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Kapitza Resistance*

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Kapitza resistance (R_K) is the thermal boundary resistance which occurs at the interface when heat flows from a solid into liquid helium. This review attempts a comprehensive presentation of experimental and theoretical knowledge since the discovery of Kapitza resistance in 1941. The experiments discussed and data presented include measurements of R_K at interfaces between liquid ^{4}He and: copper, lead, mercury, tin, indium, nickel, constantan, gold, silver, platinum, tungsten, silicon, quartz, lithium fluoride, and sapphire. The experiments between solids and liquid ^{3}He are also discussed. The treatments include discussion of the dependence of R_K on these variables: temperature, pressure, surface structure and preparation, and elastic properties of the solid. The principal experimental problems are associated with the surface properties, so these are discussed in detail. The principal theoretical discussion is of the acoustic impedance theory, following Khalatnikov and Mazo and Onsager, and the results are compared with the experiments. Modifications of the theory connected with improved matching at the interface, due, for example, to condensed He are also considered. When the theory is applied to interfaces between metals and liquid He, then it must be modified to take into account phonon electron interactions. The theory gives a temperature dependence $R_K \propto T^{-3}$, which is approximately what is experimentally, observed. However, the observed R_K's are usually an order of magnitude or more smaller than theoretical values. The source of the disagreement lies either in a lack of knowledge of the surface physics or in another, dominant, mechanism for thermal energy exchange across the interface. Evidence for each possibility and some suggestions aimed at resolving the discrepancies are offered.

CONTENTS

I. INTRODUCTION

When heat is conducted from a solid into a liquid, the temperature is not continuous at the boundary. Instead there is a small temperature difference (ΔT) across the interface. If the heat flow ($\dot{Q}$) is small, the temperature difference is proportional to it. The ratio, $\Delta T/\dot{Q}$, is effectively a thermal resistance for the boundary, and it is inversely proportional to the interfacial area A. Kapitza resistance (R_K) is the thermal boundary resistance between a solid and liquid helium. It is defined as

$$R_K = A\Delta T/\dot{Q}(\text{cm}^2\,^\circ\text{K/W}). \qquad (1)$$

The units shown are those most commonly used.

Thermal boundary resistance is probably associated with heat flow across all solid–solid as well as all solid–liquid interfaces, but Kapitza resistance is an extremely interesting special case. Generally, thermal boundary resistances are smaller, less well defined, and more difficult to measure than for interfaces between solids and liquid helium.

The study of Kapitza resistance is about 30 years old. Interest in the subject has been quickened recently for two reasons: First is for its fundamental interest in the study of the physics of solids, surfaces, and liquid helium. Second is for its application to recent experiments to attain temperatures of a few millidegrees Kelvin and below.

The phenomenon was discovered by Kapitza in 1941[1] during his classic experimental investigation of superfluidity of He II, the superfluid phase of liquid ^{4}He. In an effort to understand why the apparent thermal conductivity of He II in capillaries could be much larger than the thermal conductivity of bulk He II, Kapitza measured the temperature distribution in the neighborhood of heated metal surfaces freely suspended in He II. He observed, in the range between 1.6°K and T_λ, the lambda point temperature 2.1720°K, a temperature jump between the solid and the He II of the order of 2 m°K for each milliwatt/cm^2 of thermal flux crossing the interface. This thermal boundary resistance decreased with increasing temperature approximately as T^{-3}. From these measurements and from study of the behavior of the discontinuity when the surfaces were surrounded with emery powder, Kapitza deduced that the discontinuity took place within a few hundredths of a millimeter of the interface and not in the bulk helium.

In practice, the Kapitza resistance depends not only on temperature but also on the pressure and on the solid itself. It is also expected to be especially sensitive to the nature of the carriers of thermal energy in the

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solid, the elastic properties of the solid, and the structure of the surface as well as to the properties of liquid helium. We shall find in this study that the observed dependence of Kapitza resistance on most of these variables is surprisingly poorly understood.

This article is intended as a comprehensive review of experimental and theoretical work on Kapitza resistance since its discovery. In order to make the article useful to those with a general as well as those with a special interest in the problem, we have included background material as well as detailed presentation of techniques and results. The first question to be investigated is: Why is Kapitza resistance not better understood? We shall see that the experiments have been carried out on a wide variety of solids: metals, both normal and superconducting, a semiconductor, and insulators, so that the energy in the solids has been carried by phonons, by electrons, by quasiparticles, etc. The character of these excitations in solids can be studied experimentally and theoretically in a great variety of independent ways, and the behavior of excitations in solids at low temperature is now understood in some detail. The energy in the solids is then transferred across an interface into liquid helium, usually He II, another system in which structure and thermal energy transport have been fairly well studied. We shall see that in spite of these advantages, theoretical and experimental values of R_K are generally in disagreement by as much as one and sometimes two orders of magnitude. We will find that the observed R_K's are always less than expected from theory, i.e., there is more energy exchange across the interface than can be presently accounted for.

There is evidence that the heart of the difficulty is in the physics of the interface. Kapitza resistance has been found to vary with the roughness, crystal structure gas adsorption, and surface oxidation of the interface. The interface between a solid and He II is experimentally interesting because intimate thermal contact may be established without strain, the temperature of the liquid may be conveniently measured, and a hydrodynamic pressure may be applied at the interface. For these reasons Kapitza resistance provides a prototype study in surface physics.

The study of Kapitza resistance has, in addition, an important practical application in reaching low temperatures. Most techniques for cooling liquid He below 1°K involve first cooling some solid (for example, a paramagnetic salt or a metal) and then exchanging heat between the solid and, say, liquid He II. This flow of thermal energy from the liquid to the solid is impeded by Kapitza resistance just as the inverse flow would be, so that the He II is, during cooling, a little warmer than the cold solid. The He II may then be put in thermal contact with another solid in order to cool it. This flow of thermal energy is also impeded by Kapitza resistance, so that the second solid is a little warmer than the He II as long as there is thermal flux from the solid to the liquid. Improved understanding of Kapitza resistance would help in manipulating these low temperatures. In this context, a more general problem occurs when a solid is to be cooled by putting it in thermal contact with another, cooler, solid. This flow of energy across the solid–solid interface is also impeded by a thermal boundary resistance, similar in many ways to Kapitza resistance. This phenomenon is quantitatively better understood than Kapitza resistance, but the experimental situation is not as clear. In the solid–solid case surface problems are compounded and the problem of establishing intimate thermal contact between the solids without introducing local strains is important. We shall restrict our discussion to interfaces between solids and liquid helium.

The article is organized this way: Following the Introduction there is a detailed presentation of the experimental work (Sec. II), first for the case of solids and ^{4}He and then for solids and liquid ^{3}He. In this section emphasis has been placed on problems of the surface structure, treatment, and purity. These problems are still at the forefront of present-day research on Kapitza resistance. Section III then treats the theory of Kapitza resistance and its ramifications. In it emphasis has been placed on the interconnections between the theories in an attempt to unite them. This is natural since the theoretical work is, after all, almost all based on Khalatnikov's[2] original theory of Kapitza resistance and relies heavily on his assumptions and results. The main problems at the forefront of present-day theoretical research on Kapitza resistance are either to extend this theory so that it gives an adequate account of the experiments, or else to discover supplementary sources of thermal energy transfer across the boundaries. The final section (Sec. IV) presents the principal conclusions.

Before beginning detailed discussion of the experiments in Sec. II, we shall briefly introduce the main elements of the theories of Kapitza resistance.

The modern theoretical understanding of Kapitza resistance is that it is the result of a large impedance to passage of thermal phonons across the interface between a solid and liquid He. The important quantities in determining the reflection and transmission of phonons at the interface are the acoustic impedances of the two sides, i.e., the product of the density and the sound velocity. A typical solid may have density 5 g/cm^3 and sound velocity 5×10^5 cm/sec. Liquid He at ordinary pressures has a density of 1/7 g/cm^3 and a (first) sound velocity of 2.4×10^4 cm/sec. This means that the acoustic impedance of the solid may be more than two or three orders of magnitude greater than that of liquid He. As a result of this acoustic mismatch, a large fraction of the phonons impinging on the interface from both sides cannot pass through.

Consider a solid in equilibrium with liquid He; both are at the same temperature and there is no net flow of

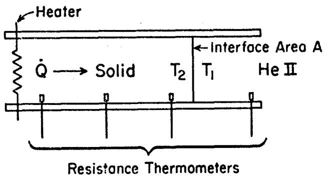

FIG. 1. A schematic diagram of a typical experiment for measuring Kapitza resistance.

heat. If the solid is now heated, the distribution of phonon energies in the entire solid changes and this results in changes in the thermal oscillations of the surface. The energy flux transferred across the interface, i.e., $\dot{Q}/A$ of Eq. (1), is determined by the efficiency with which the component of surface oscillations in the direction normal to the interface transmits energy into the liquid He. In examining the normal surface oscillations of the solid it will be necessary to consider their excitation by three possible sources. These are: longitudinal lattice waves, transverse lattice waves, and surface (Rayleigh) waves. These components of the normal oscillations and their contributions to the energy flux will each be considered in detail in Sec. III.B.1.

Another contribution to the energy flux across the interface comes from collision of excitations of the liquid He with the oscillating wall. The idea is that the excitations, principally phonons and rotons, collide with the wall, pick up energy, and then distribute that energy among the liquid's excitations. This is usually not an important source of energy transfer as we shall see in Sec. III.B.1.

One of our conclusions will be that the amount of energy flux which can be accounted for by this theory is smaller than the energy flux that is actually observed, usually by an order of magnitude or more. In terms of Kapitza resistance this means that observed R_K's are much smaller than calculated R_K's. A good deal of theoretical work has therefore been aimed at improving the acoustic match between the solid and liquid He. One important proposed source of this improved matching is in a dense layer of He adsorbed from the liquid, i.e., a layer of intermediate acoustic impedance. Another proposed source of improved matching is in the diffusion of He atoms into the solid. These refinements on the acoustic mismatch theory are discussed in detail in Sec. III.B.2.

The theory of Kapitza resistance in metals involves the interesting feature, in addition to the above, that the electrons may take part in the thermal energy transfer across the interface. An important possible contribution of electrons, for example, is that the electron–phonon interactions may increase the energy transport from surface waves into the bulk solid. Such surface waves are produced by phonons impinging from the liquid onto the interface, so that this may be an important source of energy transfer. Another important contribution that electrons may make is through direct interactions with the phonons of the liquid. These and other theoretical problems special to the Kapitza resistance between metals and liquid He are considered in Sec. III.B.3.

Unfortunately the review literature on Kapitza resistance is not as rich as the subject deserves. There are, however, several discussions that are particularly interesting of some aspects of Kapitza resistance. These works are all discussed in order in the body of the text, but a short guide to the existing literature may be helpful as introduction. An interesting early review of liquid He with a page-long summary of boundary resistance has been given by Daunt and Smith (Ref. 67) A good quantitative discussion of the theory, above 1°K, may be found in an experimental paper by Johnson and Little (Ref. 8) and, below 1°K, in an experimental paper by Anderson, Connolly, and Wheatley (Ref. 7). A more general treatment appears in a recent article by Frederking (Ref. 14), whose paper gives special discussion of thermal boundary resistance at large heat currents. A brief and clear general treatment, to 1965, is included in Wilks's recent book (Ref. 66) on liquid and solid helium.

II. EXPERIMENTS ON KAPITZA RESISTANCE

A. Interfaces between Solids and Liquid He II and other Phases of ^{4}He

A typical apparatus for determining R_K is shown schematically in Fig. 1. In equilibrium the heater supplies thermal power $\dot{Q}$ through the solid, across the interface area A, to the helium bath at the right. Loss of thermal energy through the sides is minimized by surrounding the solid with a material of low thermal conductivity. Small resistance thermometers are used to measure the temperature profile along the solid, so that the temperature on the solid side of the interface can be obtained by a short extrapolation. Since He II will not support temperature gradients (except at very low temperatures), the temperature on the liquid side of the interface may be measured anywhere in the He II. R_K is then found straightforwardly from Eq. (1) with $\Delta T = T_2 - T_1$.

Some interesting properties of Kapitza resistance are its dependences on temperature, pressure, elastic properties of the solid and liquid, and details of the excitation spectrum of the phonons and electrons in the solid. The most detailed theory of the effect is due to Khalatnikov,[2] who gives the result

$$R_K = \frac{15\hbar^3 \rho_S c_t^3}{16\pi^5 k^4 \rho c_1 F(c_l/c_t) T^3}, \tag{2}$$

in which ρ_S and ρ are, respectively, the densities of the solid and liquid; c_l and c_t are, respectively, the velocities of longitudinal and transverse waves in the solid, F is a specified function of c_l/c_t whose value is usually 1.5–2, and c_1 is the velocity of (first) sound in the liquid.

Equation (2) shows that R_K's principal dependence on the properties of the solid is in the product $\rho_S c_t^3$. This may be related to the atomic mass (M) and the Debye temperature (Θ_D) of the solid by the usual relation

$$\rho_S c_t^3 \approx \rho_S \bar{c}^3 = (4\pi k^3/3h^3) M \Theta_D^3,$$

in which $\bar{c}$ is a special average velocity of transverse and longitudinal waves. Thus Eq. (2) shows that $R_K \propto M\Theta_D^3$ is a reexpression of the theoretical dependence of R_K on the elastic properties of the solid, i.e., on $\rho_S c_t^3$, according to the acoustic mismatch theory.

1. Surface Properties

It will become clear in the experimental discussion that there are severe difficulties in obtaining reproducible data on Kapitza resistance. These difficulties are associated with properties of the surface of the solids. Solid samples which have the same purities and bulk properties may have very different surface properties and may therefore give different Kapitza resistances in essentially identical experiments. It is our lack of knowledge, of how to reproducibly prepare clean, flat surfaces which have properties characteristic of the pure bulk materials, which answers the question: Why is Kapitza resistance not better understood?

We should like here to give a brief qualitative discussion of some of the problems and techniques associated with surfaces in Kapitza resistance problems.

Kapitza resistance is determined experimentally with Eq. (1), and this means that A and ΔT near the interface must both be measured. The measurement of A involves controlling such surface properties as general roughness and asperities. If the phonon mean free paths and wavelengths satisfy certain conditions, to be discussed in detail later, then the area across which thermal energy is exchanged may be much larger than the macroscopic surface area. This will make the measured R_K much smaller than the actual R_K. The measurement of ΔT involves the thermal conductivity and the geometry near the surface. If the surface has been strained, say during cutting to expose a fresh surface, or if impurities are concentrated near the surface then the thermal conductivity may be different from the bulk material, and the measurement of ΔT may be in error. If there is an oxide layer on the surface, as frequently happens on exposure to air, or if impurities have become adsorbed on the interface, then there are two thermal boundary resistances in series between the bulk material and the liquid He, and these must each be separately accounted for. For nonisotropic solids there are also questions of whether the sample is a monocrystal and of its orientation. On the other hand if the grain size is small, or if the density of impurities or other defects is large, then the thermal conductivity may be anomalously small due to phonon scattering.

Kapitza resistance is calculated theoretically with Eq. (2), and this shows that ρ_S and c_t near the interface must both be known. These also depend sensitively on the details of the surface. For example: A surface which has been strained, or the oxide layer on a surface, or a surface with adsorbed impurities, all have locally very different elastic properties from those of the pure bulk material. These must be known and accounted for if the calculation is to correspond to an actual experiment.

The fact is, of course, that all of these factors are never taken into account. The most practical procedure is to make measurements on an experimental sample which has been prepared as carefully as possible, and to compare these measurements with the theory for a perfect surface.

Good present-day experiments start with material that is as pure as possible, say with impurities of 1 part in 10^6. Material of this kind may be prepared, for example, by zone refining a casting. The clean surface is then exposed by cutting or, if possible, by cleaving the pure sample. An improved variation of this technique is to cleave a monocrystalline sample under liquid He so that no impurities have access to the surface between preparation and experiment. Actually, as we shall discuss in detail in Sec. III.B.2, even the layers of He adsorbed on such a sample may have an important effect on R_K.

An ordinary cut sample must be annealed so that the surface material may recrystallize and the strains be removed. This is not difficult and involves the sample being held at elevated temperatures for several hours. Samples with low melting points may even anneal at room temperatures. The surface is then made smooth by careful polishing. In modern experiments this is usually electropolishing. This is intended to minimize the roughness and asperities, and to remove any further impurity layers from the surface. In the most reproducible experiments, to be discussed in Sec. II.A.2, the electropolishing was followed by ion bombardment of the surface to remove residual impurities.

In our subsequent discussions of experiments the actual procedures are to be compared to the ideal ones. However critical intercomparison of these experiments is frequently difficult. The principal sources of the difficulty are that there are large differences in the techniques of preparation of the samples by various experimenters and even larger differences in the detail with which these techniques are described in the original papers themselves. The next difficulty is that the theory is not of significant help in choosing among experiments which ones give the best results. We have attempted, using the general criteria outlined above, to pick out those results which are most characteristic of ideal surfaces, wherever the R_K for a material has been measured more than once.

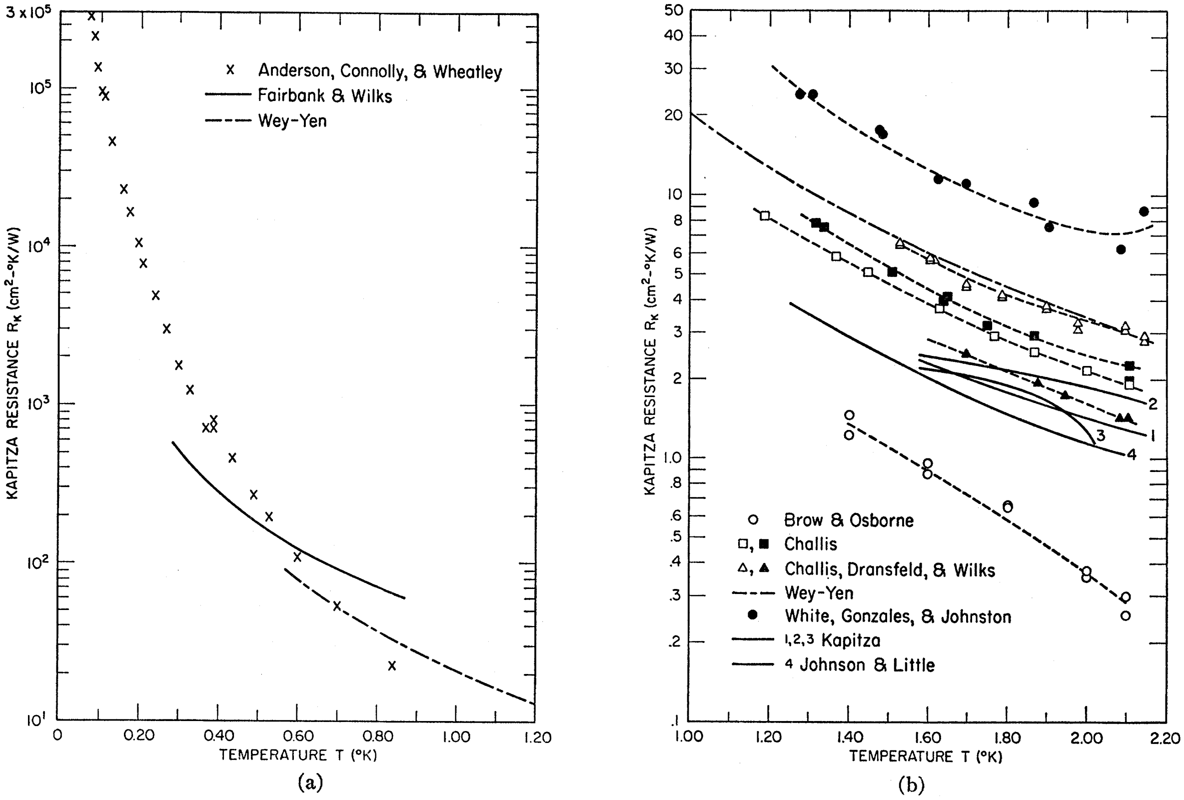

Fig. 2. (a) Experimental data on Kapitza resistance for Cu–He II interfaces from 0° to 1.2°K. These data are taken from Refs. 3, 4, and 7. (b) Experimental data on Kapitza resistance for Cu–He II interfaces from 1.0°K to T_λ. These data are taken from Refs. 1, 4, 8, 9, 12, 13, and 17. The data of Challis include measurements on surfaces of work hardened Cu (□), and of annealed Cu (■). The data of Challis, Dransfeld, and Wilks include measurements on polished Cu (△) and on surface-etched Cu (▲). The solid curves labeled 1, 2, 3 are from Kapitza's original experiments. Curve 1 is for polished Cu, curve 2 is for Cu with a surface layer of varnish, and curve 3 is for Cu surrounded by emery powder. The solid curve labeled 4 is from measurements of Johnson and Little on an oxidized Cu surface.

2. Copper

The solid whose Kapitza resistance has been most thoroughly investigated is copper. The high thermal conductivity of Cu allows the extrapolation to determine temperature on the solid side of the interface to be short and accurate. Results of much of the expermental work on Cu are shown on Figs. 2(a) and 2(b); data on R_K from about 1.2°K and below are on Fig. 2(a), and data from 1°K to T_λ are on Fig. 2(b). Kapitza resistance rises rapidly with decreasing temperature, usually approximately as T^{-3}. However, there are other observed effects of surface structure and purity, so that in the range between 0.10°K and T_λ experimental values for R_K range over six orders of magnitude. In Khalatnikov's theory of Kapitza resistance,[2] R_K is expected to vary as T^{-3} at low temperatures, as Eq. (2) shows.

