Recent progress in gauge theories of the weak, electromagnetic and strong interactions*†

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A review is presented of progress since mid-1972 in the development of gauge theories of the weak, electromagnetic, and strong interactions. A brief introduction to the history and the fundamentals of the subject is also provided.

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I. INTRODUCTION

Renormalizable gauge theories of the weak and electromagnetic interactions have been with us for six years or so, and have been popular at least since mid 1971. There are by now a number of excellent surveys of the subject, including Ben Lee's comprehensive report (1972a) at NAL and the more recent articles by Bernstein (1974), Seghal (1973b), Abers and Lee (1973), and Llewellyn Smith (1973a). I intend in the body of this review to concentrate on the exciting new developments which have occurred since the preparation of Lee's report, some of which are only a few months old. However, before getting into this new material, which is the real subject of this article, it may be well to take a brief look in this Introduction at the history and the fundamentals of gauge theories.

The history of attempts to unify weak and electromagnetic interactions goes back to Fermi's (1934) proposal of a four-vector lepton–hadron beta decay interaction, analogous to the vector interaction of electrons with the electromagnetic field. Shortly thereafter Yukawa (1935) suggested that the beta decay interaction was carried by a spin-one boson, analogous to the photon, but with a large mass. (Yukawa intended this vector boson to explain nuclear forces as well as beta decay, so he gave it what in modern terms would be called a strong interaction with hadrons and a superweak interaction with leptons, and a mass of order 100 MeV.) The subsequent discovery of a change in nuclear spin in beta decays of such nuclei as He^6, F^{18}, and Al^{26} made it clear that the beta decay interaction was *not* pure vector, and for many years the favored combination was S, T, P, which of course would quite destroy the analogy with electromagnetism.

The idea of a unified theory was revived in the late 1950's, after the form of the beta decay interaction finally settled down to a combination of vector and axial vector. Specific models were proposed by several authors, notably Schwinger (1957), Bludman (1958), Glashow (1961), and Salam and Ward (1964). However, two great obstacles stood in the way of a synthesis. One was the obvious discrepancy in mass between the photon and the intermediate vector boson W; if the W couplings are comparable with the electronic charge e, then its mass is determined by the condition that e^2/m_W^2 should be of the order of the Fermi coupling constant G_F, so that

$$m_W \approx (137 \times 10^{-5} m_p^{-2})^{-1/2} \approx 30 \text{ GeV} . \quad (1.1)$$

A less obvious though no less important difficulty had to do with the problem of high-energy behavior. Although the introduction of an intermediate vector boson ameliorated the bad asymptotic behavior of processes such as lepton–lepton scattering, there were known to be others, such as the process $\nu + \bar{\nu} \to W^+ + W^-$ considered by Gell-Mann, Goldberger, Kroll, and Low (1969), where the Born approximation amplitude grows so rapidly with energy that unitarity forces a failure of perturbation theory at energies above 300 GeV. Even worse, the occurrence of such rapidly growing matrix elements as pieces of higher-order diagrams (as in the example of $\nu - \bar{\nu}$ scattering considered by Low, 1970) invalidates perturbation theory at all energies.

In 1967 a way out of these difficulties was suggested (Weinberg, 1967b; see also Salam, 1968). It was proposed that the photon and the intermediate vector bosons should arise as quanta of the Yang–Mills vector fields[1] associated with some exact local gauge invariance of nature. In order to avoid the problem of ultraviolet divergences, it was proposed that the theory should be constructed to be renormalizable. Finally, in contrast to earlier theories of intermediate vector bosons, it was proposed that their mass should arise from a spontaneous breakdown of the gauge group.

It was already known since 1964 through the work of

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[1] The first example of a theory based on a non-Abelian gauge group was that of Yang and Mills (1954). This theory was generalized to arbitrary semisimple Lie groups by Utiyama (1956) and Gell-Mann and Glashow (1961).

Higgs (1964a, 1964b, 1966), Kibble (1967), and others (Guralnik, Hagen, and Kibble, 1964; Englert and Brout, 1964; also see Anderson, 1958), that when a local rather than a merely global symmetry group is spontaneously broken, the vector particles acquire a mass, the zero helicity states of these particles appearing in place of massless spin zero Goldstone bosons. This phenomenon was originally discovered as an interesting exception to the theorem (Goldstone, Salam, and Weinberg, 1962), that spontaneously broken summetries must always be accompanied by massless bosons.[2] This theorem had been seen as an obstacle to the application of spontaneous symmetry breaking in elementary particle theory. However, at the time the Higgs phenomenon was discovered, theorists were just beginning to get used to the idea that the pion is the Goldstone boson of a spontaneously broken approximate symmetry, and since exceptions to the Goldstone theorem were no longer being eagerly sought, interest in the Higgs phenomenon generally lapsed. (My own interest in this phenomenon arose as a result of rediscovering it as a source of $\rho - \mathrm{A1}$ mass splitting in a chiral invariant gauge field theory of strong interactions (Weinberg, 1967a, footnote 7),[3] in which the pions do not decouple because the gauge invariance is broken by an explicit degenerate ρ, A1 bare mass. It was the attempt to understand this phenomenon in a nonperturbative setting that led to the development of the spectral function sum rules.)

The motivation for bringing the Higgs phenomenon into the theory of weak and electromagnetic interactions was twofold. First, it explained the discrepancy between the photon and the intermediate vector boson masses; the photon remains massless because it corresponds to the unbroken symmetry subgroup associated with the conservation of charge, while the intermediate vector bosons get masses because they correspond to symmetries which are broken, as shown by the non-degeneracy of the lepton and baryon masses. Second, the avoidance of an explicit vector boson mass term in the Lagrangian gave at least some hope of being able to construct a renormalizable theory; it was widely believed (Komar and Salam, 1960; Umezawa and Kamefuchi, 1961; Kamefuchi *et al.* 1961; Salam, 1962) and subsequently confirmed (Veltman, 1968, 1970; Boulware, 1970) that a Yang–Mills theory with an explicit mass term is *not* renormalizable.[4] (It is interesting though that such a theory would at least have avoided some of the problems found by Gell-Mann *et al.*, 1969.)

These general guidelines were used in 1967 to construct a specific model of the weak and electromagnetic interactions of leptons (Weinberg, 1967b; see also Salam, 1968). Instead of getting immediately into the details of this model, it may be more illuminating here to describe the general class of such models, using a notation which has subsequently proved useful in a variety of different analyses. Afterwards, we will be able to approach the 1967 model of leptons as just a special case, as indeed it is.

On the basis of experience with simpler renormalizable theories, we would expect the most general renormalizable gauge-invariant theory to be described by a Lagrangian containing only terms of dimensionality four or less. The theory must then involve only the gauge fields $A_{\alpha\mu}(x)$, spin-$\frac{1}{2}$ fields $\psi_n(x)$, and spin-zero fields $\phi_i(x)$, with a Lagrangian of the form[1]

$$\mathfrak{L} = -\tfrac{1}{4}F_{\alpha\mu\nu}F_\alpha^{\mu\nu} - \tfrac{1}{2}(D_\mu\phi)_i(D^\mu\phi)_i - \bar{\psi}\gamma^\mu D_\mu\psi - \bar{\psi}\Gamma_i\psi\phi_i - P(\phi) - \bar{\psi}m_o\psi, \tag{1.2}$$

where $F_{\alpha\mu\nu}$ is the gauge-covariant curl

$$F_{\alpha\mu\nu} \equiv \partial_\mu A_{\alpha\nu} - \partial_\nu A_{\alpha\mu} - C_{\alpha\beta\gamma}A_{\beta\mu}A_{\gamma\nu}, \tag{1.3}$$

and $D_\mu\phi$ and $D_\mu\psi$ are gauge-covariant derivatives

$$(D_\mu\phi)_i \equiv \partial_\mu\phi_i - i(\theta_\alpha)_{ij}\phi_j A_{\alpha\mu}, \tag{1.4}$$

$$(D_\mu\psi)_n \equiv \partial_\mu\psi_n - i(t_\alpha)_{nm}\psi_m A_{\alpha\mu}. \tag{1.5}$$

Also, θ_α and t_α are matrices representing the Lie algebra (with structure constants $C_{\alpha\beta\gamma}$) of the gauge group

$$[\theta_\alpha, \theta_\beta] = iC_{\alpha\beta\gamma}\theta_\gamma, \tag{1.6}$$

$$[t_\alpha, t_\beta] = iC_{\alpha\beta\gamma}t_\gamma, \tag{1.7}$$

while Γ_i is a gauge-covariant Yukawa coupling matrix

$$[t_\alpha, \gamma_4\Gamma_i] = -(\theta_\alpha)_{ij}\gamma_4\Gamma_j, \tag{1.8}$$

m_0 is a gauge-invariant bare mass

$$[t_\alpha, \gamma_4 m_0] = 0, \tag{1.9}$$

and $P(\phi)$ is an arbitrary quartic polynomial satisfying the gauge invariance condition

$$[\partial P(\phi)/\partial\phi_i](\theta_\alpha)_{ij}\phi_j = 0. \tag{1.10}$$

In this very general notation all gauge coupling constants are included in the θ_α, t_α, and $C_{\alpha\beta\gamma}$, and terms involving the Dirac matrices 1 and γ_5 are allowed in t_α, Γ_j, and m_0. The lowest-order vacuum expectation value λ_i of ϕ_i is determined by the condition that $P(\phi)$ should have no term linear in the shifted field $\phi_i - \lambda_i$, or

$$\partial P(\phi)/\partial\phi_i|_{\phi=\lambda} = 0. \tag{1.11}$$

The mass matrix M_{ij} of the scalar bosons is given in lowest order by the term in $P(\phi)$ quadratic in the shifted field $\phi - \lambda$:

$$M^2_{ij} = \partial^2 P(\phi)/\partial\phi_i\partial\phi_j|_{\phi=\lambda}, \tag{1.12}$$

and, as a consequence of (1.10) and (1.11), satisfies the Goldstone theorem (Goldstone *et al.*, 1962)

$$M_{ij}^2(\theta_\alpha\lambda)_j = 0. \tag{1.13}$$

However, the nonexistence of massless scalar bosons (Higgs, 1964a, 1964b, 1966; Guralnik *et al.* 1964; Englert and Brout, 1964; Kibble, 1967; also see Anderson, 1968) is demonstrated in general by the existence of a gauge, the "unitarity gauge," in which the components of ϕ_i

[2] This theorem was discovered in specific field theoretic models by Nambu and Jona-Lasinio (1961) and Goldstone (1961). Also see Heisenberg (1959).

[3] See also Schwinger (1967); Glashow, Schnitzer and Weinberg (1967), especially Eq. (13); Weinberg (1968), Sec. VII and Footnote 24.

[4] The history of this subject is described in Veltman's talk at the 1973 Bonn Conference, to be published.

along the directions $\theta_\alpha\lambda$ vanish[5]

$$\phi_i(\theta_\alpha\lambda)_i = 0. \tag{1.14}$$

In this gauge it is easy to read off the vector boson mass matrix $\mu^2_{\alpha\beta}$ as the coefficient in $(D_\mu\phi)^2$ of $A_{\alpha\mu}A^\mu_\beta$:

$$\mu^2_{\alpha\beta} = (\theta_\alpha\theta_\beta)_{ij}\lambda_i\lambda_j. \tag{1.15}$$

Note in particular that any linear combination $C_\alpha\theta_\alpha$ of generators, which like the electric charge is unbroken, must satisfy the condition

$$C_\alpha\theta_\alpha\lambda = 0 \tag{1.16}$$

and therefore defines a linear combination of gauge fields

$$C_\alpha A_{\alpha\mu} \tag{1.17}$$

corresponding to an eigenvector of $\mu^2_{\alpha\beta}$ with eigenvalue zero. Finally, the fermion mass matrix is given in lowest order by

$$m = m_0 + \Gamma_i\lambda_i. \tag{1.18}$$

It is important here to realize that if θ_α, t_α, and Γ_i are of the order of a typical gauge coupling constant e, and if the quartic term in $P(\phi)$ is of order e^2, then the expressions (1.12), (1.15), and (1.18) give masses of zeroth order in e, because λ is of order $1/e$.

The nature of the gauge group governing the weak and electromagnetic interactions is uniquely determined if we suppose it to be as small as possible, and to have as irreducible representations the observed leptonic multiplets

$$\left(\frac{1+\gamma_5}{2}\right)\binom{\nu_e}{e^-} \qquad \left(\frac{1-\gamma_5}{2}\right)e^-$$
$$\left(\frac{1+\gamma_5}{2}\right)\binom{\nu_\mu}{\mu^-} \qquad \left(\frac{1-\nu_5}{2}\right)\mu^-. \tag{1.19}$$

The group algebra then consists of the charge Q, and a leptonic "isospin" $\mathbf{T}$ acting on the left-handed doublets. Since Q is not an "isoscalar," it is convenient to define a leptonic "hypercharge"

$$Y \equiv T_3 - Q \tag{1.20}$$

which commutes with $\mathbf{T}$. The gauge group is then the direct product of $SU(2)$ and $U(1)$, with gauge fields $\mathbf{A}_\mu$ and B_μ coupled to the generators $g\mathbf{T}$ and $g'Y$, respectively, where g and g' are independent gauge coupling constants. The charge operator is the linear combination

$$Q = (1/g)(gT_3) - (1/g')(g'Y) \tag{1.21}$$

and since this generator represents an unbroken symmetry, there must be a massless vector particle, a photon, associated with the (conventionally normalized) field

$$A_\mu = \left(\frac{1}{g^2}+\frac{1}{g'^2}\right)^{-1/2}\left[\frac{1}{g}A_{3\mu} - \frac{1}{g'}B_\mu\right] \tag{1.22}$$

(see (1.17)). Then the orthogonal linear combination of $A_{3\mu}$ and B_μ must be the field of a *massive* neutral vector boson

$$Z_\mu = \left(\frac{1}{g^2}+\frac{1}{g'^2}\right)^{-1/2}\left[\frac{1}{g'}A_{3\mu} + \frac{1}{g}B_\mu\right]. \tag{1.23}$$

The remaining gauge fields can be used to form the complex field of a charged massive vector boson

$$W_\mu = \frac{1}{\sqrt{2}}\left[A_{1\mu} + iA_{2\mu}\right]. \tag{1.24}$$

The interaction between the gauge fields and the leptons then takes the form

$$g\mathbf{T}\cdot\mathbf{A}_\mu + g'YB_\mu = g\Big\{\sin\theta QA_\mu + \cos\theta(T_3 + \tan^2\theta Y)Z_\mu + \frac{1}{\sqrt{2}}(T_1 - iT_2)W_\mu + \frac{1}{\sqrt{2}}(T_1 + iT_2)W_\mu^\dagger\Big\}, \tag{1.25}$$

where θ is the unknown A-Z mixing angle, defined by

$$\tan\theta \equiv g'/g. \tag{1.26}$$

From (1.25) we can immediately read off that the electronic unit of charge is

$$e = g\sin\theta \tag{1.27}$$

so that $g > e$. Also, taking care to get all factors of 2 right, we find that the Fermi coupling constant has the value

$$\frac{G_F}{\sqrt{2}} = \frac{1}{m_W^2}\left(\frac{g}{2\sqrt{2}}\right)^2, \tag{1.28}$$

and therefore

$$m_W = \frac{2^{-5/4}G_F^{-1/2}e}{\sin\theta} = \frac{37.3\ \text{GeV}}{\sin\theta}. \tag{1.29}$$

It was further assumed in the 1967 theory (Weinberg, 1967b; Salam, 1968) that the $SU(2)\otimes U(1)$ gauge summetry is broken in the simplest possible way, by the vacuum expectation value of a single complex doublet of scalar fields. In this case, m_Z has the value

$$m_Z = 74.6\ \text{GeV}/\sin 2\theta. \tag{1.30}$$

However, this prediction stands on a much less firm foundation than the results (1.20)–(1.29).

Unfortunately, no one in 1967 was able to prove that spontaneously broken gauge theories of the above type are in fact renormalizable. The Feynman rules for theories with unbroken gauge symmetries had recently been worked out (Feynman, 1963; DeWitt, 1964, 1967; Faddeev and Popov, 1967; Mandelstam, 1968), and were known to give a manifestly renormalizable perturbation theory, but it did not seem clear that renormalizability survives spontaneous symmetry breaking, because by shifting the scalar fields to $\phi_i - \lambda_i$ a new perturbation

[5] The existence of this gauge is proved for general theories by Weinberg (1973a). This reference also works out the canonical quantization of the theory in detail, and derives the Lorentz-invariant Feynman rules for the unitarity gauge.

series is generated. (Recall that λ is of order $1/e$.) I continued working on the problem together with students (Stuller, 1971) at M.I.T., but little progress was made. One of the obstacles was that I was not able to work out the correct Feynman rules even in the unitarity gauge until the spring of 1971, when work on a different problem (Gerstein *et al.*, 1971) introduced me to certain techniques developed a decade earlier by Lee and Yang (1962). [This derivation of the unitarity gauge Feynman rules was not published until later (Weinberg, 1973a).] However, even after deriving the Feynman rules in the unitarity gauge, it was still not clear that the theory is renormalizable.

Finally in mid-1971 the problem was cracked open by a paper of 't Hooft (1971a), which showed how, in a variety of spontaneously broken gauge models, to choose a gauge in which the Feynman rules would be manifestly renormalizable. 't Hooft's work was directed at models of strong and electromagnetic, rather than weak and electromagnetic interactions, but it immediately became apparent to a great many physicists that 't Hooft's methods could be used to test the renormalizability of unified theories of weak and electromagnetic interactions, such as the 1967 model. Detailed proofs of renormalizability were subsequently given by B. Lee (1972b) and Zinn-Justin (Lee and Zinn-Justin, 1972) and by 't Hooft and Veltman (1972a), and it was explicitly shown (Weinberg, 1971) that the introduction of the Z particle cured the bad high-energy behavior that had been found earlier by Gell-Mann *et al.* (1969).

