Fact and fancy in neutrino physics*

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This paper reviews the success of the quark model in describing deep-inelastic lepton scattering. The neutral current predictions of a variety of unified gauge models are given and it is shown how experiment may distinguish among them. All the models involve new hadronic quantum numbers (charm or fancy). Their effects at high energy are explored.

CONTENTS

These are the proceedings of an imagined round-table discussion of fact and fiction in neutrino physics, performed at Harvard on December 3, 1973. The participants are *Moderator*—an experimentalist; *Speaker*—a conservative theorist; *Model Builder*—a not-so-conservative theorist; and *Computer*—a talking computer. The reader is warned that all the participants are partisans of quarks and gauge theories, and that their discussion is not a critical review of the status of weak-interaction theory or experiment. We have divided the discussion into six sections and an appendix:

I. SPEAKER PRESENTS THE NAIVE QUARK MODEL PREDICTIONS FOR NEUTRINO EXPERIMENTS

Moderator: In recent months, we have seen rapid developments in both weak-interaction theory and experiment. We now have renormalizable theories of weak interactions (Weinberg, 1967, 1973; Salam, 1968) which make striking new experimental predictions. Neutrino experiments which can test these theories have been done and are now in progress (Musset, 1973). Our round-table discussion is concerned with these developments, with what has already been learned, and with what can be learned in the near future.

Speaker will begin the discussion with a brief talk about deep-inelastic lepton scattering in the context of the naive quark model.

Speaker: Imagine the triumphs of the naive quark model! How else can we see why hadron states occur just in those $SU(3)$ multiplets built from three quarks or a quark–antiquark pair? What simpler explanation of the observed 3/2 ratio for baryon–baryon/meson–baryon total cross sections? These are but two examples of how well the naive quark model describes strong-interaction phenomena.

This remarkable success extends to predictions for deep-inelastic lepton scattering. The model accurately predicts all inclusive charged-current neutrino and antineutrino data in terms of electroproduction information. Many of these predictions are already well known (Bjorken and Paschos, 1969, 1970).[1]

By the naive quark model, I mean the assertion that the nucleon—probed by weak or electromagnetic interactions in the deep-inelastic region—behaves as if it were composed exclusively of free pointlike p-type and n-type quarks (but no antiquarks), with a possible neutral background unseen by the probe. Deep-inelastic lepton scattering is described in terms of the quark distributions $p(x)$ and $n(x)$. They are the probability densities to find a given type of quark carrying a fraction x of the proton's longitudinal momentum, in the infinite momentum frame.[2] With this hypothesis, I can express the cross sections in terms of the distributions and the weak and electromagnetic properties of free quarks.

I assume, as do their inventors, that the quarks have fractional electric charges. Whether there is just one pair of quarks or a pair of color triplets will not matter. For muonic weak cross sections

$$\nu(\bar{\nu}) + N \to \mu(\bar{\mu}) + \cdots, \tag{1}$$

I use the conventional model of weak interaction. I interpret the recently reported muonless cross sections (Hasert *et al.*, 1973b; Benvenuti *et al.*, 1974a) as neutral-current effects

$$\nu(\bar{\nu}) + N \to \nu(\bar{\nu}) + \cdots, \tag{2}$$

and I use Weinberg-Salam (Weinberg, 1967; Salam, 1968) model to describe them. The electromagnetic coupling and the relevant effective charged and neutral weak couplings are:

$$\mathcal{L}(\text{electromag.}) = (e^2/q^2)(\bar{e}\gamma^\alpha e)(\tfrac{2}{3}\bar{p}\gamma_\alpha p - \tfrac{1}{3}\bar{n}\gamma_\alpha n), \tag{3a}$$

$$\mathcal{L}(\text{charged}) = (G/\sqrt{2})[\bar{\mu}\gamma^\alpha(1+\gamma_5)\nu][\bar{p}\gamma_\alpha(1+\gamma_5)n] + \text{h.c.}, \tag{3b}$$

$$\mathcal{L}(\text{neutral}) = (G/\sqrt{2})[\bar{\nu}\gamma^\alpha(1+\gamma_5)\nu] \times \{\bar{p}\gamma_\alpha[a(1+\gamma_5)+c(1-\gamma_5)]p + \bar{n}\gamma_\alpha[b(1+\gamma_5)+d(1-\gamma_5)]n\}. \tag{3c}$$

In the Weinberg model the quantities a, b, c, d are of order 1,

* Work supported in part by the Air Force Office of Scientific Research under contract F44620–70–C–0030 and in part by the National Science Foundation under grant GP40397X.

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[1] For a review see Llewellyn-Smith, 1972.

[2] See, for example, Feynman, 1972.

$$\begin{aligned} a &= \tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta_W, \\ b &= -\tfrac{1}{2} + \tfrac{1}{3}\sin^2\theta_W, \\ c &= -\tfrac{2}{3}\sin^2\theta_W, \\ d &= \tfrac{1}{3}\sin^2\theta_W. \end{aligned} \tag{4}$$

The neutral-current couplings involve a new dimensionless parameter $\sin^2\theta_W$ characteristic of the model. I have neglected effects proportional to $\sin^2\theta_c$, where θ_c is the Cabibbo angle. In the naive quark model, this allows me to omit all reference to strangeness and charm.

Define the usual scaling variables

$$\begin{aligned} x &= -q^2/2(pq) = -q^2/2\nu, \\ y &= (pq)/(pk) = (E - E')/E, \end{aligned} \tag{5}$$

where k, p, and q are respectively the incident lepton momentum, target momentum, and momentum transfer, and where E and E' are the laboratory energies of initial and final leptons. In terms of these variables, I find for the deep-inelastic electroproduction cross sections of protons

$$d^2\sigma(ep)/dxdy = e^4 q^{-4} mE\pi^{-1} F_2(ep)[1 + (1-y)^2], \tag{6}$$

where $F_2(ep) = x(\frac{4}{9}p + \frac{1}{9}n)$.

I denote by $\Sigma(\nu p)$ and $\Sigma(\bar\nu p)$, respectively, the muonic (charged current) cross sections for ν and $\bar\nu$ on protons in units of $G^2 mE/\pi$, and denote by $\tilde\Sigma(\nu p)$ and $\tilde\Sigma(\bar\nu p)$ the corresponding muonless cross sections. These quantities are energy-independent as a consequence of the scaling hypothesis. In the naive quark model I calculate:

$$\begin{aligned} d^2\Sigma(\nu p)/dxdy &\equiv (\pi/G^2 mE)[d^2\sigma(\nu p)/dxdy] = 2xn, \\ d^2\Sigma(\bar\nu p)/dxdy &= 2xp(1-y)^2, \\ d^2\tilde\Sigma(\nu p)/dxdy &= 2x\{[a^2 + c^2(1-y)^2]p \\ &\qquad + [b^2 + d^2(1-y)^2]n\}, \\ d^2\tilde\Sigma(\bar\nu p)/dxdy &= 2x\{[a^2(1-y)^2 + c^2]p \\ &\qquad + [b^2(1-y)^2 + d^2]n\}. \end{aligned} \tag{7}$$

Charge symmetry implies that the cross sections on neutrons can be obtained from those on protons by the interchange of p and n.

The functions $p(x)$ and $n(x)$ may be determined from electroproduction data (Bloom *et al.*, 1969; Breidenback *et al.*, 1969; Miller *et al.*, 1972; Bodek *et al.*, 1973). In terms of these functions all eight neutrino cross sections are explicitly predicted, except for the parameter θ_W. I shall now show that these predictions agree with what has been observed, concentrating first on the charged-current events.

Some predictions are independent of electroproduction data. Let

$$\Sigma(\nu d) = \Sigma(\nu p) + \Sigma(\nu n) \tag{8}$$

and

$$\Sigma(\bar\nu d) = \Sigma(\bar\nu p) + \Sigma(\bar\nu n).$$

The naive quark model predicts the ratio

$$R = \Sigma(\bar\nu d)/\Sigma(\nu d) = \tfrac{1}{3}, \tag{9}$$

to be compared with the experimental values,

$$R = \begin{cases} 0.37 \pm 0.02 & \text{Gargamelle (Eichten } et\ al.\text{, 1974)} \\ & E > 1 \text{ GeV} \\ 0.38 \pm 0.02 & \text{Gargamelle (Eichten } et\ al.\text{, 1974)} \\ & E > 2 \text{ GeV} \\ 0.34 \pm 0.03 & \text{Wide-band NAL (Benvenuti } et\ al.\text{,} \\ & \text{1974b)} \\ & \langle E\rangle \sim 40 \text{ GeV}. \end{cases} \tag{10}$$

Moreover, the differential cross section for ν scattering should be independent of y, while for $\bar\nu$ it should be proportional to $(1-y)^2$. This result is in agreement with all published data: for $\bar\nu$ at moderate energies (Perkins, 1972, see Fig. 1) and for ν at higher energies (Barish *et al.*, 1973, see Fig. 2).

The differential cross sections $d\Sigma/dx$ for both ν and $\bar\nu$ off matter (by which I mean any target consisting of approximately equal numbers of protons and neutrons) should be proportional to the sum of electroproduction structure functions $F_2(ep) + F_2(en)$. A test of this prediction with ν data (Barish *et al.*, 1973) is shown in Fig. 3.

Better tests can be obtained in terms of the variable (Myatt and Perkins, 1971)

$$xy = -q^2/2mE \cong q_\perp^2/2mE'$$

whose measurement involves only muon observables ($q_\perp$ is the laboratory transverse momentum of the muon). The $d\Sigma/d(xy)$ distributions predicted (Bjorken *et al.*, 1973) from electroproduction data are compared to the wide-band NAL results (Musset, 1973; Rubbia, 1973) for ν and $\bar\nu$ in Figs. 4 and 5.

Moments of the xy distributions can be accurately measured and compared to the predictions of the quark model. Define the average value of an observable o in lepton–hadron (lh) scattering by

$$\langle o\rangle_{lh} = [\Sigma(lh)]^{-1} \int o\, d\Sigma(lh). \tag{11}$$

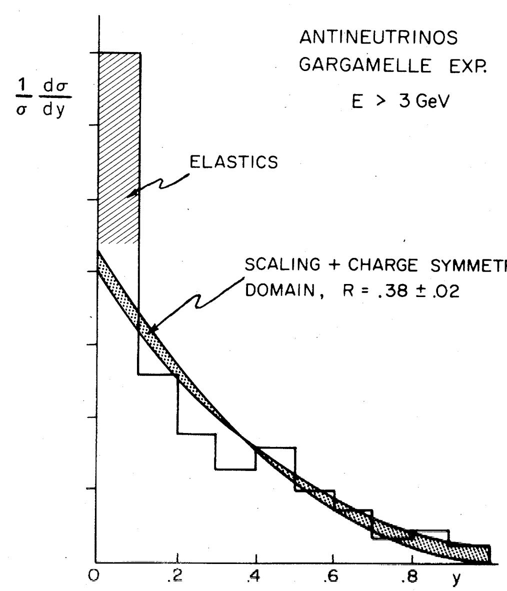

FIG. 1. Distribution of muonic events as a function of y for incident antineutrinos (Perkins, 1972).

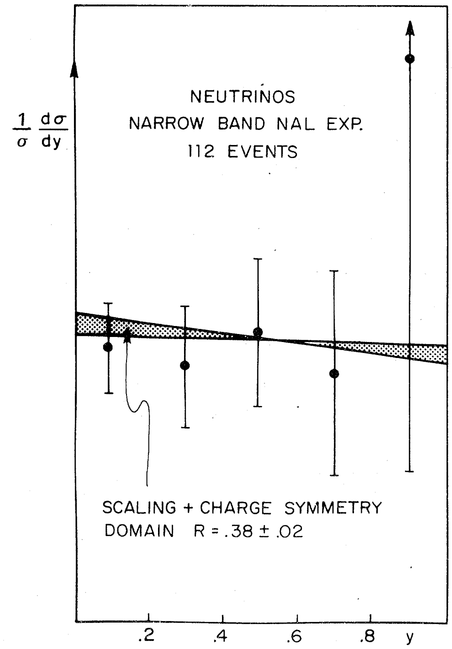

FIG. 2. Distribution of muonic events as a function of y for incident neutrinos (Musset, 1973).

