Hadrons and $SU(3)$: A critical review

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The spectroscopy of hadrons, both bosons and baryons, is critically examined with respect to the $SU(3)$ classification scheme. This is done with respect to the Gell-Mann/Okubo mass formula and the partial decay rates of member states of individual $SU(3)$ multiplet families. The agreement is exceedingly good. The baryons are catalogued into octets with $J^P = 1/2^+$, $5/2^+$, and $5/2^-$, nonets with $J^P = 3/2^-$, $7/2^-$, and $1/2^-$, and decimets with $J^P = 3/2^+$ and $7/2^+$ Among the bosons, nonet structure is well established for J^{PC} multiplets $= 0^{-+}$, 1^{--}, and 2^{++}, with less firm evidence existing for a host of other multiplets, $J^{PC} = 0^{++}$, 1^{+-}, 1^{-+}, 2^{--}, 2^{-+} and 3^{--}.

CONTENTS

I. INTRODUCTION

The first excited baryon state, the $\Delta(1238)$, was discovered by Fermi and co-workers in 1952 (Anderson, Fermi, Martin, and Nagle, 1953). In the intervening years the number of such boson and baryon resonances, now catalogued by the Particle Data Group (April, 1972), has risen to over one hundred. The $SU(3)$ classification proposed by Gell-Mann (1961) and Ne'eman (1961) for ordering these strongly interacting bosons and baryons has proved quite successful. Beyond the original $J^P = 1/2^+$ ground state [$N(940)$, $\Lambda(1115)$, $\Sigma(1190)$, and $\Xi(1320)$], the discovery of the $\eta(550)$ (Pevsner *et al.*, 1961) completed the first boson $SU(3)$ multiplet, namely that with $J^P = 0^-$. However, these early successes were somewhat negated by the complexity encountered in the $J^P = 1^-$ multiplet consisting of nine members, the $\rho(750)$, $K(890)$, $\omega(780)$, and $\Phi(1020)$ with its violation of the Gell-Mann/Okubo mass formula (Gell-Mann, 1961; Okubo, 1967), and the necessity of subsequently introducing additional parameters. The discovery of the $\Omega^-(1673)$ (Barnes *et al.*, 1964) hyperon with mass and strangeness coinciding with that expected for the tenth member of the $J^P = 3/2^+$ decimet supplied clear and unambiguous evidence for the $SU(3)$ classification scheme. Since then, several additional multiplets have been deciphered. It is the purpose of this paper to reexamine the success of $SU(3)$ as applied to the known spectra of particles (Goldberg *et al.*, 1966; Tripp *et al.*, 1967; Tripp, 1968; Levi-Setti, 1969; Plane *et al.*, 1970; Meshkov, 1970; and Samios, 1970). We propose to do this for both bosons and baryons with two main degrees of sensitivity: the first with respect to the Gell-Mann/Okubo mass formula (Gell-Mann, 1961; Okubo, 1962) and the second with respect to partial decay rates of the member states of each multiplet family. In the following section we describe the general approach adopted, as well as the assumptions behind the explicit expressions used for the $SU(3)$ formulation. This is followed in Secs. III–VI by an application of these formulae to the known boson and baryon spectra. In these same sections we examine the over-all success of $SU(3)$ in regard to all multiplets, considering the extent to which the theory is actually tested, as well as reviewing possible intermultiplet relationships in the light of $SU(6)$ and Regge relationships. Finally, Section VII contains a brief discussion of the conclusions.

II. GENERAL FORMALISM

A. Masses

The mass formulae for members of a baryon octet or decimet as derived by Gell-Mann (1961) and Okubo (1962), under the assumption that the symmetry-breaking term in the Hamiltonian transforms under $SU(3)$ as an octet, are noted below:

$$\text{Octet } (m_N + m_\Xi)/2 = (3m_\Lambda + m_\Sigma)/4 \qquad (1)$$

$$\text{Decimet } m_{\Omega^-} - m_{\Xi^-} = m_{\Xi^-} - m_{\Sigma^-} = m_{\Sigma^-} - m_{\Delta^-} \text{(equal spacing)} \qquad (2)$$

For bosons it has been suggested (Coleman and Schnit-

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zer, 1964) that the masses appearing in the above formulae should be replaced by their squares, i.e.,

$$(m_K^2 + m_{\bar{K}}^2)/2 = (3m_\eta^2 + m_\pi^2)/4. \tag{3}$$

In comparing the above relations to experimental data only members of a particular multiplet with the same charge are normally utilized, avoiding corrections due to electromagnetic effects.[1] In the case of low-mass resonances, where in general the spin parity of the individual states is well known, tests of the above formulae can often be performed in a meaningful manner. However, if two isosinglet particles with identical quantum numbers exist, a particular form of $SU(3)$ breaking, known as octet–singlet mixing (Gell-Mann, 1961; Sakurai, 1962), may occur, and the formulae (1) and (3) are considerably weakened. In this case, a mixing angle magnitude can be determined as

$$\sin^2\theta = (m_8^P - m_8^0)/(m_8^P - m_1^P) \text{ for baryons} \tag{4}$$

or

$$\sin^2\theta = [(m_8^P)^2 - (m_8^0)^2]/[(m_8^P)^2 - (m_1^P)^2] \text{ for mesons,} \tag{5}$$

where m_8^0 is the mass of the isosinglet–octet member derived from Eq. (1) or (3), m_8^P is the mass of the isosinglet particle closest in value to m_8^0, and m_1^P is the mass of the remaining isosinglet particle. Both isosinglet particles are then "mixtures" of $SU(3)$ octet and singlet states. The mass relation required for the possible formulation of such a "nonet" of particles now becomes a weak inequality: $0 < \sin^2\theta < 1$. Since mixing often occurs and since the spin and parity of some of the constituents of a proposed multiplet are not known, decay rate predictions are an important means of verifying the $SU(3)$ classification. In principle, more complicated mixing problems can occur, for instance between members of two octets. We have not included such possibilities in our subject discussion since the present data do not warrant such an extension.

B. Decay rates

In calculating partial decay rates we adopt the usual approach of writing the amplitude for a three-particle vertex as a product of a reduced coupling constant, assumed to be $SU(3)$ invariant, multiplied by an $SU(3)$ Clebsch–Gordan coefficient and, in the case of decays above threshold, a phase space factor and a spin dependent "barrier penetrationary" factor. We must point out that in doing so we are not strictly testing $SU(3)$ invariance but are also checking the validity of a whole set of additional assumptions that are not at all related to the symmetry group. These assumptions are necessary because of our inability to perform dynamical calculations in strong interaction processes. Of course, the *a posteriori* justification for the acceptance of such an approach is that the results obtained are in excellent agreement with experiment in all those cases where good experimental data are available.

A second comment is also in order at this point: We are assuming that the reduced coupling constant is an $SU(3)$ invariant (i.e., we are adopting the "unbroken $SU(3)$" approach), while we know from the large mass differences among members of a given supermultiplet that a considerable amount of symmetry breaking is at work. However, in calculating the kinematic factors we are using the physical particle masses, and this can lead us to hope that in this way most of the symmetry breaking will automatically be taken into account. Specifically, we take the partial width for a resonance decay $x \to y + z$ to be given by

$$\Gamma = \underbrace{|A_{yz}^x|^2}_{SU(3)\text{ factor}} \underbrace{(P/M)^{2l}}_{\text{barrier factor}} \underbrace{(P/m) \cdot M}_{\text{phase space factor}} \tag{6}$$

where A_{yz}^x is an amplitude containing the $SU(3)$-invariant amplitude, P is the decay momentum, and m is the mass of the decaying particle. For the barrier factor we have set the value of the interaction radius to zero, removing this quantity as a parameter (a more complete discussion of this point is given in Sec. III).[2] Here M is introduced as an arbitrary mass ($= 1\ \text{GeV}/c^2$) so that A_{yz}^x may be treated formally as a dimensionless quantity.

The structure of A_{yz}^x may be complicated by mixing, as well as by the presence of two octets in the $SU(3)$ Clebsch–Gordan series,

$$8 \otimes 8 = 1 \oplus 8^s \oplus 8^a \oplus 10 \oplus \overline{10} \oplus 27, \tag{7}$$

where 8^s and 8^a are symmetric and antisymmetric with respect to interchange of the two octets. If particle x is an isosinglet baryon nonet member of mass m_8^P, decaying to two baryons also belonging to octets, then $A_{yz}^{x=m_8^P}$ will contain three amplitudes (A_i) and three Clebsch–Gordan coefficients (C_i). This arises from (1) the coupling of the singlet part of x to the singlet state in the above expansion (C_1, A_1), (2) the coupling of the octet part of x to $8^s(C_s, A_s)$, and (3) the coupling of the octet part of x to $8^a(C_a, A_a)$. In this manner, we obtain amplitudes for all baryon nonet members of the form:

$$\begin{aligned} A_{yz}^{x=m_8^P} &= [-C_1 A_1 \sin\theta + (C_s A_s + C_a A_a)\cos\theta], \\ A_{yz}^{x=m_1^P} &= [+C_1 A_1 \cos\theta + (C_s A_s + C_a A_a)\sin\theta], \\ A_{yz}^{x=others} &= (C_s A_s + C_a A_a), \end{aligned} \tag{8}$$

where y and z belong to octets. Note that in general the form of the amplitude is given by $A_{yz}^x = \sum C_i A_i b_i$.

In the most complex case considered above, the nonet structure for baryons, there are four unknowns: the three $SU(3)$ invariants A_1, A_s, A_a and the mixing angle θ. For an unmixed octet there are two unknowns, A_s and A_a, and for a deciment only one unknown, the amplitude A_{10}. It has been found more convenient to use an alternative parameterization for octet decay rates. Instead of A_s and A_a, we define A_8 as proportional to the generally well

[1] It should be noted that in most cases the experimental uncertainties for masses of resonant states are larger than electromagnetic mass differences.

[2] Results are insensitive to the interaction radius using the indicated formalism, if the mass characterizing this radius was greater than $\sim 200\ \text{MeV}/c^2$.

TABLE I. $SU(3)$ isoscalar factors for the decay sequences octet → octet + octet and singlet → octet + octet.

$x \to yz$	Octet → octet + octet $A^x_{yz}(8 \to 8 \otimes 8)$	Singlet → octet + octet $A^x_{yz}(1 \to 8 \otimes 8)$
$N \to N\pi$	$\sqrt{3}\,A_8$	
$\to N\eta$	$[(4\alpha-1)/\sqrt{3}]A_8$	
$\to \Sigma K$	$\sqrt{3}\,(2\alpha-1)A_8$	
$\to \Lambda K$	$[(2\alpha+1)/\sqrt{3}]A_8$	
$\Lambda \to N\bar{K}$	$\sqrt{2/3}\,(1+2\alpha)A_8$	$1/2\,A_1$
$\to \Sigma\pi$	$2(\alpha-1)A_8$	$\sqrt{6/4}\,A_1$
$\to \Lambda\eta$	$(2/\sqrt{3})(\alpha-1)A_8$	$-(\sqrt{2}/4)A_1$
$\to \Xi K$	$\sqrt{2/3}\,(4\alpha-1)A_8$	$-1/2A_1$
$\Sigma \to \Sigma\pi$	$2\sqrt{2}\,\alpha A_8$	
$\to \Lambda\pi$	$(2/\sqrt{3})(1-\alpha)A_8$	
$\to N\bar{K}$	$\sqrt{2}\,(2\alpha-1)A_8$	
$\to \Sigma\eta$	$(2/\sqrt{3}\,(1-\alpha)A_8$	
$\to \Xi K$	$-\sqrt{2}\,A_8$	
$\Xi \to \Xi\pi$	$\sqrt{3}\,(2\alpha-1)A_8$	
$\to \Lambda\bar{K}$	$[(4\alpha-1)/\sqrt{3}]A_8$	
$\to \Sigma\bar{K}$	$\sqrt{3}\,A_8$	
$\to \Xi\eta$	$[(2\alpha+1)/\sqrt{3}]A_8$	
$\alpha = \frac{\sqrt{3}}{6}$	$A_8 = \frac{\sqrt{15}}{10}A_s + \frac{\sqrt{3}}{6}A_a$	

TABLE II. $SU(3)$ isoscalar factors for the decay sequences decimet → octet + octet, octet → decimet + π, decimet → octet + decimet.

$x \to yz$	Couplings Involving Decimets $A^x_{yz}(10 \to 8 \otimes 8)$
A. Decimet → Two Octets	
$\Delta \to N\pi$	$-(\sqrt{2}/2)A_{10}$
$\Delta \to \Sigma K$	$+(\sqrt{2}/2)A_{10}$
$\Sigma \to \Lambda\pi$	$-\frac{1}{2}A_{10}$
$\Sigma \to \Sigma\pi$	$(\sqrt{6}/6)A_{10}$
$\Sigma \to N\bar{K}$	$-(\sqrt{6}/6)A_{10}$
$\Sigma \to \Sigma\eta$	$+\frac{1}{2}A_{10}$
$\Xi \to \Xi\pi$	$\frac{1}{2}A_{10}$
$\Xi \to \Sigma\bar{K}$	$\frac{1}{2}A_{10}$
$\Xi \to \Lambda\bar{K}$	$-\frac{1}{2}A_{10}$
$\Xi \to \Xi\eta$	$+\frac{1}{2}A_{10}$
$\Omega^- \to \Xi\bar{K}$	$1A_{10}$
B. Octet → Decimet + π	
$N \to \Delta\pi$	$-2(\sqrt{5}/5)A_8'$
$\Sigma \to \Sigma\pi$	$-(\sqrt{30}/15)A_8'$
$\Lambda \to \Sigma\pi$	$-(\sqrt{15}/5)A_8'$
$\Xi \to \Xi\pi$	$-(\sqrt{5}/5)A_8'$
C. Decimet → Octet and Decimet	
$\Delta \to \Delta\pi$	$(\sqrt{10}/4)A_{10}'$
$\Delta \to \Delta\eta$	$+(\sqrt{2}/4)A_{10}'$
$\Delta \to \Sigma\bar{K}$	$-\frac{1}{2}A_{10}'$
$\Sigma \to \Sigma\pi$	$(\sqrt{3}/3)A_{10}'$
$\Sigma \to \Delta K$	$(\sqrt{3}/3)A_{10}'$
$\Xi \to \Xi\pi$	$(\sqrt{2}/4)A_{10}'$
$\Xi \to \Sigma\bar{K}$	$(\sqrt{2}/2)A_{10}'$
$\Omega \to \Xi\bar{K}$	$(\sqrt{2}/2)A_{10}'$

measured $N \to N\pi$ rate and related to the pion nucleon coupling constant for the $1/2^+$ octet, and we define α as proportional to the generally well measured $\Sigma \to \Sigma\pi$ rate and related to the ratio between antisymmetric (F) and symmetric (D) amplitudes. This choice yields parameters that are less correlated and that are related directly to independent experimental measurements. In this case

$$A_8 = [(15)^{1/2}/10]A_s + [(3)^{1/2}/6]A_a;\ \alpha = [(3)^{1/2}/6](A_a/A_8). \tag{9}$$

The $SU(3)$ matrix elements for octet and singlet baryon decays expressed in terms of these parameters are shown in Table I. They are all proportional to A_8 or A_1 as expected but, more important, one notes the sensitive dependence of the branching fractions on α. In Table II we display the $SU(3)$ coefficients for a baryon decimet.[3]

The situation for boson nonets is less complicated. This is due to the fact that C invariance restricts the Yukawa couplings among octets to either the symmetric (D) or antisymmetric (F) octet representation, but not both (Lipkin, 1963). The particular coupling that exists for each case will depend on the spin parity of the particles involved. A useful general rule[4] for determining whether a given decay sequence proceeds via symmetric or antisymmetric couplings is to determine the product of the C's (charge conjugation quantum numbers) of the constituent particles x, y, z. If this product of the C's is equal to (-1), then the coupling is antisymmetric (F), if $(+1)$ then it is symmetric (D). A further simplification occurs for meson nonets decaying via antisymmetric coupling where C invariance further restricts A_1 to zero. For nonet mesons decaying to octets, the amplitudes are then of the form

$$\begin{aligned} A_{yz}^{x=m_1^P} &= -C_1A_1\sin\theta + C_sA_s\cos\theta, \\ A_{yz}^{x=m_1^P} &= +C_1A_1\cos\theta + C_sA_s\sin\theta, \\ A_{yz}^{x=others} &= C_sA_s \end{aligned} \tag{10}$$

[3] Unfortunately, various conventions have been used by different authors. We use the Clebsch–Gordan coefficients of the Particle Data Group, April, 1972, adapted from deSwart (1963). Goldberg *et al.* (1966), Kernan and Smart (1966), and deSwart (1963) originally used a different convention.

Our definition of α is the same as that of deSwart, Goldberg, and Kernan and Smart. The "α" of Tripp, (1968); Levi-Setti (1969), and Plane *et al.* (1970) is $1-\alpha$ as used by deSwart, Kernan and Smart, Goldberg, and this work. In the notation of Gell-Mann, our α is $F/(D+F)$. The mixing angle as defined here is the negative of that defined by Tripp, Levi-Setti and Plane. The many combinations of different conventions that can be used may present some difficulty to those examining previous works, but at least the important parameters α and θ are simply related.

[4] We thank Prof. K C. Wali for pointing this out.

for the symmetric (D) couplings, and

$$\begin{aligned} A_{yz}^{x=m^{P8}} &= C_a A_a \cos\theta, \\ A_{yz}^{x=m_1^P} &= C_a A_a \sin\theta, \\ A_{yz}^{x=\text{others}} &= C_a A_a \end{aligned} \tag{11}$$

for the antisymmetric (F) couplings. As a result, the number of unknowns is three (A_1, A_s and θ) for the symmetric case and two (A_a, θ) in the antisymmetric case for the bosons.

C. Comparison with data

In the following sections we shall attempt to group existing baryon and meson states into $SU(3)$ multiplets, test the mass formula predictions where possible (or determine the expected Ξ particle masses where appropriate), and predict decay rates in terms of the amplitudes previously discussed. To determine these amplitudes, we minimize the χ^2 function

$$\chi^2 = \sum \chi_i^2;\ \chi_i = \frac{\Gamma^i_{SU(3)} - \Gamma^i_{\text{exptl}}}{\Delta\Gamma^i_{\text{exptl}}}. \tag{12}$$

For the ith term in Eq. (12), $\Gamma^i_{SU(3)}$ is given by (6), and Γ^i_{exptl}, $\Delta\Gamma^i_{\text{exptl}}$ are the experimentally determined partial widths and errors, respectively. The sum is taken over all partial decay rates of a given multiplet for which there is reliable quantitative information. Once the parameters have been determined, predictions can be made for decay rates which have not yet been measured and for masses of as yet unclassified Ξ states. Note that in multiplets where mixing is involved, this procedure determines the sign of the mixing angle.[5]

The closeness of fit for each particular multiplet can also be graphically displayed. The contribution (χ_i) of each partial decay rate to the χ^2 defined above can be plotted as a function of the variables involved, the A_i, θ, and α.

On such a plot a perfect fit would be represented by the curves for each χ_i intersecting at $\chi_i = 0$. In reality, the vertical displacement of each curve from the intersecting region indicates the number of standard deviations by which the rate deviates from a perfect fit; and the relative slopes of the curves near the intersecting region is a measure of the relative sensitivity of the parameter determination for each rate. The more horizontal the curve, the less the parameter is determined by the rate measurement; halving the error in the rate measurement would double the slope of the corresponding curve. As will be seen, the curves one obtains are rather smooth except in the case of the mixing angle θ. The distribution in this latter case is vastly different, exhibiting a large variation, and defining the mixing angle rather precisely from the decay rate information. In a baryon nonet this quantity may then be used either to check the Gell-Mann/Okubo mass formula if all the constituent states are known or to predict the mass of one missing state, usually the Ξ, utilizing Formulae (1) and (4).

[5] Since decay rates are quadratic in the $SU(3)$ amplitude, only the sign of $A_1 A_8 \sin 2\theta$ can be determined. We evaluate the sign of the mixing angle with the convention that A_1 and A_8 are positive.

D. Additional techniques

For completeness, we note here that for baryon resonances analyzed in s-channel formation experiments, an alternative method for testing $SU(3)$ decay rate predictions has been used.[6] To illustrate this technique, consider the reaction $y' + z' \to x \to y + z$, where x is an s-channel resonance, and y, y' belong to the same $SU(3)$ multiplet, as do z, z'. The directly measured experimental quantity in this type of experiment is not the decay rate Γ^x_{yz} but the amplitude at resonance

$$T^x_{y'z',yz} = (\Gamma^x_{yz}\Gamma^{x'}_{y'z'})^{1/2}/\Gamma. \tag{13}$$

For a set of s-channel resonances in the same multiplet, these resonant amplitudes may then be parameterized in the terms of $SU(3)$ invariants by Eqs. (6), (8), and (9). By this procedure, the various products of decay rates are subject to $SU(3)$ tests, rather than the decay rates themselves, the advantage being that the former quantities are closer to directly measured quantities than the latter, in formation experiments. However, with the current state of the data, differences in technique of $SU(3)$ comparisons are minor. For the purposes of this review, we prefer decay rate fits because of the universal applicability, especially in the case of bosons where s-channel measurements are not possible.

In formation experiments, it is often possible to extract an additional parameter relevant to $SU(3)$ tests via interference effects, as was first done by Kernan and Smart (1966). Various partial wave analyses, utilizing polarization as well as angular distribution information, can often determine the relative sign of $T^x_{y'z',yz}$ and $T^{x'}_{y'z',yz}$ for adjacent resonances (x, x'). Consider the case that (x, x') is a Σ (octet, decuplet) resonance, $\bar{K}N \to \Sigma \to \pi\Lambda$. From Tables I and II we see that

$$\begin{aligned} T^{\Sigma_8}_{\bar{K}N,\Lambda\pi} &\approx A^{\Sigma_8}_{\bar{K}N} A^{\Sigma_8}_{\Lambda\pi} = \sqrt{2}(2\alpha - 1)\frac{2}{\sqrt{3}}(1-\alpha)|A_8|^2 \\ T^{\Sigma_{10}}_{\bar{K}N,\Lambda\pi} &\approx A^{\Sigma_{10}}_{\bar{K}N} A^{\Sigma_{10}}_{\Lambda\pi} = \frac{\sqrt{6}}{6}\frac{1}{2}|A|^2. \end{aligned} \tag{14}$$

In this case the relative signs of $T^{\Sigma=\Sigma_8}$ and $T^{\Sigma=\Sigma_{10}}$ will obviously depend on α. An experimental determination of these signs will restrict α to a range of values, to be compared for consistency with the value of α obtained from decay rate fits. Note also that if $T^{\Sigma=\Sigma_8}$ and $T^{\Sigma=\Sigma_{10}}$ are opposite in sign, then both cannot be decimet members. In the case of a Λ resonance, a similar analysis can also determine if it is primarily $SU(3)$ singlet or octet.

An example of a potential source of trouble is the case where α in the above reaction has a value close to 0.5. As such, the sign of $A^{\Sigma_8}_{\bar{K}N} A^{\Sigma_8}_{\Lambda\pi}$ is unstable with respect to small changes in the values of the $(2\alpha - 1)$ term. This can arise either from the difficulty of measuring α with sufficient precision, or it can be due to the symmetry breaking which is known to exist. In essence this formalism is not relevant in these instances and in fact should not be applied. Detailed discussions of the utility of this $SU(3)$ interference phenomenon to particle resonances will be presented in the section dealing with the appropriate multiplet.

[6] See for instance Plane *et al.* (1970), and R. Levi-Setti (1969) for recent reviews.

III. DATA—BARYONS

We now discuss each $SU(3)$ multiplet in turn making comparisons between predictions and experimental observations as outlined above. The ground state ($J^P = 1/2^+$) baryon octet is dealt with first.

A. $J^P = 1/2^+$

In Table III are shown the masses of the neutral members of this octet, as well as a comparison with the Gell-Mann/Okubo mass formula. The agreement is good to 0.7%, with the quantum numbers of all the member states being well known except for the parity of the Ξ which has yet to be reliably measured.

