The hadronic structure of the photon with emphasis on its two-pion constituent*

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The hadronic structure of photons is studied by treating the electromagnetic interaction in lowest order. General consequences of this picture and its connection with generalized vector meson dominance for diffractive processes are discussed. Emphasis is given to the dipion constituent which can be regarded approximately as a superposition of two parts: a ρ^0-meson core and a loose nonresonant two-pion structure. This modification of VMD increases the dipion contribution to the real photon cross section by 10%-20% (of the ρ^0 part) and to νW_2 by several percent. The internal spatial structure of this component is shown to shrink as the photon becomes more virtual. Various views of diffractive pion pair photoproduction are reconciled. The Drell process is interpreted in terms of the dipion component and some evidence is given for its presence in inclusive π^--photoproduction data. Some speculations about the consequences of the spatial size of different photon constituents are given.

CONTENTS

I. INTRODUCTION

It is generally accepted that the hadronic components of photons play an important role in their high-energy interactions with nucleons and nuclei. The well-known qualitative similarities of photon and hadron interactions have been reviewed by Sakurai in his Erice lectures (Sakurai, 1971, 1973); the similarities of diffractive interactions have also been discussed in my Cargèse lectures (Yennie, 1975). In summary, these similarities are:

(i) Total cross sections on nucleons: Both show spectacular resonances at low energies, and above about 3 GeV they level out and become structureless, apparently tending to a constant at high energies (a logarithmic dependence at high energies is still possible). The cross section on the neutron is nearly the same as that on the proton, and the difference seems to vanish as $E \to \infty$; thus the photon cannot interact primarily with the charge of the target. In magnitude, the total photon cross section is asymptotically about 1/220 times the average of the pion cross sections, i.e., it is smaller by approximately the fine structure constant in order of magnitude.

(ii) Diffractive processes: Diffractive photoproduction is the analogue of diffractive hadronic scattering. It is about the same fraction of the total cross section ($\sim$20%) and has similar angular dependence (roughly exponential, with similar slopes). Elastic scattering of photons also appears to be primarily diffractive; in particular, the amplitude has a small real part. This is significant because, while hadronic amplitudes would be expected to be nearly imaginary due to the almost complete absorption at small impact parameters, there is no obvious reason why the photon amplitude could not have a significant refractive part. The absence of such a real part is of course related to the rapid approach of the photon cross section to a constant at high energies.

(iii) Two-body reactions: These have similar features such as peaks and dips which seem to be governed by common rules.

(iv) Inclusive cross sections: Both display sharply falling P_T^2 distributions and comparable longitudinal momentum distribution properties.

(v) Absorption by nuclei: While the total photon cross section on nuclei is of course much smaller than that of hadrons, the high-energy ($\gtrsim$ 10 GeV) A dependence is similar, corresponding to a strong shadowing effect.

All these features suggest a picture in which the photon acts like a hadron a small fraction ($\sim \alpha$) of the time. The simplest framework for describing these features, and one which works remarkably well overall, is vector meson dominance (VMD). In this model, the photon is assumed to be a well-defined linear superposition of the ρ^0, ω, and ϕ mesons before interaction (with amplitudes e/f_ρ, e/f_ω, and e/f_ϕ, respectively). With the present best values of the couplings, these mesons account for nearly all the total photon cross section; in fact, the ρ^0 alone accounts for about two-thirds of the total. Through the Orsay (Benaksas *et al.*, 1972) and Novosibirsk (Aüslender *et al.*, 1969) colliding

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beam experiments ($e^+e \to \pi^+\pi^-$), the ρ^0 is known to dominate the coupling of the photon to the pion: the values of f_ρ determined separately from the width and normalization of the data agree within the uncertainties expected from finite width corrections (Gounaris and Sakurai, 1968). The diffractive photoproduction of ρ^0, and elastic scattering of ρ^0's, as determined from photoproduction on nuclei (Alvensleben *et al.*, 1970; McClellan *et al.*, 1971; Spital and Yennie, 1974a, 1974b), are also in excellent agreement with VMD, within theoretical ambiguities at the 10–15% level. Specific processes which have a small cross section which decreases rapidly with energy, such as $\gamma N \to \pi N'$, do not agree in all details with predictions from the ρ^0-photon analogy, but at least qualitatively the vector mesons seem to play an important role in them. The shadowing effect of the total cross section in nuclei (Caldwell *et al.* 1973; Brookes, *et al.* 1972; Meyer *et al.* 1970; Heynen *et al.* 1971; Stodolsky, 1967; Gottfried and Yennie, 1969; Brodsky and Pumplin, 1969; Nauenberg, 1969; Margolis and Tang, 1969) is smaller than expected from VMD. This could indicate that a small fraction of the photon's interaction (say 20%) is not due to a hadronic component, or is due to hadronic components of such high masses that they have not yet saturated at current energies.

It is the aim of this paper and a related one (Spital and Yennie, 19++) to refine this picture, with particular emphasis on developing physical intuition about the hadronic component. It is hoped that this intuition will be useful in exploring the still controversial question about how important the hadronic components are in the various regions of inelastic electron scattering. The overall point of view is similar to that of generalized VMD (GVMD) (Gribov, 1970; Brodsky and Pumplin, 1969; Ritson, 1971; Fujikawa, 1971; Bjorken, 1972; Sakurai and Schildknecht, 1972; Bramon, Etim and Greco, 1972) in that it relates the hadronic components to the states actually observed in electron–positron annihilation (Bacci *et al.* 1973; Grilli *et al.* 1973; Bartoli *et al.* 1972; Cosme *et al.* 1972; Kurdadze *et al.* 1972; Litke *et al.* 1973; Richter *et al.* 19++). To lowest order in e, there is an exact connection which is given by perturbation theory. This formalism is developed in Sec. II and is used there for a general discussion of total cross sections and photoproduction processes. In Sec. III the two-pion (dipion) constituent of the photon is discussed in some detail. It is found to incorporate the ρ^0, and it is suggested that nonresonant pion pairs could easily enhance the usual estimate of the ρ^0 contribution to the total cross section by as much as 10 to 20% (of the ρ^0 contribution). An interesting picture in configuration space emerges in which the dipion constituent may be viewed approximately as the superposition of a ρ^0 core and a loose two-pion tail. This spatial structure is found to shrink as the photon becomes virtual with spacelike Q^2. Two models of photoproduction of pion pairs are seen there to be related naturally to this hadronic component. Section IV contains a general discussion of hadron-mediated photon interactions, with some criticism of the diagonal approximation of GVMD and an outline of the ambiguities in treating this part of the longitudinal interaction of virtual photons. An estimate of the dipion contribution to νW_2 is given, and it is found to give a small enhancement over VMD. For small x [$= (Q^2/2M\nu) \gtrsim 0.1$)] and $Q^2 \gtrsim 1.5$, the dipion, ω, and ϕ are found to account for over 80% of the experimental data. Some room is left for higher mass and other contributions, but not as much as in the usual GVMD treatments. The effects of the dipion component are sought in inclusive π^- photoproduction and appear to be present, but the analysis is too unsophisticated to give a good fit to the data. Section V contains some final remarks and speculations about the consequences of various aspects of the internal spatial structure of the photon.

II. PERTURBATION TREATMENT OF THE HADRONIC STRUCTURE OF THE PHOTON

We start with the structure of a free photon, which will be studied in lowest order of the electromagnetic coupling. The resulting hadronic structure will be expressed in terms of completely interacting hadronic states. The question of how this composite system interacts with hadronic targets will be discussed only intuitively at first, but later on we shall argue that the interaction of a physical photon takes place via two mechanisms. One is through the collision of its pre-existing hadronic components with the target, and the other is through a more direct interaction, which might be referred to as a bare photon or short-range interaction.[1]

We imagine then that the total Hamiltonian may be decomposed into two parts

$$\begin{aligned} H &= H_0 + H_1, \\ H &= H_{st} + H_{ph}, \\ H_1 &= e \int J_\mu(x) A^\mu(x)\, d^3x, \end{aligned} \tag{2.1}$$

where H_{st} is the complete hadronic Hamiltonian, H_{ph} is the Hamiltonian of the free electromagnetic field, and J_μ is the hadronic electromagnetic current. We ignore the leptonic contributions to the Hamiltonian, which would manifest themselves in radiative corrections. Using lowest order perturbation theory, we may express the physical photon state as a superposition of a bare photon term, and terms involving hadrons:

$$|\mathbf{k}_p\rangle = Z_3^{1/2}(k)\,|\mathbf{k}_B\rangle + \mathop{\mathsf{S}}_{\mathbf{P},\mathfrak{M},n_i} \frac{|\mathbf{P},\mathfrak{M},n_i\rangle\langle\mathbf{P},\mathfrak{M},n_i\,|\,H_1\,|\,\mathbf{k}_B\rangle}{\nu-(\mathfrak{M}^2+P^2)^{1/2}} + \mathop{\mathsf{S}}_{\mathbf{P},\mathfrak{M},n_i,\mathbf{k}',\mathbf{k}''} \frac{|\mathbf{P},\mathfrak{M},n_i,\mathbf{k}_B''\mathbf{k}_B'\rangle\langle\mathbf{P},\mathfrak{M},n_i\mathbf{k}_B''\mathbf{k}_B'\,|\,H_1\,|\,\mathbf{k}_B\rangle}{\nu-\nu'-\nu''-(\mathfrak{M}^2+P^2)^{1/2}}. \tag{2.2}$$

[1] As will be seen later, this separation into two mechanisms is not a relativistically invariant concept. In most cases, the separation will refer to the laboratory frame.

The subscript B or P on $\mathbf{k}$ refers to bare or physical respectively; the values of $\mathbf{k}$ are of course the same. The coefficient $Z_3(k)$ differs from unity by order e^2 and takes into account the over-all normalization of the state; it is related to the usual Z_3, but is not quite the same.[2] The hadronic states are here labeled by their total momentum $\mathbf{P}$, their total mass $\mathfrak{M}$, and other internal labels are represented by n_1. These internal labels are particle types ($\pi^+\pi^-$, $\bar{N}N$, $\pi^+\pi^-\pi^0$, $\bar{K}K$, etc.), asymptotic relative momenta of scattering states, spin projections, etc. The symbol S stands for sums and integrations over the appropriate labels. For convenience in generalizing later to virtual photons, we permit the photon energy to differ from its momentum and define

$$k^2 - \nu^2 = -q^2 = Q^2. \tag{2.3}$$

However, for the present we shall restrict the discussion to transversely polarized photons; the more subtle case of longitudinal polarization will be discussed in Sec. IV. The second term of (2.2) is the pure hadronic component we are interested in; it will turn out to be important because of the small denominator. On the other hand, the third term will always have a large denominator since $\nu = \nu'$ or ν''. It represents primarily a contribution to the self-energy of the vacuum in the presence of a photon. Although it can effect the self-energy of the photon (when $\mathbf{k}_B' = \mathbf{k}_B'' = \mathbf{k}_B$) and can contribute to Compton scattering (Brodsky, Close, and Gunion, 1972), we shall ignore it from now on.

The next step is to make some obvious simplifications on the second term of (2.2). We note first that had we originally expressed the phase space in terms of the individual particles, it would have included the invariant factor

$$\prod_i \frac{d^3p_i}{2E_i}.$$

We want to extract from this an integral over $\mathbf{P}$ and $\mathfrak{M}$. This is done by introducing the identity in the form

$$1 = \int_0^\infty d\mathfrak{M}^2 \int d^4P\delta(P^2 - \mathfrak{M}^2)\delta(P - \Sigma p_i).$$

Thus we find

$$\prod_i \frac{d^3p_i}{2E_i} \to \int_0^\infty d\mathfrak{M}^2 \frac{d^3P}{2P_0}\left\{\prod_i \frac{d^3p_i}{2E_i}\delta(P - \Sigma p_j)\right\},$$

where $P_0 = (\mathbf{P}^2 + \mathfrak{M}^2)^{1/2}$. Next we use translational invariance to rewrite the matrix element of H_1

$$\langle \mathbf{P}, \mathfrak{M}, n_i \mid H_1 \mid \mathbf{k}_B\rangle = -(2\pi)^{3/2} e\delta(\mathbf{P} - \mathbf{k})\langle \mathbf{P}, \mathfrak{M}, n_i \mid \boldsymbol{\varepsilon}\cdot\mathbf{J}(0) \mid \mathrm{vac}\rangle.$$

At this stage, the hadronic component of the photon is written

$$|\,\mathrm{HC}\rangle = -(2\pi)^{3/2} e \int_{\mathfrak{M}_t^2}^\infty \frac{d\mathfrak{M}^2}{2(k^2 + \mathfrak{M}^2)^{1/2}[\nu - (k^2 + \mathfrak{M}^2)^{1/2}]} \times \underset{n_i}{\mathsf{S}} \mid \mathbf{k}, \mathfrak{M}, n_i\rangle\langle \mathbf{k}, \mathfrak{M}, n_i \mid \boldsymbol{\varepsilon}\cdot\mathbf{J}(0) \mid \mathrm{vac}\rangle. \tag{2.4}$$

[2] The reader may wish to check for himself that in the case of the coupling (2.7) below, 1-$Z_3(k)$ has the following k dependence: $(\omega_\rho^2 + k^2)/(2\omega_\rho k)$ where $\omega_\rho = (k^2 + m^2)^{1/2}$, it becomes independent of k for $k \to \infty$.

So far no approximations have been made (to order e). From now on, however, we shall place most emphasis on the lower mass components which satisfy

$$\nu \gg (\mathfrak{M}^2 + Q^2)^{1/2}. \tag{2.5}$$

With this approximation, (2.4) simplifies to

$$|\,\mathrm{LMHC}\rangle \cong (2\pi)^{3/2} e \int_{\mathfrak{M}_t^2}^{\sim\nu^2} \frac{d\mathfrak{M}^2}{\mathfrak{M}^2 + Q^2} \underset{n_i}{\mathsf{S}} \mid \mathbf{k}, \mathfrak{M}, n_i\rangle \times \langle \mathbf{k}, \mathfrak{M}, n_i \mid \boldsymbol{\varepsilon}\cdot\mathbf{J}(0) \mid \mathrm{vac}\rangle. \tag{2.6}$$

We can begin to see the resemblance of this result to vector meson dominance (VMD). In fact, suppose that the hadronic states include only a stable vector meson and that the coupling H_1 is given by

$$H_1^{(\rho)} = +\frac{em_\rho^2}{f_\rho}\int \rho_\mu(x)A^\mu(x)\, d^3x, \tag{2.7}$$

then (2.6) reduces to

$$(e/f_\rho)[m_\rho^2/(m_\rho^2 + Q^2)] \mid \mathbf{k}, \rho\rangle \tag{2.8}$$

which is, in fact, the usual expression of VMD. In reality, there are no stable vector mesons, but we shall see later how their place is taken by resonances.

