontinuity formulae, the linear Regge trajectory, and the local form of the triple Pomeron interactions in Eq. (4.18) is only valid if we are permitted to expand them for small $(E,\mathbf{k})$. Also we treat the external particles as *sources* for the Reggeon field so n-Pomeron to m-Pomeron Green's functions $\Gamma^{(n,m)}$ also contribute (see Fig. 31). But under rather general conditions it is believed the $ig_0\Psi^+\Psi(\Psi + \Psi^+)$ interaction's contributions to $\Gamma^{(1,1)} = [G^{(1,1)}]^{-1}$ is the dominant effect of multi-Pomeron cuts (Abarbanel and Bronzan, 1974a,b).

To calculate the infrared limit of the Green's functions, Migdal, Polyakov, and Ter-Martirosyon (1974a,b) and Abarbanel and Bronzan (1974a,b) have employed the renormalization group equations of Callan–Symanzik

$$\left\{E\frac{\partial}{\partial E} - \beta(g)\frac{\partial}{\partial g} + [\alpha' - \zeta(\alpha',g)]\frac{\partial}{\partial \alpha'} + \gamma(g) - 1\right\} \times \Gamma^{(1,1)}(E,\mathbf{k},g,\alpha',E_N) = 0, \tag{4.19}$$

where $E_N < 0$ is the arbitrary renormalization point of the renormalized Green's function $\Gamma^{(1,1)}$, and the renormalized intercept $\alpha(0)$ is fixed at one. The infrared behavior is governed by a stable fixed point at $g = g^*$ determined by $\beta(g^*) = 0$. By applying the ϵ-expansion about $d = 4$ (where $d = 4 - \epsilon = 2$ transverse dimensions is physical), the anomalous dimension

$$\eta = -\gamma(g^*) = \frac{\epsilon}{12}[1 + 0.64\epsilon] + O(\epsilon^2) \simeq \tfrac{1}{3} \quad \text{at } \epsilon = 2 \tag{4.20}$$

was calculated. At $\mathbf{k} = 0$, $(\partial/\partial\alpha')\Gamma = 0$, so $[E(\partial/\partial E) + \gamma(g^*) - 1]\Gamma = 0$ or $\Gamma \sim (E)^{\eta+1}$ leading to

$$\sigma_{\mathrm{Tot}}(s) \sim (\ln s)^{\eta}. \tag{4.21}$$

Although the convergence properties of the ϵ-expansion look bad, the existence of a fixed point has probably been established.

In addition to this solution, a "weak coupling" solution has been found (Brower and Ellis, 1974) by adding a derivative interaction $i\frac{1}{2}(\lambda_0)[\Psi^+\boldsymbol{\partial}\Psi\cdot\boldsymbol{\partial}\Psi + \Psi\boldsymbol{\partial}\Psi^+\cdot\boldsymbol{\partial}\Psi^+]$. This solution has a fixed point at $(g^*,\lambda^*) = (0,0)$, and leads to constant cross sections. Now let us discuss these solutions from the standpoint of the decoupling theorems.

Arbitrariness

The most obvious shortcoming of the Gribov field theories is the obvious lack of uniqueness, corresponding to the choices for $\alpha(-\mathbf{k}^2)$ and $\mathcal{L}_{\mathrm{int}}$ One possibility is that only a small subset of these theories obeys s-channel unitarity

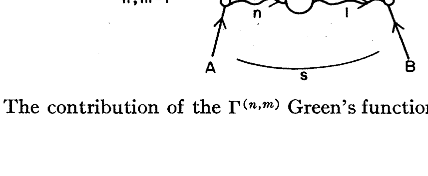


FIG. 31. The contribution of the $\Gamma^{(n,m)}$ Green's functions to $A_{\mathrm{el}}(s,t)$.

constraints. For example, we may consider the "weak coupling" solution in this light.

Weak coupling

Unitarity in the s-channel may as in the decoupling theorem demand a triple Pomeron zero. Indeed, the derivative term $i\frac{1}{2}\lambda(\Psi^{+}\partial\,\Psi\cdot\partial\Psi + \text{h.c.})$ introduced above gives precisely a nonsense wrong signature zero

$$\mathbf{k}_1\cdot\mathbf{k}_2 \sim \alpha(t) - \alpha(t_1) - \alpha(t_2) + 1$$

and in the weak coupling solution it is maintained in every order of the perturbation expansion. Hence, the objection that this solution is "unlikely" because it requires a constraint on g as a function of λ can be answered by the claim that unitarity in the s-channel forces this constraint. This position would be more attractive if we could understand the constraint as the result of an underlying symmetry on the Lagrangian.[32] While we are aware of the gauge property in the dual theory that gives rise to these zeros (see Appendix C), as yet such a gauge has not been formulated for the Gribov Lagrangian.

Further consistency of the "weak coupling" scheme with s-channel unitarity is still an open question. Although the Froissart bound is guaranteed, the conflicts with the elastic couplings presented in Secs. II and III require a knowledge of the renormalized couplings of the Pomeron and its cuts to the lower Regge trajectories and to the external particles. These extensions of the "weak coupling" solution have not yet been computed, so we are not certain whether the conflicts with s-channel unitarity will persist in the presence of the infinite number of "soft" cuts.

Strong coupling

In the strong coupling scheme there is another rebuttal for the challenge of arbitrariness. Except on a measure zero subset of interaction Lagrangians $\mathcal{L}_{\text{int}}$, there will occur in the iteration of the renormalization group the tri-linear term $i\Psi^{+}\Psi(\Psi + \Psi^{+})$, which will control the critical behavior of the theory.[33] A common "folk theorem" of critical phenomena is that the exponents (e.g., η) are independent of all but the most basic features of the theory, e.g., symmetries, dimensions of space, etc. (see Kadanoff, 1971; White, 1974; Brower *et al.*, 1975). If this be the case, some features of the "strong coupling" solution are very model independent.

This raises the vital question: Is "strong coupling" consistent with s-channel unitarity? So far even the Froissart bound ($\eta \leq 2$) has not been proven to hold for "strong coupling." However, Migdal, Polykov, and Ter-Martirosyan (1974b) have given plausible extensions of the "strong coupling" theory to production amplitudes (inclusive and exclusive) and checked the s-channel sum rules of this paper. It is encouraging that in their calculations to first order in $\epsilon = 4 - d$, they find no violation of the unitarity sum rules. Although this is far from a proof of consistency, an interesting screening mechanism is at work which may restore s-channel unitarity.

[32] In analogy with asymptotically free non-Abelian gauge theories, the special scaling properties are then a result of a special symmetry property.

[33] An exception would be $\mathcal{L}_{\text{int}}$ with only even power of Ψ, but this has a special symmetry $\Psi \to -\Psi$.

Clearly, these new results in the Gribov theory are encouraging, although more work on calculational techniques and extensions of the calculus to include multibody production in the s-channel is needed. Further work may well settle the question of whether the infrared properties of multi-Pomeron cuts do indeed resolve all conflicts with s-channel unitarity.

C. Phenomenological implications

We have discussed briefly above several theoretical alternatives. However, we would like to stress that diffractive behavior will be highly constrained by the s-channel unitarity conditions of Sec. II regardless of the precise nature of the j-plane fine structure (i.e., powers of $\ln s$ for $s \to \infty$). As an illustration of this point we discuss briefly the partial cross section $\sigma^{(n)}$ for the production of n clusters—see Sec. II.B.2, and Fig. 19.

