Recent Developments of the Regge Pole Model*

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The recent proliferation of literature on the Regge pole model has made it evident that a general review of the model is needed. This paper summarizes work done in the field prior to October 1968. It reviews the types of singularities thought to exist in the complex angular-momentum plane, the symmetries of Regge amplitudes and trajectories with special emphasis on four-dimensional symmetry associated with zero momentum transfer, properties of both meson and baryon trajectories, predictions of the model, and suggested experimental tests of various assumptions made in the model. The paper is written for the general reader with an interest in the field and for those in the field interested in areas in which they are not directly involved. There is as little use of mathematics as possible, with emphasis essentially on the results and conclusions of recent papers.

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I. INTRODUCTION

In the last two years there has been a proliferation of literature related to the Regge pole model. This paper summarizes and reviews the recent developments that have taken place in the theory and the evolution of the model as an attempt to explain experimental phenomena.[1]

Although a complete Regge pole theory for strong interactions does not yet exist, those theories which are concerned with analyticity in a complex-angular-momentum plane are described by the term. In high-energy physics, the angular-momentum variable for a cross channel is continued to complex values. The model thus attempts to describe direct-channel reactions in terms of the analytic properties of the cross-channel amplitudes as a function of the cross-channel angular

† Sections which should be of most interest to people not engaged in field.

[1] This review was prompted by and is based in part on a status discussion meeting for the Regge pole model held in March 1968 in Eugene, Ore.

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momentum. The concept of analyticity in the cross-channel angular-momentum variable, j, sets the Regge pole model apart from other peripheral models.

A cross-channel theory is desirable because experimentally high-energy reactions are characterized by a definite correlation between the peaking (or lack of it) of a reaction in the forward or backward direction and the existence (or lack of it) of particles with the quantum numbers exchanged in the respective cross channel [319].*

The simplest interpretation of the feature that reactions are governed by the exchange of particles was embodied in the one-particle-exchange (OPE) model and its later modifications. The most famous of these, the absorptive OPE model, comes close to treating angular momentum as a nondiscrete variable because absorptive corrections destroy the exchange of a definite value of angular momentum in the cross channel.

One of the failures of the OPE and related models is their inability to predict correctly the energy dependence of reactions when exchanged particles with spin of unity or greater are involved. The Regge pole model avoids this difficulty by associating particles with the same quantum numbers, but different discrete values of angular momentum, with poles in the complex-angular-momentum plane. These "Regge" poles are assumed to move or trace out trajectories as a function of the momentum transfer variable t.[2] The symbol $\alpha(t)$ designates the position of the poles as a function of t.

In a mathematical sense, the "Reggeization" of amplitudes is completely divorced from the concept of particle exchanges unless the poles are associated with trajectories of physical particles. Consequently, many people have attempted to make a connection between the asymptotic behavior of amplitudes due to Regge poles and the asymptotic behavior of the exchange of particles in the cross channel.

In particular, Van Hove [531] and Durand [214] have obtained the asymptotic behavior characteristic of Regge poles by considering the exchange of an infinite number of narrow-width particles in the cross channel. Regge asymptotic behavior of elastic amplitudes has been obtained from Feynman graphs containing the exchange of ladder diagrams in the cross channel whose rungs are reminiscent of unitarity in the direct channel [36]. Regge asymptotic behavior has also been obtained by assuming that the absorptive parts of the amplitude satisfy certain asymptotic properties [261].

Recently a crossing symmetric function which can show asymptotic Regge behavior in all channels has been found [534]. Its discontinuity across any of the axes exhibits behavior characteristic of resonances and thus illustrates how trajectories can describe both direct-channel resonance and cross-channel exchanges.

It is appropriate to mention a few of the characteristic properties of amplitudes due to the exchange of Regge poles and some of the experimental consequences. The contributions of a single Regge pole to amplitudes have the following asymptotic properties:

(a) For fixed t, the contributions to all amplitudes have an s dependence given by $s^{\alpha(t)}$.[3]

(b) The phase of all contributions is the same and is given by $1+\tau \exp[-i\pi\alpha(t)]$, where τ is the signature factor. See the Appendix for its definition.[4]

(c) The residue of a Regge pole is thought to satisfy factorization; i.e., the residue function can be written as a product of two functions, one describing the coupling of the trajectory to the initial state, and the other, the coupling to the final state in the cross channel (see Sec. VII.E).

(d) The residues also satisfy the symmetry of line reversal (see Sec. VII.F); i.e., two residues which describe cross-channel reactions with initial and/or final states related by an operation such as charge conjugation or G parity are equal up to a sign given by the respective quantum number of the trajectory.

(e) In addition to values of α corresponding to the angular momentum of physical particles, there are values of α—integral for bosons, half-integral for fermions—at which the dynamics of the trajectory determines whether its contribution is zero or finite.

If the only significant contribution to a reaction is due to a single Regge pole, the following experimental features will result:

(a) The differential cross section $d\sigma/dt$ will be asymptotic to $s^{2\alpha(t)-2}$. (From the optical theorem one would also conclude that the total cross section for all reactions resulting from the initial state will be asymptotic to $s^{\alpha(0)-1}$.)

(b) The differential cross section will shrink as a function of s at fixed t, and the degree of shrinkage will depend on the derivative of $\alpha(t)$.[5]

(c) Relationships exist between differential cross sections for reactions whose amplitudes are dominated by the same Regge pole and are related by some symmetry such as factorization or line reversal. (By using the optical theorem, one can similarly write relationships between total cross sections.)

* Numbers in brackets [ ] refer to the Bibliography, Sec. XVIII at the end of the paper.

[2] We use the Mandelstam variables s and t to designate the squares of the center-of-mass energies in the direct and cross channels, respectively.

[3] This statement must be modified when the magnitude of the cosine of the cross-channel scattering angle is near unity. This is called the "cone" effect [333].

[4] It can be argued that (b) follows from (a) by real analyticity, but the converse is not true; e.g., complicated behaviors such as $(\log s)^n s^{\alpha(t)}$ also lead to (b).

[5] Though shrinkage is usually assumed to be a pure Regge pole phenomenon, it has been obtained within the Wu and Yang diffraction model [544] for pp scattering by Durand and Lipes [218].

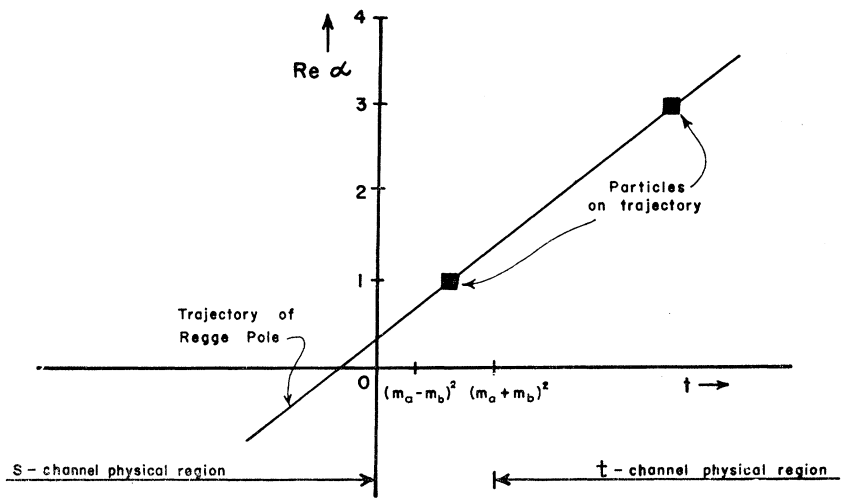

Fig. 1. Trajectory of an odd-signatured Regge pole. The kinematical points $(m_a \pm m_b)^2$ at which t-channel amplitudes may be singular are shown. The diagram, for example, could be for the ω trajectory where a and b might represent particles in the initial state such as π and ρ.

(d) Dips in differential cross sections can be explained in terms of the dynamics of the dominant Regge poles. However, the principle of factorization strongly restricts such explanations since it demands that corresponding dips occur in other reactions dominated by the same Regge pole.

(e) Once $\alpha(0)$ of the dominant trajectory is determined from the total cross section, the phase relationship allows the forward elastic cross section to be predicted exactly.

(f) The phase relationship also predicts that if only one Regge pole contributes, the polarization (P) is zero. If two Regge poles contribute, then $(d\sigma/dt)P \sim s^{\alpha_1+\alpha_2-2}$.

One should be able to associate a known particle with each trajectory (see Fig. 1). The trajectory and residue function should also extrapolate smoothly to their value at the pole of physical particles. One notable exception, the Pomeron trajectory, which is postulated to account for the apparent constancy of total cross sections at high energy, is not associated with a known particle. (See Sec. XI.B for a review of the unusual properties of the Pomeron.)

The Regge pole model avoids the difficulties associated with the exchange of elementary particles of spin greater than unity by requiring $\alpha(t) \leq 1$ for physical values of t in s-channel reactions. However, most of the concepts and properties associated with single-particle exchanges are retained. This feature makes the Regge pole model very appealing to enthusiasts of the bootstrap hypothesis.

The concept of complex angular momentum in the context of nonrelativistic potential theory was proposed by Regge in his 1959 *Nuovo Cimento* article. Theorists have continued to turn to potential theory as a guide for the analytic properties of the trajectory and residue of a Regge pole [7, 8, 9, 21, 68, 148, 338, 339, 408].

One feature of the model is both an asset and a liability: there is almost complete functional freedom of the trajectory and residue function for negative values of t. Consequently, an unlimited number of parameters can be introduced into the theory to patch up the t dependence at fixed s. The curve fitter can consider not only additional trajectories, but also other types of singularities in the j plane such as cuts and, in certain circumstances, fixed poles.

Consequently, while the basic theory is simple and explains many features of experimental data, it has considerable freedom to go to great extremes of complexity to attain agreement with experimental data. Once the basic assumptions for formulating a complex j-plane representation of amplitudes are justified, the essential question is whether such a representation is economical in terms of the number of parameters required to obtain agreement with experiment. This paper thus describes what is being done within the model to reconcile the requirements of analyticity and a simple intuitive explanation of experimental data.

The paper attempts to provide an informative survey of the recent literature and is divided into various sections, each summarizing the work on a given topic. The next section, II, is concerned chiefly with amplitudes that are used to discuss Regge pole contributions. Section III is an involved discussion of the types of singularities believed to exist in the complex j plane. It is unessential to the novice and may be avoided on a first reading. The possible dynamical choices thought to exist for residue functions at special values of the trajectory are discussed in Sec. IV. Section V is concerned with the analytic structure of helicity amplitudes at kinematic points. Though little detail is presented, it should be of interest to general readers. Section VI reviews daughter trajectories, constraint equations, a proposed higher symmetry of amplitudes and trajectories at zero momentum transfer. It is probably too involved to be of value to the novice. Section VII is a general discussion and review of symmetries of amplitudes and trajectories and should be informative to the general reader.

Because most reactions receive contributions from many Regge trajectories, it is useful to isolate individual contributions by combining results of different experiments or by performing particular experiments; methods for this are presented in Sec. VIII. Section IX discusses the backward πN scattering and the N and Δ trajectories that contribute. These trajectories are discussed in the light of MacDowell symmetry. The first two parts of this section should be very useful to the general reader.

Section X, which should also be of interest to a wide range of readers, reviews the few predictions the model makes about angular distribution and how successful these have been. Both known and speculated features of the boson trajectories are discussed in Sec. XI, which is rather detailed; the novice would be wise to skim through it rapidly. Section XII reviews various areas of research which are somewhat different from the traditional Regge pole model; it probably contains clues to the future development of the model.

Section XIII is intended primarily for experimentalists who are looking for new experiments or ways to extract more information from their present data. The experiments suggested there hopefully will help to unravel some of the confusion that exists in the present model.

Although the paper reviews a considerable number of topics related to the Regge pole model, many have not been included. In particular, it does not consider such topics as the t dependence of the trajectories (e.g., infinitely rising trajectories and universal slopes) or restrictions on the residue function (e.g., those due to unitarity). It also overlooks the large number of papers that deal with Regge pole behavior in potential theory and Feynman diagrams.[6] Undoubtedly the list of relevant topics is inexhaustible, but the author hopes that those selected illustrate the recent developments and trends in the Regge pole model.

For the reader interested in a more detailed discussion there are many books [157, 236, 424] and excellent review papers [76, 98, 247, 391, 428, 518, 539].

Although it is assumed that the reader is familiar with the basic terminology of the model, the Appendix discusses some of the terms and expressions in more detail. Readers unfamiliar with the basic concepts of the Regge pole model may want to consult one or more of the books mentioned in the previous paragraph.

A table listing various reactions, the trajectories that are thought to contribute, and references has been included (Sec. XVII). Though incomplete, it should prove helpful to people interested in particular reactions.

[6] For those interested in such topics, some recent papers are given for infinitely rising trajectories [116, 220, 269, 329, 387, 534], universal slopes [79, 312, 387], restrictions on residue functions at large values of momentum transfer [39, 476]. Regge poles in potential theory [7–9, 21, 68, 148, 338, 339, 408] and Regge pole behavior in Feynman diagrams [94, 105, 106, 254, 520].

II. CHOICE OF AMPLITUDES

Since the early days of Regge pole theory when a simple procedure for the "Reggeization" of amplitudes was put forth by Frautschi, Gell-Mann, and Zachariasen [237], many different procedures and variations have been suggested [17, 125, 188, 213, 332, 445, 523]. It is commonly agreed that the use of so many sets of amplitudes can cause great confusion and that the Reggeization procedure for helicity amplitudes proposed by Gell-Mann *et al.* [260] should be used. This general procedure is reviewed in the Appendix. Many sets of amplitudes that have special properties will continue to be used when their unique properties simplify proofs and calculations, e.g., the so-called transversity amplitudes [13, 245] for which the crossing matrix is diagonal.

There appears to be a principle of complementarity between the use of t-channel helicity amplitudes and the use of s-channel helicity amplitudes in describing Regge poles in the t channel. Factorization (see Sec. VII.E) of t-channel amplitudes is true only to leading order in s. Recent work has shown this also to be true for s-channel amplitudes [230]. Consequently, to leading order in s, the principle of factorization is equally applicable to both channels. The complementarity arises from the fact that phenomena such as dips in differential cross sections associated with certain values of α are best expressed in terms of t-channel amplitudes, while those difficulties associated with analyticity in t are best avoided by the use of s-channel amplitudes which, except for known half-angle functions, are free of kinematic singularities in t. Recent work has also shown that in certain situations, infinite sums of daughter trajectories (Sec. VI) can be avoided by using s-channel amplitudes [153, 356]. There remains the question whether the "cone" effect [333] can be easily expressed in terms of s-channel amplitudes.

III. SINGULARITIES IN THE j-PLANE

A. Poles and Families of Poles

As its name suggests, Regge pole theory is mainly concerned with simple poles in the complex-angular-momentum plane. However, there is no reason why other types of singularities should not be encountered and, in fact, other singularities such as cuts do exist. Actually, the naive picture of a single Regge pole in the complex j plane, independent of all others, has recently been rudely destroyed. A more complete description of the events that led to the abandonment of the simple picture is given in Secs. VI and VII. It is sufficient to point out here that for each trajectory there exists an infinite number of daughter trajectories, equally spaced by a unit of angular momentum at $t=0$; in certain cases there are correlations between trajectories with the same and sometimes with different quantum numbers. In Sec. VII, it is shown that if certain conditions are satisfied, MacDowell symmetry can lead to nearly coincident parity-doublet trajectories. The symmetry associated with exchange degeneracy discussed in Sec. VII.C also encourages trajectories with the same η parity (τP) and isospin to coincide. Consequently, trajectories rarely, if ever, are independent.

The existence of second-order poles, or dipoles, in addition to first-order poles, has also been suggested [350]. Such poles do not result in the same phase for all amplitudes and can therefore lead to nonzero polarization effects.

B. Cuts

One of the first serious arguments for the existence of cuts was given by Amati, Fubini, and Stanghellini [36], who proposed from a study of Feynman graphs that a cut could be generated from the simultaneous exchange of two Regge poles (see Fig. 2). Mandelstam pointed out that in the absense of the third double-spectral function, such a cut would be canceled by other diagrams corresponding to multiparticle channels [384], but that for more complicated diagrams cancellation does not appear to take place and such cuts should be present [385]. Mandelstam, using analytically continued elastic unitarity, was also able to show that these cuts invalidated the proof of the existence of essential singularities proposed by Gribov and Pomeranchuk [279] which occur at wrong signature points $[(-1)^\alpha=-\tau]$ and at which $\alpha=s_a+s_b-1$. Recently this relationship and other properties of Mandelstam cuts (e.g., signature) have been elucidated [438].

In addition to the original Mandelstam cuts which resulted from diagrams where the intermediate state consisted of a ladder (Regge pole) and an elementary particle, Fig. 3, Schwartz [489] finds the motion of the Mandelstam cuts (Type 2) must be shielded by another type of cut (Type 1) in the physical scattering region.

Although at present there is little known of the behavior of the discontinuities across cuts [113], the motion of the branch point is fairly well understood. The motion of the branch point of the first type of cut is given by

$$\alpha_{AB}^{(1)}(x)=\alpha_{AB}[x, y(x)] \tag{III.1}$$

where $x=t^{1/2}$, y is solution of

$$\delta\alpha_{AB}/\delta y=0, \tag{III.2}$$

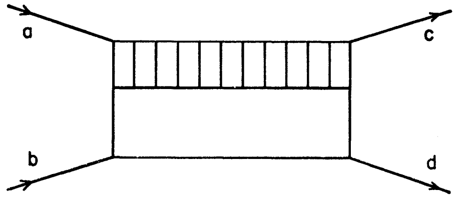

FIG. 2. Simplest diagram illustrating origin of cuts proposed by Amati *et al.* [36].

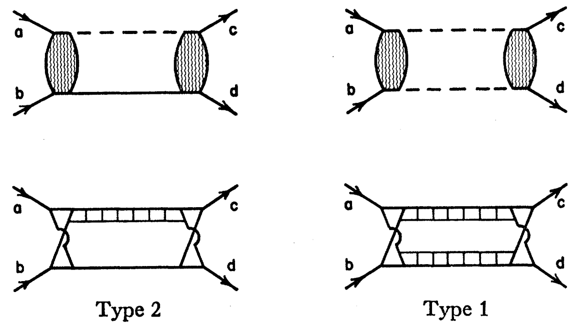

FIG. 3. Diagrams for cuts due to the exchange of a particle and a Regge pole (type 2 cut), and due to the exchange of two Regge poles (type 1 cut). Lowest-order Feynman diagrams responsible for cuts are also shown.

and $\alpha_{AB}(x, y)=\alpha_A(x-y)+\alpha_B(y)-1$. (Note $\alpha_{AB}^{(1)}$ is symmetric in A and B.) The branch point of the cuts of the second type is given by

$$\alpha_{AB}^{(2)}(x)=\alpha_A(x-m_B)+s_B-1=\alpha_{AB}(x, m_B), \tag{III.3}$$

where B is considered an elementary particle [489].

In the discussions of certain papers it will be useful to have expressions for $\alpha_{AB}^{(1)}$ in various situations. For the case of A and B being the same trajectory one has

$$\alpha_{AA}^{(1)}(t^{1/2})=2\alpha_A(t^{1/2}/2)-1. \tag{III.4}$$

For the case when trajectories can be approximated as linear functions of t, one obtains

$$\alpha_{AB}^{(1)}(t)=\alpha_A(0)+\alpha_B(0)-1+[\alpha_A'\alpha_B'/(\alpha_A'+\alpha_B')]t. \tag{III.5}$$

For the exchange of a cut and a trajectory—which is equivalent to the exchange of three trajectories—one obtains

$$\alpha_{ABC}^{(1)}(t)=\alpha_A(0)+\alpha_B(0)+\alpha_C(0)-2 +[\alpha_A'\alpha_B'\alpha_C'/(\alpha_A'\alpha_B'+\alpha_B'\alpha_C'+\alpha_A'\alpha_C')]t. \tag{III.6}$$

For n identical trajectories one obtains

$$\alpha_{nA}^{(1)}(t)=n[\alpha_A(t/n^2)-1]+1, \tag{III.7}$$

which is also true without assuming a linear t dependency [381, 384, 385]. For the simultaneous exchange of n linear trajectories of type A and one of type B, one obtains

$$\alpha_{nAB}^{(1)}(t)=n[\alpha_A(0)-1]+\alpha_B(0) +[\alpha_A'\alpha_B'/(\alpha_A'+n\alpha_B')]t. \tag{III.8}$$

Type 1 cuts, whose position is dependent on masses, are located on an unphysical j sheet for $t<(m_A+m_B)^2$ and thus do not contribute to the amplitude in the physical s channel [458, 489]. Their branch points are of the form $\log[j-\alpha(t)]$ [458, 489]. To obtain agreement with Bronzan and Jones [113] who, working with elastic unitarity, found that the discontinuity across a cut must be singular and vanish at the branch point, such log terms must be multiplied by a function which is singular and vanishes.

Several groups [328, 389, 489] have demonstrated how the Mandelstam cut for a signatured amplitude permits the Gribov–Pomeranchuk essential singularity to be replaced by a simple fixed pole which gives no contribution to the physical amplitude. The removal of the Gribov–Pomeranchuk singularity was necessary to avoid a conflict with the Froissart bound for channels that couple to states for which $s_a+s_b>2$. (See in particular the arguments of Muzinich [411] for a $\rho\rho$ cut.) Olive and Polkinghorne [418], who verify the conclusions of Schwarz [307], find that the mechanism responsible for avoiding the Gribov–Pomeranchuk essential singularity also works for states of low spin where the Froissart bound is unimportant.

The validity of certain convergence relations in the presence of these fixed poles and cuts is discussed by Schwarz [489, 490]. (See Mandelstam and Wang [389]). The Mandelstam-cut mechanism has been used to argue that the Pomeron is probably not a fixed pole [223]. The concept of a cut shielding a fixed pole has been used in postulating a fixed-pole diffraction model [416]. A theoretical treatment of the contribution of Mandelstam cuts to forward NN scattering has also been discussed [110, 212].

The generation of cuts proposed by Mandelstam, together with the assumption that the intercept of the Pomeron trajectory is identically unity, leads to an infinite sequence of Regge cuts [262]. This feature, at least for high energies, results in either vanishing total cross sections or essentially constant diffraction peaks [262]. These difficulties have encouraged the postulation that $\alpha(0)=1-\epsilon$, which would predict twisting or curving effective trajectories for $t<0$ [448, 506].

Theoretical studies which consider the effects of cuts in the Regge pole theory have also been made [209, 226, 466, 487, 515]. Papers proposing tests for cuts are discussed in Sec. XIV.

Double-charge-exchange reactions should be dominated by cuts due to exchanges such as $\rho\rho$, ρK^*, or K^*K^* [398, 432]. Consequently, superconvergence relations based on the nonexistence of a known trajectory or contributions corresponding to double charge exchange with $\alpha>0$ are in doubt.

In conclusion, it appears that the mechanism for generating cuts in the complex-angular-momentum plane is well established. In general, the importance of cuts depends on the magnitude of the third double-spectral function. Consequently, many people in the field hope that the effects due to double-spectral functions are small. As will be pointed out in the discussion of fixed poles, the famous dip in charge-exchange scattering associated with $\alpha_\rho=0$ would be absent or strongly modified if contributions from third double-spectral functions were important. Another argument for the unimportance of cuts is the experimental fact that reactions for which *no* known particles can be exchanged in the crossed channel, such as reactions with double charge exchange, are much smaller than those for which known particles can be exchanged. Perhaps the vanishing of discontinuity across the cut at the branch point [113] could be sufficient to reduce the importance of cuts in the scattering amplitude.

C. Fixed Poles and Kronecker Delta Functions

The last section reviewed how the violation of the Froissart limit by the Gribov–Pomeranchuk singularities necessitates the introduction of Mandelstam cuts and fixed poles at the position of Gribov–Pomeranchuk singularities [328, 489]. We have not yet considered that if there is a third double-spectral function, there will be Regge amplitudes of both signatures [383]. The singular contributions to an amplitude of signature, τ, from the third double-spectral function cancel at right signature values of $j[(-1)^j=\tau]$ and add at wrong signature values of $j[(-1)^j=-\tau]$. Consequently, fixed poles occur only at wrong signature values of j. Mandelstam and Wang [389] (see also Refs. [209, 328, 406, 407]) have recently studied the consequences of such singularities and concluded that at wrong signature points, residue functions for trajectories that would otherwise choose sense or nonsense have the same α dependence. The existence of such fixed poles implies that the Schwarz [490] superconvergence rules are invalidated and the question of dips at wrong signature such as πN charge exchange at $\alpha_\rho=0$ must be reconsidered. Possible experimental evidence for a fixed pole in the spin-flip amplitude of πN scattering has been found [198, 499].

In addition to the fixed poles that occur only at wrong signature points, there are arguments for fixed poles at both right and wrong signature points [5, 520, 327]. Fixed poles for weak amplitudes exist at both types of signature points and exist even in the absence of a third double-spectral function [5]. Fixed poles occurring in spin-flip amplitudes at both types of signature points were found in a perturbative field theory model without a third double-spectral function [520].

Many people have shown that nonunitary amplitudes such as Compton amplitudes have a fixed pole [5, 111, 112, 209, 389, 406, 501]. The fixed pole in the Compton amplitudes is necessary to allow the Pomeron to contribute to the total Compton cross section and to avoid having the total cross section vanish asymptotically faster than the elastic cross section [6, 410, 500]. The total cross section would vanish asymptotically because the only amplitude contributing in the forward direction is a sense–nonsense amplitude which is proportional to $[\alpha(t)-1]$ in the absence of a fixed pole. In the Compton scattering of charged photons by pions, a fixed pole was found at $j=1$ by the consideration of the Fubini sum rule [361] and the assumption of only moving Regge poles [501]. If the photons are replaced by rho mesons, the assumption of only moving Regge poles is sufficient and thus the argument for a fixed pole in Compton scattering is not applicable to purely strong interactions [501].

The possibility of a fixed pole in a Compton amplitude, e.g., $\gamma\gamma\rightarrow\pi\pi$, follows naturally [3] from considering the amplitude [4]

$$A=\bar{F}_{11}{}^{s}/(s-m^2)^2=2\bar{F}_{11}{}^{t}/(t-4m^2)^2, \qquad \text{(III.9)}$$

which is free of kinematic singularities in s and t [3, 4]; while the Born amplitude $A^B=2e^2/(s-m^2)(u-m^2)$ is finite at $t=0$, the positive-signatured amplitude $A_+{}^B=2e^2/t(m^2-s)$ diverges at $t=0$. Consequently, if one assumes that there are no fixed poles at $j=1$, which implies

$$\frac{2e^2}{t}+\int_{4m^2}^{\infty} ds \ \mathrm{Im}\ A_+(s)P_0(z)=0,$$

one obtains $2e^2/t+\beta(s/s_0)^\alpha\sim 0$ since

$$\mathop{\mathrm{Im}}_{s\to\infty} A_+\sim\beta(\alpha-1)s^{\alpha-1}.$$

Since the first term is divergent at $t=0$, one must conclude that either the residue is singular at $t=0$, i.e., $\beta\sim 1/t$, or that there is a fixed pole at $j=1$. It is the zero value of the photon mass which causes the fixed pole, and therefore a fixed pole is not needed if the photon had a nonzero mass.

Sum rules for weak amplitudes imply that the real parts of certain weak amplitudes cannot be Reggeized [74]. A mechanism for such behavior due to a fixed pole at $j=1$ has been given [112]. A fixed pole would result in a non-Regge $1/s$ asymptotic behavior for the full amplitude, while the imaginary part would have the normal Regge asymptotic behavior except for the absence of a factor of $[\alpha(t)-1]$. The possibility has been discussed of a $j=0$ fixed pole in pion photoproduction amplitudes [289]. (This is a right signature point for the π trajectory). A formalism has been proposed [334] that provides a natural description of fixed poles in photoproduction reactions.

Rubinstein *et al.* [469] use a perturbative model to discuss the fixed poles in the asymptotic behavior of weak amplitudes. They conclude that the existence of fixed poles is model dependent: it depends on whether the weakly interacting particles are considered elementary or composite. They criticize the existence of a fixed pole in pion photoproduction [289] and argue that if the pion is a composite particle, the amplitude has a normal Regge behavior. The PCAC assumption, which equates the pion field to the divergence of an axial current, would introduce a fixed pole whose effects are inconsistent with their results.

Difficulties associated with $t=0$, in the case of scattering particles with unequal mass, are usually described in the context of daughter trajectories, but fixed poles may also cancel the undesired contributions [332, 353, 434].

In conclusion, fixed poles can exist in strong and weak amplitudes as well as in reactions involving photons. Many uncertainties concerning fixed poles remain: the analytic structure of their residues, whether they are additive or multiplicative to Regge poles, and how they are related to other singularities in the complex-angular-momentum plane.

Regarding the latter question, fixed poles in spin-flip weak amplitudes at nonsense points are in general accompanied by Kronecker delta functions at sense points of spin–nonflip amplitudes [208, 283]. The existence of Kronecker-delta-function singularities has also been discussed by other authors [205, 388, 439, 507]. In particular, in the Lee model, the delta functions of elementary particles are canceled as a Regge trajectory moves to the position of the elementary particle pole [439].

IV. ALPHA FACTORS AND GHOST-KILLING MECHANISMS

In this section we review the analytic behavior of residues at integer values of α. A channel with helicity λ is normally called a sense channel if $\lambda\leq\alpha$ and is denoted by the subscript s; for $\lambda>\alpha$, it is called a nonsense channel, denoted by a subscript n. As mentioned in Sec. III.B, Mandelstam and Wang [389] have shown that if there is a nonzero third double-spectral function, then at a wrong signature point p, the residues have the following behaviors:

$$\beta_{ss}\sim 1, \quad \beta_{sn}\sim[\alpha(t)-p]^{-1/2}, \quad \beta_{nn}\sim[\alpha(t)-p]^{-1}.$$

Care must be taken in using these expressions for $p<0$; the reason is given below. It will be assumed that the α values correspond to right signature points or that the third double-spectral function is zero.

