Scattering of Polarized Leptons at High Energy*

NORMAN DOMBEY†
Department of Physics, Harvard University, Cambridge, Massachusetts

A unified treatment is given of the high-energy elastic and inelastic electromagnetic scattering of electrons and muons by hadrons in terms of the polarization density matrix of the virtual photon exchanged. It is shown that when the leptons are longitudinally polarized, the virtual photon is in a pure polarization state which is a coherent superposition of an elliptically polarized transverse state and a longitudinal state. Experiments utilizing these features are suggested and the formulas for the cross sections are obtained. The limit in which the virtual photon is almost real is investigated carefully. Finally, it is shown that the formalism is immediately applicable to neutrino scattering.

I. INTRODUCTION

Experiments where electrons or muons are scattered off nucleons and nuclei have been pursued for many years in order to discover the electromagnetic properties of the nucleons and nuclei. In elastic scattering, the quantities of interest measured by these experiments are the form factors $G_E(k^2)$ and $G_M(k^2)$, which describe the longitudinal (Coulomb) and transverse (magnetic) coupling strengths of a virtual photon of four-momentum k to the particle. The Fourier transforms of G_E and G_M in configuration space can be considered the charge and magnetic-moment spatial distribution functions of the particle (at least in nonrelativistic models). In the case of inelastic scattering,

$$l+H\to l+H', \tag{i}$$

it is convenient to consider the equivalent photon process

$$\gamma+H\to H', \tag{ii}$$

where H, H' are two different hadronic states. The photoexcitation amplitude describing the transition (ii) from H to H' at center-of-mass energy W is written as $T(W)$. Then for the process (i) the transverse transition amplitude $T(k^2, W)$ for which $T(0, W)=T(W)$ can be measured, as can the longitudinal transition amplitude $L(k^2, W)$, which is not present at $k^2=0$. If only the four-momentum of the final lepton is measured, k^2 and W are determined, but effectively all states H' of energy W are included. So in this case the quantities measured are the total absorptive cross sections $\sigma_T(k^2, W)$ and $\sigma_L(k^2, W)$. All these quantities $T(k^2, W)$, $L(k^2, W)$, $\sigma_T(k^2, W)$, and $\sigma_L(k^2, W)$ are of great interest in symmetry schemes and dynamical models in elementary particle physics and nuclear physics. If H and H' are degenerate in some symmetry scheme, the quantities $T(k^2, W)$ and $L(k^2, W)$ can be related to the form factors of H. Dynamical models (sum rules, superconvergence relations, bootstraps, for example) relate integrals of σ_T and σ_L in k^2 or W or both to zero or to the static electromagnetic properties of H, or will give the asymptotic properties of T, L, σ_T, σ_L as k^2 and W tend to infinity.[1]

* Supported in part by U.S. Air Force Office of Scientific Research AF-49(638)-1380 and Office of Naval Research NONR 1866(55) Mod. 1.

† Permanent Address: Physics Department, University of Sussex, Falmer, Sussex, England.

At present most experiments of this kind are performed with electron beams at electron accelerators. Muon beams have been formed from the decay of pions in flight at the major proton accelerator sites, and experiments have been successfully carried out recently.[2] Larger proton accelerators giving rise to more intense and higher-energy pion beams will probably be built in the next few years; this should make muon experiments of this kind more common. In particular, muon beams have one major attribute not possessed by electron beams: they are polarized. We shall always mean *longitudinally polarized* when discussing polarized leptons; this component dominates scattering at high energies.[3] In principle, it is also possible to partially polarize electron beams,[4] and so it is interesting to make an analysis of what new information can be obtained with polarized lepton beams; that is the purpose of this paper.

We begin in the next section by constructing the polarization matrix of the virtual photon initiating the reaction (assuming always that only one photon is exchanged) in the approximation where the lepton mass is neglected. In Secs. III and IV we systematically analyze elastic and inelastic scattering of these polarized, zero-mass leptons by nucleons; we choose pion production as the simplest and perhaps most immediately interesting inelastic process. In Sec. V we consider the corrections to this formalism necessitated by including the lepton mass. These corrections are most important in the forward directions where $k^2\to 0$. Finally, in Sec. VI we show that neutrino scattering can be included in this general formalism.

II. DENSITY MATRIX

We first consider the process

$$\gamma+N\to N',$$

where N' is any hadronic state. The amplitude $\mathfrak{M}$ for this process is given in terms of the hadronic electromagnetic current operator J_μ by

$$\mathfrak{M}=e\langle N' \mid J_\mu \mid N\rangle \mathcal{E}_\mu, \tag{1}$$

where $\mathcal{E}_\mu$ is the polarization of the real photon γ. The cross section for this process is given in terms of $|\mathfrak{M}|^2$:

$$|\mathfrak{M}|^2=e^2\sum_{\text{pol,spin}}\langle N'|J_\nu|N\rangle^*\langle N'|J_\mu|N\rangle\mathcal{E}_\nu^*\mathcal{E}_\mu$$
$$=e^2T_{\mu\nu}L_{\mu\nu};\tag{2}$$
$$L_{\mu\nu}=\sum_{\text{pol}}\mathcal{E}_\nu^*\mathcal{E}_\mu,\tag{3a}$$
$$T_{\mu\nu}=\sum_{\text{spin}}\langle N'|J_\nu|N\rangle^*\langle N'|J_\mu|N\rangle,\tag{3b}$$

where $\sum$ sums over final states and averages over initial states. If the Lorentz condition $k\cdot\mathcal{E}=0$ is used,

$$k_\mu L_{\mu\nu}=k_\nu L_{\mu\nu}=0.\tag{4}$$

As the hadronic current J_μ is conserved (using the shorthand $J_\mu=\langle N'|J_\mu|N\rangle$),

$$k_\mu J_\mu=k_\mu T_{\mu\nu}=k_\nu T_{\mu\nu}=0.\tag{5}$$

Equations (4) and (5) imply that it is sufficient to consider the spacelike components only of $L_{\mu\nu}$ and $T_{\mu\nu}$. The 3×3 matrix $\rho_{ij}=L_{ij}$ is called the photon-polarization density matrix. If we take the photon momentum to be in the z direction,[5]

$$k_\mu=(0,0,k,ik),\tag{6}$$

then

$$\mathcal{E}_\mu=(\mathcal{E}_x,\mathcal{E}_y,0,0),$$
$$|\mathcal{E}_x|^2+|\mathcal{E}_y|^2=1.\tag{7}$$

For linearly polarized photons in the x direction

$$\rho=(\rho_{ij})=\begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix}.\tag{8}$$

For unpolarized transverse photons averaged over x and y directions,

$$\rho=\begin{pmatrix}\frac{1}{2}&0&0\\0&\frac{1}{2}&0\\0&0&0\end{pmatrix}.\tag{9}$$

For a partially linearly polarized beam of relative strength $(1+\mathcal{E})$ in the x direction to $(1-\mathcal{E})$ in the y direction,

$$\rho=\begin{pmatrix}\frac{1}{2}(1+\mathcal{E})&0&0\\0&\frac{1}{2}(1-\mathcal{E})&0\\0&0&0\end{pmatrix}.\tag{10}$$

For right circularly polarized photons

$$\mathcal{E}_\mu=(1/\sqrt{2})(1,i,0),$$

and so

$$\rho=\begin{pmatrix}\frac{1}{2}&-i/2&0\\i/2&\frac{1}{2}&0\\0&0&0\end{pmatrix}.\tag{11}$$

We now turn to virtual photons. The matrix element for the process

$$l_1+N\rightarrow l_2+N'$$

is

$$\mathfrak{M}=e^2\langle N'|J_\mu|N\rangle\langle l_2|j_\mu|l_1\rangle/k^2,\tag{12}$$

assuming one-photon exchange, and then

$$|\mathfrak{M}|^2=(2e^4/k^2)T_{\mu\nu}L_{\mu\nu},\tag{13}$$

where now

$$2k^2L_{\mu\nu}=\sum_{\text{spin}}\langle l_2|j_\nu|l_1\rangle^*\langle l_2|j_\mu|l_1\rangle\tag{14}$$

and $T_{\mu\nu}$ is given as before.

The electromagnetic current for leptons is given by

$$j_\mu=i\bar{u}_2\gamma_\mu u_1,\tag{15}$$
$$k_\mu j_\mu=k_\mu L_{\mu\nu}=k_\nu L_{\mu\nu}=0,\tag{16}$$

and for unpolarized leptons

$$2k^2L_{\mu\nu}=\tfrac{1}{2}\operatorname{Tr}[\gamma\cdot l_1\gamma_\nu\gamma\cdot l_2\gamma_\mu]$$
$$=2(l_{1\nu}l_{2\mu}+l_{1\mu}l_{2\nu}-l_1\cdot l_2\delta_{\mu\nu})$$
$$=2(l_{1\nu}l_{2\mu}+l_{1\mu}l_{2\nu}+\tfrac{1}{2}k^2\delta_{\mu\nu}),\tag{17}$$

where the lepton mass m has been neglected. This will be a good approximation for $k^2\gg m^2$. To evaluate the right-hand side of Eq. (17), the z direction is defined as before:

$$k_\mu=(0,0,|\mathbf{k}|,ik_0)\tag{18}$$

and the x-z plane, the scattering plane, is defined by the leptons. Then, after some manipulation,

$$2L_{11}=1+(k^2/|\mathbf{k}|^2)\cot^2\tfrac{1}{2}\psi$$
$$=(1+\mathcal{E})/(1-\mathcal{E}),\tag{19a}$$
$$2L_{22}=1,\tag{19b}$$
$$2L_{33}=(k_0^2/k^2)[\mathcal{E}/(1-\mathcal{E})],\tag{19c}$$
$$2L_{13}=2L_{31}=(k_0^2/k^2)^{1/2}[2\mathcal{E}(1+\mathcal{E})]^{1/2}/(1-\mathcal{E}),\tag{19d}$$
$$L_{12}=L_{21}=L_{23}=L_{32}=0,\tag{19e}$$

where $\mathcal{E}$ is defined by Eq. (19a) and clearly measures the transverse linear polarization of the virtual photon [cf. Eq. (10)]. From Eqs. (19c) and (19d) $\mathcal{E}$ also measures the longitudinal polarization of the virtual photon and can be written as

$$\mathcal{E}=[1+2(|\mathbf{k}|^2/k^2)\tan^2\tfrac{1}{2}\psi]^{-1};\tag{20}$$

ψ is the angle of scattering between incident and final lepton.[6]