The earliest experimental data of R_K for Cu–He II interfaces in the region below 1°K come from the measurements of thermal conductivity of He II of Fairbank and Wilks.[3] Below 0.6°K, liquid helium has negligible roton concentration and it behaves like a low-temperature Debye solid whose phonons may scatter only from the boundaries. These workers took concurrent dynamic measurements of thermal conductivity and R_K in a slowly warming system between 0.2° and 1.0°K. Although they did observe a change, due to the increasing roton concentration, in the temperature dependence of thermal conductivity near 0.6°K, there was no corresponding change in R_K. This means that the rotons in the excitation spectrum of He II do not have an important role in the thermal energy exchange at the interface. Their results shown on Fig. 2(a) correspond to $R_K=45.0/T^2$ cm² °K/W. The principal uncertainty stems from determining the actual area of Cu in contact with He II, which is characteristic of Kapitza resistance measurements. A tight fit between the Cu and the German silver tube of low thermal conductivity surrounding it was used to keep He II out of contact with the sides of the specimen (see Fig. 1 for example). If the real contact area is larger than the assumed area, then experimental values for R_K will be too small. The data are in satisfactory agreement with the two other sets of experiments in the lower temperature range, as shown on Fig. 2(a). Here, R_K is apparently rather insensitive to surface details.

Figures 2(a) and 2(b) show the results for Cu–He II interfaces of Wey-Yen's[4] thorough experimental study of Kapitza resistance from about 0.6° to 2.1°K. These data correspond to $R_K = 20.8T^{-2.6\pm0.1}$ cm² °K/W and were taken with thermal fluxes in the range 0.25 to 1.0 mW/cm². In this range ΔT is still proportional to $\dot{Q}$, i.e., R_K is not a function of $\dot{Q}$. Wey-Yen also observed a reverse Kapitza resistance in these experiments. He measured heat transfer from relatively warm He II to cooler Cu to still cooler He II. From the two ΔT's, he obtained the result that at both interfaces ΔT was proportional to $\dot{Q}$ and for both interfaces $R_K \propto T^{-2.3\pm0.3}$ although the proportionality constants were different. This means that, in some sense, the phenomenon is reversible. Beenakker *et al.*[5] earlier reported a preliminary observation that the Kapitza resistance was reversible, but a subsequent search for this reversibility by Andronikashvili and Mirskaia[6] was not successful. Beenakker *et al.*[5] studied R_K at interfaces between Cu and He II from 1.24°K to T_λ as part of a study of thermal conduction in ^{3}He–^{4}He mixtures. The R_K they measured was proportional to $T^{-2.4}$, but this also includes some thermal resistance of the bulk copper in the wall which varies as T^{-1}.

The measurements at lowest temperature and highest Kapitza resistances are those reported by Anderson, Connolly, and Wheatley[7] on 99.999% pure, electropolished Cu from about 0.08° to 0.90°K as a function of temperature and pressure. Their measurements of R_K on interfaces with He II at zero pressure, shown on Fig. 2(a), have a temperature dependence of T^{-4} to $T^{-3.5}$. All the data shown on Fig. 2(a) are in agreement to within about a factor of 3, which represents satisfactory agreement.

Critical comparison of the data on Kapitza resistance for Cu displayed on Fig. 2(a) is comparatively easy. It is fair to say that the results of Anderson, Connolly, and Wheatley[7] are probably the best in this temperature region owing to the high purity and the surface smoothness of their Cu samples. The results of Fairbank and Wilks[3] have the inherent uncertainties associated with dynamic measurements as well as the difficulties, described above, with determining the contact area. Although their Cu is described as high purity, the surface structure was not clearly determined. Wey-Yen[4] unfortunately does not give details of his surface treatment of the Cu and its purity is low by present-day standards.

Data on R_K for Cu–He II interfaces in the range from 1.0°K to T_λ are shown on Fig. 2(b); the data from Kapitza's original work[1] appear as the solid curves labeled 1, 2, and 3. Kapitza studied the temperature distribution around a heated Cu surface in He II using two closely juxtaposed heater thermometers. Curve 1 is for annealed, pure, Cu whose surface had been polished, and curve 2 is for the same surface after it had been varnished. Curve 3 is for a Cu surface immersed in fine emery powder. Since this profoundly influences the hydrodynamics of the surrounding bulk He II and since the energy exchange was not correspondingly altered compared to the earlier measurements, Kapitza concluded that the temperature difference took place very close to the interface. Further investigation has supported this conclusion.

Although Kapitza observed that disturbing the clean surface increased R_K [compare curves 1 and 2 on Fig. 2(b)], the general rule for Cu–He II interfaces is that etching, oxidizing, or otherwise disturbing the surface decreases the observed Kapitza resistance, as evidenced on Fig. 2(b). Varnishing probably makes the surface smoother, i.e., decreases the interface area; this may account for the increased R_K, as Kapitza suggested. For Pb, R_K has recently been characterized better than for Cu. The lowest R_K for Pb–He II interfaces is observed for the cleanest surfaces, just the reverse of the result for Cu.

Johnson and Little[8] and Challis, Dransfeld, and Wilks[9] have observed that clean copper surfaces have larger R_K's than, respectively, oxidized and etched surfaces. Curve 4 of Fig. 2(b) represents data[8] on an oxidized Cu surface. It follows the equation $R_K = 7T^{-2.6}$ cm² °K/W; the corresponding result for a clean Cu surface was $R_K = 19T^{-3.6}$ cm² °K/W. This latter expression would fall roughly between the open squares and the filled triangles of Fig. 2(b) but has not been plotted. The data points of Challis *et al.*[9] on polished Cu are shown in Fig. 2(b) as open triangles, and their data on surface-etched Cu are shown as filled triangles. Although these two sets of data disagree by about 40% on clean surfaces and by somewhat more on oxidized and etched surfaces, the results on clean surfaces are rather distinct from and higher than the results on oxidized or etched surfaces. There is overlap only near T_λ. Unfortunately, the effect on interfacial surface area of etching, oxidizing, or varnishing the interfaces is not known quantitatively, although the dependence of R_K on A is important.

Challis's measurements[10–12] on polished, spectroscopically pure, Cu interfaces with He II support the same conclusion i.e., that undamaged surfaces have the highest Kapitza resistances. He found that R_K for a work-hardened, hammered Cu specimen was slightly smaller than the R_K for the specimen after it had been annealed. The data are shown on Fig. 2(b), respectively, as open and filled squares. White, Gonzales, and Johnston[13] were led to an early investigation of R_K on Cu–He II interfaces in an analysis of thermal conductivity data for He II. Since there can be a significant change in temperature due to Kapitza resistance at the surface of heaters and thermometers in such experiments, the actual thermal gradient applied across the He II during measurement of thermal conductivity may be less than the measured one. Their data, which represent the highest R_K for these surfaces, are shown on Fig. 2(b) and correspond to $R_K \propto T^{-2.6}$.

Frederking[14] has recently reviewed the problem of thermal transport at liquid-He II temperatures with particular emphasis on the difficulties of heat transport across interfaces. This work includes interesting discussions of Kapitza resistance, of solid–solid interfaces, and of local destruction of superfluidity of He II by large heat currents. He has included an interesting plot, analogous to Fig. 2(b) above, of experimental values of the product of Kapitza conductance ($h_K = R_K^{-1}$) and T^{-3} as a function of temperature between 1.2°K and T_λ for copper (Fig. 3 of Ref. 14). On such a figure, deviations from Khalatnikov's expected proportionality between h_K and T^3, or R_K and T^{-3}, become more readily apparent.

In another determination of thermal conductivity, Challis and Wilks,[15] observed indirectly that R_K just above T_λ was apparently about three to six times greater than just below T_λ. However it is now known that this estimate was wrongly based and that their results indicated that R_K undergoes no abrupt change at T_λ. The measurement of R_K at the interface between a solid and He I is difficult since the temperatures on both sides of the interface must be extrapolated and convection in He I may make it difficult to accurately measure the conducted heat. One expects from theory no substantial change in R_K in going through T_λ.

Ambler[16] has investigated R_K for Cu–He II surfaces in connection with the problem of cooling by contact at temperatures below 1°K. He calculates that at 1°–2°K; Kapitza resistance gives the effect of a surface layer of He I about 5×10^{-4} cm thick, in agreement with a similar calculation of White *et al.*[13] However, when extrapolated to very low temperatures, the thermal impedance can become equivalent to a layer ≥ 8 mm thick. There has been support for the idea that a condensed surface layer of helium is in immediate contact with the metal. This is discussed in Sec. III.B.2 below.

All of the measurements of R_K discussed so far have been taken at essentially steady-state heat flows with constant ΔT's of the order of millidegrees Kelvin or larger. Brow and Osborne[17] have measured R_K for Cu–He II interfaces for alternating heat flows of 500 Hz and 2 kHz with temperature differences as small as 4 μ°K. These workers examined the coupling between two second-sound resonators which were separated by a thin Cu foil. Second sound is the thermal wave motion through which thermal energy is transported in liquid He II. The coupling between the chambers was small because of the Kapitza resistance on both sides. The Cu foil was sufficiently thin and thermally diffusive so as to offer no thermal obstacle itself. The condition that must obtain for the thermal boundary resistance to exceed the bulk thermal resistance of the solid is that the solid's thickness be much smaller than the product of its thermal conductivity (λ_S) and R_K,[2] i.e.,

$$\text{thickness} \ll \lambda_S R_K.$$

The results of this experiment, shown on Fig. 2(b), represent the lowest measured values of R_K for Cu. Since the ΔT's were so small, R_K is measured in a region where properties of liquid He are linear and turbulence effects are minimal. These authors suggest that the higher R_K's observed by others may be due to nonlinear and turbulence effects which become important for large temperature gradients in He II. However, the problem of the magnitude of ac thermal boundary resistances as measured by transmission of second sound through foils is still open. Challis and Sherlock[18] have made a measurement on copper, using a technique similar to Brow and Osborne's,[17] but did not observe such a low R_K.

An earlier, more qualitative, study of R_K using second-sound pulses is due to Osborne.[19] Transmission of second sound in He II is analogous to electrical wave propagation on transmission lines[20]: $\dot{Q}/A$ corresponds to current, ΔT corresponds to voltage, and bulk liquid He II has a characteristic impedance $(\rho C c_2)^{-1}$, where C is the specific heat per unit mass and c_2 is the second-sound velocity. Osborne observed evidence for the existence of R_K as a thermal resistance to second-sound pulses at the interface between the solid and He II. From the failure of second-sound pulses to propagate through thin Cu foils, taking into account thermal diffusion in the Cu foil itself as well as the thermal boundary resistance, he concluded that a lower limit for R_K was 0.1 cm^2 °K/W at 1.2°K. The acoustic mismatch theory for the passage of second sound across metal plates has been given by Khalatnikov.[2]

From measurements of the surface absorption of second sound a quantitative estimate of R_K has been obtained in later work of Zinoveva.[21] The absorption coefficient for second sound impinging on a solid may be expressed, neglecting edge effects, as the sum of two terms, a large term due to viscosity of liquid He II and a smaller term due to thermal conduction of all kinds at the solid wall. The presence of a Kapitza resistance at the wall leads to a reduction in the thermal conduction losses. The viscosity contribution is the same at all interfaces between liquid He II and nonporous solids. However, the contribution due to thermal conduction varies with the nature of the solid. This latter term is proportional to $[(R_K/4)+(2/\rho_S C_S \lambda_S \omega)^{1/2}]^{-1}$,[21] in which C_S is the specific heat per unit mass of the solid, λ_S is the thermal conductivity of the solid, and ω is the second-sound angular frequency.

Zinoveva used this expression to analyze data on absorption of second sound at walls of glass and of copper resonators at frequencies between 20 Hz and 2.5 kHz and at temperatures from 1.0° to 2.1°K. The thermal conductivity of glass is so low that the contribution of R_K to the absorption is entirely negligible and indeed viscosity effects alone made up 98% of the absorption of second sound at the glass walls. When second sound impinges on copper, however, the term in $R_K/4$ dominates the denominator of the thermal conduction term. Zinoveva found that the absorption coefficient of second sound on copper walls was 1.5 to 2 times greater than on glass, and the supplementary absorption of copper over that of glass is then due to Kapitza resistance at the interface between Cu and He II. Thus, Zinoveva found that $R_K = 30T^{-3}$ cm^2 °K/W which, as may be seen by comparison with Fig. 2(b), is well within the range of other experiments.

There is convincing experimental evidence that a Kapitza resistance also exists between the superfluid (Rollin) He film and its substrate. Investigation of this point relies on a related experiment which we describe first. Sydoriak and Sherman,[22] in a study of vapor pressure thermometry, observed that the temperature of the liquid in a ^{4}He vapor pressure bulb thermometer could be 10 m°K higher than the temperature of the walls of the bulb. Their explanation of this is that the He film which climbs up the inside walls of the bulb thermometer evaporates when it reaches the warmer upper reaches and then recondenses into the bulb. This steady influx of heat gives rise to a thermal flow $\dot{Q}$ between the liquid in the bulb and the walls and hence to an associated ΔT. (It is now known that this difficulty can be obviated by use of an exchange gas between thermometer and bath.)

It is difficult to determine whether an R_K exists between the He film and its substrate because there is normally negligible thermal energy flow between the film and the substrate. Montgomery and Matthew[23] have indirectly observed the R_K between the film and a Cu substrate by ingeniously using the thermal energy of the recondensing warm vapor just described as a source of $\dot{Q}$. In their experiment heat flows from a ^{4}He-filled bulb into an outside bath via two paths: First, there is thermal flow from the film to the Cu wall to the colder ^{4}He bath, and second, there is a parallel thermal flow from bulk helium to wall to bath. From experiments in which flow through the second path is replaced by flow through the first as the amount of liquid in the bulb is decreased, the R_K between the film and wall was found equal to the R_K between bulk helium and the wall to within 5%. The largest uncertainty in the experiment is in the difficulty of controlling $\dot{Q}$, which is 1 mW or less. The values obtained, $R_K = 14.8$ cm^2 °K/W at 1.53°K and $R_K = 11.5$ cm^2 °K/W at 1.61°K, are high but within the range of the other data shown on Fig. 2(b). It is interesting that in these experiments the heat flow is from warm liquid ^{4}He film to cold wall, i.e., in the opposite direction from the usual case.

In a related experiment Fokkens, Taconis, and de Bruyn Ouboter[24] have obtained data on R_K between the copper walls of an adsorption vessel and unsaturated but still superfluid ^{4}He films, between about 0.8° and 1.7°K. They observed that R_K was proportional to $T^{-2.4}$. Typical results are: at 1.00°K, $R_K = 53$ cm^2 °K/W and at 1.51°K, $R_K = 19$ cm^2 °K/W. These results are in comparatively satisfactory agreement with those of Montgomery and Matthew.[23] The combined data imply that R_K may be larger between a solid wall and a He II film than between the wall and bulk He II.

In a recent experiment, Mate and Sawyer,[25,26] found evidence for a thermal boundary resistance at the interface between Cu and layers of ^{4}He adsorbed from the gas. They measured the temperature difference between two blocks of copper whose interfaces were separated by a gap of 10^{-2} mm while heat flowed from one block into the other through the gap. The gap was filled with helium gas at pressures from 0.01 to 350 Torr. They report data from 1.5° to 4.5°K on the total thermal resistance between the interfaces. At each pressure, measurements were taken down to the saturation temperature.

The total thermal resistance between the Cu interfaces may be written as the sum of three contributions:

$$R(\text{total}) = R(\text{gas}) + 2R(\text{Knudsen}) + 2R(\text{Cu–adsorbed-}^4\text{He interface}).$$

The first term which is the bulk resistance of the gas is known from other experiments. The second term is the Knudsen gas-kinetic temperature jump expected between an adsorbed layer of ^{4}He at each side of the cell and the gas from which it was adsorbed; this term can be calculated and is negligible at the higher pressures. The difference between the observed total resistance and these first two terms is assumed to be associated with the interfaces between the Cu and the adsorbed ^{4}He layers. Mate and Sawer[25,26] found evidence for a Kapitza-like resistance; their inferred R_K's were essentially pressure independent, and the magnitude was $R_K \approx 70T^{-2}$ cm^2 °K/W. This value is rather high compared to the results shown on Fig. 2(b) for Cu–liquid-He II interfaces, but it is within an order of magnitude of the other results. This is a difficult experiment since the width of the gap must be known very well to get a good value for the difference resistance $2R(\text{Cu–adsorbed-}^4\text{He interface}) = 2R_K$. In addition, it is important to know more about the adsorbed ^{4}He layer.

The spread in the observed values of R_K shown in Fig. 2(b) extends over a factor of about 20, even though the individual estimated experimental errors are only 20% or less. The large deviations among the data are probably due to differences in Cu surface structure at

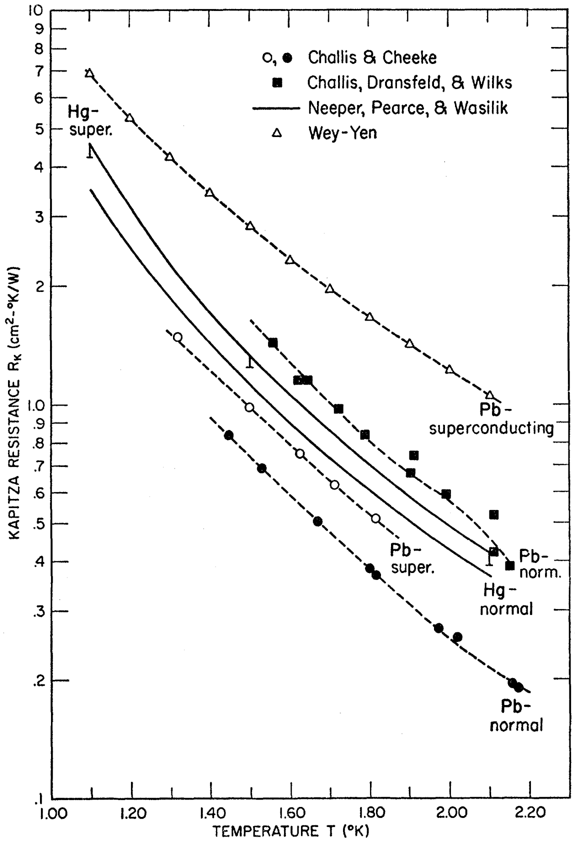

FIG. 3. Experimental data on Kapitza resistance for Pb–He II interfaces and for Hg–He II interfaces. These data are taken from Refs. 4, 9, 29, and 30. The data of Challis and Cheeke represent measurements on normal (●) and superconducting (○), annealed, electropolished, and ion bombarded Pb. The data of Challis, Dransfeld, and Wilks are for normal, surface-etched Pb. The data of Kuang Wey-Yen are for superconducting Pb after electropolishing. The data of Neeper, Pearce, and Dillinger are for Hg with 5 ppm impurity.

the interface. Theory supports the idea that surface structure should strongly influence R_K. To anticipate the discussion in Sec. III, Kapitza resistance is essentially a measure of the efficiency with which the excess thermal energy of the solid is transmitted to the liquid. In Cu and other metals the energy resides in phonons and electrons and these must interact with the He across the interface to generate phonons in the liquid He II. This interaction depends, for example, on the acoustic impedance on both sides of the interface. For surfaces which are oxidized, work hardened, varnished, etc., the acoustic impedance may be very different from that for perfect Cu surfaces.

We have seen in the preceding descriptions of the individual experiments that there are great differences in the surface treatments and in the purities of the Cu samples used. For this reason and also because of the differences in the detail with which these techniques are described by the original researchers, further critical intercomparison of the data is difficult. However, it is important to know which of the data on Fig. 2(b) are the most characteristic of pure Cu with ideal surfaces in the temperature range above 1°K. The most reliable data for this purpose are probably the results of Challis[10–12] on polished, spectroscopically pure, undamaged Cu surfaces, i.e., the data shown by the filled squares. The results of Kapitza[1] for pure, annealed polished Cu probably also give good values for R_K in this range, even though the ΔT was not measured by the extrapolation technique common to present-day experiments. Thus, Curve 1 is a practical experimental lower bound on R_K. The data of Challis *et al.*,[9] shown as open triangles corresponding to measurements on polished Cu, represent a practical upper bound.

3. *Lead and Mercury*

Lead has received the second most attention in studies of Kapitza resistance at interfaces between solids and He II. Representative data on the temperature dependence of R_K for Pb–He II interfaces are shown on Fig. 3, together with data on Hg. Pb serves as serves as a prototype for study of the effect of surface imperfections on R_K and of the effects of the superconducting transition on R_K. The theories of the effect of superconductivity on R_K will be treated quantitatively later. We shall here discuss some qualitative aspects as introduction.

Lead ($\Theta_D \approx 110$°K) and Hg ($\Theta_D \approx 80$°K), compared to other elemental superconductors, have low Debye temperatures and large ratios of T_c/Θ_D, where T_c is the critical temperature for superconductivity. Thus, in the temperature range of the experiments on R_K, the lattice, rather than the electrons, dominates the thermal properties. Kapitza resistance is expected to be larger for substances with high Debye temperatures than for those with low Debye temperatures, according to the acoustic mismatch theories. Pb and Hg behave differently from other superconductors in important respects; in particular the ratio of the electronic specific heat in the superconducting state to that in the normal state is largest for these substances. These two elements must be treated with strong-coupling theory while the other superconducting elements with which we shall be concerned, i.e., tin and indium, can be described with weak-coupling theory.