The electromagnetic mass splittings can also be predicted via $SU(3)$ as has been shown by Coleman and Glashow (1961), yielding the relation

$$m(\Xi^-) - m(\Xi^0) = m(\Sigma^-) + m(\Sigma^+) + m(P) - m(n). \tag{15}$$

If one uses the latest tabulated values (Particle Data Group, 1972), one obtains

$$\begin{aligned} 6.6 \pm 0.7 \text{ MeV} &= (7.95 \pm 0.12) - 1.29 \text{ MeV} \\ &= 6.6 \pm 0.12 \text{ MeV} \end{aligned} \tag{16}$$

The greatest uncertainty arises, not surprisingly, from the $m(\Xi^-) - m(\Xi^0)$ mass difference, but the test is clearly significant and the agreement impressive.

The experimental situation with respect to the magnetic moments of the members of this octet is not so clear. The pertinent $SU(3)$ predictions by Coleman and Glashow (1961) are[7]

$$\begin{aligned} \mu(\Lambda) &= \frac{1}{2}\mu(n) \equiv -0.96 \text{ nuclear magnetons,} \\ \mu(\Sigma^+) &= \mu(P) \equiv 2.79 \text{ nuclear magnetons,} \end{aligned} \tag{17}$$

and

$$\mu(\Xi^-) = -[\mu(P) + \mu(n)] \equiv -0.88 \text{ nuclear magnetons.}$$

Such experiments are rather difficult to perform since they require both a high magnetic field and a large number of events. Most endeavors to date have been able to amass large number of events at low fields (20 G), or few events at high fields. This is certainly true in the Σ^+ case where the compiled average of six such experiments (Particle Data Group, 1972) yields $\mu(\Sigma^+) = 2.59 \pm 0.46$ nuclear magnetons, in fine agreement with that expected for $SU(3)$. On the other hand, progress in the measurement of the Λ^0 magnetic moment has been such as to yield only two good measurements, namely: $\mu(\Lambda^0) = -0.73 \pm 0.18$ (Hill *et al.*, (1971), and $\mu(\Lambda^0) = -0.73 \pm 0.07$ nuclear magnetons, (Dahl-Jensen, 1971), to be contrasted with previous published values with errors of $\pm$ 0.5. The difference of 0.23 between experiments (−0.73) and $SU(3)$ (−0.96) is of the order of several standard deviations, in reasonable agreement considering the difficulty of the experiments and the accompanying systematic errors. More recently two measurements of the (Ξ^-) magnetic moment have also been made, although possessing relatively large errors. The values obtained were $\mu(\Xi^-) = -0.1 \pm 2.1$ (Bingham *et al.*, 1970) and $\mu(\Xi^-) = -2.2 \pm 0.8$ (Cool *et al.*, 1972). Combining these numbers gives $\mu(\Xi^-) = -1.0 \pm 0.8$, which essentially indicates a negative value for the sign of the cascade magnetic moment and deviates by 1.2σ from the expected $SU(3)$ value. It should be emphasized that mass differences within members of this multiplet are not taken into account in these relations among electromagnetic properties, so that difficulties similar to that of extracting invariant decay amplitudes exist. The discrepancy is increased for the Λ^0 magnetic moment and decreased for the Ξ magnetic moment, if one utilizes the physical masses and measures the magnetic moment in Λ^0 and Ξ magnetons, respectively. However, since errors in many magnetic moment measurements are comparable to the range of mass differences (~30%), it is not profitable to investigate these corrections at this time.

[7] The additional $SU(3)$ predictions for particle magnetic moments are $\mu(n) = \mu(\Xi^0)$ and $\mu(n) = -2\mu(\Sigma^0)$, which are not readily testable.

TABLE III. J^P $1/2^+$ octet. Masses and coupling constants.

Mass (MeV)[a]			
n	939.6		
Λ^0	1115.4 ± 0.05	$(m_n + m_\Xi)/2$	$(3m_\Lambda + m_\Sigma)/4$
Σ^0	1192.5 ± 0.1		
Ξ^0	1314.7 ± 0.7	1127.2	1134.7
$g^2_{NN\pi}/4\pi$	14.5 ± 0.4		
$g^2_{\Lambda K^- p}/4\pi$	3—15		
$g^2_{\Sigma K^- p}/4\pi$	≤ 2		
α	0.3—0.4		

[a] Particle Data Group, April 1972.

Forward dispersion relations have played an important role in determining the couplings of the members of this multiplet. The determination of the pion–nucleon coupling constant has yielded to this technique, with the current value $g^2_{NN\pi}/4\pi = 14.5$ known to within four percent (Hamilton and Woolcock, 1963; Samaramyake and Woolcock, 1965). Determinations of $\Lambda\bar{K}N$ and $\Sigma\bar{K}N$ couplings have proven much more difficult, due primarily to the existence of the complex unphysical region between the $\Lambda\pi$ and $\bar{K}N$ thresholds in $\bar{K}N$ scattering. All analyses agree that a large contribution to the above coupling constant determinations comes from this unphysical region.

In more recent analyses of low-energy $\bar{K}N$ scattering, the K-matrix effective range parameterization of Ross and Shaw (1960) has replaced the zero effective range and technique of Dalitz and Tuan (1960). There remains, however, a great deal of arbitrariness in determining the low-energy $\bar{K}N$ scattering parameters. Substantial variations exist in the choice of energy range, the number of partial waves, and the treatment of each partial wave. Thus, for example, the analysis of Kim (1967) involves 44 parameters, while that of Martin and Sakitt (1969) uses a nine-parameter formalism. Queen, Restignoli, and Violini (1969) have carried out a detailed examination of the problems involved in such coupling constant analyses. In addition to these uncertainties, we should take note of possible symmetry breaking effects due to the $K - \pi$ mass

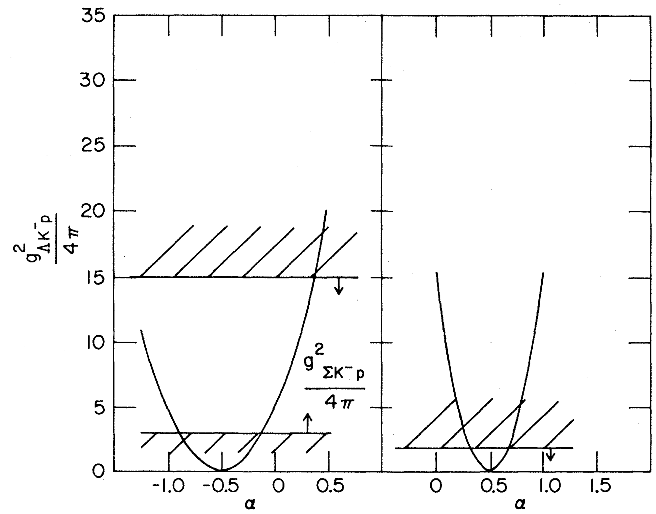


FIG. 1. Variation of $\Lambda K^- P$, and $\Sigma K^- P$ coupling constants with α.

differences which are unknown but can render all results invalid. Further, the Serpukhov data of Allaby *et al.* (1969) already indicate that the Regge-pole parametrization used to evaluate the dispersion-relation integral in the asymptotic region is inadequate. The effects of this in the coupling constant determinations are difficult to ascertain. As a first test of $SU(3)$, then, we tentatively accept the results of the analysis of Martin and Sakitt, who have studied the systematic errors inherent in their and other analyses and find $3 \lesssim g^2_{\Lambda\bar{K}N}/4\pi \lesssim 15$, $g^2_{\Sigma\bar{K}N}/4\pi \lesssim 2$. We note that virtually all results derived from forward dispersion relations (Queen, Restignoli, and Violini, 1969), in addition to other recent results from backward scattering (Martin and Michael, 1970; Hoogland *et al.*, 1970), fall within this range.

To compare these values with $SU(3)$ predictions we use appropriate amplitudes[8] of Table I to obtain

$$\begin{aligned} g^2_{\Lambda\bar{K}p}/4\pi &= [(1+2\alpha)^2/3]g^2_{NN\pi}/4\pi, \\ g^2_{\Sigma\bar{K}p}/4\pi &= (1-2\alpha)^2 g^2_{NN\pi}/4\pi. \end{aligned} \tag{18}$$

In Fig. 1 we plot each of these coupling constants as a function of α, using the known value of $g^2_{NN\pi}/4\pi = 14.5 \pm 0.4$. Applying the limits of Martin and Sakitt on $g^2_{\Lambda\bar{K}N}$ we find $-0.1 < \alpha < 0.4$ or $-1.4 < \alpha < -0.9$. The $g^2_{\Sigma\bar{K}N}$ limits yield $0.3 < \alpha < 0.7$. With these data, therefore, the only check on $SU(3)$ for this octet is that the measured value of both coupling constants is consistent with α lying within the range $0.3 \lesssim \alpha \lesssim 0.4$. Other coupling constants can, in principle, check this value, but their determination involves uncertainties at least as large as those described above. Analyses of η production by pions near threshold (Altarelli *et al.*, 1965; Deans and Holladay, 1968) agree that $g^2_{NN\eta} \lesssim 0.002$. Since from Table I we find

$$\alpha = \frac{1}{4}[1 \pm (3g^2_{NN\eta}/g^2_{NN\pi})], \tag{19}$$

these results strongly favor $\alpha = 0.25$. Investigation of backward η production from $\pi^- p$ interactions provides another means of measuring $g^2_{NN\eta}$. In particular, two groups have compared such η and η^0 production with different incoming π^- energy regions.[9] In order to extract the appropriate coupling constants the additional assumption has to be made that only the N_α (the nucleon) trajectory is exchanged. One then obtains $g^2_{NN\eta}/g^2_{NN\pi} = 0.18 \pm 0.06$, 0.45 ± 0.11, yielding $\alpha = 0.43 \pm 0.03$, 0.54 ± 0.03, respectively. Again the values of α fluctuate much more than the statistical errors, but hovering in the same region as derived earlier. It should be noted that this range of α also gives $SU(3)$ predicted values of $7 \lesssim g^2_{\Sigma\Lambda\pi}/4\pi \lesssim 10$ and $6 \lesssim g^2_{\Sigma\Sigma\pi}/4\pi \lesssim 9$. This can be made consistent with the dispersion-relation results of Chan and Smalley (1970) who find 20.9 ± 6.7 and 11.4 ± 5.5 for these quantities, respectively, if we double their purely statistical (essentially Kim's) errors. Considering the assumptions inherent in their calculations, increasing their errors may not be unwarranted.

Thus the coupling constant results for the $J^P = 1/2^+$ octet do little to confirm or reject the $SU(3)$ picture. In addition, the systematic and theoretical uncertainties inherent in the evaluation of these constants will make it difficult to test $SU(3)$ with greater precision or reliability in the near future.

B. $J^P = 3/2^-$

The constituent members of this proposed $SU(3)$ multiplet are noted in Table IV. The spin parity and decay branching fraction of the $N(1520)$, $\Lambda(1518)$, and $\Lambda(1690)$ are well established. The existence of a $\Sigma(1660)$ resonance with this spin parity $3/2^-$ has been known for some time. However, there is now clear evidence, from production experiments, for two Σ resonances with similar masses and widths (Eberhard *et al.*, 1969; Aguilar-Benitez *et al.*, 1970). For the analysis in this section we have utilized the properties of $\Sigma(1660)$ as deduced from the formation experiments where of course the spin parity is determined from partial wave analysis.

The question of agreement with the Gell-Mann/Okubo mass formula in this multiplet—as in most nonets—is ambiguous for two reasons. First, there are two Λ states with the same spin parity $3/2^-$, with the resulting singlet–octet mixing. Secondly, although there is substantial evidence for the existence of several high mass Ξ states, their spin parity is unknown, rendering their assignment to particular multiplets difficult. [In fact, the only Ξ states with known spin parity are the $J^P = 1/2^+ \Xi(1320)$ and $J^P = 3/2^+ \Xi(1530)$.] We therefore adopt the following approach in order to analyze this and similar muliplets. The decay rates of the Λ members of the nonet are used to determine the mixing angle. A knowledge of the N and Σ masses combined with mass formulae (1) and (4) then serves to predict the expected Ξ mass. If there is experimental evidence for such a particle, then it is assigned to this multiplet, and a quantitative fit is made to all the available decay rate information. In the present case, for example, a cascade with mass 1820 MeV is predicted with rather substantial experimental evidence (Smith, 1965; Badier, 1965; Alitti, 1969) for the existence of such a state

[8] Using the coupling constants defined by deSwart (1963), we have, from Table I, $|A_8|^2 = g^2_{NN\pi}$; $|A^{\Sigma}_{\bar{K}N}|^2 = 2g^2_{\Sigma\bar{K}N}$; $|A^{\Lambda}_{\bar{K}N}|^2 = 2g^2_{\Lambda\bar{K}N}$; $|A^{\Sigma}_{\Lambda\pi}|^2 = g^2_{\Sigma\Lambda\pi}$; $|A^{\Sigma}_{\Sigma\pi}|^2 = 2g^2_{\Sigma\Sigma\pi}$; $|A^{N}_{N\eta}|^2 = g^2_{NN\eta}$.

[9] At higher energies a cruder backward scattering analysis by Chase *et al.* (1969) suggests $g^2_{NN\eta}/g^2_{NN\pi} = 0.18 \pm 0.06$;

with predominant $\Lambda\bar{K}$ and $\Sigma\bar{K}$ modes. It should be kept in mind, however, that the recent work of the BMST collaboration (Hemingway *et al.*, 1970), indicates that the situation may well be more complex, in that this mass region may contain more than one resonance.

We now turn to the $SU(3)$ rate predictions. The most reliably measured rates are those of the $N(1520)$, $\Lambda(1518)$, and $\Lambda(1690)$ involving five reactions. The discovery of a second $\Sigma(1660)$ (Eberhard *et al.*, 1969; Aguilar-Benitez *et al.*, 1970) has confused a previously clear situation. Since the spin parity is automatically determined in formation experiments, the three pertinent experimental rates utilized are derived from such experiments. As noted earlier, the information concerning the Ξ state is less reliable; consequently these decay rates are not included in the fit except in a qualitative manner in that the $\Lambda\bar{K}$ and $\Sigma\bar{K}$ modes should be larger than the $\Xi\pi$ decay mode. We

TABLE IV. J^P $3/2^-$ nonet. Tabulation of masses, total widths, and experimental and $SU(3)$ predicted partial decay widths for constituent members of the multiplet.

$J^P = 3/2^-$

$\underline{8\otimes 8}$: $\chi^2 = 0.02$; $NC = 1$; $|A_8| = 41.1 \pm 1.6$

$\alpha = 0.72 \pm 0.15$; $|A_1| = 177.8 \pm 5.5$; $\theta° = 25.9° \pm 3°$

$\underline{10\otimes 8}$: $\chi^2 = 36$; $NC = 1$; $|A'_{10}| = 3.6 \pm 1.3$

M_0, Γ_{Tot} (MeV)	Decay mode $8\otimes 8$	$10\otimes 8$	Exptl Γ (MeV)	SU_3 pred Γ (MeV)
$N(1520\pm 9)$[a]	$N(1520)\to N\pi$		65 ± 8[a]	64.3
(121 ± 12)	$\to N\pi$		Observed[c]	0.1
		$\to \Delta\pi$		1.5
$\Lambda(1690\pm 3)$[c]	$\Lambda_8(1690)\to N\bar{K}$		11 ± 3.2[d]	10.8
(55 ± 15)	$\to \Sigma\pi$		32 ± 12[e]	34.3
	$\to \Lambda\eta$			0.0
		$\to \Sigma^*\pi$	1.0 ± 1.0[f]	1.0
$\Lambda(1517.8\pm 1)$[a]	$\Lambda_1(1518)\to N\bar{K}$		7.1 ± 0.6[a]	6.9
(15.5 ± 1.3)	$\to \Sigma\pi$		6.3 ± 0.6[a]	6.4
		$\to \Sigma^*\pi$	0.6 ± 0.1[g]	~ 0
$\Sigma(1671\pm 7.3)$[h]	$\Sigma(1670)\to N\bar{K}$		3.8 ± 1.2[d]	2.1
(51 ± 2.6)	$\to \Sigma\pi$		27 ± 5.8[e]	29.4
	$\to \Lambda\pi$		10 ± 6.3[i]	2.7
		$\to \Sigma^*\pi$	≤ 8.6[j]	0.2
$\Xi(1820\pm 10)$[k]	$\Xi(1820)\to \Xi\pi$		...	3.0
(35 ± 10)	$\to \Sigma\bar{K}$		...	8.7
	$\to \Lambda\bar{K}$		...	8.0
		$\to \Sigma^*\eta$		0.3

[a] Particle Data Group, April, 1972.
[b] Davies, 1967.
[c] Berley *et al.*, 1969; Armenteros *et al.*, 1969; Conforto *et al.*, 1971; Kim, 1970, 1971.
[d] Armenteros *et al.*, 1969; Kim, 1970, 1971.
[e] Berley *et al.*, 1969; Armenteros *et al.*, 1969; Kim, 1970, 1971; Barbaro-Galtieri, 1970.
[f] Prevost *et al.*, 1971.
[g] Burkhardt *et al.*, 1971; Shu-bon Chan *et al.*, 1972.
[h] Berley *et al.*, 1969; Armenteros *et al.*, 1968–1970; Kim, 1970–1971; Barbaro-Galtieri, 1970; Budgen *et al.*, 1971; Brucker *et al.*, 1970.
[i] Kim, 1970–1971; Barbaro-Galtieri, 1970; Budgen, 1971; Armenteros *et al.*, 1970.
[j] Sims *et al.*, 1968.
[k] Smith, 1965; Badier, 1965; Alitti, 1969.

have proceeded to fit these data by varying the four parameters, α, θ, A_1, and A_8, in order to minimize the χ^2 of Eq. (12). A plot of the variations of the contributions to the χ^2 of each of these variables is shown in Fig. 2. Most of the dependences are smooth except for the dramatic changes in the mixing angle θ. In this instance the $N\bar{K}$ decay of both Λ_1 and Λ_8 restricts θ to $24 \pm 3°$. The χ^2 probability for the fit is 65% with $\alpha = 0.72$. We see the results of this fit in Table IV where the $\Sigma(1660)$ rates are reproduced remarkably well and the existence of a Ξ with $J^P = 3/2^-$ and with the above-mentioned $\Xi(1820)$ pattern of predominant $\Lambda\bar{K}$, $\Sigma\bar{K}$ decays is also predicted. The decay rates of $\Lambda(1520)$, approximately equal for $\Sigma\pi$ and $N\bar{K}$, are in rough agreement with that expected for singlet coupling alone, while the preponderance of the $\Sigma\pi$ over the $N\bar{K}$ rate for the $\Lambda(1690)$ would be reversed in the absence of singlet–octet mixing. Note that the fit selects a positive mixing angle with our conventions. A significant $\Xi(1820) \to \Sigma\bar{K}$ is required independent of the value of α and has been observed as noted above. One could also have included the $\Sigma(1660)$ rates in the fit, thereby increasing the number of pieces of data by three. This was also done, the χ^2 probability being 47% in this case and yielding essentially the same values of the parameters and the Σ and Ξ predictions as before.

The *s*-channel interference technique provides supporting evidence for both the assignment of the $\Lambda(1520)$ as mainly a singlet and for the above fitted value of α. The $\Lambda(1520)$ assignment can be made by examining interference between

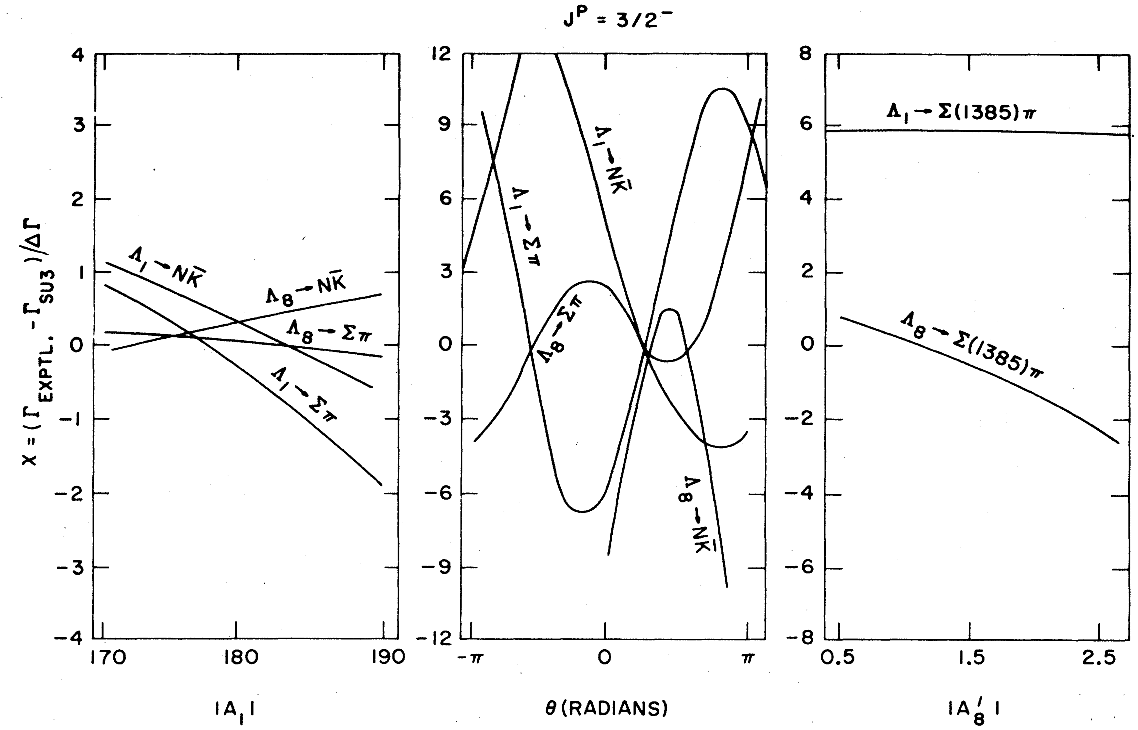
$$\bar{K} + N \to \Lambda(1520)$$
$$\Sigma\pi$$

and

$$\bar{K} + N \to \Sigma(1385)$$
$$\Sigma\pi \qquad (20)$$

since the $\Sigma(1385)$ is known to be a member of the $3/2^+$ decimet (see discussion of $3/2^+$ decimet). If the $\Lambda(1520)$ is mainly a singlet, the sign of the amplitude $T_{\bar{K}N,\Sigma\pi}^{\Lambda(1520)}$ should be opposite to $T_{\bar{K}N,\Sigma\pi}^{\Sigma(1385)}$ (see Tables I and II) *a fortiori*, since the octet coupling to $\Sigma\pi$ for the $\Lambda(1520)$ is predicted by this value of α to be small. On the other hand, the same arguments indicate that, for the $\Lambda(1690)$, the corresponding amplitude should have the same sign as for the $\Sigma(1385)$. This analysis has been carried out (Tripp *et al.*, 1968), with the results exactly as predicted above. If we assume that the $\Sigma(2030)$ is a member of a decimet,[10] a similar analysis can be performed by examining the $\Sigma(2030)$, $\Sigma(1660)$ relative signs. For the $\bar{K}N \to \Sigma\pi$ channel, the value of α derived from amplitude analysis predicts that the corresponding $\Sigma(1660)$ and $\Sigma(2030)$ amplitudes should have opposite signs, while for the $\Lambda\pi$ final states the signs should be the same. Again, the results of a partial wave analysis (Smart *et al.*, 1966) are consistent with this expectation. Such results are graphically displayed in Fig. 3 where the predicted and experimental signs of amplitudes are noted by arrows.

[10] An analysis of a possible $J^P = 7/2^+$ decimet discusses this possibility (see Section G) and the experimental consequences of such an assignment.



