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Evidence for a Nuclear Tensor Force from Mass-14 Beta- and Gamma-Ray Data

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The role of beta- and gamma-ray matrix elements in the determination of wave functions for the s^4p^{10} states of mass 14 is examined. The data considered include the Gamow–Teller (beta) matrix elements of ^{14}C and ^{14}O and the $M1$ and $E2$ matrix elements connecting the lowest four s^4p^{10} states of ^{14}N. The magnetic moment of ^{14}N is also discussed. The shell-model wave functions considered arise mainly from the s^4p^{10} configuration but include admixtures of the s^4p^8 $(2s, 1d)$ configuration. It is found that the data, considered as a whole, cannot be explained if the s^4p^{10} components of the wave functions are derived from a central plus spin–orbit interaction only, but that quite satisfactory agreement is obtained if the nuclear force includes a tensor part. The bulk of the cancellation of the ^{14}C beta decay matrix element takes place within the s^4p^{10} configuration. In general, configuration mixing is of secondary importance; its most noticeable effect is on the $M1$ decay of the first-excited state of ^{14}N.

CONTENTS

1. INTRODUCTION

Two explanations[1–5] have been advanced in recent years to explain the anomalously long lifetime for beta decay of the ^{14}C ground state to the ^{14}N ground state ($\log ft=9.03$).

In one of these, the ^{14}C and ^{14}N ground states are assumed to be well described as two-hole states in the ^{16}O doubly closed p-shell, i.e., as p^{-2} (or s^4p^{10}). A two-body interaction including a tensor force provides the right sign for cancellation of the Gamow–Teller matrix element.[1–3] Cancellation cannot be achieved in the p^{-2} model with a conventional interaction consisting of a central force plus a spin–orbit term.[3,4]

The alternate explanation, first proposed by Inglis,[4] is that both the ^{14}C and ^{14}N ground states are appreciably contaminated by admixtures from configurations other than s^4p^{10} and that the contributions from these admixtures to the Gamow–Teller matrix element cancel the contributions from s^4p^{10}. Motivated by experimental results,[6] Baranger and Meshkov suggested[5] that there may be enough mixing from the $s^4p^8(2s, 1d)$ configuration in the ^{14}C and ^{14}N ground states to provide cancellation *without* invoking any tensor force. (This notation is used for the mixed configurations $s^4p^8s^2+s^4p^8d^2+s^4p^8sd$. When no confusion should arise, we shall leave off the principal quantum numbers and the closed $1s^4$ shell.) However, this explanation depended on the *assumption* that the relative phases of the contributions from p^{10} and $(2s, 1d)$ are such that cancellation can occur.

It has been shown[1,2,3,7] that inclusion of a tensor force into the two-body interaction within the p-shell can explain the ^{14}C lifetime without invoking any configuration mixing. In Sec. 2, we investigate whether the converse is true, i.e., whether configuration mixing can cause cancellation without invoking any tensor force. More specifically: There is now firm experimental and

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theoretical evidence[6,8–16] that the only configurational admixture which may significantly contribute to the Gamow–Teller matrix element connecting the ^{14}N and ^{14}C ground states comes from the $s^4p^8(2s, 1d)$ configuration. To provide cancellation by configuration mixing, the p^{10} and $(2s, 1d)$ contributions to this matrix element must interfere destructively.

In Sec. 2.2 we investigate this question of destructive interference by basing the examination on eigenfunctions of a specific model.[9] In Sec. 2.3 we then examine the question in greater detail and ask more generally whether semi-independently of the *specific* model assumed, an interplay of p-shell and $(2s, 1d)$-shell contributions (invoking central and spin–orbit forces only) is capable of explaining the ^{14}C lifetime and, simultaneously, the lifetime of the ^{14}N 2.31-MeV state (which depends on the *same* eigenfunctions as the Gamow–Teller ^{14}C→^{14}N matrix element).

While the examination in Sec. 2.2 is straightforward with respect to phase questions because only *two-particle* eigenfunctions are involved, the more general investigations in Sec. 2.3 and the inspection of the ^{14}N gamma-ray data done in the subsequent sections require some care. This is because the generalized treatment is most conveniently done in hole formalism (i.e., the low-lying even-parity states of ^{14}C and ^{14}N are described as two holes in the ^{16}O doubly closed shell and transition matrix elements are calculated accordingly). In order to be able to compare at any time the results obtained in the hole formalism to the straightforward but limited results of the two-particle picture (when the ^{14}C and ^{14}N ground states are described as two particles in the $1p_{1/2}$, $2s_{1/2}$, and $1d_{5/2}$ orbits outside a $p_{3/2}{}^8$ core, with $p_{1/2}{}^2$ being the dominant configuration) we feel it desirable to have a well-defined and phase-fixed relationship of *all* matrix elements *throughout*. That is, we calculate all matrix elements defining gamma- and beta-transition amplitudes from one phase-defined set of eigenfunctions which, in the pure jj limit, represents the simple *two-particle* picture, and—where necessary—do particle–hole conjugations phase consistently such that the phase-fixed relationship between *all* matrix elements is never destroyed. This implies that also the ratios of gamma-emission matrix elements of different multipole order ("mixing ratios" δ) are given phase consistently, and in accord with the phase-defined development of angular-distribution theory given recently.[17] To give a description, in brief, of how the phase-fixed relationship of all matrix elements was established throughout, we shall initiate Sec. 2 by a discussion (Sec. 2.1), in which eigenfunctions and operators will be defined explicitly, and in which *two-hole* matrix elements will be related in phase to *two-particle* matrix elements, making use of the isobaric spin formalism, and thus extending the interaction multipole operators of Rose and Brink[17] into isobaric spin subspace.

The question we examine in Secs. 2.2 and 2.3 is not whether there is clear evidence for *either* a nucleon–nucleon tensor interaction *or* for configuration mixing (there is, in fact, evidence for both, cf., Refs. 6, 8–14, 18–20), but rather which of the two "mechanisms" gives rise to the bulk of cancellation and what happens when both are considered simultaneously.

The subsequent sections are devoted to discussing the significance of further gamma-decay data of the ^{14}N nucleus, which are of relevance for the cancellation "mechanism" in the beta decay of the ^{14}C nucleus. In particular we reinforce our conclusion by showing that the s^4p^{10} wave functions generated[2,3] to explain the ^{14}C lifetime, and also the wave functions derived from recent effective interaction calculations of Cohen and Kurath[19] give an excellent account of the electromagnetic transitions connecting the p^{-2} states of ^{14}N. In this comparison we keep in mind, in so far as is possible, the effects of configuration mixing. We also—to some extent—examine results which can be derived from the recent "first-principles" calculation of Becker and MacKellar.[21]

2. CONFIGURATION MIXING AND THE BETA DECAY OF ^{14}C

2.1. Phase Relations of Gamma- and Beta-Decay Matrix Elements in Particle and Hole Formalisms

The calculation of matrix elements for transitions between many particle states may sometimes be considerably simplified by using the concept of "hole" states. It is not the aim of this subsection to consider phase questions arising from particle–hole conjugation, in general, when different shells are involved. We limit ourselves to describing the over-all consistency of phases used in the present treatment of gamma- and beta-decay matrix elements between the even-parity states of mass 14. We consider these states as arising from $1s^41p^{10}$ and $1s^41p_{3/2}{}^8(2s, 1d)$, i.e.,

$$|JMTN\rangle = A^{JT}\,|\,p^{10}; JMTN\rangle + B^{JT}\,|\,p_{3/2}{}^8(2s, 1d); JMTN\rangle, \quad (1)$$

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where we have introduced N as the z quantum number of the isobaric spin total quantum number T and dropped the closed $1s_{1/2}^4$ shell. Throughout we define all eigenfunctions in terms of particle creation and annihilation operators. This means that the symbol p^{10} actually stands for a polynomial of degree 10 in terms of creation operators a_{jmn}^+ (n being again the isospin z quantum number of the single particle, dropping its total isospin quantum number $\frac{1}{2}$), which create particles in the $1p$ shell. The symbol $p_{3/2}^8$ in general stands for a polynomial of degree 8 in terms of p-shell creation operators a_{jmn}^+, but we assume that these couple to give the closed $p_{3/2}$ subshell (we refer to the closed, inert $1s_{1/2}^4 1p_{3/2}^8$ core as ^{12}C). Then the symbol $(2s, 1d)$ simply stands for a polynomial of degree 2 in terms of creation operators, which create particles outside ^{12}C in the $(2s, 1d)$ shell. ^{12}C may be considered as the vacuum state for these operators.

If p^{10} also contains the $p_{3/2}^8$ closed shell, then this polynomial also reduces to a polynomial of degree 2 in terms of the creation operators, $a_{1/2mn}^+$ and again ^{12}C may be considered as the vacuum state from which these operators create particles in the $p_{1/2}$ shell. Equation (1) then takes the very simple form

$$|JMTN\rangle = A^{JT}\,|\,a^+a^+\,|\,0\rangle; JMTN\rangle + B^{JT}\,|\,a^+a^+\,|\,0\rangle; JMTN\rangle, \quad (2)$$

with $^{12}\text{C} = |\,0\rangle$ being the vacuum state, and

$$|\,a^+a^+\,|\,0\rangle; JMTN\rangle = \sum_{m_a m_b, n_a n_b} (\tfrac{1}{2}\tfrac{1}{2} n_a n_b\,|\,TN)(j_a j_b m_a m_b\,|\,JM) \times a_{j_a m_a n_a}^+ a_{j_b m_b n_b}^+\,|\,0\rangle, \quad (3)$$

with $j_a = j_b = \frac{1}{2}$ for the first term on the right-hand side of Eq. (2). The second term in Eq. (2) symbolically stands for a sum of terms of the form (3), corresponding to the different contributions from the mixed configuration $(2s, 1d)$. This term is specified in Sec. 2.2. The only important fact at this stage is that this term is defined in particle-creation operators acting on $|\,0\rangle = {}^{12}\text{C}$. We do not make this term subject to particle–hole conjugation.

In Eq. (2) the coefficients A^{JT} and B^{JT} may be extracted from a specific model (True),[9] which generates eigenfunctions of the form (2). It is now apparent what is meant by "phase consistency throughout": Once A^{JT} and B^{JT} are picked from a certain model in "particle language" ("a^+ language") we may like to break with the restriction $j_a = j_b = \frac{1}{2}$ and investigate, for example, what happens to the transition matrix elements when the simple polynomial $p_{1/2}^2$ is replaced by the more realistic p^{10} treated in an intermediate-coupling situation. To evaluate matrix elements between p^{10} states it is most convenient to apply particle–hole conjugation and again end up with the calculation of (only) two-body matrix elements. In the jj limit an unambiguous phase relation of "two-hole" matrix elements to the two-particle matrix elements can be established because the state in the a^+ language describes (in the middle of the $p_{1/2}$-shell) the *same* physical state as the conjugate state in the "hole language" ("b^+ language").

We now define conjugate states. If a state $|\,a\rangle$ was described as a polynomial in a_{jmn}^+ operators we define the conjugate state $|\,b\rangle$ as the *same* polynomial (with the *same* quantum numbers) in b_{jmn}^+ operators. The hole-creation operators b^+ are related to particle-annihilation operators by

$$b_{jmn}^+ = (-)^{j-m+1/2-n} a_{j-m-n}. \quad (4)$$

The operators b^+ obey the same anticommutation laws as the operators a^+, namely,

$$[a_\alpha^+, a_\beta]_{(+)} = \delta_{\alpha\beta}. \quad (5)$$

The closed-shell state for the hole-creation operators in our case is

$$|\,c\rangle = {}^{16}\text{O}, \quad (6)$$

and if we always relate the closed-shell state to the vacuum state in the same manner, for example,

$$|\,c\rangle = a_{1/2\,1/2}^+ a_{-1/2\,1/2}^+ a_{1/2-1/2}^+ a_{-1/2-1/2}^+\,|\,0\rangle \quad (7)$$

(writing only the z quantum numbers as suffixes), then we obtain in the jj limit

$$|\,b^+b^+\,|\,c\rangle; JMTN\rangle = (-)^T\,|\,a^+a^+\,|\,0\rangle; JMTN\rangle. \quad (8)$$

Equation (8) connects a two-hole state with a two-particle state in the middle of the $p_{1/2}$ shell.