Thermal energy in any metal is carried by the electrons or by the elementary excitations, mainly phonons, quasiparticles, and plasmons.[27] Any change in R_K as the metal goes through a superconducting transition is due to a change in the properties of the electrons or the excitations of the metal which interact with the elementary excitations of the liquid He II at the interface.[28] Since there are fewer conduction electrons available for interaction at the interface in the superconducting state than in the normal state, the

Kapitza resistance is expected to be larger in the superconducting state. This is indeed generally observed. More precisely, it is the scarcity of quasiparticles, either thermally excited or excited by phonons incident from the liquid and of collective modes in a superconductor that is the cause of the increased R_K.[28] The problem is discussed in more detail in Sec. III.B.3.

The changes in the elementary excitations upon transition are much less marked.[27] Plasmons have energies of the order of 10 eV, so that they are not usually excited in these experiments and play only an indirect role. The phonons have mean energies of the order of 10^{-2} eV. Although many phonons are excited at these temperatures, their energies are changed very little upon the transition from the normal to the superconducting state. For our purposes the most important difference between the excitations in the two states is in the elementary quasiparticles, which may be pictured as electrons above and holes below the Fermi surface. In our subsequent discussion of R_K in superconductors and normal metals, we shall therefore be principally interested in interactions of the electrons and the quasiparticles.[28]

The only data on Kapitza resistance of Hg–He II interfaces are from the recent work of Neeper, Pearce, and Wasilik.[29] They measured R_K between 1.1° and 2.1°K for both superconducting and normal Hg on several pure samples. The nominal impurity was only 1 part in 10^6, and the normal state was obtained at these temperatures by application of a 500-Oe magnetic field. Their results for both states of one of their samples is shown in Fig. 3. The curve for superconducting Hg falls close to but above the curve for normal Hg, as expected. Comparison with the Cu curves on Fig. 2(b) shows that the R_K for Hg is only about 1/5 as large as the average for Cu. This is qualitatively as expected from acoustic mismatch theory since for Cu, $\Theta_D \approx 343°$K and is much larger than Θ_D for Hg. Strong-coupling superconductors show a rapid decrease in thermal conductivity below T_c, so there are rather large uncertainties on the curve for superconducting Hg. The extrapolation to determine the Hg interface temperature is much larger and more uncertain in the superconducting than in the normal state.

The variation of R_K among the different samples was much larger than the uncertainties shown on the Hg curves in Fig. 3; the observed limits on R_K were $2.2/T^3 < R_K < 6.3/T^3$ cm² °K/W. This is not surprising since the samples probably had different grain sizes, orientations, impurities dissolved during the experiment, etc. However, the net effect that R_K is different for the two states and larger for the superconducting state is reproducible and, therefore, probably real. These data indicate that electronic processes contribute about 30% to the energy exchange at an Hg–He II surface.

Until recently, uncontrolled effects of impurities and surface imperfections have made it very difficult to get reproducible data for the temperature dependence of Kapitza resistance of Pb–He II interfaces. A far-reaching study of some of these effects in Pb has been reported by Challis.[10–12] He measured $R_K(T)$ of normally conducting Pb, in an 800-Oe magnetic field between 1.2° and 2°K, for samples of slightly different purities, near 99.995%. His data showed that R_K varied by a factor of 4 among the samples independent of the purity of the samples. Challis also studied the effect on R_K of the superconducting transition for his purest Pb specimen. He found that R_K was considerably higher and had a stronger temperature dependence for the superconducting state than for the normal state, as expected. A rather sizable correction must be made in determining the interface temperature for superconducting Pb, just as in superconducting Hg, because of the low thermal conductivity.

These earlier values are considerably higher than the more reproducible later determinations of R_K for Pb of Challis and Cheeke[30] shown on Fig. 3. The more recent work on Pb has shown that as samples are made purer, annealed, and given cleaner surfaces by electropolishing and ion bombardment, their R_K decreases. It is interesting to notice that this is the opposite of the case for Cu in which, as we have seen, making the surfaces cleaner generally increases R_K. This brings these experimental R_K's further from theoretical values. Also in support of these ideas is an interesting study by Challis[12] of $R_K(T)$ curves of Pb as a function of time of exposure of the surfaces to the atmosphere (i.e., surface oxidation). This showed that R_K inceased by a factor of about 4 for the normal state over several months of exposure to the atmosphere.

Also plotted on Fig. 3 are the data points obtained by Challis, Dransfeld, and Wilks[9] for an interface between surface-etched Pb in the normal state and He II. The observed temperature variation is slightly stronger than T^{-3}. These authors studied $R_K(T)$ curves for several solids in an investigation of whether the theoretical prediction of the acoustic mismatch theory that $R_K \propto M\Theta_D^3$, where M is the molecular weight, was quantitatively correct. According to this result, the Kapitza resistance for Cu should be about 13 times larger than for Pb. The ratio found experimentally using Challis and Cheeke's[30] data for Pb, is about 7. A comparison of the Debye temperatures and atomic weights of Pb and Hg shows that the Kapitza resistance for Pb is expected to be about $2\frac{1}{2}$ times greater than for Hg. Actually, as Fig. 3 shows, the experimental uncertainties are too great to permit even a qualitative comparison.

To see whether the applied magnetic field of 1 kG would significantly affect the Kapitza resistance of normal Pb, Challis *et al.*[9] studied R_K for Cu in fields up to 4 kG. Although they observed no effect on Cu, Challis[10–12] has observed a magnetic field dependence of R_K in Pb. This will be discussed below.

Wey-Yen's work[4] includes an experimental study of the effects of surface roughness, adsorbed gas, and deformed surface layers on the Kapitza resistance of metals. His result, $R_K=9.09T^{-2.9}$ cm^2 °K/W, for pure ($>99.9997\%$), electroplished, superconducting Pb is shown on Fig. 3. The same surface before electropolishing had a Kapitza resistance about one-third larger than this, and when the electropolished surface was subsequently covered with a thick oxide film, its R_K increased again, all in general agreement with other experiments on Pb.

Wey-Yen[4] concludes that the principal cause of the large disagreement between observed and theoretical R_K's is an amorphous surface layer about 10^{-6} cm thick in the solids. This thickness is of the order of magnitude of the wavelength of the acoustic phonons in the experiments. Such a surface layer is formed on metals upon cold working. The deformation disappears gradually as the surface layers recrystallize during annealing; electropolishing also removes the deformed layers.

In support of this conclusion, Wey-Yen[4] showed that R_K for pure Pb could be varied from a maximum of $41.7T^{-3.2}$ cm^2 °K/W for a sample which had been freshly cold worked under the liquid-helium surface, to $R_K=9.43T^{-3}$ cm^2 °K/W for the same sample after it had been left to anneal at room temperature for four months. These effects of annealing and cold working were reproducible. The latter result for R_K is smaller than and closer to theoretical predictions than the former which is greater than theoretical predictions. His idea is that the freshly worked sample has a deep and amorphous deformed surface layer, while the annealed surface is largely recrystallized and hence has acoustic matching properties closer to the bulk material. This recrystallization at room temperature is expected to be rapid in Pb compared to other metals since its melting temperature (601°K) is comparatively low.

Challis and Cheeke[30] have continued and extended the work of Wey-Yen to normal, as well as superconducting, Pb and have succeeded in obtaining reproducible results for R_K for different Pb samples of the same purity but of different initial condition. Typical data of these workers are shown on Fig. 3 as open and closed circles for, superconducting and normal Pb, respectively, in the range 1.3°K–T_λ. These curves probably represent the best results currently available for comparison with theory. The temperature dependence for R_K in both states is about $T^{-3.5}$, and the ratio of R_K in the superconducting to that in the normal state is about 4/3. This compares with a ratio of 3 reported by Barnes and Dillinger[31] as an incidental result of their study of thermal resistances at solid–solid boundaries.

The surface treatment Challis and Cheeke[30] have developed, which leaves Pb specimens with surfaces that are reproducibly characteristic of the bulk material is to: anneal, then lightly electropolish, and finally, heavily bombard the surfaces with argon ions. Electropolishing can result in a surface layer that is chemically distinct from the bulk material. The ion bombardment is aimed at removing this and any oxide layer that may have formed during accidental exposure. Experiments in the important direction of determining the effects of purity, surface structure, and annealing on the R_K for normal and superconducting Pb have been continued by Cheeke.[32]

These workers[12,30] have also studied the dependence of R_K for Pb on applied magnetic fields up to 7 kOe. Typically, if the specimen is initially in the superconducting state at zero field, R_K stays unchanged until the field reaches $H_c/2$, where H_c is the critical field. As the applied field increases from $H_c/2$ to H_c, R_K rapidly falls to the value characteristic of the normal state. In the normal state, there is a small, negative magnetoboundary resistance. A new feature in these data is an interesting hysteretic effect: When the applied field is subsequently decreased to zero, R_K of the superconducting state is about 15% less than originally.

4. *Other Metals*

Experiments on R_K for metals in their superconducting and normal states provide tests of a theory of Little[28] dealing with the coupling between the electrons of the metal and the phonons of the He. The experimental results qualitatively support Little's conclusion that the R_K for the superconducting state should be greater than for the normal state. The idea is that the interaction with the phonons occurs through the quasiparticles in the superconductor, and the energy gap limits the number of quasiparticles. Little's theories of Kapitza resistance will be discussed in Sec. III.B.3 of this paper.

Figure 4 shows experimental results of Kapitza resistance as a function of temperature for two weak-coupling superconductors, tin and indium, and also for nickel and constantan. The curves labeled 1-Sn and 1-In are results of Gittleman and Bozowski[33,34] fitted to the theoretical expression $R_K=aT^{-3}$. They obtained for superconducting and normal Sn, respectively, $a=6.03$ and 5.48 cm^2 °K^4/W and for superconducting and normal In, respectively, $a=10.1$ and 9.54 cm^2 °K^4/W. The difference between the Kapitza resistance of the superconducting and normal states is small in both cases; in Sn the observed difference is only about 10%, and in In only 6%. This compares with the difference of about 28% observed by Challis and Cheeke in Pb.[30]

As part of a recent determination of the thermal boundary resistance at the solid–solid interface between indium and sapphire, Neeper and Dillinger[35] also measured the Kapitza resistance at the interface between In and He II between 1.1°K and T_λ. Although the In had been zone refined during casting and was spectroscopically pure, these workers found that the observed R_K was quite irreproducible. Their results on two samples, $R_K=6T^{-3}$ and $20T^{-2.7}$ cm °K/W, hold for both normal and superconducting In and bracket the earlier results[33,34] shown on Fig. 4.

Also shown on Fig. 4 are Wey-Yen's results[4] for Kapitza resistance of normal and superconducting, 99.9997% pure, Sn: $R_K=12.3T^{-2.9\pm0.1}$ cm^2 °K/W. He found the Kapitza resistance of the two states to be indistinguishable within the 2% uncertainty of his data, although his results differ considerably from Gittleman and Bozowski[34]. There is insufficient information to tell whether the disagreement is due to different purities, magnetic fields, or surface properties. Wey-Yen observed, however, that electropolishing a Sn surface has no observable effect on R_K. This is presumably because Sn oxidizes slowly and has no significant amorphous surface layer. He also observed that the heat transfer at the Sn–He II boundary was unchanged when the liquid helium was rotated up to 400 rpm.

The result $R_K \propto M\Theta_D^3$ shows that, according to the acoustic mismatch theory, R_K for Sn should be about six times larger than R_K for In. The data of Fig. 4 show roughly, due to the disagreement for Sn, that the Kapitza resistances of Sn and In actually differ by less than a factor of two and may even be the same. This theoretical proportionality also shows that the ratio of R_K for Ni to R_K for In is expected to be about 36, but the data show that the observed ratio is less than 2.

We should like to ask then: Which represents the critically best values for R_K between Sn and liquid He II? The answer is that it is not possible to choose between the data of Wey-Yen[4] and Gittleman and Bozowski.[34] This is because of differences in surface treatments and differences in the detail with which these are described. However, as may be seen from Fig. 4 the results differ only by a factor of 2. As we have come to understand, this is a comparatively minor disagreement.

A result of Wey-Yen[4] that $R_K(T)=16.1T^{-2.9\pm0.1}$ cm^2 °K/W for 99.99% pure unelectropolished Ni between 1.4° and 2.1°K, is also shown on Fig. 4. Upon electropolishing he found a dramatic increase and change in temperature dependence to $R_K=70.4\ T^{-1.5}$ cm^2 °K/W. Wey-Yen suggests that electropolishing may leave the surface with cracks which limit the local flow of He II at the surface, but the situation is unclear.

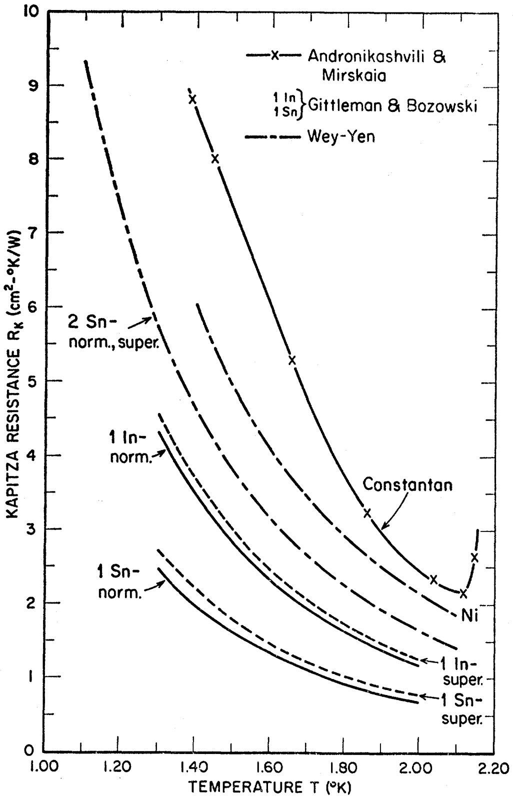

FIG. 4. Experimental data on Kapitza resistance for normal and superconducting Sn–He II interfaces, normal and superconducting In–He II interfaces, and for constantan–He II and Ni–He II interfaces. The solid and broken curves labeled 1 In are taken from data of Ref. 34 on, respectively, normal and superconducting In. The solid and broken curves labeled 1 Sn are taken from data of Ref. 34 on, respectively, normal and superconducting Sn. The curve labeled 2 Sn is taken from data of Ref. 4 on both normal and superconducting Sn. The curve which gives $R_K(T)$ for Ni is taken from data of Ref. 4, and the curve for constantan is taken from data of Ref. 6.

The data that Andronikashivili and Mirskaia[6] have obtained in an unusually interesting experiment are plotted on Fig. 4. This curve represents a lower limit on the Kapitza resistance for constantan; the geometry of the experiment prevented an absolute determination. Their results show that below 1.6°K the dependence of the surface temperature discontinuity, and hence of R_K, is as T^{-3}, and between 1.6° and 2.1°K the dependence is as T^{-4}. The minimum near 2.1°K shown in their data on Fig. 4 may be due to superheating of He II above T_λ. Indeed these workers showed that at the thermal fluxes used in these measurements (about 1 mW/cm^2), superheating could be observed in liquid He II, an interesting result.

Platinum, the original metal used by Kapitza[1] to investigate thermal boundary resistance, is especially suited to measurements of this kind because its surface does not corrode easily. The original data points are shown on Fig. 5. In the range of Kapitza's observations R_K was proportional to T^{-3}. Results of a later investigation of Wey-Yen[4] on technical-purity (99.90%) Pt between about 1.2° and 2.1°K are also shown on Fig. 5. His results follow the curve $R_K=21.3T^{-2.3\pm0.1}$ cm^2 °K/W and fall considerably higher than Kapitza's. Unfortunately, because of the comparative impurity of Wey-Yen's Pt sample, it may have had a large thermal resistance in series with the boundary resistance. This makes comparison between the two sets of data difficult.

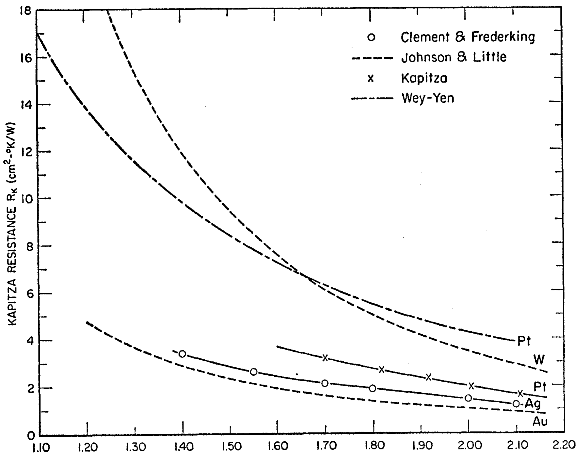

Fig. 5. Experimental data on Kapitza resistance for Pt–He II, Au–He II, W–He II, and Ag–He II interfaces. The data are taken from Refs. 1, 4, 8, and 37.

A critical evaluation of the data on Pt is again difficult. Kapitza,[1] unfortunately, does not describe surface treatment of his Pt. Wey-Yen's[4] Pt specimen was cut from a foil and then annealed at only 500°C, a glass annealing temperature. This means that his surfaces were probably far from ideal. It would be very useful for an experimental determination of R_K to be made on a well-annealed, electropolished, and perhaps ion-bombarded sample of Pt. Present-day standards of Pt purity are such that samples of 99.999% purity are obtainable, i.e., two orders of magnitude purer than Wey-Yen[4] used.

Platinum has been used by Gittleman and Bozowski[34] to test the idea of Bloch and Little[36] that energy can be transferred across the interface by coupling between the phonons of liquid helium and those parts of the electronic wave functions that reach into the liquid helium while decaying. The experiment was to apply a 10^5-V/cm peak-to-peak ac electric field at the surface of the specimen and search for a corresponding ac component in the thermal flow across the boundary. To within 1 part in 10^7, no such component could be observed. As Gittleman and Bozowski[34] note, Pt has a high work function (5.29 eV), and the effect would be small in Pt in any case.

Figure 5 also shows the experimental Kapitza resistances of polycrystalline gold (99.99% pure) and tungsten (99.95% pure) as determined by Johnson and Little.[8] For W the function $R_K = 39T^{-3.5}$ cm² °K/W is plotted, and for Au the function $R_K = 8.0T^{-3}$ cm² °K/W, a mean value, is plotted. In both cases the estimated uncertainty is $7\frac{1}{2}$%. Au is a particularly interesting metal to study because, like Pt, Au forms no stable oxides and its surface is likely to be clean. Therefore, if electron–phonon interactions in the vicinity of the surface are important in determining R_K, the effects should be manifest in Au. W is an interesting metal because according to the acoustic mismatch theory, it is expected to have a very large R_K by virtue of its large $M\Theta_D^3$. In addition, the electron–phonon interaction in W is much stronger than in Au or Cu. The ratio of the R_K's of W and Au is expected to be 13 by the acoustic mismatch theory, and the data show that the actual ratio is about 4.

Clement and Frederking[37,14] have studied the Kapitza resistance of silver from 1.40° to 2.10°K for large heat currents. Their data, connected by a smooth curve, are included on Fig. 5. In these experiments ΔT was taken from about 0.15 to 1.3 K°. Thus, ΔT is commensurate with T; the usual condition $\Delta T \ll T$ does not obtain. For higher ΔT, the thermal flux $\dot{Q}/A$ is no longer proportional to ΔT, i.e., R_K itself becomes a function of ΔT. The data points on Fig. 5 were obtained by an extrapolation which gives R_K for small ΔT's (of about 0.1 K°).

The behavior of thermal boundary resistance between solids and liquid He II as ΔT increases has important technical applications. Particularly interesting from a fundamental view are the boundary resistance when ΔT is so large that the solid's surface temperature is above T_λ, and the boundary resistance for ΔT near the vaporization limit. Particularly interesting for cryogenic technology is the maximum amount of heat that may be transported across the boundary. Work on these problems has been well reviewed recently by Frederking.[14]

5. *Silicon and Insulators*

Figure 6 shows experimental results on the temperature dependence of Kapitza resistance for silicon and for the dielectrics lithium fluoride, sapphire, and quartz. The data on Si[8] were taken on polished and etched (111) interfaces of very pure single crystals. An advantage of studying Si is that samples with different known dislocation densities may be obtained. The curve shown for Si, $R_K = 35T^{-4.15}$ cm² °K/W, is in fact the value obtained for two such samples with surface dislocation densities which differed by a factor of 10^2. This curve may be taken as characteristic of almost perfect Si crystals.

Some of the data of Figs. 4, 5, and 6 have been plotted by Frederking,[14] in a recent review, in the form $h_K T^{-3}$ versus T from about 1.2°K to T_λ (Fig. 2 of Ref. 14). Included on this plot are data of Wey-Yen[4] for Ni both before and after electropolishing.

The results shown on Fig. 6 for LiF,[8] $R_K = 23T^{-3.75}$ cm² °K/W were taken on a characteristically smooth, (100) cleavage plane for a single-crystal sample. LiF is a good insulator and provides a solid in which, at the temperatures of these experiments, thermal energy in the solid resides entirely in the lattice and not in charge carriers. The estimated experimental uncertainty in both Si and LiF is about $7\frac{1}{2}$%.