The quark model relates such moments for ν and $\bar{\nu}$ scattering to electroproduction moments. For example, it predicts

$$\langle (xy)^{-1/2} \rangle_{\nu d} = 2\langle x^{-1/2} \rangle_{ed} \sim 5.0, \tag{12a}$$

$$\langle (xy)^{-1/2} \rangle_{\bar{\nu} d} = \tfrac{16}{5}\langle x^{-1/2} \rangle_{ed} \sim 7.9, \tag{12b}$$

$$\langle (xy)^{1/2} \rangle_{\nu d} = \tfrac{2}{3}\langle x^{1/2} \rangle_{ed} \sim 0.29, \tag{12c}$$

$$\langle (xy)^{1/2} \rangle_{\bar{\nu} d} = \tfrac{16}{35}\langle x^{1/2} \rangle_{ed} \sim 0.20, \tag{12d}$$

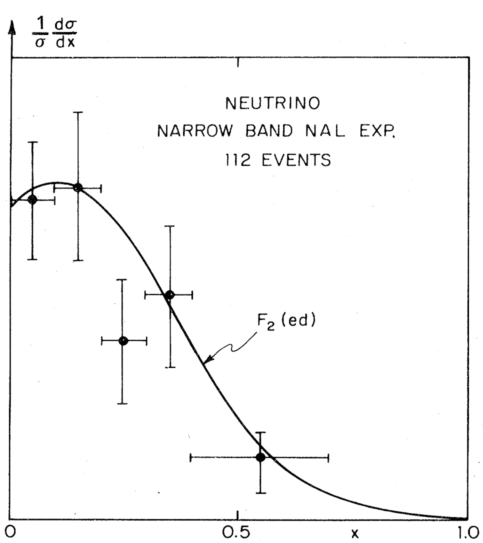

FIG. 3. Distribution of muonic events as a function of x for incident neutrinos. The solid line is the prediction from electroproduction data (Musset, 1973; Bloom *et al.*, 1969; Breidenbach *et al.*, 1969; Miller *et al.*, 1972; Bodek *et al.*, 1973).

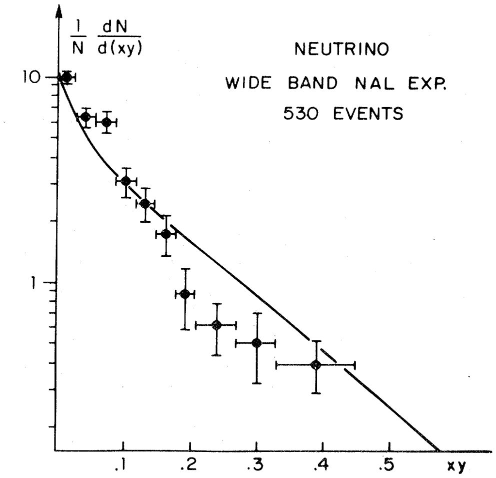

FIG. 4. Distribution of muonic events as a function of xy for incident neutrinos (Musset, 1973; Rubbia, 1973). The solid line is the prediction from electroproduction data (Bjorken *et al.* 1973).

$$\langle (xy) \rangle_{\nu d} = \tfrac{1}{2}\langle x \rangle_{ed} \sim 0.12, \tag{12e}$$

$$\langle (xy) \rangle_{\bar{\nu} d} = \tfrac{1}{4}\langle x \rangle_{ed} \sim 0.06, \tag{12f}$$

$$\langle (xy)^2 \rangle_{\nu d} = \tfrac{1}{3}\langle x^2 \rangle_{ed} \sim 0.028, \tag{12g}$$

$$\langle (xy)^2 \rangle_{\bar{\nu} d} = \tfrac{1}{10}\langle x^2 \rangle_{ed} \sim 0.008. \tag{12h}$$

Measurements of two of these moments have been published (Eichten *et al.*, 1974): are

$$\begin{aligned} \langle xy \rangle_{\nu d} &= 0.12 \pm 0.01, \\ \langle xy \rangle_{\bar{\nu} d} &= 0.07 \pm 0.01, \end{aligned} \tag{13}$$

and are in triumphant agreement with the quark model.

I can also predict the total neutrino cross section. The Gargamelle values for muonic neutrino scattering on matter (Eichten *et al.*, 1974)

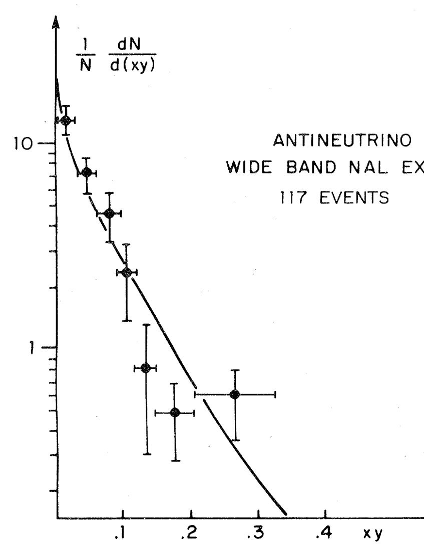

FIG. 5. Distribution of muonic events as a function of xy for incident antineutrinos (Musset, 1973; Rubbia, 1973). The solid line is the prediction from electroproduction data (Bjorken *et al.*, 1973).

$$\Sigma(\nu d) = \begin{cases} 0.904 \pm 0.025 & \text{one parameter fit}, \quad E > 1\,\text{GeV} \\ 0.904 \pm 0.038 & \text{one parameter fit}, \quad E > 2\,\text{GeV} \\ 0.855 \pm 0.088 & \text{two parameter fit}, \quad E > 1\,\text{GeV} \\ 0.942 \pm 0.113 & \text{two parameter fit}, \quad E > 2\,\text{GeV}, \end{cases} \tag{14}$$

while the quark model prediction is

$$\Sigma(\nu d) = \tfrac{18}{5} \int dx F_2(ed) = 1.10 \pm 0.06. \tag{15}$$

In summary, all of the experimental results about deep-inelastic ν or $\bar{\nu}$ charged-current scattering are correctly predicted from electroproduction data by the naive quark model. However, though a devoted fan of this model, I would not argue that these experiments need not have been done.

II. IN WHICH NEUTRAL-CURRENT EVENTS ARE CONSIDERED

Speaker: I will now discuss the neutral-current effects predicted by the Weinberg model. These effects have been recently reported and measured both at CERN (Hasert *et al.*, 1973b) and at NAL (Benvenuti *et al.*, 1974a) and they also agree with theoretical expectations.
Model Builder: What have been reported are neutrino-induced events without outgoing muons. Could these events not result from the production of heavy leptons which decay promptly into hadrons and a neutrino? These would seem like neutral-current effects.
Moderator: Any heavy lepton might be expected to have electronic decay modes. Observed electron events are few (Deden *et al.*, to be published), and are accounted for by the small ($\sim$ 1%) contamination of the neutrino beam with electron neutrinos, so it is unlikely that heavy lepton production can explain the muonless events.

For heavy leptons with the same sign of charge and lepton number, which appear in some gauge theories (Georgi and Glashow, 1972; Lee, 1972; Prentki and Zumino, 1972), I can make a stronger statement. I would expect a substantial branching ratio for decay of the heavy lepton into a muon and neutrinos. But, the muon would have the "wrong" charge: incident neutrinos would produce μ^+, while antineutrinos would produce μ^-. Events like these have not been seen, implying lower bounds for the masses of leptons of this kind:

$$\begin{aligned} M(M^+) &\geq 2.4\text{ GeV} \quad \text{(Musset, 1973)}, \\ M(M^+) &\geq 6\text{ GeV} \quad \text{(Barish, 1973)}. \end{aligned} \tag{16}$$

Speaker: Should we not take a more positive approach? The experiments have measured the distribution in $\nu = m(E_H - m)$ of the neutral-current candidates (Musset, 1973), where E_H is the energy of the hadron shower. Can simple theoretical considerations tell what these distributions should be for true neutral-current events? If the experimental data satisfies this test, it would be much harder to deny the neutral-current interpretation.
Moderator: All we need are the assumptions of scaling behavior, and of no longitudinal structure functions—well satisfied by electroproduction and charged-current data. The differential cross sections for muonless scattering of neutrinos or antineutrinos of energy E are then

$$\begin{aligned} d\tilde{\Sigma}(\nu)/d\nu &= A_R(1 - \nu/mE)^2 + A_L, \\ d\tilde{\Sigma}(\bar{\nu})/d\nu &= A_R + A_L(1 - \nu/mE)^2, \end{aligned} \tag{17}$$

where A_R and A_L are nonnegative constants. (I assume that the neutral currents are vector and axial vector). Using Eq. (17), I find for the ratio

$$\tilde{R} = \tilde{\Sigma}(\bar{\nu})/\tilde{\Sigma}(\nu) \tag{18}$$

the value

$$\tilde{R} = (3A_R + A_L)/(A_R + 3A_L) \tag{19}$$

which must lie in the range

$$3 \geq \tilde{R} \geq \tfrac{1}{3}. \tag{20}$$

With $\tilde{R}$ known, the shapes of the measured ν distributions are determined in terms of the ν and $\bar{\nu}$ fluxes.

$$\begin{aligned} d\tilde{N}(\nu)/d\nu &\propto \int_{\nu/m}^{\infty} \rho_\nu(E)[1 + A_R(1 - \nu/mE)^2/A_L]\,dE, \\ d\tilde{N}(\bar{\nu})/d\nu &\propto \int_{\nu/m}^{\infty} \rho_{\bar{\nu}}(E)[A_R/A_L + (1 - \nu/mE)^2]\,dE. \end{aligned} \tag{21}$$

Only if $A_R = 0$, do the distributions for neutral-current events have the same shape as the corresponding charged-current distributions. Otherwise, for neutrino-induced events the neutral-current distribution is more sharply peaked at low ν than the charged; while for antineutrino-induced events, it is flatter. Unfortunately,

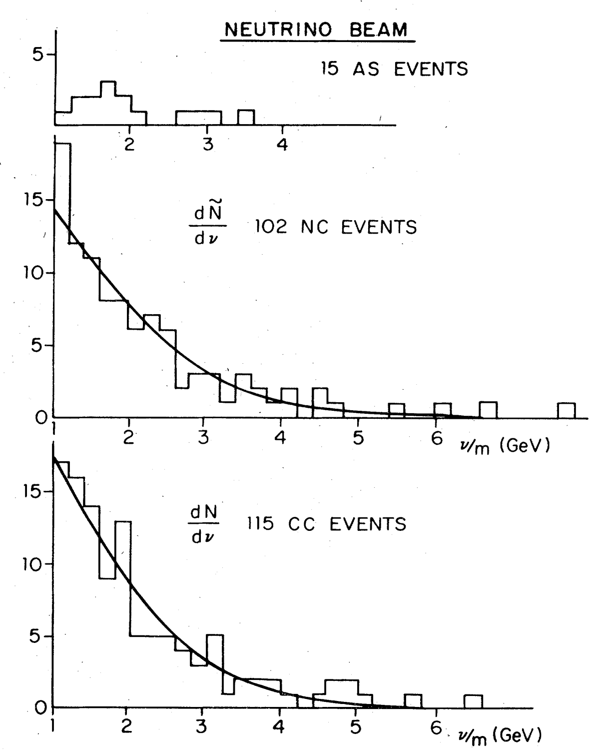

FIG. 6. Distribution of events as a function of ν for incident neutrinos (Musset, 1973). AS = "associated star" events, NC = neutral current events, CC = charged current events.

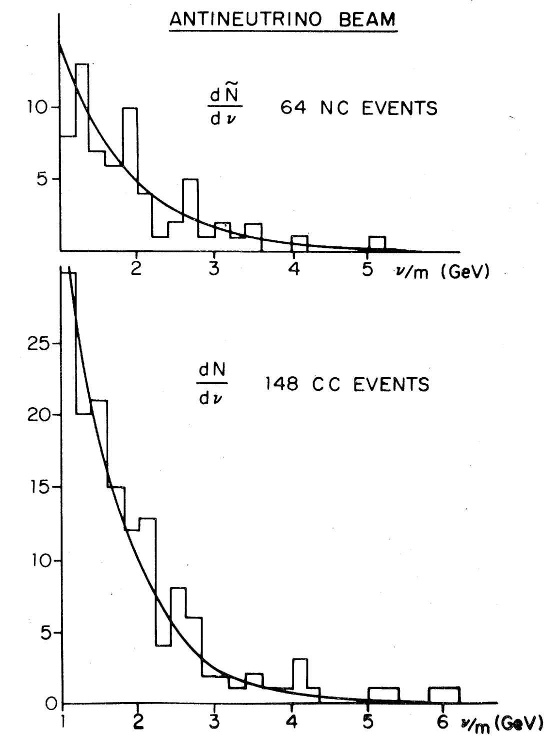

FIG. 7. Distribution of events as a function of ν for incident antineutrinos (Musset, 1973).