The problem of renormalizability was already well settled by the time of Lee's report (1972a) to the NAL conference, and will not be reviewed in detail here. However, I should mention one approach to this problem which turns out to have great advantages for practical calculations. There is a class of renormalizable gauges (Fujikawa *et al.* 1972; see also Yao, 1973; 't Hooft and Veltman, 1972b; Weinberg, 1973b, Appendix A) characterized by a free parameter ξ, in which the Feynman rules for interaction vertices are given directly by inspection of the Lagrangian (1.2), except that a complex fermion spin zero ghost field ω_α must be included (Feynman, 1963; DeWitt, 1964, 1967; Faddeev and Popov, 1967; Mandelstam, 1968; Faddeev, 1969; Fradkin and Tyutin, 1970; Mills, 1971), with couplings given by the effective Lagrangian

$$-\partial_\mu \omega_\alpha^* C_{\alpha\beta\gamma}\omega_\beta A_\gamma^\mu - \xi^{-1}\omega_\alpha^*\omega_\beta(\theta_\beta\theta_\alpha\lambda)_i\phi_i. \tag{1.31}$$

The propagators in this gauge are given by

Vectors:

$$\eta_{\mu\nu}(\kappa^2+\mu^2)^{-1}_{\alpha\beta} + (1-\xi)\kappa_\mu\kappa_\nu[(\kappa^2+\mu^2)^{-1}(\xi\kappa^2+\mu^2)^{-1}]_{\alpha\beta}, \tag{1.32}$$

Scalars:

$$(\kappa^2+M^2)^{-1}_{ij} + (\theta_\alpha\lambda)_i(\theta_\beta\lambda)_j(\kappa^2)^{-1}(\xi\kappa^2+\mu^2)^{-1}_{\alpha\beta}, \tag{1.33}$$

Spinors:

$$(i\gamma_\mu\kappa^\mu + m)^{-1}, \tag{1.34}$$

Ghosts:

$$\xi(\xi\kappa^2+\mu^2)^{-1}_{\alpha\beta}. \tag{1.35}$$

For $\xi \neq 0$, these Feynman rules satisfy the usual power-counting rules for renormalizability. (Indeed, the vector boson propagator is just what is obtained by using the old "ξ limiting" cutoff procedure of Lee and Yang, 1962.) However, these rules are derived by a path-integral quantization of the Lagrangian (1.2), so that doubts might arise as to the validity and unitarity of the results. One simple way to settle such questions is to note that the path-integral quantization procedure, while it does not guarantee the correctness of the results, does at least guarantee that the S-matrix calculated with these rules if ξ independent. For $\xi = 0$, the Feynman rules reduce to those which can be obtained (Weinberg, 1973a) by a direct canonical quantization in the "unitarity" gauge defined by Eq. (1.14), and therefore must give the correct unitary S-matrix. Hence, since the S-matrix calculated with these rules is ξ-independent and is correct for $\xi = 0$, it must be correct for all ξ. The practical advantage of this formulation is in helping to debug calculations; the results must be ξ independent, and never are until all one's errors are located.

As soon as it was recognized that spontaneously broken gauge theories are renormalizable, there was a great explosion of theoretical effort devoted to detailed calculations of higher-order weak and electromagnetic "radiative" corrections and to the construction of alternative models. The reader is directed to Lee's report (1972a) for a survey of developments in this area up to the summer of 1972; more recent developments are reviewed briefly below in Sec. II. The situation at the present moment can be summarized in the statement that while there is no known obstacle to the general idea of a renormalizable unified gauge model of weak and electromagnetic interactions, and while the original specific $SU(2) \otimes U(1)$ model of leptons may well survive as a partial description of weak interactions, neither this model nor any other specific model known is sufficiently natural and realistic to win general acceptance as a complete theory.

Given this state of affairs on the theoretical side, it is difficult to identify any one critical experiment which could tell us definitely whether the renormalizable gauge theories have anything to do with nature. Nevertheless, there are three pieces of empirical information which seem to me to offer strong encouragement:

(a) The hadronic vector and axial vector currents which appear in semileptonic weak interactions are known to be conserved or partially conserved (Feynman and Gell-Mann, 1958; Gell-Mann and Lévy, 1960; Nambu and Jona-Lasinio, 1961). In general these currents might be any vector and axial vector operators (including even second-class terms) but in a gauge theory they must be the currents associated with the gauge symmetry, and must therefore be conserved expect for the effects of spontaneous symmetry breaking.

(b) There are various empirical indications of corrections to isotopic spin conservation which, although of the same order of magnitude as the electromagnetic corrections, are *not* due to photon exchange. As discussed below in Sec. VII and VIII, these corrections can be ascribed to the weak interactions, providing that the dimensionless coupling constants of the intermediate vector bosons are of the same order of magnitude as the electronic charge e.

(c) Lastly, there is the recent discovery of neutral currents, discussed below in Section II.

Welcome as these empirical encouragements are, it remains true now, as before, that the best reason for believing in a renormalizable gauge theory of the weak and electromagnetic interactions is that it fits our preconceptions of what a fundamental field theory should be like.

II. MODELS AND EXPERIMENTS

I had thought when I started to prepare this review that I might pass over any discussion of the problem of choosing a specific gauge model for the weak and electromagnetic interactions. The progress that the theorists have made in this area recently is not such as to fill us with much enthusiasm. However, in just the last few months the experimentalists have reinvigorated us with a set of exciting new results, and I feel compelled to say a few words about the status of the various models, especially in the light of these new data.

The most direct way of testing gauge models is to look for the intermediate vector bosons. However, the expected masses in any unified theory of weak and electromagnetic interactions are of the order of $G_F^{-1/2}/\sqrt{137}$, or roughly 10–100 GeV, and present accelerator experiments are far from being able to produce anything so heavy. The best hope seems to be to produce them in proton-proton colliding beams, and look for the leptonic decay modes.[6] (In particular, the decay of a neutral intermediate vector boson into $\mu^+ + \mu^-$ would give an easily recognizable signature.) For sufficiently large values of the lepton pair center-of-mass energy, the rate of lepton pair production by intermediate vector boson decay would be about 137 times the electromagnetic production rate, because every intermediate vector boson decays, presumably half or so into leptons, while a photon needs a factor e in the matrix element to turn into a lepton pair. Even with the help of this factor of 137, it still is not clear that the structure functions hold up well enough at large momentum transfers to give an appreciable production rate. This problem is studied in the parton model in a recent paper by Jaffe and Primack (to be published).

At present, the most important empirical constraints on our choice of a gauge model have to do with the effects of virtual neutral intermediate vector bosons. Neutral vector bosons are unavoidable in any theory which treats the left-handed ν and e as an $SU(2)$ doublet, because the commutator of the $\nu \to e^-$ and $e^- \to \nu$ currents is a current that produces $\nu \to \nu$ and $e^- \to e^-$ transitions. In order to incorporate electromagnetism, it is necessary to intorduce another neutral vector field coupled to the right-handed electron $U(1)$ current. The photon field is then a linear combination of the two neutral fields, and the heavy neutral boson field Z_μ is the orthogonal linear combination. In particular, the interaction of the Z with neutrinos is

$$(ie/2 \sin 2\theta)\bar{\nu}\gamma^\mu(1 + \gamma_5)\nu Z_\mu, \tag{2.1}$$

where θ is the $\gamma - Z$ mixing angle, defined in Eq. (1.26). If you add an additional assumption, that the gauge invariance is broken in the simplest possible way by a scalar field doublet, then you have the old model of leptons (Weinberg, 1967b; Salam, 1968), and the Z mass is given by Eq. (1.30). Note that in this model you cannot get rid of the Z by making it heavy, because its couplings then become strong.

Of course, this $SU(2) \otimes U(1)$ model is only one theory out of an infinity of possibilities, but it provides a convenient framework for discussing the various experimental searches for neutral currents. Afterwards, I will come back to some of the other possible models.

From a theorist's point of view, the simplest test for neutral currents is in purely leptonic reactions, such as $\bar{\nu}_e + e \to \bar{\nu}_e + e$, $\nu_\mu + e \to \nu_\mu + e$, and $\bar{\nu}_\mu + e \to \bar{\nu}_\mu + e$, etc. The theoretical interpretation of these experiments was already well understood by the time of Lee's report (1972a), and there is not much new that needs to be added here. One proposed experiment which has been under continuing theoretical study[7] is the search for weak-interaction effects (including parity violation) in the reaction $e^+ + e^- \to \mu^+ + \mu^-$. On the experimental side, there is of course the one (count them, one!) event of the process $\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$, observed (Hasert *et al.* 1973) recently at CERN. The background expected in this experiment was only 0.03 ± 0.02 events, so this appears to be definite evidence for a neutral current, but with one event, who can tell? Taking into account the absence of observed $\nu_\mu + e^- \to \nu_\mu + e^-$ events, this experiment sets limits

$$0.1 < \sin^2\theta < 0.6. \tag{2.2}$$

These are perfectly consistent with the limits based on earlier experiments

$$\sin^2\theta < 0.6 \quad \text{from} \quad \nu_\mu + e^- \to \nu_\mu + e^-, \tag{2.3}$$

$$\sin^2\theta < 0.35 \quad \text{from} \quad \bar{\nu}_e + e^- \to \bar{\nu}_e + e^-, \tag{2.4}$$

which were discussed in Lee's review (1972a).

In order to discuss the $\Delta S = 0$ semileptonic neutral-current reactions, it is necessary to add more theoretical assumptions. The simplest approach is just to ignore strange particles altogether (Weinberg, 1971), on the grounds that nucleons are mostly made of $\mathcal{P}$ and $\mathcal{N}$ quarks, and to put the left-handed parts of the $\mathcal{P}$ and $\mathcal{N}$ quarks into a doublet like ν and e^-. Using Eq. (2.1) and (2.2), one easily finds then that the effective neutrino-nucleon Z-exchange interaction is (Weinberg, 1972b)

$$(iG_F/\sqrt{2})[\bar{\nu}_e\gamma_\lambda(1 + \gamma_5)\nu_e + \bar{\nu}_\mu\gamma_\lambda(1 + \gamma_5)\nu_\mu] \times [J_3^\lambda - 2\sin^2\theta J^\lambda], \tag{2.5}$$

where θ is the angle (1.26); J^λ is the electromagnetic current; and $\mathbf{J}^\lambda$ is the $V - A$ isospin current

$$\mathbf{J}^\lambda \equiv i(\bar{\mathcal{P}}\bar{\mathcal{N}})\mathbf{t}\gamma^\lambda(1 + \gamma_5)\begin{pmatrix}\mathcal{P}\\ \mathcal{N}\end{pmatrix}. \tag{2.6}$$

Until recently, most attention was focussed on exclusive

[6] I wish to thank S. Ting for an informative discussion on this subject.

[7] Recent work includes Godine and Hankey (1972); Love (1972); Cung *et al.* (1972); Dicus (1973); Khriplovich, to be published; Mann, Cline, and Reeder, proposal submitted to SPEAR; Budny, (1973). For earlier work, see Kinoshita *et al.* (1970).

reactions, like $\nu + p \to \nu + p$, $\nu + p \to \nu + n + \pi^+$ etc. For some of these, model-independent calculations are possible, based on using isospin invariance together with measurements of charge-exchange weak interaction processes. In other cases, heavy use must be made of dynamical assumptions, such as Δ dominance, etc. These analyses are complicated, and have not changed much since last summer, so I will simply refer you to excellent recent reviews by Baltay (1972) and Paschos (1973). It appears that none of these exclusive experiments provides conclusive evidence for or against neutral $\Delta S = 0$ currents.[8]

The major new data on neutral currents of the last few months comes from inclusive neutrino reactions[9], $\nu + N \to \nu + X$ and $\bar{\nu} + N \to \bar{\nu} + X$. There are now three separate positive results: From the CERN ν beam (Hasert *et al.*, 1973)

$$R = 0.21 \pm 0.03,$$

where

$$R \equiv \frac{\sigma(\nu + p \to \nu + X) + \sigma(\nu + n \to \nu + X)}{\sigma(\nu + p \to \mu^- + X) + \sigma(\nu + n \to \mu^- + X)}. \quad (2.7)$$

From the CERN $\bar{\nu}$ beam (Hasert *et al.*, 1973)

$$\bar{R} = 0.45 \pm 0.09,$$

where

$$\bar{R} \equiv \frac{\sigma(\bar{\nu} + n \to \bar{\nu} + X) + \sigma(\bar{\nu} + p \to \bar{\nu} + X)}{\sigma(\bar{\nu} + p \to \mu^+ + X) + \sigma(\bar{\nu} + p \to \mu^+ + X)}. \quad (2.8)$$

From the NAL mixed beam (Benvenuti *et al.*, 1974)

$$(1 - \epsilon)R + \epsilon\bar{R} = 0.29 \pm 0.09,$$

$$\epsilon = 0.19 \pm 0.05$$

In order to interpret this data, it is natural to adopt a naive quark model, which is known to work well for the reactions $\sigma(\nu + N \to \mu^- + X)$ and $\sigma(\bar{\nu} + N \to \mu^+ + X)$. Equation (2.5) then gives immediately (Albright, 1973; Seghal, 1973a, Glashow, 1974; Palmer, 1973)

$$R = \tfrac{1}{2} - \sin^2\theta + \tfrac{20}{27}\sin^4\theta \quad (2.9)$$

$$\bar{R} = \tfrac{1}{2} - \sin^2\theta + \tfrac{20}{9}\sin^4\theta. \quad (2.10)$$

The CERN and NAL data are consistent with (2.9) and (2.10) and each other, with $\sin^2\theta$ between 0.3 and 0.4. The data are also consistent with the less model-dependent analyses (1972) of Pais and Treiman (which gives $R \geq 0.24$), and of Paschos and Wolfenstein, 1973 (which gives $R \geq 0.18$ and $\bar{R} \geq 0.39$). Also see Budny (1972).

It is perhaps premature to conclude from all this that neutral currents have really at last been observed. There may be some mysterious source of background contaminating all these experiments. It is certainly too early to conclude that the old model of leptons (Weinberg 1967b; Salam, 1968) is really correct. However, there is now at least the shadow of a suspicion that something like a $SU(2) \otimes U(1)$ model, with $\sin^2\theta$ of order 0.3, may not be so far from the truth.

This would be a very disquieting conclusion to reach. The $SU(2) \otimes U(1)$ model has one virtue—it gives the simplest possible lepton spectrum. Against this, we must set severe disadvantages, with regard both to empirical and aesthetic considerations. I will remind you of some of these problems, and use them to explain the motivations for some of the other extant models

With regard to consistency with experiment, the most serious problem faced by the $SU(2) \otimes U(1)$ model has to do with the strangeness-changing neutral currents. It is impossible simply to follow Cabibbo, and make our doublet out of $\mathcal{P}$ and $\mathfrak{N} \cos\theta_c + \lambda \sin\theta_c$, because then the Z would connect $\mathfrak{N} \cos\theta_c + \lambda\sin\theta_c$ with itself, producing $\Delta S = 2$ nonleptonic and $\Delta S = 1$, $\Delta Q = 0$ semileptonic reactions of first order in G_F, in violent disagreement with experiment. In general, there seem to be just three possible ways to avoid these disasters[10]:

(i) We can add more quarks, in such a way as to cancel the $\mathfrak{N} \to \lambda$ terms in the Z coupling. The simplest possibility, actually suggested (Glashow *et al.*, 1970)[11] before the gauge revival, is to add one more quark $\mathcal{P}'$, with the same charge as the $\mathcal{P}$, so that the doublets are:

$$\begin{pmatrix} \mathcal{P} \\ \mathfrak{N}\cos\theta_c + \lambda\sin\theta_c \end{pmatrix}_L, \begin{pmatrix} \mathcal{P}' \\ -\mathfrak{N}\sin\theta_c + \lambda\cos\theta_c \end{pmatrix}_L. \quad (2.12)$$

The neutrino–hadron interaction still takes the form (2.5), but the current $\mathcal{J}^\lambda$ now receives contributions from both doublets, so that all $\bar{\mathfrak{N}}\lambda$ and $\bar{\lambda}\mathfrak{N}$ terms cancel in J_3^λ, leaving

$$J_3^\lambda = \frac{i}{2}\{\bar{\mathcal{P}}\gamma^\lambda(1 + \gamma_5)\mathcal{P} - \bar{\mathfrak{N}}\gamma^\lambda(1 + \gamma_5)\mathfrak{N} + \bar{\mathcal{P}}'\gamma^\lambda(1 + \gamma_5)\mathcal{P}' - \bar{\lambda}\gamma^\lambda(1 + \gamma_5)\lambda\}. \quad (2.13)$$

This again leads to the results (2.9) and (2.10) in the parton model, providing we again assume (as is consistent with the data on $\nu + N \to \mu^- + X$ and $\bar{\nu} + N \to \mu^+ + X$) that nucleons consist primarily of $\mathcal{P}$ and $\mathfrak{N}$ quarks, so that the $\mathcal{P}'$ as well as the λ terms may be dropped. Of course, we do not have to stop with a fourth quark—by adding various numbers of new quarks and/or leptons, we can build gauge models on almost any gauge group we like, and thereby exclude various other neutral current interactions. Most of these models were already in hand last summer, and since there is now no compelling experimental evidence that excludes neutral currents, I will just list a sample[12] of models with extra quarks. The

[8] The worst conflict here is with the data in the reaction $\nu + N \to \nu + N + \pi^0$ of W. Lee (1972). However, the theoretical analysis here is not straightforward; see B. W. Lee (1972d); Albright *et al.* (1973).

[9] Inclusive muon–nucleon scattering has also recently been considered as a test of neutral currents; see Love *et al.* (1972); Nikolaev *et al.* (1973).

[10] This discussion is based in part on invited talks given by S. L. Glashow at the January 1973 meeting of the American Physical Society, and the March 1973 meeting of the New York Academy of Sciences.

[11] The application to gauge theories was made by Weinberg (1971 and 1972b).