Johnson[38] has reported a measurement of Kapitza resistance of KCl taken on a crystal which was cleaved under liquid He. This is an effective technique for avoiding surface contamination. His result for a (100) crystal-plane interface was $R_K=12T^{-3.3}$ cm² °K/W in the range 1.2° to 2.1°K.

There are many interesting problems associated with thermal boundary resistance at interfaces between insulators and liquid He II. Besides alkali halides, the rare-gas solids seem especially well suited for such experiments because they are molecular solids with comparatively well understood intermolecular forces and properties.[39] The essential advantage of studying Kapitza resistance at insulator interfaces is that the absence of electrons simplifies the theory (Sec. III). The essential disadvantages of insulators for these purposes are the difficulties in measuring well-defined temperature distributions in the solid when the thermal conductivity is low or, at low temperatures, when the phonon mean free paths are very long. Some insulators may support second-sound[40] (i.e., phonon-density) waves. In that case R_K might be measured by a method analogous to Zinoveva's but with the phonons impinging on the solid side of the interface.

Gittleman and Bozowski[34] have measured Kapitza resistance between 1.3° and 2°K for a single-crystal sapphire rod, another substance with essentially no free electrons at these temperatures. Their result, $R_K=44.1/T^3$ cm² °K/W plotted on Fig. 6, compares with a theoretically expected value of nearly $10^4/T^3$ cm² °K/W.[34]

Two experimental curves for $R_K(T)$ of quartz are also shown on Fig. 6. The higher values are data points of Challis, Dransfeld, and Wilks[9] taken between about 1.4°–2.1°K on surface ground quartz. Their results are from 25% to 65% higher than the results of the experiments of Wey-Yen[4] taken on ground and polished quartz. His result for quartz, $R_K=17.5T^{-3.6\pm0.2}$ cm² °K/W, is also plotted on Fig. 6 for comparison. As may be seen from Fig. 6, the agreement between the two sets of data is good, so that these results may probably be taken as characteristic of quartz with ideal surfaces. More recent experiments on surface properties of dielectrics[38] use the technique of cleaving the specimen under liquid He. This is a technique that might be profitably applied to quartz also. Quartz is a dielectric with a large thermal conductivity, so that the extrapolation to get the temperature on the solid side of the interface is short and convenient. According to the acoustic mismatch theory, the R_K for quartz ($\Theta_D\approx$ 470°K) should be about two and a half times that for copper. A comparison with Fig. 2(b) shows that the R_K's for these substances are approximately equal, taking account of the uncertainty of the data.

Brodie and Mate[41] have reported evidence for a surface thermal boundary resistance between alumina powder and gaseous helium during a study of heat transfer, at helium temperatures, in gas-filled powders as a function of gas pressure, temperature, particle size, and load supported by the powder. They found that at high pressures, say from 1–100 Torr, there was a residual thermal resistance which could not be explained as thermal contact resistance between the particles or as resistance of the interstitial gas. This resistance has features which suggest that it might be a Kapitza-like resistance at the interfaces between the solid and the gas. For example, the resistance is dependent on the size of the particles as a boundary resistance should be, its estimated temperature variation is $T^{-n}(1<n<2)$, and it is independent of pressure. The magnitude of the resistance is given as 2.5±0.5 cm² °K/W at 4.2°K.

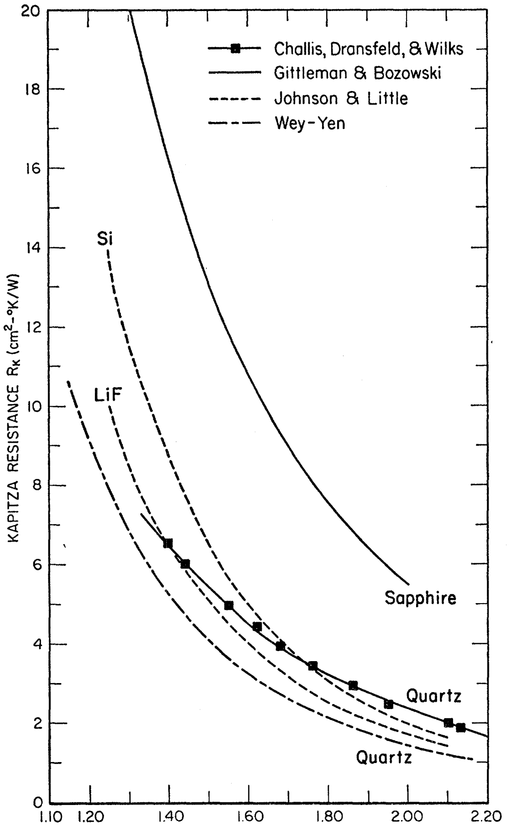

FIG. 6. Experimental data on Kapitza resistance for Quartz–He II, LiF–He II, Si–He II, and Sapphire–He II interfaces. The data are from Refs. 4, 8, 9, and 34.

The Kapitza resistance of a paramagnetic solid has recently been measured by Glättli.[42] For our purposes, a paramagnetic crystal may be considered as a combination of two communicating thermal reservoirs: the spin system and the lattice phonons. Each of these systems can be thermally excited and each has a specific heat, respectively, C_{spin} and C_{lattice}. The rates at which these systems interchange energy with each other and with a bath in which the crystal is immersed may be char-

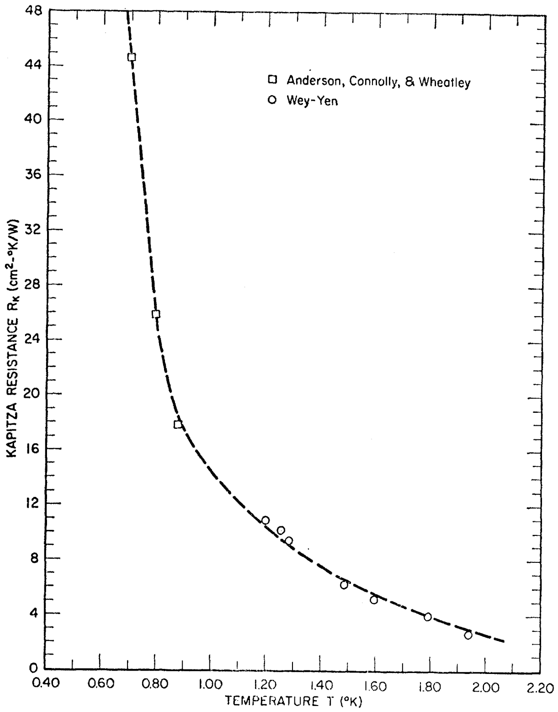

Fig. 7. Experimental data on Kapitza resistance for Cu–He II interfaces at 20 atm pressure. The data are from Refs. 4 and 7. Other experiments on Kapitza resistance at high pressures are reported in Refs. 6, 9, and 46.

acterized by relaxation times in the usual way. If the bath has a spin system of its own, as liquid ^{3}He does, then it may be possible for energy to be efficiently exchanged across the solid–liquid interface between the two spin systems (see Sec. II.B). Thus, the thermal boundary resistance associated with this energy flux is small. If the bath is liquid He II, then excess energy in the spin system goes first into the lattice and then into the bath by interaction, across the interface, of the lattice phonons and the liquid's phonons. With this phonon energy exchange an ordinary Kapitza resistance is expected.

Glättli[42] has used these ideas to measure R_K between cerium ethylsulfate (CeES), a paramagnetic solid, and liquid He II. In this solid $C_{\text{spin}} \gg C_{\text{lattice}}$, and there is a very strong spin–lattice interaction (i.e., T_1 is small). Under these conditions the spin and lattice systems attain equilibrium in a time very short compared to the relaxation time between the lattice phonons and the bath. When excess energy is put into the spin system therefore, temperature equilibrium with the lattice is established rapidly; then, both reservoirs lose energy together to the He II bath via phonon–phonon interaction across the interface. The relaxation for this process is exponential and slow, compared to the usual spin–lattice relaxation, and the relaxation time τ is a few milliseconds. This time τ is simply related to R_K in solids, such as CeES, in which $C_{\text{spin}} \gg C_{\text{lattice}}$ and in which the spin–lattice interaction is strong. The relation is

$$\tau = R_K(\text{heat capacity of solid})/(\text{interface area}).$$

Glättli[42] measured τ as a function of temperature and magnetic field by exciting the spin system to excess temperatures of only about 2 m°K with microwave pulses. Since the spins and lattice are effectively in equilibrium with each other, the spin temperature could be used as the temperature of the lattice as well. The spin temperature was determined sensitively by measurement of the Faraday rotation of the plane of polarization of visible light. This is a measurement of solid temperature that can be made without explicit recourse to the usual temperature extrapolation at the interface. The data for CeES give $R_K = 30T^{-2.4}$ cm² °K/W, between 1.4°K and T_λ. At 1.48°K, R_K was found to be independent of magnetic field, at least up to 7 kOe.

There are also measurements, by Griffiths,[43] of the temperature and magnetic field dependence of τ for PrES, a rare-earth ethylsulfate with related properties. Using the model described and these measurements, Glättli[42] has calculated that $R_K \approx 34T^{-2.4}$ cm² °K/W for PrES at 6 kOe for temperatures between 1.4°K and T_λ. Comparison with experimental value of R_K for other insulators on Fig. 6 shows that these values fall within the right range. A related theoretical analysis of the experimental data for PrES has also been given recently by Atsarkin.[44]

Vilches and Wheatley[45] have estimated the thermal boundary resistances between crystals of two magnetic salts, CrK alum and FeNH$_4$ alum, and liquid ^4He at very low temperatures. The estimates were made from specific heat data on these salts. Their result is $R_K \approx 50T^{-3}$ cm² °K/W for CrK alum in the range $0.017°\text{K} < T < 0.035°\text{K}$ and also the same R_K for higher-temperature points of FeNH$_4$ alum in the range $0.023°\text{K} < T < 0.044°\text{K}$.

6. *Pressure Dependence*

Figure 7 contains data on the Kapitza resistance of Cu–He II interfaces at a pressure of 20 atm. According to the acoustic mismatch theory,[2] the pressure dependence of R_K is principally in the input acoustic impedance of the liquid helium, ρc_1. If ρ is the liquid-helium density and c_1 is the velocity of first sound, then $R_K \propto (\rho c_1)^{-1}$. Measurements have been reported of R_K as a function of pressure up to 28 atm and from 0.3° to 2.0°K. According to theory, the ratio R_K (at saturated vapor pressure)/R_K (at 20 atm pressure)≈ 1.7 in this temperature range. We shall use this ratio as a convenient measure of how well the experiments agree with theory.

The three lowest-temperature data points on Fig. 7 are measurements of Anderson, Connolly, and Wheatley.[7] The data correspond to ratios of R_K from

1.11 to 1.06 in the range 0.7° to 0.9°K. A smooth curve connects these data with the higher-temperature data of Wey-Yen,[4] also shown on Fig. 7. Wey-Yen's results also reveal a much weaker pressure variation of R_K than that predicted by theory, in agreement with the lower-temperature data; some typical results for this ratio are 1.14 at 1.20°K, 1.08 at 1.6°K, and 1.33 at 1.94°K.

These data are all in substantial qualitative agreement with those of a thorough study of the pressure variation of R_K between about 0.3° and 2.0°K of Challis, Dransfeld, and Wilks.[9,46] Their results are not plotted on Fig. 7 since R_K's were obtained in arbitrary units owing to uncertainty in calculating the interface area. Typical values for the ratio of R_K's from their data[9] are 1.2 at 0.5°K, 1.05 at 1.1°K, and 1.0 at 2°K. In general, then, all of the experimental data give ratios of R_K significantly lower than the theoretically expected value of 1.7.

The essential reason for the theoretically expected sensitivity of R_K to pressure is that in liquid helium the elastic properties, in particular the first sound velocity, change rapidly with pressure. If the energy exchange across the interface is principally through phonon–phonon interaction, then the experimental results show that the input impedance of the liquid helium is not changing under pressure as much as expected. This has led Challis, Dransfeld, and Wilks[9] to propose that the input impedance presented to the phonons of the solid is not that characteristic of bulk helium, but rather characteristic of the more dense helium layers near the interface. Recalculation of Kapitza resistance using the new input impedance does indeed lower the theoretical pressure dependence of R_K so as to bring theory and experiment more in agreement. This theory is discussed in more detail in Sec. III.B.2.

Andronikashvili and Mirskaia[6] have examined the Kapitza resistance at a constantan–liquid-helium interface at 8 atm from 1.6°K to just above the lambda transition. They observed below T_λ that the R_K at this pressure is 10–20 times smaller than R_K at the saturated vapor pressure. No explanation as to why these results differ so sharply from other experimental results is available.

Equation (2) shows that in the acoustic mismatch theory the principal dependence of R_K on elastic properties of the solid appears in the product $\rho_S c_t^3$. We have already examined the temperature and pressure dependence of experimental R_K's in some detail. We have also studied the dependence of R_K on elastic properties of the solid by examining the proportionality $R_K \propto M\Theta_D^3$ for several materials.

We have shown several examples of the poor agreement between experimental measurements of R_K and the prediction, of the Khalatnikov acoustic mismatch theory, that R_K should be proportional to Θ_D^3. In a recent letter, Challis[47] has shown by analysis of existing experimental data that R_K has approximately linear proportionality to Θ_D. An important difficulty in finding the dependence of R_K on Θ_D, or on any other property, in this manner is the very wide spread in observed values of R_K for each material. Challis[47] chose the highest observed values of Kapitza conductance h_K, i.e., the lowest values of $R_K = h_K^{-1}$, as most representative of the value for the pure material. His plot of log h_K versus log Θ_D for 14 substances, metals and nonmetals, at 1.5°K showed that the data could be fitted approximately by the curve $R_K = \Theta_D^{0.99}/80$ cm² °K/W. There was no significant difference between metals and nonmetals. This suggests again that there is some mechanism for energy exchange across the interface which predominates over the usual acoustic mismatch energy exchange.

Neeper[48] has similarly examined experimental values of R_K, extrapolated to 1°K, as a function of $M\Theta_D^3$ for several solids. Comparison with the acoustic mismatch theory shows that the R_K of Eq. (2) comes closest to experimental values at low $M\Theta_D^3$, e.g., for Hg, In, and Pb. Even in this region the agreement is only to within an order of magnitude.

It is important to understand, however, that the quantitative dependence of R_K on Θ_D is still not certain. Frederking[14] has plotted values of $h_K T^{-3}$ as a function of Θ_D for several metals and for silicon. The range of Θ_D is from about 70° to 660°K. For each substance, i.e., for each Θ_D, there is wide variation in R_K so that any fit of the data is necessarily approximate. Frederking found that a curve of the form $R_K = 5\times10^{-4}\Theta_D^2 T^{-3}$ cm² °K/W gave an order-of-magnitude fit to the data. However, a curve of the form $R_K = 10T^{-3}$, i.e., entirely independent of Θ_D, could fit the given data as well (Fig. 4 of Ref. 14).

It is also possible to examine the dependence of R_K on elastic properties directly by comparing R_K and $\rho_S c_t^3$ for several materials. If we write $R_K = aT^{-3}$ we expect that $a \propto \rho_S c_t^3$, so that a plot of experimental values of a^{-1} as taken from R_K data *versus* $\rho_S c_t^3$ as known for crystals of each of the materials investigated should give a hyperbola of the form $(a^{-1})(\rho_S c_t^3) =$ const, if the theory is correct. Wey-Yen[4] has done essentially this. He has written $R_K = aT^{-n}$ and plotted a^{-1} *versus* $\rho_S c_t^3$ for his data on Pb, Sn, Pt, Cu, quartz, and Ni, and for Kapitza's[1] data on Pt. His study shows that over a factor of 20 in $\rho_S c_t^3$, a^{-1} is experimentally found to be essentially independent of $\rho_S c_t^3$. This disagreement with theory is somewhat offset by allowing for the difference in elastic constants between the experimental materials and perfect crystals, especially since their surfaces may be deformed. Indeed, Wey-Yen[4] showed that the values of a^{-1} obtained for electropolished Pb and Ni do fall closer to the theoretical hyperbola.

Grassman and Karagounis[49] have studied what happens at the interface between a metal wire and He I

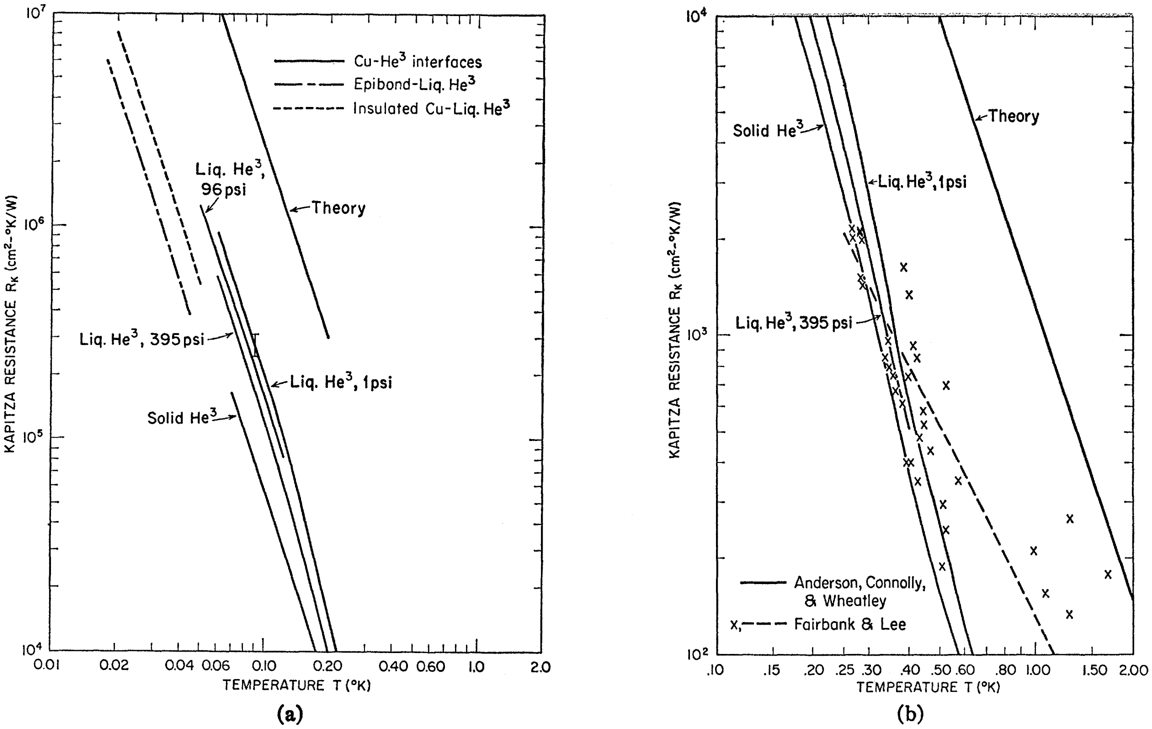

Fig. 8. (a) Experimental data on Kapitza resistance for: Cu–liquid-^{3}He, Cu–solid-^{3}He, Epibond 100-A–liquid-^{3}He, and Formex-insulated-Cu-wire–liquid-^{3}He, interfaces. The data on Cu–liquid-^{3}He and Cu–solid-^{3}He interfaces are taken from Ref. 7, the data on the Epibond 100-A–liquid-^{3}He interface are taken from Ref. 58, and the data on the Formex-insulated-Cu-wire–Liquid-^{3} He interface are taken from Ref. 60. The curve for theory comes from Eq. (3) applied to a Cu–liquid-^{3}He interface, as calculated in Ref. 7. It is discussed in Sec. III.C. (b) Experimental data on Kapitza resistance for Cu–liquid-^{3}He and Cu–solid-^{3}He interfaces. The data represented by the unbroken curves are taken from Ref. 7. The points marked × are data points of Refs. 53 and 54 on Cu–liquid-^{3}He interfaces, and the broken curve is a rough fit to these data. The curve for theory comes from Eq. (2) applied to a Cu–liquid-^{3}He interface, as calculated in Ref. 7. It is discussed in Sec. III.C. [Notice that (a) and (b) are drawn to different scales.]

when the ΔT's and thermal power fluxes become large. For $\Delta T \lesssim 0.6$°K, heat is carried away from the metal surface by free convection of He I. In this convection region $\dot{Q}$ is proportional to ΔT over most of the range of ΔT, and the effective heat transfer resistance across the boundary is 1–10 cm² °K/W. For ΔT's somewhat larger than this a transition region is observed between free convection and film boiling. In the transition region boiling is nucleated at the surface of the wire, and the heat transfer becomes much more efficient than for both smaller or larger ΔT's. Since nucleated boiling is such an efficient way to transfer energy, the transition region has the lowest effective thermal boundary resistance. For very high ΔT's, i.e., $10° < \Delta T < 720°$K, the wire is entirely covered by a film of vapor, the heat transferred is limited by the thermal conductivity of the vapor, and $\dot{Q}$ is again proportional to ΔT. The effective boundary resistance here is of the same order of magnitude as in the convection region.

A more recent study of the maximum dissipation rates of wires immersed in liquid helium has been performed by Frederking and Haben.[50] These workers report details of the experiments and results and give further references to work on related problems.