The hadronic component (2.6) is similar to any composite state as seen in a moving frame. To bring this out more clearly, we may write

$$|\,\mathbf{k}, \mathfrak{M}, n_i\rangle = U\left(\frac{\mathbf{k}}{(k^2 + \mathfrak{M}^2)^{1/2}}\right) \mid \mathbf{0}, \mathfrak{M}, n_i\rangle, \tag{2.9}$$

where $U(\mathbf{v})$ is the hadronic boost operator for velocity $\mathbf{v}$. Also, using Lorentz invariance, the matrix element of $\boldsymbol{\varepsilon}\cdot\mathbf{J}(0)$ is easily related to its rest frame value. Thus we find

$$\langle \mathbf{k}, \mathfrak{M}, n_i \mid \boldsymbol{\varepsilon}\cdot\mathbf{J}(0) \mid \mathrm{vac}\rangle = \langle \mathbf{0}, \mathfrak{M}, n_i \mid \boldsymbol{\varepsilon}\cdot\mathbf{J}(0) \mid \mathrm{vac}\rangle, \tag{2.10}$$

where we have used the fact that $\boldsymbol{\varepsilon}\cdot\mathbf{J}(0)$ commutes with the boost for transverse $\boldsymbol{\varepsilon}$. Introducing these relations into (2.6), we find for the low mass hadronic component

$$|\,\mathrm{LMHC}\rangle \cong (2\pi)^{3/2} e \int_{\mathfrak{M}_t^2}^{\sim\nu^2} \frac{d\mathfrak{M}^2}{\mathfrak{M}^2 + Q^2} U\left(\frac{\mathbf{k}}{(\mathfrak{M}^2 + k^2)^{1/2}}\right) \times \underset{n_i}{\mathsf{S}} \mid \mathbf{0}, \mathfrak{M}, n_i\rangle\langle \mathbf{0}, \mathfrak{M}, n_i \mid \boldsymbol{\varepsilon}\cdot\mathbf{J}(0) \mid \mathrm{vac}\rangle. \tag{2.11}$$

According to (2.9) the various hadronic components have boost velocities which are close to 1 for low masses. If it were not for the dependence of the boost operator on $\mathfrak{M}$, we could simply put it in front of the integral and find one universal state which is boosted to various velocities. We can still achieve this result by the following stratagem. Decompose the boost for mass $\mathfrak{M}$ into a boost to the rest frame defined by a reference mass $\mathfrak{M}_0$ followed by a boost to the final frame

$$U\left(\frac{\mathbf{k}}{(\mathfrak{M}^2+k^2)^{1/2}}\right) = U(\mathbf{v}_0)U(\mathbf{v}_r), \tag{2.12}$$

where

$$\mathbf{v}_0 = \frac{\mathbf{k}}{(\mathfrak{M}_0^2+k^2)^{1/2}} \quad \text{and} \quad \mathbf{v}_r = \hat{\mathbf{k}}\frac{\mathfrak{M}_0^2-\mathfrak{M}^2}{\mathfrak{M}_0^2+\mathfrak{M}^2}$$

and we have used $|\mathbf{k}| \gg \mathfrak{M}, \mathfrak{M}_0$. Then

$$|\mathrm{LMHC}\rangle \cong (2\pi)^{3/2} e U(\mathbf{v}_0) \int_{\mathfrak{M}_t^2}^{\sim\nu^2} \int \frac{d\mathfrak{M}^2}{\mathfrak{M}^2+Q^2} U(\mathbf{v}_r) \times \mathop{\mathsf{S}}_{n_i} |\mathbf{0}, \mathfrak{M}, m_i\rangle\langle \mathbf{0}, \mathfrak{M}, n_i | \boldsymbol{\varepsilon}\cdot\mathbf{J}(0) | \mathrm{vac}\rangle, \tag{2.13}$$

Now the integral is independent of the magnitude of $\mathbf{k}$ and therefore has an invariant internal structure. We shall call such a state composite. It consists of a superposition of various momenta along the direction of $\mathbf{k}$ with only $\mathfrak{M}_0$ at rest. The photon states of different momenta $\mathbf{k}$ are then obtained by different boosts of the hadronic components of this composite state. One may also verify that the boost necessary to change from $\mathbf{k}$ to $\mathbf{k}'$ (in the same direction) is independent of $\mathfrak{M}_0$ [if $k, k' \gg (\mathfrak{M}_0^2+Q^2)^{1/2}$] and has the same velocity as for a real photon. For fixed ν, one can find masses high enough so that these properties are not valid. Presumably in interaction these high masses will either be relatively unimportant, or they will merge with the interactions associated with the bare photons.

It is well known that the hadronic component of the photon is related to the states which are produced in the $e^+ - e^-$ annihilation experiments (Gribov, 1970; Brodsky and Pumplin, 1969; Ritson, 1971; Fujikawa, 1971; Bjorken, 1972; Sakurai and Schildknecht, 1972; Bramon, Etim, and Greco, 1972). This is seen most directly by reference to (2.11), where the matrix element is precisely the one which occurs in the annihilation reaction.

The probability of a given mass is given by

$$P(\mathfrak{M}^2, Q^2) = [\mathfrak{M}^4/(\mathfrak{M}^2+Q^2)^2]P(\mathfrak{M}^2), \tag{2.14a}$$

where

$$P(\mathfrak{M}^2) = (2\pi)^3 e^2 (1/\mathfrak{M}^4) \mathop{\mathsf{S}}_{n_i} |\langle \mathbf{0}, \mathfrak{M}, n_i | \boldsymbol{\varepsilon}\cdot\mathbf{J}(0) | \mathrm{vac}\rangle|^2$$

$$= \frac{\sigma_{\mathrm{tot}}(e^+e^- \to \mathfrak{M})}{4\pi^2\alpha}. \tag{2.14b}$$

Note also that

$$Z_3(k) = 1 - \int_{\mathfrak{M}_t^2}^{\sim\nu^2} P(\mathfrak{M}^2)\, d\mathfrak{M}^2 \tag{2.15}$$

$$- \text{high mass contributions.}$$

If ν is sufficinetly large that (2.5) is satisfied for all relevant masses, $Z_3(k)$ will reduce to Z_3; otherwise, it will vary with k.

According to (2.14b), all the debris in the reaction $e^+e^- \to X$ plays a role in the photon's structure. However, it would be misleading to propose that the localized hadronic structure of the photon bears any direct resemblance to the particles which appear asymptotically in the annihilation experiment. This localized hadronic structure will result from a Q^2- and ν-dependent superposition of all the states showing up in the annihilation experiment. Those states satisfying (2.5) will be "frozen in" as described in the previous few paragraphs, while the higher mass states will have a ν-dependent probability. In Sec. III these ideas will be illustrated by the two-pion component, which can be studied rather explicitly. The remainder of this section will be devoted to a more intuitive discussion of the hadronic component and its role in photon interactions.

The present experimental evidence (Litke *et al.*, 1973; Augustin, *et al.*, 1975) is that while $P(\mathfrak{M}^2)$ does have major low mass contributions, the annihilation cross section appears to be constant in $\mathfrak{M}^2$ rather than to decrease as $1/\mathfrak{M}^2$, as had been expected from scaling arguments. Whether or not this is a transient effect, its ultimate interpretation is likely to lead to very profound changes in our present understanding of elementary particle interactions. In the present paper we shall argue that, as is already known from vector meson dominance, the major interactions of real and nearly real photons are associated primarily with the low mass components of the photon. The mechanisms which cause the suppression of high mass contributions remain a subject of speculation.[3]

Although the vector mesons cannot appear as discrete states in the integral, various resonances will produce huge peaks in the probability as a function of mass. To the extent that these resonances can be approximated by δ functions, we would expect them to behave very much like the usual vector mesons. This will be confirmed for the two-pion constituent, which includes the ρ^0 meson in this way.

Before looking at specific constituent states in the hadronic component, let us consider qualitatively the diffractive processes which would be expected to arise due to the presence of the hadrons. Imagine a physical photon hitting a nucleon or nucleus. It is expected that the hadronic components will be very strongly absorbed. The assumption that photon absorption results entirely from this mechanism (i.e. no bare photon interaction) is known as the ***generalized VMD hypothesis*** (Gribov, 1970; Brodsky and Pumplin, 1969; Ritson, 1971; Fujikawa, 1971; Bjorken, 1972; Sakurai and Schildknecht, 1972; Bramon, Etim, and Greco, 1972). A further simplifying assumption which is often used (called the diagonal approximation) is that each component at fixed mass is absorbed independently of other components. A plausible argument in favor of this assumption is that the components of different mass are orthogonal in the initial state and they therefore should produce orthogonal final states. Thus, while different masses may interfere

[3] An interesting example of such speculations is given by Bjorken (1973). He assumes that only that part of the photon state containing particles of low transverse momenta can interact effectively. If $P(\mathfrak{M}^2) \propto 1/\mathfrak{M}^2$, his assumption could lead to scaling. Since $P(\mathfrak{M}^2)$ is more constant, a more drastic mechansim is likely to be necessary.

in individual channels, they should have no net interference in the total cross section. This plausibility argument cannot be made rigorous, and in fact there is some experimental evidence for at least a small interference effect between the ρ^0 and ω^0 contributions to the total cross section.[4] We shall return to a discussion of this point after further physical intuition has been developed through the treatment of the two-pion component. Clearly, it is difficult to test the generalized VMD hypothesis with real photons alone. One must be able to vary the "mix" of initial constituents by using virtual photons. In its most general form (without the diagonal approximation), it will be very difficult to prove or disprove the hypothesis. In the more immediate future, it seems likely that we must be content with more qualitative predictions.

Without necessarily accepting this hypothesis, let us try to visualize the nature of the state which is produced in the "shadow region" of the target, that is, in the region immediately behind the target. We expect the state to differ in three ways from the initial one:

(i) New hadrons will be present in the shadow region; they will be different in nature from the original components and will be correlated with a more or less catastrophic transformation of the target. The ultimate spreading out of these hadrons and the target fragments represents nondiffractive photoproduction.

(ii) The original hadronic components will be strongly depleted.

(iii) The bare photon component may be modified as well.

In short, the state will no longer correspond precisely to a physical photon, even one of diminished amplitude. If we re-express the part of the state corresponding to (ii) and (iii) in terms of physical states, the result is bound to be a physical photon with amplitude slightly reduced (by order α) together with a superposition of real hadronic states. Since it is the physical states which propagate with a definite wave number–energy relationship, this superposition will ultimately spread out and become separated as it moves along the beam direction. The physical hadrons will ultimately appear as diffractively photoproduced particles and the distortion of the physical photon state will result in diffractive Compton scattering. An extreme example is illustrated in Fig. 1, where it is assumed that immediately behind the target nucleon the complete ρ^0 component is removed from the photon, leaving a bare photon (other components are ignored). For a certain distance along the beam, the state in the shadow region will correspond mainly to a bare photon. However, the physical photon and physical ρ^0 components propagate with wave numbers k and $(k^2 - m_\rho^2)^{1/2}$, respectively (energy is fixed at k), and after a distance of order $2k/m_\rho^2$ they will be sufficiently out of phase so that they no longer add up to a bare photon state. While this spatial behavior cannot be observed for a photon interacting with a single nucleon, it is the mechanism which produces shadowing in nuclei. Thus, if a second nucleon happens to be immediately behind the first nucleon, the bare photon can pass right through it without interacting. In other terms, the physical photon and physical hadron are superposed in such a way that when they interact with the second nucleon their effects precisely cancel (Stodolsky, 1967; Gottfried and Yennie, 1969; Brodsky and Pumplin, 1969; Nauenberg, 1969; Margolis and Tang, 1969). The effect depends on the energy through the coherence distance $2k/m_\rho^2$. When that distance is small compared to both the nuclear radius and the mean free path of the ρ^0 in nuclear matter, the shadowing will go away. Shadowing in the total photon cross section now appears to be well confirmed (Caldwell *et al.*, 1973; Brookes *et al.*, 1972; Meyer *et al.*, 1970; Heynen *et al.*, 1971), although not precisely the amount expected by VMD.

Another nice way to think of the contribution of the hadronic component of the photon to the total cross section is that it arises from the shadowing of the vacuum polarization by the target. Then one sees that the length of time associated with a vacuum polarization fluctuation must be sufficiently long that the transitions back and forth to the bare photon state are not likely to be going on during the time interval when the photon is passing through the target. The usual uncertainty principle argument then gives

$$2\nu/(\mathfrak{M}^2 + Q^2) \gtrsim R, \tag{2.16}$$

where R is the radius of the proton ($\sim m_\pi^{-1}$). If $\nu \gtrsim 1$ GeV, masses satisfying these constraints will clearly satisfy (2.5) as well. Masses not satisfying these constraints will presumably "remember" their photon origin better and be indistinguishable from bare photons. We shall return to a discussion of photon interactions after further development of the intuitive picture of the photon in the following section.

III. THE DIPION CONSTITUENT

There are two reasons why it is important to study the dipion constitutent of the photon. The first is that it dominates the real photon cross section. Since it includes the ρ^0 contribution, we may make the simple VMD estimate

$$\sigma_\gamma^{(2\pi)} \cong (e^2/f_\rho^2)\sigma_\rho, \tag{3.1}$$

where $\sigma_\rho \cong 27$mbarns and $f_\rho^2/4\pi \cong 2.5$, leading to about 80 μbarns, out of a total cross section of $\sim$120 μbarns for $E_\gamma = 6$ GeV. We shall find that when this analysis is refined to take into account finite width corrections, the dipion constituent will account for an even larger portion of the total photon cross section. The second reason is that the dipion constituent is more amenable to analysis than some of the other constituents (although the same analysis would be directly valid for K pairs). In particular, it will be possible to find out something of the internal spatial structure of the dipion system and show explicitly the "photon-shrinking" effect with increasing Q^2 (Cheng and Wu, 1969; Bjorken, Kogut, and Soper, 1971; Kogut, 1972). This analysis will provide some intuitive guidance about the possible behavior of other constituents.

We assume a specific model for the dipion matrix element in (2.6), namely that it is given by VMD: the photon couples to the ρ^0 which propagates and finally decays to a pion pair. We assume the coupling of the photon to the ρ^0

[4] In VMD terms, the difference between total photon cross sections on protons and neutrons can be interpreted as being due to such an interference.