In the Gell-Mann *et al.* [260] Reggeization procedure, the amplitude is proportional to the product of a signature factor, a residue function, and a function $E_{\lambda\mu}{}^{\alpha}(z)$, which can be shown to have square-root singularities in α at point p satisfying $m>p\geq n$, or $-n>p\geq -m$, where $m=\mathrm{Max}\,(|\lambda|, |\mu|)$ and $n=\min\,(|\lambda|, |\mu|)$. Consequently, for an amplitude to omit undesirable branch points, the residue function must contain square-root singularities of the form $[(\alpha-p)(\alpha+p+1)]^{1/2}$ for each p.[7]

Factorization, $(\beta_{sn})^2=\beta_{ss}\beta_{nn}$, implies that square-root zeros of β_{sn} must occur as full zeros in the product $\beta_{ss}\beta_{nn}$. Since a square-root singularity in β_{ss} and β_{nn} would result in branch points in their respective amplitudes, either $\beta_{ss}\sim(1)$, $\beta_{nn}\sim(\alpha-p)$ or $\beta_{ss}\sim(\alpha-p)$, $\beta_{nn}\sim(1)$ near $\alpha=p$. The first mechanism is

[7] For a given p, only one of the points corresponds to a right signature point; consequently the discussion should be considered limited to that point if the Mandelstam argument applies. Also, only at $\alpha=p>0$ is the residue a sense–nonsense residue; a square-root singularity occurs at the negative value of α given by $\alpha=-p-1$, where the residue is a nonsense–nonsense residue. To avoid this confusion, the subscript sn will be used for α at any of the p points.

called the sense-choosing mechanism since the trajectory couples to sense–sense amplitudes, while the second mechanism is called Gell-Mann mechanism or nonsense mechanism, since the trajectory couples to nonsense–nonsense amplitudes.[8]

It is also possible to assume $\beta_{sn}\sim(\alpha-p)^{3/2}$. In this case, two of the four solutions are preferred. They are called the Chew mechanism and the no-compensation mechanism and are $(\alpha-p)$ times the sense-choosing mechanism and Gell-Mann mechanism, respectively. In summary, for $p=0$ the various mechanisms are

	β_{ss}	β_{nn}	$\beta_{sn}\alpha^{-1/2}$	
Sense-choosing	1	α	1	
Gell-Mann	α	1	1	
Chew	α	α^2	α	
No compensation	α^2	α	α.	(IV.1)

Similar discussions and expressions can be found in the literature [98, 136, 138, 209].

If a trajectory uses the same mechanism at all points within $-s\leq j<s$, $s>m$, and if the effects of the third double-spectral function can be neglected, then one obtains the general expressions:

Sense-choosing

$$\beta^s\sim[(\alpha+m)!(\alpha+n)!/(\alpha-m)!(\alpha-n)!]^{1/2},$$

Gell-Mann

$$\beta\sim[(\alpha+s)!/(\alpha-s)!](\beta^s)^{-1},$$

Chew

$$\beta\sim[(\alpha+s)!/(\alpha-s)!]\beta^s,$$

No compensation

$$\beta\sim[(\alpha+s)!/(\alpha-s)!]^2(\beta^s)^{-1}. \qquad (IV.2)$$

In the Gell-Mann *et al.* [260] method of Reggeizing there is a $(\alpha!)^{-1}$ factor which always removes the poles of $(\sin\pi\alpha)^{-1}$ for negative α's. If an unwanted pole were to occur at $\alpha=0$, the trajectory could choose any of the four mechanisms except the first. In the Gell-Mann mechanism, the pole is removed by assuming a compensating trajectory whose contribution cancels the pole [260]. A pole at $\alpha(t)=0$ for negative values of t is called a ghost state and if not canceled or "killed," would result in an infinite cross section. Ghost states and ghost-killing mechanisms have been discussed for the N/D Equation [163, 517].

V. KINEMATIC SINGULARITIES AND CONSTRAINT EQUATIONS AT NORMAL AND PSEUDOTHRESHOLDS

In addition to the many advantages of using helicity amplitudes, e.g., the ease with which angular momentum and parity properties can be written and the simplicity of their crossing relations, there are also some disadvantages. In particular, unlike invariant amplitudes which can be selected to be free of kinematic singularities [87, 229, 482], helicity amplitudes in the general spin case contain kinematic singularities.[9] Due to the appeal of helicity amplitudes to Regge pologists, there has been considerable effort to disentangle their kinematic properties [1, 18, 234, 298, 321, 345, 511, 512, 541, 545].

A. Kinematic Singularities of Helicity Amplitudes

The existence of kinematic singularities in helicity amplitudes for particular reactions can be easily illustrated by writing the helicity amplitudes in terms of invariant amplitudes [188, 548]. However, it is usually difficult to use this method to consider arbitrary reactions.

One of the most elegant methods to isolate kinematic factors is based on the analytic properties of the crossing matrix [541, 545]. The approach due to Wang [247] depends on the realization that s-channel amplitudes, except for their known half-angle functions, are free of kinematic singularities in t. By relating the t-channel amplitudes through the crossing matrix to the s-channel amplitudes, the kinematic singularities of the t-channel amplitudes can be isolated. Though this approach is simple and elegant in principle, it is anything but that in reality.

If the amplitudes of Gell-Mann *et al.* [260] are considered, the structure of the kinematic singularities can be easily understood in terms of elementary principles of angular-momentum coupling [298, 321]. In particular, for a given j value in a partial-wave expansion, the order of the singularity is determined by the product of the e function [260], which has a singularity of order $j-m$ ($m=\max(|\lambda|, |\mu|)$ and the parity-conserving partial-wave amplitude. The partial-wave amplitude reduces the singularity of the e function by an amount given by the minimum possible value of orbital angular momentum. The order of the singularity is greatest for $j\geq s_a+s_b$, where it is independent of j and is the sum of the particle spins, minus m and one or zero, depending on the parity being considered.

Explicitly, the kinematic singularity factors associated with the external particles a and b in a parity "conserving" amplitude with η parity, η, are given by

$$[t-(m_a+m_b)^2]^{-(1/2)\kappa_+}[t-(m_a-m_b)^2]^{-(1/2)\kappa_-}, \qquad (V.1)$$

with $\kappa_\pm=s_a+s_b-m-\frac{1}{2}(1-\eta\eta_a\eta_b(-1)^{s_a\pm s_b-\nu})$, where $\nu=0, \frac{1}{2}$, such that $s_a+s_b-\nu$ is an integer, s_i and η_i are the spin and intrinsic parity of particle i, and the labels a and b are assigned such that $m_a>m_b$.[10,11]

[8] The vanishing of residues of nonsense values of j has a natural interpretation when viewed from the method of Reggeization of invariant amplitudes of Jones and Scadron [331].

[9] "Kinematic" here refers to nonzero values of t associated with the masses of the particles in the initial and final states. In particular, for the t-channel reaction $ab\to cd$, the points $(m_a+m_b)^2$ and $(m_c+m_d)^2$ are referred to as threshold points, while the points $(m_a-m_b)^2$ and $(m_c-m_d)^2$ are referred to as pseudothreshold points (see Fig. 1).

[10] The restriction of $m_a>m_b$ is only important for half-integral values of j (i.e., $\nu=\frac{1}{2}$), where the equations are written for $t^{1/2}>0$. The singularities for $t^{1/2}<0$ are easily found by using MacDowell symmetry (see Sec. VII.D) and have been written out explicitly by Henyey [295].

[11] The situation when one or more of the external particles has zero mass must be treated specially [17, 164, 245, 273, 334].

B. Kinematic Constraint Equations for Helicity Amplitudes

Since physical quantities such as s-channel differential cross sections are free of kinematic singularities in t, there must be relations among the amplitudes which remove such singularities in the physical s channel [149, 150, 262, 321, 394].

The required constraint equations can be easily found for simple reactions by using invariant amplitudes [188]. The method used by Wang [541] to isolate kinematic singularities can also be inverted to find the constraint equations. In particular, by expressing the s-channel amplitude in terms of the t-channel amplitudes, the constraint equations can be obtained by demanding that s-channel amplitudes be free of kinematic singularities in t (see Sec. VI.A). However, this method has not been attempted except for constraint equations associated with the point $t=0$ [13, 238, 303]. Kinematic constraint equations are also obtained from a consideration of angular-momentum coupling in the t channel [321, 511].

The requirement that helicity amplitudes satisfy constraint equations and thus lead to physical quantities free of inverse powers of kinematic factors causes a modification of many of the expressions used previously in comparing Regge pole theory to experiment [321]. Such modifications are important when m_a-m_b is small, such as with an $N\Delta$ vertex.

The complications resulting from kinematic singularities and constraint equations have caused people to seek other amplitudes [149, 332]. In particular, s-channel helicity amplitudes [332] provide a natural way to avoid the complications associated with t-channel kinematics.

VI. CONSTRAINT EQUATIONS, DAUGHTER THEORY, AND DYNAMICAL SOLUTIONS FOR REGGE TRAJECTORIES ASSOCIATED WITH THE POINT $t=0$

A. Constraint Equation at $t=0$

The existence of constraint equations among helicity amplitudes at $t=0$ was first pointed out by Goldberger *et al.* [271] and later by Volkov and Gribov [535] for elastic NN amplitudes. These constraints are essentially due to the relation $z_s(t=0)=1$, which implies that all s-channel amplitudes for which $\lambda_s\neq\mu_s$ vanish. By expressing these vanishing s-channel amplitudes in terms of the t-channel amplitudes, one obtains constraint equations among the latter.

This section reviews the more recent work in this area [13, 150, 231, 238, 303, 335, 354]. For completeness we consider constraint equations occurring at any of the zeros of the product of $t\times TT'$ (see the Appendix). As mentioned in Sec. V, the constraint equations can easily be obtained by relating the s-channel amplitudes to the t-channel amplitudes through the crossing matrix and demanding that s-channel amplitudes with half-angle functions removed have only dynamical singularities in t.

In the following discussion, it will be useful to consider the parameter $m^2=(m_a^2-m_c^2+m_d^2-m_b^2)$ for the t-channel reaction $ab\rightarrow cd$. Consider the vector equation $f^t=Mf^s$, where M is the crossing matrix. Since s-channel amplitudes f^s are finite functions of t except for dynamical poles, f^t can only diverge at the points at which M diverges. The cosines of the crossing angles diverge at t values given by the equation $TT'=0$. If $t=0$ is a solution of $TT'=0$, the cosines of the crossing angles will diverge only if $m^2\neq0$, i.e., when only one pseudothreshold is at the point $t=0$.

In the inverted equation $f^s=M^{-1}f^t$, the left-hand side will vanish at $z_s=\pm1$ if $\lambda_s\pm\mu_s\neq0$. Only $z_s=+1$ can occur at a t value independent of s and then only for elastic scattering in the s-channel, i.e., $m^2=0$. For $m^2\neq0$, no nontrivial constraint equations exist at $z_s=1$. To understand this, it is only necessary to realize that if $m^2\neq0$, $z_s=1$ implies $z_t=1$ and thus $f^t=0$ if $\lambda_t-\mu_t\neq0$. Since $\lambda_t-\mu_t=\lambda_a-\lambda_b-(\lambda_c-\lambda_d)=\lambda_d-\lambda_b-(\lambda_c-\lambda_a)=\lambda_s-\mu_s$, the same number of amplitudes vanish in each channel and there will be no nontrivial constraint equations. For elastic scattering, i.e., $m^2=0$, vanishing of the s-channel amplitudes results in constraint equations at $t=0$ of the type found by Goldberger *et al.* [271]. For a recent discussion of this situation, see Abers and Teplitz [13].

The matrix M^{-1} has the same divergent properties as M. Since f^s is finite, the divergent terms in the product $M^{-1}f^t$ cancel. If we write $f^t=K\cdot\bar{f}^t$, where K is the most divergent factor in f^t, this cancellation implies constraint equations among the functions $\bar{f}^t$ and possibly their derivatives. The number of derivative relations will depend on the values of the helicities and spins of the reaction being considered [321]. This technique has been used in the literature for the point $t=0$ [13, 359].

In conclusion, for the various mass configurations,[12] one finds [228]:

(a) UU, general mass case ($m^2\neq0$). Constraint equations for helicity amplitudes occur only at normal and pseudothresholds and not at $t=0$.

(b) EU, only one set of masses equal ($m_a=m_b$ or $m_c=m_d$, $m^2\neq0$). The constraint equations are of the same form as above, with the pseudothreshold constraint corresponding to the equal pair of masses now occurring at $t=0$.

(c) EE, both sets of masses equal ($m_a=m_b$, $m_c=m_d$, $m^2=0$). The constraints at the normal thresholds are

[12] In the following, it is assumed no masses are zero. For constraint equations for photoproductions, see Refs. [196, 240, 273, 334, 509, and 525].

the same, but now there is only one other set of constraint equations which occurs at $t=0$.[13]

The amplitudes involved in constraint equations at $t=0$ for $m^2=0$ and $m^2\neq0$ are, in general, not the same. This has led some authors to argue that constraint equations for $m^2=0$ are not the equal mass limits of constraint equations for $m^2\neq0$ (see Refs. [13 and 303]). This seemed reasonable, since the divergence of M for $m^2=0$ arises from s-channel half-angle functions, while for $m^2\neq0$ it results from the crossing matrices. Stack [508] has shown that if all quantities are carefully expanded in terms of the mass difference, the limit of the unequal-mass constraint equations is indeed the equal-mass constraint equations.

The above method is useful for the identification of constraint equations, but is impractical for actual calculation. The use of invariant amplitudes free of kinematic singularities provides an easier method when such amplitudes can be identified. The constraint equations found by Goldberger *et al.* were found in this way.

B. Daughter Trajectories

In the general mass case, individual Regge pole contributions do not retain the asymptotic form $s^{\alpha(t)}$ for small values of t [58] because z_t $(t=0)$ is unity independent of s. Seemingly different arguments by Freedman and Wang [248] and Goldberger and Jones [272] were originally given to show that the behavior $s^{\alpha(t)}$ is maintained at small t, even though z_t approaches unity. A later paper by Freedman, Jones, and Wang [246] shows that the arguments were compatible and could be combined to give a more complete proof. The asymptotic behavior is maintained by Regge trajectories which occur in families whose intercepts at $t=0$ are equally spaced by integers. The lower or so-called "daughter" trajectories cancel the undesirable terms in the amplitude of the parent, so that the full amplitude retains the $s^{\alpha(t)}$ behavior at small t.

The proofs of the existence of daughter trajectories mentioned in the above paragraph are mathematical and not very intuitive. Consequently, various authors have proposed arguments to demonstrate that the existence of daughter trajectories is natural. Durand [215, 217] was one of the first to associate daughter trajectories with lower spin components inherent in off-mass-shell generalized Feynman propagators for bosons coupled to unequal-mass channels. He [215] pointed out that the amplitude for an intermediate boson of spin j and mass μ is given by

$$f=(2j+1)(4\bar{p}\bar{p}')^jP_j(\bar{z})/(\mu^2-t),$$

where

$$2\bar{p}=[t-2m_a^2-2m_b^2+(m_a^2-m_b^2)^2/\mu^2]^{1/2},$$

$$2\bar{p}'=[t-2m_c^2-2m_d^2+(m_c^2-m_d^2)^2/\mu^2]^{1/2},$$

and

$$\bar{z}=\tfrac{1}{4}[s-u+(m_a^2-m_b^2)(m_c^2-m_d^2)/\mu^2](\bar{p}\bar{p}')^{-1}.$$

When j is replaced by α and μ^2 in $\bar{p}\bar{p}'$, and $\bar{z}$ is replaced by t, this is the normal Regge amplitude. The factor μ^2 instead of t in the expression $\bar{p}\bar{p}'\bar{z}$ allows the dominance of the term $(s-u)$ for all t and ensures that this amplitude, unlike a Reggeized amplitude, will retain the normal asymptotic behavior of $(s-u)^j$ at small t and large s. In terms of the functions pp' and z, which are the above expressions with μ^2 replaced by t, one can write

$$(4\bar{p}\bar{p}')^jP_j(\bar{z})=(4pp')^jP_j(z)-(\mu^2-t)(m_a^2-m_b^2)\times(m_c^2-m_d^2)\mu^2t^{-1}(4pp')^{j-1}P_{j-1}(z)$$

plus terms involving lower order P_j's.

The leading term is the normal Regge contribution, and the lower-order terms have the correct $1/t$ singularities for terms due to daughter trajectories. (If either pair of external masses is equal, the dauhgter trajectories decouple.) Durand also demonstrated that the same terms were obtained in the infinite momentum limit of Regge trajectories in potential scattering.

Sugar and Sullivan [515] used the Van Hove model and amplitudes similar to those above, but with self-energy insertions in the propagators to study the motion of the daughter trajectories. They concluded that although the results were model dependent, the daughter trajectories would probably not be parallel to the parent and found that the slope of the first daughter was negative at $t=0$ if $\alpha_D(0)>-5/2$. The existence of daughter trajectories has also been demonstrated in perturbation theories containing ladder diagrams [519, 521] and in a dynamical model based on superconvergence equations and the hypothesis that amplitudes are described asymptotically by Regge poles [468]. The study of daughter trajectories in the Bethe-Salpeter equation, first pointed out by Freedman and Wang [248], is being continued [146, 147, 204, 277, 381].

Oakes [62] has proposed that, to avoid difficulties at $t=0$ for unequal-mass scattering (see also Pasches [427]), amplitudes be expanded in terms of Gegenbauer polynomials $C_n'(\Omega)$.[14] The argument of the Gegenbauer polynomials is

$$\Omega=(s-u)[(s+u)^2-(m_a^2-m_c^2+m_b^2-m_d^2)^2]^{1/2},$$

[13] Bardakci and Segre [72], who obtained constraint equations for the EE mass case by considering invariance properties of the amplitude at $t=0$, established a connection between constraint equations and superconvergence relations for amplitudes.

[14] It might be argued that since the Gegenbauer polynomials (see Bertocchi [222]) are irreducible representations of 0(4), this approach is equivalent to those discussed in Part C of this section.

which has the property that $\Omega(t\sim 0)=\frac{1}{2}(s-u)/(m_a^2+m_b^2)(m_c^2+m_d^2)$. With the asymptotic property that $C_\alpha'(\Omega)\sim\Omega^\alpha$, the normal $(s-u)^\alpha$ behavior for all t is easily obtained; Oakes then expands the Gegenbauer polynomials C_α' in terms of $P_{\alpha-n}$ and finds daughter trajectories ($n=1, \infty$) having the required residue properties. He concludes, like Durand [215], that daughters are essentially extra spin components which are present in tensor fields describing elementary particles when the intermediate particle is off its mass shell. He goes on to conclude that daughter trajectories should consequently be parallel for all t, and in analogy with free fields, whose extra spin components are eliminated by subsidiary conditions, daughter trajectories should not contribute at physical values of angular momentum. This last argument would, of course, imply that daughter trajectories would not lead to any physical particles.

The argument for daughter trajectories results from the fact that if an amplitude, which behaves for some reason as $(s-u)^{\alpha(t)}$ for all t, is expanded at fixed t in a Legendre series of argument z, the resulting series will always contain components with spin $\alpha(t)-n$ ($n=0, 1, 2, \cdots$). The coefficients in the expansion will be proportional to $[4pp'z-(s-u)]$; i.e., they will vanish if a pair of masses is equal and otherwise decrease as t moves away from zero. The question whether daughter trajectories are parallel or not is thus equivalent to the question whether the amplitude varies as $(s-u)^{\alpha(t)}$ for all t or only in a small neighborhood of $t=0$ and only approximately as $(s-u)^{\alpha(t)}$ for other values of t.

Arguments similar to this have recently been given by Sheftel *et al.* [498], who believe the hypothesis that equally spaced daughter trajectories can be built into a theory by making definite dynamical assumptions about the behavior of amplitudes as a function of α. They conclude that the existence of daughter trajectories is equivalent to assuming some type of dynamical symmetry.

The crossing symmetric amplitude proposed by Veneziano [534] which demonstrates many of the properties of Reggeized amplitudes corresponds in terms of Regge poles to an infinite sum of parallel daughter trajectories.

C. Four-Dimensional Symmetry

In the first paper by Freedman and Wang [248] in support of their conclusions on daughter trajectories, they state that daughter trajectories exist in all solutions of the Bethe–Salpeter equation which Reggeize. They stated that this follows from the four-dimensional symmetry of the Bethe–Salpeter equation at $t=0$.

Prior to this, several authors [203, 526–528] had already proposed that amplitudes for equal-mass scattering obey a four-dimensional symmetry at $t=0$. In particular, Toller [526, 527] "Reggeized" expansions of amplitudes in terms of $O(3,1)$, the homogeneous Lorentz group.[15] Toller found that a pole in the four-dimensional representation led to an infinite family of Regge poles when the amplitudes were expanded in terms of the three-dimensional representations, $D_{\lambda\mu}^j$, of $O(3)$. Poles associated with the four-dimensional group are usually referred to as Lorentz or Toller poles.

One way to understand how amplitudes could be invariant under a four-dimensional group is to consider the total four momentum $K=p_a+p_b=p_c+p_d$ of a reaction [249]. If K is timelike, it is invariant under the rotation group $O(3)$. If amplitudes are assumed to obey the symmetry of $O(3)$, their Jacob and Wick [322] expansion in terms of $D_{\lambda\mu}^j$ is a natural consequence. If K is zero, it is invariant under four-dimensional rotations in the homogeneous Lorentz group $O(3,1)$. Consequently, one might assume that for $K=0$, the amplitudes also possess the larger symmetry.

The fact that $t=K^2$ vanishes does not imply that $K=0$. Only when $m_c=m_b$ and $m_c=m_d$, does the vanishing of t correspond to $K=0$. This is the mass configuration originally treated by Toller and more recently by Freedman and Wang [247].

Freedman and Wang [249] have also discussed the equal-mass NN amplitudes and classified the various types of Regge families that contribute according to the $O(4)$ quantum numbers of their Toller poles. This classification and its relevance to solutions of the $t=0$ constraint equations is given in Part D of this section.

It appears that while the $O(4)$ symmetry is only a true symmetry in the pairwise equal-mass situation at $t=0$, the resulting classification is still true for the trajectories independent of the external-mass configuration [201, 205]. The trajectories should retain the exact symmetry classification for all external-mass configurations because although the coupling (i.e., residue function) of trajectories depends on the masses of the external particles, the trajectories themselves are the same for all reactions.

Domokos and Tindle [205] have considered Lorentz invariance of scattering amplitudes for particles of arbitrary finite mass and spin. They make a clear distinction between the "invariance group" of the amplitudes and the "classification group" of the trajectories. While the former is dependent on external masses, the latter is independent of external masses. They derive an expansion for amplitudes in terms of Toller poles for arbitrary masses which reduces to the normal expression for the equal-mass case.

Domokos [201] uses the Bethe–Salpeter equation to demonstrate how the inequality of external masses

[15] In the literature, some authors work with the group 0(4) and others with 0(3, 1). Which group is applicable depends on whether the variable s is continued to unphysical values (Wick rotation) where 0(4) symmetry applies or is kept in the physical region when the amplitudes are Reggeized. The equivalence of the two approaches has been studied by Akyeampong *et al.* [28]. In potential scattering, the analogy of the Lorentz group is the Galilei group, and the analogous daughter trajectories form a continuous sequence [148].

forbids continuing amplitudes with exact $O(4)$ symmetry to the mass shell and then letting t go to zero. He proposes a breaking of the symmetry due to mass differences and concludes that only in the limit of perfect symmetry are all trajectories of a family parallel. He also deduces a one-parameter formula to describe the deviation of the slopes of the trajectories from the exact symmetry value (see Domokos [200, 201]). Essentially the same expressions for the slopes of daughter trajectories have been obtained from a study of daughter trajectories in the Bethe–Salpeter equation [146, 147].

Schwarz [488] considers the $O(4)$ amplitudes and is able to carry out an analog of the Mandelstam form of the Sommerfeld–Watson transformation, as a consequence of the absence of Gribov–Pomeranchuk fixed-pole singularities at right signature points. He points out that the asymptotic contribution of a single Lorentz pole cannot be used near the s-channel threshold in the forward direction for elastic amplitudes and suggests a modified form to be used at intermediate energies.

Before leaving the discussion of the "Reggeization" of amplitudes with respect to parameters of the Lorentz group, one should point out that the recent work of Feldman and Mathews [221], Iverson [318], Rubin [467], Roffman [455], and Chiu and Stirling [143] have considerable bearing on the problem. The papers of Iverson [318] and Rubin [467] verify the existence of daughter trajectories as a natural consequence of the Lorentz group symmetry. Although the paper by Feldman and Mathews [221] is not directly concerned with Regge poles, it provides a manifestly covariant formalism for Regge pole theory.

The contributions of Toller poles to various reactions near the forward direction, such as pion photoproduction [289, 400, 479] and vector meson production [99, 358, 479, 480, 499] have been considered. The study [153] of the contributions of evasive Toller poles to s-channel helicity amplitudes has been extended to include exchanges of more general Toller poles for the unequal-mass situation [356]. Many of the papers consider families of Regge poles resulting from individual Toller poles and how they conspire to give contributions to forward scattering. It is useful, therefore, to consider the Toller poles that provide solutions to constraint equations.

D. Solutions of Constraint Equations and Their Classifications According to 0(4) Symmetry

Before the importance of four-dimensional symmetry to the solution of constraint equations was realized, the constraint equations at $t=0$ were known to be different from the purely kinematic constraint equations at normal and nonzero pseudothreshold points. One difference is that solutions for the $t=0$ constraint equations can involve relationships between different trajectories. Constraint equations that relate contributions from different trajectories are commonly called conspiracy relations.

In the unequal-mass case UU, the t-channel helicity amplitude is free of kinematic singularities in s, and the coefficient of the half-angle functions (i.e., the barred amplitude) can only contain singularities at $t=0$ that are removed by the powers of $\sin\theta_t$ in the half-angle functions. This results in constraint equations between "parity-conserving" combinations of the barred amplitudes. (See Frautschi and Jones [238] and Arbab and Jackson [47] for a more detailed explanation.) The constraint equations for the UU mass case thus relate either trajectories contributing to the same amplitude with opposite parity or those with the same parity whose total contribution vanished at $t=0$.

For the case where only one vertex has equal masses, the EU case, the t-channel half-angle functions are not singular at $t=0$, but the pseudothreshold corresponding to the equal-mass vertex has moved to $t=0$ and results in constraint equations between different amplitudes. When expressed in terms of "parity-conserving" amplitudes, these constraint equations can relate trajectories with either parity that contribute to different amplitudes. (See Frautschi and Jones [238].)

For the case of equal masses at each vertex, the EE case, the constraint equations result from the vanishing of s-channel half-angle functions. Here, as in the EU case, the t-channel half-angle functions are regular at $t=0$ and the constraint equations relate Regge trajectories contributing to different amplitudes which may or may not have the same η parity.

In earlier papers [231, 238, 354], three types of solutions were considered:

(1) The evasive solution for which residues satisfy certain relations but no conditions are imposed on intercepts. If all residues separately vanish at $t=0$, the solution is called a trivial evasive solution.

(2) Conspiratorial solutions in which the residues and intercepts of different Regge poles (i.e., poles with different internal quantum numbers) are related in satisfying the constraint equations.

(3) Daughterlike constraint solutions in which a parent trajectory conspires with a sequence of Regge poles with the same internal quantum numbers.

For the general mass situation, UU, if factorization of individual residues holds, then even baryon number trajectories (e.g., boson trajectories) need not conspire [354] In. contrast, for fermion trajectories MacDowell symmetry [378] implies trajectories must conspire or totally decouple at $t=0$ [354]. This latter fact is a generalization of the situation found by Gribov *et al.* [278], i.e., trajectories that couple to πN must occur in parity doublets whose trajectories intercept at $t=0$. For the EE case with arbitrary spins, the constraint equations can always be satisfied by evasion [354]. For all mass cases, the leading asymptotic contributions of each pole to the s-channel helicity amplitudes vary like $t^{1/2(m_t+n_t)}$ near $t=0$ in the absence of conspiracy [354].

Whereas the $O(4)$ results were obtained for the EE mass case, the original work on daughter trajectories was done for the UU mass case. By using factorization, these results can be used to obtain the behavior of residue functions for the EU mass case [47]. By writing the amplitudes for the EU and the UU mass cases as sums of Regge pole contributions and demanding that the amplitudes satisfy the constraint equations, one can detail the necessary daughter and conspirator trajectories. This analytic approach has been used to verify the original results of Freedman and Wang for the UU case [323]. Though this analytic method does not work in the EE mass case, if the results for the UU and EU mass cases are combined with factorization, results can be obtained for the EE mass case, and these verify the $O(4)$ conclusions [114, 190, 194, 196, 335].

Freedman and Wang [247, 249] have shown that Regge pole families of related trajectories which satisfy the $t=0$ constraint equations can be identified with Regge pole families that result from Toller poles in the $O(4)$ symmetry. Since it is fashionable to use the classification scheme of Freedman and Wang [247, 249] to discuss how various Lorentz poles that couple to the NN channel satisfy the constraint equations, we will list the three classes.

For $N\bar{N}\rightarrow N\bar{N}$ process there are only three types of Toller poles which are designated by the $O(4)$ quantum numbers M, s, and $n[n\equiv\alpha(0)]$ of the parent.[16] Any trajectory which does not completely decouple from $N\bar{N}\rightarrow N\bar{N}$ at $t=0$ must belong to one of the following classes:

Class I ($M=0$, $s=0$). This Lorentz pole results in a family of natural η-parity (i.e., $\tau P=+$) trajectories with charge conjugation equal to the Lorentz signature (i.e., $C=\tau_l$) and can only provide evasive solutions to the $N\bar{N}\rightarrow N\bar{N}$ process. Only the daughters of even order (i.e., $\tau=+\tau_l$, $\alpha=n-2, n-4, \cdots$) couple to equal-mass vertices. The odd-order daughters (i.e., $\tau=-\tau_l$, $\alpha=n-1, n-3, \cdots$) couple only to vertices of unequal mass. Candidates for the parent trajectories are P, P', ϕ, ω, Φ, and A_2 trajectories.

Class II ($M=0$, $s=1$). This Lorentz pole results in families of unnatural η-parity (i.e., $\tau P=-$) trajectories with charge conjugation equal to minus the Lorentz signature (i.e., $C=-\tau_l$) and can provide daughterlike solutions to the constraint equations for the $N\bar{N}\rightarrow N\bar{N}$ process. Both even- and odd-order daughter trajectories couple to the $N\bar{N}$ vertex. The only candidate for the parent trajectory ($C=-\tau$) appears to be the A_1 (B has $C=\tau$), while candidates for the first daughter trajectories are $\pi(1640)$, B and π.

Class III ($M=1$, $s=1$). This Lorentz pole results in families of trajectories that contain parity doublets whose charge conjugation is equal to the Lorentz signature (i.e., $C=\tau_l$) and provides conspiratorial solutions to the constraint equations. Each member of a parity doublet has its own set of even- and odd-order daughter trajectories. While both sets of even-order daughters contribute to $N\bar{N}\rightarrow N\bar{N}$, only the odd-order daughters with unnatural η parity ($\tau P=-1$) contribute to $N\bar{N}\rightarrow N\bar{N}$.

Candidates for the unnatural η-parity (i.e., $\tau P=-$) trajectories are π, η, B and A_1, though the A_1 is somewhat undesirable since it would require the existence of other high-lying trajectories. Candidates for the natural η-parity (i.e., $\tau P=+$) trajectories are $\pi(1030)$ and ρ', which are possible doublet pairs of π and B, respectively.

Jones and Shepard [335] suggest that the restriction of coupling to $N\bar{N}$ states reduces the number of Lorentz classifications of families of Regge poles. They suggest that for $M=0$ there could be leading trajectories with $C\tau$ negative for natural η parity, and leading trajectories with $C\tau$ positive for unnatural η parity. This would permit π and B mesons to be leading trajectories without having $M>0$. Similarly, they propose a $M=1$ conspiracy with leading trajectory having $C\tau$ negative. These three Lorentz poles correspond to evasive solutions for NN scattering. Mueller [405], in studying the Bethe–Salpeter equations, found, in addition to the three types of Toller poles discussed by Freedman and Wang, the three proposed by Jones and Shepard [335].

