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Electron Spin Polarization by Low-Energy Scattering from Unpolarized Targets*

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The status of theory and experiment of electron polarization resulting from spin–orbit interaction in low-energy scattering from unpolarized targets is reviewed. Polarization effects in scattering from free atoms, molecules, and solid targets at energies between a few electron volts and a few thousand electron volts are discussed. Apart from a survey of the problems which have been solved, a perspective is given of the work which would be interesting to pursue in this rapidly developing field of research.

CONTENTS

I. INTRODUCTION

A beam or other ensemble of electrons is called polarized if the spins of the electrons have a preferential orientation. The description of the electron polarization by the expectation value of the spin operator‡ $\boldsymbol{\sigma}$ and by means of density matrices has been fully discussed in earlier reviews (Tolhoek, 1956; Farago, 1965). The present survey will therefore not go into these fundamental problems, but will be concerned with a discussion of the polarization effects in low-energy electron scattering which have been investigated in recent years. By low energies, we mean energies below 10 keV.

Polarization effects in elastic electron scattering were predicted for the first time in the famous papers of Mott (1929, 1932), where the scattering process is described on the basis of the Dirac equation. The results of this theory are too complicated to be representable in a closed analytic form. Therefore, Mott derived approximate formulas for the special case of elastic electron scattering by the unscreened Coulomb field of a nucleus with the atomic number Z. He calculated numerical results for $Z=79$, since the polarization effects turned out to be significant only at large atomic numbers. Experiments for checking the theory had to be made at energies of about 100 keV or higher, since only then can the description of the scattering process as a deflection in the pure Coulomb field of the nucleus be regarded as a good approximation. Besides, it had been pointed out by Mott that unless the velocity v of the electrons is comparable with the velocity c of light, the polarization effects would become extremely small.

The considerable effort of testing Mott's results was therefore made with fast electrons. Many experiments were carried out in the two decades after the theory of "Mott Scattering" had appeared (Tolhoek, 1956), for this was a very promising way of checking Dirac's theory of the electron on which Mott's calculations were based. However, most of the experiments failed to show any polarization effect, and Dirac's theory was questioned repeatedly for this reason (see, e.g., Sommerfeld, 1939). Sound evidence of the effects predicted could not be given until 1943 when Shull, Chase, and Myers for the first time detected a genuine scattering asymmetry in a double scattering experiment. A great number of polarization experiments with fast electrons has been made since then [for pertaining papers cf. Tolhoek (1956) and Farago (1965)], and to date elaborate results are available for electrons scattered from gold targets by Mikaelyan, Borovoi, and Denisov (1963), Apalin *et al.* (1962), and especially by van Klinken (1966); these results, for energies above 100 keV and large scattering angles, are in satisfactory agreement with the theoretical findings.

Looking back, the reason for the failure of the experiments in the first one or two decades after Mott's theory can be easily seen. At that time, scattering experiments with fast electrons constituted a new field of research. Frequently one had to use β emitters with their low intensities (not knowing that one was handling polarized electrons§), and there was no experience with

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‡ The definition of the polarization as the expectation value of the spin operator in the Lorentz frame in which the electron is at rest gives rise to some difficulties resulting from the fact that $\boldsymbol{\sigma}$ does not commute with the Hamiltonian of Dirac's equation. Definitions which avoid this problem have also been reviewed Fradkin and Good, 1961).

§ The asymmetry effect in the 90°–270° position of the experiment of Chase (1930) was not due to the polarization of the electrons emitted by the radium E source, as sometimes assumed. It has the wrong sign and thus must be ascribed entirely to an instrumental asymmetry.

the pitfalls caused by plural and multiple scattering from thin films, one of which is the reflection–transmission asymmetry (Tolhoek, 1956).

On the other hand, the technique of scattering slow electrons was well developed, as shown by the detection of the Ramsauer effect (Ramsauer, 1921) and the diffraction experiments with crystals (Davisson and Germer, 1927) and gases (Arnot, 1931; a list of similar experiments is given by Kollath, 1958). As we shall see later on, the scattered electrons had a considerable polarization in many of the experiments of that kind. But in spite of the great interest in electron polarization effects, this was not realized because hardly anybody looked for polarization phenomena at low electron energies. Mott's statement that the electron velocity v must be comparable with c if any polarization was to be observed was tacitly considered by many authors to be more general than it really was. Therefore the experiments on electron polarization were generally made with fast electrons, although slow electrons could be handled much more conveniently and successfully. The few experiments on polarization effects which had been made with slow electrons (Wolf, 1929; Langstroth, 1932; Davisson and Germer, 1929; Joffé and Arseniewa, 1929) had failed. However, the reason for the failure was not the low energy, as assumed later (Richter, 1937). The negative results were due to the undeveloped theoretical understanding of the spin effects in the first work (Wolf, 1929), which was before Mott's theory, while in the papers of Langstroth (1932), Davisson and Germer (1929), and Joffé and Arseniewa (1929), where thick, solid targets were used as "polarizer" and "analyzer," presumably the overwhelming contribution of multiple scattering together with the inadequacy of the energy analysis was responsible for the failure to detect polarization.

Little attention was paid to calculations of Massey and Mohr (1941) and Mohr (1943), who considered the polarization effects in electron scattering down to energies of 100 eV, taking into account the screening of the nuclear Coulomb field by the atomic electrons and thus generalizing Mott's special results. This was the first investigation to show that appreciable polarization effects could even be expected at small energies, although the results are not in quantitative agreement with those of recent calculations which will be discussed in the next paragraph. But the experiments of the next two decades were still concentrated on the range of fast electrons, and Kollath (1949) seems to have been the first experimentalist who seriously considered elaborate polarization measurements at low energies. Lack of confidence in the earlier results of the laborious phase-shift analysis, as indicated by Mohr and Tassie (1954), made the theoretical predictions appear not very reliable.

In recent years, however, considerable progress has been made. Now the theoretical analysis of the polarization effects could be made by means of computers, and such analysis has produced very reliable data. On the other hand, the general interest in electron polarization phenomena has inspired experimentalists, so that a great number of papers on polarization effects in low-energy scattering of electrons have appeared. It is the purpose of this review to convey an over-all picture of this field.

The theoretical background of these polarization phenomena is surveyed in Sec. II, while a systematic discussion of the experimental results is the subject of Sec. III. A few possible applications are discussed in Sec. IV, which also gives a list of unsolved problems.

II. THEORETICAL BACKGROUND

A. General Relations

Though concerned with low electron energies, our considerations cannot be based on the Schrödinger wave equation. This equation does not include the spin of the electron and therefore cannot be used as a basis for describing the polarization effects. One has to use Dirac's equation, which takes account of the electron spin. The fact that this equation describes only the normal magnetic moment of the electron and neglects the anomalous part is irrelevant for the scattering processes considered here.

The physical quantities which can be measured in a scattering experiment are determined by the scattering amplitudes f and g, where f is analogous to the scattering amplitude known from the scattering theory based on Schrödinger's equation, and g describes the change of spin direction during the scattering process. Using the method of partial waves, Mott (1929) [cf. also Mott and Massey (1965)] showed that the scattering amplitudes* are

$$f(\theta)=\frac{i}{2k}\sum_{l=0}^{\infty}\{(l+1)[1-\exp(2i\delta_l)]+l[1-\exp(2i\delta_{-l-1})]\}P_l(\cos\theta),$$
$$g(\theta)=\frac{i}{2k}\sum_{l=0}^{\infty}[\exp(2i\delta_l)-\exp(2i\delta_{-l-1})]P_l^1(\cos\theta), \tag{1}$$

where P_l and P_l^1 are the Legendre polynomials and associate Legendre functions, respectively; θ is the scattering angle; and the wave number k is determined by $p=\hbar k$, where p is the relativistic momentum $m\gamma v$. In order to obtain the phase shifts δ_l, δ_{-l-1}, one has to find the asymptotic form of the regular solution for the radial part of the wave equation. The phase shift δ_l

* Needless to say, the scattering amplitudes and the quantities which will be composed from them later on depend not only on θ, as explicitly indicated, but also on the electron energy E and the atomic number Z of the scattering center.

results from the solution of the differential equation [cf. Mott and Massey (1965), Chap. IX, Eq. (22)]

$$G_l''+\{k^2-[l(l+1)/r^2]-U_l(r)\}G_l=0, \qquad (2)$$

where

$$U_l=\frac{2W}{\hbar^2c^2}V-\frac{V^2}{\hbar^2c^2}-\frac{l+1}{r}\frac{\alpha'}{\alpha}+\frac{3}{4}\frac{\alpha'^2}{\alpha^2}-\frac{1}{2}\frac{\alpha''}{\alpha},$$

$$\alpha=(\hbar c)^{-1}(W-V+mc^2); \qquad (3)$$

W is the total energy, and V is the potential energy of the electron in the field of the scattering center. For calculating δ_{-l-1}, one has to replace l by $-(l+1)$ in these equations.

All the observable quantities characterizing the scattering process contain products of two wave functions and are therefore described by quadratic terms of the complex scattering amplitudes f and g. The following quadratic terms with a straightforward physical meaning can be formed (Tolhoek, 1956; Farago, 1965; Schopper, 1959; Motz, Olsen, and Koch, 1964):

(1) The differential cross section for an unpolarized electron beam

$$d\sigma/d\Omega=I(\theta)=|f|^2+|g|^2. \qquad (4)$$

(2) The asymmetry function or Sherman function*

$$S(\theta)=i(fg^*-f^*g)/(|f|^2+|g|^2) \qquad (5)$$

describing the scattering asymmetry of a polarized electron beam: Let us assume we have a beam with a transverse polarization whose direction is indicated in Fig. 1. If $N_\uparrow$ and $N_\downarrow$ are the numbers of electrons with spin in or opposite to this direction, the degree of polarization is given by

$$P=(N_\uparrow-N_\downarrow)/(N_\uparrow+N_\downarrow). \qquad (6)$$

The scattering cross section of this beam not only depends on the angle θ but also on the azimuthal angle φ and is given by

$$I(\theta)(1-PS\sin\varphi). \qquad (7)$$

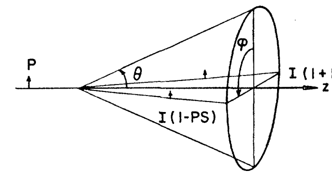
Therefore, the scattered intensity has a left–right asymmetry, maximum asymmetry being observed when the scattering planes are perpendicular to **P**.

As a result of this scattering asymmetry, an unpolarized beam can be easily seen to become polarized in the scattering process: An unpolarized beam can be considered as a mixture (incoherent superposition) of two completely polarized beams with opposite directions of the polarization.† We assume that the polarization

* Sherman (1956) was the first to calculate exact numerical values of this function $S(\theta)$ over a wide angular range for scattering by the pure Coulomb field of several nuclei ($Z=13$, 48, 80).

† Such a beam in which half the spins point in one direction and the other half in the opposite direction cannot be distinguished experimentally from a beam in which the electron spins point in all directions at random. A measurement of the spin directions would give the same result in both cases: Half the spins point in the direction defined by this measurement and the other half point in the opposite direction.

FIG. 1. Scattering asymmetry of a transversely polarized beam.



vectors of the two beams are perpendicular to the scattering plane and apply Eq. (7) to each of the beams. Then we obtain for the number of electrons scattered into a certain direction

$$N_\uparrow=I(1+S), \qquad N_\downarrow=I(1-S), \qquad (8)$$

where now $N_\uparrow$ and $N_\downarrow$ are the numbers of electrons with spin parallel or opposite to the vector

$$\mathbf{n}=(\mathbf{k}_1\times\mathbf{k}_2)/(|\mathbf{k}_1\times\mathbf{k}_2|),$$

perpendicular to the scattering plane; $\mathbf{k}_1$ and $\mathbf{k}_2$ are initial and final momentum of the electron. Here we have anticipated the result, following from Eq. (11), that the polarization vector does not change its direction during the scattering process if it is perpendicular to the scattering plane. From Eqs. (6) and (8) we find

$$\mathbf{P}_2=S\mathbf{n}, \qquad (9)$$

where $\mathbf{P}_2$ is the polarization obtained by scattering an unpolarized electron beam ($\mathbf{P}_1=0$).