The polarization matrix ρ_{ij} could be taken to be $(1-\mathcal{E})L_{ij}$. But the longitudinal components of Eq. (18) are not in their most useful form, as they appear to be singular for the real photon limit of $k^2=0$ (see Sec. VI). It helps to write out the amplitude $\mathfrak{M}$ directly in terms of the currents. Then,

$$J_\mu j_\mu=\mathbf{J}\cdot\mathbf{j}-J_0j_0=J_xj_x+J_yj_y+[1-(|\mathbf{k}|^2/k_0^2)]J_zj_z,$$

using Eqs. (5) and (16), or

$$J_\mu j_\mu = J_x j_x + J_y j_y - (k^2/k_0^2) J_z j_z. \tag{21}$$

Thus, it is possible to include the current conservation factor $(-k^2/k_0^2)$ in the lepton contribution to $\mathfrak{M}$ and ignore it in the hadronic. With the inclusion of this factor, the polarization density matrix can be written:

$$\rho_{11} = (1-\mathcal{E}) L_{11} = \tfrac{1}{2}(1+\mathcal{E}),$$
$$\rho_{22} = (1-\mathcal{E}) L_{22} = \tfrac{1}{2}(1-\mathcal{E}),$$
$$\rho_{33} = (k^4/k_0^4)(1-\mathcal{E}) L_{33} = (k^2/k_0^2)\mathcal{E},$$
$$\rho_{13} = \rho_{31} = -(k^2/k_0^2)(1-\mathcal{E}) L_{13} = -(k/k_0)[\tfrac{1}{2}\mathcal{E}(1+\mathcal{E})]^{1/2},$$
$$\rho_{12} = \rho_{21} = \rho_{23} = \rho_{32} = 0. \tag{22}$$

We have written $k = (k^2)^{1/2}$; henceforth it will only have this meaning. Finally, the longitudinal polarization is defined:

$$\mathcal{E}_L = (k^2/k_0^2)\mathcal{E}, \tag{23}$$

and now

$$\rho = \begin{pmatrix} \tfrac{1}{2}(1+\mathcal{E}) & 0 & -[\tfrac{1}{2}\mathcal{E}_L(1+\mathcal{E})]^{1/2} \\ 0 & \tfrac{1}{2}(1-\mathcal{E}) & 0 \\ -[\tfrac{1}{2}\mathcal{E}_L(1+\mathcal{E})]^{1/2} & 0 & \mathcal{E}_L \end{pmatrix}. \tag{24}$$

This is the desired result which shows that in the scattering of unpolarized leptons, the density matrix of the virtual photon is the incoherent sum of the two pure states[7]

$$\mathcal{E}_a = \{[\tfrac{1}{2}(1+\mathcal{E})]^{1/2}, 0, -\mathcal{E}_L^{1/2}\} \tag{25a}$$

and

$$\mathcal{E}_b = \{0, [\tfrac{1}{2}(1-\mathcal{E})]^{1/2}, 0\}. \tag{25b}$$

Thus, experiments using unpolarized leptons are equivalent in the small k^2 limit (but remember that $k^2 \gg m^2$; we go into this point later) to those using partially linearly polarized photons. Furthermore, the polarization $\mathcal{E}$ will be known very accurately. This result has been known for some time.[8] In general, there will also be the interference term between the longitudinal and transverse components of $\mathcal{E}_a$ which can be used to measure longitudinal amplitudes of the hadronic transition.

The main result of this paper can now be approached. Suppose our incident leptons are polarized (say, right polarized). Then the operator for zero-mass leptons which projects out the states for which

$$\boldsymbol{\sigma}\cdot\mathbf{p}\chi = |\mathbf{p}|\chi$$

is $\frac{1}{2}(1-\gamma_5)$. The corollary of Eq. (17) is now

$$2k^2 L_{\mu\nu} = \tfrac{1}{2}\,\mathrm{Tr}\,[\gamma\cdot l_1\gamma_\nu\gamma\cdot l_2\gamma_\mu(1-\gamma_5)] \tag{26}$$

or

$$L_{\mu\nu} = L_{\mu\nu}^S + L_{\mu\nu}^A, \tag{27}$$

where $L_{\mu\nu}^S$, the symmetric part of $L_{\mu\nu}$, is given by Eq. (19), but $L_{\mu\nu}^A$ satisfies

$$2k^2 L_{\mu\nu}^A = -\tfrac{1}{2}\,\mathrm{Tr}\,[\gamma\cdot l_1\gamma_\nu\gamma\cdot l_2\gamma_\mu\gamma_5]$$

or

$$k^2 L_{\mu\nu}^A = \varepsilon_{\alpha\mu\beta\nu} l_{1\alpha} l_{2\beta}; \tag{28}$$

i.e.,

$$L_{12}^A = -L_{21}^A = -\tfrac{1}{2}i[(1+\mathcal{E})/(1-\mathcal{E})]^{1/2}, \tag{29a}$$
$$L_{23}^A = -L_{32}^A = (i/\sqrt{2})(k_0/k)[\mathcal{E}/(1-\mathcal{E})]^{1/2}, \tag{29b}$$
$$L_{13}^A = L_{31}^A = 0. \tag{29c}$$

The new density matrix, defined as before, is

$$\rho = \begin{pmatrix} \tfrac{1}{2}(1+\mathcal{E}) & -\tfrac{1}{2}i(1-\mathcal{E}^2)^{1/2} & -[\tfrac{1}{2}\mathcal{E}_L(1+\mathcal{E})]^{1/2} \\ \tfrac{1}{2}i(1-\mathcal{E}^2)^{1/2} & \tfrac{1}{2}(1-\mathcal{E}) & -i[\tfrac{1}{2}\mathcal{E}_L(1-\mathcal{E})]^{1/2} \\ -[\tfrac{1}{2}\mathcal{E}_L(1+\mathcal{E})]^{1/2} & i[\tfrac{1}{2}\mathcal{E}_L(1-\mathcal{E})]^{1/2} & \mathcal{E}_L \end{pmatrix}. \tag{30}$$

This density matrix shows that the virtual photon is in the pure polarization state

$$\mathcal{E}_c = \{[\tfrac{1}{2}(1+\mathcal{E})]^{1/2}, i[\tfrac{1}{2}(1-\mathcal{E})]^{1/2}, -\mathcal{E}_L^{1/2}\}. \tag{31}$$

Thus, scattering by polarized leptons is equivalent to scattering by a photon whose polarization is a superposition of a transverse elliptic component and a longitudinal component. To extract the new information contained in interference between xy and yz components requires scattering off polarized targets, for otherwise an antisymmetric contribution to $T_{\mu\nu}$ in electromagnetic interactions cannot be obtained (as parity is conserved). Alternatively, recoil nucleon polarizations could be measured, but our analysis will be made in terms of target polarization, as this should become the more effective technique.

The rest of this paper (except Sec. VI) is devoted to a study of the scattering of polarized leptons off polarized nucleons.

III. ELASTIC SCATTERING

We will now calculate the elastic scattering of a polarized electron or muon by a polarized nucleon.[3] A formalism is developed which goes over easily into the calculation of inelastic scattering. It is most convenient to work in the Breit frame of the nucleon,[9] rather than the general frame of the last chapter. If l_1, p_1, l_2, p_2 denote the four-momentum vectors of the incident and final lepton and nucleon, the momentum transfer k is given by

$$k = l_1 - l_2 = p_2 - p_1; \tag{32}$$

in the nucleon Breit frame

$$k_0=0, \mathbf{k} \parallel \mathbf{p}_1. \tag{33}$$

From Eq. (19a)

$$(1+\mathcal{E})/(1-\mathcal{E}) = \csc^2 \tfrac{1}{2}\psi_B, \tag{34}$$

where ψ_B is the lepton angle of scattering in the nucleon Breit frame. $\mathcal{E}$ is not an invariant quantity, but it is invariant under Lorentz transformations along $\mathbf{k}$.