CERN does not directly measure $\tilde{R}$, since their data is cut so that $\nu/m \geq 1$ GeV (Musset, 1973). But, integrating Eq. (21) from $\nu/m = 1$ GeV, it should be possible to compute the values of $\tilde{R}$ or A_R/A_L in matter targets from the data, and the known ν and $\bar{\nu}$ fluxes.
Computer: I have just done that. I find

$$\tilde{R} = 0.53 \pm .15, \qquad A_R/A_L \cong 0.24. \tag{22}$$

With this result, I again use Eq. (21) to give the expected ν distributions. In Figs. 6 and 7 I compare them with the observed ν distributions of charged- and neutral-current events for both ν and $\bar{\nu}$ scattering. The agreement is satisfactory. In Fig. 6 I have also shown the experimental distribution for "associated star" events in which the neutral particle producing an interaction appears to come from a previous interaction in the chamber. These events, which are not neutrino-induced, seem to have a different ν distribution.

Similar tests can be made even when the fluxes are not well known. Let $dN(\nu)/d\nu$ and $dN(\bar{\nu})/d\nu$ denote the ν distributions of charged events. Construct the distributions $dM(\nu)/d\nu$ by weighting each neutrino event with the factor $[mE_\mu/(mE_\mu + \nu)]^2$, and $dM(\bar{\nu})/d\nu$ by weighting each antineutrino event with the inverse of this factor. In terms of these functions the neutral-current distributions are

$$\begin{aligned} d\tilde{N}(\nu)/d\nu &\propto dN(\nu)/d\nu + (A_R/A_L)dM(\nu)/d\nu, \\ d\tilde{N}(\bar{\nu})/d\nu &\propto dN(\bar{\nu})/d\nu + (A_R/A_L)dM(\bar{\nu})/d\nu. \end{aligned} \tag{23}$$

If the average neutrino and antineutrino energies are known, and if no cuts in ν are made, A_R/A_L (or $\tilde{R}$) is determined from the total numbers of neutrino and antineutrino neutral-current events.

With no cuts I can also derive flux-independent tests involving the average values of the energy transfer. The simplest examples are

$$\begin{aligned} \langle\tilde{\nu}\rangle_\nu/\langle\nu\rangle_\nu &= (17 - 3\tilde{R})/16, \\ \langle\tilde{\nu}\rangle_{\bar{\nu}}/\langle\nu\rangle_{\bar{\nu}} &= (17\tilde{R} - 3)/8\tilde{R}, \end{aligned} \tag{24}$$

where $\langle\tilde{o}\rangle$ is defined by Eq. (11) with $\tilde{\Sigma}$ replacing Σ. Eqs. (21), (23), and (24) are model-independent and apply to scattering on any target.
Speaker: We appear to have convinced ourselves that the most plausible interpretation of the muonless events is that they are neutral-current effects. I shall proceed with the predictions of the Weinberg model in the naive quark picture (Sehgal, 1973; Glashow, 1974). Consider a plot whose abcissa is the ratio of cross sections

$$\tilde{\Sigma}(\nu d)/\Sigma(\nu d) = \tfrac{1}{2} - \sin^2\theta_W + \tfrac{20}{27}\sin^4\theta_W, \tag{25}$$

and whose ordinate is

$$\tilde{\Sigma}(\bar{\nu} d)/\Sigma(\bar{\nu} d) = \tfrac{1}{2} - \sin^2\theta_W + \tfrac{20}{9}\sin^4\theta_W. \tag{26}$$

The resulting noselike curve is shown in Fig. 8. Also shown is the region allowed by the NAL experiment (Benvenuti *et al.*, 1974a) using unseparated ν and $\bar{\nu}$. I cannot show the CERN data because of their ν cut.
Computer: I can. I have corrected the CERN data (Musset, 1973; Hasert *et al.*, 1973b) in the light of our previous discussion, and superposed my deduction of the two cross sections on your figure

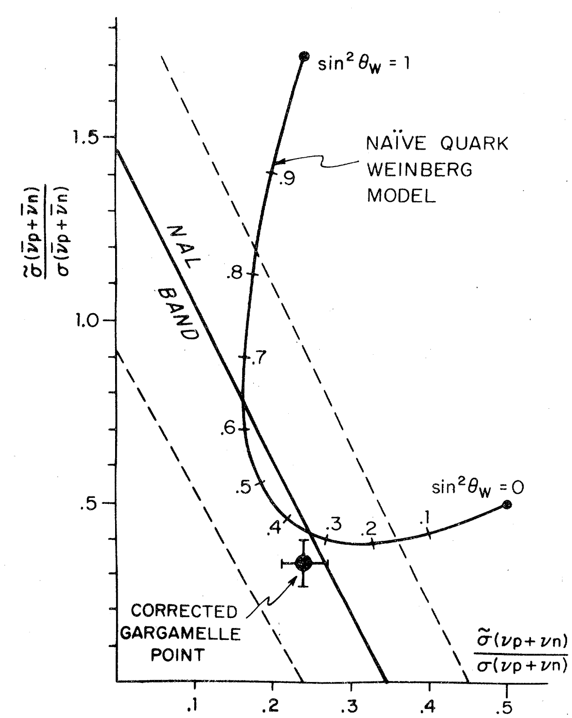

FIG. 8. Predictions for the ratios of muonless to muonic total cross sections on matter (as functions of $\sin^2\theta_W$) of the Weinberg model in the naive quark approximation. The NAL measurement is made with an unseparated neutrino and antineutrino beam (Benvenuti *et al.*, 1974a).

$$\tilde{\Sigma}(\bar{\nu}d)/\Sigma(\bar{\nu}d) = 0.33 \pm 0.07^3,$$
$$\tilde{\Sigma}(\nu d)/\Sigma(\nu d) = 0.24 \pm 0.03. \qquad (27)$$

The best value of $\sin^2\theta_W$ is

$$\sin^2\theta_W \sim 0.32. \qquad (28)$$

III. SPEAKER DISCUSSES EXPERIMENTS ON PROTON, NEUTRON, AND ELECTRON TARGETS

Moderator: You have spoken only about ν and $\bar{\nu}$ scattering from matter. What are the quark picture results for scattering from proton or neutron targets separately?
Speaker: Since there is little data to compare to, I give only predictions for the total cross sections and the average values of xy. Let me define the ratios

$$U = \Sigma(\bar{\nu}n)/\Sigma(\bar{\nu}d),$$
$$V = \Sigma(\nu p)/\Sigma(\nu d), \qquad (29)$$

of charged-current cross sections. These ratios, like R, are determined in the naive quark model, from Eq. (7). I find

$$U = V = (1+\eta)^{-1}, \qquad (30)$$

where η is the ratio of the mean momentum carried by proton quarks to that carried by neutron quarks in the proton,

$$\eta = \int_0^1 dxxp/\int_0^1 dxxn. \qquad (31)$$

The value of η is determined from electroproduction data (Bloom *et al.*, 1969; Breidenbach *et al.*, 1969; Miller *et al.*, 1972; Bodek *et al.*, 1973)

$$\eta = \int_0^1 [4F_2(ep) - F_2(en)]\,dx/\int_0^1 [4F_2(en) - F_2(ep)]\,dx \cong 1.57. \qquad (32)$$

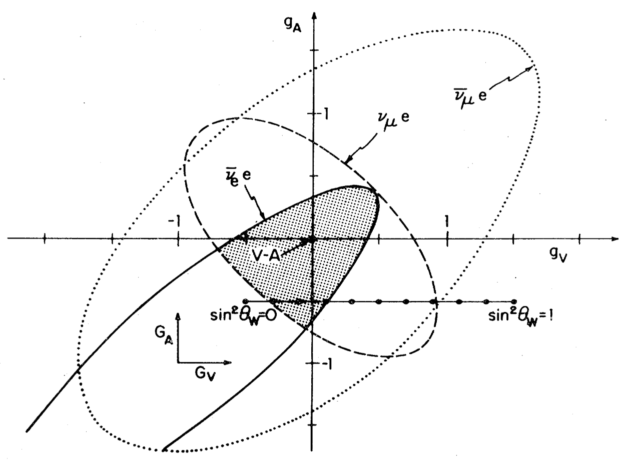

FIG. 9. Allowed domains (90% confidence level) for g_A and g_V from neutrino–electron scattering (Gurr *et al.*, 1972; Chen and Lee, 1972; Hasert *et al.*, 1973a).

[3] P. Musset informs us that this value should be reduced by about ten percent to correct for other biases of the CERN experiment.

With this result, Eq. (30) becomes

$$U = V \cong 0.39. \qquad (33)$$

Moderator: There is a published experimental value for V which agrees with this prediction. It is $V = 0.36 \pm 0.04$ (Myatt and Perkins, 1971).
Speaker: Moments of xy on separated targets are also predicted. For instance

$$\langle xy\rangle_{\nu p} = 2\langle xy\rangle_{\bar{\nu}n} = [(16+4\eta)\langle x\rangle_{en} - (4\eta+1)\langle x\rangle_{ep}]/30 \cong 0.095,$$
$$\langle xy\rangle_{\nu n} = 2\langle xy\rangle_{\bar{\nu}p} = [(16\eta+4)\langle x\rangle_{ep} - (4+\eta)\langle x\rangle_{en}]/30\eta \cong 0.13. \qquad (34)$$

Similar results apply to neutral-current cross sections. Defining the ratios

$$\tilde{U} = \tilde{\Sigma}(\bar{\nu}n)/\tilde{\Sigma}(\bar{\nu}d),$$
$$\tilde{V} = \tilde{\Sigma}(\nu p)/\tilde{\Sigma}(\nu d), \qquad (35)$$

I get

$$\tilde{U} = \frac{a^2+3c^2+\eta(b^2+3d^2)}{(1+\eta)(a^2+b^2+3c^2+3d^2)},$$
$$\tilde{V} = \frac{\eta(3a^2+c^2)+3b^2+d^2}{(1+\eta)(3a^2+3b^2+c^2+d^2)}, \qquad (36)$$

where a, b, c, d are as in Eq. (4). With $\sin^2\theta_W$ and η determined above, I predict

$$\tilde{U} = 0.49, \quad \tilde{V} = 0.48. \qquad (37)$$

In other words $\tilde{\Sigma}(\bar{\nu}p) \approx \tilde{\Sigma}(\bar{\nu}n)$ and $\tilde{\Sigma}(\nu p) \approx \tilde{\Sigma}(\nu n)$.
Moderator: Experiments on electron targets are more difficult since the expected cross sections at fixed neutrino energy are down by three orders of magnitude: the available center-of-mass energy is proportional to the target mass. On the other hand, their theoretical analysis is cleaner because the strong interactions are not involved. Can you comment on the predictions for neutrino–electron scattering?
Speaker: In any theory where the weak force is mediated by heavy vector bosons, and in particular in any $SU(2) \times U(1)$ gauge theory, the effective coupling for the elastic scattering of neutrinos from electrons is

$$\mathfrak{L}(\nu e \to \nu e) = G/\sqrt{2}[\bar{\nu}_\mu\gamma^\alpha(1+\gamma_5)\nu_\mu\bar{e}\gamma_\alpha(g_V+g_A\gamma_5)e + \bar{\nu}_e\gamma^\alpha(1+\gamma_5)\nu_e\bar{e}\gamma_\alpha(G_V+G_A\gamma_5)e]. \qquad (38)$$

If the neutrino neutral currents are invariant under the interchange of ν_μ and ν_e

$$G_V = 1 + g_V,$$
$$G_A = 1 + g_A. \qquad (39)$$

In muon–neutrino scattering, only the neutral currents participate, while electron–neutrino scattering gets contributions from both neutral and charged currents. The differential cross sections for muon neutrinos are

$$\frac{d\sigma}{dE_e}\begin{pmatrix}\bar{\nu}\\ \nu\end{pmatrix} = \frac{G^2 m_e}{2\pi}\left((g_V \mp g_A)^2 + (g_V \pm g_A)^2\left(1 - \frac{E_e}{E_\nu}\right)^2 - \frac{m_e E_e}{E_\nu^2}(g_V{}^2 - g_A^2)\right), \quad (40)$$

where E_e is the energy of the recoil electron. The last term is negligible in accelerator experiments. For electron neutrinos substitute $g_V \to G_V$, $g_A \to G_A$. Should experiment not confirm this two parameter description, the model of weak interactions would need drastic revision.