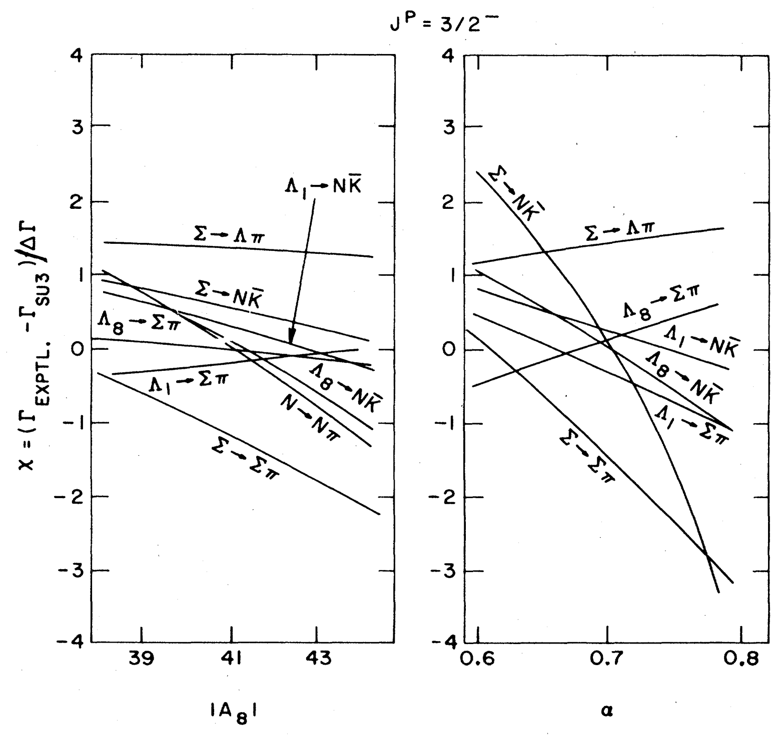


FIG. 2. $J^P = 3/2^-$ nonet; contribution of $|A_1|$, $|A_8|$, α, θ and $|A_8'|$ to χ.

Additional possible information pertaining to this $3/2^-$ nonet arises from measurement of the decay rates for $\Lambda(1520) \to \Sigma(1385)\pi$ and $\Lambda(1690) \to \Sigma(1385)\pi$. The value for the former mode is rather well known, 1.66 $\pm$ 2.4 MeV (Burkhardt *et al.*, 1971; Chan *et al.*, 1972; Mast *et al.*, 1972 a,b) and more recently a value of $1^{+2.4}_{-0.9}$ MeV has been determined for the latter (Prevost *et al.*, 1971).

Since these decays involve an $8 \to 10 \times 8$ coupling, there is only one unknown amplitude, and the rate formula yields the following relationship for these decays (using a mixing angle of 24°):

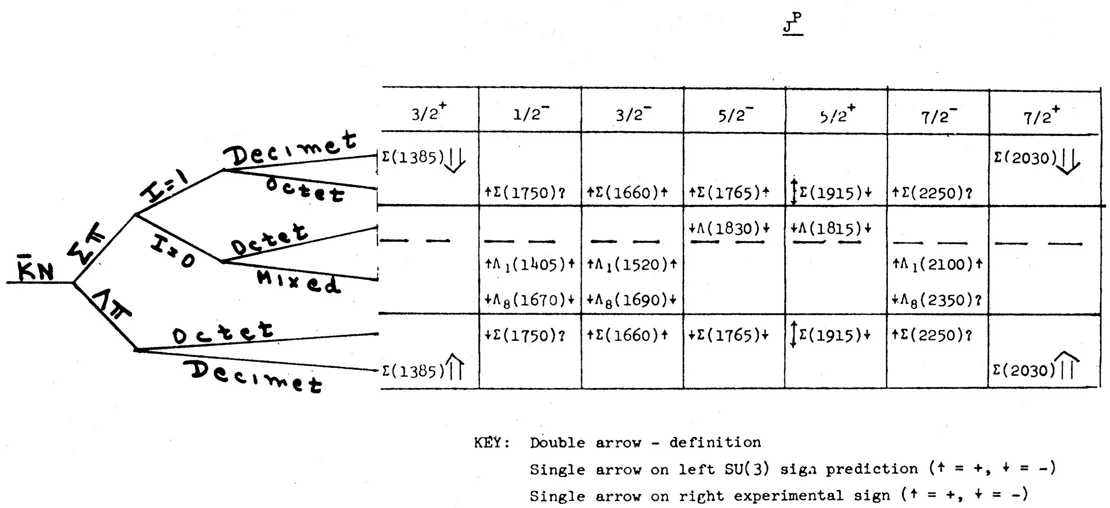


FIG. 3. Comparison of $SU(3)$ and s-channel interference determination of the relative phases of numerous resonant states.

$$\Gamma_{1520}/\Gamma_{1690} = 0.24[P_{1520}/P_{1690}]3, \qquad (21)$$

where the P's are the respective center-of-mass momenta. Clearly $P_{1520} < P_{1690}$ so that one predicts $\Gamma_{1520} \ll \Gamma_{1690}$ for the decay into $\Sigma(1385)\pi$. The measured rates seem to be comparable, but the errors are large, thereby implying an $SU(3)$ violation. We suggest that this formalism is not applicable in this case since $\Lambda(1520)$ decay is barely energetically possible. As a result one is extremely sensitive to symmetry breaking, choice of barrier factors, and other effects such as width of states, which in this approach are essentially ignored, their only cognizance being in using the physical masses. Indeed the main information from the observation of these decays is that there must be singlet mixing since a unitary singlet is forbidden to decay into a member of a decimet and octet.

Current values for the decay rates for the $3/2^-$ nonet are in remarkably good agreement with the $SU(3)$ prediction, with a rather large sensitivity. Needless to say, it is of great interest to obtain accurate $\Xi(1820)$ branching ratios as well as to solve the more difficult problem of measuring its spin parity.

C. $J^P = 3/2^+$

The resonances comprising this decimet are the $\Delta(1238)$, $\Sigma(1385)$, $\Xi(1530)$, and $\Omega(1673)$, the quantum numbers of which are well established except for the J^P of the Ω^-. For a comparison with the equal mass spacing rule, expected for a decimet, we have first used the negatively charged member of each state. Surprisingly, the mass and width values for several of these well studied resonances are imprecisely known. In fact, this seems to be a general phenomenon encountered in the study of resonant states, that is, as more experimental information is accumulated concerning any physical property, the variation is much larger than the quoted error of any particular measurements. Such discrepancies are compounded in the case of the $\Delta(1238)$ where different definitions of the resonant parameters yield different central values. For further discussion of this point, see "Review of Particle Properties" (Particle Data Group, April, 1972). Duly noting these difficulties, we proceed to examine the mass differences among members of this decimet, mainly utilizing the values obtained by fitting bumps in the effective mass spectra to achieve a uniformity of treatment. The results are shown in Table V. The weighted average mass difference is 139 MeV (a pion mass!) with a χ^2 fit of 10 for two constraints to the three individual difference values, which vary from 137 to 149 MeV. This corresponds to a 1% probability which is not too bad considering all the systematic difficulties. It is interesting to note that the largest errors occur among the

TABLE V. J^P $3/2^+$ decimet. Tabulation of masses, their differences, and experimental and $SU(3)$ predicted partial decay width for constituent members of the multiplet.

$J^P = 3/2^+$

$\chi^2 = 7.8/3C$; $|A_{10}| = 146.8 \pm 2.4$

Decay	Exptl. Γ (MeV)	SU_3 pred Γ (MeV)
$\Delta(1236) \to N\pi$	116 ± 6[a]	107.2
$\Sigma(1386) \to \Sigma\pi$	3.6 ± 1.2[b]	5.1
$\Sigma(1386) \to \Lambda\pi$	32.4 ± 5.5[b]	35.3
$\Xi(1530) \to \Xi\pi$	9.1 ± 1.3[c]	11.6

	Mass (MeV)	Differences	Mass (MeV)
Δ^-	1240.9 ± 5.0[d]		$\Delta^0 = 1236 \pm 4$[b]
Σ^-	1386.2 ± 3.2[e]	145.3 ± 6.0	$\Sigma^0 = 1383 - 1386$[b]
Ξ^-	1534.9 ± 1.1[f]	148.7 ± 3.4	$\Xi^0 = 1532.0 \pm 0.5$[g]
Ω^-	1672.3 ± 0.9[b]	137.4 ± 1.4	

Av = 139.4 ± 4.1 MeV.

[a] Carter *et al.*, 1971; Olsson *et al.*, 1965; Colton, 1972.
[b] Particle Data Group, April, 1972.
[c] Schlein *et al.*, 1963; London *et al.*, 1966; Baltay *et al.*, 1972; Kirsch *et al.*, 1972; Borenstein *et al.*, 1972; Badier *et al.*, 1972.
[d] Gidal *et al.*, 1966. This experiment determined the Δ(-)-Δ(++) mass difference of 7.9 ± 6.8 MeV Colton *et al.*, 1972.
[e] Huwe *et al.*, 1969; Armenteros *et al.*, 1965; Siegel, 1967; Habibi, private communication.
[f] London *et al.*, 1966; Kirsch *et al.*, 1972; Baltay *et al.*, 1972.
[g] Baltay *et al.*, 1972; Borenstein *et al.*, 1972; Kirsch *et al.*, 1972.

earliest resonances uncovered (Δ, Σ), with the agreement improved with a larger Σ^- mass, as suggested by the more recent determinations.

In attempting a similar analysis on the three neutral members of this decimet, one is confronted with poorer mass values; however, the mass spacing is approximately 145 MeV, comparable to that previously noted for the negative members. In fact there are few accurate measurements of either the Σ^0, which we bracket between its two charged isomultiplet partners, or the Δ^0, only its mass difference from the Δ^{++} having been determined. These values, as well as the well measured Ξ^0, are noted in Table V.

The $SU(3)$ decay rate formulation, as noted previously, is quite simple in the case of a decimet, there being only one unknown. We have adjusted this parameter $|A_{10}|^2$ to give the best possible fit to four experimental numbers, one Δ, two Σ, and the $\Xi \to \Xi\pi$ decay rates. A surprisingly large uncertainty exists regarding the width of the $\Delta(1236)$ as already discussed. We have chosen to use the results of Carter (1971), Olsson (1965), and Colton (1972), the latest and most accurate reported values from total cross section and effective-mass measurements, respectively. Due to clear systematic difficulties we have increased the width error from 3 to 6 MeV. Recent measurements of the $\Xi(1530)$ width also seem prone to these difficulties, with the range of quoted values (7.0 ± 2.0 to 16.2 ± 4.6 MeV) clearly exceeding the quoted errors. Nevertheless, the percentage errors on these decay rates are substantially better than those for the $\Sigma(1385)$.

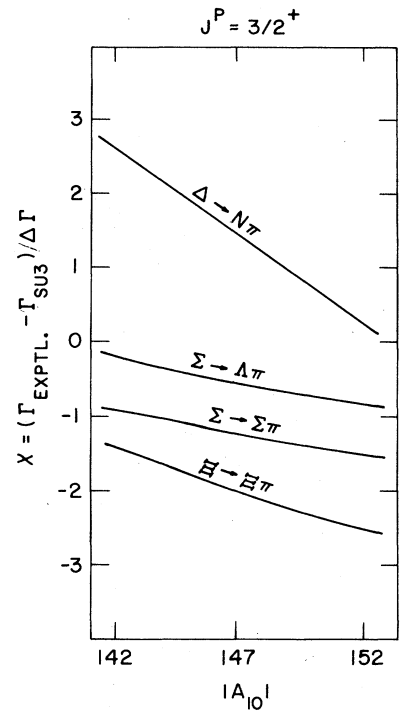


FIG. 4. $J^P = 3/2^+$ decimet; contribution of $|A_{10}|$ to χ.

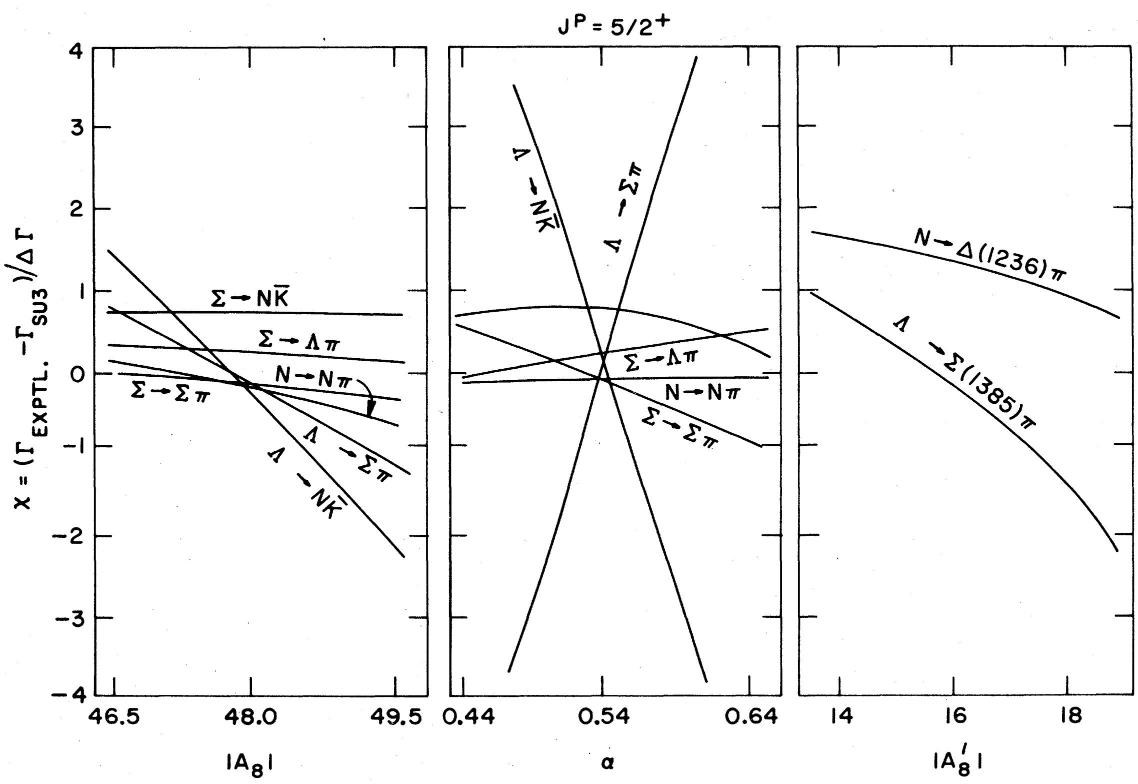


FIG. 5. $J^P = 5/2^+$ octet; contribution of $|A_8|$, α, $|A_8'|$ to χ.

The results of the $SU(3)$ fit are shown in Table V, with the resultant $\chi^2 = 7.8$ for three degrees of freedom, corresponding to 5% probability. The individual contribution (χ_i) to the χ^2 is shown in Fig. 4. We note that the parameter A_{10} is determined primarily by the $\Delta \to N\pi$ rate, with the Σ decays providing little information because of comparatively large errors. Thus the $SU(3)$ rate predictions for this multiplet are adequate, if not spectacular.[11] In Sec. IV we will review the extent to which $SU(3)$ is really tested by this theoretically simplest of multiplets.

D. $J^P = 5/2^+$

Three members of this proposed octet have been extensively investigated, with their properties being well determined, namely the $N(1688)$, $\Lambda(1815)$, and $\Sigma(1910)$. The $\Sigma(1910)$ was the first observed in total cross section measurements (Cool *et al.*, 1966) with subsequent evidence for the existence of such a state in partial wave analysis studies as well as in its observation as a resonance in production experiments. In particular the work of Ely *et al.* (1970) and the College de France, Rutherford, Saclay, Strasbourg Collaboration (1970) on the $\Sigma\pi$ and $\Lambda\pi$ decays of the $\Sigma(1910)$ has verified the $5/2^+$ spin parity assignment. We correlate the bump observed in effective mass plots with this $F_{15}(5/2^+)$ state deduced from the amplitude analyses, due both to the similarity of widths reported by experiments in both cases (50–100 MeV) and to the experimenters' agreement that the magnitude of the $\bar{K}N$ coupling is small. There exists the alternative possibility of associating the production experiment results with a reported (Barbaro-Galtieri, 1970) $3/2^-$ $\Sigma(1940)$; however, this is less appealing due both to a larger reported width ($\simeq$ 200 MeV) and to the lesser reliability of its existence, since some but not all partial wave analyses require such a state. The lack of a second Λ state with $J^P = 5/2^+$ suggests an octet rather than a nonet structure. Application of the Gell-Mann/Okubo mass formula, Eq. (1), predicts a Ξ state at a mass of $\approx$ 2030 MeV. Such a state has indeed been observed (Alitti *et al.*, 1969; Bartsch *et al.*, 1969) with predominant decays into the $\Lambda\bar{K}$ and $\Sigma\bar{K}$ modes. Accordingly, we assign it to this multiplet.

With these assumptions, we fit the 8 decay rates shown in Table VI with two unknowns, α and A_8. The results shown in the Table indicate excellent agreement, $\chi^2 = 0.6$ for four constraints. The contribution to the χ^2 as one varies α and A_8^0 are shown in Fig. 5, with the best value being $\alpha = 0.54$. These values are essentially fixed by the $N(1688)$ and $\Lambda(1815)$ well determined decay modes and require a large $\Sigma(1910) \to \Sigma\pi$ and small $N\bar{K}$ rate as indeed observed in the production experiments, as well as predominant $\Sigma\bar{K}$ and $\Lambda\bar{K}$ modes for the proposed $\Xi(2030)$, also consistent with the experiment. The extreme dependence upon α of the two competing decay modes of $\Lambda(1815) \to N\bar{K}$, $\Sigma\pi$ should be noted. In fact, these particular modes shown in Fig. 5 reveal the greatest sensitivity that has been encountered in this type of analysis.

As noted in the general discussion, caution should be used in applying the Kernan and Smart technique to multiplet constituents when α has a value such that the amplitude may flip sign with a small amount of symmetry breaking. Such is the case here for the $\Sigma(1910)$. The experimental results for the $\Lambda\pi$ amplitude are consistent with the signs predicted by $\alpha = 0.54$ and opposite to that expected for the $\Sigma\pi$ amplitude. Experimentally the $\Sigma\pi$ amplitude of the $\Sigma(1910)$ at resonance is found to have the same sign as the corresponding $\Sigma(2030)$ decay mode, while the $\Lambda\pi$ amplitude of the $\Sigma(1910)$ and $\Sigma(2030)$ have opposite signs. The analysis of the $\Lambda(1815)$ state does not present this problem, with the result that the $\Sigma\pi$ decay of the $\Lambda(1815)$ and the $\Sigma(2030)$ are in phase as expected from the $SU(3)$ fit.

TABLE VI. $J^P = 5/2^+$ octet: Tabulation of masses, total widths, and experimental and $SU(3)$ predicted partial decay width for constituent members of the multiplet.

$J^P = 5/2^+$

$8\otimes 8$: $\chi^2 = 6.2$; $NC = 4$; $|A_8| = 47.7 \pm 0.8$

$\alpha = 0.540 \pm 0.005$

$10\otimes 8$: $\chi^2 = 1.7$; $NC = 1$; $|A'_{10}| = 16.0 \pm 3.1$

M_0, Γ_0 (MeV)	Decay mode $8\otimes 8$	Decay mode $10\otimes 8$	Exptl Γ (MeV)		SU_3 pred. Γ (MeV)
$N(1687 \pm 4)$[a]	$N(1686) \to N\pi$		78 ± 15[a]		80.6
(126 ± 21)	$\to \Sigma K$		...		0.0
	$\to N\eta$		< 0.6		0.8
	$\to \Lambda K$		< 0.1		0.1
		$\to \Delta\pi$		21 ± 12[d]	6.2
$\Lambda(1817 \pm 1.1)$[e]	$\Lambda_8(1816) \to N\bar{K}$		50 ± 1.8[a]		49.7
(80 ± 2.5)	$\to \Xi K$		...		0.0
	$\to \Sigma\pi$		8.8 ± 0.8[a]		8.7
	$\to \Lambda\eta$		...		0.2
		$\to \Sigma^*\pi$		3.1 ± 2.2[a]	3.9
$\Sigma(1927 \pm 15)$[f]	$\Sigma(1910) \to N\bar{K}$		11 ± 15[a]		0.6
(90 ± 33)	$\to \Xi K$		...		0.5
	$\to \Sigma\pi$		58 ± 35[g]		64.0
	$\to \Sigma\eta$		...		0.5
	$\to \Lambda\pi$		17 ± 17[g]		13.2
	$\to \Sigma^*\pi$			...	1.6
		$\to \Delta\bar{K}$		...	4.3
$\Xi(2037 \pm 12)$[b]	$\Xi(2037) \to \Xi\pi$		(Small)		0.5
(51 ± 21)	$\to \Sigma\bar{K}$		(Large)		43.6
	$\to \Xi\eta$		...		1.7
	$\to \Lambda\bar{K}$		(Large)		13.3
	$\to \Xi^*\pi$			...	1.9
		$\to \Sigma^* K$		...	1.1

[a] Particle Data Group, April, 1972.
[b] Carreras and Donnachie, 1970.
[c] Wagner and Lovelace, 1971.
[d] Brody, Cashmore *et al.*, 1971.
[e] Conforto *et al.*, 1971; Kim, 1970, 1971; Barbero-Galtieri, 1970; Armenteros *et al.*, 1967; Bell, 1967; Bugg *et al.*, 1968; Bricman *et al.*, 1970; Cool *et al.*, 1970; Kane, 1972; Barbaro-Galtieri, 1970; Smart *et al.*, 1966; Bricman *et al.*, 1970.
[f] Kane, 1972; Cox *et al.*, 1970; Berthon *et al.*, 1970; Lichfield, 1970, 1971.
[g] Barnes *et al.*, 1969.
[h] Alitti *et al.*, 1969; Bartsch *et al.*, 1969.

[11] Upon completion of the article we noted a similar analysis of this $3/2^+$ decimet performed by A. Barbaro-Galtieri, LBL-1366, with the conclusion that unbroken $SU(3)$ gives a bad fit to the data. The main point of difference between the two analyses lies in the error assigned to the $\Xi(1530)$ width, 0.5 versus 1.3 MeV. The former is the proper statistical error of the various measurements while the latter takes into account the variation of the central values. See also Meshkov (1971).

Two decay rates to baryon decimet and meson octet [$N \to \Delta\pi$ and $\Lambda \to \Sigma^*(1385)\pi$] have been measured with reasonable precision (Particle Data Group, 1972). These are related by one unknown coupling $|A_8'|$ whose beat value of 16 ± 3 leads to a χ^2 of 1.7 for one constraint. Table VI includes these data, and Figure 4 demonstrates the sensitivity of each to $|A_8'|$.

On must conclude that the $J^P = 5/2^+$ octet fits the $SU(3)$ pattern quite well.

E. $J^P = 5/2^-$

In this multiplet, three of the four members have well determined quantum numbers and decay rates. The nucleon state is the $N(1675)$ originally deduced from phase shift analyses of πN scattering, whose presence is more clearly evident in the amplitude analysis of π^- scattering from polarized targets (Duke *et al.*, 1965). The $\Sigma(1765)$ has been extensively studied in both production and formation experiments, so that evidence for both its existence and its spin parity ranks among that for the best established states. The only difficulty is in deciphering precise values for its various partial widths. There are numerous measurements of these quantities (Particle Data Group, 1972); however, their variation is much larger, by a factor of 4–5, than most of the quoted individual errors. We have therefore increased the errors on the $\Sigma(1765)$ experimental partial widths to take account of this spread in values. It is interesting to note that such discrepancies can occur in a well endowed resonance, having been known for many years, with a width of $\simeq$ 100 MeV, and strong coupling to the $\bar{K}N$ channel. It may well be that our knowledge of the properties of many other reported states are less precisely known than indicated, especially those resonances more weakly coupled to the $\bar{K}N$ system.