The lepton four-momenta are (taking the z direction along $\mathbf{k}$)

$$l_1=\tfrac{1}{2}k(\cot \tfrac{1}{2}\psi_B, 0, 1, i \csc \tfrac{1}{2}\psi_B), \tag{35a}$$

$$l_2=\tfrac{1}{2}k(\cot \tfrac{1}{2}\psi_B, 0, -1, i \csc \tfrac{1}{2}\psi_B). \tag{35b}$$

The elements of the tensor $L_{\mu\nu}$ for an initial lepton of positive helicity are easily calculated to be

$$\begin{aligned} 2L_{11}&= \csc^2 \tfrac{1}{2}\psi_B,\\ 2L_{22}&=1,\\ 2L_{44}&=- \cot^2 \tfrac{1}{2}\psi_B,\\ 2L_{14}&=2L_{41}=i \cot \tfrac{1}{2}\psi_B \csc \tfrac{1}{2}\psi_B,\\ 2L_{12}&=-2L_{21}=-i \csc \tfrac{1}{2}\psi_B,\\ 2L_{24}&=-2L_{42}=- \cot \tfrac{1}{2}\psi_B,\\ L_{3\mu}&=L_{\mu 3}=0. \end{aligned} \tag{36}$$

The proton current in its Breit frame is very simple. In terms of two-component spinors, the nucleon current is

$$J_\mu=\chi_2{}^*\mathfrak{F}^\mu\chi_1,$$

where

$$\begin{aligned} \mathfrak{F}^1&=ikG_M\boldsymbol{\sigma}\cdot \mathbf{e}_2,\\ \mathfrak{F}^2&=-ikG_M\boldsymbol{\sigma}\cdot \mathbf{e}_1,\\ \mathfrak{F}^3&=0,\\ \mathfrak{F}^4&=2MiG_E; \end{aligned} \tag{37}$$

$\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ are unit vectors along the x, y, and z axes, and $G_E(k^2)$ and $G_M(k^2)$ are the electric (Coulomb or longitudinal) and magnetic (transverse) form factors of the nucleons. For unpolarized nucleons we can construct the symmetric tensor $T_{\mu\nu}$ given by Eq. (3b)[10]:

$$T_{\mu\nu}=\tfrac{1}{2} \operatorname{Tr} [\mathfrak{F}^{\nu+}\mathfrak{F}^\mu] \tag{38}$$

leading to

$$T_{11}=T_{22}=k^2G_M{}^2, \qquad T_{44}=-4M^2G_E{}^2; \tag{39}$$

all other $T_{\mu\nu}=0$. Hence

$$2L_{\mu\nu}T_{\mu\nu}=k^2G_M{}^2(2+ \cot^2 \tfrac{1}{2}\psi_B)+4M^2G_E{}^2 \cot^2 \tfrac{1}{2}\psi_B.$$

In terms of the angle of scattering in the laboratory frame ψ,

$$\cot^2 \tfrac{1}{2}\psi_B=(\cot^2 \tfrac{1}{2}\psi)/(1+\tau), \tag{40}$$

where

$$\tau=k^2/4M^2. \tag{41}$$

This gives the Rosenbluth formula

$$d\sigma/d\Omega= (\alpha^2/k^2) (E'/E)^2\{G_E{}^2[\cot^2 \tfrac{1}{2}\psi/(1+\tau)] +\tau G_M{}^2[2+ \cot^2 \tfrac{1}{2}\psi/(1+\tau)]\}, \tag{42}$$

where E and E' are the energies of the initial and final lepton in the laboratory frame.

Alternatively, in terms of the polarization $\mathcal{E}$, Eq. (42) can be written as

$$d\sigma/d\Omega= (2\alpha^2/k^2) (E'/E)^2(1-\mathcal{E})^{-1}(\mathcal{E}G_E{}^2+\tau G_M{}^2) \tag{43}$$

or

$$d\sigma/d\Omega= (\alpha^2/2M^2) (E'/E)^2(1-\mathcal{E})^{-1}(G_M{}^2+\mathcal{E}_LG_E{}^2), \tag{44}$$

using $\tau=k^2/4M^2=k_0^2/k^2$. Note that in this case as $k^2\to 0$, $\mathcal{E}_L\to\infty$ and the longitudinal terms dominate. This simply shows that the corresponding real photon process $\gamma+N\to N$ is forbidden.

For a polarized target, $T_{\mu\nu}{}^n$ must be added to $T_{\mu\nu}$ of Eq. (38):

$$T_{\mu\nu}{}^n=\pm\tfrac{1}{2} \operatorname{Tr} [\mathfrak{F}^{\nu+}\mathfrak{F}^\mu\boldsymbol{\sigma}\cdot \mathbf{n}] \tag{45}$$

for the nucleon polarized along (opposite) the direction defined by the unit vector $\mathbf{n}$. For large values of k^2, say $\tau\sim 1$, the term involving $G_M{}^2$ in Eq. (42) is about 40 times as big as the term $G_E{}^2$ [making the usual assumption that $G_M(k^2)\simeq G_E(k^2)$; $G_E(0)=1$, $G_M(0)=2.79$]. This means that it is basically $G_M{}^2$ which is measured in the existing experiments at large momentum transfer. The most interesting experiment is thus to measure an interference term of the form G_EG_M and so find G_E. From Eq. (37) interference between (14) and (24) components will provide terms of this form; Eq. (36), however, shows that $L_{24}=-L_{42}$, while $L_{14}=L_{41}$. So the configuration necessary will be one in which $T_{24}=-T_{42}$, and the proton must be polarized in the x direction [defined by the lepton momenta in Eq. (35)]. Now

$$T_{24}{}^x=-T_{42}{}^x=2MkG_EG_M, \tag{46}$$

so

$$2L_{\mu\nu}T_{\mu\nu}=k^2G_M{}^2(2+ \cot^2 \tfrac{1}{2}\psi_B)+4M^2G_E{}^2 \cot^2 \tfrac{1}{2}\psi_B \mp 4MkG_EG_M \cot \tfrac{1}{2}\psi_B \tag{47}$$

or

$$d\sigma/d\Omega= (\alpha^2/2M^2) (E'/E)^2(1-\mathcal{E})^{-1} \times\{G_M{}^2+\mathcal{E}_LG_E{}^2\mp 2G_EG_M[\tfrac{1}{2}\mathcal{E}_L(1-\mathcal{E})]^{1/2}\}, \tag{48}$$

where the alternative sign comes from reversing the direction of polarization. Thus, measuring the asymmetry in the cross section on flipping the target polarization will measure G_EG_M directly. Note that the sign of G_E can be determined in this measurement; as $G_E=F_1-\tau F_2$,[11] it is *a priori* quite likely that G_E becomes negative for large values of k^2. The target nucleon must be polarized perpendicular to the virtual photon in the Breit frame, i.e., perpendicular to the direction of the recoil nucleon in the laboratory and in the scattering plane.

For targets polarized in the z direction, it is easy to show that the asymmetry is proportional to G_M^2.

IV. INELASTIC SCATTERING—PION PRODUCTION

We now turn to pion production,

$$l_1+p_1\to l_2+p_2+q$$

or

$$k=l_1-l_2, \qquad k+p_1\to p_2+q, \tag{49}$$

where q is the four-momentum of the final pion. We are interested in coincidence measurements whereby two of the final particles are observed, and thus all the momenta of all the particles are determined. This is the simplest inelastic process, although its formulation is complicated enough. The methods of the last section are followed; the square of the matrix element for the process is still best evaluated in the Breit frame of the nucleon, given as before by Eq. (33). But the nucleonic system now defines its own plane containing $\mathbf{k}$ and $\mathbf{q}$. So the lepton momenta l_1 and l_2 now depend on the angle ϕ between the leptonic plane and the nucleonic plane. Hence,

$$l_1=\tfrac{1}{2}k(\cot\tfrac{1}{2}\psi_B\cos\phi, \cot\tfrac{1}{2}\psi_B\sin\phi, 1, i\csc\tfrac{1}{2}\psi_B), \tag{50a}$$

$$l_2=\tfrac{1}{2}k(\cot\tfrac{1}{2}\psi_B\cos\phi, \cot\tfrac{1}{2}\psi_B\sin\phi, -1, i\csc\tfrac{1}{2}\psi_B). \tag{50b}$$

It is convenient to break $L_{\mu\nu}$ up into symmetric $L_{\mu\nu}{}^S$ and antisymmetric $L_{\mu\nu}{}^A$ components when describing scattering by a positive helicity lepton.

We have

$$\begin{aligned}
2L_{11}{}^S&=1+\cot^2\tfrac{1}{2}\psi_B\cos^2\phi,\\
2L_{12}{}^S&=\tfrac{1}{2}\cot^2\tfrac{1}{2}\psi_B\sin 2\phi,\\
2L_{22}{}^S&=1+\cot^2\tfrac{1}{2}\psi_B\sin^2\phi,\\
2L_{14}{}^S&=i\cot\tfrac{1}{2}\psi_B\csc\tfrac{1}{2}\psi_B\cos\phi,\\
2L_{44}{}^S&=-\cot^2\tfrac{1}{2}\psi_B,\\
2L_{24}{}^S&=i\cot\tfrac{1}{2}\psi_B\csc\tfrac{1}{2}\psi_B\sin\phi,\\
L_{3\mu}{}^S&=0;
\end{aligned} \tag{51}$$

and

$$2L_{12}{}^A=-i\csc\tfrac{1}{2}\psi_B, \qquad 2L_{14}{}^A=\cot\tfrac{1}{2}\psi_B\sin\phi,$$
$$2L_{24}{}^A=-\cot\tfrac{1}{2}\psi_B\cos\phi. \tag{52}$$

First consider an unpolarized target. The nucleonic contribution $T_{\mu\nu}$ will now be symmetric; also, as the y axis is perpendicular to the Breit frame, the only symmetric tensor which includes a component in this direction is $\delta_{\mu\nu}$. Since $T_{\mu 3}=0$,

$$\begin{aligned}
2L_{\mu\nu}T_{\mu\nu}=&(1+\cot^2\tfrac{1}{2}\psi_B\cos^2\phi)T_{11}{}^B\\
&+(1+\cot^2\tfrac{1}{2}\psi_B\sin^2\phi)T_{22}{}^B-\cot^2\tfrac{1}{2}\psi_B T_{44}{}^B\\
&+i\cot\tfrac{1}{2}\psi_B\csc\tfrac{1}{2}\psi_B\cos\phi(T_{14}{}^B+T_{41}{}^B),
\end{aligned} \tag{53}$$

where $T_{\mu\nu}{}^B$ indicates that the tensor is evaluated in the Breit frame. The most convenient frame in which to evaluate $T_{\mu\nu}$ is the center-of-mass frame of the reaction $k+p_1\to p_2+q$. In order to reach this frame from the Breit frame, a Lorentz transformation is made in the z direction transforming

$$k_\mu=(0, 0, k, 0)$$

into

$$k_\mu'=(0, 0, k_c, ik_0),$$

where k_0 and k_c are functions of k^2 and the center-of-mass energy W.[9] In this frame

$$k^2T_{33}=-k_0{}^2T_{44}{}^B,$$

$$kT_{3i}=-ik_0T_{4i}{}^B, \qquad kT_{i3}=-ik_0T_{i4}{}^B; \quad i,j=1,2. \tag{54}$$