We turn now to the consideration of operators. If we have an operator of rank L acting in spin+(normal) space space and of rank K in isospin space

$$O = O_{LM} S_{Kq}, \quad (9)$$

then, in the a^+ language this operator has the expansion

$$O = \sum_{jmn, j'm'n'} \langle jm\,|\,O_{LM}\,|\,j'm'\rangle \langle \tfrac{1}{2}n\,|\,S_{Kq}\,|\,\tfrac{1}{2}n'\rangle \times a_{jmn}^+ a_{j'm'n'}, \quad (10)$$

while in the hole language it has the expansion

$$O = (-)^{(C+R+1)+(C_i+K)} \sum_{jmn, j'm'n'} \langle jm\,|\,O_{LM}\,|\,j'm'\rangle \times \langle \tfrac{1}{2}n\,|\,S_{Kq}\,|\,\tfrac{1}{2}n'\rangle b_{jmn}^+ b_{j'm'n'}, \quad (11)$$

where the phases C, R, C_i are defined by the Hermitian and time-reversal transformation properties

$$O_{LM}^+ = (-)^{C-M} O_{L-M},$$

$$\theta O_{LM} \theta^{-1} = (-)^{R-M} O_{L-M},$$

and

$$S_{Kq}^+ = (-)^{C_i-q} S_{K-q}. \quad (12)$$

Throughout we assume the standard time-reversal properties $\theta\,|\,jm\rangle = (-)^{j-m}\,|\,j-m\rangle$. K is the rank of the tensor S_{Kq} which acts in isospin subspace.

The interaction multipole operators $T_{LM}^{\langle\pi\rangle}$ of Rose and Brink,[17] making use of the isospin formalism, have the expansion in a^+ language:

$$T_{LM}^{e}=\sum(\tfrac{1}{2}-n')\langle jm \mid T_{LM}^{e}(p) \mid j'm'\rangle\delta_{nn'}a_{jmn}{}^{+}a_{j'm'n'},$$

$$T_{LM}^{m}=\sum\{(\tfrac{1}{2}-n')\langle jm \mid T_{LM}^{m}(p) \mid j'm'\rangle+(\tfrac{1}{2}+n')\times\langle jm \mid T_{LM}^{m}(n) \mid j'm'\rangle\}\times\delta_{nn'}a_{jmn}{}^{+}a_{j'm'n'}, \quad (13)$$

while, in accord with Eq. (11), their expansion in the b^+ language is

$$T_{LM}^{e}=\sum(\tfrac{1}{2}+n')\langle jm \mid T_{LM}^{e}(p) \mid j'm'\rangle\delta_{nn'}b_{jmn}{}^{+}b_{j'm'n'},$$

$$T_{LM}^{m}=\sum\{(\tfrac{1}{2}+n')\langle jm \mid T_{LM}^{m}(p) \mid j'm'\rangle+(\tfrac{1}{2}-n')\times\langle jm \mid T_{LM}^{m}(n) \mid j'm'\rangle\}\times\delta_{nn'}b_{jmn}{}^{+}b_{j'm'n'}, \quad (14)$$

because $R+C+1=$even for $T_{LM}^{\langle\pi\rangle}(p)$ and $T_{LM}^{\langle\pi\rangle}(n)$ (cf. Ref. 17).

The "nuclear part" of the Gamow–Teller interaction is usually represented by

$$G=\sqrt{2}^{-1}\tau_{1q}\sigma_{1p}, \quad (15)$$

where τ_{1q} and σ_{1p} are components of the spherical tensors

$$\tau_{1q}=(-q/\sqrt{2})(\tau_x+iq\tau_y);$$
$$\sigma_{1p}=(-p/\sqrt{2})(\sigma_x+ip\sigma_y),$$
$$\tau_{10}=\tau_z, \qquad \sigma_{10}=\sigma_z, \quad (16)$$

and $\tau_x\tau_y\tau_z$; $\sigma_x\sigma_y\sigma_z$ are the usual Pauli-spin matrices. While the operator $\boldsymbol{\sigma}$ transforms under time reversal as

$$\theta\sigma_{1p}\theta^{-1}=(-)^{1-p}\sigma_{1-p}, \quad (17)$$

the isospin operator is invariant under time reversal, i.e.,

$$\theta\tau_{1q}\theta^{-1}=\tau_{1q}. \quad (18)$$

If in a matrix element of the operator (15) we always chose to write the initial state on the left, then $q=1$ for β^- decay, while $q=-1$ for β^+ decay. The ft value of an allowed Gamow–Teller transition is related to the matrix element of the operator (15) by the following equations:

$$ft=B/\{(1-x)\langle \mathbf{F}\rangle^2+x\langle \mathbf{G}\rangle^2\}, \quad (19)$$

with $B\simeq2720$ sec, $x\simeq0.61$, and

$$\langle \mathbf{G}\rangle^2=\sum_{M_1M_2}(2J_1+1)^{-1}\mid\langle\mid G \mid\rangle\mid^2$$
$$=(T_{21}N_2q \mid T_1N_1)^2\mid\langle J_1T_1 \mid\mid \sqrt{2}^{-1}\boldsymbol{\tau}_1\boldsymbol{\sigma}_1 \mid\mid J_2T_2\rangle\mid^2. \quad (20)$$

The single-body Gamow–Teller operator may also be written in second quantization, and in the a^+ language it has the expansion

$$G=\sum\sqrt{2}^{-1}\langle jm \mid \sigma_{1p} \mid j'm'\rangle\langle\tfrac{1}{2}n \mid \tau_{1q} \mid \tfrac{1}{2}n'\rangle a_{jmn}{}^{+}a_{j'm'n'}. \quad (21)$$

The operator is readily obtained in b^+ language using Eq. (11)

$$G=(-)^1\sum\sqrt{2}^{-1}\langle jm \mid \sigma_{1p} \mid j'm'\rangle\times\langle\tfrac{1}{2}n \mid \tau_{1p} \mid \tfrac{1}{2}n'\rangle^*b_{jmn}{}^{+}b_{j'm'n'}, \quad (22)$$

since from Eqs. (16) and (17) it follows that $C=0$, $R=1$, and $C_i=0$. As $q=\pm1$ it follows that $n\neq n'$. The isospin matrix element can be worked out explicitly and is found to be real, so the asterisk may be omitted in Eq. (22). It is seen that apart from a phase factor $(-)^1$ the operator G has the same expansions in a^+ and b^+ language.

Using the definitions of eigenfunctions and operators of this section, one can explicitly work out the two-body matrix elements in a^+ and b^+ language. We obtain for the $T_{LM}^{\langle\pi\rangle}$ operators (13) in a^+ language

$$\langle\langle 0 \mid aa; J_1T_1N_1=0 \mid\mid T_{LM}^{\langle\pi\rangle}(a^+a) \mid\mid a^+a^+ \mid 0\rangle; J_2T_2N_2=0\rangle$$
$$=(-)^{T_1+T_2}[\text{Formula (4.12) of Ref. 17}] \quad (23)$$

and in b^+ language

$$\langle\langle c \mid bb; J_1T_1N_1=0 \mid\mid T_{LM}^{\langle\pi\rangle}(b^+b) \mid\mid b^+b^+ \mid c\rangle; J_2T_2N_2=0\rangle$$
$$=[\text{Formula (4.12) of Ref. 17}]. \quad (24)$$

The phase factor $(-)^{T_1+T_2}$ is irrelevant for the phase-consistent treatment of Ref. 17 and effectively results from an (arbitrary) labelling of protons and neutrons in an approach in which isospin is not used. [The labeling of Rose and Brink is "particle 1=proton" and "particle 2=neutron", cf., their Eq. (4.14). Note that this choice corresponds to INPUT data "is 1=1" and "is 2=0" if program $D3$ (Häuser *et al.*, compare section 2.2) is used for the calculation of two-particle (or two-hole) matrix elements. We have also adopted this choice here.] However, for internal consistency "throughout" of phases in this paper this factor is of relevance. Together with the dependence of the relative phase of the two-particle and two-hole eigenfunctions on T according to Eq. (8), it assures that the evaluation of the two-particle transition matrix element (23) gives the *same* result in sign and magnitude as the evaluation of the two-hole transition matrix element (24) for $\Delta T=1$ and $\Delta T=0$ gamma transitions. In this sense the gamma-transition matrix element is invariant under particle–hole conjugation.

This result is essential for establishing an internally consistent relation between the Gamow–Teller matrix element and the gamma matrix elements (23) and (24), when the latter are restricted to $M1$ transitions. According to Eqs. (21) and (22) the Gamow–Teller single-particle operator changes sign under particle–hole conjugation. This change of sign is again compensated for by Eq. (8) and, therefore, the two-particle Gamow–Teller matrix element is also invariant under particle–hole conjugation in the same sense as the gamma matrix elements are. A consistent treatment of both the Gamow–Teller and the gamma matrix elements, using isospin formalism, thus leads to the same behavior of the two types of matrix elements under particle–hole conjugation. Therefore, once a certain relative phase between them is established in particle language it is not destroyed when going to a hole description of the nuclear states involved.

In conclusion, we have shown in this section that we may ignore effects on phases due to particle–hole conjugation if we use the proper operators as defined in this section and also establish properly a phase relation between particle and hole eigenfunctions. Our proof holds for the middle of the $p_{1/2}$ shell. However, because the relationship between LS- and jj-expanded states is independent of whether particles or holes are related[22] we may extend this conclusion from the jj limit to an intermediate coupling situation, because we can collect and define phases in the jj limit, convert to LS expansion, and then go to any degree of intermediate coupling.

Finally, we quote the relationship between the matrix elements of the spin part of the $M1$ interaction multipole operators and the matrix elements of the Gamow–Teller operator (15),

$$\langle\langle 0 \mid aa; J_1 10 \mid\mid \mathbf{T}_1^{m,\sigma}(a^+a) \mid\mid a^+a^+ \mid 0\rangle; J_2 00\rangle$$
$$= -(1/2\sqrt{2})kg_s^{(-)}\beta$$
$$\times\langle\langle 0 \mid aa; J_1 11 \mid\mid G(a^+a) \mid\mid a^+a^+ \mid 0\rangle; J_2 00\rangle, \quad (25)$$

with $k=E/\hbar c$, $\beta=e\hbar/2mc$, and $g_s^{(-)}=g_s(p)-g_s(n)=9.411$ (cf. Ref. 17).

Because the orbital part of the $M1$ operator has a fixed phase relative to the spin part, we can in later equations also make use of Eq. (25) to split up the matrix element of the $\mathbf{T}_1^m$ operator into orbital and spin contributions and replace the latter by the Gamow–Teller matrix element with a definite phase relative to the orbital contribution.

2.2. True's Model and the Possibility of Cancellation of the Gamow–Teller Matrix Element

We now examine the ${}^{14}\text{C}\rightarrow{}^{14}\text{N}$, $(J^\pi, T)=(0^+, 1)\rightarrow(1^+, 0)$ Gamow–Teller matrix element and the lifetime of the 2.31-MeV state in ^{14}N, which decays to the ^{14}N ground state by a $M1$ transition and which is the isospin analog to the ^{14}C ground state. In this section we rely on the eigenfunctions generated for the ^{14}C and ^{14}N ground states by the shell-model calculations of True.[9] We recall that this model uses a purely central force and so it only tests the effects of configuration mixing.

The binding-energy calculations of Unna and Talmi[8] and the shell-model calculations of True[9] predict that the spectrum of $p^8(2s, 1d)$ levels commences at an excitation energy of $\simeq$6 MeV above the ground state of ^{14}N, which is considerably lower than for any other configuration. This is confirmed by experimental evidence[6,10–15] which also indicates that the ^{14}C and ^{14}N ground states contain admixtures from the $p^8(2s, 1d)$ configuration of approximately 10% in intensity. This estimate is consistent with the calculations of True. [In detail, his results gave intensities of 1.5% $p^8s_{1/2}^2$ and 10% $p^8d_{5/2}^2$ for the ^{14}C ground state and 0.4% $p^8s_{1/2}^2$, 3.4% $p^8d_{5/2}^2$ and 1% $p^8s_{1/2}d_{3/2}$ in the ^{14}N ground state. This is to be compared with the results of the various (p, d) and (d, t) pickup experiments,[6,12–15] which have been analyzed[5,12,13,16] to give $\simeq$0.5% $p^8s_{1/2}^2$ and $\simeq$10% $p^8d_{5/2}^2$ in the ^{14}C ground state and $\simeq$0.6% $p^8s_{1/2}^2$ and $\leq$3% $p^8s_{1/2}d_{3/2}$ in the ^{14}N ground state.]

