Doorway states and pre-equilibrium reactions

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A quantum foundation is developed for the methods employed in the calculation of the pre-equilibrium component of the energy spectrum of nuclear reaction products. It involves the application of the statistical theory of nuclear reactions to chained doorway states.

The following discussion will be concerned with the reactions induced by the collision of a particle with a nucleus. We will be most interested in the energy and angular distribution of the reaction products and what these tell us about the reaction mechanisms involved. The nature of the energy distribution is indicated in Fig. 1. The high-energy end will, with sufficient resolution, involve a series of sharp spikes corresponding to the excitation of particular levels of the residual nucleus. As the excitation energy increases the density of these levels increases so that eventually the spectrum is continuous. The description of the high-energy end of the spectrum is made usually in terms of a combination of direct and compound nuclear processes. The low-energy end is explained in terms of the statistical theory of nuclear reactions. It is often referred to as the evaporation region after a model proposed by Weisskopf (1937). In that region the Bohr independence hypothesis which roughly states that the decay of the compound system is independent of the manner in which it was formed is valid. As a consequence, the energy spectrum should not depend upon the nature of the entrance channel but only upon the excitation energy of the compound system and not separately on the energy of the incident particle. Because of the statistical assumption it can be seen that the angular distribution is symmetric about 90° (Hauser and Feshbach, 1952). Actually if the excitation energy is sufficiently high so that all levels with various values of the total angular momentum are excited with equal probability, the angular distribution is spherical (Hauser and Feshbach, 1952). This is not exactly so particularly for heavy ion projectiles. But here again a particularly simple angular distribution discussed first by Ericson and Strutinsky (1958, 1959) is valid. Experiments are in substantial agreement with these results as long as one's attention is restricted to the evaporation or "equilibrium" region.

This paper is concerned with the spectrum between the region where the residual nucleus has a discrete spectrum and where the evaporation description is valid. In this region sharp isolated doorway states may appear, such as isobar analog states. They will not be our principal concern. Rather it is the dominant background which will be of interest.

Failure of the evaporation model and of the Bohr independence hypothesis had been indicated in a number of papers in the early 1960's. The theory did not begin to attack this problem until 1966 with a number of important papers by Griffin (1966, 1967). Since that time there has been a rather massive, both experimental and theoretical, attack on this problem. I will just mention a few of the theorists and experimentalists (not all their theories are based on the Griffin model) who have been working in this vineyard. Beside Griffin, there are groups at Oak Ridge, Rochester, Columbia, and Milano. Although I had been aware of this activity, I didn't really appreciate the extensive progress that had been made until I attended a conference on "Intermediate Structure" held during September 1972 in Yugoslavia organized by N. Cindro and T. Mayer-Kuckuk. I refer the reader to the proceedings (Europhysics Study Conference, 1972) of this conference and particularly to Blann's review (1972) for the principle features of these theories.

It would of course, not be possible to repeat that review here. But a brief summary and some indication of the successes and limitations would be useful. The actual energy spectrum in the region of interest is thought to be composed of an evaporation or an "equilibrium" spectrum, plus a "pre-equilibrium" spectrum. (We shall prefer the name "pre-compound".) The model used to calculate this component is shown in the Fig. 2. The incident particle, say a nucleon, collides with a target nucleon (≡ nucleon in the target nucleus) exciting it, leading thus to a 2p-h excitation of the target. Subsequent collisions can lead to 3p-2h, 5p-4h, etc., excitations. These excitations are referred to as involving a number of excitons indicated by n. At each stage, emission into the final channel may be possible. The spectrum is then calculated as a sum of the emitted intensities from each stage. Various

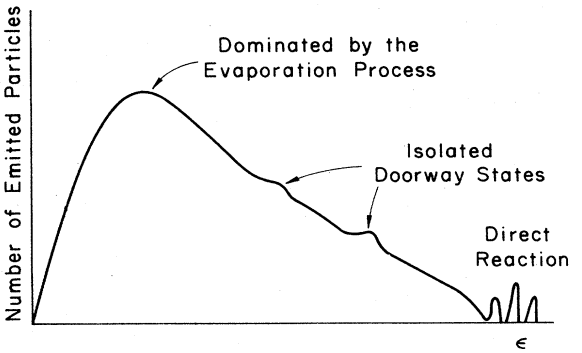

FIG. 1. Spectrum of particles emitted in a nuclear reaction.

