The electromagnetic properties of superconductors*

M. Tinkham

Department of Physics and Division of Engineering and Applied Physics, Harvard University, Cambridge, Massachusetts 02138

The London–Ginzburg–Landau concept of superconductivity as a macroscopic quantum state is reviewed. Experimental measurements are then discussed of resistance below T_c, and of enhanced diamagnetism above T_c, both caused by thermodynamic fluctuations away from the Ginzburg–Landau state of lowest free energy. Next the limitations on superconductivity at nonzero frequencies are reviewed: normal electron dissipation $\propto \omega^2$, and strong absorption above the energy gap frequency. Sum rule arguments relate the superfluid response at low frequencies to the gap; effects of strong electron–phonon coupling are also found. Finally, results of recent work on the resistive state of superconducting filaments above the critical current are summarized.

CONTENTS

I. INTRODUCTION

In 1911, Kamerlingh Onnes discovered superconductivity, i.e., that certain metals conduct an electric current without resistance when they are cooled below a characteristic critical temperature, T_c, typically in the liquid helium range of a few degrees Kelvin (Onnes, 1911). To this perfect conductivity, Meissner in 1933 added the discovery of the perfect diamagnetism of superconductors (Meissner and Ochsenfeld, 1933). These discoveries fascinated many people both because of the intellectual novelty of a sort of electronic perpetual motion machine and because of the obvious potential for practical applications. These phenomena were nicely described by a pair of equations proposed by London and London (1935); their model was generalized in a decisive way by Ginzburg and Landau (1950) to provide our present basic conceptual picture of how the phenomena may be understood in terms of a macroscopic quantum state. This phenomenological description subsequently received its microscopic foundation from the BCS theory (Bardeen, Cooper, and Schrieffer, 1957) through the work of Gor'kov (1959a,b). The rapid advance continued with such landmarks as the theoretical discoveries of type II superconductors by Abrikosov (1957a,b), and of coherent tunneling of pairs by Josephson (1962). Perhaps large scale technical applications of these concepts may be made after another twenty year interval, in the late 1970's.

From this vast panorama, how can I construct a meaningful talk in the time available? The approach I have chosen is to first describe how perfect conductivity and perfect diamagnetism are understood in terms of the Ginzburg–Landau picture. Then I shall consider a number of experiments and related concepts which have helped define the limits of validity of this basic picture, and also the limits of the superconducting domain. As is, I believe, customary for this type of talk, I shall emphasize work with which I have been associated. Specifically, I shall concentrate on fluctuation effects near T_c, effects at the energy gap frequency ω_g, and dissipative processes above the critical current I_c.

II. SUPERCONDUCTIVITY AS A MACROSCOPIC QUANTUM STATE

Fritz London introduced the idea that in the superconducting state of a metal, some of the electrons "condense" into a quantum state extending over macroscopic dimensions, or in Casimir's words "over miles of dirty lead wire." The number density of the "superconducting electrons" n_s is operationally inferred from the measured penetration depth λ of a magnetic field (within which the diamagnetic screening currents flow to give the Meissner effect) by the London relation

$$\lambda = (mc^2/4\pi n_s e^2)^{1/2}. \tag{1}$$

Ginzburg and Landau (1950) extended this concept by introducing a "wave function of the superconducting electrons" $\psi(\mathbf{r})$, such that $n_s = |\psi(\mathbf{r})|^2$. $\psi(\mathbf{r})$ was taken as a complex order parameter, going to zero continuously at T_c, where $\lambda \to \infty$. This $\psi(\mathbf{r})$ was allowed to vary in space and in response to applied fields, whereas the n_s of the London theory had been taken to be a function only of temperature. These variations were presumed to be determined by the principle of minimization of the thermodynamic free energy, with a postulated expansion of the thermodynamic free energy density in powers of $|\psi|^2$ (or n_s) and of gradients of ψ of the form

$$f_s = f_n + \alpha|\psi|^2 + \beta/2\,|\psi|^4 + 1/2m^*\,|[(\hbar/i)\nabla - (e^*/c)\mathbf{A}]\psi|^2 + B^2/8\pi. \tag{2}$$

In this β must be positive, but by definition of T_c, α must change from positive above T_c to negative below T_c; near T_c, we can set $\alpha = \alpha'(T - T_c)$. The combination of ∇ and $\mathbf{A}$ is that required for f_s to be gauge invariant. The variational principle then leads to two coupled non-linear differential equations governing ψ and $\mathbf{A}$:

$$\alpha\psi + \beta|\psi|^2\psi + (1/2m^*)[(\hbar/i)\nabla - (e^*/c)\mathbf{A}]^2\psi = 0 \tag{3}$$

* *Buckley Prize Lecture* March 1974.

$$(c/4\pi)\nabla\times(\nabla\times\mathbf{A}) = \mathbf{J} = e^*\hbar/2m^*i(\psi^*\nabla\psi - \psi\nabla\psi^*) - (e^{*2}/m^*c)\psi^*\psi\mathbf{A} \equiv |\Psi|^2 e^*\mathbf{v}_s, \quad (4)$$

where we write ψ as $|\psi|e^{i\varphi}$ and identify

$$\mathbf{v}_s = (1/m^*)[\hbar\nabla\varphi - (e^*/c)\mathbf{A}] \quad (4a)$$

as the gauge-invariant velocity of the superconducting electrons.

We now know that ψ essentially describes the center of mass motion of the Cooper electron pairs so that $e^* = 2e$, but the value of m^* depends on one's normalization convention. To the extent that we can ignore the nonlinear term in Eq. (3), it is seen that ψ obeys the Schrödinger equation for free particles of mass m^* and charge e^*, while $-\alpha$ plays the role of the energy eigenvalue. Similarly, Eq. (4) is the usual quantum mechanical current expression for such particles. Thus, although Eq. (3) can *not* be *derived* from the Schrödinger equation, for qualitative arguments we can carry over our familiarity with ordinary quantum mechanical examples.