The importance of the configurations formed by promoting a nucleon from the $1p$ shell to the $1f$ or $2p$ shell has also been estimated by True,[23] and experimental results[24] indicate as well that these configurations should influence the nuclear properties under consideration to a negligible degree. There have been no theoretical or experimental studies of the configuration in mass 14 formed by promoting nucleons out of the $1s$ shell. However, theoretical results[25] for mass 15 and consideration[26] of the $1s$–$1p$ and $1p$–$1d$ energy difference indicate that these configurations should not give any significant contributions to the wave functions of the low-lying states of mass 14.

The major weakness in the shell-model calculations of True is the assumption of a closed $p_{3/2}^8$ shell acting as a $J=0$ core for the numerical evaluation of Eq. (1) (consequently always $\mid p^{10}\rangle=\mid p_{1/2}^2\rangle$ in True's treatment). The removal of this assumption immediately introduces the complexity of a six-body problem. However, the reduction to a two-body problem seems to generate a set of wave functions (1) which approximates the problem very well. So, True's treatment[9] was successful in explaining all the bound (and some unbound) levels of ^{14}N and ^{14}C including[27] the $p_{3/2}^8p_{1/2}s_{1/2}$ and $p_{3/2}^8p_{1/2}d_{5/2}T=0$ states in ^{14}N, with the exception of those levels previously identified[11] as arising predominantly from break-up of the $p_{3/2}^8$ "core".

From True[9] we obtain the numerical values of the coefficients A^{JT} and B^{JT} of Eq. (1)

$$A^{01}=0.950 \qquad B^{01}=-0.311$$
$$A^{10}=0.966 \qquad B^{10}=0.255. \qquad (26a)$$

In Table I, first line, are collected the jj-expansion coefficients for the wave functions of the $(2s, 1d)$ mixed configuration as generated by True's model, i.e., we

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TABLE I. *jj*-expansion coefficients for the wave functions of the $s^4p^n(2s, 1d)$ components in the lowest $(J^\pi, T)=(1^+, 0)$ and $(0^+, 1)$ states of ^{14}N and ^{18}F. The coupling scheme is $\mathbf{j}_1+\mathbf{j}_2=\mathbf{J}$ and $\mathbf{l}+\mathbf{s}=\mathbf{j}$.

Calculation	$(J^\pi, T)=(0^+, 1)$ $C(s_{1/2}^2; 01)$	$C(d_{3/2}^2; 01)$	$C(d_{5/2}^2; 01)$	$(J^\pi, T)=(1^+, 0)$ $C(s_{1/2}^2; 10)$	$C(d_{3/2}^2; 10)$	$C(d_{5/2}^2; 10)$	$C(s_{1/2}d_{3/2}; 10)$	$C(d_{3/2}d_{5/2}; 10)$
True[a]	+0.391	+0.365	+0.845	+0.251	−0.514	+0.718	+0.395	+0.041
Elliott and Flowers[b]	+0.39	+0.24	+0.89	+0.55	−0.19	+0.58	−0.02	+0.57
Redlich[b]	+0.40	+0.31	+0.86	+0.46	−0.16	+0.67	+0.02	+0.56
Zamick[a]	+0.320	+0.291	+0.902	+0.492	−0.044	+0.572	−0.143	+0.640
Flowers and Wilmore[a,c]	+0.392	+0.243	+0.887	+0.350	−0.068	+0.884	−0.036	+0.300

[a] The original coupling scheme is $\mathbf{l}+\mathbf{s}=\mathbf{j}$, see Refs. 7, 9, and 35.
[b] The original coupling scheme is $\mathbf{s}+\mathbf{l}=\mathbf{j}$, see Refs. 33 and 34.
[c] The $(J^\pi, T)=(0^+, 1)$ function corresponds to $\Delta^{31}V_0=40$ MeV and $\Delta^{33}/\Delta^{31}=-1$ (in the notation of Ref. 35). For the $(1^+, 0)$ function $\Delta^{31}V_0=40$ MeV and $\Delta^{33}/\Delta^{31}=1$. This choice of parameters is adapted to give the best fit to available experimental data. We thank D. Wilmore for communicating to us the complete set of eigenfunctions underlying Ref. 35.

write

$$|\,p_{3/2}{}^8(2s, 1d); J=1 T=0\rangle = C(s_{1/2}{}^2; 10)s_{1/2}{}^2 + C(d_{3/2}{}^2; 10)d_{3/2}{}^2 + C(d_{5/2}{}^2; 10)d_{5/2}{}^2 + C(s_{1/2}\,d_{3/2}; 10)s_{1/2}\,d_{3/2} + C(d_{3/2}\,d_{5/2}; 10)\,d_{3/2}\,d_{5/2} \quad (26b)$$

and

$$|\,p_{3/2}{}^8(2s, 1d); J=0\ T=1\rangle = C(s_{1/2}{}^2; 01)s_{1/2}{}^2 + C(d_{3/2}{}^2; 01)\,d_{3/2}{}^2 + C(d_{5/2}{}^2; 01)\,d_{5/2}{}^2. \quad (26c)$$

The over-all phase of these coefficients (given in Table I) is as to reproduce with the choice $C(p_{1/2}{}^2; 10)=1$ and $C(p_{1/2}{}^2; 01)=1$ and the coefficients (26a) the correct relative phase of the two parts of Eq. (2) as yielded by True's model. The choice of the over-all phase of the right-hand side of Eq. (2) is the *only* "*free*" choice of phases available. According to Sec. 2.1 all further phases of eigenfunctions and matrix elements, then, are fixed (and—whenever numerical values are quoted—they are given accordingly and consistently throughout).

For comparison and for later use we have included in Table I some further values as obtained for mass-18 wave functions. The coupling scheme we use throughout is $\mathbf{j}_1+\mathbf{j}_2=\mathbf{J}$ and $\mathbf{l}+\mathbf{s}=\mathbf{j}$ [i.e., the respective vector-coupling coefficient is $(lsmlm_s\,|\,jm)$].

In the notation of Eqs. (1) and (20) the Gamow–Teller (reduced) matrix element $\langle\mathbf{G}\rangle$ is given by

$$\langle\mathbf{G}\rangle = A^{01}A^{10}\langle\mathbf{G}(p^{-2})\rangle + B^{01}B^{10}\langle\mathbf{G}(2s, 1d)\rangle. \quad (27)$$

With the above choice of phases and in accord with Sec. (2.1) $\langle\mathbf{G}(p^{-2})\rangle$ and $\langle\mathbf{G}(2s, 1d)\rangle$ are given by

$$\langle\mathbf{G}(p^{-2})\rangle = (\sqrt{2}/\sqrt{3})C(p_{1/2}{}^2; 01)C(p_{1/2}{}^2; 10) \quad (28a)$$

and

$$\langle\mathbf{G}(2s, 1d)\rangle = -(6)^{1/2}C(s_{1/2}{}^2; 01)C(s_{1/2}{}^2; 10) + [(6)^{1/2}/(5)^{1/2}]C(d_{3/2}{}^2; 01)C(d_{3/2}{}^2; 10) - [(14)^{1/2}/(5)^{1/2}]C(d_{5/2}{}^2; 01)C(d_{5/2}{}^2; 10) - 2[\sqrt{3}/(5)^{1/2}]C(d_{3/2}{}^2; 01)C(d_{3/2}\,d_{5/2}; 10) - 2[\sqrt{2}/(5)^{1/2}]C(d_{5/2}{}^2; 01)C(d_{3/2}\,d_{5/2}; 10). \quad (28b)$$

It is seen from Eqs. (28a) and (28b) that indeed the reduced two-particle Gamow–Teller matrix elements corresponding to transitions $s_{1/2}{}^2 \to s_{1/2}{}^2$ and $d_{5/2}{}^2 \to d_{5/2}{}^2$

TABLE II. Two-particle reduced matrix elements of the magnetic-dipole-interaction multipole operator $\mathbf{T}_1{}^m$, the magnetic dipole multipole operator $\mathbf{G}_1{}^m=\mathbf{G}_1{}^m$ (spin) $+\mathbf{G}_1{}^m$(space) for the 2.31→0 MeV, $(0^+, 1)\to(1^+, 0)$ transition in ^{14}N and the Gamow–Teller operator $G=1/\sqrt{2}\boldsymbol{\tau}\cdot\boldsymbol{\sigma}$ for the $^{14}\text{C}\to{}^{14}\text{N}(0^+, 1)\to(1^+, 0)$ beta transition.

Transition	$\langle\mathbf{T}_1{}^m\rangle$ $10^{-7}z$MeV/Gcm	$\langle\mathbf{G}_1{}^m\rangle$ Nuclear magneton β	$\langle\mathbf{G}\rangle$ Dimensionless	$\langle\mathbf{G}_1{}^m(\sigma)\rangle$ Nuclear magneton β	$\langle\mathbf{G}_1{}^m(l)\rangle$ Nuclear magneton β
$p_{1/2}{}^2\to p_{1/2}{}^2$	−5.761	−1.562	+0.816	−2.716	+1.154
$p_{1/2}{}^{-2}\to p_{1/2}{}^{-2}$	−5.761	−1.562	+0.816	−2.716	+1.154
$p^{-2}\to p^{-2}$ [a]	−4.681	−1.269	+0.709	−2.361	+1.091
$s_{1/2}{}^2\to s_{1/2}{}^2$	+30.006	+8.150	−2.449	+8.150	0.000
$d_{3/2}{}^2\to d_{3/2}{}^2$	−4.873	−1.321	+1.095	−3.644	+2.323
$d_{5/2}{}^2\to d_{5/2}{}^2$	+29.265	+7.934	−1.673	+5.567	+2.366
$d_{3/2}{}^2\to d_{3/2}d_{5/2}$	+16.993	+4.606	−1.549	+5.154	−0.547
$d_{5/2}{}^2\to d_{3/2}d_{5/2}$	+13.874	+3.761	−1.264	+4.208	−0.447

[a] Intermediate coupling $(a/K)=7$.

are considerably larger than the one corresponding to a transition within $p_{1/2}^{-2}$. However, inserting the numerical values for A^{JT} and B^{JT} from Eq. (26) and the coefficients $C(JT)$ from the first line of Table I we obtain

$$\langle \mathbf{G} \rangle = +0.918(+0.816) - 0.0797(-1.526) = 0.871 \tag{29}$$

and we see that not only is the magnitude of the $(2s, 1d)$ admixture too small for cancellations but also the sign is wrong. All contributions from $(2s, 1d)$ interfere *constructively* with $p_{1/2}^{-2}$.

We now consider the $M1$ matrix element for the ^{14}N 2.31→0 transition. The gamma width in terms of reduced matrix elements of interaction multipole operators for a transition $|J_1\rangle \rightarrow |J_2\rangle$ is defined as[17]

$$\Gamma_\gamma = 4k \sum_{L\pi} |\langle J_1 \| \mathbf{T}_L^{(\pi)} \| J_2 \rangle|^2/(2L+1), \tag{30}$$

where the wave number $k = E/\hbar c$, L is the multipolarity and $\langle\pi\rangle$ stands for either "magnetic" or "electric." Numerically Γ_γ is obtained in electron volts from

$$\Gamma_\gamma = 3.22 \times 10^{11} E_\gamma \sum_{L\pi} |\langle J_1 \| \mathbf{T}_L^{(\pi)} \| J_2 \rangle|^2/(2L+1), \tag{31}$$

when E_γ is in MeV and the matrix element is in MeV/G cm. Two-particle matrix elements of the operators $\mathbf{T}_L^{(\pi)}$ may be calculated from formula (4.12) of Ref. 17 or, more conveniently, using program $D3$ of Ref. 28. To conform with the (over-all) phases of matrix elements of the isobaric spin extended operators $\mathbf{T}_L^{(\pi)}$ and the choice of the over-all phase of the eigenfunctions as described in Sec. 2.1 we multiply the over-all phase of formula (4.12) of Ref. 17 by $(-)^{T_1+T_2}$, which does not affect the signs of the multipole-mixing ratios. These are in accord with the phase-consistent derivation of angular distribution formulas by Rose and Brink.[17]

In Table II are collected the relevant two-particle matrix elements for the 2.31→0, $(0^+, 1) \rightarrow (1^+, 0)$ transition.