In the conventional V–A model, $g_V = g_A = 0$. In the Weinberg model

$$g_V = -\frac{1}{2} + 2\sin^2\theta_W$$
$$g_A = -\frac{1}{2}. \quad (41)$$

At present only one candidate of the type $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$ has been found and there are only upper bounds for neutrino–electron scattering processes. The available information is displayed in the form of 90% confidence level constraints on g_A and g_V in Fig. 9. The $\bar{\nu}_e e$ constraint is taken directly from the reactor experiment (Gurr *et al.*, 1972; Chen and Lee, 1972). I have computed the $\nu_\mu e$ and $\bar{\nu}_\mu e$ constraints from the published CERN data (Hasert *et al.*, 1973a).

Experiment is consistent with the V–A model and with the Weinberg model for values of θ_W in the range $0.32 \geq \sin^2\theta_W \geq 0.10$. This determination of $\sin^2\theta_W$ is compatible with Eq. (28) obtained from the observations of semileptonic neutral-current phenomena. Experiments just one order of magnitude better are needed to distinguish between the models.

IV. COMPUTER INTERPRETS THE SUCCESS OF THE NAIVE QUARK MODEL

Moderator: Returning to the naive quark model, I must admit to being puzzled at its striking successes. The procedure was to determine $p(x)$ and $n(x)$ from electroproduction data, but I wonder how good the model is for that data.

Any theory in which the only charged constituents are fermions predicts that the longitudinal structure function F_L that you have been neglecting all along vanishes in the deep-inelastic regime (Callan and Gross, 1969). A constant fit to electroproduction data (Bloom, 1973) gives $F_L/F_2 \sim 0.168 \pm 0.014$. Thus, I am surprised that the quark model seems to describe neutrino processers with better than 17% accuracy.

A more specific question concerns the naive model sum rule

$$\int_0^1 [F_2(ep) - F_2(en)]\frac{dx}{x} = \frac{1}{3} \quad (42)$$

which seems to be far from saturated by data from the region $0.07 < x < 0.87$ (Bloom *et al.*, 1969; Breidenbach *et al.*, 1969; Miller *et al.*, 1972; Bodek *et al.*, 1973):

$$\int_{0.07}^{0.87} [F_2(ep) - F_2(en)]\frac{dx}{x} = 0.13 \pm 0.03. \quad (43)$$

Speaker: I can only answer your second question. In the naive quark model, integrals of the form $\int F_2(x)\,dx/x$ simply count the numbers of quarks in the proton; but the predictions I discussed more often than not depend on non-negative moments of F_2. It is conceivable that there are very many quarks in the proton carrying only very small momenta. These wee quarks would contribute to the sum rule you mention but would not significantly affect total cross sections, nor distributions away from the neighborhood of $x = 0$. Failure of the sum rule would not invalidate the predictions we have discussed, except possibly the low x moments (12a, b) and (12c, d).

Moderator: I also have a question concerning your use of the experimental information. Your electroproduction data refer to the scaling region in the (q^2, ν) plot; cuts have been implemented to eliminate the low q^2 and resonance regions. On the other hand, you use the bulk of the neutrino data, without scaling cuts. How then can you compare them in a parton model that is only supposed to apply in the scaling region?

Speaker: I would be happier if I had neutrino data restricted to the (q^2, ν) region in which electroproduction data scales. However the cross sections for elastic and resonance scattering level off with energy: these processes should not significantly affect the determination of the slope of total cross sections, at least in linear fits to $\sigma(E)$ that are not forced to go through the origin.

Moderator: I have a deeper objection to the naive quark model. What you have said depends on the existence of fractionally charged quarks. Must I believe that these unobserved particles exist in order to achieve these predictions?

Computer: I have been thinking about the same problem, and I can tell you the extent to which *Speaker*'s results can be understood without the assumption that quarks exist. Predictions almost as restrictive as those of the naive quark model can be obtained from general theoretical considerations.

Given the experimental value of R, and assuming scaling behavior and the charge symmetry of the weak current, I can deduce that $d\Sigma(\nu d)/dy$ must be approximately flat, and that $d\Sigma(\bar{\nu}d)/dy$ must be approximately proportional to $(1 - y)^2$. These results (De Rújula and Glashow, 1974), almost as strong as those of the naive quark model, are shown in Figs. 1 and 2.

To get further, I need to make additional hypotheses. The naive quark model is a special case of the general quark parton model. In this model I again describe deep-inelastic lepton scattering in terms of quark distributions, but allow for the possibility that the proton contains antiquarks and λ quarks as well as p and n quarks. Although this model also seems to treat quarks as actual consituent particles, all of its consequences have been shown to follow from light-cone algebra (Fritsch and Gell-Mann, 1971). You need not believe that quarks exist, but merely that the light cone algebra of the currents and their time derivatives is the same as if they were bilinear products of free quark fields. Assuming this, I can derive the following results . . .

Model Builder: Stop! You cannot make do with only p, n, and λ quarks. To include hadrons in the original Weinberg scheme and circumvent the appearance of strangeness-changing neutral currents you must use the mechanism of Glashow, Iliopoulos, and Maiani (1970; see also Weinberg, 1970). This requires the introduction of a fourth quark p' carrying charm (Bjorken and Glashow, 1964), a new strong interaction quantum number. The weak and electromagnetic couplings corresponding to Eqs. (3a, b, c) become

$$\mathcal{L}(\text{electromag.}) = \frac{e^2}{q^2}\{\bar{e}\gamma^\alpha e\} \times \{\tfrac{2}{3}\bar{p}\gamma_\alpha p - \tfrac{1}{3}\bar{n}\gamma_\alpha n + \tfrac{2}{3}\bar{p}'\gamma_\alpha p' - \tfrac{1}{3}\bar{\lambda}\gamma_\alpha\lambda\}, \quad (44a)$$

$$\mathcal{L}(\text{charged}) = \frac{G}{\sqrt{2}}\{\bar{\mu}\gamma^\alpha(1+\gamma_5)\nu_\mu\} \times \{\cos\theta_c\bar{p}\gamma_\alpha(1+\gamma_5)n + \sin\theta_c\bar{p}\gamma_\alpha(1+\gamma_5)\lambda + \cos\theta_c\bar{p}'\gamma_\alpha(1+\gamma_5)\lambda - \sin\theta_c\bar{p}'\gamma_\alpha(1+\gamma_5)n\} \quad (44b)$$

$$\mathcal{L}(\text{neutral}) = \frac{G}{\sqrt{2}}\{\bar{\nu}\gamma^\alpha(1+\gamma_5)\nu\} \times \{\bar{p}\gamma_\alpha[a(1+\gamma_5)+c(1-\gamma_5)]p + \bar{n}\gamma_\alpha[b(1+\gamma_5)+d(1-\gamma_5)]n + \bar{p}'\gamma_\alpha[a(1+\gamma_5)+c(1-\gamma_5)]p' + \bar{\lambda}\gamma_\alpha[b(1+\gamma_5)+d(1-\gamma_5)]\lambda\}. \quad (44c)$$

The expressions (4) for a, b, c, d remain unchanged. You can approximate $\theta_c \sim 0$ and drop the term $\bar{p}'\lambda$ in Eq. (44b) which is inoperative below the threshold for production of charmed states. For the neutral couplings (44a) and (44c), new terms survive that were dropped by hypothesis in the naive model.

Computer: I can easily include the effects of the fourth quark. From Eq. (44) I compute the cross sections for deep-inelastic lepton scattering from protons. For electron scattering, Eq. (6) is not modified but the structure function becomes

$$F_2(ep) = x[\tfrac{4}{9}(p+\bar{p}+p'+\bar{p}') + \tfrac{1}{9}(n+\bar{n}+\lambda+\bar{\lambda})]. \quad (45)$$

The neutrino cross sections are

$$\begin{aligned} d^2\Sigma(\nu p)/dxdy &= 2x[n+\bar{p}(1-y)^2] \\ d^2\Sigma(\bar{\nu} p)/dxdy &= 2x[p(1-y)^2+\bar{n}] \\ d^2\tilde{\Sigma}(\nu p)/dxdy &= 2x\{[a^2+c^2(1-y)^2](p+p') \\ &\quad + [b^2+d^2(1-y)^2](n+\lambda) \\ &\quad + [a^2(1-y)^2+c^2](\bar{p}+\bar{p}') \\ &\quad + [b^2(1-y)^2+d^2](\bar{n}+\bar{\lambda})\} \\ d^2\tilde{\Sigma}(\bar{\nu} p)/dxdy &= 2x\{[a^2(1-y)^2+c^2](p+p') \\ &\quad + [b^2(1-y)^2+d^2](n+\lambda) \\ &\quad + [a^2+c^2(1-y)^2](\bar{p}+\bar{p}') \\ &\quad + [b^2+d^2(1-y)^2](\bar{n}+\bar{\lambda})\}. \end{aligned} \quad (46)$$

In Eqs. (45) and (46) p, $\bar{p}$ etc. are non-negative functions of x. The corresponding cross sections on neutron targets are obtained from the above by interchange of the functions p and n, and of $\bar{p}$ and $\bar{n}$. As a consequence of isospin invariance and positivity, the functions satisfy the additional constraints (Nachtmann, 1972)

$$2p(x) \geq n(x), \quad 2\bar{n}(x) \geq \bar{p}(x) \quad (47)$$

for any x. Further constraints which would follow from approximate $SU(3)$ or $SU(4)$ invariance are not imposed. Although the foregoing formulas can be deduced from light-cone algebra (Callan *et al.*, 1972), I find it convenient to use the language of the quark model and to regard these functions as quark distributions, as if the general quark parton model were a mere generalization of the naive quark model. But no commitment to the existence of quarks as physical particles is implied.

Equations (45) and (46) are not independent of each other. They imply three relations among the cross sections:

$$d^2\Sigma(\nu n)/dxdy + d^2\Sigma(\bar{\nu} n)/dxdy - d^2\Sigma(\nu p)/dxdy - d^2\Sigma(\bar{\nu} p)/dxdy = 6[1-(1-y)^2][F_2(ep)-F_2(en)],$$

$$d^2\tilde{\Sigma}(\nu n)/dxdy + d^2\tilde{\Sigma}(\bar{\nu} n)/dxdy - d^2\tilde{\Sigma}(\nu p)/dxdy - d^2\tilde{\Sigma}(\bar{\nu} p)/dxdy = -6[a^2+c^2-b^2-d^2][1+(1-y)^2][F_2(ep)-F_2(en)],$$

$$d^2\tilde{\Sigma}(\nu n)/dxdy - d^2\tilde{\Sigma}(\bar{\nu} n)/dxdy - d^2\tilde{\Sigma}(\nu p)/dxdy + d^2\tilde{\Sigma}(\bar{\nu} p)/dxdy = -(a^2+d^2-b^2-c^2)[1-(1-y)^2][1+(1-y)^2]^{-1} \times \{d^2\Sigma(\nu n)/dxdy - d^2\Sigma(\bar{\nu} n)/dxdy - d^2\Sigma(\nu p)/dxdy + d^2\Sigma(\bar{\nu} p)/dxdy\}. \quad (48)$$

The first relation will be useful and may be integrated to give

$$UR = V - \tfrac{1}{2}(1-R) + 2\Sigma(\nu d)^{-1}\int_0^1 dx[F_2(ep)-F_2(en)]. \quad (49)$$

The second two relations involve data for neutral currents on separate proton and neutron targets.

I can use Eqs. (45) and (47) and existing experimental data (Bloom *et al.*, 1969; Breidenbach *et al.*, 1969; Miller *et al.*, 1972; Bodek *et al.*, 1973; Eichten *et al.*, 1974) calculate the contributions of quarks other than p and n to the total cross sections. This will give an estimate of how good an approximation the naive quark model is. At energies below charm threshold, I find

$$\int_0^1 dx\, 2x(\bar{p}+\bar{n}) = \tfrac{3}{8}(3R-1)\Sigma(\nu d) = \bar{Q} \geq 0 \quad (50)$$

and

$$\int_0^1 dx 2x[\tfrac{4}{3}(p'+\bar{p}') + \tfrac{1}{3}(\lambda+\bar{\lambda})] = 3\int_0^1 dx[F_2(ep)+F_2(en)] - \tfrac{5}{8}(1+R)\Sigma(\nu d) = S \geq 0. \quad (51)$$

In the naive quark model $\bar{Q} = S = 0$. Conversely, if $\bar{Q} = S = 0$ is found to be satisfied, then only p and n are nonzero, and the general model (based only on light cone algebra) is equivalent to the naive picture. The natural scale with which to compare $\bar{Q}$ and S is the contribution to leptonic cross sections of the naive quarks p and n

$$\int_0^1 dx 2x(p+n) = \tfrac{3}{8}(3-R)\Sigma(\nu d) = Q \geq 0. \quad (52)$$

Using the available experimental data, I find

$$Q = 0.88 \pm 0.12,$$
$$\bar{Q} = 0.052 \pm 0.024, \tag{53}$$
$$S = 0.14 \pm 0.10.$$

Moderator: As an experimentalist, I am not much enlightened by statements about the wave function of the proton in the infinite momentum frame. What are the experimental consequences of the general quark model?
Computer: From the positivity of the quark distributions, and from Eqs. (45)–(48), I can deduce the following inequalities

$$\begin{gathered} 2(3 - R) \geq 3(3V - UR) \geq 0, \\ 2(3R - 1) \geq 3(3UR - V) \geq 0. \end{gathered} \tag{54}$$

These inequalities, together with Eq. (49) replace the equalities given by the naive quark model (30). With the observed values of R, $\Sigma(\nu d)$, $F_2(ep)$, and $F_2(en)$ as input, the allowed region for U and V is shown in Fig. 10 as a small shaded region. The naive quark model point lies within.