B. Interfaces Between Solids and Liquid ^{3}He

The first observation of thermal boundary resistance was at an interface between a solid and liquid He II, i.e superfluid ^{4}He. The early theories of the effect were based on the two-fluid model and superfluid hydrodynamics of He II. In 1957, Challis and Wilks[15] were led by an experiment to suggest that a thermal boundary resistance of the same kind might exist at the interface between a solid and liquid ^{3}He. These workers measured the thermal conductivity of liquid ^{3}He between 1.3° and 3.0°K in an apparatus in which the bulk thermal resistance of the ^{3}He was measured in series with two interfaces between Cu and liquid ^{3}He. They noticed that their results[15] fell about 10% below thermal conductivity measurements of Lee, Donnelly, and Fairbank[51] which would have been unaffected by boundary resistance and suggested that the discrepancy was due to thermal resistance at the interfaces of the Cu and liquid ^{3}He.

It is now known that there is a thermal boundary resistance at the interface between a solid and liquid ^{3}He of the same order of magnitude as that at the interface between a solid and liquid He II. Both are defined by Eq. (1) and both will be called R_K here. The results of experiments on the temperature and pressure dependence of R_K between liquid ^{3}He and some solids are shown on Figs. 8(a) and 8(b).

Later theories of Kapitza resistance, in particular those of Khalatnikov,[2] and Bekarevich and Khalatnikov,[52] relate the temperature discontinuity to the acoustic mismatch between the two sides of the interface. In this model a thermal resistance should be found at the boundary between any two substances, in particular at interfaces between solids and liquid ^{3}He and at solid–solid interfaces. Application of this theory to interfaces between solids and liquid ^{3}He at sufficiently low temperatures gives the result[52]

$$R_K = \frac{5h^3 \rho_S c_t^3 m}{8\pi^5 k^4 \rho p_e [aF(c_l/c_t) + b\Phi(c_l/c_t)] T^3}. \tag{3}$$

In this equation, ρ is the density of liquid ^{3}He, m is the atomic mass of ^{3}He, p_e is the momentum of the excitations at the surface of the Fermi sphere, Φ is a function of the ratio of the velocities of longitudinal and transverse waves in the solid, and a and b are constants. Notice that Eq. (3) is very similar to Eq. (2) except that c_l of Eq. (2) is replaced by p_e/m, the Fermi velocity. Equation (3) is derived using the Fermi liquid theory of ^{3}He and may be expected to hold only for $T < 0.2$°K. In this region the thermal energy enters the ^{3}He as zero sound.[52] At higher temperatures the thermal energy is transferred across the interface by phonons, in liquid ^{3}He as well as in liquid ^{4}He, the energy exchange is limited by acoustic mismatch, and the boundary resistance is given by Eq. (2) for both substances.

In this section we shall discuss the experimental work which has been done on thermal boundary resistance in liquid ^{3}He. The relevant theories will be discussed in more detail in Sec. III.C.

The experiments and theories must both be carried through in different ways and give rather different results depending on whether liquid He II or liquid ^{3}He is involved. The principal experimental difficulty is that liquid ^{3}He itself has a thermal resistance due to its finite thermal conductivity, at least at all temperatures investigated so far. This thermal resistance is inevitably in series with the R_K. A typical measurement of R_K in liquid ^{3}He therefore involves an apparatus different from that in Fig. 1; the steady-state temperature gradient in the liquid must be measured with two or more thermometers, and the interface temperature on the liquid side must be extrapolated. Since liquid ^{3}He has both a high specific heat and a low thermal conductivity, the time required to attain equilibrium is long. In addition, the low thermal conductivity of liquid ^{3}He allows some of the thermal energy to flow down the sides of the tube which encloses it, thus changing the thermal gradient and effective interface area. The thermal conductivity of liquid ^{3}He is proportional to T^{-1} below about 0.2°K so that this correction becomes unimportant at low enough temperatures.

The first measurement of thermal boundary resistance for interfaces between a solid and liquid ^{3}He is that of Fairbank and Lee.[53,54] These workers measured $R_K(T)$ for Cu–liquid-^{3}He interfaces from 0.26° to 1.7°K. Their data are plotted on Fig. 8(b) along with the smooth curve $R_K = 130/T^2$ cm^2 °K/W which they obtained as a rough, order-of-magnitude fit. The large scatter in the data points gives an indication of the difficulty of the experiment and the associated uncertainties.

These measurements have been criticized by Jeener and Seidel[55] on the grounds that a large fraction of the heat would flow from the Cu interface down the wall of the containing tube before flowing into the ^{3}He. Since Fairbank and Lee[53,54] used only two thermometers, both in thermal contact with the tube, and since there is also a Kapitza resistance between the tube and the ^{3}He, the temperature distribution in the ^{3}He may be different from that measured. Jeener and Seidel[55] calculated what the temperature distribution in the liquid would be with their corrections and concluded that Fairbank and Lee[53,54] had underestimated R_K by a factor of 3 to 4 at 1°K and 7 to 14 at 0.25°K, and that the actual temperature dependence may be close to T^{-3}, in agreement with theory, instead of T^{-2}. Challis[11] has pointed up the importance of thermal boundary resistance and the associated temperature discontinuity at the soldered interface between the Cu plug and the containing tube. Including this temperature discontinuity in Jeener and Seidel's[55] analysis is difficult, and it is now thought that its inclusion makes their correction small.

A thorough investigation of the temperature and pressure dependence of R_K at interfaces between 99.999% pure, annealed, electropolished Cu and liquid ^{3}He has been carried through by Anderson, Connolly, and Wheatley[7] as part of a search for zero sound. Their curve on Fig. 8(b) represents the best experimental data in this region and is to be compared to the theoretical curve. Their experiment includes many important and interesting features. For example, their measurements are not open to the earlier criticism[55] since the ^{3}He came in thermal contact only with Cu and a nylon post which has negligible thermal conductivity. The interface area was well defined and the ΔT's generally were small, about 2 m °K. For these ΔT's, the thermal boundary resistance was found to be independent of $\dot{Q}$. Several of the results of this investigation are shown as smooth curves on Figs. 8(a) and 8(b).

The curve on Figs. 8(a) and 8(b) showing $R_K(T)$

measured[7] for the interface between Cu and liquid ^{3}He at 1 psi has several interesting properties: below about 0.11°K, $R_K \propto T^{-3}$ to within the experimental uncertainty, shown by the error bar. Above 0.11°K, R_K falls off more rapidly than T^{-3} with increasing temperature. In these experiments, R_K was measured in series with a very thin ($\approx$0.01 cm) layer of liquid ^{3}He, whose bulk thermal resistivity becomes significant for these measurements above 0.35°K. The curve at 1 psi was corrected for this effect, but the other curves could not be so corrected; thus, the results for R_K become imprecise in the neighborhood of, say, 0.50°K. Comparison of the 1-psi curve with the data points of Fairbank and Lee[53,54] on Fig. 8(b) shows that the agreement is satisfactory from about 0.3° to 0.5°K. Above 0.5°K, the latter R_K's are larger.

The temperature dependence of R_K between Cu and liquid ^{3}He at 96 psi was determined[7] from about 0.055° to 0.12°K, and the results are shown on Fig. 8(a). To within the experimental uncertainty, about the same as at 1 psi, $R_K \propto T^{-3}$ throughout the range investigated. The temperature dependence of R_K between Cu and liquid ^{3}He at 395 psi was measured in the same experiment[7] from about 0.07° to 0.6°K. A smooth curve extending up to 0.40°K through the uncorrected data is shown on Figs. 8(a) and 8(b).

Figures 8(a) and 8(b) also show $R_K(T)$ from 0.07°K measured[7] for the interface between Cu and solid ^{3}He. The solid ^{3}He was formed at 645 psi and was in the bcc phase. Qualitative observations of R_K for liquid and solid ^{3}He under pressure were reported earlier[56] as part of a study of the specific heat of ^{3}He under pressure.

Examination of these curves shows that $R_K(T)$ at the interfaces is proportional to T^{-3} from 0.055° to 0.11°K, independent of pressure. This agrees with the theoretical temperature dependence of Eqs. (2) and (3). An abrupt change in this temperature dependence occurs above 0.11°K. However, this change is probably not associated with production of zero sound.

Although the temperature dependence is in some agreement with theory, the magnitude of R_K for liquid ^{3}He is in poor agreement with theory. The experimental R_K's determined in these experiments for interfaces between Cu and liquid ^{3}He are about 10 times smaller in the low-temperature region than given by Eq. (3) and 60 times smaller in the high-temperature region than given by Eq. (2). Thus, the theories of Khalatnikov[2] and Bekarevich and Khalatnikov[52] appear to describe the experimental situation better as the temperature decreases.

Just as in the case of liquid He II, the theories predict more pressure dependence of thermal boundary resistance than is actually found. Theory, both in the low- and high-temperature regions, gives a ratio of R_K at 1 psi to R_K at 395 psi of about 3, but the experimental ratio is close to 1.6 (Sec. III.C).

In these experiments Anderson *et al.*[7] also found an irreversible pressure effect in R_K for liquid ^{3}He similar to the one reported in their measurements on liquid ^{4}He. When the pressure over the liquid ^{3}He was raised from 1 psi to 395 psi, the R_K decreased. But when the pressure was returned to 1 psi from 395 psi, R_K did not return to quite its original value. The effect was undetectable below 0.3°K in liquid ^{3}He and is probably associated with some irreversible change in the surface.

To the extent that acoustic mismatch theories are correct, studies of R_K give a measure of the acoustic impedance of liquid ^{3}He to thermal phonons. An interesting experimental determination of the impedance of liquid ^{3}He to acoustic phonons of 1000 Mc/sec has been reported by Keen, Matthews, and Wilks,[57] also as part of an attempt to detect zero sound. This frequency is of the order of magnitude of the frequency of thermal phonons at 0.02°K. These workers measured the reflection coefficient for such phonons impinging on the quartz side of a quartz–liquid-^{3}He interface. Since the reflection coefficient r is simply related to the ratio of acoustic impedances by $(1-r)/(1+r) = Z_{\mathrm{He}}/Z_{\mathrm{quartz}}$, they were able to determine Z_{He} in their experimental temperature range from 0.03° to 0.6°K. Their data show that Z_{He}/ρ is about 183 m/sec above 0.09°K with an abrupt rise to about 202 m/sec below 0.09°K.

Included on Fig. 8(a) are curves showing the results of two lower-temperature measurements of R_K at interfaces between solids and liquid ^{3}He. In these experiments the solids were not so well characterized but in both cases $R_K \propto T^{-3}$ was in agreement with the other low-temperature findings and with theory. Anderson, Salinger, Steyert, and Wheatley[58] have measured $R_K(T)$ from about 0.01° to 0.40°K for an epoxy resin, Epibond 100-A, cell wall in contact with liquid ^{3}He. In this experiment, measurements were taken upon cooling and warming the cell, i.e., the heat flux went both ways, from ^{3}He into the solid and from the solid into ^{3}He. They observed that the rate of heat transfer was proportional to the difference in the fourth powers of the temperatures; this means $R_K \propto T^{-3}$. From their data we calculate that $R_K = 34.7/T^3$ cm^2 °K/W for this interface. This is the equation for the curve of their results plotted on Fig. 8(a). Epibond 100-A has a density of 1.17 g/cm^3 and the velocity of sound in it may be estimated as 3×10^5 cm/sec.[58]

Another study of R_K between liquid ^{3}He and Epibond 100-A has been described by Salinger,[59] who measured the total thermal resistance extending across three interfaces in series between liquid ^{3}He and chrome alum. By estimating two solid–solid boundary resistances, i.e., R (Epibond 100-A to Cu) and R (Cu to chrome alum), he was able to get an indirect measure of R_K between liquid ^{3}He and Epibond 100-A at 0.03°K, as a function of pressure up to 32 atm.

Typical results (from Fig. 16 of Ref. 59) are: For $P=0$ atm, $R_K=20/T^3$ cm^2 °K/W, for $P=16$ atm, $R_K=17/T^3$, and for $P=32$ atm, $R_K=7/T^3$. These values are subject to uncertainties due to the estimation as well as to experimental scatter. As part of the same study, the boundary resistance between solid ^{3}He and Epibond 100-A was estimated as $R_K \approx 2/T^3$ cm^2 °K/W near 0.03°K.

The results of measurements of Anderson, Salinger, and Wheatley[60] on $R_K(T)$ from about 0.026° to 0.04°K for interfaces between Formex-insulated Cu wires and liquid ^{3}He are also shown on Fig. 8(a). The result is plotted as $R_K=67/T^3$ cm^2 °K/W.

There have been important qualitative data obtained by Wheatley[61] and Abel *et al.*,[62] of an anomalously low thermal boundary resistance between cerium magnesium nitrate (CMN) and pure liquid ^{3}He, in experiments between about 2 and 15 m°K. These workers noticed that the thermal relaxation time of a mixture of powdered CMN and liquid ^{3}He was anomalously short and the temperature dependence was different from the T^{-5} or T^{-2}, depending on temperature, which is expected if the usual $R_K \propto T^{-3}$ were present at the boundaries. The R_K they obtained for this interface is only qualitative since it involves an estimate of the total interface area, etc.[62] The value at 2 m°K is about 200 times smaller than the extrapolated value of R_K for an epoxy–liquid ^{3}He interface shown on Fig. 8(a). This anomalously low boundary resistance has far reaching practical consequences for low-temperature experiments, and there is much interest in understanding why this R_K is so small. A conjectured answer is that the resistance is so low because energy may be rapidly transferred across the interface between the spin system of the liquid ^{3}He and the spin system of the CMN.[63] This is reasonable for this case for special reasons: First, the heat capacity of CMN at these temperatures results from spin interactions. In addition, recent theories which treat liquid ^{3}He as a nearly ferromagnetic Fermi liquid and in which persistent spin fluctuations in the ^{3}He are important give good qualitative explanations relating to transport properties of liquid ^{3}He.[63,64]

III. THEORIES OF KAPITZA RESISTANCE

A. Theories Based on the Two-Fluid Model of He II

The first theoretical approaches to the problem of Kapitza resistance were in terms of the two-fluid model of He II, since the first measurements[1] of the effect were made between solids and He II. This model had successfully quantitatively explained many other phenomena in He II, such as the Fountain effect and first and second sound.[65] It is still a remarkably accurate model, based on quantum-mechanical first principles, for study of properties of He II.[66] These theories of Kapitza resistance are therefore of interest for themselves as well as historically, even though they are rather qualitative and do not explain the experiments. They were discussed by Daunt and Smith in an interesting early review of liquid-helium problems.[67]

Since the observation of Kapitza resistance between solids and liquid ^{3}He,[53,54] it has become clear that the special two-fluid properties of He II play only a small or negligible part in Kapitza resistance in He II. A comparison of R_K between copper and He II and between copper and liquid ^{3}He in the temperature range from about 0.1° to 0.6°K at low pressure may be obtained from examination of Figs. 2(a) and 2(b) with Figs. 8(a) and 8(b). This comparison shows that the R_K's for the two cases are the same to well within the experimental uncertainties in this temperature range. Unfortunately, the ^{3}He data are not sufficiently precise to afford good comparison at higher temperatures. It is from measurements at higher temperatures, say above 1.2°K, that more direct knowledge of the two-fluid model contribution can be obtained. Although He II is in a sense pure superfluid below 0.6°K, the long mean free paths for excitations control its thermal conductivity, and the superfluid properties are just not manifest in the same way as at the higher temperatures.

Another good test of the importance of two-fluid effects is an experimental study of the temperature dependence of R_K from below T_λ to above it. In this range the superfluid properties of liquid ^{4}He change dramatically, but the acoustic impedance changes only slightly except in the immediate region of the lambda transition. Precise experimental data of this kind are very difficult to obtain since in He I thermal conductivity, as well as convection currents, must be considered in measuring ΔT.

The theory of Gorter, Taconis, and Beenakker[68] is an analysis of thermal boundary resistance in terms of the two-fluid model under these three assumptions: (1) Thermal energy flow into liquid He II from a heated solid wall can be described as $\dot{Q}/A=-\lambda\nabla T$, in which λ is a thermal conductivity of bulk He II, (2) temperature is continuous at the boundary between the solid and He II, and (3) the interconversion of normal fluid and superfluid components takes place at a finite rate. The idea was that in the steady state some thermal flux $\dot{Q}/A$ enters the He II from the wall which is at temperature $T_0+\Delta T$, compared to the temperature T_0 of the bulk He II far away from the wall. Near the wall superfluid is being converted to normal fluid at a finite rate so that the temperature falls off exponentially, in distance, from $T_0+\Delta T$ to T_0.

The situation is described by two equations. If normal fluid of relatively density ρ_n/ρ in equilibrium at T_0 flows into a region where the temperature is T, and the rate of increase in ρ_n/ρ is taken as $\alpha(T-T_0)$, then the equation for conservation of normal-fluid density in the steady state may be written

$$(\nabla\cdot\rho_n\mathbf{v}_n/\rho)+\alpha(T-T_0)=0. \qquad (4)$$

In this equation α is a rate constant and $\mathbf{v}_n$ is the normal-fluid velocity. The equation for conservation of energy for the two-fluid model under these assumptions may be written

$$\rho TS\mathbf{v}_n-\lambda\nabla T=\text{const}. \qquad (5)$$

The term $\rho TS\mathbf{v}_n$ is the energy flux associated with convective flow of the two fluids, and S is the entropy per gram of bulk He II. Actually the formulation of Gorter *et al.*[68] is in terms of the Gibbs free energy $G(p, T, \rho_n/\rho)$, and in that case the energy conservation equation involves derivatives of G. Equation (5) has been simplified by using[65]

$$\frac{\partial^2 G}{\partial(\rho_n/\rho)\partial T}=-\frac{\partial S}{\partial(\rho_n/\rho)} \quad\text{and}\quad \frac{(\rho_n/\rho)\partial S}{\partial(\rho_n/\rho)}=S.$$

In the case of heat flow from a plane solid surface into a He II bath, Eqs. (4) and (5) are just one-dimensional equations. Let z be the coordinate perpendicular to the wall; then differentiating Eq. (5) with respect to z, neglecting higher-order terms, and eliminating dv_n/dz from Eqs. (4) and (5) gives

$$\lambda(d^2T/dz^2)+(\rho^2TS\alpha/\rho_n)(T-T_0)=0. \qquad (6)$$

The solution for $T(z)$, the steady-state temperature distribution, is then given approximately by

$$T(z)-T_0=\Delta T\exp(-z/\delta), \qquad (7)$$

with

$$\delta=(\rho_n\lambda/\rho^2TS\alpha)^{1/2}. \qquad (8)$$

The T which appears in Eq. (8) is to be taken as an average temperature between T_0 and $T_0+\Delta T$. According to this model, Eqs. (7) and (8) show that the temperature difference falls off exponentially with a characteristic length δ which depends on λ and α.

The Kapitza resistance at a boundary between a solid wall and He II as defined by Eq. (1) becomes

$$R_K=\frac{\Delta T}{-\lambda(dT/dz)_{z=0}}=\frac{\delta}{\lambda}=\left(\frac{\rho_n}{\rho^2TS\alpha\lambda}\right)^{1/2} \qquad (9)$$

and depends on the product $\alpha\lambda$.

It is now known that the assumptions on which this model is based do not describe the real physical situation. Liquid He II in the temperature range of interest will not support a temperature gradient, and heat flow in it normally takes place through second-sound waves instead of by thermal conduction. This means that the thermal conductivity λ cannot be defined in the usual sense.[69] It is also now thought that the temperature is discontinuous at the boundary because of acoustic mismatch between the solid and He II, as we shall discuss in Sec. III.B. In addition, modern experiments[23,24] show that the Kapitza resistance between a solid wall and a He II film is about the same as for bulk He II, so that any characteristic length δ would have to be smaller than the film thicknesses. Starting with Kapitza's original work,[1] direct experimental attempts to find a surface layer in bulk He II in which the temperature difference takes place have not been successful.

The theory of Kronig, Thellung, and Woldringh[70,71] is an analysis of thermal boundary resistance which depends on the viscosity of the normal fluid as well as its thermal conductivity. In the earlier work Kronig and Thellung[70] showed that according to the two-fluid equations, liquid helium could support four kinds of waves near material boundaries. Starting from the equations of motion of normal fluid and of superfluid and the equations of conservation of mass and energy, expressions for the four complex wavenumbers $k(\omega)$ could be found as functions of frequency and properties of He II. The first two wave modes were first and second sound, respectively, and the fourth mode is a viscosity wave that is also found in ordinary liquids. However, their third wave mode becomes important near boundaries and interests us here.

In the very-low-frequency limit and at temperatures for which neither ρ_{super} nor ρ_n are too small, the square of the wavenumber of this interesting mode becomes [Eqs. (42) and (43) of Ref. 70]

$$k^2{}_{\text{Kr-Th}}\approx-\rho_n\rho^2c_2{}^2C_V/\rho_{\text{super}}\eta_n\lambda_n. \qquad (10)$$

In this expression C_V is the specific heat per unit mass at constant volume of bulk He II; λ_n and η_n are, respectively, the thermal conductivity and the viscosity of the normal fluid component; ρ_{super} is the superfluid density. The wavenumber for this case, $k_{\text{Kr-Th}}$, is large in magnitude and pure imaginary, so that any excitation of the mode damps out in a distance of 10^{-5} or 10^{-6} cm.

Waves in the mode described by Eq. (10) are essentially a surface phenomenon and would be a natural energy sink near interfaces. Indeed in subsequent work, Kronig[72] showed that excitation in this mode could be responsible for a temperature discontinuity at the interface when thermal energy at vanishing frequency flows from a solid into He II.