Multiplying the transition matrix elements given in Table II with the amplitudes A^{JT} and B^{JT} and the respective coefficients from line 1 of Table I we have

$$\begin{aligned} &\langle\langle 0 \mid aa; 0^+1 \| \mathbf{T}_1^m(a^+a) \| a^+a^+ \mid 0\rangle; 1^+0\rangle \\ &= \langle\langle c \mid bb; 0^+1 \| \mathbf{T}_1^m(b^+b) \| b^+b^+ \mid c\rangle; 1^+0\rangle \\ &= A^{01}A^{10}\langle T_1^m(p_{1/2}^2)\rangle + B^{01}B^{10}\langle T_1^m(2s, 1d)\rangle \\ &= 0.918(-5.761) - 0.079(+22.390) \end{aligned} \tag{32}$$

in 10^{-7} MeV/G cm. Using formula (31) this gives $\Gamma_\gamma(M1) = 125.1 \times 10^{-3}$ eV. If the ^{14}N 2.31-MeV state and ground state were described purely as $p_{1/2}^2$ we would have obtained $\Gamma_\gamma(M1) = 82.1 \times 10^{-3}$ eV.

The weighted average of four measurements[29–32] of the radiative width of the ^{14}N 2.31→0 transition is $\Gamma_\gamma(M1)_{\rm exp} = (8.1 \pm 1.4) \times 10^{-3}$ eV.

In conclusion, basing the examination of the question of the vanishing Gamow–Teller matrix element due to a destructive interplay of different configurations on True's model, we find that the cancellation cannot be achieved within the framework of this otherwise quite successful model. Also the lifetime of the 2.31-MeV state in ^{14}N cannot be explained within the framework of this model. Consequently either the model is nonrealistic or the mechanism of cancellation cannot be due to an interplay of different configurations, as long as the bulk of cancellation has not already been achieved in the p shell. Therefore, in the next section, we try to repeat the examination in a model semi-independent way and examine whether the Gamow–Teller matrix element can be made to vanish and simultaneously the $M1$ width brought into agreement with experiment by an interplay of the two configurations or vice versa.

2.3. Model Semi-independent Examination of the $(0^+, 1) \rightarrow (1^+, 0)$ Beta- and Gamma-Decay Matrix Elements

To make conclusions independent (or at least semi-independent) of the specific model which was used in Sec. 2.2, we now proceed in two steps as explained below.

(i) Because the major weakness in the shell-model calculations of True[9] was the assumption of a closed $p_{3/2}^8$ shell, we shall remove this assumption and insert for $|p^{10}; JMTN\rangle$ of Eq. (1) the full set of two-hole wave functions $|p^{-2}; JMTN\rangle$ rather than $|p_{1/2}^2; JMTN\rangle$. The phase relationship of hole and particle descriptions has been established in Sec. 2.1. We expand $|p^{-2}; JMTN\rangle$ in LS coupling and go into an intermediate-coupling situation as produced by a conventional central plus spin–orbit interaction. That is we write

$$\begin{aligned} |p^{-2}; J=1\ T=0\rangle &= C_S^{10}\,{}^3S_1 + C_P^{10}\,{}^1P_1 + C_D^{10}\,{}^3D_1 \\ |p^{-2}; J=0\ T=1\rangle &= C_S^{01}\,{}^1S_0 + C_P^{01}\,{}^3P_0. \end{aligned} \tag{33}$$

For the $p_{3/2}^8(2s, 1d)$ wave functions we consider results of shell-model calculations[7,33–35] for mass 18, i.e., $p^{12}(2s, 1d)$ as well as those of True.[9] We also consider pure jj coupling of $(2s, 1d)$, i.e., pure $d_{5/2}^2$. The coefficients resulting from these various calcula-

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[35] B. H. Flowers and D. Wilmore, Proc. Phys. Soc. (London) **83**, 683 (1964).

TABLE III. LS-expansion coefficients for the wave functions of the lowest three s^4p^{10} states of ^{14}N.

Calculation[a]	Ground state $(J^\pi, T)=(1^+,0)$			2.31-MeV level $(J^\pi, T)=(0^+,1)$		3.95-MeV level $(J^\pi, T)=(1^+,0)$		
	C_S	C_P	C_D	C_S	C_P	C_S	C_P	C_D
jj coupling	−0.192	+0.471	+0.861	+0.577	+0.817	+0.770	−0.471	+0.430
Soper	−0.204	+0.358	+0.911	+0.638	+0.770	+0.907	−0.281	+0.314
Elliott	+0.077	+0.179	+0.981	+0.805	+0.593	+0.98	−0.22	−0.05
Visscher and Ferrell	+0.173	+0.355	+0.920	+0.764	+0.646	+0.813	−0.580	+0.073
Cohen and Kurath I	+0.088	+0.261	+0.962	+0.865	+0.502	+0.963	−0.272	−0.014
Cohen and Kurath II	+0.136	+0.240	+0.962	+0.880	+0.474	+0.961	−0.267	−0.070

[a] The calculations are as follows: Soper, intermediate coupling $L/K=6$, $a/K=7$, Ref. 36; Elliott Ref. 2; Visscher and Ferrell, Ref. 3; Cohen and Kurath I, (8–16)2BME, $\epsilon=5.67$ MeV, Ref. 19; Cohen and Kurath II, (8–16)2BME, $\epsilon=5.15$, Ref. 19.

tions are given in Table I. The reason for considering these $p^{12}(2s, 1d)$ results as well as those for $p^8(2s, 1d)$ is to examine the sensitivity of the predictions to the particular wave functions assumed. The relative phases of the individual terms contributing to these wave functions have been chosen to conform with the coupling scheme used in this work, namely, $\mathbf{l}+\mathbf{s}=\mathbf{j}$ and $\mathbf{j}_1+\mathbf{j}_2=\mathbf{J}$. The choice of over-all phase in lines 2–5 of Table I is made to conform to line 1. As mentioned before, the choice of over-all phase of line 1 for two-*particle* eigenfunctions is the only arbitrary choice available.

The LS-expansion coefficients C_L^{JT} of Eq. (33) for an intermediate-coupling situation $(a/K)=7$ as obtained by Soper[36] are given in Table III. The choice of over-all phase here, is again such that the functions (33) describe two-*particle* states. As apparent from Sec. 2.1, the phase of the matrix elements of the Gamow–Teller and the electromagnetic transition operators is invariant under particle–hole conjugation.

The above substitutions of wave functions for the p-shell and $(2s, 1d)$-shell contributions in Eq. (1) will, at this stage, still be done without changing True's results for the coefficients A^{JT} and B^{JT}. This is done for the simple reason that all p^{-2} wave functions for the ^{14}C and ^{14}N ground states which give a reasonable description of experiment have a $p_{1/2}^{-2}$ component of $\geq 80\%$, so that the inclusion of the small $p_{3/2}^{-1}p_{1/2}^{-1}$ and $p_{3/2}^{-2}$ admixtures will have little influence on the small $p_{3/2}^{8}(2s, 1d)$ admixtures and vice versa. We also assume that the restriction of a closed $p_{3/2}^{8}$ core in $p^8(2s, 1d)$ can be relaxed to that of a $J^\pi=0^+$ p^8 core state without undue influence on the B^{JT}.

(ii) In a final step of generalization we shall then make the numerical values of A^{JT} and B^{JT} subject to variation. That is, we determine and inspect the set of A^{JT} and B^{JT} which yields, by an interplay of p-shell and $(2s, 1d)$-shell contributions, a vanishing Gamow–Teller matrix element and reproduces the $(0, 1)\rightarrow(1, 0)$ experimental gamma width. Because there are four coefficients A^{JT}, B^{JT} entering the matrix elements, and because

$$\langle \mathbf{G} \rangle = 0, \tag{34}$$

$$\langle 01 \,||\, \mathbf{T}_1^m \,||\, 10 \rangle = \langle 01 \,||\, \mathbf{T}_1^m \,||\, 10 \rangle_{\exp}, \tag{35}$$

and the two normalization conditions provide four equations for them, it is obvious that a set of A^{JT}, B^{JT} can always be found which, in principle, provides an interplay of the configurations in question so that the experimental facts (34) and (35) are described. However, we show that the specific solution so obtained requires that either the ^{14}C or the ^{14}N ground state would have to arise almost purely from the $(2s, 1d)$ configuration. Because this is quite unreasonable, we then conclude—model independently (or at least semi-independently)—that the bulk of cancellation of the ^{14}C beta-decay matrix element cannot be explained without the use of a tensor force, i.e., as being due to an interplay of different configurations only.

We do not discuss the magnetic-dipole or electric-quadrupole moments of the ^{14}N ground state at this time since these moments are found to be rather insensitive to the presence of both a tensor interaction and admixtures from $(2s, 1d)$ and give essential agreement with experiment with or without these refinements to the wave functions. Neither do we discuss at this time the electromagnetic transitions originating from the higher-lying ^{14}N states of p^{-2}; the reason being that the other two bound states of p^{-2} which we shall consider later arise predominantly from $p_{3/2}^{-1}p_{1/2}^{-1}$ and thus the coefficients tabulated in Table I no longer give us a quantitative guide to the $p^8(2s, 1d)$ eigenfunctions.

We now consider step (i). In LS-coupling Eq. (28a) is to be replaced by

$$\langle \mathbf{G}(p^{-2}) \rangle = +\sqrt{2}\{C_P^{01}C_P^{10} - \sqrt{3}C_S^{01}C_S^{10}\}, \tag{36}$$

and $\langle \mathbf{G}(2s, 1d) \rangle$ is still obtained from Eq. (28b).

Experimentally, the magnitude of the Gamow–Teller matrix element for ^{14}C is obtained from the ft value[3]

$$\langle \mathbf{G} \rangle \,| = 0.002 \text{ for } ^{14}\text{C}, \tag{37}$$

[36] J. M. Soper (private communication).

while from the positron decay of ^{14}O it was obtained[37]

$$|\langle \mathbf{G}\rangle| = (0.013\pm0.004) \text{ for } ^{14}\text{O}. \quad (38)$$

The magnitude of $\langle \mathbf{G}\rangle$ is about seven times larger for ^{14}O than it is for ^{14}C. This difference is presumably due[3] to dynamic distortion effects related to the change of two neutrons into protons in going from ^{14}C to ^{14}O. However for present purposes both $\langle \mathbf{G}\rangle$ values differ negligibly from zero and can be taken as such. Thus, we neglect such charge-dependent effects and consider Eq. (27) to give identical results for all three $(0^+, 1)$ members of the isobaric triplet.

We now evaluate Eq. (27) using the intermediate-coupling results of Soper[36] for $\langle \mathbf{G}(p^{-2})\rangle$, the wave functions of Table I for $\langle \mathbf{G}(2s, 1d)\rangle$, and the A_0^{JT} and B_0^{JT} of True[9]. This gives

$$\langle \mathbf{G}\rangle = 0.918\langle \mathbf{G}(p^{-2})\rangle - 0.079\langle \mathbf{G}(2s, 1d)\rangle. \quad (39a)$$

The value of $\langle \mathbf{G}(p^{-2})\rangle$ is found to be quite insensitive to variation of the intermediate-coupling parameter a/K. This is illustrated in Fig. 1. The value of a/K which best fits the energy spectra and other properties of the mass-14 nuclei is $\sim$7. Figure 1 shows that $\langle \mathbf{G}(p^{-2})\rangle$ is quite insensitive to a/K in the region from $\sim$4 to 9. Thus we take the value for $a/K=7$ which is $\langle \mathbf{G}(p^{-2})\rangle = +0.709$.

The values of $\langle \mathbf{G}(2s, 1d)\rangle$ corresponding to the five sets of wave functions of Table I are listed in Table IV. $\langle \mathbf{G}(2s, 1d)\rangle$ is insensitive to the variations in these wave functions so that we can use True's wave functions with some confidence. Combining these results we have,

$$\langle \mathbf{G}\rangle = (0.918)(+0.709) - (0.079)(-1.527)$$
$$= (0.651+0.122) = 0.773,$$

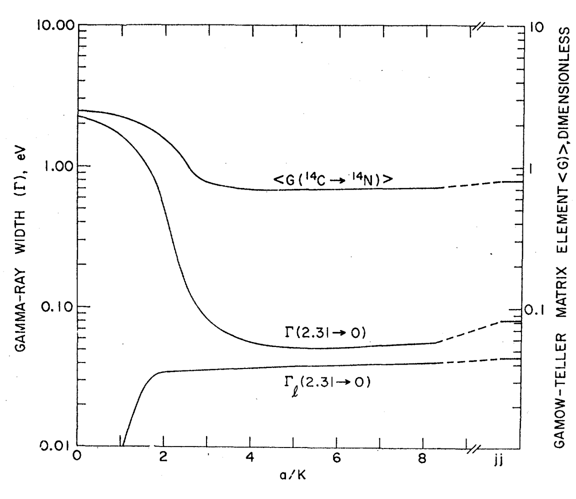

FIG. 1. Total width Γ and its orbital contribution Γ_l of the 2.31 MeV→gs $M1$ transition in ^{14}N vs the intermediate coupling parameter a/K. The GT matrix element, $\langle \mathbf{G}\rangle$, is also plotted.

