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Density Effect in the Ionization Energy Loss of Fast Charged Particles in Matter

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Recent experimental work and its theoretical interpretation concerned with the density effect on the energy loss of charged particles in matter is reviewed with particular reference to electrons and muons. Recent proposals that radiative corrections should result in a significant reduction in the ionization loss at very high energies are analyzed in some detail. It is concluded that such an effect has not been substantiated experimentally, and that the predicted radiative corrections are likely to be small ($\sim 1\%$).

CONTENTS

1. INTRODUCTION

The saturation of the ionization energy loss of very energetic charged particles in matter, first predicted by Swann (1938) and Fermi (1940), is by now a well established experimental fact, and its origin in the polarization of the medium by the incident charged particle (the density effect) is at least qualitatively understood. However, from the quantitative point of view, there is still room for doubt as to the exact magnitude of the effect in the various detectors, and so detailed comparison with theory has not so far been possible. The comparisons which have been made and which will be described below show that theory and experiment are in fair agreement.

In addition, Tsytovitch (1962) and Zhdanov *et al.* (1962) have suggested that a reduction in the saturation (plateau) value should occur at the highest energies as a result of higher-order corrections in the fine-structure constant, an effect which has not so far been substantiated by other workers. On examining the expected magnitude of this effect, it became clear to the authors that it could only be expected to occur in the ionization loss as a result of a misunderstanding of the loss mechanism in dielectric media, and its interpretation in terms of the classical Fermi formula.

Briefly, the classical formulas, at least in transparent media, predict that the relativistic rise in energy loss up to the plateau should escape as Cerenkov radiation (Schönberg, 1951; Messel and Ritson, 1950). If absorption using a realistic model of the dielectric is included, it turns out that the energy loss should be deposited close to the path of the particle. This is largely because in the presence of strong absorption, the condition for Cerenkov radiation production may not be satisfied over the whole of the possible frequency range (Sternheimer, 1953a, 1967), and also because the Cerenkov radiation that is produced may itself be strongly absorbed (Sternheimer, 1953a; Budini, 1953). However the energy loss is still assumed to be accurately predicted by the classical formula, so that all that is involved by the introduction of absorption is a redistribution of the spatial region in which the energy loss appears. Tsytovitch has calculated radiative corrections to the classical formula in transparent media, and at first glance they would appear to be relevant to ionization loss. However we believe that in strongly absorbing media, his treatment, which makes explicit use of the Cerenkov condition, fails.

The following section discusses in detail the various theoretical procedures which have been applied to the problem. In addition, a method which links immediately with the results on isolated atoms and which may be used to calculate directly the radiative corrections to the ionization loss is described.

2. THEORY OF THE DENSITY EFFECT

2.1 Introduction

The modern theory of energy loss dates from the quantum mechanical treatment of Møller (1932), Bethe (1930, 1933), and Bloch (1933a, b) in which the dielectric properties of the medium were ignored. The first author to avoid this approximation in evaluating the results, and whose work is therefore our starting point, was Fermi. This was in fact the first occasion in which the density effect was discussed in detail. The method used was a classical one (see also Wick, 1943; Halpern and Hall, 1948) which has been extensively employed, particularly in a series of papers by Sternheimer (1952–1967), in most of the quantitative work since. The results of this work have been reviewed by Uehling (1954), Fano (1963), and Sternheimer (1961), who give details of the numerical procedures used, together with further references.

It is convenient to begin by describing the Møller method since this will allow us to introduce the effect of the dielectric in a rather direct way. Later, we will explain the Tsytovitch method and show first how it is related to the Fermi–Sternheimer approach, and second to the Williams (1935)–Weizsäcker (1934) approach as developed by Budini (1953), and in less detail by Fowler and Jones (1953). This allows us to present the arguments which we believe lead to the conclusion mentioned in the introduction concerning the reduction in the energy loss.

2.2 Calculation of the Energy Loss Neglecting the Density Effect

The interaction energy between the incident particle and an atom of the medium is given in relativistic notation by

$$H_I = -(c)^{-1}\int J_\mu(x) A_\mu(x)\ d^4x \qquad (\mu=1,\cdots,4), \quad (1)$$

where $J_\mu(x)$ is the atomic electron four-current density at the space–time point labeled by x, and $A_\mu(x)$ is the four-vector potential produced by the incident particle. The explicit expression for the latter may be found directly from Maxwell's equations following Møller. Thus A_μ satisfies

$$\partial^2 A_\mu(x)/\partial x_\nu^2 = -j_\mu(x), \qquad (2)$$

where $j_\mu(x)$ is the four-current density, corresponding to the incident particle, which may be written in terms of the initial and final incident particle wave functions as:

$$j_\mu(x) = e\bar{\psi}_f(x)\gamma_\mu\psi_i(x)$$

(assuming that the incident particle is a Dirac particle without anomalous moment). This describes the effective four-current density for an electron making a transition from an initial state i to a final state f, and γ_μ is the Dirac matrix which is equivalent to v/c in the nonrelativistic limit. For the present applications it is sufficient to use the plane-wave approximation to $\psi(x)$, so that

$$\psi_i(x) = U_s(P)\ \exp(iP_\mu x_\mu/\hbar)$$

and

$$\psi_f(x) = U_{s'}(P')\ \exp(iP_\mu' x_\mu/\hbar),$$

where P_μ and P_μ' are the initial and final four-momenta, respectively, and $U_s(P)$ is a Dirac spinor. The field due to the current j_μ has, therefore, only the Fourier components corresponding to $P_\mu - P_\mu'$, i.e., the four-momentum transfer, and so we may write for the Fourier transform of Eq. (2)

$$q^2\tilde{A}_\mu(\mathbf{k},\omega) = U_{s'}(P')\gamma_\mu U_s(P), \qquad (3)$$

where

$$q^2 = (\mathbf{P}-\mathbf{P}')^2/\hbar^2 - (E-E')^2/\hbar^2c^2 \equiv \mathbf{k}^2 - \omega^2/c^2.$$

In the absence of sources, i.e., when $j_\mu \equiv 0$, a nontrivial solution of Eq. (3) requires that $q^2=0$, which is the condition satisfied by the frequency and wave number of propagating physically real electromagnetic waves or photons. When sources are present, then $q^2 \neq 0$, and the electromagnetic field which they produce may be represented by a stream of so-called virtual photons. Thus the interaction Hamiltonian is

$$H_I = -\frac{e}{c} \times \int \frac{J_\mu(x)\,U_{s'}(P')\gamma_\mu U_s(P)\ \exp[i(P-P')_\nu x_\nu/\hbar]}{q^2}\,d^4x, \qquad (4)$$

and the corresponding differential cross section may be written

$$\partial^2\sigma/\partial q^2\partial\omega = (\pi e^2/q^4)(\mu^2/mc)\,\Gamma_{\mu\nu}K_{\mu\nu}, \qquad (5)$$

where

$$e^2\Gamma_{\mu\nu} = \tfrac{1}{2}\sum_{s'}\sum_{s}\langle P' \mid j_\mu \mid P\rangle\langle P' \mid j_\nu \mid P\rangle^*,$$

$$K_{\mu\nu} = (2s+1)^{-1}\sum_{s}\sum_{s'}\langle f \mid J_\mu \mid i\rangle\langle f \mid J_\nu \mid i\rangle^*,$$

and m and μ are the electron and incident-particle masses, respectively. Also,

$$\langle f \mid J_\mu \mid i\rangle = \frac{e}{\hbar c}\left(\frac{2m}{\pi^3}\right)^{1/2}\int \Psi_f^*(x) \times \exp\left(\frac{i(P-P')_\nu x_\nu}{\hbar}\right) O_\mu \Psi_i(x)\ d^4x,$$

with

$$O_\mu \equiv \mathbf{P}/m,\ E/mc,$$

and the $\Psi_{i,f}$ represent atomic electron wave functions.

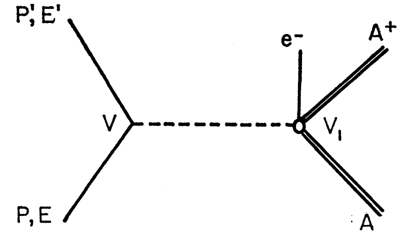

FIG. 1.

The process described in this way is commonly represented by a Feynman diagram, as shown in Fig. 1, where the two vertices V and V_1 refer to the two factors $U_{s'}(P')\gamma_\mu U_s(P)$ and $\int J_\mu(x) \exp[i(P-P')_\nu x_\nu]\ d^4x$, respectively, and the wavy line refers to the factor q^{-2}. This describes the transfer or propagation of a virtual photon from the incoming particle P to the atom A, resulting in an outgoing particle P', and an ion pair A^+, e^-.

In what follows we shall be interested only in those collisions in which atomic binding is important, the so-called "distant collisions," so that Formula (4) introduces form-factor effects dependent on the details of the charge distribution of the atomic states involved. The reason for the restriction to "distant collisions," i.e., those for which the projectile is always distant from the target atom, is that only in such cases will the energy transfer generally be comparable with the atomic energies involved, and it is only in such cases that dielectric polarization and hence screening effects are expected to be important.

For the application of Formula (5) to the density effect, it is necessary to separate the single photon exchange process into longitudinal and transverse parts as we shall see in Sec. 2.3(i). To do this we define the polarization vectors of the virtual photon by three mutually orthogonal four-vectors $e_\mu^{(\lambda)}$ which satisfy $e_\mu^{(\lambda)}\cdot q_\mu=0$ and $e_\mu^{(\lambda)}\cdot e_\mu^{(\lambda')}=0$, if $\lambda\neq\lambda'$, where $\lambda=1, 2, 3$. Two are chosen to be spacelike, satisfying $\mathbf{e}^\lambda\cdot\mathbf{k}=0$, and are denoted by $\mathbf{e}_\perp$; the third denoted by $e_{\parallel\mu}$ is timelike, being given by

$$e_{\parallel\mu}=(q^2)^{-1/2}\left(\frac{(\omega/c)\mathbf{k}}{|\mathbf{k}|},\ |\mathbf{k}|\right).$$

The cross section for absorption of a virtual photon of polarization λ is now

$$\sigma_\lambda=(\pi^2/|\mathbf{k}|)(\hbar/mc)\,e_\mu^{(\lambda)}K_{\mu\nu}e_\nu^{(\lambda)}.$$

In the following discussion, σ_λ with $\lambda=1, 2$ will be referred to as σ_T, and σ_λ with $\lambda=3$ will be referred to as σ_L.

It may be shown that

$$\Gamma_{\mu\nu}=(2\mu^2c^2)^{-1}[P_\mu P_\nu'+P_\nu P_\mu'+(\hbar^2/2)q^2\delta_{\mu\nu}],$$

and so we have

$$\frac{\partial^2\sigma}{\partial q^2\partial\omega}=\frac{e^2}{2\pi\hbar c}\frac{\hbar^2}{\mathbf{P}^2c}\frac{|\mathbf{k}|}{q^4}\left[\left(q^2-\frac{2\mu^2c^2}{\hbar^2}\right)\sigma_T(q^2,\omega)+\frac{q^2}{\mathbf{k}^2}[\sigma_T(q^2,\omega)+\sigma_L(q^2,\omega)]\left(\frac{2EE'}{\hbar^2c^2}-\tfrac{1}{2}q^2\right)\right]. \quad (6)$$

In the present case, the transverse and timelike components of J_μ satisfy

$$J_0^2(x)=(c^2\mathbf{k}^2/\omega^2)J_T^2(x),$$

so that σ_L may be eliminated in favor of σ_T. Finally, we introduce the cross section for absorption of real photons, $\sigma_T^R(\omega)$, by introducing a form factor $\chi(q^2)$ according to

$$\sigma_T(q^2,\omega)=\chi(q^2)[\omega/(c|\mathbf{k}|)]\sigma_T^R(\omega).$$

The factor $\omega/|\mathbf{k}|$ here, simply takes into account the difference in kinematics between real and virtual photons.

Introducing these relations into (6) gives

$$\frac{\partial^2\sigma}{\partial q^2\partial\omega}=\frac{\alpha}{2\pi}\frac{\chi(q^2)\omega\hbar^2}{|\mathbf{P}^2|q^4c^2}\left[\left(q^2-\frac{2\mu^2c^2}{\hbar^2}\right)\sigma_T^R(\omega)+\frac{q^2}{|\mathbf{k}^2|}\left(\sigma_T^R(\omega)+\frac{q^2c^2}{\omega^2}\sigma_T^R(\omega)\right)\left(\frac{2EE'}{\hbar^2c^2}-\tfrac{1}{2}q^2\right)\right],$$

where $\alpha=e^2/\hbar c$. The effect of the finite atomic size is introduced by assuming that for the form factor $\chi(q^2)$

$$\chi(q^2)=1,\qquad q^2\leq q_{\max}^2,$$
$$=0,\qquad q^2>q_{\max}^2,$$

with $q_{\max}^2=\mathbf{k}_{\max}^2-\omega^2/c^2$, and $\mathbf{k}_{\max}$ is defined in terms of $b_{\min}$, the atomic radius, by $|\mathbf{k}_{\max}|\,b_{\min}=1$. We shall also assume that $\mathbf{k}_{\max}^2\gg\omega^2$. This assumes that the photon cross section rapidly falls to zero for large momentum transfers to the atomic electron, a behavior which is to be expected for distant collisions or collisions for which the atomic binding is involved. The integration over q^2, ω being fixed, may now be carried out giving

$$\frac{\partial\sigma}{\partial\omega}=\frac{\alpha}{\pi\omega}\frac{c^2}{v^2}\sigma_T^R(\omega)\left(\frac{\ln|\mathbf{P}^2|}{\mu^2\omega^2b_{\min}^2}-\frac{v^2}{c^2}\right),$$

and the energy loss per centimeter becomes,

$$W=\frac{N\alpha}{\pi}\frac{\hbar c^2}{v^2}\int_{w_0}^{E/\hbar}d\omega\left(\ln\frac{v^2}{(1-v^2/c^2)\omega^2b_{\min}^2}-\frac{v^2}{c^2}\right)\sigma_T^R(\omega),$$

where v is the incident-particle velocity, and N is the number of atoms per cubic centimeter. This expression is, of course, the usual result for relativistic particles.

2.3 The Density Effect

2.3(i) The Modified Møller Method

The simplest way to modify (4) to take into account the effect of the medium on the electromagnetic field is to modify the propagator q^{-2}. To do this we must consider the transverse and longitudinal parts separately.