It is easy to trace the origin of the behavior $\sigma^{(n)} \sim (\ln s)^{n-1}$ in Eq. (2.57). The multi-Regge phase space contributes $(\ln s)^{n-2}$ through the ζ_i integrals, the integrals over the cluster masses μ_i give $(\ln s)^n$, and the integrals over the t_i give $(\ln s)^{n+1}$ coming from the shrinkage of the diffraction peaks. In order to satisfy $\sigma^{(n)} \leq \sigma^{\text{Tot}}$, the growth of $\sigma^{(n)}$ caused by increasing phase space must be compensated for by the shrinkage of the diffraction peaks and/or zeroes in the amplitude at $t_i = 0$. Suppose now there is some complicated fine structure in the j-plane leading to a $\sigma^{(n)}$ which behaves as $\zeta_i^{2\eta_i}$ for large $\zeta_i = \ln s_{i,i+1}$, behaves as $\mu_i^{\bar{\eta}_i}$ for large $\mu_i = \ln M_i^2$, and which has diffraction peaks shrinking as $(\zeta_i - \mu_i - \mu_{i+1})^{-\nu_i}$ and vanishing as $t_i^{2\lambda_i}$ as $t_i \to 0$. It is easy to see that we then have $\sigma^{(n)} \sim (\ln s)^k$, where

$$k = \sum_{i=1}^{n-1} (2\eta_i + 1) + \sum_{i=1}^{n} (\bar{\eta}_i + 1) - \sum_{i=1}^{n-1} \nu_i(2\lambda_i + 1) - 1 \tag{4.22}$$

for $n \geq 2$. The constraint on the parameters η_i, $\bar{\eta}_i$, λ_i, and ν_i which follows from (2.49) is very general and thus rather obscure. As a first illustration, suppose there is a weak generalized factorization and the parameters are independent of i and n. Then we have

$$(n-1)[2\eta + \bar{\eta} + 2 - \nu(\lambda + 1)] + \eta \leq \eta \tag{4.23}$$

which for $n \geq 2$ becomes

$$2\eta + \bar{\eta} + 2 \leq \nu(\lambda + 1). \tag{4.24}$$

Thus if η and $\bar{\eta}$ are not negative a certain amount of shrinkage and/or vanishing at $t_i = 0$ is required.[34] For the weak coupling solution $\eta = \bar{\eta} = 0$, the inequality is satisfied with shrinkage ($\nu = 1$) and a linear zero in the triple Pomeron ($\lambda = 1$). For the strong coupling scheme, $\eta = \bar{\eta} = \epsilon/12$ and $\nu = 1 + \epsilon/24$ but the effective triple Pomeron vanishes just so as to satisfy the inequality. Indeed this iteration is effectively the renormalization of the propagator that gives rise to the anomalous dimension η. As a second illustration, suppose $\sigma_{\text{Tot}} \sim \ln^2 s$ and

$$d^2\sigma/d\mu dt \sim (\zeta - \mu)^{\eta}\mu^{\bar{\eta}} f(x,t)$$

with $\bar{\eta} = 2$ so that $d^2\sigma/d\mu dt \sim \ln^2 s$ for fixed x and

$$\int dt\, f(x,t) \simeq 1/(\zeta - \mu)^2$$

[34] Consistency for the single triple Pomeron contribution $\sigma^{(1)}$ assuming $\eta = 0$ has also been studied by Sivers (1973).

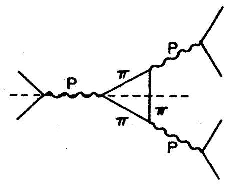


FIG. 32. Pion pole contribution to inclusive sum rule.

so that the shrinkage is $\ln^2 s$ as for the elastic cross section. Then $\sigma^{(1)} < \sigma_{\rm Tot}$ requires that $\eta = 1$. For $\eta = 1$, the cross section for single diffractive dissociation for fixed M^2 goes like $\ln s$. This is the same behavior given by Cheng and Wu (1971). On the other hand, Caneschi and Schwimmer (1972a,b) found $\eta = -1$ in a specific self-consistent model for absorptive effects on the model (Finkelstein and Zachariasen, 1971).

We would also like to stress that the constraints on diffractive processes have direct bearing on the validity of the Chew–Low extrapolation. Thus if the pion pole does give a good approximation in production amplitudes, the existence of multiple pion exchange and diffractive pion–pion scattering implies multiple diffractive behavior (e.g., multiple Pomeron exchange) in the production amplitudes. One is then directly led to the Finkelstein–Kajantie decoupling problem. The same reasoning applies to the inclusive sum rules; thus the pion pole gives a contribution of the form Fig. 32 to the sum rule (2.34) or (2.36).[35] Therefore, as we have mentioned in the discussion of alternative (ii) above, circumvention of the decoupling theorems may force us to abandon simple properties like the Chew–Low extrapolation for multiparticle amplitudes.

The discussion above has been essentially devoted to theoretical questions about the asymptotic behavior of diffractive scattering. The alternatives we have discussed are distinguished by the detailed nature of the Pomeron fine structures, i.e., powers of $\ln s$ as $s \to \infty$. However, for $\ln s$ large but finite these alternatives may be difficult to distinguish experimentally. For example, we can expand the eikonal form (4.12) in $\epsilon \ln s$ and $(\alpha'/a) \ln s$ to get

$$\sigma_{\rm Tot} \sim 4\pi^2 \left[\lambda(1 - \epsilon \ln s) - \frac{\lambda^2(1-2\epsilon \ln s)}{4(a + \alpha' \ln s)} \right]$$

$$\sim 4\pi^2 \lambda \left[1 + \left(\frac{\lambda\alpha'}{4a} + \frac{\lambda\epsilon}{2a} - \epsilon \right) \ln s \right] + \cdots . \quad (4.25)$$

Since $s^{-\epsilon} \simeq 1 - \epsilon \ln s$, and $[1 + (\alpha'/a) \ln s]^{-1} \simeq 1 - (\alpha'/a) \ln s$ the precise nature of the j-plane singularity may not be phenomenologically important. For example, temporarily rising cross sections can be obtained for $\epsilon < 0$; $\epsilon = 0$, and $\epsilon > 0$.

Indeed the recent data from ISR seems to indicate that we are in such a nonasymptotic regime. For example, $\alpha'/a \simeq \frac{1}{10}$ and $\ln s \simeq 8$. Therefore the s-channel unitarity constraints may not yet be coming into force in the asymptotic form in which we have discussed them. Since $(\alpha'/a) \ln s$ is not large, shrinkage of the diffraction peak is not yet playing a major role. This changes the role of various contributions to $\sigma_{\rm Tot}$ considerably. Thus the triple Pomeron contribution (2.8) is growing like $\ln s$. This growth is a result of the increasing phase space for the process and is independent of the detailed behavior of the amplitude, including whether or not $f_{\hat{P}PP}(0)$ vanishes [only the domination of the shrinkage factor causes $t = 0$ to dominate in (2.18)]. Similarly the cross sections for multiple diffractive dissociation $\sigma^{(n)}$ grow like $\ln s^{2(n-1)}$. A very interesting phenomenology has grown up based on the identification of the $\ln s$ triple Pomeron term with the apparent growth of $\sigma_{\rm Tot}$ seen at ISR (Amati *et al.*, 1973; Finkelstein, 1973; Capella *et al.*, 1973; Kronenfeld, 1973).

Thus we conclude by concurring with the point of view which is becoming more and more popular: because of the smallness of multiple-diffractive dissociation, the detailed nature of the Pomeron may be unimportant for understanding phenomenology at present energies. The same phenomenology can result from $\epsilon < 0$, $\epsilon = 0$, or $\epsilon > 0$.

Thus for phenomenological purposes for the present nonasymptotic energies, an expansion in powers of $(\ln s)^{-1}$ about $\sigma_{\rm Tot} =$ const (i.e., a Laurent expansion about $j = 1$) may be more appropriate than an expansion about $\sigma_{\rm Tot}(\infty)$. If the relative strengths of the terms in such an expansion are adequately constrained by unitarity, a predictive scheme might evolve without a commitment on the exact singularity structure in the j-plane.

ACKNOWLEDGMENTS

We wish to acknowledge many fruitful conversations with H. Abarbanel, M. Baker, J. Bjorken, J. Bronzan, J. Cardy, L. Caneshi, M. Ciafaloni, A. White, and F. Zachariasen. Also one of us (R.C.B.) wishes to express his gratitude for the support of the Santa Cruz physics group.

APPENDIX A. STRUCTURE OF THE TRIPLE REGGE VERTEX

In this appendix we discuss in more detail the conditions imposed on the triple Regge vertices by the Schwartz inequalities discussed in Sec. II.A.2. In order to do this we need to study the analytic structure of the triple Regge vertex in some detail.

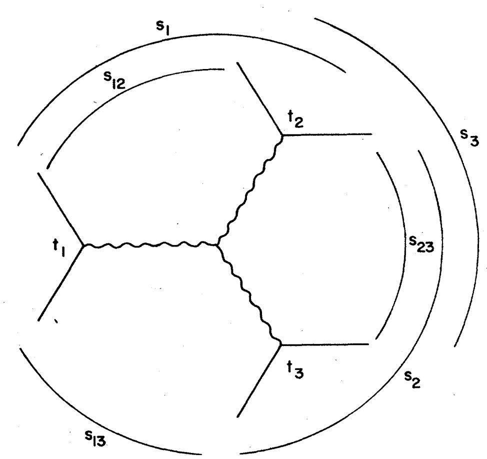


FIG. 33. Variables for the triple Regge vertex. The lines can represent groups of particles.

[35] However, the pion pole does not necessarily give a lower bound for $f_{\hat{P}PP}$. As we noted in Sec. III.B, the contribution of an exchange degenerate trajectory, Eq. (3.25), can cancel against it.