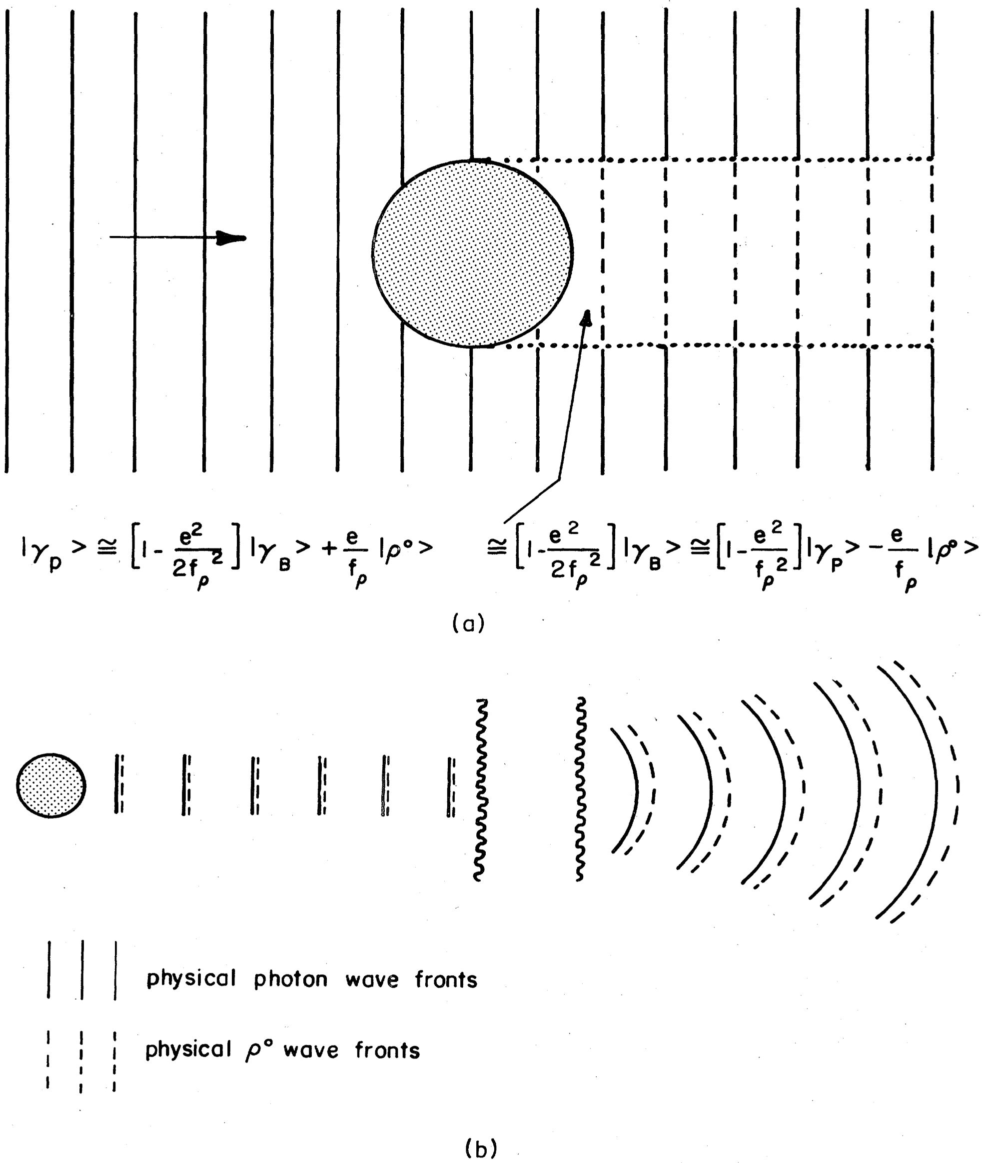

FIG. 1. Compton scattering and diffractive photon production of ρ^0 mesons in a simple model. (a) shows the situation in the vicinity of the target nucleon. The ρ^0 component of the physical photon is absorbed out, leaving a bare photon. The bare photon state may be re-expressed in terms of physical photon and physical ρ^0 states; the coefficient of the physical photon state is fixed by the condition that the state be correctly normalized to order e^2/f_ρ^2. (b) shows the modified portion of the wave re-expressed in terms of physical particles. Immediately behind the target nucleon, the two components are in phase and combine with the incident wave to reproduce a bare photon. Further along they become disentangled and represent real Compton scattering and ρ^0 production.

is constant, but permit the ρ^0 decay to have a form factor (normalized to 1 at $\mathfrak{M}^2=0$). The ρ^0 propagator is modified to take into account the vacuum polarization bubbles due to the dissociation of the ρ^0 into pion pairs. Thus we have

$$\langle \mathbf{q}_+\mathbf{q}_- - \mid \boldsymbol{\varepsilon}\cdot\mathbf{J}(0)\mid \mathrm{vac}\rangle = \boldsymbol{\varepsilon}\cdot(\mathbf{q}_+ - \mathbf{q}_-)F_\pi(\mathfrak{M}^2)/(2\pi)^3,$$

where

$$F_\pi(\mathfrak{M}^2) = \{-[m_\rho^2 - \Pi(0)]/[\mathfrak{M}^2 - m_\rho^2 + \Pi(\mathfrak{M}^2)]\} \times F_{\rho\pi\pi}(\mathfrak{M}^2). \qquad (3.2)$$

The coupling of the ρ^0 to the pion pairs is given by $f_\rho F_{\rho\pi\pi}(\mathfrak{M}^2)$, while the coupling of the photon to the ρ^0 is $-e[m_\rho^2 - \Pi(0)]/f_\rho$ which is adjusted so that the pion form factor is one at zero momentum transfer. The vacuum polarization effects are incorporated in $\Pi(\mathfrak{M}^2)$, whose imaginary part gives the width, according to

$$\mathrm{Im}\Pi(\mathfrak{M}^2) = m_\rho\Gamma_\rho(\mathfrak{M}^2) = \frac{1}{3}\frac{f_\rho^2}{4\pi}\frac{q_\pi^3}{\omega_\pi}\mid F_{\rho\pi\pi}\mid^2 \qquad (3.3)$$

where $\omega_\pi = \frac{1}{2}\mathfrak{M}$ and $q_\pi = (\omega_\pi^2 - m_\pi^2)^{1/2}$. The real part of Π is related to the imaginary part by a twice subtracted dispersion relation chosen so that the propagator has a simple normalization at the mass of the ρ^0

$$\mathrm{Re}\Pi(m_\rho^2) = \mathrm{Re}\Pi'(m_\rho^2) = 0. \qquad (3.4)$$

The model is fairly general in that it assumes only the following: (i) unsubtracted dispersion relations for F_π; (ii) a single resonance in the π–π channel; and (iii) inelastic channels, (4π, etc.) are unimportant for the pion form factor and the ρ^0 resonance is in turn unimportant for such channels. This means that the ρ^0 contribution to the photon propagator is associated entirely with the π–π channel. These assumptions permit $F_{\rho\pi\pi}$ to vary with mass due to the nonresonant final state interactions of the pions. If one neglects this possible variation, it is of course possible to evaluate Π explicitly. This has been done by Gounaris and Sakurai (1968) (with somewhat different normalization conventions), and it turns out that the additional factor $[1 - \Pi(0)/m_\rho^2]$ relative to VMD is precisely the normalization correction they obtained in their analysis of $e^+e^- \to \pi^+\pi^-$. This explicit treatment is outlined in the Appendix and is applied later to an improved estimate of the total photon cross section and the two pion contribution to νW_2.

Our first application of (3.2) is to evaluate the total probability associated with the dipion component. Remarkably, this can be carried through completely, without reference to the explicit form of $F_{\rho\pi\pi}$. Carrying out the internal integrations and taking note of (3.3), we find for this probability

$$P_{2\pi} = \frac{e^2}{\pi f_\rho^2}(m_\rho^2 - \Pi(0))^2 \times \int_{\mathfrak{M}_{\text{th}}^2}^{\infty} \frac{d\mathfrak{M}^2}{(\mathfrak{M}^2 + Q^2)^2} \frac{\operatorname{Im}\Pi(\mathfrak{M}^2)}{|\mathfrak{M}^2 - m_\rho^2 + \Pi(\mathfrak{M}^2)|^2}.$$

This integral may be evaluated easily by contour integration. We note the fact that the integrand contains as a factor the discontinuity of $[\mathfrak{M}^2 - m_\rho^2 + \Pi(\mathfrak{M}^2)]^{-1}$ across the branch cut running from $4m_\pi^2$ to ∞, so that the integral may be replaced by

$$P_{2\pi} = \frac{e^2}{-2\pi i f_\rho^2}[m_\rho^2 - \Pi(0)]^2 \times \int_C \frac{d\mathfrak{M}^2}{(\mathfrak{M}^2 + Q^2)^2} \frac{1}{\mathfrak{M}^2 - m_\rho^2 + \Pi(\mathfrak{M}^2)},$$

where the contour runs from $+\infty$ below the branch cut and then back to $+\infty$ above the branch cut. Barring bizarre properties of the ρ^0 propagator, the contour may be transformed into one surrounding the pole at $\mathfrak{M}^2 = -Q^2$ and the result is simply

$$P_{2\pi}(Q^2) = (e^2/f_\rho^2)[1 + \Pi'(-Q^2)][F_\pi(-Q^2)]^2. \tag{3.5}$$

If we assume the relativistic p-wave width ($F_{\rho\pi\pi} = 1$), Π and Π' are easily evaluated and for $Q^2 = 0$, $P_{2\pi}$ turns out to be approximately

$$P_{2\pi}(0) = (e^2/f_\rho^2)(1 + 0.57\Gamma_\rho/m_\rho),$$

which amounts to a 10% enhancement of the VMD result! Details are given in the Appendix. For $Q^2 \neq 0$, the gross features are the same as those for VMD (aside from finite width corrections of order Γ_ρ/m_ρ), namely

$$P_{2\pi}(Q^2) \approx \frac{e^2}{f_\rho^2}\left(\frac{m_\rho^2}{m_\rho^2 + Q^2}\right)^2. \tag{3.6}$$

It should be remarked that had we ignored the existence of the ρ^0 resonance, $P_{2\pi}$ would have turned out to be infinite or, more precisely, logarithmically dependent on the cutoff suggested by (2.16) for any finite energy. Thus the resonance causes the dipion contribution to saturate at relatively low energies. However, it will become increasingly clear that it is not correct to assume that the nonresonant dipion continuum has been completely replaced by the ρ^0. This is somewhat evident from the 10% enhancement of the dipion probability over the VMD value [note incidentally that $(e^2/f_\rho^2)\Pi'(0)$ is independent of f_ρ^2]. The new picture is that the dipion contribution should be thought of mainly as a ρ^0 meson with a little bit of nonresonant two-pion state attached.

To bring out this physical picture more clearly, we next study the internal spatial structure of the dipion state. There is of course no way to do this at extremely small distances, where the strong interactions will completely obscure the picture. Instead, we make the assumption that outside some radius the pions are sufficiently well separated that it makes sense to use a two-pion scattering wave function. This might become reasonable when the distance between pions is greater than about 1F. In the center-of-mass frame for the two-pion system, the P-state wave function takes the form

$$|\,\mathbf{q}_+\mathbf{q}_-\,\pm\rangle \sim 2\omega_\pi \hat{\mathbf{q}}_\pi\cdot\hat{\mathbf{r}} \exp(\pm i\delta_1)\frac{\partial}{\partial(q_\pi r)}\frac{\sin(q_\pi r + \delta_1)}{q_\pi r},$$

where the factor $2\omega_\pi$ has been introduced because of the relativistic normalization used in (2.13), and r is the spatial distance between the pions. We note that the pion form factor has the phase factor $\exp(\mp i\delta_1)$, so the result is the same whether we use + or − scattering states. Next we make an approximation for reasons of expediency. Each mass should receive a different boost in (2.13). However, our result will suggest that the main contribution comes from relatively low masses, so we approximate all the rest frames by the one applying only at the threshold mass. With these approximations, the spatial wave function in this particular rest frame reduces to (normalization arbitrary)

$$[m_\rho^2 - \Pi(0)]\boldsymbol{\varepsilon}\cdot\nabla\int_{\mathfrak{M}_t^2}^{\infty}\frac{d\mathfrak{M}^2}{\mathfrak{M}^2 + Q^2} \times \frac{\sin(q_\pi r + \delta_1)\,|F_{\rho\pi\pi}(\mathfrak{M}^2)|}{r\,|\mathfrak{M}^2 - m_\rho^2 + \Pi(\mathfrak{M}^2)|}$$

$$= [m_\rho^2 - \Pi(0)]\boldsymbol{\varepsilon}\cdot\nabla\int_C\frac{d\mathfrak{M}^2}{\mathfrak{M}^2 + Q^2} \times \frac{\exp(iq_\pi r)F_{\rho\pi\pi}}{2ir[\mathfrak{M}^2 - m_\rho^2 + \Pi(\mathfrak{M}^2)]},$$

where the contour is the same one that occurred in the evaluation of $P_{2\pi}$. Since the integrand is exponentially damped at ∞, the integral may again be evaluated by the residue at $-Q^2$. The remarkable result is

$$F_\pi(-Q^2)\,\boldsymbol{\varepsilon}\cdot\nabla\{\exp[-(m_\pi^2+\tfrac{1}{4}Q^2)^{1/2}r]/r\}. \tag{3.7}$$

To see the significance of this result, we first set $Q^2 = 0$ and note that the resulting wave function is completely independent of the ρ^0 meson! Physically, this means that the probability associated with the ρ^0 is contained entirely inside the interaction region. This result could have been anticipated from the theory of decaying states, in which a superposition of energy eigenfunctions from the vicinity of a resonance leads to a localized state. If we had neglected the strong interactions of the pions, the result (3.7) would have been valid at all radii (except that the assumption about frames would have been less justified at smaller radii where higher masses would be much more important). The divergence of $P_{2\pi}$ in that case is undoubtedly associated with the strong singularity in the wave function. As a result of the interactions, this singular part of the wave function is somehow eliminated in favor of the ρ^0 structure. This reinforces the approximate picture that the dipion constituent of the photon may be regarded as a ρ^0 meson core surrounded by a two pion cloud. However, the dipion constituent is a complete unit and there is no way to make a unique physical separation into these two parts.

The next point of interest is the photon shrinking which is evident in (3.7) (Cheng and Wu, 1969; Bjorken, Kogut, and Soper, 1971; Kogut, 1972). As Q^2 increases, the two-pion tail of the photon is rapidly drawn in. The range is again independent of the ρ^0, but the strength depends on the ρ^0 through the pion form factor. While it cannot be proved by this type of argument, it seems plausible that the whole structure may continue to shrink as Q^2 increases. This means that although one may choose to regard the ρ^0 as a well-defined object of definite internal size, it is still possible to superpose states, including the ρ^0, which have a spatially smaller structure. (Analogy: the ground state of the hydrogen atom is well spread out yet we may form a superposition, including the ground state, which is very well localized). *If this view is correct, then the hadronic component of the photon cannot be regarded a sum of separately interacting constituents. Rather, all the constituents would have to act in unison, corresponding to an object whose structure, and hence interactions depends on Q^2 in a nontrivial way.* This would be in distinction to the diagonal assumption of the generalized VMD hypothesis, which has each component interacting independently in the total cross section. In a sense, it is possible that as far as interactions are concerned *the whole is less than the sum of its parts.*

Let us write this portion of the photon's hadronic component as

$$|\,2\pi,\mathbf{k}\rangle = [P_{2\pi}(Q^2)]^{1/2}\,|\,``\rho", Q^2,\mathbf{k}\rangle, \tag{3.8}$$

where $P_{2\pi}$ is given by (3.5) or (3.6). The state $|\,``\rho"\rangle$ is normalized to unity (δ function). We may think of it roughly as a ρ-meson state whose properties vary with Q^2. Physically, it is a rather extended structure for small Q^2; but as Q^2 increases, it shrinks into a tighter unit. Possible consequences of this varying behavior on the electroproduction of ρ^0's from nucleons and nuclei, and on photon shadowing as a function of Q^2, will be discussed later and will be studied in more detail in a separate paper.

What are the experimental consequences of the dipion part of the photon's structure? It has already been pointed out that it gives approximately two-thirds of the total photon cross section. If the whole dipion constituent were absorbed like the ρ^0, we would expect this to be enhanced by several percent, due to the $\Pi'(0)$ term in (3.5). Actually, it seems plausible that the two-pion tail should experience a somewhat larger absorption than the ρ^0 core. A calculation of this effect is extremely model dependent, of course, but it is trivial to construct reasonable models which could raise the dipion contribution to 90–100 μbarns. Such a model is presented in the Appendix; it yields a total cross section contribution of 94 μbarns. Since the VMD estimate for the ω and ϕ contributions together also accounts for about 15 μbarns, the amount which remains for other constituents is very small and very uncertain.