Consider first the case of a superconducting wire of length L, closed on itself to form a superconducting ring carrying a persistent current. With x measured around the ring, the wave function will vary as $\exp(ikx)$. For ψ to be single valued, the phase $\varphi = kx$ can change only by $2\pi n$ if $x \to x + L$. Thus, $k_nL = 2n\pi$, and the currents in the ring can have only values such that [integrating Eq. (4a)] the quantity Φ' has a quantum value

$$\Phi' \equiv \Phi + (m^*c/2e)\oint \mathbf{v}_s\cdot d\mathbf{s} = n\Phi_0, \quad (5)$$

where

$$\Phi_0 \equiv hc/2e \approx 2.07\times 10^{-7}\ \text{gauss-cm}^2, \quad (5a)$$

and where $\Phi = \oint \mathbf{A}\cdot d\mathbf{s}$ is the magnetic flux enclosed by the path. This Φ' is the *fluxoid* of the circuit, as introduced and defined by F. London[1,2]; it differs from the magnetic flux Φ by the term in $\oint \mathbf{v}_s\cdot d\mathbf{s}$. London also remarked, in a famous footnote,[1] that the fluxoid should be quantized as we have found here, but lacking the pairing theory, he assumed $\Phi_0 = hc/e$. It is obvious that fluxoid quantization is based on the same elementary argument used for quantizing L_z in an atom: the requirement that the phase be unchanged modulo 2π in going around one cycle. But in the present context, the quantum condition is applied over macroscopic dimensions.

In the special case of a wire thick compared to the penetration depth, $v_s \to 0$ in the interior. Then Eq. (5) implies that the flux Φ itself is quantized in integral multiples of Φ_0, as was shown experimentally by Deaver and Fairbank (1961) and by Doll and Näbauer (1961).

The same principle allows us to see how persistent currents can be understood. Because of the quantization, the current cannot decay continuously, as in a normal conductor, but only in quantum jumps in which the total phase change around the ring changes by a multiple of 2π. [In the atomic analogy, this corresponds to a jump from a d state to a p state, for example]. If no such quantum jump occurs, there is *no* resistance, not just a small resistance; our task is then to show that we can account for the low probability of these quantum jumps. We shall turn to that in a moment.

But first, let us note that the perfect diamagnetism discovered by Meissner also has a qualitative explanation from this macroscopic quantum picture. The Meissner state corresponds simply to the state with fluxoid quantum number $n = 0$ for all possible circuits in the medium. Since the flux Φ is then zero for all paths except those involving the surface penetration layer where $v_s \neq 0$, it follows that $B = 0$ everywhere except in this surface layer. In this context, also, the atomic analogy is suggestive. The restriction to $n = 0$ corresponds to considering an atom whose ground state is an s-state. Then the (orbital) magnetic moment is zero in the absence of a field, but is governed by the Langevin–van Vleck[3] susceptibility

$$\chi \equiv M/B = -1/6(N/V)(e^2/mc^2)\langle r^2\rangle \quad (6)$$

in the presence of a field. With (N/V) replaced by $|\psi|^2$, this will approach $-\infty$ for our "macroscopic atom" in which $\langle r^2\rangle$ reflects the sample size rather than an atomic dimension. Thus $B = H + 4\pi M \to 0$ while $M \to -H/4\pi$.

Now let us turn to some experiments.

III. RESISTANCE IN ONE-DIMENSIONAL SUPERCONDUCTORS

The explanation of perfect conductivity above utilizes the special case of a closed superconducting ring. If instead we consider a superconducting wire fed by normal leads, the relative phase $\Delta\varphi_{12}$ at the two ends depends on the length of the wire and on the current through it. But as Josephson emphasized, since the phase of ψ evolves as $\exp(-i2\mu t/\hbar)$, where μ is the electrochemical potential of the paired electrons, the phase difference $\Delta\varphi_{12}$ will evolve according to

$$d(\Delta\varphi_{12})/dt = 2eV_{12}/\hbar \quad (7)$$

so that $\Delta\varphi_{12}$ will be constant if there is no voltage difference and hence no dissipation. [In the special case of a closed superconducting ring, the phase change around the ring is constrained to have the particular constant value zero, modulo 2π]. As pointed out by Little (1967) and by Langer and Ambegaokar (1967) the elementary dissipative event is a phase slip by 2π at some localized point along the wire, made possible by having $|\psi(x)|$ go momentarily to zero at that point. Such a phase slip requires a voltage pulse $V(t)$ such that

$$\int V(t)\,dt = h/2e \approx 2\times 10^{-15}\ \text{volt-sec} \quad (8)$$

[1] F. London, 1950, *Superfluids, Vol. 1* (J. Wiley, New York), see footnote on page 152.

[2] For a later discussion of fluxoid quantization in connection with the vortex state of superconductors, and its implications concerning the angular dependence of the critical field of thin superconducting films, see M. Tinkham, 1963, Phys. Rev. **129,** 2413 and M. Tinkham, 1964, Rev. Mod. Phys. **36,** 268.

[3] See, for example, J. H. Van Vleck, 1932, *Theory of Electric and Magnetic Susceptibilities* (Oxford University Press, Oxford), p. 91.

to build $\Delta\varphi_{12}$, and the current, back up to their original values. Thus, the dc average voltage for constant current will be given by $(h\nu/2e)$, where ν is the net number of phase slip events per second.

Well below the critical current $I_c(T)$, phase slips can occur only by thermally activated fluctuation processes, which effectively cause a short section of the wire momentarily to go normal, i.e., have $\psi(x) \to 0$. The minimum length in which such a fluctuation can occur is the *GL* coherence length $\xi(T) \propto (1-t)^{-1/2}$, where $t = T/T_c$. The free energy cost per unit volume is $H_c^2/8\pi \propto (1-t)^2$. Thus, the activation energy is proportional to $(1-t)^{3/2}$, and the probability of resistance-producing fluctuations should fall exponentially as T is reduced below T_c. Neglecting the temperature dependence of the attempt frequency (given in the theory of McCumber and Halperin, 1970), we expect the resistance to vary as

$$R \propto \exp[-C(T_c - T)^{3/2}/T], \tag{9}$$

where the constant C is proportional to the cross sectional area of the wire.

To get this resistance to be large enough to be observed, experiments have been done on tin whiskers about 1 μm in diameter: at Cornell by Webb, Warburton, and Lukens (1970), and at Harvard by Newbower, Beasley, and myself (1972). Even with these whisker samples and using a superconductive femtovoltmeter, one must work very near to T_c to see resistance. Some of Newbower's data are shown in Fig. 1. Note the expected exponential fall extending over some six orders of magnitude. (The "foot" at the bottom appears to be due to contact problems). At the lowest voltage shown, phase slip events are occurring at a rate of only 100 per sec. Extrapolating down in temperature another millidegree, the phase slips are expected only once in 1000 years, and in another millidegree once in 10^{19} years. Thus, in about 3 millidegrees we can trace the full transition from the normally resistive state above T_c to a state showing not a single bit of dissipation in the age of the universe!