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TABLE IV. The Gamow–Teller matrix element and the space part of the $M1$ matrix element connecting the $s^4p^n(2s, 1d)$ components in the lowest $(J^\pi, T) = (0^+, 1)$ and $(1^+, 0)$ states of ^{14}N and ^{18}F.

Calculation[a]	$\langle \mathbf{G}(2s, 1d)\rangle$	$\langle \mathbf{G}_1^{m,l}(2s, 1d)\rangle$
True	−1.527	+0.976
Elliott and Flowers	−2.293	+0.813
Redlich	−2.347	+0.937
Zamick	−2.282	+0.830
Flowers and Wilmore	−2.115	+1.658
jj coupling	−1.673	+2.366

[a] The origins of the first five calculations are given in Table I. *jj*-coupling refers to an assumption of pure $d_{5/2}^2$.

and we see that as in Sec. 2.2 not only is the magnitude of the $p^8(2s, 1d)$ admixture too small for cancellation but more importantly the sign is wrong.

We now consider again the $M1$ matrix element

$$\langle 01 \| \mathbf{T}_1^m \| 10\rangle$$
$$= A^{01}A^{10}\langle \mathbf{T}_1^m(p^{-2})\rangle + B^{01}B^{10}\langle \mathbf{T}_1^m(2s, 1d)\rangle. \quad (39b)$$

From Fig. 1 it is seen that the p^{-2} intermediate-coupling prediction is $\Gamma_\gamma = 55\times10^{-3}$ eV, in disagreement with experiment by a factor of approximately 6. If we, again, take the effect of admixtures into account and use the coefficients of (26a), Table II, and the $(2s, 1d)$ eigenfunctions as generated by True's model, we obtain $\Gamma_\gamma = 103\times10^{-3}$ eV. Likewise the inclusion of admixtures from $(2s, 1d)$ when these are described by the other wave functions enlisted in Table I [and only the coefficients of (26a) are retained from True's model] yield an increased gamma width compared to the pure p^{-2} intermediate-coupling result. This is simply because the spin contribution to the $M1$ transition is dominating the contribution from the space part due to the relatively large magnitude of $g_s^{(-)}$. Because the matrix elements of the Gamow–Teller operator and the spin part of the $M1$ operator have a fixed-phase relation [Eq. (25)], there will always be constructive interference of the $M1$ amplitudes when there is constructive interference of the Gamow–Teller matrix elements as long as $\langle \mathbf{T}_1^{m,\sigma}\rangle$ dominates in magnitude $\langle \mathbf{T}_1^{m,l}\rangle$. In other words: as long as $\langle \mathbf{G}\rangle$ cannot be brought close to cancellation, also the gamma width of the $(0^+, 1)\rightarrow(1^+, 0)$ transition cannot be brought close to its experimental value. The constructive interference with which we are left is still caused by the sign of the amplitudes A^{JT} and B^{JT} being taken from True's model. As long as these amplitudes can be believed to be yielded correctly in sign by the model and at least to be roughly approximated in magnitude by True's model, the variation of $p_{1/2}^{-2}\rightarrow p^{-2}$ and the replacement of True's $(2s, 1d)$ wave functions does not lessen the disagreement between experiment and the theoretical results obtained from an "interplay model" of configurations excluding a tensor force.

We now, consequently, proceed to the last step (ii) of generalizing the "interplay model": that is, we no longer take the amplitudes A^{JT} and B^{JT} from True's model but instead choose them as to fit the experimental results. By definition then, the amplitudes A^{JT} and B^{JT} are determined so as to make the Gamow–Teller matrix element vanish. Consequently the $M1$ matrix element does not involve contributions from the spin part of the operator $\mathbf{T}_1^m$ and we need consider only those contributions as arising from $\mathbf{T}_1^{m,l}$. The following conclusions could be drawn just as well from a consideration of the matrix element of the (total) $M1$ operator $\mathbf{T}_1^m$ as entering Eq. (35). The investigation is, however, more transparent when we consider the space part only and replace Eq. (35) by

$$\langle 01 \parallel \mathbf{G}_1^{m,l} \parallel 10\rangle = \langle 01 \parallel \mathbf{G}_1^m \parallel 10\rangle_{\text{exp}}, \qquad (40)$$

where we have introduced for convenience the magnetic multipole operator $\mathbf{G}_1^m$, which is related to the corresponding interaction multipole operator $\mathbf{T}_1^m$ by (cf. Ref. 17)

$$T_{1M}^m = kG_{1M}^m. \qquad (41)$$

[When in general replacements of interaction multipole operators $T_{LM}^{(\pi)}$ by multipole operators $G_{LM}^{(\pi)}$ and replacements of the corresponding matrix elements are made, great care has to be exercised regarding the phases of the matrix elements of $G_{LM}^{(\pi)}$. This is especially true when particle–hole conjugations are involved (cf. Ref. 17). In the case of the magnetic multipole operator under consideration no "danger" is involved.]

The numerical relation between the reduced matrix elements of these two operators is

$$\langle 01 \parallel \mathbf{G}_1^m \parallel 10\rangle = 6.26 \times E_\gamma^{-1} \langle 01 \parallel \mathbf{T}_1^m \parallel 10\rangle, \quad (42)$$

when the right-hand side matrix element is inserted in eV/G cm, the gamma-ray energy E_γ in MeV, and the left-hand side in units of the nuclear magneton β.

In Table II, we have included the reduced matrix elements of the multipole operator $\mathbf{G}_1^m$ and also split up its matrix-element phase consistently into the two contributions from $G_1^{m,\sigma} = G_1^m(\text{spin})$ and $G_1^{m,l} = G_1^m$ (space). The Gamow–Teller matrix element has also been included phase consistently. From Eqs. (41) and (25) it is easily shown that the relation is

$$\langle \mathbf{G}\rangle = (-9.411/2\sqrt{2})\langle \mathbf{G}_1^{m,\sigma}\rangle, \qquad (43a)$$

when $\langle \mathbf{G}_1^{m,\sigma}\rangle$ is inserted in units of β.

For p^{-2} in LS coupling the matrix elements of the operator

$$\mathbf{G}_1^m = \mathbf{G}_1^{m,\sigma} + \mathbf{G}_1^{m,l}$$

are given by

$$\langle 01 \parallel \mathbf{G}_1^m \parallel 10\rangle = -\langle \mathbf{G}(p^{-2})\rangle (g^{(-)}/2\sqrt{2}) + 6^{-1/2}$$
$$\times \{(5)^{1/2} C_P^{01} C_D^{10} - 2C_P^{01} C_S^{10} + 2\sqrt{3} C_S^{01} C_P^{10}\}, \quad (43b)$$

and for $(2s, 1d)$

$$\langle 01 \parallel \mathbf{G}_1^m \parallel 10\rangle = -\langle \mathbf{G}(2s, 1d)\rangle [g^{(-)}/2\sqrt{2}]$$
$$+\{[2(7)^{1/2}/(5)^{1/2}] C(d_{5/2}{}^2; 01) C(d_{5/2}{}^2; 10)$$
$$+[(3\sqrt{3}/5^{1/2})] C(d_{3/2}{}^2; 01) C(d_{3/2}{}^2; 10)$$
$$-(5)^{-1/2} C(d_{5/2}{}^2; 01) C(d_{3/2}\, d_{5/2}; 10)$$
$$-(\sqrt{3}/\sqrt{2}5^{1/2}) C(d_{3/2}{}^2; 01) C(d_{3/2}\, d_{5/2}; 10)\}. \quad (43c)$$

Using Tables I and II, the numerical values for the Gamow–Teller and the space part of the $M1$ matrix element are easily obtained. They are displayed in Table IV from which it is seen that $\langle \mathbf{G}_1^{m,l}(2s, 1d)\rangle$ retains the same sign (and also does not vary appreciably in magnitude). As mentioned earlier the same is true for $\langle \mathbf{G}(2s, 1d)\rangle$. Furthermore, from Fig. 1 it is seen that $\langle \mathbf{G}_1^{m,l}(p^{-2})\rangle$ and $\langle \mathbf{G}(p^{-2})\rangle$ are insensitive to a/K in the whole of the intermediate-coupling region of interest and retain their signs throughout this region. We adopt the values given for $a/K=7$ in Table II.

From $\Gamma_\gamma(M1)_{\text{exp}} = 8.1 \times 10^{-3}$ eV, we extract

$$\langle 01 \parallel \mathbf{G}_1^m \parallel 10\rangle_{\text{exp}} = 0.490. \qquad (44)$$

We can now determine the sets of A^{JT}, B^{JT} which make the Gamow–Teller matrix element vanish and reproduce the experimental gamma width. Equations (34) and (40) lead to

$$A^{01}A^{10}\langle \mathbf{G}(p^{-2})\rangle + B^{01}B^{10}\langle \mathbf{G}(2s, 1d)\rangle = 0 \qquad (45)$$

$$A^{01}A^{10}\langle \mathbf{G}_1^{m,l}(p^{-2})\rangle + B^{01}B^{10}\langle \mathbf{G}_1^{m,l}(2s, 1d)\rangle = \langle \mathbf{G}_1^m\rangle_{\text{exp}}, \qquad (46)$$

and, in addition, we have the equations

$$(A^{01})^2 + (B^{01})^2 = 1; \qquad (47a)$$

$$(A^{10})^2 + (B^{10})^2 = 1. \qquad (47b)$$

Substituting the numerical values for the matrix elements using Tables II and IV and Eq. (44) (the final result being not dependent on the sign of $\langle \mathbf{G}_1^m\rangle_{\text{exp}}$), we obtain the result that *either* the $(0^+, 1)$ *or* the $(1^+, 0)$ state must arise almost purely from the $(2s, 1d)$ configuration, its intensity being between 85% and 95% from $(2s, 1d)$ for *any* of the numerical sets of coefficients tabulated in Table IV.

To summarize: the "mechanism" which causes this result is a "mechanism of phases." Our phase-consistent examination yielded opposite signs for $\langle \mathbf{G}(p^{-2})\rangle$ and $\langle \mathbf{G}(2s, 1d)\rangle$, which enter Eq. (45). Consequently, in order to satisfy Eq. (45), the products $A^{01}A^{10}$ and $B^{01}B^{10}$ must have the *same* sign. *Destructive* interference is necessary to make $\langle \mathbf{G}\rangle$ vanish. On the other hand, for any degree of intermediate coupling which could be considered reasonable and for any reasonable variation in the $(2s, 1d)$ wave functions the matrix elements $\langle \mathbf{G}_1^{m,l}(p^{-2})\rangle$ and $\langle \mathbf{G}_1^{m,l}(2s, 1d)\rangle$ have the *same* sign. Consequently, because the relative sign of the products $A^{01}A^{10}$ and $B^{01}B^{10}$ is fixed from $\langle \mathbf{G}\rangle = 0$, we have *con-*

structive interference in the gamma width. The result is that in order to satisfy Eq. (46) (i.e., to fit the low experimental gamma width) it is necessary that in either of the two products $A^{01}A^{10}$ and $B^{01}B^{10}$ one factor must be very small, which—because of the normalization—implies that either the ^{14}C or the ^{14}N ground state can *not* arise predominantly from p^{-2}. We conclude, then, that the beta-decay rate of ^{14}C and the $2.31 \rightarrow 0$ radiative width cannot simultaneously be explained by a conventional central force plus spin–orbit interaction together with configuration mixing. We now turn to a consideration of the decay of the second $(1^+, 0)$ level of p^{-2} at an excitation energy of 3.95 MeV in ^{14}N, this empirical data providing further evidence against an accounting of the p^{-2} wave functions by a conventional intermediate-coupling calculation.