The dipion component also shows up as diffractive photoproduction of $\pi^+\pi^-$ pairs. This is a very important process experimentally as it accounts for about 15% of the total photon cross section, or about 20% of the isovector part of the total photon cross section. This corresponds reasonably well to the usual ratio of hadronic total elastic cross section to total cross section. In more detail, the observed mass distribution of pion pairs is qualitatively similar to that in the original structure of the photon. Examples are shown in Fig. 2; the data is from the DESY–MIT group.[5] The curve labeled ρ^0 represents the pure ρ^0 part of the data as fitted by R. Spital and Yennie.[6] It is obvious that the resonance peak is strongly skewed toward lower masses by an interfering background. The simplest model for such a mass distribution would be that each mass component in the photon is individually absorbed by the target (i.e., the diagonal assumption). If all components experienced the same absorption, the resulting amplitude for pair production would be

$$\begin{aligned}\text{M.E.} &\propto \frac{1}{\mathfrak{M}^2}\frac{-m_\rho^2+\Pi(0)}{\mathfrak{M}^2-m_\rho^2+\Pi(\mathfrak{M}^2)}F_{\rho\pi\pi}\\ &\approx \frac{1}{\mathfrak{M}^2}\frac{-m_\rho^2}{\mathfrak{M}^2-m_\rho^2+\Pi(\mathfrak{M}^2)}\,.\end{aligned} \tag{3.9}$$

The square of this leads to the phenomenological Ross–Stodolsky formula for the mass distribution (Ross and Stodolsky, 1966). This is shown in Fig. 2 as the curves labeled R–S. This distribution has fallen into disfavor because of its apparently questionable derivation. However, it is now apparent that it does incorporate much of the correct physics, under the assumption that each mass component acts separately, as was also assumed by Ross and Stodolsky. One can see how it might be improved by permitting the absorption to vary across the mass distribution. One might expect, for example, that the lower masses which correspond to the two-pion tail should have a larger am-

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[5] I thank Professors U. Becker and S. Ting for generously supplying this data.

[6] The fitting procedure is described in Spital and Yennie (1974b). In the present fits, the mass and width of the ρ^0 were held fixed at 770 and 140 MeV, respectively, while the strength of the ρ^0 and the $\rho-\omega$ interference and background terms were varied to obtain a best fit. In the two cases shown in Fig. 2, the values of χ^2 were 54 and 46 for 47 and 39 degrees of freedom respectively. I wish to thank Dr. R. Spital for obtaining these fits for me.

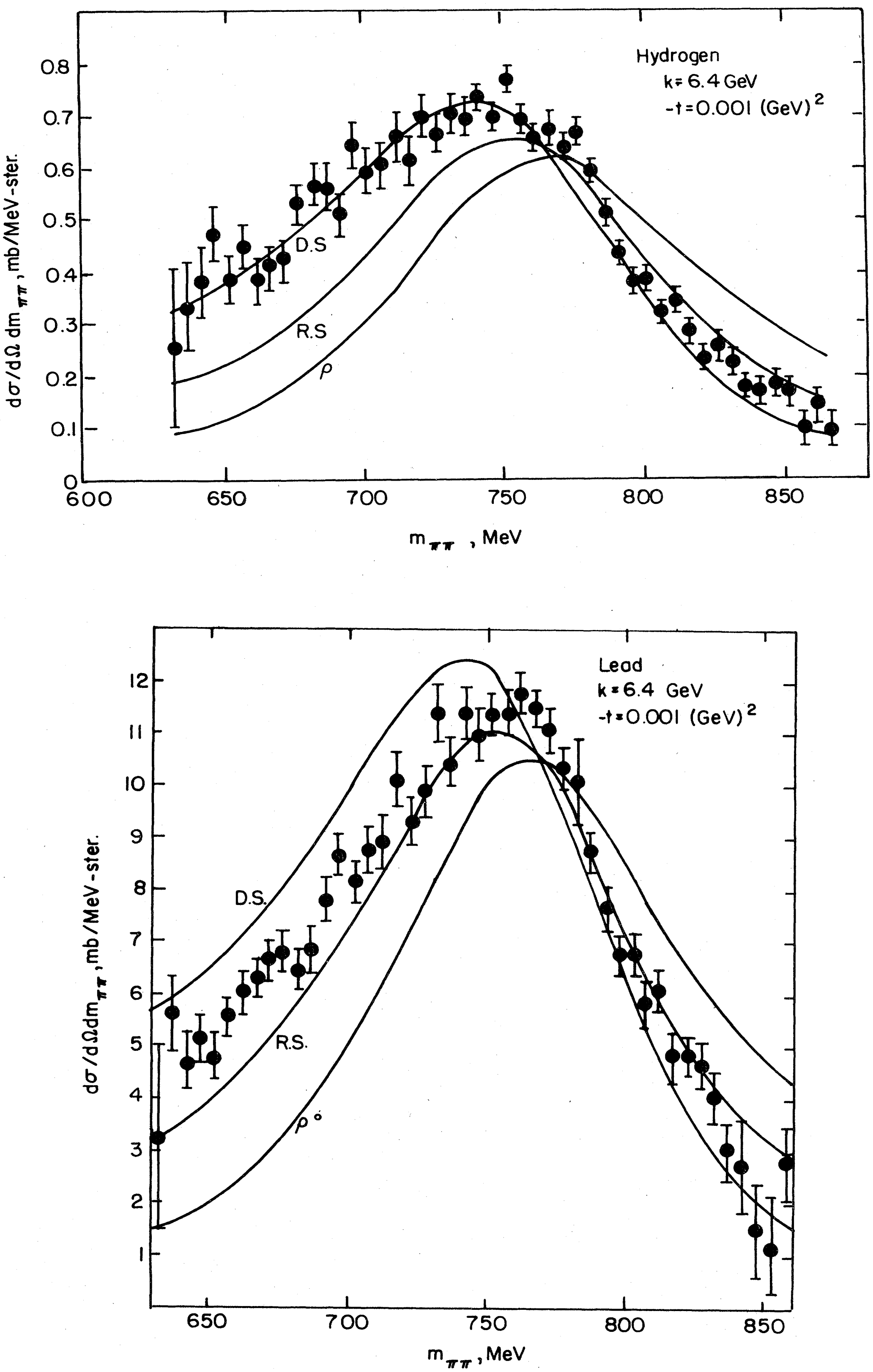

FIG. 2. Two examples of mass distributions in pion pair photoproduction. The curve labeled ρ^0 is the pure ρ^0 part of a best fit with an adjustable background (Spital and Yennie, 1974b). The curves labeled R-S and D-S refer to phenomenological models of Ross and Stodolsky (1966) and Söding (1966). The data are from the DESY-MIT group.[5]

plitude than is given by (3.9). This appears to happen. Further, the low mass components should have a larger interaction radius than the ρ^0 core and they should therefore have a greater slope (in t) than does the ρ^0. This is also an apparent feature of the data (not shown in Fig. 2).

We may also see the connection of (3.9) with another popular treatment of the pair mass distribution, namely the Söding model (Söding, 1966). A simple rearrangement of (3.9) gives

$$\text{M.E.} \propto \frac{-1}{\mathfrak{M}^2 - m_\rho^2 + \Pi(\mathfrak{M}^2)} + \frac{1}{\mathfrak{M}^2}\frac{\mathfrak{M}^2 - m_\rho^2}{\mathfrak{M}^2 - m_\rho^2 + \Pi(\mathfrak{M}^2)}. \tag{3.10}$$

The first term of (3.10) may be interpreted as the pure ρ^0 term. For example, it does not contribute to the two-pion tail in configuration space; if we go through the steps leading to (3.7) with this term alone, there is no longer a pole at $-Q^2$ ($=0$ now) and the result is zero. In configuration space, the entire contribution from the first term of (3.10) is contained inside the interaction region. (The physical interpretation is a little unclear, however, since the integral giving the probability may diverge unless it is cut off strongly by $F_{\rho\pi\pi}$). The second term of (3.10) closely resembles the usual Drell amplitude (Drell, 1960) for pair production as modified by the double counting correction of Bauer (1971) and Pumplin (1970). Because of their different spatial structure, we would expect these two contributions to be differently absorbed. The Söding model accounts for this by assigning the two amplitudes independent strengths, interpreted as being proportional to σ_ρ and $\sigma_{\pi^+} + \sigma_{\pi^-}$, respectively; in fact, the second term does turn out experimentally to be enhanced relative to the first. The curves labeled D–S in Fig. 2 correspond to this model, with $\sigma_{\pi^+} + \sigma_{\pi^-}$ taken to be $2\sigma_\rho$ for simplicity. The D–S curve fits the hydrogen data quite well, but the lead data lies between the R–S and D–S curves. In fitting experimental data with the Söding model, the nonresonant amplitude is found to become relatively less important as $|t|$ increases. This is the same phenomenological behavior noted in the preceding paragraph as being due to the more extended structure of the nonresonant two-pion component.

An actual calculation of the pair production from the hadronic component has not been made directly, but the Söding model, particularly as elaborated by Bauer (1971) incorporates its physical ideas. The usual Drell amplitude corresponds to one-pion diffracting through the target while the other is simply released with its original momentum. The factor from the pion propagator is related to the energy denominator in the present discussion. However, only in the case of forward production, where the momentum transfer to the target is extremely small, is the mass of the hadronic component nearly equal to the mass of the pion pair ultimately observed. For finite angles of production, a superposition of initial masses produces each final mass. There is another contribution to the amplitude where both pions interact with the target. As shown by Bauer (1971), this tends to reduce the amplitude relative to the Drell amplitude alone. Physically it is very similar to the Glauber shadowing correction in deuterium: the total cross section for the pion pair to interact is necessarily less than the sum of the cross sections of the individual pions.

Using Bauer's ideas, it is possible to understand the qualitative difference between Figs. 2a and 2b. He finds for lead that the Drell part of the amplitude is reduced by nearly a factor of 2 due to the shadowing correction. That is, the total cross section of the two nonresonant pions is comparable to the total ρ^0 cross section. This would tend to make R–S the correct curve to compare with the data. However, there is another important effect which must be taken into account. At finite energies, the cross section is reduced because of a form factor effect due to the momentum transfer ($= m_{\pi\pi}^2/2\nu$) necessary to turn a photon into a pion pair. This reduction is smaller at lower masses, and hence the data rises above the Ross-Stodolsky curve in that region.[7]

It is clear that this general picture gives a good account of the main features of pion pair photoproduction from nucleons and nuclei. However, it is equally clear that there is very little hope of constructing a theory which will predict the mass spectrum perfectly as a function of energy, momentum transfer, and nucleus. Such a theory would treat the dipion component as a unit rather than make an artificial distinction between a pure ρ^0 and nonresonant pions. Therefore, it has been necessary to resort to a phenomenological description for the purpose of interpreting experiments (Spital and Yennie, 1974a,b). This description concentrates on determining the ρ^0 cross section and gives up any attempt to extract useful quantitative information from the background region.

How should the discussion be modified to include electroproduction? The most obvious effect is to modify (3.9) and (3.10) by changing the energy denominator

$$\begin{aligned}\text{M.E.} &\propto \frac{1}{\mathfrak{M}^2 + Q^2}\frac{-m_\rho^2}{\mathfrak{M}^2 - m_\rho^2 + \Pi(\mathfrak{M}^2)} \\ &= \frac{\mathfrak{M}_\rho^2}{m_\rho^2 + Q^2}\left[\frac{-1}{\mathfrak{M}^2 - m_\rho^2 + \Pi(\mathfrak{M}^2)}\right. \\ &\qquad \left. + \frac{1}{\mathfrak{M}^2 + Q^2}\frac{\mathfrak{M}^2 - m_\rho^2}{\mathfrak{M}^2 - m_\rho^2 + \Pi(\mathfrak{M}^2)}\right].\end{aligned} \tag{3.11}$$

Comparing this with (3.10), we see that the low mass region is rapidly suppressed relative to its real photon behavior as Q^2 increases from zero to m_ρ^2 and beyond. On the other hand, because the pion tail is spatially drawn in, the t dependence of the background should become broader as Q^2 increases. In fact, it has been speculated that the whole t distribution, *including the* ρ^0, may become broader as Q^2 increases (Cheng and Wu, 1969; Bjorken, Kogut, and Soper, 1971; Kogut, 1972). The idea behind this speculation is that the whole physical size of the dipion component given by (3.7) becomes smaller as Q^2 increases. Any small mass range, such as that primarily associated with the ρ^0, may have an extended structure, but through superposition the net structure is compact. As a consequence, the shadow region behind a target nucleon will have a smaller transverse extension and the particles which are photoproduced will diffract out to wider angles. Bauer has calculated such an effect, but it turns out to be rather small (Bauer, 1973). The reason is apparently that his model includes only the

[7] Another physical effect which influences the ρ^0 shape in photoproduction from nuclei (at lower energies) is discussed in Gottfried and Julius (1970).

shrinking two-pion tail rather than the shrinking of the whole dipion structure. As the dipion component shrinks, it is conceivable that it can interact only with the more central regions of the nucleon, where the harder partons reside. If so, the shadow region could shrink to a smaller size than the nucleon itself,[8] and the slope parameter in the t distribution could become very small. The mass distribution of the material shadowed out (and hence diffractively photoproduced) will probably be approximated by (3.11), but different masses could have a somewhat different impact parameter and t dependence. If this picture is correct, the total cross section for electroproducing ρ^0's should decrease relative to VMD and the t dependence should be broadened. The shrinking photon effect has perhaps been confirmed by electroproduction of ρ^0 mesons (Ahrens *et al.*, 1974; Dakin *et al.*, 1973; Eckardt *et al.*, 1973) (however, the experimental results could be in part due to threshold effects)[9]. The data can also be interpreted as showing that the total cross section for $|``\rho," Q^2\rangle$ on a nucleon is a decreasing function of Q^2.[10]

This picture would have interesting consequences for processes in nuclei, particularly if the energy is high enough for the different mass components to remain in phase after production. In that case the value of σ_ρ, as determined by the A dependence of electroproduction, might be a decreasing function of Q^2. That is, the dipion state could penetrate nuclear matter more freely. Similar effects have been observed from hadronic reactions on nuclei ($\pi \rightarrow 3\pi$), (Bemporad *et al.*, 1971; Beusch, 1972) where the produced particles act as a unit with about the same cross section as the incident particle. Another effect would be a drastic reduction in photon shadowing in nuclei as Q^2 increases. While such an effect has been seen (Kendall, 1972; Friedman and Kendall, 1972),[11] it is not yet known whether it can be completely accounted for in this manner. There is not yet enough information from other experiments to carry through a convincing calculation for photon shadowing, but preliminary estimates indicate that one can fit the data with a reasonable model.

IV. INTERACTIONS OF PHOTONS

Until now we have described the interaction of photons intuitively; that is, we assumed that the photon with its hadronic component encountered a nucleon and this component then interacted more or less like an ordinary hadron, while the bare photon component might also interact in some manner. We picture this most easily in configuration space where we imagine the photon dissociating into hadrons some distance before reaching the target nucleon. The *formation distance* or *time* for some component is given by the uncertainty principle argument to be $2\nu/(\mathfrak{M}^2+Q^2)$. This space–time view will be elaborated in another paper (Spital and Yennie, 1975) where it will be shown that in the case of narrow resonances the actual evolution of the hadronic component may take much longer than this formation time indicates. In reality, the formation distance is of the nature of a coherence length. Only those parts of the hadronic component produced within such an interval can act together coherently. When the formation time is of the order of the collision time (e.g., the radius of the proton, R_p) or smaller, it is clear that it becomes meaningless to visualize the interaction as due to a pre-existing hadronic component. Therefore the higher mass components ultimately merge with the direct interaction of the photon.