The triple Regge vertex is a function of the three momentum transfers and two Toller angles. It is convenient to use the three variables η_{12}, η_{23}, η_{31} instead of the Toller angles, since the former are simply related to the invariants:

$$\eta_{ij} = \frac{1}{\kappa_{ij}} = \frac{s_{ij}}{s_i s_j} \sim \frac{2(t_i t_j)^{\frac{1}{2}}\cos\omega_{ij} + t_k - t_i - t_j}{\lambda(t_i,t_j,t_k)}, \qquad \text{(A1)}$$

where the invariants are defined in Fig. 33 and the three η_{ij} are related by the constraint

$$\omega_{12} + \omega_{23} + \omega_{31} = 0.$$

The triple Regge contribution to a signatured amplitude which has only right-hand cuts in the s_i and s_{ij} [36] is then given by

$$A^{\tau_1\tau_2\tau_3} \sim \prod_{i=1,2,3} [\Gamma(-\alpha_i)(-s_i)^{\alpha_i}] V(t_i,t_2,t_3;\eta_{12},\eta_{23},\eta_{31}). \qquad \text{(A2)}$$

The absence of simultaneous discontinuities in overlapping invariants in the physical region (Weis, 1972) allows only the configurations of simultaneous discontinuities in the s_i and s_{ij} shown in Fig. 34. The vertex V must have singularities in the η_{ij} to satisfy this requirement. The nature of these singularities is easily obtained by requiring consistency of Eq. (A2) and Fig. 34 (DeTar and Weis, 1971).

$$\begin{aligned} V(t_1,t_2,t_3;\,\eta_{12},\eta_{23},\eta_{31}) &= (-\eta_{12})^{\alpha_2}(-\eta_{31})^{\alpha_3} V_{23}(t_1,t_2,t_3;\,\eta_{12},\eta_{23},\eta_{31}) \\ &+ (-\eta_{23})^{\alpha_3}(-\eta_{12})^{\alpha_1} V_{31}(t_1,t_2,t_3;\,\eta_{12},\eta_{23},\eta_{31}) \\ &+ (-\eta_{31})^{\alpha_1}(-\eta_{23})^{\alpha_2} V_{12}(t_1,t_2,t_3;\,\eta_{12},\eta_{23},\eta_{31}) \\ &+ (-\eta_{12})^{\frac{1}{2}(\alpha_1+\alpha_2-\alpha_3)}(-\eta_{23})^{\frac{1}{2}(\alpha_2+\alpha_3-\alpha_1)}(-\eta_{31})^{\frac{1}{2}(\alpha_3+\alpha_1-\alpha_2)} \\ &\times V_{12}(t_1,t_2,t_3;\,\eta_{12},\eta_{23},\eta_{31}), \end{aligned} \qquad \text{(A3)}$$

where the V_{ij} and V_{ijk} do not have such singularities in the η_{ij}.

The decoupling results of Sec. II apply to a discontinuity of the vertex across one of the Reggeons. The discontinuity across α_1 is the discontinuity in the energy s_1 of that Reggeon. From Fig. 34 we see that only the V_{23} contribution has such a discontinuity. Its contribution is

$$\frac{\sin\pi(\alpha_1-\alpha_2-\alpha_3)}{\sin\pi\alpha_1}\,\eta_{12}{}^{\alpha_2}\eta_{31}{}^{\alpha_3}V_{23}. \qquad \text{(A4)}$$

This is to be inserted in an amplitude with the full Regge propagators $\xi_2\Gamma(-\alpha_2)s_2{}^{\alpha_2}$ and $\xi_3\Gamma(-\alpha_3)s_3{}^{\alpha_3}$ and the cut propagapator $[\pi/\Gamma(\alpha_1+1)]s_1{}^{\alpha_1}$.

Let us now discuss the structure of V_{23} in more detail. Only V_{23} can contribute when both α_2 and α_3 are positive integers, since only it gives a residue which is a polynomial. The further poles for α_1 integral can only be accommodated by singularities at $\alpha_1-\alpha_2-\alpha_3$ integral. Requiring that the

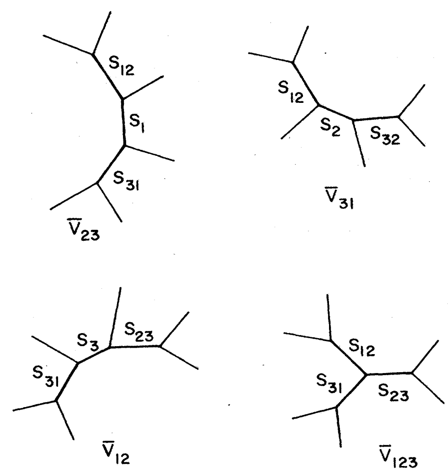


FIG. 34. Possible simultaneous discontinuities in asymptotic invariants in the triple Regge limit.

residues be polynomials gives (DeTar and Weis, 1971)

$$\begin{aligned} V_{23} = [\Gamma(-\alpha_1)\Gamma(-\alpha_2)\Gamma(-\alpha_3)]^{-1} &\sum_{i,j,k=0}^{\infty} \Gamma(-\alpha_2+i+k) \\ \times\,\Gamma(-\alpha_3+j+k)&\Gamma(-\alpha_1+\alpha_2+\alpha_3-i-j-2k) \\ \times\,\frac{1}{i!j!k!}\,\eta_{12}{}^{-i}\eta_{31}{}^{-j}&\left(\frac{\eta_{31}\eta_{12}}{\eta_{23}}\right)^{-k}\beta_{ijk}(t_1,t_2,t_3), \end{aligned} \qquad \text{(A5)}$$

where $\beta(t_1,t_2,t_3)$ is regular for t_i below threshold.[37,38]

We now consider the implications of the decoupling theorems. Suppose all three trajectories are Pomerons, and $t_1,t_2,t_3 \to 0$. From Eq. (A1) we see that η_{12} and η_{31} become infinite so only the term $i=j=k=0$ in (A5) remains. The decoupling of the Pomeron vertex (A4) then gives

$$(\alpha_1-\alpha_2-\alpha_3+1)\beta_{0,0,0} = 0 \qquad \text{(A6)}$$

for $t_1 \simeq t_2 \simeq t_3 \simeq 0$. This will be satisfied as long as $\beta_{0,0,0}$ is finite at $t_1=t_2=t_3=0$. Neglecting Regge cuts, this is just the requirement of no multiplicative fixed poles (DeTar and Weis, 1971). The helicity of the Pomeron α_1 is $\alpha_2+\alpha_3=2$ so $\alpha_1=1$ is a nonsense wrong-signature point and the *vanishing can arise from the nonsense wrong-signature zero* at $\alpha_1-\alpha_2-\alpha_3+1=0$.

The Schwartz inequalities give the vanishing of Eq. (A4) for the much more general situation where only one of the (uncut) trajectories is a Pomeron at zero momentum transfer, say $t_2=0$, $\alpha_2=1$. Only terms in Eq. (A5) with $i+k \le 1$ contribute. Furthermore, from Eq. (A1) we have

$$\eta_{12} = -\eta_{23} = (t_3-t_1)^{-1}$$

$$\eta_{31} = \frac{2(t_1t_3)^{\frac{1}{2}}\cos\omega_{13} - t_1 - t_3}{(t_1-t_3)^3}.$$

[36] In order to analyze the structure of the vertex we must *assume* that such signatured amplitudes exist. The contribution to the full amplitude is then a sum of the eight terms obtained by including terms with the pair of lines at each external vertex interchanged. Here we neglect the further signatures associated with the Toller angles which must be included to obtain the most general vertex; these will not affect the conclusions below. A complete discussion of signature is given for the simpler case of the double Regge vertex in Sec. III.B.

[37] Equations (A3) and (A5) have been explicitly verified in the ordinary dual model by DeTar and Weis (1971) and the nonlinear dual model by Sukhatme (1972).

[38] β can have simple pole singularities in the α_i if fixed poles are present. These do not affect our discussion. We have also assumed that the trajectories have Toller quantum number $M=0$, which excludes $(t_i)^{\frac{1}{2}}$ singularities.

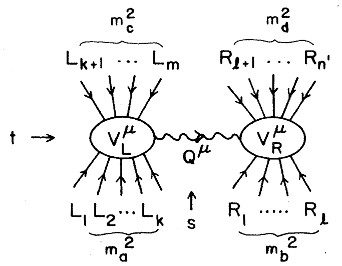


FIG. 35. Single-particle exchange in a many-particle amplitude.