I can also work out the allowed region for the average values of xy in the scattering of ν's or $\bar{\nu}$'s off matter. It is shown in Fig. 11. Figure 12 is an expanded version of Fig. 11 showing the naive quark model prediction and the CERN experimental point (Eichten *et al.*, 1974).

Let me also show you what is predicted for the neutral cross sections on matter targets by the Weinberg theory in the general light-cone algebra picture. Making use of experimental values of $\Sigma(\nu d)$, R, and electroproduction data, I find that the allowed values for $\tilde{\Sigma}(\bar{\nu}, d)/\Sigma(\bar{\nu}, d)$ and $\tilde{\Sigma}(\nu d)/\Sigma(\nu d)$ lie in the shaded region in Fig. (13). Also shown in this figure is the prediction of the Weinberg model in the naive quark picture. Here, the possible presence of λ or $\bar{\lambda}$ quarks in the nucleon has a significant effect upon the strength of my predictions.[4]
Moderator: Let me summarize your remarks. The predictions of light-cone algebra, which are the same as those of the general quark model, do not depend on the physical existence of quarks. Experiment tells us that the p and n quark contributions are dominant. Because of this remarkable fact, the interpretation of inclusive deep-inelastic lepton scattering becomes much like that of νe scattering, where the target is pointlike. On the other hand, for exclusive processes, like elastic νp scattering, the theoretical analysis depends on the details of hadron dynamics as well as on the structure of the hadron current.

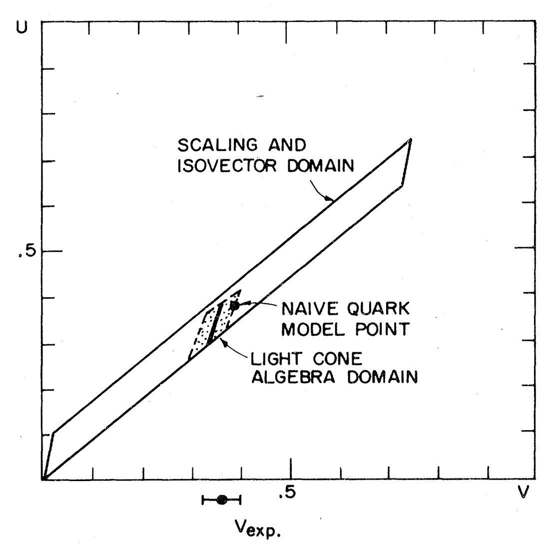

FIG. 10. Predictions for U and V. The shaded region is the light-cone algebra prediction including one-standard deviation-errors in experimental input. The larger parallelogram is the domain allowed by scaling and the isovector nature of the charged current (De Rújula and Glashow, 1973). A published measurement of V is shown (Myatt and Perkins, 1971).

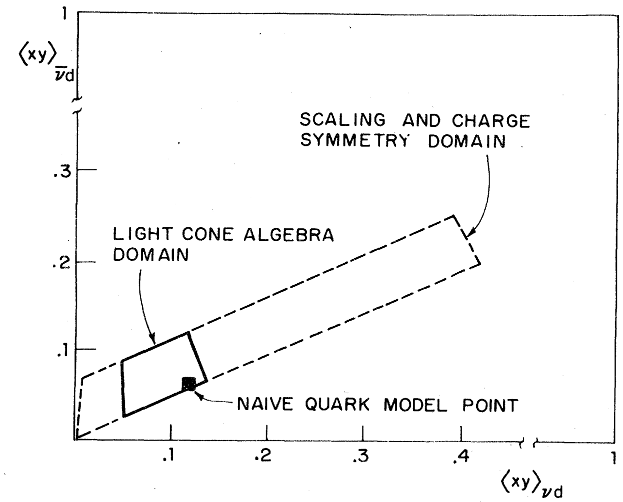

FIG. 11. Predictions for $\langle xy \rangle_{\nu d}$ and $\langle xy \rangle_{\bar{\nu} d}$. The shaded region is the domain allowed by the light-cone algebra and the central experimental values of $\Sigma(\nu d)$, R, and electroproduction data. The larger region is the domain allowed by scaling and charge symmetry (De Rújula and Glashow, 1974).

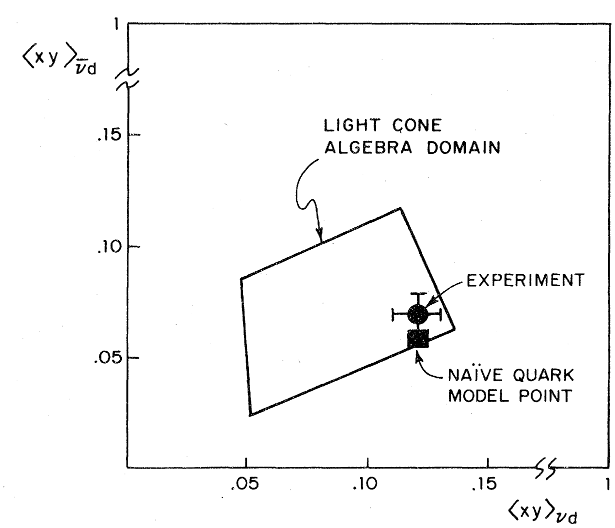

FIG. 12. Blow up of Fig. 11 showing the CERN measurement (Eichten *et al.*, 1974).

[4] Model-independent bounds on the neutral-current effects in Weinberg's theory have also been derived by other authors. See, example, Pais and Treiman (1972); Paschos and Wolfesnstein (1973).

V. MODEL BUILDER BUILDS MODELS

Model Builder: Let me comment on the predictions of neutral-current effects. Should experiment fall on the curve, or within the shaded region of Fig. 13, it would of course be a triumph of the Weinberg model. Otherwise, it would be necessary to change the model of weak interactions. The simplest possibility is to introduce extra scalar mesons which change the strength of the neutral current's effects, but not its structure.

Moderator: Since you mention the possibility that the predictions of the Weinberg theory may not be borne out, I would like to turn the discussion to possible alternatives to it. What about other renormalizable models like those of Georgi and Glashow (1972), or of Lee (1972); and Prentki and Zumino (1972)?

Speaker: Those models were designed to avoid neutral currents. If it is established that neutral currents are comparable in strength to charged-current effects, they lose all but archeological interest. Also, they use integer charge quarks and are incompatible with the naive quark model. Baryon configurations must be $qq\bar{q}$ rather than qqq, and many of the good quark-model predictions are lost, both in hadron physics and in the comparison of neutrino and electroproduction data.

Model Builder: But there are other renormalizable models–infinitely many—which coincide with the conventional theory for charged currents, and are consistent with the naive quark model, yet give different predictions for neutral currents. Let me construct a variety of such models, all based on the gauge group $SU(2)\times U(1)$. In the Weinberg model, the left-handed fermions are put into weak $SU(2)$ doublets

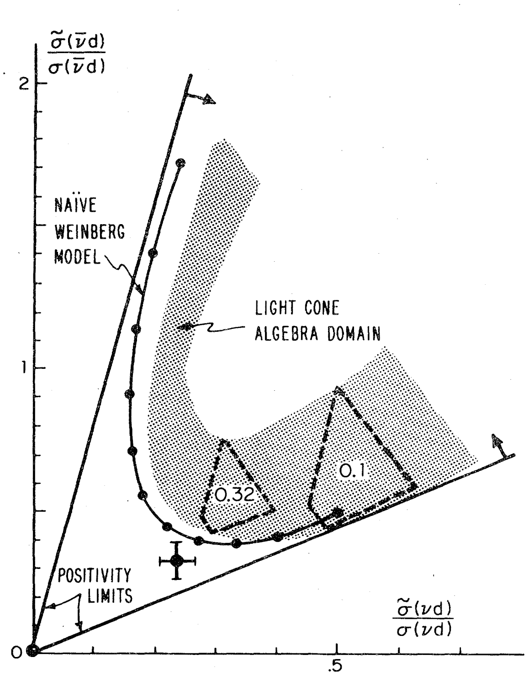

FIG. 13. Predictions for the ratios of muonless to muonic total cross sections on matter. The shaded region is the domain allowed by the Weinberg model and light-cone algebra. The domains for $\sin^2\theta_W = 0.32$ and $\sin^2\theta_W = 0.1$ [the maximum and minimum values allowed by neutrino–electron scattering (Gurr *et al.*, 1972; Chen and Lee, 1972; Hasert *et al.*, 1973a)] are also shown.

$$\begin{pmatrix} p \\ n \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L ; \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \tag{55}$$

while the right-handed fermions are singlets

$$p_R;\ n_R;\ \mu_R;\ e_R. \tag{56}$$

We could change these assignments. Indeed the models to which *Moderator* referred use triplets and singlets rather than doublets and singlets.

I consider a class of models in which n_R and p_R (together with other new quarks) transform according to nontrivial representations of $SU(2)$. The assignments of the left-handed quarks and the leptons are left as they are in the Weinberg model. Of course, n_R and p_R must be put into different $SU(2)$ multiplets, otherwise universality and the $V-A$ form of the isovector current are lost. Changes of the left-handed current are also conceivable, but I restrict myself to conservative innovations on the right.

A particular model of this class is specified by the assignment of weak isospin T and its third component τ to p_R and n_R. For example, with $T_p = T_n = 1/2$ and $\tau_p = -\tau_n = 1/2$, the weak $SU(2)$ multiplets are

$$\begin{pmatrix} p \\ n \end{pmatrix}_L ; \begin{pmatrix} p \\ x \end{pmatrix}_R ; \begin{pmatrix} y \\ n \end{pmatrix}_R ;\ldots \tag{57}$$

where the dots indicate additional multiplets which do not involve p and n. The charged weak current becomes

$$\bar{p}\,\gamma_\alpha(1+\gamma_5)n + \bar{p}\,\gamma_\alpha(1-\gamma_5)x + \bar{y}\gamma_\alpha(1-\gamma_5)n + \cdots. \tag{58}$$

The extra quarks x and y have electric charges $-1/3$ and $2/3$.

All these models need extra quarks, unnecessary to describe the known hadrons. New quarks mean new strong-interaction quantum numbers. I refer to these new quantum numbers, collectively, as fancy.

Below threshold for the production of fancy states the modified charged weak current is indistinguishable from the conventional current. On the other hand, even the naive quark contributions to the neutral current are radically changed:

$$J_z^{\alpha} = \bar{p}\gamma^\alpha(1+\gamma_5)p - \bar{n}\gamma^\alpha(1+\gamma_5)n + 2\tau_p\bar{p}\gamma^\alpha(1-\gamma_5)p + 2\tau_n\bar{n}\gamma^\alpha(1-\gamma_5)n - 4\sin^2\theta_W J^{\alpha}_{em}. \tag{59}$$

These models also differ from the original Weinberg model in that the Z mass and thus the strength of the neutral-current couplings is not theoretically constrained. The relationship $M_Z\cos\theta_W = M_W$ is a specific property of the Weinberg model requiring the original weak isospin assignments $T_p = T_n = 0$, and depending on an assumption about just how the gauge symmetry is broken: the scalar mesons whose vacuum expectation values break the symmetry must be weak $SU(2)$ doublets. My new models require other scalar meson multiplets.

I would like to know which of my infinite number of models is compatible with experiment, and what further experiments are necessary to decide which one, if any, is correct.

Speaker: You are comparing a simple, elegant model, Weinberg's, with an infinite class of ugly models involving extra quarks, unnecessary new quantum numbers, and an extra parameter. Is there any justification for your models beyond their mere existence?