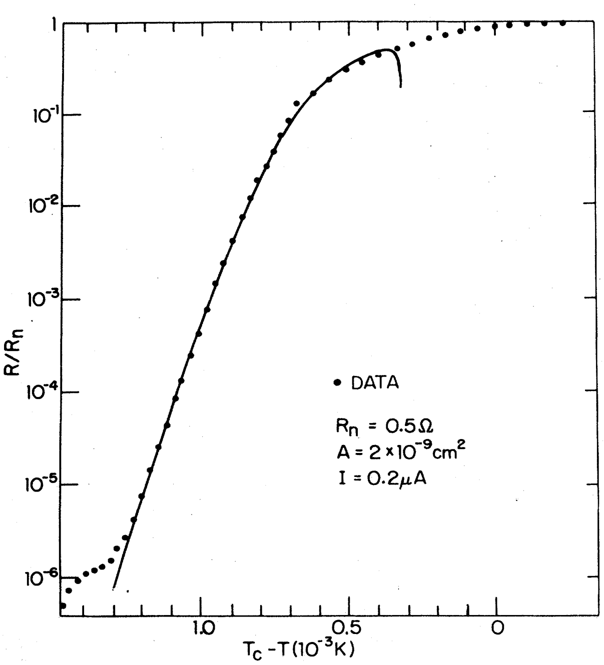

FIG. 1. Comparison of McCumber–Halperin theory (1970) with experimental data points of Newbower, Beasley, and Tinkham (1972) for resistance of tin whisker below T_c due to thermodynamic fluctuations. The small "foot" is believed due to contact effects.

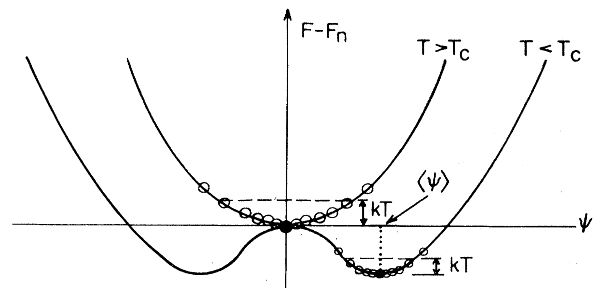

FIG. 2. Schematic representation of fluctuation effects above and below T_c in the Ginzburg–Landau theory (1950). Solid circles represent equilibrium values of ψ, giving minimum free energy; hollow circles represent typical fluctuations.

IV. SUPERCONDUCTIVITY ABOVE T_c

While speaking of fluctuation effects, it is appropriate also to consider the experimental manifestations of the evanescent droplets of superconducting electrons created *above* T_c by similar thermal fluctuations. Above T_c, where $\alpha > 0$, the GL free energy (2) has its minimum for $|\psi| = 0$, as illustrated in Fig. 2. But, just as below T_c fluctuations could take ψ from $\langle\psi\rangle$ down to zero, allowing a phase slip, so above T_c the thermal energy kT allows fluctuations of ψ about zero with $(\delta\psi)^2 \propto kT/\alpha \propto kT/(T-T_c)$, taking the fluctuating volume to be fixed, for simplicity. Since this fluctuating ψ has no long-range phase coherence in space or time, there is no possibility of macroscopic persistent currents. Rather the decay time of the fluctuations sets a limit on the free acceleration time of the superconducting electrons, just as one is set by the scattering time in a normal conductor. Thus, there is only a finite enchancement of the conductivity, which tends to diverge as T_c is approached from above. This enhancement was first observed by Glover (1967) in thin films, and has since been extensively studied.

In our group we instead probed the *spatial* coherence of these fluctuations by measuring the enhancement of the normal Landau diamagnetism above T_c. In view of Eq. (6), this diamagnetism should be proportional to $(\delta\psi)^2$ and to a mean square radius $\langle r^2\rangle$ over which coherence is maintained. Since the fluctuating droplets of superconducting electrons have a radius $\sim\xi(T) \sim 10^4$ Å, whereas the normal diamagnetism corresponds to only an atomic radius, even a small density of superconducting electrons can produce a sizeable increase in χ above the normal value. As shown by H. Schmidt (1968)[4] and by A. Schmid (1969), for very weak fields, GL theory predicts

$$\chi = -(\pi/6)[kT\xi(T)/\Phi_0^2] \approx -10^{-7}(T_c/T-T_c)^{1/2}. \tag{10}$$

To detect this tiny susceptibility, Gollub, Beasley, and I

[4] A numerical error of a factor of 4 occurs in this calculation.

