# Magnetic flux structures in superconductors—a conference summary*

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The proceedings of the International Conference on Magnetic Structures in Superconductors held at Argonne National Laboratory, Argonne, Illinois on 5–8 September 1973 are reviewed and summarized. Recent experimental and theoretical developments in the field of magnetic flux structures in both type-I and type-II superconductors are described, current research directions are discussed, and expected future developments are outlined.

## CONTENTS

## I. INTRODUCTION

Forty years after the discovery of the Meissner effect, magnetic flux structures in superconductors were the subject of an international conference[1] held at Argonne National Laboratory, 5–8 September 1973, and attended by 120 scientists from ten countries. The conference program focused on fluxoids, flux tubes, and normal domains in type-I and type-II superconductors, including experiments and theory relating to their internal structure, dynamics, and interaction.

When a superconductor is placed in a weak magnetic field, the field is completely expelled from the superconducting material except for a layer at the surface with a thickness given by the penetration depth. This phenomenon is generally referred to as the Meissner effect (Meissner and Ochsenfeld, 1933) and represents perhaps the most fundamental aspect of superconductivity. Here the dimensions of the superconductor perpendicular to the applied magnetic field are assumed to be relatively small, such that demagnetizing effects can be neglected. If the applied field is sufficiently strong and if demagnetization becomes appreciable, for example in a thin superconducting plate oriented perpendicular to the field, then magnetic flux penetrates through portions of the superconductor, and the material splits up into normal and superconducting domains. The geometrical detail of the domain structure depends, of course, sensitively on the wall energy associated with the interface between the normal and superconducting domains. This wall energy is directly proportional to the difference between the coherence length and the penetration depth of the superconductor.

In type-I superconductors the wall energy is positive, the amount of flux contained in an individual domain can be large (many flux quanta), and the domain configuration is called the intermediate state. In the normal domains the magnetic field is equal to the thermodynamic critical field $H_c$, whereas in the superconducting domains it is zero. This domain configuration is attained in the magnetic field range $H_c(1 - D) < H < H_c$, where $D$ is the demagnetization coefficient. Type-II superconductors are characterized by a negative wall energy. In this case magnetic flux is dispersed through the material in the form of single flux quanta of flux $\varphi_0 = hc/2e = 2.07 \times 10^{-7}$ G cm$^2$, the smallest possible unit of magnetic flux. This distribution of flux is referred to as the mixed state, attained in the field range $H_{c1}(1 - D) < H < H_{c2}$, where $H_{c1}$ and $H_{c2}$ are the lower and upper critical field, respectively.

The rapid progress achieved recently in the field of magnetic flux structures in both type-I and type-II superconductors to a large extent can be attributed to the application of novel experimental techniques and more sophisticated versions of some traditional methods. Outstanding examples are the flux decoration technique invented by Träuble and Essman (1966) based on the combination of a Bitter method with electron microscopy, the high-resolution magneto-optical technique developed by Kirchner (1971a) utilizing for the first time magneto-optical films of some rare earth compounds with a specific Faraday rotation several orders of magnitude larger than the previous materials, Sharvin's point contact probe (Sharvin, 1965; Sharvin and Landau, 1970) by which changes in the contact resistance indicate the passage of normal or superconducting domains under the contact, as well as electron tunneling, electron wave interference, neutron diffraction, and nuclear magnetic resonance.

Many experiments on the classical transport properties of superconductors and on their close relation to the dynamics of the magnetic flux structures in these materials have provided a rich source for the development and growth of the field. The discovery and investigation of

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[1] The International Conference on Magnetic Structures in Superconductors was sponsored by the American Physical Society, Argonne National Laboratory, the International Institute of Refrigeration, the International Union of Pure and Applied Physics, and the National Science Foundation. The program, the list of participants, and the abstracts of all papers of this conference are included in an Argonne National Laboratory report, ANL-8054, available from the National Technical Information Service, U. S. Department of Commerce, 5285 Port Royal Road, Springfield, Virginia 22151. Authors cited in the text of ths paper, but not listed in the References, were participants in this Conference where their papers were presented.

the motion of magnetic flux structures induced by the Lorentz force of an electric current or by the thermal force of a temperature gradient has been an important development of the past decade. Shoenberg (1952) noted that the intermediate-state structure in a type-I superconductor would become unstable in the presence of an electric current and that the domains would be expected to move in a direction perpendicular to the current and the magnetic field. He appears to have been the first to suggest the phenomenon of current-induced flux motion. Gorter (1957) first proposed flux flow as a resistive mechanism in a type-I superconductor. This concept of flux motion was later extended by Gorter (1962) and Anderson (1962) to flux lines in type-II superconductors. The first experimental evidence for the appearance of flux-flow resistance in the mixed state was obtained by Kim, Hempstead, and Strnad (1963, 1965). Subsequently, the Hall, Peltier, Ettinghausen, and Nernst effects induced in a superconductor by the motion of magnetic flux structures were also observed (Huebener, 1972).

The flow of single-quantum flux lines or of multiple-quantum flux tubes through a superconducting medium results in an electric field oriented perpendicular to the direction of flux motion and to the flux-line axis. The flux-flow voltage is a direct consequence of the Josephson (1962) relation, $\partial\varphi/\partial t = 2eV/\hbar$, which connects the time derivative of the phase difference $\varphi$ of the superconducting wave function (order parameter) between two points of the superconductor with the chemical potential difference between these points. Here $V$ is the voltage, $e$ the elementary charge, and $\hbar$ Planck's constant divided by $2\pi$. Dissipation associated with flux flow may be regarded as arising from eddy-current damping in the moving normal core.

The first theory of the domain structure in the intermediate state was set forth by Landau (1937) more than 35 years ago. Assuming a superconducting plate of thickness $d$ placed perpendicularly in a magnetic field and a periodic arrangement of straight and parallel superconducting and normal domains (laminae), Landau obtained for the periodicity length $a$ from center to center between two neighboring normal or superconducting domains, $a = [\Delta d/\Phi(h)]^{1/2}$. Here, the (positive) wall energy parameter $\Delta$ is approximately equal to the coherence length minus the penetration depth, and $\Phi(h)$ is a dimensionless numerical function of the reduced field $h = H/H_c$ (Haenssler and Rinderer, 1967). Despite the fact that in type-I superconducting films or plates the domains are usually curved and randomly arranged, the above theory, called the nonbranching model, agrees reasonably well with experiment (Haenssler and Rinderer, 1967; Huebener, Kampwirth, and Rowe, 1972; Huebener and Kampwirth, 1972) if the specimen thickness does not become very large. In rather thick samples of type-I superconductors, the normal domains divide into branches as they approach the specimen's surface (Haenssler and Rinderer, 1967; Kirchner, 1971b; Träuble and Essmann, 1966). Such behavior was predicted in Landau's branching model (Landau, 1938a, 1943). The transition from the nonbranching to the branching domain structure as the specimen's thickness increases has been theoretically examined recently by Hubert (1967).

The first theory of type-II superconductivity was developed by Abrikosov (1957) from the general phenomenological theory of Ginzburg and Landau (1950). He proposed a structure of the mixed state consisting of a two-dimensional lattice of vortex lines, each vortex containing a single flux quantum. The most direct experimental proof of the Abrikosov structure has been given by Essmann and Träuble (1967) with their high-resolution been confirmed by many experiments (Serin, 1969). Since the Abrikosov theory was shown by Gor'kov (1958, 1959) to be valid only in a restricted regime near the transition temperature, it has been extended in various ways by numerous authors, the generalizations being applicable to different regimes of temperature, magnetic field, and mean free path of the electrons (Fetter and Hohenberg, 1969). When the coherence length is much smaller than the penetration depth, an isolated vortex line may be regarded as a narrow tube of normal phase, of radius about equal to the coherence length, surrounded by superconducting phase. The local magnetic field is rather large in the normal core but, because of shielding supercurrents that circulate around the axis within a penetration depth of the core, decays almost exponentially in the superconducting phase. Although this phenomenological model, called the London model, is quite useful in providing a qualitative understanding of many of the properties of type-II superconductors (Fetter and Hohenberg, 1969; de Gennes, 1966), one must, of course, return to the extensions of the Abrikosov theory to obtain accurate quantitative results. Current theoretical work on the static vortex structure includes detailed numerical solutions of the microscopic vortex structure (Kramer and Pesch, 1973; Kramer, Pesch, and Watts–Tobin, 1974) and comparison with experimental results from, say, neutron diffraction. These calculations are based on Eilenberger's reformulation (1968) of the Gor'kov theory (1958, 1959) in terms of simpler transportlike equations.

An interesting recent development has been the discovery of a new type of magnetic flux structure consisting of a mixture of flux-free domains (Meissner phase) and domains containing a flux-line lattice (Shubnikov phase) with constant lattice parameter (Träuble and Essmann, 1967; Krägeloh, 1970; Sarma, 1968). The new behavior is accompanied by a first-order phase transition at $H_{c1}$, and is explained in terms of a long-range attractive interaction between vortices (Jacobs, 1973).

According to the Landau model of the intermediate state, as the sample thickness decreases, the domain size of the magnetic structure decreases, and the positive wall energy contributes more and more to the free energy of the superconductor. As seen from the expression for the periodicity length $a$, since the function $\Phi(h)$ is typically of order unity, this length becomes of the order of the wall energy parameter $\Delta$ itself when the film thickness $d$ is about equal to $\Delta$. Furthermore, the critical field perpendicular to the superconducting film decreases with decreasing film thickness and, from the Landau model, would become zero when the film thickness is about equal to the wall energy parameter $\Delta$. To resolve these difficulties, Tinkham (1963) pointed out that films of a type-I superconductor with a thickness less than the coherence length assume a vortex state in a perpendicular magnetic field similar to the mixed state in type-II superconductors. Tinkham's theory subsequently was extended by others (Maki, 1965; Pearl, 1964; Lasher, 1967; Fetter and Hohenberg, 1967), and the vortex structure in thin film type-I superconductors has been observed directly with a high-resolution Bitter method (Barbee, 1969; Dolan and Silcox, 1973; Boersch, Kunze, Lischke, and Rodewald, 1973).