Blankenbecker *et al.* [106] used Feynman diagrams to study conspiracies for equal- and unequal-mass configuration and concluded that factorization forbids Class II and III conspiracies to occur in the same diagrams. A study of the contribution of an $M=1$ Toller pole in a field-theoretic model concludes that parity doubling of boson trajectories is natural [30].

E. Solution of Constraint Equations Continued Away from $t=0$

There are two reasons for the interest in the behavior of solutions to the constraint equations at nonzero values of t. The first reason is that for the unequal-mass situation, forward scattering does not correspond to $t=0$, and one wants to know how different Toller poles contribute to forward scattering for such reactions. The second reason results from attempts to learn how the parent and daughter trajectories behave away from $t=0$.

To review the first question, let us consider the conclusions that Sawyer [477] (also Sawyer and Shepard [481]) obtained concerning the leading asymptotic contribution of Toller poles to s-channel helicity amplitudes. For the s-channel process $ab\rightarrow cd$, described by the amplitude $f_{cd;ab}{}^s$, the amplitude $f_{ab;ab}{}^s$ is referred to as a nonflip amplitude, $f_n{}^s$, while $f_{a\pm1,b\pm1;ab}{}^s$ is referred to as a double-flip amplitude, $f_{df}{}^s$. In the forward direction, only amplitudes with $\lambda_s=\mu_s$ are nonvanishing; i.e.,

[16] For a given reaction, only Toller poles with $M\leq\min(s_a+s_b, s_c+s_d)$ can contribute; see Bertocchi [222].

spin-flip amplitudes $f_{a\pm1,b;ab}{}^s$ and $f_{a,b\pm1;ab}{}^s$ vanish. Sawyer [477] found that with any mass configuration for a Toller pole with $M=0$

$$f_{\rm nf}{}^s \sim s^\alpha; \qquad f_{\rm df}{}^s \sim s^{\alpha-1}$$

and for a Toller pole with $M=1$

$$f_{\rm nf}{}^s \sim s^{\alpha-1}; \qquad f_{\rm df}{}^s \sim s^\alpha.$$

The same result can be written in the more compact form [165]:

$$f_{cd,ab}{}^s(s,\theta_s=0)\sim\delta_{a-c;b-d}s^{\alpha-|M-|a-c||}.$$

It is surprising that these results could be extended from the equal-mass case (EE) to both the equal–unequal (EU) and the unequal–unequal (UU) mass cases, since between the point $t=0$ and the point $t_{\rm min}$ defined by $\theta_s(t_{\rm min})=0$, z_t varies rapidly. For example, in the UU case, z_t has unit magnitude but opposite sign at the two points [58, 471]; consequently, t-channel amplitudes with $\lambda_t=\mu_t$ will contribute at one t value and not at the other and conversely for those amplitudes with $\lambda_t=-\mu_t$.

Some insight into the unequal-mass situation has been given by recent papers [99, 499] in which Sawyer's results were obtained by considering the behavior of t-channel kinematic factors in the residues along the curve $\theta_s=0$ for large s. The approach is easy to understand. For the EU or UU mass case, $z_s=1$ implies $z_t=\pm1$, where the correct sign depends on the inequalities between masses and the phase convention. Since the argument is essentially the same either way, we assume the plus sign. For $z_t=1$ the only nonzero t-channel helicity amplitudes are those for which $\mu_t=\lambda_t$. For $\theta_s=0$, all crossing angles are either 0 or π; i.e., independent of s. Consequently, along the curve $t(s)$ where $\theta_s=0$,

$$\begin{aligned} f^s(\lambda_s=\mu_s)=\bar{f}^s=Mf^t(\lambda_t=\mu_t)=M\bar{f}^t(\lambda_t=\mu_t)\\ =M\sum\beta_\lambda{}^\alpha[t(s)]s^{\alpha-\lambda}t,\end{aligned}$$

where the sum is over all Toller poles. The residue functions $\beta(t)$ for each Toller pole contain the correct kinematic singularities in t and satisfy the $t=0$ constraint equations in the manner appropriate to that Toller pole. Along the curve $t(s)$ the only s dependence on the right-hand side is in the residues and in the terms $s^{\alpha-\lambda}$. Solving the equation $z_t(t,s)=1$ for t (i.e., $t\sim1/s$ for UU and $t\sim1/s^2$ for EU) and knowing the kinematic structure of the residues for each Toller pole, one easily obtains Sawyer's results. For the reaction $\pi N\rightarrow\rho N$, one obtains:

Class II:

$$f_{0+,0+}{}^s=f_{\rm nf}{}^s\sim s^\alpha, \qquad f_{1+;0-}{}^s=f_{\rm df}{}^s\sim s^{\alpha-1};$$

Class III:

$$f_{0+,0+}{}^s=f_{\rm nf}{}^s\sim s^{\alpha-1}, \qquad f_{1+;0-}{}^s=f_{\rm df}{}^s\sim s^\alpha.$$

Class I poles cannot contribute to this reaction since Class I trajectories only appear as poles in the NN partial-wave amplitude, $f_{11}{}^j$, which has $\lambda_{N\bar{N}}=0$, and states of positive η parity cannot couple to $\pi\rho$ states with $\mu_{\pi\rho}=0$ [499]. Consequently, Class I trajectories cannot have $\lambda_t=\mu_t$ and thus will not contribute to forward scattering. This is slightly paradoxical since ω is thought to be a Class I trajectory, but it contributes to $f_{10+-}{}^t$, which does not vanish in the forward direction.

The above work illustrates how the same results can be obtained by using either analyticity properties of amplitudes or group theory, as does the use of analytic techniques to verify the $O(4)$ conclusions for daughter trajectories in the EE mass case [114, 190, 194, 196, 335]. The two approaches complement each other when they overlap and supplement each other when one method cannot be easily applied [335].

A second reason for considering the solution of the constraint equations continued to nonzero values of t is to obtain information on the motion of trajectories. The problem has been studied with group theory and also with a method based on analyticity and factorization. (For a more general discussion of daughter trajectories, see Part B of this section. For a review on the work of particular trajectories, such as the pion, see Sec. XI.)

Those using the group-theoretic approach include Domokos [200, 201, 202, 204], Cosenza *et al.* [164, 165], and Delbourgo *et al.* [174, 175]. Domokos argues that the inequality of external mass breaks the exact $O(4)$ symmetry and uses a symmetry-breaking term to obtain a one-parameter formula to describe the deviation of the slopes from the exact symmetry value. Cosenza *et al.* [164, 165] present a formalism which permits one to discuss families of conspiring Regge poles for nonzero values of t. They find a solution which is not the most general since it predicts parallel daughter trajectories. Delbourgo *et al.* [175] obtain expansions of amplitudes with arbitrary external masses and spins in terms of the homogeneous Lorentz group. Their expressions satisfy $O(3,1)$ constraint equations and should provide a useful formalism if trajectories are approximately parallel. Many groups have used these formalisms, particularly that of Delbourgo *et al.*, to study various reactions [40, 41, 42, 344, 379, 412].

As mentioned in Part D of this section, analyticity and factorization can be used to verify the group-theory description at $t=0$ for the equal-mass case. Di Vecchia *et al.* [193] and Bronzan and Jones [114] have also used the approach to obtain expressions for the expressions relate the positions of possible Regge recurrences, they are generally referred to as "mass formulas."

After reviewing the work on the complications associated with the point $t=0$, the comment of Leader [354] seems appropriate. In discussing whether there should be phenomenological consequences resulting from the various solutions to the constraint equations, he comments: "Our present feeling is that the requirements [imposed by solutions of constraint equations] are so artificial, and even arbitrary, that it is much more likely that their existence is simply a manifestation of a weakness in our standard method of Reggeizing relativistic problems!"

It is certainly unfortunate that of all the kinematic singularity points, the one at $t=0$, which is the closest to the physical region, should be the one at which Regge theory encounters so many complications.

VII. SYMMETRIES OF REGGE AMPLITUDES AND TRAJECTORIES

A. Four-Dimensional Symmetry and Daughter Trajectories

As the discussion in the last section has indicated, amplitudes for the equal-mass case are believed to satisfy four-dimensional symmetry for $t=0$. This symmetry is apparently broken by mass difference terms even at $t=0$ for the EU or UU mass cases [201, 205].

The spectra of Regge poles, i.e., intercepts of trajectories, at $t=0$ obey an $O(4)$ symmetry independent of external masses. The external-mass situation determines only which trajectories are coupled to a reaction at $t=0$. Toller poles resulting from $O(4)$ symmetry decompose into families of Regge trajectories with related residues and $t=0$ intercepts equally spaced by a unit of angular momentum. The original daughter trajectories of Freedman and Wang are members of a family of Regge poles from the same Toller pole which couple in the unequal-mass case UU.

The possible Toller assignments of particular trajectories are discussed in Sec. XI. For a general classification see Ahmadzadeh and Jacob [25]. If daughter trajectories are considered as extra spin components resulting from the exchange of off-mass-shell particles [215, 415], it is reasonable to expect equal spacing of trajectories for nonzero values of t. Work using $O(4)$ arguments gives credence to at least an approximately equal spacing [201]. Perhaps arguments for the absence of physical particles on daughter trajectories, like the one presented by Oakes (see Sec. VI.B), could explain the difficulty found in identifying daughters. Though identification of daughter trajectories is very difficult due to the probable existence of secondary nondaughter trajectories [232], a tentative identification of some meson daughter occurrences has been made [503]. The situation for baryon daughter occurrences is somewhat more promising and is discussed in Sec. IX.

B. $SU(3)$ and Other Possible Internal Symmetries

The large number of free parameters associated with residues and trajectories has encouraged attempts to relate parameters through exact or approximate symmetries. $SU(3)$ symmetry has been known for several years to be an approximate symmetry of strong interactions that could be used to relate couplings and, with suitable symmetry breaking, to relate masses within multiplets. Mesons have been classified into singlets and octets, or, in the case of symmetry mixing between a singlet and an octet, into nonet representations of $SU(3)$, while baryons have been classified into singlet, octet, and decouplet representations. In addition to the generally assumed meson classifications of π, η, K into a 0^- octet and ρ, ω, ϕ, K^* into a 1^- nonet, A_2 (1320), f (1250), f' (1500), K^* (1430) have been assigned to a 2^+ nonet and P to a 2^+ singlet [85, 184, 477]. There has also been interest in assigning A_1 (1080) or B (1220) and K^* (1175) to a 1^+ octet [22].

$SU(3)$ predictions of residue functions have been compared to experimental total cross sections via the optical theorem. Deviations due to symmetry breaking [517] were found to be on the order of 15%–20% [84, 85, 86]. For example, the Pomeron couplings to $\pi\pi$ and $\bar{K}K$ differed by 20% from that expected of a pure $SU(3)$ singlet [86]. This small amount of disagreement between experiment and exact $SU(3)$ symmetry is quite surprising considering the large breakdown of $SU(3)$ symmetry found in bootstrap calculations and comparisons of decay widths. It is possible though that $SU(3)$ symmetry is a better symmetry at $t=0$ than at the poles of physical particle [401].

A study of K^* (1^-) and K^{**} (2^+) trajectories found agreement between trajectory intercepts determined from forward hypercharge-exchange reactions and those determined from total-cross-section data by assuming the Pomeron had a small $SU(3)$, $I=0$ octet component [473]. This symmetry breaking can also explain the suppression of the reaction $\gamma p \rightarrow \phi p$ [473].

Bootstrap calculations for entire $SU(3)$ multiples have been attempted [14, 392]. By using exchange degeneracy it is possible to obtain a mass relationship between the vector, tensor, and pseudoscalar octets [14, 392]. Photoproduction of pseudoscalar, vector, and tensor mesons has been discussed with the assumptions of exact $SU(3)$ symmetry for the residues and broken $SU(3)$ symmetry for the trajectories [26]. As is quite popular [51, 56], this study also considers the consequences of assuming exchange degeneracy between various $SU(3)$ multiplets. The simplification of expressions due to exchange degeneracy is discussed in Part C of this section.

In analogy with mass splitting, it is generally assumed that trajectories and, in particular, their intercepts obey a broken $SU(3)$ symmetry. Present experimental data are incapable of answering such questions as whether the members of $SU(3)$ multiplets have the same ghost-killing mechanism and whether $SU(3)$ multiplets result from $SU(3)$ multiplets of Toller poles (i.e., if they satisfy $t=0$ constraint equations in the same manner). The latter question was proposed for the π and K conspiratory mechanism in a study of photoproduction [296].

Before leaving this section on internal symmetries, some of the work involving symmetries other than $SU(3)$ should be mentioned. The assumption that

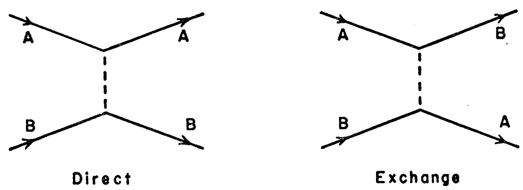

FIG. 4. Example of diagrams in potential theory that would contribute to the direct and exchange potentials, respectively.

residue functions possess the algebraic structure of $[U(3)\otimes U(3)]_\beta$ greatly reduces the number of free parameters in the couplings of vector and axial vector nonets to baryons and pseudoscalor mesons [123, 270]. Chiral $U(3)\otimes U(3)$ has been used to study relations between residues at zero momentum transfer for Δ production in np and nn interactions and has made predictions [401] for Δ production from pp scattering and forward charge exchange in np scattering in reasonable agreement with experimental data. The concept of a $U(3)\otimes U(3)$ algebra of residues applied to meson–baryon and baryon–baryon processes is able to reproduce quark model results and other relations that are in good agreement with existing experimental data [413].

Delbourgo *et al.* [172, 173] have suggested that quark numbers, in analogy to angular momentum, be Reggeized. Perfect quark symmetry would lead to two master trajectories, one with baryon number zero and the other with baryon number unity. In such a scheme quarks would no longer need to be thought of as physical entities. A study [265] of the Regge pole model and the bootstrap hypothesis supports the assertion that residues obey a chiral $SU(2)\otimes SU(2)$ algebra. Baryonic recurrences imply that there is some experimental justification for assuming that baryon trajections belong to a representation of $U(6)\otimes U(6)\otimes O(3)$ whose residues satisfy $U(6)\otimes O(2)$ symmetry [251].

It is evident that Regge pole theory has revitalized the quest for higher symmetries whose predictions are in reasonable accord with experiment.

C. Exchange Degeneracy

Exchange degeneracy can be understood most easily through potential-scattering theory, where one speaks of direct and exchange potentials [424]. These potentials result in angular decompositions of the form

$$f^t(s,t)=\sum_j [F_D{}^j(t)P_j(z_t)+F_E{}^j(t)P_j(-z_t)]$$
$$=\sum_j [F_D{}^j+(-1)^jF_E]P_j(z), \qquad \text{(VII.1)}$$

where $F_D{}^j$ and $F_E{}^j$ are partial-wave amplitudes resulting from the direct and exchange potentials, respectively (see Fig. 4). In order to avoid the bad behavior of $(-1)^j$ for complex j, one Reggeizes the so-called signatured amplitudes $f^\pm$, defined by

$$f^\pm(z,t)=\sum (F_D{}^j\pm F_E{}^j)P_j(z). \qquad \text{(VII.2)}$$

The full amplitude is given by

$$f^t(s,t)$$
$$=\tfrac{1}{2}[(f^+(z,t)+f^-(z,t))+(f^+(-z,t)-f^-(-z,t))]$$
$$=\tfrac{1}{2}[(f^+(z,t)+f^+(-z,t))+(f^-(z,t)-f^-(-z,t))]$$
$$=\tfrac{1}{2}(1+\exp(-i\pi\alpha_+))f^+(z,t)$$
$$+\tfrac{1}{2}(1-\exp(-i\pi\alpha_-))f^-(z,t), \qquad \text{(VII.3)}$$

where the final expression results from the assumption that f^+ is dominated by a Regge pole at α_+, and f^-, by the Regge pole at α_-.

Exchange degeneracy is the statement that f^+ and f^- are equal. This, of course, is true if the exchange potential is zero. In such a case the equality of odd- and even-signatured amplitudes implies that there is only one trajectory (i.e., $\alpha=\alpha_+=\alpha_-$) and only one residue function (i.e., $\beta=\beta_+=\beta_-$). Approximate exchange degeneracy would result if the exchange potential were small compared to the direct potential, or if the exchange potential were not very small, but had a shorter range than the direct potential [19]. In such a situation both the residues and the trajectories of even and odd signature would be approximately equal.

In relativistic scattering, one does not work with potentials but with discontinuity and spectral functions. Dispersion relations for the barred amplitudes are written in the form (suppressing helicity indices)

$$\bar{f}^t(z,t)=\pi^{-1}\int\frac{A^s(t,s(x))}{x-z}dx+\pi^{-1}\int\frac{A^u(t,u(-x))}{x+z}dx, \qquad \text{(VII.4)}$$

where A^s and A^u are discontinuity functions across the s and u cuts, respectively [157, 523]. The functions A^s and A^u play much the same role as the direct and exchange potentials in potential theory. A partial-wave decomposition of the above expression contains the same undesirable factor $(-1)^j$ as a coefficient of the contribution from A^u. Consequently one again defines even- and odd-signatured amplitudes and Reggeizes each separately.

Exchange degeneracy implies A^u is zero. Using the Mandelstam representation, the functions A^s and A^u can be written as sums of two dispersion integrals involving the double-spectral functions. While A^s depends on the double-spectral functions ρ_{su} and ρ_{st}, A^u depends on ρ_{su} and ρ_{tu} [157]. Thus, exchange degeneracy is equivalent to the hypothesis that the contributions from the spectral functions ρ_{su} and ρ_{tu} are negligible compared to that from ρ_{st} [53].

The above argument used unsubtracted dispersion relations for simplicity. Supposedly, exchange degeneracy could be formulated in the presence of subtraction terms and conceivably might not depend on the existence of nonzero double-spectral functions.

The implications of exact or even approximate exchange degeneracy are very strong. In addition to even and odd trajectories being coincident, the equality of the residues implies that they must have the same ghost-killing mechanism and result from exchange-degenerate Toller poles. The connection between ghost-killing factors and M classification of Toller poles, as explained by Toller at Coral Gables, implies that trajectories resulting from exchange-degenerate Toller poles do have the same α factors.

There has been considerable interest in establishing approximate exchange degeneracy between the pairs [$\pi(0^-)$, $B(1^+)$], [$K^*(1^-)$, $K^{**}(2^+)$], and [$\rho(1^-)$, $A_2(2^+)$]. Since Sec. XI is concerned with the properties of particular trajectories, only a few comments will be made about these trajectories in this section. Exchange-degenerate pairs of trajectories have the same values of isospin and η parity (τP); the difference in their parities implies that their trajectory and residue functions have different threshold behaviors.

Many authors have used exchange degeneracy to reduce the number of free parameters in fitting data. Exchange degeneracy for (ρ, A_2) and (K^*, K^{**}) and $SU(3)$ symmetry have been used to predict high-energy photoproduction branching ratios [26] and to relate various total and differential cross sections such as $\sigma_{pp}=\sigma_{pn}$ and $\sigma_{K+p}=\sigma_{K+n}$, which are in reasonable agreement with experiment [19, 20, 23]. The same assumptions have also been used [24] to obtain

$$\sigma_{\pi-p}+\sigma_{K-n}=\sigma_{\pi+p}+\sigma_{K-p}$$

and

$$2d\sigma(K^-p\rightarrow\bar{K}^0n)=d\sigma(\pi^-p\rightarrow\pi^0n)+3d\sigma(\pi^-p\rightarrow\eta n)$$

which are in very good agreement with available data [24, 309]. The choice $\alpha_\rho=\alpha_{A2}\sim0.5$ and $\alpha_{K^*}=\alpha_{K^{**}}\sim0.35$ give reasonable energy dependence for the total cross sections.

Arnold [51] has done an extensive study of charge and hypercharge reactions assuming $SU(3)$ symmetry and exchange-degenerate pairs [(ρ, A_2) and (K^*, K^{**})]. By assuming linear trajectories for each exchange-degenerate pair, he had only two free parameters and was able to obtain agreement with all available high-energy small-momentum-transfer data for 18 different inelastic reactions with initial states of either $\pi^\pm p$, $K^\pm p$, or K^+n. The energy dependences of his fit involve no free parameters and are in very good agreement with the data. An $SU(3)$ exchange-degenerate Regge pole model has also been used to describe and predict polarization data for elastic πp, Kp, $\bar{p}p$, pp reactions [56]. A recent study [22] classifies conspiring trajectories according to $SU(3)$ symmetry, Lorentz symmetry, and exchange degeneracy [516]. A classification of nearly all known mesons in terms of exchange degeneracy and $SU(3)$ symmetry has also been made [224].

The proposed (ρ, A_2) exchange degeneracy is the easiest to verify experimentally and consequently has caused the most controversy [232]. In particular, consider the results of a study [85] of the reactions $\pi^-p\rightarrow\pi^0n$ and $\pi^-p\rightarrow\eta n$, which found $\alpha_\rho=0.57\pm0.03$ and $\alpha_A=0.34\pm0.03$. This is a significant deviation from exchange degeneracy. Recently two types of exchange degeneracy have been proposed: strong exchange degeneracy (i.e., equal residues and trajectories) and weak exchange degeneracy (only trajectories equal) [76]. The former type of exchange degeneracy requires an α factor in the nonflip residue corresponding to the ghost-killing α factor needed by the A_2 residue [76]. The suggestion [76] that this would cause an absolute zero in $d\sigma(\pi^-p\rightarrow\pi^0n)$ at the value of t for which α is zero is not correct since polarization measurements imply the existence of a contribution in addition to the ρ, which would produce the experimentally observed dip. Since the ρ is dominant, one would still expect to see a strong dip in the polarization where α_ρ is zero. An exchange degeneracy in which residues are not identical, but have the same value at their respective lowest recurrence, has also been suggested [463].

Exchange degeneracy implies the same $SU(3)$ f to d ratio for both members of a degeneracy [23]. The f to d ratios for ρ and A_2 are found [84, 473] to be essentially the same and thus give some support to their exchange degeneracy. Recently, it has been found [313] that both the A_2 and ρ residues appear to have zeros for $t\sim-0.2$ which explain the crossover effect in $K^\pm p$ and $\pi^\pm p$ differential cross sections [232]. A study of various known Regge trajectories has shown that exchange degeneracy appears to hold for leading trajectories, but not for secondary or daughter trajectories [232].

Before concluding the discussion on exchange degeneracy, we should point out that $U(3)\otimes U(3)$ has been used to study the possible dynamic origin and consequences of exchange degeneracy; it was concluded that exchange degeneracy [53] may not hold for two meson intermediate states, since such states give equal contributions to the spectral functions ρ_{st} and ρ_{tu} and thus would split even and odd trajectories [401].

In view of the values of ρ and A_2 intercepts, i.e., $\alpha_\rho\sim0.57$, $\alpha_A<0.4$ [85], it appears that exchange degeneracy like $SU(3)$ symmetry is not an exact symmetry but is fairly well satisfied and a good approximation.

D. MacDowell Symmetry and Baryonic Parity Doublets

Considerable recent experimental data suggest that baryonic trajectories occur in parity doublets [75] related to a symmetry of amplitudes first proposed by MacDowell [378]. This symmetry, which considers half-integral j amplitudes as functions W instead of $t(=W^2)$, relates partial-wave amplitudes with opposite parities but the same quantum numbers by $F^{j+}(W)=-F^{j-}(-W)$ [75, 142, 182, 260]. This implies that if F^{j+} were dominated by a Regge pole at W, $F^{j+}(W)=\beta_+(W)/[j-\alpha_+(W)]$, then a corresponding trajectory

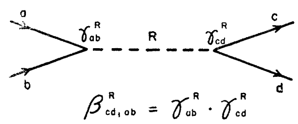

FIG. 5. Factorization for contribution to t-channel reaction $ab\to cd$ due to trajectory R.

must occur for F^{j-} at $-W_j$ such that $\beta_+(W) = -\beta_-(-W)$ and $\alpha_+(W)=\alpha_-(-W)$ near W_j.

The point $-W_j$ corresponds to negative energy, and since physical particles occur at positive energies, the existence or nonexistence of physical particles depends on the continuation of α_- to positive values of W. It is becoming evident that trajectories are even functions of W, at least for positive values of W^2 [78]. This implies that $\alpha(W)\simeq\alpha(-W)$ or $\alpha_+(W)\simeq\alpha_-(W)$ and thus that baryonic trajectories differing only in parity should be approximately degenerate.

The fact that baryonic resonances occur as approximately degenerate trajectories which are linear in W^2 has recently been demonstrated [75, 79] by the assignment of all known baryonic resonances to four sets of trajectories. For each of the two values of isospin, 1/2 and 3/2, there are four types of trajectories: (α) $J^P=1/2^+, 5/2^+, \cdots$; (β) $J^P=1/2^-, 5/2^-, \cdots$; (γ) $J^P=3/2^-, 7/2^-, \cdots$; and (δ) $J^P=3/2^+, 7/2^+, \cdots$. The first two and the last two types are found to be nearly degenerate [79]. Considerable evidence also indicates that MacDowell parity doublets exist for complete $SU(3)$ baryonic multiplets and possibly for secondary trajectories which are equally spaced by a unit of angular momentum below the leading trajectory [79]. This work and other related work are discussed in more detail in Sec. IX.

E. Factorization

Consider the t-channel reaction $ab\to$(Regge Pole)$\to cd$ (see Fig. 5). The factorization theorem states that the residue function for a Regge pole can be written as

$$\beta_{cd;ab}=\gamma_{cd}\gamma_{ab}, \tag{VII.5}$$

where the function γ_{ab} (or γ_{cd}) for a given Regge pole is the same for all reactions that involve particles a and b (or c and d) [135, 230, 259, 280, 493, 536, 537, 546].

The theorem, as given, is for a single Regge pole. Recently it has been concluded that residue functions of at least the first daughter trajectory factorize [335]. However, absorption corrections destroy factorization which is easily understood by considering them as multiple exchanges of Regge trajectories.

Recently, the factorization proof has been generalized [341] to amplitudes resulting from any singularity in the j plane that can be written for j near α as

$$f_{cd;ab}{}^j=\beta_{cd;ab}f^j+g_{cd;ab}{}^j,$$
$$\lim_{j\to\alpha}\,(f^j/g_{cd;ab})\to\infty. \tag{VII.6}$$

As mentioned in Sec. VI.D, factorization is a main ingredient in the analytic approach to the solution of the constraint equations and the identification of Regge poles resulting from Toller poles. Factorization has been used to argue the necessity of boson and fermion trajectories to conspire or evade [354], and to relate analytic properties of residues for various mass configurations (see Sec. VI.E).

Factorization imposes stringent limitations on the parameterization of related residues and complicates the explanation of certain phenomena (discussed below). However, proofs of factorization are, in general, based on some form of the unitary equation and are difficult to discredit.

The recent argument given by Le Bellac [357] which assumes a simple pion conspiracy leads to an apparent contradiction between experiments and the principle of factorization.[17] He considers the pion residue function in the t-channel reactions $\pi\rho\to N\bar{N}$, $\pi\rho\to\pi\rho$, $N\bar{N}\to N\bar{N}$. With the states $N\bar{N}$ and $\pi\rho$ designated by a and b respectively, factorization implies

$$(\beta_{\frac12\frac12;00}{}^{ab})^2=(\beta_{\frac12\frac12;\frac12\frac12}{}^{aa})(\beta_{00;00}{}^{bb}) \tag{VII.7}$$

The residue $\beta_{\frac12\frac12 00}{}^{ab}$ is known to vanish as $t^{1/2}$ independent of the Toller classification of the pion. If the π belongs to a Class III Toller pole, then $\beta_{\frac12\frac12,\frac12\frac12}{}^{aa}$ as indicated by the strong forward peaking in $np\to pn$, is finite at $t=0$ and consequently $\beta_{00,00}{}^{bb}\to t$ near $t=0$. Le Bellac then considers the factorization relation between the residues of the reactions

$$\pi\rho\to\Delta\bar{N},\qquad \pi\rho\to\pi\rho\qquad \Delta\bar{N}\to\Delta\bar{N}.$$

For $c\equiv\Delta\bar{N}$,

$$(\beta_{00,\frac12\frac12}{}^{bc})^2=(\beta_{00,00}{}^{bb})(\beta_{\frac12\frac12,\frac12\frac12}{}^{cc}). \tag{VII.8}$$

The residue function $\beta_{00\frac12\frac12}{}^{bc}$ is analytic at $t=0$. The residue $\beta_{\frac12\frac12\frac12\frac12}{}^{cc}$ cannot have a $(1/t)$ kinematic behavior at $t=0$, and one is led to conclude

$$\beta_{00\frac12\frac12}{}^{bc}\sim t. \tag{VII.9}$$

Though this should lead to a dip in the forward direction for the s-channel reaction $\pi p\to\rho\Delta$, the broad widths of the ρ and Δ complicate the effect [253]. The argument can be similarly applied to $\pi N\to f^0\Delta$ and $KN\to K^*\Delta$. Recently a dip in $\pi^+p\to\rho^+p$ has been found, but the reaction $\pi^+p\to\rho^0\Delta^{++}$, which is apparently dominated by pion exchange, does not exhibit a dip in the forward direction [18]. This implies a breakdown of the simple pion conspiracy plus factorization hypothesis used in LeBellac's argument.

A study of the pion residue function for various production processes has been made to see if the pion conspires (Class III) to produce forward peaks in these reactions [253]. This work concluded that in the j plane there must be singularities more complicated

[17] Arbab and Jackson (47) explain how the Le Bellac argument will fail if the pion has $M=0$. The suggestion [46, 243, 244, 478, 514] that there are two trajectories, one with $M=0$ and the other with $M=1$, which have the quantum numbers of the pion also leads to a breakdown of Le Bellac's argument.

than a simple pole, associated with the pion in order to avoid a violation of the principle of factorization. One such mechanism could be for a simple evasive pion to interfere with a Class III parity doublet with the same quantum numbers [46].

A recent interpretation of Mandelstam's argument concerning a $M=1$ Toller pole assignment of the π and PCAC has been given in terms of factorization [47].

The simplest mechanisms for explaining crossover effects (see Sec. X.A) encounter inconsistencies when the principle of factorization is applied. Consider the crossover point $t=t_x$, at which the difference between the pp and $p\bar{p}$ elastic differential cross sections changes sign. There appears to be a similar crossover in K^-p and K^+p near the value of t_x [102, 168, 197]. The simplest explanation is that the residue function of the ω trajectory vanishes at t_x [434]. By factorization, this zero should manifest itself as a dip in the reactions $\pi N\rightarrow\rho N$ and $\gamma N\rightarrow\pi N$. There seems to be no evidence of a dip in the reaction $\gamma p\rightarrow\pi^0 p$ [83, 195]. The combination $d\sigma(\pi^+p\rightarrow\rho^+p)+d\sigma(\pi^-p\rightarrow\rho^-p)-d\sigma(\pi^-p\rightarrow\rho^0 n)$, which depends only on trajectories with quantum numbers of the ω, shows no evidence for a zero near t_x [158].

