Abnormal nuclear states and vacuum excitation*†

T. D. Lee

Physics Department, Columbia University, New York, New York 10027

We examine the theoretical possibility that at high densities there may exist a new type of nuclear state in which the nucleon mass is either zero or nearly zero. The related phenomenon of vacuum excitation is also discussed.

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I. INTRODUCTION

In this talk, I would like to discuss some of my recent theoretical speculations, made in collaboration with G. C. Wick. As you shall see, these speculations suggest the possible existence of some rather interesting physical objects, hitherto unobserved.[1] An effective way to search for these new objects is through the use of high-energy heavy ions, which is the subject matter of this meeting.

To begin with, we assume the existence of a strongly interacting neutral spin 0 even parity meson field $\phi(x)$. Such a field may simply be a phenomenological description of a composite 0+ state of other particles, say $\pi\pi$, or $K\bar{K}$.[2] Through the transformation $\phi(x) \to \phi(x) +$ constant, we can always choose for the normal vacuum state

$$\langle \text{vac} \mid \phi(x) \mid \text{vac} \rangle = 0 \quad \text{everywhere.} \tag{1}$$

The state $|\ \rangle$ that we are interested in is an excited state; it has an abnormal expectation value of $\phi(x)$ in a relatively large volume Ω:

$$\langle \mid \phi(x) \mid \rangle \begin{cases} = \text{constant} \neq 0 & \text{inside } \Omega \\ = 0 & \text{outside } \Omega \\ \text{has rapid variation near the surface of } \Omega. \end{cases} \tag{2}$$

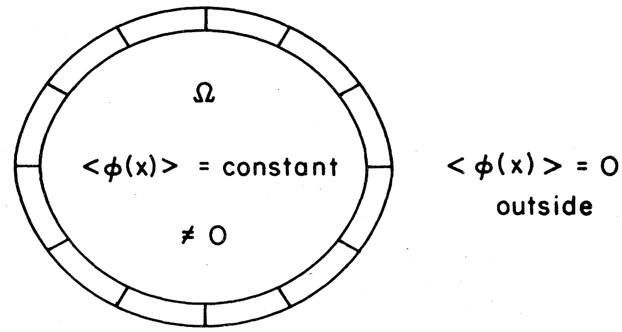

FIG. 1. A pictorial representation of an excited state in which $\langle \phi(x) \rangle$ differs from its vacuum value inside a macroscopic volume Ω.

Pictorially, we may visualize the expectation value $\langle \phi(x) \rangle$ in such an excited state as represented by Fig. 1. The linear dimension of Ω is assumed to be much larger than the usual microscopic length in particle physics (say, $\Omega^{1/3}$ is $\sim$, or $>$, 10^{-12} cm). Thus, much of the description of the field becomes almost classical.

In the following, we shall discuss two different circumstances:

(i) Ω is filled with nuclear matter, and

(ii) Ω does not contain any nuclear matter.

As we shall see, case (i) may lead to abnormal nuclear states and case (ii) to pure vacuum excitation states.

II. ABNORMAL NUCLEAR STATES

We first consider case (i). For definiteness, the Lagrangian density is assumed to be

$$\mathcal{L} = -\tfrac{1}{2}(\partial\phi/\partial x_\mu)^2 - U(\phi) - \psi^\dagger\gamma_4[\gamma_\mu(\partial/\partial x_\mu) + (m_N + g\phi)]\psi, \tag{3}$$

where

$$U(\phi) = \tfrac{1}{2}m_\phi^2\phi^2 + \cdots, \tag{4}$$

m_ϕ is the 0+ meson mass, m_N is the nucleon mass, and ψ is the nucleon field. In (4) the precise form of "$\cdots$" depends on the theory; for a renormalizable theory, it contains cubic and quartic powers of ϕ.

Let the volume Ω of the state $|\ \rangle$, given by Eq. (2), be simply that of a super-heavy nucleus. Outside the nucleus, $\langle \phi(x) \rangle = 0$ and the nucleon mass is m_N. But inside the

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[1] For some earlier speculations on related subjects, cf. E. Feenberg and H. Primakoff, 1946, Phys. Rev. **70**, 980; A. R. Bodmer, 1971, Phys. Rev. D **4**, 1601; A. B. Migdal, 1972, Zh. Eksp. Teor. Fiz. **63**, 1993; Y. Ne'eman, 1972, in *Physics of Dense Matter* I. A. U. Symposium, Boulder, Colorado.

[2] From a theoretical point of view, one knows that at least in the low and intermediate energy region, the chiral $SU_2 \times SU_2$ symmetry is a reasonably good approximation, as supported by the Adler–Weisberger relation, the Goldberger–Treiman relation, and the various soft-pion relations. The chiral symmetry leads naturally to either $\pi\pi$ correlations or a 0+ field, such as in the σ-model.

nucleus, the "effective" nuclear mass is m_{eff}, determined by

$$m_{eff}^2 = [m_N + g\langle\phi\rangle]^2, \tag{5}$$

which may be quite different from m_N. In particular, if we assume

$$\langle\phi(x)\rangle \cong -m_N/g \qquad \text{inside } \Omega, \tag{6}$$

then the "effective" nucleon mass inside the nucleus would be $\cong 0$. The energy difference Δ between such an "abnormal" state and the normal state may be estimated approximately

$$\Delta \sim -Nm_N + U(-m_N/g)\Omega + \text{surface energy} \tag{7}$$

where N is the total number of nucleons.[3] For a sufficiently heavy nucleus, the surface energy may be neglected. Since the negative term in Eq. (7) is proportional to N, while the positive term is proportional to the volume Ω, the energy difference Δ in this simple system becomes negative if the nucleon density $n \equiv N/\Omega$ is sufficiently high. To have a rough idea of the order of magnitude, we may take $U(\phi) \sim \frac{1}{2}m_\phi^2\phi^2$; the abnormal state becomes the lower energy state if the nucleon density n is greater (or much greater) than a critical value n_c where

$$n_c \sim m_\phi^2 m_N/2g^2. \tag{8}$$

If we assume $m_\phi \sim m_N$ and its coupling g to be of the same order as the $\pi - N$ coupling $(4\pi)^{-1}g^2 \sim 15$, then at the critical density, the internucleon distance is

$$n_c^{-1/3} \sim 1.5 \times 10^{-13} \text{ cm} \tag{9}$$

which is of the same order as the distance between nucleons in the existing nuclei.[4]

Of course, the above estimation, Eq. (8) is quite crude, since it neglects the nuclear forces, the relativistic motion of nucleons, etc. While a complete analysis is difficult, some simple model calculations can be readily made.