The transverse propagator is

$$q_{\epsilon_T}^{-2}=[\mathbf{k}^2-(\omega^2/c^2)\epsilon(\omega)]^{-1},$$

which follows immediately from Maxwell's equations suitably modified to allow for the dielectric properties of the medium, with $\epsilon(\omega)$ the frequency-dependent dielectric function. The longitudinal propagator is

$$q_{\epsilon_L}^{-2}=[\mathbf{k}^2\epsilon(\omega)]^{-1},$$

which is the usual modification of the static potential. With Eqs. (4) and (5) it is more convenient to use

$$q_{\epsilon_L}^{-2}=[q^2\epsilon(\omega)]^{-1}.$$

The corresponding Feynman diagram is shown in Fig. 2. In order to include these modifications it is only necessary to recall that σ_T and σ_L refer to transverse and longitudinal effects, respectively, so that the required result may be obtained directly from (6),

$$\frac{\partial^2\sigma}{\partial q^2\partial\omega}=\frac{\alpha}{2\pi}\chi(q^2)\frac{\sigma_T{}^R(\omega)\hbar^2}{\mathbf{P}^2c^2}\left\{\frac{\omega}{|q_{\epsilon_T}{}^2|^2}\left[q^2-\frac{2\mu^2c^2}{\hbar^2}+\frac{q^2}{\mathbf{k}^2}\left(\frac{2EE'}{\hbar^2c^2}-\frac{q^2}{2}\right)\right]+\frac{q^4}{|q_{\epsilon_L}{}^2|^2}\left(\frac{(2EE'/\hbar^2c^2)-\frac{1}{2}q^2}{\mathbf{k}^2\omega}\right)\right\}$$

$$\simeq\frac{\alpha}{2\pi}\chi(q^2)\frac{\sigma_T{}^R(\omega)\hbar^2}{\mathbf{P}^2c^2}\left\{\frac{\omega}{|q_{\epsilon_T}{}^2|^2}\left(\frac{2EE'q^2}{\hbar^2c^2\mathbf{k}^2}-\frac{2\mu^2c^2}{\hbar^2}\right)+\frac{q^4}{|q_{\epsilon_L}{}^2|^2}\frac{2EE'}{\hbar^2c^2\mathbf{k}^2\omega}\right\}. \qquad (6')$$

The limits of integration over q^2 give the following limits on $q_{\epsilon_{T,L}}{}^2$:

$$q_{\epsilon_T}{}^2;\quad (\omega^2/v^2)-\omega^2\epsilon(\omega)\le q_{\epsilon_T}{}^2\le(b_{\min}{}^2)^{-1}$$

$$q_{\epsilon_L}{}^2;\quad (\mu^2\omega^2/\mathbf{P}^2)\epsilon(\omega)\le q_{\epsilon_L}{}^2\le\epsilon(\omega)/b_{\min}{}^2.$$

The result for the energy loss, following the same procedure used to derive the earlier result, is:

$$W=\frac{N\alpha\hbar c^2}{\pi v^2}\left(\int_{w_0}^{E/\hbar}d\omega\frac{\sigma_T{}^R(\omega)}{|\epsilon(\omega)|^2}\left\{\ln\left(\frac{v^2}{|1-[v^2\epsilon(\omega)/c^2]|\,\omega^2b_{\min}{}^2}\right)\right.\right.$$

$$\left.\left.-\left(\frac{\mathrm{Re}\,\epsilon(\omega)-(v^2/c^2)\,|\epsilon(\omega)|^2}{\mathrm{Im}\,\epsilon(\omega)}\right)\left[\tfrac{1}{2}\pi-\tan^{-1}\left(\frac{1-(v^2/c^2)\,\mathrm{Re}\,\epsilon(\omega)}{(v^2/c^2)\,\mathrm{Im}\,\epsilon(\omega)}\right)\right]\right\}\right). \qquad (7)$$

The ionization produced is given by replacing $\sigma_T{}^R$ by $\sigma_T{}^R/\omega$ in (7). This expression is the same as that given by Budini (see below) if we replace the cross section σ by $\mathrm{Im}\,\epsilon(\omega)$ through

$$N\sigma=(\omega/c)\,\mathrm{Im}\,\epsilon(\omega), \qquad (7')$$

and we replace $b_{\min}$ by $(\gamma/2)b_{\min}=0.9b_{\min}$, where $\ln\gamma=0.577$.

In fact, the calculations reported by Budini were valid in the approximation $\mathrm{Im}\,\epsilon(\omega)\ll1$, a condition which facilitates the extraction of the Cerenkov contribution in the classical limit.

From the derivation of (7) it is plain that only energy transferred to the medium in the form of excitation or ionization is included. We shall discuss this further in connection with the classical calculation of the density effect [see Sec. 2.3(iii)]: It will also become clear that the methods of the present section could be used to calculate higher order corrections to (7). Indeed Fowler and Hall (1965) have estimated the correction due to two, density-corrected, photon exchanges, shown in Figs. 3(a) and 3(b), as being of the order of 1%, with opposite signs for positive and negatively charged particles. (This is of course just the second Born approximation for this process).

This compares with a value of 5%–10% predicted by Tsytovitch for the reduction in the plateau value in ionization loss. At the present time the experimental results appear to exclude a 5%–10% reduction in the plateau value.

We now discuss alternative methods of calculating the energy loss from which it will emerge that the Tsytovitch conclusion, as to the magnitude of the radiative corrections, is not in fact relevant to the energy loss deposited directly in the medium in the form of ionization and excitation.

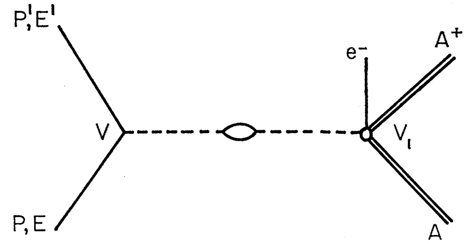

FIG. 2.

2.3(ii) *Mass Operator Method*

The self-energy or mass operator of the incident charged particle in lowest order may be written down according to the Feynman prescription as

$$\Sigma_1(\mathbf{P}, E)=\frac{2\alpha}{(2\pi)^4}\frac{i\hbar^2}{c}\int_{-\infty}^{\infty}d\omega$$
$$\times\int d\mathbf{k}\left(\frac{4\pi(v\wedge\mathbf{k})^2}{[\mathbf{k}^2-\omega^2\epsilon(\omega)+i\delta]\mathbf{k}^2}-\frac{4\pi}{\mathbf{k}^2\epsilon(\omega)}\right)$$
$$\times(E-\epsilon_{\mathbf{P}-\hbar\mathbf{k}}-\hbar\omega+i\delta)^{-1}\equiv\int_{-\infty}^{\infty}\Sigma_1(\omega)\,d\omega,\quad (8)$$

where $\epsilon_{\mathbf{P}-\hbar\mathbf{k}}=|\mathbf{P}-\hbar\mathbf{k}|$ in the extreme relativistic approximation. This corresponds to the diagram of Fig. 4.

The calculation of the energy loss by this method consists in evaluating

$$W=(2N/v)\int_0^{\infty}\omega\ \mathrm{Im}\ \Sigma_1(\omega)\,d\omega,\quad (9)$$

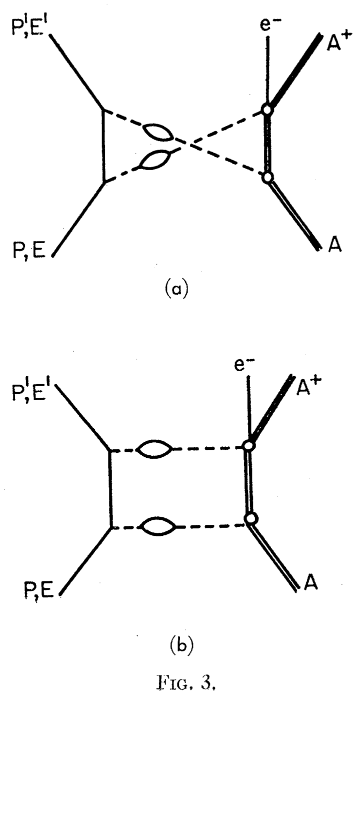

FIG. 3.

by placing both particle and photon on their respective mass shells so that the photon propagator is replaced according to

$$(\mathbf{k}^2-\omega^2\epsilon(\omega)+i\delta)^{-1}\rightarrow -i\pi\delta[\mathbf{k}^2-\omega^2\epsilon(\omega)]\quad (10a)$$

and

$$[\mathbf{k}^2\epsilon(\omega)]^{-1}\rightarrow-(i\pi/\mathbf{k}^2)\delta[\epsilon(\omega)]\quad (10b)$$

(which are legitimate in the case of zero damping), and the particle propagator is replaced according to

$$(E-\epsilon_{\mathbf{P}-\hbar\mathbf{k}}-\hbar\omega+i\delta)^{-1}\rightarrow -i\pi\delta(E-\epsilon_{\mathbf{P}-\hbar\mathbf{k}}-\hbar\omega).\quad (11)$$

If in (8) we replace E by $\epsilon_{\mathbf{P}}$ (ignoring any mass shift due to the dielectric medium), we find that for a relativistic incident particle, taken to be approximately undeviated, (11) becomes

$$(E-\epsilon_{\mathbf{P}-\hbar\mathbf{k}}-\hbar\omega+i\delta)^{-1}\rightarrow -i\pi\delta(\hbar\omega-\hbar\mathbf{k}\cdot\mathbf{v}).\quad (11')$$

It is helpful to discuss replacements (10a) and (10b) in more detail at this point.

On introducing cylindrical coordinates k_ρ, k_z, ϕ with the z axis along the incident direction, we see from

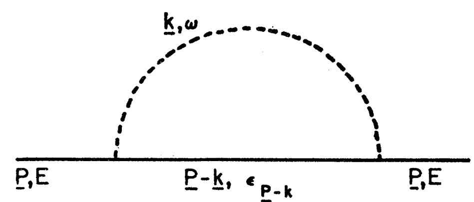

FIG. 4.

(10a) and (11′) that only frequencies satisfying

$$\mathbf{k}^2=(\omega^2/c^2)\epsilon(\omega)\quad (12)$$

or

$$k_\rho{}^2=(\omega^2/c^2)\epsilon(\omega)-(\omega^2/v^2)\quad (12')$$

or

$$(\omega^2/v^2)[1-(v^2/c^2)\epsilon(\omega)]<0\quad (12'')$$

can contribute to the energy loss.

If $\epsilon(\omega)$ has the usual Kramers–Heisenberg form, then Eq. (12) is the condition satisfied by the dynamical excitons discussed by Hopfield (1958) and Fowler (1964), and (12′) is the condition for Cerenkov radiation. It follows that the lower frequency dynamical exciton mode can be excited as Cerenkov radiation [since (12′) holds when $(v^2\epsilon(\omega)/c^2)>1$]. The condition (10b) is satisfied by the longitudinal collective excitation of the medium corresponding to the familiar plasmon in the electron gas case.

The preceding is the basis of Tsytovitch's procedure, and it is clear that this gives the contribution to the energy loss arising from Cerenkov radiation (and plasmon creation) alone in the case of transparent media. Radiative corrections to this result may be obtained by including the contribution from higher-order diagrams to the self-energy, and these have been evaluated by Tsytovitch again assuming a transparent medium. As has been mentioned a reduction of approximately 5% to 10% of the plateau value was found. It is, of course, true that the Tsytovitch result agrees with the classical value for the energy loss so that the radiative corrections which he finds should be regarded as genuine corrections to the classical result. If they are not confirmed experimentally, this may be regarded as a refutation of the classical formula as applied to ionization loss. However, in the presence of strong absorption, the use of (10a) is no longer legitimate over an important part of the region of integration, and indeed $\mathbf{k}^2-(\omega^2/c^2)\,\mathrm{Re}\,\epsilon(\omega)$ may have no real roots according to Sternheimer. Consequently, we must conclude that the Tsytovitch prediction is inapplicable to ionization-loss studies.

It should be said, and we discuss this point further in the next section, that in the classical calculations of energy loss, the dielectric function $\epsilon(\omega)$ used contains no damping terms, even in the strongly absorbing case. Nevertheless, this should not be taken to imply that 10(a) is relevant.

We examine next the connection between the mass operator method described above and the classical impact-parameter approach given by Fermi.

2.3(*iii*) *Classical Impact-Parameter Method*

The connection between (9) and this method can be found by substituting (11′) in (8), and introducing cylindrical polar coordinates. This gives

$$\tilde{\Sigma}_1=\frac{\alpha v\hbar}{2\pi c}\left[\int_{-\infty}^{\infty}d\omega\left(\int_0^{\infty}\frac{k_\rho^2\,d(k\rho)^2}{(k_\rho^2+\omega^2/v^2)(k_\rho^2+\tilde{z})}-\frac{c^2}{v^2}\times\int_0^{\infty}\frac{d(k_\rho^2)}{(k_\rho^2+\omega^2/v^2)\epsilon(\omega)}\right)\right],\qquad(13)$$

where

$$\tilde{z}=(\omega^2/v^2)[1-(v^2/c^2)\epsilon(\omega)],$$

and $\tilde{\Sigma}_1$ differs from Σ_1 in that the intermediate charged particle is taken to be on its mass shell. The process considered is then one in which energy and momentum are exchanged between two charged particles so that

$$\tilde{\Sigma}_1=\frac{\alpha v\hbar}{2\pi c}\int_{-\infty}^{\infty}d\omega\int_0^{\infty}d(k_\rho^2)\,\frac{[v^2\epsilon(\omega)-c^2]}{v^2\epsilon(\omega)}\,(k_\rho^2+\tilde{z})^{-1}$$

$$=\frac{\alpha v\hbar}{2\pi c}\int_{-\infty}^{\infty}d\omega\left[\frac{[v^2\epsilon(\omega)-c^2]}{v^2\epsilon(\omega)}\ln\left(\frac{k_{\rho_{\max}}^2+\tilde{z}}{\tilde{z}}\right)\right].$$

If $k_{\rho_{\max}}^2+\tilde{z}\approx k_{\rho_{\max}}^2=(b_{\min}^2)^{-1}$, we find for the energy loss

$$W=\mathrm{Im}\,\frac{\alpha c\hbar}{\pi v^2}\int_0^{\infty}\omega\,d\omega\left(\frac{v^2}{c^2}-\frac{1}{\epsilon(\omega)}\right)\times\ln\frac{v^2}{\omega^2\,|\,1-[\epsilon(\omega)\,v^2/c^2]\,|\,b_{\min}^2},$$

which agrees well with Fermi's result in the limit $(v/c)\rightarrow 1$.

As pointed out in Sec. 2.3(ii), the result for zero damping should reduce to the contribution from Cerenkov radiation and plasmon excitation alone. A comparison with Fermi's result shows that this is indeed true in the impact-parameter formulation [c.f. Eqs. (4–6) of Fermi].

In order to account for the observed relativistic increase in ionization as distinct from Cerenkov radiation, Sternheimer (1953) considered the effect of strong absorption separately. Using a model for the polarizability of the medium derived from the theory of x rays, he was able to show that in this case the range of frequencies over which the Cerenkov condition is effectively satisfied is much reduced, so that in fact the loss of energy at large distances from the track through Cerenkov radiation, is very much reduced. This conclusion is supported by Budini (1953) who used a model for $\epsilon(\omega)$ with large damping constants. However, in the actual evaluation of the energy loss using the classical model, dielectric functions in which there is no damping are used, and the ionization continuum is represented by discrete lines at or about the ionization levels, so that the numerical results ought in principle to refer to the case in which Cerenkov radiation escapes. The use of these functions in interpreting the ionization loss is therefore based on two assumptions: first, that the energy loss per ion pair is constant and independent of incident particle energy (see Sec. 3.2); and second, (and more important for the purposes of the present discussion), that the classical formula for the energy loss is correct, and that the consistent introduction of an ionization continuum in $\epsilon(\omega)$ in place of discrete levels would not change the numerical results significantly. On the other hand, from the point of view of the physical interpretation of the formulas, the difference is important, particularly if one wishes to calculate radiative corrections to the ionization loss. It seems possible, therefore, that a direct calculation of ionization energy loss as in Sec. 2.3(i), besides being free of ambiguity, may give different results from the classical theory.