Model Builder: The Weinberg model is said to unify weak and electromagnetic interactions, but it involves two independent dimensionless coupling constants $e/\cos\theta_W$ and $e/\sin\theta_W$. A true unification would require a simple gauge group with just one coupling constant. Attempts in this direction have been made by Weinberg (1972) and by Georgi and Glashow (1973; see also Georgi and Glashow, 1972; B. W. Lee, 1972; Prentki and Zumino, 1972). Any construction of a simple gauge theory requires radical changes of the original model, perhaps from the direction I have sketched. What you find ugly and complicated about my models may be just what is needed for there to be underlying simplicity.

Moderator: There is another virtue to these crazy models. As an experimentalist, I wonder how accurately neutrino experiments must be done in order to verify the Weinberg theory, or any other model. The existence of this infinite class of models provides a partial answer. We need experimental data which is at least good enough to distinguish among them.

Computer: I have just worked out some of the predictions of these models in the naive quark picture.

The neutral hadronic current depends just on τ_p and τ_n and not on the total weak isospins of p_R and n_R. The effective neutral coupling, except for its undetermined overall strength, is given by Eq. (3) with the replacements

$$a \to a, \qquad c \to c + \tau_p, \qquad b \to b \qquad d \to d + \tau_n. \tag{60}$$

Additional terms referring to the fancy quarks x, y, etc. are ignored in the naive quark picture because there are no such quarks in the naive nucleon.

I first describe the predictions of these models depending on the structure of this neutral current, but not on the strength of its coupling. The simplest experimental quantity to consider is $\tilde{R}$. I find

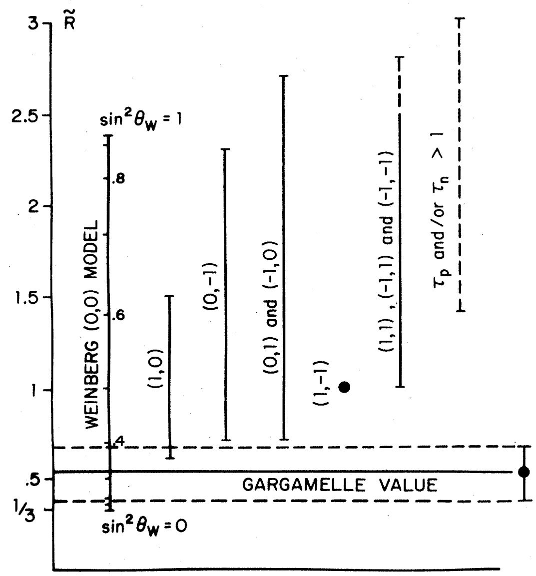

FIG. 15. Allowed ranges of $\tilde{R}$ for various theories.

$$\frac{A_R}{A_L} = \frac{3\tilde{R} - 1}{3 - \tilde{R}} = \frac{(\tau_P - \frac{2}{3}\sin^2\theta_w)^2 + (\tau_n + \frac{1}{3}\sin^2\theta_w)^2}{\frac{1}{2} - \sin^2\theta_w + \frac{5}{9}\sin^4\theta_w} \tag{61}$$

and display this result on a two-dimensional lattice. Each lattice point corresponds to a choice of integral values of $(2\tau_p, 2\tau_n)$. With the corrected Gargamelle result for $\tilde{R}$ [Eq. (22)] the allowed region is shown in Fig. 14. Evidently only a few of the lattice points correspond to empirically acceptable models.

Similar information is displayed in another way in Fig. 15, which shows the possible range of R for various assignments $(2\tau_p, 2\tau_n)$. We see from these two figures that the only assignments which are compatible with the Gargamelle data are the five possibilities $(0,0)$, $(\pm1,0)$, and $(0,\pm1)$, with $(\pm1,\pm1)$ perhaps also viable.

Model Builder: If it turns out that $\tilde{R}$ is less than 0.63, then only the choice $\tau_p = \tau_n = 0$ survives. This possibility

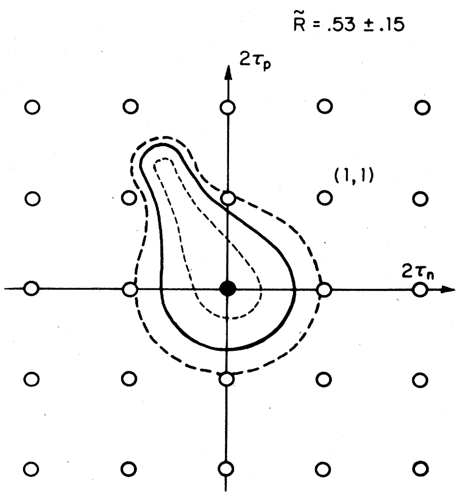

FIG. 14. Values of $(2\tau_p, 2\tau_n)$ allowed by the CERN data. The region enclosed by the solid curve is the allowed region for $\tilde{R} = 0.53$. Dashed curves correspond to one-standard-deviation errors.

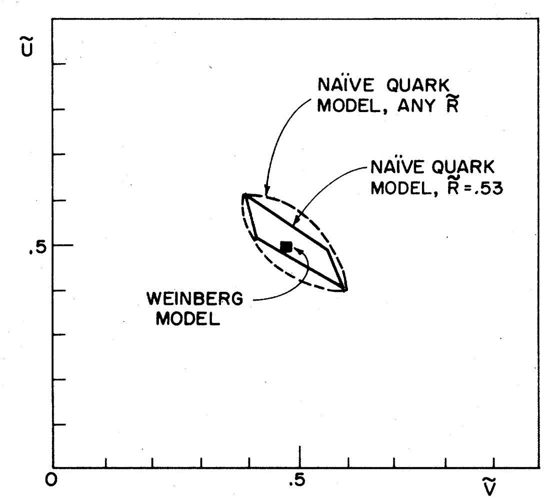

FIG. 16. In the naive quark approximation, the values of $\tilde{U}$ and $\tilde{V}$ must lie in the region enclosed by the dashed line. For $\tilde{R} = 0.53$, they must lie within the inscribed parallelogram.

includes the Weinberg model, but also other models in which p_R and n_R transform nontrivially under weak $SU(2)$ as central members of integer-spin multiplets.
Moderator: Can measurements of neutral-current cross sections on other targets distinguish among these models?
Computer: When ν and $\bar{\nu}$ scattering is done on separated proton and neutron targets, $\tilde{U}$ and $\tilde{V}$ can be measured, Eqs. (36) remain valid with the substitutions of Eqs. (60). Certainly, the predicted values of $\tilde{U}$ and $\tilde{V}$ will depend on the choice among *Model Builder's* models. However, quite independently of the structure of the neutral current, I can show that the naive quark model requires that $\tilde{U}$ and $\tilde{V}$ lie within the region bounded by the hyperbolae

$$9(\eta - 1)^2 = [8(1+\eta)\tilde{V} + \eta - 9][8(1+\eta)\tilde{U} + \eta - 9],$$
$$9(\eta - 1)^2 = [8(1+\eta)\tilde{V} + 1 - 9\eta][8(1+\eta)\tilde{U} + 1 - 9\eta], \tag{62}$$

with η defined as in Eq. (31). This region is the interior of the "football" shown in Fig. 16. It is a relatively small region, so that it will be necessary to have precise determinations of $\tilde{U}$ and $\tilde{V}$ for any discrimination to be made among the new models.

Once $\tilde{R}$ is measured, the allowed region shrinks to the interior of the parallelogram

$$3\eta + \tilde{R} \geq (1+\eta)[3\tilde{V} + \tilde{U}\tilde{R}] \geq 3 + \eta\tilde{R},$$
$$3\eta\tilde{R} + 1 \geq (1+\eta)[\tilde{V} + 3\tilde{U}\tilde{R}] \geq 3\tilde{R} + \eta. \tag{63}$$

This is also shown in Fig. 16 for $\tilde{R} = 0.53$.

Of *Model Builder's* many models, only the Weinberg model is consistent with $\tilde{R} = 0.53$. Its predicted values of $\tilde{U}$ and $\tilde{V}$ are shown in Fig. (17), along with error bars indicating the experimental uncertainty of $\tilde{R}$. Also shown are the predictions of the four other models compatible with $\tilde{R} \leq 0.83$. (They are computed at the lowest value of $\tilde{R}$ compatible with each model.)
Speaker: This seems to be an excellent test of the naive quark model. Whatever model of neutral currents is used, the experimental results are constrained to lie in a small range.

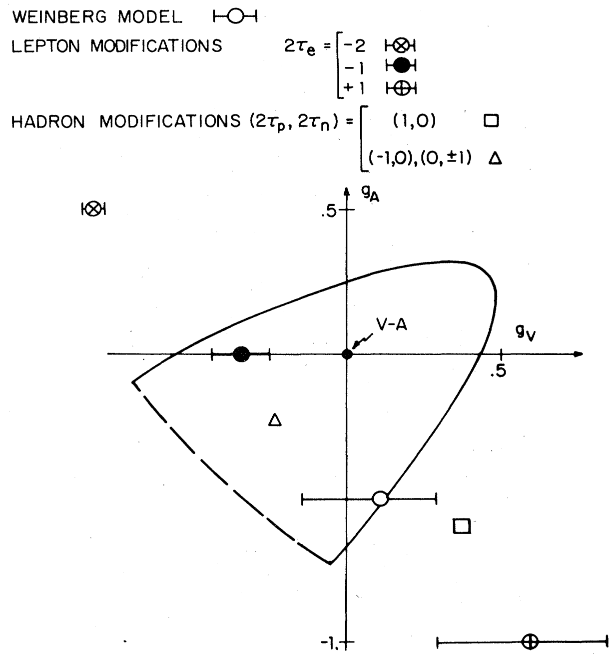

FIG. 18. Predictions of g_A and g_V in various models. The experimentally allowed domain is also shown (Gurr *et al*, 1972; Chen and Lee, 1972; Hasert *et al.*, 1973a).

Moderator: On the other hand, it is going to be very difficult to distinguish among the different models: their predictions for $\tilde{U}$ and $\tilde{V}$ are not so very different. Even if we had good experimental data, I doubt that the approximation of the quark model is sufficiently reliable. Perhaps the best way to distinguish among the models is in their predictions for purely leptonic phenomena.
Computer: Certainly, the predictions of purely leptonic phenomena are different for the various models. The over-all strengths of g_V and g_A depend on M_Z, which is a free parameter; and g_V depends on $\sin^2\theta_W$. For the likeliest alternative models, I have adjusted both θ_W and M_Z to fit the CERN neutral-current data (Eichten *et al.*, 1974). The resulting predictions for purely leptonic phe-

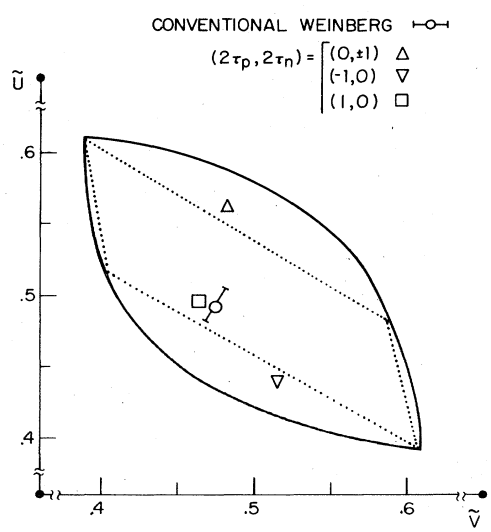

FIG. 17. Blow up of the football of Fig. (16), showing the predictions of the Weinberg theory and some fancy models.

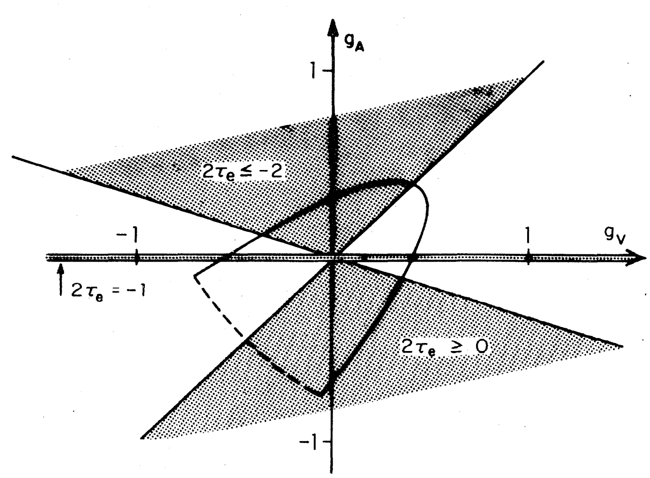

FIG. 19. Allowed values of g_A and g_V when the right-handed electron is assigned a unique value of weak isospin.