III. σ-MODEL

For definiteness, let us assume the well-known σ-model for the meson field.[5] In this model, besides the usual isovector 0− pion field $\boldsymbol{\pi}(x)$ there is also an isoscalar 0+ field $\sigma(x)$. In terms of the customary notation of $\sigma(x)$ used in the literature, the aforementioned field $\phi(x)$ is given by

$$\phi = \sigma - (m_N/g), \tag{10}$$

and its mass m_ϕ = σ-meson mass m_σ. In the normal vacuum, we adopt the convention $\langle \text{vac} \mid \phi(x) \mid \text{vac}\rangle = 0$ as before; consequently,

$$\sigma_0 \equiv \langle \text{vac} \mid \sigma(x) \mid \text{vac}\rangle = (m_N/g). \tag{11}$$

Since $\boldsymbol{\pi}(x)$ is a pseudoscalar field, we have

$$\langle \text{vac} \mid \boldsymbol{\pi}(x) \mid \text{vac}\rangle = 0. \tag{12}$$

In the σ-model, the potential energy density U_σ is given by

$$U_\sigma = \tfrac{1}{4}\lambda^2(\sigma^2 + \boldsymbol{\pi}^2 - \mu^2/\lambda^2)^2 - (m_\pi^2\sigma_0)\sigma, \tag{13}$$

where σ_0 is given by Eq. (11), and the constants λ and μ are related to the σ-mass m_σ and the π-mass m_π by

$$2\mu^2 = m_\sigma^2 - 3m_\pi^2 \qquad \text{and} \qquad 2\lambda^2\sigma_0^2 = m_\sigma^2 - m_\pi^2. \tag{14}$$

If one neglects m_π, the σ-model is symmetric under the chiral $SU_2 \times SU_2$ transformation. So far as the meson fields are concerned, this chiral transformation is the same as the four-dimensional orthogonal transformation between $\sigma(x)$ and $\boldsymbol{\pi}(x)$. Consequently, the σ-nucleon coupling is equal to the π-nucleon coupling. One has

$$g^2/4\pi \cong 15. \tag{15}$$

Thus, in the σ-model there is *only one unknown parameter* m_σ.

Let us apply the σ-model to the problem of abnormal states in a large nucleus, considered in the previous section. The effective nuclear mass is now given by

$$m_{eff} = g(\langle\sigma\rangle^2 + \langle\boldsymbol{\pi}\rangle^2)^{1/2}, \tag{16}$$

where $\langle\sigma\rangle$ and $\langle\boldsymbol{\pi}\rangle$ are the expectation values of σ and $\boldsymbol{\pi}$ inside the nucleus, and both are assumed to be constants.[6] For simplicity, let us assume the nucleons to be described by a degenerate Fermi distribution with a top Fermi momentum k_F. In the simple case of an equal number of protons and neutrons, k_F is related to the nucleon density n by

$$k_F = (3\pi^2 n/2)^{1/3}. \tag{17}$$

The kinetic energy density of nucleons is given by

$$U_N = (2/\pi^2)\int_0^{k_F} k^2(k^2 + m_{eff}^2)^{1/2}\, dk. \tag{18}$$

In addition, there is the usual short-range nuclear interaction. As a first model calculation, we shall assume that because of the short-range interaction (especially if the repulsive force is particularly strong) the nuclear matter resembles an incompressible fluid. Therefore, we may keep the nucleon density *n fixed*; the energy density of the system is then assumed to be given by

$$\mathcal{E} \equiv U_\sigma + U_N - nm_N \tag{19}$$

plus an additive constant that may depend on the fixed parameter n.[7]

[3] For simplicity, we neglect the relativistic correction here; a detailed calculation is given in the next section.

[4] Here, as well as in the following, the critical density n_c depends only on the ratio m_ϕ^2/g^2.

[5] For references on the σ-model, see B. W. Lee, 1972, *Chiral Dynamics*.

[6] As we shall see, in order to minimize energy, $\langle\boldsymbol{\pi}\rangle = 0$, and therefore $m_{eff}^2 = g^2\langle\sigma\rangle^2$ which reduces to Eq. (5) because of Eq. (10).

[7] Later, in a more realistic model-calculation, the short-range repulsive interaction will be considered explicitly. See the discussion given in this section after Eq. (23) and in the Appendix.