[12] For some models with extra quarks, see Georgi and Glashow (1972b); B. W. Lee (1972c); Prentki and Zumino (1972); Bouchiat *et al.* (1972); Beg and Zee (1973b); Achiman, 1973; Georgi and Glashow (1973a); Rawls and Yu (1973); Yu (1973); etc. Further references are listed below.

situation may become clarified experimentally by the discovery of direct effects of the new quarks; note that the $\mathcal{P}'$ cannot be too heavy or else $\Delta S = 2$ nonleptonic processes will be insufficiently suppressed (Lee *et al.*, 1973; Carlson and Freund, 1972). A number of recent theoretical studies indicate very promising possibilities for finding effects of exotic quarks in neutrino reactions (De Rújala and Glashow, 1973; Beg and Zee, 1973a).

(ii) We can take $(\mathcal{P},\mathfrak{N})$ and $(\mathcal{P},\lambda)$ as doublets with respect to two different commuting $SU(2)$ gauge groups, so that one neutral vector boson interacts with $\bar{\mathfrak{N}}\mathfrak{N}$ and the other with $\bar{\lambda}\lambda$, but none with $\bar{\mathfrak{N}}\lambda$. However, this is impossible if the $\mathcal{P}$ in the two doublets are the same, because then the two different $SU(2)$ currents will not commute, and the commutator will include $\mathfrak{N} \leftrightarrow \lambda$ terms. Georgi and Glashow (1973c) have suggested a "pseudoCabibbo" theory in which the $\mathcal{P}$ in the two doublets are different because one doublet, say $(\mathcal{P}, \mathfrak{N})$, is left handed and the other, $(\mathcal{P}, \lambda)$, is right handed. The $\Delta S = 0$ semileptonic weak interactions would then have the usual $V - A$ form, while the $\Delta S = 1$ interactions would be $V + A$. This does not seem to agree with experiment (Oehme *et al.*, 1973), which is a pity because the pseudoCabibbo idea would have otherwise provided an extremely economical solution to the problem of neutral strangeness-changing currents.

(iii) We can try to change the rules, so that the weak interactions of the hadrons are not directly determined by their transformation properties under the gauge group. This is the case in the models studied by Bars, Halpern, and Yoshimura (1972, 1973)[13], where the weakly interacting vector bosons do not couple directly to the quarks or baryons, but through mixing with a set of strongly interacting gauge bosons ρ, $A1$, K^*, etc. Such theories allow more than enough flexibility to avoid any conflict with experiment. In particular, it is possible to keep the $SU(2) \otimes U(1)$ description of leptons and weakly interacting intermediate vector bosons, and still choose the scalar field vacuum expectation values so that neutral semileptonic interactions have $\Delta S = 0$ but not $\Delta S = 1$. Such a theory would behave just like the old model of leptons (Weinberg, 1967b; Salam, 1968) in most experiments, including the neutrino experiments discussed above. I have certain reservations about models of this general type, which I will explain below in Sec. VIII.

Although there are clearly several ways of saving the $SU(2) \otimes U(1)$ model from the problem of neutral strangeness-changing currents, and although it faces no other conclusive contrary experimental data at present, there are still aesthetic reasons for being dissatisfied with *any* $SU(2) \otimes U(1)$ model. The group $SU(2) \otimes U(1)$ is not simple, so there is no principle which imposes any special relation between the gauge coupling constants g and g'. This would not bother us if the theory were merely to be taken as an interim phenomenological model, but if it is truly a fundamental theory, it ought not to involve an arbitrary mixing angle like (1.26). One way to eliminate this arbitrariness is to reduce the group from $SU(2) \otimes U(1)$ to just $SU(2)$, as in the Georgi–Glashow model (1972b). In such a theory the single neutral vector boson must be the photon, so the confirmation of neutral current effects would close off this route. Another approach is to enlarge the $SU(2) \otimes U(1)$ group to some simple group of which $SU(2) \otimes U(1)$ is a subgroup. Specifically, it has been suggested (Weinberg, 1972c) that the full weak and electromagnetic gauge group might be $SU(3) \otimes SU(3) \otimes P$ (where P is parity), with the left- and right-handed parts of the triplet (μ^+, ν, e^-) forming the representations (3,0) and (0,3), respectively. The trouble of course is that any such theory contains a great many unobserved weak interactions, arising from couplings of the various gauge bosons to currents such as $(\bar{\nu}\gamma^\lambda(1 - \gamma_5)e^-)$, $(\bar{\nu}\gamma^\lambda(1 + \gamma_5)\mu^+)$, and $(\bar{\mu}^+\gamma^\lambda(1 \pm \gamma_5)e^-)$. Since there is supposed to be only one gauge coupling constant, the only way to suppress the unwanted processes is to assume that the corresponding symmetries are broken very strongly, so that these gauge bosons have unusually large masses. The relatively weakly broken symmetries would form a subgroup which would approximately describe the observed weak and electromagnetic interactions. If this subgroup happened to be $SU(2) \otimes U(1)$, we would expect the old model of leptons to work pretty well, but with the mixing angle fixed by the larger $SU(3) \otimes SU(3) \otimes P$ structure to have the value (Weinberg, 1972c) $\sin^2\theta = 0.25$. It is intriguing that this falls in the range suggested by experiment. However, it is difficult to take this "success" very seriously, because the charge operator appears in the leptonic $SU(3) \otimes SU(3)$ group in an entirely different way than in the usual hadronic $SU(3) \otimes SU(3)$, so that it appears impossible to extend this theory directly to hadrons. Nevertheless, the aesthetic argument against $SU(2) \otimes U(1)$ as a fundamental gauge group seems to me so strong that I suspect we may have to come back to some theory with a larger simple gauge group and a hierarchy of weak interaction strengths associated with a hierarchy of spontaneous symmetry breaking. For this reason, I would urge the experimentalists to be on the lookout for small violations of "accepted" truths about the weak interactions, such as the two-component neutrino and the conservation of muon number.

Another problem with $SU(2) \otimes U(1)$ is the possible presence of Adler–Bell–Jackiw anomalies. There are other gauge groups which never have this problem, but the anomalies can be cancelled in any gauge theory by a judicious choice of fermion multiplets. Many of the papers listed in Footnote 12 were at least in part motivated by the need to cancel anomalies. This problem was thoroughly understood by the time of Lee's review (1972a), so I will not discuss it further here.

The $SU(2) \otimes U(1)$ model is also deficient in that it does not immediately lead to CP violation. Mohapatra (1972) has shown how CP violation can be introduced in the four-quark version of the $SU(2) \otimes U(1)$ model, but the strength of the violation is arbitrary. More recently, Pais (1973) and T. D. Lee (unpublished), have proposed interesting and rather different mechanisms for spontaneous violation of CP invariance,[14] in which the observed CP violating effects are automatically small. The differ-

[13] Also see Bardakci and Halpern (1973); Georgi (1973); Georgi and Goldman (1973); Horvath and Acharya, to be published; De Wit, to be published; etc. There are certain similarities between these theories and that of Schwinger (1973). Schwinger does not work in the framework of spontaneously broken gauge symmetries, so a detailed comparison is difficult.

[14] Also see Cheng, to be published, and Pais and Primack, 1973.

ence is that in the model of T. D. Lee, *CP* violation appears through spontaneous symmetry breaking, while in that of Pais, it is present in the Lagrangian from the beginning, and is needed for Cabibbo's version of weak interaction universality. Subsequent work by Georgi and Pais (1974) shows that *CP* violation may arise purely spontaneously in models of the type originally studied by Pais (1973), and that in this way weak universality can be achieved naturally.

There is also the old problem of the $\Delta I = 1/2$ rule in nonleptonic weak decays. B. Lee and Treiman (1973) have suggested that the dominant nonleptonic interactions arise from "Higgs" scalar boson exchange rather than vector exchange, and point out that the scalar exchange can have a pure $\Delta I = 1/2$ structure. (Also see Beg, 1973; DeWit, to be published; Pais, 1973; Cheng, to be published). However, the complications caused by the strong interactions, including the possibility of octet enhancement, prevent a straightforward assessment of these ideas.

In my view, the most important criticism of the $SU(2) \otimes U(1)$ model, and also of all other existing gauge models, is that none of these theories is sufficiently *natural*. That is, the parameters in these theories have to be carefully rigged so as to achieve even a qualitative agreement with experiment. In particular, these models all contain small parameters, such as m_e/m_μ or $(m_{\mathcal{P}} - m_{\mathcal{N}})/m_{\mathcal{P}}$, which we feel ought to be calculable in any fundamental theory, but which in the existing theories have to be put in by hand. This problem is discussed in detail in Sec. VII and VIII, and various kinds of natural gauge theory are described there, but so far none of them is very realistic. We need a theory that is both natural and realistic, but so far this has eluded us.

III. DERIVATIONS OF GAUGE INVARIANCE FROM HIGH-ENERGY CONSTRAINTS

A number of authors have independently carried out calculations which shed light on the physical significance of spontaneously broken gauge symmetry. It is well known (Gell-Mann *et al.*, 1969) that the tree graphs in both the Fermi and the intermediate vector boson theories grow rapidly with energy, so that in order to save unitarity we must abandon perturbation theory above energies of order 300 GeV. It is also well known[15] that when the couplings and particle spectrum satisfy the constraints imposed by a spontaneously broken gauge theory, there appear wonderful cancellations which save perturbative unitarity at all energies. (Indeed, the best way to convince oneself that gauge theories may have something to do with nature is to carry out some specific calculation and watch the cancellations occur before one's very eyes.) The new point made by Cornwall, Levin, and Tiktopoulos (1973), Llewellyn Smith (1973b), and Joglekan (to be published) is that this argument may be turned around, so that by requiring these cancellations one can recover all the facts about the couplings and spectrum which were previously derived directly from the broken gauge symmetry.

For instance, suppose that we have a set of massive spin-one bosons W_α^μ and spin-zero bosons Φ_i, with an interaction Lagrangian of the form

$$\begin{aligned}\mathcal{L}' = {} & A_{\alpha\beta\gamma}\epsilon^{\mu\nu\lambda\rho}(\partial_\rho W_{\alpha\mu})W_{\beta\nu}W_{\gamma\lambda} + B_{\alpha\beta\gamma\delta}\epsilon^{\mu\nu\lambda\rho}W_{\alpha\mu}W_{\beta\nu}W_{\gamma\lambda}W_{\delta\rho} \\ & + C_{\alpha\beta\gamma}(\partial_\nu W_{\alpha\mu})W_\beta^\mu W_\gamma^\nu + D_{\alpha\beta\gamma\delta}W_{\alpha\mu}W_\beta^\mu W_{\gamma\nu}W_\delta^\nu \\ & + \Phi - W \text{ couplings} \\ & + \Phi \text{ self couplings}\end{aligned} \tag{3.1}$$

where A, B, C, D are general real coefficients. Then the demand for perturbative unitarity in the process $WW \to WW$ imposes the relations

$$A_{\alpha\beta\gamma} = 0, \tag{3.2}$$

$$B_{\alpha\beta\gamma\delta} = 0, \tag{3.3}$$

$$C_{\alpha\beta\gamma} \text{ totally antisymmetric,} \tag{3.4}$$

$$C_{\alpha\beta\epsilon}C_{\gamma\delta\epsilon} + C_{\gamma\alpha\epsilon}C_{\beta\delta\epsilon} + C_{\beta\gamma\epsilon}C_{\alpha\delta\epsilon} = 0, \tag{3.5}$$

$$D_{\alpha\beta\gamma\delta} = \tfrac{1}{8}C_{\gamma\alpha\epsilon}C_{\beta\delta\epsilon} + \tfrac{1}{8}C_{\beta\gamma\epsilon}C_{\alpha\delta\epsilon}, \tag{3.6}$$

plus a complicated relation among the C's, the W masses, and the $WW\Phi$ couplings. The conditions (3.4) and (3.5) simply tell us that the C's are the structure constants of some Lie group G; Eqs. (3.2), (3.3), and (3.6) require that the W self-interactions are the same as in Yang–Mills Lagrangian[1] based on the gauge group G; and the final condition relates the strength of the ΦWW coupling to the strength of the violation of local G invariance by the vector masses. Other constraints in the $\Phi - W$ couplings and $\Phi - \Phi$ couplings are derived by making similar demands on the reactions $\Phi W \to \Phi W$, $\Phi\Phi \to \Phi\Phi$, etc., and of course still more constraints are generated if we include fermions.

All these constraints are satisfied[15] if the Lagrangian is governed by a spontaneously broken gauge symmetry G. In special cases it has been shown (Cornwall, Levin, and Tiktopoulos, 1973; Llewellyn Smith, 1973b; Joglekan, to be published) that such gauge theories provide the *only* solution (aside from the rather trivial exception that gauge bosons associated with the Abelian subgroups of G can be given arbitrary masses), but this has not been proved in general. Even if this were proved, the argument for a spontaneously broken gauge theory would still be a bit weak, becasue in writing down the original Lagrangian (3.1) only superficially renormalizable terms were included; it might be that terms with more derivatives or fields could be included at the cost of changing the constraints. Nevertheless, despite these words of caution, it seems to me to be highly likely that the only theories of massive spin-one particles which satisfy the requirements of perturbative unitarity are (aside from Abelian gluons) just those described by a spontaneously broken gauge symmetry.

A somewhat different approach is taken by Sucher and Woo (1973). They introduce an unphysical scalar field, in such a way as to cancel the longitudinal part of the vector meson propagator. The condition that unphysical particles not be produced in collisions of ordinary particles

[15] See, for example, Weinberg (1971); Vainshtein and Khriplovich (1971). To the best of my knowledge, the first calculation which showed the necessity of "Higgs" scalars was that of $3W$ production in lepton–antilepton annihilation by H. R. Quinn, unpublished. Recently it has been shown that scalar mesons are also required in $2W$ production when the lepton and antilepton have equal helicity; see Schecter and Ueda, to be published. The proof of cancellation in the general case is given by Bell, to be published.

then leads to the constraints characteristic of a spontaneously broken gauge symmetry. There is little doubt that this approach is equivalent to that of Cornwall *et al.*, Llewellyn Smith, and Joglekan, and just corresponds to starting out in a renormalizable gauge and demanding unitarity, rather than starting out in a unitary gauge and demanding renormalizability. However, Sucher and Woo go on to argue against the existence of any physical gauge symmetry in such theories. It seems to me the point is moot; unless someone can find a difference between the results of Cornwall *et al.*, Llewellyn Smith, and Joglekan and the results of spontaneously broken gauge theories, it is just a matter of words whether one says that there "really" are spontaneoulsy broken gauge symmetries in these theories. Of course, it would be extremely interesting and important if such a difference could be found, and this is not out of the question, because as far as I know there is no general theorem proving the equivalence of renormalizability and perturbative unitarity

Apart from the fundamental question it raises, the work of Cornwall *et al.*, Llewelyn Smith and Joglekan also suggests an explanation for the observation by Gervais, Neveu, and Scherk (Gervais and Neveu, 1972; Neveu and Scherk, 1972) that dual models in the limit of zero Regge slope give the same results as gauge models for processes like $WW \to WW$. In the zero slope limit the dual models essentially just say that the scattering amplitude is a sum of a finite number of tree graphs with asymptotic behavior no worse than given by Regge exchange. These are just the assumptions needed in Cornwall *et al.*, Llewelyn Smith, and Joglekan to derive the constraints characteristic of a gauge theory.

IV. REGGEIZATION OF GAUGE THEORIES

A group at Brandeis (Grisaru *et al.*, 1973a, b)[16] has discovered a further remarkable connection between spontaneously broken gauge symmetry and high-energy behavior. Some years ago, it was noted (Gell-Mann, Marx *et al.*, 1964; Gell-Mann, Singh *et al.*, 1964, Cheng and Wu, 1966)[17] that a particle can be elementary, in the sense that its field appears in the Lagrangian, and yet can also lie on a Regge trajectory. That is, if we write the trajectory function $\alpha(s)$ as a power series in the coupling constant with zeroth-order term equal to the particle spin j, then the Regge-pole formulas for scattering amplitudes may be consistent order-by-order with the results of perturbation theory. Of course, if the lowest-order diagram is a tree graph with a spin j particle in the s channel, the lowest-order partial wave amplitude $A(s,J)$ will have a Kronecker delta factor δ_{Jj}, which must be interpreted as the zeroth-order term in the quantity $[\alpha(s) - j]/[\alpha(s) - J]$. A necessary condition for Reggeization is that the residue at the pole at $\alpha(s) = J$ should factorize into a product of factors depending only on initial and final state helicities, respectively. It turned out that this factorization condition is in fact satisfied for spin-1/2 particles in a renormalizable theory in which the fermions interact with massive neutral vector mesons. However, very few such examples could be found. In particular, in a massive Yang–Mills theory with isospin-1/2 spin-1/2 fermions, the factorization condition was satisfied for the fermion pole in fermion-vector scattering, but not for the vector pole in vector–vector scattering (Dicus *et al.*, 1971; also see Abers *et al.*, 1970).

Inspired in part by the connection between duality and gauge invariance mentioned in the last section, Grisaru, Schnitzer and Tsao (1973a, b)[16] have now examined a renormalizable Yang–Mills theory, based again on the isospin gauge group, but with the vector mesons receiving their mass from the vacuum expectation values of a scalar multiplet. They find that the factorization conditions are now satisfied for the fermion pole in fermion–vector scattering and also for the vector pole in vector–vector scattering. The factorization condition is not satisfied for the spin-zero pole in vector–vector scattering, but this may be a function of the particular choice of multiplet. Apparently the whole apparatus of the spontaneously broken gauge theory including the scalar exchange terms is necessary to produce the delicate cancellations needed for factorization in vector–vector scattering.

All this only goes to show that certain *necessary* conditions are met. Mandelstam (1965; see also Abers and Teplitz, 1967) has proposed certain criteria as *sufficient* conditions for Reggeization. These conditions are met here for the vector and spinor particles (but not for the scalar) so we may conclude that Reggeization really does occur here. Incidentally the Mandelstam conditions are also satisfied by the vector mesons in the ordinary massive Yang–Mills theory, which certainly does not Reggeize, but the failure of Reggeization there is believed (Grisaru *et al.*, 1973a, b) to be due to the nonrenormalizability of the theory.