Following the discovery of flux flow as a dissipative process in type-II superconductors, phenomenological theories of flux flow were proposed by Bardeen and Stephen (1965), van Vijfeijken (1968), and Nozieres and Vinen (1966). All three models assume a rather simple structure of the vortex core, which is treated as being fully normal and where dissipation takes place via quasiparticle scattering by the lattice. Although these theories are fairly successful in their predictions for the flux-flow resistivity, they generally do not account for the experimentally observed Hall effect data, nor do they explain the thermomagnetic transport phenomena. A more fundamental theoretical approach for treating the dynamics of vortex structures and for obtaining an understanding of flux flow and its role in the various transport properties is based on time-dependent extensions of the Ginzburg–Landau theory (Cyrot, 1973). Unfortunately for the dynamic case, expansions in powers of the order parameter near the transition temperature cannot legitimately be carried as far as in the static case. The difficulties are associated with the existence of a gap in the energy spectrum and the interconversion between normal excitations and superfluid. Although in the general case an extension beyond the linear term of the Ginzburg–Landau theory is impossible, such an extension is possible for a gapless superconductor containing magnetic impurities, where the difficulties are not so serious (Gor'kov and Eliashberg, 1968). Presently the time-dependent Ginzburg–Landau theory provides much insight into vortex dynamics, and semiquantitatively accounts for many transport phenomena in type-II superconductors including flux-flow resistivity, the Hall effect, and the thermomagnetic effects. However, compared with the theory of the static vortex structure, the time-dependent theory is in a relatively primitive stage, and considerably more work needs to be done.

It appears that the static vortex structure in homogeneous type-II superconductors is reasonably well understood and that, in principle, the static vortex properties of any isotropic type-II material can be reliably calculated. However, important areas for further development are anisotropic or inhomogeneous superconductors, the microscopic structure of interfaces or domain boundaries, bound states of excitations, and the influence of the metallurgical microstructure. As for our understanding of the time-dependent behavior and the dynamics of flux structure, we are just at the beginning, and a truly satisfactory microscopic model of dissipation still remains to be worked out. Electric and magnetic instabilities and the microscopic mechanism for breakdown of superconductivity are also important subjects for future development. Among the anticipated experimental advances, one can foresee techniques providing substantial increases of the resolution in time, space, and magnetic field for studying magnetic flux structures. Advances in time resolution will make a particularly strong impact, since many of the present techniques are limited to stationary conditions.

A deeper knowledge of the behavior of magnetic flux in superconductors is, of course, important for practical applications of superconductivity. Generally speaking, one wishes to prevent or reduce the energy losses associated with the nucleation of flux-containing normal regions and their motion within the superconductor. Thus, a basic understanding of these dissipative effects and of their prevention through flux pinning is desired.

In the following sections we describe in more detail the recent advances in the field of magnetic flux structures in superconductors, as they became apparent during the Argonne conference.

## II. METHODS FOR FLUX DETECTION AND THE INTERMEDIATE STATE

A most fruitful recent advance has been the high-resolution magneto-optical method developed by Kirchner (1971b) for the direct visualization of magnetic flux structures in superconductors. This technique is a more sophisticated version of the magneto-optical experiments of Alers (1957) and DeSorbo (DeSorbo and Healy, 1964), which utilized the Faraday rotation of a beam of polarized light in a magneto-optic glass due to a magnetic field along the direction of the light propagation. Kirchner's method is based on magneto-optic materials such as EuS or EuSe, usually in the form of a deposited film in intimate contact with the superconductor. Because of the very high specific Faraday rotation of these substances at low temperatures, films with a thickness of the order of 2000 Å yield sufficient magneto-optical contrast. As in any optical probe, the spatial resolution is limited by the wavelength of the light, yielding about 0.5 $\mu$m resolution for light in the visible range. Perhaps the greatest asset of the magneto-optical method is its capability (in contrast to the Bitter technique) of producing a continuous image also under nonstationary conditions, i.e., for flux structures changing rapidly with time. The magnetic sensitivity of the Kirchner method is presently about 30 Oe. The time resolution of the magneto-optical detection is limited by the light output and the sensitivity of the recording photographic film. Whereas the intensity of the light beam directed on the specimen cannot be increased arbitrarily because of heating effects, the light output of the system can be enhanced considerably using electronic image intensification. In this way a time resolution of 1–10 msec is currently obtained. It appears feasible that higher spatial resolution may be attained in the future through operation in the ultraviolet range. However, further developments of the magneto-optical materials and electronic image conversion would be necessary.

Although the magneto-optical technique has been applied with great success to the intermediate state, it so far has been unable to detect individual flux quanta in type-II superconductors. This task may require operation close to the transition temperature, where the flux-line diameter is sufficiently large to be spatially resolved. However, here the modulation of the magnetic field in the superconductor becomes rather small. By proper optimization of conditions, the direct visualization of single flux quanta with the magneto-optical technique may be achieved in the near future. Such a development would make a strong impact upon the field of magnetic structures in superconductors.

The Lorentz force experienced by electrons in a magnetic field has been utilized in various electron-optical attempts (electron shadow microscopy, electron mirror microscopy) to visualize flux structures in superconductors. However, these techniques suffer, so far, from

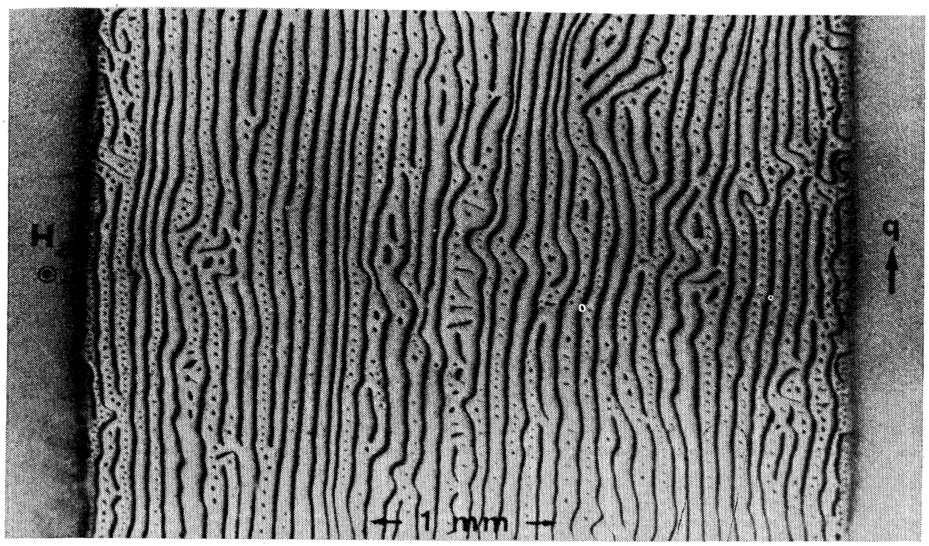

FIG. 1. Intermediate-state static lamellar structure observed when passing a heat current $q$ through a lead slab subjected to a transverse magnetic field. Superconducting domains are black; sample thickness $d = 0.5$ mm; reduced magnetic field $h = H_a/H_c = 0.91$; structure parameter $a = 100\ \mu$; mean temperature $T = 3.2$ K; averaged heat flow $q = 3.3$ Watt/cm$^2$; averaged temperature gradient $\nabla T \simeq 0.16$ K/cm. (Rinderer, paper 1:2. The paper number in all figure captions refers to the Argonne report of Footnote 1.)

inferior spatial resolution compared with the magneto-optical method, or from very restricting requirements regarding the surface quality of the specimens. Further improvements in electron-optical flux detection in the future are conceivable, such that high resolution may be combined with the possibility of studying strongly time-dependent phenomena.

An intriguing method for flux detection utilizes the phase shift of an electron wave caused by a vector potential (Bohm-Aharanov effect) in an interference experiment. The method has strong similarities to the Josephson effect and was applied first to flux quanta in superconductors by Wahl (1970). The motion of single flux quanta can easily be detected (see Sec. IX). However, no direct information about the spatial flux distribution can be obtained with this technique.

The usefulness of the magneto-optical flux detection method was demonstrated again by a beautiful motion picture study presented at the conference by Rinderer (paper 1:2 of the Argonne report of Footnote 1), continuing a tradition established at previous conferences by Kirchner (1968) and by Solomon and Harris (1971). Rinderer's experiments include investigations of the nucleation of flux tubes at the sample edge during flux penetration, as well as investigations of flux motion in the presence of an electrical transport current or a temperature gradient. It is the first time that magneto-optical experiments with superconductors under the application of a temperature gradient were reported. Figure 1 shows the intermediate-state structure in a lead slab subjected to a temperature gradient in the direction $-q$. Laeng, Rothen, and Rinderer have investigated magneto-optically the dynamics of the intermediate state in a lead slab near the critical field with both an electric field and a temperature gradient applied in the same direction and with the magnetic field oriented perpendicular to the slab. Here, the domain structure is generally characterized by straight superconducting laminae. At low temperatures the structures are static, and for zero electric field the laminae are oriented parallel to the temperature gradient. If an electric field and a temperature gradient are present, the orientation of the laminae depends upon the ratio of the densities of the electrical current and the heat current. At high temperatures domain motion occurs. The results can be qualitatively understood from the theory of Andreev and Dzhikaev (1971).