Kronig *et al.*[71] generalized this treatment to take account of the finite conversion rate between normal fluid and superfluid of Gorter *et al.*[68] Their work includes the combined effects of viscosity, thermal conductivity, and a finite relaxation time τ which characterizes the conversion. Since the two-fluid equations in this formulation are in terms of the Gibbs function G, this relaxation time may be elegantly included. However, the new expression corresponding to Eq. (10) for $k^2{}_{\text{Kr-Th}}$ includes partial derivatives of G which must be estimated [Eq. (75) of Ref. 71].

The result for the temperature distribution near the interface is the same as Eq. (7) except that there is a new expression for the characteristic decay length. In the result of this treatment,[71] δ of Eq. (7) is replaced by $(\delta'^2+\delta''^2)^{1/2}$, where

$$\delta'^2=\eta_n\lambda_n \Big/ \rho_n^2\left[\frac{\partial^2 G}{\partial(\rho_n/\rho)\partial T}\right]^2 T \tag{11}$$

and

$$\delta''^2=-\lambda_{nT} \Big/ \rho\left(\frac{\partial^2 G}{\partial T^2}\right) T. \tag{12}$$

The contribution of δ' is due to the excitation of Kronig and Thellung's[70] wave mode of Eq. (10), and the contribution of δ'', which is proportional to $\tau^{1/2}$, arises from the finite relaxation time. From measurements of the frequency and temperature dependence of first-sound velocity and attenuation, τ was estimated to be less than 10^{-12} sec.[71]

B. The Acoustic Impedance Theory for Liquid ^{4}He

1. The Work of Khalatnikov and of Mazo and Onsager

The theories which come closest to quantitatively explaining the experimental measurements of Kapitza resistance, discussed in Sec. II, are those based on the acoustic impedance or acoustic mismatch theory. This explanation was first put forth by Khalatnikov[2] and later, independently, by Mazo and Onsager.[73] It is important to keep in mind that values of R_K calculated from this theory are characteristically higher than experimental R_K's, usually by an order of magnitude.

The idea is easy to explain qualitatively.[74] In a typical solid the velocity of sound is an order of magnitude higher than in liquid He. Therefore, the phonon momenta are very different in the solid from in the liquid He at the same temperature. Phonons impinging on the interface cannot pass freely across since energy and momentum cannot be conserved at arbitrary angles of incidence. The transfer of phonon energy across the interface is thus impeded, and this impedance is reflected in a temperature jump at the interface whenever thermal energy is transported across it. Khalatnikov's original paper[2] is a clear and elegant analysis of this idea, so we examine it in some detail.

Khalatnikov[2] considered a solid at temperature T' in contact with liquid He II at a slightly lower temperature T. Heat exchange between the two can be effected in two ways: (a) Emission of energy of lattice vibrations from the surface of the solid, i.e., radiation of phonons and rotons and (b) transfer of energy upon collision of phonons and rotons with the solid wall. First the energy flux and Kapitza resistance associated with radiation are considered. This part of the problem has also been treated in essentially the same way by Mazo.[73]

Suppose that the interface is the plane $z=0$, that $z>0$ is filled with liquid He II and $z<0$ with some isotropic solid. If the solid surface undergoes small oscillations whose displacement may be described as $u_z(\omega)$, i.e., oscillations perpendicular to the interface and of frequency ω, the rate at which energy will be radiated into the liquid He II is given by[2,73]

$$W(\omega, T)=\rho c_1\int|\dot{u}_z(\omega)|^2\,dA, \tag{13}$$

the integration is over the interface area. Equation (13) applies when the wavelengths of the emitted excitations are small compared to the dimensions of the solid. The corresponding energy flux for a sufficiently large plane interface is then

$$W(\omega, T)/A=\rho c_1|\dot{u}_z(\omega)|^2. \tag{14}$$

The oscillations of the solid surface $\dot{u}_z(\omega)$ are manifestations of normal lattice vibrations of the solid (Debye waves) or phonons which are excited in a broad range of frequencies. At these low temperatures we shall be especially interested in long-wavelength phonons.

The net energy flux from the solid to the liquid radiated over all frequencies, $\dot{Q}_{\mathrm{rad}}/A$, can be calculated directly. Direct calculation of the energy radiated by the liquid into the solid is more difficult but, since the two fluxes are equal at equilibrium (when $T=T'$), we may take the latter flux to be the same function of temperature as the former.

For $T\neq T'$ [2]

$$\dot{Q}_{\mathrm{rad}}/A=W(T')/A-W(T)/A$$
$$=\int\left(\frac{W(\omega, T)}{A}\right)[n(T')-n(T)]\,d\tau_\omega, \tag{15}$$

where the integration is over frequency space, and $n(T)=[\exp(\hbar\omega/kT)-1]^{-1}$ is the Bose–Einstein distribution function for phonons. Equation (15) is in the form of a difference, i.e., energy flux radiated over all frequencies from the solid into the liquid, $W(T')/A$, minus energy flux radiated from the liquid to the solid, $W(T)/A$. Since the radiated fluxes are the same function of temperature, we shall have to calculate the function only once.

The major problem in finding how much energy is radiated into the liquid is then to find the motion of the interface due to lattice vibrations. Khalatnikov[2] and Mazo[73] consider normal oscillations of the surface resulting from three kinds of lattice waves: longitudinal waves impinging on the surface from the solid (which give a displacement u_{zl}), transverse waves with displacement u_{zt}, and surface waves with displacement u_{zs}. Each of these contribution is discussed briefly. Each $|\dot{u}_z|^2$ gives a contribution in the form of the integral in Eq. (15), and from the net thermal flux the R_K may be calculated. We shall be concerned here only with radiation of phonons and neglect radiation of rotons, because the excitation of a roton requires a minimum energy $\Delta/k=8.7°\text{K}$.[66] At He II temperatures, phonons energetic enough to excite rotons are rare.

First, the effect on the solid surface of a longitudinal displacement plane wave which impinges on it from below is considered. In general, both a longitudinal and a transverse plane wave will be produced upon reflection from the surface. If the x–z plane is the plane of incidence, then it is also the plane of reflection and the plane in which the longitudinal and transverse oscillations take place. The full displacement vector of the incident and reflected waves in the solid may be written[2]

$$\mathbf{u}(\mathbf{r},t)=[A_0\mathbf{n}_0\exp(i\mathbf{k}_0\cdot\mathbf{r})+A_l\mathbf{n}_l\times\exp(i\mathbf{k}_l\cdot\mathbf{r})+A_t(\mathbf{n}_t\times\mathbf{e}_y)\times\exp(i\mathbf{k}_t\cdot\mathbf{r})]\exp(-i\omega t). \quad (16)$$

The coefficients A_0, A_l, and A_t are amplitudes, respectively, of the incident longitudinal wave, the reflected longitudinal wave, and the reflected transverse wave. The vectors $\mathbf{n}_0$, $\mathbf{n}_l$, $\mathbf{n}_t$ and $\mathbf{k}_0$, $\mathbf{k}_l$, $\mathbf{k}_t$ are correspondingly unit vectors in the propagation directions and wave vectors. The magnitudes of these wave vectors and the associated angles of incidence (θ_0) and reflection (θ_l and θ_t) are determined by the elastic properties of the solid. If c_l and c_t are defined as the velocities of longitudinal and transverse waves, $k_0=k_l=\omega/c_l$, $k_t=\omega/c_t$ and

$$\theta_0=\theta_l, \quad c_t\sin\theta_0=c_l\sin\theta_t. \quad (17)$$

Since c_t is usually smaller than c_l, the transverse wave is reflected closer to the normal than the longitudinal wave.

The ratios A_l/A_0 and A_t/A_0 are now determined from the boundary conditions. Since liquid He II has a small density ($\rho\approx1/7\ \text{g/cm}^3$) compared to solids, the appropriate boundary conditions may be taken as those for a stress-free interface between a solid and a vacuum. These are that the normal component of displacement and the normal components of the stress are continuous at the interface.[74]

Expressed in terms of the stress tensor σ_{ik}, the stress boundary conditions are $\sigma_{ik}n_k=0$. For our problem this becomes $\sigma_{zz}=\sigma_{xz}=0$.[2] When the boundary conditions are applied to Eqs. (16) and (17), the amplitudes of the reflected longitudinal and transverse waves become[2]

$$A_l=A_0\frac{c_t^2\sin2\theta_t\sin2\theta_0-c_l^2\cos^2 2\theta_t}{c_t^2\sin2\theta_t\sin2\theta_0+c_l^2\cos^2 2\theta_t}, \quad (18)$$

$$A_t=-A_0\frac{2c_lc_t\sin2\theta_0\cos2\theta_t}{c_t^2\sin2\theta_t\sin2\theta_0+c_l^2\cos^2 2\theta_t}. \quad (19)$$

Mazo[73] has used somewhat different boundary conditions. There is a normal stress at the boundary equaling the reaction of the liquid to compression, i.e., if χ is the compressibility of the liquid and $\Delta V/V$ is the relative volume change of the compressed liquid, then $\sigma_{zz}=-\chi^{-1}(\Delta V/V)$. In this formulation the liquid is characterized by its density, compressibility, and velocity of sound waves. The equations of motion of the longitudinal and transverse displacements in the solid then become elegant connections between the displacement vector and the stress and strain tensors. Equations analogous to (18) and (19) are then derived by considering solutions to the equations of motion.

If Eqs. (18) and (19) are substituted back into Eq. (16) to find the normal displacement u_{zl} at the surface, $z=0$, the result is

$$u_{zl}(\omega)=A_0\frac{2c_l^2\cos\theta_0\cos2\theta_t}{c_t^2\sin2\theta_t\sin2\theta_0+c_l^2\cos^2 2\theta_t}\exp(-i\omega t). \quad (20)$$

To make the connection from lattice waves to phonons, Khalatnikov[2] sets the total energy (i.e., twice the kinetic energy) of the incoming plane wave equal to $\hbar\omega$, the phonon energy. Then,

$$\int\rho_S|\dot{u}_l|^2dV_S=\rho_S|A_0|^2\omega^2V_S=\hbar\omega, \quad (21)$$

from which

$$|A_0|=(\hbar/\rho_S\omega V_S)^{1/2}, \quad (22)$$

where the integral is taken over the solid volume.[73]

Finally we have for $|\dot{u}_{zl}(\omega)|$, the absolute magnitude of the normal surface velocity due to longitudinal waves,[2]

$$|\dot{u}_{zl}(\omega)|=\left(\frac{\hbar\omega}{\rho_SV_S}\right)^{1/2}\left|\frac{2c_l^2\cos\theta_0\cos2\theta_t}{c_t^2\sin2\theta_t\sin2\theta_0+c_l^2\cos^2 2\theta_t}\right|. \quad (23)$$

When substitued into Eq. (14), this will give the contribution of longitudinal waves in the solid to $W(\omega,T)/A$.

Transverse displacement plane waves in the solid may be treated in an analogous manner. Since we are concerned only with oscillations in the z direction of the interface, we may confine our interest to transverse waves whose displacements are in the x–z plane, the plane of incidence. As before, we call the amplitude of the incident transverse wave A_0 and the angle of incidence θ_0. Upon reflection there will be generated a longitudinal wave with amplitude A_l at angle θ_l and a reflected transverse wave with amplitude A_t at angle θ_t. The connections between the angles for the incident transverse wave case are

$$\theta_0=\theta_t, \quad c_t\sin\theta_l=c_l\sin\theta_0, \quad (24)$$

analogous to Eq. (17). Since generally $c_t<c_l$, Eq. (24)

may give $\sin\theta_l > 1$ for some $\sin\theta_0$, and hence $\cos\theta_l$ becomes imaginary. This means that the reflected longitudinal wave travels along the interface and is exponentially attenuated. Just as in the longitudinal case, the boundary conditions make determination of A_l/A_0 and A_t/A_0 possible. From these, the displacement at the surface due to transverse waves, $u_{zt}(\omega)$, may be calculated. After quantization in the manner described above we have[2]

$$|\dot{u}_{zt}(\omega)| = \left(\frac{\hbar\omega}{\rho_S V_S}\right)^{1/2} \left|\frac{2c_t^2\cos\theta_0\sin 2\theta_l}{c_t^2\sin 2\theta_l\sin 2\theta_0 + c_l^2\sin^2 2\theta_0}\right|. \qquad (25)$$

The third source of excitations in the solid for radiation of phonons into the liquid is surface displacement waves, i.e., Rayleigh waves. Associated with these are surface displacements in both the longitudinal and transverse directions, i.e., u_{zs} and u_{xs}, a velocity $c_{\text{surf}} = \xi(c_t/c_l)\ c_t$ in which $\xi(c_t/c_l)$ is a function of the velocity ratio,[74] and a wavenumber $k_{\text{surf}} = \omega/c_{\text{surf}}$. The displacement vectors and the propagation properties of these waves have been discussed in detail,[75] so we shall confine our interest to the quantitative results needed to calculate R_K.

The quantization condition for these waves, corresponding to a total energy $\hbar\omega$, becomes

$$\int_0^\infty \rho_S(|\dot{u}_{xs}|^2 + |\dot{u}_{zs}|^2)A\,dz = \hbar\omega. \qquad (26)$$

The boundary conditions are the same as before, $\sigma_{ik}n_k = 0$, and the resultant $|\dot{u}_{zs}(\omega)|$, the magnitude of the normal surface velocity due to surface waves, is[2]

$$|\dot{u}_{zs}(\omega)| = [(k^2_{\text{surf}} - \kappa_t^2)/2k_{\text{surf}}]\,(\hbar\omega/\rho_S A f)^{1/2}. \qquad (27)$$

In Eq. (27) $\kappa_t = \omega(c^{-2}_{\text{surf}} - c_t^{-2})^{1/2}$ and f is a known function of k_{surf}, c_l, and c_t [cf. Eq. (1.24) of Ref. 2].

The effect of surface waves on thermal radiation from a solid into liquid He II has been reexamined by Little,[76] who observed that these make no important contribution, except in special cases. He observed that there are no matrix elements coupling the phonons of a semi-infinite solid to the quantized surface waves of Eqs. (26) and (27) (which he calls epiphonons) when the anharmonicity in the solid is small. When anharmonicity was considered, the contribution of surface waves to the thermal flux was still small.[76] According to this, the term for surface waves in Eq. (28) below should be omitted; the calculated $W(T)$ is therefore reduced by about 30%. In Sec. III.B.3 we shall discuss the coupling between the phonons and surface waves through electrons in normal metals.

To find the net thermal flux $\dot{Q}/A$ from the solid to the liquid, three integrals of the form Eq. (15) must be calculated; in each $W(\omega, T)/A$ is given by Eq. (14) with the appropriate displacement velocities: $|\dot{u}_{zl}(\omega)|$ from Eq. (23), $|\dot{u}_{zt}(\omega)|$ from Eq. (25), and $|\dot{u}_{zs}(\omega)|$ from Eq. (27). The resultant expression must then be integrated over solid angles to account for all possible angles of incidence θ_0 of longitudinal and transverse waves. As remarked above, we need only do this for one part of Eq. (15). Then for $W(T)/A = \int [W(\omega, T)/A]\,n(T)\,d\tau_\omega d\Omega$,

$$\frac{W(T)}{A} = \int \rho c_1[\exp(\hbar\omega/kT) - 1]^{-1}d\omega \times \left[\int |\dot{u}_{zl}|^2 \frac{\omega^2 V_S d\Omega}{2(2\pi c_l)^3} + \int |\dot{u}_{zt}|^2 \frac{\omega^3 V_S d\Omega}{2(2\pi c_t)^3} + |\dot{u}_{zs}|^2 \frac{2\pi\omega A}{(2\pi c_{\text{surf}})^2}\right]. \qquad (28)$$

The terms in [ ] are contributions to the emitted energy, respectively, of longitudinal, transverse, and surface waves, and $d\Omega = 2\pi\sin\theta_0 d\theta_0$.

The integrations over $d\Omega$ are difficult and the net result for the emitted energy may be written

$$\frac{W(T)}{A} = \frac{4\pi^5\rho c_1(kT)^4}{15\rho_S(hc_t)^3}\,F\left(\frac{c_l}{c_t}\right). \qquad (29)$$

$F(c_l/c_t)$ is a known but rather complicated integral expression which is always of order one.[74,77] Khalatnikov[2] has calculated for glass and platinum that $F_{\text{glass}}(c_l/c_t = 1.71) \approx 2.5$ and $F_{\text{Pt}}(c_l/c_t = 2.22) \approx 2.0$; however, his c_l/c_t for Pt is too high.

Equation (29) shows that the energy radiated from one body to the other is proportional to the fourth power of the absolute temperature, an interesting and important result. If the temperature difference $T' - T$ is small, then for the net thermal flux from the solid to the liquid He II[2]

$$\frac{\dot{Q}}{A} = \frac{W(T')}{A} - \frac{W(T)}{A} = \frac{16\pi^5\rho c_1 k(kT)^3}{15\rho_S(hc_t)^3} \times F(c_l/c_t)\,(T' - T). \qquad (30)$$

Equation (30) shows that when the direction of energy flow is reversed, the magnitude of $\dot{Q}/A$ stays essentially the same. This means that the Kapitza boundary resistance is quantitatively reversible in this model.

Finally, the Kapitza resistance due to radiation of phonons is, from Eqs. (1) and (30),[2]

$$R_K^{\text{rad}} = \frac{15h^3\rho_S c_t^3}{16\pi^5 k^4 \rho c_1 F(c_l/c_t)\,T^3}. \qquad (2)$$

This is, in fact, the most important source of R_K according to the acoustic mismatch theory.

Equations (30) and (2) are surprisingly insensitive to the properties of the liquid. The only liquid property that appears is the product ρc_1, and this is not strongly dependent on temperature. The radiated energy and hence the R_K associated with radiation depends very little on the liquid because, for one reason, all the $|\dot{u}_z|^2$ contributions have been calculated for a stress-free interface, i.e., an interface between a solid and a vacuum.[2] Actually, Mazo[73] allowed a normal stress to appear at the interface. His result for R_K is similar to Eq. (2) but does not depend on c_1 [cf. Eq. (1.31) of Ref. 73].

Another reason is that the liquid was only introduced into the problem with Eqs. (13) and (14), in which it was assumed that the oscillating solid surface forces oscillations in the liquid.[2,73] The product ρc_1 in Eqs. (30) and (2) comes directly from Eqs. (13) and (14). A last reason is to be found in the discussion preceding Eq. (15): It has not been necessary to find the energy radiated from the liquid to the solid in terms of the properties of the liquid because of the equilibrium condition. The energy radiated will not change severely as we go through T_λ. If radiated energy were the only contribution to R_K, then we would expect that the same would be true of R_K. The theoretical situation is not clear, however, because there is a contribution to the energy flux due to collisions of the liquid He II excitations with the walls. Above T_λ this contribution is much more difficult to assess because of the more complicated nature of excitations in liquid He I.

In Eqs. (30) and (2) the principal pressure dependence of $\dot{Q}/A$ and of R_K is also contained in the product ρc_1 since liquid He is so much more compressible than typical solids. Therefore, to a good approximation,

$$R_K \text{ (at saturated vapor pressure)}/R_K \text{ (at 20 atm pressure)} = (\rho c_1)_{20\text{ atm}}/(\rho c_1)_{\text{s.v.p.}} = 1.7.$$

This ratio has been used in Sec. II.A.6 for comparisons with Fig. 7 and related data.

We turn now, still following Khalatnikov,[2] to the contribution to the energy flux due to collisions of excitations in the liquid He II with the solid wall. The source of energy transfer which is second in importance to Eq. (30) is $\dot{Q}_{\text{phon}}/A$, the energy flux due to phonons colliding with the oscillating solid which itself produces a phonon field. When phonons and rotons are reflected from the wall, both energy and the component of momentum parallel to the interface are conserved. Since the velocity of sound in the solid is much larger than in liquid He II, for phonons of the same energy the momentum of a phonon in the solid may be neglected compared to the momentum of the incident and reflected He II phonons. Rotons have momenta near $p_0 = 2\times10^{-19}$ g·cm/sec,[66] i.e., larger than liquid-He II phonon momenta, so the solid phonon momenta may also be neglected compared to the roton momenta. This simplifies the calculation.

This calculation of the interaction of phonons and rotons with the oscillating wall holds, strictly speaking, only for the case that the excitation wavelength is large compared to the interatomic spacing in the solid.[2] If this condition is satisfied, then the interface appears locally smooth to the phonons and the angles of incidence and reflection are well defined. If we calculate a typical phonon wavelength in liquid helium as $\lambda = hc_1/3kT$, with $c_1 = 238$ m/sec,[66] we obtain $\lambda = 38$ Å at 1°K and $\lambda = 380$ Å at 0.1°K as wavelengths for phonons important in heat exchange at these temperatures. Making the same calculation for Cu with sound velocity 5000 m/sec, we obtain $\lambda = 800$ Å at 1°K and $\lambda = 8000$ Å at 0.1°K.[7] These wavelengths are all large compared to the lattice spacing. However, typical roton wavelengths are of the order of $h/p_0 = 3$ Å,[78] which is commensurate with the lattice spacing, so care must be used in interpreting the calculation of the interaction of rotons with the phonon field of the wall, as Khalatnikov has pointed out.[2]

The results of Montgomery and Matthew[23] on R_K between Cu and liquid-He II films are consistent with the acoustic mismatch model in this sense: The R_K between a Cu wall and the film is about the same as for bulk He II because the important phonon wavelengths in He II at the temperature of their experiment (25 Å at 1.5°K) are small compared to the experimental film thickness (300 A).[23] The mean free path for phonons is about 1000 Å at 1.5°K[66] so the explanation is not clearcut.