Equating powers of the free variable η_{31} then gives

$$\frac{\Gamma(\alpha_1+1)\Gamma(-\alpha_3+j)}{\Gamma(\alpha_1-\alpha_3+j+1)\Gamma(-\alpha_3)}\eta_{12}\eta_{31}{}^{\alpha_3-j}[(\alpha_3-\alpha_1-j)\beta_{0,j,0} - (t_3-t_1)\beta_{1,j,0}+j\beta_{0,j-1,1}]=0. \quad \text{(A7)}$$

This condition will generally be satified by a relationship between the β_{ijk}. The nonsense wrong-signature zero clearly is not sufficient to satisfy Eq. (A7). In Appendix B we show that *the condition (A7) can be interpreted as the requirement that the Pomeron couple like a conserved vector current at $t=0$.*

We remark that Eqs. (A6) and (A7) are satisfied in the ordinary planar dual model. In this case all $\beta_{ijk}=1$. See Appendix C for further discussion.

APPENDIX B. KINEMATICS FOR THE POMERON AND CONSERVED VECTOR COUPLINGS

In this appendix we show that many of the Pomeron decoupling theorems (weak theorems) can be interpreted as the requirement that the Pomeron couple like a conserved vector current at $t=0$.

We first discuss the kinematics of vector particle exchange in an arbitrary amplitude (Fig. 35) at high energy s and $t=Q^2=0$. We group the momenta to form the quasi-four-particle amplitude for $a+b\to c+d$. In the rest frame of particle b

$$\begin{aligned} p_a &= (E_a,\mathbf{0},p_a),\\ p_b &= (M_b,\mathbf{0},0),\\ p_c &= (E_c,\mathbf{p}_\perp,p_{\parallel c}),\\ p_d &= (E_d,-\mathbf{p}_\perp,p_{\parallel b}). \end{aligned} \quad \text{(B1)}$$

For large s, one can show that $t=Q^2=(p_a-p_c)^2=0$ implies $\mathbf{p}_\perp=0$. Then

$$\begin{aligned} p_a &\simeq E_a(1,\mathbf{0},1),\\ p_b &\simeq M_b(1,\mathbf{0},0),\\ p_c &\simeq E_c(1,\mathbf{0},1),\\ Q &= p_a+p_c\simeq(E_a+E_c)(1,\mathbf{0},1). \end{aligned} \quad \text{(B2)}$$

Since $M_d^2=(Q-p_b{}^2)=(p_a+p_c-p_b)^2=M_b{}^2-2M_b(E_a+E_c)$, we have

$$E_a+E_c=(M_d{}^2-M_b{}^2)/2M_b.$$

For $M_b{}^2\neq M_d{}^2$ we thus have the important relation

$$p_a\sim p_c\sim Q \quad \text{(B3)}$$

for $s\to\infty$, $t=0$.

A vector exchange contribution will be an arbitrary linear polynomial in the overlapping invariants

$$A=\sum_{i=1}^{n}\sum_{j=1}^{n'}C_{ij}P_{Li}\cdot P_{Rj}, \quad \text{(B4)}$$

where C_{ij} is a function only of $P_{L_1}\cdot P_{L_m}$, $P_{R_1}\cdot P_{R_m}$, and Q^2. Choosing p_a to be the momentum P_{L_i} from (B3) we have

$$\frac{P_{Li}\cdot P_{Rj}}{P_{Li}\cdot P_R}=\frac{Q\cdot P_{Rj}}{Q\cdot P_R}, \quad \text{(B5)}$$

where P_R is a fixed one of the P_{R_j}. This holds as long as

$$P_R{}^2\neq(Q+P_R)^2,\quad P_{R_j}{}^2\neq(Q+P_{R_j})^2. \quad \text{(B6)}$$

Similarly

$$\frac{P_{Li}\cdot P_R}{P_L\cdot P_R}=\frac{P_{Li}\cdot Q}{P_L\cdot Q} \quad \text{(B7)}$$

for

$$P_L{}^2\neq(Q-P_L)^2,\quad P_{L_i}{}^2\neq(Q-P_{L_i})^2, \quad \text{(B8)}$$

where P_L is a fixed momentum in the set P_{L_i}. Combining Eqs. (B6) and (B7) we have the fundamental result

$$P_{Li}\cdot P_{Rj}=(Q\cdot P_{Li})(Q\cdot P_{Rj})\frac{P_L\cdot P_R}{(Q\cdot P_L)(Q\cdot P_R)}. \quad \text{(B9)}$$

Factorization of the vector exchange requires

$$C_{ij}=C_i(P_{L_k})C_j(P_{R_l}). \quad \text{(B10)}$$

Combining Eqs. (B4), (B9), and (B10) we have

$$A\sim[Q\cdot\sum_i C_iP_{Li}][Q\cdot\sum_j C_jP_{Rj}]\frac{P_L\cdot P_R}{(Q\cdot P_L)(Q\cdot P_R)}. \quad \text{(B11)}$$

Equation (B11) shows that the vector exchange amplitude has the remarkable property that the divergences of the vertices $V_L{}^\mu=\Sigma C_i\cdot P_{L_i}{}^\mu$ and $V_R{}^\mu=\Sigma C_i\cdot P_{R_i}{}^\mu$ are what determine the asymptotic behavior as $s\to\infty$ at $t=0$. Equation (B11) holds as long as momenta P_R and P_L can be chosen with

$$P_R\neq(Q+P_R)^2,\quad P_L{}^2\neq(Q-P_L)^2,$$

i.e., $Q\cdot P_R\neq 0$, $Q\cdot P_L\neq 0$.[39] This is always possible unless the set L or R consists of two equal mass particles. For example, if L consists of two equal mass particles we then have

$$A\sim C_{L_1}[Q\cdot\sum_j C_jP_{R_i}]\frac{P_L\cdot P_R}{Q\cdot P_R}, \quad \text{(B12)}$$

of, if both consist of only two equal mass particles,

$$A\sim C_{L_1}C_{R_1}P_{L_1}\cdot P_{R_1}. \quad \text{(B13)}$$

From Eq. (B11) or (B12) we see that decoupling of a vector exchange contribution in an inelastic process requires that the couplings of the particle satisfy the current conservation divergence condition

$$Q_\mu V_R{}^\mu(P_{R_l})=0\quad\text{for}\quad Q^2=0. \quad \text{(B14)}$$

[39] The conditions $P_{R_j}{}^2\neq(Q+P_{R_j})^2$ are generally satisfied except for isolated values of the invariants associated with the cluster. We can stay away from such points.

Conversely, current conservation implies the vanishing of the leading asymptotic term at $t = 0$ except for elastic processes (B13). Thus we recover the well-known property of photon exchange, that only elastic cross sections have the Coulomb singularity for large s.

Since the Pomeron has wrong signature at $\alpha = 1$, it does not correspond to a pure vector particle exchange and the decoupling theorems cannot generally be translated into divergence conditions (B14). However, we can isolate pieces of the Pomeron exchange amplitude which do have the same structure as a simple vector particle exchange. For example, in Sec. III.A. we showed that the Pomeron–Reggeon–particle vertex gave a contribution to the five-particle amplitude

$$A_5 \sim s^{\alpha_P} s_R^{\alpha_R - \alpha_P} \xi_P \xi_{RP} \Gamma(-\alpha_P)\Gamma(-\alpha_R) V_P + s^{\alpha_R} s_P^{\alpha_P - \alpha_R} \xi_R \xi_{PR} \Gamma(-\alpha_P)\Gamma(-\alpha_R) V_R. \tag{B15}$$

The first term has the signature factor ξ_P and thus only the residue of the pole in $\Gamma(-\alpha_P)\Gamma(-\alpha_R)V_P$ contributes. This residue shares all the properties of a normal vector exchange amplitude. Thus the vanishing of this part of the vertex can be translated into a divergence condition like (B14).