The problem may also be discussed by using the Williams–Weizsäcker method to which we now turn.

2.3(*iv*) *Williams–Weizsäcker Method*

This approach is closely related to the previous one, but whereas that is essentially a classical calculation of the Poynting vector representing energy lost by the particle, the W–W method refers explicitly to transitions in which an atom of the medium is directly excited or ionized, and the method may therefore be used in a quantum mechanical calculation.

The energy loss actually deposited in the medium per unit length of path at an impact parameter b is written in the form

$$\frac{dW(b)}{db}=\frac{N}{v}\int_0^\infty S(b,\omega)\sigma(\omega)\,d\omega, \qquad (14)$$

where $S(b,\omega)/\omega$ is the virtual photon flux at b corresponding to the incident particle, and $\sigma(\omega)$ is the experimental photo cross section for energy transfer ω. The quantity $S(b,\omega)$ may be evaluated from the Fermi expressions for the electric and magnetic fields. If this is done, then using (7′), expression (14) is identical to the derivative, with respect to the impact parameter b, of Fermi's result.

Hence the energy loss corresponding to all impact parameters $b \gg b_{\min}$ is now given by Fermi's result, i.e.,

$$W=\frac{\alpha\hbar c}{\pi v^2}\int_0^\infty \omega\,d\omega\,\frac{\operatorname{Im}\epsilon(\omega)}{|\epsilon(\omega)|^2}\ln\frac{4v^2}{\gamma^2\omega^2|1-[\epsilon(\omega)v^2/c^2]|\,b_{\min}^2}+\frac{\alpha\hbar c}{\pi v^2}\int_0^\infty \phi\left(\frac{\operatorname{Re}\epsilon(\omega)}{|\epsilon(\omega)|^2}-\frac{v^2}{c^2}\right)\omega\,d\omega, \qquad (15)$$

where

$$\tan\phi=-\frac{(v^2/c^2)\operatorname{Im}\epsilon(\omega)}{1-(v^2/c^2)\operatorname{Re}\epsilon(\omega)}.$$

It is important to realize that (15) contains a contribution from Cerenkov radiation which is not contained in (14). To exclude it we may assume that $|\phi|\ll 1$ which is true if $\operatorname{Im}\epsilon(\omega)\ll 1-(v^2/c^2)\operatorname{Re}\epsilon(\omega)$, i.e., $1-(v^2/c^2)\operatorname{Re}\epsilon(\omega)\neq 0$.

The energy-loss expression is then

$$W=\frac{e^2}{\pi v^2}\int_0^\infty \omega\,d\omega\,\frac{\operatorname{Im}\epsilon(\omega)}{|\epsilon(\omega)|^2}\times\left(\ln\frac{4v^2}{\gamma^2\omega^2|1-(v^2/c^2)\epsilon(\omega)|\,b_{\min}^2}-\frac{v^2}{c^2}\operatorname{Re}\epsilon(\omega)\right), \qquad (16)$$

which has been used by Budini (1953) to evaluate the ionization loss.

2.4 Discussion and Conclusions

The first point to note about (16) is that in the limit of zero damping the factor $\operatorname{Im}\epsilon(\omega)/|\epsilon(\omega)|^2$ is represented by a sum of δ functionlike singularities at the zeros of $\epsilon(\omega)$, so that (16) would show no relativistic increase at all in this approximation. This simply reflects the fact that, when damping is neglected, all the relativistic increase in energy loss is contained in the Cerenkov radiation. This conclusion is changed when a more realistic continuum model for $\epsilon(\omega)$ is considered. As has been mentioned this leads to the disappearance of the Cerenkov radiation.

From the Williams–Weizsäcker point of view we may represent the more realistic case by including damping constants in the usual multilevel formula for $\epsilon(\omega)$, following Budini; the ionization loss given by (16) then shows a relativistic increase. This leads again to the point discussed in Sec. 2.3(iii): that is that the classical calculation is used to predict the ionization loss with zero damping constants but (16), which is entirely equivalent to the classical calculation with Cerenkov loss subtracted, requires nonzero damping if a relativistic increase in ionization loss is to be observed. Thus the increase predicted by (16) must be matched by a reduction in Cerenkov radiation so that the classical formula predicts a relativistic increase in energy loss which is essentially independent of damping constants. In fact, a close scrutiny of the classical result even including damping [Sternheimer (1952), Eqs. (38)–(46) with $l=0$] shows that the terms which depend on damping do not contribute to the relativistic increase in energy loss. This is consistent with earlier remarks. The point being made here, as elsewhere, is not that the classical formula evaluated with zero damping is incorrect, but rather that it contains the assumption that, in the presence of absorption, the Cerenkov loss falls, and the ionization loss increases *pari passu*, so that the classical formula gives the correct result. Indeed, the rather good agreement between Sternheimer's calculations and experiments show that this is generally true, but it should be said that the parameters appearing in the expression for $\epsilon(\omega)$ are generally chosen to give the correct mean excitation potential appearing in the Bethe–Bloch formula. It may be that the Sternheimer approach corresponds to a best-fit classical formula. In the comparison with experiment, we shall use it as a theoretical yardstick.

The more direct method of Sec. 2.3(i) which is capable of giving, in addition, the radiative corrections, requires rather detailed information on the photo-ionization cross sections and form factors. The calculations have so far only been performed approximately for oxygen, with results which will be described in a later section.

3. THE IONIZATION PROCESS

3.1 Fluctuation Theories and the Most-Probable Energy Loss

Before discussing the experimental results in detail, it is necessary to explain the precise relationship between what is measured and what is predicted theoretically. In the first place, the theoretical section has been concerned with the density effect on the mean energy loss of a charged particle. However Bohr (1913, 1915), Williams (1929), and Landau (1944) have pointed out that since the energy lost by a particle passing through matter is the result of a large number of independent events, the process is a statistical phenomenon, i.e., no unique value for the energy loss is obtained. It was shown that the resultant energy-loss distribution is negatively skewed—the high energy-loss tail being due to those collisions in which a large amount of energy is transferred to the target electron in a single collision. In a "thin" absorber, the small probability of such collisions results in a relatively large random statistical variation in their number, and thus fluctuations in the total energy loss occur. Several theories, which are reviewed in the Appendix, predict the probability distribution of energy loss. Of these, that of Vavilov (1957) is probably the most comprehensive.

It should be noticed that fluctuations are only important over the range $0.01<K<1$ (see Appendix), and the vast majority of experimental results presented in this review fall outside this range. Furthermore, because the difference between mean and most-probable loss is related to the close collisions, for which no density effect is involved, all the discussion and conclusions of the theoretical section are relevant when the most-probable rather than the mean energy loss is being considered. In fact, the high-energy transfer close collisions are also generally difficult to record satisfactorily. Such events may be rejected (as in cloud chambers or emulsions), undetected (the escape of high-energy knock-on electrons from scintillators), distorted (saturation of detector or electronics), or may be simply spurious (a simultaneous air shower). Hence what is liberated and what is detected in the medium need not be the same thing; it is important to note that the basic theory is concerned with the former.

The most-probable energy loss, which corresponds to the peak of the measured experimental distribution, can be calculated from the theory of Landau. Justification for following Landau's approach when considering relativistic particles is given in the Appendix. In terms of the notation used by Sternheimer (1953b, 1956), we have for a particle of mass μ, momentum P, and velocity v on traversing an absorber:

$$E_p=(Ax/\beta^2)[B+1.06+2\ln(P/\mu c)+\ln(Ax/\beta^2)-\beta^2-\delta(\beta)], \quad (17)$$

where

$$A=2\pi ne^4/mc^2\rho_0=0.1536(Z/A_0)\ \text{MeV g}^{-1}\,\text{cm}^2 \quad (18)$$

and

$$B=\ln[mc^2(10^6\ \text{eV})/I^2]. \quad (19)$$

Here n is the number of atomic electrons (mass m) in a cubic centimeter of the material of thickness x g cm^{-2}, and density ρ_0 g cm^{-3}; Z, A_0, and I refer to the atomic number, atomic weight and mean *excitation* potential of the material; $\delta(\beta)$ is the correction factor for the density effect produced by local polarization of the material by the incident particle as described in the theoretical section, and β is v/c.

Few accurate experimental measurements of the mean excitation potential have been made, and in the past a rough empirical rule has been $I=13Z$ eV. However a best fit to the experimental data is given by the semiempirical expression proposed by Sternheimer (1966, see also Turner, 1964) for $Z\geq 13$ which is

$$I/Z=9.76+58.8Z^{-1.19}\ \text{eV}. \quad (20)$$

For compounds, the value of I is given by the logarithmic average of the I values of the constituent atoms. Thus,

$$\ln I=\sum_k f_k \ln I_k, \quad (21)$$

where f_k is the fractional number of electrons of the kth atomic species with excitation potential I_k.

Analytical expressions for $\delta(\beta)$ are given by Sternheimer (1956) as

$$\delta(\beta)=4.606X+C_1+a(X_1-X)^{m'}, \quad \text{for} \quad X_0<X<X_1 \quad (22a)$$

and

$$\delta(\beta)=4.606X+C_1, \quad \text{for} \quad X>X_1, \quad (22b)$$

where X is given by log $(P/\mu c)$, and a, m', and C_1 are constants which depend on the substance; X_0 is the value of X which corresponds to the minimum below which $\delta(\beta)=0$ [see Eq. (9a), Sternheimer, 1952]; and X_1 corresponds to the momentum above which the relation between $\delta(\beta)$ and X can be considered linear (see Figs. 1 and 2, Sternheimer, 1952). Substituting for X in Eqs. (22a) and (22b), we obtain for the same operating conditions:

$$\delta(\beta)=2\ln(P/\mu c)+C_1+a(X_1-X)^{m'} \quad (23a)$$

and

$$\delta(\beta)=2\ln(P/\mu c)+C_1. \quad (23b)$$

The important problem of locating the appropriate mode has troubled many workers. A number of methods the description of which falls outside the scope of this article have been used. Those interested should consult Ghosh *et al.* (1954), Bowen (1954), Price (1955), Barnaby (1961), and Simpson (1964).

3.2 The Energy Loss Per Ion Pair

A more serious problem arises when the measured quantity is actually the ionization produced by the charged particle. In this case the use of the classical formulae for the total ionization energy loss involves some assumption concerning W, the average energy lost by an ionizing particle per ion pair created. The value of W is of the order of 30 eV for all gases, and is often assumed to be constant for a given material. Knowledge of its magnitude is necessary for a comparison between the experimental results (number of ion pairs per centimeter) and theory (energy loss per centimeter). Jesse and Sadauskis (1955), Weiss and Bernstein (1956), and Jesse (1961) have tabulated values of W (see Table I).

One difficulty is that there is no fully reliable theoretical calculation of W values other than a possible one for helium. A recent review of the subject by Doust and Harris (1968) has spotlighted the scarcity of reliable experimental data. Experiments to determine W contain three parameters of interest. These are the type of gas used, the type of particle used to ionize the gas, and the energy of the primary ionizing particle.

TABLE I. Experimental values of W_β (in eV).

Gas	Weiss and Bernstein	Jesse and Sadauskis	Jesse
Air	33.9	34.1	33.8
H_2	36.3±0.7	36.3	
He	40.3±0.8	42.3	
Ne	35.3±0.7	36.7	
A	25.8±0.5	26.4	
Kr	24.7±0.5	24.2	
Xe	22.0±0.4	22.2	
O_2	31.2±0.6	30.9	
N_2	34.6±0.7	34.7	35.0
C_2H_4	26.4±0.5	26.3	26.2

It is now well known that to a first-order approximation, W is independent of the type of gas, and is about twice the first ionization potential $I_1(Z)$: Platzman (1961) gives $W/I_1(Z)=1.73$ for the noble gases. Air, nitrogen, argon, and helium have all been extensively studied, though W values in pure helium are still very much in doubt with possible errors up to 10%. This is probably due to the great importance of very small quantities of impurity in the gas.

Most experimental determinations of W have been obtained with α particles from radioactive isotopes. This limits the range over which the quantity W_α is measured to 4–9 MeV. It would appear incorrect to *automatically* use W_α values in experiments where the primary particles are betas, muons, pions or protons. For example, Jesse and Sadauskis showed that the ratio W_α/W_β differs from unity by only a few percent for the noble gases and hydrogen, but by about 6% for molecular gases. However, as the energy of the α particle increases, then W_α does tend to approach W_β. (It is interesting that $W_\alpha > W_\beta$ for gases, while $W_\alpha < W_\beta$ for silicon semiconductor detectors.)

Values of W for beta particles are very difficult to obtain owing to the greater range of these particles and the complex nature of the energy spectrum emitted from radioactive isotopes. The corresponding value of W for protons (W_p) is also of interest. In the 2-MeV region, $W_p \simeq W_\alpha$, whereas at 340 MeV, $W_p \simeq W_\beta$. Leake (1967) has concluded that W is a function of either velocity or specific ionization, rather than of particle type. Accurate W-value work on such particles as muons is still a long way off.

It has been suggested that statements indicating that W is an energy independent constant should really read that W is independent of energy to within certain limits and within a given energy range. Although the classical work of Jesse and Sadauskis with α particles in argon has been challenged by Leake, their conclusion that W_β is always independent of energy has found general acceptance.

It would be of interest to use very high energy α particles (or protons) to see if $W_\alpha = W_\beta$ for all gases at high energies.

4. INTRODUCTION TO REVIEW OF EXPERIMENTAL RESULTS

Although Cousins and Nash (1962), Fano (1963), and Sternheimer (1961) made brief mention of some of the experiments on ionization loss, there has not been a detailed experimental review since that of Price (1955). The intention of the present review is to build on Price's work, but first some general remarks are appropriate.

The validity of ionization results when using cosmic radiation depends directly on the accuracy of the momentum measurement for the incident particle. High particle energies require the use of magnetic spectrographs which consist of counter-hodoscope arrays placed several meters apart on either side of a strong magnetic field in which the particle is deflected [see Fig. 5(a)]. Vertical spectrographs were already in use before 1955 (e.g., Hyams *et al.*, 1950), but have been successfully developed to incorporate arrays of neon flash tubes and trays of Geiger tubes, to facilitate a more accurate location of the particle's trajectory (Hayman and Wolfendale, 1962). A further advance has been the use of a solid iron magnet in place of an air gap magnet to produce magnetic fields (Bull *et al.*, 1965). This gives an increased magnetic field and magnetic volume—hence an increased particle flux—as well as a muon beam uncontaminated by the nuclear active component. Such magnets have also been used with horizontal spectrographs (see Ashton and Wolfendale, 1963). These devices were constructed taking into account the fact that the sea level flux of muons of energy above 400 GeV at zenith angles approaching 90° should be much greater than the corresponding vertical flux (Jakeman, 1956).