There is also the question, mentioned earlier, of the effect of macroscopic irregularities on the surface. Anderson *et al.*[7] estimated that electropolished Cu surfaces have asperities of several hundred angstroms. Little[79,80] has quantitatively discussed the effect of surface roughness on thermal boundary resistance between two solids, a closely related problem. There are two important cases: If the mean free path of excitations is comparable to the amplitude of the roughness, then the area across which energy is exchanged depends on the energy. For phonons whose wavelengths are large compared to the roughness, the effective area is just the macroscopic interface area. However, for phonons whose wavelenghts are small compared to the roughness, the effective area is the larger microscopic area. Therefore, the area for energy exchange increases with increasing energy,[79] and roughness of the interface may appreciably affect the thermal exchange.

If the mean free path for phonons is greater than the amplitude of the roughness of the interface between a solid and liquid He, then the effect of the roughness on R_K is small. Little's[80] analysis of this problem, for phonons of wavelength comparable to the roughness, takes account of the roughness by modifying the phase of the impinging phonons appropriately. He showed that only those phonons incident upon the solid from the liquid He which are scattered into a critical cone about the local normal can transmit energy across the interface. It is because this cone is so narrow, the sine of its half-angle is c_l/c_t, that the thermal exchange is so little affected by roughness for this case.

An excitation in the liquid He II which finds itself in the phonon field of the oscillating solid wall can absorb energy and undergo transition to a higher energy state.[2] The transition probability for this is

$$w=(2\pi/\hbar)\ |H_{if}|^2. \tag{31}$$

In Eq. (31),

$$H_{if}=\int_0^\infty \psi_f V \psi_i dz$$

is the matrix element for the transition between the initial and final states under the operator

$$V=-\tfrac{1}{2}(\mathbf{p}\dot{u}+\dot{u}\mathbf{p}) \tag{32}$$

for the interaction of the excitation with the phonon field.

Choosing suitably normalized ψ functions for the phonon case,[2,74] Khalatnikov obtained the transition probability of a phonon incident on the solid from the liquid with momentum p at angle θ to a phonon reflected with p_1 at $\theta_1(p\sin\theta \approx p_1 \sin\theta_1)$:

$$w=32p^4\cos^3\theta\ |\dot{u}_z(\omega)|^2/(p_1^2-p^2)^2c_1^2\cos\theta_1. \tag{33}$$

In order to get the net energy flux from the transition probability, Eq. (33) must be multiplied by the field phonon energy $\hbar\omega$ and integrated over all incident momenta and angles and over all oscillation frequencies for the wall; careful account must be taken of the distribution functions for the incident and reflected phonons in the liquid and for the transmitted phonon in the solid. The result must be obtained for each of the three contributions $|\dot{u}_z(\omega)|^2$ corresponding to Eqs. (23), (25), and (27). The net result for the energy flux from the solid to the liquid due to phonon collisions with the wall is[2]

$$\frac{\dot{Q}_{\text{phon}}}{A}=\frac{256\pi^2(kT)^8}{h^6\rho_S c_t^3 c_1^4 T}F\left(\frac{c_l}{c_t}\right)G_{\text{phon}}(T'-T), \tag{34}$$

in which G_{phon} is an integral whose value is approximately 7!/2. The disturbance due to a reflected phonon extends into the solid to a distance of about one-tenth of the phonon wavelength. Surface structure and lattice imperfections this close to the interface will affect the energy exchange.[76]

The ratio of the contribution to energy flux due to phonon radiation [Eq. (30)] to the phonon collision contribution [Eq. (34)] is

$$\frac{(\dot{Q}_{\text{rad}}/A)}{(\dot{Q}_{\text{phon}}/A)}\approx 50T^{-4}. \tag{35}$$

Thus, the radiation contribution is some three times larger than the phonon contribution at T_λ, and the ratio increases rapidly with decreasing temperatures.

By choosing suitably normalized wave functions for roton states, Khalatnikov[2] calculated the transition probability from Eqs. (31) and (32) for an incident roton of momentum p, velocity v, at angle θ to a reflected roton of p_1, v_1, and θ_1. The transition probability for rotons, analogous to Eq. (33) for phonons, is

$$w=\frac{32p^2p_1^2\cos\theta\cos\theta_1\,|\dot{u}_z(\omega)|^2}{(p_1^2-p^2)^2vv_1}. \tag{36}$$

The net energy flux due to roton collisions with the walls is obtained, analogously with the phonon case, by multiplication by $\hbar\omega$, integration over p, θ, and ω, use of the appropriate distribution functions, and the summation of contributions due to $\dot{u}_{zl}$, $\dot{u}_{zt}$, and $\dot{u}_{zs}$. The result is[2]

$$\frac{\dot{Q}_{\text{rot}}}{A}=\frac{16\pi^2k(T)^4p_0^4\exp(-\Delta/kT)}{h^6\rho_S c_t^3 T}\times F(c_l/c_t)G_{\text{rot}}(T'-T), \tag{37}$$

in which G_{rot} is an integral whose value is approximately 50.

The ratio of the contribution to energy flux due to phonon collisions [Eq. (34)] to the roton collisions contribution [Eq. (37)] is

$$\frac{(\dot{Q}_{\text{phon}}/A)}{(\dot{Q}_{\text{rot}}/A)}\approx 8\times10^{-4}T^4\exp(\Delta/kT). \tag{38}$$

Examination shows that the phonon and roton contributions are approximately equally important at 2°K, but the ratio increases to 10 at 1°K.

We conclude then that according to the acoustic mismatch theory, the energy exchange between a solid body and liquid He II is principally the result of radiation (and absorption) of phonons at the interface. The thermal flux given by Eq. (30) is proportional to the temperature difference, in agreement with experiment. R_K of Eq. (2) is therefore the Kapitza resistance according to Khalatnikov's acoustic mismatch theory. According to Eq. (2) $R_K \propto T^{-3}$, in substantial agreement with experimental temperature dependences.

On Fig. 9, R_K given by Eq. (2) is plotted for several solids using appropriate values of c_l, c_t, ρ_S, and $F(c_l/c_t)$.

Comparison of the theoretically predicted R_K's of Fig. 9 with the experimentally determined R_K's of Figs. 2(a) through (6) shows that the agreement is only qualitative. The theoretical R_K's are about an order of magnitude or more larger than measured R_K's. For Pb the disagreement is less than this, as discussed in Sec. II.A.3 and Ref. 30.

There are several possible sources of error in the experiments, e.g., surface roughness, parasitic thermal fluxes, and the unknown elastic properties of surface

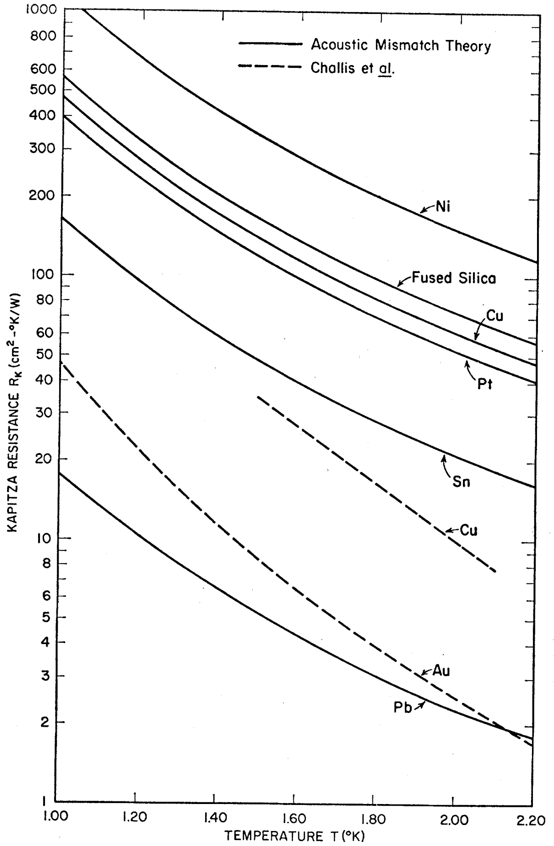

Fig. 9. Theoretical curves for Kapitza resistance as a function of temperature for interfaces of Pb, Sn, Pt, Cu, fused silica, and Ni, with He II. The solid curves are $R_K(T)$ from Eq. (2) for the acoustic mismatch theory. The corresponding curve for Au, not shown, lies slightly below the Sn curve. Values of ρ_S and c_t were obtained from Tables 3f-1 and 3f-2 of the *American Institute of Physics Handbook*, D. E. Gray, Coordinating Ed. (McGraw-Hill Book Co., New York, 1963) and are taken as temperature independent. Values of ρ and c_l were obtained from Ref. 66. For Pb, Au, Sn, fused silica, and Ni, $F(c_l/c_t)$ has been taken as 1.5 following Refs. 8, 9, and 11. For Pt, $F(c_l/c_t)=2.0$ following Ref. 2 and for Cu, $F(c_l/c_t)=1.6$ following Ref. 7. The broken curves are $R_K^{\mathrm{CDW}}(T)$ from Eq. (39) for the acoustic mismatch theory as modified by Challis *et al.* (Ref. 9). The curve for Cu is taken from Ref. 9 and the curve for Au is taken from work of Johnson and Little (Ref. 8).

layers, and all cause differences between the measure boundary resistance and the ideal boundary resistance. Near T_λ the effects of phonon and roton collisions with the solid wall may become important enough to increase the thermal flux 1.5 to 2 times.[2] However, it is difficult to avoid the conclusion that some other energy exchange process is also present and unaccounted for.

2. *Improved Calculations of Acoustic Impedance*

The evidence is, to anticipate some of our conclusions, that if phonon radiation and acoustic impendance are the determining factors of thermal energy transfer and the associated R_K, then some important modifying effects must be present. The acoustic impedance theory of Sec. III.B.1 predicts an R_K that is much too large and too pressure dependent to explain the experiments described in Sec. II.A. The theoretical R_K also does not agree with the observed dependence on the elastic properties of the solid (Sec. II.A.6).

Khalatnikov[2,74] treated the interface as a plane with acoustic impendances of ρc_l on the liquid-He II side and $\rho_S c_l$ on the solid side. This large acoustic mismatch in the theory gives rise to the large calculated R_K. Challis, Dransfeld, and Wilks[81,9] have refined the calculation of the acoustic impedance between the liquid and the solid by smoothing out this discontinuity. They took account of the improved matching due to increased density of liquid He II near solid walls. The existence of solid layers of He in films adsorbed on cold walls has recently been reviewed by Manchester,[82] who concluded that the evidence in favor is not strong. However, the question is still open and the requirements of the theory of Challis *et al.*[81,9] are not too stringent.

The two main ideas are[81,9]: (1) The dense layer has an impedance intermediate between that of liquid He II and the solid; this will result in a better acoustic matching and hence a decrease in R_K. (2) The properties of the dense layer will be comparatively independent of pressure; this will result in an R_K that is itself also less pressure dependent.

Challis *et al.*[81,9] calculated the complex input impendance $Z_i=X_i+iY_i$ from the solid into liquid He II at 0°K through a lossless condensed interface region which extends out about 15 Å. The condensed He layer has a characteristic impedance $Z_0(z)$, which is a function of the distance z from the wall since both the density and sound velocity are functions of z. The calculation of Z_i when the condensed region is terminated at $z=15$ Å by bulk liquid proceeds by the usual techniques of electromagnetic transmission line theory applied to thermal energy transfer in liquid He II.[20]

To get $Z_0(z)$, Challis *et al.*[81,9] first calculated $\rho(z)$ from a knowledge of the force between a He atom and the wall.[83] The He density at the interface, $z=0$, is taken as about twice the bulk density; then with increasing z, it decreases continuously to the bulk value at $z=15$ Å. The characteristic impedance $Z_0(z)$ may then be calculated from $\rho(z)$ and experimental values of c_l as a function of ρ. It varies by a factor of 7 over these 15 Å. For the condensed region to be lossless its thickness must be less than the phonon mean-free path, and for the region to provide appreciable matching its thickness must be an appreciable fraction of a phonon wavelength. In the present case these conditions are satisfied. A numerical computer calculation of the real part of the acoustic input impedance, $X_i(T)$ was carried out by dividing the condensed He layer into about 20 sections. The result is given on Table I of Ref. 9.

To modify Khalatnikov's model with the present one, $X_i(T)$ replaces ρc_1 in Eq. (29) for $W(T)/A$. The net thermal flux is obtained, as in Eq. (30), as $\dot{Q}/A = \{d[W(T)A^{-1}]/dT\}\Delta T$, and the Kapitza resistance then becomes

$$R_K{}^{\mathrm{CDW}} = \frac{15h^3\rho_S c_t{}^3}{4\pi^5 k^4(4+m^{\mathrm{CDW}})X_i(T)F(c_l/c_t)T^3}, \quad (39)$$

in which $m^{\mathrm{CDW}} = d \ln X_i(T)/d \ln T$. Equation (39) is to be compared with Eq. (2). In the case that $X_i(T) = \rho c_1$, the equations are the same.

A plot of $R_K{}^{\mathrm{CDW}}$ taken from Fig. 9 of Ref. 9 for the interface between Cu and He II is shown on Fig. 9. Also shown is a plot of $R_K{}^{\mathrm{CDW}} = 47\ T^{-4.2}$ cm^2 °K/W obtained by Johnson and Little[8] for Au. Comparison of these broken curves, respectively, with Figs. 2(b) and (5) shows that $R_K{}^{\mathrm{CDW}}$ is much closer in magnitude to the observed Kapitza resistance than is R_K of Eq. (2). Johnson and Little[8] have also calculated $R_K{}^{\mathrm{CDW}}$ for a Si crystal and a LiF crystal. In both of these cases the calculated magnitudes are in better agreement with experiment than the R_K calculated from Eq. (2).

The resistance $R_K{}^{\mathrm{CDW}}$ given in Eq. (39) also shows a pressure dependence that is closer to observed values (Sec. II.A.6), as expected. The ratio $R_K{}^{\mathrm{CDW}}$ (at saturated vapor pressure)/$R_K{}^{\mathrm{CDW}}$ (at 20 atm pressure) is a smooth function of temperature and is about 1.1 at 2°K, 1.25 at 1°K, and rises more steeply with decreasing temperatures to 1.64 at 0°K.

The improved agreement with experiment of the magnitude and pressure dependence of $R_K{}^{\mathrm{CDW}}$ supports the existence of a condensed He layer, as proposed by Challis *et al.*,[81,9] or some other improved acoustic matching at the interface. However, $R_K{}^{\mathrm{CDW}}$ of Eq. (39) has a characteristic temperature dependence of about $T^{-4.2}$ above 1°K. This is considerably stronger than the experimentally observed temperature dependence, except for some qualitative similarity to observations of Anderson *et al.*[7]

Improvements in this model depend on better understanding of the interaction between liquid He and solid walls, a fundamental problem with interesting ramifications in the physics of liquid-helium films.[84,85]

Recently, Abbe[86] has examined the effect on the acoustic mismatch theory of diffusion, or other penetration at the interface, of He atoms into the solid. The idea is that this interface region also would have elastic properties intermediate between those of the liquid and solid and, if thick enough, could significantly improve the acoustic match. This would decrease calculated R_K's in analogous fashion to the model of Challis *et al.*[81,9]

Abbe[86] calculated the phonon transport across the solid interface region by application of simple kinetic theory, using a Debye model of the mixed solid to estimate the specific heat. In this model, calculation of R_K depends on several quantities which are difficult to obtain, e.g., the thermal conductivity, sound velocity, temperature, and phonon mean-free path all as functions of position in the solid. The calculated R_K has the characteristic T^{-3} dependence, in agreement with experiment, and is linearly proportional to the quotient $L_0/\lambda_{\mathrm{phon}}$, where L_0 is a penetration depth of He atoms into the solid and λ_{phon} is the mean-free path of phonons in the interface region. Using $L_0/\lambda_{\mathrm{phon}}$ as an adjustable parameter, Abbe was able to fit experimental data on R_K for Cu, Au, and Pb; the respective values are $L_0/\lambda_{\mathrm{phon}} = 32$, 42, and 116.

The theory in its present form is difficult to evaluate since very little is known about L_0 and λ_{phon}. However, the basic idea that R_K is influenced by the change in structure at the interface due to penetration of He atoms into the solid is a good one. Whether the effect is large enough to explain the experiments can be better judged after further investigation.

3. Applications to Interfaces between Metals and Liquid He II

At the interface between a metal and liquid He II, the electrons may also aid in thermal energy transfer. In the discussion in Sec. II.A.3 and in Figs. 3 and 4, it was shown that there is a decrease in R_K in going from the superconducting to the normal state. This is the most direct evidence for interactions between the electrons of the normal metal and the phonons of the liquid. There have been several mechanisms proposed for this electron–phonon interaction and the theoretical conclusions depend on whether strong or weak coupling is assumed, the nature of the model assumed for the metal, the relative magnitudes of the electron mean free path and the phonon wavelength, and the relative magnitudes of the electron relaxation time and the phonon period. Most of the theories have been lucidly unified and quantitatively compared in a recent study of Challis and Cheeke[87] so that a detailed account is not necessary here, but we shall briefly describe and compare the main theories and their results.

The most important effect of electrons on the acoustic mismatch theory (Sec. III.B.1) is that electrons may interact with the surface waves and make possible absorption of surface wave energy into the bulk solid. The terms of interest appear in the expression for $\dot{u}_{zs}(\omega)$ of Eq. (27) and the corresponding surface wave contribution to the thermal flux, $W(T)/A$ of Eq. (28). We recall that in general[76,79] these surface waves cannot interact with the phonons of the solid and therefore make no contribution to the thermal flux; this is the case for interfaces between liquid He II and dielectrics or superconducting metals. In a normal metal, however, thermal energy in the surface waves may be transported to the phonon system through interaction with normal electrons.

We consider the function $F(c_l/c_t)$ of Eqs. (29) and (30) as having two parts, in the manner of Challis and Cheeke[87]:

$$F(c_l/c_t)=F_1(c_l/c_t)+F_2(c_l/c_t); \qquad (40)$$

F_1 represents the contribution to thermal energy flow across the interface of longitudinal and transverse phonons in the solid, and F_2 is the surface wave contribution. Khalatnikov's[2] calculation for $\dot{Q}/A$ of Eq. (30), with $F(c_l/c_t)=F_1+F_2$, assumes that the surface wave energy is strongly coupled to the bulk solid and therefore quickly absorbed into the phonon system. If this were the case for a normal metal, then the ratio of R_K in the superconducting state to R_K in the normal state would be $(F_1+F_2)/F_1$. Challis and Cheeke[87] have calculated F_1 and F_2 as functions of c_l/c_t, and their results are displayed on Table 1 of Ref. 87. For Pb ($c_l/c_t=2.84$), the ratio $(F_1+F_2)/F_1=2.4$, is the maximum ratio. If all the surface wave energy is not absorbed, then the theoretical ratio of the R_K's becomes closer to unity. We shall see below that this is indeed what happens in the general theoretical case.

Little[79] first suggested that electron–phonon interactions might contribute to the thermal flow between a metal and liquid He II, as part of a detailed consideration of heat transport across solid–solid interfaces. The energy transport arises this way[28,36]: Because of the large acoustic mismatch between liquid He II and solids, almost all phonons incident on the interface from the liquid are reflected. Actually, only those phonons whose angle of incidence is less than $\sin^{-1} c_l/c_t$ can be transmitted into the solid.[87,88] Phonons with larger incident angles excite surface waves at the interface but are ordinarily totally reflected, since the surface waves cannot generally exchange energy with the phonon system of the solid. In a normal metal, however, the surface waves may interact with the electrons which may then exchange energy with the phonons.

Little[28] considered phonons incident on the interface from the liquid in terms of the scalar displacement potential of quantized plane waves in the liquid. The scalar and vector potentials of the quantized plane waves produced in the solid are obtained by application of boundary conditions such as Mazo's[73] on the displacements and stresses at the interface. For a large range of angles of incidence no wave is propagated into the solid, but rather the potentials correspond to surface waves which decay exponentially in the normal direction into the solid. Little treated the solid as a Sommerfeld metal in whose ionic lattice the displacement waves produced density changes. These could be treated as perturbations of the electron Hamiltonian, and the new eigenstates for the electrons were calculated in this way.

To obtain the actual transfer of thermal energy across the interface, time-dependent perturbation theory is used to calculate matrix elements for processes such as an electron scattered and a phonon created or destroyed. Challis and Cheeke[87] have shown that Little's calculation[28] is carried out, implicitly, for the condition $\omega_{\text{phon}}\tau_{\text{el}}\gg1$, where ω_{phon} is the phonon angular frequency and τ_{el} is the electron relaxation time. We shall consider the inverse case below. In Little's calculation only electrons in characteristic regions of the Fermi surface contribute to the energy flow. The associated integrations are somewhat simplified since electrons near the Fermi surface have energies much greater than the phonon energies and by geometrical considerations, but they are still difficult.