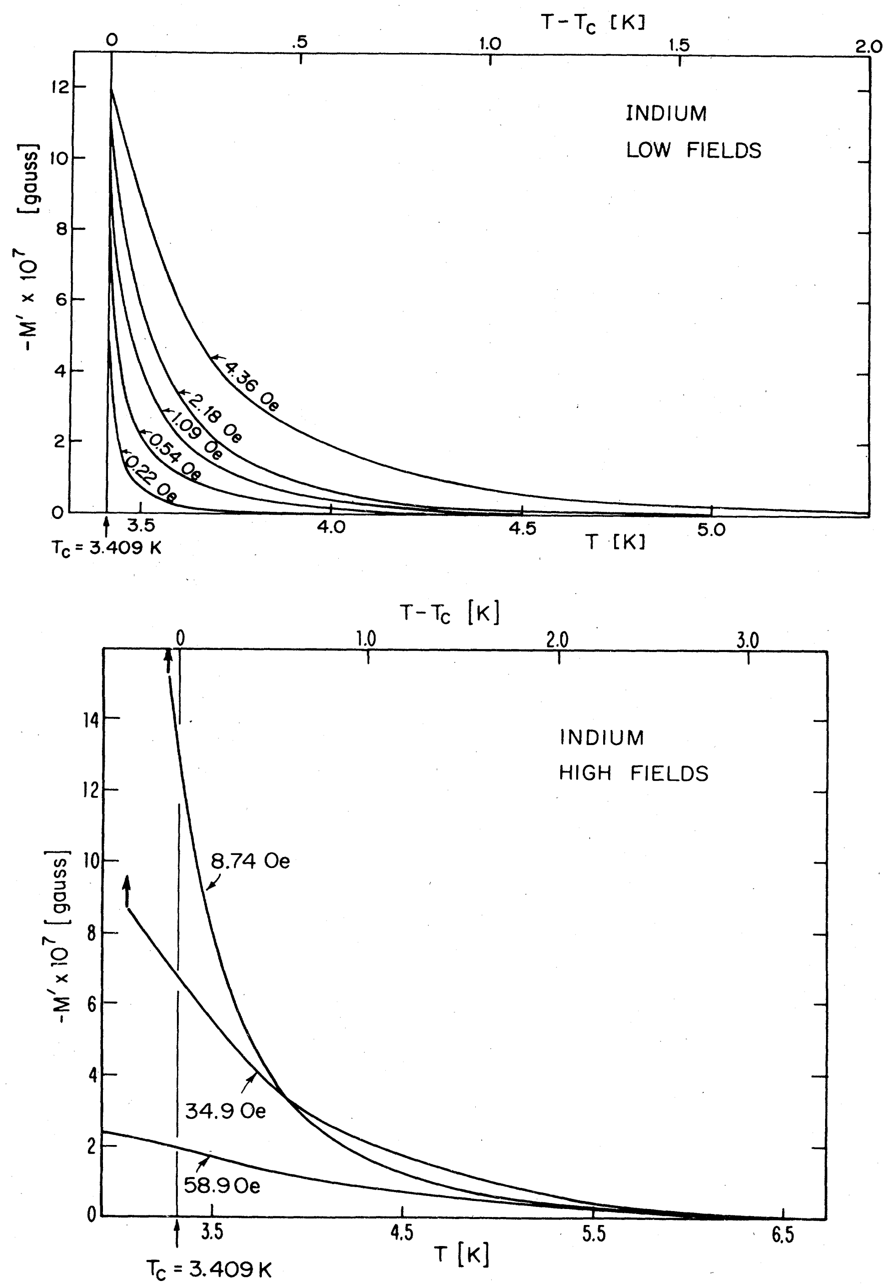

FIG. 3. Data of Gollub *et al.*, (1970, 1973) on diamagnetism of indium above T_c due to thermodynamic fluctuations. Measurements were made using a SQUID magnetometer.

(1969, 1970) used a sensitive SQUID (Superconducting Quantum Interference Device) magnetometer to measure the change in magnetic moment with temperature of a pencil-shaped sample held in the absolutely constant field of a superconducting coil carrying a persistent current.

Some typical data on indium are shown in Fig. 3. The upper part shows results in relatively low fields; M' increases with H, but less than linearly. The lower part shows higher field data; here M' *decreases* as H increases, because the higher fields are rapidly extinguishing the fluctuations. Note the discontinuous jump indicated at the left end of the curve for $H = 34.9$ Oe. At this point, M jumps by *five orders of magnitude* to the Meissner effect value. But since it is a first-order transition, there is no divergence anticipating the jump. As suggested by this figure, a temperature-dependent M' can be observed out to about $2T_c$. In fact, with lead, M' could readily be following out to 16°K, the highest temperature at which the apparatus worked well. Since there is no chance that any strains or impurities could give lead such a high T_c, it is clear that we are observing intrinsic effects, not just some sort of inhomogeneity of the sample.

To compare these results with theory, it was obviously necessary to generalize the Schmidt–Schmid result [Eq. (10)] to finite fields. This was done exactly (within the framework of the GL theory) by Prange (1970), who found that $M'/H^{1/2}T$ should be a universal function of the scaled temperature difference $(T-T_c)/H(dH_{c2}/dT)^{-1}$. But a suitable plot of the experimental data showed that M' fell further and further below this "universal" prediction as H

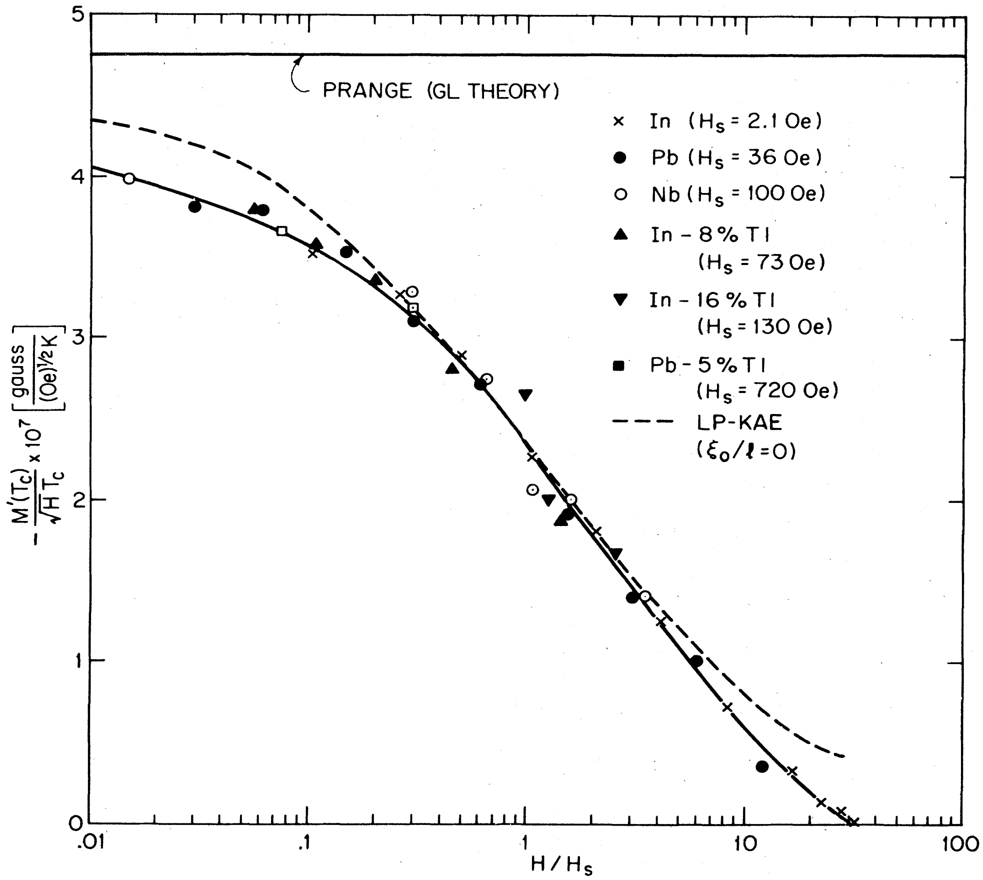

FIG. 4. Field dependent decrease of fluctuation magnetization at T_c below that predicted by GL theory. Experimental results of Gollub, *et al.*, (1970, 1973) are compared with the theory of Lee and Payne (1971) and of Kurkijärvi, Ambegaokar, and Eilenberger (1972).