(3) The terms

$$T(\theta)=(|f|^2-|g|^2)/(|f|^2+|g|^2),$$

$$U(\theta)=(fg^*+f^*g)/(|f|^2+|g|^2) \qquad (10)$$

describe the rotation of the polarization vector during the scattering process. The polarization vector after the scattering $\mathbf{P}_2$ can be expressed in terms of the polarization vector before the scattering ($\mathbf{P}_1=\mathbf{P}_{1t}+\mathbf{P}_{1p}$) in the following way:

$$\mathbf{P}_2=\frac{(P_{1t}+S)\mathbf{n}+T\mathbf{P}_{1p}+U[\mathbf{n}\times\mathbf{P}_{1p}]}{1+P_{1t}S}, \qquad (11a)$$

where $\mathbf{P}_1$ has been decomposed into a transverse component perpendicular to the scattering plane ($\mathbf{P}_{1t}=P_{1t}\cdot\mathbf{n}$) and a component parallel to the scattering plane ($\mathbf{P}_{1p}$) (Fig. 2). If $U\neq 0$, there is a component of the polarization $\mathbf{P}_2$ perpendicular to the plane ($\mathbf{n}$, $\mathbf{P}_{1p}$) [which is identical with the plane ($\mathbf{P}_{1t}$, $\mathbf{P}_{1p}$)], i.e., U describes the rotation of the polarization out of its initial plane. T is seen from Eq. (11a) to describe the change of the polarization component parallel to the scattering plane. If the scattering amplitude g were zero, T would be equal to 1 while S and U would be zero; thus, there would be no change of the polarization during the scattering process.

For polarization measurements it is more useful to

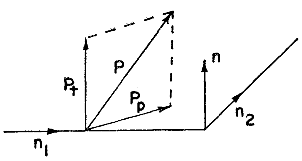


Fig. 2. Decomposition of arbitrary initial polarization.

express P_2 by the unit vectors $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_2 \times \mathbf{n}$:

$$\mathbf{P}_2 = \{\mathbf{n}(S+\mathbf{P}_1\cdot\mathbf{n})+\mathbf{n}_2(L\mathbf{P}_1\cdot\mathbf{n}_1+R\mathbf{P}_1\cdot[\mathbf{n}_1\times\mathbf{n}]) + [\mathbf{n}_2\times\mathbf{n}](L\mathbf{P}_1\cdot[\mathbf{n}_1\times\mathbf{n}]-R\mathbf{P}_1\cdot\mathbf{n}_1)\}/[1+S(\mathbf{P}_1\cdot\mathbf{n})], \quad (11b)$$

where $L=U \sin\theta+T\cos\theta$, $R=U\cos\theta-T\sin\theta$.

The results given here show that the scattering amplitudes f and g provide a complete description of the intensity distribution and the behavior of the polarization in the scattering process. The problem is to find reliable values for the complex functions f and g.

B. Elastic Scattering by Strongly Screened Coulomb Field

For scattering in the pure Coulomb field, f and g have been calculated by Sherman (1956) for certain combinations of the relevant parameters: Z (atomic number), E (electron energy), and θ. Figure 3 reminds us of the essential results given by the theory for scattering in the Coulomb potential. The Sherman function S, which has very low values at small Z, has appreciable values for heavy nuclei. These larger values (up to about 0.5 for Au) are found preferentially at large scattering angles, whereas S is very small at small angles. The angular and energy dependences of S are given by curves which are relatively smooth.

These theoretical values for S could be confirmed experimentally (Mikaelyan, Borovoi, and Denisov, 1963; Apalin *et al.*, 1962; van Klinken, 1966) for gold targets if, as in Fig. 3, the energies and scattering angles were chosen high enough so that the description of the scattering as a deflection in the pure Coulomb field of the nucleus was a good approximation. At somewhat lower energies where the screening of the nucleus by the atomic electrons begins to play a role, the agreement between theory (Bonham, 1962; Lin, Sherman, and Percus, 1963; Lin, 1964; Holzwarth and Meister, 1964a; 1964b) and experiment is doubtful (van Klinken, 1966), even if screening is taken into account theoretically.

As for numerical evaluations of f and g at low electron energies, where the influence of screening is crucial, extensive calculations for elastic scattering have been made in recent years. Numerical results for the scattering amplitudes and/or their products have been published down to 100 eV [cross sections even lower (Schonfelder, 1966)] for some heavy elements (mainly $Z=80$ and 79, some data also for $Z=81$ and 83) by several authors (Holzwarth and Meister, 1964a; 1964b; Schonfelder, 1966; Bunyan, 1963; Bunyan and Schonfelder, 1965); these show that the few early calculations mentioned in Sec. I (which had to be made without the aid of computers) gave a proper qualitative description of the polarization. While this article was being prepared, numerical evaluations of the polarization have been published for iodine at 400 eV by Yates (1968a). Similar calculations which have not yet been published were made for iodine by Bühring* at 200, 300, 400, and 600 eV and at 300 eV by Walker,*† who also made unpublished calculations for bismuth at 300, 400, 500, 700, 900, and 1200 eV. Coulthard* made unpublished calculations for mercury at 10, 15, 20, 25, 30, 40, 50, 60, 70, 75, 80, and 90 eV and for argon, krypton, and xenon at 40, 1000, and 300 eV, respectively.†

As an example of the polarization effects predicted by the new calculations, Fig. 4 gives the function S which describes the polarization of an electron beam resulting from the scattering of an unpolarized beam [Eq. (9)]. Contours $S(\theta, E)=$ const for Hg with 100 eV $\leq E \leq 3$ keV are shown‡ in a logarithmic scale. One can see that there are several areas where $|S|$ has high values of more than 0.8. According to the numerical tables (Holzwarth and Meister, 1964b) and a paper by Bühring (1968b) to be discussed later, values close or equal to 1 should be attainable at certain combinations of energy and scattering angle in the middle of the peaks of Fig. 4. The potential used in calculating these results was the relativistic Hartree potential for mercury computed by Mayers (1957). Several effects which are known to be important in

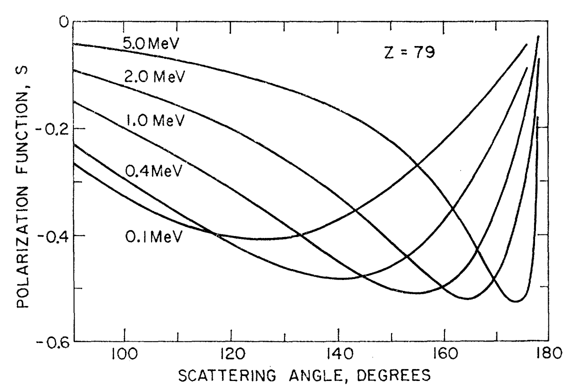


FIG. 3. Dependence of the asymmetry function S on the scattering angle for a gold target at various energies (from Motz, Olsen, and Koch, 1964). These data are valid for scattering in the Coulomb field of a point nucleus with no screening.

* The author is indebted to Dr. W. Bühring, Dr. M. A. Coulthard, and Dr. D. W. Walker for kindly making available their unpublished results.

† See the addendum.

‡ The author is indebted to Dr. K. Jost for kindly providing this figure.

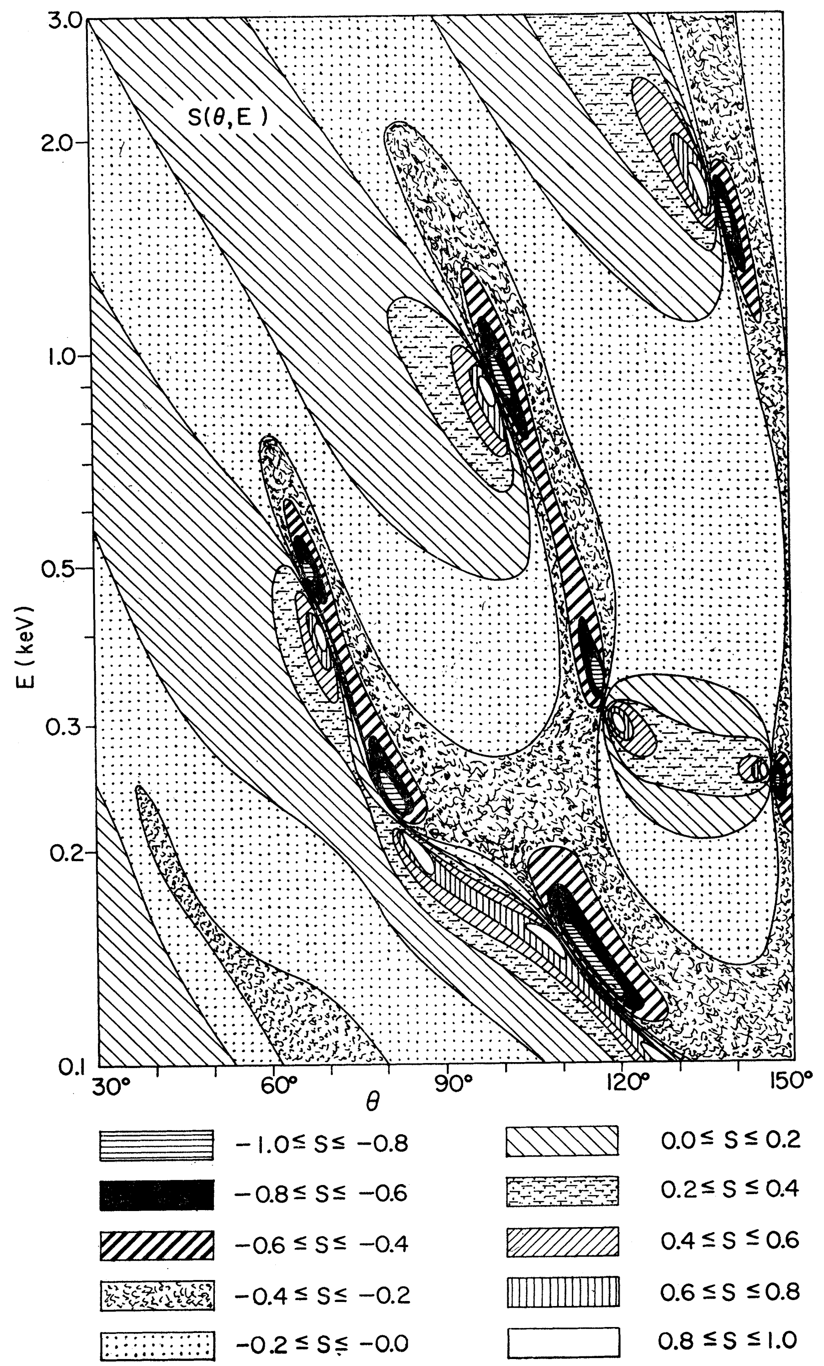


FIG. 4. Contours of $S(\theta, E) = \text{const}$ for Hg. For the special case of scattering of an unpolarized electron beam, $S=P$, the polarization of the scattered electrons. Logarithmic energy scale.

slow electron scattering, like exchange effects and electrical polarizability of the atom by the scattered electron [cf. Mott and Massey (1965), Chap. XVIII], have not been considered in the theories (Holzwarth and Meister, 1964a; 1964b; Schonfelder, 1966; Bunyan, 1963; Bunyan and Schonfelder, 1965), so that there has been some concern about the reliability of the theoretical results. As we shall see in Sec. III, the agreement of theory and experiment in the energy range shown in Fig. 4 turns out to be very good despite these approximations.

Marked polarization effects at low energies, like those shown in Fig. 4, could not be anticipated by a simple extrapolation of the facts known from scattering in the pure Coulomb field, for in that case the polarization of the scattered electrons decreases when the

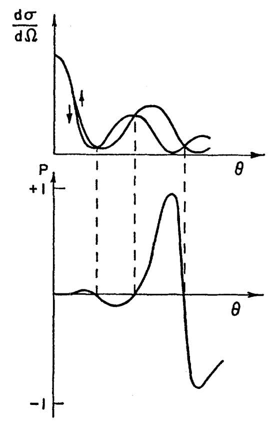


Fig. 5. Relation between the cross sections for spin-up and spin-down electrons and the polarization of an electron beam obtained by scattering an unpolarized beam.

electron energy and the atomic number of the scattering nucleus becomes smaller. In the case under discussion, the electrons are very slow and the "effective atomic number" is very low because of the strong influence of screening at these energies. Therefore, from a naive point of view, it seemed obvious that little polarization was to be expected here.

In order to get a clear understanding of the origin of the polarization phenomena, it is expedient to use an optical model, although nearly all the quantitative calculations have been made by the method of partial waves. A simplified qualitative explanation of the polarization effects can be given as follows:

Although the electron beam is scattered by an electrostatic field (magnetic moments of the atom have not been considered by the aforementioned theories), the magnetic moment of the electron $\boldsymbol{\mu}$ plays a role in the scattering process because in the rest frame of the electron there is a magnetic field

$$\mathbf{H} = -(\mathbf{v}/c) \times \mathbf{E} \qquad (12)$$

interacting with $\boldsymbol{\mu}$. The interaction energy is $-\boldsymbol{\mu}\cdot\mathbf{H}$ which, after substituting $\mathbf{H}$ from Eq. (12) and $\boldsymbol{\mu} = (e/mc)\mathbf{S}$, is seen to be the spin–orbit energy

$$(2m^2c^2)^{-1} r^{-1} (dV/dr) \mathbf{S}\cdot\mathbf{L} \qquad (13)$$

except for a factor of 2 caused by the Thomas precession (Thomas, 1926; Farago, 1967). Qualitatively, the resulting scattering potential is the sum of the screened Coulomb potential and the spin–orbit potential and is thus dependent on the relative direction of spin $\mathbf{S}$ and orbital angular momentum $\mathbf{L}$ of the incident electron.