We recall that vanishing of V_P in Eq. (B15) was obtained from the Schwartz inequality constraints as well as from the inclusive sum rule constraints where the vanishing of V_R was also obtained. Quite generally the decoupling theorems obtained from the Schwartz inequalities are equivalent to divergence conditions. These theorems involve an amplitude like that in Fig. 36. If we assume the existence of signatured amplitudes with only right-hand cuts in s and either right- or left-hand cuts in the s_i, we can write the full amplitude as

$$A \sim \beta(-s)^{\alpha_P} V(s_i/s,\ P_i \cdot P_j) + \tau_P \beta s^{\alpha_P} \times V(s_i/s e^{i\phi},\ P_i \cdot P_j), \tag{B16}$$

where $\phi = 0$ if s_i has a right-hand cut and $\phi = -2\pi$ if s_i has a left-hand cut. If we decompose V into a piece V_P which has no singularities in s_i/s and a remainder piece V_R, we have

$$A \sim \beta \xi_P s^{\alpha_P} V_P(s_i/s,\ P_i \cdot P_j) + \beta s^{\alpha_P} [e^{-i\pi\alpha_P} V_R(s_i/s,\ P_i \cdot P_j) + \tau_P V_R(s_i/s e^{i\phi},\ P_i \cdot P_j)]. \tag{B17}$$

The V_P piece has the signature factor ξ_P so only the residue of the pole at $\alpha = 1$ contributes and $V_P = 0$ is equivalent to the divergence condition (B14). The V_R piece does not have the structure of a vector particle exchange. The discontinuity of the amplitude in M^2 (Fig. 36) by the Steinman relations should be a regular function of the s_i and thus be entirely in V_P. Since the Schwartz inequalities require only the vanishing of the discontinuity in M^2 they are equivalent to divergence conditions.

The Schwartz inequalities only require the vanishing of the M^2 discontinuity for large M^2. Let us discuss briefly, however, the consequences of a Pomeron with conserved vector couplings for all M^2. By conserved vector coupling we mean the vanishing of the V_P contribution in Eq. (B17) or, more generally, those contributions to Pomeron exchange which are proportional to ξ_P and thus pure spin one. The fact that these requirements are equivalent to divergence conditions immediately suggests a Pomeron–photon analogy. Such an analogy has been proposed by Ravndal (1971).

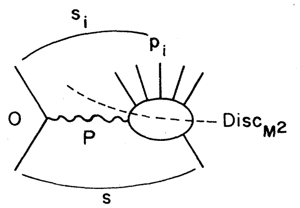


FIG. 36. Decoupling theorem from Schwartz inequality.

Since he has developed it only for quasi-two-body reactions (e.g., resonances in M^2) which clearly have only a V_P part, his use of the analogy is permissable. Independent of the validity of the Pomeron–photon analogy and the consequent relations between electromagnetic and diffractive processes, the conserved vector coupling of the Pomeron implies the vanishing of the diffractive production of resonances at $t = 0$. Furthermore, at $t = 0$ the Deck mechanism would receive contributions only from the real parts of the exchanges in the blob on the right of Fig. 36 (e.g., the parts with left-hand cuts in the s_i). Thus diffractive production would take place only through real Deck-like mechanisms and not resonance production at $t = 0$. [See discussion in Sec. IV, and Appendix C, Eq. (C19).]

APPENDIX C. GAUGE INVARIANT (POMERON) COUPLINGS IN THE DUAL MODEL

We have maintained throughout that the decoupling theorems fall into two classes: strong theorems that conflict with elastic couplings and weak theorems that do not conflict with these couplings. In Sec. III we showed that the analyticity requirements for Regge residues force this conflict between strong theorems and elastic couplings. Here we show that no conflict *based on analyticity* can be formulated for weak theorems. To do this we construct an explicit example of the Pomeron couplings which satisfies all weak theorems with nonzero elastic couplings and proper analytic properties.

Our example is provided by the dual resonance model (Veneziano, 1972) with a trajectory at $\alpha(0) = 1$ of positive signature. The existential question of whether this *is* the Pomeron is not relevant here, since we only wish to find a realization of the weak decoupling theorems. When and if a dual model is found with an output (second order) diffractive pole at $\alpha(0) = 1$ (present models give $\alpha^{\text{out}}(0) = 2$!), we may investigate its decoupling properties.

Our example is the standard dual beta function (Veneziano, 1972) $B_N(p_1, p_2, \ldots, p_N) = B_N(\alpha_{ij})$ for N particles with momenta p_i and trajectories

$$\alpha_{ij} = \alpha' s_{ij} + \alpha_{ij}(0) \tag{C1}$$

in the channel $s_{ij} = (p_i + p_{i+1} + \cdots + p_j)^2$. The full amplitude $A_N(p_i, \ldots, p_N)$ with singularities in all channel invariants is given by summing over the permutations of the external lines. (See Fig. 37.) B_N is given by

$$B_N = \int_{z_i < z_j} \prod_{i=1}^{N} \frac{dz_i}{dV} \prod_{i<j} (z_j - z_i)^{-\Delta_{ij}}$$

$$\Delta_{ij} = \alpha_{i,j} + \alpha_{i+1,j-1} - \alpha_{i,j-1} - \alpha_{i+1,j}$$

$$dV = \frac{dz_a dz_b dz_c}{|z_a - z_b|\,|z_b - z_c|\,|z_c - z_b|}, \text{ any } a, b, c. \tag{C2}$$

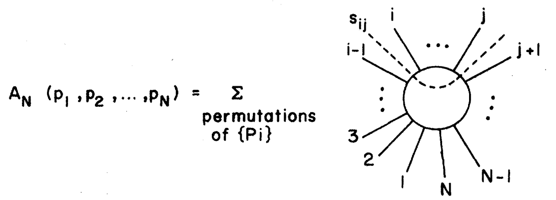


FIG. 37. N-particle amplitude $A_N(p_i, \ldots, p_N)$ as a sum over permutations of $B_N(p_1, p_2, \ldots, p_N)$ with poles in subenergies of adjacent lines.

These amplitudes are a remarkably simple laboratory for multi-Regge theory compared to Φ^3 theory where each multi-Regge limit requires the summation over a *different* infinite set of graphs. Here one $N-3$ dimensional integral possesses all multi-Regge limits!

It is well known that when all $\alpha_{ij}(0) = 1$, the dual model has strong gauges that remove all ghost states (Brower, 1972; Goddard and Thorn, 1972); however, earlier it was realized that less stringent constraints give leading trajectory gauge conditions (Brower and Weis, 1971). (This fact was used by us in an attempt to introduce conserved currents into the model.) Taking the "Pomeron" to have $\alpha(0) = 1$ and all other trajectories with like quantum numbers to have the same intercepts $\alpha(0) < 1$, the state at $j = 1$ in the Pomeron channel is a gauge vector particle.

For example consider the diffraction production amplitude of Fig. 38, with $\alpha_{12}(0) = \alpha_P(0) = 1$ and channels that differ by the Pomeron (vacuum) line to have like intercepts

$$\alpha_{1j}(0) = \alpha_{3j}(0);\ j = 4, \ldots, N-1. \tag{C3}$$

We symmetrize the amplitude in p_1 and p_2 to give positive signature to the Pomeron trajectory $\alpha_{12} = \alpha_P$. By inserting this into the left-hand side of the Schwartz inequality of Eq. (2.24), one can see that *all* the weak decoupling theorems involve *discontinuities* of this amplitude in $M^2 = s_{3,N-1}$ for large $s = s_{1N}$.

We shall now show that this amplitude vanishes as $t_{12} = t \to 0$. The B_N function in the configuration of Fig. 39 [choosing $(z_a, z_b, z_c) = (z_{N-2}, z_{N-1}, z_N) = (1, \infty, 0)$] is

$$\begin{aligned} B_N = \int_0^1 & dx_1 dx_2 \cdots dx_{N-3} I(x_3, \cdots, x_{N-3}) \\ & \times [x_1^{-\alpha_{N1}-1}(1-x_1)^{-\alpha_P(t)-1}(1-x_1x_2)^{-\Delta_{13}} \cdots \\ & (1-x_1\cdots x_{N-3})^{-\Delta_{1,N-2}}] \\ & \times [x_2^{-\alpha_{N2}-1}(1-x_2)^{-\alpha_{23}-1}(1-x_2x_3)^{-\Delta_{24}} \cdots \\ & (1-x_2\cdots x_{N-3})^{-\Delta_{1,N-2}}]. \end{aligned} \tag{C4}$$

In the Regge limit we make the substitution $x_1 = 1 + y/s$ for $s = s_{N1} \to -\infty$. We note that $\Delta_{2j} = -\Delta_{1j} + (\Delta_{1j} + \Delta_{2j})$

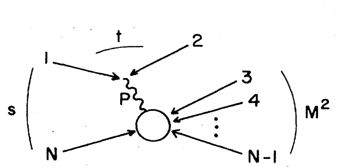


FIG. 38. Dual amplitude B_N with $\alpha_{12}(t) = \alpha_P$, and a nonzero discontinuity in M^2.

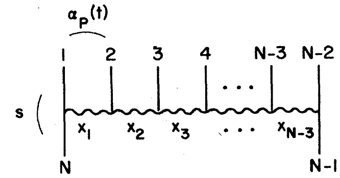


FIG. 39. The B_N configuration for the calculation of the Regge limit.