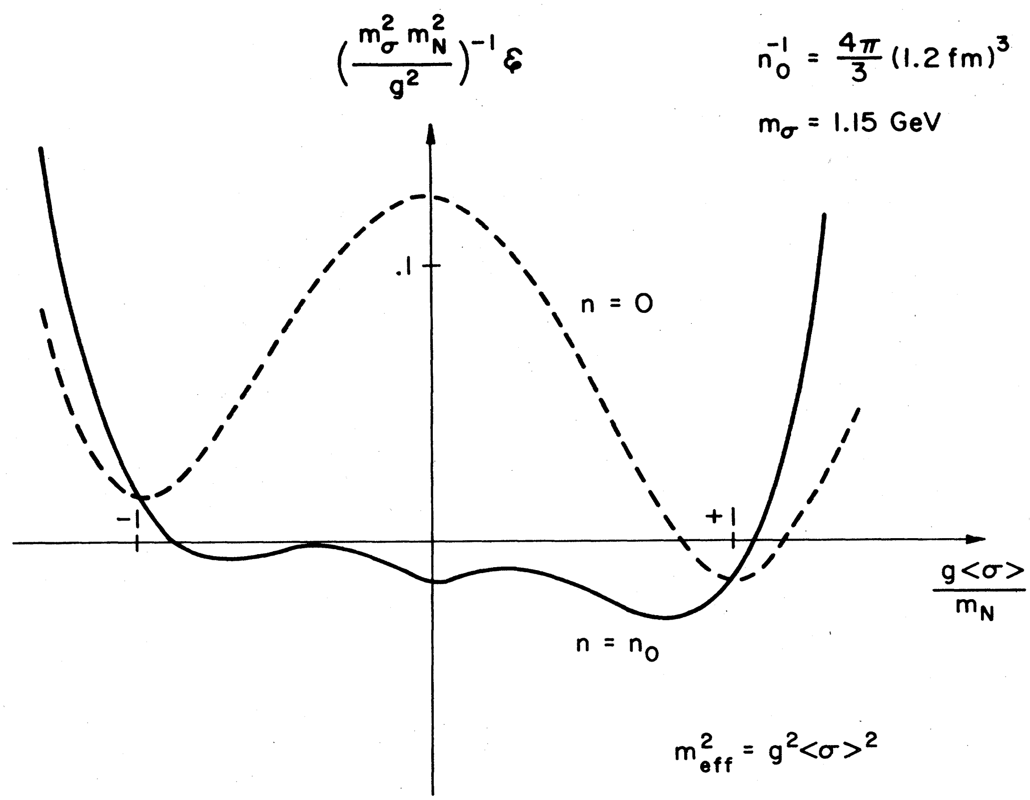

FIG. 2. Energy density ϵ in the σ model vs $g\langle\sigma\rangle/m_N$.

We shall assume the volume Ω of the nucleus to be sufficiently large so that the surface energy can be neglected. The solution that we are interested in is one in which $\langle\sigma\rangle$ and $\langle\boldsymbol{\pi}\rangle$ are constants inside the nucleus. Because of $m_\pi \neq 0$, from Eq. (13) it follows that in the minimum energy state the expectation value $\langle\boldsymbol{\pi}\rangle = 0$; therefore

$$m_{\text{eff}}^2 = g^2\langle\sigma\rangle^2. \tag{20}$$

At a fixed nucleon density n, we may plot the energy density $\mathcal{E}$ against $g\langle\sigma\rangle/m_N$ (see Fig. 2).

In this figure m_σ is assumed to be 1.15 GeV.[8] The dashed line is for zero nucleon density $n = 0$, and the solid line is for n equal to the density in the existing heavy nuclei. The slight asymmetry between $\langle\sigma\rangle$ positive and $\langle\sigma\rangle$ negative is due to $m_\pi \neq 0$. One sees that at zero nucleon density, the minimum of $\mathcal{E}$ is at $m_{\text{eff}} = m_N$. As n increases, the minimum slowly decreases from $m_{\text{eff}} = m_N$ to $m_{\text{eff}} \sim .8m_N$ at $n = [(4\pi/3)(1.2\text{ fm})^3]^{-1}$. The point $m_{\text{eff}}/m_N \cong 0$ is a local maximum of $\mathcal{E}$ at $n = 0$, but it becomes a local minimum of $\mathcal{E}$ when n increases to near the density of the existing heavy nuclei.

In the above figure, the unit of energy density is $(m_\sigma m_N/g)^2$. At the density $n = [(4\pi/3)(1.2\text{ fm})^3]^{-1}$, this unit energy density corresponds to $\sim$5.2 GeV/nucleon; the energy difference between the normal state ($m_{\text{eff}} \cong 0.8\, m_N$) and the abnormal state ($m_{\text{eff}} \cong 0$) is about 80 MeV/nucleon. There is a local maximum of $\mathcal{E}$ at about $m_{\text{eff}} \cong .25m_N$. The energy difference between this local maximum and the abnormal state is about 15 MeV/nucleon which is quite a sizable potential barrier. Thus, the abnormal state $m_{\text{eff}} \cong 0$ is a metastable state, if n is constrained at the fixed value $[(4\pi/3)(1.2\text{ fm}^3)]^{-1}$. At the same fixed value of nuclear density, in order to reach the abnormal state from the normal state, one has to pass through a potential barrier of maximum height $\sim 15 + 80 = 95$ MeV/nucleon.

If the nucleon density can be further increased to exceed a critical value n_c,[9] then the abnormal state becomes the absolute minimum state. Consequently for the minimum energy state, the value m_{eff}/m_N makes a discontinuous jump at $n = n_c$ (see Fig. 3). In the abnormal state ($n > n_c$), the effective nucleon mass m_{eff} is almost zero, but not exactly zero; this is due to $m_\pi \neq 0$. In the zero pion-mass limit, $m_{\text{eff}} = 0$ in the abnormal state.

The conditions for the production of such abnormal nuclear states are

1. $N^{1/3} \gg 1$, so that the surface energy can be neglected, and

[8] As we shall see, in the hard-sphere model calculation given in the Appendix, this value for m_σ gives about the correct binding for the normal nuclear state. See T. D. Lee and M. Margulies, Phys. Rev. D (in press) for results in which different values of m_σ and g are assumed.