The $\Lambda(1830)$ has also been extensively studied, mainly in formation experiments (Armenteros *et al.*, 1968) where it is seen as a clear effect in the $\Sigma\pi$ channel. It should be noted that this decay pattern with dominant $N\bar{K}$ mode for the $\Sigma(1765)$ and $\Sigma\pi$ mode for the $\Lambda(1830)$ is the reverse of the comparable states in the $J^P = 5/2^+$ octet. An application of the Gell-Mann/Okubo mass formula predicts a Ξ state with a mass of 1953 ± 11 MeV. Again there is rather good evidence for such a particle with a mass in this range and width $\approx$100 MeV (Badier *et al.*, 1965; Alitti *et al.*, 1968; Oxford Bubble Chamber Group, 1970; and Apsell *et al.*, 1970). It has been observed to decay in the $\Xi\pi$ and possibly $\Xi(1530)\pi$ modes. Since the branching ratios are very poorly known (only upper limits available on the other modes) the cascade information is omitted from the $SU(3)$ fit. The results of the fit for this $5/2^-$ multiplet are shown in Table VII where 6 experimental partial widths are reasonably well reproduced by varying two parameters α and A_8, yielding $\alpha = -0.16$ with a $\chi^2 = 5.1$ for four constraints. The contribution of the various decay rates to the χ^2 is displayed in Fig. 6 for each of the two parameters A_8 and α. One notes that the derived values are rather sensitive to the $\Sigma(1765)$ decay rates and rather insensitive to those of $\Lambda(1830)$. In addition, all rates which are known to be small, e.g., $N(1670) \to N\eta$ are indeed predicted to be small. The $\Xi(1950)$ pattern is also well reproduced, the ($\Xi\pi$) decay mode expected to be dominant over both $\Sigma\bar{K}$ and $\Lambda\bar{K}$ decay modes.[12]

Data on the decays of $5/2^- \to 3/2^+$(decimet) + 0^-(octet) have also been included in the table. There are three measured rates, as shown in Table VII, to be fit by one parameter $|A_8'|$. The χ^2 for this fit (2 for 2 constraints) is good, with the prediction of a 15 MeV partial width of the $\Xi(1940) \to \Xi(1530)\pi$ mode. This addition makes the expected Ξ total width $\simeq$ 80 MeV quite consistent with that experimentally measured.

TABLE VII. $J^P = 5/2^-$ octet. Tabulation of masses, total widths, and experimental and $SU(3)$ predicted partial decay width for constituent members of the multiplet.

$J^P = 5/2^-$

$8\otimes 8$: $\chi^2 = 5.1$; $NC = 4$; $|A_8| = 22.8 \pm 1.4$

$\alpha = -.16 \pm .01$

$10\otimes 8$: $\chi^2 = 2.0$; $NC = 2$; $|A'_{10}| = 156 \pm 12.3$

M_0, Γ_{Tot} (MeV)	Decay mode $8\otimes 8$	Decay mode $10\otimes 8$	Exptl Γ (MeV)	SU_3 pred. Γ (MeV)
$N(1674 \pm 8)$[a]	$N(1673) \to N\pi$		60 ± 12[a]	52.6
(143 ± 26)	$\to N\eta$		$0.8 - 2.5$[b]	2.1
	$\to \Lambda K$		> 0.1[c]	0.02
		$\to \Delta\pi$	90 ± 22	71.4
$\Lambda(1829 \pm 5)$[e]	$\Lambda_8(1830) \to N\bar{K}$		4.3 ± 1.3[f]	4.4
(97 ± 14)	$\to \Xi K$		...	0.01
	$\to \Sigma\pi$		22 ± 24	54.3
	$\to \Lambda\eta$		...	3.3
		$\to \Sigma^*\pi$	27 ± 26[h]	55.1
$\Sigma(1767 \pm 1.4)$[a]	$\Sigma(1765) \to N\bar{K}$		49 ± 11[a]	32.5
(113 ± 26)	$\to \Sigma\pi$		1.4 ± 0.6[a]	1.4
	$\to \Sigma\eta$		27 ± 7	0.03
	$\to \Lambda\pi$		16 ± 4.1[a]	19.8
		$\to \Sigma^*\pi$	5.2 ± 2.6[a]	5.8
		$\to \Delta K$	...	0.9
$\Xi(1945 \pm 15)$[i]	$\Xi(1950) \to \Xi\pi$		(Large)	47.3
(100 ± 30)	$\to \Sigma\bar{K}$		...	14.7
	$\to \Xi\eta$		...	0.04
	$\to \Lambda\bar{K}$		...	8.7
		$\to \Xi^*\pi$	...	15.6
		$\to \Sigma^*\bar{K}$	...	2.7

[a] Particle Data Group, April, 1972.
[b] Carreras and Donnachie, 1970; Deans and Wooten, 1969; Botke, 1969.
[c] Burkhardt *et al.*, 1971; Shu-bon Chan *et al.*, 1972.
[d] Brody, Cashmore *et al.*, 1971.
[e] Conforto *et al.*, 1971; Kim, 1970, 1971; Barbaro-Galtieri, 1971; Armenteros *et al.*, 1968; Bell, 1967.
[f] Conforto *et al.*, 1971; Armenteros *et al.*, 1968; Bricman *et al.*, 1970.
[g] Barbaro-Galtieri, 1970; Kim, 1970, 1971; Armenteros *et al.*, 1968; Bell, 1967.
[h] Prevost *et al.*, 1971.
[i] Badier *et al.*, 1965; Alitti *et al.*, 1968; Oxford Bubble Chamber Group, 1970; Aspell *et al.*, 1970.

[12] No experimental evidence exists for further Λ states with $J^P = 5/2^-$. However, the two $\Lambda(1830)$ measured decay rates may be used to determine two parameters $|A_1|$ and θ required for singlet–octet mixing. In this way a χ^2 of 3 for 2 constraints is obtained, and none of the other rates is seriously altered. Using mass of 1950 MeV for the Ξ state, the value $\theta = 36° \pm 20°$ predicts a predominantly singlet Λ mass of about 1800 MeV with $\Sigma\pi$ partial width $\sim$30 MeV, but negligible KN coupling. Consequently, it is unlikely that such a state would have been observed.

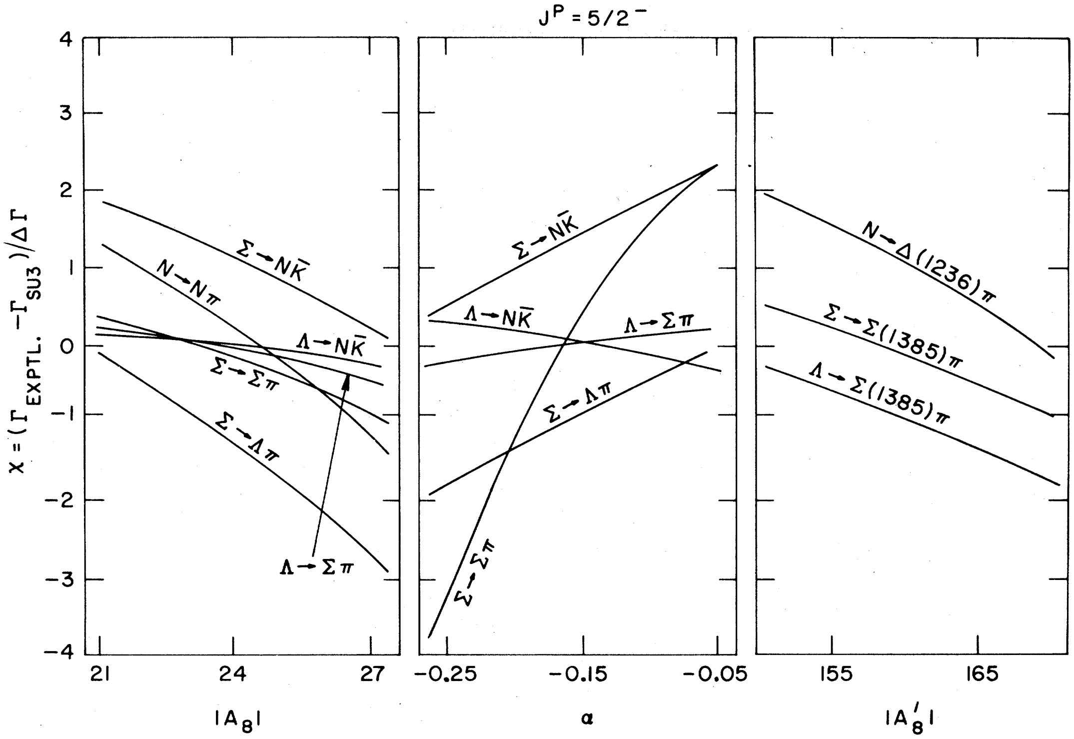


FIG. 6. $J^P = 5/2^-$ octet; contribution of $|A_8|$, α, and $|A_8'|$ to χ.

The small negative value of α for the octet leads to the prediction that the amplitudes at resonance for the $\Sigma(1765)$ in the $\bar{K}N \to \Lambda\pi$ and $\bar{K}N \to \Sigma\pi$ channels should be opposite in sign to those for the $\Sigma(2030)$, again assuming the latter is a member of a decimet. A partial wave analysis in this mass region (Barbaro-Galtieri, 1970; Kane, 1972) indicates that this is indeed the case. Analysis of the $\Lambda(1830)$ in the $\bar{K}N \to \Sigma\pi$ channel yields the same sign for $T^{1830}_{\bar{K}N,\Sigma\pi}$ as for $T^{2030}_{\bar{K}N,\Sigma\pi}$, which is also consistent with that expected utilizing the parameters derived for the multiplet. In all, the $5/2^-$ seems to be a well established multiplet, but with parameters, namely α, quite different from most other $SU(3)$ families.

F. $J^P = 7/2^-$

We now turn to a more speculative multiplet, in that the spin parity of only two members is definitely known, and few of the possible decay rates of any of the other conjectured states have been measured. Nevertheless, there is a grouping of higher-mass Λ, Σ, and Ξ states which have masses and gross decay features matching the pattern expected in a $7/2^-$ nonet. These are the $N(2185)$ and $\Lambda(2100)$, both with measured spin parity $7/2^-$, the $\Lambda(2350)$, $\Sigma(2250)$, and $\Xi(2430)$. There is conflicting evidence for the spin parities of the $\Lambda(2350)$[72,87] ($7/2^-$ and $9/2^+$) and for the $\Sigma(2250)$[88] ($7/2^-$ and $9/2^-$), the main conclusion being that they are probably high-spin states with $7/2^-$ being a possibility for both. Noting this uncertainty, we proceed to analyze this multiplet utilizing the seven decay rates associated with the nucleon, two Λ and one Σ states, again the Ξ data being too poorly known to influence the fit. The data for the first three come from measurements made in both total cross-section and formation experiments, while evidence for the Σ state is presented by total cross-section and production experiments, the latter yielding the $\Sigma\pi$ and $\Lambda\pi$ partial widths.

There are seven experimental numbers to be fit by four parameters, α, A_1, A_8, and θ, where the mixing angle has been left free. The results, shown in Table VIII, are $\chi^2 = 2.3$ for 3 constraints, with $\alpha = 0.83$ (similar to the α value for the $3/2^-$ nonet), and $\theta = 30.9 \pm 2.5°$, again similar to the 24° for the $3/2^-$ nonet. The contribution of these decay rates to the χ^2 in the determination of those four parameters is shown in Figure 7. All curves are rather smooth and slowly varying except in the case of the mixing angle θ. Here the singlet decay rates severely restrict the permissible values of this parameter as noted above. The similarity of the $7/2^-$ and $3/2^-$ nonets is striking. This value of the mixing angle yields an $SU(3)$ mass for the $I = 0$, $Y = 0$ member of the octet which, combined with the proposed N, Σ and Λ members, gives an estimate of 2370 ± 230 MeV for the expected Ξ mass. Most of the uncertainty in this mass arises from the poorly known $N(2185)$ mass. We associate this predicted Ξ with the recently discovered $\Xi(2430)$ state (Alitti *et al.*, 1969; Bartsch *et al.*, 1969), since the pattern for its decay is well reproduced: large ΛK^-, $\Sigma\bar{K}$, and smaller $\Xi\pi$, $\Xi\eta$ modes. Some of the more interesting and verifiable predictions for this multiplet are the following: a large $\Lambda(2530) \to \Sigma\pi$ rate, which should be observable in production experiments at high momenta and should also be evident in formation experiments at K^- momenta of $\simeq 2.5$GeV/c since its $N\bar{K}$ coupling is 39 MeV; the observation of the $\Lambda(2350) \to \Xi K$ mode which is predicted to be 18 MeV; a 15% branching fraction for $N(2185) \to N\eta(550)$; and four possible and substantial Ξ decay modes. In the study of the $\bar{K}N \to \Sigma\pi$ channel for

TABLE VIII. $J^P = 7/2^-$ nonet. Tabulation of masses, total widths, and experimental and $SU(3)$ predicted partial decay width for constituent members of the multiplet.

$J^P = 7/2^-$

$8 \otimes 8$: $\chi^2 = 2.3$; $NC = 3$; $|A_8| = 13.5 \pm 1.0$

$\alpha = .83 \pm .02$; $|A_1| = 43.1 \pm 2.8$; $\theta° = 31° \pm 3°$

M_0, Γ_{Tot} (MeV)	Decay mode $8 \otimes 8$	Exptl Γ (MeV)	SU_3 pred. Γ (MeV)
$N(2184 \pm 91)$[a]	$N(2185) \to N\pi$	85 ± 29[a]	98.3
(275 ± 63)	$\to \Sigma K$	...	4.1
	$\to N\eta$	...	22.8
	$\to \Lambda K$	...	13.0
$\Lambda(2350 \pm 6)$[b]	$\Lambda_8(2350) \to N\bar{K}$	42 ± 17[b]	39.6
(195 ± 74)	$\to \Xi K$	...	17.7
	$\to \Sigma\pi$	...	33.8
	$\to \Lambda\eta$	...	1.6
$\Lambda(2099 \pm 9)$[a]	$\Lambda_1(2100) \to N\bar{K}$	44 ± 6.5[d]	43.6
(143 ± 20)	$\to \Xi K$	...	0.02
	$\to \Sigma\pi$	8.0 ± 3.6[e]	8.4
	$\to \Lambda\eta$	< 4.3	1.4
$\Sigma(2252 \pm 6)$	$\Sigma(2250) \to N\bar{K}$	14 ± 7.6[b]	16.2
(163 ± 59)	$\to \Xi K$	...	2.2
	$\to \Sigma\pi$	125 ± 59[g]	60.9
	$\to \Sigma\eta$	...	0.1
	$\to \Lambda\pi$	24 ± 26[g]	0.7
$\Xi(2430 \pm 20)$[h]	$\Xi(2370) \to \Xi\pi$	...	21.9
	$\to \Sigma\bar{K}$	...(75 ± 40)[h]	46.4
	$\to \Xi\eta$	...	11.4
	$\to \Lambda\bar{K}$	...(75 ± 40)[h]	42.3

[a] Particle Data Group, April, 1972.
[b] Bugg *et al.*, 1968; Bricman *et al.*, 1970; Cool *et al.*, 1970.
[c] Barbaro-Galtieri, 1970; Kane, 1972; Lichfield, 1971.
[d] Lichfield, 1971.
[e] Kane, 1972; Lichfield, 1971.
[f] Bugg *et al.*, 1968; Bricman *et al.*, 1970; Cool *et al.*, 1970; Berthon *et al.*, 1970; Aguilar-Benitez *et al.*, 1970; Lu *et al.*, 1970.
[g] Berthon *et al.*, 1970.
[h] Alitti *et al.*, 1969; Bartsch *et al.*, 1969.

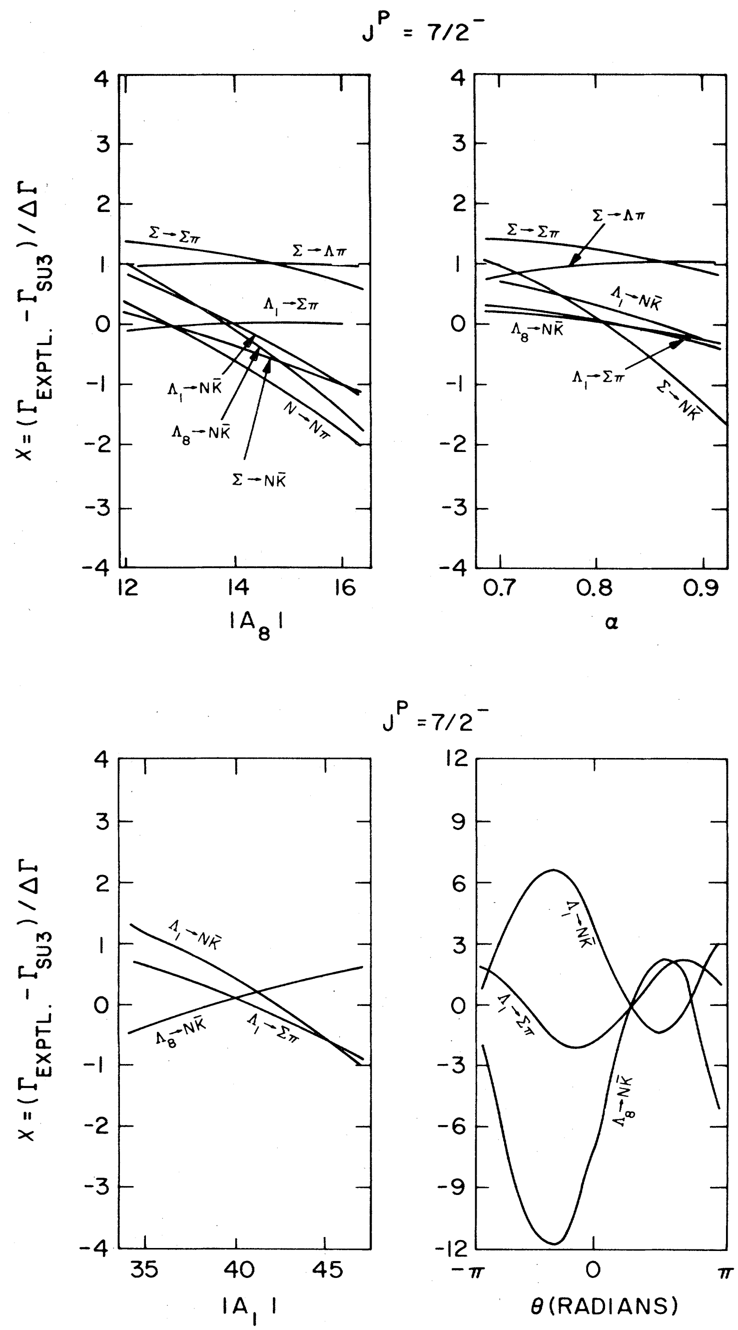


FIG. 7. $J^P = 7/2^-$ nonet; contribution of $|A_8|$, $|A_1|$, α and θ to χ.

the $\Lambda(2100)$, the amplitude at resonance has been found to be opposite in sign to the corresponding $\Sigma(2030)$ amplitude (Barbaro-Galtieri, 1970; Kane, 1972; Berthon *et al.*, 1970; and Lichfield, 1971), as expected from the $SU(3)$ analysis of this predominantly singlet state (see Figure 3). It is clear that the $7/2^-$ is a most exciting multiplet, with much work yet to be done in deciphering the spin parity of the remaining states as well as their widths and branching ratios; however, the $SU(3)$ pattern seems to fit rather well.

G. $J^P = 7/2^+$

This is the second of the conjectured decimets. Two states with spin parity $7/2^+$ have been known for some time, the $\Delta(1930)$ and $\Sigma(2030)$. The former cannot be a member of an octet, the lowest representation available to it being a decimet. The simplest options for the $\Sigma(2030)$ are for it to be a member of an octet or decimet. As noted earlier, most of the s-channel interference arguments depended crucially on the assignment of the $\Sigma(2030)$ to a decimet. On the basis of this over-all consistency as well as the $7/2^+$ spin parity value [the $\Delta(1930)$ being $7/2^+$] a decimet assignment is strongly suggested; we thus assign it and explore the consequences. Since each of these states has several well determined rates, this preliminary assignment is subject to some verification. The equal mass spacing rule for decimets predicts the existence of a $7/2^+ \Xi$ at a mass of 2130 MeV, and an Ω^- at a mass of 2230 MeV. We apply Eq. (6) to the five known rates for a member of a decimet decaying into two particles, each a member of an octet. This involves one unknown parameter $|A_{10}|$. The results are shown in Table IX yielding a $\chi^2 = 7.6$ four constraints, a satisfactory fit to the available data. An equally interesting feature is the prediction of a Ξ width of 40–50

TABLE IX. $J^P = 7/2^+$ decimet. Tabulation of masses, total widths, and experimental and $SU(3)$ predicted partial decay width for constituent members of the multiplet.

$J^P = 7/2^+$

$8\otimes8$: $\chi^2 = 7.6$; $NC = 4$; $|A_{10}| = 51.6 \pm 3.3$

$10\otimes8$: $\chi^2 = 4.8$; $NC = 1$; $|A'_{10}| = 79.7 \pm 7.1$

M_0, Γ_{Tot} (MeV)	Decay mode $8\otimes8$	Decay mode $10\otimes8$	Exptl Γ (MeV) $8\otimes8$	Exptl Γ (MeV) $10\otimes8$	SU_3 pred. Γ (MeV)
$\Delta(1930\pm18)$[a] (199 ± 25)[b]	$\Delta(1930)\to N\pi$		88 ± 18[b]		83.1
	$\to \Sigma K$		3.7 ± 0.7[c]		2.9
		$\to \Delta\pi$		26 ± 6.5[b]	25.7
		$\to \Delta\eta$		3.1 ± 1.4[d]	0.4
		$\to \Sigma^*K$		···	0.04
$\Sigma(2031\pm12)$[e] (138 ± 6)	$\Sigma(2030)\to N\bar{K}$		28 ± 8[f]		18.5
	$\to \Xi K$		<2.7		0.5
	$\to \Sigma\pi$		5.4 ± 2.8[g]		11.4
	$\to \Sigma\eta$		···		2.5
	$\to \Lambda\pi$		27 ± 7.5		27.1
		$\to \Sigma^*\pi$		···	12.2
		$\to \Delta\bar{K}$		···	7.7
	$\Xi(2130)\to \Xi\pi$		···		13.3
	$\to \Sigma\bar{K}$		···		9.4
	$\to \Xi\eta$		···		1.5
	$\to \Lambda\bar{K}$		···		16.9
		$\to \Xi^*\pi$		···	2.6
		$\to \Xi^*\eta$		···	0.01
		$\to \Sigma^*K$		···	5.3
	$\Omega(2230)\to \Xi K$		···		26.7
		$\to \Xi^*\bar{K}$		···	2.0

[a] Particle Data Group, April, 1972, Brisson *et al.*, 1961; Devlin *et al.*, 1965.
[b] Particle Data Group, April, 1972.
[c] Feverbacher and Holladay, 1970; Kalmus *et al.*, 1970; Chinowsky *et al.*, 1968.
[d] Chinowsky *et al.*, 1968.
[e] Barbaro-Galtieri, 1970; Bugg *et al.*, 1968; Bricman *et al.*, 1970; Cool *et al.*, 1970; Lichfield, 1971; Smart, 1968.
[f] Bugg *et al.*, 1968; Bricman *et al.*, 1970; Cool *et al.*, 1970; Lichfield, 1971; Campbell *et al.*, 1971.
[g] Kane, 1972; Lichfield, 1971.
[h] Barbaro-Galtieri, 1970; Cox *et al.*, 1970; Berthon *et al.*, 1970; Lichfield, 1970; Smart, 1968.

MeV roughly equally divided into the $\Lambda\bar{K}$, $\Sigma\bar{K}$, and $\Xi\pi$ modes, and and Ω^- width $\simeq 30$ MeV mainly for the $\Xi(1320)\bar{K}$ mode. The threshold for producing such an Ω^- via $K^-p \to \Omega^- K^+ K^0$ reaction is 5 GeV/c. For completeness we have also analyzed the sequence, decimet decaying into octet plus decimet, again only requiring one unknown constant. The results are included in Table IX where the two known $\Delta(1930)$ decays have been used to predict several possible Σ, Ξ, and Ω decays, the most noteworthy being $\Sigma(2030) \to \Sigma(1385)\pi$, 12 MeV; $\Sigma(2030) \to \Delta(1238)K$, 8 MeV, $\Xi(2130) \to \Sigma(1385)\bar{K}$, 5 MeV; and, $\Omega(2230) \to \Xi(1530)K$, 2 MeV. The properties of the proposed Ξ and Ω states are clearly accessible to production experiments utilizing $\bar{K}$ nucleon interactions as well as Σ and Ξ nuclear interactions via the hyperon beams presently being brought into operation. Figure 8 shows the sensitivity of the two pertinent parameters $|A_{10}|$ and $|A'_{10}|$ to the decay rates. One notes that the curves are rather smooth and not changing rapidly in the former case while the single decay mode $\Delta(1930) \to \Delta(1236)\pi$ serves to restrict severely the allowed value of $|A'_{10}|$ in the latter case.