was increased. The explanation of this failure of the simple GL prediction was first suggested qualitatively by Patton, Ambegaokar, and Wilkins (1969), and then worked out quantitatively by Lee and Payne (1971, 1972), and by Kurkijärvi, Ambegaokar, and Eilenberger (1972). Basically, it is this: The GL theory is based on an expansion in powers of $|\psi|^2$ and of $|(\nabla + i2\pi \mathbf{A}/\Phi_0)\psi|^2$. In the regime above T_c, $|\psi|$ itself is small, but the gradient term is not. For example, in a spherical droplet of radius R, $\nabla\psi \approx \psi/R$, whereas in a field H, $A\psi \approx \frac{1}{2}RH\psi$. The combination takes on a minimum value if $\pi R^2 H \approx \Phi_0$, i.e., the cross section of the fluctuating volume embraces roughly one quantum of flux. Thus, as stronger and stronger fields are applied, the most favorable fluctuation size shrinks smaller and smaller, and the one term expansion in derivatives of ψ becomes a poorer and poorer approximation.

This argument suggests that there should be a characteristic field for each material, at which the fluctuations are squeezed down to a size at which the GL expansion starts seriously to break down. Since this breakdown should occur for a dimension $\sim\xi(0)$, which can be thought of as the size of a Cooper pair, this field should be of the order of $H_{c2}(0) \approx \Phi_0/2\pi\xi^2(0)$. Experimentally, we found that there was indeed a universal dependence of $M'/H^{1/2}T$ on the two variables: Prange's scaled temperature, and a scaled field H/H_s, where we defined the scaling field H_s as the field at which M' is reduced at T_c to half the Prange (or GL) value. [While H_s did turn out to be $\sim\frac{1}{2}H_{c2}(0)$ for alloys, we were surprised to find $H_s \approx (1/20)H_{c2}(0)$ for pure superconductors. This extremely low scaling field for pure samples is in fact predicted by the microscopic theory, where it can be traced back to the effects of nonlocal electrodynamics.] Fig. 4 shows the cut through this universal function in the plane $T = T_c$, comparing the experimental data on a wide variety of materials with the results of the microscopic theory, as well as with the GL approximation. This comparison shows that GL theory becomes a good approximation (as it should) in very weak fields at T_c, where the dimension of the dominant fluctuations is becoming very large compared to the characteristic length $\xi(0)$; but in stronger fields (and also at higher temperatures), where the fluctuations shrink in size, it is inadequate to make the GL approximation of retaining only the leading term of a power series expansion of the true free energy.

V. HIGH-FREQUENCY LIMITATIONS ON SUPERCONDUCTIVITY

If one is dealing with a time varying current, the velocity $\mathbf{v}_s$ of the superconducting electrons [Eq. (4a)] varies in response to an applied electric field as expected from Newton's law

$$d(m^*\mathbf{v}_s)/dt = e^*\mathbf{E}. \tag{11}$$

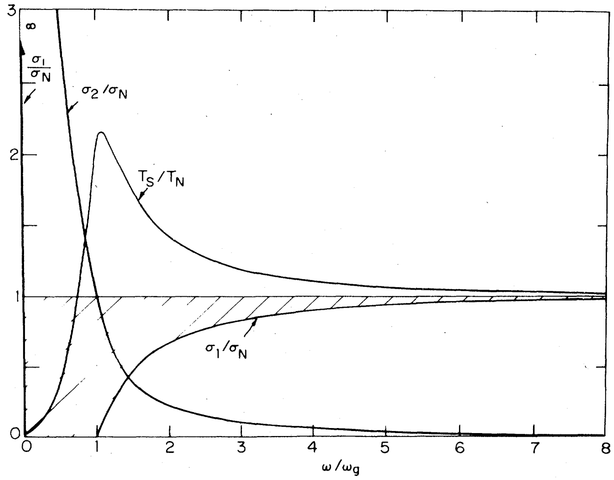

FIG. 5. BCS prediction for σ_1/σ_N and σ_2/σ_N, and transmissivity ratio T_S/T_N computed from these for a typical thin film with normal resistance $377/(n+1)$ ohms per square, where n is the index of refraction of the substrate. The shaded area represents the part of the normal metal oscillator strength which is missing at finite frequencies in the superconducting state, appearing instead as the superfluid response δ function at $\omega = 0$.

Thus, in any ac situation, there must be an electric field in the superconductor. But so long as $T > 0$, there will be quasiparticles or "normal" electrons present, as well as the "superconducting" electrons condensed into the macroscopic quantum state. These normal electrons will also be accelerated by the electric field, but by making collisions they will dissipate the energy gained. Thus, inevitably, there will be some dissipation in a superconductor at any finite frequency. Since the electric field to drive a given current is proportional to the frequency [according to Eq. (11)], the power dissipated $\sigma_1 E^2$ will vary as ω^2, in so far as σ_1 (the real or in-phase conductivity due to quasiparticles) can be taken independent of ω. This argument shows that *perfect* conductivity is possible *only* at $\omega = 0$.

But because of the BCS energy gap Δ, the quasiparticle excitations are rapidly frozen out at low temperatures, and σ_1/σ_n (σ_n is the conductivity in the normal state) approaches zero as $\exp[-\Delta(0)/kT] = \exp(-1.76T_c/T)$. Thus, well below T_c, the dissipation is still much less than in the normal state, even up to microwave frequencies. This allows construction of superconducting microwave cavities (Turneaure and Viet, 1970) with Q's up to 10^{11}, compared to $Q \approx 10^5$ even for the best normal metal cavities.