[9] Here n_c is $\sim 1.16 \times [(4\pi/3)(1.2\text{ fm})^3]^{-1}$ for $m_\sigma = 1.15$ GeV, and $(4\pi)^{-1}g^2 = 15$

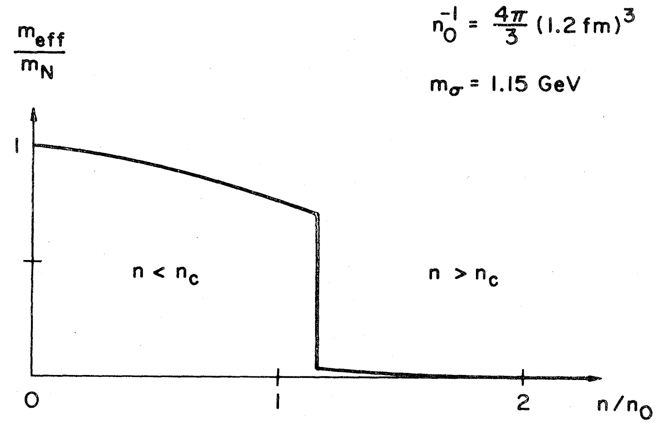

FIG. 3. A discontinuous transition in the effective mass m_{eff} between the normal state ($n < n_c$) and the abnormal state ($n > n_c$).

2. $n > n_c$. The precise value of n_c depends on m_σ and g; neither is really known with any degree of certainty. For different values of m_σ and g, the critical density n_c varies as m_σ^2/g^2, or more explicitly

$$n_c \cong 12(m_\sigma/m_N)^2(4\pi/g^2)n_0, \tag{21}$$

where

$$n_0^{-1} = (4\pi/3)(1.2\text{ fm})^3. \tag{22}$$

Thus, one expects that by using *high energy collisions between heavy ions* (say Pb on Pb at $\sim\frac{1}{2}$ GeV per nucleon in the center-of-mass system), one may double the nucleon density, and thereby produce such abnormal states, provided that m_σ is not too much higher than 1 GeV.

So far we have discussed only the minimum energy state at a fixed nucleon density n. Next (still assuming $N^{1/3} \gg 1$), we would like to examine the optimum nucleon density n, and inquire whether the minimum energy state is a normal state $n < n_c$, or an abnormal state $n > n_c$?[10] In order to answer this question, one must leave the "incompressible fluid" model. The real physics problem is tied closely to the strength and the range of the short-range repulsive force between nucleons.[11]

For definiteness, let us assume that the attraction between nucleons is provided by the long-range interaction through $\langle\sigma\rangle$, and the short-range repulsion between two nucleons (proton or neutron) is of the simple form

$$u(r) = u_0 \quad \text{for } r < a,$$
$$\qquad 0 \quad \text{for } r > a, \tag{23}$$

where r is the relative distance between the two nucleons. We shall consider in the following, two extreme cases (i) $\frac{1}{2}$ GeV $\gg u_0 \gg 20$ MeV and (ii) $u_0 = \infty$. The correct physical situation probably lies somewhere in between.

(*i*) *Soft-core interaction*

We assume Eq. (23) with $\frac{1}{2}$ GeV $\gg u_0 \gg 20$ MeV for the short-range interaction. Thus, so far as the normal nuclear states are concerned, the repulsive interaction resembles the hard-sphere interaction with a = diameter of the hard sphere. However, in the abnormal state, since one deals with energy change $\sim O$ (1 GeV) per nucleon, as a zeroth approximation this repulsive interaction can be neglected. As shown in the Appendix, in order for the normal nuclear state to have a nucleon density $n = [(4\pi/3)(1.2\text{ fm})^3]^{-1}$ and a (volume) binding energy $\sim$16 MeV per nucleon,[12] one finds

$$a \cong 0.56\text{ fm} \quad \text{and} \quad m_\sigma \cong 1.15\text{ GeV.}^{13} \tag{24}$$

For the abnormal state, one has $m_{eff} \cong 0$ and therefore the total energy E is given by

$$E/N = \tfrac{3}{4}k_F + m_\sigma^2 m_N^2/8ng^2 + \text{surface energy} \tag{25}$$

where, as before, N is the total number of nucleons and n is the nucleon density, related to the top Fermi momentum k_F by Eq. (17). In Eq. (25), the first term $\frac{3}{4}k_F$ is the average kinetic energy per nucleon, and the second term $m_\sigma^2 m_N^2/8ng^2$ is simply the value of U_σ/N at $\sigma = \boldsymbol{\pi} = 0$ where U_σ is given by Eq. (13). For simplicity, we have set $m_\pi = 0$ in Eq. (25) and also neglect the soft-core repulsion on account of $u_0 \ll \frac{1}{2}$ GeV. The minimum of Eq. (25) is at $(\partial/\partial n)(E/N) = 0$, which implies that, after neglecting the surface energy,

$$\tfrac{1}{4}k_F - m_\sigma^2 m_N^2/8ng^2 = 0; \tag{26}$$

this gives for $g^2/4\pi = 15$

$$(k_F/m_N) \cong 0.45(m_\sigma/m_N)^{1/2}. \tag{27}$$

From Eqs. (25) and (26) one sees that

$$E/N = k_F, \tag{28}$$

and therefore the binding energy per nucleon is $(m_N - k_F)$. If one assumes $(4\pi)^{-1}g^2 \cong 15$ and $m_\sigma \cong 1.15$ GeV, as given by Eq. (24), then one finds for the abnormal state

$$\text{binding energy} \cong 475\text{ MeV/nucleon}$$

and

$$n^{-1} \cong (4\pi/3)(0.65\text{ fm})^3. \tag{29}$$

Since the short-range repulsion is completely neglected in this calculation, the above large binding energy can only be regarded as an *upper bound* of the actual value.

(*ii*) *Hard-core interaction*

Next, we consider the other extreme case that the short-range interaction is given by Eq. (23) with $u_0 = +\infty$. The description of the normal nuclear state is the same as that

[10] The condition $N^{1/3} \gg 1$ excludes practically all existing nuclei.

[11] For the abnormal state, since one deals with a large-scale energy change $\sim$O (GeV) per nucleon, the usual short-range attractive force does not play any important role.