Interesting heat conductivity experiments in the intermediate state of tin were reported by Suter, Rothen and Rinderer. Using Sharvin's inclined field method (1957), they produced a regular laminar domain structure, and investigated the heat conductivity parallel to the interphase boundaries. Additional heat resistance due to Andreev scattering (1964) of excitations at the normal-superconducting boundary has been detected. By extending Andreev's model to the case of finite electron mean free path, satisfactory agreement with the experimental results was obtained.

Sharvin's point contact method (1965; Sharvin and Landau, 1970) for studying the motion of magnetic flux structures in superconductors has recently been employed by Farrell (1972), who eliminated flux-pinning effects by placing ancillary field coils at both sample ends. Extremely regular motion of the Landau domain structure in an inclined magnetic field has been observed under the application of both an electric field and a temperature gradient (see Fig. 2). From such experiments the velocity of the Landau domains can be obtained directly. The point contact method has recently been applied also to superconductors with low transition temperature, such as zinc and aluminum, by Farrell and coworkers. It appears feasible that with sufficient time

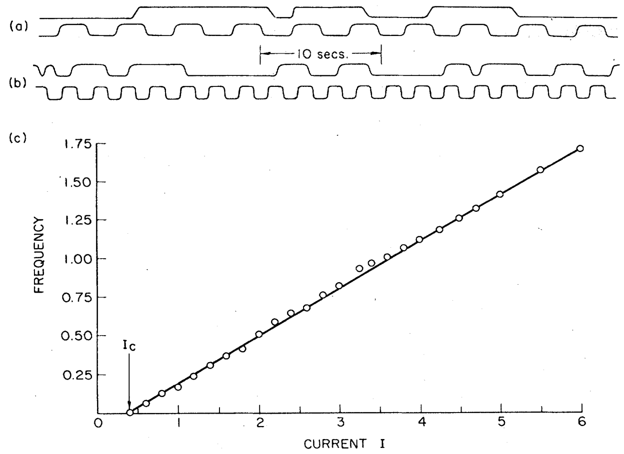

FIG. 2. (a), (b) Resistance fluctuations, observed as domains pass under a single point, before (upper) and after (lower) ancillary field is applied. The vertical axis for each trace records resistance, with the maxima corresponding to material under the point being completely normal, and minima to its being superconducting. Time increases to the right along the horizontal axis. For all traces, $\beta = 10°$. (a) $T = 3.34$ °K, $H = 55$ Oe, $I = 2.0$ A. (b) $T = 1.60$ °K, $H = 216$ Oe, $I = 3.30$ A. (c) Frequency (number of domains passing under a point per second) as a function of transport current $I$ (in amperes), with the ancillary field turned on. $T = 1.82$ °K and $H = 202$ Oe. (Farrell, paper 2:1)

resolution the point contact probe may yield interesting details about the normal-superconducting boundary as it passes underneath the point contact.

Details of the growth of the Landau domain structure in an inclined magnetic field and the conditions for obtaining highly regular structures have recently been studied magneto-optically in single-crystal type-I superconductors by Farrell, Huebener, and Kampwirth. The same authors also investigated the domain structure at the small end face of a long rectangular plate. For a sufficiently small angle between the applied field and the broad sample face, a superconducting "monodomain" was observed, surrounded by normal material and with its normal-superconducting interface nearly parallel to the broad sample faces (Farrell, Huebener, and Kampwirth, 1973).

## III. EXPERIMENTS ON VORTEX STRUCTURE

One of the most interesting recent developments has been the discovery of a new type of magnetic flux structure consisting of a mixture of flux-free domains (Meissner phase) and domains containing a flux-line lattice (Shubnikov phase) with constant lattice parameter. This new structure was first revealed by the decoration technique in experiments by Träuble and Essmann (1967), Sarma (1968), and Krägeloh (1970). The new behavior is accompanied by a first-order phase transition at $H_{c1}$. It is observed in low-$\kappa$ type-II superconductors ($\kappa \approx 1/\sqrt{2}$, where $\kappa$ is defined as the ratio of the penetration depth to the coherence length). These results are explained in terms of a long-range attractive interaction between vortices. So far the experimental work (decoration technique, magnetization measurements, and neutron diffraction) includes the low-$\kappa$ metals or alloy systems: Nb, V, Pb–Tl, Pb–In, Ta–N, and Nb–N. The different techniques agreed satisfactorily when applied to the same material. For the system TaN a complete phase diagram for the new state versus $\kappa$ and temperature has been constructed by Auer and Ullmaier (1973) from their experimental data.

An additional characteristic feature of low-$\kappa$ type-II superconductors is the correlation between the orientation and structure of the flux-line lattice and the crystal lattice of the superconductor. Figure 3 shows the coexistence of the flux-tube configuration and the anisotropic laminar structure as observed by Essmann with the decoration method in a single-crystalline lead foil of about 25 $\mu$m thickness. A systematic study of the influence of the crystal anisotropy on the orientation and symmetry of the flux-line lattice has been performed by Obst (1971) for single crystals of lead and alloys of lead with thallium and indium, using the decoration technique. The correlation between the anisotropy of the superconducting crystal and the orientation of the flux-line lattice has also been shown in neutron diffraction experiments, as will be discussed further below. Two theoretical explanations of these phenomena have been proposed: one, based on the anisotropy of the Fermi surface, by Takanaka (1973), and the other, based on the anisotropy in the elastic properties, by Ullmaier, Zeller, and Dederichs (1973).

The Träuble–Essmann decoration technique continues to be a most important tool for studying vortex structures in superconductors under static conditions. Its spatial resolution of about 100 Å is very impressive, and the method is now utilized in an increasing number of laboratories.

Since the pioneering experiments by Cribier, Jacrot, Rao, and Farnoux (1967), neutron diffraction in recent years has become an important technique for studying

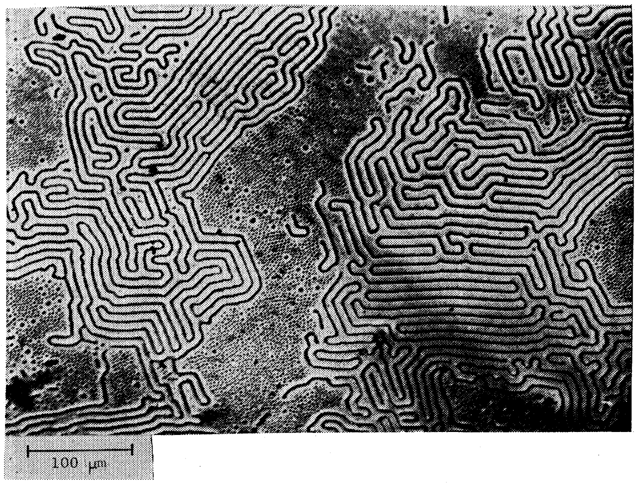

FIG. 3. Intermediate-state structure in a single-crystalline lead foil with 23–27 $\mu$m thickness and (357) orientation; $T = 1.2$ °K, $h = 0.19$. The circular flux tubes contain about 47 flux quanta. The normal laminae (black) are parallel to the projections of the (lll) directions into the plane of the foil. (Essman, paper 7:1)

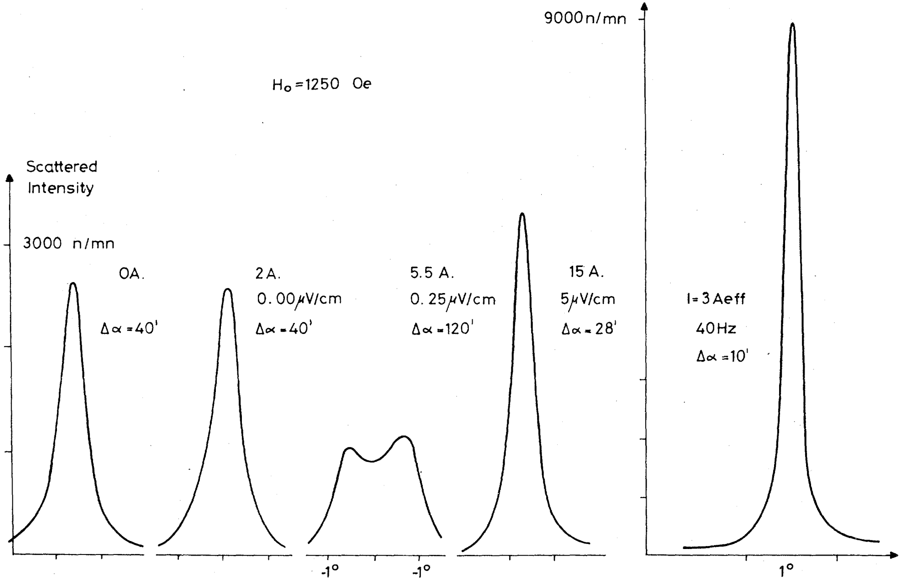

FIG. 4. Different shapes of rocking curve on a vortex crystal in monocrystalline Nb with a magnetic field of 1250 Oe. In each case, the integrated scattered intensity, i.e., the area under the rocking curves, remains constant. (a) The first curve is done on a vortex crystal created by increasing the field above $H_{c2}$, and decreasing it to 1250 Oe. The full-width-half-maximum of the rocking curve $\Delta\alpha$ is a measurement of the mosaicity of this vortex crystal. (b) The second curve is done with a transport dc current of 2 A in the zero voltage region of the I–V characteristics (no change in shape). (c) The third curve is for a transport current of 5.5 A (above the critical current = 4.5 A) in the nonlinear region of the I–V characteristics described by the flux creep model (strong broadening of the peak and details of the shape not very reproducible). (d) The fourth curve is for an applied current of 15 A, in the flux flow region. The width of 28′ is the limiting value when the current is increased to 30 A. (c) The last curve is for an applied ac current of 40 Hz and 3 A effective magnitude, the peak value being lower than the dc critical current (rocking curve is very sharp, with $\Delta\alpha = 10'$, representing the highest quality obtained for this sampe; note change of scales). (Thorel and Kahn; paper 9:2)

flux-line structures in superconductors. Here one utilizes the interaction between the magnetic moment of the neutron and the magnetic field modulation of the vortex structure. The dominant part of the experimental work has been performed in Saclay and Grenoble, France, and in Jülich, Germany. The neutron beam is directed perpendicular to the direction of the flux lines. The scattering angle $\theta$ of the neutron beam is given by the Bragg equation, $2d\sin(\theta/2) = \lambda$, where $d$ is the spacing of the flux-line lattice, and $\lambda$ the neutron wavelength. Neutron beams with reasonable intensity up to about 10 Å wavelength are available. Since the spacing of the flux-line lattice is typically about 1000 Å, a small-angle scattering facility is required. With drastic collimation of the incident beam, scattering angles as small as 12′ have been measured.