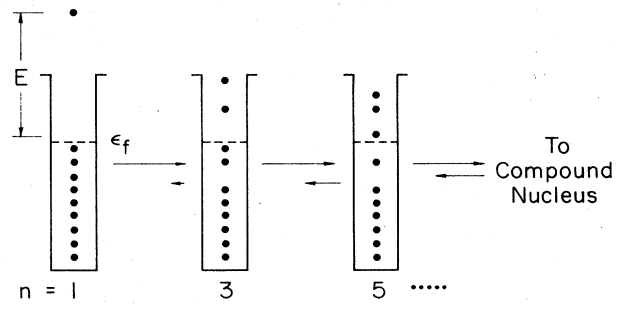

FIG. 2. Stages in the development of a compound nucleus.

theories differ according to how the transition probabilities between these stages and the attenuation because of emission in preceding stages are evaluated. The formula used by Blann is

$$\sigma_x(\varepsilon) = \pi\lambda^2 \sum_{l=0}^{\infty} (2l+1)T_l \sum_{n=n_0}^{n} p_x \frac{\rho_n(U,\varepsilon)}{\rho_n(E^*)} \frac{D_{n,l}\lambda_c(\varepsilon)}{\lambda_c(\varepsilon)+\lambda_n(\varepsilon)}. \quad (1)$$

The elements of this expression are fairly obvious. The first factor is the transmission factor for the lth partial wave, the term with the level densities ρ_n is the ratio of the density of levels with the emitted particle in the continuum and the residual nucleus with excitation U to the total n exciton level density in the compound nucleus with excitation E^*. U is the excitation energy of the residual nucleus. The factor $D_{n,l}$ equals the fraction of the initial population of the lth partial wave surviving to the nth exciton state. The ratio of the λ's measure the relative probability of the emission to the probability of going on to the next more complex reaction. The factor p_x gives the number of excitons which are of type x. As in the case of evaporation theory the densities ρ_n play a decisive role.

A few examples taken from Blann's review will show the quality of the fits to experimental data obtained with this expression, or similar ones. It should be remarked that there are other approaches [see Europhysics Study Conference, *Proceedings*, 1972] beside the Griffin–Blann one. In Fig. 3 and Fig. 4 excitation curves (Birattari *et al.*, 1971) are given for the reactions ^{169}Tm(p, $3n$) and ^{169}Tm(p, $4n$), respectively. The solid line gives the theoretical prediction. It is composed of the evaporation spectrum given by the dashed line and a pre-compound component indicated by the dot-dashed line. Figure 5 gives the excitation spectrum (Blann and Lanzafame, 1970) for ^{197}Am(α, xn) reactions. The heavy solid curve gives the experimental results, the dashed the total theoretical prediction while the light line gives the evaporation contribution. Finally in Fig. 6 a set of spectra for various (α,p) reactions is compared with theory given by the solid line (West, 1966; Blann *et al.*, 1972). The agreement is really quite good—of course its a semilog scale—but still the calculated excitation curve and spectrum often agree with experiment, and when there is a deviation it is not an order of magnitude, but factors of two or three.

Of course, there are some *ad hoc* elements in the use of the above formulas particularly when one of the particles is not a nucleon. The angular distribution is not calculated. The value of n_0 in Eq. (1) is treated as an empirical parameter. The free nucleon–nucleon cross section is used to describe the transition from one state of excitation to another. The kinematics of a free collision is used to determine the final energies of the interacting particles. The whole procedure reminds one of the status of the Weisskopf evaporation theory before a quantum mechanical justification which was provided by a systematic application of the statistical theory of nuclear reactions. It is the aim of this presentation to provide a corresponding discussion for the entire spectrum.[1] It should include the pre-compound distribution, the evaporation spectrum, and the present day explanation of the high-energy components. Finally the angular distribution should be predicted.

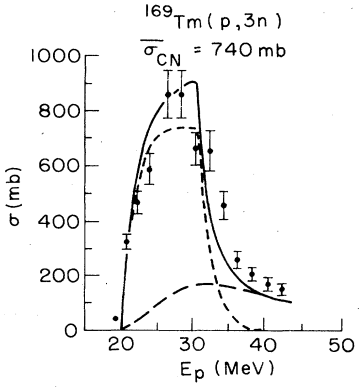

FIG. 3 Excitation function for the reaction ^{169}Tm(p, $3n$). Broken line gives pre-compound contribution, the dotted line the evaporation component. The solid line is the theoretical total.