[12] See, e.g., A. Bohr and B. R. Mottelson, 1969, *Nuclear Structure*, Vol. 1, p. 142.

[13] One notes that if the hard sphere has a radius $\sim(\omega\text{-mass})^{-1}$, then its diameter a is $\sim$.5 fm.

in the previous case of a soft-core repulsive interaction, and therefore Eq. (24) holds. As shown in the Appendix, the binding energy and the nucleon density of the abnormal state can be calculated approximately. For a hard-sphere interaction with a diameter $\cong 0.56$ fm, $g^2/4\pi \cong 15$ and $m_\sigma \cong 1.15$ GeV, one finds for the abnormal state

binding energy $\cong 130$ MeV/nucleon,

and

$$n^{-1} \cong (4\pi/3)(0.91 \text{ fm})^3. \tag{30}$$

In either case, if $N^{1/3}$ is sufficiently large so that one may neglect the surface energy, then the abnormal state is stable; it is the lowest energy state with a substantially larger binding energy per nucleon than that in the normal state.

Remarks

1. The above calculations are approximations serving only to illustrate the general features of the abnormal states. Within such approximation, one may ask: suppose that m_σ is >1.15 GeV; would the abnormal state remain the lowest energy state? If one assumes a soft-core repulsion, then from Eqs. (27) and (28), one sees that for $(4\pi)^{-1}g^2 = 15$, the abnormal state is stable provided that m_σ is less than $\sim 4.9 m_N$, and $N^{1/3} \gg 1$. In the case of hard-core repulsion, for $(4\pi)^{-1}g^2 = 15$, the abnormal state is stable only if $m_\sigma \lesssim 1.5 m_N$; the corresponding range for metastable abnormal states is, of course, much wider. Since only the ratio m_σ^2/g^2 enters into the calculation, the above range in m_σ also gives the corresponding latitude in g^2.

2. So far, we have assumed the short-range repulsive interaction to be an isoscalar. There is, in addition, an isovector part due to, e.g., ρ exchange. This isovector part makes the short-range repulsion between nn and pp larger than that between np. This difference in repulsion plus the role of statistical weight lead one to expect that in the abnormal state the average number Z of protons is $\sim\frac{1}{2}N$, so as to minimize the total energy.[14]

3. Keeping $Z \sim \frac{1}{2}N$, one finds the Coulomb energy to be $\sim(1/10)m_N$ when $N \sim 10^4$. Thus, for $N \gg 10^4$, the abnormal state is most likely unstable.[15] When N is of astronomical size, the abnormal state may again be stable because of gravitation.

4. The general feature of the "abnormal nuclear states" is not sensitive to the details of the σ-model; it depends only on the existence of a strongly interacting 0+ resonance, whose long wavelength aspect may be represented by a field. Since we are only interested in the long wavelength limit, the microscopic details of the 0+ resonance should not be important. This is analogous to the phenomena of Bose-Einstein condensation and superfluidity. He^4 is a composite composed only of Fermions p, n and e^-, yet it exhibits superfluidity and undergoes Bose-Einstein condensation. Through the B.C.S. pair-correlation, the electrons in metals exhibit similar phase transitions that give rise to superconductivity. Recently, it has been observed that even He^3 exhibits superfluidity. Likewise, the abnormal state that we are interested in is a similar *condensed phase* of the long wavelength limit of a 0+ field.

5. As remarked earlier, in order to produce such abnormal states it is best to collide heavy ions on heavy ions at high energy, so as to maximize both $N^{1/3}$ and n. If such abnormal states do exist, one may observe in the final state a *stable, or metastable, nucleus of very large baryon number*, say $N \gtrsim 400$. Both the binding energy and the radius of these abnormal states are quite different from the usual extrapolations derived from the normal states. The nucleons in the abnormal states behave like zero-mass particles, and that should produce rather distinct physical characteristics in the dynamics of these abnormal states (e.g., in its interaction with the electromagnetic field).

6. It may be of interest to examine a strongly interacting 0+ field theory different from the σ-model. As an extreme example, one may assume the potential $U(\phi)$ in Eq. (3) to consist of only $\frac{1}{2}m_\phi^2\phi^2$ without any nonlinear interaction. In such a case, as the nuclear density n increases, the transition from the normal state $m_{\text{eff}} \sim m_N$ to the abnormal state $m_{\text{eff}} \sim 0$ becomes continuous, though rapid. Once away from the transition region, even in this extreme example, the overall characteristics of the nuclear state remain similar to those in the σ-model; e.g., as seen from Eq. (8), when n is $\gg m_\phi^2 m_N/2g^2$, the effective nucleon mass does become near zero and the nuclear state becomes abnormal, just as in the σ-model.

IV. PURE VACUUM EXCITATION

We now turn to our next topic; a pure vacuum excitation state. For definiteness, let us consider again a strongly interacting 0+ field ϕ, with a Lagrangian density given by Eq. (3). Like the σ model, the potential $U(\phi)$ is of the form (see Fig. 4). However, unlike in the σ-model, we assume the potential U to have a *second local minimum* at $\phi = \phi_{\text{vex}} \neq 0$, as shown in Fig. 4.[16]

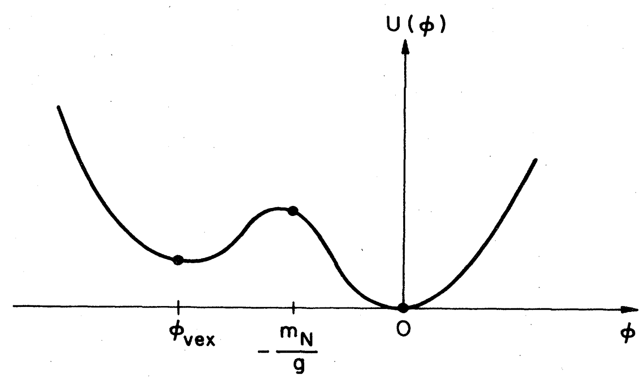

FIG. 4. An example of a potential function $U(\phi)$ with two minima.