We shall now give a heuristic discussion to justify this picture of the photon interaction through two different mechanisms.[12] We hasten to emphasize that such a separation is not a unique, but only an approximate, concept. In the first place, it is Lorentz frame dependent (things may seem quite different in the proton's infinite momentum frame!). In the second place, the separation is not gauge invariant. In spite of these two related defects, the approximate separation may be conceptually useful for interpreting experimental results. At extremely high energies, the hadronic component surely dominates the real photon total cross section and many of the individual channels. It probably also dominates inelastic electron scattering in the small x (i.e., $Q^2/2M\nu$) region where νW_2 appears to have a limiting value. This limiting value (of order 0.3) is associated with the longest longitudinal range interaction of the virtual photon, which is clearly due to its hadronic component (suri and Yennie 1972). At larger x, the hadronic and direct interactions merge and it is meaningless to think of them as separate entities. However, it is significant that νW_2 decreases with increasing x, suggesting that the role of the direct interaction may in large part be a short-range weakening of the effect of the hadronic component.

Consider the electromagnetic matrix element $\langle f- | J_\mu | N\rangle$ (with $E_f = E_N + \nu$, $\mathbf{P}_{f'} = \mathbf{P}_N + \mathbf{k}$) for a real or virtual photon interaction with a nucleon to produce a final state $|f-\rangle$. We shall argue that this matrix element should be decomposable into the form[12]

$$\begin{aligned}\langle f- | J_\mu(0) | N\rangle &= -(2\pi)^{3/2} e \int \frac{d\mathfrak{M}^2}{2P_0(\nu - P_0)} \\ &\quad \times \mathop{\mathsf{S}}_{n_i} \langle f- | H_I(0) | N, \mathbf{k}, \mathfrak{M}, n_i+\rangle_{\text{free}} \\ &\quad \times \langle \mathbf{k}, \mathfrak{M}, n_i+ | J_\mu(0) | \text{vac}\rangle + \langle f- | J_\mu(0) | N\rangle_{\text{dir}}, \end{aligned} \tag{4.1}$$

where $P_0 = (\mathfrak{M}^2 + \mathbf{k}^2)^{1/2}$. The meaning of the two terms must be stated carefully. The separation is based on time ordered perturbation theory, the first term containing all contributions in which the photon transforms into hadrons before interacting and the second representing direct interaction of the photon with hadrons already present in the physical nucleon. The subscript "free" means that the nucleon and hadronic component of the photon are completely dressed but have not yet interacted with each other. Their initial interaction is represented by $H_I(0)$, and $\langle f-|$ contains all subsequent final state interactions. This matrix

[8] This disagrees with a view expressed by Nieh (1972).

[9] A dissenting view is given by Talman (1974).

[10] For the purposes of Fig. 3 below, the Q^2 dependence of σ_ρ was guessed to be $[1+0.3Q^2/M^2]^{-1}$.

[11] According to R. Talman (private communication), radiative corrections, whose effects were largely expected to cancel out in this experiment, may lead to a slower dropoff in shadowing than the preliminary analysis had indicated.

[12] This argument is examined in more detail in Spital and Yennie (1975).

element would agree with the T-matrix element except for the fact that it is not on the energy shell ($E_f \neq E_n + P_N$); in fact, the deviation from the energy shell is just the energy denominator $\nu - P_0$, which is small for small Q^2 and $\mathfrak{M}^2$.

We note the lack of Lorentz and gauge invariance of the separation. The first is obvious since a particular frame is singled out to define $\mathbf{k}$ and ν; however, for masses satisfying (2.5), the explicit frame dependence drops out. But (2.5) is frame-dependent also; it is not valid in the infinite momentum frame usually employed in the parton model. The fact that the individual terms are not gauge invariant is seen from the identity

$$k^\mu \langle \mathbf{k}, \mathfrak{M}, n_i + \mid J_\mu(0) \mid \text{vac} \rangle = (\nu - P_0) \langle \mathbf{k}, \mathfrak{M}, n_i + \mid J_0(0) \mid \text{vac} \rangle. \tag{4.2}$$

This means either that the hadronic photon contributions compensate each other or that we may ignore the direct term only at the expense of violating gauge invariance. As will be seen later, the treatment of longitudinal interactions requires special care. Any prescription we may choose in treating the hadronic component alone will amount to some assumption about the properties of the direct term. However, because of the small factor $\nu - P_0 \approx (\mathfrak{M}^2 + Q^2)/2\nu$ on the r.h.s. of (4.2), gauge invariance is almost satisfied by the hadronic component alone. We shall find later that it is possible for the hadronic component to dominate the interaction even though a small direct term must be present. At the same time, one should be aware that it makes no sense whatever to take effects of relative order $\nu - P_0$ seriously without at the same time including contributions from the direct term. Since only the lower masses should behave like ordinary hadrons, we rearrange (4.1) to combine the high mass contributions with the direct terms:

$$\begin{aligned} \langle f - \mid J_\mu(0) \mid N \rangle &= (2\pi)^{3/2} e \int^{\mathfrak{M}_1^2} \frac{d\mathfrak{M}^2}{\mathfrak{M}^2 + Q^2} \\ &\quad \times \underset{n_i}{\mathsf{S}} \langle f - \mid H_I(0) \mid N, \mathbf{k}, \mathfrak{M}, n_i + \rangle_{\text{free}} \\ &\quad \times \langle \mathbf{k}, \mathfrak{M}, n_i + \mid J_\mu(0) \mid \text{vac} \rangle + \langle f - \mid J_\mu(0) \mid N \rangle_{\text{bare}}, \end{aligned} \tag{4.3}$$

where "bare" combines the high mass components with the usual direct interaction. In this form, the two terms are (approximately) separately gauge invariant.

Equation (4.3) is presented more for intuitive discussion than as a practical starting point for calculations. It is meant to suggest that each photon interaction can be conceived to take place via two mechanisms. This is of course only an approximate idea and it is likely to be useful only if one or the other mechanism dominates a given process. The separation mass $\mathfrak{M}_1$ could depend on the process as well as on Q^2 and ν. For example, if a given process is believed to have a very short collision time for some reason (e.g., interaction with most central region of a nucleon), $\mathfrak{M}_1$ could be larger than for a typical reaction in which the collision time is of order R_P.

To the extent that the second term of Eq. (4.3) may be neglected for a given process, we may interpret the equation as an expression of generalized vector meson dominance. $\langle \mathbf{k}, \mathfrak{M}, n_i + \mid J_\mu(0) \mid \text{vac} \rangle$ gives the amplitude for the photon to convert to a $J^{PC} = 1^{--}$ system; $1/(Q^2 + \mathfrak{M}^2)$ corresponds to the propagator of the 1^{--} system; and $\langle f - \mid H_I(0) \mid N, \mathbf{k}, \mathfrak{M}, n_i + \rangle_{\text{free}}$ represents the interaction of the 1^{--} system with the target (N) to produce the final state f. Without further assumptions and approximations, this formula is of little practical utility. One such approximation is to replace the integral over $\mathfrak{M}^2$ and sum over n_i by a discrete sum over a few vector mesons ($\rho, \omega, \phi, \cdots$). Then some information about the matrix element may be obtained by studying the inverse reaction $f \to NV$, provided that $\mid f \rangle$ is sufficiently simple to serve as a realizable incident state. The qualitative and quantitative successes of VMD will not be reviewed here,[13] but they have been sufficient so that there seems little doubt that the 1^{--} resonances play an important role in the hadronic structure of the photon. On the other hand, it seems clear that beyond providing a reasonable account of certain processes, the more complete physics represented by Eq. (4.3) is difficult to incorporate into any simple improvement on VMD. Unfortunately, the first term of Eq. (4.3) is so general that it could dominate photon interactions without being contradicted by any known experimental results, including deep inelastic electron scattering. This will be elaborated in another paper (Spital and Yennie, 1975) where it will also be discussed how GVMD can merge with the parton model. It may then be that, aside from estimating and relating certain specific processes, GVMD will play its most important role in providing a qualitative understanding of various features of photon processes.

Let us briefly consider some processes from the point of view of GVMD. The simplest, and currently most important, is the total photon cross section as a function of Q^2. In the present work, we shall make some simplifying assumptions and then treat the dipion contribution to νW_2. Before doing this, let us discuss some of the conceptual and logical questions in applying (4.3) to the total cross section. The first question concerns the use of the diagonal approximation described in Sec. II. This approximation assumes that while different photon constituents ($\mathfrak{M}^2, n_i$) may interfere in specific final channels (f), their net interference in the total cross section is unimportant. It should be emphasized that there is no property of (4.3) which would lead to such a conclusion. However, it does seem plausible that if two different components ($\mathfrak{M}_1^2, n_{i1}$ and $\mathfrak{M}_2^2, n_{i2}$) are sufficiently different in nature—spatial size, particle configurations, etc.—their interference in $\sigma_{\gamma\text{tot}}$ will be small. Thus for small Q^2, a spatially extended component associated with a narrow resonance might not interfere appreciably with a component having a broad mass distribution and a very localized structure. Then as Q^2 increases and the complete hadronic structure becomes very localized, it is likely that interference between different mass components becomes more significant. It is inconceivable that the various components which may separately be spatially larger

[13] A good overview of vector meson dominance is given in Sakurai (1969).

than their superposition will fail to interfere in a manner which reflects the physical properties of that superposition. For example, it is possible that a broad mass distribution of different (asymptotically defined) components might correspond to a localized state that is primarily a simple quark pair. In that case the interaction could be described more simply in terms of the quark pair than in terms of the physical particles labeled by $\mathfrak{M}^2$ and n_i.

Another troublesome problem is how to treat longitudinal cross sections. We avoided discussing longitudinal interactions in Sec. II, but we may now note that (2.10) is replaced by

$$\begin{aligned}
&\langle \mathbf{k}, \mathfrak{M}, n_i + \mid J_0(0) \mid \text{vac}\rangle \\
&\quad = (k/\mathfrak{M})\langle \mathbf{0}, \mathfrak{M}, n_i + \mid J_3(0) \mid \text{vac}\rangle \\
&\langle \mathbf{k}, \mathfrak{M}, n_i + \mid J_3(0) \mid \text{vac}\rangle \\
&\quad = \frac{(k^2+m^2)^{1/2}}{\mathfrak{M}}\langle \mathbf{0}, \mathfrak{M}, n_i + \mid J_3(0) \mid \text{vac}\rangle,
\end{aligned} \tag{4.4}$$

where $\mathbf{k}$ is taken in the 3-direction, and we have used Lorentz invariance and current conservation. The polarization vector ϵ^μ multiplying this current satisfies $k_\mu \epsilon^\mu = 0$, yielding

$$\begin{aligned}
&\epsilon^\mu \langle \mathbf{k}, \mathfrak{M}, n_i + \mid J_\mu(0) \mid \text{vac}\rangle \\
&\quad = \frac{k^2 - \nu(k^2+m^2)^{1/2}}{\nu\mathfrak{M}}\epsilon^3\langle \mathbf{0}, \mathfrak{M}, n_i + \mid J_3(0) \mid \text{vac}\rangle \\
&\quad \approx \frac{Q^2 - \mathfrak{M}^2}{2\nu\mathfrak{M}}\epsilon^3\langle \mathbf{0}, \mathfrak{M}, n_i + \mid J_3(0) \mid \text{vac}\rangle.
\end{aligned} \tag{4.5}$$

Thus a constituent of mass $\mathfrak{M}$ apparently fails to interact when the photon's (spacelike) four-momentum Q^2 happens to equal $\mathfrak{M}^2$. On the other hand, the complete amplitude is gauge invariant so that

$$k^\mu \langle f - \mid J_\mu(0) \mid N\rangle = 0$$

and

$$\epsilon^\mu \langle f - \mid J_\mu(0) \mid N\rangle = Q^2/\nu^2)\epsilon^3\langle f - \mid J_3(0) \mid N\rangle. \tag{4.6}$$

Thus the hadronic component might be taken to be proportional to

$$(Q^2/\nu\mathfrak{M})\epsilon^3\langle \mathbf{0}, \mathfrak{M}, n_i + \mid J_3(0) \mid \text{vac}\rangle \tag{4.7}$$

in place of (4.5). It is easy to see that the direct term contribution, when added to (4.5), reproduces (4.7). Gauge invariance yields

$$\begin{aligned}
&k^\mu \langle f - \mid J_\mu(0) \mid N\rangle \\
&\quad = 0 \cong -(2\pi)^{3/2} e\int^{\mathfrak{M}_1^2} (d\mathfrak{M}^2/2\mathfrak{M}) \\
&\quad\quad \times \underset{n_i}{\mathsf{S}}\,\langle f - \mid H_I(0) \mid N, \mathbf{k}, \mathfrak{M}, n_i + \rangle_{\text{free}} \\
&\quad\quad \times \langle \mathbf{0}, \mathfrak{M}, n_i + \mid J_3(0) \mid \text{vac}\rangle \\
&\quad\quad + k^\mu \langle f - \mid J_\mu(0) \mid N\rangle_{\text{bare}}.
\end{aligned} \tag{4.8}$$

Now assume that the bare contribution is, component for component, smaller than the hadron-mediated contribution to (4.3). For example, assume that either the 0 or 3 component of the bare term is negligible. Then use (4.8) to solve for the 3 or 0 component. When this contribution is added to that of (4.5), the result obtained is (4.7), to relative order Q^2/ν^2. In addition to this part of $\langle f - \mid J_\mu \mid N\rangle_{\text{bare}}$ required by gauge invariance, there is of course an indeterminant part orthogonal to k^μ. What we have shown is that hadron-mediated dominance is compatible with gauge invariance; we have *not* shown that the bare term is actually small.

Having mentioned these difficulties, we now proceed to ignore them and calculate the dipion contribution to νW_2. The form (4.7) will be used in estimating the longitudinal part. This amounts to defining

$$\sigma_L/\sigma_T = (Q^2/\nu^2)(W_{33}/W_{11}), \tag{4.9}$$

where $W_{\mu\nu}$ is the function used in inelastic electron scattering. The details of the calculation are given in the Appendix. The model used incorporates the diagonal assumption and also takes the total absorption cross section of the dipion to decrease from $2\sigma_\rho^{\text{tot}}$ ($\approx \sigma_{\pi^+} + \sigma_{\pi^-}$) at small masses, to σ_ρ^{tot} at m_ρ^2, and to zero at high masses. As was previously mentioned, this increases the dipion contribution to the real photon total cross section from 80 μbarns (VMD) to 94 μbarns. It also increases the dipion contribution relative to VMD as a function of Q^2. The results for νW_2 are shown in Fig. 3 and compared with the data in the small x region (Friedman and Kendall, 1972). The ω and ϕ contributions as estimated by VMD are also incorporated.

The noteworthy features of this figure are the following: (i) The data are a function primarily of Q^2, and only slightly a function of x. This is precisely the general behavior expected in the small x region where the diffractive hadronic components should dominate (suri and Yennie, 1972). (ii) The data rise from 0 at $Q^2 = 0$ to about 0.33 at $Q^2 = 1$. Thus the maximum value of νW_2 at small x can be attributed primarily to the hadronic component of the photon. In the parton model, this behavior is instead associated with a dx/x distribution in the longitudinal momentum. In our view, it is more natural to consider this small x cross section, which is also related to a long longitudinal range,[14] to be due to the hadronic constituents of the photon. The subsequent fall of the data with increasing x could be attributed to a weakening of the strength of the hadronic interaction as the formation distance $[\sim 2\nu/(Q^2 + M^2)]$ becomes smaller than the nucleon size. In a later paper (Spital and Yennie), it will be shown how the hadronic component can give scaling with a general x dependence. (iii) The dipion, plus the ω and ϕ, may account for the main part of the data in the small x region. It is, however, more likely that the present model gives a significant overestimate of the dipion component with increasing Q^2. There is evidence that electroproduction of ρ^0's falls more rapidly with Q^2 than is predicted by VMD. This would fit in with the shrinking photon concept that the dipion acts as a unit, as suggested by (3.8). A guess as to how that might change the dipion contribution is shown in the figure. (iv) The model gives a

[14] This connection was noted by Ioffe (1969) and was studied more explicitly by Pestieau, Roy, and Terazawa (1970).