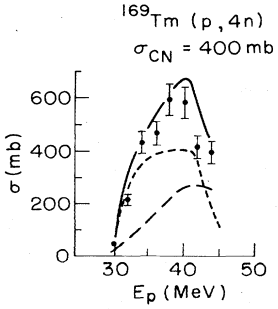

FIG. 4. Excitation function for the reaction ^{169}Tm(p, $4n$).

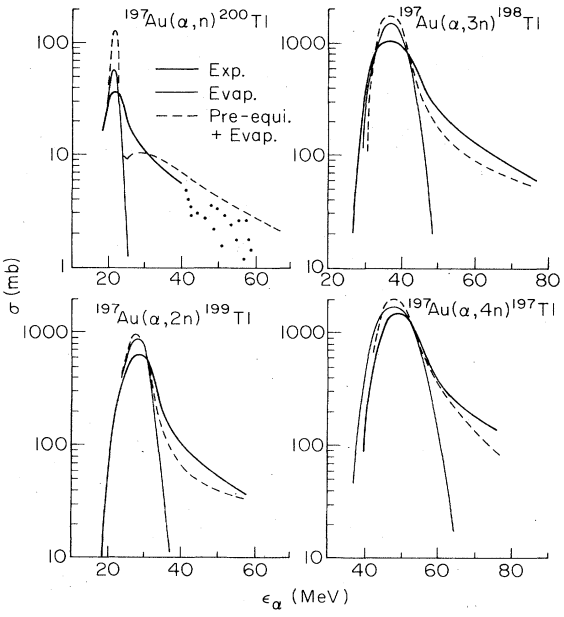

FIG. 5. Excitation function for the reaction ^{197}Au(α, xn).

[1] This work is being done in collaboration with A. K. Kerman and S. E. Koonin.

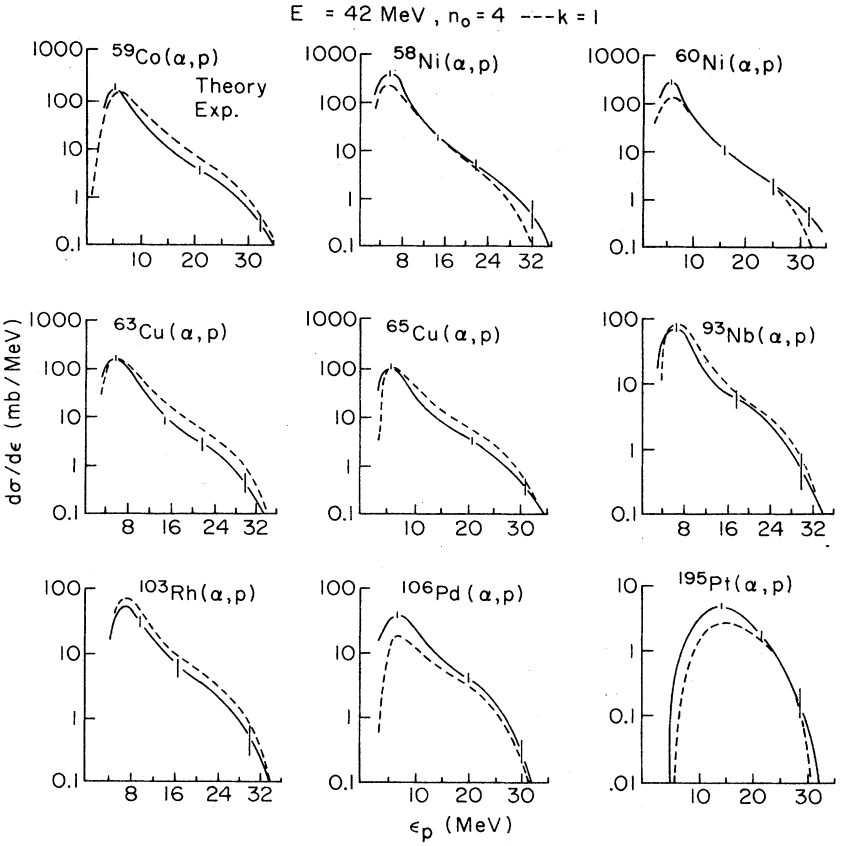

FIG. 6. Proton spectra for 42 MeV α particles incident upon various nuclei.