[14] See also the discussion given at the end of the Appendix.

[15] When Coulomb energy becomes important, the abnormal state can create e^+e^- pairs; the e^+ will be sent to infinity, but the e^- will be kept within the abnormal nucleus. As Z increases, the number of e^- also increases. The interplay between the added Fermi energy of e^- and the Coulomb energy may eventually bring the abnormal state to the point of instability.

[16] In the σ-model, because of the π field, U_σ has only one local minimum at $\sigma \simeq (m_N/g)$. The point $\sigma \simeq -(m_N/g)$ is a saddle point, *not* a local minimum.

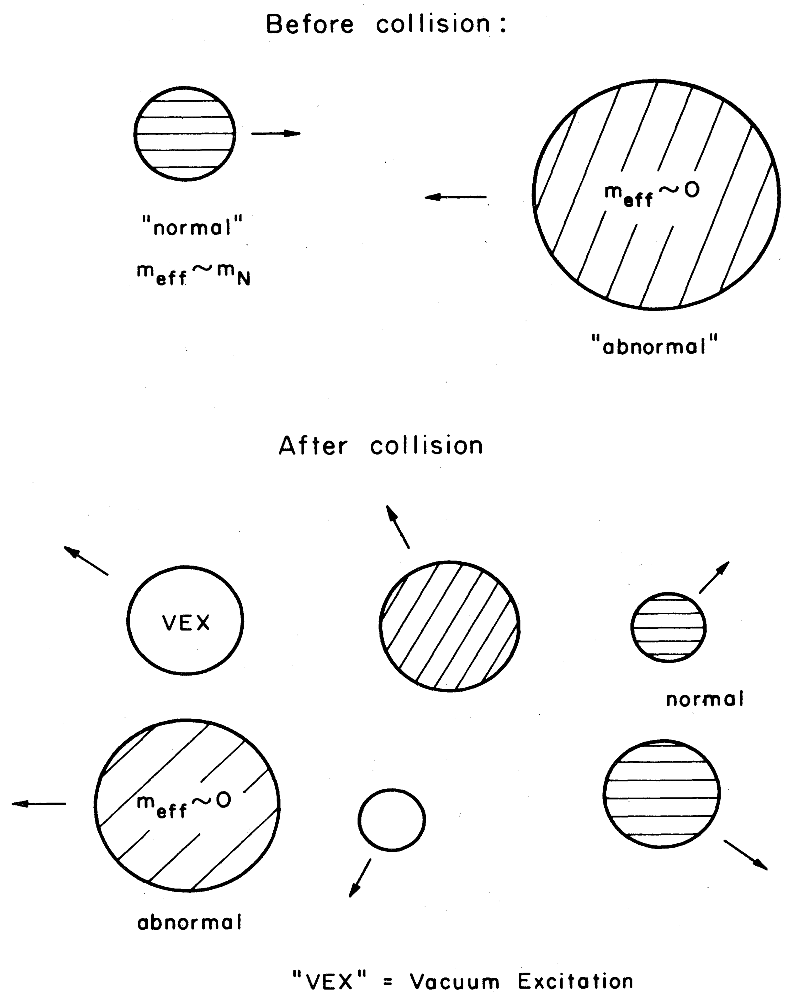

FIG. 5. Possible production of a vacuum excitation state from collisions between normal and abnormal nuclear states.

From our previous discussion, one sees that the condition for producing the "abnormal" nuclear state in the present case is identical to that in the σ model. Now, suppose that we have created such an "abnormal" nuclear state in which $m_{\text{eff}} \sim 0$, and therefore $\langle\phi\rangle \sim -m_N/g$. Let us consider the collision between an "abnormal" nucleus ($\langle\phi\rangle \sim -m_N/g$) and a normal nucleus ($\langle\phi\rangle \sim 0$) (see Fig. 5). After the collision, there will be various fragments, some normal and some abnormal. However, among the fragments, occasionally there can appear a new physical object called the "vacuum excitation" state. The vacuum excitation state occupies a volume Ω, and it carries a 4-momentum P_μ. Inside Ω, one has

$$\langle\phi\rangle = \phi_{\text{vex}} \tag{31}$$

except near the surface region. Outside Ω, one has the normal vacuum $\langle\phi\rangle = 0$. The rest mass M_{vex} of the vacuum excitation state is given by

$$M_{\text{vex}}^2 = -P_\mu^2 = [\Omega U(\phi_{\text{vex}}) + \text{surface energy}]^2. \tag{32}$$

The baryon number of the vacuum excitation is zero. Thus, if one wishes, one may view the vacuum excitation state as a gigantic meson "blob." The linear dimension $\Omega^{1/3}$ is assumed to be much greater than any of the usual microscopic lengths in particle physics.

Next, we examine the question of the width of the vacuum excitation state. It is important to differentiate two cases:

1. Degenerate case

We assume in this case $U(\phi)$ at $\phi = \phi_{\text{vex}}$ to be degenerate with that at $\phi = 0$; i.e.,

$$U(\phi_{\text{vex}}) = 0 \tag{33}$$

in which we have adopted the usual convention that for the normal vacuum $\phi = 0$ one sets $U(0) = 0$. Thus, the rest mass M_{vex} is entirely due to the surface energy. The vacuum excitation state can decay through meson emission by contracting its surface. From relativity, one knows that the lifetime τ is of the order