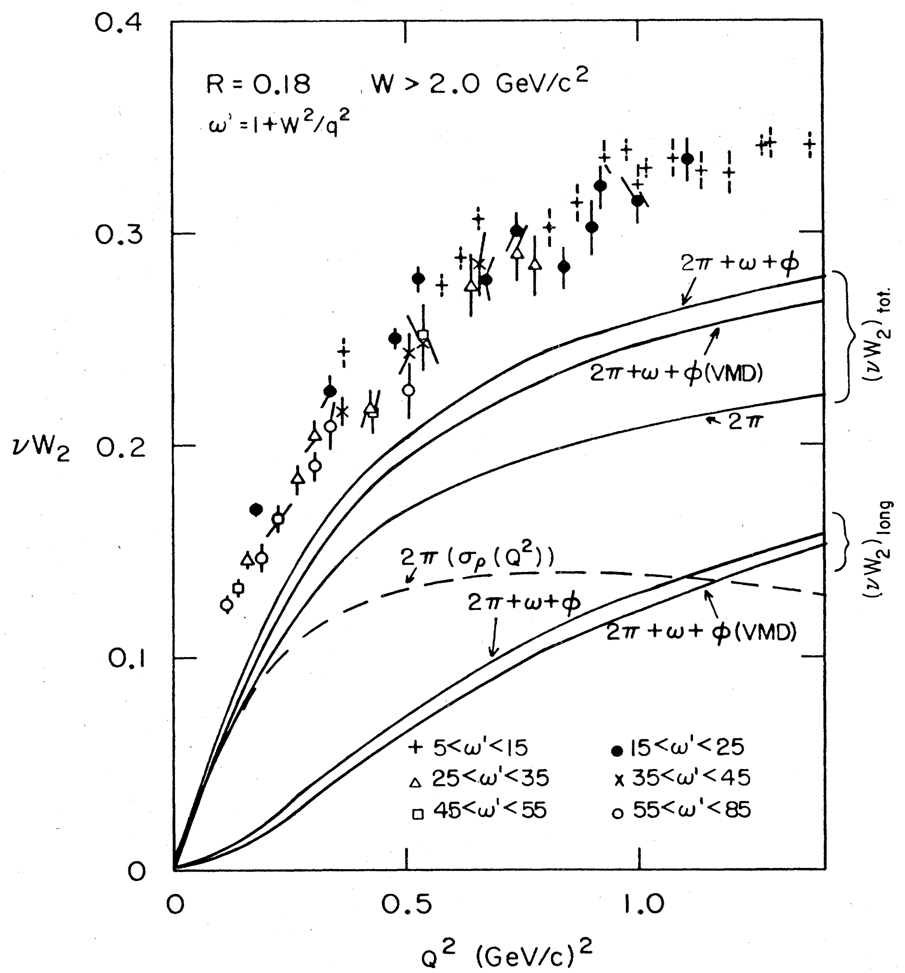

FIG. 3. νW_2 in the small $x(=Q^2/2M\nu)$ region. The two pion contribution is as treated in the text and the ω and ϕ are estimated using VMD.

rather large value for σ_L/σ_T. While this ratio does appear to rise experimentally for decreasing Q^2,[15] it remains well below the value suggested by the model. More data is needed in this region.

Until now we have used the intuitive idea of (4.3) to discuss diffractive processes. The treatment of specific final channels is much more problematic. Here there is certainly likely to be interference between different masses, and different types of constituents. Photoproduction of single pions is a case in point. The usual VMD discussion indicates that the vector mesons play a role in the process, but in terms of the present picture the correct treatment is far from obvious. Intuitively, in addition to the ρ^0 contribution, there should also be a contribution in which one of the pions in the tail is captured by the target while the other is released. It is hard to decide how much of this latter contribution is already contained in the VMD terms. In any case, the hadronic structure plays a role in the process, even though the details are too delicate to calculate from (4.1).

A possibly more tractable process to calculate is the contribution of the loose two-pion structure to the inclusive pion spectrum. The picture is that one pion hits the target and interacts in some way which is not detected while the other flies free and is detected. This is reminiscent of the original Drell process (Drell, 1960) which is illustrated in Fig. 4(a). It is easy to see that Drell's result corresponds to precisely this picture. His original formula was

$$d\sigma_{\gamma\to\pi^-}=\frac{\alpha}{2\pi}\frac{\sin^2\theta}{(1-\beta\cos\theta)^2}\frac{d\Omega}{4\pi}\frac{\omega(k-\omega)}{k^3}\,d\omega\sigma_{\pi^+}{}^{\text{tot}}. \tag{4.10}$$

Let us re-express this in the terms of the intermediate two-pion total mass. Let $q_{\parallel}$ and $\mathbf{q}_{\perp}$ be the components of the pion momentum $\mathbf{q}_-$ parallel and perpendicular, respectively, to the direction of k; then this mass is given by

$$\begin{aligned}\mathfrak{M}^2 &= [(q_{\parallel}{}^2+q_{\perp}{}^2+\mu^2)^{1/2}\\ &\quad+((k-q_{\parallel})^2+q_{\perp}{}^2+\mu^2)^{1/2}]^2-k^2\\ &\cong (q_{\perp}{}^2+\mu^2)k^2/q_{\parallel}(k-q_{\parallel})\\ &\qquad [q_{\parallel},\,k-q_{\parallel}\gg(q_{\perp}{}^2+\mu^2)^{1/2}]. \end{aligned}\tag{4.11}$$

In this language, Drell's formula becomes

$$d\sigma_{\gamma\to\pi^-}\cong\frac{\alpha}{2\pi}\frac{d\mathfrak{M}^2}{\mathfrak{M}^4}\,q_{\perp}{}^2\,\frac{dq_{\parallel}}{k}\,\sigma_{\pi^+}{}^{\text{tot}}, \tag{4.12}$$

[15] The data [Riordan *et al.* 1974)] for σ_L/σ_T appear to have a maximum of about 0.6 for $Q^2\simeq 2$. The values of x $(=Q^2/2M\nu)$ of about 0.1 are probably too high for comparison with the model.

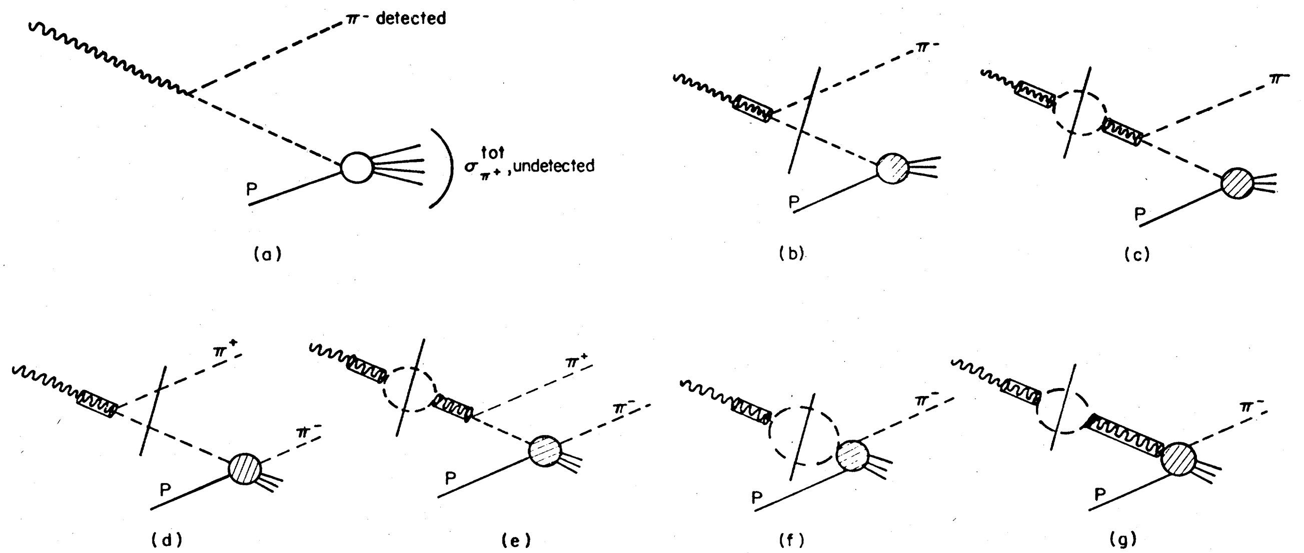

FIG. 4. Various contributions to π^- inclusive electroproduction resulting from the dipion constituent of the photon. (a) is the usual Feynman graph for the Drell process. (b) represents a contribution in which the π^+ part of the dipion interacts directly with the nucleon. In (c), the pions rescatter before the π^+ interacts. The remaining diagrams illustrate various other contributions to the amplitude.

which is precisely what we would have anticipated from (4.3), omitting the effect of the ρ^0.

If we continue to ignore the ρ^0 and take this result literally, it apparently leads to an unreasonable contribution to the total photon cross section. Integrating over the allowed kinematic range and multiplying by two (for both pions), we find

$$\sigma_{Dr} = \frac{\alpha}{6\pi}\,\sigma_\pi{}^{\rm tot}\left[\ln\frac{2Mk}{\mu^2} - \frac{8}{3}\right]$$
$$\cong 50\ \mu\text{barns} \quad \text{for } k = 10 \text{ GeV}. \tag{4.13}$$

This yields an unexpectedly large fraction of $\sigma_\gamma{}^{\rm tot}$, which increases logarithmically with k. It should be reduced to take into account the double counting when both pions strike the target. More important, we can hardly believe the mass spectrum implied by (4.12) for masses which are large compared to the ρ^0 mass. If we arbitrarily restrict the integration to masses smaller than m_ρ^2, the cross section is reduced to about 10 μbarns.

This raises the question, "How should we properly calculate the Drell process taking into account the realistic hadronic structure of the photon?" A field theoretic approach with ρ dominance would agree with Drell's expression (aside from terms neglected in both approaches) since the intermediate ρ_0 propagator would be evaluated at $Q^2 = 0$. If, instead, we try to use (4.3), we would find the types of contributions illustrated in Fig. 4(b)–(g). Only the contributions represented by Fig. 4(b) and 4(c) correspond to the same approximations used by Drell (for leading π^- production). The contribution from Fig. 4(b) by itself would have an enormous peak at the ρ^0 mass which would be in complete disagreement with experimental data. In fact, it is obvious that if $\mathfrak{M}^2$ is in the neighborhood of m_ρ^2, the rescattering of the pion pair represented by Fig. 4(c) will be quite important. It turns out that this contribution can be easily evaluated by contour integration and when the two contributions are added they exactly reproduce the contribution of Fig. 4(a). The simple details are given in the Appendix. A feature of this exercise is that it makes one suspicious about using Drell's expression for large $\mathfrak{M}^2$ ($\gtrsim$ 2 GeV^2, say) without providing any guidance as to the mechanism which cuts it off. It also restores our confidence in the possibility that the Drell process could be a very significant part of the 94 μbarns which we attribute to the dipion component of the photon but (4.13) is still likely to be an overestimate.

Next we turn to the question of confirming the presence of this process in the data. So far this has been done only in a very rough way but it should be possible to improve on the treatment which will be described here. The main point is to pick an experimental region where the last four contributions of Fig. 4 are of lesser importance. It seems intuitively clear that this is the region where the π^- carries off most of the energy of the photon. For example, Figs. 4(d) and 4(e) would correspond to $\pi^- p \to \pi^- X$ and since the π^+ is already carrying off some energy, we may hope that this does not give an important contribution to high energy π^- mesons. Little can be said about Figs. 4(f) and 4(g) except that they do not contain the large contribution from small $\mathfrak{M}^2$ which lead to Drell's original hope that Fig. 4(a) would be dominant (or at least important). In any case, if the process does not show up in an important way for high-energy π^- mesons, there is certainly no hope that it could be important for lower energy ones. There are simply too many mechanisms for producing π^- mesons with a small fraction of the photon's momentum whether or not we believe in complete dominance by the hadronic component of the photon.

In order to compare theory and data, it is also necessary to take into account the special contribution from diffrac-

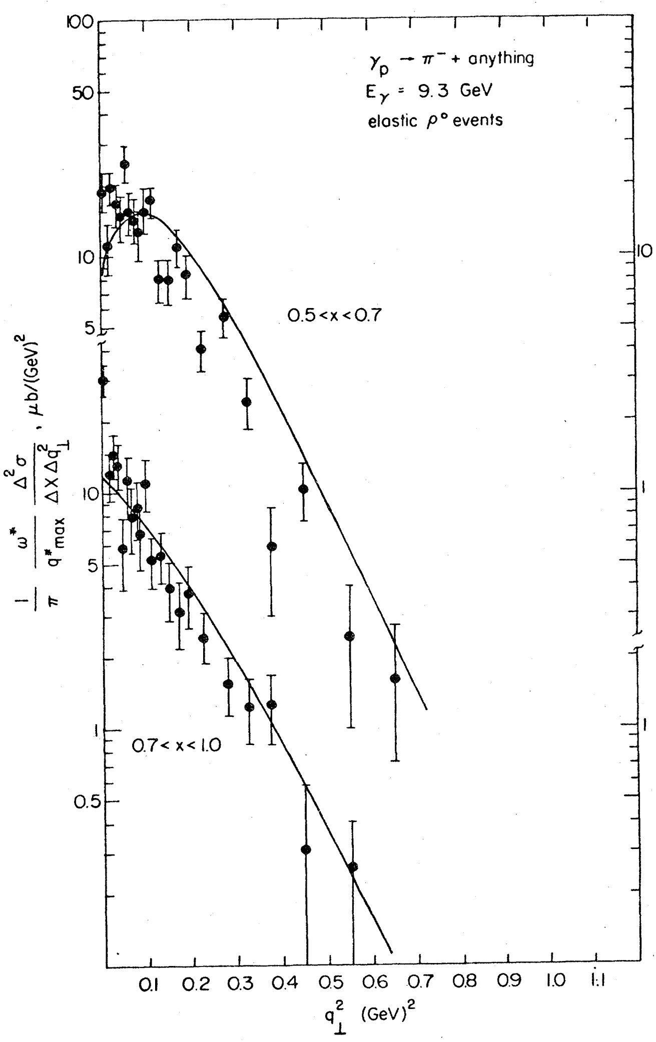

FIG. 5. Inclusive π^- photoproduction: elastic ρ^0 events. The data from Ref. 40 consist of events with $\pi^+\pi^-P$ in final state, with $m_{\pi\pi} < 1$ (GeV). The theoretical curve treats the ρ^0 as having a definite mass.