The neutron diffraction work focuses on the determination of form factors, flux-line lattice parameters, rocking curves, and correlations between orientation and shape of the unit cell of the flux-line lattice and the crystallographic structure of the superconducting metal. From the form factors, the microscopic magnetic field distribution in a vortex line can be obtained. The width of the rocking curve is determined by the misorientations of flux-line single crystallites in the symmetry plane of the lattice, and serves as an indicator for the quality of the vortex crystal (mosaic spread).

Thorel and Kahn recently performed neutron diffraction experiments on niobium single crystals to investigate the influence of various factors on the quality of the flux-line lattice. Their results are summarized in Fig. 4. From the width of the rocking curve, they found that with increasing magnetic field the vortex crystal becomes more perfect, as expected from the enhanced vortex–vortex interaction. An interesting correlation was found between the vortex crystal quality and the flux-flow behavior under the application of a direct electric current. At the onset of flux motion, a broadening of the rocking curve was observed, which was evident throughout the nonlinear regime of the flux-flow voltage. The rocking curve sharpened again for higher currents where the voltage-current relation became linear. These results indicate the motion of a rather perfect vortex crystal through the superconductor in the linear flux-flow regime. The orientation of the moving flux-line lattice relative to the crystallographic axes of the niobium crystal remained unchanged following a change in current direction by 90°. For the studied range of flux-flow velocities up to about 1 cm/sec no measurable effect on the magnetic field profile around a vortex core could be detected. A drastic sharpening of the rocking curve was observed following the application of a low-frequency (~ 40 Hz) alternating current or of a number of current pulses of sufficient amplitude. The frequency dependence of the effectiveness of these methods for growing vortex crystals is closely related to the skin depth for the penetration of the oscillatory field.

Thorel and Kahn have proposed two experiments for the direct measurement of the flux-flow velocity in the bulk by neutron diffraction. The first utilizes the fact that a neutron wave travelling with speed $v_n$ through a moving flux-line lattice encounters flux lines with their apparent positions displaced in the direction of the flux-line velocity $v_L$. This displacement produces a shift of order $v_L/v_n$ in

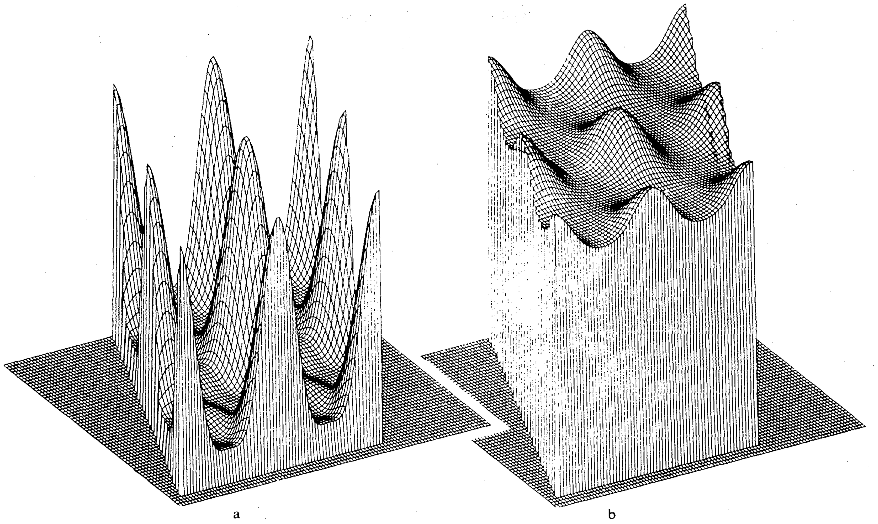

FIG. 5. Three-dimensional views of the microscopic flux distribution in a niobium single crystal; (a) for a flux density $B = 560$ G $= B_0$ and (b) for $B = 2200$ G $= 0.7H_{c2}$. In (a) the nearest-neighbor distance between flux lines is 2060 Å; the maximum field at the flux line centers is 2270 G. The corresponding numbers in Fig. 3b are 1040 Å and 2550 G, respectively. (Ullmaier, Schelten, and Lippmann, paper 10:1)

the Bragg scattering angle. Since experimental shifts in excess of $10^{-2}$ radians would be necessary, observation of the effect would require flux-line speeds of, say, $v_L = 1$ m/sec and slow neutrons with $v_n = 100$ m/sec or 40 Å wave length. In the second experiment, one would detect the Doppler shift in wavelength or speed of the neutron beam resulting from its interaction with the moving vortex lattice. By using a neutron spin-echo technique, flux-line speeds as low as 5 cm/sec would be detectable with neutrons of 20 Å wave length.

Correlations between orientation and shape of the unit cell of the flux-line lattice and the crystal lattice of the superconductor have been observed in neutron diffraction experiments by Thorel and Kahn (papers 9:2 and 10:2) on single crystals of Nb and Pb–Tl alloys and by Ullmaier, Schelten, and Lippmann (paper 10:1) and Weber, Schelten, and Lippmann (paper 10:3) for pure and impure Nb, and for Pb alloys and Tc. Depending on the crystal orientation of the sample, deformation of the hexagonal unit cell of the flux-line lattice by several degrees were found. Since these effects persisted in niobium up to rather high impurity concentration, the anisotropy of the energy gap observed in pure samples and absent in the dirty limit appears to be unable to explain the interaction between the flux-line lattice and the crystal lattice.

Weber, Schelten, and Lippmann determined the average tilting angle between the flux lines and the direction of the applied magnetic field in Nb single crystals from neutron depolarization experiments. With increasing magnetic field the tilting angle decreased from 18° near $H_{c1}$, approaching 0° at the same field for which the mosaic spread of the vortex crystallites reached its saturation value.

Using neutron diffraction, the Jülich group recently determined the microscopic magnetic field distribution in a vortex line for pure and impure Nb and for $Nb_{0.73}Ta_{0.27}$ and $Pb_{0.95}Bi_{0.05}$ alloys. The experiments with niobium confirmed the attractive vortex interaction and the resulting impossibility of exceeding a maximum lattice parameter. In the vicinity of the upper critical field, the experimental form factors were in rather poor agreement with theoretical values obtained by Eilenberger (1967) from Abrikosov's solution. A theoretical treatment by Delrieu (1972), also valid only near $H_{c2}$, agrees much better with the experimental results. Recent exact numerical calculations by Kramer and Pesch for pure superconductors with arbitrary $\kappa$ values, discussed in Sec. IV, yield satisfactory agreement. In Fig. 5 we show a three-dimensional plot of the microscopic field distribution for Nb at two flux densities, as obtained by neutron diffraction. From such plots the maximum, minimum, and saddle point fields and their dependence on the flux density and the impurity parameter have been determined. For the alloys $Nb_{0.73}Ta_{0.27}$ and $Pb_{0.95}Bi_{0.05}$ the microscopic magnetic field distribution of an isolated flux line has been obtained. The field maximum in the center of the flux line was found to be approximately equal to twice the lower critical field $H_{c1}$, as expected from theory. The Jülich group is now extending its measurements to InBi and InPb alloys.

Nuclear magnetic resonance also makes possible the direct measurement of the magnetic field distribution in the vortex structure through the broadening of the resonance line caused by the field inhomogeneity. Such experiments were performed recently by Redfield (1967), Kung (1970), and Delrieu and Winter (1966). NMR is particularly suitable for determining directly the maximum, minimum, and saddle point field value of the flux-line lattice, without requiring any assumption on the sign of the form factors, in contrast to neutron diffraction. A weakness of the NMR experiments lies in the fact that they can be performed only on powder or thin foils and wires.

As shown by Delrieu (1973), a nuclear magnetic resonance method may be used to measure the speed of the flux-line lattice. If the nuclear spin is exposed to an oscillating magnetic field, the resonance frequency is frequency modulated. For high flux-line speeds this results in a narrowing of the line width, thus providing a measure of the flux-line speed.

Vortex motion in superconductors can be investigated by frequency-analyzing the flux-transport voltage across a contact pair attached to the sample. The dc component of this voltage is simply proportional to the rate at which vortices pass between the contacts. The ac components, on the other hand, depend upon the time-dependent behavior of individual moving vortices, and upon the correlations between them. At the conference, Heiden reported on his measurements of the flux-flow voltage noise power spectrum in Nb and V foils. His results indicate that during flux flow vortices are moving across the foil in a rather well ordered array and that the time-dependent voltage fluctuations arise chiefly from local fluctuations in the average vortex velocity.