Although the crossover effect in $\pi^\pm p$ can be explained by a cancellation between the ρ flip and nonflip amplitudes, the simplest explanation assumes that the ρ nonflip residue vanishes. There appears to be no dip at t_x in the reaction $\pi^-p\rightarrow\omega n$ to substantiate a vanishing of the ρ nonflip residues [199, 442]. One would also expect to see a dip due to factorization in the angular distribution of $\pi^-p\rightarrow\omega n$ at $\alpha_\rho=0$ corresponding to the dip in $\pi^-p\rightarrow\pi^0 n$ [98].

Tests for factorization involving polarization measurements have been suggested [442]. There appear to be several ways to avoid difficulties with factorization. One of the most popular is to assume the existence of secondary trajectories such as ω', ρ', and π'. Absorptive corrections that do not involve double counting may also introduce nonfactorizing contributions [197]. Perhaps other j-plane singularities such as square-root branch points which do not factorize could account for small violations of factorization [487]. Whatever the modifications, hopefully they will be small enough not to negate a great amount of work which relies heavily on factorization, e.g., the application of Regge poles to multiperipheral reactions.

F. Line Reversal

Factorization, when combined with Regge poles having definite quantum numbers, leads to a symmetry usually referred to as line reversal [428, 537]. This symmetry results when two states are related by an operation such as G or charge conjugation. In such a case, the corresponding vertex functions for a given Regge pole will be related by the appropriate eigenvalue of the Regge pole (e.g., G or C). For example, in reversing a set of proton lines (i.e., $p\bar{p}$ to $\bar{p}p$), the phase between the two vertices will be the charge-conjugation eigenvalue, C, of the Regge pole. For the case of spinless particles (e.g., $\pi^+\pi^-$ to $\pi^-\pi^+$), the channel spin of the two particles is zero and $C=(-1)^L=(-1)^j=\tau$; i.e., the phase is just the signature factor. (The relation $C=\tau$ holds for all nonstrange mesons except possibly A_1.) An example of two states related by G conjugation are $n\bar{n}$ and $p\bar{p}$. A very beautiful application of line reversal has been made to KN scattering involving the P, P', ρ, and A_2 trajectories [428]. The essential ingredient in this example is that the relative signs between the amplitudes for K^-p, K^+n, K^-n compared to K^+p are $C=+\tau$, $G=(-)^I\tau$, and $CG=(-)^I$ for the various trajectories. Vertices such as π^0K^+ and $\pi^+\bar{K}^0$ are also related by the signature factor τ of the Regge pole involved [537].

VIII. METHODS OF ISOLATING CONTRIBUTIONS OF INDIVIDUAL TRAJECTORIES

Few, if any, reactions receive contributions from only one Regge pole. Consequently it is difficult to obtain information from a given reaction about a particular trajectory without making assumptions about contributions from other trajectories. By using charge conjugation and isospin invariance (i.e., line reversal) or higher symmetries such as $SU(3)$, one can form linear combinations of experimental quantities (e.g., total or differential cross sections) to isolate contributions from trajectories with definite sets of quantum numbers.

In the following, we will designate total cross sections by σ, differential cross sections by $d\sigma$, and the density matrix for a resonance by ρ_{mn}. The argument of σ will be the initial state in the s-channel reaction, while that of $d\sigma$ will be a particular s-channel reaction. We will consider meson trajectories for which quantum numbers are isospin I, signature τ, charge conjugation C, and η parity $(\eta=\tau P)$.

We will first consider total cross sections and illustrate how linear combinations can isolate contributions with definite C and I values [76, 84]. Because only meson states with $C=P=\tau$ contribute to the spin-averaged total cross section [84], only mesons with natural η parity are involved. It is useful to define the symbols $\Sigma(AB)$ and $\Delta(AB)$ by

$$\Sigma(AB)=\sigma(\bar{A}B)+\sigma(AB),$$
$$\Delta(AB)=\sigma(\bar{A}B)-\sigma(AB). \qquad \text{(VIII.1)}$$

By line reversal, one finds that trajectories contributing to Σ and Δ have C values of plus and minus, respectively.

Using Clebsch–Gordan coefficients and the isospin crossing matrices [69], one can write πN amplitudes in terms of the definite t-channel isospin contributions T^I,

$$f(\pi^+p)\equiv f(\pi^+p\rightarrow\pi^+p)=-\tfrac{1}{2}T^1+1/6^{1/2}T^0,$$
$$f(\pi^-p)\equiv f(\pi^-p\rightarrow\pi^-p)=\tfrac{1}{2}T^1+1/6^{1/2}T^0,$$

or

$$T^0=(3/2)^{1/2}[f(\pi^-p\to\pi^-p)+f(\pi^+p\to\pi^+p)], \qquad T^1=f(\pi^-p\to\pi^-p)-f(\pi^+p\to\pi^+p). \tag{VIII.2}$$

By the optical theorem, $\Sigma(\pi^+p)$ has $I=0$ and $C=+$, while $\Delta(\pi^+p)$ has $I=1$ and $C=-$. In a similar manner, the following classification [84] can be obtained:

$$\begin{array}{llllll} I=1, & C=- & \Delta(\pi^+p); & \Delta(K^+p)-\Delta(K^+n); & \Delta(pp)-\Delta(pn) & \rho \\ I=1, & C=+ & & \Sigma(K^+p)-\Sigma(K^+n); & \Sigma(pp)-\Sigma(pn) & A_2 \\ I=0, & C=- & & \Delta(K^+p)+\Delta(K^+n); & \Delta(pp)+\Delta(pn) & \phi\omega \\ I=0, & C=+ & \Sigma(\pi^+p); & \Sigma(K^+p)+\Sigma(K^+n); & \Sigma(pp)+\Sigma(pn) & P, P' \end{array} \tag{VIII.3}$$

These combinations also can be obtained easily by using line reversal. From $f(AB)=Cf(\bar{A}B)$, it is apparent that C is positive for Σ and negative for Δ. By using charge and G conjugation successively on the nucleon vertex, one has $f(Ap)=(-1)^I f(An)$. These expressions, together with $f(\pi^-n)=f(\pi^+p)$ and $f(\pi^+n)=f(\pi^-p)$, give the above result. Several of these entries can be related by $SU(3)$ invariance; i.e., for large s $\Delta(K^+p)-\Delta(K^+n)=\Delta(\pi^+p)$ [76, 84].

Unlike total cross sections, differential cross sections can only be related to squares of amplitudes; consequently it is more difficult to write combinations that isolate definite sets of quantum numbers. As an example, we list the expression used to isolate the $I=0$, $C=-$ contribution to ρ production used to study the ω trajectory [76, 138, 158]:

$$d\sigma(I=0)=d\sigma(\pi^+p\to\rho^+p)+d\sigma(\pi^-p\to\rho^-n)-d\sigma(\pi^-p\to\rho^0n). \tag{VIII.4}$$

This can be obtained by considering that the amplitudes $f(\pi^+p)$ and $f(\pi^-p)$ have the same isospin decomposition as those in ρ production and $f(\pi^-p\to\rho^0n)=1/\sqrt{2}T^1$. By replacing the ρ by a π, one can use the new expression to isolate the $I=0$, $C=+$ contribution to πN scattering, $P+P'$. The definite value of C for the two cases is fixed by the meson vertex.

In resonance production, it is also possible to isolate contributions from particular trajectories by considering certain helicity decay matrix elements. For example, in vector meson production, because of the unnatural η parity ($\tau P=-$) of the pion–vector state, only trajectories with unnatural η-parity contribute to $\rho_{00}\,d\sigma$. Similarly, for the same reaction one can show that $(\rho_{11}+\rho_{1-1})\,d\sigma$ and $\rho_{10}d\sigma$ receive contributions from only unnatural η-parity states, while $(\rho_{11}-\rho_{1-1})\,d\sigma$ receives contributions from only natural η-parity states (231).

Ringland and Thews [447] have derived a set of relations that the density-matrix elements should satisfy if the reaction is dominated by a t-channel exchange with a definite set of quantum numbers. As they demonstrate, these expressions are useful in establishing the presence of other contributions. Ader *et al.* [16] have extended this work and demonstrated that differential cross sections and density-matrix data can be used to separate natural ($\tau P=+$) and unnatural ($\tau P=-$) η-parity contributions. They conclude that if either initial particle is spinless, one need only consider simple linear combinations of ρ_{mm} and ρ_{m-m}; otherwise one has to perform a single, well-defined polarization experiment to achieve the separation.

The methods used to isolate contributions of trajectories mentioned so far, except for resonance production, involve reactions with unpolarized beams and targets. Recently it has been suggested that polarized photons in photoproduction of mesons act as a "parity filter" in that only trajectories with a given value of η parity contribute for certain polarizations [159, 161].

In summary, contributions of trajectories with definite values of charge conjugation and isospin can be isolated from total and differential cross sections. Contributions with definite η parity τP can be isolated by considering combinations of density matrices for reactions that involve resonance production. It is possible to restrict the types of trajectories contributing to a reaction by the use of polarized beams or targets.

IX. BACKWARD πN SCATTERING AND BARYONIC TRAJECTORIES[18]

A. Analysis of $\pi^\pm p$ Scattering Data in the Backward Hemisphere[18]

Backward πN elastic scattering provides an opportunity to study the N and Δ Regge trajectories. The elastic reactions $\pi^\pm p\to\pi^\pm p$ are both peaked in the backward direction. There is a pronounced dip in the angular distribution of π^+p scattering, while there seems to be no such structure in π^-p scattering. Since both the N_α and the Δ_δ trajectories contribute to π^+p and only the Δ_δ trajectory contributes to π^-p, this dip is attributed to a vanishing of the contribution due to the N_α trajectory. In support of this hypothesis, in the region of the dip, the ratio of the differential cross section for π^-p to that of π^+p is about 9 to 1, as expected for pure Δ_δ exchange [78].

The backward $\pi^\pm p$ data have recently been analyzed [78, 128, 142, 414]. A rather complete discussion has

[18] In this section we discuss the data in the asymptotic energy region. For a discussion of the fits to the data in the intermediate energy region, see "The Interference Model," Sec. XII.A.

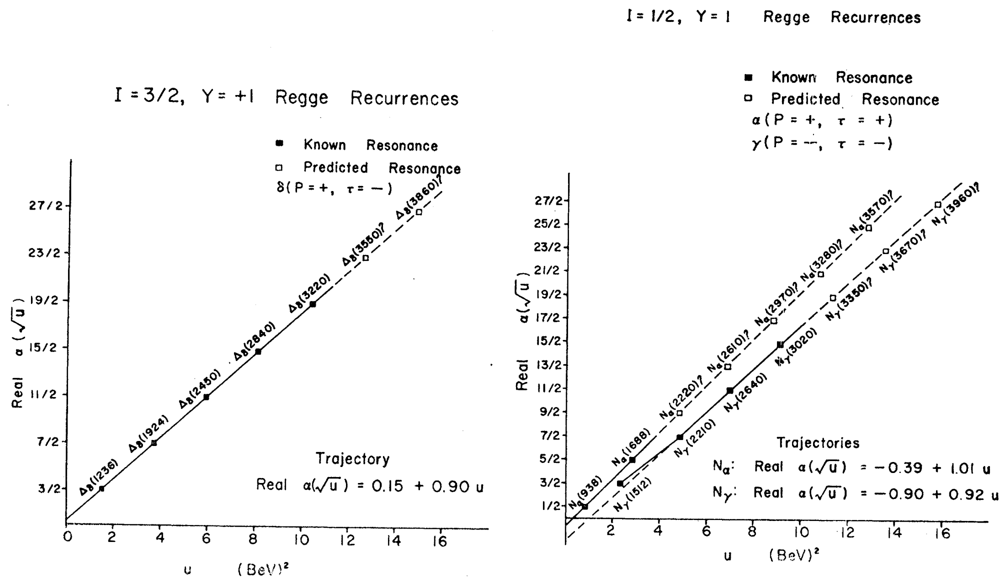

FIG. 6. Chew–Frautschi diagram for baryon trajectories supporting linearity of trajectories with $W^2(u)$ (Ref. [81]).

been given of the amplitudes used to describe the reaction [142]. By demanding that the amplitudes satisfy MacDowell symmetry, the expressions essentially include the parity-doublet trajectories (N_α, N_β) and (Δ_γ, Δ_δ) (see Sec. VII.D).

The contribution due to the N trajectory vanishes when it is assumed that $\alpha_N=-\frac{1}{2}$ in the region of the dip. This is a wrong signature point for the N trajectory, and the amplitude vanishes in the absence of fixed poles in the j plane (see Sec. III.C or Ref. [389]). Most fits to the π^+p data [80, 142, 414] include a zero in the residue of the N trajectory at the value of α where the parity doublet of the nucleon would otherwise be expected.

In general, the theoretical fits to the data are good, though there is some arbitrariness in the determination of the Δ_δ parameters. If the residues for the N_α and Δ_δ trajectories are extrapolated to the respective poles, then parametrizations, which assume the residue functions to be constant except for the zeros due to the missing parity doublets, do not give the Born value [80] whereas those which included exponential factors do give the Born values [142].

The trajectories used in the fits to the data are

$$\alpha_\Delta=0.15+0.90W^2, \qquad \alpha_N \text{ (not needed)};$$

$$\alpha_\Delta=0.15+0.90W^2, \qquad \alpha_N=-0.33+0.11W+1.06W^2;$$

$$\alpha_\Delta \text{ (not given)}, \qquad \alpha_N=-0.34+0.09W+1.05W^2;$$

$$\alpha_\Delta=0.19+0.87W^2, \qquad \alpha_N=-0.38 \qquad +0.88W^2,$$

for Carnahan [128], Noirot *et al.* [414], Chiu and Stack [142], and Barger and Cline [78], respectively. These should be compared to the trajectories obtained from Chew–Frautschi plots [78]:

$$\alpha_\Delta=0.15+0.90W^2, \qquad \alpha_N=-0.39+1.01W^2.$$

Barger and Cline [78] suggest that the slight difference in slopes and intercepts is due to a flattening out of the trajectories for $W^2<0$.

If the trajectories are approximately even functions of W, MacDowell symmetry implies the existence of parity doublet trajectories (see Sec. VII.D). The fact that the amplitudes for the N trajectory in $\pi^\pm p$ backward scattering could vanish implies that odd powers of W in α must be small, since they contribute to Im α and the amplitude is proportional to $(\alpha+\frac{1}{2})\sim\text{Im}\,\alpha$ in the region of the dip. This is reflected in the smallness of the coefficient of W in the trajectories found in fitting the data.

One would expect [78] a similar dip in the differential cross section for π^-p scattering where the Δ_δ trajectory passes through a wrong signature point; e.g., at $\alpha=-3/2$. If one extrapolates the Δ_δ trajectory to $-3/2$, the dip should occur at about $W^2=-1.9$; but there is no indication of dip in the π^-p data (Ref. [78], also see Ref. [44]). If the mechanism that allows the contribution from the N trajectory to vanish is assumed to work similarly for the Δ trajectory, the only conceivable conclusion appears to be that the trajectories do not continue to decrease as rapidly with $W^2<0$ [78].

Igi *et al.* [317] have found that the π^-p backward-peak data suggests a zero of the Δ residue at wrong signature sense point $\alpha_\Delta=\frac{1}{2}$. They conclude that the Δ_δ trajectory should favor the Chew or Gell-Mann ghost-killing mechanism. Zeros in the Δ residue can be confirmed by experiment on backward π^-p charge-exchange data since it should be sensitive to the relative sign between the N and Δ contributions.

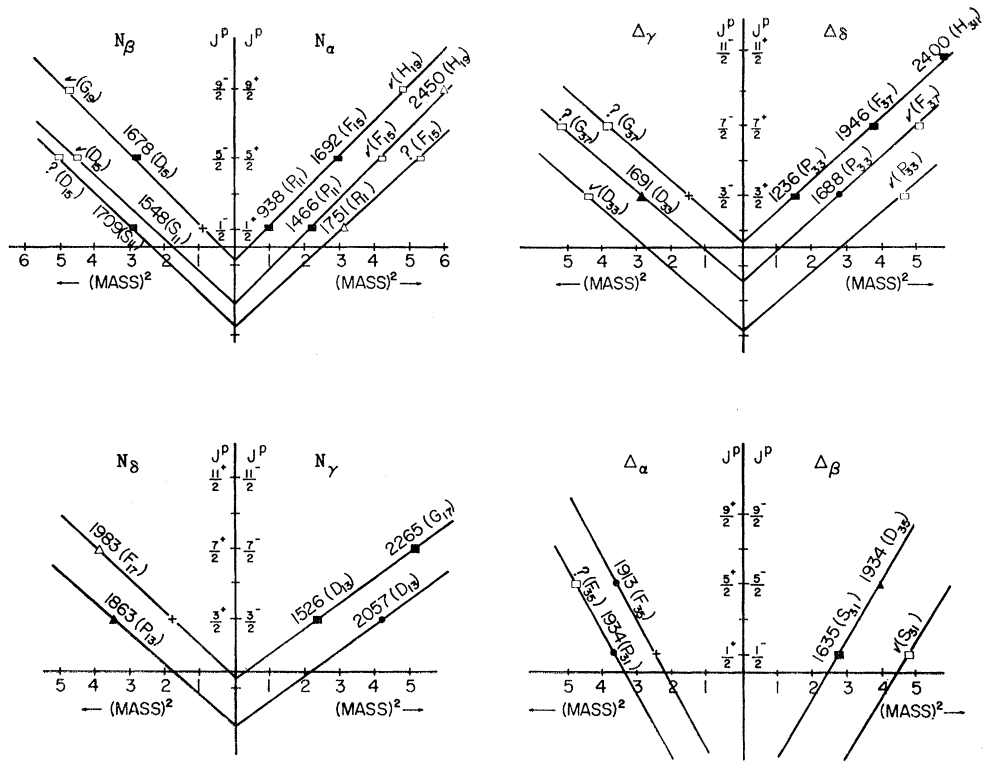

FIG. 7. Proposed classification of πN resonances on MacDowell-symmetric Regge trajectories. The notation for the resonances is (■) established, (●) probable, (▲) interpretation in doubt, (△) unconfirmed, (√) indication of increasing absorption in partial wave, (□) predicted, (+) extinguished state, (?) no information available (Ref. [79]).

B. Classification of Baryonic Resonances as Regge Trajectories

Recent πN phase-shift analyses have revealed the existence of many new inelastic baryonic resonances (see [79]). Barger and Cline [75, 79] assign these and other baryonic resonances to a very systematic classification scheme. For each value of isospin, $\frac{1}{2}$(N-type trajectories) and 3/2 (Δ-type trajectories), there are four trajectories: (α) $J^P=1/2^+, 5/2^+\cdots$; (β) $J^P=1/2^-$, $5/2^-$, $\cdots$; (γ) $J^P=3/2^-$, $7/2^-$, $\cdots$; and (δ) $J^P=3/2^+$, $7/2^+$, $\cdots$. Previously, only resonances on the N_α, N_γ, Δ_β, and Δ_δ trajectories had been assigned (see Fig. 6). Of these, the assignment of four and five resonances to the N_γ and Δ_δ trajectories, respectively, gives clear evidence that trajectories are approximately linear in W^2. New phase-shift data has allowed resonances to be assigned to the N_β, N_δ, Δ_α and Δ_γ trajectories [75, 79].

Because the resonances assigned to N_β, N_δ, Δ_α, and Δ_γ are inelastic and thus do not dominate πN elastic scattering as strongly as N_α, N_γ, Δ_β, and Δ_δ, they are referred to as nondominant trajectories; the latter set of trajectories are referred to as dominant trajectories. Nondominant trajectories obey the selection rule $j-l=-\frac{1}{2}(-1)^{I+1/2}$, while dominant trajectories obey the selection rule $j-l=+\frac{1}{2}(-1)^{I+1/2}$.

The classification scheme as given in Fig. 7 has several very striking features. One is that trajectories with the same isospin and signature but opposite parity are approximately degenerate; this is suggested by MacDowell symmetry for trajectories that are approximately even functions of W (see Sec. VII.D). If the trajectories of a parity doublet are extrapolated to $W=0$, (i) they have the same intercept[19]; (ii) the value of the slope and intercept for the $N_{\alpha,\beta}$ and $\Delta_{\delta,\gamma}$ trajectories are approximately the same as those determined experimentally from $\pi^\pm p$ data (see Barger and Cline [78]): (iii) secondary trajectories which are roughly parallel to the leading trajectories have intercepts equally spaced by a unit of angular momentum. Point (iii) is very interesting in terms of the theory of daughter trajectories (see Sec. VI.B). The orderly arrangement of trajectories appears to be evidence that the trajectories belong to a Toller trajectory with half-integral M.

Barger and Cline [75] have investigated the consequences of $SU(3)$ symmetry. Figure 8 shows the predictions and some confirmations of the assumption that multiplets of $SU(3)$ lie on trajectories whose mass splitting is the same for higher recurrences (i.e.,

[19] Actually, as Leader [354] has shown, solutions to the $t=0$ constraint equations for fermions demand that these trajectories conspire and occur in parity doublets and thus have to have the same intercepts.

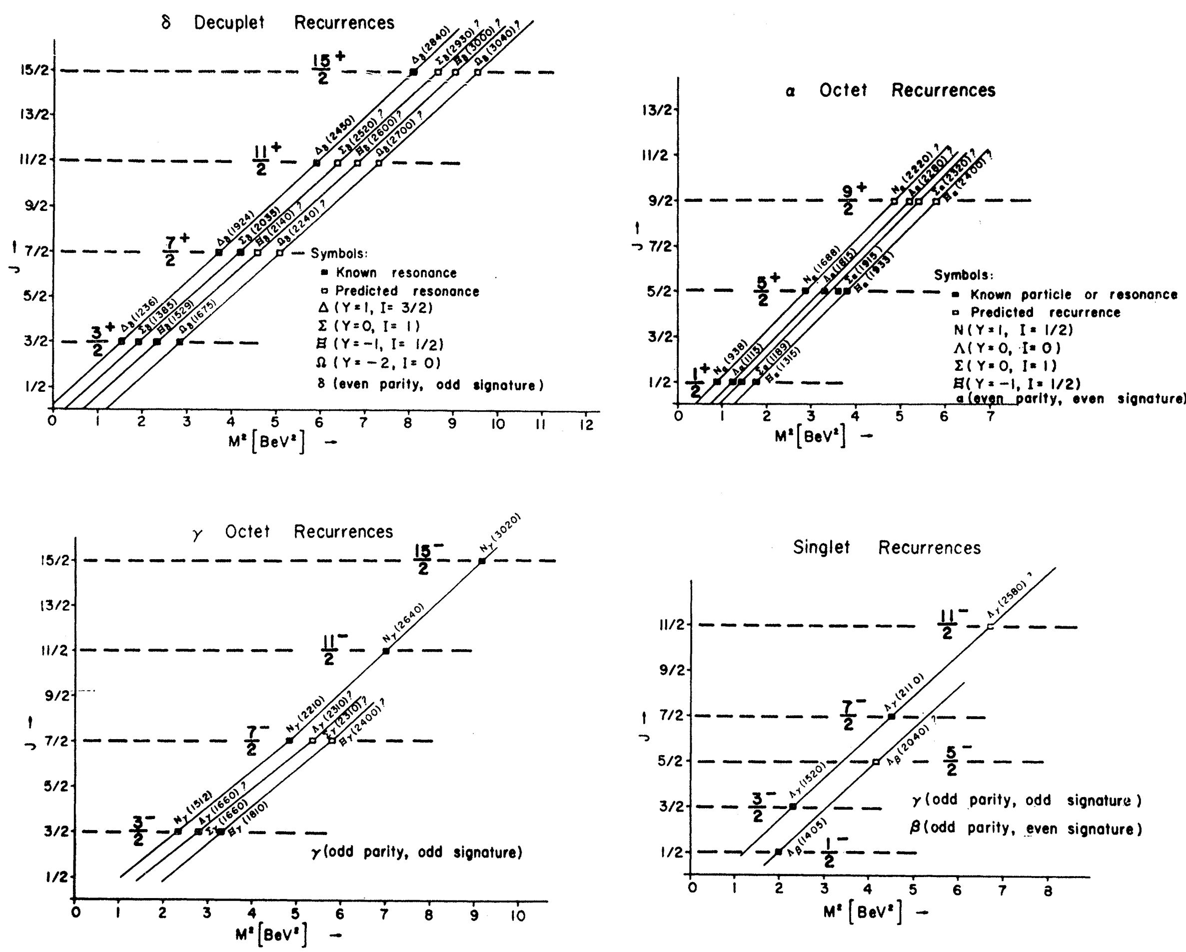

FIG. 8. Evidence and predictions for $SU(3)$ symmetric baryon trajectories (Ref. [77]).

trajectories of members of a multiplet are parallel). It appears (see Fig. 9) that at least for the leading trajectories, those with opposite parities occur as approximate parity doublets. Since trajectories in the α octet are approximately parallel to those of the δ decuplet, some encouragement is given to the suggestion [251] that these multiplets are numbers of a representation of $U(6)\otimes U(6)\otimes O(3)$.

Several models have been used to generate N and Δ trajectories. One interesting attempt [130] used a bootstrap model to explain the trajectories. The positions and widths of known resonances have been used in a doubly subtracted dispersion relation to generate the trajectories for negative W [185]. A calculation using a broken $O(3, 1)$ symmetry [201, 204] found that the N and Δ trajectories were rather symmetric in W [202].

C. Absence of Lowest Nondominant States

One striking feature in both Figs. 7 and 9 is the absence of the lowest state of each leading nondominant trajectories; i.e., N_β (940, 1/2⁻), N_δ (1500, 3/2⁺), and Δ_γ (1240, 3/2⁻). Several explanations are proposed for their absence; in particular, that these resonances may be too inelastic to be observed by present measurements, that their residue functions vanish at mass values where they would otherwise occur [75, 79], and that the trajectories have cusplike behavior for small values of W^2 which cause the resonance to be more massive than otherwise expected [330, 377]. It is possible, but unlikely, that their measured masses are in error due to effects of background phases in phase-shift analysis. The latter possibility may also explain [213] the break in the N_γ trajectory at small values of W.

The most promising of the explanations, is that the absence is due to a zero of the residue function [75, 79]. That this zero is the one whose existence was proven by Desai [182] is doubtful; Desai argued, using unitarity and MacDowell symmetry, that if the potential theory result that $\operatorname{Im}\alpha(W)>0$ in the neighborhood of the threshold is applicable, then a zero should occur for W^2 less than the square of the threshold energy; e.g., for the N trajectory, $W^2<(m+\mu)^2$. Recent analyses [80, 142, 414] of all available backward $\pi^\pm p$ elastic data, in

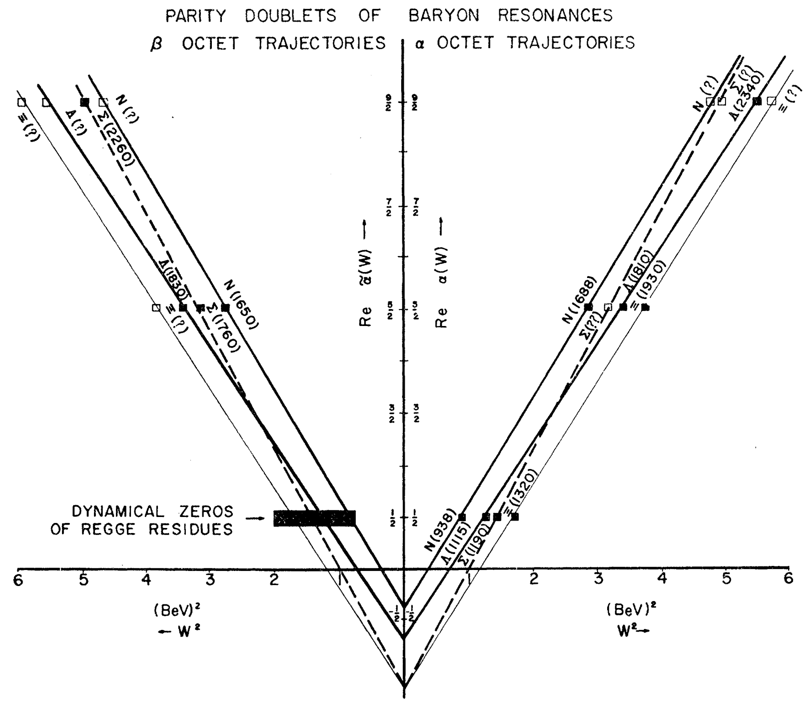

FIG. 9. An assignment of baryon resonances to MacDowell parity doublets, $SU(3)$ symmetric trajectories (Ref. [75]).

which the residues were assumed to contain this zero, have been reasonably successful. In fact, one fit assumed the N and Δ residues could be parametrized by $\beta(W)=(W-W_0)\gamma$, and it gave reasonable fits if $W_0=-M_N$ and $W_0=-M_\Delta$ for the N and Δ trajectories, respectively [80].

The explanation that trajectories are asymmetric functions of W and have cusplike behaviors as a function of W^2 is based on the difference in threshold behaviors due to the parity difference of members of a MacDowell doublet [330, 377]. Such cusplike behavior should be apparent, at least for the N trajectories, in abnormal parity trajectories like N_β [330]. This explanation makes it seem accidental that the trajectories, when extrapolated to $W=0$, have the same slope and intercept values obtained from fits to experimental $\pi^\pm p$ data [78]. Mixed support for the argument is given by a doubly subtracted dispersion-relation calculation which generated a symmetric N_α and an asymmetric Δ_γ trajectory [185, 441].

In conclusion, we have seen that the $\pi^\pm p$ backward elastic-scattering data could be reproduced with the N_α and Δ_δ trajectories and their MacDowell symmetric pair N_β and Δ_γ, respectively. Baryonic resonances can be fit into a very elegant trajectory scheme in which trajectories occur as approximate parity doublets and trajectories of each set are approximately parallel with equally spaced intercepts. Dominant and nondominant trajectories have $j-l=\frac{1}{2}\epsilon(-)^{I+1/2}$, where ϵ is plus or minus, respectively.

X. CROSSOVERS, DIPS, AND PEAKS AND THEIR REGGE THEORY EXPLANATIONS

Generally speaking, the Regge pole model makes predictions about the dependence on energy, s, of high-energy reactions, but says very little about their dependence on momentum transfer, t. The theory does provide definite ways in which amplitudes should vary when a trajectory passes through special values, i.e., integers for boson and half-integers for fermions (see Sec. IV). Consequently once the trajectory is known as a function of t, the behavior of amplitudes is restricted for values of t corresponding to special values of α. Also, once the t dependence of a vertex function describing a Regge pole coupling to two particles is known, factorization demands that the behavior of the vertex function be the same for all reactions it enters. In this section we review some t-dependent effects that Regge pole theorists have recently attempted to explain. Harari [291] has given an excellent summary of reactions that the Regge pole model has difficulty describing.

A. The Crossover Effect

If the differential cross sections for $AB\to AB$ and $\bar{A}B\to\bar{A}B$ are plotted together as a function of t, it is usually observed experimentally that the two curves cross over each other for some small value of t. This phenomenon is called the "crossover" effect and the value of t at which it takes place is called the crossover point (t_x). If we identify the antiparticle–particle system $\bar{A}B$ with the two-particle system which couples to the greater number of direct channels (e.g., π^-p, K^-p, $\bar{p}p$) and AB with the system which couples to the lesser number of direct channels (e.g, π^+p, K^+p, pp), then the difference

$$D(AB) = d\sigma(\bar{A}B) - d\sigma(AB)$$

is positive for small values of t and negative for $-t > -t_x$.