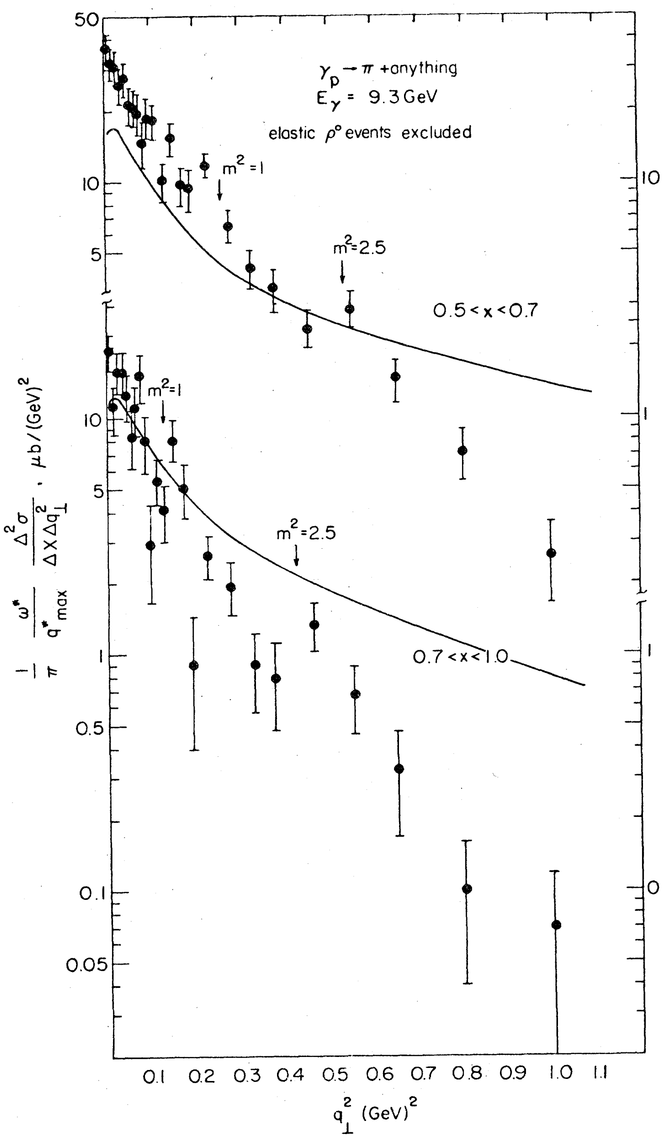

FIG. 6. Inclusive π^- photoproduction: elastic ρ^0 excluded. The theoretical curve is the Drell process. A few typical values of $\mathfrak{M}^2$ of the contributing dipion are indicated.

tive production of pion pairs, particularly through the ρ^0. This is not illustrated explicitly in Fig. 4. The process

$$\gamma + P \to \rho^0 + P, \quad \rho^0 \to \pi^+ + \pi^- \tag{4.14}$$

actually contributes about half the pions seen near the upper end of the spectrum. To make a reasonable comparison with the data for this channel, one should take into account the ρ^0 width and also the interference with (and contribution from) the diffractive pion scattering in Figs. 4(b)–4(e). Instead, the process (4.14) has been calculated under the crude assumption that the ρ^0 has a definite mass and does not interfere with other contributions. This badly distorts the $q_\perp^2$ dependence of π^- mesons from this process, but yields a reasonable estimate of the x ($= q^{cm}/q_{max}^{cm}$) dependence.

The simplified calculation for the ρ^0 and Drell contributions is given in the Appendix. Comparison with 9.3 GeV data from the SLAC–Berkeley–Tufts bubble chamber experiment (Moffeit *et al.* 1972)[16] is shown in Figs. 5 and 6.) While this cannot be regarded as a striking fit to the data, it does certainly indicate the presence of the dipion component of the photon. One may hope that when the defects of the present treatment are removed, agreement with the π^+–π^- channel will be obtained and a reasonable understanding of other channels will be found in this x region.

V. DISCUSSION

The importance of the hadronic structure of the photon seems to be well confirmed by many features of the experimental data. However, except in situations where VMD works reasonably well, it has generally not yet been possible to use the concept to make detailed predictions of experimental results. It seems worthwhile now to make some

[16] I wish to thank Dr. Moffeit for supplying the data separated into ρ^0 events and with ρ^0 excluded. ρ^0 events are defined as events in which the final state is $\pi^+\pi^-P$, with the mass of the pion pair less than 1 GeV.

further qualitative speculations about possible consequences of this picture.

The physical size of the photon seems to play an important role in its interactions. We have seen in Sec. III that the constituent with the lowest threshold has a very extended structure for real photons. Presumably the higher mass constituents have a much tighter structure. However, since our arguments were not valid inside the region of interaction, we may only speculate on the behavior of the high masses. It is suggestive that if this is a correct view, the decreasing size of the photon for higher mass components could be connected with their smaller cross section for interaction, which permits the real photon cross section to be dominated by low mass constituents. Although it was not discussed here, such a decrease of interaction cross section with increasing mass of the constituent is necessary for scaling of the hadronic component in GVMD (Gribov, 1970; Brodsky and Pumplin, 1969; Ritson *et al.* 1971; Fujikawa, 1971; Bjorken, 1972; Sakurai and Schildknecht, 1972; Bramon, Etim and Greco, 1972). Perhaps the high mass components can penetrate the peripheral region of the target nucleon with very small chance of interaction. They may interact primarily with the central core of the nucleon. Such a picture could account for two pieces of experimental data. One is that the diffractive photoproduction of high mass states has a significantly broader t distribution than that of low masses (the ρ^0, for example).[17] Another is that about 20% of the real photon cross section is not significantly shadowed in nuclei (Caldwell *et al.* 1973; Armstrong *et al.* 1972; Meyer *et al.* 1970; Heynen *et al.* 1971). The higher mass constituent of the photon may be able to penetrate the nucleus with little chance of interaction if it does not hit the nucleon core.

Some features of electroproduction may also be accounted for by the shrinking photon effect. It is possible that even for moderate values of Q^2 (say, 0.5–1 GeV^2) the photon has had an over-all shrinkage which makes its cross section considerable smaller and more localized in the nucleon than is true for real photons. As mentioned earlier, this leads to a smaller slope in the electroproduction of ρ^0's (Ahrens *et al.* 1974; Dakin *et al.* 1973; Eckardt *et al.* 1973).[9] One would also expect the slope to be a decreasing function of $\mathfrak{M}^2$, which is also confirmed experimentally (Ahrens *et al.*, 1974). It may turn out that the hadronic structure as a whole has a much smaller absorption cross section than its individual constituent due to the destructive interference which cancels out the hadronic material outside a small radius. This could account for the rapid decrease in the shadowing effect in nuclei with increasing Q^2.

Another possible consequence which has not yet been looked for is the Q^2 and A dependence of electroproduction of ρ^0's on nuclei. This may take place in the following way. The virtual photon hits a nucleon and has its hadronic component absorbed out in a very localized region. Immediately behind the individual nucleon, this results in the real presence of a spatially small chunk of hadronic matter. This chunk of matter is a superposition of many constituents, including the ρ^0. This superposition may propagate through the remainder of the nucleus before the various constituents disentangle and appear as real particles. Thus, the outgoing absorption might be smaller than that expected for the ρ^0 alone. The effect would show up in the A dependence of the cross section, in that the cross section for production from heavy nuclei would be larger than otherwise expected. It will be interesting to check on this possible effect.

[17] It should be emphasized, however, that the behavior of the high masses (1.05 to 1.63 GeV) is very nondiffractive in character. The slope and strength vary quite rapidly with s.

ACKNOWLEDGMENTS

My ideas have been shaped by discussions with many colleagues, especially the audience of a seminar series in the Cornell Laboratory of Nuclear Studies. I would particularly like to thank Professor Kurt Gottfried, Dr. Robin Spital, and Dr. Theodore Bauer for discussions and their helpful criticism of the manuscript. Conversations with Dr. Bauer played an essential role in clarifying the discussion of gauge invariance in Sec. IV.

APPENDIX

For convenience, we summarize here some formulas and details of analysis used to obtain the results given in the main text.

A. Formulas involving Π, assuming relativistic p-wave width

We have

$$\mathrm{Im}\,\Pi(\mathfrak{M}^2) = C\,\mathrm{Im}f(z), \tag{A1}$$

where

$$C = \frac{2}{\pi} m_\rho \Gamma_\rho \frac{z_\rho^{1/2}}{(z_\rho - 1)^{3/2}}, \quad z = \frac{\mathfrak{M}^2}{4m_\pi^2}, \quad z_\rho = \frac{m_\rho^2}{4m_\pi^2},$$

and

$$\begin{aligned} f(z) &= \frac{(1-z)^{3/2}}{(-z)^{1/2}} \ln[(-z)^{1/2} + (1-z)^{1/2}], \quad z<0, \\ &= \frac{(1-z)^{3/2}}{z^{1/2}} \tan^{-1}\left[\left(\frac{z}{1-z}\right)\right]^{1/2}, \quad 0<z<1, \\ &= -\frac{(z-1)^{3/2}}{z^{1/2}} \left[\ln[z^{1/2} + (z-1)^{1/2}] - \frac{i\pi}{2}\right]. \end{aligned} \tag{A2}$$

$f(z)$ is analytic outside the cut from $z = 1$ to ∞. To satisfy the conditions (3.4), we define

$$\Pi(\mathfrak{M}^2) = C\bar{f}(z) \tag{A3}$$

with

$$\bar{f}(z) = f(z) - \mathrm{Re}f(z_\rho) - (z - z_\rho)\,\mathrm{Re}f'(z_\rho). \tag{A4}$$

For $m_\rho^2 = 0.593$, we find $\Pi(0) = -0.52 m_\rho \Gamma_\rho$ and $\Pi'(0) = 0.57\Gamma_\rho/m_\rho$. Here $\Pi(0)$ gives the Gounaris-Sakurai (1960) normalization correction and $\Pi'(0)$ gives the enhancement of $P_{2\pi}$ over VMD. We note that the denominator $\mathfrak{M}^2 - m_\rho^2 + \Pi(\mathfrak{M}^2)$ has a spurious zero at a large negative value of $\mathfrak{M}^2$; in further work we ignore this nonphysical singularity.

To calculate the total photon absorption as a function of Q^2, we need a model for the cross section of the dipion constituent as a function of $\mathfrak{M}^2$. If this cross section were independent of $\mathfrak{M}^2$, the resulting (transverse) total photon cross section would be

$$\sigma_{\gamma T}{}^{(2\pi)}(Q^2) = (e^2/f_\rho{}^2)[1+\Pi'(-Q^2)][F_\pi(-Q^2)]^2\sigma_\rho, \quad [\sigma_{2\pi}(\mathfrak{M}^2) = \sigma_\rho]. \quad \text{(A5)}$$

For $Q^2 = 0$, this amounts to a 10% enhancement over VMD. A more realistic treatment requires a dipion cross section of order $\sigma_{\pi^+} + \sigma_{\pi^-}$ ($\approx 2\sigma_\rho$) for low masses. A simple model for such a dependence is

$$\sigma_{2\pi}(\mathfrak{M}^2) = \sigma_\rho[2m_\rho{}^2/(\mathfrak{M}^2 + m_\rho{}^2)]. \quad \text{(A6)}$$

This particular analytic form has no physical significance, but is has the advantage of yielding an integral which is easily evaluated by contour integration:

$$\begin{aligned}\sigma_{\gamma T}{}^{(2\pi)}(Q^2) &= \int_{m_\pi{}^2}^\infty P_{2\pi}(\mathfrak{M}^2)\sigma_{2\pi}(\mathfrak{M}^2)\, d\mathfrak{M}^2 \\ &= \frac{e^2}{f_\rho{}^2}[m_\rho{}^2 - \Pi(0)]^2 2m_\rho{}^2\left[\frac{1}{(m_\rho{}^2-Q^2)^2}\right. \\ &\quad\times\left(\frac{1}{D_1(m_\rho{}^2)} - \frac{1}{D_1(Q^2)}\right) \\ &\quad\left. + \frac{1}{(m_\rho{}^2-Q^2)}\frac{1+\Pi'(-Q^2)}{[D_1(Q^2)]^2}\right]\sigma_\rho,\end{aligned} \quad \text{(A7)}$$

where

$$D_1(\lambda) = \lambda + m_\rho{}^2 - \Pi(-\lambda). \quad \text{(A8)}$$

In spite of its appearance, this expression has no singularity at $Q^2 = m_\rho{}^2$. For a real photon, this becomes

$$\begin{aligned}\sigma_{\gamma T}{}^{(2\pi)}(0) &\approx \frac{e^2}{f_\rho{}^2}\left(1 + 2\Pi'(0) + \frac{\Pi(-m_\rho{}^2)}{2m_\rho{}^2}\right)\sigma_\rho \\ &= \frac{e^2}{f_\rho{}^2}(1.174)\sigma_\rho = 94\ \mu\text{barns}\end{aligned} \quad \text{(A9)}$$

(using $f_\rho{}^2/4\pi = 2.5$, $\sigma_\rho = 27$ mbarns). The longitudinal cross section is given by (in diagonal approximation):

$$\begin{aligned}\sigma_{\gamma L}{}^{(2\pi)}(Q^2) &= \xi\int_{m_\pi{}^2}^\infty P_{2\pi}(\mathfrak{M}^2)\sigma_{2\pi}(\mathfrak{M}^2)\, d\mathfrak{M}^2 \\ &= \frac{e^2}{f_\rho{}^2}[m_\rho{}^2 - \Pi(0)]^2 2Q^2 m_\rho{}^2\left[\frac{1}{(m_\rho{}^2-Q^2)^2}\right. \\ &\quad\times\left(\frac{1}{Q^2D_1(Q^2)} - \frac{1}{m_\rho{}^2D_1(m_\rho{}^2)}\right) \\ &\quad + \frac{1}{(m_\rho{}^2-Q^2)}\left(\frac{-[1+\Pi'(-Q^2)]}{Q^2D_1(Q^2)}\right. \\ &\quad\left.\left. - \frac{1}{Q^4D_1(Q^2)}\right) + \frac{1}{Q^4m_\rho{}^2D_1(0)}\right]\xi\sigma_\rho,\end{aligned} \quad \text{(A10)}$$

where ξ is the ratio of longitudinal to transverse dipion cross sections. There is no reason why this ratio could not be a function of $\mathfrak{M}^2$, but we have simply taken it to be a constant. There is a geometrical reason why it is not surprising that $\xi < 1$; namely, in the longitudinal state the two pions tend to be aligned in the direction of k. We have taken $\xi = 0.6$, which is compatible with data on electroproduction of ρ^0.

B. Hadronic photon estimate of the Drell process

The purpose of this section of the Appendix is to show that the two contributions of Fig. 4(b) and (c), which take the ρ^0 resonance into account, yield the same result as that of Fig. 4(a), which ignores this resonance. Common factors associated with the lower vertex are ignored. The contribution from Fig. 4(a) is

$$4(\text{a}) = e\boldsymbol{\varepsilon}\cdot(\mathbf{q}_+ - \mathbf{q}_-)/\mathfrak{M}^2, \quad \text{(B1)}$$

where $\mathfrak{M}^2$ is given by (4.11). The contribution from Fig. 4(b) is

$$4(\text{b}) = \frac{-e(m_\rho{}^2 - \Pi(0))\,\boldsymbol{\varepsilon}\cdot(\mathbf{q}_+ - \mathbf{q}_-)}{\mathfrak{M}^2(\mathfrak{M}^2 - m_\rho{}^2 + \Pi^*(\mathfrak{M}^2))}. \quad \text{(B2)}$$

If this term contributed by itself, it would give a huge peak at the ρ^0 mass, of order $(m_\rho/\Gamma_\rho)^2$ times the simple Drell cross section. There is absolutely no experimental evidence for such a peak.