We naturally think of Reggeization as having to do with strong interactions, but the above considerations may become applicable to the weak and electromagnetic interactions as well, if these really are described by non-Abelian gauge theories. Schnitzer (to be published) notes that the electron would be expected to lie on a trajectory with slope of the order of the Fermi coupling G_F, so that the trajectory would cross $J = 3/2$ at an energy of order 300 GeV!

V. DYNAMICAL BREAKDOWN OF GAUGE SYMMETRIES

Usually we suppose that the spontaneous breakdown of gauge symmetries occurs because certain scalar fields in the theory develop nonvanishing vacuum expectation values. However, these scalar fields are extremely troublesome when it comes to making models of the real world. For one thing, after elimination of the Goldstone bosons by an appropriate choice of gauge, there will be left over certain physical scalar particles, none of which have been seen in experiments. We saw in the last section that these scalars do not Reggeize, at least in the models so far examined, and we shall see in the following sections that they can mess up desirable properties such as parity conservation and asymptotic freedom. Worst of all from the point of view of the model builder is the too great freedom that they give us—even after choosing a gauge group and a fermion multiplet, we can do almost anything we like with the physical content of the theory by an appropriate choice of scalar multiplet and scalar self-couplings. For all these reasons, it is interesting to see

[16] Also see S. Y. Lee, Rawls, and Wong, to be published.
[17] The first article, Gell-Mann, Marx, *et al.* (1964), contains references to earlier work.

how far we can return to the original spirit of Nambu and Jona-Lasinio (1971) and understand the spontaneous breakdown of gauge symmetries in purely dynamical terms. A major step in this direction has been taken in the independent work of Jackiw and Johnson (1973) and Cornwall and Norton (1973). In order to see the key points in this work, let us consider the case of an Abelian gauge group, with a single gauge field A_μ coupled to a conserved current of some sort or other. Current conservation requires the proper self-energy part $\Pi_{\mu\nu}$ to take the form

$$\Pi_{\mu\nu}(q) = (q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2). \tag{5.1}$$

The complete vector propagator will then be

$$\Delta'_{\mu\nu}(q) = [g_{\mu\nu}/q^2(1 - \Pi(q^2))] + q_\mu q_\nu \text{ terms} . \tag{5.2}$$

(The $q_\mu q_\nu$ terms are of course gauge dependent). This appears to have a pole at $q^2 = 0$, but as pointed out long ago by Schwinger (1962a, 1962b) the pole can be killed if $\Pi(q^2)$ itself has a pole at $q^2 = 0$.

But why should $\Pi(q^2)$ have such a pole? The only known answer is that $\Pi(q^2)$ will pick up a pole at $q^2 = 0$ if the global symmetry associated with A_μ is spontaneously broken, so that a massless Goldstone boson appears in the A channel. In this case, we will have

$$\Pi(q^2) \to (-F^2/q^2) \quad \text{for } q^2 \to 0, \tag{5.3}$$

where F is the coefficient (analogous to F_π) describing the coupling of the A field to the Goldstone boson state. Near zero momentum the A propagator will then behave as

$$\Delta'_{\mu\nu}(q) \to g_{\mu\nu}/F^2 + q_\mu q_\nu \text{ terms} \tag{5.4}$$

so there is no more massless vector particle.

However, we still have to worry about the effects of the massless Goldstone boson. Suppose we consider some reaction $i \to f$ which could have the Goldstone boson and the A_μ vector boson as single-particle intermediate states in the s channel. The scattering amplitude can be decomposed into three terms:

$$T_{fi} = T_{fi}^{(1)} + T_{fi}^{(2)} + T_{fi}^{(3)}, \tag{5.5}$$

where $T_{fi}^{(1)}$ is the sum of all graphs with neither vector boson nor Goldstone boson poles in the s-channel; $T_{fi}^{(2)}$ is the sum of all graphs with a Goldstone boson pole but no vector pole in the s channel, $T_{fi}^{(3)}$ is the sum of all graphs with a vector boson pole in the s channel.

We can also write

$$T_{fi}^{(2)} = \Gamma_f(1/q^2)\Gamma_i, \tag{5.6}$$

where Γ_f and Γ_i are the sums of all vertex graphs connecting the final and initial states with the Goldstone boson, excluding graphs with a vector or a Goldstone boson pole in the s-channel, and are therefore regular at $q = 0$. Further, we have

$$T_{fi}^{(3)} = \Gamma_f^\mu \Delta'_{\mu\nu}(q)\Gamma_i^\nu, \tag{5.7}$$

where Γ_f^μ and Γ_i^ν are the sums of all vertex graphs connecting the final and initial states with the vector boson, excluding graphs with a vector boson in the s channel. The crucial point now is that $\Gamma_{f\mu}$ and $\Gamma_{i\nu}$ *do* have Goldstone boson poles, so they are not regular at $q = 0$. Instead, we have

$$\Gamma_{f\mu} = \tilde{\Gamma}_{f\mu} + \Gamma_f(1/q^2)Fq_\mu, \tag{5.8}$$

where $\tilde{\Gamma}_{f\mu}$ is the sum of the graphs which do not contain the Goldstone pole, and Γ_f and F are the quantities appearing above in Eqs. (5.6) and (5.4). Keeping in mind that the current is conserved, so that $q^\mu\Gamma_{f\mu}$ vanishes, we find after a little algebra that

$$T_{fi}^{(3)} = [1/q^2(1 - \Pi(q^2))]\{\tilde{\Gamma}_{f\mu}\tilde{\Gamma}_i^\mu - F^2(\Gamma_f\Gamma_i/q^2)\}. \tag{5.9}$$

Equation (5.3) then shows that the singularity in (5.9) at $q^2 = 0$ cancels the singularity in (5.6), and the Goldstone boson therefore does not appear as a true intermediate state.

All this applies to any theory in which the gauge symmetry can be spontaneously broken as a global, as well as a local, invariance. This includes two-dimensional electrodynamics with vanishing bare "electron" mass, which was the example considered by Schwinger (1962a, 1962b). There is also a wide variety of four-dimensional theories which meet this criterion, including axial-vector electrodynamics (i.e., with photons coupled to $\bar{\Psi}\gamma_5\gamma_\mu\Psi$) which was the example considered by Jackiw and Johnson (1973), and an $SO(2)$ gauge theory with two fermions, which was the example considered by Cornwall and Norton (1973). However, it does not appear possible for the photon in ordinary quantum electrodynamics to get a mass in this way, because the spontaneous appearance of an electron mass would break chirality but not charge conservation, so that the Goldstone bosons would have the wrong parity.

So far, these considerations are very general, and do not depend on the mechanism by which the symmetry is spontaneously broken. In the familiar models there is a multiplet ϕ_i of scalar fields, some of which may develop vacuum expectation values and thereby break the symmetry. It is well known [for instance, from the σ model (Gell-Mann and Lévy, 1960)] that the coupling F of a Goldstone boson to the corresponding current is then proportional to the product of the gauge coupling constant g times some scalar field vacuum expectation value λ with $F \approx g\lambda$, so Eq. (5.4) gives a vector meson mass $\mu^2 \approx F^2 \approx g^2\lambda^2$, in agreement with the Higgs result (1964a, 1964b, 1966).

Of course, the real point of the analysis described above is that it allows us to deal with theories in which the Goldstone boson is *not* represented by a field in the Lagrangian but is a true bound state. However, this poses a problem if we want to apply these ideas to theories of the weak and electromagnetic interactions, because we would normally not expect zero-mass bound states to form unless the forces were sufficiently strong. There is one possible solution, suggested independently by Jackiw and Johnson (1973) and Cornwall and Norton (1973). In a theory like axial-vector electrodynamics, the kernel of the Bethe–Salpeter equation for fermion–antifermion scattering is not compact in the ladder approximation, so there is no theorem that says that the zero-energy bound states occur for only a discrete set of sufficiently strong coupling constants. Instead it is found that zero-energy bound states exist for *any* values of the coupling, providing only that there is a vector particle exchanged in the cross channel with a coupling larger than the axial-vector coupling (Baker and Johnson, 1971; also see Johnson *et al.*, 1964). This is not an entirely satisfactory solution, though, because if the Bethe–Salpeter kernel really allows such a homogeneous solution at zero mass, it allows any number of solutions at any masses we like so that almost no predictive power could be left in the original Lagrangian. So far, the problem has only been studied in the ladder approximation, which may give misleading results.

It has not yet been possible to develop a complete perturbative formalism for doing calculations in these theories. For this reason, it is perhaps premature to ask whether massive gauge theories without elementary scalars are renormalizable. Nevertheless, I doubt that they are. As discussed earlier, in order to have reasonable asymptotic behavior in the tree approximation, it is absolutely necessary to have poles for scalar particles other than the Goldstone bosons. Whatever sort of perturbation theory is developed, it presumably starts with something like a sum of tree graphs, and it is hard to see how these can have an acceptable asymptotic behavior without nonGoldstone scalar poles.

Indeed, we can make a pretty good guess that when a perturbation theory is developed for these new theories, it will look like a theory with a set of elementary spin-zero fields which form a minimal nonlinear realization of the gauge group [such as the pion triplet in the nonlinear σ-model (Gell-Mann and Lévy, 1960)] so that no spin-zero fields are left after transforming to a unitarity gauge.[18] Such theories are known not to be renormalizable in any ordinary sense.

Of course, massive nonGoldstone scalar particles may appear as composite particles as well as Goldstone bosons. It is not clear whether this would restore renormalizability. We usually think of renormalizability as having to do with the elementary particles in the theory. (Otherwise, what would we do with the high spin states of positronium?) However, there is no way of settling such questions until a systematic method of calculation is developed for these theories.

VI. NEUTRAL CURRENTS AND ASTROPHYSICS

Weak interactions play a fundamental role in astrophysical processes. Indeed, the first step in the reaction chain that produces the energy of the sun is the weak process $p + p \to d + e^+ + \nu$. For the most part, the weak interactions of importance in astrophysics are of the charge-exchange type, like the *pp* reaction, and are therefore unaffected by the recent developments in weak interaction theory. However, here and there in the universe there are phenomena which could be significantly affected by neutral-current weak interactions.

One place where the neutral currents might be expected to be important is in the early universe. In the usual "big bang" models,[19] neutrinos and antineutrinos are kept in thermal equilibrium at very early times through reactions such as $\nu_\mu + e^- \leftrightarrow \nu_e + \mu^-$, $\nu_e + e^\pm \leftrightarrow \nu_e + e^\pm$, etc. The muon-type neutrinos and antineutrinos are then supposed to go out of equilibrium at a temperature 1.3×10^{11} K, when the number of particles with energies of order m_μ becomes very small, while the electron neutrinos stay in thermal equilibrium until the electron pairs disappear, at around 3×10^9 K With neutral currents, we have reactions like $\nu_\mu + e^\pm \leftrightarrow \nu_\mu + e^\pm$, so the muon neutrinos behave more or less like electron neutrinos, and in addition the electron neutrino cross sections themselves are somewhat changed, so that both muon and electron neutrino can go out equilibrium at temperatures somewhat above or below 3×10^9 K. However, it turns out that this makes essentially no difference to the thermal history of the early universe. The reason is that during the whole period from when the temperature drops below 10^{12} K until it reaches about 10^4 K the entropy of the universe is overwhelmingly in the form of extremely relativistic particles (photons, neutrinos, etc., so that the temperature-volume relation is the same ($T \propto V^{-1/3}$) as for a gas of free massless particles, and it makes no difference whether the neutrinos are in thermal equilibrium or not. In particular, the cosmological production of He^4 is hardly at all affected by any reasonable change in the leptonic reaction rates (Hecht, 1971).

Neutral currents can play a more important role in determining the rate of loss of energy by stars. Of course, photon production rates are always much greater than neutrino production rates, but photons can go only a tiny distance in a star, while neutrinos leave the stellar core without feeling any effects of the surrounding mass. Aside from nuclear reactions, the dominant neutrino production mechanisms are expected to be the reactions $e^+ + e^- \to \nu + \bar{\nu}$, $\gamma + e^- \to e^- + \nu + \bar{\nu}$, and plasmon decay into $\nu + \bar{\nu}$. Conventionally it is assumed that only electron neutrinos can be produced this way, but if neutral currents exist then muon neutrinos can also be emitted, and the ν_e production rates are somewhat altered. Dicus (1972) finds that the neutrino energy loss rate from an electron gas is changed by the neutral currents by a factor which, over a broad range of temperatures and densities, is between 8.5 and 0.5. (Also see Biswas *et al.*, 1973.) This is within the range allowed by observation of the temperature distribution of white dwarfs (Stothers, 1970). It would be very interesting to see what effect the neutral currents would have in other contexts where neutrino cooling is important, such as the evolution of stars whose cores have evolved mostly to carbon.

Perhaps the most important astrophysical implications of weak neutral currents lie in the context of supernova explosions. According to current ideas,[20] a presupernova star consists of a relatively small, highly evolved core, consisting of a few solar masses of iron (or perhaps carbon, for less massive stars), surrounded by a much more massive envelope consisting of tens of solar masses of less evolved matter. For one reason or other the core becomes unstable and starts to implode. If there were no envelope, we would expect it to "bounce" when it becomes small enough (say 10 km radius) for neutron degeneracy pressure to become important, and it would then settle down to form a neutron star. However, with

[18] Such a theory has recently been discussed by Faddeev, to be published.

[19] For a general introduction, see Weinberg (1972a), Chap. XV.

[20] For general background, see Wheeler (1973); Imshennik and Nadezhin (1971). I am grateful to Dr. Wheeler for his guidance in the preparation of this section.

the huge envelope falling onto the core this is impossible, and if nothing intervenes, the core and envelope will continue to collapse, forming a black hole. In 1966 Colgate and White suggested that the envelope might be blown off by neutrinos, which are produced thermally in the extremely high temperatures ($\sim 10^{12}$K) of the core or the surrounding shock wave, and are then stopped in the envelope. (Note that although the "optical depth" of the sun for 1 MeV neutrinos is of order 10^{-10}, if we compress the sun by a factor 10^5 to neutron star dimensions its optical depth becomes of order 1 for 1 MeV neutrinos, and even smaller for 100 MeV neutrinos). The result would then be a supernova with a left-over neutron star, but no black hole. A number of calculations (Arnett, 1966, 1967; Ivanova *et al.*, 1967; Schwartz, 1967; Wilson, 1971) have been carried out to test these ideas, but the hydrodynamics is extremely complicated, and it is not yet clear whether neutrinos get stopped at the right time and place to blow off the envelope. If we believe in neutral currents, then these calculations must be redone to take account of the changed neutrino opacity, especially the great increase in the muon neutrino opacity. One particular case has been studied by Wilson[21]; he finds that with or without neutral currents, the envelope in this case does not explode.

VII. NATURAL SYMMETRIES AND SYMMETRY BREAKING

The problem of approximate symmetries has been with us ever since it was observed that the neutron has very nearly the same mass as the proton. From the beginning, there has also seemed to be an obvious solution: the neutron and proton masses are nearly but not exactly equal because the bare masses and nuclear forces are charge independent while the electromagnetic interactions are not. Later, when it was discovered that strangeness, parity, and charge conjugation are very weakly broken symmetries, it seemed natural to suppose that the strong and electromagnetic interactions preserve these symmetries while the weak interactions do not.

The trouble with this sort of explanation is that it seems to run afoul of ultraviolet divergences in actual calculations.[22] With the advent of renormalization theory, it became clear that these divergences could in some cases be absorbed into a redefinition of the bare masses and coupling constants. However, this requires that the bare masses and couplings themselves have to violate the symmetry in order to provide enough free parameters to absorb all infinities. For instance, in order to calculate electromagnetic corrections to isotopic spin conservation, we would have to introduce unequal bare masses for the neutron and proton. This merely shifts the mystery—why do the bare masses *nearly* satisfy symmetries such as isospin? In any case, once we agree to absorb the infinite part of the electromagnetic or weak self-energy into the bare masses, we give up any possibility of calculating the finite corrections to the approximate symmetries.

The new picture of the weak and electromagnetic interactions, based on spontaneously broken gauge symmetries, has forced us to think again about this old problem. Not only do we feel that the approximate symmetries ought to be explicable within this picture, but we are encouraged to seek the explanation within renormalized perturbation theory, rather than in some mysterious future cutoff.

At the same time, the consideration of renormalizable field theories with spontaneously broken gauge symmetry has provided us with an insight which may lead to the correct explanation of the approximate symmetries. A theory of massive mutually coupled spin-one intermediate bosons is unrenormalizable unless the vector boson masses arise from the spontaneous breakdown of an *exact* gauge symmetry. However, once we specify this exact gauge symmetry, and specify the elementary spin-zero and spin-1/2 fields which enter in the Lagrangian, there is not a great deal of freedom left in choosing the most general possible renormalizable and gauge invariant Lagrangian. It sometimes happens that the structure of the Lagrangian is so restricted that when the gauge group is spontaneously broken, the resulting masses and coupling constants are found *for all possible values of the parameters in the Lagrangian* to obey in zeroth order certain exact symmetry relations which do not correspond to any "unbroken" subgroup of the original invariance group of the Lagrangian. These zeroth-order symmetry relations will in general not survive in higher-order calculations, and, if the gauge coupling constants are of order e, there will appear corrections of order α, α^2, etc. However, these higher-order corrections must be *finite*, precisely because we have assumed that the symmetry relations in question are satisfied for all possible values of the parameters of the Lagrangian. This assumption prevents us from introducing counter-terms in the Lagrangian which could absorb divergences in corrections to the zeroth-order symmetry relations, so if such divergence did occur, they could not be removed by renormalization and so they can *not* occur, because the theories we deal with are in fact renormalizable.[23] (This is just the same argument that explains why the scattering of light by light must come out finite in quantum electrodynamics).

A theory of this sort, in which all approximate symmetries appear in zeroth order as exact consequences of the gauge invariance, field content, and renormalizability of the Lagrangian, rather than of some particular choice of parameters, may justly be called a *natural* theory of approximate symmetry. Not only is such a theory natural in an aesthetic sense, in a way that existing models of the weak and electromagnetic interactions are not, but it also is natural in the important technical sense, that the corrections to the symmetry arising from higher-order effects are finite.