The fact that the differential elastic cross section for $\bar{A}B$ (e.g., π^-p) is larger at $t=0$ and more sharply peaked than that for AB (e.g., π^+p) seems [353] to be a diffractive effect associated with the larger number of channels open to the $\bar{A}B$ system. Consequently, an explanation of this phenomenon in terms of cross-channel exchanges such as Regge poles is difficult. *A priori* the Regge pole theory cannot predict at what point the two differential cross sections should be equal.

Line reversal implies that the same trajectories contribute to both processes and the contribution of trajectories with positive values of charge conjugation is the same in both processes, while the contribution of those with negative values of charge conjugation changes sign. Thus D(AB) can be written

$$D(AB) = 4 \text{ Re } (T_+{}^*T_-),$$

where a sum over all helicity states is understood and T_c is the sum of all contributions from Regge poles with charge conjugation C; e.g., $T_+ = P + P' + A_2 + \pi$, $T_- = \rho + \omega + \phi$. Thus, to explain the crossover effect in terms of Regge poles some mechanism must be found which will allow this interference term to vanish at t_x. Another unpleasant feature for Regge pole theorists is that while the dominant trajectories for $\pi^{\pm}p$, Kp, $\pm$ pp, and $\bar{p}p$ are not the same, the value of t_x seems to be essentially the same for all of the reactions; i.e., $0.10 \leq -t_x \leq 0.15$.

To underscore the confusion resulting from the crossover effect, we review the situation. It is generally argued that only the imaginary part of the helicity nonflip amplitude of T_+ is important; and thus $D(AB)$ can change sign only if the helicity nonflip part of T_- changes sign [83]. It is thought that the coupling of the ϕ to $N\bar{N}$ is very weak and if both ρ and ω contribute to a reaction, the ω exchange is dominant [83].

In the crossover effect for Kp and pp, it is believed that the nonflip contribution of the ω changes sign at t_x. Factorization implies that ω amplitudes in every reaction should also vanish at t_x [442]. From studies of residue functions for the ω in Kp scattering using finite energy sum rules, FESR (see Sec. XII.B), there is evidence for a zero in both the nonflip (A') and flip (B) helicity amplitudes at t_x [168, 197]. But, on the other hand, the ω contribution to $\pi N \rightarrow \rho N$ shows no evidence for a zero or dip near t_x [158].[20] Similarly FESR studies and high-energy fits to π^0 photoproduction fail to indicate a zero in the ω contributions [97].

Thus, the explanation of crossovers for $D(pp)$ and $D(K^+p)$ is in trouble. Perhaps reliable intermediate-energy phase-shift data for pp and $\bar{p}p$ will help swing the evidence to one side, but the present inconsistencies must be resolved. A solution could be provided by an ω' for which Im $(\omega+\omega')$ vanishes and Re $(\omega+\omega')$ is finite for the nonflip amplitudes [83, 291]. This could also resolve difficulties with the absence of dips in $\gamma p \rightarrow \pi^0 p$ [195].

The situation for $D(\pi^+p)$ also appears to be confused. In this case the only major contribution to T_- is from the ρ trajectory. A FESR study [199] of πp data has found a zero in the spin-nonflip amplitude $A'^{(-)}$ near $t_x \sim 0.1$, consistent with a fit to the high-energy πp data (442). However, an interference model fit to $\pi^- p \rightarrow \pi^0 n$ has failed to find a change in sign for the spin-flip amplitude $b_1(t)$ in the region of t_x [549]. A ρ' similar to the ω' has been suggested to avoid difficulties [76, 83, 232]. In general, only some sort of conspiracy mechanism such as this is likely to give a reliable explanation to the crossover phenomenon [76, 83]. This particular mechanism would modify present Regge pole predictions and could be tested [76, 83].

The crossover phenomenon could also be due to cut corrections that represent multiple scattering [242] or absorption [49, 54, 297] effects. In such models the total contribution of cuts and Regge poles would cause $D(AB)$ to vanish without requiring the residues of individual trajectories to vanish. For the crossover in $D(\pi^+p)$ the cut contributions are estimated to be too small to explain the effect at t_x [452] although an eikonal absorption model [54] has produced a crossover between -0.3 and -0.4.

B. Dips of Differential Cross Sections Associated with Special Values of α

In addition to the zeros of residues that seem to be imposed by the crossover phenomena, there are some natural zeros associated with special values of α appearing in α factors as discussed in Sec. IV. In the following all special values of α are assumed to be integral; i.e., for fermions we work with $\alpha = \alpha_F - \frac{1}{2}$. To determine whether an amplitude is infinite, finite but nonzero, or zero at a particular value, one has to consider the α factors coming from three sources: (i) from the signature factor $[1+\tau \exp(-i\pi\alpha)]/\sin \pi\alpha$, which is finite at wrong signature points [i.e., $\exp(-i\pi\alpha) = -\tau$] and infinite at right signature points [i.e., $\exp(-i\pi\alpha) = +\tau$]; (ii) from the residue function which, in addition to the four usual ghost-killing mechanisms, may have fixed poles at wrong signature points (see Sec. IV); and (iii) from the Gell-Mann *et al.* [260] E-functions. Of these three sources only the α dependence of the residue function is effected by dynamics.

[20] The latter conclusion is based on curves that are subject to large errors, both statistical and systematic; e.g., normalization errors in cross sections.

Instead of giving tables of α factors for general cases, we will review the literature on supposed α-dependent dips in differential cross sections associated with various trajectories.

The classic example of a dip associated with a special value of α, the dip at $t\sim -0.6$ in the reaction $\pi^- p\rightarrow \pi^0 n$, is explained by the ρ choosing sense at $\alpha_\rho(-0.6)=0$ (see Refs. [76 and 353] for references). This is a wrong signature point and would not be a possible explanation if fixed poles were important. Similarly one would expect dips in $\pi^+ n\rightarrow \omega p$ and $\pi^+ p\rightarrow \omega\Delta^{++}$ due to ρ exchange, but they do not seem to appear [41, 353]. One argument used to explain their absence is that the contribution of the B trajectory is large enough to fill in the dips. The zero near $t\sim -0.5$ of the πN spin-flip amplitude $B^{(-)}$ was found using FESR's, in agreement with the findings from fits of high-energy data [199].

In reactions dominated by the ω trajectory one would expect a dip similar to those due to the ρ trajectory. Whereas ρ exchange fails to give a dip in ω production, ω exchange does give a dip in ρ production. A dip in the ω contribution to ρ production has been found at $t\sim -0.5$ [158]. The reaction $\gamma p\rightarrow \pi^0 p$ is dominated by the ω (and B) and has the expected dip [353, 366]. The point $\alpha_\omega=0$ is a wrong signature point and one needs to explain the absence of fixed pole effects. In contrast, for the Kp data for which ω is important, there is no zero in the nonflip amplitude (at least for $-t<0.8$ [168]) although the helicity-flip and nonflip amplitudes do vanish near -1.0 [197].

The A_2 trajectory contributions to $\pi^- p\rightarrow \eta n$, $K^- p\rightarrow \bar{K}^0 n$, $K^+ p\rightarrow K^0\Delta^{++}$ [43, 348, 433], and $\pi N\rightarrow \eta\Delta$ [40] have been recently studied. The latter reaction does not exhibit a dip at $\alpha_A=0$ [347]. The point $\alpha_A=0$ is a right signature point, but it is uncertain which ghost-killing mechanism the A_2 uses [43].

Recently the $\pi^\pm p$ elastic data has been fitted; it was assumed that the dip seen in each at $t\sim -0.8$ is due to the P' trajectory [138, 442]. The point $\alpha_{P'}=0$ is a right signature point, and in order to produce a sizeable dip, it was necessary to have the P' choose the no-compensation mechanism. To associate the dips with the P' trajectory of the two solutions previously obtained in studying πN data [434], only the solution with a large slope for the P' trajectory was possible [138, 442].

Fits to the pp and $\bar{p}p$ data need to explain the existence of a dip in the elastic $\bar{p}p$ data at $t\sim -0.5$ and the lack of a corresponding dip in the elastic pp data [138, 442]. Since both pp and $\bar{p}p$ receive contributions from the same trajectories and only differ in the relative sign of the $C=-1$ contributions, a delicate interference effect must take place between trajectories with $C=+$ and $C=-$. In particular, such an interference could exist between the P and P' amplitudes and the ω amplitude [442]. If the P' and ω trajectories are degenerate, one can use $\beta_{P'}\sim \sin^2(\frac{1}{2}\pi\alpha)$, $\beta_\omega\sim \cos^2(\frac{1}{2}\pi\alpha)$ as a crude mechanism to explain the effect [89, 300].

A pronounced dip is found in $\pi^+ p$ but not in $\pi^- p$ in the backward elastic data (see Sec. IX). In that case the dip is attributed to the passing of the N trajectory through the wrong signature point $-\frac{1}{2}$. But there appears to be no dip in $\pi^- p$ associated with the passing of the Δ trajectory through $-\frac{3}{2}$, also a wrong signature point.

Barger and Phillips [89] summarize the latest experimental evidence for dips or inflection points in various reactions. Barger [76] also gives a very good review of the recent work done on explaining dips in differential cross sections.

In conclusion, the dip mechanism associated with special values of α has had some remarkable successes, but also some discouraging failures. The recent work of Mandelstam and Wang [389] clouds the theoretical picture at a time when the experimental picture is also confused. Assuming the difficulties can be resolved, dips may provide a useful way to determine how trajectories fall with t. Barger and Phillips [89] suggest that if their interpretation of primary and secondary dips is correct, the existence of the dips constitutes the first experimental evidence for linearly falling trajectories.

C. Forward Peaks and Regge Pole Theory

For many reactions a simple Regge pole model cannot explain the forward peak of the differential cross sections. A more complicated mechanism, such as conspiracy (see Sec. VI.D) or absorptive corrections (see Sec. XII.E) or combinations of both, is necessary to obtain agreement between theory and experiment.

Some of the reactions which require complicated Regge explanations are $pn\rightarrow np$, $p\bar{p}\rightarrow nn$, $\pi N\rightarrow \rho N$, $\pi N\rightarrow \rho\Delta$, $\gamma p\rightarrow \pi N$; these have been discussed theoretically [162, 210, 479, 480] and fitted with conspiracy schemes [35, 46, 311], (Also see Sec. XI.F which discusses π conspiracy.)

The Regge pole model does not provide as simple an explanation of peaks as do models like the OPE model with absorptive corrections. In the above reactions the pion would normally be expected to be the dominant contributor. Whereas the OPE model with absorptive corrections worked best when pion exchange was possible, in the Regge pole model the pion is almost forgotten in the ensuing conspiracy between other trajectories (see Refs. [44] and [46]).

In conclusion, the Regge pole model cannot easily explain the angular dependence of many reactions. Its greatest success, that of correlating dips in angular distributions with special values of the trajectories, has encountered difficulty both experimentally and theoretically. The model has explained too much experimental data to be easily dismissed, but it is being severely harassed by the complications mentioned in this section.[21]

[21] See Harari [291] for a review of the difficulties a simple Regge pole model encounters for particular reactions.

XI. ACCUMULATED FOLKLORE ON BOSON TRAJECTORIES

In this section we will review properties of the dominant trajectories and will discuss what is known about their participation in reactions; their proposed intercepts and slopes, their choice of α factors; their $O(4)$ classification; and whether they are exchange degenerate with other trajectories.

A. The ρ and ρ' Trajectories ($\tau=-, C=-, P=-, I=1$)

Of the major Regge poles ($P, P', \rho, \omega, A_2, \pi$) the ρ is the only one that can participate in the charge-exchange reaction $\pi^-p\rightarrow\pi^0n$, where it appears to be responsible for an α-dependent dip at $t\sim-0.6$; i.e., $\alpha_\rho(-0.6)=0$ [45, 138, 304, 369, 434, 442]. The ρ trajectory is also thought to be a leading participant in other charge-exchange reactions such as $pn\rightarrow np$, $p\bar{p}\rightarrow nn$, $K^-p\rightarrow \bar{K}^0n$, $\pi N=\omega\Delta$, $\gamma p\rightarrow\rho n$, $\pi p\rightarrow\pi^0\Delta^{++}$. There have been many fits to these reactions [41, 43, 46, 51, 85, 141, 180, 227, 275, 284, 307, 309, 325, 367, 369, 375, 412, 436, 442, 460, 461, 463, 493, 494]. The participation of the ρ trajectory in sum rules, including FESR's, has also been studied [34, 199, 307, 316, 395, 411, 493, 522]. The ρ trajectory is apparently linear in t though the quoted values of the intercept vary between 0.52 and 0.65 [307], and those of the slope, between 0.64 and 1.0, The most popular parametrization seems to be $\alpha_\rho=0.58+0.90t$. A ρ trajectory with a slight curvature has also been obtained from a doubly subtracted dispersion-relation calculation [2].

Since the earliest papers on the ρ trajectory, its role in the crossover for πp elastic cross sections (see Sec. X.A) has been uncertain. Usually the crossover is attributed to the vanishing of the spin–nonflip ρ amplitude. The ρ trajectory is capable of being bootstrapped [282, 314, 485] and appears to be a Class I ($M=0$) trajectory [29, 41, 83, 232, 249] (see Sec. VI.D).

Though the question of which α-factor mechanism the ρ chooses at $\alpha=0$ is still unresolved [43, 138, 232, 349, 452], it is generally thought that the ρ uses the sense-choosing mechanism. The residue behavior found by Mandelstam and Wang [389] is hopefully assumed to be unimportant at $\alpha=0$, since the connection between the dip in $d\sigma$ ($\pi^-p\rightarrow\pi^0n$) and the vanishing of α_ρ [45, 304, 434] would otherwise be destroyed. Supposedly this dip can be explained with any of the α-factor mechanisms if other poles (e.g., ρ') that are needed for nonzero values of the polarization are included [485]. Perhaps the new πp charge-exchange data [284] will help to resolve what is happening at the dip. The energy dependence at the dip should determine whether any amplitudes due to the ρ contribute at $\alpha=0$. The energy dependence of the bump after the dip that occurs at about $t=-1.0$ decreases very rapidly with increasing energy [502]. If the dip is due to the vanishing of certain ρ amplitudes, the energy dependence of the bump should be given by the ρ trajectory.

The ρ trajectory is often considered exchange degenerate with the R or A_2 (see Sec. VII.C). Exact degeneracy requires that the residues of the A_2 and ρ trajectories be equal. Since there cannot be a ghost pole at $\alpha=0$, either a lower compensating trajectory exists [260], or all the residues for the A_2 must contain a ghost-killing factor of α, which similarly must be contained in the residues for the ρ. This would imply that the ρ has any of the α factors discussed in Sec. IV except that corresponding to sense-choosing. The relations between total and differential cross sections resulting from this exchange degeneracy have been compared to experimental data [19, 20, 22, 23, 26, 51, 309, 494]. Experimentally determined values of residues and trajectories for ρ and A_2 (e.g., $\alpha_\rho(0)=0.58$, $\alpha_A(0)=0.34$) suggest that the symmetry is broken [76, 85]. Some similarities of their residues have been found, and these suggest at least an approximate exchange degeneracy (313). The approximate equality of the two trajectories $|\alpha_A-\alpha_\rho|\lesssim0.1$ for $-0.6<t<-0.2$ [305, 433] and their dominance in photoproduction have been used to relate differential cross sections for pion photoproduction to vector meson production [189].

There has been considerable interest in a second trajectory (i.e., ρ') with the same quantum numbers as the ρ. Its physical manifestation could [370] be the δ (965 MeV). It was first introduced to account for a discrepancy between the $\rho+A_2$ model and the experimental differential cross sections for forward pn and $\bar{p}p$ charge exchange [301]. An intercept of $\alpha_{\rho'}(0)\sim-0.6$ gives the necessary corrections [301].

Since the A_2 cannot contribute to $\pi^-p\rightarrow\pi^0n$, the $\rho+A_2$ model predicts that polarization should be zero. Many groups have used a ρ' to account for the nonzero experimental value of the polarization [92, 98, 107, 256, 370, 493, 495]. With $\alpha_\rho(0)$ constrained to be 0.58, values of $\alpha_\rho'(0)$ have been found ranging from -0.5 to 0.25 [256] and even as high as 0.34 ± 0.10 [493]. A positive intercept is needed to obtain agreement with the experimental polarization which is about the same at 5.9 GeV/c and 11.2 GeV/c [107].

Several recent papers consider the necessity of a ρ' contribution in sum rules [74, 119, 168, 198, 419, 493]. A study of πN data using FESR found $\alpha_{\rho'}(t)\sim\alpha_\rho(t)-0.4$ [119, 198], while a similar calculation using CMSR found $\alpha_{\rho'}(0)\sim0$ [421]. The residue function of the ρ' at $t=0$ was also found to be less than one-tenth that of the ρ [421]. Studies of πN data using CMSR's show that the ρ' decouples at $t=0$ [415, 421, 495] or does not exist [493]. A sum rule for πN scattering demonstrates that within experimental accuracy the ρ' contribution is unnecessary [316]. This encouraged a recalculation [256] of earlier results [370] which found that two solutions (i.e., $\alpha_{\rho'}(0)=-0.5$ and $\alpha_{\rho'}(0)=+0.2$) gave agreement with the sum rule result.

Fitting the high-energy differential cross sections for

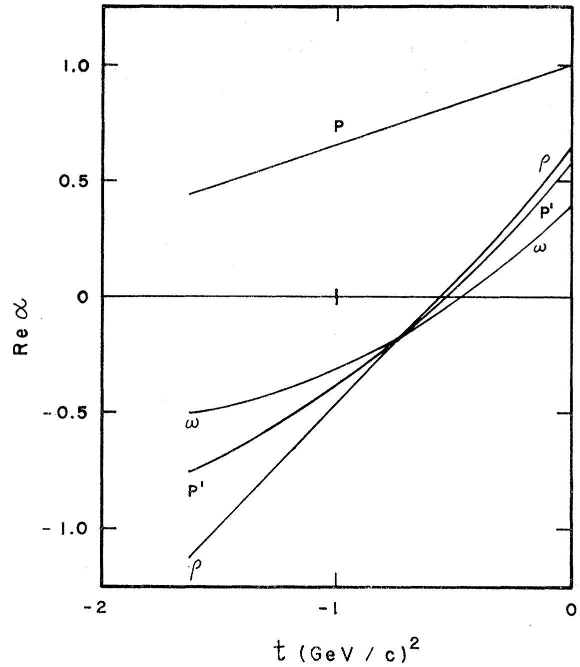

Fig. 10. Typical example of trajectories found in fitting experimental data (Ref. [138]).

the reactions $K^+n \rightarrow K^0p$, $\pi^-p \rightarrow \pi^0 n$, $K^-p \rightarrow \bar{K}^0 n$, and $\pi^- p \rightarrow \eta n$ required addition of a ρ' to the $\rho + A_2$ model with $\alpha_{\rho'} = -0.48 + 1.44t$ [491]. This result was criticized [168], at least for the reaction $K^+n \rightarrow K^0 p$, since a fit can also be obtained within the $\rho + A_2$ model if the residue of the A_2 is allowed to change sign.

The ρ' could be used to explain the crossover effect in πN scattering [76, 83]. The α-factor mechanism of the ρ' might result from a Mandelstam–Wang [389] fixed pole [232]. In an $O(4)$ classification the ρ' and the B are assumed to form a Class III ($M=1$) parity doublet [22, 98, 232, 495]. A ρ' has also been postulated to avoid having the ρ be a mixture of two Toller poles [29].

B. The P and P' Trajectories ($\tau = +, C = +, P = +, I = 0$)

To explain certain features of high-energy reactions—such as the apparent constancy of total cross sections as a function of energy—in terms of simple Regge poles, a trajectory which is called the Pomeron or Pomeranchuk trajectory P, is postulated. For constant total cross sections, the optical theorem demands $\alpha_p(0) = 1$. Since many elastic amplitudes involve a two-pion vertex, the Pomeron must have $\tau P = +$ and $G = +$ and thus $\tau = +$ [518]. This naturally implies that the Pomeron contribution to an elastic amplitude is imaginary at $t = 0$; this is consistent with a diffraction point of view.

Pomeranchuk has provided several rules [236] for the high-energy behavior of reactions. One of these rules implies that cross sections in the limit of high energy are independent of isospin, e.g., $\sigma_{AB}{}^{I_s} = \sigma_{AB}{}^{I_s'}$. This implies that the Pomeron has zero isospin. Another of the rules implies $\sigma(AB) \rightarrow \sigma(\bar{A}B)$ at high energies. Then by line reversal, the Pomeron must have positive charge conjugation. This also follows from $I=0$ and $G=+$ or from the fact that only mesons with $C = \tau$ contribute to spin-averaged total cross section [76].

Morrison [404] recently considered the contribution of the Pomeron to inelastic reactions such as $pp \rightarrow pN^*$ and $\pi^{\pm}p \rightarrow \pi^{\pm}N^*$. He noted that while other inelastic cross sections decreased with energy, there is one constant-energy case: that in which an N^* is produced from an initial nucleus for which the change in isospin is zero and the change in parity (ΔP) and the change in spin (ΔJ) obey $\Delta P = (-1)^{\Delta J}$. In terms of N and Δ trajectories this would imply that the Pomeron could play a role in the formation of an N_α or N_γ but not an N_β, N_δ, or any Δ trajectory from a nucleon (i.e., N_α trajectory). Similarly for meson resonance production Morrison found that for reactions where the change in quantum numbers between the resonance and the initial meson obey $\Delta I = 0$ and $\Delta P = (-1)^{\Delta J}$, the cross sections are nearly constant; e.g., $K^{\pm}p \rightarrow K^{*\pm}p$, K^* (1320), or $K^*(1790)$.[22]

The Pomeron is generally assumed to be a $SU(3)$ singlet [76]. Recently, however, it has been argued [473] that some deviations from exact $SU(3)$ symmetry can be explained if the Pomeron has a small $I=0$ octet component. In the Freedman and Wang $O(4)$ classification, the Pomeron is assumed to be a Class I trajectory.

So far the only experimental fits involving the Pomeron have been to elastic scattering [102, 108, 138, 181, 183, 227, 326, 356, 442, 456]. The Pomeron has been used in a $P + P' + \rho$ model [141] to fit total cross sections for $\pi^{\pm}p$ scattering and in a $P + P' + \omega$ model [227] to make a study of the NN and $N\bar{N}$ elastic data. (For a discussion of P', see below.) Recently these two models have been used in a combined study of πp, pp, and $p\bar{p}$ elastic data [442].

To explain the absence of shrinkage in πp, the slope of α_P needs to be rather small (e.g., ~ 0.3), in contrast to the more or less universal value of 1.0 for other trajectories [102, 181, 183]. In an analysis of $\pi^{\pm}p$ total-cross-section data, two solutions were found [141]: (a) $\alpha_P' = \alpha_{P'}' = 0.34$, and (b) $\alpha_P' = 0.23$ and $\alpha_{P'}' = 0.93$. The study of NN and $N\bar{N}$ elastic data found $0.25 < \alpha_P'(0) < 0.4$ and $1.1 < \alpha_{P'}'(0) < 1.4$, with typical values for a best fit of $\alpha_P = 1 + 0.3t$, $\alpha_{P'} = 0.7 + 1.21t$, and $\alpha_\omega = 0.5 + 0.7t$ [227]. The combined study of πp, pp, and $p\bar{p}$ elastic data found that solution (a) could not give agreement with the data [442]. In fact, four reasonable

[22] If the reactions are crossed into the t channel, the $\Delta P = (-1)^{\Delta J}$ law is equivalent to $(-1)^{j_{ab} - l_{ab}} = (-1)^{s_a + s_b}$, where j_{ab} and l_{ab} are any total and orbital angular-momentum values contributing in a partial-wave decomposition. Written in this way the rule implies that the Pomeron couples to whichever parity state allows the smallest value of l_{ab} for a given value of total angular momentum, j_{ab} (when $j_{ab} \geq s_a + s_b$).

fits were found which gave ranges on the slope for the Pomeron trajectory of $0 \leq \alpha_{P'}(0) \leq 0.29$, and parametrizations such as $\alpha_{P'} = 0.75 + 1.5t$ and $\alpha_{P'} = 0.57 + 2.17t$ for the P' trajectory [442].

The residue and trajectory of the Pomeron have been studied using dispersion relations, sum rules, and FESR's [87, 120, 167, 168, 233, 264, 267, 275, 276, 306, 372, 420, 493] with results, in general, consistent with those mentioned above. Similarly the incorrectness of solutions (a) for $\pi^{\pm}p$ data [94] was found using inverse dispersion relations [306]. The B^+ amplitude used in the combined study of πp, pp, and $\bar{p}p$ does not satisfy the sum rules [87]; but since the high-energy fits are not very sensitive to B^+, the parametrization of the trajectories is unaffected. The intercept of the Pomeron trajectory, which cannot be accurately obtained from high-energy data, has been determined from an application of continuous moment sum rules to low-energy πN total-cross-section data [44] to be unity within an uncertainty of 0.02–0.03.

There has recently been considerable discussion of the implications of the unit intercept and small slope of the Pomeron. The unit value of the intercept has been used to derive two high-energy theorems for amplitudes [362]. A recent discussion of the analytic properties of trajectories and their residues as the mass of their first physical occurrence approaches zero suggests that the small value of the slope is closely tied to the fact that the intercept is unity [423]. In order for residues to obey the algebra of $U(3) \otimes U(3)$, the intercept must be less than one (i.e., $\alpha_P(0) = 0.93$) which implies that all cross sections tend asymptotically to zero [124, 308]. A calculation [117] of the shift due to electromagnetic interactions of the intercept, which was assumed to be unity in the absence of such interactions, found $\alpha_P(0) = 0.94$ [117]. A value of $\alpha_P(0) < 1$ would also avoid the inconsistency mentioned in Sec. III.C associated with a factor of $(\alpha_P - 1)$ in Compton scattering [5, 500].

As mentioned in Sec. III.B, a Pomeron can be exchanged simultaneously with another trajectory to produce a cut [110, 223, 242, 262, 310, 357, 432, 440, 487, 493, 504, 506]. If the intercept of the Pomeron is unity the branch point of the cut will have the same intercept as the other trajectory and a slope less than that of either trajectory, i.e., $\alpha_B = \alpha_1'\alpha_2'/(\alpha_1' + \alpha_2')$ (see Sec. III.A). To avoid the accumulation of the branch points of an infinite number of cuts at $j = 1$, it has been suggested that $\alpha_p(0) = 1 - \epsilon$ [262, 506]. The cut mechanism is not sufficient to allow the slope of the Pomeron to be zero and, consequently, it is unlikely that the Pomeron is a fixed pole [223]. The Pomeron may actually represent some types of singularities in the j plane other than a simple pole [487].

Perhaps the Pomeron does not bootstrap from cross-channel resonances because it is the result of the nonresonating background in the cross channel, in contrast to other trajectories that appear to result from cross-channel resonances [88, 155, 291, 293, 351]. Similarly the Pomeron is perhaps the Regge pole mechanism for describing diffractive scattering [291, 293]. If this were correct, the Pomeron trajectory would not pass through physical particles. It has been argued that if trajectories pass through physical particles and if some technical assumptions are valid, then the slopes of the trajectories would be the same; i.e., there would be a universal slope [312]. This implies that the unusual value of the slope of the Pomeron is related to the absence of its physical occurrences. The usual argument that the 2^+ meson, f (1250 MeV), is on the Pomeron trajectory is incorrect [76, 184].

Discussions of the role of multiple Pomeron exchange in multiperipheral reactions [66, 225, 226, 551] show that in the absence of cuts multiple Pomeron exchange leads to a violation of the Froissart high-energy bound [66], even if the Pomeron has a nonzero slope [551].

In conclusion, it appears that the Pomeron is a rather unusual trajectory and that we have just begun to understand its role in high-energy scattering.

The unusual properties of the Pomeron P require the existence of a second trajectory P' with a lower intercept and a more normal slope [315]. It has the same quantum numbers as the Pomeron and presumably the same $O(4)$ classification (Class I). Unlike the Pomeron, it is thought to belong to an octet representation of $SU(3)$ and pass through the f (1250 MeV) 2^+ meson instead of the f' (1520 MeV) 2^+ meson as once thought [184]. Since the P' has the same quantum numbers as the P, most authors who fit the experimental data or use FESR to study the Pomeron also include the parameterization of the P' trajectory.

Although the small slope of the Pomeron prevents a determination of its α-factor mechanism, a study of πN and N scattering shows that the P' probably chooses the no-compensation mechanism [138]. This conclusion has also been reached from studies of πN [87] and KN [168] scattering amplitudes. A dip in the $\pi^{\pm}p$ elastic cross section at $t = -0.8$ has been attributed to the vanishing of the helicity nonflip amplitude of the P' at $\alpha_{P'} = 0$ [138]. Similar dip–bump phenomena in K^-p at $t = -1.0$ and $\bar{p}p$ at $t = -0.5$ are thought to be due to the P' trajectory [76, 89, 353]. These dips do not occur at the same value of t possibly because the more slowly varying contribution of the Pomeron tends to shift the position of the dip outward as the energy increases [138]. An example of this has been given where $\alpha_{P'}$ passes through zero at -0.5, while the dip occurs at -0.8 [138]. Perhaps contributions of the P' and ω interfere to produce dips in $\bar{p}p$ but not in pp elastic scattering [89]. Equal intercepts for the ω and P' trajectories can explain the constancy of the total cross section for pp scattering [183, 227, 443]. Such proposals require approximate degeneracy between the ω and P' trajectories (see Sec. XI.C). Recent work [138] using the no-compensation mechanism for the P' suggests that the trajectory of the P' is close to that of the ρ and ω for negative t.

In general, the parameters obtained for the P' trajectory depend on what assumptions are made about those of the Pomeron. For example, Logan and Razmi (Ref. [373]; also see Ref. [372]) assume $\alpha_P(0)=1$ and find $\alpha_{P'}(0)=0.67$; but if they assume $\alpha_P(0)=0.93$, as suggested by Cabibbo *et al.* [124], they find $\alpha_{P'}(0)=0.64$ with a poorer fit.

There has been some speculation on the existence of a P'' [420]. It is difficult to distinguish whether contributions are due to a P'' Regge pole or to a cut [493].

In summary, the P' trajectory appears to be similar to other trajectories in that it appears to be responsible for dips in differential cross sections, to pass through physical particles, and to have a slope and intercept very similar to that of ω or ρ [138].