2.4. The Decay of the ^{14}N 3.95-MeV Level

The empirical data relating to the decay of the second excited state of ^{14}N at 3.95 MeV have recently been combined to give[38]

$$\Gamma_\gamma(M1)_{\text{exp}} = (0.140 \pm 0.013) \text{ eV} \qquad (48a)$$

for the $3.95 \rightarrow 2.31$, $(1^+, 0) \rightarrow (0^+, 1)$ transition and

$$\Gamma_\gamma(M1)_{\text{exp}} = (5.8 \pm 1.2) \times 10^{-4} \text{ eV} \qquad (48b)$$

and

$$\Gamma_\gamma(E2)_{\text{exp}} = (4.81 \pm 0.33) \times 10^{-3} \text{ eV}, \qquad (48c)$$

for the $3.95 \rightarrow 0$ transition. Furthermore, the $E2/M1$ mixing ratio, which is defined (cf. Ref. 17) by

$$\delta = \frac{\langle J_1 \| T_L^{(\pi)} \| J_2 \rangle / (2L+1)^{1/2}}{\langle J_1 \| T_{\tilde{L}}^{(\tilde{\pi})} \| J_2 \rangle / (2\tilde{L}+1)^{1/2}}, \qquad (49)$$

with $\tilde{L}$ being the lowest-order multipolarity, is given (cf. Ref. 38 and further references therein) by

$$\delta(E2/M1)_{\text{exp}} = -(2.87 \pm 0.27). \qquad (50)$$

The evidence that the ^{14}N 3.95-MeV level is the second $(1^+, 0)$ level of (predominantly) p^{-2} is quite conclusive. Assuming a wave function for this level of the form of Eq. (1), the $M1$ and $E2$ matrix elements are of the form of Eq. (39b). Numerically the $3.95 \rightarrow 2.31$ matrix element can be obtained in eV/G cm, when E_γ is inserted in MeV and $\langle \mathbf{G}_1{}^m \rangle$ in units of the nuclear magneton β from

$$\langle 10 \| T_1{}^m \| 01 \rangle = (-1/\sqrt{3}) \times 0.159 \times E_\gamma \langle 01 \| \mathbf{G}_1{}^m \| 10 \rangle, \qquad (51)$$

and inserting for $\langle 01 \| \mathbf{G}_1{}^m \| 10 \rangle$ the matrix elements of Eqs. (43b) and (43c) and the coefficients $C_L{}^{JT}$ of Table III representing the 3.95-MeV level in intermediate coupling. The results are tabulated in Table V. Since True's calculation does not provide any quantitative guide for the admixtures from $(2s, 1d)$ in

[38] J. W. Olness, A. R. Poletti, and E. K. Warburton, Phys. Rev. **154**, 971 (1967).

TABLE V. Two-particle matrix elements of the magnetic-dipole-interaction multiple operator for the ^{14}N, $3.95 \rightarrow 2.31 (1^+, 0) \rightarrow (0^+, 1)$ transition.

Transition	$\langle \mathbf{G}_1{}^m \rangle$	$\langle \mathbf{T}_1{}^m \rangle$	$C(ll'; 0^+1) \langle \mathbf{T}_1{}^m \rangle$
Units	β	10^{-7} MeV/G cm	
$p_{1/2}{}^{-2} \rightarrow p_{1/2}{}^{-2}$	+0.901	+2.351	
$p_{1/2}{}^{-1} p_{3/2}{}^{-1} \rightarrow p_{1/2}{}^{-2}$	−2.803	−7.310	
$p^{-2} \rightarrow p^{-2}$ [a]	−2.959	−7.716	
$s_{1/2}{}^2 \rightarrow s_{1/2}{}^2$	−4.705	−12.270	−4.797
$d_{3/2}{}^2 \rightarrow d_{3/2}{}^2$	+0.762	+1.989	+0.725
$d_{5/2}{}^2 \rightarrow d_{5/2}{}^2$	−4.580	−11.944	−10.093
$d_{3/2} d_{5/2} \rightarrow d_{3/2}{}^2$	−2.659	−6.935	−2.531
$d_{3/2} d_{5/2} \rightarrow d_{5/2}{}^2$	−2.171	−5.663	−4.785

[a] Intermediate coupling $a/K = 7$.

the 3.95-MeV state we have included in Table V the products of the $(2s, 1d)$ matrix elements with the corresponding amplitudes taken from Table I for the ^{14}N 2.31-MeV state as given by True's model. Using the $M1$ matrix element for $a/K = 7$ and assuming the 3.95-MeV state to arise purely from p^{-2} we arrive at $\Gamma_\gamma(M1) = 0.105$ eV for the $3.95 \rightarrow 2.31$ transition. Again, this value is found to be insensitive to the intermediate coupling parameter a/K in the region of interest. We see that the experimental value of the $3.95 \rightarrow 2.31$ $M1$-transition width is roughly accounted for by the p-shell contributions. Applying purely empirical knowledge, this is not surprising, since the transition strength is 1.47 Weisskopf units,[39] and therefore should be rather insensitive to small $p^8(2s, 1d)$ admixtures in both the 3.95- and the 2.31-MeV states. The experimental matrix element which we may extract from $\Gamma_\gamma(M1)_{\text{exp}}$ is

$$|\langle 10 \| T_1{}^m \| 01 \rangle|_{\text{exp}} = 8.944 \times 10^{-7} \text{ MeV/G cm},$$

which, considering the experimental errors, may be as low as 8.53×10^{-7}. This value is to be compared to the pure p-shell result 7.72×10^{-7}.

Certainly the discrepancy is not of great significance. For the over-all picture it is, however, interesting to inspect whether it is likely that admixtures from $(2s, 1d)$ can completely remove the disagreement. From the first line of Table I it is seen that the $d_{3/2}\, d_{5/2}$ component in the $(2s, 1d)$ part of the ground-state eigenfunction is smallest. As the 3.95-MeV state must be orthogonal to the ground state we may, therefore, assume that this component is large in the 3.95-MeV eigenfunction without influencing the ground-state wave function unduly. An upper limit on the influence of contributions from $(2s, 1d)$ can then be estimated in a reasonable way from inspection of the last column of Table V, from which it is seen that the contribution from $(2s, 1d)$ to the transition rate will be close to its maximum value if we assume the $p^8(2s, 1d)$ part of the 3.95-MeV eigenfunction to be pure $p^8 d_{3/2}\, d_{5/2}$. We

[39] D. H. Wilkinson, in *Nuclear Spectroscopy*, F. Ajzenberg-Selove, Ed. (Academic Press Inc., New York, 1960), Part B, pp. 852–889.

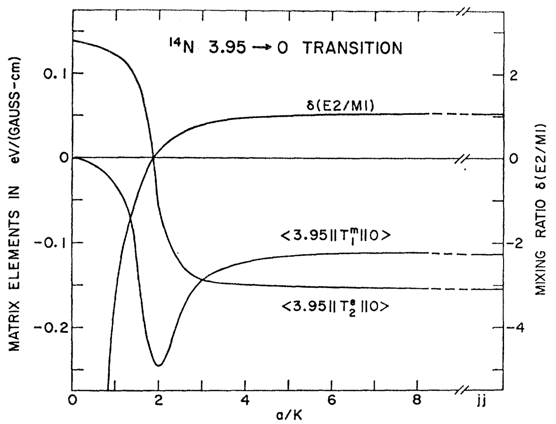

FIG. 2. Matrix elements of the $\mathbf{T}_2^e$ and $\mathbf{T}_1^m$ operators and the $E2$–$M1$ mixing ratio δ of the 3.95 MeV→gs transition in ^{14}N as obtained from the eigenfunctions of Soper.[36]

also know (cf. Sec. 2.3) that we can rely with some confidence on the eigenfunctions of True for the 2.31-MeV state. Using True's values A^{01} and B^{01} [Eqs. (26a)] for the 2.31-MeV state and Table V, we then have

$$\langle 10 \,||\, \mathbf{T}_1^m \,||\, 01\rangle = [1-(B_{3.95}{}^{10})^2]^{1/2}(0.950)(-7.716) + B_{3.95}{}^{10}(-0.311)(-7.316).$$

For an approximately 10% admixture of $p^8(2s, 1d)$ in the 3.95-MeV state we obtain a maximum value for the transition rate which is not larger than the pure p-shell value. In fact, if we accept a configurational admixture of ~10% in the 3.95-MeV state as reasonable, we have

$$\langle 10 \,||\, \mathbf{T}_1^m \,||\, 01\rangle = [-6.955 \pm 0.720] \times 10^{-7}\ \text{MeV/G cm}.$$

We now consider the $(1^+, 0) \rightarrow (1^+, 0)$ transition of the 3.95-MeV level to the ground state. In LS-coupling the magnetic-dipole and electric-quadrupole matrix elements as arising from p^{-2} can be obtained numerically in 10^{-7} MeV/G cm from

$$\begin{aligned}\langle 10 \,||\, \mathbf{T}_1^m \,||\, 10\rangle &= 7.836 C_S{}^{10\prime} C_S{}^{10} + 4.453 C_P{}^{10\prime} C_P{}^{10} + 2.761 C_D{}^{10\prime} C_D{}^{10} \\ &= 3.383\{C_S{}^{10\prime} C_S{}^{10} - \tfrac{1}{2} C_D{}^{10\prime} C_D{}^{10}\}, \end{aligned} \tag{52}$$

$$\begin{aligned}\langle 10 \,||\, \mathbf{T}_2^e \,||\, 10\rangle = \{&0.772 C_P{}^{10\prime} C_P{}^{10} \\ &-0.690(C_S{}^{10\prime} C_D{}^{10} + C_D{}^{10\prime} C_S{}^{10}) \\ &-0.540 C_D{}^{10\prime} C_D{}^{10}\}(1+2\alpha), \end{aligned} \tag{53}$$

where the primed coefficients correspond to the 3.95-MeV level.

For the calculation of the $E2$-matrix elements a radial fall-off parameter 0.595 has been used. This corresponds in the p shell to a radial integral $\langle r^2\rangle = 7.06 f^2$, a value which has formerly been used.[3,11] It agrees fairly well with the value of $(6.4 \pm 0.5) f^2$ recently extracted[40] from various experimental data by Wilkinson. [Wilkinson's recommended value for $\langle r^2\rangle$ is dependent on the binding energies involved. For the transitions considered here it is recommended that this value be increased by 5% or 10%, which would bring it into agreement with our adopted value of $7f^2$ (private communication from D. H. Wilkinson).]

The parameter α appearing in the expression (53) for the electric-quadrupole matrix element is the effective charge in units of the proton charge, resulting from the use of the effective charge parametrization. In comparing the calculations to experiment we shall take α to be 0.5. The use of the effective charge parametrization is the most uncertain point in our calculation of $E2$ rates. However, it has been shown that some sort of collective enhancement of the $E2$ rates is clearly necessary and recent calculations[41] indicate that $\alpha = 0.5$ gives agreement with experiment for the stronger, less model-dependent $E2$ rates in the $1p$ shell. The 3.95→0 $E2$ transition is relatively strong, 3 Weisskopf units,[39] thus indicating that small configurational admixtures will have little influence on it.

In Table VI we have tabulated the $M1$ and $E2$ matrix elements as they result from a phase-consistent calculation. For the p-shell contributions we again use $a/K = 7$. It is shown in Fig. 2 that (again) in the region of interest the p^{-2} contributions are insensitive to a/K. Furthermore, we see from Fig. 2 that the mixing ratio δ is opposite in sign to the observed one and is, as are the $E2$ and $M1$ matrix elements, almost independent of a/K in the whole region of interest. The calculated $E2$ width corresponding to $a/K = 7$, considering p-shell contributions only, is $\Gamma_\gamma(E2) = 5.82 \times 10^{-3}$ eV, which agrees within the errors with the experimental value. For the $M1$ width, however, we obtain (for $a/K = 7$) $\Gamma_\gamma(M1) = 5.19 \times 10^{-3}$ eV which is an order of magnitude larger than the experimental value. Because of the agreement of the experimental $E2$ width and the disagreement of the $M1$ width with the calculated value from p^{-2} we may and shall assume that the $E2$-matrix element is well represented (i.e., in magnitude and phase) by the p^{-2} calculations and the $M1$-matrix element is not.

We now ask if $p^8(2s, 1d)$ admixtures can resolve these discrepancies. To reproduce theoretically the measured sign of the mixing ratio, we must inspect whether we can change the sign of the $M1$ matrix element by an "interplay" of p^{-2} and $(2s, 1d)$ contributions. In fact, we even have to "overcompensate" the p^{-2} matrix element by contributions from $(2s, 1d)$ in order to arrive at the correct sign and at the finite experimental $M1$ width. Remembering the requirement of orthogonality of the 3.95-MeV state to the ground state, examination of column 3 of Table VI shows that

[40] D. H. Wilkinson, Nucl. Phys. **85**, 114 (1966).
[41] A. R. Poletti, E. K. Warburton, and D. Kurath, Phys. Rev. **155**, 1096 (1967).