We can regard the incident unpolarized beam as being made up of equal numbers of electrons with spins parallel and antiparallel to the normal of the scattering plane* ("spin-up" and "spin-down" electrons). The sign of the spin–orbit potential [see Eq. (13)] and thus to the resulting scattering potential is different for spin-up and spin-down electrons of the same **L**. In other words, the "effective radius" of the atom (which can be defined as that radius where the potential has dropped to a certain value) is different for spin-up and spin-down electrons of the same **L**.

The elastic scattering can be regarded as a diffraction of the incident electron wave by the atom. At the energies discussed here, the electron wavelength λ is comparable with the atomic radius R, and one gets a typical interference pattern with distinct maxima and minima of the scattered intensity (quantitatively shown in Fig. 17). Since the positions of the minima and maxima are determined by λ/R and the effective radius R is different for spin-up and spin-down electrons, there is a relative shift of the diffraction patterns for the two spin directions as illustrated in Fig. 5. Apart from this shift, there will also be some difference in the shape of the cross sections because of the somewhat different scattering potential. For these reasons, the cross sections for spin-up and spin-down electrons are usually different from one another at a certain angle θ, i.e., there are different numbers of spin-up and spin-down electrons in the scattered beam so that this beam is polarized. Recalling the definition of the polarization [Eq. (6)] and the fact that the numbers $N_\uparrow$ and $N_\downarrow$ in the scattered beam are proportional to the corresponding cross sections, we find

$$P = \frac{(d\sigma/d\Omega)_\uparrow - (d\sigma/d\Omega)_\downarrow}{(d\sigma/d\Omega)_\uparrow + (d\sigma/d\Omega)_\downarrow} \qquad (14)$$

and can readily construct the polarization curve from the cross section curves as indicated in Fig. 5. Whenever one of the two cross sections has such a deep minimum that its value is very small compared to that of the other cross section at the same angle, $|P|$ is close to 1 because then there are virtually only electrons of one spin direction in the scattered beam.

Figure 6 shows a quantitative example for these relations between the cross sections and the polarizations. One can see that the mutual shift and the change in the shape of the cross section are not very great even for a heavy atom like mercury.

The angular dependences of the cross sections for fast electron scattering (energies as given in Fig. 3) have no minima and maxima any more, because λ is now much smaller than the atomic dimensions. The cross sections for spin-up and spin-down electrons drop monotonically with increasing θ, so that the construction of the polarization from these cross sections (analogous to Fig. 5) results in the smooth curves of Fig. 3. There are no high polarization peaks because there are no deep minima in the cross sections.

The optical analogy used here was very suitable for our heuristic discussion. However, for obtaining

* This special decomposition of the unpolarized beam has the advantage that the spin directions are not changed in the scattering process because the magnetic field [Eq. (12)] is perpendicular to the scattering plane, too.

quantitative results on the polarization effects, an optical model has been used only in the investigation made by Goldberg (1963). He calculated cross sections and spin polarizations of the scattered electrons for $Z=18$ (argon) and energies between 0.01 and 100 eV. The optical model was based on a Thomas–Fermi–Dirac potential obtained from the screening function tables of Abrahamson (1961) plus the standard spin-orbit term and an additional term

$$V_p(r)=e^2\alpha/(r^2+R_p^2)^2, \tag{15}$$

which takes into account the electrical polarizability α of the atom; R_p is an adjustable parameter. Figure 7 gives some results for two electron energies (4 eV, 20 eV) and three values of R_p, including the case $R_p=\infty$ or $V_p=0$. One can see that the results are similar to those obtained for mercury by other methods. $S(\theta)$, the polarization $P(\theta)$ of the scattered electrons for the special case of scattering of an unpolarized electron beam, is an oscillating curve whose structure is very sensitive to the exact form of the scattering potential, especially in the case of the lower electron energy, where the electrical polarizability of the atom plays an important role. The polarization curve does not go up to values close to 1 (or 100%) in Fig. 7; also in Fig. 4 only a small fraction of the possible curves $S(\theta, E=\text{const})$ go through high peaks. Thus, one cannot say very much about the polarization peaks for argon from the few curves calculated so far.*

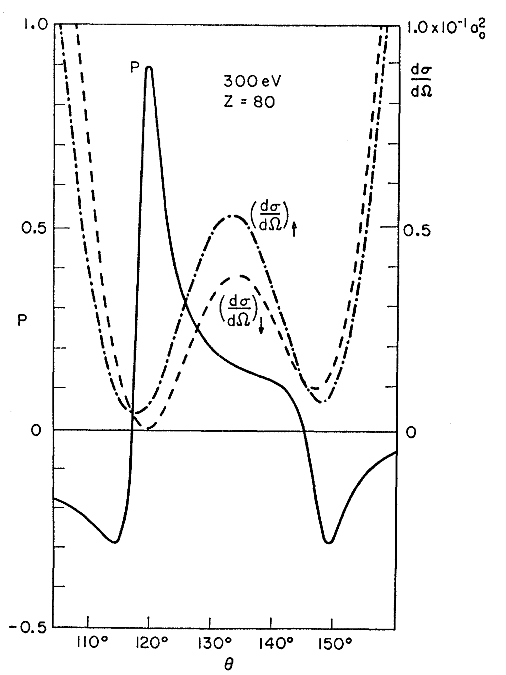


Fig. 6. Quantitative example for the relation between the cross sections for spin-up and spin-down electrons and the polarization. (a_0=Bohr radius in hydrogen.) (Data from Holzwarth and Meister, 1964b.)

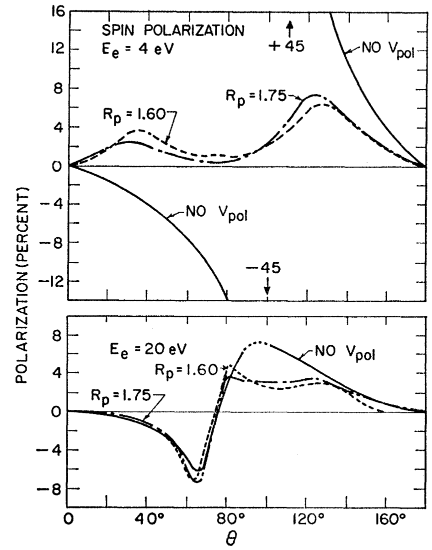


Fig. 7. $S(\theta)$ for argon at 4 and 20 eV calculated with an optical model (reproduced from Goldberg, 1962). (For the meaning of S cf. the caption of Fig. 4 or the text.)

For the other elements, which we did not mention, no theoretical polarization data are available at present. In view of their importance (see, e.g., Sec. III), these data should be calculated at least for such a variety of atomic numbers that the remaining data can be interpolated.

It is interesting to note that there is a close analogy between the polarization effects in electron scattering and those of nucleon scattering which have been investigated for a much longer period and more extensively. At energies of about 10 to 100 MeV, the de Broglie wavelengths of proton, neutron, and the light nuclei are comparable to the nuclear radii, just as the electron wavelength is comparable to the atomic radii in the 100-eV range. Thus, there is a direct analogy between the oscillating cross sections and polarization curves found in nuclear scattering [here we quote only one typical example (Satchler, 1967) from the many papers treating these phenomena] and the corresponding results in slow electron scattering. The only difference is that the nature of the forces which bring about these results is different (electromagnetic and nuclear forces, respectively). Therefore, in nuclear scattering there are relations between the differential cross section and the polarization very similar to those discussed before. Rodberg (1960), Hüfner and De Shalit (1965), and De Shalit (1966) showed how to express these relations more quantitatively with certain approximations.

Extending the results of De Shalit (1966), Bühring

* See the addendum.

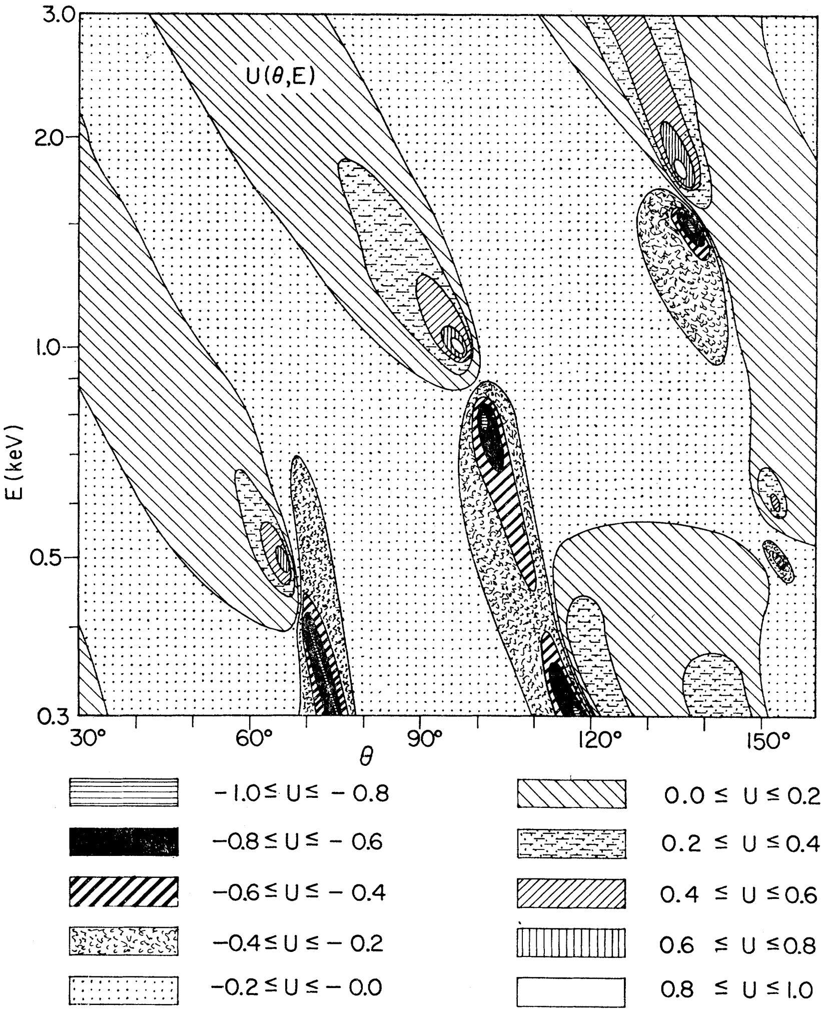


FIG. 8. Contours of $U(\theta, E)$ = const for Hg. U describes the rotation of the polarization out of its initial plane. Logarithmic energy scale.

(1968) showed that valuable information on the electron polarization in low-energy electron scattering can be obtained when the scattering amplitude $f(z)$ (with $z = \cos\theta$) is continued analytically into the complex z plane, and its properties in the neighborhood of the real axis are studied. In particular, he studied the trajectories in the complex plane on which the zeros of $f(z)$ move when the energy is varied. This led to some relations which allow one to interpolate the extrema and the positions of the zeros of the polarization curve for a certain energy from the cross sections and polarizations near the intersections of these trajectories with the real axis. By these relations the combinations of energy and scattering angle for which total polarization occurs can be easily found. Bühring applied the results of his calculations to mercury, iodine, krypton, argon, and neon with the unexpected consequence that even for the lightest of these atoms, total polarization should be attainable. Another interesting result of his considerations is the significant quantitative dependence of the polarization on the atomic model used to describe the screened Coulomb field.

Apart from the function S discussed so far, the functions T and U defined in Eq. (10) are necessary for a complete description of the polarization effects in electron scattering. These quantities, which describe the rotation of the polarization vector during a scattering process, have seldom been discussed in theoretical papers on electron polarization and never in experimental papers.* The reason is that a measurement of T and U would be at the margin of what can be done presently. Such a measurement would have to be a triple scattering experiment if the primary beam is unpolarized, whereas measurements of S are made by double scattering experiments (cf. Sec. III). By the first scattering, a beam of well-defined polarization P_1

* See the addendum.

would be produced, the second scattering would transfer P_1 to P_2 (the process to be studied), while the third scattering would be necessary to analyze P_2.

From the tables of Holzwarth and Meister (1964b), values of $T(\theta)$ and $U(\theta)$ could be determined for the energies given there. Interpolation for the other energies leads to the contours of $U(\theta, E)=$ const for Hg given in Fig. 8. They cover a smaller energy range than the curves $S(\theta, E)=$ const in Fig. 4 because fewer data weie available, particularly at the lowest energies considered. Comparison of Figs. 4 and 8 shows that the function U has its peaks near those areas in the E–θ diagram where $|S|$ also has very high values.