TABLE I. Asymptotic value of R for naive quark models.

$2\tau_p$ \ $2\tau_n$	1	0	−1
1	7/3	4/3	1
0	4/3	1/3	1/4
−1	1	1/4	1/5

nomena are shown in Fig. 18, along with the experimentally allowed domain.

Model Builder: It may not be the best idea to use data from purely leptonic scattering to tell one of these models from another. After all, I could change the lepton current, just as I changed the hadron current, by assigning nonzero weak isospin to e_R. How would this affect the predictions?

Computer: Denote by τ_e the third component of weak isospin of e_R. Figure 19 shows the allowed values of g_V and g_A for all values of τ_e, whatever the form of the hadron current. The predictions for several simple choices of τ_e (with $\tau_p = \tau_n = 0$), again fitted to the CERN neutral-current data, are also shown in Fig. 18. Some models are excluded by the data.

VI. IN WHICH SURPRISES AT HIGH ENERGY ARE PREDICTED

Model Builder: There may be a more dramatic way to find out which, if any, of my models is the right one. In order to assign weak isospin to p_R and n_R it was necessary to introduce fancy quarks. Surely, fancy hadron states should exist, although I cannot guess at what mass they appear. When the ν or $\bar{\nu}$ energy exceeds threshold for the production of these states, I would expect a peculiar behavior of the muonic cross sections. Once produced, these states would decay weakly but with short lifetimes ($< 10^{-11}$ sec) because of their high mass. Their nonleptonic decays would seem like ordinary inelastic muonic events. Their semileptonic decays would yield final states with two oppositely charged leptons. These would also seem like ordinary inelastic muonic events if only the energetic, forward-produced muon were detected. Of course, it would be more exciting if semileptonic decays were copious, and di-lepton events were seen. Then, the experimenter would have the challenging problem of telling whether he had discovered fancy, the intermediate W boson, or a heavy lepton!

If the energy is well above fancy threshold, I can use the naive quark model to compute the additional contributions to ν and $\bar{\nu}$ cross sections due to fancy hadron production. At this energy, I would no longer expect the ratio R of charged cross sections to equal the magic value of 1/3.

Let me examine only the simplest possibilities with $2\tau_p, 2\tau_n = \pm 1, 0$, and specify the charged currents of the models by assuming $2T_p, 2T_n = 1, 0$. That is, all the right-handed quarks are either in weak doublets of singlets. The naive asymptotic value of R for each of these models is given in Table I.

In models like this, where the weak current is not charge symmetric, R is not constrained to be between 1/3 and 3. A dramatic change in R at high energy would be an indication that one of my models is correct.

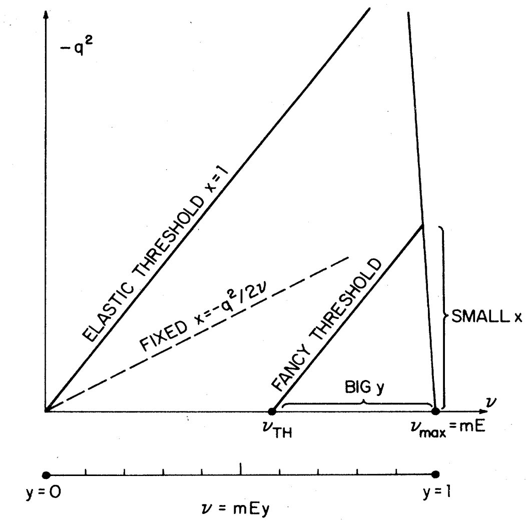

FIG. 20. The q^2, ν plot, showing a fancy (or charm) threshold.

Moderator: For R to change significantly at high energies would surely be spectacular. However, such a change could be due to the opening up of new lepton channels as well as new hadron channels. Are there more specific effects to be expected if the extra events are due to the production of fancy hadrons?

Model Builder: Certainly. Suppose the energy is somewhat above fancy threshold, as indicated on the (q^2, ν) plot in Fig. 20. Fancy hadrons may be produced only in the region of small x and large y:

$$\begin{aligned} 1 \geq y \geq E_{\text{th}}/E, \\ 1 - E_{\text{th}}/E \geq x \geq 0, \end{aligned} \tag{64}$$

where E_{th} is fancy threshold

$$E_{\text{th}} = (M^2 - m_P^2)/2m_P \tag{65}$$

and M is the mass of the lightest fancy hadron state with unit baryon number. As the energy increases further, the extra events creep in from $y = 1$ and $x = 0$. Only at energies considerably beyond E_{th} does scaling behavior reassert itself in the post-fancy regime.

This kind of behavior is not expected for heavy lepton production. Moreover, if the data are displayed as a distribution in hadron mass W, the extra events due to fancy should appear as a bump or rise beginning at $W = M$. Events due to heavy leptons would not show this behavior either.

Computer: I can guess what to expect for the x and y distributions and for the total cross sections of ν and $\bar{\nu}$ scattering on matter as energy passes fancy threshold, again making use of the naive quark model.

Consider, for example, the (1,0) model which involves the charged current

$$\bar{p}\gamma_\alpha(1 + \gamma_5)n + \bar{p}\gamma_\alpha(1 - \gamma_5)x, \tag{66}$$

where x is a fancy quark. Below fancy threshold, the cross section is

$$d\Sigma(\bar{\nu}d)/dxdy = (1 - y)^2 \tfrac{18}{5} F_2(ed, x). \tag{67}$$

Above fancy threshold, the extra term contributes to $\bar{\nu}$

(but not ν) scattering. At sufficiently high energy, the cross section becomes

$$d\Sigma(\bar{\nu}d)/dxdy = [(1-y)^2 + 1]\tfrac{18}{5}F_2(ed,x). \quad (68)$$

I can only guess how the transition between these forms comes about, but its qualitative nature is given by kinematics. Let me consider two extreme possibilities:

1. "Fast Rescaling": The structure functions rescale immediately and the threshold effect is purely kinematical. The interpolated cross section is simply

$$d\Sigma(\bar{\nu}d)/dxdy = [(1-y)^2 + \theta(W-M)]\tfrac{18}{5}F_2(ed,x). \quad (69)$$

2. "Slow Rescaling": In addition to the kinematic θ function, I replace x by a new scaling variable x', in which the origin of the energy variable is displaced to fancy threshold

$$x' = -q^2/2(\nu - \nu_{\mathrm{th}}), \quad (70)$$

where $\nu_{\mathrm{th}} = mE_{\mathrm{th}}$. In this case, the interpolated cross section is

$$d\Sigma(\bar{\nu}d)/dxdy = (1-y)^2\tfrac{18}{5}F_2(ed,x) + \theta(W-M)\tfrac{18}{5}F_2(ed,x'). \quad (71)$$

Figure 21 shows the behavior of $\Sigma(\nu d)$ and $\Sigma(\bar{\nu}d)$ as functions of energy (measured in units of E_{th}) for the different theories. Figure 22 shows the y distributions which may be expected for $\bar{\nu}$ in the (1, 1) model and for ν in the (−1, −1) model with either rescaling hypothesis.

Figure (23) shows how fancy production affects the x distributions.

Moderator: Fancy resonances might be produced just above fancy threshold. At low q^2, hence low x, these would enhance the distributions estimated by *Computer*.

Model Builder: Some of my models involve more than one extra quark, and hence more than one kind of fancy. There may be several thresholds at which new hadron states begin to appear. This could confuse the experimental situation even more.

Moderator: Your new models involve new quarks, which may mean new hadrons. It is the opening of these new channels that leads to the weird effects you have described. But charm is also a new hadron quantum number associated with an extra quark. What is the difference between fancy and charm?

Model Builder: Both charm and fancy are new conjectured hadronic quantum numbers. Charm seems to be necessary: without it I cannot construct models of weak interactions which make sense. But, in the naive quark model, the effects of charm are only of order θ_c^2, because there are only p and n quarks in the naive nucleon. On the other hand, fancy is a completely *ad hoc* invention which allows the right-handed quarks to participate in weak interactions. It is neither necessary nor clearly desirable. But if it does exist, it will make sizable effects on neutral-current cross sections, and on charged-current cross sections at high energies.

Moderator: It is only in the naive quark model that the charmed current $\bar{p}'\gamma_\alpha(1+\gamma_5)[n \sin\theta_c - \lambda \cos\theta_c]$ will

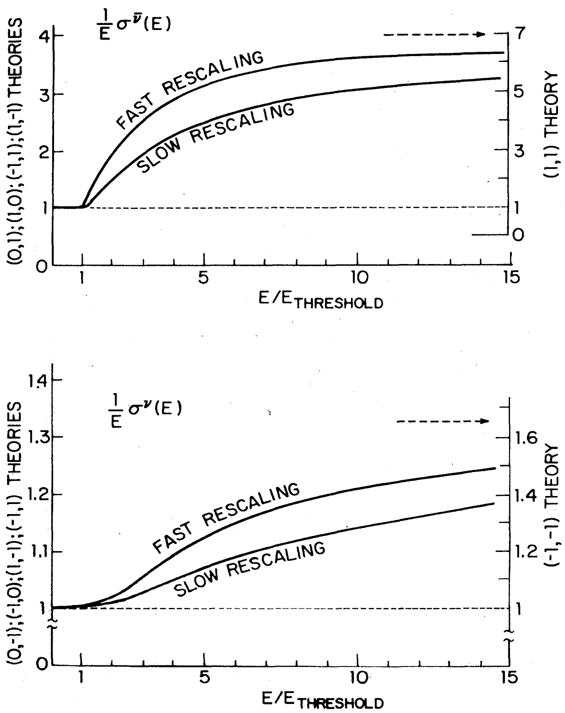

FIG. 21. Anticipated energy dependence of the slopes of muonic cross sections for neutrinos and antineutrinos in a variety of fancy theories. The dotted lines are the sloped below fancy threshold normalized to one. The dashed arrows indicate the asymptotic slopes.

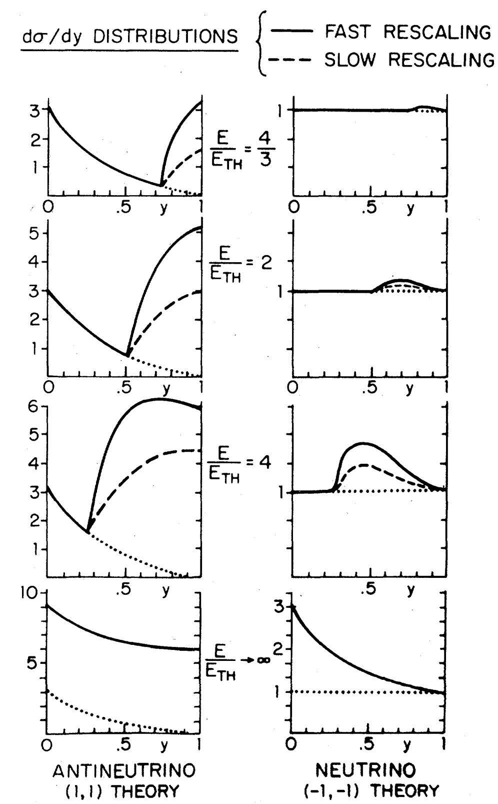

FIG. 22. Anticipated y distributions for neutrino and antineutrino scattering on matter in fancy theories at various energies. The dotted line shows the distribution below fancy threshold.

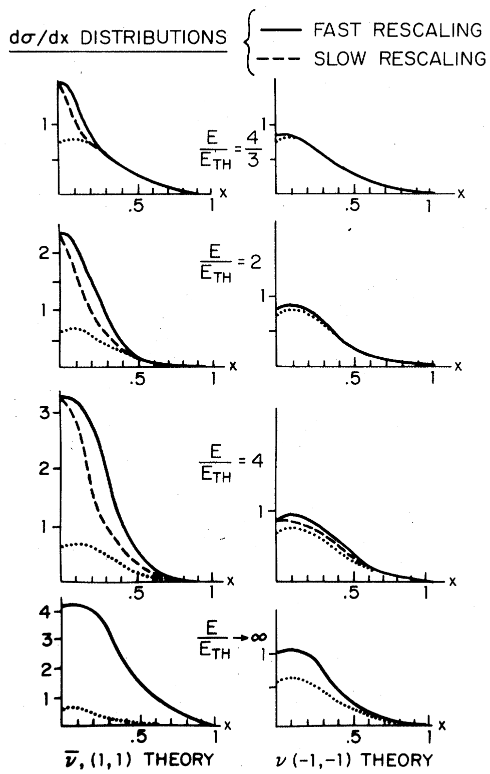

FIG. 23. Anticipated x distributions for neutrino and antineutrino scattering on matter in fancy theories at various energies. The dotted line shows the distribution below fancy threshold.

have little effect. But this model is only approximate. Didn't *Computer* show us that there could be a significant admixture of λ or $\bar{\lambda}$ quarks in the nucleon? Above charm threshold, wouldn't these quarks lead to effects similar to the effects of fancy?