A. The energy gap

Naturally if the photon energies reach the energy gap, a new dissipative mechanism appears in which bound pairs are broken, creating two quasiparticles. As a result σ_1 rises steeply from ~ 0 (for $T \ll T_c$), as soon as $\hbar\omega$ exceeds 2Δ. Although in hindsight this is very straightforward, the situation was quite different in the pre-BCS days of 1956 when Glover and I were making the first crude far infrared experiments (Glover and Tinkham, 1956, 1957) in an attempt to see whether there *was* an energy gap of the order of kT_c as had been speculated by various earlier authors. The first successful experiments measured the far infrared transmissivity of thin lead films as a function of frequency. This must approach zero as $\omega \to 0$ because of the supercurrent response [Eq. (11)], and it was known to approach a limiting value indistinguishable from that in the normal state when ω reached the near infrared. The question was, would some anomaly appear for $\hbar\omega \approx kT_c$, which would reflect the onset of absorption across the gap. To our great pleasure, there *was* an anomaly: on decreasing the infrared frequency, the transmitted fraction rose to a rather sharp maximum for $\hbar\omega \approx 3.5kT_c$ before dropping smoothly toward zero as ω was decreased further.

B. Complex conductivity

To give a quantitative interpretation of this data, we introduced the general complex conductivity function $\sigma(\omega) = \sigma_1(\omega) - i\sigma_2(\omega)$, as illustrated in Fig. 5. (This figure actually shows the BCS theoretical results). For $T \ll T_c$, $\sigma_1(\omega) \approx 0$ for $\omega < \omega_g$, while σ_1 rose sharply to approach σ_n, the normal conductivity, well above ω_g. On the other hand, the imaginary conductivity σ_2 reflects the supercurrent response as described by Eq. (11). In fact, for n_s^* superconducting pairs (or n_s superconducting electrons) per unit volume,

$$\begin{aligned} i\omega J_s = dJ_s/dt = n_s^* e^* (dv_s/dt) &= (n_s^* e^{*2}/m^*)E \\ &= (n_s e^2/m)E. \end{aligned}$$

Thus we can write

$$J_s = -i\sigma_2 E, \tag{12}$$

with

$$\sigma_2 = n_s e^2/m\omega \tag{12a}$$

for frequencies low enough that n_s can be taken to have the same value as for dc.

C. Sum rule

In fitting the measured data, we had essentially three parameters, which could be taken to be σ_n, ω_g, and $\omega\sigma_2 = n_s e^2/m$. We were intrigued by the fact that our fits gave values such that $\omega\sigma_2$ was always approximately equal to the product $\sigma_n\omega_g$, i.e., that $\sigma_2/\sigma_n \approx \omega_g/\omega$. In other words, the strength of the superfluid response σ_2 could apparently be computed from the normal response σ_n and the gap frequency ω_g. By using the Kramers–Kronig relations and the oscillator strength sum rule, Ferrell, Glover, and I [Ferrell and Glover (1958); Tinkham and Ferrell (1959)] were able to throw this conjecture into quantitative form. Using this approach, we were able to give a new interpretation of Pippard's discovery (Pippard, 1953) that measured penetration depths usually exceed those predicted by the London formula for λ_L, if λ_L (for $T = 0$) is computed using $n_s = n$, the total density of conduction electrons. The central point in our view is that only that fraction of the conduction electron density corresponding to the fraction of the oscillator strength [as measured by $\sigma_{1n}(\omega)$] lying below $\sim\omega_g$ which is "missing" in the superconducting state [i.e., the shaded area in Fig. 5], appears there as n_s (i.e., as a δ function at $\omega = 0$); the higher frequency part is essentially unaffected by the transition. For example, in superconducting alloys in which the scattering rate τ^{-1} exceeds ω_g, $n_s/n \approx \omega_g\tau \approx l/\xi_0$, so that $\lambda \approx \lambda_L(\xi_0/l)^{1/2}$, as had been found by Pippard.

D. Later experiments

The early far infrared experiments on thin films of Glover and myself were soon superseded by the work in my lab of Ginsberg (Ginsberg and Tinkham, 1960), who continued with thin films, and of Richards (Richards and Tinkham, 1958, 1960), who worked with bulk samples formed into nonresonant far infrared cavities. Their use of helium temperature bolometers to replace the room temperature detectors used in the early work gave enough greater sensitivity to improve the accuracy of the gap determinations, and to extend the measurement to additional materials.

The measurements were pushed to a new level of quantitative accuracy by Palmer (Palmer and Tinkham, 1968), who developed a complex apparatus allowing simultaneous measurement of reflectivity and transmissivity of thin lead films, as a function of far-infrared frequency. From these two independent measurements, we could directly compute the real and imaginary parts of the conductivity at each frequency, without any auxiliary assumptions. The results obtained for $\sigma_1(\omega)$ were close to those expected from BCS theory, as is shown by Fig. 6. Note particularly the decrease in gap frequency and the rise in σ_1 below the gap due to thermally excited quasiparticles as T is raised closer to T_c ($=7.2°$K). But $\sigma_2(\omega)$ turned out to be about 25% low. On the face of it, this seemed impossible, since $\sigma_1(\omega)$ and $\sigma_2(\omega)$ are related by the Kramers–Kronig relations, and can not be independently varied. The dilemma was resolved by recognizing that lead is a superconductor with unusually strong electron–phonon coupling, while the BCS theory is a weak coupling theory. The oscillator strength missing from the superfluid response appears instead well above the gap, spread inconspicuously thinly over the wide frequency region characteristic of the phonon spectrum of lead. Calculations of Nam (1967a,b,c), including this effect, gave good agreement with the experimentally observed reduction of σ_2 (or n_s), as is illustrated in Fig. 7.

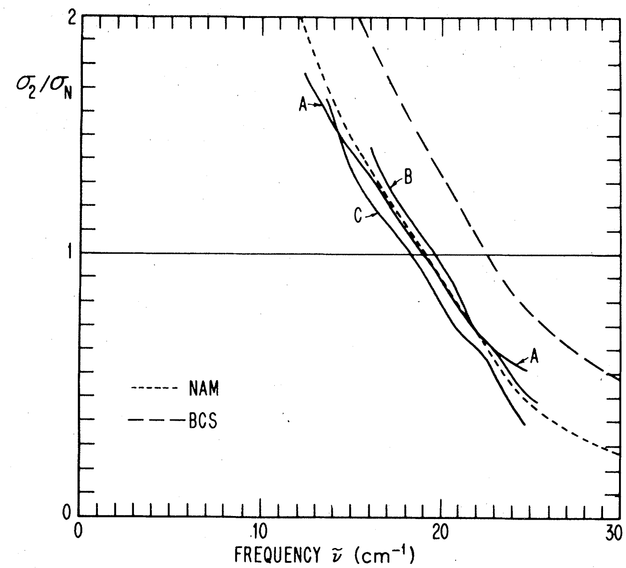

FIG. 7. Measurements of the imaginary part of the normalized conductivity of three thin lead films (A,B,C) at 2°K. Curve labeled BCS is the weak-coupling result, while that labeled Nam includes the strong-coupling effects.