The contours $T(E, \theta)=$ const are not displayed here, since they can be determined from the corresponding figures for S and U using the relation $S^2+T^2+U^2=1$, which follows from Eqs. (5) and (10).

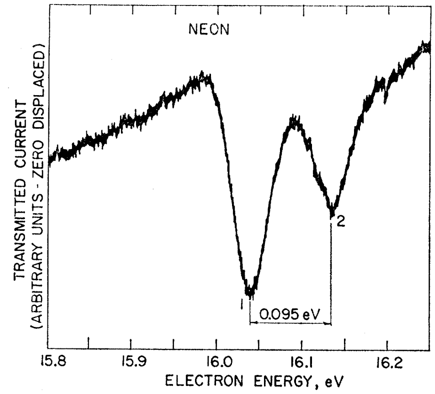


FIG. 9. Transmission of electrons by neon, showing the first two resonances in the total cross section located about 0.5 eV below the first excited states of neon (from Kuyatt, Simpson, and Mielczarek, 1965).

C. Resonance Scattering

Again, the effect to be discussed has a direct analogy to a phenomenon which is well known in nuclear scattering. Although spin polarization in resonance scattering of nucleons was predicted as early as 1946 (Schwinger, 1946) and investigated since then (Wolfenstein, 1949; Faissner, 1959; de Facio and Gammel, 1966), resonance scattering of electrons has been detected only in recent years. Schulz (1963) reported a strong resonance in the cross section for electron elastic scattering by He occurring at 19.3 eV, i.e., about 0.5 eV below the first excitation threshold. Similar and more-detailed results were found in many other cases [for a list of pertinent references cf. the review of K. Smith (1966)] confirming that the resonance results from the temporary formation of a negative ion in a compound state analogous to the compound nuclei formed in nucleon scattering.

In the case of neon, Simpson and co-workers (Kuyatt, Simpson, and Mielczarek, 1965) were the first to show that one can find a doublet structure of the resonance if the energy resolution is high enough (Fig. 9). [More recent measurements have been made by Andrick and Ehrhardt (1966).] This structure has been interpreted by Simpson and Fano (1963) as corresponding to a fine-structure splitting of the compound ion with the configurations $(1s^22s^22p^53s^2)P_{3/2,1/2}$. Since there is not much overlap between the cross sections of these two configurations, which differ by the electron spin direction, it seems plausible to expect an appreciable polarization of the electrons passing through one of these resonances, analogous to the polarization of neutrons and protons observed in resonance scattering from helium (Schwinger, 1946; Wolfenstein, 1949; Faissner, 1959; de Facio and Gammel, 1966). This has been predicted by Franzen and Gupta (1965), who calculated the polarization to be expected.

It seems worth noting that maximum polarization is not to be expected when only one state (for instance $P_{3/2}$) contributes to the scattering cross section. On the contrary, in that case the polarization, which is a result of the interference between the potential scattering and the resonance scattering of the p waves from the split level, is zero. This general fact can be easily proved in the special case where all the partial waves with $l\geq 2$ can be neglected. (The higher partial waves are not very important, anyway, at the small electron energies where resonance scattering plays a role.) Then we get from Eq. (1)

$$f=(i/2k)\{1-\exp(2i\delta_0)+[3-2\exp(2i\delta_1^+)-\exp(2i\delta_1^-)]\cos\theta\},$$
$$g=(i/2k)\sin\theta[\exp(2i\delta_1^+)-\exp(2i\delta_1^-)], \qquad (16)$$

where we use notation δ_1^+ and δ_1^- to show that the phase shifts δ_l and δ_{-l-1} in Eq. (1) belong to the partial waves with $j=l+\frac{1}{2}$ and $j=l-\frac{1}{2}$ (cf. Bethe and Salpeter, 1957). Substitution of Eq. (16) in Eqs. (5) and (9) yields, for the polarization of an initially unpolarized beam,

$$P=[2\sin\theta\sin(\delta_1^+-\delta_1^-)/k^2I(\theta)]\times[\sin\delta_0\sin(\delta_0-\delta_1^+-\delta_1^-)-3\sin\delta_1^+\sin\delta_1^-\cos\theta], \qquad (17)$$

where $I(\theta)$ is defined in Eq. (4).

We see that the polarization disappears not only for $\delta_1^+=\delta_1^-$, which is a trivial case because the level splitting disappears (Wolfenstein, 1949; Franzen and Gupta, 1965), but also if only one of the phase shifts is different from zero.

A theoretical analysis of electron polarization by resonance scattering in neon was made by Franzen and Gupta (1965). The authors calculated the scattering amplitudes in the vicinity of the resonance. They determined phase shifts by making use of the results

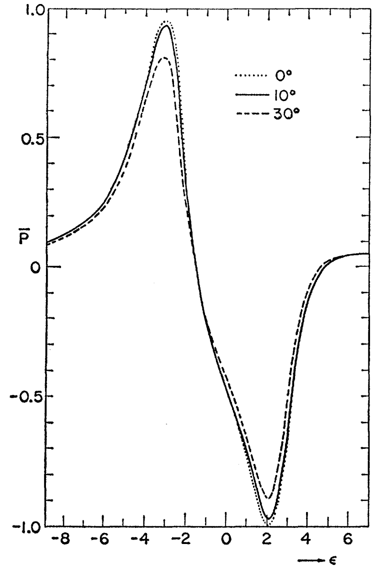


FIG. 10. Electron polarization $\bar{P}$ averaged over three different solid angles as a function of electron energy for elastic scattering by 90° from neon in the vicinity of the scattering resonance at $E_0=16.079$ eV. Here $\epsilon=(E-E_0)/\frac{1}{2}\Gamma$, where $\Gamma=0.038$ eV (from Franzen and Gupta, 1965).

of Simpson (Fig. 9) and of the absolute differential cross sections for the scattering of 15.9-eV electrons by neon measured by Ramsauer and Kollath (1932). From the semiempirical scattering amplitudes thus obtained, the authors calculated the polarizations for scattering into finite solid angles centered around $\theta=90°$, which are given in Fig. 10. The electrons are assumed to be scattered into the solid angle subtended by a pyramid of square cross section whose axis is perpendicular to the initial direction ($\theta=90°$) and whose faces make the indicated angles with its axis.

The average polarization $\bar{\mathbf{P}}$, which is perpendicular to the incident beam and to the axis of the pyramid, has very high peak values for the three solid angles indicated. It is interesting to note that $\bar{P}$ changes very rapidly with the electron energy but depends only slightly on the solid angle. This behavior is contrary to that of the polarization curves shown in Fig. 4. Thus, in the latter case the energy resolution can be 100 times worse than in the resonance scattering without affecting the measured polarization curve, while the angular resolution must be about ten times better than in the resonance case.

The curves of Fig. 10 are the only theoretical results on the polarization in resonance scattering of slow electrons which are available at present. Further investigations are desirable.

D. Related Effects

Slow electrons can be polarized by exchange scattering from polarized atoms, too, but here it is not the scattering process as such but the polarization of the target which gives rise to the electron polarization. Effects of this kind are only loosely related to the problems to be discussed here and will be mentioned only briefly.

Exchange scattering of electrons from atoms with a preferential population of certain spin states [which may be produced by the Stern–Gerlach experiment or optical pumping with polarized light (Kastler, 1954)] results in a transfer of polarization from the atoms to the electrons. The theoretical background of this polarization transfer has been discussed by Byrne and Farago (1965); experiments have been made by Farago and Siegmann (1966). Since no exact theoretical data are available, one has to consult the few experimental investigations existing so far (Dehmelt, 1958; Franken, Sands, and Hobart, 1958; Collins, Goldstein, Bederson, and Rubin, 1967; Lichten and Schultz, 1959) for the values of the spin exchange cross sections.

Naturally, the exchange effect does not result from an explicit spin dependence of the scattering potential, but from the fact that the incident and the atomic electron are identical particles of spin $\frac{1}{2}$, requiring an appropriate symmetrization of the wave functions. The results of Burke and Schey (1962), who treated the influence of the spin directions of incident and atomic electrons in electron–hydrogen scattering, show that the effective spin dependence of the forces due to the Pauli principle can have quite remarkable effects. [This is also known from Møller scattering (Frauenfelder and Steffen, 1965).] Recently, these results have been extended by Goldwire (1967) to scattering from He^+, but much work is left to be done since no data are available for other elements and for electron energies higher than the lowest excitation energy.

III. EXPERIMENTAL RESULTS

Intensive experimental studies of the polarization effects connected with potential scattering in the strongly screened Coulomb field have been made in the present decade. It is the main purpose of this section to survey these experiments, their theoretical aspect having been discussed in Secs. II.A and II.B. Electron polarization in scattering from single atoms, molecules, and solid targets has been studied and will be treated successively.

Polarization in resonance scattering has been studied in only one experiment so far (Reichert and Deichsel, 1967) (cf. Sec. III.A.3) though other work is in preparation (Franzen).

A. Scattering by Atoms

The theories which have been discussed in Sec. II deal with scattering from single atoms. Therefore, any experiment with slow electrons which is to be compared with these theories is made preferably with targets of gas atoms of low density. Besides, the theoretical results on the polarization effects hold for elastic scattering only; thus, the inelastically scattered electrons must be eliminated in the experiments to make possible a direct comparison. Electron polarization in inelastic scattering has been studied neither theoretically nor experimentally, although this would be an interesting subject.

1. Mercury

Extensive measurements of electron polarization in low-energy elastic scattering from mercury have been made in two laboratories during the past few years: In the group (Deichsel, 1961; Deichsel and Reichert, 1965; Steidl, Reichert, and Deichsel, 1965; Deichsel, Reichert, and Steidl, 1966) of Kollath, who published the first ideas on measurements of this kind (1949), at the University of Mainz, and in the author's group (Jost and Kessler, 1965; 1966; Eitel, Jost, and Kessler, 1967; 1968) at the University of Karlsruhe. In both cases double scattering experiments were made, the polarization due to the first scatterer having been analyzed by the second scattering process. As an example for the basic experimental arrangement, Fig. 11 gives a schematic diagram of the apparatus used by the Karlsruhe group.

A beam of electrons crosses an atomic beam which scatters some of the electrons. The scattering angle θ could be varied continuously between 0° and $\pm 150°$ by rotating the electron gun about the axis of the atomic beam. The scattered-electron beam is expected to be polarized, the polarization P being perpendicular to the scattering plane [cf. Eq. (9)] as indicated in Fig. 11.

The measurement of the polarization was performed by a "Mott detector" which, because of the many investigations on polarization in β decay (van Klinken, 1966; Frauenfelder and Steffen, 1965; Schopper, 1966), has become the most accurate device for this purpose. The best results with this method are obtained for electron energies above 100 keV. Therefore, the polarized electrons which passed through a filter lens (Simpson and Marton, 1961; Kessler and Lindner, 1964) for removing inelastically scattered electrons were accelerated to 120 keV before they entered the Mott detector. There they were scattered for a second time. In the geometrical arrangement of Fig. 11, the ratio of N_l and N_r, the number of electrons counted in the left and right counter, respectively, is, according to Eq. (7), given by

$$N_l/N_r = (1+PS)/(1-PS). \qquad (18)$$

Here S is the asymmetry function of the target in the Mott analyzer and is well known at $Z=79$ (gold) and electron energies E above 100 keV, particularly at 121 keV where van Klinken (1966) made very accurate measurements. Also, at this energy and $\theta=120°$, there is only a slight dependence of S on E and θ (cf. Fig. 3), so that $Z=79$, $E=120$ keV, and $\theta=120°$ were chosen as parameters for the Mott detector in Fig. 11. From the known value of S together with the measured ratio N_l/N_r, P can be determined.

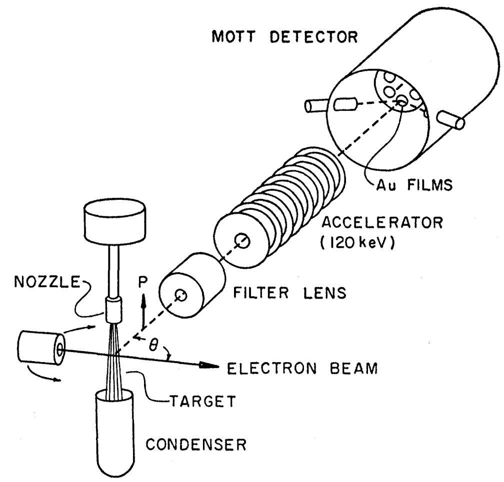


FIG. 11. Schematic diagram of double scattering experiment for measuring the polarization (from Jost and Kessler, 1966).