Putting all this together gives the well-known result

$$\frac{d^3\sigma}{dE'd\omega_l d\Omega_\pi}=\frac{\alpha}{2\pi^2}\frac{E'}{E}\frac{|\mathbf{K}|}{k^2}(1-\mathcal{E})^{-1}\frac{d\sigma_v}{d\Omega_\pi} \tag{55}$$

for the differential cross section for scattering into the lepton solid angle $d\omega_l$ measured in the laboratory and into the pion solid angle $d\Omega_\pi$ measured in the center of mass. E, E', and $|\mathbf{K}|$ are measured in the laboratory frame; i.e., in this frame $k_\mu=(\mathbf{K}, iK_0)$, $K_0=E-E'$. $d\sigma_v/d\Omega$ is the cross section in the center-of-mass frame for pion production by a virtual photon and can be written from Eqs. (53) and (54) as

$$\begin{aligned}
(k_c/|\mathbf{q}|)(d\sigma_v/d\Omega)=&\tfrac{1}{2}(T_{11}+T_{22})+\tfrac{1}{2}(T_{11}-T_{22})\mathcal{E}\cos 2\phi\\
&+\mathcal{E}_LT_{33}-[\tfrac{1}{2}\mathcal{E}_L(1+\mathcal{E})]^{1/2}(T_{13}+T_{31})\cos\phi.
\end{aligned} \tag{56}$$

The first term is just the cross section for pion production by an unpolarized, transverse virtual photon; the second term comes about from interference between transverse states and is the ϕ-dependent term which occurs in photoproduction using linearly polarized photons; the third term is the cross section for pion production by a longitudinal photon, and the fourth term represents interference between longitudinal and transverse components. The expressions for the T_{ij} in terms of the conventional amplitudes $\mathfrak{F}_1\cdots\mathfrak{F}_6$[12] are given in Eq. (A4) of Appendix A. By varying ϕ and $\mathcal{E}$ for fixed k^2, W, and θ, each of the four terms can be measured separately.

We now consider scattering off a polarized target First, let us consider what can be learned in pion photoproduction experiments using polarized transverse photons and polarized targets.[13] There are four complex functions $\mathfrak{F}_1$, $\mathfrak{F}_2$, $\mathfrak{F}_3$, $\mathfrak{F}_4$ and hence seven independent quantities describing pion photoproduction for each pair of values of W and θ (the pion scattering angle). If linearly polarized photons are scattered by an unpolarized target, two quantities T_{11} and T_{22} are measured, one for each direction of polarization. Linearly polarized photons scattered by a polarized target give two amplitudes for each direction of polarization of the target; hence, six independent quantities can be measured (the two amplitudes measured on an unpolarized target are, of course, not independent of these six). Thus, this is still insufficient to specify the problem; these measurements must be supplemented by measurements of a circularly polarized photon scattered by a polarized target, which give two extra quantities and provide a complete set of measurements.

Furthermore, the measurements using circularly polarized photons involve the real part of certain products of amplitudes $\mathfrak{F}_i$, while the measurements using linearly polarized photons involve the imaginary part of the same products of amplitudes. Hence these measurements are complementary.

Proceeding to the calculation with a polarized target, we write down the formulas for the target polarized in the x, y, and z directions: The x and y directions are defined so that the pion momentum q is given by

$$\mathbf{q}=|\mathbf{q}|(\sin\theta,0,\cos\theta)$$

or

$$\mathbf{q}/|\mathbf{q}|=\mathbf{e}_1\sin\theta+\mathbf{e}_3\cos\theta;\qquad \mathbf{e}_2=\mathbf{e}_3\times\mathbf{e}_1. \tag{57}$$

We define

$$T_{ij}{}^n=\tfrac{1}{2}\,\mathrm{Tr}\,[\mathfrak{F}^{j+}\mathfrak{F}^i\boldsymbol{\sigma}\cdot\mathbf{n}]. \tag{58}$$

From measurements [using Eq. (55)] of the pion-production cross section $d\sigma_v/d\Omega$ for polarization along and opposite to the direction $\mathbf{n}$, we can form the sum and differences

$$2(d\sigma_v/d\Omega)_S{}^n=(d\sigma_v/d\Omega)^{\mathbf{n}}+(d\sigma_v/d\Omega)^{-\mathbf{n}}$$

and

$$2(d\sigma_v/d\Omega)_D{}^n=(d\sigma_v/d\Omega)^{\mathbf{n}}-(d\sigma_v/d\Omega)^{-\mathbf{n}}. \tag{59}$$

$(d\sigma_v/d\Omega)_S{}^n$ is just $d\sigma_v/d\Omega$ of Eq. (56) for unpolarized particles. The asymmetry in the cross sections for polarization along the x and z directions is given by

$$\begin{aligned}(d\sigma/d\Omega)_D{}^n=&\tfrac{1}{2}[T_{12}{}^n+T_{21}{}^n)\mathcal{E}\sin 2\phi\\&-(i/2)(1-\mathcal{E}^2)^{1/2}(T_{12}{}^n-T_{21}{}^n)\\&-[\tfrac{1}{2}\mathcal{E}_L(1+\mathcal{E})]^{1/2}(T_{23}{}^n+T_{32}{}^n)\sin\phi\\&-i[\tfrac{1}{2}\mathcal{E}_L(1-\mathcal{E})]^{1/2}(T_{23}{}^n-T_{32}{}^n)\cos\phi.\end{aligned} \tag{60}$$

For polarization along the y direction,

$$\begin{aligned}(d\sigma/d\Omega)_D{}^y=&\tfrac{1}{2}(T_{11}{}^y+T_{22}{}^y)+\tfrac{1}{2}(T_{11}{}^y-T_{22}{}^y)\mathcal{E}\cos 2\phi\\&+\mathcal{E}_LT_{33}{}^y-[\tfrac{1}{2}\mathcal{E}_L(1+\mathcal{E})]^{1/2}(T_{13}{}^y+T_{31}{}^y)\cos\phi\\&+i[\tfrac{1}{2}\mathcal{E}_L(1-\mathcal{E})]^{1/2}(T_{13}{}^y-T_{31}{}^y)\sin\phi.\end{aligned} \tag{61}$$

All four terms in Eq. (60) and all the interference terms in Eq. (61) can be measured by varying the azimuthal angle ϕ. To decouple the transverse and longitudinal terms $(T_{11}{}^y+T_{22}{}^y)$ and $T_{33}{}^y$, $\mathcal{E}$ must also be varied. The expressions for $T_{ij}{}^n$ are given by Eqs. (A5)–(A7) of Appendix A.

As seen from the form of these expressions, the symmetric terms $T_{ij}{}^n+T_{ji}{}^n$ (which could be measured with an unpolarized beam scattered on a polarized target) do give the imaginary part of products of amplitudes whose real part is given by the antisymmetric terms. The transverse terms $(i,j=1,2)$ of Eqs. (60) and (61) are precisely those which appear in the photoproduction expressions involving linearly and circularly polarized photons. Thus, working at low k^2 (in view of the condition $k^2\gg m^2$, this is in practice restricted to polarized electron scattering), it is possible to make a complete set of measurements of the photoproduction amplitudes in one experiment, as the elliptically polarized virtual photon is a superposition of linearly and circularly polarized components.

By the same argument as before, there are six complex functions of W, θ, k^2 describing the general case; hence, 11 independent real observables must be measured. The symmetric parts of Eqs. (56), (60), and (61) give 12 quantities, thus in principle specifying the system. The five extra antisymmetric quantities involved in using polarized leptons overdetermine the system, but in practice would be very useful because they involve the real part of functions whose imaginary part is known.

A simple example is the first pion–nucleon resonance, the $N_{1236}{}^*(J=\frac{3}{2},I=\frac{3}{2})$. We consider the production of π^0 (in order to avoid the pion pole) at resonance and include only s and p waves of the final π–N system, using the partial-wave analysis of Eq. (A8).

The dominant excitation in this region is the magnetic dipole M_{1+}. The goal is to measure the less important terms at resonance. These form two groups:

(i) The electric and longitudinal quadrupole excitations E_{1+} and L_{1+} of the N^* which are in phase with M_{1+}.[14]

(ii) The background terms E_{0+}, L_{0+}, M_{1-}, L_{1-} whose phase is small, as the $s_{1/2}$ and $p_{1/2}$ π–N phase shifts are small at this energy.[15]

The formulas are approximated by only including terms quadratic and linear in M_{1+}. The term $|M_{1+}|^2$ is the dominant contribution and has already been measured[16,17] in unpolarized electroproduction experiments. M_{1+} is pure imaginary at resonance, so the real parts of the interference terms of group (ii) with M_{1+} and the imaginary parts of interference terms in group (i) can be omitted. For the transverse terms using $\mathfrak{F}_4=0$,

$$T_{12}{}^z+T_{21}{}^z\text{ determines Im }(M_{1-}M_{1+}{}^*), \tag{62a}$$

$$T_{12}{}^x+T_{21}{}^x\text{ determines Im }(E_{0+}+3M_{1-}\cos\theta)M_{1+}{}^*, \tag{62b}$$

$$T_{12}{}^z-T_{21}{}^z\text{ determines }(3\cos^2\theta-2)\,|M_{1+}|^2+6E_{1+}M_{1+}{}^*. \tag{62c}$$

We can choose θ appropriately in (62b) and (62c) and thus provide clean measurements of all three transverse nonresonant terms.