$= -\Delta_{1j} + 2\alpha' q \cdot p_j$ for $q = p_1 + p_2$ because of the trajectory condition $[\alpha_{1j}(0) = \alpha_{3j}(0)]$, also that as $|s| \to \infty$, $\Delta_{1j} \sim 2\alpha' p_i \cdot p_j \sim -s\alpha' \eta_j$ for η_j fixed $(0 < \eta_j < 1)$. Taking $\alpha' = 1$, and $\rho_j = x_2 x_3 \cdots x_{j-1}$ with the approximation

$$\begin{aligned} & (1 - x_1\rho_j)^{-\Delta_{1j}}(1-\rho_j)^{-\Delta_{2j}} \\ & \simeq \left(1 - \frac{y\rho_j}{s(1-\rho_j)}\right)^{s\eta_j}(1-\rho_j)^{-2q\cdot p_j} \\ & \simeq \exp\left[-\frac{y\eta_j\rho_j}{1-\rho_j}\right](1-\rho_j)^{-2q\cdot p_j}, \end{aligned} \tag{C5}$$

we can do the y integral from 0 to ∞ to get

$$\begin{aligned} B_N = & (-s)^{\alpha_P(t)}\Gamma[-\alpha_P(t)]\int dx_2 \cdots dx_{N-3} I(x_3, \ldots, x_{N-3}) \\ & \times \left(1 + \eta_3 \frac{x_2}{1-x_2} + \eta_4 \frac{x_2x_3}{1-x_2x_3} + \cdots \right. \\ & \left. + \eta_{N-2}\frac{x_2\cdots x_{N-3}}{1-x_2\cdots x_{N-3}}\right)^{\alpha_P(t)} [x_2^{-q^2-2q\cdot p_2-1}(1-x_2)^{-2q\cdot p_3} \\ & \times (1-x_2x_3)^{-2q\cdot p_4}\cdots(1-x_2\cdots x_{N-3})^{-2q\cdot p_{N-2}}]. \end{aligned} \tag{C6}$$

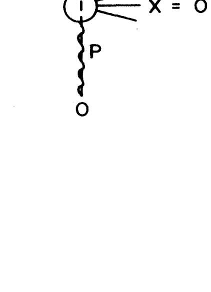
The factor $(-s)^{\alpha_P}$ exhibits the right-hand cut in s. Adding the term with $p_1 \leftrightarrow p_2$ replaces $(-s)^{\alpha_P}$ by $(s)^{\alpha_P}(1 + e^{-i\pi\alpha_P}) = s^{\alpha_P}\xi_P$. Now as we take $t_{12} = t \to 0$ $(\alpha_P \to 1)$ we pick out the residue of the pole in $\Gamma(-\alpha_P)$, which is zero by a gauge condition. From Appendix B (B5) we have the kinematic condition as $t = q^2 \to 0$ that

$$\eta_j = -\frac{p_i \cdot p_j}{p_i \cdot p_N} \simeq -\frac{q\cdot p_j}{q \cdot p_N} \tag{C7}$$

so that the residue at $\alpha_P(t) = 1 (q^2 = 0)$ is a perfect differential in x_1.

$$\begin{aligned} B_N \sim \int_0^1 & dx_2 \frac{d}{dx_2}[x_2^{-2q\cdot p_2-1}(1-x_2)^{-2q\cdot p_3} \\ & \cdots (1-x_2\cdots x_{N-3})^{-2q\cdot p_{N-2}}] = 0. \end{aligned} \tag{C8}$$

This proves *all* the weak decoupling theorems in our dual constructs, since they all follow from taking a discontinuity in M^2 as $M^2 \to \infty$, with $\alpha_{N,N-1} = \alpha_P$.



(C9)

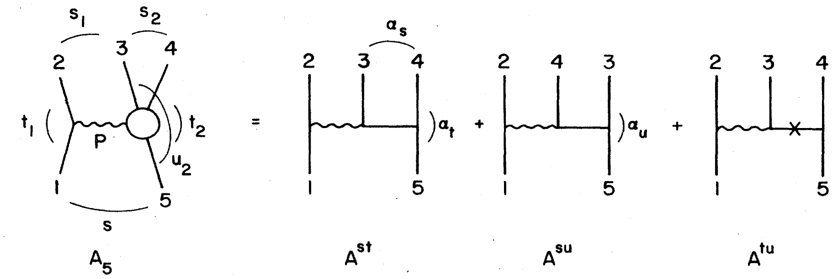


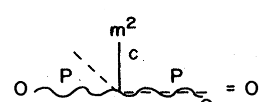
FIG. 40. The three contributions to Pomeron exchange in A^5: A^{st}, A^{su}, and A^{ut} have poles in (s_2,t_2), (s_2,u_2), and (u_2,t_2), respectively. (Dualists use the cross to indicate the twist operator that exchanges 4 and 5.)

In this example the Pomeron does not completely decouple at $t = 0$, because there are contributions involving the exchange of the p_N and p_i ($i = 3, \ldots, N-1$) lines in the dual function. These have no poles in M^2, so they do *not* contribute to discontinuity in M^2 (of the Schwartz inequality), but they *do* give contributions at $t = 0$. To see this one must take care in defining the signature phase in the physical limit [see Appendix B, Eq. (B16)].

The factor $[1+\eta_3 x_2/(1-x_2)+\cdots]^{\alpha_P}$ gives cuts in the η planes which can in general give additional phases not included in $\xi_P = 1+e^{-i\pi\alpha_P}$. We considered the sum of two terms which were real as $s\to(\mp\infty)$ and $u_j = 2p_i\cdot p_j \to (\pm\infty)$, respectively, so that all $\eta_j = -u_j/s$ are held positive and fixed throughout. However, if p_N and p_k are interchanged, the functions would be real for $s\to(\pm\infty)$ and $u_{j\neq k}\to(\pm\infty)$, so we have real amplitudes for $\eta_j < 0$ and continue in the η plane to the physical region. Using the $+i\epsilon$ prescription gives $\eta\to|\eta|\pm i\epsilon$ for the two terms [(1,2,...) and $1\leftrightarrow 2$ interchanged] so, as we show below in the case $N = 5$, phases from discontinuities in η contribute. For these pieces we cannot use the gauge identities relating to the wrong-signature pole in $\Gamma(-\alpha_P)$ at $\alpha_P = 1$.

It is instructive to consider the special cases [Eqs. (2.27)–(2.29)] that follow from the general theorem (C9) as they are realized in the dual theory.

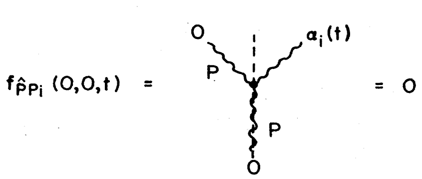
The first decoupling theorem in the dual model was discovered by Gordon (1971) for $X = (p_3, p_4, \ldots, p_{N-1})$ replaced by a single particle.



(C10)

Here the restriction to $\alpha_{1j}(0) = \alpha_{3j}(0)$ forces c to lie on the Pomeron trajectory ($\alpha' m_c^2 = -1, 1, 3, \ldots$).

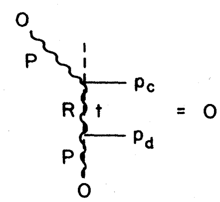
For X replaced by a Reggeon



(C11)

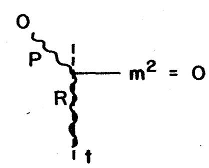
we obtain decoupling for the case $\alpha_i(t) = \alpha_P(t)$. In Appendix A, the general manner in which this is satisfied is thoroughly discussed.

For $X = (p_c, p_d)$ in the limit $s_{cd}\to\infty$ we have



(C12)

for c and d on the trajectory α_R [$m_c^2 = m_d^2 = -\alpha_R(0)/\alpha'$]. As discussed in the text this leads immediately to



(C13)

The importance of the discontinuity in these theorems may be easily studied in this example. We consider Eq. (C6) for $N = 5$ ($x = 1 - x_2$) symmetrized in p_1 and p_2 (see Fig. 40 for invariants)

$$A_5^{st} = s^{\alpha_P}\xi_P\Gamma(-\alpha_P)\int_0^1 dx \times [x+\eta(1-x)]^{\alpha_P} x^{-\alpha_t(t_2)-1}(1-x)^{-\alpha_s(s_2)-1}, \qquad \text{(C14)}$$

where $\xi_P = (1+e^{-i\pi\alpha_P})$. Since $\eta = \eta_3 = s_1/s > 0$, the entire phase comes from the factor ξ_P. As $t_2\to 0$ ($\alpha_P\to 1$), we have

$$\eta\to -\frac{q\cdot p_3}{q\cdot p_5} = \frac{M_3^2 - t_2}{s_2 - M_1^2} = -\frac{\alpha_t}{\alpha_s}$$

and the integrand is an exact differential, so $A^{st}\to 0$.