$$\tau \sim O(\Omega^{1/3}). \tag{34}$$

Thus, the ratio of the width to the mass is

$$(\text{width/mass})_{\text{vex}} \sim O(1/m_\phi{}^3\Omega) \ll 1 \tag{35}$$

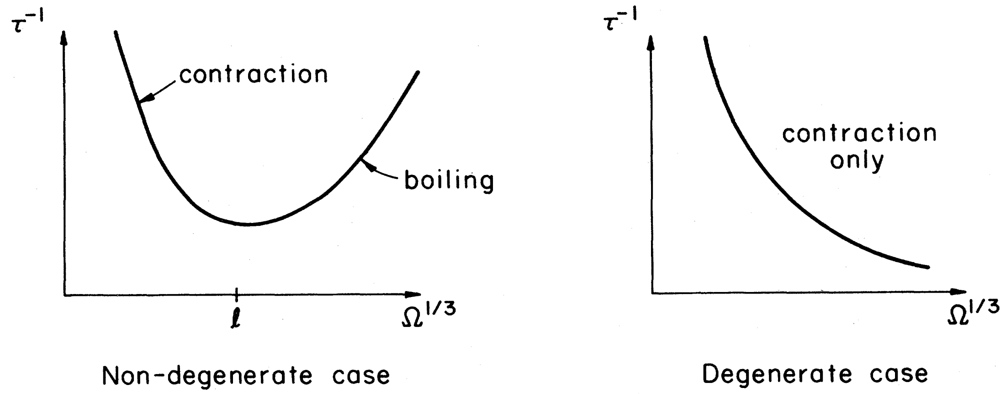

FIG. 6. Lifetime τ of a vacuum excitation state vs its linear dimension $\Omega^{1/3}$.

in which, as well as in the following, we use m_ϕ^{-1} as a typical microscopic length in the problem. From Eq. (35), one sees that the larger the volume Ω is, the sharper is the vacuum excitation state defined. There is no limit to the volume Ω; it can even be of astronomical size.

2. Non-degenerate case

In this case, we have instead of (33)

$$U(\phi_{\text{vex}}) > 0. \tag{36}$$

It turns out that for $\Omega^{1/3} \gg m_\phi^{-1}$, but $\ln(\Omega^{1/3}m_\phi)$ not too large, the vacuum excitation state decays via the same "contraction" mechanism as in the degenerate case, in which through surface contraction, mesons are emitted near the surface. However, when $\ln(\Omega^{1/3}m_\phi) \gg 1$, there is another decay mechanism, called "boiling" in which mesons are produced in the interior of the volume, on account of Eq. (36). In this case there is a limit to the volume Ω. We may plot schematically the width τ^{-1} versus the linear size $\Omega^{1/3}$ for these two cases (see Fig. 6).

In the non-degenerate case, the minimum width occurs at $\Omega^{1/3} = l$ where $\ln(lm) \gg 1$. In some typical examples, l is found to be $\sim O$ (mm). Thus, in both cases, the size of the vacuum excitation can reach macroscopic dimensions.[17]

V. REMARKS

1. In the literature, there have been extensive theoretical discussions of the "spontaneous symmetry breaking" mechanism and other related topics, such as Goldstone bosons, Higgs phenomenon, etc. In all these discussions, the interaction is assumed to be symmetric under a certain group of transformations (or nearly symmetric, as in the case of the σ model). The observed asymmetry is due to the specific expectation value of a certain spin 0 field $\phi(x)$ in the physical vacuum state. In such a theory, it is necessary that there must exist other states which are degenerate (or nearly degenerate) with the physical vacuum state. A natural question to ask is what happens to the other degenerate (or nearly degenerate) states in which $\langle\phi(x)\rangle$ is different from its normal vacuum value. It has been often argued that since we are dealing with an infinite system, only one vacuum state can be realized in our world, all the other states are unphysical, even if they are degenerate with the physical vacuum state.

This situation is rather similar to the example of Heisenberg's infinite ferromagnet. The existence of a ferromagnet clearly defines a specific direction in space, but it does not imply a violation of rotational symmetry of the physical law. If the ferromagnet is of infinite dimension, then it will be physically impossible to rotate all the spins of an infinite ferromagnet.

However, as is well known, there can be "domain structure." In the case of the ferromagnet, such a "domain" may be created by applying a local magnetic field over a relatively large region in space. Here, we may make the analog:

$$\langle\phi(x)\rangle \leftrightarrow \text{spin}$$

$$\psi^\dagger\gamma_4\psi \leftrightarrow \text{magnetic field.}$$

Since nuclear matter interacts linearly with $\phi(x)$, by having a sufficient amount of nuclear density over a large volume in space, we may hope to create a similar "domain structure" with respect to the vacuum state.

2. The question whether we live in a "medium" or in a "vacuum" dates back to the beginning of physics. From relativity, we know that the "vacuum" must be Lorentz-invariant, but that does not mean the "vacuum" is simple. From Dirac's hole theory, one has learned that the vacuum, though Lorentz-invariant, can be rather complicated. However, so long as all of its properties cannot be changed, so long as, e.g., the value of vacuum polarization cannot be modified, then it is purely a question of semantics whether the vacuum should be called a medium or not.

What we try to suggest is that if we do indeed live in a medium, then there should be ways through which we may change the properties of that medium.

Hitherto, in high-energy physics we have concentrated on experiments in which we distribute a higher and higher amount of energy into a region with smaller and smaller

[17] For further details and the question of quantum fluctuations, see T. D. Lee and G. C. Wick, 1974, Phys. Rev. D 9, 2291.

dimensions. In order to study the question of "vacuum," we must turn to a different direction; we should investigate some "bulk" phenomena by distributing high energy over a relatively large volume. *The fact that this direction has never been explored should*, by itself, *serve as an incentive for doing such experiments*. As we have discussed, there are possibilities that abnormal states may be created, in which the nucleon mass may be very different from its normal value. It is conceivable that inside the volume of the abnormal state, some of the symmetry properties may become changed, or even that the usual roles of strong and weak interactions may become altered. If indeed the properties of the "vacuum" can be transformed, we may eventually be led to some even more striking consequences than those that have been discussed in this lecture.