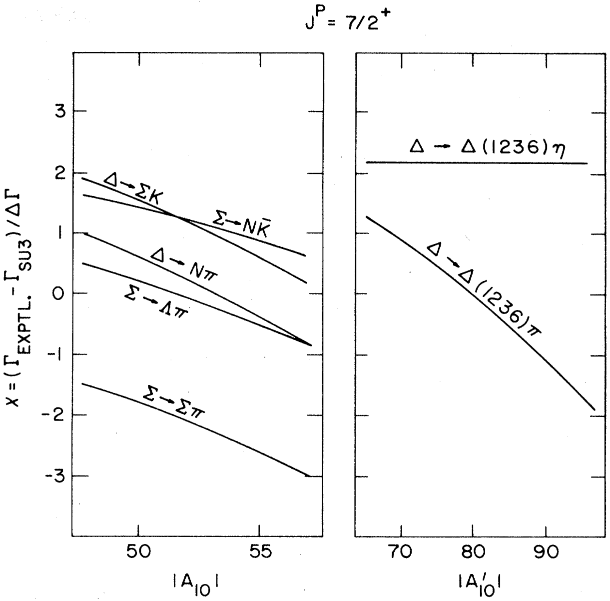


FIG. 8. $J^P = 7/2^+$ decimet; contribution of $|A_{10}|$, $|A_{10}'|$ to χ.

The existence of a $7/2^+$ decimet as outlined here is a very exciting possibility. Of equal interest to the existence of possible Δ, Σ, Ξ, and Ω states with the same spin parity would be the dynamic details such as mass and decay rate values. If the equal mass spacing rule and the predicted $SU(3)$ decay rates are indeed satisfied for this multiplet existing at such a relatively highly excited state ~2 GeV with large interference possibilities, it may suggest that the baryon spectroscopy is simpler than one would have expected.

H. $J^P = 1/2^-$

This comprises the last of the conjectured $SU(3)$ baryon multiplets. There is firm evidence for the $N(1525)$, $\Lambda(1405)$, and $\Lambda(1670)$, all with spin parity $1/2^-$, and rather weaker evidence for a $\Sigma(1750)$, also $1/2^-$. No Ξ candidates for this multiplet have yet been isolated. In this respect, it is worthwhile to review briefly the status of possible Ξ states in the 1800 MeV mass region. As noted in the section dealing with the $3/2^-$ nonet, there is good evidence for a Ξ state at a mass of 1820 MeV with dominant $\Lambda\bar{K}$, $\Sigma\bar{K}$ decay modes. Recently the BMST collaboration (Hemingway *et al.*, 1970) has presented evidence for Ξ resonances at a similar mass 1800–1820 MeV, but with dominant $\Xi\pi$ and $\Xi(1530)\pi$ decay modes offering further evidence for $\Lambda\bar{K}$ and $\Sigma\bar{K}$ modes at a higher Ξ mass, namely 1860–1870 MeV. The experimental situation is certainly confusing, with many possibilities ranging from the existence of one resonance with multiple decay modes to three resonances with restricted decay modes. The conjecture is that the $J^P = 1/2^-$ Ξ state is to be found among this evidence.

The $\Lambda(1405)$ was one of the first hyperons to be found (Alston *et al.*, 1961), and it has been extensively studied both as a bump in production experiments and via the utilization of the K-matrix formalism (or varieties thereof) and the extrapolation of $\overline{K}N$ amplitudes below threshold (Ross and Shaw, 1960; Dalitz and Tuan, 1960; Kim, 1967; Martin and Sakitt, 1969; Queen *et al.*, 1969). We take the mass and width values from the former since the uncertainties involved in the latter as noted earlier are rather large, depending on the precise variation of the formalism adopted, i.e., constant scattering length, K matrix, etc. The $N(1530)$, on the other hand, suffers from imprecise knowledge of the decay modes as well as total width, although there is general agreement that the πN coupling is large and a dominant $N\eta$ mode is characteristic of the inelastic channels (Ayed *et al.*, 1970). This is reflected in the large errors that have to be assigned to these partial decay rates, a main and crucial qualitative feature being that the $N\eta$ mode is not larger than the $N\pi$, although it could be comparable. The $\Lambda(1670)$ is another resonance whose existence is well established but whose properties are not well determined. There is agreement upon substantial decays into $\overline{K}N$, $\Sigma\pi$, and $\Lambda\eta$; however, the branching fractions vary by as much as 50% from experiment to experiment and also by factors many times larger than the quoted errors (see Table X for references). The numbers listed in Table X take these uncertainties into account. As noted earlier, the $\Sigma(1750)$ is not on a firm footing. It was originally detected through its decay into the $\Sigma\eta$ mode with additional evidence having since been presented for $\overline{K}N$ and $\Lambda\pi$ modes (Cline *et al.*, 1964). The values for the various decay rates are all poorly known, and as a result we only include that for the $N\overline{K}$ in the fit, noting that the other modes have been observed. Seven decay rates are fit by four parameters, A_1, A_8, α, and θ. The results are shown in Table X, the fit

TABLE X. $J^P = 1/2^-$. Tabulation of masses, total widths, and experimental and $SU(3)$ predicted partial decay width for constituent members of the multiplet.

$J^P = 1/2^-$

$8\otimes 8$: $\chi^2 = 3.2$; $NC = 3$; $|A_8| = 5.2 \pm 0.5$

$\alpha = -0.28 \pm 0.06$; $|A_1| = 26.2 \pm 1.8$; $|\theta|° = 16.5° \pm 5°$

$10\otimes 8$: $\chi^2 = 0.0$; $NC = 0$; $|A'_{10}| = 262 \pm 58$

M_0, Γ_{Tot} (MeV)	Decay mode $8\otimes 8$	Decay mode $10\otimes 8$	Exptl Γ (MeV)	SU_3 pred. Γ (MeV)
$N(1531 \pm 27)$[a] (106 ± 38)	$N(1530) \to N\pi$		40 ± 20[a]	24.4
	$\to N\eta$		51 ± 30[b]	5.0
		$\to \Delta\pi$	31 ± 19[c]	31
$\Lambda(1675 \pm 2.7)$[d] (25 ± 7)	$\Lambda_8(1677) \to N\overline{K}$		7 ± 3[e]	7.5
	$\to \Sigma\pi$		15 ± 6[f]	15.7
	$\to \Lambda\eta$		6 ± 3[g]	5.5
		$\to \Sigma^*\pi$	∘∘∘	16.3
$\Lambda(1402.4 \pm 35)$[a] (38.1 ± 3.9)	$\Lambda_1(1402) \to \Sigma\pi$		38 ± 4[a]	38.1
$\Sigma(1774 \pm 6.5)$[b] (62 ± 2.5)	$\Sigma(1774) \to N\overline{K}$		29 ± 21[i]	37.6
	$\to \Sigma\pi$		3[j]	4.6
	$\to \Sigma\eta$		Observed[k]	5.6
	$\to \Lambda\pi$		Observed[l]	17.4
		$\to \Sigma^*\pi$	∘∘∘	18.5
		$\to \Delta\overline{K}$	∘∘∘	6.1
	$\Xi(1835) \to \Xi\pi$		•••	45.1
	$\to \Sigma\overline{K}$		•••	9.0
	$\to \Lambda\overline{K}$		•••	9.0
		$\to \Xi^*\pi$	∘∘∘	5.7

[a] Particle Data Group, April, 1972.
[b] Davies, 1967; Carreras and Donnachie, 1970; Deans and Wooten, 1969; Bradsdet *et al.*, 1965; Michael, 1966; Uchiyama *et al.*, 1966; Delcourt *et al.*, 1969; Diem *et al.*, 1970.
[c] Diem *et al.*, 1970.
[d] Berley *et al.*, 1969; Armenteros *et al.*, 1969; Kim, 1970, 1971; Barbaro-Galtieri, 1970; Armenteros *et al.*, (CHS Collaboration), 1969.
[e] Smith, 1965; Badier, 1965; Alitti, 1969; Kim, 1970, 1971.
[f] Berley *et al.*, 1969; Armenteros *et al.*, 1969; Kim, 1970, 1971; Barbaro-Galtieri, 1970.
[g] Kim, 1970, 1971; Berley *et al.*, 1965.
[h] Conforto *et al.*, 1971; Kim, 1970, 1971; Armenteros *et al.*, 1970.
[i] Conforto *et al.*, 1971; Kim, 1970, 1971.
[j] Kim, 1971.
[k] Cline *et al.*, 1964.
[l] Kim, 1970, 1971; Armenteros *et al.*, 1970.

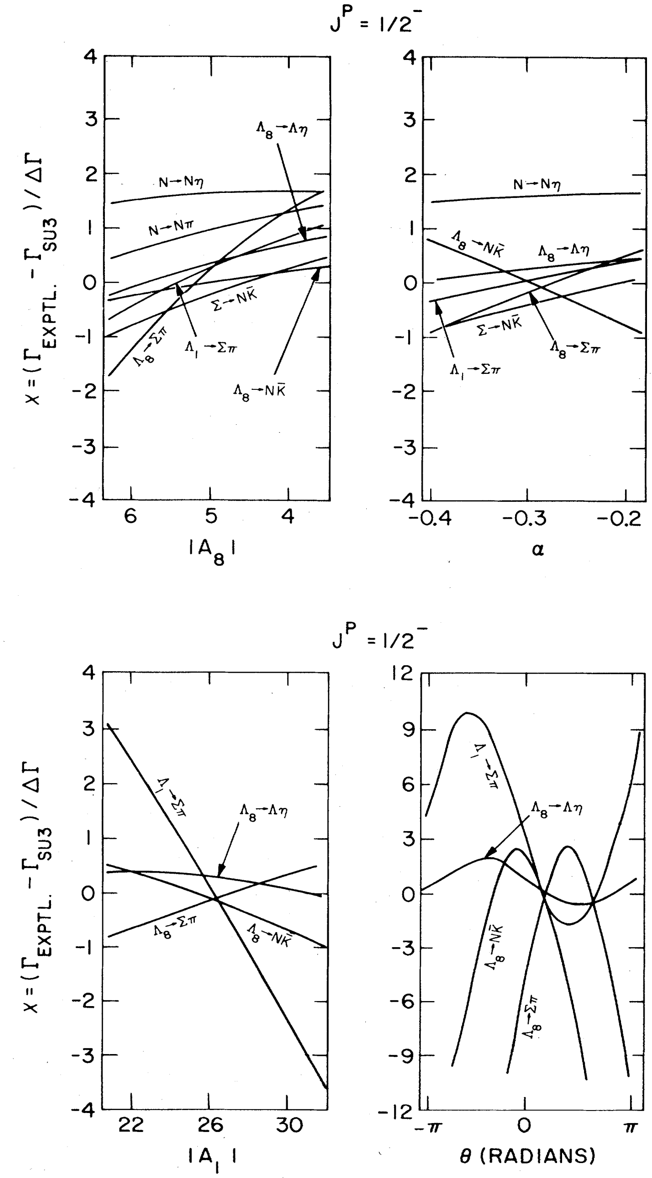


FIG. 9. $J^P = 1/2^-$ nonet; contribution of $|A_8|$, $|A_1|$, α and θ to χ.

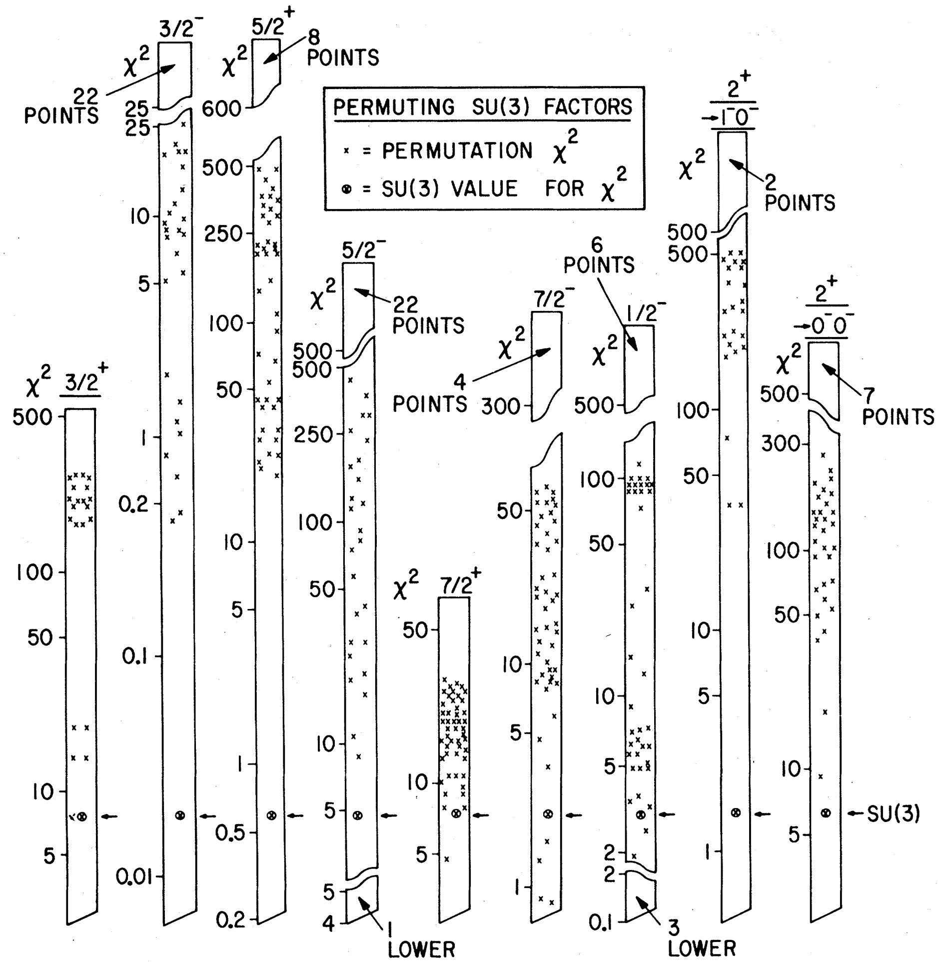


FIG. 10. Pictorial display of the χ^2 variation resulting from randomly permuting $SU(3)$ isoscalar factors for both baryon and boson multiplets.

yielding a $\chi^2 = 3.2$ for three constraints, more than adequate but largely reflecting substantial errors for most of the data. The single measured $8 \otimes 10$ decay of $N(1530) \to \Delta\pi$ leads to predictions for similar decays included in the Table. Among the most interesting of these, are appreciable $\Sigma(1385)\pi$ rates for both Λ states and a measurable $\Xi(1530)\pi$ mode for the Ξ. The fitted value mixing angle $\theta = 16.5 \pm 5°$ predicts, via the mixing formalism and the Gell-Mann/Okubo mass formula, the existence of a Ξ resonance with mass $\simeq 1835 \pm 160$ MeV. The expected properties of such a state are also included in Table X, the dominant feature being a width of $\simeq 75$ MeV with a large $\Xi\pi$ decay mode. The value obtained for $\alpha(-0.28)$ is similar to that for the $5/2^-$ octet. The lack of definite measurement of the $\Sigma(1750) \to \Sigma\pi$ decay rate, together with a relatively large A_1, leads to an alternate possibility, namely $\alpha = +1.3$. The latter solution, however, predicts a $\Sigma(1750) \to \Sigma\pi$ decay width of 110 MeV compared to 4.6 MeV in the former case. That such a large rate should probably have been observed tends to rule out this latter possibility. The χ^2 contribution for the various decay rates to the determination of the four parameters is shown in Fig. 9. Again the curves associated with the octet variables, $|A_8|$, α are rather slowly varying with shallow slopes, to be contrasted with the contribution to $|A_1|$ and θ which display very rapid variations. The singlet decay rates determine these two parameters rather precisely, the $\Lambda(1405) \to \Sigma\pi$ value being one of the more sensitive quantities in this evaluation.

The amplitude phases of the $\Sigma\pi$ decays of the $\Lambda(1405)$, $\Lambda(1670)$ are shown in Fig. 3 where they are seen to agree with those expected from $SU(3)$. Though this multiplet reasonably fits an $SU(3)$ nonet pattern, there exists a clear need for more precise determinations of decay rates, and an understanding of the Ξ region near 1800 MeV for both this and the $3/2^-$ multiplets.

IV. SIGNIFICANT SU(3) TESTS

The general success of $SU(3)$ in fitting partial decay rates leaves open the question of whether these fits are meaningful. In a few of the cases investigated there were several rates within the multiplet that were rather poorly measured, and with up to four free parameters to fit a multiplet, it is possible that many models with a similar number of parameters can give adequate fits. To investigate this possibility, we have carried out the following analysis: in each multiplet we have generated up to 50 random permutations of the $SU(3)$ Clebsch-Gordan coefficients associated with those rates. In this way, we create 50 "pseudo-$SU(3)$" models per multiplet, each of which has the same number of free parameters as the original fit. If the $SU(3)$ fits are not meaningful in the sense described above, we should expect about half of the pseudomodels to give equally good fits. The results of this procedure are shown in Fig. 10 and clearly indicate that

TABLE XI. Test of the sensitivity of the $SU(3)$ formalism to various kinematic factors as determined by comparison with the experimental and predicted partial width for the Δ and Ξ members of the $J^P = 3/2^+$ decimet.

Test of Kinematic Factor for $3/2^+$ Decuplet Form of kinematic factor	χ^2	Predicted $\Gamma(\Delta\to N\pi)$	Predicted $\Gamma(\Xi\to\Xi\pi)$
(a) $\frac{p}{m}\left(\frac{p}{M}\right)^{2l} M$	7.77	107.2	11.6
(b) $\frac{p}{m}\left(\frac{p}{M}\right)^{2l} (E_B + m_B/M)^P \cdot M$	32.2	94.3	14.3
(c) $\frac{p}{m}\left(\frac{p}{M}\right)^{2l}\left[B_1\left(\frac{p}{x}\right)\right]^l M$	7.77	107.2	11.6 } Best value $x\to\infty$
(d) $\frac{p}{m}\cdot\left(\frac{p}{M}\right)^{2l}\cdot B_l\left(\frac{p}{x}\right)\cdot\left(\frac{M}{m}\right)\cdot M$	1.57	113.0	9.9 } Best value $x\to\infty$
(e) $\left(\frac{p}{m}\right)^{2l+1}\cdot M$	21.2	99.5	13.4

$P = +1$ even parity

$= -1$ odd parity

the $SU(3)$ fits are indeed meaningful. For example, consider the $J^P = 3/2^+$ decimet. All of the twenty pseudomodels give fits with higher χ^2 than the $SU(3)$ fit; in fact, sixteen of these have χ^2 of over 100. (Permutation of the $\Sigma \to \Lambda\pi$ and $\Xi \to \Xi\pi$ coefficients does not alter χ^2.) This pattern is generally repeated in the remaining multiplets, with only eleven of the 400 pseudomodels achieving better fits than $SU(3)$. In the case of $3/2^-$, $5/2^+$ and $3/2^+$ multiplets the $SU(3)$ fit the best, for the $5/2^-$ and $7/2^+$ multiplet there is only one permutation (out of a possible 50) that is better than the $SU(3)$ fit while for the $7/2^-$ and $1/2^-$ multiplet there are on the order of five better fits. It is noteworthy that those multiplets with the best measured rates have fewer alternate fits in the acceptable range.

A test such as that described above can be performed for many currently popular models to see whether the particular model is sensitive to experimental measurements. For example, those who wish to employ $SU(3)$ breaking, thereby introducing additional parameters, can thereby determine if their fits are sensitive to the breaking parameters, or if fits are achieved simply because sufficient parameters are available.

Another consideration bearing on the validity of our $SU(3)$ tests is the form of the kinematics, or barrier factors of Eq. (6). To test the effects of different kinematic factors, we again utilize the $J^P = 3/2^+$ decimet, for which only one $SU(3)$ parameter is involved, and which has the best measured rates. Table XI summarizes the "performance" of kinematic factors used by various authors.[13] The factors (c) and (d) include an additional parameter, an inverse interaction radius X. As long ago as 1966 (Leitner *et al.*, 1966), it was known that fits were insensitive to this parameter, with the best value of X large, and in the limit $X \to \infty$ the factor (c) reduces to the factor (a), which has been used in the above analysis.

A glance at Figure 4 indicates that the $SU(3)$ fit would be better if the predicted $\Delta(1236)$ decay rate was increased while the predicted $\Xi(1530)$ rate was reduced. This can be simply achieved by multiplying the right-hand side of Eq. (6) by (M/m). Essentially, this is what leads to the improved χ^2 of the factor (d). In the remaining multiplets, the factor (c) yields results close to the factor (a). The factors (b) and (e) are relatively poor. If we compare the χ^2 spread due to these factors with that due to permuting the $SU(3)$ amplitude, we find that the form of the factors does not substantially mask the sensitivity to $SU(3)$. It is therefore quite clear that before invoking symmetry breaking, it would be wise to investigate whether adjusted kinematic factors can achieve a better fit to the data.

V. BARYON SYSTEMATICS

An examination of a large fraction of the reported baryon resonances has shown the existence of numerous

[13] The forms for $B_l(p/x)$ are as given by Blatt and Weisskopf, *Theoretical Nuclear Physics* (1952).

TABLE XII. Summary of the baryon and boson $SU(3)$ fits.

Summary of Multiplet Fitting J^P	Decay products	χ^2	NC	$\lvert A_8\rvert$, $\lvert A_8'\rvert$, $\lvert A_{10}\rvert$ or $\lvert A_{10}'\rvert$	α	$\lvert A_1\rvert$	$\theta°$
$1/2^+$	...	...	...	...	0.3 ± 0.4	...	...
$1/2^-$	$1/2^+0^-$	3.2	3	-5.2 ± 0.5	-0.28 ± 2.4	26.2 ± 1.8	16.5 ± 5
$3/2^+$	$1/2^+0^-$	7.8	3	147.0 ± 2.4	...	...	...
$3/2^-$	$1/2^+0^-$	0.02	1	41.1 ± 1.6	0.72 ± 0.03	178.0 ± 5.5	26.0 ± 3
$5/2^+$	$1/2^+0^-$	6.2	4	47.7 ± 0.7	0.540 ± 0.005	...	...
$5/2^-$	$1/2^+0^-$	5.1	4	22.8 ± 1.3	-0.16 ± 0.02	...	...
Baryons							
$7/2^+$	$1/2^+0^-$	7.6	4	51.6 ± 2.1	...	...	...
$7/2^-$	$1/2^+0^-$	2.3	3	13.5 ± 1.0	0.83 ± 0.02	41.3 ± 2.8	30.9 ± 3
$1/2^-$	$3/2^+0^-$	0	0	262.0 ± 58	...	...	...
$3/2^-$	$3/2^+0^-$	36	1	3.6 ± 1.3	...	...	...
$7/2^+$	$3/2^+0^-$	4.8	1	79.7 ± 7.1	...	...	...
Bosons							
1^-	0^-0^-	0.6	2	$A_a=61\pm1$	...	...	31 ± 3
2^+	1^-0^-	1.5	2	$A_a=160\pm3.9$	...	...	48 ± 45
2^+	0^-0^-	6.4	4	$A_s=47.2\pm1.3$	...	85.5 ± 5.4	35 ± 8

TABLE XIII. Partial list of reported resonant baryon states not accommodated in any $SU(3)$ multiplet.