A motion picture of simulated vortex motion, photographed by Meissner, was shown at the conference. Vortices were simulated by magnetic needles in buoyant spheres floating vertically in a cupric sulfate solution. The Lorentz force was provided by a current flowing through the solution, and pinning barriers were provided at intervals by magnetized strips. Although the analogy with vortex motion in superconductors is not perfect, the two situations have many qualitative features in common. Such simulation experiments are helpful in illustrating several properties of vortex motion that previously have been inferred from indirect measurements in superconductors.

Microwave surface impedance studies of type-II superconductors, in a magnetic field parallel to the specimen surface, by Walton, Rosenblum, and Bridges have led to the postulation of lines of zero order parameter (nascent vortices) at the surface parallel to the applied field. Nascent vortices act as nucleation sites for vortices. Further supporting evidence for this model can be derived from magnetization and tunneling experiments. A structure at the surface very similar to the nascent vortex state has also been found in a theoretical study by Kramer (1973). Most recently, the hysteresis of the microwave surface impedance during changes in the applied field and in the presence of an electric transport current with changing magnitude has been investigated. The results can be understood by accounting for both the effect of nascent vortices and the motion of interior vortices near the sample surface.

## IV. THEORY OF STATIC VORTEX STRUCTURE

Our present theoretical understanding of the vortex structure in type-II superconductors has its foundation in the theory of Ginzburg and Landau (1950). They first expressed the free energy functional of a superconductor containing a magnetic field as an expansion of powers of the order parameter. Then, using a variational technique, they derived the now famous Ginzburg–Landau equations, which relate the order parameter to the supercurrent density and the magnetic field. The next major development was Abrikosov's solution (1957) of the Ginzburg–Landau equations for a periodic array of singly quantized vortices (fluxoids) in a magnetic field close to the upper critical field $H_{c2}$. Then came Gor'kov's reformulation (1958, 1959) of the BCS theory in terms of Green's functions, and his application of this new formalism to the properties of a superconductor containing an inhomogeneous magnetic field distribution. He showed that, for both very pure and very impure superconductors, the Ginzburg–Landau–Abrikosov theory follows from the microscopic theory in the limit as the temperature $T$ approaches the transition temperature $T_c$, provided that the order parameter is small, and that the spatial variation of both the order parameter and the magnetic field is slow. The results and concepts of the Ginzburg–Landau–Abrikosov–Gor'kov theories, taken together, are now often referred to as the GLAG theory.

Since the original formulation of the GLAG theory, there have been four main paths taken to derive, from the Gor'kov equations, results that are valid outside the limited range of temperatures near $T_c$ to which the GLAG theory is confined. Along the first path, one relaxes the requirement that the order parameter be small, but expands in powers of the spatial variation of the order parameter about the equilibrium BCS value. This procedure, developed by Werthamer, Tewordt, Maki, Tsusuki, Eilenberger, and others during the period 1963 to 1966 (Werthamer, 1969), leads to a deeper understanding of the validity of the Ginzburg–Landau equations but does not produce a theory valid for the mixed state of type-II superconductors except close to $T_c$.

Along the second path, one relaxes the requirement that the spatial variations be slow, but expands in powers of the order parameter. This method applies to the mixed state of type-II superconductors at magnetic fields close to the upper critical field $H_{c2}$, where a second-order transition to the normal state occurs. The resulting theories, developed by Helfand and Werthamer, Maki and Tsuzuki, Eilenberger, de Gennes, and others during the period 1964 to 1967 (Werthamer, 1969; Fetter and Hohenberg, 1969), extend the GLAG theory to all temperatures and mean free paths, but are limited to fields near $H_{c2}$.

Along the third path, one expands all terms to higher order in powers of $(T_c - T)$, the deviation from the transition temperature. This procedure, developed primarily by Neumann and Tewordt (Tewordt, 1964 and 1965; Neumann and Tewordt, 1966a and 1966b), extends the theory to a wider range of temperatures near $T_c$ and, within this limitation, is appropriate for arbitrary magnetic fields and mean free paths.

Along the fourth path, one relaxes all requirements in order to obtain theoretical results valid for all temperatures, magnetic fields, and mean free paths. This approach was initiated by the work of Eilenberger (1968), who transformed the Gor'kov equations into a set of transportlike integro-differential equations. In their appropriate ranges of validity, all previous known results could be shown to follow from Eilenberger's equations. The price paid for the generality of these equations, however, is that they must be solved numerically with the aid of high-speed computers.

At the conference, newly developed iterative methods for obtaining self-consistent numerical solutions of the Eilenberger equations were described. Kramer and Pesch discussed applications of these methods to compute the order parameter and magnetic field near an isolated vortex in clean superconductors as a function of temperature. With decreasing temperature, the calculated magnetic field distribution was found to sharpen into a nearly conical profile with a rather sharp point at the vortex axis, in agreement with recent experimental results.

As shown by Usadel (1970), for the case of very impure superconductors (dirty limit), the Eilenberger equations can be transformed into much simpler diffusionlike equations. At the conference, Watts-Tobin, Kramer, and Pesch described recent self-consistent, numerical solutions of the Usadel equations and the resulting thermodynamic properties predicted for dirty superconductors containing a low density of vortices.

Although it is valid only close to $T_c$, the GLAG theory is remarkably successful in describing the qualitative behavior of type-II superconductors. Probably the most noticeable qualitative shortcoming of the theory, however, is its failure to account for an attractive interaction between vortices, which has been seen using decoration techniques, in magnetization measurements, and in neutron diffraction experiments in superconductors having Ginzburg–Landau parameters $\kappa \sim 1/\sqrt{2}$. As was discussed by Jacobs at the conference, at a given temperature below $T_c$, the attractive interaction occurs only for a window of values of $\kappa$ near $1/\sqrt{2}$. As the temperature approaches $T_c$, the height of the window vanishes linearly with $T_c - T$. A complete theory of the attractive interaction remains to be developed. It appears that such a theory will be based upon the Eilenberger equations, will require a careful treatment of nonlocal electrodynamics, and probably also will involve a detailed numerical analysis of the competing magnetic field-dependent and order parameter-dependent terms in the Gibbs free energy of an array of interacting vortices.

Several theoretical groups currently are at work developing methods for the numerical solution of the Eilenberger equations. In the near future we may expect to see numerous reports of calculations of the thermodynamic properties, including the heat capacity and magnetization, of type-II superconductors for arbitrary temperatures, magnetic fields, and mean free paths. It probably will be some time, however, before satisfactory theoretical explanations are developed for the experimentally observed effects of strong electron-phonon coupling and anisotropy of the phonons, the Fermi surface, and the electron-phonon interaction.

## V. THEORY OF DYNAMIC VORTEX STRUCTURE

By comparison with the theory of the static vortex structure, the theory for the dynamic case is still in a relatively primitive stage. One would have liked for nature to give us time-dependent Ginzburg–Landau equations (a set of differential equations involving only first- or second-order space and time derivatives of the order parameter) for a wide range of temperatures, magnetic fields, and mean free paths. But nature has not been so kind, and a simple set of time-dependent Ginzburg–Landau equations holds only under highly restrictive conditions. For example, such a set of equations has been shown to be valid very close to $T_c$, where the order parameter is small. Further, as shown by Gor'kov and Eliashberg (1968), the space and time variation of the order parameter must not be too slow, since the dynamic expansion of those terms proportional to frequency $\omega$ proceeds in powers of the order parameter divided by the frequency plus the diffusion constant times the wave vector squared $[\Delta/(\omega + Dk^2)]$. The resulting equation for the order parameter then resembles a diffusion equation, since it contains a first-order time derivative and a second-order space derivative.

Three main paths have been taken in efforts to extend the range of validity of the time-dependent Ginzburg–Landau theory. Along the first path, one relaxes the requirement that the order parameter be small, and considers variations of small amplitude about the BCS equilibrium state. Abrahams and Tsuneto (1966), for example, showed that for superconductors obeying local electrodynamics there are two temperature regimes for which time-dependent Ginzburg–Landau equations can be derived from the microscopic theory: first, at temperatures near absolute zero, where the resulting equations are wavelike, and second, at temperatures near $T_c$, where the equations are diffusionlike. At other temperatures, time-dependent Ginzburg–Landau equations cannot be derived. Although this theory was not directly applicable to the mixed state of type-II superconductors, except near $T_c$, it suggested that future dynamical theories of the mixed state for arbitrary temperatures, magnetic fields, and mean free paths were doomed to be quite complex and could not be expressible in terms of simple time-dependent differential equations.

Along the second path, one considers the mixed state of type-II superconductors at magnetic fields close to the upper critical field $H_{c2}$, where a second-order phase transition to the normal state occurs, and the order parameter is small. Such theoretical work by Schmid, Caroli, Maki, Thompson, Takayama, Ebisawa, and others during the period 1966 to the present (Cyrot, 1973) has been quite fruitful, yielding many predictions for the temperature dependence of the flux-flow resistivity, Hall coefficient, radio-frequency surface impedance, and various thermo-galvano-magnetic coefficients at fields near $H_{c2}$. However, since most of these results are confined either to the clean or dirty limit, further work is needed to bridge the gap between these cases.