The contribution from Fig. 4(c) is seemingly more complicated as it involves production of a meson pair of mass $\mathfrak{M}'$, followed by their resonant scattering and then interaction with the target. This contribution is

$$\begin{aligned}4(\text{c}) &= \frac{1}{(2\pi)^3}\int\frac{d\mathfrak{M}'^2}{\mathfrak{M}'^2}\frac{d^3q_+'}{2\omega_+'}\frac{d^3q_-'}{2\omega_-'} \\ &\quad\times\frac{ef_\rho{}^2[m_\rho{}^2 - \Pi(0)]\boldsymbol{\varepsilon}\cdot(\mathbf{q}_+' - \mathbf{q}_-')}{\mathfrak{M}^2 - m_\rho{}^2 + \Pi^*(\mathfrak{M}'^2)} \\ &\quad\times\sum_{\boldsymbol{\varepsilon}_\rho}\frac{\boldsymbol{\varepsilon}_\rho\cdot(\mathbf{q}_+' - \mathbf{q}_-')\,\boldsymbol{\varepsilon}_\rho\cdot(\mathbf{q}_+ - \mathbf{q}_-)}{\mathfrak{M}^2 - m_\rho{}^2 + \Pi(\mathfrak{M}'^2)}\frac{1}{\mathfrak{M}^2 - \mathfrak{M}'^2 - i\epsilon} \\ &\quad\times\delta[\omega_+' + \omega_-' - (\mathfrak{M}'^2 + k^2)^{1/2}]\delta(\mathbf{q}_+' + \mathbf{q}_-' - \mathbf{k}).\end{aligned} \quad \text{(B3)}$$

The integration over the internal pion momentum is most easily carried out in the rest frame defined by the four vector $[(\mathfrak{M}'^2 + k^2)^{1/2}, \mathbf{k}]$. Noting that $\boldsymbol{\varepsilon}\cdot\mathbf{k} = 0$, we find

$$\frac{1}{(2\pi)^3}\int\frac{d^3q_+'}{\omega_+'^2}\boldsymbol{\varepsilon}\cdot\mathbf{q}_+'\boldsymbol{\varepsilon}_\rho\cdot\mathbf{q}_+'\delta(2\omega_+' - \mathfrak{M}') = \frac{q_+'^3}{12\pi^2\omega_+'}\boldsymbol{\varepsilon}\cdot\boldsymbol{\varepsilon}_\rho'$$

and 4(c) becomes

$$\begin{aligned}4(\text{c}) &= \frac{1}{\pi}\int\frac{d\mathfrak{M}'^2}{\mathfrak{M}'^2}\frac{\text{Im}\Pi(\mathfrak{M}'^2)}{|\mathfrak{M}'^2 - m_\rho{}^2 + \Pi(\mathfrak{M}'^2)|^2} \\ &\quad\times\frac{e(m_\rho{}^2 - \Pi(0))\,\boldsymbol{\varepsilon}\cdot(\mathbf{q}_+ - \mathbf{q}_-)}{\mathfrak{M}^2 - \mathfrak{M}'^2 - i\epsilon}.\end{aligned}$$

As usual, this is evaluated using the same contour integration technique and picking up contributions from the poles at $\mathfrak{M}'^2 = 0$ and $\mathfrak{M}'^2 = \mathfrak{M}^2 - i\epsilon$. The desired result is obtained, namely

$$(4c) = (4a) - (4b). \tag{B4}$$

C. The ρ^0 contribution to π^- inclusive photoproduction

A very simple model is used in which the ρ^0 is assumed to have a definite mass. Since the $q_\perp^2$ distribution is sensitive to the ρ^0 mass, this is not likely to give very accurate results; and a more careful treatment would be desirable. The probability for a ρ^0 to be produced in a given direction is taken to be

$$\frac{f_0^2 k_\rho^2}{\pi} \exp(Bt)\, d\Omega_\rho, \tag{C1}$$

where f_0^2 is assumed to be 100 μb/GeV^2, which is typical of experimental values. When this is folded into the decay distribution, the resulting inclusive cross section is found to be

$$q_- \frac{d^3\sigma}{dq_-^3} = \frac{3f_0^2 k_\rho}{2\pi m_\rho^2[1-(4m_\pi^2/m_\rho^2)]^{3/2}} \frac{(\Delta q)^2}{\omega_-} \times \exp[-2Bk_\rho^2(1-\cos\theta_0\cos\theta_\rho)] I_0(2Bk_\rho^2 \sin\theta_0 \sin\theta_\rho), \tag{C2}$$

where

$$(\Delta q)^2 = m_\rho^2/k_\rho^2[4\omega_-(\omega_\rho - \omega_-) - m_\rho^2] - 4\mu^2,$$
$$\sin^2\theta_\rho = (\Delta q)^2/4k_+^2, \qquad \sin^2\theta_0 = q_{-\perp}^2/q_-^2,$$

and I_0 is the modified Bessel function: $I_0(z) = J_0(iz)$. In the curves shown in Fig. 5, B was taken to be 7.

REFERENCES

Ahrens, L., K. Berkelman, G. S. Brown, D. G. Cassel, W. R. Francis, P. H. Garbincius, D. Harding, D. L. Hartill, J. L. Hartmann, R. L. Loveless, R. C. Rohlfs, D. H. White, and A. J. Sadoff, 1974, Phys. Rev. D **9**, 1894.

Alvensleben, H., U. Becker, W. F. Bertram, M. Chen, K. J. Cohen, T. M. Knasel, R. Marshall, D. J. Quinn, M. Rohde, G. H. Sanders, H. Schubel, and S. C. C. Ting, 1970, Nucl. Phys. B **18**, 333.

Aüslender, V. L., G. I. Budker, E. V. Beter, U. G. Gulyamov, Yu N. Pestov, V. A. Siderov, A. N. Skrinskii, and A. G. Khabaklysashev, 1969, Sov. J. Nucl. Phys. **9**, 69.

Augustin, J.-E., A. M. Boyarski, M. Breidenbach, F. Bulos, J. T. Dakin, G. J. Feldman, G. E. Fischer, D. Fryberger, G. Hanson, B. Jean-Marie, R. R. Larsen, V. Lüth, H. L. Lynch, D. Lyon, C. C. Morehouse, J. M. Paterson, M. L. Perl, B. Richter, R. F. Schwitters, F. Vannucci, G. S. Abrams, D. Briggs, W. Chinowsky, C. E. Friedberg, G. Goldhaber, R. J. Hollebeek, J. A. Kadyk, G. H. Trilling, J. S. Whitaker, and J. E. Zipse, Phys. Rev. Lett. (to be published).

Bacci, C., G. Pensi, G. Salvini, B. Stella, R. Baldini-Celio, G. Capon, C. Mencuccini, G. P. Murtas, M. Spinetti, and A. Zallo, 1973, Phys. Letters **44B**, 533.

Bartoli, B., F. Felicetti, H. Ogren, V. Silvestrini, G. Marini, A. Nigro, and F. Vanoli, 1972, Phys. Rev. D **6**, 2374.

Bauer, T. H., 1971, Phys. Rev. D **3**, 2671.

Bauer, T. H., 1973, Nucl. Phys. B **57**, 109.

Bemporad, C., W. Beusch, A. C. Melissinos, E. Polgar, D. Websdale, J. D. Wilson, J. P. Dufey, K. Freudenreich, R. Frosch, F. X. Gentit, P. Muehlemann, J. Codling, J. G. Lee, M. Letheren, G. Bellini, M. Di Corato, and G. Vegni, 1971, Nucl. Phys. B **33**, 397.

Benaksas, D., G. Cosme, B. Jean-Marie, S. Jullian, F. Laplanche, J. LeFrancois, A. D. Liberman, G. Parrour, J. P. Repellin, and G. Sauvage, 1972, Phys. Letters **39B**, 289.

Beusch, W., 1972, Acta Physica Polonica B **3**, 679.

Bjorken, J. D., 1972, in *Proceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energies*, edited by N. Mistry (Laboratory of Nuclear Studies, Cornell University Press, Ithaca, N.Y.), p. 281.

Bjorken, J. D., 1973, in *Proceedings of Summer Institute on Particle Physics*, SLAC Report No. 167, Vol. 1, p. 1.

Bjorken, J. D., J. B. Kogut, and D. E. Soper, 1971, Phys. Rev. D **3**, 1382.

Bramon, A., E. Etim, and M. Greco, 1972, Phys. Letters **41B**, 609.

Brodsky, S. J., F. E. Close, and J. Gunion, 1972, Phys. Rev. D **5**, 1384.

Brodsky, S. J., and J. Pumplin, 1969, Phys. Rev. **182**, 1794.,

Brookes, G. R., A. S. Clough, J. H. Freeland, W. Galbraith, A. F. King, T. A. Armstrong, W. R. Hogg, G. M. Lewis, A. W. Robertson, W. R. Rawlinson, N. R. S. Tait, J. C. Thompson, D. W. L. Tolfree, 1973, Phys. Rev. D **8**, 2826.

Caldwell, D. O., V. B. Elings, W. P. Hesse, R. J. Morrison, F. V. Murphy, and D. E. Yount, 1973, Phys. Rev. D **7**, 1362.

Cheng, H., and T. T. Wu, 1969, Phys. Rev. **183**, 1324.

Cosme, G., B. Jean-Marie, S. Jullian, F. Laplanche, J. LeFrancois, A. D. Liberman, G. Parrour, J. P. Repellin, G. Sauvage, and G. Szklarz, 1972, Phys. Letters **40B**, 685.

Dakin, J. T., G. J. Feldman, W. L. Lakin, F. Martin, M. L. Perl, E. W. Petraske, and W. T. Toner, 1973, Phys. Rev. D **8**, 687.

Drell, S. D., 1960, Phys. Rev. Lett. **5**, 278.

Eckardt, V., H. J. Gebauer, P. Joos, H. Meyer, B. Naroska, D. Notz, W. J. Podolsky, G. Wolf, S. Yellin, H. Dau, G. Drews, D. Greubel, W. Meincke, H. Nagel, and E. Rabe, 1973, Nucl. Phys. B **55**, 45.

Friedman, J., and H. Kendall, 1972, Ann. Rev. Nucl. Sci. **22**, 203.

Fujikawa, K., 1971, Phys. Rev. D **4**, 2794.

Gladding, G. E., J. J. Russell, M. J. Tannenbaum, J. M. Weiss, and G. B. Thomson, 1973, Phys. Rev. D **8**, 3735.

Gottfried, K., and D. I. Julius, 1970, Phys. Rev. D **1**, 140.

Gottfried, K., and D. R. Yennie, 1969, Phys. Rev. **182**, 1595.

Gounaris, G. J., and J. J. Sakurai, 1968, Phys. Rev. Lett. **21**, 244.

Gribov, V. N., 1970, Sov. Phys.—JETP **30**, 709.

Grilli, M., E. Iarocci, P. Spillantini, V. Valente, R. Visentin, B. Borgia, F. Ceradini, M. Conversi, L. Paoluzi, R. Santonico, M. Nigro, L. Trasatti, and G. T. Zorn, 1973, Nuovo Cimento **13A**, 593.

Heynen, V., H. Meyer, B. Naroska, and D. Notz, 1971, Phys. Letters **34B**, 651.

Ioffe, B. L., 1969a, Sov. Phys.—JETP Letters **9**, 97.

Ioffe, B. L., 1969b, Phys. Letters **30B**, 123.

Kendall, H. W., 1972, *Proceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energies*, edited by N. Mistry (Laboratory of Nuclear Studies, Cornell University Press, Ithaca, N.Y.), p. 247.

Kogut, J. B., 1972, Phys. Rev. D **5**, 1152.

Kurdadze, L. M., A. P. Onuchin, S. I. Serednyakov, V. A. Sidorov, and S. I. Eidelman, 1972, Phys. Letters **42B**, 515.

Litke, A., G. Hanson, A. Hofmann, J. Koch, L. Law, M. E. Law, J. Leong, R. Little, R. Madaras, H. Newman, J. M. Paterson, R. Pordes, K. Strauch, G. Tarnopolsky, and R. Wilson, 1973, Phys. Rev. Lett. **30**, 1189.

Margolis, B., and C. L. Tang, 1969, Nucl. Phys. B **10**, 329.

McClellan, G., N. Mistry, P. Mostek, H. Ogren, A. Silverman, J. Swartz, and R. Talman, 1971, Phys. Rev. D **4**, 2683.

Meyer, H., B. Naroska, J. H. Weber, M. Wong, V. Heynen, E. Mandelkow, and D. Notz, 1970, Phys. Letters **33B**, 189.

Moffeit, K., J. Ballam, G. B. Chadwick, M. Della-Negra, R. Gearhart, J. J. Murray, P. Seyboth, C. K. Sinclair, I. O. Skillicorn, H. Spitzer, G. Wolf, H. H. Bingham, W. B. Fretter, W. J. Podolsky, M. S. Rabin, A. H. Rosenfeld, R. Windmolders, G. P. Yost, and R. H. Milburn, 1972, Phys. Rev. D **5**, 1603.

Nauenberg, M., 1969, Phys. Rev. Lett. **22**, 556.

Nieh, H. T., 1972, Phys. Letters **38B**, 100.

Pestieau, J., P. Roy, and H. Terazawa, 1970, Phys. Rev. Lett. **25**, 402.

Pumplin, J., 1970, Phys. Rev. D **2**, 1859.

Riordan, E. M., A. Bodek, M. Breidenbach, D. L. Dubin, J. E. Elias, J. I. Friedman, H. W. Kendall, J. S. Poucher, M. R. Sogard, and

D. H. Coward, 1973, Phys. Rev. Lett. **33**, 561.
Ritson, D. M., 1971, Phys. Rev. D **3**, 1267.
Ross, M., and L. Stodolsky, 1966, Phys. Rev. **149**, 1172.
Sakurai, J. J., 1969, in *Proceedings of the Fourth International Symposium on Electron and Photon Interactions at High Energies*, edited by D. W. Braben (Daresbury Nuclear Physics Laboratory, Daresbury near Warrington, Lancashire, England), p. 91.
Sakurai, J. J., 1971, Erice Lectures, UCLA/71/TEP/39.
Sakurai, J. J., 1973, Erice Lectures, UCLA/73/TEP/89.
Sakurai, J. J., and D. Schildknecht, 1972a, Phys. Letters **40B**, 121.
Sakurai, J. J., and D. Schildknecht, 1972b, Phys. Letters **41B**, 489.
Sakurai, J. J., and D. Schildknecht, 1972c, Phys. Letters **42B**, 216.
Söding, P., 1966, Phys. Letters **19**, 702.
Spital, R., and D. R. Yennie, 1974a, Phys. Rev. D **9**, 138.
Spital, R., and D. R. Yennie, 1974b, Phys. Rev. D **9**, 126.
Spital, R., and D. R. Yennie, 1975, Cornell Laboratory of Nuclear Studies preprint (to be published).
Stodolsky, L., 1967, Phys. Rev. Lett. **18**, 135.
suri, A., and D. R. Yennie, 1972, Ann. Phys. (N.Y.) **72**, 243.
Talman, R., 1974, in *Proceedings of the 6th International Symposium on Electron and Photon Interactions at High Energies*, edited by H. Rollnik and W. Pfeil (North-Holland Publ. Co., Amsterdam and London), p. 145.
Yennie, D. R., 1975, *1972 Cargèse Lectures*, edited by M. M. Lévy (Gordon and Breach, Paris), in press.