In addition to the A^{st} contribution above with poles in s_2 and t_2, there is a *second* contribution with poles in s_2 and u_2

$$A_5^{su} = s^{\alpha_P}\xi_P\Gamma(-\alpha_P)\int_0^1 dx \times [x+\eta'(1-x)]^{\alpha_P} x^{-\alpha_u(u_2)-1}(1-x)^{-\alpha_s(s_2)-1} \qquad \text{(C15)}$$

with $\eta' = -p_1\cdot p_4/p_1\cdot p_5 > 0$ and the phase given entirely by ξ_P; and there is a *third* contribution with poles in u_2 and t_2 (see Fig. 40). In the third term,

$$\eta\to\eta'' = -p_1\cdot p_3/p_1\cdot p_4 \simeq \frac{s_1}{s_1 - s} = \frac{\eta}{\eta - 1} < 0 \qquad \text{(C16)}$$

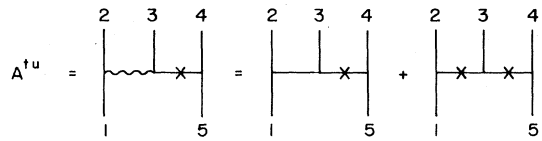


FIG. 41. The two contributions to the positive signature Pomeron, $A^{tu} = B(p_1,p_2,p_3,p_5,p_4) + B(p_2,p_1,p_3,p_5,p_4)$, respectively.

so that care must be taken in defining the phase of $[x + \eta''(1-x)]^{\alpha_P}(\pm\eta s/\eta'')^{\alpha_P} = (\pm s)^{\alpha_P}(\eta - x)^{\alpha_P}$. The $(-s)^{\alpha_P}$ term (see Fig. 41) coming from $B(p_1,p_2,p_3,p_5,p_4)$ is real for $s > 0$ and $s_1 < 0$, so the phase by the $+i\epsilon$ prescription coming from the cut in η is $s^{\alpha_P}[x - (\eta + i\epsilon)]^{\alpha_P}$. But the piece coming from $B(p_2,p_1,p_3,p_5,p_4)$ is real for $s < 0$ and $s_1 > 0$, so the phase is $e^{-i\pi\alpha_P}s^{\alpha_P}[x - (\eta - i\epsilon)]^{\alpha_P}$, Consequently the correct expression for the A^{tu} term [in contrast to that given in the literature (Doren *et al.*, 1971)] is

$$A_5{}^{tu} = s^{\alpha_P}\Gamma(-\alpha_P)\left[\xi_P\int_0^1 dx[x-\eta+i\epsilon]^{\alpha_P}x^{-\alpha_t(t_2)} \times (1-x)^{-\alpha_u(u_2)-1} + 2i\sin\alpha_P e^{-i\pi\alpha_P} \times\int_0^\eta dx[\eta - x]^{\alpha_P}x^{-\alpha_t(t_2)-1}(1-x)^{-\alpha_u(u_2)-1}\right]. \quad \text{(C17)}$$

In the full expression for $A_5 = A_5{}^{st} + A_5{}^{su} + A_5{}^{tu}$, the last integral in Eq. (C17) does not vanish as $t_1 \to 0$ ($\alpha_P \to 1$). Instead we get as $t_1 \to 0$,

$$A_5 = \frac{2\pi i s\alpha'}{\alpha_t + \alpha_u}\int_0^\eta dx\frac{d}{dx}[x^{-\alpha_t}(1-x)^{-\alpha_u}] = 2\pi i\frac{\alpha' s_1}{\alpha_t}\left(1+\frac{\alpha_u}{\alpha_t}\right)^{\alpha_t}\left(1+\frac{\alpha_t}{\alpha_u}\right)^{\alpha_u}. \quad \text{(C18)}$$

In the double Regge limit with $\alpha_u(u_2) \sim -\alpha' s_2 \to \infty$

$$A_5(\alpha_P = 1) \simeq 2\pi i(\alpha' s_1)(\alpha' s_2)^{\alpha_t}\frac{e^{\alpha_t}(-\alpha_t)^{\alpha_t}}{\alpha_t}. \quad \text{(C19)}$$

For a nearby (pion) pole, this gives us a *real* Regge power $(s_2{}^{\alpha_\pi})$ and an exponentially damped (pion) pole exchange $[\alpha_\pi{}^{-1}\exp(\alpha_\pi + \alpha_\pi\log(-\alpha_\pi))]$, with *no* recurrences! The diffractive production is *pure* Deck mechanism.

Indeed in the dual model the Pomeron at $t = 0$ couples to the elastic state and *only* the elastic state. This decoupling from all the inelastic resonances is not required by the weak theorems, but it is certainly an attractive possibility for a gauge invariant Pomeron theory (Brower and Weis, 1972; Ravndal, 1971). In spite of the subtlety of gauge invariance for the Pomeron, we still find it an attractive hypothesis that it plays *some* role in an $\alpha_P(0) = 1$ self-consistent theory.