Along the third path, one considers superconductors heavily doped with paramagnetic impurities. Primarily because the excitation spectrum is then gapless, a complete set of time-dependent Ginzburg–Landau equations may be derived from the microscopic theory, as was demonstrated by Gor'kov and Eliashberg (1968). During the period 1971 to the present, Thompson and Hu (1973) have shown that the resulting equations may be applied to the dynamical mixed state of type-II superconductors for arbitrary magnetic fields in the limit of very low temperatures.

At the conference, Ebisawa reviewed some recent developments obtained along the second path, and reported on his theoretical progress towards the solution of the problem of the Hall effect in type-II superconductors. For the mixed state near $H_{c2}$, he finds two contributions to the Hall angle. The first term is associated with the dynamics of the quasiparticles in the vortex cores and has a value equal to the normal-state Hall angle. The second term is associated with the dynamics of the order parameter, whose magnitude is in turn an implicit function of the magnetic field. This term arises from the so-called fluctuation part of the conductivity tensor, whereas the first term arises from the so-called static or regular part. The calculated field dependence of the Hall angle was found to be in qualitative agreement with some recent experimental results of Noto, Shinzawa, and Muto.

In reviewing developments obtained along the third path, Thompson and Hu discussed the solution of the Gor'kov–Eliashberg equations for a lattice of vortices moving through a flat superconductor containing many paramagnetic impurities. They predicted that, as a result of the nonlinear response of vortices to strong electric fields, thin films of such superconductors should exhibit dynamical instabilities: the voltage along the film is expected to jump discontinuously from a lower flux-flow value to a higher normal-state value when the transport current is increased beyond the maximum value the flux-flow state can support.

A complete theory of the dynamical properties of superconductors, yet to be developed, necessarily will be quite complex, in order to account properly for such phenomena as the interconversion of normal fluid and superfluid, the exchange of energy between electrons and phonons, the diffusion of quasiparticles, and the production of temperature gradients. Nevertheless, in noting analogies with the static case, we may hope that a time-dependent generalization of the Eilenberger equation approach will someday provide a compact and elegant theoretical framework for the dynamical properties of the mixed state of type-II superconductors for arbitrary temperatures, magnetic fields, and mean free paths

## VI. THIN FILMS

Since the famous paper by Tinkham (1963), the transition to type-II behavior with decreasing film thickness, for thin films of type-I superconductors, has been the subject of a series of experimental and theoretical studies. The experimental work includes the direct observation of the flux structures with the Träuble-Essmann decoration technique, as well as measurements of the parallel and perpendicular critical fields, the flux-flow behavior, and the current hysteresis in the electrical resistance. An important aspect of these studies is the determination of the critical film thickness below which single-quantum flux lines appear in the materials.

Beautiful experiments performed with the Träuble-Essmann decoration technique were reported by Dolan and by Boersch, Lischke, and Rodewald. An investigation of the critical film thickness, $d_c$, by Dolan indicates $d_c$ to be much smaller than the values derived previously from critical field measurements. On the other hand, the new values of $d_c$ are in good agreement with theoretical calculations by Lasher (1967) and Maki (1965). Boersch *et al.* also found single-quantum flux lines in Pb and Sn films with thickness less than 1500 Å. An interesting result shown in both papers is the clustering of flux lines near grain boundaries, and the influence of the metallurgical microstructure on the flux-line distribution. Figure 6 shows the results by Boersch and co-workers. The combination of the Träuble-Essmann decoration method with the electron microscopic study of the metallurgical structure of the thin film samples has become a useful tool for investigating the interaction between individual flux lines and inhomogeneities in the superconducting crystal. The extension of decoration experiments in the future to single-crystal films of varying thickness can be expected to produce interesting results regarding the possibility of multiquanta flux lines predicted by Lasher (1967).

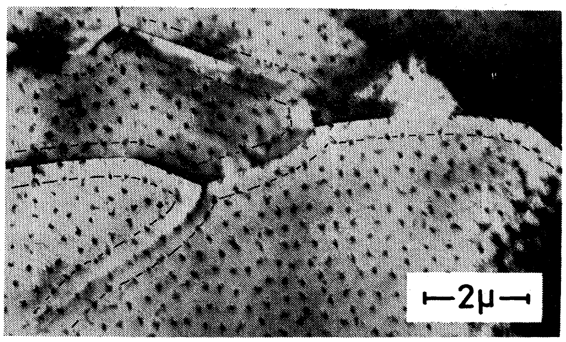

FIG. 6. Simultaneous observation of the flux line distribution and of the crystalline structure of a thin Pb film ($d = 1500$ Å) as detected by decoration microscopy. To accentuate the arrangement of the flux lines parallel to the grain boundaries, caused by flux pinning at the grain boundaries, some of them are connected by dashed lines. $H$ raised from 0 to 160 A/cm, $T = 1.2$°K. (Boersch, Lischke, and Rodewald, paper 14:4).

Recently tunneling in a magnetic field perpendicular to the junction has been used as a new tool for studying magnetic flux structures in thin film superconductors. Gray reported tunneling studies of the transition from type-I to type-II superconductivity in thin films of indium and lead. The density of states and the critical field, as found from the tunneling conductivity, indicate that the transition from type-I to type-II behavior occurs at the critical thickness $d_c = 2700$Å for In, and at $d_c = 4000$Å for Pb. In the films thicker than $d_c$, the flux-tube diameter apparently increases smoothly with thickness, and supercooling characteristic of type-I behavior is observed. The films thinner than $d_c$ show a density of states independent of thickness and are reversible, indicating type-II behavior. The critical thickness values found in the tunneling experiments are somewhat larger than those reported by the two groups which had studied the same materials with the decoration technique. However, since the different experiments were not performed at the same temperature and magnetic field, an exact comparison may be difficult. Comparing his results on Pb and In with measurements on type-II Al films, Gray noted an interesting correlation between the energy dependence of the density of states and the diameter of the vortex core, the energy dependence becoming stronger with decreasing core diameter.

The dilute vortex state in thin films of type-I superconductors was recently investigated in tunneling experiments by Donaldson and Band. The studies were performed at temperatures as low as 30 mK, such that the only tunneling below the gap voltage was due to excitations in the vortex core. For an $S$–$N$ junction (Al-Ag) the tunneling conductance is consistent with the result expected from the usual tunneling equations and the theoretical density of states. For a symmetric $S$–$S$ junction (Al–Al) the tunneling conductance is smaller than for the $S$–$N$ case by a factor of 20. Since the vortex cores in the two films can be shown to be in good registration with each other, the reduction in the tunneling conductivity may be explained by the application of selection rules conserving angular momentum of excitations in vortex-vortex tunneling. Nonsymmetric $S$–$S$ junctions (Al–Sn and Al–Pb) also show extremely small tunneling conductance. A puzzling feature of the latter junctions, not yet explained, is the proportionality of the conductance to the square of the magnetic field.

The tunneling technique, now in an early stage of development, shows promise of providing important information about various aspects of flux structures and in particular about vortex core excitations. We expect interesting new results in the future, when this technique is extended to nonstationary conditions, such as the flux-flow state. The use of magnetically coupled superconducting films may assist in obtaining additional information in such experiments.

Experimental observations of the magnetic coupling properties of vortex arrays in adjacent superconducting films separated by a thin insulating layer were reported at the conference by Ekin, Serin, and Clem. Type-II superconducting films of granular aluminum were prepared by evaporating aluminum in the presence of a partial pressure of oxygen. By proper oxygen-doping, very small depinning currents were achieved, making possible an investigation of the vortex coupling characteristics in a low-current regime free of heating instabilities. Measurements of the system's current-voltage characteristics were made not only at relatively low currents, where the vortices in the two films move at the same velocity because of their coupling, but also at currents more than an order of magnitude larger than the current at which the two vortex lattices cease to move at the same velocity.

The current-voltage characteristics, including the effects of currents applied simultaneously to both films, are well described by a simple periodic coupling-force model, as reported by Clem. The main features of the time-averaged primary and secondary flux-flow voltages versus the primary and secondary current depend upon only five parameters: the primary and secondary critical depinning currents, the primary and secondary flux-flow resistances, and a coupling current, which is directly related to the maximum coupling force exerted on a secondary vortex by the displaced primary vortex lattice. Shown in Fig. 7 is a comparison of the experimental and theoretical current-voltage characteristics. Further experimental and theoretical work remains to be done on the dependence of the maximum coupling force upon the magnetic field, the temperature, and the film thicknesses.