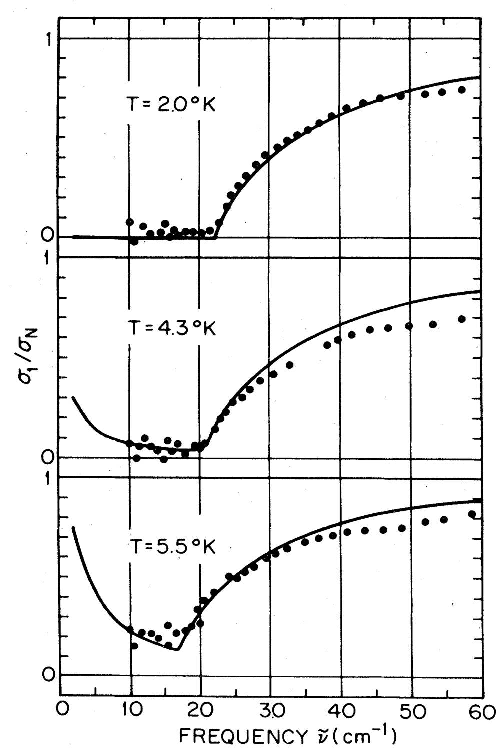

FIG. 6. Temperature and frequency dependence of normalized conductivity σ_1/σ_N in a thin lead film as measured by Palmer and compared with BCS theory. The gap frequency was fitted only for the low temperature limit. Note decrease of gap and increase of σ_1/σ_N below the gap as the temperature is increased.

VI. SUPERCONDUCTIVITY ABOVE THE CRITICAL CURRENT

As my final topic, I want to return to the simple case of filamentary or one-dimensional superconductors, and tell you about some recent developments in our understanding of exactly how they recover their normal resistance above the critical current I_c, for temperatures far enough below T_c that the fluctuation effects discussed in Sec. III are negligible. In short, we find that well below T_c, heating effects dominate; but near T_c, quantum effects dominate, giving rise to the "steps" in the I–V curves reported first by Webb and Warburton (1968).

The critical current I_c of a superconducting wire can be defined theoretically as the highest current for which a time-independent, nondissipative, equilibrium solution of the GL equations exists. Experimentally speaking, I_c is the highest current which can be carried before a voltage develops, apart from the minute voltage due to thermal fluctuation effects.

In a thick wire of type I superconductor

$$I_c = caH_c/2, \tag{13}$$

where a is the radius of the wire; this limit is set by Silsbee's rule that superconductivity must be destroyed if the magnetic field at the surface set up by the current exceeds the thermodynamic critical field H_c. Above this current, the magnetic pressure at the surface would exceed the condensation energy per unit volume of the superconducting state, leading to an instability. As a result of this instability, at I_c there is a discontinuous jump to an "intermediate state" containing both superconductive and resistive regions (F. London, 1950). The resistance jumps to about half the normal value, and rises gradually toward the full normal resistance with further increase in current.

In a thick wire of ideal (no pinning) type II superconductor, the same qualitative behavior is expected, with H_{c1}, the field for first entry of quantized flux-carrying vortices, taking the place of H_c in Eq. (13). But instead of a discontinuous jump in resistance, the resistance rises continuously between the currents producing surface fields of H_{c1} and H_{c2}, where H_{c2} is the highest field at which bulk superconductivity is possible. On the other hand, if there is strong pinning, the vortex rings created at the surface can only contract by a slow thermally activated flux creep process, so the resistance may remain unobservably small until currents far above $caH_{c1}/2$ are reached.

Because of this diversity of behaviors for the thick wire case, it is of particular interest to consider thin wires, of radius small compared to both λ and ξ. Then, because $a \ll \lambda$, the current density will be uniform over the cross section, and the magnetic field energies produced by the current will be negligible compared to the kinetic energy of the electrons; because $a \ll \xi$, no significant variation of $|\psi|$ can occur transverse to the wire, and the distinction between type I and II materials becomes irrelevant. The classic realization of this thin one-dimensional superconductor is the tin whisker, mentioned earlier, but long narrow microbridges cut from a thin film also satisfy the requirements, and they facilitate variation of experimental parameters. I shall conclude my talk by summarizing the results to date of an investigation of the resistive properties of such microbridges, being carried on in my group by W. J. Skocpol.

Because dissipation of energy is inherent in the finite voltage regime, heating effects are critical in understanding the observed I–V curves, except very near T_c. To see how this comes about, consider a cycle in which the current is increased from zero to $I_c(T_b)$, T_b being the bath temperature, above which a voltage develops. If the dissipated power IV related to this voltage produces enough Joule heat to raise the temperature of part of the sample from T_b to above T_c, it will become fully normal and stay normal even if the current is subsequently reduced below $I_c(T_b)$; thus hysteresis appears. Even lesser amounts of heating will obviously complicate the I–V curves by introducing temperature gradients. Now, since $I_c(T_b) \propto (T_c - T_b)^{3/2}$, I^2R heating will tend to increase as $(T_c - T_b)^3$, while the temperature rise needed to sustain the normal state will increase only as $(T_c - T_b)$. Thus, paradoxically, heating effects are more serious the *further below* T_c one is operating. For typical geometries, the crossover to heating domination occurs roughly 0.1° below T_c. Below this crossover, there will typically be a self-heated normal hotspot (Skocpol, Beasley, and Tinkham, 1974a,b) in the center of the bridge,

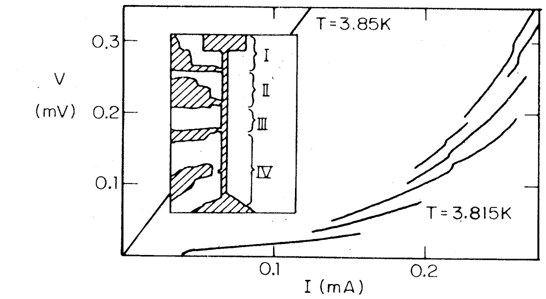

FIG. 8. Data of Skocpol on the I–V curve of a superconducting tin microbridge 4 μm $\times$ 140 μm in size. Voltage tabs shown in inset allow localized voltage drop measurements.

and superconducting coherence is lost if this expands beyond an effective coherence length of about 1 μm.