How is it possible for a symmetry relation to arise in zeroth-order perturbation theory and yet receive corrections in higher order? The simplest example of this phenomenon is provided by the class of theories in which the Lagrangian contains no scalar fields which have the right quantum numbers to allow renormalizable Yukawa-type interactions with the fermions. (For instance if the gauge group is isospin, and all scalar fields have isospins

[21] J. R. Wilson, private communication.

[22] See, for example, Weisskopf (1939).

[23] The proof that the infinities in gauge theories obey all the natural relations of the zeroth-order parameters of the theory appears as an essential part of the proof of the renormalizability of such theories by B. W. Lee and Zinn-Justin (1972). For an earlier discussion of the same point in the context of the σ model, see B. W. Lee (1969); Gervals and B. W. Lee (1969).

greater than twice the fermion isospin, then no Yukawa interactions are possible). In this case, the zeroth-order fermion mass matrix is just the bare mass matrix, which must be invariant under the gauge group of the theory. However, even though the fermion masses are invariant under the gauge group in zeroth order, the gauge invariance of the theory will in general be broken by the vacuum expectation values of the various scalar fields, which introduce zeroth-order violations of the gauge symmetry into the vector boson masses even though they cannot directly affect the fermion masses. The emission and absorption of virtual intermediate vector bosons will thus produce corrections to the gauge invariance of the fermion masses in higher order, and, by the above arguments, these corrections must be finite. For instance, if the zeroth-order mass relation tells us that $m_p = m_n$, then the second-order corrections will typically be of the form

$$\delta m_p = c\alpha m \ln(\Lambda/\mu), \tag{7.1}$$

$$\delta m_n = c\alpha m \ln(\Lambda/\mu'), \tag{7.2}$$

where c is a numerical constant [such as $(3/16\pi)$], m is the common zeroth-order mass, Λ is an ultraviolet cutoff, and μ and μ' are functions of the various zeroth-order intermediate vector boson masses. Although both δm_p and δm_n are infinite, the correction to the zeroth-order relation is necessarily finite; in this case, we have

$$\delta m_p - \delta m_n = c\alpha m \ln(\mu'/\mu). \tag{7.3}$$

One important feature of this sort of result is that even though μ and μ' may be very large, the mass shifts are typically of order αm. About this, more later.

In the general case, zeroth-order fermion mass relations may arise even if there are some scalar fields in the theory with quantum numbers which allow Yukawa coupling to the fermions. The situation is then much more complicated because the zeroth-order fermion mass matrix m receives contributions from the Yukawa couplings $\bar{\psi}\Gamma_i\psi\phi_i$ as well as from the bare mass m_0:

$$m = m_0 + \Gamma_i\lambda_i, \tag{7.4}$$

where λ_i is the lowest-order vacuum expectation value of ϕ_i, determined as the value of ϕ_i for which the quartic polynomial $P(\phi)$ appearing in the Lagrangian is stationary:

$$\partial P(\phi)/\partial\phi_i = 0 \text{ at } \phi = \lambda. \tag{7.5}$$

(See Sec. I). In particular, the zeroth-order fermion mass relations will *not* simply require the fermion masses to respect the gauge group of the theory.

A good deal of effort over the past year has been put into the task of formulating a general catalog of natural zeroth-order symmetries. As a result of agreements between Georgi and Glashow and myself, there has been developed a tripartite classification of natural zeroth-order fermion mass relations:

(1) Relations arising (Weinberg, 1972d)[24] (as in the examples discussed above) because not all of the scalar fields which could have Yukawa couplings actually appear in the theory.

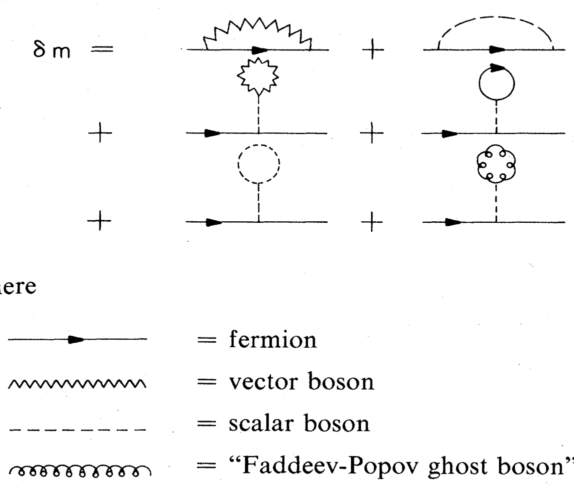
(2) Relations arising (Weinberg, 1972e)[25] because the polynomial $P(\phi)$, solely by virtue of its being quartic and gauge invariant, is necessarily invariant under global symmetries other than those of the original gauge group, some of which symmetries are not broken by the vacuum expectation values λ:

(3) Relations arising (Georgi and Glashow, 1972a, 1973b) from constraints on λ_i other than those of type (2).

The hard part here is to analyze the catch-all "type 3." Important contributions have been made by B. W. Lee (unpublished), Duncan (unpublished), and Schattner (to be published), but more work is needed for a really general understanding.

The most exciting result which hopefully will come out of these considerations is an understanding of the electron–muon mass ratio. That is, we hope to find a theory in which the muon mass is some typical zeroth-order fermion mass, but in which the electron mass is forced to vanish in zeroth order by some natural zeroth-order symmetry of the sort discussed above. If we are lucky, intermediate vector boson exchange would then produce an electron mass of order αm_μ. Such a theory has actually been constructed by Georgi and Glashow (1973b), but their model is not a realistic one, and is intended for illustrative purpose only.

Even though we do not have a really thorough understanding of the various kinds of zeroth-order mass relation which may arise, it is possible to carry out a completely general analysis of the second-order corrections to such relations. The Feynman diagrams are shown below:



(In calculating these graphs, it is extremely convenient to use a general formalism derived by Fujikawa, Lee and Sanda (1972), described in Sec. I.) It turns out that although δm is generally infinite, the terms in δm which can produce corrections to *natural* zeroth-order mass relations are always finite (Weinberg, 1973b). Several authors have carried out detailed calculations along these

[24] An example of a "type 1" mass relation was presented earlier by 't Hooft (1971b), but was not described in these terms.

[25] The "type 2" mass relations show some intriguing features, related to peculiarities discovered in a different context by Coleman and E. Weinberg (1973).

lines in specific illustrative models.[26]

What does all this have to do with the problem with which we started, that of isospin breaking? Will not the presence of strong interactions invalidate any conclusions based on simple one-loop Feynman graphs? The problem of the strong interactions is considered in the next two sections, with special attention to their effect on order α corrections to natural symmetries. In the end, we will see that for a large class of strong interaction theories, the answers given by perturbation theory are in fact correct.

VIII. STRONG INTERACTIONS AND HADRONIC SYMMETRIES

Even if we regard the weak and electromagnetic interactions as the natural province of gauge theories, it is necessary in applying these theories to hadrons to have some idea of what produces the strong interactions. If in the attempt to hang on to the attractive features of the gauge theory of weak and electromagnetic interactions, we are led to a specific model of strong interactions, so much the better. In fact, this is just what happens—it does not take very much thought along these lines before one arrives at a gauge theory of the strong as well as the weak and electromagnetic interactions.

The requirements that we would like to have satisfied by any field theory of strong interactions may be listed as follows:

(1) *Renormalizability.* The matrix elements of currents should have an asymptotic behavior sufficiently similar to that expected in the absence of strong interactions to assure the renormalizability of the weak and electromagnetic interactions.

(2) *Natural zeroth-order symmetries.* It should be natural (in the sense described in the last section) that after spontaneous breaking of the weak and electromagnetic gauge group, the theory should conserve parity, strangeness, isospin, etc. to zeroth order in e but to all orders in the strong interaction.

(3) *Natural order α symmetries.* It has already been emphasized that the emission and absorption of intermediate vector bosons will, in general, despite their large mass, produce corrections to natural zeroth-order symmetries which are of order α rather than of order $G_F \sim \alpha/\mu_W^2$. We do not mind (and indeed we welcome) such order α violations of isotopic spin conservation, but it would be a disaster if parity or strangeness conservation were violated in order α.

(4) *Asymptotic freedom of strong interactions.* The meaning and importance of this condition is discussed in the next section. For the moment, you may substitute "Bjorken scaling" for "asymptotic freedom," although the latter is a much more far-reaching property.

It turns out that these conditions can be satisfied for any renormalizable gauge model of weak and electromagnetic interactions, provided we turn on the strong interactions in a certain way. Rather than try to derive the necessary features of the strong interactions deductively from conditions A–D, I will instead simply present a description of a class of theories which satisfies these conditions, and leave it to the subsequent discussion to explain in what sense this class of theories is unique. The rules are as follows (Weinberg, 1974a):

(1) The whole theory of strong, weak, and electromagnetic interactions is described by a renormalizable Lagrangian with a gauge invariance group $G_S \otimes G_W$, given by the direct product of a strong gauge group G_S and a weak and electromagnetic gauge group G_W. The gauge coupling constants associated with G_S and G_W are presumed to be of order 1 and e, respectively.

(2) The group G_S is semisimple. This condition, which is required only for the purpose of assuring asymptotic freedom, rules out a simple Abelian neutral vector gluon model.[27]

(3) The group G_S is also nonchiral.

As an example of a model satisfying 1, 2, and 3, we may take the "colored quark" model (Bardeen *et al.*, to be published; also see Dalitz, 1965) in which the fundamental fermions form a matrix

$$\begin{vmatrix} \mathcal{P}_R & \mathcal{P}_W & \mathcal{P}_B \\ \mathcal{N}_R & \mathcal{N}_W & \mathcal{N}_B \\ \lambda_R & \lambda_W & \lambda_B \end{vmatrix}.$$

The G_W transformations act vertically, and may form any subgroup of chiral $U(3) \otimes U(3)$, while the G_S transformations act horizontally, and would have to form the nonchiral group $SU(3)$. Note that the "intermediate vector bosons" associated with G_W are neutral under G_S, and therefore have no strong interactions, while the "gluons" associated with G_S are neutral under G_W, and therefore have no weak or electromagnetic interactions. If G_S is $SU(3)$, there is an octet of such purely neutral vector gluons.

(4) There is a set of scalar fields, neutral under G_S, which have no strong interactions, but whose vacuum expectation values break G_W.

(5) There are no strongly interacting scalar fields.

Before entering into the properties of these theories, there is one obvious question that must be addressed: in the absence of strongly interacting scalars, how is G_S broken? One possible answer is provided by the dynamical mechanisms for spontaneous symmetry breaking discussed above in Sec. V. A very different answer is suggested by the failure of experimentalists to find free

[26] These include Duncan and Schattner (1973); Fayyazuddin and Riazuddin, 1973a,b; Love and Ross, to be published; Freedman and Kummer (1973).

[27] Weinberg (1974a and 1973d). In an earlier version of the same work, asymptotic freedom was not demanded, and the strong interations were described by an Abelian gluon theory; see Weinberg (1973c). A very similar analysis, covering both Abelian and non-Abelian gluon models, was independently developed by Mohapatra, Pati, and Vinciarelli, 1973. Also see Kummer and Lane (1973); Mohapatra and Vinciarelli (1973a, 1973b); Fayyazuddin and Riazuddin (1973); Hoh, Minamikawa, and Miura, to be published. (These methods have been applied to the calculation of "radiative" corrections to semileptonic decays by Mohapatra and Sakakibara, 1974.) There is a related but somewhat different model of Pati and Salam (1973), which satisfies all these conditions except asymptotic freedom.

quarks. If there were some systematic reason[28] why quarks can *never* be produced in collisions of G_S neutral particles such as ordinary hadrons, then we might expect the same mechanisms also to prevent the production of gluons, which (for G_S semisimple) are, like the quarks, nonneutral under G_S. In this case, it might be possible that the G_S gauge group is not broken at all, and that the unobservable neutral vector gluons have zero mass!

A mechanism for preventing the production of free quarks or gluons, based specifically on the masslessness of the gluons, is discussed at the end of the next section.

Of the four conditions listed at the beginning of this section, the first, renormalizability, is obviously satisfied. Detailed calculations (Weinberg, 1973c, and 1974a) show how the cancellation of divergences in second-order corrections to natural zeroth-order symmetry relations follows from the conservation and commutation properties of the weak and electromagnetic currents.

In order to explore the symmetries of the strong interactions, we note that to zeroth order in e the strong interactions are described by a Lagrangian $\mathcal{L}_S$ which may be obtained from the over-all Lagrangian of the theory by dropping all terms proportional to powers of e, except where they appear multiplied by scalar-field vacuum expectation values, which are of order $1/e$. We then have

$$\mathcal{L}_S = -\bar{\psi}\gamma^\mu D^S_\mu \psi - \tfrac{1}{4}F_{\alpha\mu\nu}F^{\mu\nu}_\alpha - \bar{\psi}m\psi, \qquad (8.1)$$

where D^S_μ is the G_S-covariant derivative, which of course involves the gluon field; $F_{\alpha\mu\nu}$ is the G_S-covariant nonlinear curl of the gluon field; and m is the zeroth-order fermion mass. The symmetries of this Lagrangian are determined by the structure of m, which may be fairly complicated, because as discussed in the last section, m receives contributions from the vacuum expectation values of all those scalar fields which can participate in Yukawa interactions with the fermions. In particular, m may include terms which involve the Dirac matrix γ_5, so it is not immediately obvious that parity is conserved. There is however a theorem (Weinberg, 1973c and 1974a) which states that *it is always possible to redefine ψ in such a manner that, without changing the $\bar{\psi}\gamma^\mu D_\mu\psi$ term, the matrix m is made real, diagonal, and free of γ_5 terms.* Once this is done, the symmetries of the strong interactions (aside from G_S itself) consist of:

(1) *Conservation of parity* with the understanding that all fermions are taken to have equal parity and all gluons are taken as polar vectors.

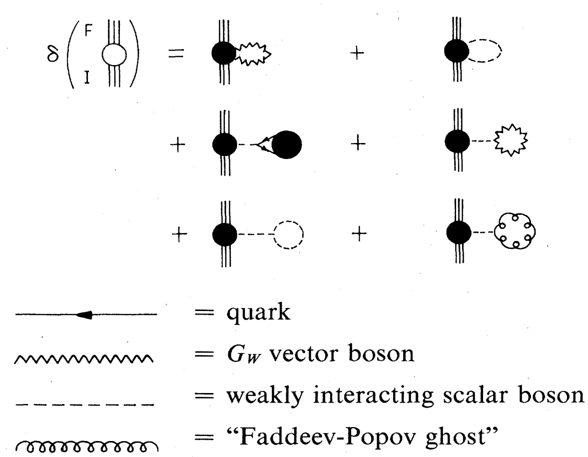
(2) *Conservation of strangeness, charge, and baryon number.* In general the number of quarks minus antiquarks in each row (i.e., summed over color) is conserved. This is equivalent to conservation of strangeness, charge, and baryon number, defined by

$$S \equiv -\sum_{\text{color}} N_\lambda,$$
$$Q \equiv \sum_{\text{color}} \left[\tfrac{2}{3}N_{\mathcal{P}} - \tfrac{1}{3}N_{\mathcal{N}} - \tfrac{1}{3}N_\lambda\right], \qquad (8.2)$$
$$B \equiv \sum_{\text{color}} \left[\tfrac{1}{3}N_{\mathcal{P}} + \tfrac{1}{3}N_{\mathcal{N}} + \tfrac{1}{3}N_\lambda\right].$$

If there is a fourth quark row then a fourth quantum number, the charm, must also be conserved.

(3) *Conservation (perhaps) of isospin* If there is some zeroth-order mass relation of the sort discussed in the last section which requires that $m_{\mathcal{P}} = m_{\mathcal{N}}$, the whole of the strong interaction Lagrangian $\mathcal{L}_S$ will conserve isotopic spin, with the understanding that the degenerate doublet has isotopic spin-1/2 and all other quarks have isotopic spin-0. (There are other possible symmetries which can arise in this way. These will be discussed below.)

Now, what about the breaking of these symmetries in second order? In an arbitrary transition from an initial hadron state I to a final hadron state F, the change in the zeroth-order T-matrix is given to second order in e by the diagrammatic formula



and a darkened blob now represents an infinite sum of strong-interaction graphs. By using the commutation and conservation properties of the vector and scalar currents, it can be shown[27] that the sum of diagrams gives a gauge-invariant result, and that the corrections to natural first-order symmetry relations are finite, just as in the absence of strong interactions.

In general, the presence of strong interactions would prevent us from being able to calculate the second-order corrections to T_{FI}. However, considerable progress can be made if we are willing to concentrate on those terms in δT_{FI} which are *not* suppressed by the large mass of the intermediate vector bosons. Such terms are called of "order α," to distinguish them from the suppressed terms, which are of order $G_F \sim \alpha/\mu_W^2$. A straightforward analysis[27] shows that the order α terms consist of certain "tadpole" terms, taking the form of insertions in fermion lines, plus the integral

$$\delta T_{FI} = \int d^4k\, \mathcal{T}^{FI}_{\alpha\beta}(k)(k^2 + \mu_W^2)^{-1}_{\alpha\beta}, \qquad (8.3)$$

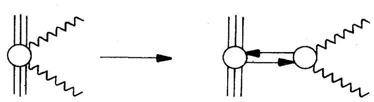
where $\mathcal{T}^{FI}_{\alpha\beta}(k)$ is the Fourier transformed matrix element between states F and I of the time-ordered product of the

[28] It was suggested by Bardeen, Fritsch, and Gell-Mann, to be published, that ordinary hadrons are all color singlets, and that collisions of such particles can never produce quarks. The suggestion that gluons form a color octet, and therefore also cannot be produced in collisions of ordinary particles, was made by Fritsch and Gell-Mann (1973), following an unpublished communication by J. Wess. Specific mechanisms for suppression of quark production, not directly related to the masslessness of the gluons, have been suggested by Casher *et al.* (1973) and by K. Johnson, to be published.

weak currents $J_{\alpha\mu}$ and $J_{\beta\nu}$, contracting vector indices μ and ν. This looks like it should be suppressed by the large term μ_W^2 in the denominator but there are two exceptions. First, not all elements of μ_W^2 are large, because μ_W^2 has an eigenvalue zero corresponding to the photon. It is very convenient to write the photon propagator in the form

$$\frac{1}{k^2} = \left[\frac{1}{k^2} - \frac{1}{k^2 + \Lambda^2}\right] + \frac{1}{k^2 + \Lambda^2}, \tag{8.4}$$

where Λ is a large but otherwise arbitrary mass. The bracketed quantity must be treated separately; it just gives the usual photon exchange contribution, with a natural cutoff Λ. The remaining term can be put back in the vector boson propagator, with the effect that the matrix μ_W^2 is replacpd with $\mu_W'^2$, which is just the same as μ_W^2 except that the photon mass is given the artificial value Λ. All the eigenvalues of $\mu_W'^2$ are large, so the only way that an integral involving $(k^2 + \mu_W'^2)^{-1}$ can avoid being suppressed by the large denominator is for the coefficient function to vanish as $k \to \infty$ no faster than $1/k^2$. The question is, what terms in $\mathfrak{T}_{\alpha\beta}^{FI}(k)$ vanish as $k \to \infty$ no faster than $1/k^2$? Leaving aside terms which produce no corrections to zeroth-order symmetries, the only such terms in $\mathfrak{T}$ arise from diagrams with a fermion–antifermion bridge connecting the currents to the hadron states I and F.