Computer: What you say is interesting. My result allows a lot of room for λ or $\bar{\lambda}$ quarks in the nucleon. Using the experimental determination of S including errors [Eq. (53)], I can calculate the allowed values of $\Sigma(\nu d)$ and $\Sigma(\bar{\nu} d)$ above charm threshold. These are shown in Fig. (24).

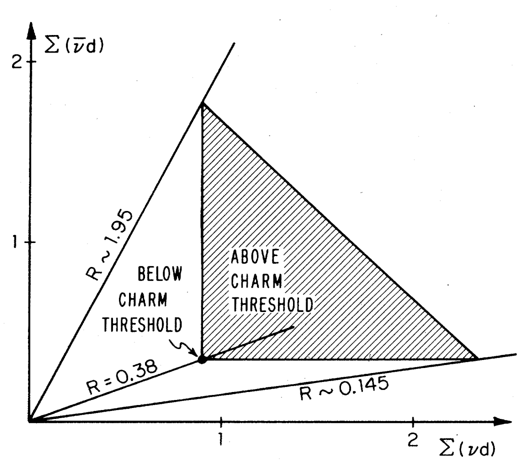

FIG. 24. The shaded triangle shows the allowed domain for $\Sigma(\nu d)$ and $\Sigma(\bar{\nu} d)$ above charm threshold, allowing for one standard deviation error in the experimental input.

Model Builder: That's remarkable It seems that high-energy neutrinos might produce more charmed states than uncharmed states. This would lead to effects on x and y distributions not unlike those *Computer* showed us.

Speaker: Such a large strange quark distribution may be ruled out experimentally. It would sharply increase the predictions for neutral-current cross sections, since λ and $\bar{\lambda}$ quarks contribute to[these but not to charged-current cross sections below charm threshold. However, the Gargamelle measurement of neutral-current effects lies below the fat nose in Fig. 13.

Computer: You may be right. My estimates were based on only charged-current data. I was reluctant to use the neutral-current data to improve my estimates for two reasons. The experiments which discovered neutral-current effects may be no more than preliminary measurements of their magnitudes. Furthermore, in order to use these data to improve my estimate of the strange quark distribution, I must take the Weinberg model quite literally, including its prediction of the Z mass. With those reservations, I deduce the following result

$$\int_0^1 dx 2x(\lambda + \bar{\lambda}) = (1 - \tfrac{8}{9}\sin^2\theta_w)^{-1}$$
$$[2(1 + \tilde{R})\tilde{\Sigma}(\nu d) - (\tfrac{3}{8} - \tfrac{1}{3}\sin^2\theta_w)(1 + R)\Sigma(\nu d)$$
$$- (3 - 8\sin^2\theta_w + \tfrac{32}{3}\sin^4\theta_w)\int_0^1 dx F_2(ed)]. \quad (72)$$

Putting in Gargamelle neutral current data (Eichten *et al.*,1974), I find that the rhs (which must be positive) is $\sim -0.20 \pm 0.12$. A possible explanation for this result is that the Weinberg model is wrong, perhaps only in its prediction of the overall strength of neutral-current couplings. Another explanation—if my result is considered to be compatible with zero—is that there is hardly any room at all in the nucleon for λ or $\bar{\lambda}$ quarks.

Moderator: Since the experimental and theoretical status of neutral currents is not yet settled, let us forget them for the moment. Suppose the effects we have been discussing are observed: have we fancy or have we charm?

Model Builder: By looking at the final state, it might be possible to tell fancy from charm. Because of the structure of the current, I would expect charmed, but not fancy, final states to decay often into states with two particles of opposite strangeness. Moreover, charm production must be the same on proton or neutron targets, while fancy production need not.

Moderator: The hour is late and our listeners weary. Let me summarize the major points of our discussion. First, *Speaker* convinced us that the naive quark model is a very useful tool for interpreting inclusive deep-inelastic lepton scattering. It correctly predicts all of the measured charged-current effects in deep-inelastic ν or $\bar{\nu}$ scattering. Combined with the Weinberg model, it adequately describes the neutral-current effects. We saw how most of the naive quark model results could be understood with the weaker assumptions of light-cone algebra. *Model-Builder* invented renormalizable models of weak and electromagnetic interactions in which the right-handed quark field transform nontrivially under weak $SU(2)$. These models necessarily involve one or more of new strong-interaction quantum numbers called fancy. Below threshold for the production of fancy states the charged-current effects are not changed, but the neutral-current effects are. Those models not already ruled out by the CERN experiment may be distinguished from the Weinberg model with precise neutral-current data. Above fancy threshold, the models predict striking and specific new effects. If these effects are seen, it is not necessarily evidence for fancy, because similar effects might occur above charm threshold. In either case, be it fancy or be it charm, a new strong-interaction quantum number will have been discovered.

APPENDIX: COMPUTER REVEALS TECHNIQUES FOR DERIVING OPTIMAL SETS OF INEQUALITIES

Moderator: Computer has stated many results in the form of inequalities following from light-cone algebra or other more general assumptions. Are the inequalities the best that can be derived from a given set of hypotheses? How are they obtained?

Computer: The inequalities are optimal. They may be mechanically derived with a well known technique of "linear programming." Suppose you have a set of quantities $\sigma_i (i = 1, \ldots, I)$, which are linear combinations of a set of variables $p_k (k = 1, \ldots, K)$. The latter are only constrained to be non-negative:

$$\sigma_i = \sum_{k=1}^{K} A_{ik} p_k; \qquad p_k \geq 0. \tag{A1}$$

The problem is to find the complete set of inequalities between the σ's. In practice the σ's are the observables. If I am deriving consequences of the light-cone algebra (or general quark model) assumptions, the p_k are moments of the non-negative distribution functions for the different quarks $[2p(x) - n(x), n(x), \lambda(x)$, etc.] and the coefficients A_{ik} are explicitly known.

Suppose that the σ_i are linearly independent (all possible equalities between the σ_i have been taken care of). There are then two possibilities:

1. The case $K = I$ is trivial. Equation (A1) is invertible and the inequalities are

$$p_k = \sum_{i=1}^{I} (A)^{-1}_{ki} \sigma_i \geq 0. \tag{A2}$$

2. The case $K > I$ is more interesting. In p space one is constained by hypothesis to the first "quadrant" in Fig. 25(a). Equation (A1) maps each axis in p space into a ray in σ space. In Fig. 25(b) rays are shown for the case $K = 4$, $I = 3$. The allowed domain in σ space is bounded by the exterior rays, those that are not linear combinations with non-negative coefficients of others. In this figure, rays 1, 2, and 3 are external.

Moderator: I see. The rays are like a bouquet of flowers. You must wrap it up, throw away the flowers, and save the wrapping paper, which is the boundary of the allowed domain. Can you work out an example?

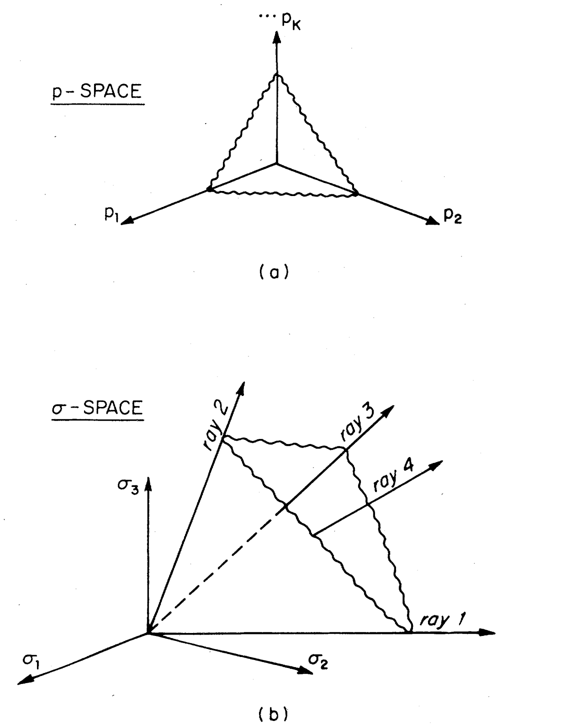

FIG. 25. Allowed domains in p space and σ space. Ray 4 is interior.

Computer: Yes. Consider the light-cone algebra domain for the quantities $\tilde{\Sigma}(\nu d)/\Sigma(\nu d)$ and $\tilde{\Sigma}(\bar{\nu} d)/\Sigma(\bar{\nu} d)$. Define the observables

$$\begin{aligned}
\sigma_1 &= \tfrac{2}{3} \int_0^1 dx F_2(ed) - \tfrac{5}{36}[\Sigma(\nu d) + \Sigma(\bar{\nu} d)], \\
\sigma_2(\sin^2\theta_W) &= \tilde{\Sigma}(\nu d) + \tilde{\Sigma}(\bar{\nu} d) \\
&\quad - \left(\tfrac{1}{2} - \sin^2\theta_W + \tfrac{10}{9}\sin^4\theta_W\right)[\Sigma(\nu d) + \Sigma(\bar{\nu} d)], \\
\sigma_3(\sin^2\theta_W) &= \tilde{\Sigma}(\nu d) - \tilde{\Sigma}(\bar{\nu} d) \\
&\quad - \left(\tfrac{1}{2} - \sin^2\theta_W\right)[\Sigma(\nu d) - \Sigma(\bar{\nu} d)].
\end{aligned} \tag{A3}$$

By construction, these depend only on integrals of charmed and strange quark distributions

$$p_1 = \frac{2}{3}\int_0^1 2xp'(x)\,dx \tag{A4}$$

and analogously for p_2, p_3, and p_4 in terms of $\bar{p}'(x)$, $\lambda(x)$, and $\bar{\lambda}(x)$ respectively. In this example, Eq. (A1) becomes:

$$\begin{pmatrix} \sigma_1 \\ \sigma_2(x) \\ \sigma_3(x) \end{pmatrix} = \begin{pmatrix} 4/9 & 4/9 & 1/9 & 1/9 \\ 1 - 4x + 8x^2 & 1 - 4x + 8x^2 & 1 - 2x + 2x^2 & 1 - 2x + 2x^2 \\ \tfrac{1}{2} - 2x & -\tfrac{1}{2} + 2x & \tfrac{1}{2} - x & -\tfrac{1}{2} + x \end{pmatrix} \begin{pmatrix} {}_1p \\ p_2 \\ p_3 \\ p_4 \end{pmatrix},$$

where $x = 2/3 \sin^2\theta_W$. The corresponding inequalities are messy and depend on $\sin^2\theta_W$, but they can be mechanically derived with the method outlined above. At $\sin^2\theta_W = 0$, for instance, all rays are external and the inequalities are

$$\begin{aligned} -g\sigma_1 + 4\sigma_2(0) &\geq 0, \\ \sigma_1 - \sigma_2(0) &\geq 0, \\ \sigma_2(0) - 2\sigma_3(0) &\geq 0, \\ \sigma_2(0) + 2\sigma_3(0) &\geq 0. \end{aligned} \tag{A6}$$

The example of the average values of xy on matter targets is much more cumbersome. I must consider first and second x moments of all quark distributions. I derive ten inequalities. Four of them are shown in Figs. 11 and 12. They are

$$\begin{aligned} &\tfrac{35}{32}(3R - 1) \geq 6R\langle xy\rangle_{\bar{\nu}d} - \langle xy\rangle_{\nu d} \geq 0, \\ &\Sigma(\nu d)\{1.7\langle xy\rangle_{\nu d} + 3R\langle xy\rangle_{\bar{\nu}d}\} \\ &\qquad \geq \tfrac{105}{2}\int_0^1 x[F_2(ep) - F_2(en)]\,dx, \\ &\Sigma(\nu d)\{\langle xy\rangle_{\nu d} + R\langle xy\rangle_{\bar{\nu}d}\} \leq \tfrac{21}{10}\int_0^1 x[F_2(ep) + F_2(en)]\,dx. \end{aligned} \tag{A7}$$

The remaining six inequalities do not imply any further constraints, given the experimental values of $\Sigma(\nu d)$, R and the electroproduction data.

Speaker, Model Builder and Moderator:

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