It is not necessary to discuss here the elimination of systematic errors like instrumental asymmetries or the change of S due to finite thickness of the gold foil; this is now standard technique and can be found in many original papers and reviews (Frauenfelder and Steffen, 1965). The statistical error of the polarization is given by*

$$\Delta P = [(S^{-2}-P^2)/(N_l+N_r)]^{1/2}, \qquad (19)$$

which follows from Eq. (18) together with the law of error propagation. In most cases $1/S^2 \gg P^2$ is fulfilled and we obtain

$$\Delta P \approx [S(N_l+N_r)^{1/2}]^{-1}. \qquad (20)$$

The apparatus of the Mainz group differs from that of Fig. 11 mainly in the second scattering process: The polarization is analyzed by scattering the electrons at *low* energies from a mercury atomic beam by $\pm 90°$ (in the newer papers). The asymmetry function for mercury at 90° and low energies, which is needed in order to calculate the polarization according to Eq. (18), is determined by the authors themselves: If scattering angle, electron energy, and target are the same in both scattering processes, we see from Eqs. (18) and (9) that

$$N_l/N_r = (1+S^2)/(1-S^2), \qquad (21)$$

and $|S|$ can be determined by a measurement of the left–right asymmetry.

The main advantage of this arrangement is that the electrons do not have to be accelerated to high energies (>100 keV) after the first scattering, whereas in the arrangement of Fig. 11 either the scattering chamber

* The plus sign in Jost and Kessler (1966) in the numerator of this formula is a misprint.

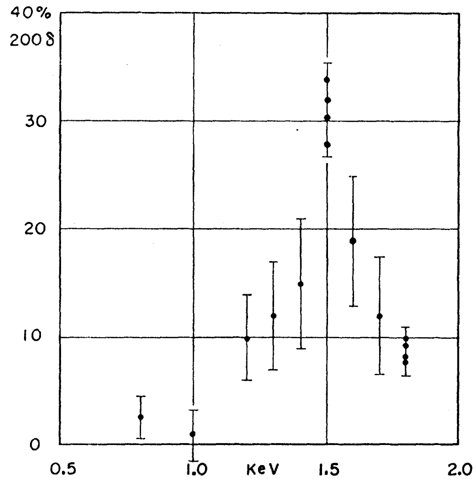


Fig. 12. The first experimental proof of electron polarization in low-energy scattering. Energy dependence of the scattering asymmetry $200\delta(\delta=S^2)$ for double 90° scattering by mercury (from Deichsel, 1961).

or the Mott detector must be at high potential (in the experiments of the Karlsruhe group, it was the Mott detector).

On the other hand, it appears worthwhile to face this complication because of the advantages of the conventional Mott detector. In this case the asymmetry function S has been determined very accurately by various authors, so that Eq. (18) yields very accurate values for P, and the efficiency of the detector* is much larger. The ratio of the numbers of incident and scattered electrons was about 10^{-4} in our case (S was 0.26) and even 6×10^{-4} in van Klinken's experiment (1966). The large efficiency of the Mott detector makes it possible to analyze even the polarization of low-intensity beams quite accurately. Therefore, the polarization curves could be measured with high angular resolution ($\pm1°$ in most cases) which is equivalent to low intensity of the polarized beam because of the small angular divergence of primary and scattered beam. High angular resolution is crucial, especially for measurements near the higher values of the polarization, which changes very rapidly with θ as shown in Fig. 4.

The first experimental proof that low-energy electrons can be polarized by scattering was given by Deichsel (1961) and is reproduced in Fig. 12. The left–right asymmetry was measured in a double scattering experiment with $\theta=90°$ and $Z=80$ in both scattering processes, yielding S^2 according to Eq. (21). Although quantitatively the results for $\delta=S^2$ do not agree for all energies with the theoretical and experimental data found later on (cf. Bunyan, 1963, for a thorough discussion of these very first measurements), the peak at 1.5 keV is compatible with the results of subsequent work. This can be seen from Fig. 13, which shows more recent experimental data by Eitel, Jost, and Kessler (1967) together with theoretical results (Holzwarth and Meister, 1964a; 1964b; Schonfelder, 1966; Bunyan, 1963; Bunyan and Schonfelder, 1965); $P(E, 90°)$ is shown for mercury in the energy range 50–2500 eV covered by the measurements of the Karlsruhe group. Since we are concerned with the polarization P arising from the scattering of an unpolarized beam throughout this section, we have $P\equiv S$. Therefore, 200 P^2 should give the data of Fig. 12, which is about right for $E>1300$ eV. The agreement between the new experimental data and the theoretical predictions is remarkably good, as Fig. 13 shows. This is a general feature of the results for mercury and will be discussed later in this section.

The angular dependence of the polarization has also been measured extensively for mercury. In these experiments, the energy has been varied in small steps between 3.5 eV (Deichsel, Reichert, and Steidl, 1966) and 1700 eV (Jost and Kessler, 1966). The following figures show a few typical examples of these measurements. Practically all the experiments have been made in the angular range 30° to 150°, for below 30° the polarization is so small that measurements appeared not very appealing, whereas measurements far beyond 150° were not possible with the layout of the present scattering chambers.

Figure 14(a) shows $P(\theta)$ for 1700 eV, the highest energy for which a complete polarization curve from 30° to 150° has been measured. At this energy, only a few theoretical values are available; they agree quite well with the experimental results.

The 900-eV curve shown in Fig. 14(b) has even more pronounced minima and maxima. At this energy the highest polarization measured so far was observed: $P=-85\%$ at $\theta=101°$. It is quite obvious that in Fig. 14 and some of the other examples given here, the experimental peak values of the Karlsruhe group are slightly lower than the theoretical ones. However, this is not a failure of the theory. Because of the finite

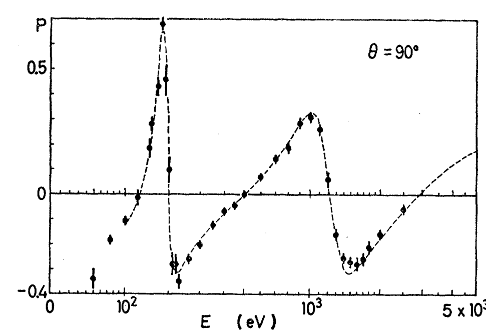


Fig. 13. Energy dependence of the polarization P for $\theta=90°$, $Z=80$. Solid circles, experimental values with statistical errors; dashed line, theory. Pseudologarithmic energy scale defined by log $(1+E/100$ eV) (E in electron volts) (from Eitel, Jost, and Kessler, 1967).

* As can be seen from Eq. (20), an adequate measure for the efficiency of a Mott detector is the ratio $S^2(N_l+N_r)/N$, where N is the number of incident electrons.

angular resolution of the measurement, the experimental data are average values over an angular range of $\pm 1°$. This angular resolution is almost everywhere good enough to make possible a direct comparison of the experimental results with the theoretical polarization curves which have been calculated for an ideal angular resolution; the only exceptions are peaks of the very

(a)

(b)

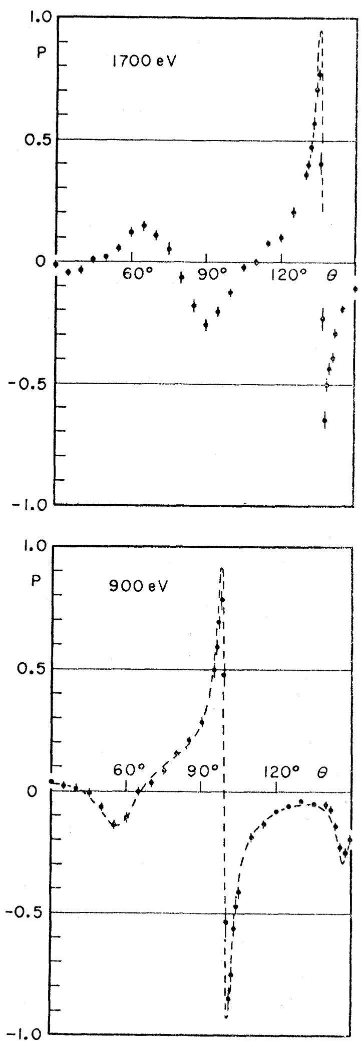


FIG. 14. (a, b) Angular dependence of the polarization P for 1700 and 900 eV, $Z=80$. Experimental and theoretical values (from Jost and Kessler, 1966).

narrow maxima and minima with widths comparable to $\pm 1°$. This fully explains the somewhat lower experimental peak values which occasionally occur.

The theoretical values used in this section are either those of the Munich group (Holzwarth and Meister, 1964a; 1964b) or of the Melbourne group (Bunyan, 1963; Bunyan and Schonfelder, 1965), depending on what was available at the energies considered. For those energies where data of both groups are available, they

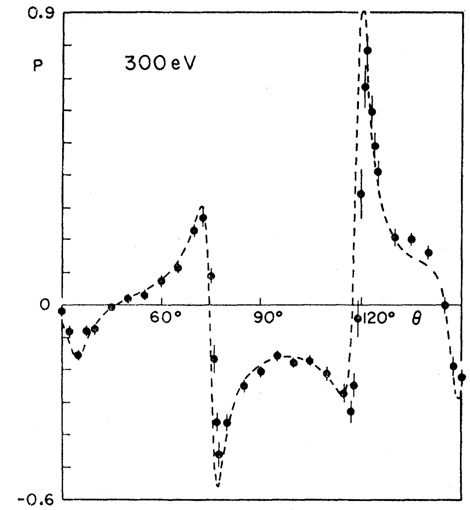


FIG. 15. Angular dependence of the polarization P for 300 eV, $Z=80$. Experimental and theoretical values (from Jost and Kessler, 1966).

are practically identical. The theoretical curves plotted in the papers of the Mainz and Karlsruhe groups, respectively, look different only because in the former work the theoretical data have been convoluted with the angular resolution of $\pm 3°$. This gives rise to an appreciable modification of the curves near the peaks.

Although the agreement between experimental and theoretical polarizations is generally very good, some slight deviations exist. An example is given in Fig. 15 which shows $P(\theta)$ for 300 eV. The experimental curve is shifted by about 2° towards larger angles. Some of the following figures show a similar behavior at other energies. The angular shift can be seen even better in Fig. 16, where the strong energy dependence of the polarization peak in the neighborhood of 300 eV is shown. At the time when these measurements were published (Jost and Kessler, 1966), no theoretical

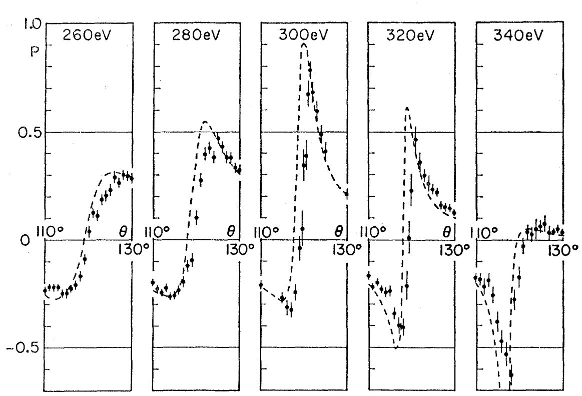


FIG. 16. Energy and angular dependence of the polarization P for $Z=80$ in the neighborhood of 300 eV, 120°. Experimental values from Jost and Kessler (1966); dashed line, unpublished theoretical values by Coulthard [see Footnote (*), p. 16].

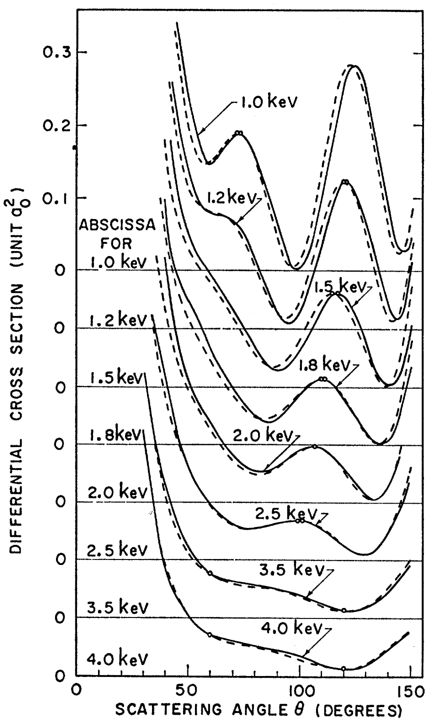


Fig. 17. Differential cross sections for electron scattering by mercury between 1000 and 4000 eV. Solid line, experiment; dashed line, theory. At the points marked by a circle, the ordinate of each experimental curve has been normalized to the corresponding theoretical curve. (a_0=Bohr radius in hydrogen.) (From Kessler and Lindner, 1965.)

results for 260, 280, 320, and 340 eV were available. The theoretical data of Fig. 16 have been calculated subsequently by Coulthard.*

An angular shift has been found in the cross-section measurements as well and is demonstrated in Fig. 17. Again some of the experimental curves [full line, plotted continuously by an $X-Y$ recorder (Kessler and Lindner, 1965)] are shifted by about 2° to the right.† More experimental cross-section curves can be found in some of the papers on polarization measurements (Deichsel, 1961; Deichsel and Reichert, 1965; Steidl, Reichert, and Deichsel, 1965; Deichsel, Reichert, and Steidl, 1966; Eitel, Jost, and Kessler, 1968) and also in Kessler and Lindner (1965), where comparison with theory is made. The relations between the cross section and polarization curves which have been discussed in Sec. II.B (polarization maxima near cross section minima, etc.) have been confirmed in all the experiments. Because of these close relations, it is quite natural that angular shifts between the theoretical and experimental curves, if any, occur in both the polarizations and the cross sections. It is worth noting that Fig. 17 shows very clearly how the interference minima and maxima disappear with increasing energy because the electron wavelength becomes less and less comparable to the atomic radius.