The effect of such terms is, like the tadpoles, simply to produce an insertion into a fermion line. We conclude then that the complete "order α" corrections to the T-matrix consists of the usual electromagnetic terms, with a cutoff Λ, plus a correction to the fermion mass matrix.

$$\delta m = \int_0^\infty U_{\alpha\beta}(\kappa)(\kappa^2 + \mu_W'^2)^{-1}_{\alpha\beta}\kappa^3\, d\kappa + \text{tadpoles}, \tag{8.5}$$

where $U_{\alpha\beta}(\sqrt{k^2})$ is the coefficient function for the operator $\bar{\psi}\psi$ in the Wilson operator product expansion (K. Wilson, 1969)[29] of the two currents $J_{\alpha\mu}$ and $J_{\beta\nu}$, contracted over μ and ν and averaged over directions in momentum space. It should be noted that, at least in perturbation theory, $U_{\alpha\beta}(\kappa)$ has the asymptotic behavior for large κ

$$U_{\alpha\beta}(\kappa) \sim \kappa^{-2} \times \text{powers of } \ln \kappa, \tag{8.6}$$

so the integral in δm does not converge term-by-term, but only through cancellation of different terms. (The divergent part involves the trace $U_{\alpha\alpha}(\kappa)$ which can be shown[27] to make no contribution to corrections to "natural" zeroth-order symmetries.) It is for this reason that the answer is proportional to powers of $\ln \mu_W'^2$, not $\mu_W'^{-2}$, and is therefore "of order α." Although finite, the answer does depend on the fictitious photon mass Λ, and this Λ dependence cancels the cutoff dependence of the photon exchange terms.

The problem of calculating δm will be taken up again at the end of the next section. However, even if we make no attempt actually to calculate δm, this result, that "order α" corrections to natural strong-interaction symmetries consist solely of ordinary photon exchange terms plus shifts in the quark mass matrix, has extremely important corollaries. First, note that just as we earlier defined ψ so that m is real, diagonal, and γ_5 free, we can now redefine ψ so that $m + \delta m$ is real, diagonal, and γ_5 free. As already indicated, this means that corrections to the quark mass matrix can introduce no corrections to parity and strangeness conservation, though they can of course violate isospin conservation. The photon-exchange terms also conserve parity and strangeness, so we may conclude that *parity and strangeness are automatically conserved in order* α. However, they are violated by the terms of order $\alpha/\mu_W'^2$, which in general can *not* be expressed in terms of quark mass shifts.

At this point we may look back and ask what we would have found if we had chosen a theory of strong interactions with strongly interacting scalars and/or chiral gauge fields. In general, such a theory could violate parity and strangeness conservation in zeroth order, through various Yukawa interactions, scalar self-interactions, or V-A gauge field mixing. (It is not possible to make a completely general statement here. For instance, if the theory involves strongly interacting scalar fields which belong to representations of G_S which forbid direct Yukawa coupling to the fermions, then parity is still naturally conserved, although asymptotic freedom may be lost.) When we say that "such a theory could violate parity and strangeness conservation" we mean that these conservation laws are not natural, and can only be achieved by careful adjustment of parameters in the original Lagrangian. Such theories also can have "order α" violations of parity and strangeness, appearing through other operator terms in the Wilson expansion (1969), such as $\bar{\psi}\psi\phi$, ϕ^2, ϕ^3, ϕ^4, and (in chiral theories) $F_{\mu\nu}^V F^{A\mu\nu}$. It is true, as recently emphasized by Bars (to be published), that such "order α" corrections to parity and strangeness conservation always take the form of corrections to the zero-order parameters of the theory, and, therefore, whenever parity and strangeness are not natural zeroth-order symmetries, these violations of parity and strangeness conservation can be made as small as we like by adjustment of the parameters in the Lagrangian. This is in particular the case for theories of the Bars-Halpern-Yoshimura type (see Sec. II), where parity conservation is not natural, but has to be achieved by choosing the parameters in the Lagrangian as suitable power series in α. There is some disagreement about whether this rules out the Bars-Halpern-Yoshimura theory as a fundamental field theory of strong interactions. To me, it seems completely unacceptable to suppose that nature chooses the parameters in the Lagrangian just so that violations of parity and strangness are not only finite but tiny. It is as if the neutron-proton mass difference were 1 eV, and we tried to explain this mass difference in conventional electrodynamics by supposing that the isotopic spin violation in the bare nucleon masses cancelled not only the infinite part of the electromagnetic self energy but also almost all of the finite part!

Returning now to the theories described at the beginning of this section, for which parity and strangeness conservation are natural, we note that isotopic spin will in general be violated in order α, through the appearance of shifts in the various quark masses. However, this

[29] Many of the remarks below were anticipated in Wilson's analysis of the divergences in electromagnetic self-energies.

isotopic spin violation is controlled by the isotopic spin content of the quark multiplet. As already indicated, the quarks *must* consist solely of isotopic spin doublets and singlets, because if $n > 2$ quarks in each column are degenerate in zeroth order then the strong interaction symmetry would be $SU(n)$, not $SU(2)$. It follows that the nonelectromagnetic violation of isotopic spin conservation produced by the quark mass shifts must take the form of a pure $\Delta I = 1$ perturbation.

This conclusion fits in very well with what is known of the phenomenology of isotopic spin breaking.[30] Those mass shifts of the pure $\Delta I = 2$ type, such as $m(\pi^+) - m(\pi^0)$ or $2m(\Sigma^0) - m(\Sigma^+) - m(\Sigma^-)$, should receive no contribution from quark mass shifts, and therefore should be calculable in terms of photon exchange alone, as indeed they are. On the other hand, a $\Delta I = 1$ quantity like $m(p) - m(n)$ receives contributions from both photon exchange and quark mass shifts, so it cannot be calculated in purely electromagnetic terms. *It is the weak interactions that cancel the divergence in the electromagnetic self-energy difference of the nucleons, and that may also be responsible for the fact that the neutron is heavier than the proton.* Finally, there is the $\Delta I = 1$ process $\eta \to 3\pi$ which is forbidden (in the soft pion limit) to go by ordinary photon exchange,[30] so since this process is not particularly slow, we must conclude that it is produced almost entirely by the weak interactions.[31] This is not to say that $\eta \to 3\pi$ should show P or C violating anomalies, however, for as emphasized above, the effects of the weak interactions are limited in order α to quark mass shifts, and these cannot produce a P or C violation.

The general picture of order α weak corrections arising through quark mass shifts can also be used to justify various detailed dynamical calculations. One obvious example is the parton model calculations, of the sort carried out recently by Gunion (1973). Another example is provided by the current algebra calculations carried out over the last five years, many of which depend on detailed assumptions as to the nature of symmetry-breaking terms in the effective Lagrangian.[32] In order to discuss the latter class of calculation, it is necessary first to say a few words about the status of the strongly broken unitary or chiral symmetries in our general theoretical framework.

First, it is possible that the zero-order quark masses obey exact natural symmetry relations other than $m_{\mathcal{P}} = m_{\mathcal{N}}$. For instance, we may have $m_{\mathcal{P}} = m_{\mathcal{N}} = 0$, in which case the natural zeroth-order symmetries of the strong interactions include a chiral $SU(2) \otimes SU(2)$ group and a chiral $U(1)$ group. These symmetries will in general be broken in order α by the weak and electromagnetic interactions, and in addition may be broken spontaneously by the strong interactions, producing "pseudo-Goldstone" bosons[33] with masses of order $\sqrt{\alpha}$ times ordinary hadron masses.[34]

Alternatively, it is possible that the zeroth-order quark masses exhibit *approximate* symmetries. For instance, they might all be in some sense "small," in which case there would be an approximate $SU(3) \otimes SU(3) \otimes U(1)$ global symmetry of strong interactions. Any such approximate symmetry can also be spontaneously broken by the strong interactions, yielding pseudo-Goldstone bosons of relatively small mass. For instance, approximate $SU(3) \otimes SU(3) \otimes U(1)$ might be spontaneously broken down to an approximate $SU(3)$ symmetry, with the appearance of a pseudoscalar nonet of relatively light bosons.

It is not clear which picture describes the real world, although the rather large mass of the K and η mesons suggests that the second alternative may be closer to the truth, at least for the λ quark mass. The chief practical difference between the two approaches is that in the first case the π, K, etc. masses may themselves be calculated in terms of spectral function integrals,[35] while in the second case it is only the splitting within the pseudoscalar isospin multiplets that can be so calculated.[36] In either case, however, the breaking of $SU(3) \otimes SU(3) \otimes U(1)$ symmetry in the strong interactions is entirely due to photons plus quark mass terms (whether zeroth order or of order α) in agreement with the assumptions generally made in current algebra analyses.[32]

IX. ASYMPTOTIC FREEDOM

The renormalization group method, either in the original version of Gell-Mann and Low (1954), or in the modified version of Callan (1970) and Symanzik (1970), provides a technique for estimating the asymptotic behavior of Green's functions or Wilson coefficient functions in the limit of large Euclidean momenta. In general, any such function $\Gamma(\kappa, g_1)$ will be given asymptotically by

$$\Gamma(\kappa, g_1) \to \left(\frac{\kappa}{\kappa_0}\right)^{D_\Gamma} \exp\left[-\int_{\kappa_0}^{\kappa} \gamma_\Gamma(g(\kappa))\frac{d\kappa}{\kappa}\right] \Gamma[\kappa_0, g(\kappa)], \tag{9.1}$$

where κ is a variable factor which specifies the scale of all

[30] For a review, see Zee (1972).

[31] There is a problem in this explanation of η decay, pointed out by Georgi, Glashow and Jackiw, private communication, and by Mohapatra and Pati, 1973. Briefly, in the approximation in which the pions are massless, the $\Delta I = 1$ quark mass term is a total divergence and therefore cannot contribute to $\eta \to 3\pi$ in the soft pion limit. This problem is actually very complicated and has been recently studied in unpublished work by Georgi and Glashow. See also Cicogna *et al.* (1973).

[32] For example, Glashow and Weinberg (1968); Gell-Mann, Oakes, and Renner (1968); Glashow *et al.* (1969), etc.

[33] "Pseudo-Goldstone bosons" are the bosons of small mass which arise when some global symmetry of *part* of the Lagrangian is spontaneously broken. Originally the term was coined to describe the low mass bosons which arise when the polynomial $P(\phi)$ in the Lagrangian has a natural symmetry group larger than that of the whole Lagrangian; see Weinberg (1972e). It is used here in a somewhat different sense.

[34] In theories without strong interactions, where the pseudo-Goldstone bosons arise from the spontaneous breakdown of a "type 2" symmetry (See Sec. VIII), the masses of these bosons turn out to be of the order of $\sqrt{\alpha}$ times the gauge boson masses, which of course is much too big. See Weinberg (1973b) and S. Y. Lee, Rawls, and Yu, to be published. The first example of a theory in which a ratio of scalar and vector masses is calculated to be proportional to a gauge coupling constant is that of Coleman and E. Weinberg (1973).

[35] The pion mass has been calculated in this way for theories of the "Berkeley" type by Bars and Lane (1973b). Also see Bars and Lane (1973a); Bars, Halpern, and Lane, to be published.

[36] Dicus and Mathur (1973); Weinberg (1974b). The method used here is essentially that of Das *et al.* (1967), with the difference that cancellations between weak and electromagnetic interactions yield finite results whether or not the second spectral function sum rule is satisfied. In cases like the pion electromagnetic mass difference, where there are no order α weak contributions, the second spectral function sum rule must be, and in fact is, satisfied. See also K. Wilson (1969).

momenta; κ_0 is any fixed value of κ; D_Γ is the naive dimensionality of Γ, which would determine the asymptotic behavior of Γ if there were no renormalizations to be performed; $\gamma_\Gamma(g)$ is an "anomalous dimension" function, arising from the wave function renormalizations associated with the external lines of Γ; and $g(\kappa)$ is an effective coupling constant, given by a differential equation of the form

$$\kappa(d/d\kappa)g(\kappa) = \beta(g(\kappa)), \tag{9.2}$$

with the initial condition

$$g(\kappa_0) = g_1, \tag{9.3}$$

where g_1 is the physical renormalized coupling constant.

In most field theories $\beta(g)$ has the same sign as g for g near zero, so even if the renormalized coupling constant g_1 is small, $g(\kappa)$ increases in absolute value with increasing κ, either approaching some finite point g_0 where $\beta = 0$ or else approaching infinity as $\kappa \to \infty$. In particular, the perturbative expression of $\beta(g)$ shows that this is the case for quantum electrodynamics. When $g(\kappa)$ behaves in this way, there is not much that can be said about the asymptotic behavior of Γ as $\kappa \to \infty$, except to note that if $g(\kappa)$ approaches a constant g_0 as $\kappa \to \infty$, then

$$\Gamma \propto \kappa^{D_\Gamma - \gamma_\Gamma(g_0)} \qquad \text{for} \qquad \kappa \to \infty \tag{9.4}$$

with an unknown proportionality constant. In order for Bjorken scaling to occur in deep inelastic electron scattering it would be necessary for an infinite number of γ's to vanish at g_0, and it can be shown that this would imply that the theory with $g = g_0$ is a free field theory. (Parisi, 1973; Callan and Gross, to be published).

Now suppose on the other hand that $\beta(g)$ and g were of opposite sign for small g. As long as g_1 is not too large, $g(\kappa)$ would then decrease in absolute value as κ increases, approaching zero for $\kappa \to \infty$. Such a theory would automatically exhibit Bjorken scaling up to logarithms of κ.

The exciting new thing that has happened recently is that Gross and Wilczek (1973a, b, 1974) and Politzer (1973) have shown that $\beta(g)$ is of opposite sign to g for small g in non-Abelian Yang–Mills theories based on semisimple gauge groups (I am informed that similar results were earlier obtained by G. 't Hooft but not published.) In such theories, $\beta(g)$ has the perturbative expansion

$$\beta(g) = bg^3 + cg^5 + \cdots \tag{9.5}$$

and explicit calculation shows that $b < 0$. The negative sign of b persists if we add a not too large number of fermion and/or scalar multiplets. However, the addition of scalar fields complicates the analysis through the appearance of new coupling constants, and it does not appear that is is possible to add enough scalar fields to break the gauge symmetry without losing asymptotic freedom. Coleman and Gross (1973) have shown on the other hand that, apart from examples known to have negative energy problems, the only asymptotically free field theories are those with non-Abelian gauge fields.

Asymptotic freedom does not mean that the strong interactions can simply be ignored at high momenta. Equations (9.2) and (9.5) have the asymptotic solution (for $b < 0$)

$$g(\kappa) \propto (\ln \kappa)^{-1/2} \qquad \text{for} \qquad \kappa \to \infty. \tag{9.6}$$

Since $\gamma(g) \propto g^2$ for small g, Eq. (9.1) gives

$$\Gamma(\kappa, g_1) \to \left(\frac{\kappa}{\kappa_0}\right)^{D_\Gamma} \times [\ln(\kappa/\kappa_1)]^{A_\Gamma} \exp\left\{-\int_{\kappa_0}^{\kappa_1} \gamma_\Gamma[g(\kappa)]\frac{d\kappa}{\kappa}\right\}\Gamma(\kappa_0, 0), \tag{9.7}$$

where κ_1 is large but otherwise arbitrary, and A_Γ is a calculable number. The integral from κ_0 to κ_1 runs over values of κ where $g(\kappa)$ may not be small, so even though $\Gamma(\kappa_0, 0)$ can be calculated in the Born approximation, the asymptotic form of $\Gamma(\kappa, g_1)$ still contains an unknown numerical factor. In particular, in electroproduction it is not possible to use these results to calculate the functions νW_2 and W_1 completely, even when the theory is asymptotically free (Georgi & Politzer, 1974; Gross & Wilczek, 1973b, 1974).

Fortunately there are a few cases where the anomalous dimension γ_Γ vanishes, so that asymptotic freedom really does lead to a completely calculable asymptotic form

$$\Gamma(\kappa, g_1) \to (\kappa/\kappa_0)^{D_\Gamma}\Gamma(\kappa_0, 0) \qquad \text{for} \qquad \kappa \to \infty. \tag{9.8}$$

One such example is provided by the reaction $e^+ + e^- \to$ hadrons, where γ vanishes because the electromagnetic current has canonical dimensions. The result here is that the annihilation cross section is simply equal asymptotically to the Born-approximation cross section for $e^+ + e^- \to$ quark plus antiquark (Applequist and Georgi, 1973; Zee, 1973).