An important spectrograph parameter is its "maximum detectable momentum" (mdm) which has been arbitrarily defined as the momentum at which the rms error in the deflection measurement equals the magnetic deflection of the particle. Values for the mdm vary over the range 150–5000 GeV/c, and are being constantly improved; [For a review of magnetic spectrographs see Wolfendale (1967).] For ionization-loss studies it is essential that the mdm be well above the value at which the so-called Fermi plateau is reached.

The alternative approach of using machine-produced particles, as shown in Fig. 5(b), has been used extensively with nuclear emulsions but less frequently in conjunction with other detectors. Here the main problem involves the production of a weak enough flux of particles or low beam currents. This difficulty is clearly brought out by Aggson and Fretter (1962) who found that the undesirable recombination of the ions in the gas only became unimportant if the beam current was sufficiently low (10^{-12} A). Unfortunately, for such low values of beam currents, the accelerator became difficult to regulate, resulting in erratic pulsing.

In connection with the theoretical predictions, the most satisfactory method of calculating the ionization loss, described in Sec. 2.3(i), depends upon a knowledge of the theoretical cross sections for ionization by virtual photons, or at least of the experimental cross sections by real photons. Since these are known in few cases, a satisfactory detailed comparison of theory and experiment is not generally possible. In these circumstances we have used the Sternheimer results as a measure of the theoretical predictions in the following sections, which review the most recent experimental results on the density effect, and in fact the agreement with experiment of the Sternheimer theory is generally reasonable. It may be, however, that difficulties arise when a detailed understanding of the mechanisms of the energy loss is required.

The review of the experimental work is subdivided according to the type of detector used. (Since 1955 there have been no experiments with low-pressure Geiger tubes.) Finally, Sec. 9 contains a brief summary of the conclusions, and suggestions for further work.

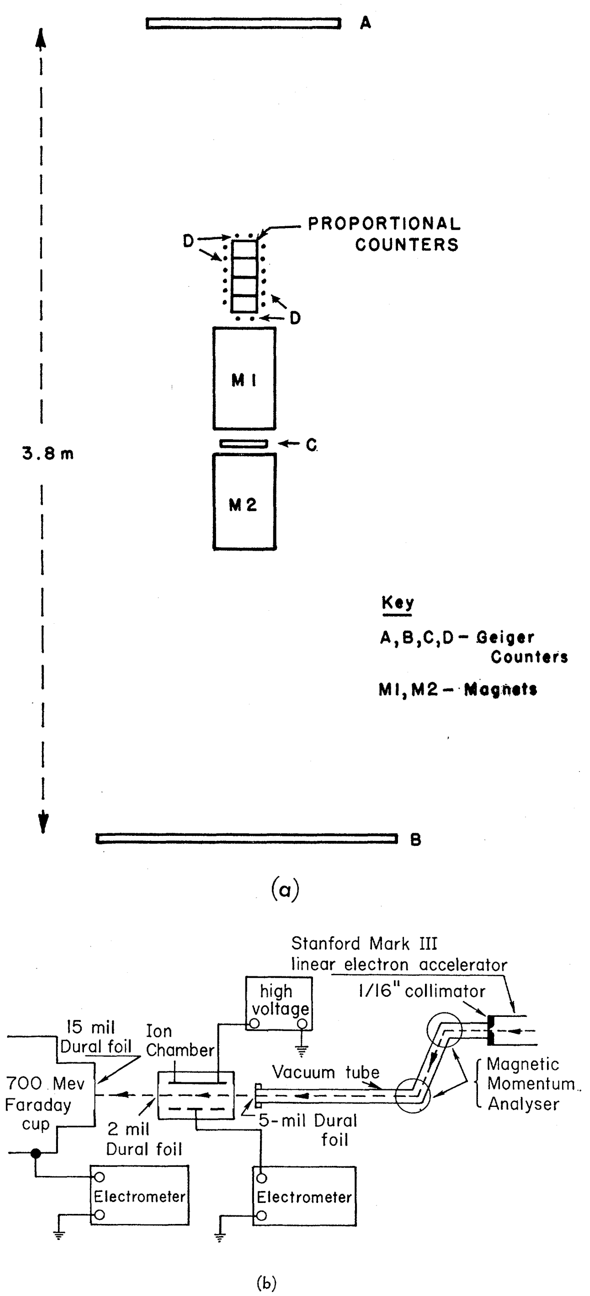

FIG. 5. (a) A magnetic spectrometer (after Lanou and Kraybill, 1959); (b) Experimental arrangement with accelerator (after Aggson and Fretter, 1962).

5. RESULTS OBTAINED USING GASEOUS DETECTORS

Originally it was the possibility that the large relativistic increase in ionization might be used to measure the velocity of high-energy particles that stimulated interest in this field. However, up until 1955 only Ghosh *et al.* (1952, 1954) and Eyeions *et al.* (1955) had convincingly detected the relativistic rise in ionization and also the existence of the Fermi plateau. Contemporary interest in the subject has been sustained as a result of discrepancies between theory and experiment, and indeed between some of the experimental results themselves. Care must be taken when comparing experiments with each other since the onset of the density effect (δ) varies with: (a) the gas pressure. Sternheimer (1952) has shown that the relation giving δ_p as a function of the pressure p (in atmospheres) is

$$\delta_p(P)=\delta_1(Pp^{1/2}), \tag{24}$$

where P is the momentum at which δ_p is evaluated, and δ_1 is the density effect at one atmosphere; (b) the atomic number Z. This result is expected as a consequence of the tighter binding energies of the electrons in materials of large Z [see Fig. 4 of Sternheimer (1952)].

5.1 Proportional Counters

In designing a proportional-counter experiment the following experimental details should be considered: (a) care should be taken, not to saturate the counter by using too high a gas amplification, and also to reduce the variation in the counter sensitivity due to recombination; (b) the necessity of using x rays for energy calibration (Jones *et al.*, 1963); (c) the use of more than one counter to increase the number of observations N^1 as the uncertainty in identifying the most-probable energy loss is proportional to $1/(N^1)^{1/2}$; (d) to design the counters for use with pressures

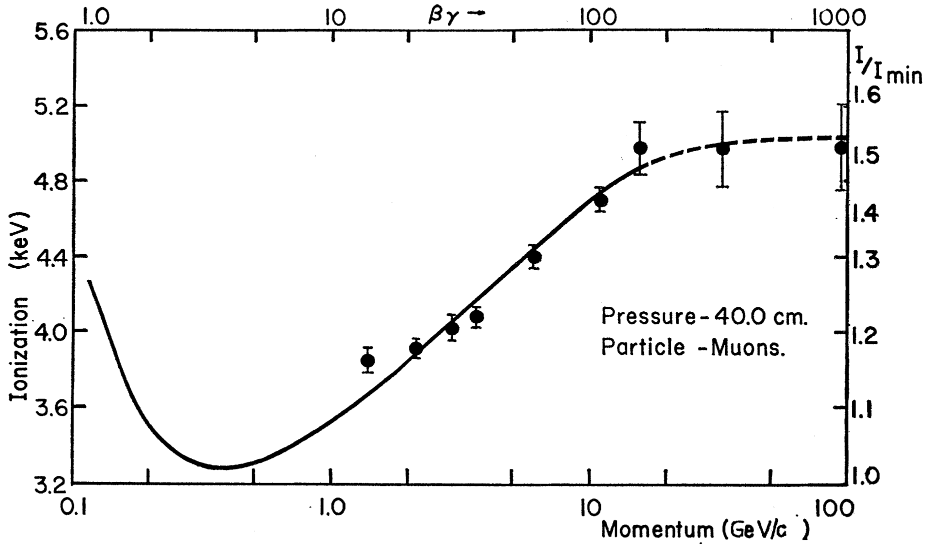

FIG. 6. The most-probable ionization in neon (after Eyeions *et al.*, 1955).

(p atm.) and particle path lengths (t cm) which satisfy the condition $pt \gg 2$.

With reference to (d), Bradley (1955) showed experimentally that, if the value of the product pt is about 70, then the observed distribution, for any gas filling, begins to approximate Landau's curve. This would give experimentalists greater confidence in comparing their observed most-probable ionization with Landau's calculated value for the energy loss.

Until 1955 only Parry *et al.* (1953) and Eyeions *et al.* (1955) had investigated the high-energy plateau region. Figure 6 shows that the latter group confirmed the existence of the plateau.

Results since 1955 have been rather disappointing. Jones *et al.* using the Durham vertical spectrograph and a neon–methane mixture examined the ionization by cosmic-ray muons over the range $3 \leq \beta\gamma \leq 300$. The plateau was not reached. Their results agreed with theory over the limited momentum range investigated and indicated, as did Eyeions *et al.* with the same mixture, a somewhat smaller rate of increase than the predicted value.

The results of Lanou and Kraybill (1959) for $31 \leq \beta\gamma \leq 1300$ were obtained with four helium counters. Their gas pressure was 2.7 atm—the value of pt was 20.7—and hence the onset of the density effect was observed at lower particle energies. Although the measured rise ($28\% \pm 4\%$) was less than the expected value, the results were in general agreement with the Landau treatment of the collision loss, corrected by Sternheimer (1953, 1956) for the density effect. The theoretical curve is sensitive to the value for the ionization potential $I(Z)$, and their results indicate a preference for the value extrapolated by Sternheimer from Bakker and Segrè (1951), as opposed to that of Williams (1937). The authors do not discount the possibility that more of the energy loss escapes as Cerenkov radiation than is allowed for by the Sternheimer theory.

More recently, Smith and Stewart (1966) have examined the ionization loss of electrons in the range $50 \leq \beta\gamma \leq 300$ in an argon–methane mixture. Their results indicate the onset of the density effect, but unfortunately, as in the case of Jones *et al.*, the momentum-range examined was not extensive enough to quantitatively measure the relativistic rise, or to comment conclusively on the validity of the Tsytovitch correction.

5.2 Cloud Chambers

Kepler *et al.* (1958) and Rousset *et al.* (1959) have studied the relativistic increase of ionization in various gases by the method of drop counting in a cloud chamber. These workers used a magnetic spectrograph, as did Ghosh *et al.* (1954), to deduce the momentum of cosmic-ray muons and electrons.

Both groups delayed the expansion of their chambers by a few hundred milliseconds, thus allowing the ions time to separate by diffusion and to form the nucleus of separate drops which could be readily counted. As Kepler *et al.* indicate, the rate of drop growth is limited mainly by two things. First, by diffusion, because as the drop grows it depletes the vapor in the immediate vicinity and can grow only as rapidly as vapor diffuses to it. Second, by thermal conductivity, since as the vapor condenses heat is liberated, and the faster this heat escapes the faster the drop will grow; hence the worse the thermal conductivity of the gas the longer the delay required between expansion and illumination.

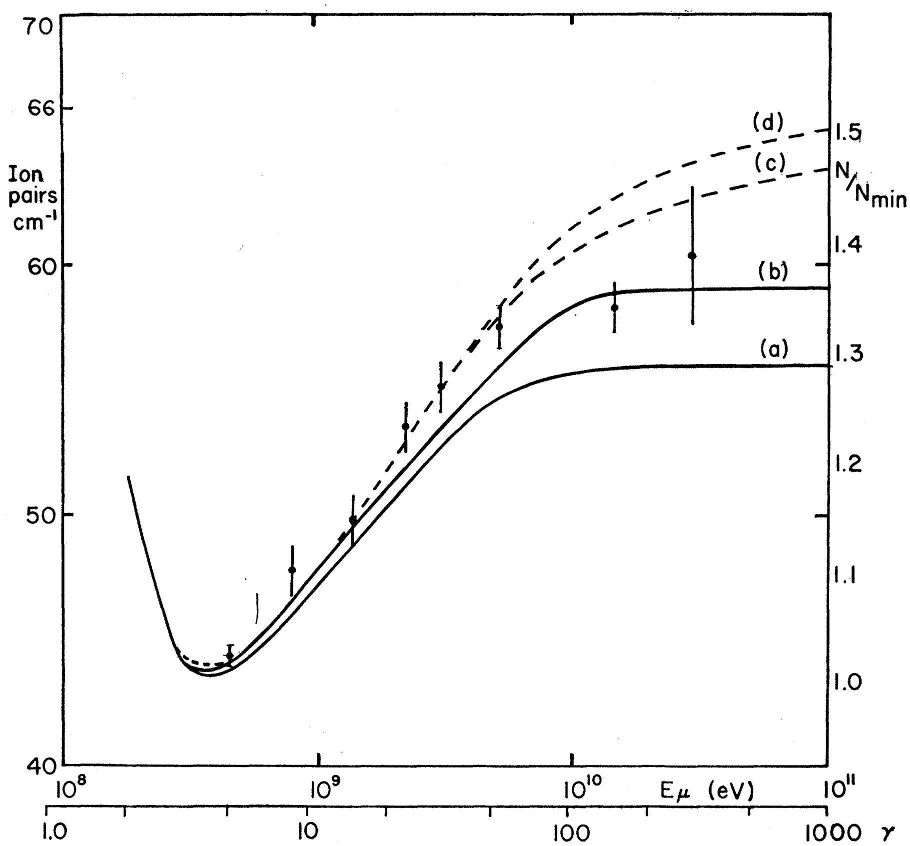

FIG. 7. Ionization of muons in oxygen. (a) Theoretical prediction of Budini (1953) [based on Eq. (16) of text]; (b) present work; (c) theoretical prediction of Sternheimer (1952, 1953) corrected for Cerenkov loss; (d) theoretical prediction of Sternheimer including Cerenkov loss. Experimental points are those of Ghosh *et al.* (1952).

For example, typical values in practice would be about 140 and 250 msec for helium and argon, respectively.

With this technique, the effect of the relatively infrequent high-energy collisions (see Sec. 3.1) is observed as a large cluster (or blob) of overlapping drop images. The impossibility of deducing the number of drops in a cluster dictates the measurement of the most-probable energy loss, as opposed to the average energy loss, which remains unknown. The effect of such clusters was eliminated from both experiments under consideration by first estimating the number of drops in a cluster and rejecting an event if it contained more than 40 drops; this is a measure of the maximum energy transfer (η) in a single collision. Values of η used were 700 eV (Rousset) and 960 eV (Kepler), but the observed value of ionization is not sensitive to the exact magnitude of this limiting value.

Using the CERN 600-MeV Synchrocyclotron, Ballario *et al.* (1961) adopted a different approach. Specific primary-ionization measurements were made on postexpansion electron tracks ($5 \leq \beta\gamma \leq 680$) in a helium–alcohol mixture at 1.07 atmospheres total pressure. With postexpansion operation the time interval between the passage of the particle and the photographic recording is shorter, and counter control is not essential. Thus, momentum measurements should be more precise (by virtue of narrower tracks and reduced track distortion due to gas movement), and measurements of the primary ionization should be much less affected by fluctuations in the negative-ion condensation efficiency (Hazen 1944), and by the Landau fluctuations. Apparently, the reduction in the effect of Landau fluctuations is due to the fact that with postexpansion operation one observes sets of drops close to the particle track, and there is therefore greater accuracy in measuring the actual number of primary ionizations.

It is desirable that throughout the duration of the experiment, the fraction of ions acting as condensation nuclei remains constant. As drops form more easily on positive than on negative ions, the condensation efficiency may be obtained by examining the tracks of ions of either charge which have been separated by the clearing field. Nielsen (1941, 1942) showed that effectively all the positive ions act as condensation nuclei if the expansion is controlled so as to give a

TABLE II. Values for the relativistic rise in helium at various pressures.