The few examples shown here, together with the more comprehensive results of the original papers, give evidence of the good correspondence between theoretical and experimental results if we regard the angular shift as a minor difference. On the other hand, discrepancies between theory and experiment are to be anticipated if the electron energy is low enough so that the conditions of experiment are no longer compatible with the assumptions of theory. As we pointed out in Sec. II.B, the present theories treat the scattering by a static potential without taking into account that the electron cloud of the atom is distorted during the scattering process if the incident electrons are slow enough. Furthermore, exchange and correlation effects due to electron spin have been neglected. No doubt, these effects have an appreciable influence at lower energies; the question is at what energy does this influence begin.

Discrepancies between theory and experiment have been claimed for energies below 500 eV by Schonfelder (1966), whose considerations are based on some of the results of one of the experimental groups (Steidl, Reichert, and Deichsel, 1965). Schonfelder proposed to explain these discrepancies by the above-mentioned theoretical approximations. However, this disagreement could not be confirmed in extensive measurements of our own (Eitel, Jost, and Kessler, 1967) where the polarization below 500 eV has been studied. The measurements were made down to 100 eV, which was the lowest energy for which numerical evaluations of the present theories were available. Figure 18 gives a comparison of the experimental and theoretical results for this energy, showing that the correspondence is as good as at the higher energies discussed before. We found this to be true for all the energies studied between 100 and 500 eV (Eitel, Jost, and Kessler, 1967).

These results are considered to be particularly reliable for two reasons. First, the angular resolution

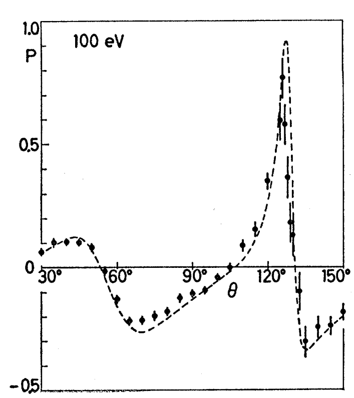


Fig. 18. Angular dependence of the polarization P for 100 eV, Z=80. Experimental and theoretical values (from Eitel, Jost, and Kessler, 1967).

* The author is indebted to Dr. Coulthard for kindly making available his unpublished results.

† While the ordinate of each experimental cross-section curve has been normalized to the corresponding theoretical curve, no such adaption has been made with the polarizations; they have been measured absolutely.

was high enough to make possible a direct comparison with the theoretical results. Therefore, with the pronounced structure of the curves, any discrepancies should have been even more distinct than in a presentation where the theoretical curves are convoluted with a lower angular resolution. Second, it was ascertained that at each energy the density of the mercury atomic beam was low enough to avoid plural scattering for which the results would no longer be comparable with the theoretical values calculated for single scattering. This turned out to be particularly important at the lower energies. Figure 19 gives an example, showing the rapid change of the polarization curve for 300 eV with increasing temperature of the mercury oven, i.e. increasing density of the atomic beam. Under the conditions of the experiment (Eitel, Jost, and Kessler, 1968), the results of Fig. 15 could be obtained only for oven temperatures up to 160°C. At higher temperatures the shape of the curve changes drastically because of a strong reduction of the polarization peaks by plural scattering.

Since the maxima of a polarization curve are close to the minima of the corresponding cross-section curve, it can be easily understood that plural scattering has this effect. As shown in Fig. 19(b), plural scattering begins to affect the differential cross section mainly by filling up the minima (for further discussion see Eitel, Jost, and Kessler, 1968). This means that electrons are scattered into the angular range of a polarization peak which, under the conditions of single scattering, would reach adjoining angles. The polarization connected with these adjoining angles is much smaller than the peak polarization or even has opposite sign, so that the maximum polarization is reduced considerably by plural scattering.

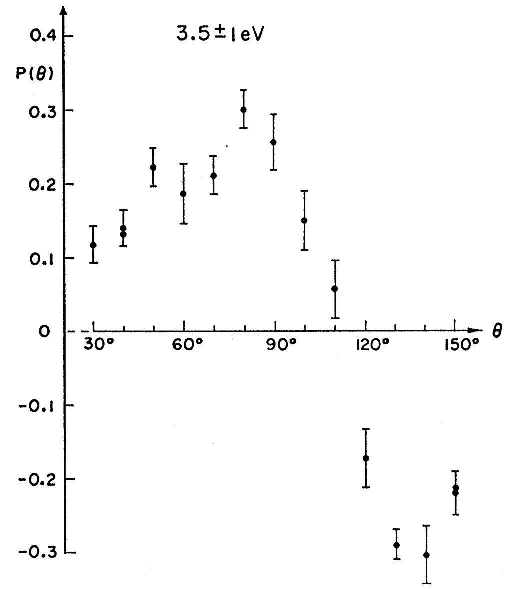


FIG. 20. Angular dependence of the polarization P for 3.5 eV, $Z=80$ (from Deichsel, Reichert, and Steidl, 1966).

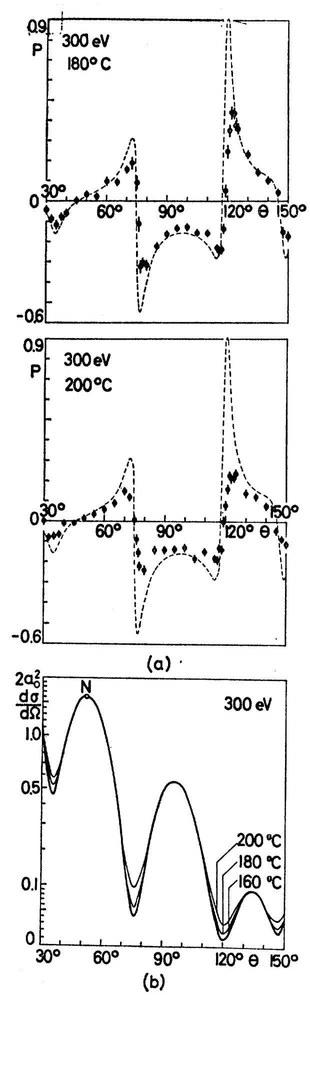


FIG. 19. (a) Effect of plural scattering on polarization curve for 300 eV, $Z=80$ and different oven temperatures. Dashed line, theoretical values for single scattering. (b) Effect of plural scattering on differential cross section for 300 eV, $Z=80$ and various oven temperatures (N = normalization point). The values for 160°C correspond to single scattering (from Eitel, Jost, and Kessler, 1968).

Summarizing the results obtained between 100 and 1700 eV for mercury, we can say that scattering by a static Hartree potential is a good description of the scattering process in this energy range. No appreciable influence of atomic polarizability, exchange and correlation effects can be found. Theoretical investigations are being made (Walker, private communication) to find out whether the slight angular shift between theory and experiment can be explained by these effects.

There is a great interest in theoretical studies which take these effects into account. Experimental results on polarizations (Deichsel, Reichert, and Steidl, 1966) and cross sections (Deichsel, Reichert, and Steidl, 1966; Eitel, Jost, and Kessler, 1968) at energies far below 100 eV, where this improvement of the theory is expected to be important, are already available. The measurements of the Mainz group have been made down to 3.5 eV, the lowest energy for which a polarization curve has been measured. The result is shown in Fig. 20. The polarization, which reaches values up to 0.3, changes less rapidly with the scattering angle because at these

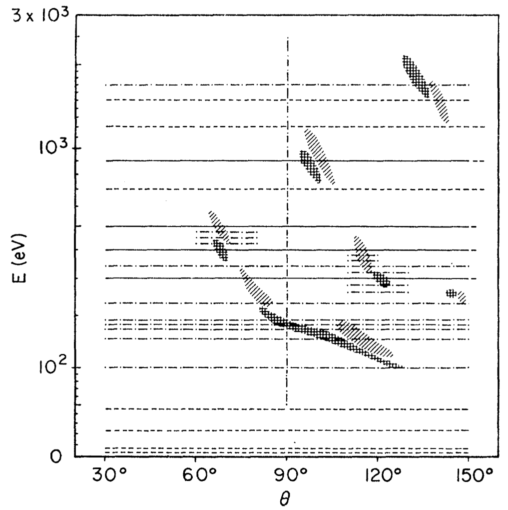


Fig. 21. Survey of angular and energy range covered by measurements of electron polarization in elastic low-energy scattering from mercury. Dashed lines, measurements of the Mainz group (Deichsel, 1961; Deichsel and Reichert, 1965; Steidl, Reichert, and Deichsel, 1965; Deichsel, Reichert, and Steidl, 1966); dash-and-dot lines, measurements of the Karlsruhe group (Jost and Kessler, 1965; 1966; Eitel, Jost, and Kessler, 1967; 1968); solid line, overlapping measurements of both groups. For better orientation areas $P>0.5$ (checkered) and $P<0.5$ (hatched) are indicated.

low energies only a few partial waves of lowest order contribute to the scattering process. Theoretical results for comparison do not exist as yet.

Since the present review can display only a few examples from the many measurements which have been made for mercury, Fig. 21 gives a survey of the energies and angles where electron polarization in elastic low-energy scattering from mercury has been measured. The measurements cover the whole region thoroughly enough to provide consistent information on the validity of the theory between 100 and, say, 2000 eV. At the energies below 100 eV, calculations and experiments were made at different energies so that a comparison is not feasible.

The calculations of Holzwarth and Meister (1964a, 1964b) have been made for a large energy range extending up to several hundred thousand electron volts. Since these higher energies are of no particular interest in the present paper, we merely mention that at these energies and at large scattering angles the agreement of the theoretical results with new experimental cross section data (Kessler and Weichert, 1968) is not as good as at low energies. For further discussion cf. Kessler and Weichert (1968) and Bühring (1968).

2. *Other Elements*

We have discussed spin polarization in elastic electron scattering from only mercury, by far the most investigated atom to date. Information on the other elements is insufficient. The only other free atom for which such measurements have been made is argon*; the results of Mehr (1967) for $P(\theta)$ at 40 eV are reproduced in Fig. 22. These were obtained with the Mainz apparatus; the first mercury atomic beam was replaced by the argon beam. The theoretical data inserted into this figure are those of Coulthard.†

For other free atoms, experimental polarization data are not available*; the theoretical situation is not much better, as pointed out in Sec. II.B. Not only are the polarization data missing, but in many cases there are no data even for cross sections at low energies. The situation at higher energies (large range around 100 keV) is more favorable. Calculations of polarization have been made for several elements scattered over the periodic table, and it is easier to make interpolations because of the smoother dependence of P on the atomic number. Cross sections, at least theoretical ones, can be found for all elements across the periodic table. Although these cross sections are based on the Born approximation (Motz, Olsen, and Koch, 1964) in many cases,‡ they are, within certain limits (Fink and Kessler, 1966), accurate enough for most purposes. This more favorable situation at higher energies is due to the fact that electron diffraction in this energy range has been a valuable tool in the study of matter for a long time.

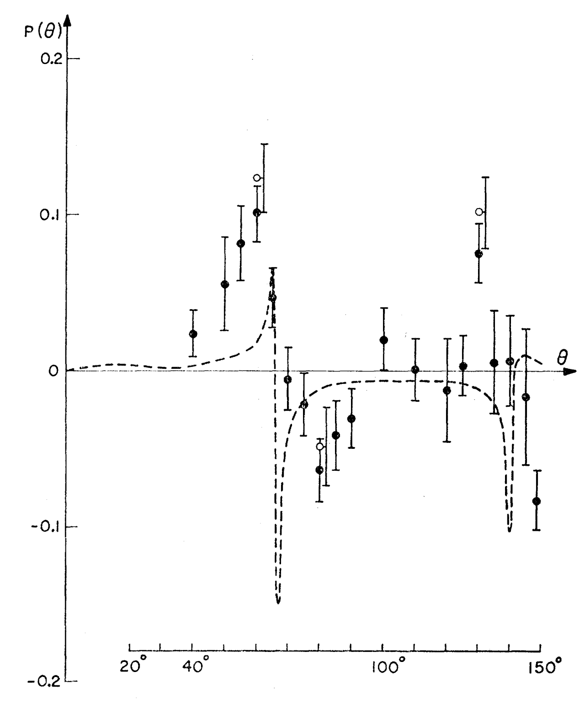


Fig. 22. $P(\theta)$ for 40 eV, $Z=18$. Experimental values from Mehr (1967), theoretical values by Coulthard for ideal angular resolution.