To obtain a regime in which heating effects are small enough to allow the quantum properties of the superconducting state to be observed relatively cleanly in our thin-film microbridges, one must operate within about 30 millidegrees of T_c, and at low voltages. In this regime, as one increases the current from zero, there is a step rise in voltage at I_c, to a value below that corresponding to the fully normal state. With further increase in current, the voltage rises linearly, with a differential resistance typically equal to that which would result if a length $\sim$10 μm of the bridge were normal. With still further increase of current, this step terminates with a jump to a second step, whose differential resistance is about twice that of the first. This process continues until heating gradually takes over, causing an upward curvature until the bridge becomes fully normal. On decreasing the current, a similar structure is seen, but the downward steps occur at lower currents, indicating some hysteresis. Some data illustrating these features are shown in Fig. 8. (In this figure, the full metastable range of each step has been traced out). By using the voltage tabs shown in the inset, Skocpol was able to show that the increment of voltage associated with each step was localized to just one segment of the microbridge. These curiously regular steps were first seen by Webb and Warburton in their experiments on fluctuation resistance in whiskers, and they have subsequently been reported by many other workers. The piece wise linearity of the steps is particularly evident in the extensive data of Meyer and v. Minnigerode [Meyer and Minnigerode (1972); Meyer (1973)] on pure tin whiskers. where heating effects are minimized by the good thermal conductivity of the pure tin, and because the sharp intrinsic transitions allow working very close to T_c.

The interpretation we have developed (Skocpol, Beasley, and Tinkham, 1974a,b,c) of these steps is that each one corresponds to the establishment of a localized "phase-slip center," depicted schematically in Fig. 9. In the "core" region of the center, the order parameter ψ executes a relaxation oscillation, the phase slipping by 2π in each cycle when $|\psi|$ drops to zero. Such an oscillatory cycle was proposed for short weak links by Notarys and Mercereau (1971), and discussed in more detail for that case by Rieger, Scalapino, and Mercereau (1971, 1972). The frequency of the oscillation is given by the Josephson relation $\nu = 2eV/h$, V being the voltage across a single center. This

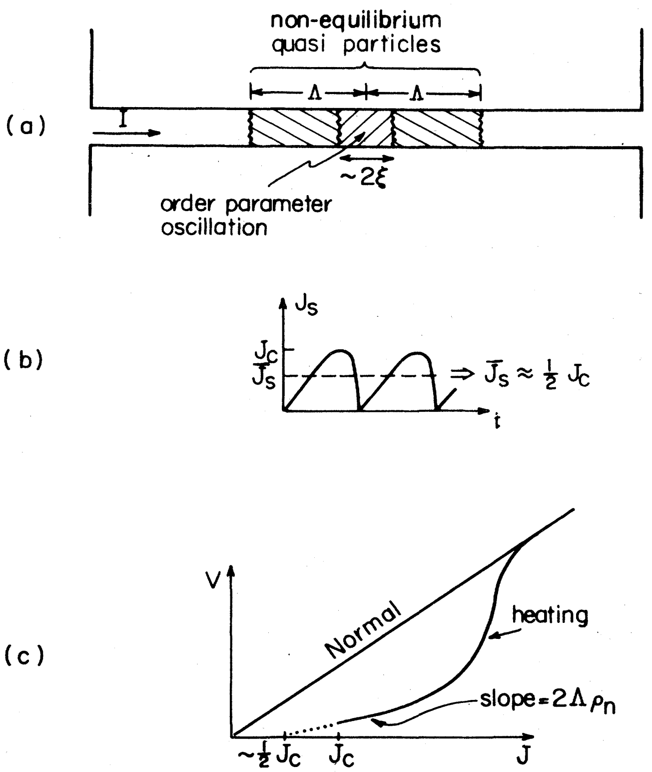

FIG. 9. (a) Schematic diagram of model of phase-slip center. The relaxation oscillation of the order parameter occurs in a core length $\sim 2\xi$, while the nonequilibrium quasiparticles diffuse a distance $\sim\Lambda$ in either direction. (b) The oscillatory supercurrent in the core region, with an average value of $\sim\frac{1}{2}J_c$. (c) Schematic I–V curve of a bridge (or segment thereof) containing only a single phase-slip center.

voltage, in turn, is governed in our model by

$$V \approx 2\rho\Lambda(J - \bar{J}_s), \tag{14}$$

where ρ is the normal resistivity, Λ is the quasiparticle diffusion length, and $\bar{J}_s \approx \frac{1}{2}J_c$ is the supercurrent density in the center averaged over the relaxation oscillation cycle. Qualitatively, this formula reflects the fact that the current density in excess of $\bar{J}_s$ must pass through the center as a normal current, and that this normal current can only convert back to supercurrent over a distance allowing pair–quasiparticle conversion by inelastic scattering processes. Based on the original one-dimensional discussion of Pippard, Shepherd, and Tindall (1971), we take this distance to be $\Lambda = (\frac{1}{3}ll_2)^{1/2}$, i.e., the distance quasiparticles diffuse by a random walk motion while traveling a total path length l_2 (the mean free path for *inelastic* scattering) with a mean free path l between scattering events. Thus, the differential resistance of the steps is determined by purely normal state properties, and should not depend on $T - T_c$, in agreement with the experimental results. Moreover, the time dependence of the model is confirmed by the experimental fact that small constant-voltage steps can be induced by the presence of microwave radiation, indicating synchronization of the relaxation oscillation of ψ with the microwave field.

Despite the success we have had with this model, it is clearly approximate and incomplete. For example, we plan to probe the relation of our model to conventional time-dependent GL theory, which has only been justified theoretically (Gor'kov and Eliashberg, 1968a,b; Eliashberg, 1968, 1969) in the restricted case of gapless superconductivity, and which is incapable of accounting for a temperature-independent length such as Λ. But for the present, many questions concerning time dependent, nonequilibrium superconductivity remain open, and the subjects of active research.

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