Author	Detector	Gas pressure (atm)	Average ionization potential (eV)	Relativistic increase (%) Exptl	Theory—predicted by:	
Ballario *et al.*	Cloud chamber	1.07	24.59	50±3	~36	Budini and Taffara
Kepler *et al.*	Cloud chamber	1.3	49.4		45	Sternheimer
					42	Budini
				42±3		
			35.7		~39	Sternheimer
					34	Budini
Lanou and Kraybill	Proportional counters	2.7	44.0	28±4	~40[a]	Sternheimer
			26.8		34[a]	
Barber	Ionization chamber	10.0	27.0	17±1	17.9	Sternheimer

[a] These values refer to curves which do not include any contribution from Cerenkov radiation.

negative/positive ratio greater than about 0.2:1 [a later communication of Ghosh *et al.* (1954) quotes a figure of 0.3]. Values of this ratio from the experiments of Kepler *et al.* and Ballario *et al.* appear satisfactory, while Rousset *et al.* do not quote any figures.

Although Ghosh *et al.* obtained accurate experimental results on the relativistic rise (using muons in the range $5 \leq \beta\gamma \leq 280$) and demonstrated clearly the existence of the density effect, their evidence as to the existence of a plateau was inconclusive; the same may be said of Ballario *et al.*

The results of Ghosh *et al.* have been compared by Fowler and Hall with the predictions of Eqs. (7) and (16), using experimental data on the photoionization cross sections of oxygen, and also with the Sternheimer theory. The results are shown in Fig. 7 and would appear to favor Eq. (7), although in view of the large experimental errors and the approximations made in evaluating the theoretical curves, this may not be significant.

The experimental ionization results of Ballario *et al.* with helium seem significantly greater (~20%) than those expected using the theory of Budini and Taffara (1956). It is noteworthy, however, that the theoretical curves based on (7) and (16) differ from each other in much the same way as the results of Ballario *et al.* differ from (16), so that perhaps calculations on helium based on (7) may give better agreement with experiment.

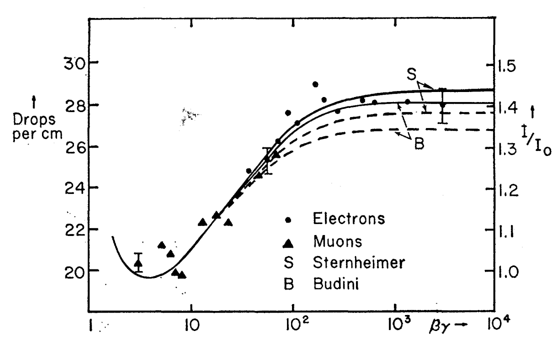

FIG. 8. Ionization loss in helium. The theoretical curves are normalized to the muons of $\beta\gamma < 30$, and are calculated using values of 49.4 and 35.7 eV (dotted curves) for the average ionization potential for the gas (after Kepler *et al.*, 1958).

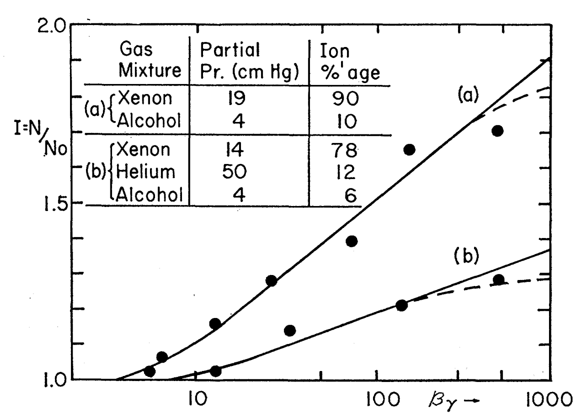

FIG. 9. Ionization loss in xenon and xenon–helium mixture. The solid curve is the best fit of all the experimental data below a $\beta\gamma$ of 500. The dotted curve is drawn by a more elaborate fitting method to account for a possible curvature in the upper part of the curve (after Rousset *et al.*, 1959).

The same might be said of Kepler's helium results compared to Budini's curve, when the latter is evaluated using the average ionization potential $I(Z)$ deduced from Sternheimer (1952). On the other hand, excellent agreement exists when Sternheimer's 1956 figures are substituted; this is clearly shown in Fig. 8, which also shows curves based on Sternheimer's theory. It is evident that the results are especially sensitive to the value of $I(Z)$ that is assumed. Table II indicates that the values used by the various workers differed in some cases by a factor of nearly 2 ! The preferred value of Berger and Seltzer (1964) is 42.0 eV for helium which is close to the old value of 44.0 eV, calculated by Williams (1937).

The results in Fig. 8 (Kepler *et al.*, 1958) strikingly demonstrate the presence of a plateau for values of $\beta\gamma$ from about 200 to 3000. Kepler *et al.* were not so successful, in this sense, when using argon, argon–helium, and xenon–helium. These workers concluded that the slope of the relativistic rise in the heavier gases was significantly smaller than that predicted theoretically; this is in direct contradiction with the results of Rousset *et al.* who, for example, detected a 69%±5% rise for xenon. This latter group, by adding helium to xenon, demonstrated convincingly a reduction in the relativistic rise and showed that the disturbing effect of helium increases in the same way as the partial pressure of the helium (see Fig. 9), a result which agrees with that of Kepler *et al.* using the same mixture. Rousset (1958) has endeavored to explain the effect as due to the presence of helium absorption lines in the region of the ionizing frequencies of xenon; the ionization potentials of helium and the $5p$ electrons of xenon are taken to be 24.5 and 12 eV, respectively. If this helium absorption contributes strongly to the polarization, then the $5p$ electrons of xenon may be prevented from participating in the ionization increase. Interestingly enough, the addition of hydrogen [$I(Z)$ of 13 eV] to xenon did not reduce the magnitude of the relativistic rise.

5.3 Ionization Chambers

This technique has been used by Barber (1955, 1956), Hall (1959), and Aggson and Fretter (1962). As in the case of the cloud chambers, recombination is a possible source of experimental error. This effect may be minimized by: (a) employing gases of low electronic density to keep the ion density as low as possible, and (b) using primary beams of large cross-sectional area and low intensity. Care must be taken to ensure that the beam cross section is not so large as to strike the aperture of the chamber and thus cause an electron cascade. With these precautions in mind, Barber (1956) calculated the residual recombination effect and found it to be about 2% or 3%.

As in all energy-loss studies, a knowledge of η (maximum energy transfer) is necessary for any comparison with theory. For ion-chamber investigations this is taken as the energy of a knock-on electron whose range is such that the electron just traverses the length of the chamber. Such an electron has an energy of 15 keV in hydrogen at one atmosphere. Since η occurs within the logarithm of the energy-loss equation an accurate knowledge of its magnitude is not essential. Aggson and Fretter estimated their value of η from the range–energy curves of Aron *et al.* (1949). Their apparatus has been referred to above [Fig. 5(b)], and the techniques and method of experimentation employed were similar to that of Barber. Also, both experiments used the Stanford 35-MeV linear accelerator to produce their primary electrons.

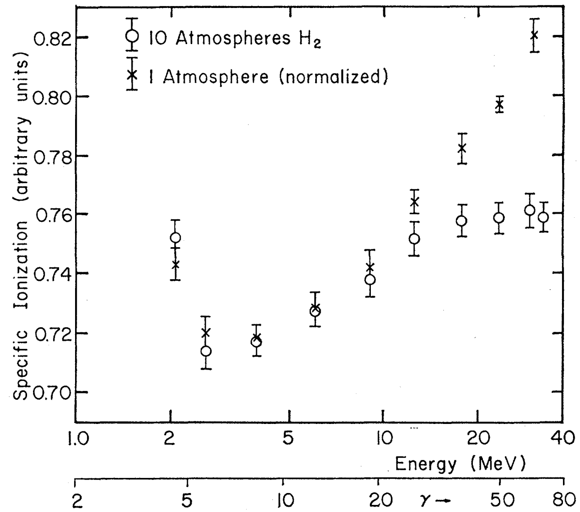

FIG. 10. Relative specific ionization in hydrogen as measured with a thick-window ion chamber (after Barber, 1956).

Hall's investigation need only be mentioned briefly since his results with argon (3.9 atm, 0°C) and cosmic-ray muons ($3.1<\beta\gamma<6.2$) only examined the minimum ionization region. Barber measured the specific ionization in both helium and hydrogen at 1 and 10 atmospheres pressure. No density effect was detected at the lower pressure as expected, but at the higher value the familiar plateau feature was observed (see Fig. 10). The measured rise after correcting for scattering by the ion-chamber window hardly differed from that expected from the calculations of Sternheimer, i.e. the differences were (1.5±1)% and (0.3±1.3)% in hydrogen and helium, respectively.

Aggson and Fretter designed their experiment both to extend Barber's measurements to higher energies and also to verify the reduction in ionization reported from the cloud-chamber experiments when helium was added to xenon (Sec. 5.2). Their results for hydrogen are plotted in Fig. 11 along with those of Barber and a

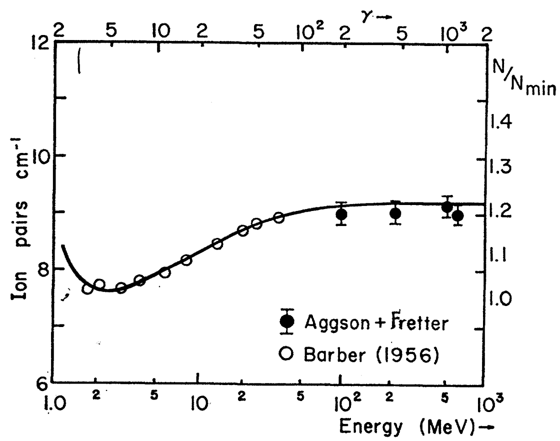

Fig. 11. Specific ionization in hydrogen at NTP by electrons. The curve plotted was calculated from Budini's model I (1953) using $W = 38$ eV/ion pair, and $I(Z) = 15.5$ eV (after Aggson and Fretter, 1962).

theoretical curve from Budini (1953). Good agreement was obtained. From the figure, the minimum number of ion pairs per centimeter is about 7.5 which is roughly 1.4 times the value found by using low-pressure Geiger counters (McClure 1953). Results for the xenon–helium mixtures were in good agreement with theory and did not show the substantial decrease in ionization that was expected on the basis of the cloud-chamber work. One tempting explanation for this discrepancy, advanced by Aggson and Fretter, is that the correction for overlapping droplets in the cloud-chamber work was underestimated. It was later established, however, that although a truer estimate of this correction was in the right direction, it was not large enough to bring the cloud-chamber results into accord with other gas-counter results and theory.

5.4 Discussion

Although much more experimental data is needed, particularly with heavier gases and with mixtures involving helium, some tentative conclusions may be reached:

(1) The existence of the plateau has now been reasonably well established, e.g., by Eyeions *et al.*, Barber (1956), Kepler *et al.*, and Aggson and Fretter (1962).

(2) There has been no indication of a decrease in ionization after reaching the Fermi plateau, as was reported by some workers with nuclear emulsions.

(3) (a) With the exception of Rousset *et al.*, using xenon, the measured relativistic rise to the plateau has been found to be less than that predicted by theory. (b) This is especially so with some of the cloud-chamber results in which mixtures involving helium were used. Here the effect is so pronounced as to be in conflict not only with theory but also with results using other detectors.

Possible explanations for 3(a) suggested by some of the workers involved are:

(i) The magnitude of η may depend on the atomic number. Instead of η being a fixed value, dependent on the number of drops in a cluster, it should be deduced by averaging the various values of η for the different electron shells. Kepler *et al.* substituted this average value in the theory of Sternheimer, and found that the slope was reduced, but not sufficiently to account for the observed discrepancy, e.g., in xenon the reduction was 3% at a $\beta\gamma$ of 100. Nevertheless, the effect would appear worth taking into account whenever energy-loss calculations are made.

(ii) The ratio of the energy lost by ionization to the energy lost by excitation may not be energy independent, and may vary with atomic number. In the case of mixtures, the proportionality between both types of collision loss might not hold at high energies due to the complexity of secondary ionization (Rousset *et al.*).

(iii) Jones *et al.* (1963) have drawn attention to the possibility of electronic differentiation which would result in a smaller pulse from the muon. With a proportional counter this effect would be undetected when calibration x rays are used, since their ionization does not lie along such an extended track as does the muon.

The disagreement between the cloud- and ionization-chamber observations remains unresolved at present. An explanation may lie in a physical phenomenon outlined by Rousset in Sec. 5.2 or in some unknown imperfections in the cloud-chamber experimental technique. Various workers using helium *alone* have often detected a low value of the relativistic rise to the plateau (see Table II).

Finally, attention should be paid to the role of Cerenkov radiation, even if experimenters decide after consideration that its contribution is insignificant (as did Kepler *et al.* 1958). Sternheimer (1953) has pointed out that its importance in the relativistic increase arises from the fact that the total ionization loss includes Cerenkov radiation. Hence, the energy deposited in the region of impact parameters $<b$ can be obtained from the total ionization loss by subtracting the Cerenkov component (W_b) for impact parameters $>b$; for cloud-chamber experiments half the width of the track should be used (i.e., $b \simeq 0.1$ cm). An estimation of W_b is given by Eqs. (35) and (36) of Sternheimer (1953). These equations indicate that W_b may be appreciable for light gases, while being negligible for heavier gases ($Z \gtrsim 10$). For example, taking $\beta = 1$, the values of W_b in H_2, He, O_2, and Xe are 0.130, 0.085, 0.017, and 0.006 MeV g^{-1} cm^2, respectively (Sternheimer, 1956).

6. RESULTS OBTAINED USING SCINTILLATION COUNTERS

6.1 Organic and Inorganic Devices

It is conventional to divide scintillation counters into two main categories: organic and inorganic counters.

There have been remarkably few experiments with organic crystals, which is somewhat surprising in view of the fact that an appreciable logarithmic rise—about 11%—may be observed.

In the more easily polarizable organic materials, the density effect should be greater than that found for the inorganic scintillators and therefore the relativistic increase should be practically eliminated. When this was established interest in using these materials waned. However, the availability of large area plastic phosphors (e.g., NE 102A), and the development of very large area liquid scintillators mainly based on xylene or paraffin has revived interest considerably. These devices ensure a good counting rate, and have been used successfully with cosmic-ray spectrographs.

With scintillation counters, whether organic or inorganic, it is the light output that is measured. Price (1955) draws attention to the fact that organic crystals and liquids show a nonlinear response for heavily ionizing tracks but, that this nonlinearity is fortunately negligible for the more lightly ionizing relativistic particles. Chou (1952) using pions and protons from the Chicago cyclotron showed that most scintillators are nearly linear up to three or four times the minimum value of ionization. This result was put in doubt by the results of Baskin and Winckler (1953) who, using an organic liquid counter, found that the ionization rise at low muon energies was much less rapid than Chou's results indicated. A similar result may be inferred from the results of Barnaby (1961) as originally plotted; though such an inference depends very much on the particular point of normalization of the experimental points to the theoretical curve. The results of Crispin and Hayman (1964) do not show any effect at low muon energies.