$N(1470)$ $1/2^+$	$\Delta(1650)$ $1/2^-$
$N(1700)$ $1/2^-$	$\Delta(1670)$ $3/2^-$
$N(1780)$ $1/2^+$	$\Delta(1890)$ $5/2^+$
$N(1860)$ $3/2^+$	$\Delta(1910)$ $1/2^+$
$\Lambda(1575)$ $1/2^+$	$\Xi(1630)$
$\Sigma(1700)$ $1/2^-$	
$\Lambda(1750)$ $1/2^+, 3/2^+$	
$\Lambda(1850)$ $1/2^+, 3/2^+$	
$\Sigma(1900)$ $1/2^+$	
$\Sigma(1940)$ $3/2^-$	

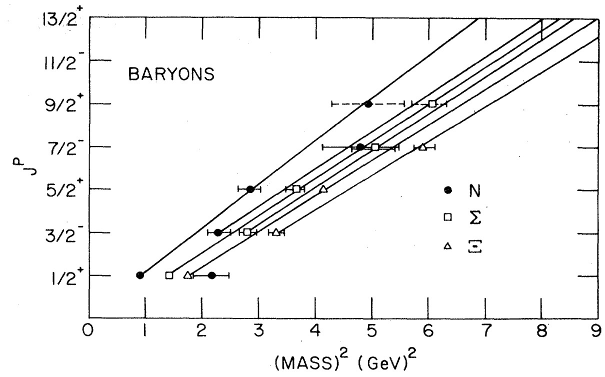


FIG. 11. Plot of J^P vs M^2 for the N, Σ, and Ξ states of the $1/2^+$, $3/2^-$, $5/2^+ \cdots$ baryon sequence.

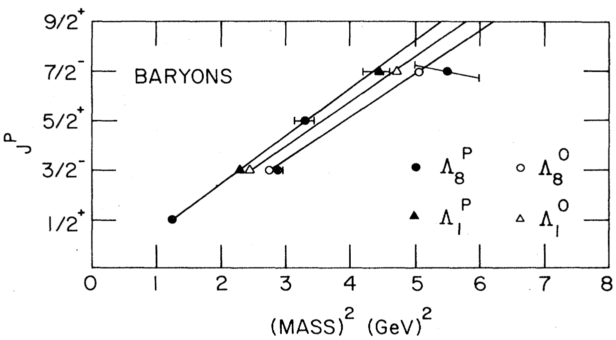


FIG. 12. Plot of J^P vs M^2 for the Λ states of the $1/2^+$, $3/2^-$, $5/2^+ \cdots$ baryon sequence.

$SU(3)$ multiplets. In nearly all cases at least three of the constituent members of a particular multiplet had well determined spin parities, and in all instances there were a sufficient number of measured decay rates to perform a meaningful rate analysis. Several of the pertinent parameters of these multiplets, α, θ, and A_i's, are summarized in Table XII. In this assemblage there are three octets, three nonets and two decimets. The octets are those with J^P of $1/2^+$, $5/2^+$, and $5/2^-$ with the α being $\simeq 0.4-0.5$ for the first two multiplets and having much different value, $\alpha=-0.16$ for the $5/2^-$ multiplet. The nonets occur among the $J^P = 3/2^-$, $7/2^-$, and $1/2^-$ multiplets again with the former two having similar α's $= 0.7-0.8$ in contrast to the value of $\alpha=-0.2$ for the $1/2^-$ nonet, this being the same value as for the $5/2^-$ multiplet. The mixing angles θ show a similar pattern, yielding values of $\approx 30°$ for the $3/2^-$, $7/2^-$ nonet, and $15-20°$ for the $1/2^-$ case. These values for the mixing angle yield a singlet mass value which satisfies the Gell-Mann/Okubo mass formula for the $3/2^-$ and $7/2^-$ cases utilizing existing Ξ resonances (of unmeasured spin parity but whose decay patterns fit those predicted for the particular multiplets to which they were assigned) and predicts another Ξ resonance at a mass of 1835 MeV with $J^P = 1/2^-$.

A summary of the relative sign of resonant amplitudes as determined both experimentally and via $SU(3)$, utilizing the Kernan–Smart technique, was displayed in Fig. 3. Several states have been investigated in both the $\Lambda\pi$ and $\Sigma\pi$ modes but the predominant work has been carried out for the $\Sigma\pi$ final state. As mentioned in the individual sections, all phases agree with those expected from $SU(3)$. It is important to note that the predominant information is derived from interference with either the $\Sigma(1385)$ or $\Sigma(2030)$, both taken to be members of a decimet. If, for instance, the $\Sigma(2030)$ resonance actually turns out to be a member of an octet, then a major portion of the conclusions derived from this analysis is negated.

There have been many more resonances reported than have been discussed. These include the $N(1470)$ and numerous Δ, Λ, and Σ resonances mainly derived from phase-shift analysis. As yet they do not seem to form any $SU(3)$ patterns. However, they are considerably weaker states than the ones discussed previously and either their partners have yet to be uncovered or they may indeed not exist. A partial list of resonances is given in Table XIII. They span mass values from 1000–2000 MeV, and spin from 1/2 to 7/2. The discussion of such states awaits the accumulation of more experimental data.

Further baryon systematics can be explored utilizing a Chew–Frautschi plot (Chew and Frautschi, 1961), where one examines the correlation between the J^P and mass squared of each of the resonant states. The spin parity assignment of states in which these quantities have yet to be determined was set via the $SU(3)$ analysis described in the previous sections. For instance, the $\Xi(1930)$ was assigned $J^P = 5/2^+$; the $\Sigma(2250)$ $7/2^-$ and the $\Omega^-(1672)$ was assigned $J^P = 3/2^+$. Such information for the $1/2^+$, $3/2^-$, $5/2^+$, $7/2^-$ sequence is shown in Fig. 11 for the N, Σ, Ξ, and one Ω states. The indicated straight lines are drawn between similar states of same parity. It is apparent that these lines are all roughly parallel with slopes approximately equal to one. On the other hand, they do not overlap and therefore are not exchange degenerate. Extrapolation of the highest N and Σ trajectories intersects the $9/2^+$ axis at masses of established N and Σ resonances but of unmeasured spin parity as shown in the same figure. This may indeed be the beginning of yet another multiplet, the $9/2^+$. The parallelism of the N, Σ, Ξ trajectories of the Chew–Frautschi plot would predict a $\Xi(2560)$ partner to the $N(2220)$, $\Sigma(2450)$. To avoid confusion, the corresponding $\Lambda°$ resonances have been plotted in Fig. 12. In this instance the $SU(3)$ singlet and octet masses for the $3/2^-$ and $7/2^-$ have been plotted in addition to the actual physical masses where the singlet bare mass is given by the expression $m_1^0 = m_8^P - m_8^0 + m_1^P$. When such Λ states of similar parity are joined by straight lines one notes that the $1/2^+$, $5/2^+$ trajectory is

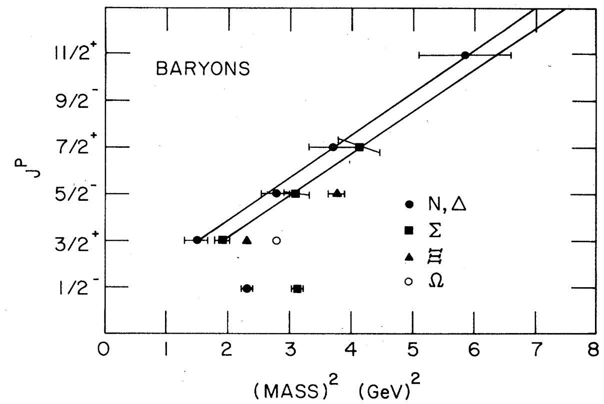


FIG. 13. Plot of J^P vs M^2 for the N, Δ, Σ, Ξ, and Ω states of the $1/2^-$, $3/2^+$, $5/2^-$ $\cdots$ baryon sequence.

degenerate with the physical singlet masses of the $3/2^-$, $5/2^-$ sequence. On the other hand, the two trajectories for the $SU(3)$ singlet and octet masses are parallel and have slopes of $\simeq 1$.

Data for a similar investigation of the $1/2^-$, $3/2^+$, $5/2^-$, $7/2^+$ sequence are shown in Fig. 13. There the Δ trajectory connecting the $3/2^+$ and $7/2^+$ decimet states extrapolates through another known Δ state with mass^2 5.82 GeV^2 and spin $11/2^+$. Similar behavior is observed for the Σ sequence, both these trajectories possessing slopes of 1. This observed parallelism of trajectories can be utilized also to predict the masses of a possible $9/2^-$ multiplet from the observed $5/2^-$ octet. This yields $N(2.23)$, $\Sigma(2.32)$, and $\Xi(2.45)$. In essence an examination of spin parity and mass distribution of the well established multiplets indicates recurrences and parallelism but not degenerate trajectories.

Although the emphasis of this paper has been on the comparison of the experimentally observed spectroscopic data with $SU(3)$, it is of further interest to explore the matching of the multiplet structure to that predicted via $SU(6)$ where the spin is included as an integral part of the symmetry, in addition to the hypercharge and isospin (Gursey and Radicati, 1964; Sakita, 1964). The possible multiplets are the 56, 70, and 20 representation and their excitations. Their decomposition into $SU(3)$ multiplets within any given $SU(6)$ multiplet increases dramatically, so much so that the number almost exceeds the list of reported multiplets. In view of this, firm statements can be made only on the lower states, with a wide range of speculation available for the higher states. The first assignment made was that for the positive parity ground state (also shown in Table XIV)

$$\underline{56}(L = 0) \rightarrow (8; 1/2^+) + (10; 3/2^+) \qquad (22)$$

where in fact the observed masses satisfy the $SU(6)$ mass relation $m = m_0 + bS(S+1) + CY + d[I(I+1) - (Y^2/4)]$ which is a generalization of the Gell-Mann/Okubo mass formula (Gursey and Radicati, 1964). Such a choice is dictated by the fact that only the $\underline{56}$ contains a decimet with spin 3/2 (as a ground state). Such an assignment however has the theoretical difficulty that a quark picture of such a representation has the three quarks in an S state, a symmetric situation where the Pauli principle would require an antisymmetric configuration if quarks obeyed Fermi statistics as should normal spin 1/2 particles. One can invoke parastatistics or ignore quarks.

The negative parity multiplets can then be accommodated in either $\underline{70}L = 1$ or $\underline{56}L = 1$. The choice is not absolutely clear cut; however, the $\underline{70}L = 1$ is preferred experimentally. This is due to the evidence for both $1/2^-$, $3/2^-$ nonets and a $5/2^-$ octet (in a similar mass range) all conveniently accommodated in this $SU(6)$ multiplet and not in the $\underline{56}L = 1$. The breakdown into $SU(3)$ submultiplets is also shown in Table XIV. Those underlined are among the well established $SU(3)$ multiplets as noted in previous section. This classification predicts the existence of four additional multiplets, a second $1/2^-$ and $3/2^-$ octet, and two decimets, $1/2^-$ and $3/2^-$. The recent evidence for two Σ(1660) states, most probably both with $3/2^-$, lends credence to this scheme. In addition, there is a Δ(1650), $1/2^-$ and Δ(1670), $3/2^-$ reported which could be the beginning of the proposed $1/2^-$ and $3/2^-$ decimets. Very little can be said concerning the second $1/2^-$ octet.

TABLE XIV. Listing of $SU(6)$ multiplets and their $SU(3)$ subcomponents.

$SU(6)$	$[SU(3);S]$
56→	(8;1/2) + (10;3/2)
70→	(1;1/2) + (8;1/2) + (10;1/2) + (8;3/2)
20→	(8;1/2) + (1;3/2)
$SU(6)$	$[SU(3);J^P]$
56 $L=0$;	$(8;1/2^+)^a$ $(10;3/2^+)^a$
70 $L=1$;	$(1;1/2^-)^a$ $(8;1/2^-)^a$ $(1;3/2^-)^a$ $(8;3/2^-)^a$ $(8;5/2^-)^a$ $(10;1/2^-)$ $(10;3/2^-)$ $(8;1/2^-)$ $(8;3/2^-)$ ↓ Σ(1660) $(10;1/2^-)$ ↓ Δ(1650); $(10;3/2^-)$ ↓ Δ(1670)
56 $L=2$;	$(8;3/2^+)(8;5/2^+)^a$ $(10;1/2^+)$ $(10;3/2^+)$ $(10;5/2^+$ $(10;7/2^+)^a$ $(10;1/2^+)$ ↓ Δ(1910); $(10;3/2^+)$ ↓ Δ(1890)
70 $L=3$;	$(1;5/2^-)$ $(8;5/2^-)$ $(10;5/2^-)$ $(8;3/2^-)$ $(8;5/2^-)$ $(8;9/2^-)$ $(1;7/2^-)^a$ $(8;7/2^-)^a$ $(10;7/2^-)$ $(8;7/2)$

[a] Known $SU(3)$ multiplets.

The placement of the odd parity states in the $\underline{70}L = 1$ also provides a simple explanation of why the $5/2^-$ is only an octet and not a nonet as one would normally expect from the $SU(3)$ recurrence pattern (i.e., $1/2^-$ is a nonet). Only when one reaches $\underline{70}L = 3$ does one expect a nonet structure for the $5/2^-$. From a total of nine $SU(3)$ multiplets in this $\underline{70}L = 1$ representation there is firm evidence for five members, some evidence for a few members of the three others, and no information on the last.

In reviewing the above, one notes a possible pattern emerging, namely positive parity states being connected with $\underline{56}$ representation and odd parity with $\underline{70}$. We have ignored the $\underline{20}$ since its members cannot decay into baryon–meson final states from which most of the experimental data arises but instead are correlated to baryon–boson–boson decay modes.

The higher excited states of the $\underline{56}$ and $\underline{70}$ representations are noted in Table XIV. This includes six additional $SU(3)$ multiplets for the $\underline{56}L = 2$ case, of which two have been established, $(8; 5/2^+)$ and $(10; 7/2^+)$, with little or no data for the remaining four. We note possible Δ members for two of the decimets. The situation is even more vague for the $\underline{70} = 3$ multiplet where only the $7/2^-$ nonet has been conjecturally identified with an additional eight more predicted multiplets. If such an $SU(6)$ scheme does indeed describe the spectroscopy of particles, then the number of baryons yet to be detected with masses less than 2.5 GeV is rather large, with the bulk having rather low spins, $\leq 7/2$. Furthermore, if this were indeed the case, it is somewhat difficult to understand the apparent success of both $SU(3)$ in fitting the decay rates, the Gell-Mann/Okubo mass formula in fitting the mass spectra. Higher order mixing (between Λ's, Σ's, etc.) could occur since the masses of many of the constituent states with the same spin-parity should lie in a similar mass range.

We conclude this section by further noting that $SU(6)$ makes unique prediction for the D/F ratio for the various $SU(3)$ submultiplets of each $SU(6)$ multiplet. We note these in Table XV. If one assumes no mixing, then one can test these predictions for the deciphered $SU(3)$ multiplets. The agreement for the positive parity states, namely $\alpha = 0.4$, is quite good. In the negative parity states there is a basic ambiguity since there are two $SU(3)$ multiplets with the same spin parity in the $\underline{70}L = 1, 3$ $SU(6)$ multiplets. The $3/2^-$ value of 0.72 is quite close to the expected 0.625, while the $1/2^-$ value of -0.23 is closer to the $= -0.5$ expected from the (8.3/2) excited state, and distant from the +0.625 expected from (8,1/2) excited state. The $7/2^-$ multiplet fits rather nicely into the $\underline{70}L = 3$ with its value of $\alpha = 0.83$ close to the noted ground state 0.625. In essence there is qualitative agreement with the limited data and it will be quite interesting to determine the α values of the other conjectured multiplets. In particular the other $3/2^-$ should have $\alpha = -0.5$, the higher massed $5/2^-$ should have $\alpha = +0.625$, with the 9.2^- having $\alpha = -0.5$. If there are deviations, one can also invoke mixing; however, this scheme then loses some of its simplicity and thereby some of its attraction.

VI. DATA–BOSONS

Progress in the classification of bosons into $SU(3)$ representations has been slow and rather unsteady in comparison with that for baryons. For many years the only known families have been the $J^P = 0^-$, 1^-, and 2^+ nonets. Resonant states with other quantum numbers have been uncovered, but not in sufficient quantities to convincingly establish further multiplets. This is especially true for the 0^+, 1^+, and 3^- categories. With this in mind, we propose to perform a review of the well known nonets as well as to speculate on the existence of other possible $SU(3)$ groupings.

TABLE XV. $SU(6)$ predicted and observed values of α for numerous $SU(3)$ submultiplets of $SU(6)$.

Represent.	$[SU(3), S]$	α	
56	(8, 1/2)	+0.4	
70	(8, 1/2)	0.625	
	(8, 3/2)	−0.5	
$SU(6)$	$[SU(3); J^P]$	α (Observed)	α (Predicted)
56 $L=0$	$(8; 1/2^+)$	0.3–0.4	0.4
$L=2$	$(8; 3/2^+)$	...	...
	$(8; 5/2^+)$	0.54	0.4
$L=4$	$(8; 7/2^+)$	...	0.4
	$(8; 9/2^+)$	...	0.4
70 $L=1$	$(8; 1/2^-)$	...	0.625
	$(8; 3/2^-)$	0.72	0.625
	$(8; 1/2^-)$	−0.28	−0.5
	$(8; 3/2^-)$	...	−0.5
	$(8; 5/2^-)$	−0.16	−0.5
70 $L=3$	$(8; 5/2^-)$	...	...
	$(8; 7/2^-)$	0.83	0.625
	$(8; 3/2^-)$	...	−0.5
	$(8; 7/2^-)$	...	−0.5
	$(8; 9/2^-)$	...	−0.5

A. $J^P = 0^-$

The members of this ground state are the $\pi(140)$, $K(495)$, $\eta(550)$, and $\eta(960)$. All the states have well established spin parities of 0^-. The first two states to be observed, the π and K, were in fact used by Gell-Mann to predict the existence of the $\eta(550)$ using the mass formula (3). The detailed properties of the $\eta(960)$ were unclear until quite recently. The study of the production and decay correlations of this resonance, as investigated at a variety of K^-p energies, indeed shows no evidence for any spin, all production and decay distributions being isotropic (Danburg *et al.*, 1972). This is in contrast to strong correlations observed for nonspinless particles such as the ρ and ω. The $\eta(960)$ is now definitely $J^P = 0^-$. With two $I = 0$ states with the same spin parity, it is natural to expect octet–singlet mixing. The Gell-Mann/Okubo mass formula works reasonably well, its agreement being $\simeq 13\%$, quite adequate considering the observed mass splittings. However, this is much larger than the 1% agreement observed in the case of baryon octets. If mixing is invoked, the observed masses yield $|\theta| \simeq 10°$ using Eq. (5). If instead one utilizes a linear mass relationship, then $|\theta| \simeq 23°$.

In principle, $SU(3)$ can yield predictions for the electromagnetic decay rates of members of this multiplet. If one considers a γ ray as a U spin zero ($U = 0$) member of an octet, then one can relate the three decay modes, $\pi^\circ \to \gamma\gamma$, $\eta(550) \to \gamma\gamma$, and $\eta'(960) \to \gamma\gamma$ with two unknown parameters. The formalism is well known (Gourdin, 1967); therefore only the pertinent relations are noted.

Process $0^- \rightarrow \gamma\gamma$

$$\begin{aligned} A_{\eta(550)} &= -\sin\theta A_1 + \cos\theta A_8, \\ A_{\eta'(960)} &= \cos\theta A_1 + \sin\theta A_8, \\ A_{\pi^\circ\rightarrow 2\gamma} &= \sqrt{3} A_{8\rightarrow 2\gamma}, \\ \Gamma_{0^-\rightarrow 2\gamma} &= (\alpha^2 |A|^2 m^3)/64\pi. \end{aligned} \tag{23}$$

We note that the ratio of masses of the heaviest to the lightest state in this multiplet is $\simeq 7$ and further that the rate for these electromagnetic decays involves powers of the decaying particle mass up to three (other variations having even higher powers). The rather large uncertainties associated with determining such essentially phase space factors serve as a signal to view these particular conjectures with some caution.

As far as the experimental situation is concerned, both the π° and $\eta(550)$ widths (lifetimes) have been measured; that for the π° via both a direct determination of its life time in emulsions (Stamer *et al.*, 1966) (K_{π_2} decay) and the Primakoff effect (Bellettini *et al.*, 1970; Kryshkin *et al.*, 1970), while the $\eta(550)$ width has only been measured by the latter technique (Bemporad *et al.*, 1967).[14] The values obtained were:

$$\frac{\Gamma_{\eta^\circ(550)\rightarrow\gamma\gamma}}{\Gamma_{\eta^\circ\rightarrow\gamma\gamma}} = \frac{1}{3}\left(\cos\theta - \sin\theta\frac{A_1}{A_8}\right)^2 \left(\frac{m_\eta}{m_{\eta^\circ}}\right)^3 \approx \frac{1\text{ keV}}{7-8\text{eV}} \approx 128. \tag{24}$$

It is clear that mixing is required (i.e., $\theta \neq 0$) since $1/3(m_\eta/m_\eta)^3 \simeq 20$. Since the rate relationships are quadratic in the amplitudes, there are two possible solutions with two resultant predictions for the $\eta' \rightarrow 2\gamma$ partial width. Using the value $\theta = 10°$ derived from the mass relationship these are:

$$A_1/A_8 \simeq -9; \quad \Gamma_{\eta'(960)\rightarrow 2\gamma} = 60\text{ keV},$$

and

$$A_1/A_8 \simeq 20; \quad \Gamma_{\eta'(960)\rightarrow 2\gamma} = 360\text{ keV}. \tag{25}$$

This latter possibility can be eliminated since the branching ratio $\eta'(960) \rightarrow 2\gamma/\text{All} \simeq 2\%$ (Bollini *et al.*, 1968; Basik *et al.*, 1971; Harvey *et al.*, 1971), yielding a predicted total width of 360 keV/0.02 $\simeq$ 18 MeV, which is much larger than the 4 MeV upper limit recently reported by several experiments at the Philadelphia Meson Conference, 1972 (AIP Conference Proceedings #8). The recent experimental results of the Cornell Group (Brownman *et al.*, 1973) have altered the situation somewhat. If the $\Gamma(\eta(550) \rightarrow \gamma\gamma)$ partial width is indeed lower by a factor of ~ 2.7, this has the effect of decreasing the ratio $A_1/A_8 \approx -3$ as well as the predicted $\Gamma(\eta'(960) \rightarrow 2\gamma)$ to 10 keV and the total $\Gamma_{\eta'}$ width to 0.5 MeV. The extreme sensitivity of the total $\eta'(960)$ width to the $\eta(550) \rightarrow \gamma\gamma$ partial width should be noted; a change of a factor of 2.7 in the latter quantity resulting in a factor of 8 reduction in the former 4 MeV to 0.5 MeV. It is therefore of great importance as well as interest to verify the $\eta(550) \rightarrow \gamma\gamma$ partial width and to measure both the $\eta'(960)$ partial width into the $\gamma\gamma$ final states as well as the total $\eta'(960)$ width.

[14] More recently A. Browman *et al.* reported at the International Symposium on Electron and Photon Interactions, Bonn, August 1973, a much lower value for the partial width $\Gamma_\eta \rightarrow \gamma\gamma = 374 \pm 60$ eV, to be contrasted with the previously accepted value of 1000 ± 220 eV.