The resultant net thermal flux from the metal to the liquid He II contains two terms according to these calculations; the first term is proportional to $T^3\Delta T$; the second term is proportional to $T^5\Delta T$ and is much smaller at helium temperatures.[28,36] The general expressions for $\dot{Q}/A$ appear as Eqs. (3.12) and (3.15) of Ref. 28. Little has evaluated the result for a solid whose properties are close to those of Pb and obtained $\dot{Q}/A=5.4\times10^{-4}T^3\Delta T$ W/cm^2 as the principal contribution. This corresponds to a very large effective R_K for this process, about 1850 T^{-3} cm^2 °K/W. This contribution to $\dot{Q}/A$ is small, and the associated R_K is large, compared to the acoustic-mismatch calculated values and the observed values of net flux and R_K. This means that in the $\omega_{\text{phon}}\tau_{\text{el}}\gg1$ limit, and for the model assumed, electron–phonon interactions make only a small contribution to the thermal flux, i.e., the energy of surface waves is only weakly coupled into the bulk solid.

Andreev[88] has also calculated the effect of conduction electrons in transferring thermal energy from surface waves into the bulk solid in an elegant way. Near the interface the solid may be expected to be rich in strains, impurities, and other defects so that the electron relaxation time is short. Andreev calculated the effect for the condition $\omega_{\text{phon}}\tau_{\text{el}}\ll1$ for this reason. This calculation finds the amplitude for surface and lattice displacements, which are produced when a plane sound wave is incident on the interface from the liquid side, by considering suitable scalar and vector potentials for the velocity fields. Although similar to Little's[28] approach, Andreev's calculation is essentially classical. It is divided into two parts depending on the relative values of the electron mean free path and the wavelength of sound.

When the electron mean free path is much less than the phonon wavelength, and $\omega_{\text{phon}}\tau_{\text{el}}\ll1$, Andreev[88] showed, by using the concept of electron viscosity, that the fraction of energy transmitted from the surface wave to the solid was small. The fraction is itself approximately the same as the ratio of the electron mean free path to the phonon wavelength so that the electrons have a negligible effect on R_K.

In the case that the electron mean free path is much larger than the phonon wavelength and $\omega_{\text{phon}}\tau_{\text{el}}\ll 1$, Andreev[88] found the interaction classically by very elegantly solving the Boltzmann equation for the electron distribution function in the presence of the electric field produced by a longitudinal and transverse sound field in the solid. The force on the boundary due to the electrons may be calculated from the distribution function, and then the reflection and transmission coefficients for incident sound waves may be obtained. Under the present formulation and assumptions it can be made manifest, as in Eq. (23) of Ref. 88, that the reflection coefficient for phonons incident on the solid is less than unity for all angles of incidence. That is, the thermal energy in the surface waves excited by phonons incident outside the critical cone is absorbed into the solid and contributes something to the net thermal flux.

The expression Andreev[88] obtained for the additional net thermal flux due to electron interactions, as re-expressed by Challis and Cheeke,[87] is

$$\frac{\dot{Q}}{A}=\frac{16\pi^5\rho c_1 k(kT)^3\Delta T}{15\rho_S(hc_t)^3}\,\frac{B}{B+1}\,F_2\left(\frac{c_l}{c_t}\right). \qquad (41)$$

In Eq. (41), B is a function of the Fermi energy, velocity, and momentum for the metal considered, and also depends on several integrals which are themselves functions of c_l/c_t. In the strong-coupling limit, $B\gg 1$ and $\dot{Q}/A$ of Eq. (41) becomes the same as the contribution of surface waves to Khalatnikov's net thermal flux given in Eq. (30).[87] Therefore Khalatnikov's calculation[2] which assumes strong coupling of the surface waves to the bulk solid, gives the same result as Andreev's[88] treatment in this limit.

Challis and Cheeke[87] have evaluated and tabulated the relevant functions for B and have obtained $B\approx 0.06$ for a monovalent metal, and $B\approx 0.13$ for Pb. These values correspond to weak coupling and give a ratio of R_K for superconducting Pb to R_K for normal Pb of $[F_1+B(B+1)^{-1}F_2]/F_1\approx 1.2$. This means that the contribution due to electron–phonon coupling is small in the limit $\omega_{\text{phon}}\tau_{\text{el}}\ll 1$ investigated by Andreev[87,88] for the electron mean free path much larger than, as well as much smaller than, the phonon wavelength. Andreev has estimated that $B\approx 1$. The disagreement apparently depends on the integrals in B, which have been evaluated by Challis and Cheeke.[87]

In their recent examination and synthesis of theories of R_K in metals, Challis and Cheeke[87] have extended Andreev's[88] classical approach from the $\omega_{\text{phon}}\tau_{\text{el}}\ll 1$ region into the region of Little's[28] quantum-mechanical calculation, $\omega_{\text{phon}}\tau_{\text{el}}\gg 1$. They have shown that these two calculations are in essential agreement and give similar answers in this region. They have also re-expressed the equations for $\dot{Q}/A$ of these workers and of Khalatnikov[2] so that all may be quantitatively intercompared by use of an interesting and complete table of relevant functions (Table 1 of Ref. 87). In the $\omega_{\text{phon}}\tau_{\text{el}}\gg 1$ region, the calculated thermal flux is shown to be smaller by a factor of $(c_t/c_l)^4$ than the thermal flux for $\omega_{\text{phon}}\tau_{\text{el}}\ll 1$, and the electron mean free path large compared to the phonon wavelength. This is the result of certain cancellations which depend on the details of the electron phonon interaction. Although it is still too soon, due to experimental uncertainties, for a critical comparison between these theories and experimental data, the agreement among the different theories in their common regions of application strongly supports them.

Johnson and Little[8] have examined their experimental data on R_K for Au, Cu, and W for evidence of the electron–phonon interaction calculated by Little.[28] According to this calculation, the part of R_K due to the interaction should be approximately proportional to $\rho_S c_{l,t}^3/\rho$ (Fermi energy)^2. However, the data do not show ordering of this kind, as expected, since the electron–phonon contribution to the net thermal flux is small in the theory.

There have been several other interesting suggestions and calculations of interactions between liquid-helium phonons and electrons in the metal. Little[28,36] has noticed that an impinging phonon interacts with the electrons not only by changing the ionic density in the bulk metal, but also through the periodic surface oscillations. Since the potential at the interface must vanish, oscillations of the surface modulate the potential in the region of the interface. The contribution to $\dot{Q}/A$ of this interaction is proportional to ΔT and has been calculated[28] to be of the order of $2\times 10^{-3}\,T^3$ W/cm^2 °K. However, more detailed knowledge of the interactions in the solid are necessary for improved evaluation of the importance of this mechanism.

An especially interesting and possibly very large contribution to the thermal flux between liquid He II and metals has been suggested by Bloch.[8,36] The idea is that the electron wave functions extend a short distance beyond the metal surface and may in this region interact directly with phonons in the liquid. The resultant thermal flux is expected to be proportional to the product (Fermi energy)$^3\times$(Fermi velocity)$\times$(work function)$^{-3}\,T^4$.[8] The ratio of $\dot{Q}/A$ due to this mechanism to the $\dot{Q}/A$ calculated from the acoustic mismatch theory is about $10T^2$, so that the effect is potentially very important and, if present, should be observable as a T^5 dependence in experimental R_K's. No such T^5 dependence was observed by Johnson and Little,[8] or other workers, however, and a special attempt by Gittleman and Bozowski[34] to observe evidence for Bloch's mechanism in Pt was not successful (Sec. II.A.4). Electron tunneling depends critically on the structure of the surface and it may be that the effect exists but has escaped detection so far because of surface imperfections and impurities.

There has been an interesting suggestion by Johnson and Little[8] that understanding of the problem may be improved by considering the energy exchange across the interface associated with adsorption and subsequent desorption of He atoms at the solid surface. A calculation of the size of the effect and importance of this mechanism would be very interesting.

C. The Acoustic Impedance Theory for Liquid ^{3}He

The theory of Kapitza resistance at interfaces between solids and liquid ^{3}He is due to Bekarevich and Khalatnikov[52,89] and is based on the acoustic-mismatch approach of Khalatnikov,[2] discussed in Sec. III.B.1. However, several interesting changes must be made to take account of the properties of liquid ^{3}He as a Fermi fluid.[90] The relevant experimental data have been discussed in Sec. II.B and displayed on Figs. 8(a) and 8(b). In this section we outline the main features of the theory and compare them with experimental results, principally on Cu. We shall see that the modifications that must be made in the theory are far reaching in principle, but the theoretically predicted T^{-3} dependence, as well as several other features, is retained.

To find the thermal boundary resistance at an interface between liquid ^{3}He and a solid, Bekarevich and Khalatnikov[52] proceed in the familiar way by finding the work done by the thermally vibrating solid against the pressure of the liquid ^{3}He as a function of the temperature and the frequency of vibration. If the solid surface has velocity components $\dot{u}_i(\omega)$, then the rate at which work is being done and energy radiated is

$$W(\omega, T) = -\tfrac{1}{2} \sum_i \int \Pi_{iz} \dot{u}_i(\omega)\, dA, \qquad (42)$$

in which the integration is taken over the interface. The factor of $\frac{1}{2}$ in Eq. (42) takes account of a time average of W over a period of vibration of the wall.[52] In a recent rederivation of some of these results, Gavoret[91] has suggested that this factor should be unity instead.

The role of liquid ^{3}He as a Fermi liquid comes into the problem with calculation of the tensor of momentum flux in the liquid, Π_{ij}, in terms of the distribution function, excitation energies, and other Fermi fluid properties.[90] These considerations will be made later but we shall anticipate the result now. Calculating Π_{ij} from the Landau theory of Fermi fluids and applying suitable boundary conditions, the magnitude of the energy flow from the solid into the liquid becomes, from Eq. (42),[52]

$$|W(\omega, T)| = \tfrac{3}{2} p_e \frac{\rho}{m} \int (a\,|\dot{u}_z(\omega)|^2 + b\,|\dot{u}_x(\omega)|^2)\, dA, \qquad (43)$$

analogous to Eq. (13) of the liquid-^{4}He case. In Eq. (43) ρ and m are the density and atomic mass for liquid ^{3}He, and p_e is the momentum of excitations at the Fermi surface, as in Eq. (3). The characteristic constants a and b are obtained from the boundary conditions. The calculation of radiated energy and Π_{ij} in this way, and the associated energy flux [Eq. (44)] and R_K [Eq. (3)] holds only for $T<0.2$°K, the temperature region in which the specific heat of liquid ^{3}He is approximately proportional to T.

From Eq. (43) we see that oscillations in both the normal and tangential directions contribute to the energy flow into the liquid. As expected from our earlier considerations, these contributions must be taken for three kinds of oscillations: longitudinal, transverse, and surface wave modes in the solid, with due account taken of the distribution function for the phonons in the solid. When this is done and the energy flow is integrated over all frequencies and solid angles and Khalatnikov's[2] earlier results for quantization of the surface vibrations are applied, Bekarevich and Khalatnikov obtain, for the energy flux from the solid to liquid ^{3}He,[52]

$$\frac{W(T)}{A} = \tfrac{3}{2} p_e \frac{4\pi^5 \rho (kT)^4}{15 \rho_S m (h c_t)^3} \left[aF\left(\frac{c_l}{c_t}\right) + b\Phi\left(\frac{c_l}{c_t}\right) \right]. \qquad (44)$$

In Eq. (44), $\Phi(c_l/c_t)$ is, like $F(c_l/c_t)$, a characteristic function of the ratio of velocities of longitudinal and transverse waves in the solid.

The net flux $\dot{Q}/A$ from the solid into the liquid for small temperature differences is calculated by the methods used earlier for Eqs. (29) and (30), so that the result for the Kapitza boundary resistance between a solid and liquid ^{3}He at low temperatures becomes[52]

$$R_K = \frac{5 h^3 \rho_S c_t^3 m}{8\pi^5 k^4 \rho p_e [aF(c_l/c_t) + b\Phi(c_l/c_t)] T^3}. \qquad (3)$$

At higher temperatures, say $T>0.2$°K, the detailed Fermi fluid properties of liquid ^{3}He do not play an important role in the energy radiated from the solid into the liquid. As for the ^{4}He problem, the energy is radiated principally as phonons[2] and $\dot{Q}/A$ is given by Eq. (30) with the product ρc_1 now understood to be the acoustic impedance for liquid ^{3}He, i.e., the product of the density of liquid ^{3}He and the velocity of sound in liquid ^{3}He. The R_K for an interface between a solid and liquid ^{3}He for $T>0.2$°K is given by Eq. (2) with the same replacement of acoustic impedances.[52] Thus, the ratio of Kapitza resistances in this region is

$$R_K(^3\mathrm{He}) / R_K(^4\mathrm{He}) = (\rho c_1)_{^4\mathrm{He}} / (\rho c_1)_{^3\mathrm{He}} \approx 2,$$

independent of the solid.

Since liquid ^{3}He is a Fermi liquid, its interaction with the solid wall in the low-temperature region ($T<0.2$°K) must be analyzed in detail in terms of a suitable distribution function and excitation energy.

The Landau theory of Fermi liquids has been applied to liquid ^{3}He by Abrikosov and Khalatnikov,[90] and a thorough discussion of the relevant functions, and their definitions and interconnections, can be found in their work. Liquid ^{3}He is thus characterized by a distribution function of excitations, which is itself a function of momentum, position, and time, and an excitation energy, which is in turm a functional of the distribution function. The distribution function and the excitation energy are connected together by the usual kinetic equation.[52,89] In our problem for the low-temperature case, the product of the angular frequency of the solid's oscillations ω_{exc} and the mean collision time for excitations in the liquid τ_{exc} satisfies the condition $\omega_{\text{exc}}\tau_{\text{exc}}\gg1$, i.e., collisions are not important here.[90] This makes the collision integral simple, and the kinetic equation is therefore more tractable.

Bekarevich and Khalatnikov[52] found R_K for the Fermi liquid ^{3}He by solving this appropriate form of the kinetic equation, subject to the boundary conditions that apply when the excitations collide with the wall and are scattered, and relating the resultant distribution function to Π_{ij} of Eq. (42). The calculation is mathematically difficult and we refer back to the original work for the details. The idea is to linearize the kinetic equation by considering the difference between the real distribution function and that of a Fermi liquid at 0°K. After a judicious succession of physical arguments, changes of variable, expansion in a series of spherical functions, and relinearization, the kinetic equation may be re-expressed as an integral equation for the part of the distribution function which determines Π_{ij}.[52,89,90]

The boundary conditions that obtain at the interface are conservation of energy and momentum and the additional condition that the number of incident excitations must equal the number of reflected excitations. The interaction of the excitations with the wall also depends on the nature of the reflection, i.e., whether diffuse or specular, and on the kinds of oscillation undergone by the wall. Bekarevich and Khalatnikov[52] found the radiated energy assuming: (a) completely diffuse reflection, for both transverse and longitudinal wall oscillations, using a Wiener–Hopf technique to solve the integral equation, and (b) completely specular reflection, using a Fourier integral technique. One of their results is that Π_{ij}, and hence R_K, depends only weakly on the nature of the reflections.

In the $\omega_{\text{exc}}\tau_{\text{exc}}\gg1$ region, the only kind of sound that may be propagated in liquid ^{3}He is zero sound,[90] and indeed Bekarevich and Khalatnikov[52] showed that this was the nature of the collective oscillations in liquid ^{3}He which were excited under conditions of specular reflection and of diffuse reflection with longitudinal wall oscillations. Gavoret[91] has developed a simpler rederivation of the R_K for interfaces between solids and liquid ^{3}He based on the earlier work.[52] We have already noticed that the resultant energy transfer depends only weakly on the nature of the reflections of excitations. Gavoret solved the problem by showing that a plane wave of zero sound emitted into liquid ^{3}He by the oscillating wall could satisfy both specular and diffuse reflection boundary conditions. The motion of the oscillating solid surface is taken after Khalatnikov's result,[2] described in Sec. III.B.1; the work done by the solid against the liquid is the product $\Pi_{zz}\dot{u}_z$, as in Eq. (42) without the factor of $\frac{1}{2}$. This work produces the zero sound wave which is considered to carry an appropriate increment in free energy away from the solid at the zero sound velocity.

The results of Gavoret[91] for R_K are essentially the same as those of Bekarevich and Khalatnikov[52] except for the factor of 2 mentioned above. In the low-temperature region Gavoret's theoretical R_K's are smaller by this factor.

Comparison of the theoretical R_K for liquid ^{3}He, as given by Eq. (3) for the low-temperature region ($T<0.2$°K) and by Eq. (2) with ρc_1 the acoustic impedance of liquid ^{3}He for higher temperatures, may be made with the data of Figs. 8(a) and 8(b). Theory shows that R_K should vary as T^{-3} in both regions, but with different coefficients. The ratio of the coefficients R_K (low T, ^{3}He)$/R_K$ (high T, ^{3}He) has been estimated as about 2 or 3[7,52] and, more recently,[66] as about 0.9.

This implies that near 0.2°K, there should be some transition region in which R_K changes magnitude. Applying Eq. (3) to the interface between liquid ^{3}He and Cu, Anderson *et al.*[7] obtained $R_K=2400\ T^{-3}$ cm^2 °K/W using $\Phi=1$, $a=0.38$, and $b=0.05$. This theoretical curve has been plotted on Fig. 8(a). Comparison with the low-pressure, low-temperature experimental results for Cu shows that although the T^{-3} dependence is correct for all the R_K curves shown, the magnitude of the theoretical R_K is 12 times larger than the relevant experimental one. The acoustic mismatch theory thus predicts the correct temperature dependence for all the interfaces at low T, but this is as far as the quantitative agreement with experiment extends.

The higher-temperature experimental R_K's are no longer proportional to T^{-3} so that the ratio of experimental R_K's at low and high T is temperature dependent and not constant as expected from theory. We have plotted on Fig. 8(b) $R_K=1200\ T^{-3}$ cm^2 °K/W as given by Eq. (2) for a liquid-^{3}He interface with Cu at low pressure and high temperatures.[7] The theoretical R_K's of Eq. (2) for the high-temperature region are about 60 times higher than experimental values of Anderson *et al.*[7] at 0.8°K, so that the disagreement here is even greater than at low temperatures. These experiments did reveal an abrupt transition at 0.11°K, independent of pressure, in the temperature dependence for R_K for liquid ^{3}He (Sec. II.B).

It is not clear whether this is to be associated with the theoretical transition at 0.2°K, because of the anomalous behaviour of R_K on the high-temperature side of the transition.

A further test of Bekarevich and Khalatnikov's[52] theory is obtained by comparing the theoretical pressure dependence with the observed one. For the low-temperature region, an increase in pressure from 1 psi to 395 psi decreases the theoretical R_K of Eq. (3) by a factor of about 3. The observed[7] decrease in R_K for this same pressure increase below 0.1°K is a factor of only 1.58 for liquid ^{3}He in contact with Cu walls or epoxy resin walls. In the high-temperature region, Eq. (2) gives about the same ratio of the low pressure to high pressure R_K as at low temperatures, but the observed decrease in R_K near 0.4°K is only 1.3.[7] These results are similar to the pressure dependence of R_K for the ^{4}He case. There, too, the pressure dependence of R_K is observed to be less than theoretically expected.

IV. CONCLUSIONS

Kapitza resistance, in conclusion, is a phenomenon which is understood only qualitatively at best. The experiments show that the energy exchange across the interface between a solid and liquid He generally takes place an order of magnitude more efficiently than theories predict. The problem is: How does this energy exchange occur? There are many directions in which solutions to the difficulties may lie. Probably the best indicators are the results of previous work, and we have tried to point some of these out in the text.

There are two possibilities: The first is that the acoustic-mismatch idea and associated theory are at the heart of the thermal energy exchange, i.e., there is no other dominant mechanism. The second possibility is that some other, more important mechanism is also present. In support of the first idea are the observed T^{-3} temperature dependence of R_K and the generally improved agreement between theory and experiment that results when the acoustic mismatch theory is modified. The problem is a difficult one but, in balance, this seems to be the more likely possibility. There is much evidence for the large variations in surface structure in solids, and we think that most of the disagreement between theory and experiment will be resolved by use of better characterized surfaces and by better understanding of the surface physics.

To examine this problem, improvements in both theory and experiment are needed. For example: A better knowledge of the structure of liquid He near the interface is important. Further experiments in the manner of Challis and Cheeke on Pb[30] would be helpful on other metals, especially well-oriented single crystals. Can surface treatments of this kind be found which will give reproducible results on clean surfaces of Cu, Hg, Sn, Ni, and other solids? Since the surface structure, as we have noted, is likely to be more disordered than the bulk, perhaps the extrapolation in measuring the surface temperature, even in good conductors, can be improved. Can the temperature profile in the solid near the interface, say within microns, be measured in more detail? A clearcut experiment on whether R_K is quantitatively reversible would also be helpful: Is the thermal boundary resistance for energy flow from the liquid He into the solid the same as from the solid into the liquid He?

It would also be interesting to study the thermal boundary resistance as a function of frequency, perhaps in an experiment similar to Brow and Osborne's[17] or Zinoveva's.[21] We might then be able to test whether the theoretical microscopic understanding of the interaction between the surface and an impinging phonon of well-defined frequency and direction is correct. There are several important diagreements between the theoretical and experimental dependence of R_K on elastic properties of the solids, e.g., the recent observation that R_K is closely proportional to Θ_D, rather than to $\Theta_D{}^3$.[47] These suggest that some new mechanism is present.

The second possibility is that some other mechanism, not yet accounted for, dominates over acoustic mismatch. Examination of this problem awaits better and deeper understanding of solids and of liquid helium.

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