C. The ω and ω' Trajectories ($\tau=-, C=-, P=-, I=0$)

The ω trajectory is assumed to be a member of a Class I $O(4)$ trajectory and to belong to a vector octet of $SU(3)$. Since its isospin ($I=0$) allows it to play a dominant role in elastic scattering, the ω trajectory is usually studied along with the P and P' trajectories (see the previous section for references). It is assumed to play the dominant role, along with the A_1, A_2, and π, in ρ production. (The π trajectory appears to be important only at very small values of $-t$, on the order of 2 or $3m_\pi^2$ [44, 158, 298, 299].) In ρ production the dip in the differential cross section is assumed to be due to α_ω passing through zero in the same way as α_ρ does in π^-p charge exchange. This assumption is supported by the observance of a dip in the ω contribution to ρ-production data [158]. There is also a dip in $\gamma p \to \pi^0 p$ which is apparently due to the ω trajectory [38, 195, 292, 366]. A study of this reaction shows that the ω trajectory chooses nonsense at $\alpha_\omega=0$ [51]. The ω trajectory is possibly responsible for the crossover effect in NN and KN reactions (see Refs. [76, 83, 158, 195, 263], and Sec. X.A). For small t at high energies the reaction $K_2^0 p \to K_1^0 p$ is dominated by the ω trajectory and should supply useful information on the ω trajectory [263].

The ω is thought to be approximately exchange degenerate with the P' trajectory. This degeneracy is usually based on one or more of the following experimental features:

(a) The total cross section for pp scattering is nearly constant in s, whereas that for $\bar{p}p$ decreases toward the common asymptotic value [183, 227, 443].

(b) The total cross sections for K^+n and K^+p are approximately equal and nearly constant in s [86].

(c) There are no dips in the pp differential cross section corresponding to those in $\bar{p}p$ [89].

Interference of the ω with the P and P' trajectories can explain the shrinkage in pp and antishrinkage in $\bar{p}p$ differential cross sections [86]. Such effects would suggest that not only the residues, but also the trajectories, for P' and ω should be related. Evidence for this is rather controversial [86]. In particular, a study of KN scattering using FESR's found that the residues, but not the trajectories, are approximately degenerate [168]. Typical trajectories [227] used in fitting NN data (i.e., $\alpha_{p'}=0.7+1.25t$, $\alpha_\omega=0.5+0.7t$) also imply that the trajectories are not degenerate although those found in a combined study [138] of πN and NN data are similar (see Fig. 10). A fit of πp, pp and $p\bar{p}$ data [442] showed a large curvature and variation of parameters for the ω trajectory (i.e., $\alpha_\omega(0)$ from 0.21 to 0.47, $\alpha_\omega'(0)$ from 0.31 to 1.66) which could result from lower trajectories (e.g., ϕ) being included in the effective ω amplitude. One recent calculation [326] found intercepts of 0.5 and 0.75, while another [302] found intercepts of 0.52 and 0.50 for the ω and P', respectively. A study of $\bar{p}p$ total cross sections using FESR's found $\alpha_\omega=0.22$ and $\alpha_{p'}=0.58$ [120]. The present conclusion is that if there is exchange degeneracy between the P' and ω, it is at most only approximate. Consequently, the explanation of the experimental features that lead to the proposal of some sort of degeneracy must be more complicated.

The existence of an ω' [22, 83] could remove some difficulties present in a single ω model; such a difficulty is the crossover phenomena in pp and $\bar{p}p$ elastic scattering. Of course, such a trajectory may just be another way of describing contributions from less important trajectories such as the ϕ, or from cuts due perhaps to $P\omega$ or $\rho\pi$ exchanges.

D. The A_2 Trajectory ($\tau=+, C=+, P=+, I=1$)

The A_2 or R trajectory is associated with the $J^{PG}=2^{+-}$ (1310 MeV) meson and is assumed to belong to the same $SU(3)$ octet as the P' and to be a member of a Class I Toller pole family. It is assumed to be the dominant trajectory in $\pi^-p\to\eta n$ [43, 51, 85, 302, 374, 435, 446, 492, 550] and $\pi N\to\eta\Delta$ (40, 51), to be important in photoproduction [115, 189, 233, 533], and to share a dominant role with the ρ trajectory in charge-exchange reactions like $K^-p\to\bar{K}^0 n$, $K^+n\to K^0 p$ [43, 51, 85, 115, 361, 446], and $\pi^+p\to\pi^0\Delta^{++}$, $K^+p\to K^0\Delta^{++}$ [51, 523], and to play a minor role with the ρ in $pn\to np$ and $p\bar{p}\to n\bar{n}$ [115, 227, 361].

A few of the values proposed for the A_2 and ρ trajectories $\alpha_A(0)=0.34$ $\alpha_\rho(0)=0.34$ [85]; $\alpha_A=0.8+3.5t+3.5t^2$, $\alpha_\rho=0.62+0.56t$ [523]; $\alpha_A=0.5+0.96t$, $\alpha_\rho=0.58+1.11t$ [115]; $\alpha_A=0.5+0.9t=\alpha_\rho$ [51]; and $\alpha_A=0.40+0.49t$ [550], where we have listed α_ρ to emphasize the status of their exchange degeneracy. It is clear that the A_2 trajectory is not well known. A study of the first three of these reactions found that for fixed α_ρ, the slope of the A_2 is uncertain within a factor of 2 [43]. Recent data on $\pi^-p\to\pi^0 n$ and $\pi^-p\to\eta n$ may help to determine the parameters of the A_2 and ρ independently [284].

As discussed in Sec. VII.C, the A_2 trajectory has been thought to be exchange degenerate with the ρ trajectory. Evidently, this degeneracy at most is only approximate.

There have been attempts to determine the α-factor mechanism of A_2 at $\alpha=0$. Because this is a right signature point, one would expect the trajectory to choose either the Chew mechanism, the no-compensation mechanism, or the Gell-Mann mechanism to avoid a ghost state at $\alpha=0$ [274]. The α-factor mechanism of A_2 has been studied with FESR's by using the KN and $\bar{K}N$ elastic-scattering [274, 393] data and charged-pion-photoproduction data [114, 533]. Uncertainties in the early data and coupling constants prevented a clear decision but suggested that the A_2 uses either the Chew or the no-compensation mechanism. A more recent analysis of the KN data [274] gives strong evidence for the A_2 using the no-compensation mechanism.

E. The A_1 and B Trajectories ($\tau=-$, $C=+,-$, $P=+$, $I=1$)

The A_1 and B trajectories are essentially the peons of the Regge pole hierarchy. They are held responsible when no other respectable pole accounts for certain physical phenomena. Very little is actually known about their existence, but a brief review of some of their features is appropriate.

The A_1 trajectory is associated with a meson peak recently separated [403] from the A_2 at about 1090 MeV with $J^G=1^-$. At one time the A_1 peak was not thought to represent a resonance, since it could be explained by the Deck effect which involves only cross-channel exchanges. The new "Dolen–Horn–Schmid duality" [198] again leads to a resonance interpretation by arguing that cross-channel exchanges are closely tied up with direct-channel resonance effects [137].

The A_1 is usually assumed [249] to be a member of a Class II Toller pole family with a $J^{PG}=0^{--}$ daughter which might be the π (1640 MeV). The assignment of the A_1 to a Class III Toller pole cannot be ruled out as recently demonstrated by a study of $\pi^+p\rightarrow\pi^0\Delta^{++}$ and $\pi^+p\rightarrow\rho^+p$ reactions [213]. Theoretical discussions of the contribution of the A_1 and its daughter to the forward direction have been given for ρ production [99, 480]. The A_1 trajectory has also been discussed in the context of a field theory model [520].

If the A_1 has unit isospin, it should be important in charge-exchange reactions. The A_1 and its daughter have been included in fits to pn and $\bar{p}p$ charge-exchange data [46, 115] and also in ρ production [523]. The A_1 and π trajectories are thought to dominate the reaction $\pi^+p\rightarrow\rho^0\Delta^{++}$ at small t values [287].

The A_1 and B are assumed to be members of $SU(3)$ octets. The B trajectory is associated with a meson resonance at 1220 MeV with a probable J^G assignment of 1^+ [57]. The B is usually considered a member of a Class II Toller pole, but it has been suggested that if the B contributes to scattering at $t=0$, it could be a member of a Class III Toller pole [249]. The B trajectory contributes to photoproduction of pions [27, 70, 109, 115, 292, 297] where a typical parametrization for its trajectory is $\alpha_B=-0.2+0.8t$ [115]. It is also assumed to play a minor role in pn and $\bar{p}p$ charge-exchange reactions. A fit to these two reactions gives $\alpha_B=-0.4+0.9t$ and $\alpha_\pi=-0.025+1.25t$ [115]. The B trajectory can also explain the decay density matrix in ω production [91, 298] where an intercept of about $+0.05$ gives reasonable agreement with the data [91]. The B trajectory may explain the lack of dips in $\pi^+n\rightarrow\omega p$ and $\pi^+p\rightarrow\omega\Delta^{++}$ at $\alpha_\rho=0$ [41, 253] and the dip in π^0 photoproduction at $\alpha_\omega=0$ which fades away at high energies [38]. Recently, a $\rho+B$ model has been used to study $\pi^-p\rightarrow\pi n$, $\pi\Delta$, ωn, $\omega\Delta$, A_1n, Δ_1A, A_2n, and $A_2\Delta$ data [232].

As mentioned in Sec. VII.C, π and B are sometimes considered exchange degenerate trajectories. At present, except for a few crude estimates of the trajectory of the B, no parameters are available to prove or disprove the assertion. In the Regge pole model the pion has rather exotic properties. It is difficult to believe that even one trajectory could be so complicated.

F. The π and π' Trajectories ($\tau=+$, $C=+$, $P=-,+$, $I=1$)

The role of the pion trajectory in Regge pole theory has always been a little confusing. The recent work on Toller poles and conspiring trajectories has caused an enormous proliferation of literature related to the pion. This section reviews the course of the recent development and shows how complex the situation is becoming.

Consider the role the pion played in the OPE model with absorption corrections [320].[23] The single pion exchange with absorption connections very adequately describes resonance production in pseudoscalar nucleon interactions (e.g., $\pi N\rightarrow\rho N$, $\rho\Delta$, $f^0\Delta$, and $Kp\rightarrow K^*\Delta$) in the few-gigaelectron-volt region for small values of t. It predicts both differential cross sections and decay angular distributions for resonances. The model also successfully uses pion exchange to describe reactions such as $np\rightarrow pn$, $\gamma p\rightarrow\pi^+n$ and $\bar{p}p\rightarrow\bar{\Delta}\Delta$ [216].

The importance of the pion in the OPE model resulted from the smallness of its mass, which gave a pole near the physical region, and its strong coupling to the $N\bar{N}$ and $N\bar{\Delta}$ channels [287]. In the Regge pole model, the nearness of the pole to $t=0$ is no longer an important consideration in determining which trajectory dominates. In the following discussion of fits to experimental data with a pion Regge pole, it will be clear that the magnitude of the coupling constant at the pion pole is not a help but a hindrance to the fits. To be sure, the unmodified Born amplitude gave incorrect angular distributions in the OPE model. The absorption corrections were necessary to reduce the size of lower partial-wave components, i.e., bring them in line with the unitary limit and consequently produce the experimen-

[23] For additional references see Arnold [50] and Durand [216]. For a discussion of the role of the pion in the "droplet" model, see Byers [121] and Byers and Thomas [122].

tal angular distribution. In the Regge pole model it is not clear how much absorption corrections are included in Reggeization. It is clear, however, that the contribution of a simple Reggeized pion (say, in NN scattering) will still vanish at $t=0$, whereas the absorption corrections in an OPE model modify the s wave sufficiently that the contribution of the pion no longer vanishes at $t=0$.

To obtain a nonvanishing contribution at $t=0$ and reasonable decay distributions when resonances are produced, the Regge pole theory must be modified in some way, for instance, by the introduction of cuts or conspiring trajectories.

Several reactions have been described by a Regge pole calculation involving the pion trajectory and requiring no conspiring trajectories [239, 287, 299, 451, 523]. In a (ρ, ω, A_2, π) model calculation for $K^+p \to K^0\Delta^{++}$, $\pi^+p \to \pi^0\Delta^{++}$, ρ^+p, and $\pi^-p \to f^0n$, it was found that $\alpha_\pi = -0.08+0.69t$ gives a good fit; but the pion residue, when evaluated at the pole, is smaller than the known coupling constants; e.g., by a factor of 2 for $\pi^+p \to \rho^+p$ and by a factor of 10 for $\pi^-p \to f^0n$ [523]. A study of reactions involving scattering of π, K, and $\bar{p}$ from p that result in Δ production found [287] that trajectories for negative τP such as the π or A_1 dominate at small values of t and $\alpha_\pi = (1.50 \text{ to } 1.75)(t-\mu^2)$. The difference between these two parametrizations of α_π is essentially due to the pion contribution being important only over a range in t of a few μ^2 so that estimates of its slope are statistically difficult [44, 158, 298, 299].

Comment on the various attempts made by Frautschi and Jones [239] in fitting $\pi N \to \rho A$ is worthwhile. They demanded that their value of the pion residue at the pole be the value of the Born residue. Two of their attempts are particularly interesting. In the first, they assumed the residue function has the same t dependence as the Born residue and found the predicted cross section to be much larger than the experimental value. This, of course, was the same difficulty mentioned above [523] and results from the fact that a Reggeized evasive pion for small t is approximately equal to an elementary pion and that an elementary pion exchange requires absorption corrections to fit experiment. In the second, the residue function was assumed proportional to $(t-b)$; a good fit was obtained with $b \sim 0$. Actually this zero of the residue function results from the failure of their amplitudes to satisfy kinematic constraint equations and is unnecessary when the constraint equations are taken into account [321].

Frautschi and Jones [239] point out that their model does not help in understanding other reactions which have a large forward peak such as $pn \to np$ and $\gamma p \to \pi^+n$. In addition to these reactions, the reactions $\pi N \to \rho\Delta$, $\bar{p}p \to \bar{\Delta}\Delta$, and perhaps $\pi N \to \rho N$ have behaviors near the forward direction that are difficult to understand without the introduction of conspiracy between trajectories [429]. In particular, the forward cross section for pion photoproduction with a single-pion exchange will have a forward dip unless conspiracy takes place [210]. A Class II type pion is also insufficient and the data require a Class III type conspiracy [400].

The Toller pole classification of the pion is uncertain. Since the pion has unnatural η parity, i.e., $\tau P = -$, it cannot result from a Class I Toller pole. If the residue of the pion were nonzero at $t=0$, it could be a member of a Class III Toller family [249]. If this were true, the nonexistence of a physical parity doublet π' in the neighborhood of the pion would imply the trajectory for the π' does not remain parallel to that of the pion or that it chooses nonsense at $\alpha_{\pi'}=0$. In the limit of zero mass, $\alpha_\pi(0)=0$ and a zero-mass pion cannot be a member of a Class III Toller family since this representation of $O(4)$ does not contain a zero-spin representation of $O(3)$. Such a pion would be a member of a Class II Toller family. Since the mass of the physical pion is very small, one might expect it too would belong to a Class II Toller family. But there are certain complications associated with the pion being a Class II conspiracy. Since the quantum numbers of the pion ($CP = -$) correspond to those of an odd-daughter trajectory, there should be a parent trajectory with $J^{PG} = 1^{+-}$ at a mass approximately that of the pion; this is not observed [384]. A possible classification for the pion (Class IV) has been suggested in which the pion has a nonvanishing off-shell residue even for $t=0$ but vanishes on shell for zero four-momentum transfer [479].

Theoretical discussions have been given on how the amplitudes for $\pi N \to \rho N$ should vary as a function of energy in the forward direction for Class II and III pions [99, 231, 479, 480]. At high energies a Class II conspiracy produces only longitudinal ρ's, whereas a Class III conspiracy produces only transverse ρ's [231, 480] (see Sec. VI.E). Experimentally it appears that longitudinal ρ's are produced in the forward direction [177, 547]. Discussion of π conspiracy have been given for pion photoproduction [26, 400]. Le Bellac [357] has presented an argument using factorization to show that a Class III pion conspiracy would result in the reactions $\pi N \to \rho\Delta$, $KN \to K^*\Delta$, and $\pi N \to f^0\Delta$ exhibiting a dip in the forward direction (see Sec. VII.E).

Mandelstam [384] has related the Adler self-consistency condition imposed by PCAC on hadron amplitudes and the conspiracy classification of the pion (see Ref. [469] also). In particular he has shown that the Adler self-consistency condition is true without reference to currents if the pion trajectory is a member of a conspiracy with $M \geq 1$. In proving this, Mandelstam showed that the vertex function coupling two equal-mass particles vanishes at $t=0$ for a zero-mass pion coupling. The vertex function at $t=0$ for a zero-mass pion must vanish for $M=1$ since the $M=1$ representation of $O(4)$ does not contain states with $j=0$. Since PCAC assumes that amplitudes are smoothly varying functions of the pion mass, one would expect this zero of the vertex functions to be in the neighborhood of $t=0$ for the coupling of equal-mass particles to a Reggeized pion whose physical mass is not zero but μ^2. Consequently, many authors include a factor of $(t-\lambda\mu^2)$ in the pion residue function [70, 115, 192].

An $M\geq 1$ pion results not only in the vanishing of the amplitudes for soft pions (equal-mass scattering), but also of those for hard pions (unequal-mass scattering) and vanishes in the zero-pion-mass limit [47, 478]. The decoupling of the pion from all reactions in the limit of zero mass would cast considerable doubt on many current-algebra results [478]. To avoid this, one would conclude that $M=0$ for pions. It is also possible that the limit of zero pion mass, which is needed to obtain current-algebra results, is either not continuous [423] or is at least exceedingly complicated.

The Bethe–Salpeter equation has been used to study how a pion could change from a $M=1$ trajectory at $t=0$ to a physical $j=0$ particle at the nearby point $t=\mu^2$ [104, 514]. In these solutions, the pion residue function is proportional to $\alpha(0)$ for a nonzero-mass pion and to t for a zero-mass pion [104, 514]. This would imply a factor like $(t-\lambda\mu^2)$ in the residue. The zero of the residue function in the interval $1/3\leq -t/\mu^2\leq 3/5$ requires a negative λ. The trajectory for the parity doublet is found to have a negative slope at $t=0$, which suggests that it dominates in the cross-channel reaction. The $O(4)$ classification for massless pions in the Bethe–Salpeter equation is complicated and implies that a $M=1$ pion is very unlikely [405].

The classification of the pion has been studied in an off-mass-shell generalization of $O(4)$ symmetry; it is found that a pion with $M=1$ at $t=0$ requires trajectory mixing in the region $0<t<\mu^2$ to permit the trajectory to be pure $M=0$ at $t=\mu^2$ [244, 363].

A trajectory with the quantum numbers of the pion was found in a field theory model using Feynman diagrams [520] and appears to be a Class II and not a Class III trajectory.

Many fits to experimental data use conspiracy schemes involving the π trajectory [18, 27, 44, 46, 70, 115, 159, 188, 253, 296, 311, 429, 478, 492]. A study of the reactions $NN\rightarrow N\Delta$ and $\pi N\rightarrow\rho\Delta$ favors an evasive pion and definitely rules out a zero in the pion residue function near $t=-0.02$ commonly associated with a Class III conspiracy [253]. In order to avoid a violation of the principle of factorization in these reactions, singularities more complicated than a conspiring pion pole must exist in the j plane [253]; e.g., conspiring poles with the quantum numbers of the pion [46, 115]. The experimental data for $\pi p\rightarrow\rho p$, $\rho\Delta$ have been analyzed with both a Class II and a Class III pion trajectory [47, 231]. A class II conspiracy between an A_1 and its daughter is included with a Class III pion whose residue function has a zero at negative t [47]. The reaction $\pi^-p\rightarrow\rho^0n$ has been examined for small values of t, and there is no clear evidence for assigning the pion to a Class III Toller pole [492]. A dip has been found in the forward direction for the reaction $\pi^+p\rightarrow\rho^+p$, but the reaction $\pi^+p\rightarrow\rho^0\Delta^{++}$, which is dominated at small t by the pion, shows no dip near the forward direction [18]. This result cast doubt on Le Bellac's [357] arguments (see Sec. VII.E) for a Class III pion.

Photoproduction of pions and K mesons has been considered with Class II and Class III pions [27, 70, 109, 159, 192, 219, 296, 297, 478, 529]. Though both a Class II [296] and a Class III [70] K trajectories have been used, the data seem to require a Class III pion whose residue function vanishes at some value of negative t, e.g., for pions $t=-0.85\mu^2$ [70]. Studies of low-energy π-photoproduction data using sum rules and the assumption that only Regge poles dominate have found evidence of a conspirator and of rapid variations in the pion residue function with a zero at small values of t; these support the class III assignment of the pion [101, 114, 192, 465].

A combined study of photoproduction of pions and pn and $p\bar{p}$ charge-exchange data has been done using a Class III pion with a full zero in the $N\bar{N}$ vertex function at $t=-2\mu^2$ [115]. The pn and $p\bar{p}$ charge-exchange data were considered with both a Class III pion and a Class II pion with a A_1 daughter conspiracy. A Class III pion gave the better fit [46]. These data have also been fitted with a Class II pion and a set of conspiring cuts, i.e., ρP and A_2P [311]. The Class III pion fit [46] has been criticized [311] because it requires rapid variations of the residue functions at small values of t.

Such rapid variations may indicate that the assumption of a Class III pion is incorrect and the variations are actually due to interference between a Class II pion and a Class III trajectory [419, 478]. Any zero of the pion contribution could result from a cancellation between the unabsorbed pion contribution and absorption corrections [224]. From this point of view, the unabsorbed pion would not conspire, but the absorption correction which contributes to both parity states would conspire.

There is far more speculation about the pion than any other trajectory, except possibly the Pomeron, but unfortunately the properties of the pion are different to measure and are not known with any degree of certainty. Perhaps a $\rho\omega$ cut with the pion quantum numbers and approximate intercept could exist and might be responsible for some of the present confusion [26].

G. The K, K^*, and K^{**} Trajectories ($\tau=+,-,+$, $P=-,-,+$, $I=\frac{1}{2}$)

Because the K (0^-), K^* (1^-), and K^{**} (2^+) have nonzero strangeness, they are expected to be the dominant contribution to hypercharge-exchange reactions. In an early application of the exchange of a K^* trajectory [459, 462] to the reactions $p\bar{p}\rightarrow\Lambda\bar{\Lambda}$, $(\Lambda\bar{\Sigma}+\Sigma\bar{\Lambda})$, $\bar{\Sigma}^+\Sigma^-$ obtained a trajectory of $\alpha_{K^*}=0.4+0.7t$. A recent calculation using a $SU(3)$ breaking expression for the trajectories gave $\alpha_{K^*}=0.35+0.96t+0.16t^2$ and $\alpha_{K^{**}}=0.276+0.41t$ from the trajectories for ρ and A_2, respectively (446). These trajectories gave adequate fits to

$\pi^- p \to K^0\Lambda$, $K^0\Sigma^0$ and $K^- p \to \pi^-\Sigma^+$, $\pi^0\Lambda$ [446]. A K^* and K^{**} model has also been used to analyze hypercharge-exchange reactions involving meson (e.g., K^-, $\pi^\pm$) scattering by protons that result in a baryon or baryon resonance [51]. This model used $\alpha_{K^*}=\alpha_{K^{**}}=0.25+0.9t$, in analogy to exchange degeneracy in ρ and A_2 trajectories. An attempt to determine intercepts for K^* and K^{**} from total-cross-section data using $SU(3)$ symmetry relations obtained values of 0.24 for both trajectories [473]. Analyses of K-meson-photoproduction data [70, 159, 243, 296] have been made using both a conspiring [70] and an evasive [296] K^* exchange. The K trajectory may dominate over the K^* trajectory in the forward direction [70]. An attempt to do a Reggeized bootstrap calculation of the K^* meson from the $K\pi$ channel has met with reasonable success [532].

In this section, we reviewed what is known about prominent and some not-so-prominent boson trajectories. Our present knowledge falls short of a complete understanding of Regge trajectories. We discussed the necessity for a Pomeron trajectory with unit intercept and considered the speculation that its lack of physical occurrences might be associated with its being a Regge pole description of diffractive scattering. We noticed how secondary trajectories such as the ρ', ω', π' seem to be necessary from either experimental or theoretical reasons. We saw how the mighty work horse of the OPE model, the pion, has become extremely complicated and endowed with almost mystical properties in the Regge picture. One of the most satisfying features of the model—evidence for trajectories that pass through known particles—certainly is emerging from comparison of theory to experiment.

XII. BY-PRODUCTS OF REGGE POLE THEORY AND RELATED MODELS

This section is concerned with various fields of research that incorporate aspects of Regge pole theory. Whether such topics as multiperipheral reactions and FESR should be discussed here or in another section is open to debate. Many people working in these newer areas have also done considerable work in the mainstream of Regge pole theory and would argue that these topics represent new trends in the development of the more general theory. The main criterion for considering the topics here is that they involve assumptions additional to those of Regge pole theory.

A. The Interference Model

Low-energy scattering data (incident momentum of 1 or 2 GeV/c) is in general discussed in terms of phase-shift analysis and direct-channel resonances, whereas a description of data in terms of cross-channel exchanges, i.e., Regge pole model, is expected to be valid only at high energies (incident momentums in general greater than 4 GeV/c). There have been many attempts to select and combine the best aspect of both approaches to describe the data at energies intermediate between the two regions.

By considering the work of Van Hove [531] and Durand [214], who show that the sum of an infinite number of single-particle exchange diagrams gives a Regge pole behavior, one might be led to believe that a complete description should also include sums of direct-channel-resonance diagrams. The fallacy of this reasoning is obvious when one realizes that the s-channel and t-channel amplitudes each give a complete description of a reaction, and the Regge pole expansion is an approximate description of the t-channel amplitudes. Nevertheless, one might hope that the direct-channel resonances represent terms in the t-channel amplitude which are neglected in making an expansion in terms of a finite number of Regge poles; that is, they would be included in a background integral. Similar situations hold in potential scattering [213]. An equivalent statement would be that the Regge exchange terms form a background for the leading poles or resonances in the direct channels.

Such "resonance-plus-Regge-pole" models are generally referred to as interference models, since the two contributions interfere in such reactions as πN scattering to give nonzero predictions for polarization. In the early applications of the model, the direct-channel-resonance contributions were parameterized by using Breit–Wigner approximations. In particular, the model has been used to describe neutron polarization in $\pi^- p \to \pi^0 n$ [33, 375] and in $\pi^- p \to \eta n$ [374, 550], backward $\pi^\pm p$ scattering [77], and charge exchange and hypercharge exchange in pseudoscalar meson–baryon scattering [430, 446]. (See Refs. [213] and [374] for a more complete description of the model and its application.)

From a theoretical point of view the model has several serious defects. One of the most apparent is the use of Breit–Wigner approximations for the direct-channel resonances. The Breit–Wigner approximation results in resonance contributions that decrease slowly as a function of energy. The sum of such contributions, the so-called Breit–Wigner tails, constitutes a background. Consequently there is double counting when these contributions are included with the Regge pole contribution. Possibly, by using a Khuri representation for the direct-channel poles and by including energy dependence in the elasticity parameters for each resonance to account for multichannel effects, one could avoid the double-counting problem [185]. This approach has been used to study πp charge-exchange polarization [185]; but, in general, such calculations still fail to predict correctly the energy dependence of the polarization [285]. The large number of uncertain parameters associated with the resonances makes such an approach somewhat dubious. For example, a study of backward $\pi^- p$ elastic scatterings [189] using the same set of 15 resonances as Barger and Cline [77] found, by varying a few parameters slightly, that a Regge pole background was not necessary to fit the data. The model also failed to fit the angular distribution of the data away from 180° [189]. A fit to the backward $\pi^- p$ data [399] without a Regge pole contribution predicts polarization different from that of the interference-model predictions [77]. A description of the angular distribution for backward $\pi^- p$ scattering using an interference model, in which the Regge contribution was modified to account for a background term, met with moderate success [128].

The discussion in Part XII.B on finite energy sum rules, FESR, shows that in some sense the leading Regge pole contribution must be the local average in energy of the full amplitude. Consequently, in such cases as the spin-flip amplitude in $\pi^- p$ scattering, $B^{(-)}$, where the resonances all contribute with the same sign, the inclusion of both direct-channel resonances and Regge exchange poles amounts to double counting [59, 82, 140, 267].

Thus there are three strong criticisms of the interference model in its early formulation. First, the use of the Breit–Wigner approximations for resonances does not give an adequate description of resonances away from their poles and leads to a conflict with the assumption that the background is due to a Regge pole. Second, the use of large numbers of direct-channel resonances introduces too many uncertain parameters into the calculation. And, finally, unless the resonance contributions either are small or enter with opposite signs, there is definite double counting in including both resonances and Regge poles.

The latter objection can be rephrased as an attack on the assumption that the Regge pole contribution is a good approximation to the lower-order s-channel poles that make up the direct-channel background. The discussion of FESR shows that the Regge pole contribution appears to be related to the dominant s-channel resonances, in contrast to the assumption of the interference model. Perhaps the Pomeron trajectory is not related to s-channel resonances and can be correctly added to direct-channel resonance to obtain the full amplitude [264, 291, 293].

Durand [213] proposes several modifications to avoid the first two defects of the original model and argues, using an analogy to potential theory, that the Regge exchange contribution can be used as a background for the direct-channel amplitude. Consequently, there would be no serious difficulty with double counting. He suggests that the direct-channel resonances be described in terms of their Regge trajectories rather than in terms of individual resonance contributions. In addition, he proposes (similar to Desai [185]) the use of a representation for which the resonance terms, i.e., direct-channel trajectories, contribute significantly only for Re $\alpha \sim j$. Previous calculations have been redone using these suggestions and the results are essentially unchanged [73].

One way to avoid double counting in the interference model is to do a partial-wave analysis of the various contributions and to insist that the resonances only contribute to fluctuations about the smooth background given by the Regge exchange poles [73]. The fluctuations would give a very small contribution to an energy average of the amplitude, and the only significant contribution would be the Regge exchange pole as suggested by the FESR results.

Concluding the discussion of the interference model, it appears that the model should provide a useful method to study reactions and the resonances that contribute in the intermediate energy range (1 to 5 GeV/c) if contributions of resonances and Regge poles can be combined in a manner that avoids double counting.

B. From Dispersion Relations to Finite Energy Sum Rules

One of the most rapidly developing areas of research in Regge pole theory has been the merging of dispersion theory and the analytic properties of amplitudes predicted by the Regge pole model. In addition to the familiar terms like dispersion relations, sum rules, and superconvergence relations, such terms as generalized superconvergence relations, continuous moment sum rules, and finite energy sum rules are becoming more prevalent in the literature. Though these terms are sometimes used interchangeably, this section will review the various relations and discuss how they have been used in connection with Regge pole theory.

In general, one assumes for an amplitude free of kinematic singularities in s and u, that a fixed-t dispersion relation can be written in the variable $\nu=(s-u)/4m$:

$$F(\nu)=\int_{-\infty}^{\infty}\frac{\operatorname{Im}F(\nu')\,d\nu'}{\nu'-\nu}, \qquad \text{(XII.1)}$$

where the integration includes contribution from both the cuts and the pole terms. Recently dispersion relations of this form for Reggeized amplitudes [61] have been used with πN charge-exchange data to study the ρ trajectory [60].