B. $J^P = 1^-$

The vector meson nonet contains the first observed excited boson states. These are $\rho(750)$, $K(890)$, $\omega(780)$, and $\varphi(1020)$, all with a well determined spin parity of 1^-. The difference between the square of the $K(890)$ and $\rho(750)$ masses is 0.22 $(\text{GeV})^2$, the same as that for the corresponding members of the pseudoscalar nonet. It was immediately apparent that the Gell-Mann/Okubo mass formula was badly violated, and the concept of mixing was first introduced by Gell-Mann and Sakurai (Gell-Mann, 1961; Sakurai, 1962) to correct this defect. The mixing angle obtained from the known masses is $|\theta| = 40°$. The only verification of $SU(3)$, therefore, lies in a determination of the decay rates. As shown in Table XVI, there are three modes consisting of a 1^- state decaying into two pseudoscalar $(0^- + 0^-)$ mesons and therefore involving only the antisymmetric coupling (Eq. (11)). Thus there is one unknown, A_a, to be determined by three decay rates. The $\omega - \varphi$ mixing only adds a $\cos^2\theta$ factor to the decay $\varphi \rightarrow K\bar{K}$ since the coupling to the bare singlet vanishes in this case. As shown in Table XVI, the fit is rather good. It is not, however, a very sensitive test of the $SU(3)$ formalism since the $SU(3)$ coefficients are similar in value. It is interesting to note that if the coupling were symmetric the large $\rho \rightarrow \pi\pi$ rate would be forbidden. In fact, no other set of $SU(3)$ couplings lead to approximately equal Clebsch–Gordan coefficients. Note also that if there were no mixing, i.e., $\cos\theta = 1$, then the predicted width for $\varphi \rightarrow K\bar{K}$ would be 5.8 MeV (to be compared with the experimental value of 4.4

TABLE XVI. $J^P = 1^-$. Tabulation of masses, total widths, and experimental and $SU(3)$ predicted partial decay widths for the constituent members of the multiplet.

$J^P = 1^-$ $8\otimes 8$: $\chi^2 = 0.6$; $NC = 2$; $\lvert A_a^8 \rvert = 6 \pm 1$; $\theta = 31° \pm 3°$; $\theta_{GMO} = 40° \pm 1°$			
M^0, Γ_0 (MeV)	Decay mode $8\otimes 8$	Exptl. Γ (MeV)	$SU(3)$ pred. Γ (MeV)
$1^- \rightarrow 0^-0^-$			
$\rho(765 \pm 10)$ [a]	$\rho(750) \rightarrow \pi\pi$	135 ± 20	148
135 ± 20	$(\sqrt{6}/3) A_a^8]$		
$K(892 \pm 0.5)$ [a]	$K(890) \rightarrow K\pi$	50 ± 1	50
50 ± 1	$(1/\sqrt{2}) A_a^8]$		
$\phi(1019 \pm 0.5)$ [a]	$\phi(1020) \rightarrow K\bar{K}$	4.4 ± 0.3	4.4
4.4 ± 0.3	$(1 \cos\theta_1 A_a^8)$		
$1^- \rightarrow 1^-0^-$			
$\phi(1019 \pm 0.5)$	$\phi(1020) \rightarrow \rho\pi$	0.6 ± 0.3	...
	$\{\sqrt{3}(1/8)^{1/2} A_1 \sin\theta_1 - (1/5)^{1/2} A_s^8 \cos\theta_1\}$		

[a] Particle Data Group, April 1972.

$\pm$ 0.3 MeV)^2 so that mixing does indeed improve the agreement. The only other pertinent two-body decay mode is $\varphi \to \rho\pi$ decay, where both the octet and singlet couplings exist (see Table XVI). The observed rate of $\approx$ 1 MeV occurs if $A_1 \approx A_s^8$ (a reasonable possibility) with the lack of other decay modes precluding any verification of this result. Therefore $SU(3)$ provides a natural means for producing a small rate which on general grounds (i.e., phase space, barrier factors) would be expected to be much larger.

More recently evidence has been presented for the possible existence of a higher-mass vector particle, the $\rho'(1600)$. This has come from studies of both e^+e^- annihilation (Bacci *et al.*, 1972) and photoproduced final states (Davier *et al.*, 1971; SLAC-PUB-666, 1969). In particular the analysis of the mass and decay distribution of four-pion (4π) systems recoiling against a proton as produced by 9.3 GeV linearly polarized photons interacting in hydrogen (Bingham *et al.*, 1972) is consistent with that expected for a $J^P = 1^-$ state. The reported width is $\sim$ 500 MeV with the main decay being $\rho\pi\pi$. Although a broad enhancement is indicated by the data, much additional work remains to be done before it is demonstrated that it is a resonance with $J^P = 1^-$.

The electromagnetic decay of vector mesons into a pseudoscalar meson and a γ ray can be treated in analogous manner to the two-photon decay of pseudoscalar mesons. Once again, considering the photon as a $U = 0$ member of an octet and applying C (charge conjugation) and U invariance, one can relate numerous such decays. In this case there are three unknown constants corresponding to the three possible couplings of octet and singlet members of the vector and pseudoscalar multiplets. The rate for such decays depends on the third power of the masses (Gourdin, 1967) via the relation:

$1^- \to 0^-\gamma$

$$\Gamma = \frac{\alpha}{3}|A|^2\left[\frac{(m_{1^-})^2 - (m_{0^-})^2}{2m_{1^-}}\right]^3$$

$V(1^-) \to PS(0^-)\gamma$

$$\begin{array}{ccc} A_{88} & 8 & 8 \\ A_{18} & 1 & 8 \\ A_{81} & 8 & 1 \end{array} \qquad (26)$$

with $A = C_{88}A_{88} + C_{18}A_{18} + C_{81}A_{81}$.

The coefficients C_{ij} for a variety of processes are noted in Table XVII.

Since there are three unknowns, the first three reactions, namely $\omega^\circ \to \pi^\circ\gamma$ (1$-$MeV width), $\phi \to \pi^\circ\gamma$ ($\sim$ 0.01$-$MeV width), and $\phi \to \eta\gamma$ (0.1$-$MeV width) were used to determine these numbers. The result was $A_{18}/A_{88} \simeq 1.1$ and $A_{81}/A_{88} \simeq -2$ with $A_{88}^2 = (4.3/\alpha) \times 10^{-3}$ GeV^{-2}. Predictions for the remaining listed decays could then be made, and these have been duly noted. A measurement of any or all of these processes would be very interesting. There is a qualitative consistency check on the $\eta' \to \rho\gamma$ with several qualifications. As noted in the section dealing with the $SU(3)$ discussion of the decay of pseudoscalar mesons into two γ's, present evidence requires the total η' width to be 3 MeV [60 keV $\times$(1/0.02)]. This in turn, with a knowledge of the $\eta' \to \rho\gamma$ branching ratio, yields a partial width of 1.1 MeV for this mode, to be compared with the expected 0.5 MeV. This is really an $SU(3)$ consistency check which would be meaningful if the η' total width indeed measured $\simeq$ 3 MeV.

Further predictions have already been noted in the previous tabulation for $\rho \to \pi\gamma$ (16 MeV), $\omega \to \eta\gamma$ (0.01 MeV), and $K^{*0} \to K^0\gamma$ (0.24 MeV), and $K^{*+} \to K^+\gamma$ (0.06 MeV). The prediction of a factor 4 difference between the neutral and charged electromagnetic decay of the $K^*(892)$ is a sensitive test of $SU(3)$, since it arises from the couplings and not from phase space or barrier factor considerations. Again the method of Primakoff should provide a means of measuring several of these processes.

Another approach would be to search for processes forbidden by U-spin considerations. Unfortunately, since only octets have been observed for the bosons, no such direct tests are available in this category. However, it is worth noting that this restriction does not exist for baryons since decimets are clearly evident which contain $U = 3/2$ members (i.e., Ξ^{*-}, Y^{*-}, Ω^-, and Δ^-) and *cannot*, therefore, decay into a member of an octet plus a γ. Searches for $\Xi^*(1530) \to \Xi^-\gamma$, and $\Sigma^-(1385) \to \Sigma^-\gamma$ are therefore of great importance.

C. $J^P = 2^+$

The constituent members of this tensor multiplet have been known and well established for several years. These are the $A_2(1320)$, $I = 1$; $K(1420)$, $I = 1/2$; and $f(1250)$, $f'(1515)$, $I = 0$. The main controversy has revolved

TABLE XVII. Tabulation of $SU(3)$ isoscalar factors for various electromagnetic decays of vector mesons. The observed and predicted widths of several states are also noted.

Exptl. width [a]		C_{88}	C_{18}	C_{81}	Predicted width
$\simeq$1 MeV	$\omega \to \pi^0\gamma$	$\sqrt{3}\sin\theta_1$	$\sqrt{3}\cos\theta_1$		
$\simeq$0.01 MeV	$\phi \to \pi^0\gamma$	$\sqrt{3}\cos\theta_1$	$\sqrt{3}\sin\theta_1$		
$\simeq$0.1 MeV	$\phi \to \eta\gamma$	$-\cos\theta_1\cos\theta_0$	$-\cos\theta_1\sin\theta_0$		
0.6$\pm$0.3 MeV	$\omega \to \eta\gamma$	$-\sin\theta_1\cos\theta_0$	$-\sin\theta_1\sin\theta_0$	$-\sin\theta_1\sin\theta_0$	0.01 MeV
	$\rho \to \pi\gamma$	1			0.16 MeV
($\sim$1 MeV)	$\eta' \to \rho\gamma$	$\sqrt{3}\sin\theta_0$		$\sqrt{3}\cos\theta_0$	0.5 MeV
	$K^{*0} \to \bar{K}^0\gamma$	-2			0.24 MeV
	$K^{*\pm} \to K^\pm\gamma$	1			0.06 MeV
	$\rho \to \eta\gamma$	$\sqrt{3}\cos\theta_0$		$-\sqrt{3}\sin\theta_0$	0.09 MeV
				$\theta_0 = 10°$ $\theta_1 = 35°$	

[a] Particle Data Group, April 1972.

around any possible structure in the region of the $A_2(1320)$ boson. Such a possibility was first reported by the CERN MM experiment in which the mass spectrum recoiling against the proton (consisting mainly of the 3π final state) in the reaction $\pi^- p \to p\, MM$ showed a statistically significant dip in the center of the A_2 peak (Chikovani *et al.*, 1967). Subsequent experiments exploring both the KK (Foley *et al.*, 1971; Grayer *et al.*, 1971) and 3π (Bowen *et al.*, 1971) decay modes are devoid of such structure, the observed mass spectra being well described by a Breit–Wigner shape. As such, in this discussion, the properties of the $A_2(1320)$ will be those derived from a single resonance. The spin parities of all the member states have been measured to be 2^+, and the differences in the square of the $K(1420)$ and $A_2(1320)$ masses to be equal to 0.27 (GeV)^2, slightly larger than the 0.22 (GeV)^2 obtained for the 0^- and 1^- nonets. For these higher mass states there are two categories of decay modes, the $2^+ \to 1^-0^-$, and $2^+ \to 0^-0^-$ sequences. As shown in Table XVIII, the former involves one unknown A_a^8, and the latter two unknowns, A_s^8 and A_1. Singlet–octet mixing is again present with θ_2 and θ_1 being the mixing angles for the 2^+ and 1^- nonets, respectively. Using the Gell-Mann/Okubo mass formula and the known masses, we obtain $|\theta_2| \simeq 30°$, similar to the value for the 1^- multiplet. There are five known rates for the $2^+ \to 1^-0^-$, and six for the $2^+ \to 0^-0^-$ sequences. The comparisons with the $SU(3)$ predictions are displayed in Table XV, the resultant χ^2 probabilities being both $\simeq$ 15%, rather good with the $f(1250)$ and $f(1515)$ decays into 2π and $K\bar{K}$ modes being a sensitive test of mixing. This is due to the fact that, in spite of identical quantum numbers and similar masses, the $f(1250)$ decays predominantly into the $\pi\pi$ mode and very little into the $K\bar{K}$ mode, with the $f(1515)$ performing in the directly opposite manner. In addition, all the low rate modes are well reproduced. The individual contributions to the χ^2 are noted in Figure 14 for both the $2^+ \to 0^-0^-$ and $2^+ \to 1^-0^-$ sequences. In the former, it is seen that the $f° \to \pi\pi$ decay rate serves to determine $|A_1|$ and that it, coupled with the rate for $f'(1515) \to K\bar{K}$, determines the mixing angle rather precisely, the value being $35 \pm 9°$ in excellent agreement with that derived from the mass formula. On the other hand, the smaller A_2 decay modes, $\pi\eta$ and $K\bar{K}$, are the most sensitive to the $|A_s|$, while its dominant $A_2 \to \pi p$ mode plays a similar role for $|A_a|$. This is to be contrasted with the shallower slopes for the various $K(1420)$ decay rates.

The sensitivity of the decay rates of the 2^+ multiplets to the isoscalar factors has also been investigated via the permutation of these $SU(3)$ factors in a manner similar to that discussed for the baryons. This has been done for both decay sequences, $2^+ \to 1^-0^-$ and $2^+ \to 0^-0^-$, the results displayed graphically in Figure 10. The $SU(3)$ combination is preferred in both instances by rather large and significant factors. In essence, the $J^P = 2^+$ nonet behaves in an ideal manner from the $SU(3)$ point of view.

D. $J^P = 0^+$

We now turn to the speculative segment of the $SU(3)$ investigation of bosons. As in the case of baryons, the difficulties arise from the lack of knowledge of the spin parities of many of the constituent states, but here they are further compounded by the question of their actual

TABLE XVIII. $J^P = 2^+$. Tabulation of masses, total widths, and experimental and $SU(3)$ predicted partial decay widths for the constituent members of the multiplet.

$J^P = 2^+$

$8\otimes 8$: $2^+ \to 1^-0^-$; $\chi^2 = 1.5$; $NC = 2$; $|A_a^8| = 159.8 \pm 3.3$; $\theta = 48° \pm 45°$

$8\otimes 8$: $2^+ \to 0^-0^-$; $\chi^2 = 6.4$; $N = 4$; $|A_s^8| = 47.2 \pm 1.3$; $\theta = 34.5° \pm 9°$

$|A_1| = 85.5 \pm 5.4$; $\theta =_{GMO} = 30° \pm 10°$

M_0, Γ_0 (MeV)	Decay mode	Exptl. (MeV) $\to 1^-0^-$	$\to 0^-0^-$	SU_3 Pred. Γ (MeV)
K^{**}: 1421.3 ± 26 [a]	$K^{**} \to Kp$	10 ± 3.5 [b]		8.0
	$\to K\pi$		63 ± 7.5 [b]	62.5
	$\to \pi K^*$	30 ± 6 [b]		27.6
	$\to K\omega$	3.2 ± 3 [b]		2.4
	$\to K\eta$		< 2.6 [b]	2.0
f': 1513 ± 7	$f' \to K^*\bar{K} + \bar{K}^*K$	8.3 ± 8.3 [c]		8.3
73 ± 23	$\to K\bar{K}$		55 ± 22 [c]	55.1
	$\to \pi\pi$		< 14	0
	$\to \eta\eta$		< 37	14.3
f^0: 1277 ± 4 [d]	$f^0 \to K\bar{K}$		...	6.0
183 ± 15	$\to \pi\pi$		150 ± 12 [e]	149.9
	$\to \eta\eta$		...	0.2
A_2: 1312 ± 7 [f]	$A_2 \to \pi p$	81 ± 6 [a]		83
106 ± 8	$\to K\bar{K}$		6.1 ± 1.0 [a]	7.7
			17 ± 2.0 [a]	13.8
	$\to \pi\eta'$		1.2 ± 1.2 [a]	0.02

[a] Particle Data Group, April 1972.
[b] Aguilar-Benitez *et al.*, 1971.
[c] Barnes *et al.*, 1967; Ammar *et al.*, 1967.
[d] Flatte *et al.*, 1971.
[e] Beaupre *et al.*, 1970; Oh *et al.*, 1971.
[f] Grayer *et al.*, 1971; Foley *et al.*, 1971; Bowen *et al.*, 1971.

existence. There are few if any bona fide identified members of a 0^+ multiplet, although there are many candidates. The more recent developments have occurred with respect to the possible $I = 0$ members, the so-called S^* mesons. Studies of $(K_1^0 K_1^0)$ mass distributions clearly indicate the presence of a resonance in the mass region 1040–1070 MeV (Crennell *et al.*, 1966; Beusch *et al.*, 1967; Alitti *et al.*, 1968; Phildelphia Conference on Experimental Meson Spectroscopy, 1968 and 1970) with a poorly determined width of 40–100 MeV. The continuing and more detailed studies concerning $\pi\pi$ phase shifts now indicate the presence of a pole in the $\pi\pi$ scattering

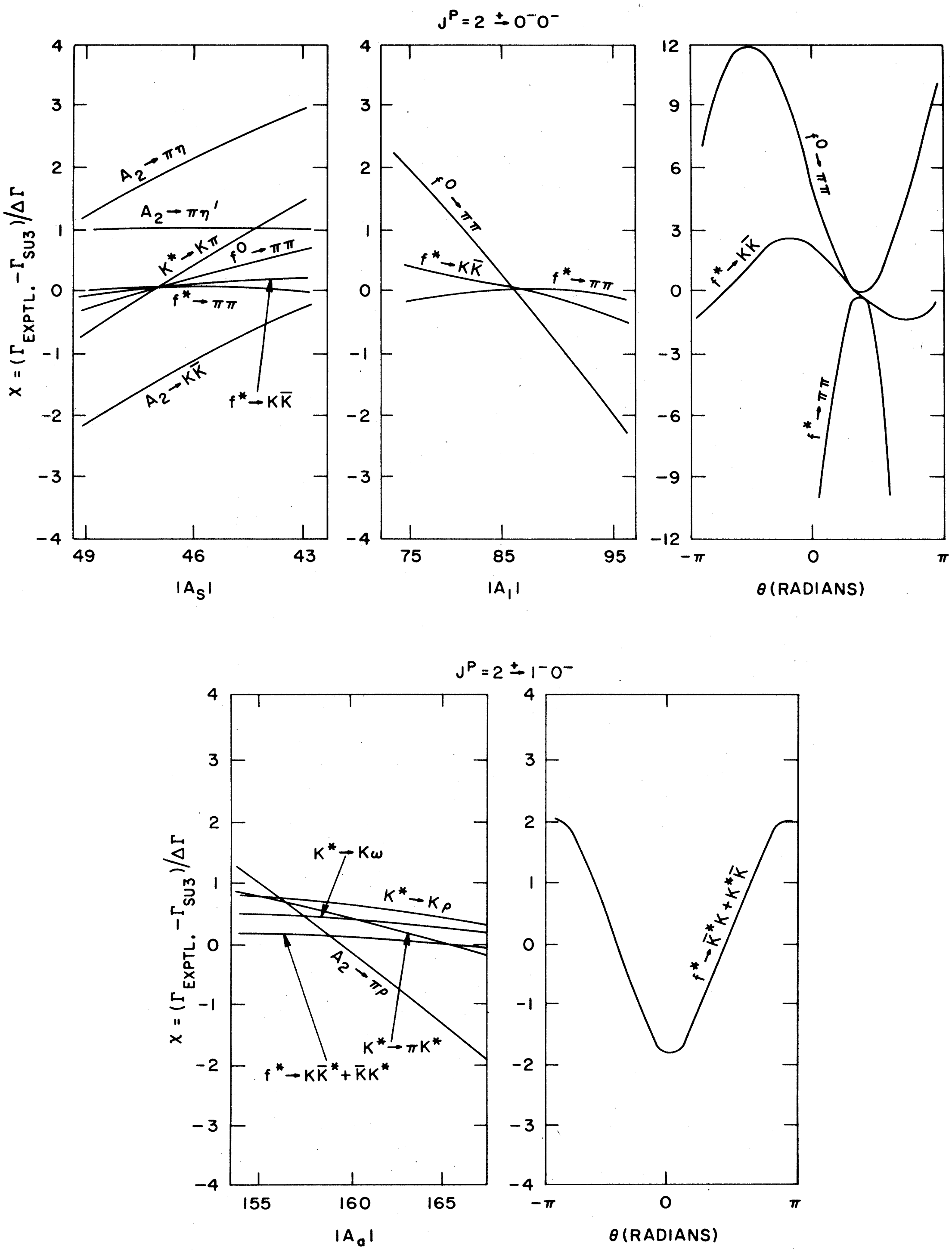


FIG. 14. $J^P = 2^+$ nonet; contribution of $|A_8|$, $|A_1|$ and θ to the χ for the $2^+ \to 0^-0^-$ sequence; contribution of $|A_8|$, θ to χ for the $2^+ \to 1^-0^-$ sequence.

amplitude at a mass $\simeq 980$ MeV and a width $\simeq 40$ MeV with errors of $\simeq \pm 10$ MeV on this latter quantity (Protopopescu, 1973). There is a remote possibility that these are indeed alternate decay modes of the same object; however the difference in the observed mass values strongly suggests that there are two distinct resonances, one above and one below the $K\bar{K}$ threshold.

The existence and properties of a possible $I = 1$, $\delta(960)$ meson have been controversial for many years. The first evidence for such a state was presented by the CERN *MM* experiment (Chikovani *et al.*, 1967) followed by several contradictory experiments involving a variety of reactions. Supporting evidence has since been presented from several bubble chamber studies involving both K^-p and $\bar{p}p$ interactions. Three separate bubble chamber experiments (Barnes *et al.*, 1969; Ammar *et al.*, 1970; Mulvey) with incoming K^- momenta $\simeq 4-5$ GeV/c observe a $3\sigma\,\delta^- \to \pi^- n(550)_N$ peak recoiling against a $\Sigma^+(1385)$ in the reaction $K^-p \to \Lambda\pi^+\pi^- n_N(550)$. There is also some evidence for the $D(1285)$ (see later discussion on 1^+ bosons) decaying into a $\pi\delta$ final state in $\bar{p}p$ annihilations. In summary, there is reasonable but not overwhelming evidence for an $I = 1$ $\delta(960)$ rather narrow resonance $\Gamma \leq 30$ MeV, with a dominant $\pi\eta(550)$ decay mode and $J^P = 0^+$.

The question then arises, where is the K member of the multiplet? If one accepts the three above noted states as members of a possible 0^+ nonet, then utilizing Eqs. (3) and (5) one expects the K mass to lie in the mass range $975-1045$ MeV as the mixing angle varies from $0°$ to $90°$. Furthermore, the width is expected to be similar to the δ member as deduced from a comparison of the isoscalar factors for the δ and K decays

Sequence $0^+ \to 0^-0^-$

$$\begin{aligned} \delta(960) &\to \pi\eta^0(\sqrt{10}/5)A_S^8\cos\theta_0, \\ K &\to K\pi(3/\sqrt{10})A_S^8. \end{aligned} \tag{27}$$

For any reasonable mixing angle $0°-45°$ the K should be 2.25 to 4.5 times broader than the δ width; therefore $\Gamma \leq 135$ MeV. A systematic study of $K\pi$ phase shifts in a manner analogous to that performed for the $\pi\pi$ system in the $1-$GeV mass region is likely to be a fruitful approach for uncovering such a resonance. The expected width, however, dictates the necessity for large quantities of data for enhancing the chances of success. Bump hunting is an alternate but less enticing approach, since the production cross section is expected to be small, $\simeq 10\mu b$, comparable to that for producing the other members (S^*, δ), and since the $S^*(955)\pi\pi$ state itself has not been observed as a bump in the $\pi\pi$ mass spectrum.

Considering all the difficulties, it will be quite some time before a 0^+ multiplet achieves a status comparable to the 0^-, 1^-, and 2^+ nonets already discussed.

E. $J^P = 1^+$

Any 1^+ multiplet is expected, on general grounds, to have its members in a mass region between the 0^- and 2^+ nonets. Several candidates have been reported in the literature in various categories, $I = 0$ nonstrange and $I = 1/2$ strange mesons with varying degrees of persuasion. The actual $J^P = 1^+$ particle mass spectra may indeed be more complex since two varieties are expected to exist: $J^{PC} = 1^{++}$ and 1^{+-}. Because of this, the simple Gell-Mann/Okubo mass formula with singlet octet mixing will not necessarily be obeyed. The masses of the members of these multiplets should lie near one another, in particular the K members, which can thereby interfere with each other. This possibility does not seem to occur for the negative parity vector states, 1^-, since only the 1^{--} nonet has been observed, no example of a 1^{-+} state being yet unearthed probably due to its exotic nature $(0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$, etc.,) multiplets cannot be accomodated in a simple quark model classification. (See later discussion, Sec. VI.) With these preliminaries we proceed to discuss the possible composition of axial vector multiplets.