It is well known that the experimentally obtained pulse height distributions are not purely "Landau" in shape, but somewhat broader. Here the resolution of a counter is defined as the peak width at half-maximum height of the differential pulse height distribution, expressed as a percentage of the pulse height corresponding to the peak position. The width of this peak is due to a number of different factors which considering a large area device viewing the cosmic radiation, may be listed as follows: (a) the statistical variation in the energy absorbed from the radiation by the phosphor, i.e., the "Landau" effect [see Landau (1944)]; (b) the statistical variation in the number of electrons emitted by the photocathode; (c) variations in the proportion of light collected from different regions of the scintillator; (d) variations in the path length of cosmic rays inclined to the vertical; (e) the composite nature of the cosmic rays; and (f) the momentum distribution of the cosmic rays.

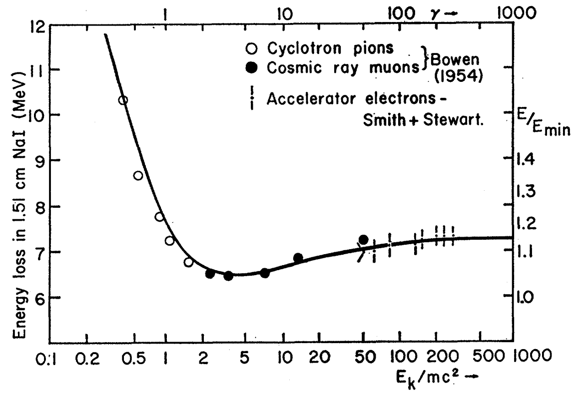

FIG. 12. The most-probable ionization loss in inorganic crystals plotted as a function of kinetic energy (E_K) in units of rest mass. The curve is calculated from the theory of Sternheimer, to which the results of Smith and Stewart (1966) have been normalized at $E_K/mc^2 = 49$.

The width of the Landau distribution for relativistic muons in an absorber of a few g cm^{-2} thickness is about 17%. If the effect of the variation in muon energies is added, this figure is increased to about 19%. The second largest contributor is the photomultiplier effect. Briefly, fluctuations in the number of photoelectrons will arise from (i) variations in the number of photoelectrons emitted from the photocathode; and (ii) variations in the number of electrons emitted from the various dynode surfaces. The experimental arrangement of Barnaby (1960) will serve to indicate the orders of magnitude of factors (b), (c), and (d) which were found to be 14%, 16%, and 9.5%, respectively. The total resolution can now be found by adding quadratically the above independent contributions. The exact combined total will depend on the type of counter used since some of the individual contributions will vary in magnitude and relative importance. Hence 33%±3% would be a reasonable figure for a large area plastic counter (Barnaby and Barton 1960), while the corresponding value for a liquid device would approach 60%. Finally, for small crystals, bombarded by a narrow electron beam, only factors (a) and (b) are significant.

Until 1953, scintillation-counter experiments were very few and very restricted in the energy ranges over which the ionization loss had been measured. Results obtained since then are tabulated in Table III.

Concerning inorganic crystal scintillators, Bowen (1954) using sodium iodide found that the relativistic increase was (10.9±1.0)% for energies up to 5 GeV. The momentum range investigated by Smith and Stewart (1966) with a cesium iodide crystal was much

TABLE III. Summary of scintillation counter results. (αNPO = α-naphthylphenyloxazole, Acc = accelerator, CR = cosmic rays).

Author	Counter	Dimensions (cm)	Incident particles Type	Incident particles Source	Particle velocity	Relativistic increase (%)	Comments
Bowen, 1954	NaI crystal	$5.97\times3.47\times1.51$	π, μ μ	Acc CR	$1.0<\gamma<50.5$	10.9 ± 1.0	Plateau not quite reached
Smith and Stewart, 1966	CsI crystal	2.5 diam$\times$0.25	e	Acc	$50<\beta\gamma<300$	···	
Bellamy *et al.*, 1967	NaI crystal	7.6 diam$\times$0.62	μ	Acc	$4.7<\beta\gamma<99$	~13	Crystal calibrated using γ source
Barnaby, 1961	Plastic—NE 102	$55\times17.5\times3.8$	μ	CR	$2.9<\beta\gamma<95$	<1.0, if any	
Crispin and Hayman, 1964	Plastic—NE 102	$55\times17.5\times3.7$	μ	CR	$3.8<\beta\gamma<975$	3.0 ± 1.0 at 100 GeV/c	
Smith and Stewart, 1966	Plastic—NE 102A	2.5 diam$\times$0.31	e	Acc	$50<\beta\gamma<300$		Incorrect theoretical curve
Jones *et al.*, 1968	Plastic—NE 102A	$43.7\times37.6\times2.5$	μ	CR	$3.4<\beta\gamma<930$	1.2 ± 0.7 at $P>10$ GeV/c	
Baskin and Winckler, 1953	Liquid, p-diphenylbenzene plus 10 mg diphenylhexatriene per 10^{-3} cc of xylene	20.3 diam$\times$5.1	μ	CR	$3.7\lesssim\gamma\lesssim23$	≤2.0	Plateau not reached
Bowen, 1954	Liquid, phenylcyclohexane	?	μ π, μ	CR Acc	$1.0<\gamma<50.5$	<2.0	No graph shown; few details
Millar *et al.*, 1958	Liquid, p-terphenyl in x-triethylbenzene plus 50 mg l^{-1} αNPO	$75.6\times75.6\times5.1$	p μ	CR CR	$\beta\gamma=3.7$ and 20.8	~1.5	Plateau not reached; two points only but of high statistical accuracy
Ashton and Simpson, 1963	Liquid, 0.5 g l^{-1} p-terphenyl in medicinal paraffin plus 0.008 g l^{-1} of POPOP	$130\times95\times17.5$	μ	CR	$18\leq\beta\gamma\lesssim1750$	~1.5	

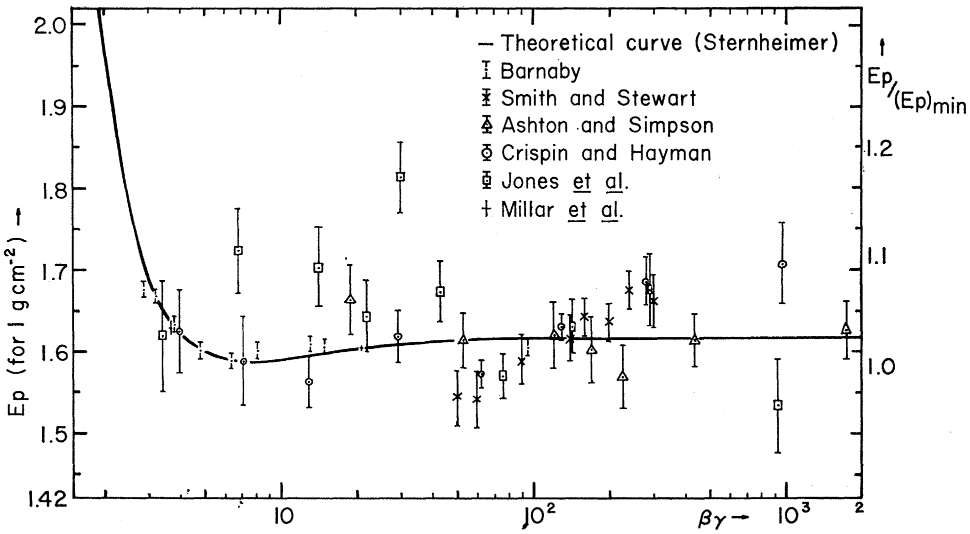

FIG. 13. Variation of E_p in organic scintillator as a function of $\beta\gamma$. Values of $\beta\gamma$ at which the respective results have been normalized are Crispin and Hayman (7.1), Smith and Stewart (140), Barnaby (6.4), Ashton and Simpson (53.0), and Millar *et al.* (20.8).

higher—$50<\beta\gamma<300$. Their electron results demonstrate a relativistic rise and, if combined with those of Bowen, assuming that the results are consistent, give a total rise of about $(11.3\pm1)\%$ from the minimum (see Fig. 12). The excellent agreement with Sternheimer, as seen in this figure, has been substantiated by Bellamy *et al.* (1967) who measured absolute values for the most-probable energy loss of muons passing through a thin sodium iodide absorber. Their results agreed, with the theoretical values within a 1% experimental error.

After some initial uncertainty, the position regarding organic scintillators is becoming clearer. The early work of Bowen and Roser (1952) using anthracene and cosmic-ray muons showed that between 0.3 and 3.0 GeV the rise in the most-probable energy loss was less than 2%. This investigation unfortunately suffered from poor statistics. Bowen (1954), using a liquid scintillator over the same energy range, found that the relativistic increase did not exceed 2%. This agreed with previous results on similar organic materials (Baskin and Winckler, 1953; Meshkovskii and Shebanov, 1952).

Since 1955 experiments with plastic phosphors have been performed by Barnaby (1961), Crispin and Hayman (1964), Smith and Stewart (1966) and more recently by Jones *et al.* (1968). Investigations with large area organic liquid counters have been performed by Millar *et al.* (1958) and Ashton and Simpson (1965). The results of these workers are shown in Fig. 13, along with the theoretical curve [Eq. (17)] which predicts a 1.5% rise over the range $7.5\leq\beta\gamma\leq99$, and no rise for particles of $\beta\gamma$ greater than, say, 100.

The experiments of Barnaby and of Crispin and Hayman may be conveniently grouped together because both used a large-area plastic counter of practically identical size, shape and mounting. Barnaby operated his apparatus at 55 mwe underground and estimated the energies of his cosmic-ray muons from their range in lead placed between trays of Geiger counters. With the maximum amount of lead absorber in the telescope, the minimum energy of the particles which passed through completely was 1.55 GeV. The energy spectrum at this depth shows that the median energy was then 10 GeV. Barnaby concluded that his results were reasonably consistent with Sternheimer's prediction that the density effect in organic materials is so large that it practically eliminates the relativistic rise of the most-probable energy loss. The results show that if there is a rise it must be less than 1%. Crispin and Hayman utilized the vertical Durham cosmic-ray spectrograph to extend the investigation to a $\beta\gamma$ of about 950. Their results are compatible, within experimental error, with those of Barnaby in the momentum region in which they overlap. They also suggest that at a $\beta\gamma$ of 950, the most-probable energy loss may be about $3\%\pm1\%$ above the predicted rise.

Smith and Stewart, mentioned above and in Sec. 5.1, also examined the effect of relativistic electrons with a NE 102A plastic disc. They state that the Tsytovich correction should become appreciable at $\beta\gamma\sim50$, and reach its asymptotic value at a $\beta\gamma$ of 300. No such effect was detected. Indeed, the *trend* of their results gave a much steeper rise than that allowed for theoretically. However, the results of Jones *et al.* (1968) with muons do not confirm such a trend, though they do tentatively suggest that there is some evidence for fine structure in the variation of energy loss—notably a rise of about $(9\pm3)\%$ in the muon momentum range 5–30 GeV/c. Their average E_p value for muons of

$\beta\gamma > 76$ has been normalized to the plateau value of Fig. 13. The shape of the best-fitting straight line, in the region above 2 GeV/c (ignoring the anomalously high result at 3.18 GeV/c, or $\beta\gamma=30$), gave $-(1.5\pm1.1)\%$ per decade of momentum; a value not inconsistent with zero. Jones *et al.* improve on the precision of their experimental results by combining their data with those of Crispin and Hayman, who covered the same momentum range and used virtually identical scintillator material. The value of the combined energy-loss data for muon momentum greater than 10 GeV/c is quoted as being $(1.24\pm0.69)\%$ above the theoretical plateau value. This conclusion lends no support to Tsytovich's prediction.

Millar *et al.* used a liquid counter to study the effect of cosmic-ray protons and muons. Concerning the latter, their results at the two energy values considered, i.e., 0.30 and 2.2 GeV, gave good agreement with the density-corrected energy-loss theory; inevitably, sampling the ionization loss at two particle energies only is not entirely satisfactory. In comparison, the momentum range investigated by Ashton and Simpson corresponded to $20\leq\beta\gamma<1750$. Their counter was mounted vertically and placed in the Durham horizontal spectrograph. They also make special reference to the radiative corrections of Tsytovich. The authors estimated the magnitude of the expected effect in a typical organic scintillator to be about 6% for muon energies of $E\gg8$ GeV ($\beta\gamma\gg7.5$). Initially, they concluded that their preliminary results confirmed the existence of the density effect up to the highest momenta measured and were not inconsistent with a small decrease in ionization loss at muon momenta $\gtrsim50$ GeV/c (Ashton and Simpson, 1963). Even so, this decrease seemed unlikely to be as large as that suggested from a straightforward substitution in Tsytovich's theory. Any decrease observed by Ashton and Simpson would have suggested in fact that a similar decrease could have been expected in the plastic scintillator results. These preliminary results were, however, superseded and it is their final corrected values that are shown in Fig. 13. The conclusion, drawn from these final results, is that there is no evidence for a decrease. Also, it is legitimate to interpret their results as confirming the existence of the plateau as far as a $\beta\gamma$ of 1750 which would seem to substantiate the uncorrected Sternheimer theory.

6.2 Discussion

(1) Virtually no decrease in ionization energy loss due to radiative corrections has been observed

(2) Referring specifically to organic scintillators. (a) The results are compatible with the Sternheimer density correction to the ionization-loss theory of Bethe–Bloch up to a $\beta\gamma$ of 200. At this value the logarithmic rise is predicted to be 1.5% or less. (b) Above $\beta\gamma=200$, the position is rather unclear. There is good agreement between the results of Ashton and Simpson and the Sternheimer theory up to a $\beta\gamma$ of 1750, but Crispin and Hayman suggest that there may be a small increase of 0.6–4.0% above the predicted value at $\beta\gamma=950$. Jones *et al.* (1968) do not consider their very low value at a $\beta\gamma$ of 930 as substantiating any decrease in energy loss (see earlier discussion).

(3) For inorganic crystals, good agreement exists between the theory and the few experimental results available. The relativistic rise from the minimum to the plateau is about 11.3%±1%.

7. THE USE OF SEMICONDUCTOR RADIATION DETECTORS

These comparatively recent devices possess very high energy resolution, linearity of response, high stopping power, and excellent signal-to-noise ratio even for minimum ionizing particles. This last feature is due to the deep depletion regions ($\geq$5-mm thick) which can be produced by the lithium-drifted process. After Miller *et al.* (1961) and others had shown the usefulness of semiconductor detectors for energy-loss measurements with fast charged particles, these devices seemed the best means of evaluating the various fluctuation theories (referred to in the Appendix). More recently, Aitken *et al.* (1969) used a lithium-drifted silicon detector to investigate the ionization loss of electrons over the range $300\leq\beta\gamma\leq1500$. Their observations provided additional confirmation of the density correction to the relativistic rise, but gave no evidence to support any radiative correction to the collision-loss theory. Unfortunately, since no electron measurements were made in the minimum region, the magnitude of the relativistic rise to the plateau was not obtained.