If the amplitude is odd under crossing $\nu\rightarrow-\nu$ [i.e., $\operatorname{Im}F(\nu)=\operatorname{Im}F(-\nu)$ above the cuts] and if $\nu F(\nu)$ goes to zero as ν goes to infinity, one obtains the superconvergence relation (SCR)[24]

$$\int_0^{\infty}\operatorname{Im}F(\nu')\,d\nu'=0. \qquad \text{(XII.2)}$$

Amplitudes for t-channel reactions, free of kinematic singularities, obey such a relation if $\alpha(t)-m+1$ is less than zero for the leading trajectory, where m is the maximum t-channel helicity flip [530]. By demanding that the residues of fixed poles in the amplitudes vanish, such relations have also been derived for $\nu^n \operatorname{Im}F(\nu)$, where n is integral and satisfies $0>n>m-\alpha(t)-1$

[24] There is a simple connection between conspiracy and superconvergence properties of amplitudes [72].

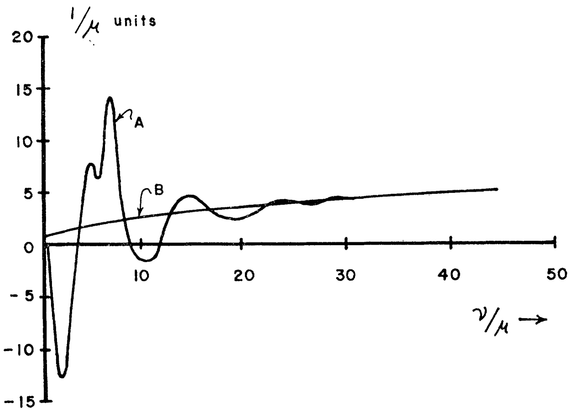

FIG. 11. Plot of the integrand of the superconvergence relation [Eq. (XII.3)]. The values of $(\nu^2-\mu^2)^{1/2}[\sigma_{\pi^-p}(\nu)-\sigma_{\pi^+p}(\nu)]$ [curve A] are taken from experiment, while the values of $4\pi\beta P_\alpha(\nu/\mu)$ [curve B] are calculated using $\alpha_\rho=0.54$ and $\beta_\rho=5.98\times10^{-2}\mu^{-1}$ (μ is the pion mass) (Ref. [316]).

[490]. A recent consideration of the relations imposed on elastic amplitudes by the constraint equation at $t=0$ has shown that whereas the t-channel amplitudes and thus their SCR are related at $t=0$, there exists a set of amplitudes which leads to independent SCR's [359, 361].

Recently SCR's have been used extensively to study properties of Reggeized amplitudes [289, 294, 307, 354, 411, 444, 450, 451, 463, 468, 496, 504], in particular to investigate NN amplitudes in the backward direction [354]; to determine the ρ parameters from πp charge-exchange data [307]; to investigate the pion contribution in pion photoproduction [289]; to consider the existence of fixed poles in Compton scattering [504]; and to study the existence of an $I=2$ cut due to the exchange of two ρ's in various reactions [411].

If one writes a dispersion relation for $F'=F-F_R$, where F_R is a sum of Regge poles that give the asymptotic behavior of F, then it might not be possible to write a SCR for F, but it will always be possible to write a SCR for F' by including a sufficient number of Regge poles in F_R.[25]

As an example, consider the SCR

$$4\pi f^2-(2\pi)^{-1}\int_\mu^\infty d\nu[(\nu^2-\mu^2)^{1/2}[\sigma_{\pi^-p}(\nu)-\sigma_{\pi^+p}(\nu)] -4\pi\sum_i\beta_iP_{\alpha_i}(\nu/\mu)]=0, \quad \text{(XII.3)}$$

where the optical theorem has been used to relate $\text{Im} f(\nu)$ to the total cross sections for $\pi^\pm p$ and the pole term has been put in explicitly ($f^2=0.081$) [316, 376]. (See Fig. 11.) Such expressions are normally called generalized superconvergence relations (GSCR) but some authors refer to them as sum rules.

The use of Eq. (XII.3) with $\pi^\pm p$ data implies that the ρ trajectory is sufficient, and additional contributions such as a cut or ρ' are not needed [316]. A similar expression has been used to study the ρ contribution to πp charge-exchange data [307] and the P and P' contributions to πN elastic data [276, 357].

By assuming for sufficiently large energies (say, $\nu>N$) that the contribution of Im $(F-F_R)$ to the integral is negligible, one can truncate the integration at N and write

$$\int_0^N \text{Im}\,(F-F_R)\,d\nu=0 \quad \text{(XII.4)}$$

or

$$S_0\equiv N^{-1}\int_0^N \text{Im}\,F\,d\nu=\sum\beta N^\alpha(\alpha+1)^{-1}. \quad \text{(XII.5)}$$

If $\nu^{n+1}(F-F_R)$ for some nonnegative integer n goes to zero as ν goes to infinity, one can write the finite energy sum rules (FESR)

$$S_n\equiv\frac{1}{N^{n+1}}\int_0^N \nu^n\,\text{Im}\,F\,d\nu=\sum\beta N^\alpha(\alpha+n+1)^{-1}. \quad \text{(XII.6)}$$

Similar sum rules that suppress the high-energy contributions by using decreasing weight functions have been derived by using unitarity [48].

By using the experimental low-energy data, one can determine the residues and trajectories of the dominant Regge poles with these expressions. Since N is in the asymptotic region, the importance of lower trajectories, cuts, etc. is the same as in a fit to high-energy data [199]. The advantage of FESR's is that one can calculate Regge parameters directly, using low-energy data, and need not work with the meager available high-energy data or attempt to fit differential cross sections where the contributions of trajectories with different quantum numbers must be considered simultaneously.

A tremendous number of papers utilize FESR's to study Regge trajectories, but in general there are three areas of endeavor: (1) the use of low-energy πN and KN data to study P, P', ρ, N_α, and A_2 contributions; (2) the use of pion-photoproduction data to study π and A_2 contributions; and (3) the use of FESR as a new type of bootstrap mechanism. Exceptions to this classification are: (1) an analysis of NN data to determine the intercepts of the P', ω, p, and A_2 trajectories [120] and (2) a study of the importance of the Pomeron and other trajectories in nucleon Compton scattering [167, 233].

Low-energy πN data and FESR's have been used to study ρ contributions [34, 166, 199, 376, 395, 422], the A_2 contributions [393], the P and P' contributions [62, 87, 372, 373, 375, 376], and the P residue (357).

Charged-pion-photoproduction data and FESR's have been used to study the A_2 [114, 465] and π [101, 114, 465] trajectories. The π-trajectory results give

[25] The exact α dependence used in F_R [e.g., $P(\nu/\mu)$ or ν^α] is arbitrary as long as it gives the normal asymptotic behavior.

support to its Class III assignment since the pion residue does not vanish at $t=0$ as would the residue of an evasive pion [101, 114, 465], but varies rapidly near $t=0$ and vanishes at $t=-0.03$.

By using the known Born terms and the resonance spectra to saturate the integral in the FESR, one can use the FESR to generate bootstrap trajectories in the cross channel [199]. This concept of saturating the FESR with narrow resonance states has been used to calculate decay widths of resonances [14, 15, 250, 392].

Reasonable results have been obtained in bootstrapping the ρ and other trajectories in $\pi\pi$ scattering where the t-channel Regge poles (ρ, f, g) are also the s-channel resonances [484]. While generation of the ρ from the πN resonance spectrum was successful [88, 199], the Pomeron could not be similarly bootstrapped [88]; this is consistent with the interpretation of the P mentioned in Sec. XI.B. Unlike the Pomeron, the ρ can bootstrap itself in a dynamical scheme [387] which uses, in addition to crossing relations imposed by the Dolen–Horn–Schmid superconvergence relations, a finite number of Regge poles in all channels. Unitarity and FESR have been combined to give a bootstrap scheme that permits not only ratios but also absolute values of masses and coupling constants to be calculated [314].

This new bootstrap approach, sometimes called the "Dolen–Horn–Schmid" duality [198, 199], has been used to argue that the connection between direct-channel resonances and cross-channel exchange implies that if the A_1 is due to a peripheral reaction (the Deck effect), the A_1 is a true resonance or, at least, cannot be distinguished from a true resonance [137].

Generation of the ρ trajectory through the use of πN resonances in the direct channel has strong implications on the interference model [88].[26] It essentially implies that by including the direct-channel resonances, one has included the cross-channel exchanges, and thus that the ρ cannot be a background for the resonances. The expression

$$\int_0^N \nu^n \operatorname{Im}(F-F_R)\, d\nu=0$$

implies that in some sense the Regge contribution must give the average contribution to the amplitude over the energy range. Since this is only true for certain moments, a simple formulation of the local average effect is difficult. A simple version of the interference model in terms of Regge poles plus resonances will not work for $d\sigma_{\pi^+p}$ (180°), $d\sigma_{\pi^\pm p}$ (0°), or in polarization for π^+p where resonances contributions are large and enter with the same sign; in these cases there would be double counting [143].

Continuous moment sum rules are obtained by considering dispersion relations for $\nu^\gamma \exp(-i\pi\gamma/2)F'(\nu)$ [493] or $(\nu^2-\mu^2)\exp(-i\pi\gamma)F'(\nu)$ [365, 419, 421], where γ is considered a continuous variable in contrast to n which was an integer. By using continuous moments, it is possible to obtain a continuous curve for S_n instead of a few discrete points and thus to obtain more information from the data. Continuous moment sum rules (CMSR) allow both the real and imaginary parts of the amplitude to enter into the calculation.

Continuous moment sum rules have been applied to low-energy πN data to study the ρ and ρ' trajectories [419, 421, 493] in order to determine the intercept of the P trajectories [493] and the contributions of vacuum trajectories (in addition to the P and P' trajectories) [420, 493]. Applications to pion-photoproduction data have been used to study the π trajectory and its conspirators [192] and to study the A_2 trajectory [533].

C. Schmid Loops in Argand Diagrams

Recently Schmid [485] has demonstrated that if the ρ contribution to the helicity-flip πN charge-exchange amplitude is subjected to an s-channel partial-wave analysis, loops are found in the Argand diagrams. The masses and widths of these "resonances" are in rather good agreement with the physical values of the known N^* resonances. Of course the Regge pole contribution is finite at the N^* poles, but Schmid uses this to argue that the Regge pole contribution is simply the leading term in an asymptotic expansion of the exact amplitude which has poles at the masses of the resonances. (See Ref. [534].)

This hypothesis that partial-wave projections of Regge pole amplitudes can be related directly to resonance states has stirred a great amount of controversy. In particular, it is in direct contradiction to the basic assumption of the interference model.

One of the most compelling attacks on the "Schmid" hypothesis has been given by Collins *et al.* [155]. They agree with Schmid that partial-wave projections of Regge pole amplitudes result in loops in Argand diagrams, but argue that such loops should not be associated with physical resonances because (1) Schmid "resonances" do not cause peaks in amplitudes as functions of energy (i.e., partial-wave components compensate each other to give smooth Regge behavior) and (2) by unitarity, true resonances occur in all processes with the same s-channel quantum numbers, whereas Schmid "resonances" are due to t-channel quantum numbers. For example, both $\pi\pi\to\pi\pi$ and $N\bar{N}\to\pi\pi$ have the same s-channel but different t-channel quantum numbers. Thus identification of resonances by phase-shift analysis is now questionable [155].

Point (2) has also been emphasized by Allessandrini and Squires [32], who point out that since Regge pole theory does not have unitarity built in, it cannot be

[26] By contrast, it has been suggested that for the πN charge-exchange amplitude at $t=0$, the ρ is generated solely by the nonresonant background [420].

expected to account for resonances due to "forces" in other channels which couple to the s channel.

Alessandrini *et al.* [31] have obtained similar conclusions by considering partial-wave projections of a Regge pole amplitude with a linear trajectory. In particular, they find "Schmid resonance" with small masses and high values of angular momentum. They argue that either (1) such resonances exist, or (2) there is a theoretical way to distinguish physical resonances from Schmid loops, or (3) one must be very careful in identifying Argand loops at low energy and high values of j as physical resonances [31]. A similar calculation found that linear trajectories in the t channel resulted in s-channel trajectories rising as $s^{1/2}$ [420]. The Schmid approach also generates parallel daughter trajectories [31, 468].

Kreps *et al.* [349] have made partial analysis of Regge pole amplitudes for πN charge-exchange scattering and concluded that there is a lack of correspondence between Schmid loops and measured resonances. This conclusion is in strong contrast to the results of Lipshutz [364], who also analyzed πN charge-exchange data. Lipshutz was able to identify a large number of Schmid loops and measured resonances. He was even able to make several predictions about yet-unmeasured properties of some of the resonances.

The hypothesis [15] that a sum of s-channel, finitely spaced Regge trajectories cannot result in asymptotic behavior characteristics of a t-channel Regge pole has been disproved by a counter example [266].

The crossing-symmetric, Regge-behaved amplitudes recently proposed [534] have also given support to Schmid hypothesis in that they demonstrate that there is no interference between Regge pole amplitudes and direct-channel resonance.

D. The Use of the Asymptotic Properties of Regge Poles in Bootstrap Calculations

One of the difficulties of traditional bootstrap calculations is that the energy dependence of the poles used results in divergent integrals unless a cutoff is used. With the advent of Regge pole theory and the reasonable asymptotic energy behavior which it predicts, this difficulty can be avoided. In Part B of this section, we pointed out that the "Dolen–Horn–Schmid" duality provides a new bootstrap approach. Previous to this work there were a number of papers using Regge poles in bootstrap calculations [10, 11, 61, 64, 129, 154, 239, 241, 265, 524, 532].

Bootstrapping of entire Regge trajectories to avoid the cutoff problem has been suggested [241]. In certain situations the bootstrap hypothesis can generate the Regge trajectories of external particles [524]. The bootstrapping of Regge trajectories has been considered in a modification of the strip approximation [61, 64, 154] and has led to the interesting feature that, unlike Born amplitudes, the residue functions for Regge poles must oscillate in sign [154]. Higher boson resonances R, S, T, and U are incompatible with the bootstrap hypothesis if they are on the ρ and A_2 trajectories [156].

A bootstrap calculation for the ρ [10] and K^* [532] trajectories was done using a modified Cheng representation [11]. The N and Δ trajectories have been bootstraped using a generalization of Chew's reciprocal bootstrap model in which virtual transitions like $\pi N \rightarrow \rho N'$ are the driving forces for Regge trajectories of inelastic resonances [129, 130]. Bootstrap theory combined with Regge poles can support arguments for a noninvariance algebra for meson isobar residues [265].

In many ways the Regge pole concept has helped enormously to revive the bootstrap theory. The solution of how to treat particles with large spins has enabled calculations to proceed where they were previously questionable because of the use of cutoff parameters.

E. Absorptive Corrections and Regge Poles

As mentioned in the discussion of the pion trajectory, Sec. XI.F, the Regge pole model has difficuties explaining phenomena which were easily explained by the one-particle-exchange model with absorptive corrections. Except for the introduction of amplitudes forbidden by angular-momentum conservation, the Reggeization of helicity amplitudes at fixed s is more analogous to the modification of Born amplitudes through the use of Ferrari–Selleri type form facts than the modification resulting from absorptive corrections. For example, whenever a Born amplitude is evasive at $t=0$, an amplitude for a simple Regge pole is also evasive, whereas a Born amplitude with absorptive corrections contributes at $t=0$. There is some absorption introduced by Reggeizing amplitudes, and one must be very careful in introducing absorptive corrections to avoid double counting. Just as in the one-particle-exchange model, where the use of both form factors and absorptive corrections introduced too many arbitrary parameters into the theory, the use of Regge poles with absorptive corrections can involve numerous parameters that must be determined from experimental data.

At present there are two closely related methods of introducing absorptive corrections into the Regge pole model. The first, proposed by Arnold [49, 50, 52, 54, and 103] and others [69, 242 and 504], uses an eikonal approximation; it generalizes the potential theory concept of an impact parameter to modify Regge pole amplitudes by using eikonals obtained from elastic-scattering data. This approach attempts to incorporate the effects of unitarity into inelastic reactions by considering corrections due to elastic scattering in the initial and final states. This method, identical to the earlier approach used to introduce absorptive corrections into the one-particle exchange model, has been extended to consider photoproduction reactions [457].

The other approach, proposed by Cohen-Tannoudji *et al.* [152, 402] (also see Refs. [297] and [449]), also attempts to incorporate unitarity, and leads to a multichannel model in which the amplitude is written as

$$T(s,z_s)=A(s,z_s)+R(s,z_s)+2i\int d^2\hat{p}A(s,z_i)R(s,z_f), \tag{XII.7}$$

where $z_s=\hat{p}_i\cdot\hat{p}_f$ and $z_i=\hat{p}_i\cdot\hat{p}$ and $z_f=\hat{p}_f\cdot\hat{p}$. The matrix A is diagonal and is a purely imaginary shadow scattering amplitude, thus associated with the Pomeron trajectory. The matrix R is a Regge-type amplitude containing all trajectories whose intercept is less than unity. The integral can thus be understood as due to cuts which come from the simultaneous exchange of the Pomeron and other Regge poles important in the reaction. Although it does not appear to be necessary, the Pomeron is assumed to be a branch point singularity. Though only evasive Regge poles are used, the amplitude for any inelastic reaction (e.g., pn charge exchange) will receive a contribution from the integral, i.e., from the cut, and need not vanish in the forward direction. Application of the model to πN, KN, and $\bar{K}N$ scattering gave good agreement with all available experimental data [152, 402].

One obvious objection to these models is the assumption that the unmodified amplitude is equal to the Regge pole amplitude: the old problem of double counting seems to arise when this assumption is made.[27] Another objection is that the integral requires knowledge of Regge pole contributions for much larger ranges of momentum transfer than is normally needed in a pure pole calculation. One of the most appealing features of the model is that it provides a simple way to parametrize contributions due to cuts.

Several authors [224, 242, 291, 452, 458, 504, 531] have discussed the connection between cuts and absorption corrections. While the contribution of cuts in inelastic amplitudes is small and resembles the effect obtained by the application of absorptive corrections to Reggeized amplitudes, contribution due to cuts should dominate the high-energy elastic scattering [504].

In conclusion, a Regge pole model consisting only of simple evasive trajectories is not sufficient to explain many features of high-energy data; some modification, such as absorption corrections, cuts (which may be equivalent to absorption corrections), or complicated families of conspiring poles is necessary.

F. Multiperipheral Reactions

The description of reactions that lead to three or more particles is complicated. Various papers attempt to describe such reactions in terms of the exchange of Regge poles (see Refs. 1, 63, 65, 66, 93, 95, 105, 126, 127, 131–134, 225, 226, 254, 337, 397, 430, 437, 453, 454, and 551]). At present the approach is still largely theoretical, but there is some experimental evidence that Regge poles play a role in multiperipheral reactions in much the same way as in two-body reactions.

The formalism necessary to describe multiproduction reactions has been discussed [1, 65, 66, 126, 454, 551]. The value of using the variables introduced by Toller in a group-theoretic analysis of kinematics is that, in addition to the range of each variable being independent of the values of the others, these variables lead naturally to a description of the asymptotic behavior of amplitudes [65, 66]. The continuation of amplitudes in a three-body model to complex angular momentum has been considered [1]. Crossing relations [229], which are similar to relations for two-body amplitudes, have been derived [126]. Multiparticle amplitudes do develop the $O(4)$ symmetry as the total four momentum vanishes [63]. A recent formalism for three-body reactions treats spin properly and can be used to describe resonances between two of the final particles [127].

If the Pomeron is a Regge pole, then the total cross sections are constant asymptotically [551] since the particle multiplicity grows asymptotically like $\ln s$, and the cross section—to produce a given number of particles—falls off like $1/\ln s$. This, or course, is consistent with the conclusion obtained using the optical theorem and with the fact that asymptotically elastic amplitudes are dominated by the Pomeron. If the trajectory of the Pomeron is flat, i.e., a fixed pole, the exchange of the Pomeron leads to a violation of the Froissart bound [66]. This violation remains even if the Pomeron has a finite slope, provided that cuts are not present [225, 226]. Further, either the trajectory of the Pomeron is not flat or the number of times a Pomeron can be exchanged is limited [336].

For the reaction $\pi N\rightarrow\pi\rho N$ a deck calculation using Regge pole exchanges has explained the enhancement of the $\pi\rho$ mass in the region of the A_1 in terms of multiperipheral exchanges [93]. The enhancement increased when the exchanged pion was Reggeized [93]. That a Reggeized pion should work better is consistent with the argument that the A_1 is probably a true resonance. This argument uses the "Dolen–Horn–Schmid" duality between direct-channel resonances and cross-channel Regge exchanges to suggest that Regge amplitudes give a rough representation of resonances even though the amplitudes do not contain the resonance poles [137]. This duality also results in a great simplification in multiperipheral calculations, since one no longer need consider direct-channel resonances when using Regge exchanges [137]. The πN enhancement at 1400 MeV/c in the reaction $pp\rightarrow pn\pi$ has been similarly investigated [95].

A Feynman diagram model with multiperipheral Regge exchanges has been used to discuss the origin

[27] It might be argued that since the existence of cuts cannot be denied, there is as much danger in half-counting as in double counting; this model actually introduces fewer parameters than a pure pole model since the input Regge poles are evasive (i.e., conspiring trajectories are not required) and have smoothly varying reduced residue functions (i.e., the contribution of cuts avoids the necessity of rapidly varying residue functions).

of conspiracies, ghost-killing mechanisms, and the kinematic structure of Regge pole amplitudes [105].

The multiperipheral Regge model has been successful in explaining experimental data [95, 97, 131, 133, 134, 437, 453]. In fitting reactions with more than three particles in the final state, the structure of nonresonant low-mass clusters is sometimes assumed to be governed only by phase space [131, 392]. In multiperipheral calculations the same trajectories dominate as in two-body reactions; the values of their intercepts and slopes and the behavior of their residues are consistent with those of two-body reactions [131, 392].

In this section we discussed the rationale for an interference model to describe reactions in the intermediate-energy region where the asymptotic Regge behavior is not dominant and direct-channel resonances are important. Various relations, such as superconvergence relations and finite energy sum rules, can be used to study the behavior of residues and trajectories of Regge poles. The "Dolen–Horn–Schmid" duality and its implications on the interference model and on a new type bootstrap theory were discussed. In particular, this duality destroys the assumptions necessary for a simple resonance plus Regge pole type of interference model. We also discussed the effect of the Regge pole concept on the bootstrap model and described attempts to introduce absorption corrections into the Regge pole model and how such corrections simulate the effects of cuts in the complex j plane. For those who would like a relatively simple model, the use of absorption corrections is no more appealing than the alternate choices: i.e., of cuts or the use of large numbers of complicated Regge poles. Some mention was made of the heroic work of those groups attempting to describe multiperipheral reactions using Regge poles.

The Regge pole concept can obviously be useful in many areas of research. In addition to the uses mentioned in this section, the asymptotic behavior of Regge poles can be used to calculate electromagnetic mass differences [417, 505]. The success of the concept of complex angular momentum might suggest that spin and isospin should be on equal footings, and one should consequently Reggeize isospin [472]. This, of course, amounts to a higher symmetry and should lead to considering the N_i and Δ_i trajectories ($i=\alpha, \beta, \gamma, \delta$) as isospin recurrences on isospin trajectories.

XIII. SUGGESTED EXPERIMENTAL TEST OF ASSUMPTIONS USED IN REGGE POLE THEORY

Probably no experiments can deal a death blow to the theory of Regge poles, but many can help to test basic assumptions and resolve ambiguities that confront theorists. In this section, we review some of the suggested experiments and discuss what may be learned from performing them. All of the experiments need to be done at energies above the resonance region so that the high-energy (Regge) behavior dominates.

Perhaps a good test of line reversal (See Sec. VII.F) would be a measurement of the Σ^+ polarization in the two reactions $\pi^+p\rightarrow K^+\Sigma^+$ and $K^-p\rightarrow\pi^-\Sigma^+$ [51, 428]. In these reactions, the polarization can be easily determined by the decay of the Σ^+. One would expect that the only contributions are the exchange of the K^* [890] and K^{**} (1400) trajectories. Line reversal says that the relative sign between the two contributions changes for the two reactions, and thus their polarizations multiplied by the respective differential cross sections should be equal in magnitude but opposite in sign. Similarly, for the reactions $\pi^-p\rightarrow K^0\Lambda$ and $K^-p\rightarrow\pi^0\Lambda$, the same $K+K^*$ model predicts

$$P_\Lambda d\sigma(K^-p\rightarrow\pi^0\Lambda) = -\tfrac{1}{2}P_\Lambda d\sigma(\pi^-p\rightarrow K^0\Lambda),$$

where the factor of $\frac{1}{2}$ comes from isospin Clebsch–Gordon coefficients [51]. The data for these reactions are also very meager and do not provide a true test of the assumption of line reversal [51].

As mentioned in Sec. VI.C, there is some controversy about whether the ρ and A_2 trajectories are exchange degenerate. A measurement of the polarization of the recoil neutron in $K^-p\rightarrow\bar{K}^0n$ could provide a test of how much exchange degeneracy is violated since the polarization would be zero if their trajectories or residues were degenerate.

An accurate measurement of the polarization in the reaction $\pi^-p\rightarrow\pi^0n$ for $-t<0.1$ should help to determine whether the ρ' is a conspiring or evasive trajectory [83, 495].

Several experiments have been proposed to determine whether the pion is a member of a Class II or Class III Toller family. For example, in the reaction $\pi N\rightarrow\rho N$, the t dependence of the decay density matrix for the ρ at small t differs depending on the pion classification. If the pion is Class II, ρ_{00}, ρ_{01}, $d\sigma\rho_{1-1}$, and $d\sigma\rho_{11}$ are approximately 1, $(-t)^{3/2}$, t, and t, respectively; if the pion is Class III they are approximately <1, $(-t)^{1/2}$, t, and a constant [231]. Asymptotically in the forward direction, ρ_{00} would be 1 and 0, respectively [480]. The ρ data at 8.0 GeV/c seem to imply $\rho_{00}\rightarrow1$ and ρ_{1-1}, $\rho_{01}\rightarrow0$ in the forward direction, which would mean the pion is evasive [177, 434]. The data at 4.2 GeV/c seem to be consistent [547]. What is needed is an accurate measurement of the density matrix near the forward direction over a range of t of 1 or $2\mu^2$. Some results on this reaction for small values of t have been reported [18, 492].[28] The use of polarized photons in pion photoproduction [160, 162, 206] and Coulomb interference in pp scattering [119] could also help to determine the $O(4)$ classification of the pion although the predictions are based on the assumption that only Regge poles dominate.

[28] The experimental determination of the M value of the pion may be difficult if the suggestion of Sawyer [478], Frazer *et al.*, [243, 244] and Sugar and Blankenbecker [514] is true. They suggest that there must be both $M=0$ and $M=1$ trajectories with the quantum numbers of the pion which interfere.

By considering forward scattering of reactions involving nucleons, such as $\pi N \to \rho N$, ωN, σN; $\gamma N \to \pi N$, $A_1 N$, and $NN \to NN$, and conspiracy classes contributing one should be able to conclude which conspiracies nature is using [479]. A measurement of the reactions $pp \to pp$, $Kp \to K^* p$, or $\pi p \to A_2 p$ in the forward direction might help to determine whether conspiring doublets (Class III) or cuts contribute to forward cross sections [212].

A measurement of the second-rank polarization tensor for $NN \to NN$ in the nonforward direction should determine the presence of cuts [431]. If cuts are an important contribution, polarization measurements should fall off like $(\ln s)^{-1}$ [170]. (A $(\ln s)^{-1}$ behavior, of course, would be difficult to observe experimentally.) Measurements of the differential cross section for $\gamma N \to \pi^0 N$ or $\gamma N \to \pi^0 \Delta$ for photons polarized in the scattering plane should determine whether cuts are important [161, 292, 297].

Reactions in which two units of charge are exchanged have no known Regge pole contributions and should provide a means of determining the importance of cuts [398, 432]. Examples of such reactions are $K^- p \to K^+ \Xi^-$, $\pi^- p \to K^+ \Sigma^-$, and $K^- p \to \pi^+ \Sigma^-$. Since the same cuts (e.g., ρK^*) contribute to the second and third reactions, they should have the same energy dependence. With values for ρ and K^* intercepts, $(\ln s)^2 d\sigma$ should vary as s^0 and $s^{-0.4}$ for the first two, respectively [432.]

As mentioned in Sec. VII.E, the principle of factorization places stringent limitations on the explanation of particular phenomena. Factorization says that if a vertex function has certain properties in one reaction, then it must have the same properties in all other reactions it enters. A test of factorization requires that a particular property of a vertex function be determined without making any assumptions. This is where tests of factorization break down: One invariably makes assumptions concerning the contributions of other trajectories, etc., in concluding that a vertex function has a certain property. Consequently, tests of factorization without any assumptions are difficult to propose.

The approximate mirror symmetry of polarizations for $\pi^\pm p$ has led to the conclusion that the contribution of the P and P' tarjectories is small [442]. Since factorization requires the P and P' contribution be small also for NN scattering, the polarization for pp and $\bar{p}p$ should also be mirror symmetric (similarly for KN system). If the assumptions are correct, measurements of the polarization for pp (and also $K^\pm p$) should provide a test of factorization [442].

By using Le Bellac's arguments, one can test either factorization or the Class III pion explanation of the forward peak in NN scattering by determining whether the reactions $\pi N \to \rho \Delta$, $KN \to K^* \Delta$, $\pi N \to f^0 \Delta$ exhibit a dip in the forward direction. The reaction $\pi N \to \rho \Delta$ fails to show the predicted dip in the forward direction [18].

Other reactions in which accurate experimental measurements could help to resolve certain ambiguities of the Regge pole model are ones whose differential cross sections are thought to have dip–bump phenomena associated with the α factors of amplitudes. (See Sec. X.B.) In particular, accurate measurements of the pp and $\bar{p}p$ differential cross sections in the t region of -1.4 to -1.0 and incident momentum range of 2.0 to 8.0 GeV/c should determine the mechanisms responsible for the dip in $\bar{p}p$ and its absence in pp [138].

Polarization measurements of the recoil proton in vector photoproduction would be useful in determining the existence of fixed poles [382]. With the use of polarized photons in ω photoproduction, it should be possible to study the pion residue function [160]. The reaction $K_2^0 p \to K_1^0 p$ could yield information on the ω trajectory such as its α dependence and its role in the crossover effect [263]. Other polarization experiments testing assumptions used in Regge pole models have been suggested [56, 399].

There are many experiments that can unravel ambiguities in the Regge pole model and test some of its basic assumptions. Among the most useful experiments from the Regge pole point of view are those at small t that help establish if and how trajectories conspire, those which determine the importance of cuts such as double-charge exchange reactions, and those which measure differential cross sections in regions of t where certain amplitudes may have dynamical zeros (α factors, crossover zeros, residue of Class III pions, etc.). When more complicated polarization experiments (e.g., C_{NN}) are possible, many of the present ambiguities of Regge pole theory may be resolved.

XIV. SUMMARY

This paper has attempted to review the recent developments in the Regge pole model. In addition to the simple poles related to known particles, it has become necessary to postulate the existence of other poles, families of poles, and various other j-plane singularities such as cuts and possibly fixed poles. There appear to be very good theoretical arguments that cuts and fixed poles at wrong signature points should give important contributions; but experimentally there presently seems to be no compelling evidence of their existence unless one interprets the rapid variations of residue functions required in pure pole models as indicative of cuts.