TABLE VI. Two-particle matrix elements of the interaction-multipole operators $\mathbf{T}_1^m$ and $\mathbf{T}_2^e$ for the ^{14}N, 3.95→0 MeV, $(1^+, 0)\rightarrow(1^+, 0)$ transition (units: 10^{-7} MeV/G cm).

Transition	$\langle \mathbf{T}_1^m \rangle$	$\langle \mathbf{T}_2^e \rangle$	$C(ll'; 10)_{\mathrm{g.s.}} \times \langle \mathbf{T}_1^m \rangle$	$(1+2\alpha)\, C(ll'; 10)_{\mathrm{g.s.}} \times \langle \mathbf{T}_2^e \rangle$ [b]
$p_{1/2}^{-2}\rightarrow p_{1/2}^{-2}$	+3.326	0		
$p_{1/2}^{-1}p_{3/2}^{-1}\rightarrow p_{1/2}^{-2}$	−1.128	−0.772		
$p^{-2}\rightarrow p^{-2}$ [a]	−1.108	−0.758		
$s_{1/2}^2\rightarrow s_{1/2}^2$	+7.839	0	+1.967	0
$d_{3/2}^2\rightarrow d_{3/2}^2$	+3.777	+0.865	−1.941	−0.889
$d_{5/2}^2\rightarrow d_{5/2}^2$	+5.131	+0.989	+3.684	+1.420
$d_{3/2}^2\rightarrow d_{3/2}d_{5/2}$	−1.436	−0.458	−0.058	−0.037
$d_{3/2}d_{5/2}\rightarrow d_{3/2}^2$	−1.436	−0.458	+0.738	+0.470
$d_{5/2}^2\rightarrow d_{3/2}d_{5/2}$	+1.791	−0.245	+0.073	−0.020
$d_{3/2}d_{5/2}\rightarrow d_{5/2}^2$	+1.791	−0.245	+1.285	−0.352
$d_{3/2}d_{5/2}\rightarrow d_{3/2}d_{5/2}$	+6.146	−0.540	+0.252	−0.044
$s_{1/2}d_{3/2}\rightarrow s_{1/2}d_{3/2}$	+2.761	−0.540	+1.090	−0.427
$s_{1/2}d_{3/2}\rightarrow d_{3/2}^2$	0	−0.309	0	+0.317
$s_{1/2}d_{3/2}\rightarrow s_{1/2}^2$	0	−0.977	0	−0.490
$s_{1/2}^2\rightarrow s_{1/2}d_{3/2}$	0	−0.977	0	−0.772

[a] Intermediate coupling $a/K=7$. [b] $\alpha=0.5$ throughout.

the $M1$ matrix element for the 3.95→0 transition is close to its maximum value if the $p^8(2s, 1d)$ admixture in the 3.95-MeV level is mostly $p^8\, d_{3/2}\, d_{5/2}$. Using pure $p^8\, d_{3/2}\, d_{5/2}$ for the $p^8(2s, 1d)$ part of the wave function for the 3.95-MeV level and True's wave function for the ^{14}N ground state we obtain with the help of Table VI

$$\langle 10 \,\|\, \mathbf{T}_1^m \,\|\, 10\rangle = A_{3.95}^{10}(0.966)\,(-1.108) + B_{3.95}^{10}(0.255)\,(2.093). \quad (54)$$

The 3.95-MeV level would need to have a p^{-2} intensity smaller than 20% and a $p^8(2s, 1d)$ component larger than 80% in intensity before $\langle \mathbf{T}_1^m \rangle$ would change sign to give agreement with experiment. In fact, the experimental value of $\langle T_1^m \rangle$ is $+0.370\times10^{-7}$ MeV/G cm, thus requiring $B_{3.95}^{10}$ to be of the same (positive) sign as $A_{3.95}^{10}$ while to resolve completely by "interplay" the discrepancies found for the 3.95→2.31 transition, they would have to be of opposite sign.

We conclude that the disagreement with experiment of the intermediate coupling predictions for the mixing ratio δ of the 3.95→0 transition are nearly as significant as that for the ^{14}C beta decay: admixtures of $p^8(2s, 1d)$ cannot reconcile these disagreements and again a strong modification of the p^{-2} wave functions is demanded.

3. MASS-14 WAVE FUNCTIONS FROM MODELS INCLUDING A TWO-BODY TENSOR TENSOR INTERACTION

3.1. Historical

Inglis[4] first showed that a s^4p^{10} model with a two-body central interaction and a single-particle (or hole) spin–orbit interaction was incapable of causing cancellation of the Gamow–Teller matrix element [Eq. (36)]. Visscher and Ferrell[3] generalized this proof to hold for a general two-body spin–orbit force.

Jancovici and Talmi[1] pointed out that the inclusion of a tensor interaction into the two-body interaction could bring about the necessary modifications in the wave functions so that cancellation can be achieved in the Gamow–Teller matrix element. However, the tensor force considered by Jancovici and Talmi[1] was large enough to cause severe distortion of the ^{14}N spectrum. Further work by Visscher and Ferrell[3] and by Elliott[2] succeeded in reducing the strength of the necessary tensor interaction to an acceptable level. These authors gave the matrix elements of the interaction which includes a tensor force. The various quantities entering these matrix elements were determined approximately from first principles.[3] Adjustments of the matrix elements to fit the level scheme of ^{14}N, to make the Gamow–Teller matrix element vanish (and the use of some further quantities known from p-shell data), lead to a tensor force contribution to the two-body interaction which is in reasonable agreement with the information from two-nucleon data.

The wave functions of Elliott and of Visscher and Ferrell for the three lowest levels of mass 14 are represented by the LS-coupling coefficients given in Table III. The other bound p^{-2} state of ^{14}N has $(J^\pi, T) = (2^+, 0)$ and is described uniquely in LS coupling by $^{13}\mathrm{D}_2$. We identify this state with the ^{14}N 7.03-MeV level.

The LS-coefficients of Soper for $a/K=7$ are also given in Table III. These represent the intermediate coupling wave functions discussed in Sec. 2. The Cohen and Kurath[19] wave functions are discussed in Sec. 3.3. From Eq. (36) and Table III we see how the introduction of a tensor force can cause the Gamow–Teller matrix element to vanish. For the central plus spin–orbit interaction of Soper,[36] C_S^{10}/C_P^{10} is always negative and C_S^{01}/C_P^{01} is always positive; while introduction of a tensor force causes C_S^{10}/C_P^{10} but not C_S^{01}/C_P^{01} to change sign.

The three treatments we have briefly discussed here show quite clearly that the introduction of a tensor force can explain the longevity of ^{14}C and ^{14}O. However, it might be argued that the introduction of the necessary tensor interaction was, to some extent, arbitrary. Since the time of these early treatments the evidence for a nucleon–nucleon tensor interaction has been considerably strengthened (see, for example, Ref. 18) and recent calculations[7,19,21] indicate that the necessary tensor force is a general attribute of effective two-body forces in nuclear matter.

3.2. The Two-Body Matrix Elements of Becker and MacKellar

Recently Kuo and Brown[42] and Becker and MacKellar[21] have extracted two-body matrix elements, V_{ij}, from a first principles calculation based on the Hamada–Johnston potential.[18] This interaction has central, tensor, spin–orbit, and quadratic spin–orbit terms and results from a fit to two-nucleon scattering data up to several hundred MeV. Zamick[7] investigated the ^{14}C beta decay in relation to the matrix elements of Kuo and Brown[42] assuming pure p^{-2} states and showed that the longevity of ^{14}C followed quite reasonably from them. It is our intent to show that similar results follow from the Becker–MacKellar matrix elements. For our investigation, using the reaction matrix elements V_{ij} of Becker and MacKellar the $(1^+, 0)$ wave functions are obtained in a jj-coupling basic set ($p_{1/2}^{-2}$, $p_{1/2}^{-1}p_{3/2}^{-1}$, $p_{3/2}^{-2}$) by diagonalizing

$$(V_{ij})+\tfrac{1}{3}\epsilon\begin{Bmatrix}-4 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 2\end{Bmatrix}, \tag{55}$$

and similarly the $(0^+, 1)$ wave functions are obtained from diagonalizing (basic set $p_{1/2}^{-2}$, $p_{3/2}^{-2}$)

$$(V_{ij})+\tfrac{1}{3}\epsilon\begin{Bmatrix}-4 & 0\\ 0 & 2\end{Bmatrix}, \tag{56}$$

where ϵ is the $p_{3/2}-p_{1/2}$ splitting, equal to $3a/2$ (where a is the strength of the single-particle spin–orbit force).

Zamick[7] treated ϵ as a free parameter (as a/K is in intermediate coupling) and showed that the ^{14}C Gamow–Teller matrix element went through zero at $\epsilon=9.2$ MeV. Zamick[7] remarks that the fact that this value of ϵ is larger than the experimental splitting of $\sim$6.3 MeV (in mass 15) indicates that the tensor part of the Hamada–Johnston potential is too large. He also showed that using the Kallio–Kolltveit[43] potential, which is purely central, cancellation of the ^{14}C Gamow–Teller matrix element could not be achieved. This is predicted by the general theorem of Inglis[4] and Visscher and Ferrell.[3]

We have made an investigation similar to that of Zamick[7] but using the Becker–MacKellar[21] matrix elements, the purpose being to see if this model is capable of explaining simultaneously the Gamow–Teller matrix element and the $E2/M1$ mixing ratio of the ^{14}N 3.95→g.s. transition at the same time as it gives a reasonable level scheme for ^{14}N. It is at present still an inherent feature of this first principles approach that the binding energies (as well as ϵ) come out rather high. Therefore, agreement in detail for mass 14 should not be expected to the same extent as in those treatments[2,3] specific to ^{14}C–^{14}N.

The results are displayed in Table VII for different radial fall-off parameters α. Table VII shows that, in principle, one can produce for a particular ϵ cancellation of the Gamow–Teller matrix element and obtain simultaneously agreement with $\delta(E2/M1)$ for the 3.95→gs transition with $\alpha=0.5$. However, the experimental-level scheme is reproduced better at higher values of ϵ indicating that the tensor force is too strong in agreement with the work of Zamick.

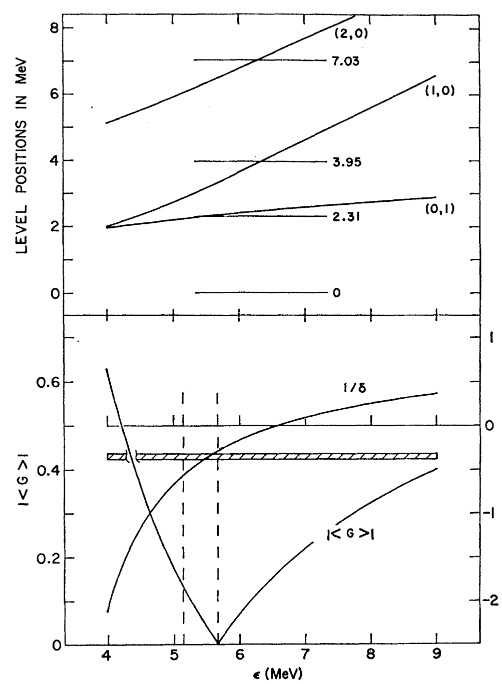

FIG. 3. Results obtained from the (8–16)2 BME matrix elements of Ref. 19. In the upper half of the figure the calculated level positions of ^{14}N are shown vs the $p_{3/2}$–$p_{1/2}$ splitting ϵ together with the experimental-level sheme. In the lower half $|\langle \mathbf{G}\rangle|$ and the reciprocal of δ(3.95 MeV→gs) is plotted vs ϵ. The hatched area corresponds to the experimental value $\delta=(2.87\pm0.27)$. The other possible value of δ corresponding to $-4.3\leq 1/\delta\leq -2.3$ is not shown. The meaning of the dashed lines is explained in the text.

[42] T. T. S. Kuo and G. E. Brown, Phys. Letters **18**, 54 (1965).
[43] A. Kallio and K. Kolltviet, Nucl. Phys. **53**, 87 (1964).

TABLE VII. The ^{14}N spectra; ^{14}C Gamow–Teller matrix element $\langle G \rangle$ and the $E2/M1$ mixing ratio δ for the ^{14}N 3.95→0 transition obtained from the two-body matrix elements of Becker and MacKellar (Ref. 21).