It is relevant here to refer to the phenomenon of "particle channelling." Erginsoy *et al.* (1964) and Gibson *et al.* (1965) reported observing anomalously low energy losses of charged particles (3-MeV and 4.85-MeV protons, respectively) when passing these through thin single crystals. Basically this phenomenon is explained in terms of a "channelling" of the incident particles between rows of crystal atoms. Particles thus trapped sample a lower electron density during their passage than would particles passing through at random (Lindhard 1964, Erginsoy 1965). Thus, whenever an incident particle beam is aligned within a certain critical angle $C\psi_1$ with a row of atoms in a lattice, an anomalously low energy loss should occur. The angle ψ_1 is given by $(2Z_1Z_2e^2/dE)^{1/2}$, where Z_1 and Z_2 are the atomic numbers of the incident particle and target atom, E is the particle energy, and d is the interatomic distance; C is numerically about 1.5.

However, it seems unlikely that the results of Aitken *et al.* and Bowen (with sodium iodide) are affected by this phenomenon for the following reasons: (a) The angular definition of the incident beam required for channelling is extremely small for relativistic muons and electrons, i.e. about 0.06° and 0.7° respectively in sodium iodide ($\bar{Z}_2=32$). (b) The likelihood of channelling is greatly decreased for incident energies greater than 10 MeV per nucleon. (Maccabee *et al.*, 1968). (c) No significant increase in the number of very low energy loss traversals was recorded. An anomalous high energy loss has also been observed by Erginsoy *et al.* and others, but this again would seem unimportant for relativistic particles.

8. RESULTS OBTAINED USING NUCLEAR EMULSIONS

8.1 A Review of Experimental Data

Experiments using photographic emulsions are numerous, and there are several review articles dealing specifically with this method of detection (e.g., Shapiro, 1958; O'Ceallaigh, 1965). The difficulties involved in the use of this technique should be appreciated before an assessment of any results can be made. They have been enumerated by Herz (1964) and others and are listed below.

(a) Nuclear emulsions present a complex inhomogeneous material.
(b) They are sensitive to temperature and humidity, which cause fading of the latent images and affect their density and shrinkage factor.
(c) The tracks made by particles are subject to distortion that can be confused with scattering.
(d) Great care must be taken in processing if reproducible results and uniform sensitivity are to be obtained.
(e) The variation of blob density with depth in the emulsions is a possible source of error.

A further, previously unsuspected, source of error which has been mentioned by Herz is that the water content of emulsions may vary from batch to batch or even from package to package. This is due to the inability of the manufacturers to allow the necessary time for the emulsions to reach equilibrium at the standard 50% relative humidity at which they are normally dried and packed. Variations in emulsion density arise chiefly from changes in their water content (see, in particular, Oliver, 1954 and Barkas *et al.*, 1958).

Thus the sources of variations and fluctuations in blob density are many and must be reduced or eliminated if the detection of small differences in rates of energy loss is to be made with any reliability. This is particularly important when looking for the effect predicted by Tsytovich. The effect is not large and to check it the rate of energy loss, or the quantity proportional to it, must be measured to an accuracy of better than $\pm1\%$. Such precision has not as yet been achieved in ionization measurements using emulsions. As Herz states, a 1% change in blob density means in practice a $\frac{1}{4}$ blob in each 100 micron (μ) interval, which is not a noticeable change in the normal type of emulsion experiment.

Before reviewing the ionization results, a brief comment must be made concerning the magnitude of the energy loss in emulsion due to Cerenkov radiation. When considering Cerenkov radiation, the impact parameter, b, should be taken as the mean grain radius. The Cerenkov loss W_b for particles of $\beta=1$ and for $b=0.13\,\mu$ is quoted by Sternheimer (1953) as being 2.10×10^{-3} MeV g^{-1} cm^2, which is small compared to the relativistic rise of the total ionization loss, $(1/\rho)\ (dE/dx)$, of 0.12 MeV g^{-1} cm^2.

Prior to 1962, all experimental evidence pointed to the existence of a plateau value for the ionization loss up to the highest momenta measured. Some early investigations by Occhialini (1949) and Corson and Keck (1950) of electron tracks in emulsions established the existence of a plateau at high energies, although they did not detect any variation of grain density for electrons from 7.5 to 500 MeV, and from 10 to 180 MeV, respectively. The same is true for the results of McDiarmid (1951) and Morrish (1952). However, as their investigations were confined to electrons, their measurements were not extended down to the minimum of ionization and hence could not yield a value for the magnitude of the relativistic rise. This is because electrons in the region of the theoretical minimum have energies less than 3 MeV, and are therefore strongly scattered. It is this scattering which imposes severe limitations on the accuracy of the determinations of the grain or blob densities (see Stiller and Shapiro, 1953). Nevertheless workers such as Pickup and Voyvodic (1950), Voyvodic (1952), and Daniel *et al.* (1952) reported differences of 8–10% between the plateau value for extreme relativistic particles and the minimum value. This rise in the energy loss has been substantiated by Stiller and Shapiro (1953), Michaelis and Violet (1953), Fleming and Lord (1953), Jauneau and Trembley (1955), Alexander and Johnston (1957), Jongejans (1960), and Congel and McNulty (1968). With the exception of Michaelis and Violet, all the last mentioned authors obtained values of 10–16% for the relativistic rise, i.e., generally greater than those previously obtained. Indeed Patrick and Barkas (1962) have reported observing an 18% effect, although their data, on examination, could be interpreted as supporting a value more like $(15.5\pm2.5)\,\%$.

Both Price (1955) and Shapiro point out that the low values for the relativistic rise reported in some of the early work, and also the spread in values, can be accounted for by the method of grain-density (g) normalization employed. That is, in order to correct for variations in g between (a) various plates, (b) different areas of the same plate, and (c) as a function of depth in the emulsion, the observed g was divided by the

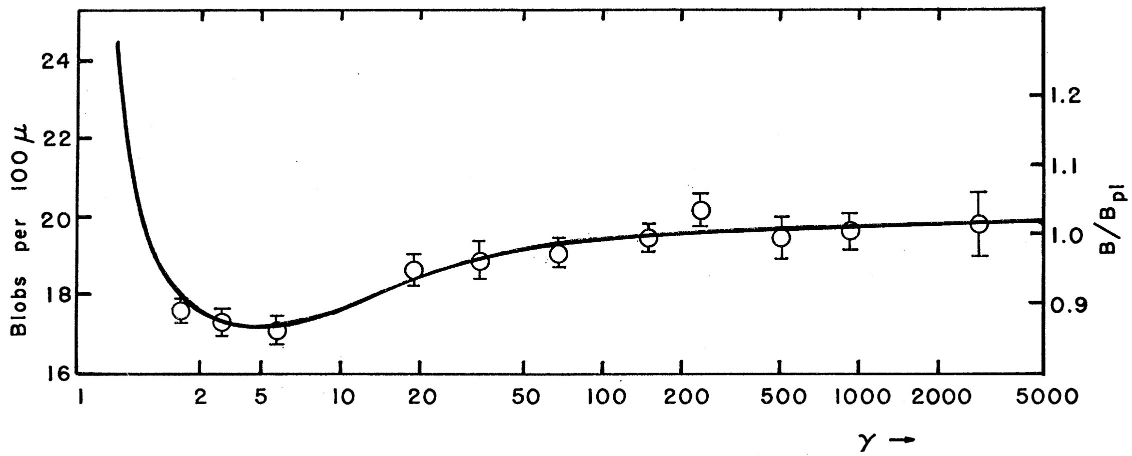

FIG. 14. Relativistic rise of the rate of energy loss by ionization in emulsion. Blob density is plotted against the total energy of singly charged particles in rest-mass units. The scale at the right shows the normalization of the blob count to the Fermi plateau (after Stiller and Shapiro, 1953).

density, g_0, of a nearby reference track, and the ratio g/g_0 plotted against Pv. It was usual to select reference particles from those primaries of $\gamma > 10$ which produced stars in the plate as these were assumed to ionize at the plateau value. Since this γ value is now considered to be nearer the minimum than the plateau, it follows that g_0 was too low, resulting in an underestimation of the relativistic rise and the observation of a rapid rise of the grain density to the saturation value. Thus any assumptions made regarding the energy at which the plateau is reached are of paramount importance, as they are reflected in the experimental results. It is obviously desirable to select primaries in the far relativistic region ($\gamma > 100$ say) but even so the possibility of including primaries of lower energy is not negligible (see Price).

The particles most readily available having plateau ionization are either electrons of energy $E > 50$ MeV resulting from pair production in the emulsion or primaries which produce showers of high multiplicity. Other suitable reference primaries, apart from their scarcity, are the singly charged particles, usually protons or pions, which comprise narrow-angled jets.

O'Ceallaigh has reported that statistically significant evidence has been shown for a decrease in the magnitude of the relativistic rise with increasing degree of development. A similar conclusion was reached by Brown (1953) in considering the effect of fluctuations in the energy loss. This effect could account for the low values observed by Michaelis and Violet and others whose plates were heavily developed. However Shapiro argues that it is still valid to compare measurements of g with ionization-loss values since their proportionality is maintained for the usual intensities of development (Fowler 1950). In any case, slight changes in the plateau value affect the magnitude of the logarithmic rise significantly.

The situation up until 1963 was one of good general agreement between experiment and theory. Figure 14 shows a typical result of Stiller and Shapiro. By exposing their plates to the cosmic radiation, they made blob counts on long tracks of electrons, muons, and protons. The value obtained for the logarithmic rise was $(14\pm3)\%$, and they concluded that the rate of ionization loss in AgBr saturates at $\gamma > 100$, and maintains the plateau value at least as far as a γ of 3400. This particular investigation has the merit not only of obtaining data in the ultrarelativistic region, but also in the region of the minimum, a desirable feature when quantitatively examining the logarithmic rise to the plateau value, but one which is unfortunately absent from some recent investigations.

In 1962 a new effect was detected by the work of Zhdanov *et al.* This Russian experiment with electrons in emulsions indicated that the energy loss began to decrease again at a point ($\gamma \simeq 100$) soon after the plateau value had been reached; this was in good agreement with the prediction of Tsytovich (1962a, 1962b, 1962c) which has been described elsewhere. Any such decrease is in conflict with both other theories and existing experimental data. In particular, neither Stiller and Shapiro nor Jongejans gave any indication of a decrease. Stiller and Shapiro's results have already been referred to above, while Jongejans concludes that his results agree with the theory of Sternheimer, and that they maintain the plateau value as far as $\gamma \approx 1000$.

In Zhdanov's experiment, precise data were obtained first, by using NIKFI R.10-type emulsions exposed to a proton beam of 8.7 GeV at Dubna, and second by using ILFORD G.5 emulsions exposed to a 19-GeV proton beam at CERN. The relative blob density along secondary electron tracks was measured. For calibration purposes the blob density along the tracks of primary

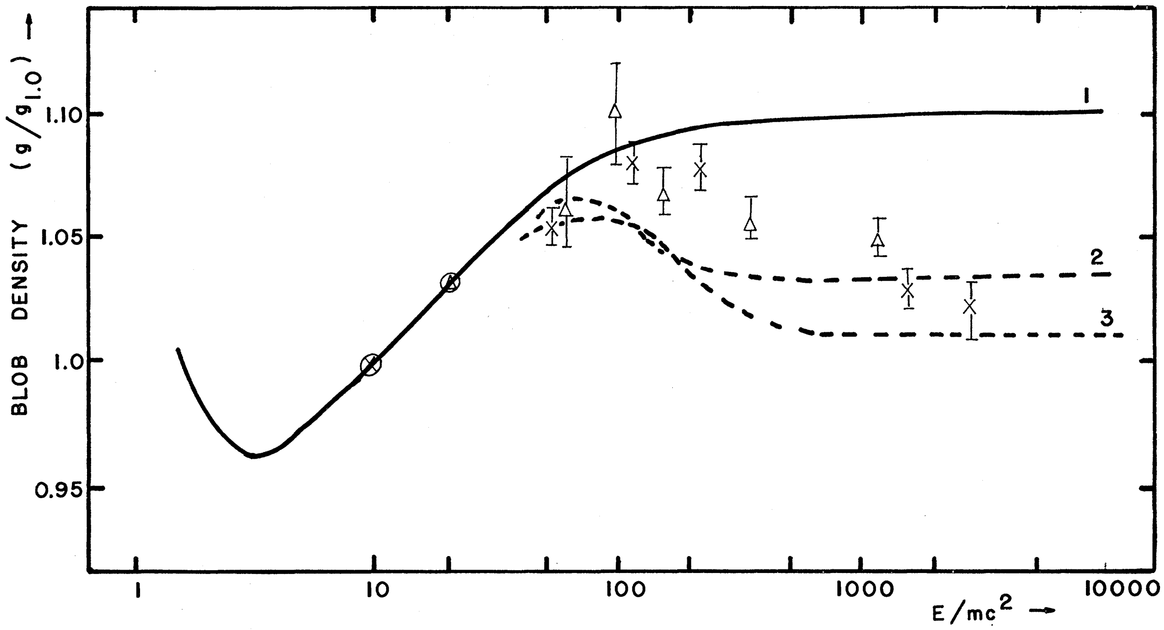

FIG. 15. Results of Zhdanov *et al.* using NIKFI (X) and Ilford G-5 (Δ) showing blob density along the track plotted as a function of electron energy. [Curve 1 indicates the theoretical curve neglecting radiative corrections (Jongejans, 1960), while 2 and 3 are asymptotic theoretical curves taking radiative corrections into account. Experimental calibration points for each emulsion are encircled.]

protons crossing the same region of emulsion was measured. The electron energies were deduced from multiple scattering. Their preliminary results are shown in Fig. 15, which, they conclude, agree satisfactorily with the predicted effect of the radiative "Tsytovich corrections"; this result was confirmed later by the same group while investigating the ionization-momentum dependence of electrons and positrons (Zhdanov *et al.*, 1964). Alekseeva *et al.* (1963) obtained a similar result with the same types of emulsion.

Since 1963, a number of experimental groups have worked on this problem. Not only has it a bearing on measurements in nuclear emulsions but of equal importance is its possible bearing on energy-loss theory; and consequently on the range-energy relationship. In order that a valid test of the possible effect be made, it is desirable that comparable results should extend far enough along the plateau; a γ of at least 1000 would

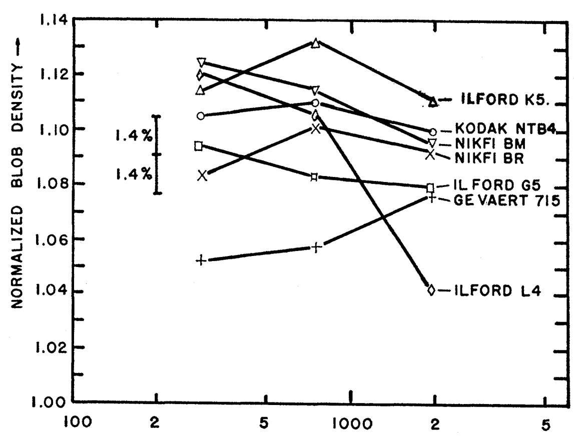

FIG. 16. Normalized blob density plotted as a function of γ. The 1.4% represents the expected standard deviation in the blob-density distribution at a fixed γ, when using various emulsions (after Herz and Stiller, 1964).

TABLE IV. Summary of average normalized blob densities.