The need for α factors in residue functions and the various proposed α-factor mechanisms was reviewed. It is unclear why some dips in differential cross sections can be explained in terms of α factors at wrong signature points, when the existence of fixed poles should invalidate such explanations.

A discussion of the intuitive origin of kinematic singularities of t-channel amplitudes at normal and pseudothreshold points was given, and it was pointed out that the amplitudes must also obey constraint equations at these points. The solutions of the constraint equation then give physical cross sections that are free of kinematic polelike factors.

Many complications arise from demanding that Regge amplitudes have the correct analytic properties at $t=0$. In particular, the concept of daughter trajectories seems to be a natural consequence of a higher symmetry obeyed by the trajectories at zero four momentum. If large numbers of correlated trajectories are postulated, the analyticity of the resulting amplitudes permits contributions to cross sections in the forward directions that were not allowed for simple poles. The obvious objection to the introduction of daughter and conspiring trajectories is that they destroy the simple, appealing Regge pole model that needs only a few simple poles related to physical particles. Since the introduction of such poles is due to mathematical complications and has no physical basis, one is led to believe that their need implies a basic fallacy in the mathematical statement of the Regge pole model.

A review of the basic symmetries that Regge trajectories and amplitudes are assumed to obey—such as internal symmetries like $SU(3)$, exchange degeneracy, factorization, and line reversal—was presented with a brief discussion of their implications. Most of these symmetries are common to other pole models and are not fundamental to the basic theory of Regge poles. The importance of factorization, its restrictions on the present model, and the difficulties in formulating a definite test of factorization were pointed out. Symmetries such as line reversal and charge independence can be used to isolate the contributions of various trajectories to a given set of reactions. The example of how the ω contribution to $\pi N \rightarrow \rho N$ has been isolated was mentioned.

Backward πN scattering reactions can be explained in terms of the N and Δ trajectories. The identification of recurrence of N and Δ trajectories appears to be definite evidence for MacDowell-symmetric trajectories and indicates that lower trajectories are parallel and equally spaced by a unit of angular momentum from each other. This is very reassuring and indicates that physical particles actually lie on Regge trajectories that are approximately linear functions and have slopes and intercepts consistent with those obtained from scattering experiments.

The success of the Regge pole in explaining features of differential cross sections such as dips, crossovers, and forward peaks was discussed. Of the three, only the dip mechanism associated with α factors had a simple explanation in terms of Regge poles. Because of the recent work on fixed poles at wrong signature points, even this correlation between dips and α factors must be viewed with skepticism.

In a discussion of the more important trajectories, it was seen that the pion has a difficult time fitting into the Regge pole model and seems to play a role which is greatly diminished compared to its role in other models and one's intuitive feeling of its fundamental role in nature. Those who feel the pion has a complicated behavior can point to recent fits of experimental data and studies of its residue using finite energy sum rules (FESR). Their arguments are rather circular since their fits incorporate the present Regge pole model of the pion and FESR owes its very existence to the assumption that Regge poles provide an accurate description of high-energy phenomena.

Apparently the Pomeron is unlike any other trajectory. In particular, it has a rather low slope, appears to have no physical occurrences, and seems to be incapable of fitting into a bootstrap model. A dynamical model to describe the Pomeron appears to be emerging; it relates these and other features which distinguish it from other trajectories.

The interference model which has grown out of a merging of the Regge pole concept and the use of direct-channel resonance has several theoretical difficulties. Suggested modifications to avoid these objections were discussed.

FESR's are important as a tool for studying the residues and trajectories of Regge poles. In general, they give results consistent with Regge pole fits; e.g., they find that the pion is conspiring and that its residue has a zero quite close to the forward direction. The FESR are derived with the assumption that above some energy the amplitude is given exactly by the Regge pole contribution. It would be a dreary world if cross sections failed to show any features at high energies. Perhaps some fluctuations due to higher resonances must always be included; this seems quite natural, even from a Regge point of view, since trajectories appear to have higher recurrences. FESR's are possible only if these effects average out at higher energies. Only then could a duality such as that of Dolen–Horn–Schmid be made.

Partial-wave projections of Regge amplitudes have caused considerable controversy about the meaning of the resulting loops in Argand diagrams and concern about the identification of loops in Argand diagrams with resonances.

The Regge pole concept has been applied to other problems such as multiperipheral reactions and bootstrap calculations. Attempts to incorporate absorption corrections into the Regge pole model have been made with rather successful results.

In the last section there are suggestions for experiments that might help to resolve various difficulties in the present Regge pole interpretation of experimental features. In particular, there are experiments that could test the symmetry of line reversal, the importance of cuts, and the mechanism responsible for the presence or lack of certain dips in differential cross sections associated with α factors.

In conclusion, the Regge pole theory has certainly accounted for a large amount of high-energy data and has given new life to other fields of endeavor such as the bootstrap theory. It remains to be seen whether the model will continue to increase in complexity as it attempts to explain new data or whether a new formulation of the model will emerge that will avoid the difficulties of the present model and provide a simple explanation of experimental data. Either way, the elegance of a Regge pole approach to strong interactions has definitely been established and should remain with us for some time to come.

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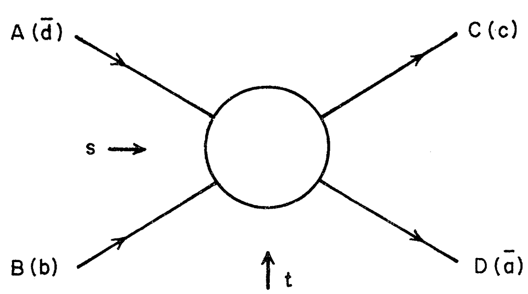
XV. APPENDIX

The Regge pole theory attempts to describe an s-channel reaction $A+B\to C+D$ or $\bar{d}+b\to c+\bar{a}$ by considering the amplitudes for the reaction as seen from a cross channel, designated here as the t channel. We will use the notation $a+b\to c+d$ to designate the t-channel reaction.



The mass, spin, intrinsic parity, and helicity of the ith particle will be denoted by m_i, s_i, η_i and λ_i, respectively. For the simplicity in labeling amplitudes the helicity will be denoted by i.

A. Kinematic Notation

It is customary to define the symbols

$$S_{xy}=[(s-(m_x+m_y)^2)(s-(m_x-m_y)^2)]^{1/2},$$
$$T_{xy}=[(t-(m_x+m_y)^2)(t-(m_x-m_y)^2)]^{1/2}, \tag{A.1}$$

where s and t are the usual Mandelstam variables. It is also customary to use the symbols $S(T)$ and $S'(T')$ for $S_{xy}(T_{xy})$ when x and y designate particles in the initial or final states, respectively, of the s channel (t channel). In terms of these functions the initial and final center-of-mass momentum (p and p') in the s and t channels can be written as

$$p_s=S_{bd}/2s^{1/2}=S/2s^{1/2},$$
$$p_s'=S_{ac}/2s^{1/2}=S'/2s^{1/2},$$
$$p_t=T_{ab}/2t^{1/2}=T/2t^{1/2},$$
$$p_t'=T_{cd}/2t^{1/2}=T'/2t^{1/2}. \tag{A.2}$$

The cosines of the center-of-mass scattering angles for the s and t channels can be written as

$$z_s\equiv\cos\theta_s=[s(t-u)+(m_a^2-m_c^2)(m_b^2-m_d^2)]/SS',$$
$$z_t\equiv\cos\theta_t=[t(s-u)+(m_a^2-m_b^2)(m_c^2-m_d^2)]/TT'. \tag{A.3}$$

By using the Kibble boundary function $\phi(s,t,u)$ where

$$\phi(s,t,u)=stu-s(m_a^2m_c^2+m_b^2m_d^2)-t(m_a^2m_b^2+m_c^2m_d^2)$$
$$-u(m_a^2m_d^2+m_b^2m_c^2)+2m_a^2m_b^2m_c^2m_d^2$$
$$\times(1/m_a^2+1/m_b^2+1/m_c^2+1/m_d^2) \tag{A.4}$$

and $s+t+u=m_a^2+m_b^2+m_c^2+m_d^2$, one can write

$$\sin\theta_s=2[s\phi]^{1/2}/SS',\ \sin\theta_t=2[t\phi]^{1/2}/TT',$$

and

$$\cos\chi_a=[-(s+m_a^2-m_c^2)(t+m_a^2-m_b^2)-2m_a^2m^2]/S'T,$$
$$\cos\chi_b=[\ \ (s+m_b^2-m_d^2)(t+m_b^2-m_a^2)-2m_b^2m^2]/ST,$$
$$\cos\chi_c=[\ \ (s+m_c^2-m_a^2)(t+m_c^2-m_d^2)-2m_c^2m^2]/S'T',$$
$$\cos\chi_d=[-(s+m_d^2-m_b^2)(t+m_d^2-m_c^2)-2m_d^2m^2]/ST',$$
$$m^2=(m_a^2-m_b^2)-(m_c^2-m_d^2); \tag{A.5}$$

χ_i is the crossing angle for the ith particle with reaction and helicity amplitudes for the s and t channels designated by

$$s\text{ channel}\quad \bar{d}+b\to c+\bar{a},\ f_{c\bar{a};\bar{d}b}{}^s,$$
$$t\text{ channel}\quad a+b\to c+d,\ f_{cd;ab}{}^t.$$

The crossing relation is

$$f_{ca;db}{}^s=\sum_{a'b'c'd'}M_{ca;db}{}^{c'd';a'b'}f_{c'd';a'b'}{}^t \tag{A.6}$$

or simply

$$f^s=Mf^t,$$

where the cross matrix M is given by

$$M_{ca;db}{}^{c'd';a'b'}=d_{a'a}{}^{sa}(\chi_a)d_{b'b}{}^{sb}(\chi_b)d_{c'c}{}^{sc}(\chi_c)d_{d'd}{}^{sd}(\chi_d). \tag{A7}$$

B. Reggeization of Helicity Amplitudes

In the following we do not attempt to present a new approach but only to introduce the notation commonly used in the literature. In so doing we follow the Reggeization procedure outlined by Gell-Mann *et al.* [260]. For the reader who is interested in the details of Reggeization we also suggest the excellent papers by Abers and Teplitz [13], Ader *et al.* [17], Calogero *et al.* [125], Drechsler [213], Jones and Scadron [332], Rebbi [445], and Thews [523].

In the Jacob and Wick [322] helicity formalism the t-channel helicity amplitude has a partial-wave expansion given by

$$f_{cd;ab}{}^{t}=\sum_{j}(2j+1)F_{cd;ab}{}^{j}d_{\lambda\mu}{}^{j}(\theta_t),\qquad\text{(A8)}$$

where $\lambda=a-b$, $\mu=c-d$. (Note that our amplitude differs from that of Jacob and Wick by a factor of $-8\pi(tT/T')^{1/2}$, which also results in a corresponding difference in our F^j's.)

Since the d functions can be written as

$$d_{\lambda\mu}{}^{j}(\theta_t)=(1+z_t)^{|\lambda+\mu|/2}(1-z_t)^{|\lambda-\mu|/2}P_{\lambda\mu}{}^{j}(z_t),\qquad\text{(A9)}$$

where $P_{\lambda\mu}{}^{j}(z)$ is a polynomial of $z_t=\cos\theta_t$, the barred amplitudes $\bar{f}$ defined by

$$\bar{f}_{cd;ab}=(1+z_t)^{-|\lambda+\mu|/2}(1-z_t)^{-|\lambda-\mu|/2}f_{cd;ab}{}^{t}\qquad\text{(A10)}$$

contain only dynamic singularities in s.

By using the parity properties of F^j, one can define a partial-wave amplitude $F^{j\eta}$ that corresponds to scattering between states of definite parity P by

$$F_{cd;ab}{}^{j\eta}=F_{cd;ab}{}^{j}+\eta\eta_c\eta_d(-1)^{s_a+s_d-\nu}F_{-c-d;ab}{}^{j};\qquad\text{(A11)}$$

the η-parity factor is given by

$$\eta\equiv P(-1)^{j-\nu}=P\tau,\qquad\text{(A12)}$$

where ν is 0 or $\frac{1}{2}$ depending on whether j is integral or half-integral and $\tau[=(-1)^{j-\nu}]$ is called the signature factor.

Using the e functions of Gell-Mann *et al.* [260], one can define a so-called "parity-conserving" amplitude f^η by

$$\begin{aligned}f_{cd;ab}{}^{\eta}&\equiv\bar{f}_{cd;ab}+(-1)^{m+\lambda}\eta\eta_c\eta_d(-1)^{s_c+s_d-\nu}\bar{f}_{-c-d;ab}\\&=\sum_{j}(2j+1)(e^{j+}F_{cd;ab}{}^{j\eta}+e^{j-}F_{cd;ab}{}^{j-\eta}),\qquad\text{(A13)}\end{aligned}$$

where $m=\max(|\lambda|,|\mu|)$.

One normally assumes that the amplitudes $F^{j\eta}$ have poles in the complex j plane (i.e., Regge poles) with η parity, η (e.g., poles due to trajectories such as the ρ and A_2 could occur in F^{j+}, while trajectories such as the π and A_1 could occur in F^{j-}).

In order to Reggeize the amplitudes $F^{j\eta}$, one must have an expression for $F^{j\eta}$ that can be continued to complex values of j. This is usually done by assuming $\bar{f}$ or f^η obeys dispersion relations involving its discontinuity across the s and u cuts [125, 213, 523]. In doing so, one finds that the contribution from the cut in u introduces a factor $(-1)^{j-\nu}$ which is not well behaved for complex j. Consequently one cannot continue $F^{j\eta}$, but must define two new functions, $F^{j\eta\tau}(\tau=\pm)$, by replacing $(-1)^{j-\nu}$ by τ in the expression for $F^{j\eta}$. The functions $F^{j\eta\tau}$ are then continued to complex angular momentum and are assumed to have Regge poles with definite η parity, η, and signature τ.

When the "parity-conserving" amplitudes f^η are expressed in terms of the $f^{j\eta\tau}$ amplitudes and a Sommerfeld–Watson transformation is made, one expects Regge pole contributions of the form

$$\begin{aligned}f_{cd;ab}{}^{\eta}=&\sum_{\tau\alpha}\beta_{cd;ab}{}^{\eta\tau}\left(\frac{1+\tau\exp(-i\pi\alpha')}{(\sin\pi\alpha')}\right)E_{\lambda\mu}{}^{\alpha+}\\&+\sum_{\tau\alpha}\beta_{cd;ab}{}^{-\eta\tau}\left(\frac{1+\tau\exp(-i\pi\alpha')}{(\sin\pi\alpha')}\right)E_{\lambda\mu}{}^{\alpha-},\qquad\text{(A14)}\end{aligned}$$

where $\alpha=\alpha_\tau{}^\eta$, $\alpha'=\alpha-\nu$, and the $E_{\lambda\mu}{}^{j\pm}$ are the continuations of the functions $e_{\lambda\mu}{}^{j\pm}$ to complex j.

C. Glossary

$s=-(A+B)^2=-(a-c)^2$ $t=-(A-C)^2=-(a+b)^2$ $u=-(A-D)^2=-(a-d)^2$	Mandelstam variables written in terms of the four momenta of the s-channel reaction ($AB\to CD$) and the t-channel reaction ($ab\to cd$), respectively
$z_s=\cos\theta_s$ $z_t=\cos\theta_t$	Cosines of the center-of-mass scattering angles in the s and t channels, respectively
m_x, s_x, η_x, λ_x (or x)	Mass, spin, intrinsic parity, and helicity of particle, respectively
$\lambda_t\equiv a-b$ $\mu_t\equiv c-d$ $\lambda_s\equiv\bar{d}-b$ $\mu_s\equiv c-\bar{a}$	The t-channel and s-channel helicity differences in the initial and final states, corresponding to the components of angular momentum along the incoming and outgoing directions in the respective centers of mass

j	Angular momentum
$\nu=(s-u)/4m$	Energy variables used in dispersion relations for πN scattering
ν	Factor equal to 0 or $\frac{1}{2}$ which makes $j-\nu$ integral for physical values of j
$\tau=(-1)^{j-\nu}$	Signature factor [$\tau=+(-)$ for trajectories which can lead to physical particles at values of j such that $(-1)^{j-\nu}=+(-)$]
C, G, I, and P	Possible quantum numbers of particles denoting charge conjugation, G parity, isospin, and parity, respectively
$\eta=P\tau$	The η-parity factor [the symbol σ is sometimes used when working with $O(4)$]
$\alpha(t)$	Trajectory function which describes the location of a Regge pole in the complex j plane
$\beta(t)$	Residue of partial-wave amplitude at the Regge pole described by $\alpha(t)$
$f_{c\bar{a};\bar{d}b}{}^{s}$	Helicity amplitude for s-channel reaction $\bar{d}+b\rightarrow c+\bar{a}$
$f_{cd;ab}{}^{t}$	Helicity amplitude for t-channel reaction $a+b\rightarrow c+d$
$\bar{f}^s$ or $\bar{f}^t$	Bared amplitudes formed by removing the respective half-angle functions from f^s or f^t
f^η	"Parity-conserving" amplitude
F^j	Partial-wave amplitude corresponding to definite angular momentum j
$F^{j\eta}$	Partial-wave amplitude corresponding to definite angular momentum j and η parity, η
$F^{j\eta\tau}$	Partial-wave amplitude corresponding to definite angular momentum j, η parity, and signature τ

A "right signature point" is one at which $(-1)^\alpha=+\tau$

A "wrong signature point" is one at which $(-1)^\alpha=-\tau$

A "sense value of α" is one that $\alpha\geq\max(|\lambda|, |\mu|)$

A "nonsense value of α" is one that $\alpha<\max(|\lambda|, |\mu|)$

An "evasive trajectory" is one whose amplitudes satisfy the $t=0$ constraint equations independent of other trajectories

A "conspiring trajectory" is one whose amplitudes satisfy the $t=0$ constraint equations with the assistance of other trajectories; i.e., their intercepts and residues are related at $t=0$

SR	Sum rule
SCR	Super convergent relation
FESR	Finite energy sum rule
CMSR	Continuous moment sum rule
PCAC	Partially conserved axial current
OPE	One particle exchange

Units for s, t, and u are $(\mathrm{GeV}/c)^2$ or $(\mathrm{GeV})^2$

Units for $d\sigma/dt$ are millibarns per $(\mathrm{GeV}/c)^2$

Units for m are GeV

Units ($\hbar=c=1$)

XVII. REACTIONS, PRINCIPAL TRAJECTORIES, AND REFERENCES

Reaction	*Principal trajectory*	*References*
πP		
$\pi p \to \pi p$	Cut P, P', ρ, ω, ρ'	56, 76, 87, 89, 102, 138, 141, 181, 183, 242, 275, 276, 300, 306, 316, 317, 391, 396, 421, 440, 442, 456, 493, 522, 548
$\pi^- p \to \pi^0 n$	ρ, ρ'	22, 33, 49, 51, 58, 76, 85, 92, 107, 138, 141, 179, 185, 211, 224, 256, 284, 285, 316, 369, 370, 375, 420, 446, 491, 493, 495, 502, 513, 518, 538, 549
$\pi p \to \rho N$	A_2, ω, A_1, π	18, 44, 99, 158, 207, 216, 231, 238, 239, 291, 298, 333, 479, 480, 489, 492, 523
$\to f^0 \Delta^{++}$	π	90, 239, 287
$\to \rho\Delta$	π	16, 44, 52, 76, 216, 231, 239, 287
$\to \eta\Delta$	A_2	44, 51
$\to \pi\Delta$	ρ	51, 391, 412, 461, 523
$\to n\Delta$	ρ, ρ'	298, 333, 428
$\to \omega\Delta$	B, ρ	461, 495
$\to f^0 n$	A_2, π	239, 333, 523
$\to \eta n$	A_2	22, 51, 75, 85, 211, 284, 285, 286, 367, 374, 426, 428, 446, 491, 538
$\to K\Lambda$, $\pi\Sigma$, $\eta\Lambda$, $\pi\Lambda$	K^*, K^{**}	51, 446
$A_2 p$	π	333
Crossover $D(\pi^+ p)$	ρ, cuts	242, 297, 442
NN and $N\bar{N}$		
$pp \to N\Delta$	π	287
$pp \to pp$	P, P', ω, ρ, π, A_2	23, 76, 89, 102, 118, 138, 181, 183, 227, 242, 310, 400, **442**, 443, 463, 470, 539
$p\bar{p} \to p\bar{p}$	P, P', ω, ρ, π, A_2	23, 76, 89, 102, 138, 183, 227, 242, 405, 442, 443, 539
$\bar{p}n \to \bar{p}n$	P, P', ω, ρ, π, A_2	23, 227
$pn \to pn$	P, P', ω, ρ, π, A_2	23, 76
$\bar{p}n \to \bar{p}n$	B, A_2, π, ρ	227
$pn \to np$	B, A_2, π, ρ	22, 46, 116, 227, 239, 311, 478
$p\bar{p} \to \Delta\bar{\Delta}$	K, K^*	216, 287
$p\bar{p} \to \Lambda\Lambda$	π	462
$p\bar{p} \to n\bar{n}$	B, A_2, π	46, 115, 311
$p\bar{p} \to \pi^+\pi^-$	N_α, Δ_δ	74
Crossover $D(pp)$	ω, cuts	22, 82, 286, 391, 422, 442

KN		
$Kp(n) \rightarrow Kp(n)$	P, P', ω, A_2	23, 76, 89, 102, 181, 274, 428
$K^+n \rightarrow K^0p$	A_2, ρ, ρ'	51, 168, 284, 491
$K^-p \rightarrow K^0n$	ρ, A_2	51, 76, 85, 367, 371, 446, 460, 491
$Kp \rightarrow K^*\Delta$	π	239, 287
$Kp \rightarrow K\Delta$	ρ	51, 391, 461
Crossover $D(K^+p)$	ω, cuts	49, 168, 242
γN		
Compton	P	5, 6, 112, 167, 233, 294, 382, 500
$\gamma p \rightarrow \pi^+n(\pi^0p)$	π, B, ω, ρ	6, 16, 22, 26, 27, 35, 38, 97, 109, 115, 118, 159, 162, 189, 191, 192, 195, 210, 216, 219, 289, 291, 292, 297, 334, 400, 436, 469, 478, 529, 533
$\rightarrow VN$	π	26, 118, 160, 196, 273, 382, 436
$\rightarrow K\Lambda, K\Sigma$	K^*	70, 296, 525

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XIX. NOTE ADDED IN PROOF

Since the completion of this review in October 1968 there have been many developments related to the Regge pole model. The most significant of these has been the Veneziano model, but other topics such as duality, multi-Regge bootstrap models, and non-double-counting interference models have created their share of interest. Much of this work has been reviewed by Jackson (1969) at the Lund Conference and we will only briefly discuss some of the developments and mention papers which should be informative.

In the past year the Veneziano model has eclipsed the traditional Regge pole model as a source of new ideas. Its main attraction lies in it being a simple compact expression (Euler beta function) that has crossing symmetry, shows resonant structure and, with the assumption of infinitely rising trajectories, shows asymptotic Regge behavior in all channels and is capable of being generalized to multiparticle processes. The two classic papers on the Veneziano model are the original paper of Veneziano (1968) and that of Lovelace (1968). In the first, Veneziano proposes a supplementary condition on trajectories which predicts slopes and intercepts for the ρ and A_2 trajectories in reasonable agreement with experiment, derives $\pi\pi$ scattering lengths, and demonstrates that FESR's are satisfied. Lovelace recognized that the Veneziano model provided an expression for describing three-particle decays and for continuing external pion masses to zero as required by the Adler-self-consistency condition and other current algebra constraints. More recent work on two-body reactions such as $\pi\pi$ scattering has been done by Abers and Teplitz (1969), Goldberg and Srivastava (1969), Igi (1968), Igi and Storrow (1969), Kawarabayaski, *et al.* (1969), Lovelace (1969), Phillips and Ringland (1969), Roberts and Wagner (1969), Shapiro (1969), Shapiro and Yellin (1968), three-body decays by Jengo and Remiddi (1969), and on current algebra constraints by Ademollo, *et al.* (1969), Arnowitt, *et al.* (1969), and Osborn (1969). The results of these papers and others have been discussed by Jacob (1969) and Yellin (1968, 1969).

The Veneziano model was first generalized to reactions involving five particles (i.e., the five-point function) by Bardakci and Ruegg (1969 I) and Vinasoro (1969) and was studied for particular situations by Bardakci and Ruegg (1969 II), Białas and Pokorski (1969), and Burnett and Schwarz (1969). Extension to N particles was made by Chan and Tsun (1969), Goebel and Sakita (1969), and Hopkinson and Plahte (1969). A general review of the generalized N-particle Veneziano model has been given by Chan (1969).

The merging of field theory concepts and the Veneziano N-particle amplitude has created a great deal of interest in an approach analogous to that in QED, for which the amplitude is written as a product of vertex functions and propagators (Fubini, Gordon, and Veneziano, 1969; Fubini and Veneziano, 1969; and Kikkawa, Sakita, and Virasoro, 1969). Kikkawa, Sakita, and Virasoro propose a perturbative approach in which the Veneziano amplitude plays the role of the Born term and which should preserve crossing symmetry and satisfies unitarity.

There have been several other attempts to unitarize the Veneziano model. In particular, Roberts and Wagner (1969 I and II) and Wagner (1969) use a K-matrix formation with the Veneziano amplitude as the K matrix to obtain $\pi\pi$ scattering results comparable with experiment. Attempts to incorporate the Veneziano amplitude in an N/D approach have been attempted by Atkinson, *et al.* (1969) and Balazs (1969). Unfortunately, these attempts sacrifice crossing symmetry. The new approach by Atkinson and Balazs using the strip approximation overcomes this drawback. Huang (1969) and Martin (1969) have shown by averaging or smearing the Veneziano amplitude that cuts can be introduced to allow the poles to move to the second sheet, and have given suggestions for unitarizing the new amplitudes. This procedure, unfortunately, does not give the normal asymptotic Regge behavior, but this may be the price paid for a model with cuts in the j plane necessary for unitarity.

Interest has continued in the concept of duality in the sense of a relationship between the resonances of one channel and the cross-channel Regge trajectories as first suggested by the Dolin–Horn–Schmid interpretation of FESR's (see Sec. XII.B) and later extended by Schmid (see Sec. XII.C). Many of the recent developments have been reviewed by Jacob (1969) and only a few will be mentioned here. The interpretation and discussion of loops in Argand diagrams has continued (Chin and Kotonski, 1968; Coulter, Ma, and Shaw, 1969; and Schmid, 1969) and it is generally agreed (Horari, 1968) that a loop signifies the existence of a resonance.

The various interpretations of the term duality are discussed in several papers (Childers, 1969; Lichtenberg, Newton, and Predazzi, 1969; Mandula, Weyers, and Zweig, 1969; and Oehme, 1969). Oehme has give support to the concept of duality by proving that an amplitude in the narrow width approximation (i.e., amplitude is meromorphic in s and t) cannot have complete Regge behavior, $(-s)^{\alpha(t)}$, in a given channel without having an infinite number of resonances in the same channel.

Duality has been very successful in correlating the resonance structure in one channel with the degree of exchange degeneracy (Sec. VII.C) of the Regge trajectories in a cross channel. In particular, if there are no nonexotic resonances coupling to a given channel at lower energies, duality implies the imaginary part of the amplitude is zero for all energies. Consequently, the amplitude for either cross channels will receive contributions from only one discontinuity function [see Eq. (VII.4)] and will predict exchange degenerate trajectories in those channels. The argument works similarly in the other direction. The conditions for which this situation leads to approximate exchange degeneracy are discussed by Mandula, Weyers, and Zweig (1969).

Harari (1969) and Rosner (1969) have combined duality with the elementary quark model to give a simple graphical method of determining when the imaginary part of an amplitude is zero. These "duality diagrams" are also useful in connection with unitarized theories (Freund and Rivers, 1969; and Kikkawa, Sakita, and Virasoro, 1969) and with production amplitudes (Fubini, Gordon, and Veneziano, 1969). In particular, Freund and Rivers, in unitarizing an amplitude (i.e., putting duality diagrams together), obtain cuts and a singularity that is built up from nonresonant background much like the Pomeron should according to Harari [see Sec. XI.B and the recent work of Dance and Shaw (1968) and Gilman, Harari, and Zarmi (1968)].

There has been interest in formulating interference models (Sec. XII.A) which avoid the double counting problem. Jengo (1969 I and II) has proved the validity of a generalized interference model and gives an example which does not show duality. Moffatt (1969) has given an interference model for $\pi\pi$ scattering which is able to give the Weinberg scattering lengths and to satisfy the Adler self-consistency condition. Coulter and Shaw (1969) have argued from the behavior of terms in a Veneziano amplitude that one can avoid the problem of double counting by replacing the Euler beta function, which has poles in s and t, by a sum of direct-channel resonances and keeping the asymptotic form of the other contributions. This amounts to replacing the part of a signatured Regge amplitude which contains the exponential by a sum of resonances. This is quite reasonable, since this term results from the contribution of the direct channel [see Eq. (VII.4)], and is the term which generates the Schmid loops. The model thus represents a self-consistent way of adding contributions due to cross-channel Regge trajectories and direct-channel resonances without double counting. In the model the resonances and Regge pole contributions are not independent, since the residue function for the Regge poles can be determined by saturating the FESR's with the resonances. This model overcomes the difficulties of pure resonance or pure Regge pole fits and gives a good fit to πN backward scattering (Sec. IX.A).

The use of absorptive corrections (Sec. XII.E) has become common recently, presumably because of the complexity of conspiracy schemes and the inability to distinguish experimentally between cuts and conspiring trajectories [see Jackson and Quigg (1969)]. The controversy of whether absorptive corrections and unitarity or rescattering corrections have opposite signs (Finkelstein and Jacob, 1968; and Rivers and Saunders, 1968) has been settled by Caneschi (1969) who showed that a consideration of absorptive corrections to inelastic amplitudes effectively changes the sign of the unitarity correction.

An especially interesting paper by Avni and Harari (1969) has considered pion contributions to forward scattering and, except in photoproduction, concluded that structure from pion exchange exists only if there are non-helicity-flip t-channel amplitudes.

With the availability of production data and recent theoretical developments, the multi-Regge model has become increasingly popular. The experimental support for the model has been reviewed by Ranft (1969), while some of the theoretical results have been reviewed by Chan (1969). The most interesting theoretical development has been the multiperipheral bootstrap model of Chew and Pignotti (1968) [see also Caneschi and Pignotti (1969)]. The model, based on unitarity and the multi-Regge hypothesis, yields self-consistency equations for Regge trajectories. A similar model has also been presented by Halliday (1969) and Halliday and Saunders (1969). The role of the Pomeron (Sec. XI.B) in multiperipheral reactions has been studied by Satz (1969), Caneschi and Pignotti (1969), and Chew and Pignotti (1969). The latter two papers found a Pomeron type singularity which is generated by lower-meson trajectories and consequently has a slope similar to that of other trajectories.

This discussion does not come close to including the many developments of the last year, but it should serve as a guide to the current trends of the Regge pole model.

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