* See the addendum.

† The author is indebted to Dr. M. A. Coulthard for kindly making available his unpublished results.

‡ Recently, R. A. Bonham and H. L. Cox (1967) calculated elastic electron scattering amplitudes for atoms using the partial-wave method between 10 and 100 keV from $Z=1$ to $Z=54$.

For interpretation of such experiments, knowledge of at least the cross sections was necessary. Now the technique of low-energy electron diffraction is becoming more and more important, and data of cross sections and polarizations in low-energy electron scattering are not only of fundamental interest (appealing enough by itself) but also of great value for the study of matter. Sections B and C will give examples. A strong effort to overcome the lack of data in this field is needed.

3. Resonance Scattering from Neon

The theoretical predictions by Franzen and Gupta (1965) on spin polarization by elastic resonance scattering from neon (cf. Sec. II.C) have been confirmed by Reichert and Deichsel (1967). A schematic diagram of their apparatus is given in Fig. 23. An electron beam of 2×10^{-8} A with an energy half-width $\Delta E=40$ meV and an angular divergence of $\pm2°$ emerges from a 127° cylindrical monochromator and is scattered by a neon atomic beam. The electrons scattered by $\theta=90°$ are accelerated to 300 eV and scattered by a mercury atomic beam in order to analyze their polarization by the method described in Sec. III.A.1. It is the same polarization analyzer used by the authors in earlier work.

Figure 24 gives the experimental results. The resonance of the cross section for $\theta=90°$ is shown in Fig. 24(a). It was recorded with the detector "north," the mercury beam having been switched off. Figure 24(b) shows the energy dependence of the spin polarization. The measurements have been checked by rotating primary beam and monochromator by 180°. The points thus obtained are labeled by K.

Comparison of the experimental results with the theoretical predictions of Franzen and Gupta (Fig. 10) shows good agreement. Since the resonance peaks are only 95 meV apart, the curves calculated by Franzen and Gupta for an ideal energy resolution can be checked only with an energy half-width $\Delta E\ll95$ meV. Polarization measurements with increased energy resolution are highly desirable but very difficult because of intensity problems; an experiment with this goal is being done by Franzen and Gupta (private communication).

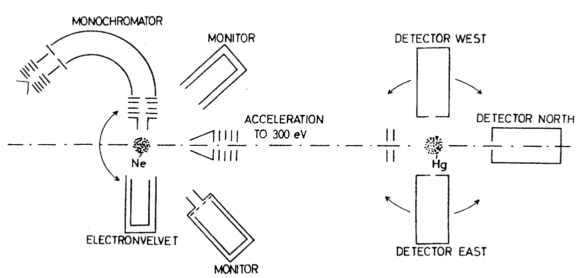


FIG. 23. Schematic diagram of apparatus for measuring polarization in resonance scattering from neon (from Reichert and Deichsel, 1967).

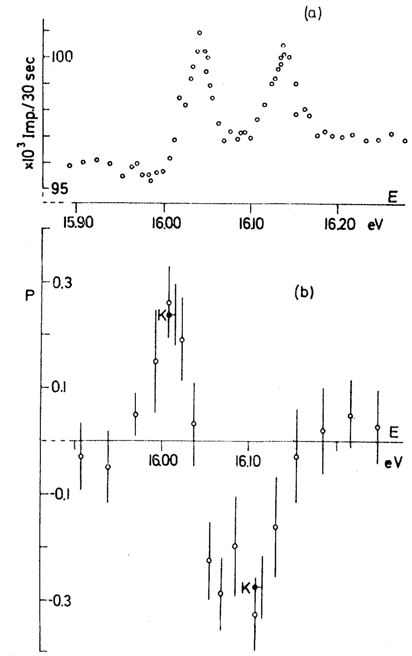


FIG. 24. (a) Experimental results for the resonances of the cross section for $\theta=90°$ in neon. (b) Energy dependence of the polarization in resonance scattering from neon for $\theta=90°$ (from Reichert and Deichsel, 1967).

B. Scattering by Molecules

Experiments on spin polarization in electron scattering from molecules have been started recently (Hilgner and Kessler, 1967), prompting interesting theoretical studies (Yates, 1968a; 1968b)* in this direction.

According to Eqs. (5) and (9), where no special assumptions on the nature of the scattering center have been made, the polarization P obtained by the scattering of an unpolarized electron beam by a molecule is

$$P=i(FG^*-F^*G)/(|F|^2+|G|^2), \qquad (22)$$

where F and G are the amplitudes of the wave scattered by the molecule. They can be expressed in terms of f_j and g_j, the amplitudes of the wave scattered by the jth atom, as

$$F=\sum_j f_j \exp(i\mathbf{s}\mathbf{r}_j),$$
$$G=\sum_j g_j \exp(i\mathbf{s}\mathbf{r}_j), \qquad (23)$$

as shown in the textbooks on electron diffraction (see, e.g., Pinsker, 1953); $\mathbf{s}$ is the momentum transfer in units of $\hbar$, $s=(4\pi/\lambda)\sin\frac{1}{2}\theta$; $\mathbf{r}_j$ is the radius vector from the origin of coordinates to the center of the jth atom. Equations (23) hold under the assumption that the molecule is composed of free atoms which are independent of each other. Substitution of Eqs. (23) into

* See the addendum.

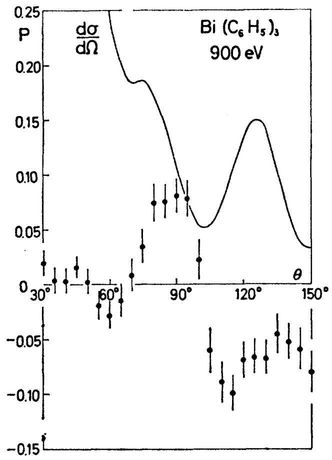


FIG. 25. Experimental results for differential cross section and polarization of 900-eV electrons scattered by $Bi(C_6H_5)_3$ (from Hilgner, Kessler, and Steeb, to be published).

Eq. (22) gives the polarization of an electron beam scattered from a molecule with a certain spatial orientation. The experiments to be discussed here were made with beams of unoriented molecules. The mean value of the polarization for arbitrary orientation of the molecules ($\bar{P}$) is obtained by a straightforward integration of Eq. (22) analogous to the evaluation of the mean scattered intensity in electron diffraction from molecules (Pinsker, 1953). The result is (cf., e.g., Hilgner and Kessler, 1969)

$$\bar{P}=\sum_{j,k} P_{jk}\sigma_{jk}/\sum_{j,k}\sigma_{jk}, \qquad (24)$$

where

$$P_{jk}=[i(f_jg_k^*-f_k^*g_j)]/(f_jf_k^*+g_jg_k^*),$$

$$\sigma_{jk}=(f_jf_k^*+g_jg_k^*)(\sin sr_{jk}/sr_{jk}).$$

If the molecule consists of atoms of the same kind, all P_{jk} are identical, $P_{jk}=P_A$, and Eq. (24) reduces to

$$\bar{P}=P_A\sum_{j,k}\sigma_{jk}/\sum_{j,k}\sigma_{jk}=P_A, \qquad (25)$$

i.e., the molecule gives the same polarization as a single atom.

The apparatus used for the polarization experiments with molecular targets was basically the same as that used for the measurements with mercury targets (Fig. 11). So far, measurements have been made (Hilgner, Kessler, and Steeb, 1969) for I_2, C_2H_5I, H_2O, CCl_4, $Bi(C_6H_5)_3$, and C_6H_6.

As a typical example for the results obtained with a molecule, Fig. 25 gives experimental curves of polarization and differential cross section for electron scattering from $Bi(C_6H_5)_3$ at 900 eV. One can clearly see polarization peaks near the minima of the cross section. The polarization curve has exactly the same structure as that of the bismuth atom calculated by Walker,* the difference being that the polarization values for the bismuth atom are much higher. In $Bi(C_6H_5)_3$, the polarization of the electrons scattered from Bi is "diluted" by those scattered from the low-Z part of the molecule, which have a very small polarization. The latter fact was verified by the measurements made with molecules consisting of light atoms, like H_2O, which showed that the polarizations are not greater than 0.01. A more thorough evaluation of the measurements with $Bi(C_6H_5)_3$ (which could not be completed before conclusion of this article) shows that the influence of the bismuth atom increases at higher electron energies, whereas the light atoms play an important role at lower energies. For full details see Hilgner, Kessler, and Steeb (1969).

As an example of what one can learn from electron polarization measurements with molecules, Fig. 26 gives a comparison of the polarization curves for ethyl iodide and iodine at the same electron energy. Minima and maxima are at the same angles in the two cases but are more pronounced for I_2. Since the electrons scattered from the light atoms, C and H, are virtually unpolarized, we can consider the scattered intensity in the case of C_2H_5I as being composed of two parts. One part comes from the iodine atom and is marked by its polarization, given by the values of Fig. 26(b). (The iodine molecule gives rise to the same polarization as a single atom if certain conditions are fulfilled.) It can therefore be distinguished from the unpolarized part coming from C and H. The resulting polarization is determined by the ratio of the polarized and unpolarized fractions of the scattered intensity. So, an old question (Hughes and McMillen, 1933) in slow electron scattering can be

* The author is indebted to Dr. D. W. Walker for kindly making available his unpublished results.

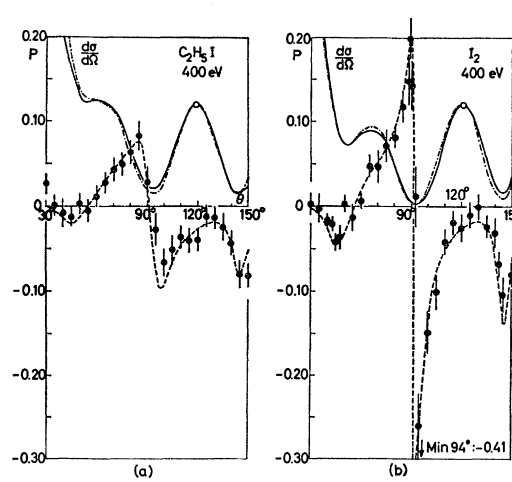


FIG. 26. (a, b) Angular dependence of cross section and polarization at 400 eV for C_2H_5I and I_2. Solid lines, experimental cross sections (open circles, normalization points); dash-and-dot lines, theoretical cross sections; solid circles, experimental polarizations; dashed lines, theoretical polarizations (from Hilgner and Kessler, 1969).

easily answered in the example of C_2H_5I; namely, How much of the scattered intensity comes from one part of the molecule and how much comes from another part?

A quantitative theoretical analysis of these measurements has been made by Yates (1968a; 1968b), whose results are also shown in Fig. 26. The good agreement of theoretical and experimental data shows that it was a good approximation to make the following assumptions in the calculations: The molecule consists of free atoms which are independent of each other (the influence of binding is negligible), and scattering of an electron does not occur more than once within the molecule. Unpublished theoretical results of Walker*,† and Bühring* at other energies also agree satisfactorily with the experimental results, though yielding higher polarization peaks for iodine. However, according to Bühring, there is a significant dependence of these peaks on the scattering potential used for the calculations.

According to the calculations, the mixed terms $j \neq k$ in Eq. (24) are relatively unimportant for the molecules and the angular range considered here. This can be quite different for molecules consisting of medium or heavy atoms, where not all the mixed products contain small factors due to light atoms. Theoretical estimates show (Bonham, private communication) that these molecules can give rise to an enhancement of the polarization over that obtained from the single atoms.

Many other interesting points have not yet been studied experimentally because investigations on polarization in electron–molecule scattering began only recently. Another point which will be studied in the near future is the following: Since the polarization is very sensitive to plural and multiple scattering (Eitel, Jost, and Kessler, 1968), effects of multiple elastic intramolecular scattering which were treated theoretically by many authors (Bonham, 1965) and are expected to be quite important at low energies in molecules containing several heavy atoms should be easily detectable by polarization measurements. Actually, multiple intramolecular scattering is probably responsible for the small deviations found between theoretical and experimental polarization peaks. This is discussed by Hilgner and Kessler (1969).

Massey and Burhop (1952) write in their well-known book, *Electronic and Ionic Impact Phenomena* that "much interesting information about the outer fields of molecules could be derived from a systematic study of the elastic scattering of slow electrons in conjunction with approximate theories, but a much wider range of molecular types would have to be investigated." This statement is underlined by the fact, not known when that book was written, that measurements of the polarization of the scattered electrons give additional information on the molecules which complements the knowledge obtained by cross section measurements (cf. Sec. IV). For evaluation of future measurements with molecules, it is necessary to know the polarization curves for the atoms. Calculations across the periodic table are desirable to overcome the lack of data mentioned in Sec. III.A.2.