REFERENCES

Aachen-CERN-Geneva-Harvard-Turin collaboration, 1972, XVI International Conference on High Energy Physics, Batavia, Ill.
Abarbanel, H. D. I., G. F. Chew, M. L. Goldberger, and L. M. Saunders, 1971, Phys. Rev. Lett. **26**, 937.
Abarbanel, H. D. I., and A. Schwimmer, 1972, Phys. Rev. D **6**, 3018.
Abarbanel, H. D. I., and J. B. Bronzan, 1974a, Phys. Lett. B **48**, 345.
Abarbanel, H. D. I., and J. B. Bronzan, 1974b, Phys. Rev. D **9**, 2397.
Abarbanel, H. D. I., S. D. Ellis, M. B. Green, and A. Zee, 1972a (unpublished).
Abarbanel, H. D. I., V. N. Gribov, and O. V. Kanchelli, 1972b (unpublished).
Amaldi, U., R. Biancastelli, C. Gosio, G. Matthiae, J. V. Allaby, W. Bartel, G. Cocconi, A. N. Diddens, R. W. Dobinson, and A. M. Wetherell, 1973, Phys. Lett. B **44**, 112.
Amati, D., S. Fubini, A. Stanghellini, 1962a, Nuovo Cimento **26**, 869.
Amati, D., S. Fubini, and A. Stanghellini, 1962b, Phys. Lett. **1**, 29.
Amati, D., L. Caneschi, and M. Ciafaloni, 1973, Nucl. Phys. B **62**, 173.
Baker, M., and J. H. Weis (unpublished).
Bartenev, V., *et al.*, 1973, USSR-USA Collaboration, NAL-Pub. 73/54-EXP.
Bertocchi, L., S. Fubini, and M. Tonin, 1962, Nuovo Cimento **25**, 626.
Bishari, M., G. F. Chew, and J. Koplik, 1974, Nucl. Phys. B **72**, 61.
Bishari, M., and J. Koplik, 1974, Nucl. Phys. B **72**, 93.
Blankenbecler, R., 1973, Phys. Rev. Lett. **31**, 964.
Blankenbecler, R., J. R. Fulco, and R. L. Sugar, 1974, Phys. Rev. D **9**, 736.
Bronzan, J. B., 1972a, Phys. Rev. D **4**, 1097.
Bronzan, J. B., 1972b, Phys. Rev. D **6**, 1130.
Bronzan, J. B., 1973, Phys. Rev. D **7**, 480.
Brower, R. C., 1972, Phys. Rev. D **6**, 1655.
Brower, R. C., and J. H. Weis, 1971, Phys. Rev. D **3**, 451.
Brower, R. C., and J. H. Weis, 1972, Phys. Lett. **41**, 631.
Brower, R. C., and J. H. Weis, 1973, Univ. of Wash. preprint RLO-1388-655, and ANL/HEP 7372.
Brower, R. C., R. Cahn, and J. Ellis, 1973a, Phys. Lett. B **44**, 81.
Brower, R. C., M. B. Einhorn, M. B. Green, A. Patrascioiu, and J. H. Weis, 1973b, Phys. Rev. D **8**, 2524.
Brower, R. C., A. Mueller, A. Sen, and J. H. Weis, 1973c, Phys. Lett. B **46**, 105.
Brower, R. C., and J. Ellis, 1974, Phys. Lett. B **51**, 242.
Brower, R. C., J. Ellis, R. Savit, and W. Zakrzewski, 1975, CERN preprint.
Brower, R. C. and F. Zachriasen (unpublished).
Caneschi, L., and A. Schwimmer, 1972a, Nucl. Phys. B **44**, 31.
Caneschi, L., and A. Schwimmer, 1972b, Nucl. Phys. B **48**, 519.
Capella, A., M.-S. Chen, M. Kugler, and R. D. Peccei, 1973, Phys. Rev. Lett. **31**. 497.
Cardy, J. L., and A. R. White, 1973a, Phys. Lett. B **47**, 445.
Cardy, J. L., and A. R. White, 1973b, CERN TH-1740.
Chang, S.-Y., D. Gordon, R. E. Low, and S. B. Treiman, 1971, Phys. Rev. D **4**, 3055.
Chen, M. S., R. R. Kinsey, T. W. Morris, R. S. Panvini, L. L. Wang, T. F. Wong, S. L. Stone, T. Ferbel, P. Slattery, B. Werner, J. W. Elbert, and A. R. Erwin, 1971, Phys. Rev. Lett. **26**, 1585.
Cheng, H., and T. T. Wu, 1970, Phys. Rev. Lett. **24**, 1456.
Cheng, H., and T. T. Wu, 1971, Phys. Lett. B **26**, 357.
Chew, G. F., and A. Pignotti, 1968, Phys. Rev. **176**, 2112.
Chou, T. T., and C. N. Yang, 1970, Phys. Rev. Lett. **25**, 1072.
Ciafaloni, M., and G. Marchesini, 1974, Nucl. Phys. B **71**, 493.
DeTar, C. E., 1974, M.I.T. preprint CTP 421.
DeTar, C. E., D. Z. Freedman, and G. Veneziano, 1971, Phys. Rev. D **4**, 906.
DeTar, C. E., C. E. Jones, F. E. Low, C.-I. Tan, J. H. Weis, and J. E. Young, 1971, Phys. Rev. Lett. **26**, 1016.
DeTar, C. E., and J. H. Weis, 1971, Phys. Rev. D **4**, 3141.
Doren, J. D., V. Rittenberg, and D. Jaffe, 1971, Nucl. Phys. B **30**, 306.
Drumond, I. T., P. V. Landshoff, and W. J. Zakrzewski, 1969, Phys. Lett. B **28**, 676.
Ellis, J., J. Finkelstein, and R. D. Peccei, 1972, Nuovo Cimento **12**, 763.
Finkelstein, J., 1973, Phys. Rev. D **8**, 4176.
Finkelstein, J., and K. Kajantie, 1968a, Nuovo Cimento A **56**, 659.
Finkelstein, J., and K. Kajantie, 1968b, Phys. Lett. B **26**, 305.
Finkelstein, J., and F. Zachariasen, 1971, Phys. Lett. B **34**, 631.
Froissart, M., 1961, Phys. Rev. **123**, 1053.
Giacomelli, G., 1972, Rapporteur talk, in *Proceedings of the XVI International Conference on High Energy Physics*, Batavia, Ill., Vol. 3, p. 219.
Goddard, P., and A. R. White, 1971, Nuovo Cimento A **1**, 645.
Goddard, P., and C. B. Thorn, 1972, Phys. Lett. B **40**, 235.
Goddard, P. and A. R. White, 1972, Phys. Lett. B **38**, 93.
Gordon, D., 1971, Nuovo Cimento A **6**, 107.
Gordon, D., 1972, Phys. Rev. D **5**, 2102.
Gribov, V. N., 1967, Zh. Eksp. Teor. Fiz. **53**, 654 [Sov. Phys.-JETP **26**, 414 (1968)].

Gribov, V. N., 1972, in *Proceedings of the XVI International Conference on High Energy Physics*, Batavia, Ill., Vol 3, p. 491.
Gribov, V. N., Y. Ya Pomeranchuk, and K. A. Ter-Martirosyan, 1965, Phys. Rev. B **139**, 184.
Gribov, V. N., and A. A. Migdal, 1968a, Yad. Fiz. **8**, 1002, 1003 [Sov. J. Nucl. Phys. **8**, 583, 703 (1969).]
Gribov, V. N., and A. A. Migdal, 1968b, Zh. Eksp. Teor. Fiz. **55**, 1498 [Sov. Phys-JETP **28**, 794 (1969)].
Halliday, I. G., 1971, Nucl. Phys. B **33**, 285.
Halliday, I. G., and C. T. Schrajda, 1973, Phys. Rev. D **8**, 3598.
Hasslacher, B., G. M. Cicnta, D. K. Sinclair, and R. L. Sugar, 1970, Phys. Rev. Lett. **25**, 159.
Henyey, F., and W. K. Zakrzewski, 1973, Phys. Lett. B **42**, 431.
Horn, D., and F. Zachariasen, 1973, *Hadron Physics at Very High Energies* (Benjamin, Reading, Mass.).
Iwasaki, Y., and S. Yazaki, 1973, Phys. Lett. B **45**, 263.
Jengo, R., 1974, Phys. Lett. B **51**, 143.
Jones, C. E., F. E. Low, and J. E. Young, 1971, Phys. Rev. D **4**, 2358.
Jones, C. E., F. E. Low., S. H. Tye, G. Veneziano, and J. E. Young, 1972 Phys. Rev. D **6**, 1033.
Kadanoff, L. P., 1971, in *Proceedings of the International School of Physics "Enrico Fermi"*, edited by N. Green (Academic, New York).
Kronenfeld, R., 1973, Stanford Preprint ITP-438.
LeBellac, M., 1971, Phys. Lett. B **37**, 413.
Lee, B. W., and R. F. Sawyer, 1962, Phys. Rev. **127**, 2266.
Lee, H., 1973, Phys. Lett. B **46**, 117.
Leith, D. W. G. S., 1972, in *Proceedings of the XVI International Conference on High Energy Physics*, Batavia, Ill., Vol. 3, p. 321.
Mehta, C. H., and D. Silverman, 1973a, Nucl. Phys. B **52**, 77.
Mehta, C. H., and D. Silverman, 1973b, Nucl. Phys. B **59**, 460.
Migdal, A. A., A. M. Polyakov, and K. A. Ter-Martirosyan, 1974a, Phys. Lett. B **48**, 239.
Migdal, A. A., A. M. Polyakov, and K. A. Ter-Martirosyan, 1974b, ITEP (Moscow), preprint 102.
Misheloff, M. N., 1969, Phys. Rev. **184**, 1732.
Moen, I. O. and A. R. White, 1972, Phys. Lett. **42B**, 75.
Moen, I. O., and W. J. Zakrzewski, 1973, Nucl. Phys. B **65**, 346.
Moen, I. O., and W. J. Zakrzewski, 1974, Nuovo Cimento A **19**, 34.
Mueller, A., 1970, Phys. Rev. D **2**, 2963.
Mueller, A. H., and T. L. Trueman, 1972, Phys. Rev. D **5**, 2115.
Muzinich, I. J., F. E. Paige, T. L. Trueman, and L. L. Wang, 1972, Phys. Rev. D **6**, 1048.
Neff, T. L., 1973, Phys. Lett. B **45**, 349.
Oehme, R., 1973, Phys. Lett. B **45**, 279.
Rajaramann, R., 1972, Phys. Lett. B **42**, 271.
Ravndal, R., 1971, Phys. Lett. B **37**, 300.
Regge, T., 1959, Nuovo Cimento **14**, 951.
Regge, T., 1960, Nuovo Cimento **18**, 947.
Roth, R., 1972, Phys. Rev. D **6**, 2823.
Roy. S. M., 1972, Phys. Rep. **5**, 125.
Sivers, D., 1073, Phys. Rev. D **7**, 2790.
Stapp, F. P., 1971, Phys. Rev. D **3**, 3177.
Sukhatme, U. P., 1972, Phys. Rev. D **6**, 2765.
Sukhatme, U. P., and J. N. Ng, 1974, Nucl. Phys. **B70**, 229.
Veneziano, G., 1972, Phys. Rep. C **9**, 4.
Veneziano, G., 1974, Nucl. Phys. B **69**, 317.
Weis, J. H., 1972, Phys. Rev. D **6**, 2823.
Weis, J. H., 1973, Phys. Lett. B **43**, 487.
Weis, J. H., 1974, Nucl. Phys. B **71**, 342.
White, A. R., 1972, Nucl. Phys. B **39**, 432, 461.
White, A. R., 1973, VIII Rencontre de Morind, CERN TH-1646.
White, A. R., 1974, in *Proceedings of the Stanford Summer Institute on Particle Physics*, Vol. II, SLAC Report No. 179.