The clue for such a development has been provided by Grimes, Anderson, Pohl, McClure, and Wong in two papers (Grimes *et al.*, 1971). Grimes *et al.* propose to describe the pre-compound distribution in terms of the statistical theory of doorway states. Such a theory was proposed by Kerman and Lemmer and myself (1967) but the applicability to the theory of the pre-compound component was not indicated. Actually the statistical theory was used in the paper with Block (Block and Feshbach, 1963) in discussing the neutron strength function. Griffin based his discussion on the doorway concept. Grimes *et al.*'s suggestion was based on the following qualitative evidence indicated in Fig. 7 and Fig. 8. There are other examples. In each of these U is the excitation energy in the residual nucleus, large U corresponds to low neutron energy. At the low-energy side, the angular distribution is isotropic, and then becomes anisotropic but remains symmetric about 90°. In the isotropic component the nuclear temperature was independent of angle and approximately independent of the bombarding energy. The effective temperature for the symmetric component differs markedly from this value being much greater indicating a smaller slope in the semilog plot. The symmetry about 90° is just what would be expected from a statistical theory of doorway state reactions. That theory uses the concept of the doorway state illustrated in Fig. 9. In the paper with Kerman and Lemmer (Feshbach, Kerman, and Lemmer, 1967) we emphasized that after averaging over the effect of the complex states the doorway state resonances could be treated like ordinary compound nuclear resonances with the difference that the total width of the state contains not only the width due to the decay of the resonance into open channels but also an average "down" width describing the possibility of its decay into more complex states. Hence making the usual assumptions of statistical theory it becomes possible to state (1) that the angular distribution will be symmetric about 90°, and (2) that the cross section is given by

$$\sigma_{\alpha\beta} = \sigma_\alpha(\Gamma_\beta/\Gamma_d), \tag{2}$$

where σ_α is the absorption cross section for particle α, Γ_β the average emission width, Γ_d the average doorway width. By standard methods and approximations used in the compound nuclear statistical theory this formula can be written as follows:

$$\sigma_{\alpha\beta} \sim \frac{(2S+1)\sigma_\alpha(E)\sigma_\beta(\varepsilon)\varepsilon\rho(E-\varepsilon)}{\rho_d(E^*)\Gamma_d(E^*)}\,d\varepsilon, \tag{3}$$

where σ_β is the inverse cross section for the formation of doorway state by particle β, $\rho_d(E^*)$ is the density of doorway states in the compound nucleus at the appropriate energy. This formula is very similar to the one given by Blann. To this cross section one adds the standard evaporation result. Grimes *et al.* analyze their results to obtain the density of states in the residual nucleus, the

density of doorway states in the compound system and their average width. The form assumed for the latter is U^n. It is important to note that no particular model for the doorway state has been assumed. The results appear to be entirely reasonable. However, the above equation gives just one term in the Blann series. The other terms are very similar in character to the one indicated above, but have the additional factor of the probability of reaching the required excitation state.

We now turn to the required generalization to obtain the full series. This involves now a dissection of the complex component of the wave function into a set of less complex and more complex components. This is shown in Fig. 10, which it should be recognized, appears very similar to Fig. 2 taken from Blann with a major difference that no attempt has been made to specify the nature of the various stages of complexity except to say that one gets to a given stage of complexity only from one of one degree less complex nature. This simplifying chaining assumption can be lifted. But at this first exploration of the problem, it helps to make it. In the figure we consider an emission occuring at the nth stage taking full account of the possibility that it can occur at earlier or later stages. The total amplitude is necessarily a sum of these individual amplitudes, i.e., a sum over n. Actually the q component should be really labelled q_n. But this n dependence is not too serious. The higher-order doorways have been observed in, for example, the case of the giant dipole resonances, and in a few isobar analog states.