Another example with $\gamma = 0$, which is closer to the scope of this review, is provided by the order α quark mass shifts discussed in the last section. The Wilson coefficient function $U_{\alpha\beta}(\kappa)$ has zero anomalous dimension because the γ term associated with the operator term $\bar{\psi}\psi$ of the operator product expansion is canceled by a similar term which arises because $U_{\alpha\beta}(\kappa)$ is asymptotically proportional to the quark masses (Weinberg, 1974c; 1974a). Thus, as long as $g(\kappa)$ approaches a constant g_0 as $\kappa \to \infty$, whether or not $g_0 = 0$, the Wilson function is asymptotically proportional to $1/\kappa^2$. (This incidentally implies the convergence of all integrals appearing in the second spectral function sum rules, though the sum rules themselves may not be true.) In this case, the κ integral in δm can be carried out explicitly, and we have:

$$\delta m = \{\mathcal{C}\gamma_4[t_\alpha t_\beta, \gamma_4 m] + \mathfrak{D}\gamma_4[t_\alpha, [t_\beta, \gamma_4 m]]\}(\ln \mu_W'^2)_{\alpha\beta} + \text{known tadpoles}, \tag{9.9}$$

where t_α is the matrix to which the G_s gauge field couples, and $\mathcal{C}$ and $\mathfrak{D}$ are dimensionless constants. All the complications due to the strong interactions appear here only in $\mathcal{C}$ and $\mathfrak{D}$. If the theory is asymptotically free, and if the $1/\kappa^2$ asymptotic behavior sets in below κ of order μ_W, then $\mathcal{C}$ aed $\mathfrak{D}$ must have the values which they would have in the absence of strong interactions

$$\mathcal{C} = -3/32\pi^2 \qquad \mathfrak{D} = 1/8\pi^2. \tag{9.10}$$

Thus δm is entirely calculable, the rule being simply to write down all graphs of second order in e and ignore the strong interactions (Weinberg, 1973d and 1974a). This result was suggested earlier on the basis of the observation of Bjorken scaling (Jackiw *et al.*, 1970; Pagels, 1969; Jackiw and Schnitzer, 1972; Gunion, 1973), so it is perhaps not surprising that it follows from the same field theoretic assumptions that lead to Bjorken scaling. Of course, even though δm may be calculated, the result does not directly give the mass of any physical particle, but must be used as an input to parton-model or current algebra calculations.

I find it difficult to express strongly enough my enthusiasm for the discovery of asymptotically free theories of strong interactions. Ever since the disappointing failure of field theory to account for the new facts of meson physics in the early 1950's, progress in elementary particle physics has been impeded by the inadequacy of perturbation theory to deal with strong interactions. Now in asymptotically free theories we see the fog of the strong interactions lifting here and there, revealing the underlying spectrum of the elementary hadrons.

There is yet another possible implication of asymptotic freedom, which however lies on a much less sound mathematical foundation than the foregoing applications. In the last section I mentioned the possibility that the strong gauge group G_s is not broken, so that the gluons are massless, and that the masslessness of the gluons might be responsible for some dynamical mechanism which prevents the production of free "colored" particles, either quarks or gluons, in collisions of ordinary hadrons. You will recall that the infrared phenomenon in ordinary electrodynamics introduces a factor $\exp(-\infty)$ in the matrix element for production of any definite number of charged particles in photon–photon collisions. Of course, this does not prevent reactions like $\gamma + \gamma \rightarrow e^+ + e^-$, but only requires that the charged particles are accompanied with an indefinite number of very soft photons. The infrared divergence problem is much more complicated in non-Abelian gauge theories, and no one knows how to sum up the divergent graphs, but there are arguments (Kinoshita, 1962; T. D. Lee and Nauenberg, 1964) to the effect that in perturbation theory the total reaction rates, summed over suitable numbers of outgoing soft particles, are always finite. If these arguments apply to non-Abelian gauge theories, then in order to obtain the desired suppression of colored particle production we must look for specifically non-perturbative effects of the massless gluons. Such effects are in fact to be expected in any asymptotically free theory. When we separate two G_s non-neutral particles such as quarks or gluons by a distance r, their mutual interaction presumably is governed by the effective coupling $g(\kappa)$ with κ of order $1/r$, so that with $\beta(g)$ of opposite sign to g, the interaction increases as r increases. It may even increase without limit if $\beta(g)$ has no zeros between g_1 and infinity. Hopefully this makes it impossible to separate quarks and/or gluons by large distances,[37] although it certainly would not prevent the separation of G_s-neutral ordinary hadrons. In order to verify the suppression of quark and gluon production in asymptotically free field theories, it would be necessary to show that the amplitudes for reactions among G_s-neutral particles are free of cuts corresponding to intermediate states containing quarks or gluons. This has not yet been done, so the above remarks are at present mere speculations. However, the renormalization-group methods which allow us to sum up powers of ln κ to determine *asymptotic* behavior in κ can also be used to sum up powers of ln κ to explore *analytic* structure in κ, so there is a good prospect that these questions may be answered before too long.

To carry these speculations one step further, if quarks must appear only in composite systems for any value of the strong coupling constant, then it is possible that the strong interactions are not really very strong for most purposes, becoming strong only when we try to pull the quarks apart. Something of this sort may be required to explain why Bjorken scaling appears to set in at such surprisingly low energies. Is the strong interaction gauge coupling constant, like the weak and electromagnetic couplings, really of order e?

[37] This suggestion was made independently by Weinberg (1973d and 1974a) and by Gross and Wilczek, 1973b. The suggestion that quark and gluon production is suppressed by infrared divergences associated with the massless gluons was also made independently by Fritsch, Gell-Mann, and Leutwyler, 1973, but was not connected with asymptotic freedom.

X. CONCLUSION

The general idea of a renormalizable gauge theory of weak and electromagnetic interactions seemed (at least to some) to be so intrinsically attractive that extensive theoretical investigations were carried out without the slightest support from experimental data. At the same time, no one specific model seemed very compelling, and it was not clear what sort of experiment would provide a really crucial test of these general ideas. The recent experimental discovery of neutral currents has not substantially changed this situation, although of course it provides us with no end of encouragement, and also suggests that the simple $SU(2) \otimes U(1)$ model may at least be a part of the final answer. We still do not know of any overall model which is realistic, in the sense that it agrees with all existing data, and is also natural, in the sense that the parameters in the theory do not have to be carefully rigged to achieve even a qualitative resemblance to nature. However, we do now at least have a good general idea of how the natural symmetries of the strong interactions could arise in certain gauge theories of weak, electromagnetic and strong interactions. Indeed, the apparent empirical need for nonelectromagnetic violations of isotopic spin still seems to me to be the best argument for a unified theory in which weak as well as electromagnetic couplings are of order e. The discovery that some of these combined gauge theories are asymptotically free fits in very well with this general picture, both because it lends support to the non-Abelian gauge theories of strong interactions, and because asymptotic freedom would actually allow us to calculate the nonphoton corrections to symmetries like isospin. All in all, we are increasingly confident that if we had the correct weak and electromagnetic gauge theory, we would know how to use it. To me, the continued failure of theorists, despite all this progress, to come up with a satisfactory detailed model indicates that we are probably missing something fundamental, perhaps new fermions, or new kinds of weak interactions, or both. In particular, the *a priori* prejudice for simple rather than merely semisimple Lie groups suggests that $SU(2) \otimes U(1)$ may be part of a larger gauge group which generates weak interactions too weak to have been seen yet. I would bet that the experimentalists will solve these problems for us by discovering superweak interactions which violate more of the currently accepted truths about weak interactions. However, in the meantime, there is evidently a good deal for even theorists to do.

Note Added in Proof: Recent progress in gauge theories of the weak, electromagnetic, and strong interactions

In the period since the preparation of this report there have been a number of developments which bear on the points previously discussed. The following is a brief account of these latest developments.

1. Neutral currents in neutrino experiments

The neutral currents have continued as the object of active experimental effort. A second candidate for the process $\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$ has been found at CERN (Musset, 1974). The NAL neutral current results have now been published, with a note added in proof reporting a reduction in the estimated ratio R of neutral to charged current events from 0.29 ± 0.09 to 0.23 ± 0.09 for the NAL mixed beam, as a result of a reduction in the calculated muon detection efficiency (Benvenuti *et al.*, 1974). Further observations by the same consortium yield a ratio $R = 0.20 \pm 0.05$ (Aubert *et al.*, 1974). The theoretical background for these and other neutrino experiments has been dramatically reviewed by De Rujula *et al.* (1974).

The exclusive process $\nu + N \to \nu + N + \pi^0$, which had seemed to provide the strongest evidence against neutral currents, has been under further theoretical study. First, Adler (1974a) reconsidered the π^0 production process on free nucleons, with results essentially consistent with the earlier work of Lee (1972d). According to these results, the old model of leptons (Weinberg, 1967b) is at best barely consistent with the data of W. Lee (1972). However, it had been suggested by Perkins (1972) that the observed rate of π^0 production might be seriously affected by charge exchange in the nucleii, such as ^{27}Al, used as targets. This suggestion has now been confirmed by the detailed calculations of Adler *et al.* (1974) and Adler (1974b). They find that charge exchange typically halves the π^0 production rate, thus removing any clear discrepancy between theory and experiment.

An ingenious test for neutral currents by neutrino excitation of nuclear energy levels has been proposed by Donnelly *et al.* (1974). They note that the reaction $\bar{\nu}_e + \mathrm{Li}^7 \to \bar{\nu}_e + L_i{}^{7*}$ could lead to a few observed de-excitation γ rays per day per kg of target at a reactor of the Savannah River type.

2. Neutral currents in supernovae

It has been observed by Freedman (1974) that if neutral currents exist then coherent scattering can lead to anomalously large neutrino opacity in heavy nuclei, such as ^{56}Fe. This effect allows the neutrinos produced in the collapse of the core of a massive star to deposit their momentum more efficiently in the outer layers of the core and the inner layers of the envelope. Wilson (1974) has repeated his computer calculations of stellar collapse, now taking coherent neutrino scattering into account, and finds that the enhanced neutrino pressure can at least in some cases blow off the envelope, thereby producing a supernova. It has become a matter of some urgency for experimental particle physicists to supply astrophysicists with precise information as to the existence and the strength of neutral current weak interactions.

3. Gauge symmetries at high temperature

It had been suggested by Kirzhnits and Linde (1972) that at a sufficiently high temperature there is a phase transition in which the gauge symmetry underlying the unified theory of weak and electromagnetic interactions becomes unbroken, so that the W and Z intermediate bosons become massless, like the photon. This has now been confirmed by detailed calculations (Weinberg, 1974; Dolan and Jackiw, 1974). The leading effect for weak coupling and high temperature is a change in the effective bare scalar mass; the phase transition occurs when this mass vanishes. This may provide some sort of answer to the question discussed in Sec. III; whether a spontaneously broken gauge symmetry should be regarded as a true symmetry.

4. Unified gauge theories of strong, weak, and electromagnetic interactions

Georgi and Glashow (1974) have offered a unified model of the strong, weak, and electromagnetic interactions based on the simple group $SU(5)$. In this model, $SU(5)$ suffers a superstrong spontaneous symmetry breakdown (Weinberg, 1972c) to the "observed" gauge group, taken as $SU(2) \times U(1)$ and color $SU(3)$. This proposal provides a concrete realization of the rather vague suggestion discussed in Sec. IX, that the strong interactions are "really" of the same strength as the weak and electromagnetic interactions, becoming stronger only at low energies or large distance. However, a problem with this specific model is that the baryon number is not strictly conserved.

The logarithmic renormalization effects which make the strong interactions strong at ordinary energies have been calculated for theories of this general type by Georgi *et al.* (1974). It turns out that the masses of the superheavy vector bosons in the $SU(5)$ model are sufficiently high to suppress the baryon decay rate below observable levels.

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REFERENCES

Abers, E., R. A. Keller, and V. L. Teplitz, 1970, Phys. Rev. D **2**, 1757
Abers, E., and V. L. Teplitz, 1967, Phys. Rev. **158**, 1365.
Abers, E. S., and B. W. Lee, 1973, Phys. Rep. **9**, 1.
Achiman, Y., 1973, Nucl. Phys. B **56**, 635.
Albright, C. H., B. W. Lee, E. A. Paschos, and L. Wolfenstein, 1973,

Phys. Rev. D **7**, 2220.
Albright, C. H., 1973, Phys. Rev. D **8**, 3162.
Anderson, P. W., 1958, Phys. Rev. **112**, 1900 (1958).
Applequist, T., and H. Georgi, 1973, Phys. Rev. D **8**, 4000.
Arnett, W. D., 1966, Can. J. Phys. **44**, 2553.
Arnett, W. D., 1967, Can. J. Phys. **45**, 1621.
Baker, M., and K. Johnson, 1971, Phys. Rev. D **3**, 2516.
Baltay, C., 1972, Recent Experimental Results on Neutrino Interactions, invited paper presented at the December 1972 meeting of the American Physical Society.
Bardakci, K., and M. B. Halpern, 1973, Phys. Rev. D **6**, 696.
Bardeen, W., H. Fritsch, and M. Gell-Mann.
Bars, I., M. B. Halpern, and M. Yoshimura, 1972, Phys. Rev. Lett. **29**, 969.
Bars, I., M. B. Halpern, and M. Yoshimura, 1973, Phys. Rev. D **7**, 1233.
Bars, I., and K. Lane, 1973a, Phys. Rev. D **8**, 1169.
Bars, I., and K. Lane, 1973b, Phys. Rev. D **8**, 1252.
Bars, I., to be published.
Bars, I , M. B. Halpern, and K. Lane, to be published.
Beg, M. A. B., and A. Zee, 1973a, Phys. Rev. D **8**, 2334.
Beg, M. A. B., and A. Zee, 1973b, Phys. Rev. Lett. **30**, 675.
Beg, M. A. B., 1973, Phys. Rev. D **8**, 664.
Bell, J., High Energy Behaviour of Tree Diagrams in Gauge Theories, to be published.
Benvenuti, A., *et al.* (Harvard-Pennsylvania-Wisconsin-NAL collaboration), 1974, Phys. Rev. Lett. **32**, 800.
Bernstein, Jeremy, 1974, Rev. Mod. Phys. **46**, .
Biswas, S. N., R. N. Chaudhuri, and J. K. S. Taank, 1973, J. Phys. A **6**, 1751.
Bludman, S. A., 1958, Nuovo Cimento **9**, 433.
Bouchiat, C., J. Iliopoulos, and P. Neyer, 1972, Phys Lett. B **38**, 519.
Boulware, D., 1970, Ann. Phys. (N.Y.) **56**, 140.
Budny, R., 1972, Phys. Lett. B **39**, 553.
Budny, R., 1973, Phys. Lett. B **45**, 340.
Callan, C. G., 1970, Phys. Rev. D **2**, 1541
Callan, C. G., and D Gross, .
Carlson, C. E., and P. G. O. Freund, 1972, Phys. Lett. B **39**, 349.
Casher, A., J. Kogut, and L. Susskind, 1973, Phys. Rev. Lett. **31**, 792.
Cheng, H., and T. T. Wu, 1966, Phys. Rev. **140**, Part B, 465.
Cheng, T. P., "An $O(4)$ Gauge Theory with Three-Triplet Quarks," to be published.
Cicogna, G., F. Strocchi, and R. Vergara Caffarelli, 1973, Phys. Lett. B **46**, 217.
Coleman, S., and D. J. Gross, 1973, Phys. Rev. Lett. **31**, 851.
Coleman, S., and E. Weinberg, 1973, Phys. Rev. D **7**, 1888.
Colgate, S. A., and R. H. White, 1966, Astrophys. J. **143**, 626.
Cornwall, J. M., D. N. Levin, and G. Tiktopoulos, 1973, Phys. Rev. Lett. **30**, 1268; **31**, 572(E).
Cornwall, J. M., and R. E. Norton, 1973, Phys. Rev. D **8**, 3338.
Cung, V. K., A. K. Mann, and E. A. Paschos, 1972, Phys. Lett. B **41**, 355.
Dalitz, R. H. D., 1965, In Grenoble Université, École d'été Physique Théorique, Les Houches, p. 287.
Das, T., G. S. Guralnik, V. S. Mathur, F. E. Low, and J. E. Young, 1967, Phys. Rev. Lett. **18**, 759.
De Rújula, A., and S. L. Glashow, 1973, Phys. Lett. B **46**, 377, 381.
De Wit, B., "Gauge Field Theory and the Cabibbo Angle," to be published.
DeWitt, B. S., 1964, Phys. Rev. Lett. **12**, 742.
DeWitt, B. S., 1967, Phys. Rev. **162**, 1195.
Dicus, D. A., 1972, Phys. Rev. D **6**, 941.
Dicus, D. A., 1973, Phys. Rev. D **8**, 890.
Dicus, D. A., D. Z. Freedman, and V. L. Teplitz, 1971, Phys. Rev. D **4**, 2320.
Dicus, D. A., and V. S. Mathur, 1973, Phys. Rev. D **7**, 525.
Duncan, H. A., and P. Schattner, 1973, Phys. Rev. D **7**, 1861.
Duncan, H. A., unpublished.
Englert, F., and R. Brout, 1964, Phys. Rev. Lett. **13**, 321.
Faddeev, L. D., 1969, Theor. and Math. Phys. **1**, 3 (English translation Theor. and Math. Phys. **1**, 1)
Faddeev, L. D., Gauge Invariant Model of Electromagnetic and Weak Interactions of Leptons, to be published in Dokl Akad. Nauk SSSR
Faddeev, L. D., and V. N. Popov, 1967, Phys. Lett. B **25**, 29.
Fayyazuddin and Riazuddin, 1973a, Phys. Rev. D **8**, 510.
Fayyazuddin and Riazuddin, 1973b, Phys. Rev. D **8**, 4635.
Fermi, E., 1934, Z. Phys. **88**, 161.
Feynman, R. P., 1963, Acta Phys. Pol. **24**, 697.
Feynman, R. P., and M. Gell-Mann, 1958, Phys. Rev **109**, 193.
Fradkin, E. S., and I. V. Tyutin, 1970, Phys. Rev. D **2**, 2841.
Freedman, D. Z., and W. Kummer, 1973, Phys. Rev. D **7**, 1829.
Fritsch, H., and M. Gell-Mann, 1973, in *Proceedings of the XIV International Conference on High Energy Physics* (National Accelerator Laboratory, Batavia, Ill.).
Fritsch, H., M. Gell-Mann, and H. Leutwyler, 1973, Physics Lett. B **47**, 365.
Fujikawa, K., B. W. Lee, and A. I. Sanda, 1972, Phys. Rev. D **6**, 2923.
Gell-Mann, M., and F. E. Low, 1954, Phys. Rev. **95**, 1300.
Gell-Mann, M., L. Goldberger, N. M. Kroll, and F. E. Low, 1964, Phys. Rev. **179**, 1518.
Gell-Mann, M., M. L. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, 1964, Phys. Rev. **133**, Part B, 145.
Gell-Mann, M., M. L. Goldberger, F. E. Low, V. Singh, and F. Zachariasen, 1964, Phys. Rev. **133**, Part B, 161.
Gell-Mann, M., and M. Lévy, 1960, Nuovo Cimento **16**, 705.
Gell-Mann, M., R. J. Oakes, and B. Renner, 1968, Phys. Rev. **175**, 2195.
Gell-Mann, M., and S. L. Glashow, 1961, Ann. Phys. (N.Y.) **15**, 437.
Georgi, H., 1973, Phys. Rev. D **7**, 1258.
Georgi, H., and H. D. Politzer, 1974, Phys. Rev. D **9**, 416.
Georgi, H., and S. L. Glashow, 1972a, Phys. Rev. D **6**, 2977.
Georgi, H., and S. L. Glashow, 1972b, Phys. Rev. Lett. **28**, 1494.
Georgi, H., and S. L. Glashow, 1973a, Phys. Rev. D **7**, 561.
Georgi, H., and S. L. Glashow, 1973b, Phys. Rev. D **7**, 2457.
Georgi, H., and S. L. Glashow, 1973c, Phys. Lett. B **44**, 191.
Georgi, H., S. Glashow, and R. Jackiw, private communication.
Georgi, H. and A. Pais, 1974, Phys. Rev. D (to be published).
Gerstein, I. S., R. Jackiw, B. W. Lee, and S. Weinberg, 1971, Phys. Rev. D **3**, 2486.
Gervais, J. L., and A. Neveu, 1972, Nucl. Phys. B **46**, 381.
Gervais, J. L., and B. W. Lee, 1969, Nucl. Phys. B **12**, 627.
Glashow, S. L., 1961, Nucl. Phys **22**, 579.
Glashow, S. L. 1974, Proceedings of the 1973 Erice Summer School on Elementary Particle Physics.
Glashow, S. L., H. Schnitzer, and S. Weinberg, 1967, Phys. Rev. Lett. **19**, 139.
Glashow, S. L., J. Iliopoulos, and L. Maiani, 1970, Phys Rev. D **2**, 1285.
Glashow, S. L., R. Jackiw, and S. Shei, 1969, Phys. Rev. **187**, 1416.
Glashow, S. L., and S. Weinberg, 1968, Phys. Rev. Lett **20**, 224.
Godine, J., and A. Hankey, 1972, Phys. Rev. D **6**, 3301.
Goldstone, J., 1961, Nuovo Cimento **19**, 154.
Goldstone, J., A. Salam, and S. Weinberg, 1962, Phys. Rev. **127**, 965.
Grisaru, M. T., H. J. Schnitzer, and H. S. Tsao, 1973a, Phys. Rev. Lett. **30**, 811.
Grisaru, M. T., H. J. Schnitzer, and H. S. Tsao, 1973b, Phys. Rev. D **8**, 4498.
Gross, D. J., and F. Wilczek, 1973a, Phys. Rev. Lett. **30**, 1343.
Gross, D. J., and F. Wilczek, 1973b, Phys. Rev. D **8**, 3497.
Gross, D. J., and F. Wilczek, 1974, Phys. Rev. D **9**, 980.
Gunion, J., 1973, Phys. Rev. D **8**, 517 (1973)
Guralnik, G. S., C. R. Hagen, and T. W. B. Kibble, 1964, Phys. Rev. Lett. **13**, 585.
Hasert, F. J., et al., 1973 (Aachen-Brussels-CERN-Ecole Polytechnique-Milan-Orsay-London collaboration), Phys. Lett. V **46**, 121, 138.
Hecht, H., 1971, Astrophys. J. **170**, 401.
Heisenberg, W., 1959, Z. Naturforsch. **14**, 441.
Higgs, P. W., 1964a, Phys. Lett. **12**, 132.
Higgs, P. W., 1964b, Phys. Rev. Lett **13**, 508.
Higgs, P. W., 1966, Phys. Rev. **145**, 1156.
Hoh, C., T. Minamikawa, and K. Miura, to be published.
Horvath, Z., and R. Acharya, A Unified Bardakci-Weinberg Model, to be published.
Imshennik, V. S., and D. K. Nadezhin, 1971, in *Relativistic Astrophysics*, by Ya. B. Zeldovich and I. D Novikov (Univ. of Chicago Press), Vol. 1, Sec. 11.4.
Ivanova, L. N., V. S. Imshennik, and D. K. Nadezhin, 1967, Sci. Ins. Astr. Council USSR Acad. Sci. **13**.
Jackiw, R., and H. Schnitzer, 1972, Phys. Rev. D **5**, 2008.
Jackiw, R., and K. Johnson, 1973, Phys. Rev. D **8**, 2386.
Jackiw, R., R. Van Royen, and G. B. West, 1970, Phys. Rev. D **2**, 2473.
Jaffe, R. L., and J. R. Primack, Production of Weak Interactions Bosons in High Energy Proton-Proton Collisions, to be published.
Joglekan, S., SUNY-Stony Brook preprint, to be published.
Johnson, K., M. Baker, and R. Willey, 1964, Phys. Rev. B **136**, 1111 (1964)
Johnson, K., to be published.
Kamefuchi, S., L. O'Raifeartaigh, and A. Salam, 1961, Nuc. Phys. **28**, 529 (1961)