For the longitudinal terms the procedure is similar.

Starting with the expression for unpolarized particles [Eq. (56)]:

$T_{13}+T_{31}$ determines Re $(L_{0+}+6L_{1+}\cos\theta)M_{1+}^*$. (63a)

This term has been measured[17] and presumably is a measurement of L_{1+}. Also,

$T_{23}^x+T_{32}^x$ determines

$$\mathrm{Im}\,[L_{0+}\cos\theta+L_{1-}(3\cos^2\theta-2)]M_{1+}^*, \quad (63b)$$

and L_{0+} and L_{1-} can be isolated by taking $\cos\theta=0$ and $\frac{2}{3}$. The higher π–N resonances can be treated similarly but they are more complicated.

We now look at inelastic lepton scattering integrating over the pion angles θ and ϕ. If Eq. (56) for unpolarized leptons and nucleons is integrated, we obtain, using Eq. (55),

$$d^2\sigma/dE'd\omega_l=(\alpha/2\pi^2)(E'/E)(|\mathbf{K}|/k^2)(1-\mathcal{E})^{-1}\times[\sigma_T(k^2,W)+\mathcal{E}_L\sigma_L(k^2,W)] \quad (64)$$

for the total virtual single pion photoproduction cross sections σ_T and σ_L. Here

$$\frac{k_c}{|\mathbf{q}|}\sigma_T=\pi\int_{-1}^{1}dx(T_{11}+T_{22})$$

and

$$\frac{k_c}{|\mathbf{q}|}\sigma_L=2\pi\int_{-1}^{1}dxT_{33}, \quad (65)$$

where $x=\cos\theta$, for virtual single pion photoproduction. But clearly, Eq. (64) is independent of the final hadronic state N' in the reaction $l_1+N\rightarrow l_2+N'$, where only l_2 is observed. So, in general, σ_T and σ_L measure the total virtual-photon absorption cross sections. Now, Eq. (60) gives the asymmetry in lepton scattering off a nucleon polarized along and opposite to the direction of the virtual photon:

$$(d^2\sigma/dE'd\omega_l)^{\mathrm{n}=\mathrm{e}_3}-(d^2\sigma/dE'd\omega_l)^{\mathrm{n}=-\mathrm{e}_3}$$
$$=\frac{\alpha}{2\pi^2}\frac{E'}{E}\frac{|\mathbf{K}|}{k^2}\left(\frac{1+\mathcal{E}}{1-\mathcal{E}}\right)^{1/2}[\sigma_P(k^2,W)-\sigma_A(k^2,W)], \quad (66)$$

where

$$\frac{k_c}{|\mathbf{q}|}(\sigma_P-\sigma_A)=-2\pi i\int_{-1}^{1}dx(T_{12}^z-T_{21}^z). \quad (67)$$

By comparison with Eq. (11) for the density matrix of a right circularly polarized photon, we see that σ_P and σ_A measure the total pion production cross sections for virtual transverse photons of spin parallel and antiparallel to the spin of the nucleon. Again, Eq. (66) can be generalized to apply to all final hadronic states N'.

The absorptive cross sections $\sigma_P(W)$, $\sigma_A(W)$ for $k^2=0$ are those which appear in the Drell–Hearn–Gerasimov[1] sum rule. They can thus be measured directly using polarized electron beams (see below for the use of polarized muon beams in this connection) to obtain the limit $k^2\rightarrow 0$. This means scattering at almost forward angles, so that

$$[(1-\mathcal{E})/(1+\mathcal{E})]^{1/2}\rightarrow(E-E')/(E+E'). \quad (68)$$

Although this factor is inverted in Eq. (66), it represents the ratio of the difference to the sum of the transverse cross sections. Hence it should be as large as possible in order to separate the asymmetric effects. For a given energy E, the largest values of W are the best to measure.

V. FORWARD SCATTERING

We now consider the case where k^2 is not large compared with m^2. This is especially important for muon beams. For simplicity, only forward scattering, $\psi=0$, is considered and now m^2 is not neglected.

To first order in m^2

$$k^2=m^2(E-E')^2/EE'. \quad (69)$$

As this is nonzero and positive, the Breit-frame formalism can still be applied. Thus, the lepton tensor $L_{\mu\nu}$ including the mass terms must be calculated with $\frac{1}{2}(1+\sigma_z)$ in place of $\frac{1}{2}(1-\gamma_5)$ for the lepton spin-projection operator. Then,

$$2k^2L_{\mu\nu}=\tfrac{1}{2}\,\mathrm{Tr}\,[(\gamma\cdot l_1+im)\gamma_\nu(\gamma\cdot l_2+im)\gamma_\mu(1+\sigma_z)], \quad (70)$$

and for $L_{\mu\nu}=L_{\mu\nu}{}^S+L_{\mu\nu}{}^A$,

$$k^2L_{\mu\nu}{}^S=(l_{1\nu}l_{2\mu}+l_{1\mu}l_{2\nu}+\tfrac{1}{2}k^2\delta_{\mu\nu}) \quad (71)$$

and

$$L_{\mu\nu}{}^A=-(i/2)(\delta_{1\mu}\delta_{2\nu}-\delta_{1\nu}\delta_{2\mu}), \quad (72)$$

so

$$L_{12}{}^A=-L_{21}{}^A=-i/2. \quad (73)$$

The lepton momenta are given by

$$l_1=[0,0,\tfrac{1}{2}k,i(m^2+\tfrac{1}{4}k^2)^{1/2}],$$
$$l_2=[0,0,-\tfrac{1}{2}k,i(m^2+\tfrac{1}{4}k^2)^{1/2}]. \quad (74)$$

So for the nonzero elements of $L_{\mu\nu}{}^S$,

$$L_{11}{}^S=L_{22}{}^S=1/2,$$
$$L_{44}{}^S=-2m^2/k^2. \quad (75)$$

Thus, $L_{\mu\nu}$ is an incoherent superposition of a completely right circularly polarized transverse state (for incident positive-helicity muons) plus a pure longitudinal state. In terms of $\mathcal{E}$ we find that for the transverse contribution, $\mathcal{E}$ is zero, while for the longitudinal state, $\mathcal{E}=2EE'/(E^2+E'^2)$, which is the forward-angle limit of Eq. (20) neglecting m. If the lepton were of zero mass, it must conserve helicity, and the lepton current is now proportional to k_μ for forward scattering.[18] Hence, if the hadron current is conserved there is no forward scattering in this approximation. Including mass, this theorem can be generalized: If the lepton scatters preserving helicity in the forward direction, it is still true to first order in m/E that j_μ is proportional to k_μ. Thus, all the scattering is helicity-flip, and the virtual photon in a forward helicity-flip transition must have spin 1. Hence, we have the above result.

We now obtain

$$\frac{d^3\sigma}{dE'd\omega_l d\Omega_\pi}=\frac{\alpha}{2\pi^2}\frac{E'}{E}\frac{|\mathbf{K}|}{k^2}\frac{d\sigma_v}{d\Omega}, \tag{76}$$

where for unpolarized nucleons

$$(k_c/|\mathbf{q}|)(d\sigma_v/d\Omega)=\tfrac{1}{2}(T_{11}+T_{22})+(2m^2/k_0^2)T_{33}. \tag{77}$$

For right-polarized muons and a nucleon polarized parallel and antiparallel to the beam direction, integrating over pion angles gives

$$(d^2\sigma/dE'd\omega_l)^P-(d^2\sigma/dE'd\omega_l)^A$$
$$=\frac{\alpha}{2\pi^2}\frac{E'}{E}\frac{|\mathbf{K}|}{k^2}[\sigma_P(k^2,W)-\sigma_A(k^2,W)]. \tag{78}$$

Thus, forward muon scattering where the muons are polarized, provides a beam of almost real, completely circularly polarized photons, for k^2 can be made very small by using very high-energy muon beams; $\sigma_P(W)-\sigma_A(W)$ for almost real photons can thus be measured.

VI. NEUTRINO SCATTERING

We now consider the process of neutrino scattering on nucleons

$$\nu+N\rightarrow l^-+N', \tag{79}$$

where N' is any state which can be reached through first-order weak interactions.

A high-energy neutrino beam is an inevitable complement to a high-energy muon beam. The major difficulty of performing experiments with this beam, apart from the very small cross section, is that the incident neutrino energy is not known. With better methods of particle detection and with a more intense neutrino beam, it may be possible to detect all final state particles of Reaction (79) and thus determine the over-all energy-momentum configuration. Expressions for the relevant cross sections will be written assuming this to be the case.

The matrix element for Reaction (79) is given by standard weak-interaction theory to be

$$(G'/2^{1/2})\langle N'|J_\mu{}^w|N\rangle\langle l|j_\mu{}^w|\nu\rangle, \tag{80}$$

where $G'=G\cos\theta_c$ or $G\sin\theta_c$, depending on whether strangeness is conserved or not, and θ_c is the Cabibbo angle.[19]

The weak leptonic current is

$$j_\mu=i\bar{u}_2\gamma_\mu(1+\gamma_5)u_1; \tag{81}$$

thus,

$$|\mathfrak{M}|^2=\tfrac{1}{2}G'^2T_{\mu\nu}E_{\mu\nu}, \tag{82}$$

where

$$T_{\mu\nu}=\sum\langle N'|J_\nu{}^w|N\rangle^*\langle N'|J_\mu{}^w|N\rangle \tag{83}$$

as in Eq. (3b), and

$$E_{\mu\nu}=\mathrm{Tr}[\gamma\cdot l_1\gamma_\nu(1+\gamma_5)\gamma\cdot l_2\gamma_\mu(1+\gamma_5)]$$
$$=2\,\mathrm{Tr}[\gamma\cdot l_1\gamma_\nu\gamma\cdot l_2\gamma_\mu(1+\gamma_5)]. \tag{84}$$

Comparing Eq. (84) with Eq. (26), it is apparent that

$$E_{\mu\nu}=8k^2L_{\mu\nu}{}^T, \tag{85}$$

where $L_{\mu\nu}{}^T$ is the transpose of $L_{\mu\nu}$. This shows that the formalism of the previous chapters is applicable to the neutrino case, provided neutrinos are compared with left-polarized muons or electrons, and antineutrinos with right-polarized muons or electrons.