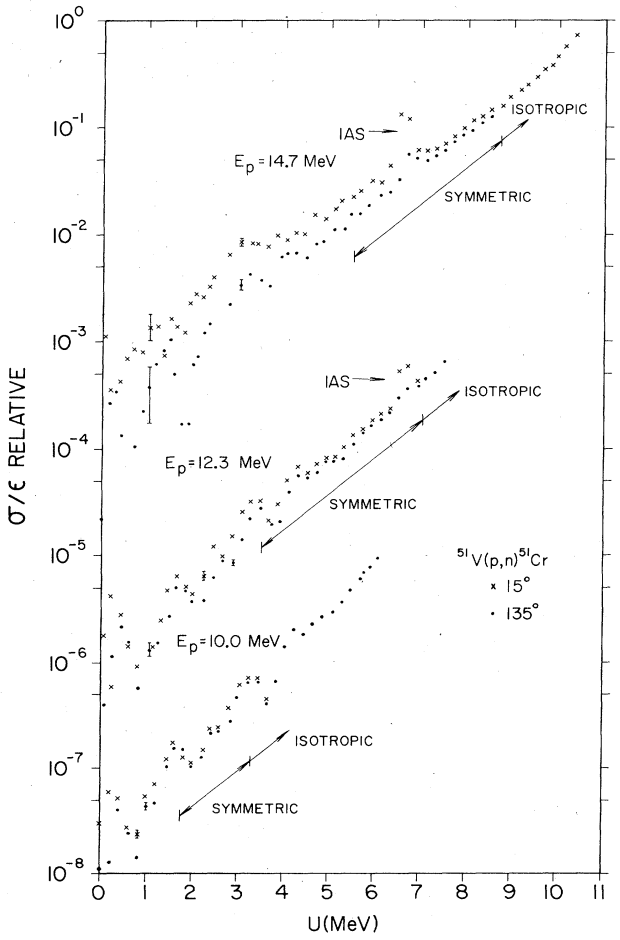

FIG. 7. Neutron spectra form $^{51}V(p,n)^{51}C$ at three bombarding energies and at ten angles. Those labeled x are at 15°; those labeled · at 135°. The portions of the spectra in which the angular distribution is isotropic or symmetric are also indicated.

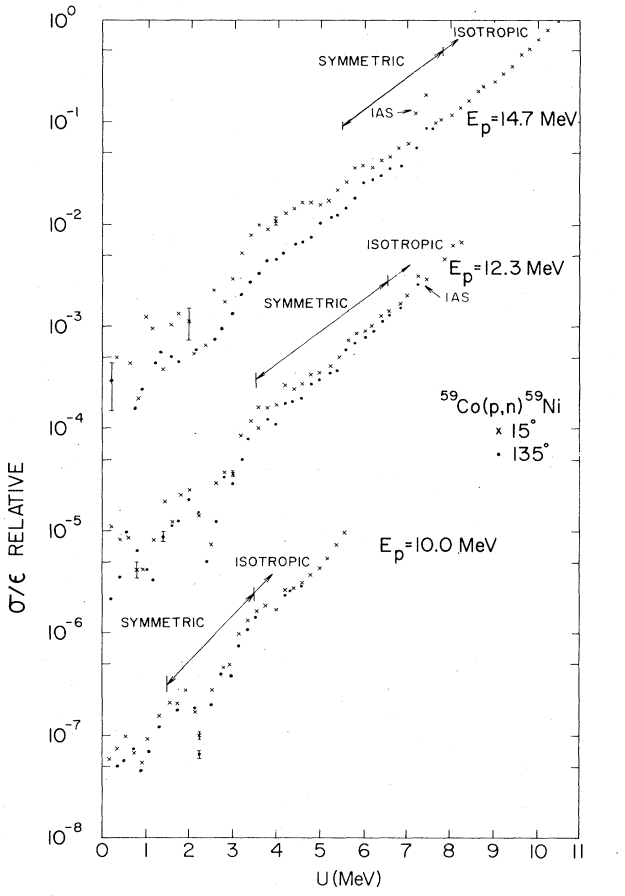

FIG. 8. Neutron spectrum from $^{59}Co(p,n)^{59}Ni$. See Fig. 7 for description.

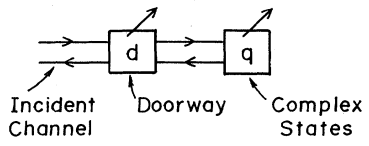

FIG. 9. Simple doorway.

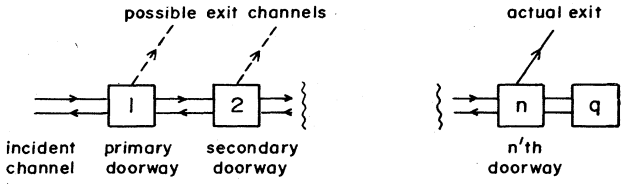

FIG. 10. Doorway states as stages in development of the compound nucleus.