C. Scattering by Solid Targets

From what has already been said about scattering from atoms and molecules, one will readily anticipate that slow electrons scattered from solid surfaces are polarized, too. For instance, in low-energy electron diffraction (LEED) experiments, one should obtain electrons of appreciable polarization and intensity whenever a diffraction spot appears under an angle for which $P(\theta)$ for the single atom has a maximum. Apart from the very first attempt by Davisson and Germer (1929), polarization in LEED with single crystals has not been investigated. But polarization studies were made with foils of tungsten, platinum, and gold (Loth, 1967) and a solid mercury target (Eckstein, 1967).

These experiments have been made with the apparatus of the Mainz group (cf. Sec. III.A.1) after replacing the mercury atomic beam in the first scattering chamber by the solid target under investigation. The metal foils could be heated in order to clean their surfaces, whereas the mercury was condensed continuously on a liquid-air-cooled supporting substrate. The evaporation rate of the mercury was chosen low enough to make the intensity of the electrons scattered by mercury vapor into the angular range studied small compared to the intensity scattered by the solid target. The angular resolution of the polarization measurement is given by the authors as $\Delta\theta = 8°$. Angles between 65° and 155° were covered. With the mercury target, the electron energies were between 300 and 900 eV, while 900-eV electrons were used for the other targets. As in all the experiments discussed in this review, only elastically scattered electrons were observed.

Polarization of the scattered electrons was found with all these solid targets. As an example, the points in Fig. 27 give the measured polarization values at 900 eV for the solid mercury target. The measurements with this target have the advantage that they can be compared with the results obtained for free mercury atoms. For this purpose the dashed line in Fig. 27 represents the theoretical polarization curve [which was confirmed experimentally (Deichsel and Reichert, 1965; Jost and Kessler, 1966)] for the free atom convoluted with the angular resolution of $\Delta\theta = 8°$ of the experiment under discussion [for the unconvoluted curve cf. Fig. 14(b)]. The two results of Fig. 27 have the same qualitative angular dependence. Most of the polarization values for the solid target are smaller, however, than those for

* The author is indebted to Dr. W. Bühring and Dr. D. W. Walker for kindly making available their unpublished results.

† See the addendum.

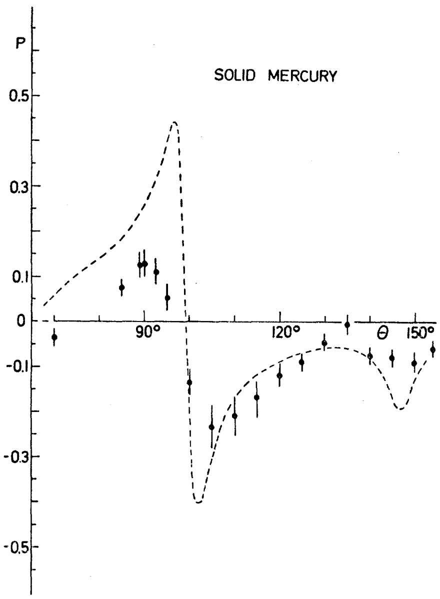


FIG. 27. Experimental results for the polarization of 900-eV electrons scattered by a solid mercury target. Dashed line, theoretical curve for free mercury atoms convoluted with the experimental angular resolution $\Delta\theta=8°$ (from Eckstein, 1967).

the free atom, this difference being particularly pronounced near the peaks of the curve in Fig. 27. The same follows from a comparison of the corresponding results for the other energies studied.

This behavior of the polarization curve for the solid target can be easily understood if we recall the differences in scattering from atomic beams of low and high density presented in Fig. 19. Plural scattering in the beams of high density resulted in deviations from the theoretical curve which were significant near the polarization peaks and of the same kind as those of Fig. 27. Since plural and multiple scattering undoubtedly play an important role in the experiments with the solid targets, no agreement with the results for free atoms can be expected.

The results for tungsten, platinum, and gold look very similar. Figure 28 gives an example. The measurement was made at a temperature of 1200°C of the platinum target. This was important since the results turned out to depend very sensitively on the purity of the target surfaces. Also in the measurements with the solid mercury target, impurities are indicated by the author to be an additional reason for the differences shown in Fig. 27.

Polarization studies in LEED are planned in several laboratories. Single-crystal targets would be of particular interest for this purpose. It would also be very helpful for these experiments to have as a guide theoretical polarization data for a large number of elements.

IV. APPLICATIONS AND OUTLOOK

Electron scattering experiments have been an invaluable tool for studying the structure of matter. One of the fundamental quantities measured in these experiments was the differential cross section for an unpolarized beam

$$d\sigma/d\Omega = |f|^2 + |g|^2, \tag{26}$$

which gives information on the structure of the scattering center.

If one observes the polarization of the scattered electrons, one gets additional information on the scattering center which is independent of that obtained by cross section measurements: The polarization of an electron beam resulting from scattering of an unpolarized beam is given by

$$P = i(fg^* - f^*g)/(|f|^2 + |g|^2). \tag{27}$$

One can easily see that a measurement of P gives information on the complex functions f and g and, thus, on the scattering center, which basically cannot be obtained by a measurement of the cross section Eq. (26).

Since the complex scattering amplitudes f and g are equivalent to four independent real functions, measurements of the two quantities $d\sigma/d\Omega$ and P still do not give all the information attainable about the f and g. However, this information could be obtained by measurements of T and U defined by Eq. (10), i.e., by measuring the rotation of the polarization in a scattering process. Because of the relation $S^2+T^2+U^2=1$, it is impossible to make four independent measurements for determining the f and g. This is a consequence of the fact that the phase φ_1 of the wave function $\psi = e^{i\varphi_1}[|a|\psi_{1/2} + |b| \exp(i\varphi_2)\psi_{-1/2}]$ which describes an electron beam of arbitrary spin direction ($\psi_{1/2}$ and $\psi_{-1/2}$ mean orthogonal spin functions) cannot be determined; only the three parameters $|a|$, $|b|$, and φ_2 are accessible to measurement. Measurements of the rotation of P are at the margin of what can be done presently, as we saw in Sec. II.B.

On the other hand, measurements of the polarization produced by scattering can be made with great accu-

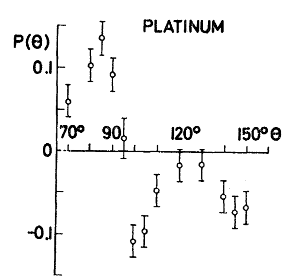


FIG. 28. Experimental results for the polarization of 900-eV electrons scattered by a heated platinum target (from Loth, 1967).

racy. They not only provide information which is independent of that obtained by cross-section measurements but also make possible a very accurate check of the models used to describe the scattering center; for the theoretical polarizations depend very sensitively on these models. (Cf. Meister and Weiss, 1968.) This can be made plausible by recalling the fact shown in Fig. 6; P is determined by the difference of two cross sections which are not very much different from one another so that small changes of the cross sections due to different theoretical models can result in great changes of P.

The advantages mentioned suggest that it is rewarding to make many more studies of the spin polarization in electron scattering, thus obtaining information which cannot be obtained by the more conventional investigations of cross sections.

Not only do measurements of the polarization of the scattered electrons yield interesting knowledge of the scattering center, but also the polarized electrons can be used as probes for studying matter. Measurements of the scattering asymmetry or of the change of the polarization vector in a scattering process would give new insight into many problems of atomic, molecular, and nuclear physics. This is one of the reasons for the great many attempts made in various laboratories to construct sources of polarized electrons* (Müller, Obermair, and Siegmann, 1967; Hofmann, Regenfus, Schärpf, and Kennedy, 1967; Hughes, Lubell, Posner, and Raith, 1967; Donally, Raith, and Becker, 1968; Krisciokaitis and Robinson, 1965; McCusker, Hatfield, Kessler and Walters, 1968). So far, the working sources yield either small polarization and satisfactory current or high polarization and rather small current. One of the best results is that of Hughes, Lubell, Posner, and Raith (1967), who obtained 85% polarization with 3×10^7 electrons per pulse by photoionization of a polarized lithium atomic beam. As long as entirely new approaches resulting in much higher intensities are not found, polarization by low-energy scattering compares favorably with the methods existing. It is a simple method giving a reasonable direct current of polarized electrons: In 1965, Steidl, Reichert, and Deichsel obtained currents of 10^{-9} A with 17% polarization. More favorable results are attainable by producing the primary electron beam with high-intensity guns such as those used in electron accelerators. In contrast to foils, there are no limits for the power of the primary beam hitting the gaseous target. The use of gaseous targets is possible (and necessary) because of the large scattering cross section of the slow electrons. A source of polarized electrons along this line is being prepared at Stanford University and intended for use with an accelerator.

Since we were concerned with slow electrons throughout this article, we will not go into the question of which problems can be attacked with polarized electrons in high-energy physics. Instead, we mention a few interesting applications at low energies.

Pepinsky (1966) discussed the possibility of studying effects of ordered arrangements of magnetic atoms at crystal surfaces by low-energy electron diffraction (LEED) experiments with polarized electrons. Since scattering due to electron spins (magnetic dipole scattering and exchange scattering) is small compared to Coulomb scattering, it is generally masked by the latter. However, in "spin-only" diffraction maxima (which can occur when the chemical unit cell and the magnetic structure cell differ from one another), Coulomb scattering is repressed by interference; reflections arising solely from the magnetic dipole distribution should be detectable even when the incident beam is unpolarized. But by analogy with neutron-scattering (Bacon, 1966), more information will be attainable through use of polarized electrons. For example, magnetic form factors describing the distribution of unpaired electron spin density in the atom or directions of magnetic moments in the sample could be obtained. Since exchange scattering makes an essential contribution to the "spin-only" maxima of the LEED pattern, these experiments will also be an interesting way of examining the exchange interaction.

There is an even more direct way of studying the electron exchange interaction: Polarized electrons are scattered by atoms and the change of the polarization is observed. It was not until a few years ago that a general method for exploring electron exchange collisions was developed (Collins, Goldstein, Bederson, and Rubin, 1967; Lichten and Schultz, 1959) based on the use of polarized atomic beams. The use of polarized electron beams would be a different approach to this problem, feasible also for elements which are inappropriate for producing polarized atomic beams. On the other hand, in those cases where polarized atomic beams can be produced, the use of polarized electron beams would make possible experiments with electrons which can be distinguished from each other; one has only to choose the direction of polarization in electron beam and atomic beam opposite to one another.

It should be one of the objectives of a review article to give perspective to the work which would be interesting or even exciting to pursue. Pertinent remarks are scattered throughout the present survey, but it may be useful to compile in a condensed list the problems the attack of which seems rewarding:

(1) Calculation of the scattering amplitudes f and g across the periodic table using a static potential and/or better approximations including polarizability of the atoms, exchange, and other effects which play a role at low energies. (Cf. Secs. II.B, III.A.2, III.B, and III.C).

(2) Polarization measurements at lower energies in

* Literature on this subject up to 1965 is quoted by Farago (1965).

conjunction with approximate theories to find out the influence of polarizability of the atom, exchange, etc.; measurements for various Z (cf. Secs. III.A.1 and III.A.2).

(3) Experimental check of *all* the results predicted by theory by studying not only the polarization of a scattered beam or the scattering asymmetry of a polarized beam but also the change of the polarization vector in scattering (cf. Secs. II.B and IV).

(4) Connection between chemical binding in a molecular target and polarization of scattered electron beam; range of validity of independent-atom model for molecules; influence of molecular structure on polarization; enhancement of polarization by molecules over that by single atoms; indication of internal multiple scattering in molecules by decrease of polarization peaks (cf. Sec. III.B).

(5) Direct measurements of cross sections for exchange scattering; influence of spin directions of incident and atomic electrons on scattering; extension of the results of Burke and Schey (1962) to higher energies and other elements (cf. Secs. II.D and IV).

(6) Polarization in resonance scattering; improved measurements for neon; calculations and experiments for other targets (cf. Secs. II.C and III.A.3).

(7) Theoretical and experimental work on polarization effects in inelastic scattering of slow electrons (cf. Sec. III.A).

(8) Polarization in scattering from solid targets, particularly single crystals (cf. Sec. III.C).

(9) Magnetic structures of surfaces (cf. Sec. IV).

V. ADDENDUM

After completion of this article the following relevant work appeared: D. W. Walker, Phys. Rev. Letters **20**, 827 (1968) (calculation of polarizations for C_2H_5I and I_2 at 300 eV); K. Schackert, Z. Physik **213**, 316 (1968) (measurement and comparison with theory of polarizations and cross sections for noble gases between 40 and 150 eV); R. J. v. Duinen, J. W. G. Aalders, Nucl. Phys. **A115**, 353 (1968) (triple scattering of electrons by gold at 261 keV).

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