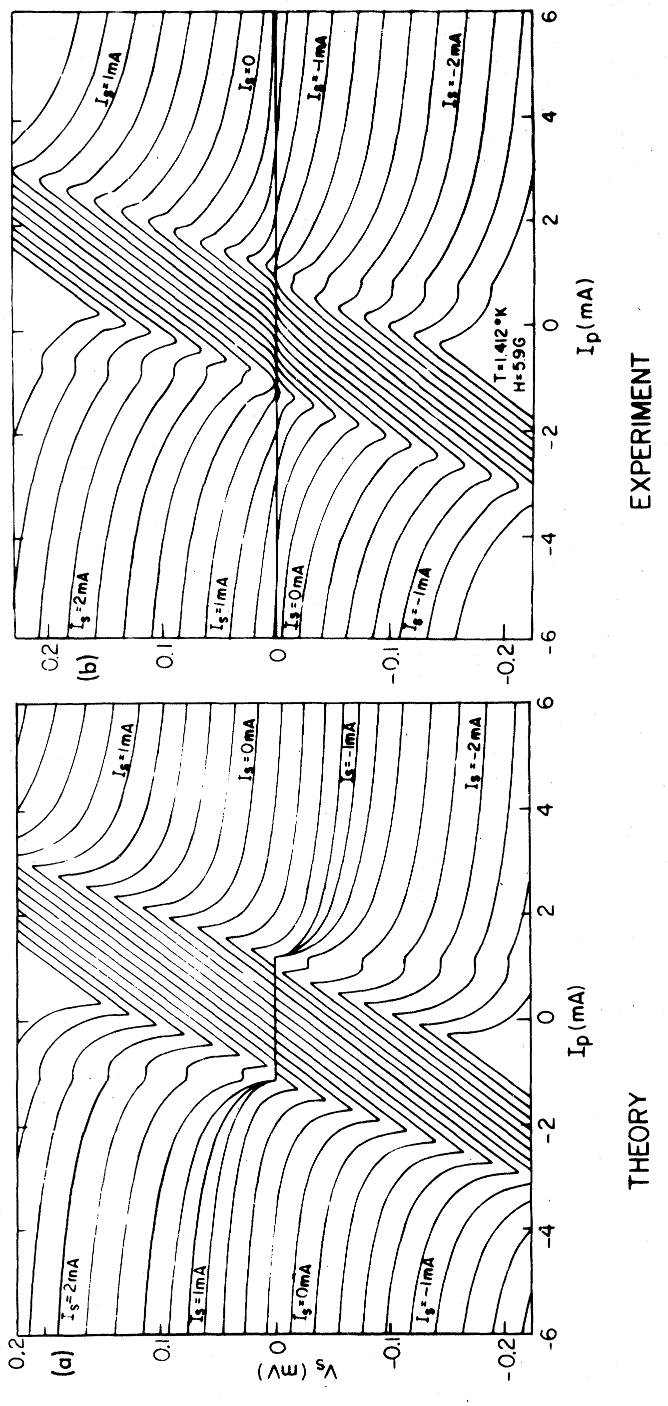

FIG. 7. Theoretical and experimental V–I characteristics for the superconducting dc transformer ($V_s$ is the average secondary voltage, $I_p$ and $I_s$ and the primary and secondary currents, respectively). The theoretical curves were generated from a periodic magnetic-coupling-force model. The experimental curves were obtained from a granular aluminum dc transformer in which critical current densities had been minimized. (J. Ekin, Serin, and Clem, paper 13:3).

Information about the structure and interactions of vortices in superconducting films can be obtained from quantum interference experiments, as was reported by Fiory. Steps in the flux-flow current-voltage characteristics are induced by a superimposed rf current when the frequency is a harmonic or subharmonic of the ratio of the vortex speed to the vortex-lattice parameter. Fiory's (1973) analysis of the experimental results for granular aluminum films in terms of a phenomenological theory by Schmid and Hauger (1973) has yielded the Fourier transform of the pinning potential correlation function, the local free energy density in the vicinity of a vortex, and the modulus of shear deformation of the vortex lattice. The shear modulus was found to be in satisfactory agreement with the theory for thin films.

Progress toward a theory of intermediate quantum resistance states in type-I superconducting films was reported at the conference by Kümmel. States of quantized resistance were first observed by Chen, Hayler, and Kim (1973) in 4000 Å films in the surface sheath regime in parallel magnetic fields. To explain these results Kümmel suggested that, when the applied current is above a certain critical value, quasiparticles confined chiefly to the normal lamina between the two surface sheaths are excited, which subsequently decay in the superconducting regions, leading to a voltage along the film. In order to complete the description of the resistance states, however, many details of the theory remain to be worked out.

## VII. CURRENT-INDUCED TRANSITIONS

Step structure in the voltage-current characteristics of superconducting whiskers or thin-film microbridges has been observed in recent years by Webb and Warburton (1968), Rochlin (1968), and Meyer and Minnigerode (1972). In some cases these experiments were performed in "one-dimensional" samples with the dimensions perpendicular to the current direction smaller than or about equal to the superconducting penetration depth and coherence length. Three papers at the conference contributed to a more complete understanding of these phenomena.

Fink proposed spatially periodic solutions of the time-dependent Ginzburg–Landau equations for a long one-dimensional superconductor, resulting in spatially periodic super and normal transport currents flowing parallel to each other. At certain points, where the order parameter becomes zero, phase slippage occurs between singly-connected superconducting regions resulting in a dc voltage. This new current-carrying state is expected above the critical current of the homogeneous solution and below the upper critical current at which the superconductor becomes completely normal.

The current-induced breakdown of superconductivity in thin-film microbridges of tin was investigated by Skocpol, Beasley, and Tinkham. The microbridges were typically 0.1 $\mu$m thick, 4 $\mu$m wide, and 150 $\mu$m long. A series of several voltage tabs permitted measurements of the voltage distribution along the bridge. Again, step structure in the voltage-current behavior was observed. The data taken with different pairs of voltage leads indicate that the steps correspond to the creation of isolated, spatially localized, and very similar phase-slip centers at a current equal to the local critical current. The critical current varies along the bridge because of inhomogeneities and variations in $T_c$. An important result is the fact that the differential resistance is approximately equal for all centers in the same bridge and independent of temperature. Apparently the equivalent normal length of each phase-slip center is related to the quasiparticle diffusion length of Pippard, Shepherd, and Tindall (1971). The quantum phase-slip process operates at the Josephson frequency, and this dynamic behavior was verified through synchronization with microwave radiation. Some of the experimental evidence obtained by Tinkham's group disagrees with Fink's model.

The extension of these studies on the resistive behavior of very small "one-dimensional" superconducting links to the case of a three-dimensional type-I superconductor was the subject of a paper by Huebener, Watson, and Kampwirth. They observed voltage steps in superconducting film strips of Pb and In, with dimensions perpendicular to the current direction much larger than the coherence length or penetration depth (see Fig. 8). As indicated by high-resolution magneto-optical experiments and other evidence, the voltage steps can be attributed to the nucleation of trains of flux tubes moving rapidly from the edge to the center of the strips. In the center, opposite tubes arriving from opposite edges annihilate each other. From a simple eddy-current damping model the number of flux tubes existing simultaneously in a single flux-tube train can be estimated. As indicated by their dynamic behavior, type-I films exhibiting voltage steps are the three-dimensional analogs of weak-link Josephson junctions and, in particular, of the Anderson–Dayem bridge, the moving flux-tube trains playing the role of the single flux quanta in such a bridge.

As predicted by Landau (1938b), the current-induced breakdown of type-I superconductivity in hollow cylinders requires a mixture of superconducting and normal regions in the cylinder wall, a mixture called the two-dimensional mixed state. This state was discovered and experimentally studied by Landau and Sharvin (1969, 1972). At the conference, Bestgen reported on theoretical work on this state done with Andreev and Landau. Using the time-dependent Ginzburg–Landau equations, they calculated the current dependence and surface impedance of the two-dimensional mixed state in the presence of magnetic fields either applied longitudinally or produced by a current-carrying wire threading the cylinder

## VIII. INHOMOGENEOUS MATERIALS

The influence of the metallurgical microstructure on the superconducting properties and the magnetic flux structure is an important subject. Experiments performed under a controlled variation of the microstructure are still relatively rare. Two papers on this subject were reported at the conference.

Nemoz investigated a supersaturated aluminum–silver alloy (16.7 at. % Ag) and the influence of precipitation after isothermal aging. Transition temperatures and critical fields were deduced from magnetization measurements. Two kinds of regions of precipitation were observed: first, regions of intragranular precipitation consisting of silver-rich plates oriented in (111) directions and

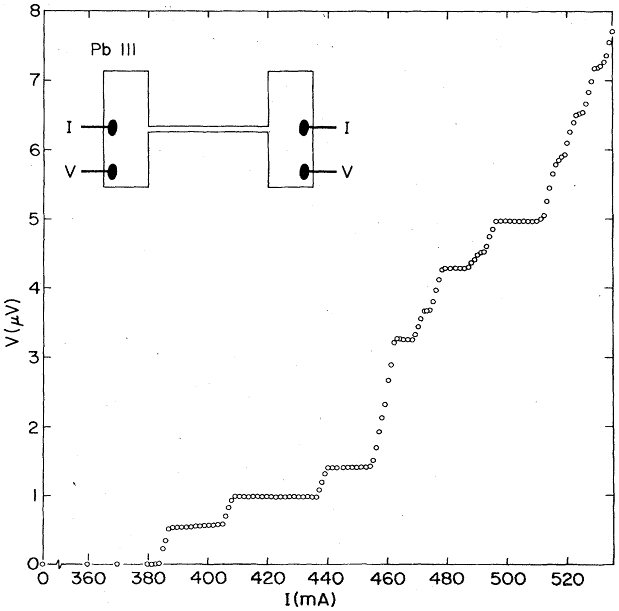

FIG. 8. Voltage versus current in a Pb strip of 3.2 $\mu$m thickness, 100 $\mu$m width, and 6.3 mm length at 4.2°K at the onset of the current-induced resistive state. (R. P. Huebener, H. L. Watson, and R. T. Kampwirth, paper 11:3).

Guinier-Preston zones, and second, regions of alternate lamellae of nearly pure Al and of the equilibrium precipitation. The first and second regions behave as type-II and type-I superconductors, respectively.

Raffy, Renard, and Guyon studied the properties of Pb–Bi alloy films with a periodic modulation of composition. The modulation was achieved by co-evaporation of the two materials from two sources, with the Pb deposition rate kept constant, and the Bi deposition rate varied periodically. For the fresh films the extremes of the Bi concentration were 2% and 20%. The period of the concentration profile ranged between 750 and 8000 Å, being of the same order as the characteristic lengths of the superconductor. Total film thickness was about 6 $\mu$m, corresponding to 60 layers for a case with 1000 Å periodicity. For a magnetic field parallel to the alloy film, and a current direction perpendicular to the field and parallel to the planes of equal composition, the critical current $J_c$ shows peaks at certain field values. The detailed structure of $J_c(H)$ depends on the periodicity length of the concentration profile and on the amplitude of the concentration modulation. The peak structure decreases when the angle $\theta$ between the film and the magnetic field is increased from zero, and disappears for $\theta \approx 50°$. The results strongly indicate a geometrical matching effect between the concentration profile and the magnetic flux distribution. Inhomogeneous materials similar to those studied by Raffy and co-workers may yield interesting results in experiments on other properties of superconductors, such as bound states of excitations.