	Unprocessed grain diameter (μ)	$B_{\rm pl}/B_{\rm min}$
Kodak NRB4	0.4	1.105
Ilford G5	0.27	1.085
K5	0.20	1.119
L4	0.14	1.090
NIKFI BR	0.28	1.092
BM	0.14	1.111
Gevaert 715	0.15	1.062

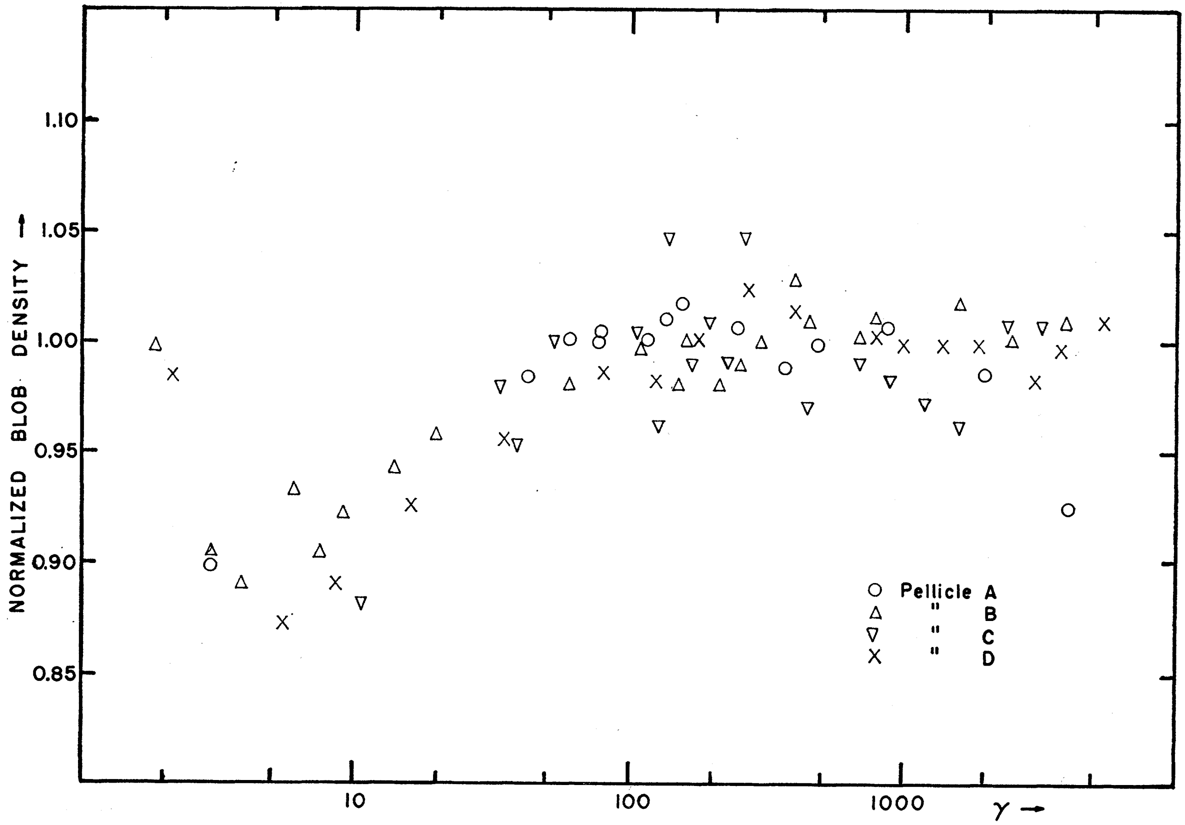

FIG. 17. Normalized blob densities for electrons ($\gamma > 100$) and pions plotted as a function of γ. The data were gathered from 4 pellicles of 600 μ thick Ilford K.5 emulsion (after Buskirk *et al.*, 1964).

seem to be an appropriate value. Experiments which satisfy such a criterion are those of Stiller (1963), Buskirk *et al.* (1964), and Herz and Stiller (1964).

Stiller, using a composite stack, examined blob counts from electrons of energies 100, 210, 450, and 1000 MeV, and negative pions of energy 450 MeV. The emulsions used were the same as those indicated in Fig. 16 apart from NIKFI BM and Gevaert 715. The magnitudes of the relativistic rise were in general low. In particular the result of 7% with Ilford G5 seemed in contradiction with the earlier work of Stiller and Shapiro which gave 14%. At the time it was felt that the low G5 value was due to the emulsion packets not having sufficient time to attain a constant temperature before exposure to the electron beam. Debeauvais-Wack (1960) and Nikitin *et al.* (1960) have provided evidence that emulsion sensitivity increases 10% or more in going from $-20°C$ to $+20°C$, which is sufficient, for example, to cause the normalized electron blob density ($B_{\mathrm{pl}}/B_{\mathrm{min}}$) to go from 1.06 to 1.16. However, from Stiller's text it is not immediately apparent whether the application of this correlation would in fact explain his anomalously low result for the G5 emulsion; for a full discussion, see O'Ceallaigh, page 93.

The work of Herz and Stiller was practically an extension of the above. One negative pion and three electron exposures were obtained with $\gamma = 4$, 293, and 1953 respectively. From their results (see Fig. 16) they concluded the following: First, the rates of energy loss at the three highest γ values were constant to within about 1%; no explanation for the behavior of Ilford L4 was given. Second, the magnitude of the relativistic rise is not the same in all emulsions, though the more recent results of Congel and McNulty are in disagreement with this conclusion. Table IV gives the average values, quoted by Herz and Stiller, of the normalized blob densities at the same three values of γ. The observed variation bore no apparent correlation with unprocessed grain diameter but might be due to differences in the fading behavior or the response of the emulsions. Third, the average observed relativistic rise of about 10% was rather less than had been commonly found. Again fading provides a possible explanation, particularly if it were established that rapid fading occurs during the first few days after exposure. In this case the pion tracks, from which were derived the minimum blob densities, faded for two days less than the other tracks.

Buskirk *et al.* exposed Ilford K5 emulsions to a 16-GeV/c negative pion beam. They made careful blob counts on pion and electron tracks over the range $2 \leq \gamma \leq 4000$; velocities were estimated from multiple-scattering measurements. Their results are shown in Fig. 17 where the uncertainty in blob density for each data point is $<2\%$. No significant evidence was found for a departure from a flat plateau at $\gamma > 100$.

8.2 Discussion

Sternheimer (1963) has suggested that, if the effect of Tsytovich is real, it is possible that Stiller and Shapiro may have missed it due to the uncertainty in their momentum determination. It seems unlikely, however, that the same applies to Jongejans, Stiller, Buskirk *et al.*, Herz and Stiller and others.

The apparent disagreement remains unsettled and awaits a decisive experiment. In conclusion it may be said that:

(1) When comparing nuclear emulsion results, it is imperative to consider such factors as the actual type of emulsion used, its degree of development, the time which has elapsed between exposure and development, and the temperature conditions.

(2) Apart from the work of Zhdanov and his collaborators, there has been no evidence for the existence of the Tsytovich effect.

9. GENERAL CONCLUSIONS

First, as stated above, apart from Zhdanov *et al.* no significant decrease in the ionization loss after the "plateau" is reached has been definitely detected, though some results, within the experimental errors, could be interpreted as indicating a small effect (1%–2%). However, even in such cases the suggested decrease did not approach the magnitude predicted by Tsytovich. Second, although all results demonstrate the logarithmic rise—and subsequent plateau—some workers have reported a smaller value for the percentage rise.

As ever, there is still a need for experiments of a high standard of technique which yield data of good statistical accuracy. Some further investigations which have suggested themselves to the authors are as follows. (a) The accurate measurement of the density effect for "plateau" value of γ, in media whose photoelectric cross-sections, electronic ionization potentials, and oscillator strengths are *well known*. The extension of our knowledge of these parameters over a wider range of relevant media would also be useful. (b) There is a need to quantitatively resolve the energy loss into its separate components of ionization, excitation, and Cerenkov radiation, although the experimental means of accomplishing this is not obvious. If such a resolution were successful it would make possible a more direct comparison between the results and the various theories.

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APPENDIX

A heavy charged particle passing through a thin absorber loses its energy by collisions with the atomic electrons of the material. Individual collisions are independent events so that energy losses may vary. We therefore have a statistical phenomenon. Parameters of the resultant characteristic energy-loss distribution which are of interest are (a) its shape, (b) the average energy loss E_a, (c) the most-probable energy loss E_p and (d) the full width at half-maximum of the distribution, Δ_{fw}.

It is unnecessary to examine the various discussions of the theory of the energy-loss fluctuations as they have been reviewed exhaustively elsewhere, e.g., Fano (1963). Here the main features of the various investigations will be dealt with, thus enabling the experimentalist to apply the most relevant distribution to his results.

The most significant observation is that the parameters β and K uniquely define a spectrum for a given particle type and energy, and for a given material. The parameter K is defined by

$$K = \xi / \epsilon_{\max}, \tag{A1}$$

where

$$\xi = (2\pi e^4 z^2 N x / m v^2)(Z/A_0), \tag{A1$'$}$$

where z is the charge of the incident particle, N the number of atoms per cubic centimeter of the absorber material, and the remaining terms have their usual meaning. The quantity $\epsilon_{\max}$ is the maximum possible energy transfer in a heavy particle–electron collision which is given approximately by

$$\epsilon_{\max} = 2mv^2/(1-\beta^2) \tag{A2}$$

when

$$Mc^2/(1-\beta^2)^{1/2} \ll M^2c^2/m,$$

where M is the heavy-particle mass.

Substituting (A1′) and (A2) in (A1) gives

$$K = [0.15029 z^2 x Z (1-\beta^2)]/A_0\beta^4 \tag{A3}$$

or

$$K = [0.30058 z^2 x Z m c^2]/\beta^2 A_0 \epsilon_{\max}. \tag{A4}$$

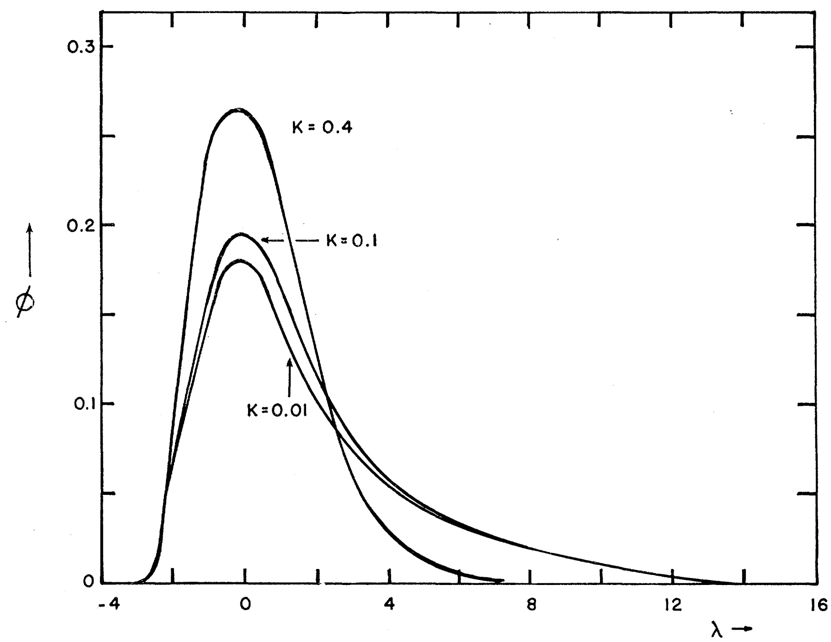

FIG. 18. The shape of the Vavilov distribution for various values of the parameter K (after Skyrme, 1967).

Considering a singly charged incident particle and rearranging terms, we obtain

$$K \approx Ax/\beta^2\epsilon_{\max}, \tag{A5}$$

where A is the constant defined by Sternheimer in his discussion of the density effect (see Eq. 18). Here K may be thought of as a measure of the ratio of the total energy loss to the maximum possible energy loss in a single collision, i.e., an estimate of the number of large energy-loss collisions suffered by the particle in passage.[1] There are three basic cases to be considered.

(a) $K \gtrsim 1$: Here the number of collisions in each energy-loss interval is large and the effect of fluctuations negligible. The distribution is Gaussian with a width or variance, according to Bohr (1915, 1948) of

$$\sigma^2 = 4\pi e^4 z^2 NxZ, \tag{A6}$$

and a Δ_{fw} of 2.35σ. Expression (A6) has been corrected slightly by Williams (1932); see also Cranshaw (1952). There is good experimental verification of the Bohr theory, with modifications by Livingston and Bethe (1937), particularly with α particles and low-energy protons for which $K \gtrsim 1$.

(b) $K \lesssim 0.01$: This is the opposite case where the number of collisions in each energy-loss interval is small. This problem has been solved by Landau (1944). The resulting distribution is asymmetric with a long high-energy tail and a broad peak. The value of Δ_{fw} is about 30% of E_p or 3.98ξ. E_p is significantly less than E_a and is given by Eq. (17).

Two conditions for the validity of the Landau theory have been mentioned by Maccabee *et al.*, (1968). First, if ϵ_0 is approximately the mean binding energy of the atomic electrons then $\xi \gg \epsilon_0$; thus the theory breaks down in the limit of a very thin absorber. Second, the collision spectrum must be directly proportional to ϵ^{-2}. Departure from this latter condition has been discussed by Blunck and Leisegang (1950), Blunck and Westphal (1951) and Shulek *et al.* (1966).

There has been good general agreement with the Landau theory [for a recent precise investigation, see Bellamy *et al.* (1967)].

(c) $0.01 < K < 1.0$: This intermediate region was first investigated by Symon (1952) who obtained a more general expression than Landau's for the probability distribution, $f(\epsilon)$, which he solved under the conditions,

$$M \gg m, \qquad Mc^2 \leq T \leq 10Mc^2, \tag{A7}$$

where T is the kinetic energy of the incident particle. Thus he was able to link approximately the Gaussian and Landau regions. However, as pointed out by Skyrme (1967), the application of Symon's solution to specific cases needs considerable manipulation and extrapolation of his published results.

The transport equation describing the energy-loss distribution has been more rigorously solved by Vavilov

[1] The spectrum of the energy transferred in single collisions, i.e., the collision spectrum, may be divided into an arbitrary number of intervals.

(1957). His precise calculation uses integrals which have been tabulated by Seltzer and Berger (1964). For $K \lesssim 0.01$ and $K \gtrsim 1.0$ the distributions are practically Landau and Gaussian in shape, as is partially shown in Fig. 18. In this figure ϕ is a measure of the probability that a particle will lose an energy of $\Delta - \Delta + d\Delta$ in transversing an absorber, and λ, the Landau factor, is given by $(\Delta - E_p)/\xi - 0.423 - \beta^2 - \ln K$. Thus Vavilov's approach has the advantage of being applicable over the whole range of K as has been established by Maccabee *et al.*

Vavilov's concluding remarks fittingly serve here as a summary, i.e., for

(a) $K \gtrsim 1.0$, his Eq. (13) may be used to give a Gaussian distribution;

(b) $0.01 < K < 1.0$, his exact solution (Eq. 16) must be used;

(c) $K \lesssim 0.01$, Landau's approximation is valid.

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