The neutrino analogs of Secs. III, IV, and V are now briefly described.

A. Charge-Exchange Scattering

We consider the processes

$$\nu+n\rightarrow p+l^-,$$
$$\bar{\nu}+p\rightarrow n+l^+.$$

In the nucleon Breit frame, $L_{\mu\nu}$ is given by Eq. (36), and so $E_{\mu\nu}$ is determined.

The weak nucleon current

$$J_\mu{}^w=J_\mu{}^v+J_\mu{}^A, \tag{86}$$

where by the conserved current theory $J_\mu{}^v$ is just the nucleon isovector electromagnetic current and[20]

$$\langle p|J_\mu{}^A|n\rangle=i\bar{u}_2\gamma_\mu\gamma_5u_1\alpha(k^2)+k_\mu\bar{u}_2\gamma_5u_1\beta(k^2). \tag{87}$$

As $k_\mu L_{\mu\nu}=0$ in our approximation, the second term does not contribute, so there is just one extra form factor $\alpha(k^2)$ to consider.

As before $J_\mu{}^w$ is written in terms of two-component spinors, $J_\mu{}^w=\chi_2{}^*\mathfrak{F}^\mu\chi_1$, where in the Breit frame

$$\mathfrak{F}^1=ikG_M\boldsymbol{\sigma}\cdot\mathbf{e}_2-2\omega\alpha\boldsymbol{\sigma}\cdot\mathbf{e}_1,$$
$$\mathfrak{F}^2=-ikG_M\boldsymbol{\sigma}\cdot\mathbf{e}_1-2\omega\alpha\boldsymbol{\sigma}\cdot\mathbf{e}_2,$$
$$\mathfrak{F}^3=-2M\alpha\boldsymbol{\sigma}\cdot\mathbf{e}_3,$$
$$\mathfrak{F}^4=2MiG_E; \tag{88}$$

$\mathfrak{F}^3$ is nonzero showing that $J_\mu{}^A$ is not conserved. The energy of the nucleon in its Breit frame is $\omega=(M^2+k^2/4)^{1/2}=M(1+\tau)^{1/2}$. We are not interested here in polarizing the target nucleon, so $T_{\mu\nu}$ is given as in Eq. (38) and

$$T_{11}=T_{22}=k^2G_M{}^2+4\omega^2\alpha^2,$$
$$T_{44}=-4M^2G_E{}^2,$$
$$T_{12}=-T_{21}=-4ik\omega G_M\alpha. \tag{89}$$

Thus, the neutrino analog to the Rosenbluth formula for the differential cross section in the laboratory is[21] [cf. Eq. (42)]

$$\frac{d\sigma}{d\Omega}=\frac{G'^2k^2}{8\pi^2}\left(\frac{E'}{E}\right)^2\left\{G_E^2\frac{\cot^2\frac{1}{2}\psi}{1+\tau}+[\tau G_M^2+(1+\tau)\alpha^2]\times\left(2+\frac{\cot^2\frac{1}{2}\psi}{1+\tau}\right)\pm 4G_M\alpha\left(\frac{E}{M}-\tau\right)\right\},\quad (90)$$

or in terms of $\mathcal{E}$, as

$$d\sigma/d\Omega=(G'^2k^2/4\pi^2)(E'/E)^2(1-\mathcal{E})^{-1}\times\{\tau G_M^2+\mathcal{E}G_E^2+(1+\tau)\alpha^2\pm 2G_M\alpha[\tau(1+\tau)(1-\mathcal{E}^2)]^{1/2}\}.\quad (91)$$

The positive sign is for incident neutrinos.

B. Weak Pion Production

The processes we consider are

$$\nu+N\rightarrow l^-+N+\pi,$$
$$\bar{\nu}+N\rightarrow l^++N+\pi.$$

By analogy with the electromagnetic case, the weak pion production cross section $d\sigma_w/d\Omega$ can be defined by

$$\frac{d^3\sigma}{dE'd\omega_l d\Omega_\pi}=\frac{G'^2k^2}{4\pi^3}\frac{E'}{E}\frac{|\mathbf{K}|}{1-\mathcal{E}}\frac{d\sigma_w}{d\Omega},\quad (92)$$

where the notation is the same as in Eq. (55). The general form for $d\sigma_w/d\Omega$ for an unpolarized target can be written down in terms of the $\mathfrak{F}_i$ and G_i of Appendices A and B, but this would not be very useful. Instead, we consider the production of the N_{1236}^* resonance including only the dominant terms M_{1+}, the magnetic dipole transition arising from the vector part of the weak current, and $\bar{E}_{1+}$, $\bar{L}_{1+}$, $\bar{S}_{1+}$, the electric, longitudinal and scalar dipole transitions from the axial vector part of the weak current. The form of these dipole operators is taken from the partial-wave analyses [Eqs. (A8) and (B2)].

Both L and S (suffixes are no longer necessary) for the axial current must be considered because it is not conserved. So, in place of Eqs. (54),

$$k^2T_{44}^B=-k_0^2T_{33}-ik_0k_c(T_{34}+T_{43})+k_c^2T_{44},$$
$$kT_{i4}^B=-ik_0T_{i3}+k_cT_{i4},$$
$$kT_{4i}^B=-ik_0T_{3i}+k_cT_{4i},\qquad i=1,2.\quad (93)$$

Similar equations for $T_{3\mu}^B$ are not needed here because $L_{\mu3}=L_{3\mu}=0$. From Eqs. (93), the quantity

$$\Lambda=L-(k_c/k_0)S\quad (94)$$

alone describes the longitudinal and scalar transitions, provided that the lepton current is conserved.

From Eqs. (51) and (52) for $L_{\mu\nu}$, the cross section for producing a pion at the first resonance can now be written:

$$(k_c/|\mathbf{q}|)(d\sigma_w/d\Omega)=\tfrac{1}{2}[|E|^2+|M|^2\pm 2(1-\mathcal{E}^2)^{1/2}E^*M](2+3\sin^2\theta)+\tfrac{3}{2}\mathcal{E}(E^2-M^2)\sin^2\theta\cos 2\phi+6(k_0/k)\{[\tfrac{1}{2}\mathcal{E}(1+\mathcal{E})]^{1/2}E^*\Lambda\pm[\tfrac{1}{2}\mathcal{E}(1-\mathcal{E})]^{1/2}M^*\Lambda\}\sin\theta\cos\theta\cos\phi+(k_0^2/k^2)\mathcal{E}(1+3\cos^2\theta)|\Lambda|^2.\quad (95)$$

In the simplest Chew–Low-type theories, E and L are proportional to $\alpha(k^2)$ and $S=0$.[22]

Note that the effective longitudinal polarization is now given by $(k_0/k)\mathcal{E}^{1/2}$, rather than by $\mathcal{E}_L^{1/2}=(k/k_0)\mathcal{E}^{1/2}$ as in the electromagnetic case. If the weak current were conserved,

$$k_cL-k_0S=0,$$

so that

$$\Lambda=-(k^2/k_0^2)L,\quad (96)$$

and the old formalism is regained. But as it is not conserved, Eqs. (92) and (95) show that as $k^2\rightarrow 0$, only the longitudinal term involving $|\Lambda|^2$ remains. In addition, in this limit $k_0\Lambda=k_0L-k_cS\rightarrow k_cL-k_0S$, which is just (apart from angular factors) the matrix element of $\partial_\mu J_\mu^A$ between N and N_{1236}^*. If the usual assumption is made, i.e., that this matrix element is proportional to that of ϕ_π (ϕ_π is the pion field operator) for k^2 close to $-m_\pi^2$, in particular for $k^2=0$,[23] Adler's result is obtained[18]—for forward neutrino scattering, $d\sigma_w/d\Omega$ is always proportional to the cross section for the corresponding process with an initial charged pion replacing the weak current.

Finally, the appropriate value of $\mathcal{E}$ in the $k^2\rightarrow 0$ limit must be found as in Sec. V. At forward angles

$$k^2=m^2(E-E')/E'\quad (97)$$

for final lepton mass m. $L_{\mu\nu}$ is now

$$k^2L_{\mu\nu}^S=[l_{1\mu}l_{2\nu}+l_{1\nu}l_{2\mu}+\tfrac{1}{2}(k^2+m^2)\delta_{\mu\nu}];\quad (98)$$

and $L_{\mu\nu}^A$ is ignored, as only the longitudinal and scalar couplings are important for small k^2. The lepton momenta in the Breit frame are

$$l_1=[0,0,\tfrac{1}{2}(m^2+k^2)/k,\tfrac{1}{2}i(m^2+k^2)/k],$$
$$l_2=[0,0,\tfrac{1}{2}(m^2-k^2)/k,\tfrac{1}{2}i(m^2+k^2)/k],\quad (99)$$

and so

$$k^2L_{33}=-k^2L_{44}=\tfrac{1}{2}m^2(k^2+m^2)/k^2,$$
$$k^2L_{34}=k^2L_{43}=\tfrac{1}{2}im^2(k^2+m^2)/k^2.\quad (100)$$

Note that the $k^2\rightarrow 0$ limit is here much more singular than the limit was in Sec. V, for now neither the weak lepton current nor the weak hadronic current is conserved.