The $J^{PC} = 1^{++}$ sequence has several candidates (Particle Data Group, 1972). These are the $A_1(1100)I = 1$; $D(1285)$, $E(1420)$, $I = 0$; and a host of K's, the most prominent being the $K(1240)$, $K(1320)$, and $Q(1300)$. A rather broad 3π resonance recoiling against a proton in πp interactions was first reported by Goldhaber *et al.* (1964), with its subsequent delineation into a broad A_1 and a narrower A_2 by the Aachen-Berlin-Birmingham-Cern collaboration (Aderholz *et al.*, 1964) and Chung *et al.* (1964).

Such a broad structure has been observed in numerous other experiments with the additional feature of the possible presence of a narrower resonance in the same mass region. The main difficulty has been the nonresonant type of behavior of the 1^+ partial wave amplitude. Although this mass region (1100 MeV) of the $\rho\pi(3\pi)$ system is dominated by the 1^+ amplitude, its energy variation is similar to that of other background partial waves and thus not resonancelike. Most other accepted resonances show rapid phase changes in a particular partial wave as one varies the mass value under investigation. Therefore, it is still an open question as to whether the $A_1(1100)$ is a resonance; however, this behavior is not an isolated case. The $A_3(1645)$ to be discussed in the 2^- section and $Q(K\pi\pi)$ structure exhibit similar behavior, and in this they may indicate a new phenomenon coupled with resonance formation.

Both the $D(1285)$ and $E(1420)$ are rather well established resonances with $J^P = 1^+$ being the more favored assignment. In the case of the $E(1420)$ (Baillon *et al*., 1967; Lörstad *et al.*, 1969) the analysis of decay distributions also allows for $J^P = 0^-$ assignment but invokes a low-mass $K\bar{K}$ enhancement, which may or may not exist. Since the $\eta(960)$ has been clearly shown to be 0^-, we prefer a 1^+ assignment for the $E(1420)$ on the grounds that the 0^- nonet is already complete and that the 1^+ assignment is equally favorable. Neither width is well defined, but they lie in the region 30–80 MeV, respectable values for resonances, with the $K\bar{K}\pi$ decay mode common to both.

As noted earlier, there are three prominent K candidates, the $K(1240)$, $K(1320)$, and $Q(1300)$. The $K(1240)$, otherwise known as the C meson, has been mainly observed in $\bar{p}p$ annihilation experiments (Armenteros *et al.*, 1964), where it decays roughly equally into $K(890)\pi$ and $K\rho(750)$ final states. Analysis of branching ratios and angular correlations indicates $I = 1/2$ and $J^P = 1^+$. The evidence for the $K(1320)$ arises from a study of the reaction $\pi^-p \to \Lambda(K\pi\pi)$ (Crennell *et al.*, 1969), which has the dual assets of establishing $I = 1/2$ for the $K\pi\pi$ system and of being a nondiffractive process relatively free from purely kinematic enhancements. Again, J^P

$= 1^+$ is consistent, although not conclusive, with the observed correlations. The Q bump, $K\pi\pi$ enhancement at a mass of $\simeq 1300$ MeV, and a rather broad width $\simeq 200$ MeV, is another prominent 1^+ candidate. Its general properties are similar to the $A_1(1100)$ decay correlations favoring $J^P = 1^+$, produced diffractively with part or a large fraction of the effect possibly to be interpreted as a dynamical consequence, i.e., double Regge exchange.

Accepting the $A_1(1100)$, $D(1285)$, and $E(1420)$ resonances as members of a 1^{++} nonet, application of the Gell-Mann/Okubo mass formula and octet–singlet mixing restricts the mass of the K member to lying between 1240 and 1350 MeV as the mixing angle is varied between 0° and 90°. This range is sufficiently large to encompass the above K candidates so that the unraveling of the details of this multiplet is dependent on accurate partial width measurements. As noted earlier, full widths and branching ratios are still rather imprecise. This fact, coupled with the sparseness of decay modes, i.e., the $D(1285)$ cannot decay into $K(890)K$, and the conjectured K is close to threshold for its decay into $K\rho$, thereby having only a $K(890)\pi$ mode for comparison makes even this type of analysis difficult. In essence there would be one observed mode per resonance, and therefore four rates to be fit with two unknowns—A_a the antisymmetric coupling and θ the mixing angle—not a sensitive test considering all the uncertainties.

The difficulty with a possible $J^{PC} = 1^{+-}$ multiplet is simply the lack of candidates. The $I = 1$, $B(1220)$ is the only such well established resonance. Detailed studies of its $\pi\omega$ decay mode clearly fix the $J^{PC} = 1^{+-}$ (Particle Data Group, 1972; Chung, 1973). The K member can be any of the K candidates noted in the previous (1^{++}) discussion or a K resonance as yet to be uncovered. The question is whether each of these is one broad resonance composed of one broad enhancement with an accompanying narrower resonance, or even more complex and composed of multiple resonances. One method of attacking this question is to search for structure in these mass intervals via a host of nondiffractive processes, with good resolution and large statistics. We complete this discussion by noting that the $I = 0$ nonstrange counterparts with 1^{+-} are completely missing.

F. $J^P = 2^-$

The analogy of possibly 2^- multiplets with the previously discussed 1^+ resonances is rather strong. They both come in two varieties with opposite C parities, $J^{PC} = 2^{--}, 2^{-+}$; the expected masses of both multiplets are quite similar, 1650–1800 MeV; the K masses can interfere via octet–octet mixing; the A_1, Q phenomenon has a counterpart in the $A_3(1645)$, "$L(1750)$" enhancements; and finally there are few well established states in this 2^- category.

We begin with the 2^{-+} multiplet. The $A_3(1640)$ meson decaying primarily into $\pi f^0(1250)$ final state, $I = 1$, with a rather broad width of ≈ 200 MeV, has been known for some time. The interesting new developments arise from a detailed amplitude analysis of the production and decay correlation of the A_3 as produced in πp interactions at high energies, 16 and 40 GeV (Ascoli *et al.*, 1973). Essentially the phase of the 2^- amplitude does not vary with respect to any other partial amplitudes as the 3π effective mass is varied through the resonance region, although the amplitude itself has a peak at the A_3 mass. This is not the behavior expected from a normal resonance. Any conjectured K partner has not been as thoroughly investigated due to more limited data available in K compared to π exposures. However, there is evidence for an "L" meson at a mass of 1750 MeV produced diffractively and coupled to the $K\pi\pi$ final state and not $K\pi$. This suggests that the spin parity belongs to the 0^-, 1^+, 2^- series and its mass value is appropriate for the 2^- assignment. Again, there may be two phenomena occurring, one a broad enhancement, width ≈ 250 MeV, coupled to the $K(1420)\pi$ channel (Barbaro-Galtieri *et al.*, 1969), and the other, a narrower bump, width ≈ 100 MeV, with several decay modes, mainly variations of the $K\pi\pi$ final state (Barsch *et al.*, 1966; Aguilar-Benitez *et al.*, 1970; Colley *et al.*, 1971). As such the former could be associated with the A_3, both broad and only decaying into a (0^-2^+), i.e., $A_3 \to \pi f^0(1250)$ and "L" (1750) $\to \pi K(1420)$ final state. The narrower $L(1750)$ then could be placed in the 2^{--} multiplet to be discussed next.

The only relevant candidate for the 2^{--} multiplet is the $I = 1 F_1(1540)$. It has been studied mainly in $\bar{p}p$ annihilations (Aguilar-Benitez *et al.*, 1969) where the correlations in its $K\bar{K}\pi$ decay modes suggest $J^P = 1^+, 2^-$. The C parity is not well known. The present meager evidence favors $C = +1$, contrary to this assignment; however, it is far from convincing. The resolution of this question awaits the observation of the F_1 decay into multiple pion states which directly determines the C parity. As noted earlier, a likely candidate for the K member of the 2^{--} multiplet is the narrower $L(1750)$ which is mainly observed to decay into $K\pi\pi$ final state, not $\pi K(1420)$. The determination of the spin parities of these possible $L(1750)$ resonances awaits the examination of the decay correlations in reactions where the signal-to-background ratio is propitious. Unfortunately, the field of $I = 0$ candidates for either the 2^{-+} or the 2^{--} multiplets is barren, even though there should be four states in the mass region 1650–1800 MeV.

G. $J^P = 3^-$

Although higher in spin value than the 1^+, 2^- categories, the experimental situation in this 3^{--} multiplet appears less ambiguous. The $I = 1$, $g(1640)$ meson has been known and well established for years. Its 2π decay has been extensively studied yielding a width of ≈ 150 MeV and a $J^{PC} = 3^{--}$.[2] The unresolved problems revolve around other possible decay modes. Evidence has been presented for $\omega\pi$, $K\bar{K}$, and $\bar{K}K^*$ decays with varying degrees of confidence and with masses and widths consistent with those for 2π decay. The existence of such modes and firm values for the branching fractions have now to be determined. More recently a possible additional $K(1750)$ resonance has been uncovered, produced in K^-p interactions but in a nondiffractive reaction and therefore not associated with the $L(1750)$ (Carmony *et al.*, 1971; Firestone *et al.*, 1971; Aguilar-Benitez *et al.*, 1973). Its reported width is $\simeq 60$ MeV with enhancements in both the $K\pi$ and $K\pi\pi$ final states, the latter being mainly $K(890)\pi$. A study of the decay correlation of the $K\pi$ mode indicates $J^P = 3^-$; however the limited data and relatively large backgrounds yield large uncertainties in all these numbers. Evidence for at least one possible $I = 0$ member is also rather firm, this being the $\omega(1680)$ observed to decay into $\rho\pi$ (Mathews *et al.*, 1971). Its width is rather broad, $\Gamma = 150$ MeV, and an analysis of the 3π Dalitz plot favors 1^-, 2^+, 3^- spin parity assignments. We assign it to this 3^- multiplet on the basis of its mass value. Not unexpectedly, this group of particles does not satisfy the Gell-Mann/Okubo mass formula, the singlet mass, for instance, predicted to be 1.80 GeV instead of the observed 1.68 GeV. In view of this one can invoke singlet–octet mixing with the resultant φ partner expected to have a mass between 1.80 GeV and 1.90 GeV as the mixing angle varies from 0° to 45°. (If one varies the angle from 45° to 90° then the mass extends to ∞; however, one can redefine which singlet is a member of the octet, thereby restricting the mass to the same range as noted above.) The incompleteness of the data precludes any meaningful $SU(3)$ decay analysis (Graham and Yoon, 1972); the fit is almost guaranteed to be good. The interesting feature is to search for the missing member, the (1800-1900), which extrapolating from the $g(1680) \rightarrow 2\pi$ decay should have an appreciable $\varphi \rightarrow K\bar{K}$ rate ≈ 20 MeV. The total width of this φ state could be larger due to the possible contribution of the $\varphi \rightarrow KK(890)$ mode. In essence this 3^- multiplet, although far from established, is clearly tenable with good prospects of realizing respectability.

VII. BOSON SYSTEMATICS

Although there is fragmentary information for numerous boson $SU(3)$ families, there are still only three multiplets which are complete and well established, namely those with $J^{PC} = 0^{-+}$, 1^{--}, and 2^{++}. All these display a nonet structure with mixing angles of 10°, 40°, and 30°, respectively. As noted in the previous sections, there are several established singlet ($I = 1$) states, the $B(1220)$, 1^{+-}; $g(1640)$, 3^{--} with the status of the $A_1(1100)$, 1^{++}; $A_3(1640)$, 2^{-+} being somewhat uncertain, and that of the $\delta(960)$, 0^{++}; $\rho(1600)$, 1^{--}, $F_1(1540)$, 2^{--} very uncertain. As already discussed, the A_1, A_3 [as well as $Q(1200)$, $L(1750)$] regions may indeed be more complex in that diffraction dissociation phenomena, in addition to resonance formation, may be occurring in these same mass regions. With these reservations there still seems to be evidence for nonet structure for the 0^{++} and 1^{++} multiplets, there being clear singlet candidates, with the K members yet to be clearly deciphered. The 3^{--} multiplet is almost complete, the ϕ member which should lie between 1800 and 1900 MeV being missing. The evidence for boson states is summarized in Table XIX, in which members of various spin parities are noted. The order is that predicted by the simple quark model in which all meson states are assumed to correspond to quark–antiquark system with increasing angular momentum.

Of possible equal interest to the noted resonances are those that are missing, namely bosons with $J^{PC} = 0^{+-}$, 1^{-+}, 2^{+-}, 3^{-+}, and 0^{--}, all forbidden by this simple quark picture. The other type of nonexistent "exotic" resonant state is that with (isospin, hypercharge) (I, Y) also not allowed by the simple quark model, or equivalently corresponding to a higher $(SU3)$ representation than 1 or 8. Examples are $(I, Y) \equiv (2,0)(1,2)(3/2,1)$ states, all sought as $\pi^+\pi^+$, K^+K^+, $K^+\pi^+$ effective masses and not observed. As such, these results complement the baryon study in the lack of exotics and the validity of the simple quark model.

For completeness we also discuss the possibility of higher symmetries for the boson spectrum. When one includes spin in addition to isospin and hypercharge, namely combining spin with $SU(3)$, this leads to $SU(6)$. The expected representations are $1 + 35$, where the $SU(3)$ decomposition of the 35 is $[SU(3), J] = (8 + 1, 1) + (8, 0)$. It is natural to associate the 0^- octet and 1^- nonet with this representation with the $\eta(960)$ being the lone singlet. Since the spin 1 combination consists of a singlet plus an octet, there is mixing, and the relevant mass formulas are (Gursey and Radicati, 1964, O'Raifertaigh, 1968):

$$\begin{aligned} &\text{(a)} \qquad (m_{K^2} - m_{\rho^2})_{1^-} = (m_{K^2} - m_{\pi^2})_{0^-}, \\ &\text{(b)} \qquad m_{\omega^2} = m_{\rho^2}, \\ &\text{(c)} \qquad 2(m_{K^2})_{1^-} = (m_{\rho^2})_{1^-} + (m_\phi)^2_{1^-}, \end{aligned} \tag{28}$$

where (a) and (c) are satisfied to better than 5%; the largest deviation being the inequality of the ρ and ω masses. In essence, the correlation between masses in the 0^- and 1^- multiplets is well reproduced [relation (a)]as is the 1^- mixing angle [relation (b) and (c) being equivalent to $\cos^2\theta = 1/3$]. Such mass formulae are by no means unique, depending upon assumptions concerning the transformation properties of the mass matrix, in this case the rather plausible assumption that it transforms as a 35 representation of $SU(6)$. In this particular case the predictions are equivalent to those derivable from the simple quark model. The higher spin members are then obtained by adding angular momentum to the quark–antiquark system, thereby attaining:

$$\begin{array}{ll} L = 1 & J^{PC}\,2^{++}\,1 + 8 \\ & 1^{++}\,1 + 8 \\ & 0^{++}\,1 + 8 \\ & 1^{+-}\,8 \\ & 1^{+-}\,1 \\ L = 2 & J^{PC}\,3^{--}\,1 + 8 \\ & 2^{--}\,1 + 8 \\ & 1^{--}\,1 + 8 \\ & 2^{-+}\,8 \\ & 2^{-+}\,1 \end{array} \tag{29}$$

and so on, duplicating the expected states previously noted in Table XIX for the bosons. The lack of completed $SU(3)$ submultiplets makes further detailed mass examination unprofitable.

Possible boson systematics can be further explored via the Chew–Frautschi plots (1961). Such a natural spin parity series is displayed in Fig. 15 including both the strange and nonstrange members. One notes that the trajectories for J vs M^2 are indeed linear, with a slope of $\simeq 1$. For the $I = 1$ category there are three well-known points, the ρ, g masses and the intercept at $m^2 = 0$ whose value is 0.5 as deduced from an analysis of the charge exchange reaction $\pi^- p \rightarrow \pi^0 n$ (Sonderegger, *et al.*, 1965 and 1966). As is well known, the trajectory connecting

TABLE XIX. Listing of reasonably well established boson states according to their J^P and I spin.

J^P \ I	1	1/2	0	0	$\|\theta\|$
0^{-+}	$\pi(140)$	$K(495)$	$\eta(550)$	$\eta'(960)$	10°
1^{--}	$\rho(750)$	$K(895)$	$\omega(785)$	$\phi(1020)$	40°
1^{+-}	$B(1220)$	$[K(1320)]$			
0^{++}	$\delta(960)$	$[K(975-1045)]$	$S(980)$	$S^*(1060)$	
1^{++}	$A_1(1100)$ $\hookrightarrow \pi\rho$	$[K(1240-1350)]$ Q	$D(1285)$	$E(1420)$	
2^{++}	$A_2(1320)$	$K(1420)$	$f(1210)$	$f'(1175)$	35°
2^{-+}	$A_3(1640)$ $\hookrightarrow \pi f^\circ$	$L'(1750)$ $\hookrightarrow \pi K(1420)$			
1^{--}	$\rho(1600)$ $\hookrightarrow 4\pi$				
2^{--}	$F_1(1540)$ $\hookrightarrow 2\pi$	$K(1750)$ $\rightarrow K\pi\pi$			
3^{--}	$g(1640)$ $\hookrightarrow 2\pi$	$K(1760)$ $\hookrightarrow K\pi$	$\omega(1680)$	$[\phi(1800-1900)]$	

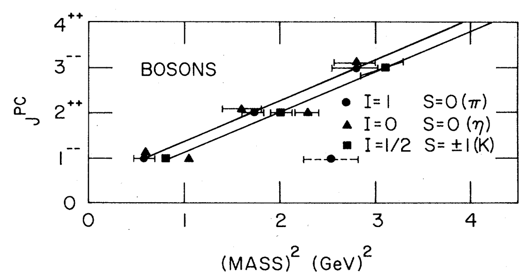


FIG. 15. Plot of J^P vs M^2 for the members of the 1^{--}, 2^{++}, 3^{--}, 4^{++} ··· boson sequence.

these points intersects the 2^+ axis at the A_2 mass, i.e., exchange degenerate. A similar pattern exists for the $I = 1/2$, K components, the slope of this trajectory being parallel to that for the ρ, A_2, g, and equal to 1, and exchange degenerate in that it intersects the $K(1420)$ at $J = 2^+$. Although the singlet ($I = 0$) information is not as complete, the $\omega(785)$, $f^0(1250)$, $\omega(1680)$ trajectory lies rather close and parallel to that previously noted for $I = 1$; the $\phi(1020)$, $f'(1515)$ sequence strongly suggesting the existence of a $\phi(1815)$, with 3^{--}. For completeness we include the possibility of the $\rho'(1600)$, 1^{--}, which, if it exists, is easily accommodated in the d-shell quark structure (see Table XIX). The advent of the Veneziano formalism (Veneziano, 1968), with the accompanying possibility of daughter trajectories, necessitates a few additional comments. The daughters of the ρ, A_2, g sequence would be $I = 1$, $J^{PC} = 0^{+-}$, 1^{-+}, 2^{+-} states, all exotic and nonobserved. The granddaughters of the A_2, g resonances would possess $J^{PC} = 0^{++}$, 1^{--} values, respectively. As such, the $\rho'(1600)$ could also be viewed as a descendant of the g meson, this being the only such possibility recorded to date.

For completeness we include similar graphs for the π, B, A_3 sequence (Figure 16) as well as the A_1, F_1 resonances (Figure 17). The slopes for those $I = 1$ cases is $\simeq 0.8$ with the πA_3 being degenerate with the $B(1220)$. The

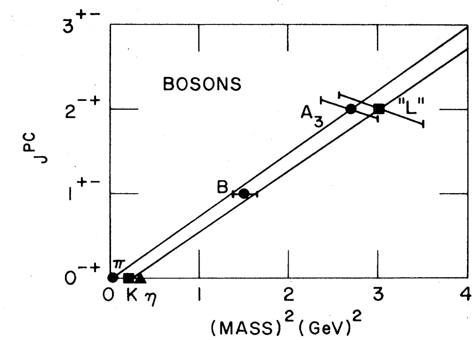


FIG. 16. Plot of J^P vs M^2 for the members of the 0^{-+}, 1^{+-}, 2^{-+} ··· boson sequence.

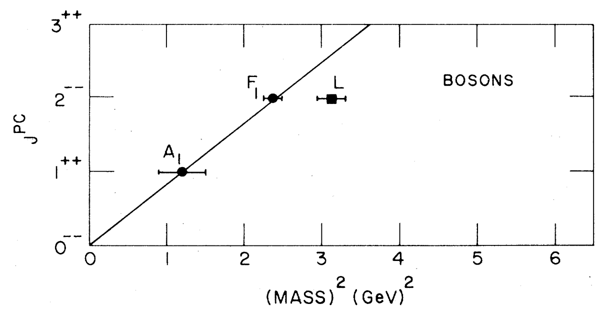


FIG. 17. Plot of J^P vs M^2 for members of the 0^{--}, 1^{++}, 2^{--}, 3^{++} ··· boson sequence.

fragmentary evidence for the K members indicates that these trajectories will have similar slopes and behave in a manner similar to their $I = 1$ partners.

Major efforts are clearly required in this lower mass region, < 2 – GeV mass, to attain an understanding of the boson spectra. These include investigation into a possible 0^{++} multiplet; search for singlet ($I = 0$) and $K(I = 1/2)$ members of 1^+, 2^-, and 3^- multiplets; the question of the existence of exotics, possibly not coupled to two pseudoscalar mesons; detailed measurement of rates for electromagnetic decays of pseudoscalar and vector particles; and an understanding of the A_1A_3, $Q'L'$ phenomena. Quite a task.

VIII. CONCLUSIONS

An examination of the spectra of known boson and baryon states shows clear evidence for numerous $SU(3)$ multiplets. The detailed study of mass relationships, decay rates, and interference phenomena shows remarkable agreement with that expected from the most simple unbroken $SU(3)$ symmetry scheme. Among the baryons there is good evidence for three nonets and two octets, with strong support for the existence of an additional two nonets. The boson category has fewer well established multiplets, namely three, but strong prospects for the completion of six more multiplets. The data accumulated to date is of sufficient accuracy and depth to establish that the observed agreement with $SU(3)$ is not fortuitous. In fact, the mass agreements, where testable, are good to an accuracy of a few percent, much better than expected considering the wide range of mass values within a given multiplet. Therefore the introduction of broken symmetry schemes, considering the present state of the data, certainly seems premature. Extensions of these $SU(3)$ results to higher symmetries such as $SU(6)$ and the simple quark model appear promising; however, many more multiplets have to be uncovered before sensitive tests of such possibilities can be performed. In this regard several comments are in order:

(1) Searches for exotic resonances have proved negative. The only possible candidates are the so-called Z^* baryons, strangeness plus one, $I = 0, 1$, bumps found in the total cross section measurements. A proper amplitude analysis through this mass region demonstrating resonant behavior, although crucial, has yet to be done.

(2) Although partial amplitude analyses have been performed for the pion–nucleon and kaon–nucleon systems, only resonances with reasonably large KN and πN couplings have been derived with a high degree of confidence. A host of other resonances, highly inelastic, have been reported with very little overlap in their properties as determined by different groups. The problem of deciphering resonant states with small elastic couplings has to be resolved before further progress is achieved in this field.

(3) General investigation of the electromagnetic decays of baryon and boson states would seem to be extremely profitable. This range from searches for electromagnetic Ξ and Σ decays to precise measurement of K^* and $\eta'(960)$ decay rates.

In reviewing this host of data it is apparent that the number of resonances with mass less than 2.5 GeV is far from exhausted; on the contrary, the main interest is to delineate further their number and properties in this still relatively low mass range. To this end, both theoretical and experimental techniques must be improved, the former in establishing convincing and unique methods of extracting the resonances and their parameters from the wealth of data accumulated, and the latter in increasing the resolution of detecting devices as well as the data in order to clearly exhibit resonant structures, preferably in reactions where signal-to-noise is favorable. Needless to say, these tasks are formidable; however, their resolution may pay enormous dividends since in retrospect an examination of the spectrum of atomic hydrogen in the early 20's was sufficient in principle to provide for an understanding of atomic structure, thereby leading to the formulation of quantum mechanics. Hopefully an understanding of these particle spectra in this limited mass range could lead to a similar gold mine of insight into natures's workings.

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