APPENDIX: HARD-SPHERE GAS MODEL

In this appendix, we simply approximate the short-range nuclear force by the hard-sphere interaction of diameter a. The attraction between nucleons is assumed to be provided by the long-range interaction through the expectation value of the σ field. The result should give us at least a qualitative understanding of the abnormal state. For self-consistency of the model, the parameters a and m_σ are to be determined by fitting the known properties of normal nuclear states.

A. Normal nuclear states[18]

In the normal state, we may consider the nucleons to be nonrelativistic; furthermore, the nonlinear aspect of the σ field may be neglected, since σ does not deviate too much from its vacuum expectation value σ_0. The energy per nucleon is given by

$$E/N = m_{\rm eff} + (1/2m_{\rm eff})\tfrac{3}{5}P_F^2 + \tfrac{1}{2}m_\sigma^2(\sigma - \sigma_0)^2(\Omega/N) \qquad \text{(A1)}$$

where, for simplicity, we have neglected the surface energy,

$$\sigma_0 = m_N/g, \qquad m_{\rm eff} = g\sigma, \qquad \text{(A2)}$$

Ω is the nuclear volume

$$\Omega = (4\pi/3)r^3N, \qquad \text{(A3)}$$

and P_F is the top Fermi momentum for the hard sphere gas, related to the nuclear density $n = N/\Omega$ by

$$P_F = [(3\pi^2/2)n]^{1/3}[r/(r - 0.8a)]. \qquad \text{(A4)}$$

The above Fermi momentum differs from that of a free gas[19] by a factor $r/(r - 0.8a)$, showing that the effective nuclear radius available to the hard spheres is smaller than r by a factor $1 - 0.8(a/r)$. The coefficient 0.8 is chosen, so that for a dilute system of hard spheres, the first-order energy correction in $(n^{1/3}a)$ agrees with the *exact* calculation; the second order energy correction turns out to be too large, about 1.5 times the exact value, which implies that this "Van der Waals type" approximation perhaps overestimates the repulsive energy.[20]

By setting

$$(\partial/\partial\sigma)(E/N) = 0 \quad \text{and} \quad (\partial/\partial r)(E/N) = 0,$$

we derive

$$m_N(1 - m_{\rm eff}^{-1}T) = (m_N - m_{\rm eff})^{-1}2m_N u_\sigma \qquad \text{(A5)}$$

and

$$T = \tfrac{3}{2}u_\sigma[1 - 0.8(a/r)], \qquad \text{(A6)}$$

where T is the average kinetic energy per nucleon, and u_σ is the average σ-field energy per nucleon;

$$T = 3P_F^2/10m_{\rm eff} \qquad \text{(A7)}$$

and

$$u_\sigma = \tfrac{1}{2}m_\sigma^2(\sigma - \sigma_0)^2\Omega/N. \qquad \text{(A8)}$$

The binding energy (b.e.) per nucleon is given by

$$\text{b.e.} = m_N - N^{-1}E. \qquad \text{(A9)}$$

If we set b.e. $\cong$ 16 MeV, $r \cong 1.2$ fm, and $(4\pi)^{-1}g^2 = 15$, then we obtain

$$a \sim .56 \text{ fm}, \qquad m_{\rm eff} \sim .85m_N \qquad \text{(A10)}$$

and

$$m_\sigma \sim 1.15 \text{ GeV}. \qquad \text{(A11)}$$

The corresponding value of the average kinetic energy T is $\sim$60 MeV per nucleon and the average field energy $u_\sigma \sim 65$ MeV per nucleon, which together with $m_{\rm eff} - m_N \sim -141$ MeV lead to the 16 MeV (bulk) binding energy per nucleon.[21]

B. Abnormal nuclear states

In the abnormal state, one has $\sigma = 0$ and $m_{\rm eff} = 0$. For the hard-sphere interaction, upon neglecting the surface energy and setting $m_\pi = 0$, one has, instead of Eq. (A1),

$$E/N = \tfrac{3}{4}P_F + m_N^2m_\sigma^2/8g^2n, \qquad \text{(A12)}$$

where P_F is given by (A4) and $n = N/\Omega = [(4\pi/3)r^3]^{-1}$.[22] By setting

$$(\partial/\partial r)(E/N) = 0,$$

[18] I wish to thank R. Serber for discussions. For further details, see T. D. Lee and M. Margulies, Phys. Rev. D (in press). Cf., also S. A. Chin and J. D. Walecka, 1974, Phys. Lett. **52B**, 24 for a related but independent calculation of the normal nuclear states. I wish to thank Dirk Walecka for sending me a copy of his paper before publication.

[19] Compare Eq. (17).

[20] See, for example, p. 256 of Bohr and Mottelson (1969).

[21] Compare pp. 142 and 245 of Bohr and Mottelson (1969).

[22] If one has the soft-core repulsion, then P_F is replaced by p_F, and (A11) becomes (25).

one finds for the abnormal state

$$\text{b.e.} \sim 130 \text{ MeV/nucleon} \tag{A13}$$

and

$$r \sim .91 \text{ fm}. \tag{A14}$$

The corresponding value of $\frac{3}{4}P_F$ is $\sim$490 MeV and the field energy per nucleon is $\sim$320 MeV; together, they lead to a binding energy of about 130 MeV per nucleon, provided that the surface energy can be neglected.

If one is dealing with a pure neutron system ($Z = 0$), then instead of Eq. (A4), P_F is given by

$$P_F = (3\pi^2 n)^{1/3}[r/(r - 0.8a)]. \tag{A15}$$

Because of the isovector part of the repulsive force (e.g., due to ρ exchange), one expects the hard-core diameter between neutrons to be greater than that given by Eq. (A10). As an illustration, we may assume $a > .6$ fm for a pure neutron system; then by using Eq. (A12), we find the minimum of (E/N) for the corresponding abnormal state to be greater than $m_N + 25$ MeV; consequently, the abnormal state for a pure neutron system has a *higher* energy than the (unbound) normal state.

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