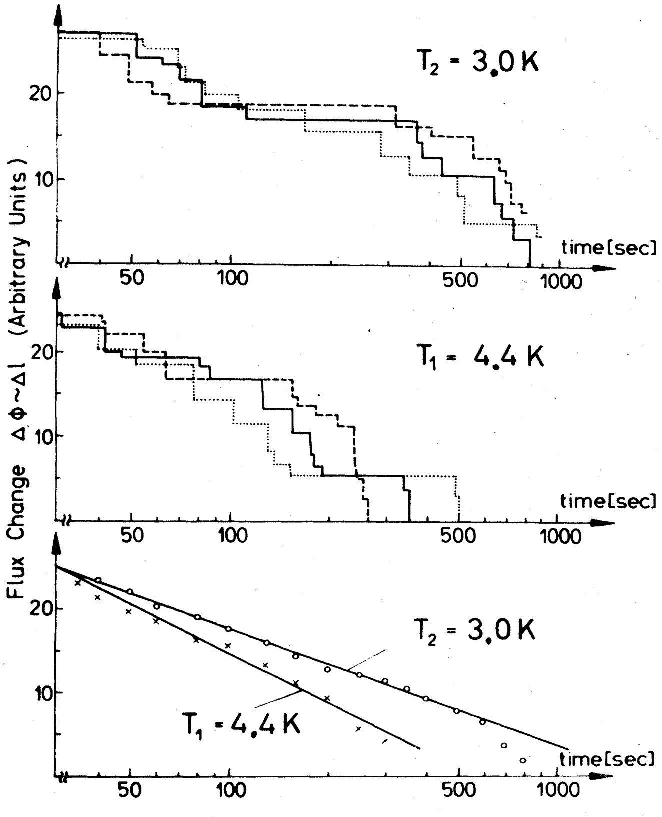

FIG. 9. Creep of single flux quanta measured with a vortex microscope. For details see text. A flux change of 10 arbitrary units corresponds to a flux decrease of one flux quantum in a cylinder of 10 $\mu$m length (Boersch, Lischke, and Söllig, paper 12:3).

## IX. FLUX JUMPS AND FLUX CREEP

Experimental studies of the kinetics of flux jumps (events in which large numbers of vortices overcome local pinning forces and move rapidly and irreversibly into a specimen) were reported by two groups at the conference. LeBlanc reported some collaborative work with Bussiere and Boyer in which they measured the velocity of propagation of flux jumps along wires of Nb and V carrying steady transport currents and magnetized in longitudinal fields. By monitoring the propagation with two small pickup coils that embraced the wire and were spaced along the length, they were able to deduce that a localized flux jump proceeds rapidly down the length of the wire along a helical path that corresponds to the configuration of flux lines near the wire's surface.

Harrison and Wright reported on magneto-optical and high-speed photographic studies of flux jumps into a disc of Nb–25% Zr. They found that the distance of advance of the flux front from its point of origin was a simple exponential function of time. Mukherjee presented a phenomenological theory that yields the experimentally observed time dependence and relates both the time constant and the final distance of flux-front advance to the pinning force and the flux-flow viscosity.

A remarkable motion picture illustrating the kinetics of flux creep (flux motion following thermal excitation out of pinning sites) for single flux quanta has been prepared by Boersch, Lischke, and Söllig, and was narrated at the conference by Rodewald.. For these studies, flux quanta are trapped within a hollow superconducting microcylinder. Electron waves incident upon the cylinder undergo the analog of Fresnel diffraction and produce interference fringes, which are observed with the help of an image intensifier. Whether the central fringe is a maximum or a minimum depends upon whether there are an even or an odd number of flux quanta trapped in the cylinder. Using this apparatus, which the authors called a vortex microscope, it is possible to observe the motion, along the cylinder length, of single flux quanta with velocities down to about 100 Å/sec. Figure 9 shows the flux change as a function of time for two temperatures in a Pb–In cylinder with only one flux quantum trapped in the bore. The flux change was recorded after switching off the external field. In the upper diagrams only three curves are shown from a series of many experiments. In the lower diagram the mean values of the flux change are plotted, demonstrating an enhanced creep rate with increasing temperature. The results agree with Anderson's model of flux creep (1962).

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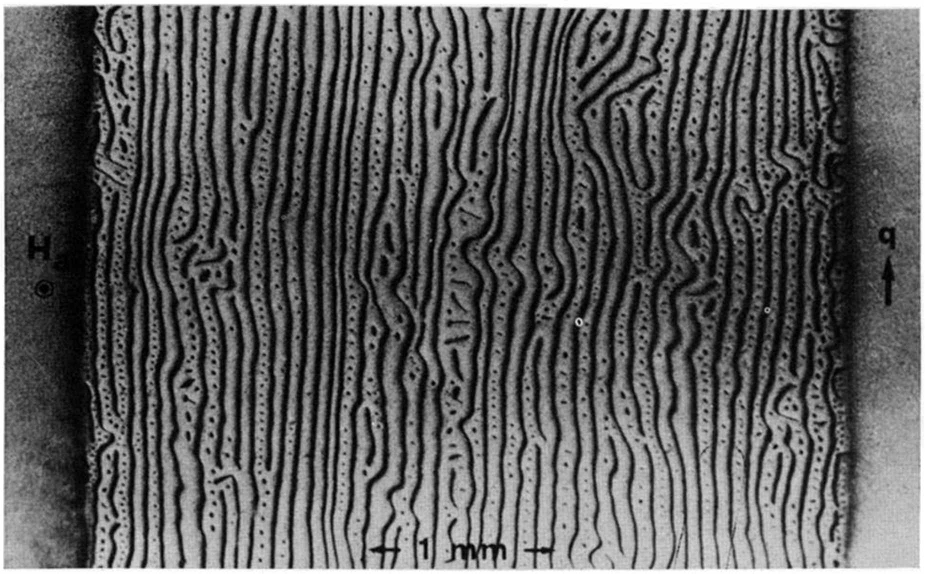

FIG. 1. Intermediate-state static lamellar structure observed when passing a heat current $q$ through a lead slab subjected to a transverse magnetic field. Superconducting domains are black; sample thickness $d = 0.5$ mm; reduced magnetic field $h = H_a/H_c = 0.91$; structure parameter $a = 100\ \mu$; mean temperature $T = 3.2$ K; averaged heat flow $q = 3.3$ Watt/cm²; averaged temperature gradient $\nabla T \simeq 0.16$ K/cm. (Rinderer, paper 1:2. The paper number in all figure captions refers to the Argonne report of Footnote 1.)

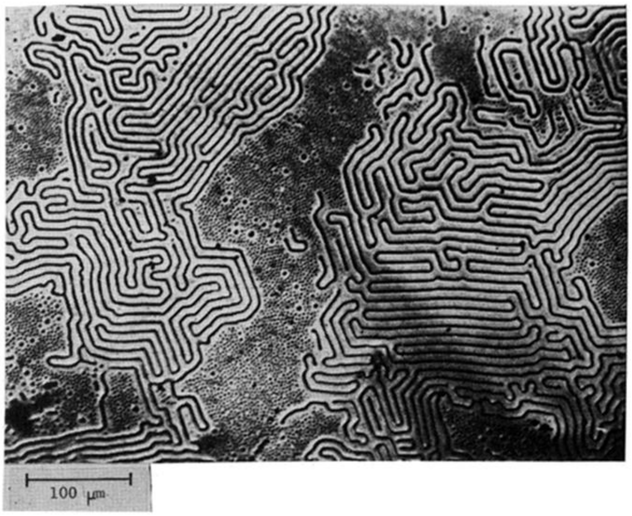

FIG. 3. Intermediate-state structure in a single-crystalline lead foil with 23–27 µm thickness and (357) orientation; $T = 1.2$ °K, $h = 0.19$. The circular flux tubes contain about 47 flux quanta. The normal laminae (black) are parallel to the projections of the (lll) directions into the plane of the foil. (Essman, paper 7:1)

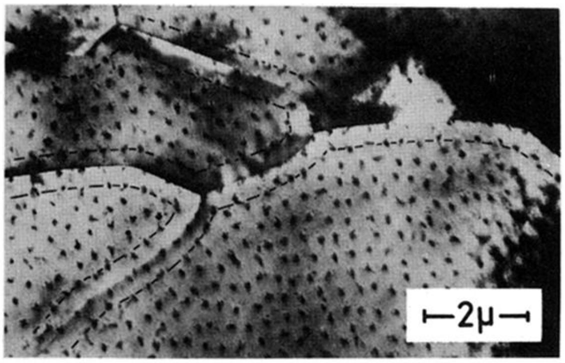

FIG. 6. Simultaneous observation of the flux line distribution and of the crystalline structure of a thin Pb film ($d = 1500$ Å) as detected by decoration microscopy. To accentuate the arrangement of the flux lines parallel to the grain boundaries, caused by flux pinning at the grain boundaries, some of them are connected by dashed lines. *H* raised from 0 to 160 A/cm, $T = 1.2$°K. (Boersch, Lischke, and Rodewald, paper 14:4).