This description then gives rise to the following amplitude:

$$T_n = \langle \chi_f^{(n)} | \hat{H}(f,n)(E - \hat{H}_n)^{-1} \hat{H}(n,n-1)(E - \hat{H}_{n-1})^{-1} \cdots \hat{H}_2(2,1)(E - \hat{H}_1)^{-1} \hat{H}(1,i) | \chi_i \rangle. \quad (4)$$

In this formula $\chi_f^{(n)}$ is the final state, χ_i the initial state. I give only the part which is statistical in nature. The direct interactions and any special states such as the isobar analog or other doorways have been separated out and put into T_D. It will be assumed that $\langle T_D^* T_n \rangle = 0$. It will also be assumed that the evaporation component does not interfere with T_D or T_n The assumption has been clearly made that the emission at the nth stage is tied directly to nth order doorway, the coupling terms $\hat{H}(n, n-1)$ give the part of the Hamiltonian which connects the doorway of $(n-1)$st order to the doorway of the nth doorway. It is seen that this expression represents quite clearly the picture of the phenomena as the system proceeds from the original simple state to those of successively greater complexity. The factors $(E - \hat{H}_n)^{-1}$ simply give the propagation factors as one proceeds from one state to the next.

To complete the story one needs the definitions of the Hamiltonian $\hat{H}_n$ and the coupling Hamiltonians. There are several possibilities of which the following is one example.

$\hat{H}_\nu$ is defined by a set of recurrence relations

$$\begin{aligned} \hat{H}_\nu &= \hat{H}_{\nu\nu} + W_\nu, \\ W_\nu &= \hat{H}(\nu,\nu-1)(E - \hat{H}_{\nu-1})^{-1} \hat{H}(\nu-1,\nu), \\ &= \hat{H}(\nu,\nu-1)(E - \hat{H}_{\nu-1,\nu-1} - W_{\nu-1})^{-1} \hat{H}(\nu-1,\nu), \text{ etc.} \end{aligned} \quad (5)$$

$\hat{H}$ is not the exact H but one which results when all open channels have been removed. This does not mean that the effect of the open channels is not present since

$$\hat{H} = QHQ + QH[p/(E^+ - H_{pp})]HQ, \quad (6)$$

where p now refers to the open channels. Hence $\hat{H}_{\nu\nu}$ contains the effect of the possible emission from the νth channel while W_ν contains the width due to the possible decay to the $\nu - 1$ doorway. Similarly, $\hat{H}(\nu, \nu-1)$ contains not only the effect of the transition to the next doorway but the possible coupling of ν to $\nu + 1$ through the open channels. It is possible, of course, to run relations (5) in the opposite direction. In either procedure, the final *form* is the same. To this $T^{(n)}$ must be added, the more complex state contributions and the direct interaction.

The next stages in the development are quite standard. The inverse operators are expanded in terms of the eigenfunctions of $\hat{H}_\nu$, i.e., in terms of the doorway states of various orders. The resulting cross section is averaged over the energy width of the incident beam, and the random phase assumption is made with regard to the matrix elements between doorway states. It is assumed that the direct and special state contribution, the doorway contributions for each value of n, and the more complex component contribution, do not interfere. The result for the pre-compound component for the case of a given n is

$$\langle |T_n|^2 \rangle = \frac{2\pi}{D_n} \frac{\Gamma(fn)}{\Gamma_n} \frac{\Gamma(n,n-1)}{\Gamma_{n-1}} \cdots \frac{\Gamma(2,1)}{\Gamma_1} \Gamma(1i), \quad (7)$$

where Γ_{fn} is the average width for formation of the final state from the nth order doorway $\Gamma(n, n-1)$ is the width for the transition $n - 1 \rightarrow n$, while Γ_n is the average width for the nth order doorway.

$$\Gamma(\nu,\nu-1) = 2\pi |\langle \psi_\nu | \hat{H}(\nu,\nu-1)\psi_{\nu-1} \rangle|^2. \quad (8)$$

The factors $(2\pi/D_n)$ $[\Gamma(fn)/\Gamma_n]$ and $\Gamma(1i)$ of the above expression give rise to the formula of Grimes *et al.* described earlier. The remaining factors give the probability of reaching doorway state n from the primary doorway state.

These results need to be appropriately summed over n and the other contributions added. This sum is rapidly convergent.

With these results we have made a first step toward reconciling pre-compound theory with standard theory.

To conclude, what relevance do these studies have for nuclear physics beside the very important one of the understanding of the reaction phenomena involved? One of the objects of nuclear studies is to obtain a better understanding of the nuclear wave function. We have already mentioned the doorway width, as well as the density of states of a given complexity. This is fundamental information. It might be hoped that from these results a contribution toward the understanding of the more complex components of the nuclear wave function will be obtained.

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