The terms $T_{3\mu}^B$ and $T_{\mu3}^B$ must now be calculated; for $k^2\rightarrow 0$

$$T_{33}^B=-T_{44}^B=\tfrac{1}{2}i(T_{34}^B+T_{43}^B),\quad (101)$$

so

$$\mathcal{E}/(1-\mathcal{E})=2m^2(k^2+m^2)/k^4=2EE'/(E-E')^2,\quad (102)$$

and with this substitution, Eqs. (92) and (95) give the forward cross section. It is perhaps surprising that this value of $\mathcal{E}$ is again the forward angle limit of Eq. (20) neglecting m, although the weak current is not conserved.

ACKNOWLEDGMENTS

I would like to thank K. Berkelman and Richard Wilson for interesting me in this problem, and D. Imrie and C. Mistretta for useful discussions on pion electroproduction.

APPENDIX A

The form of the matrix element $\langle N\pi | \mathbf{J} | N\rangle$ for the process $\gamma+N\rightarrow\pi+N$, where γ is a virtual photon in the center-of-mass frame, is given by

$$\chi_2^*\mathfrak{F}\chi_1=i\boldsymbol{\sigma}\mathfrak{F}_1+\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}\mathfrak{F}_2+i\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}\hat{\mathbf{q}}\mathfrak{F}_3+i\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}\hat{\mathbf{q}}\mathfrak{F}_4+i\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}\hat{\mathbf{k}}\mathfrak{F}_5+i\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}\hat{\mathbf{k}}\mathfrak{F}_6, \quad \text{(A1)}$$

where $\hat{\mathbf{q}}$ and $\mathbf{k}$ are unit vectors in the directions of $\mathbf{q}$ and $\mathbf{k}$. It is also useful to define

$$\mathfrak{F}_5'=\mathfrak{F}_1+\mathfrak{F}_3\cos\theta+\mathfrak{F}_5,$$
$$\mathfrak{F}_6'=\mathfrak{F}_4\cos\theta+\mathfrak{F}_6, \quad \text{(A2)}$$

so

$$\mathfrak{F}^1=i\boldsymbol{\sigma}\cdot[(\mathfrak{F}_1-\mathfrak{F}_2\cos\theta+\mathfrak{F}_4\sin^2\theta)\mathbf{e}_1+(\mathfrak{F}_2+\mathfrak{F}_3+\mathfrak{F}_4\cos\theta)\sin\theta\,\mathbf{e}_3],$$
$$\mathfrak{F}^2=-\mathfrak{F}_2\sin\theta+i\boldsymbol{\sigma}\cdot\mathbf{e}_2(\mathfrak{F}_1-\mathfrak{F}_2\cos\theta),$$
$$\mathfrak{F}^3=i\boldsymbol{\sigma}\cdot[\mathfrak{F}_6'\sin\theta\mathbf{e}_1+(\mathfrak{F}_5'+\mathfrak{F}_6'\cos\theta)\mathbf{e}_3]. \quad \text{(A3)}$$

Then, starting with the formulas for unpolarized targets, we have

$$T_{11}=T_{22}+[|\mathfrak{F}_2|^2+|\mathfrak{F}_4|^2+2\,\mathrm{Re}\,(\mathfrak{F}_2^*\mathfrak{F}_3)+2\,\mathrm{Re}\,(\mathfrak{F}_1^*\mathfrak{F}_4)+2\,\mathrm{Re}\,(\mathfrak{F}_3^*\mathfrak{F}_4)\cos\theta]\sin^2\theta,$$
$$T_{22}=|\mathfrak{F}_1|^2+|\mathfrak{F}_2|^2-2\,\mathrm{Re}\,(\mathfrak{F}_1^*\mathfrak{F}_2)\cos\theta,$$
$$T_{33}=|\mathfrak{F}_5'|^2+|\mathfrak{F}_6'|^2+2\,\mathrm{Re}\,(\mathfrak{F}_5'^*\mathfrak{F}_6')\cos\theta,$$
$$T_{13}+T_{31}=2\sin\theta\,\mathrm{Re}\,[\mathfrak{F}_5'^*(\mathfrak{F}_2+\mathfrak{F}_3+\mathfrak{F}_4\cos\theta)+\mathfrak{F}_6'^*(\mathfrak{F}_1+\mathfrak{F}_3\cos\theta+\mathfrak{F}_4)]. \quad \text{(A4)}$$

For polarized targets, first $\mathbf{n}=\mathbf{e}_1$, we have

$$T_{12}^x+T_{21}^x=2\,\mathrm{Im}\,[\mathfrak{F}_2^*(\mathfrak{F}_1+\mathfrak{F}_3\cos\theta+\mathfrak{F}_4)-\mathfrak{F}_1^*(\mathfrak{F}_2+\mathfrak{F}_3+\mathfrak{F}_4\cos\theta)]\sin\theta,$$
$$T_{23}^x+T_{32}^x=2\,\mathrm{Im}\,[\mathfrak{F}_5'^*(\mathfrak{F}_1-\mathfrak{F}_2\cos\theta)+\mathfrak{F}_6'^*(\mathfrak{F}_1\cos\theta-\mathfrak{F}_2)],$$
$$T_{12}^x-T_{21}^x=-2i\,\mathrm{Re}\,[\mathfrak{F}_2^*(\mathfrak{F}_1+\mathfrak{F}_3\cos\theta+\mathfrak{F}_4)-\mathfrak{F}_1^*(\mathfrak{F}_2+\mathfrak{F}_3+\mathfrak{F}_4\cos\theta)]\sin\theta,$$
$$T_{23}^x-T_{32}^x=-2i\,\mathrm{Re}\,[\mathfrak{F}_5'^*(\mathfrak{F}_1-\mathfrak{F}_2\cos\theta)+\mathfrak{F}_6'^*(\mathfrak{F}_1\cos\theta-\mathfrak{F}_2)],$$
$$T_{11}^x=T_{22}^x=T_{33}^x=T_{13}^x=T_{31}^x=0. \quad \text{(A5)}$$

Next, for $\mathbf{n}=\mathbf{e}_3$

$$T_{12}^z+T_{21}^z=2\,\mathrm{Im}\,(\mathfrak{F}_1^*\mathfrak{F}_4+\mathfrak{F}_2^*\mathfrak{F}_3)\sin^2\theta,$$
$$T_{23}^z+T_{32}^z=-2\,\mathrm{Im}\,(\mathfrak{F}_5'^*\mathfrak{F}_2+\mathfrak{F}_6'^*\mathfrak{F}_1)\sin\theta,$$
$$T_{12}^z-T_{21}^z=-2i[T_{22}+\mathrm{Re}\,(\mathfrak{F}_1^*\mathfrak{F}_4+\mathfrak{F}_2^*\mathfrak{F}_3)\sin^2\theta],$$
$$T_{23}^z-T_{32}^z=2i\,\mathrm{Re}\,(\mathfrak{F}_5'^*\mathfrak{F}_2+\mathfrak{F}_6'^*\mathfrak{F}_1)\sin\theta,$$
$$T_{11}^z=T_{22}^z=T_{33}^z=T_{13}^z=T_{31}^z=0. \quad \text{(A6)}$$

Now, for $\mathbf{n}=\mathbf{e}_2$

$$T_{11}^y=2\,\mathrm{Im}\,(\mathfrak{F}_1^*-\mathfrak{F}_2^*\cos\theta+\mathfrak{F}_4^*\sin^2\theta)\times(\mathfrak{F}_2+\mathfrak{F}_3+\mathfrak{F}_4\cos\theta)\sin\theta,$$
$$T_{22}^y=-2\,\mathrm{Im}\,(\mathfrak{F}_1^*\mathfrak{F}_2)\sin\theta,$$
$$T_{33}^y=-2\,\mathrm{Im}\,(\mathfrak{F}_5'^*\mathfrak{F}_6')\sin\theta,$$
$$T_{13}^y+T_{31}^y=-2\,\mathrm{Im}\,[\mathfrak{F}_5'^*(\mathfrak{F}_1-\mathfrak{F}_2\cos\theta+\mathfrak{F}_4\sin^2\theta)-\mathfrak{F}_6'^*(\mathfrak{F}_2-\mathfrak{F}_1\cos\theta+\mathfrak{F}_3\sin^2\theta)],$$
$$T_{13}^y-T_{31}^y=2i\,\mathrm{Re}\,[\mathfrak{F}_5'^*(\mathfrak{F}_1-\mathfrak{F}_2\cos\theta+\mathfrak{F}_4\sin^2\theta)-\mathfrak{F}_6'^*(\mathfrak{F}_2-\mathfrak{F}_1\cos\theta+\mathfrak{F}_3\sin^2\theta)],$$
$$T_{12}^y=T_{21}^y=T_{23}^y=T_{32}^y=0. \quad \text{(A7)}$$

The multipole expansion for the $\mathfrak{F}_i$ is, with $x=\cos\theta$,[24]

$$\mathfrak{F}_1=\sum\{(lM_{l+}+E_{l+})P_{l+1}'(x)+[(l+1)M_{l-}+E_{l-}]P_{l-1}'(x)\},$$
$$\mathfrak{F}_2=\sum[(l+1)M_{l+}+lM_{l-}]P_l'(x),$$
$$\mathfrak{F}_3=\sum[(E_{l+}-M_{l+})P_{l+1}''(x)+(E_{l-}+M_{l-})P_{l-1}''(x)],$$
$$\mathfrak{F}_4=\sum(M_{l+}-E_{l+}-M_{l-}-E_{l-})P_l''(x),$$
$$\mathfrak{F}_5'=\sum[(l+1)L_{l+}P_{l+1}'(x)-lL_{l-}P_{l-1}'(x)],$$
$$\mathfrak{F}_6'=\sum\{[lL_{l-}-(l+1)L_{l+}]P_l'(x)\}. \quad \text{(A8)}$$

Here, $M_{l\pm}$, $E_{l\pm}$, and $L_{l\pm}$ are the magnetic, electric, and longitudinal multipole amplitudes leading to a final π–N state with orbital angular momentum l and $J=l\pm\frac{1}{2}$.