Khriplovich, I. B., Charge Symmetry in the Process $e^+e^- \to \mu^+\mu^-$, to be published.
Kibble, T. W. B , 1967, Phys Rev. **155**, 1554.
Kinoshita, T., 1962, J. Math. Phys. **3**, 650.
Kinoshita, T., J. Pestiaau, P. Roy, and H. Terezawa, 1970, Phys. Rev. D **2**, 910.
Komar, A., and A. Salam, 1960, Nucl. Phys. **21**, 624.
Kummer, W., and K. Lane, 1973, Phys. Rev. D **7**, 1910.
Lee, B. W., 1969, Nucl. Phys. B **9**, 649.
Lee, B. W., 1972a, in *Proceedings of the XVI International Conference on High Energy Physics*, ed. by J. D. Jackson and A. Roberts (National Accelerator Laboratory, Batavia, Illinois), Vol. IV, p. 249.
Lee, B. W., 1972b. Phys. Rev. D **5**, 823.
Lee, B. W., 1972c, Phys. Rev. D **6**, 1188.
Lee, B. W., 1972d, Phys. Lett. B **40**, 420.
Lee, B. W., J. R. Primack, and S. B. Treiman, 1973, Phys. Rev. D **7**, 510.
Lee, B. W., and J. Zinn-Justin, 1972, Phys. Rev. D **5**, 3121, 3137, 3155.
Lee, B. W., and S. B. Treiman, 1973, Phys. Rev. D **7**, 1211.
Lee, B. W., unpublished.
Lee, S. Y., J. M. Rawls, and D. Y. Wong, "Reggeization of Spin 1 Particles in General Lagrangian Models," to be published.
Lee, S. Y., J. M. Rawls, and L-p. Yu, Pseudo-Goldstone Boson Masses in One-Loop Approximation, to be published.
Lee, T. D., "A Theory of Spontaneous *T* Violation," to be published.
Lee, T. D., Spontaneous *T* Violation, talk presented at the June, 1973 Topical Seminar on Weak Interactions at Trieste, Italy, to be published.
Lee, T. D., and C. N. Yang, 1962, Phys. Rev. **128**, 885.
Lee, T. D., and M. Nauenberg, 1964, Phys. Rev. **113**, Part B, 1549.
Lee, W., 1972, Phys. Lett. B **40**, 423.
Llewellyn-Smith, C. H., 1973a, "An Introduction to Renormalizable Models of Weak Interactions and Their Experimental Consequences," CERN report Ref. TH. 1710.
Llewellyn-Smith, C. H., 1973b, Phys. Lett. B **46**, 233.
Love, A., 1972, Nuovo Cimento Lett. **5**, 113.
Love, A., G. O. Ross, and D. V. Nanopoulos, 1972, Nucl. Phys. B **49**, 513.
Love, A., and G. O. Ross, to be published.
Low, F. E., 1970, in *Lectures on Weak Interactions* (Tata Institute of Fundamental Research, Bombay), p. 11.
Mandelstam, S., 1965, Phys. Rev **137**, Part B, 949.
Mandelstam, S., 1968, Phys Rev. **175**, 1580, 1604.
Mann, A. K., D. B. Cline, and D. D. Reeder, "Search for Forbidden Angular Configurations in the Reaction $e^+e^- \to \mu^+\mu^-$," proposal submitted to SPEAR.
Mills, R., 1971, Phys. Rev. D **3**, 2969.
Mohapatra, R. N., 1972, Phys. Rev. D **6**, 2023.
Mohapatra, R. N., and J. C. Pati, 1973, Phys. Rev. D **8**, 4212.
Mohapatra, R. N., J. Pati, and P. Vinciarelli, 1973, Phys. Rev. D **8**, 3652.
Mohapatra, R. N., and P. Vinciarelli, 1973a, Phys. Rev. D **8**, 481.
Mohapatra, R. N., and P. Vinciarelli, 1973b, Phys. Rev. Lett. **30**, 804.
Mohapatra, R. N., and S. Sakakibara, 1974, Phys. Rev. D **9**, 429.
Nambu, Y., and Jona-Lasinio, 1961, Phys. Rev. **122**, 345.
Neveu, A., and J. Scherk, 1972, Nucl. Phys. B **36**, 155.
Nikolaev, N. N., M. A. Shifman, and M. Zh. Shmatikov, 1973, Zh. Eksp. Teor. Fiz. Pis'ma Red. **18**, 70 (1973) (English translation JETP Letters **18**, 39 (1973))
Oehme, R., E. C. Swallow, and R. Winster, 1973, Phys. Rev. D **8**, 2124.
Pagels, H., 1969, Phys. Rev **185**, 1990.
Pais, A., 1973, Phys. Rev. D **8**, 625.
Pais, A., and J. R. Primack, (1973), Phys. Rev. D **8**, 3063.
Pais, A., and S. B. Treiman, 1972, Phys. Rev. D **6**, 2700.
Palmer, R. B., 1973, Phys. Lett. B **46**, 240.
Parisi, G., 1973, Nuovo Cimento Lett. **7**, 84.
Paschos, E. A., Theoretical Interpretation of Neutrino Experiments, invited talk presented at the January, 1973 meeting of the American Physical Society (NAL-CONF.-73/27-THY)
Paschos, E. A., and L. Wolfenstein, 1973, Phys. Rev. D **7**, 91.
Pati, J. C., and A. Salam, 1973, Phys. Rev. D **8**, 1240.
Politzer, H. D., 1973, Phys. Rev. Lett. **30**, 1346.
Prentki, J., and B. Zumino, 1972, Nucl. Phys. B **47**, 99.
Quinn, H. R., unpublished.
Rawls, J. M., and L. P. Yu, 1973, Phys. Rev. D **7**, 2463.
Salam, A., 1962, Phys. Rev. **127**, 331.
Salam, A., 1968, in *Elementary Particle Physics*, edited by N. Svartholm (Almguist and Wiksells, Stockholm), p. 367.
Salam, A., and J. C. Ward, 1964, Phys. Lett. **13**, 168.
Schattner, P., to be published.
Schecter, J., and Y. Ueda, Higgs Mesons and High Energy Behavior, to be published.
Schnitzer, H., invited paper presented at the New York Academy of Science Conference on Recent Advances in Elementary Particle Physics, New York, March 1973, to be published.
Schwartz, R. A., 1967, Ann Phys. (N.Y.) **43**, 42.
Schwinger, J., 1957, Ann. Phys. (N.Y.) **2**, 407.
Schwinger, J., 1962a, Phys. Rev. **125**, 397.
Schwinger, J., 1962b, Phys. Rev. **128**, 2425.
Schwinger, J., 1967, Phys. Lett B **24**, 473 (1967)
Schwinger, J., 1973, Phys. Rev. D **8**, 960.
Seghal, L. M., 1973a, Nucl. Phys. B **65**, 141.
Seghal, L. M., 1973b, Unified Theories of Weak and Electromagnetic Interactions: an Elementary Introduction, to be published.
Stothers, R., 1970, Phys. Rev. Lett. **24**, 538.
Stuller, L., 1971, M.I.T. Ph.D. thesis, unpublished
Sucher, J., and C. H. Woo, 1973, Phys. Rev. D **8**, 2721.
Symanzik, K., 1970, Commun. Math Phys. **18**, 227.
't Hooft, G., 1971a, Nucl. Phys. B **33**, 173.
't Hooft, G., 1971b, Nucl. Phys. B **35**, 167.
't Hooft, G., and M. Veltman, 1972a, Nucl. Phys. B **44**, 189.
't Hooft, G., and M. Veltman, 1972b, Nucl. Phys. B **50**, 318.
Umezawa, H., and S. Kamefuchi, 1961, Nucl. Phys. **23**, 399.
Utiyama, R., 1956, Phys. Rev. **101**, 1597.
Vainshtein, A. I., and I. B. Khriplovich, 1971, Sov. J. Nuc. Phys. **13**, 111.
Veltman, M., 1968, Nucl. Phys. B **7**, 637.
Veltman, M., 1970, Nucl. Phys. B **21**, 288.
Veltman, M. 1973.
Weinberg, S., 1967a, Phys. Rev. Lett. **18**, 507.
Weinberg, S., 1967b, Phys. Rev. Lett. **19**, 1264.
Weinberg, S., 1968, Phys. Rev. **166**, 1568.
Weinberg, S., 1971, Phys. Rev. Lett. **27**, 1688.
Weinberg, S., 1972a, *Gravitation and Cosmology* (Wiley, New York), Chapt. XV.
Weinberg, S., 1972b, Phys. Rev. D **5**, 1412.
Weinberg, S., 1972c, Phys. Rev. D **5**, 1962.
Weinberg, S., 1972d, Phys. Rev. Lett. **29**, 388.
Weinberg, S., 1972e, Phys. Rev. Lett. **29**, 1698.
Weinberg, S., 1973a, Phys. Rev. D **7**, 1068.
Weinberg, S., 1973b, Phys. Rev. D **7**, 2887.
Weinberg, S., 1973c, Phys Rev. D **8**, 605.
Weinberg, S., 1973d, Phys. Rev. Lett. **31**, 494.
Weinberg, S., 1974a, "Current Algebra and Gauge Theories II: Non-Abelian Gluons," to be published.
Weinberg, S., 1974b, "Current Algebras and Gauge Theories III" paper in preparation.
Weinberg, S., 1974c, Phys. Rev. **D8**, 3494.
Weisskopf, V. F., 1939, Phys. Rev. **56**, 72.
Wess, J., unpublished communication.
Wheeler, J. C., 1973, Am. Sci. **61**, 42.
Wilson, J. R., 1971, Astrophys. J. **163**, 209.
Wilson, K., 1969, Phys. Rev. **179**, 1499.
Yang, C. N., and R. L. Mills, 1954, Phys. Rev. **96**, 191.
Yao, Y. P., 1973, Phys. Rev. D **7**, 1647.
Yu, L. P., 1973, Phys. Rev. D **7**, 2521.
Yukawa, H., 1935, Proc. Phys.-Math. Soc. Jap. (3) **17**, 48.
Zee, A., 1972, Phys. Rep. **3** C, 127.
Zee, A., 1973, Phys. Rev. D **8**, 4038.

NEW REFERENCES

Adler, S. L. 1974a, Phys. Rev. D **8**, 229.
Adler, S. L. 1974b, IAS Report, to be published.
Adler, S. L., S. Nussinov, and E. A. Paschos, 1974, NAL Report Publ.-73/86 THY, to be published.
Aubert, B., *et al.*, 1974 (Harvard–Pennsylvania–Wisconsin–NAL collaboration), to be published.
De Rujula, A., H. Georgi, S. Glashow, and H. R. Quinn, 1974, Rev. Mod. Phys. **46**, 111.
Dolan, L. and R. Jackiw, 1974, M.I.T. Rept. to be published.
Donnelly, T. W., D. Hitlin, M. Schwartz, J. D. Walecka, and S. J. Wiesner, 1974, Phys. Lett. B **49**, 8.
Freedman, D. Z., 1974, NAL Rept. Pub.–73/76 THY, to be published.
Georgi, H. and S. L. Glashow, 1974, Phys. Rev. Lett. **32**, 438.

Georgi, H., H. R. Quinn, and S. Weinberg, 1974, Harvard Report in preparation.
Kirzhnits, D. A., and A. D. Linde, 1972, Phys. Lett. B **42**, 471.
Musset, P., 1974, private communication.
Perkins, D. H., 1972, in *Proceedings of the XVI International Conference on High Energy Physics*, Vol. 4, p. 198.
Weinberg, S., 1974, Phys. Rev. D, 1974, to be published.
Wilson, J. R., 1974, Phys. Rev. Lett. **32**, 849.
Weinberg, S., 1974a, Phys. Rev. **D8**, 4482.
Weinberg, S., 1974b, "Current algebras and gauge theories III" paper in preparation.
Weinberg, S., 1974c, Phys. Rev. **D8**, 3494.

Additional Note Added in Proof:

The existence of neutral currents has been further confirmed by an experiment in which a 12-foot liquid hydrogen–deuterium bubble chamber was exposed to the neutrino beam from the Zero Gradient Synchotron of the Argonne National Laboratory. This experiment provides positive evidence for the production of single π^+ and π° mesons in neutrino–proton collisions. See S. J. Barish *et al.*, Argonne Report ANL/HEP 7411, May 1974, to be published.