APPENDIX B

The form of the matrix element $\langle N\pi | J_\mu^w | \pi\rangle$ in the center-of-mass frame is

$$\chi_2^*(\mathfrak{F}^\mu+iG^\mu)\chi_1,$$

where

$$\mathfrak{F}^\mu=(\mathfrak{F},0),$$

and $\mathfrak{F}$ is given in Appendix A. The most general form for G^μ is[22]

$$\mathbf{G}=G_1\hat{\mathbf{q}}+G_2\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}\boldsymbol{\sigma}+G_3\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}\hat{\mathbf{q}}+G_4\boldsymbol{\sigma}\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}+G_5\hat{\mathbf{k}}+G_6\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}\hat{\mathbf{k}},$$
$$G^4=iG_7+iG_8\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}. \quad \text{(B1)}$$

In the multipole expansion of the G_i it is not clear which multipoles to call electric and which magnetic.

We will choose the convention that the electric multipoles still have the same quantum numbers as the longitudinal and scalar multipoles, i.e., they carry the natural spin-parity assignments for an axial vector field. So with this convention and otherwise using the same notation as the multipole analysis of $\mathfrak{F}_i$ we obtain

$$\begin{aligned}
G_1&=\sum[(\bar{M}_{l+}-\bar{E}_{l+})P_{l+1}''(x)-(\bar{M}_{l-}-\bar{E}_{l-})P_{l-1}''(x)],\\
G_2&=-\sum[(l+2)\bar{M}_{l+}+(l-1)\bar{M}_{l-}-\bar{E}_{l+}+\bar{E}_{l-}]P_l'(x),\\
G_3&=\sum(\bar{E}_{l+}-\bar{M}_{l+}+\bar{M}_{l-}-\bar{E}_{l-})P_l''(x),\\
G_4&=-\sum[(l+1)\bar{M}_{l+}P_{l+1}'(x)+l\bar{M}_{l-}P_{l-1}'(x)],\\
G_5'&=\sum[\bar{L}_{l+}P_{l+1}'(x)-\bar{L}_{l-}P_{l-1}'(x)],\\
G_6'&=\sum(\bar{L}_l-\bar{L}_{l+})P_l'(x),\\
G_7&=\sum[\bar{S}_{l+}P_{l+1}'(x)-\bar{S}_{l-}P_{l-1}'(x)],\\
G_8&=\sum(\bar{S}_{l-}-\bar{S}_{l+})P_l'(x),
\end{aligned} \tag{B2}$$

where

$$G_5'=G_5+G_1\cos\theta+G_4,$$
$$G_6'=G_6+G_2+G_3\cos\theta. \tag{B3}$$

APPENDIX C

There are unfortunately as many notations as authors in this field. Equation (64) for inelastic scattering detecting only the final lepton is

$$\frac{d^2\sigma}{dE'd\omega_l}=\frac{\alpha}{2\pi^2}\frac{E'}{E}\frac{|\mathbf{K}|}{k^2}(1-\epsilon)^{-1}[\sigma_T+\epsilon_L\sigma_L].$$

Akerlof *et al.*[7] use basically the same notation but they include the gauge-invariant factor (k^2/k_0^2) in their longitudinal cross section $\sigma_L^{(A)}$ rather than in the virtual photon polarization density matrix. So

$$\sigma_L^{(A)}=(k^2/k_0^2)\sigma_L. \tag{C1}$$

Gilman[25] introduces

$$\sigma_{\text{trans}}=\sigma_T,\qquad \sigma_{\text{long}}=-\sigma_L^{(A)}. \tag{C2}$$

Hand[6] writes Eq. (64) in terms of the equivalent real photon energy K

$$K=(W^2-M^2)/2M; \tag{C3}$$

so

$$\sigma_T^{(H)}=(|\mathbf{K}|/K)\sigma_T,$$
$$\sigma_S^{(H)}=(|\mathbf{K}|/K)\sigma_L^{(A)}. \tag{C4}$$

In terms of the inelastic form factors[26] $\alpha(K_0,k^2)$ and $\beta(K_0,k^2)$

$$d^2\sigma/dE'd\omega_l=(4\alpha^2E'^2/k^4)[2\sin^2\tfrac{1}{2}\psi\,\alpha(K_0,k^2)+\cos^2\tfrac{1}{2}\psi\,\beta(K_0,k^2)], \tag{C5}$$

where $K_0=E-E'$. Then

$$\alpha(K_0,k^2)=(|\mathbf{K}|/4\pi^2\alpha)\sigma_T,$$
$$\beta(K_0,k^2)=(4\pi^2\alpha)^{-1}(k^2/|\mathbf{K}|)(\sigma_T+\sigma_L^{(A)}). \tag{C6}$$

Drell and Walecka[27] use the form factors W_1 and W_2,

$$\alpha=W_1/M,\qquad \beta=W_2/M. \tag{C7}$$

REFERENCES

[1] Examples of sum rules are: S. B. Gerasimov, J. Nucl. Phys. (USSR) **2**, 598 (1965) [Sov. J. Nucl. Phys. **2**, 430 (1966)]; S. D. Drell and A. C. Hearn, Phys. Rev. Letters **16**, 908 (1966), for $k^2=0$ integrating over W; and W. N. Cottingham, Am. Phys. **25**, 424 (1963), for integrating over both k^2 and W. For consequences of the bootstrap model, see J. D. Stack, Phys. Rev. **164**, 1904 (1967).

[2] M. L. Perl, *Proceedings of the International Symposium on Electron and Photon Interactions at High Energy* (Stanford Linear Accelerator Center, 1967) reviewed the current situation.

[3] J. H. Scofield, Phys. Rev. **113**, 1599 (1959).

[4] R. J. Krisciokaitis and K. W. Robinson, Trans. IEEE Nucl. Sci. NS-14 **3**, 110 (1967).

[5] We use the metric $a_\mu=(\mathbf{a},a_4)=(\mathbf{a},ia_0)$; $a^2=\mathbf{a}\cdot\mathbf{a}=a_\mu a_\mu=\mathbf{a}^2-a_0^2$. For spinors we normalize so that $\bar{\psi}\psi=2m$.

[6] The polarization $\mathcal{E}$ was first introduced by L. Hand, Phys. Rev. **129**, 1834 (1963); see Appendix C.

[7] This discussion is based on that given by C. W. Akerlof, W. W. Ash, K. Berkelman, A. C. Lichtenstein, A. Ramanauskas, and R. H. Siemann, Phys. Rev. **163**, 1482 (1967); see Appendix C.

[8] M. Gourdin, Nuovo Cimento **21**, 1094 (1961).

[9] L. Hand and R. Wilson, Summer Study Rept., Pt. 2, SLAC Rept. No. 25, 1963 (unpublished); H. F. Jones, Nuovo Cimento **40**, 1018 (1965).

[10] The usual difficulties arise in taking the adjoints of time components in our metric.

[11] The form factors $F_1(k^2)$, $F_2(k^2)$ are defined by

$$\langle N\,|\,J_\mu\,|\,N\rangle=i\bar{\psi}_2\gamma_\mu\psi_1F_1-(ik_\nu/2M)\bar{\psi}_2\sigma_{\mu\nu}\psi_1F_2.$$

[12] R. Blankenbecler, S. Gartenhaus, R. Huff, and Y. Nambu, Nuovo Cimento **17**, 775 (1960).

[13] G. T. Hoff, Phys. Rev. **122**, 665 (1961). See also the exhaustive treatment by P. L. Csonka, M. J. Moravcsik, and M. D. Scadron, Rev. Mod. Phys. **39**, 178 (1967).

[14] The N_{1236}^* is an elastic resonance and so the Watson theorem applies. For inelastic resonances it will be approximately true that the various electromagnetic multipoles to the resonance are in phase at resonance. See Y. C. Chau, N. Dombey, and R. G. Moorhouse, Phys. Rev. **163**, 1632 (1967).

[15] See F. A. Berends, A. Donnachie, and D. L. Weaver [Nucl. Phys. **B4**, 54 (1968)] for a recent tabulation of the low-energy π–N phase shifts.

[16] W. W. Ash, K. Berkelman, C. A. Lichtenstein, A. Ramanauskas, and R. H. Siemann, Phys. Letters **24B**, 165 (1967).

[17] D. Imrie, C. Mistretta, and R. Wilson, Phys. Rev. Letters **20**, 1074 (1968).

[18] S. L. Adler, Phys. Rev. **135**, B963 (1964).

[19] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[20] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 354 (1958). In conventional weak-interaction terminology, $\alpha(0)=-G_A/G$.

[21] Y. Yamaguchi, Progr. Theoret. Phys. (Kyoto) **23**, **1117** (1960).

[22] N. Dombey, Phys. Rev. **127**, 653 (1962).

[23] J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo Cimento **17**, 757 (1960).

[24] P. Dennery, Phys. Rev. **124**, 2000 (1961).

[25] F. J. Gilman, Phys. Rev. **167**, 1365 (1968).

[26] S. L. Adler, Phys. Rev. **143**, 1144 (1966).

[27] S. D. Drell and J. D. Walecka, Ann. Phys. (N.Y.) **28**, 18 (1964).