α[a] (Fermi)	ϵ (MeV)	BE (MeV)	Best spectrum Excitation energies (01)	(10)	(20)	$\langle G \rangle^2$	$\delta(E2/M1)$
0.400	9.7	−16.6	2.32	4.30	6.75	0.211	+2.4
0.4348	10.2	−17.6	2.58	4.30	6.50	0.104	+5.2
0.4696	10.9	−18.7	2.98	4.70	6.35	0.048	+26
0.500	11.7	−20.0	3.20	4.50	6.40	0.050	−8
Experiment		−13.2	2.31	3.95	7.03	4.2×10^{-6}	$-(2.87\pm0.27)$
Vanishing Gamow–Teller matrix element							
0.400	7.2	−13.5	2.36	2.05	4.45	$<10^{-5}$	−2.6
0.4348	8.4	−15.4	2.64	2.70	4.80	$<10^{-5}$	−3.0
0.4696	9.5	−17.1	2.95	3.45	5.05	$<10^{-5}$	−3.0
0.500	10.5	−18.4	3.20	3.60	5.30	$<10^{-5}$	−2.8

[a] α is the harmonic oscillator radial fall-off parameter, ϵ the $p_{3/2}$–$p_{1/2}$ energy splitting, BE is the binding energy of ^{14}N. The excitation energies of the three lowest (J, T) states of ^{14}N are given in MeV.

3.3. The Two-Body Matrix Elements of Cohen and Kurath

Cohen and Kurath[19] have presented effective p-shell matrix elements for the two-body interaction obtained from an over-all least-squares fit of 35(44) data from $A=8$ through $A=16$ ($A=6$ through $A=16$). They have made two separate fits to each of these sets of data (POT and 2BME). In general, there are 15 matrix elements and 2 single-particle energies which ought to be determined from least squares fits. In the POT case symmetry relations between the nondiagonal matrix elements are used in addition, and only 11 parameters from the interaction remain while the 2BME case represents a 15+2 parameter case.

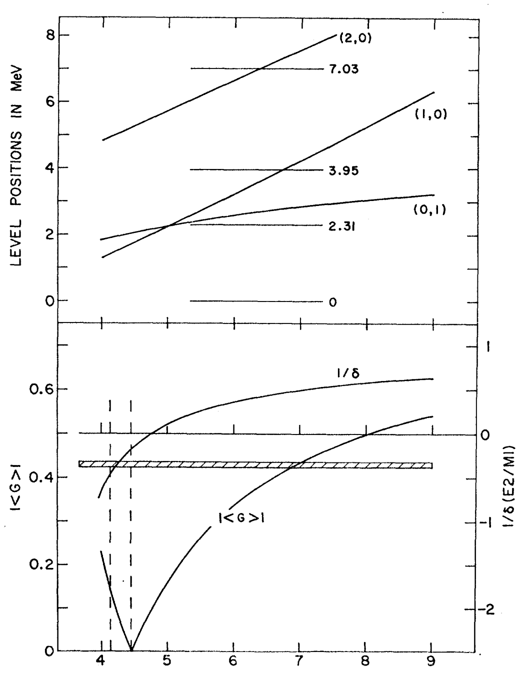

FIG. 4. Results obtained from the (8–16) POT matrix elements of Ref. 19. Again, the level positions, the modulus of the GT matrix element, and $1/\delta$ of the 3.95 MeV→gs transition are shown vs ϵ (compare caption, Fig. 3).

Following the same procedure as in Sec. 3.2 but now starting off with the two-body matrix elements of Cohen and Kurath[19] we arrive at the results displayed in Fig. 3, which gives results for (8–16)2BME, and Fig. 4, which is for (8–16)POT. The (6–16)POT and (6–16)2BME matrix elements of Cohen and Kurath[19] were not used. These latter matrix elements correspond to a fit of the p-shell data with a larger χ^2. Furthermore, it seems more reasonable for a specific investigation within $A=14$ to rely on the $A=8$ through $A=16$ data rather than to include $A=6$ and $A=7$ data.

In Fig. 3 the modulus of the ^{14}C Gamow–Teller matrix element and the reciprocal of $\delta(E2/M1)$ for the ^{14}N 3.95→0 transition are plotted vs the spin–orbit splitting $\epsilon=3a/2$. The dependence of the energy positions of the three lowest excited states from p^{-2} is also plotted vs ϵ. The Gamow–Teller matrix element vanishes at $\epsilon\simeq-5.7$ MeV. The inverse of the mixing ratio, $1/\delta$, has the right sign for $\epsilon<6.5$ MeV. For $\epsilon\approx5.6$ MeV it agrees with the experimental value $1/\delta=-(0.35\pm0.03)$, i.e., $\delta=-(2.87\pm0.27)$, if an effective charge of $\alpha=0.5$ is used for the $E2$ rate. The experimental level scheme is almost identically reproduced at $\epsilon=6.2$ MeV. For this value of ϵ the binding energy also has the correct value (−13.21 MeV).

The same procedure was repeated starting from the (8–16)POT matrix elements. The results are displayed in Fig. 4 and in principle the same conclusions can be drawn as from Fig. 3 although a comparison of these two figures shows that the (8–16)POT results are not quite as striking as the ones arrived at from the (8–16)2BME case.

When a decomposition of the matrix elements of Cohen and Kurath[19] is made in terms of the Hamada–Johnston potential[18] and Table I of Ref. 19 is used, one can extract $\epsilon=7.5$ MeV. This value again seems to reflect that the strong tensor force in the Hamada–Johnston potential leads to too large a spin–orbit force (cf. Sec. 3.2). Probably a smaller value ($\epsilon\approx 6$ MeV) is better to describe the mass-14 data, because the $p_{3/2}$–$p_{1/2}$ splitting seems to be rather well represented (near $A=15$) by the splitting of the $3/2^-$ and $1/2^-$ states in mass 15 (cf. Ref. 44). However, a strict interpretation of ϵ as the experimental $p_{3/2}$–$p_{1/2}$ energy splitting in mass 15 or 14 is probably misleading. In both the first-principles calculations described in Sec. 3.2 and in the effective interaction calculation of Cohen and Kurath, ϵ is an *effective* parameter which bears part of the burden of masking our ignorance of the forces acting in nuclear matter; thus it should not surprise us when slightly different values of ϵ are extracted from comparison with different measured quantities.

Figures 3 and 4 show that for a pure p^{-2} model the experimental data considered here can be almost exactly reproduced within a small range of variation of ϵ between 5.6 and 6.2 MeV. This was already anticipated by Cohen and Kurath[19] who did not vary ϵ but did note that only a small perturbation of the mass-14 wave functions was necessary in order to have $\langle \mathbf{G}(p^{-2})\rangle=0$.

Now let us consider the effects of admixtures of $p^8(2s, 1d)$. Eqs. (27) and (29) show that our model for configuration mixing yields $\langle \mathbf{G}(p^{-2})\rangle=-0.132$ if the contributions from p^{-2} and $p^8(2s, 1d)$ are to cancel. The value of ϵ resulting in this value $\langle \mathbf{G}(p^{-2})\rangle$ is indicated by the vertical dashed line in Figs. 3 and 4. For the (8–16)2BME case it is $\epsilon=5.15$ MeV. At this value of ϵ the magnitude of $\delta(E2/M1)$ for the ^{14}N $3.95\rightarrow 0$ transition is too small by about a factor of 2. This discrepancy is well within the possible range of variation due to the effects of $p^8(2s, 1d)$ admixtures on this parameter. To show this we note that for the (8–16)2BME case with $\epsilon=5.15$ MeV the $3.95\rightarrow 0$ $M1$ matrix element analogous to Eq. (54) is

$$\langle 10 \,||\, \mathbf{T}_1^m \,||\, 10\rangle = A_{3.95}{}^{10}(0.966)(0.555) + B_{3.95}{}^{10}(0.255)(2.093), \quad (57)$$

where the first part of the term is the p^{-2} contribution, i.e., (8–16)2BME, and the second part is our estimate of the largest feasible contribution from $p^8(2s, 1d)$. Inserting for $A_{3.95}{}^{10}$ and $B_{3.95}{}^{10}$ the values corresponding to a 10% (in intensity) contamination of the 3.95-MeV state from $(2s, 1d)$ we obtain

$$\langle 10 \,||\, \mathbf{T}_1^m \,||\, 10\rangle = 0.509\pm 0.169. \quad (58)$$

It is clear that the $p^8(2s, 1d)$ admixtures can have a large effect on this $M1$ rate and thus on $\delta(E2/M1)$. In fact, the numerical value of the $M1$ matrix element necessary to fit the experimental $M1$ width and the mixing ratio, was $+0.370$ (cf. Sec. 2.4), which is well within the range of values permitted by Eq. (58). We thus feel that the fact that the right sign of $\delta(E2/M1)$ is predicted and that the magnitude is of the right order of magnitude constitutes satisfactory agreement with experiment for this highly sensitive parameter.

The wave functions extracted from the (8–16)2BME two-body matrix elements of Cohen and Kurath[19] using $\epsilon=5.67$ MeV $[\langle \mathbf{G}(p^{-2})\rangle=0]$ and $\epsilon=5.15$ MeV $[\langle \mathbf{G}(p^{-2})\rangle=-0.132]$ are listed in Table III as I and II, respectively.

4. COMPREHENSIVE COMPARISON WITH EXPERIMENT

4.1. ^{14}N Electromagnetic Transitions

In this subsection we compare the predictions of the various p^{-2} models (Table III) with the known properties of the electromagnetic transitions connecting the bound ^{14}N states of p^{-2}. Actually the three states already considered and the $(2+, 0)$ state at 7.03 MeV in ^{14}N are the only mass-14 states which are known to be predominantly s^4p^{10}. The next highest state of this configuration is expected to be a $(2^+, 1)$ state. This state appears to be highly mixed with a $(2^+, 1)$ state from $p^8(2s, 1d)$ so that two levels—those at 9.17 and 10.43 MeV in ^{14}N—share the properties of the $p^{-2}(2^+, 1)$ state about equally.[11,24]

The decay of the $(2^+, 0)p^{-2}$ state to the three lower p^{-2} states is described by

$$\langle 20 \,||\, \mathbf{T}_1^m \,||\, 10\rangle = 0.0576 E_\gamma C_D{}^{10} \quad (59a)$$

and

$$\langle 20 \,||\, \mathbf{T}_2^e \,||\, 10\rangle = 0.632\times 10^{-3} E_\gamma{}^3 \times[0.782 C_D{}^{10} - C_S{}^{10}](1+2\alpha) \quad (59b)$$

for the $(2^+, 0)\rightarrow(1^+, 0)$ transitions, and

$$\langle 20 \,||\, \mathbf{T}_2^e \,||\, 01\rangle = 2.465\times 10^{-3} E_\gamma{}^3 C_P{}^{01} \quad (60)$$

for the $(2^+, 0)\rightarrow(0^+, 1)$ transition. The matrix elements are obtained in eV/G cm when E_γ is inserted in MeV. The weak-surface coupling approximation predicts no

[44] H. J. Rose and J. S. Lopes, Phys. Letters **22**, 601 (1966).

TABLE VIII. Magnetic-dipole radiative widths (10^{-3} eV) connecting the four lowest s^4p^{10} states of ^{14}N.

Calculation	2.31 → 0[a] (1)	2.31 → 0[a] (2)	3.95→2.31	3.95→0	7.03→0	7.03→3.95
Soper	54.9	103	105	5.256	103	1.035
Elliott	16.9		159	0.491	120	0.026
Visscher and Ferrell	23.7		137	0.559	106	0.056
Cohen and Kurath I	18.0		180	0.408	115	0.002
Cohen and Kurath II	41.5	9.60	185	1.321	115	0.052
Experiment	8.1±1.4		140±13	0.58±0.12	91±13	≤(1.1±0.3)

[a] For the 2.31→0 transition column (1) is calculated from the wave functions of Table III, using Eq. (43b), and column (2) includes the effects of admixtures of the $p^8(2s, 1d)$ configuration as explained in the text.

collective enhancement for $\Delta T=1$ $E2$ transitions in self-conjugate nuclei.[45] (However, see Ref. 41.) Thus, no enhancement is allowed for in the $(2^+, 0)\rightarrow(0^+, 1)$ transition.

The $M1$ and $E2$ transition rates calculated from Eqs. (43), (51), (52), (53), (59), and (60) for the decay of the three excited ^{14}N s^4p^{10} states in question are presented in Tables VIII and IX, and the $E2/M1$ mixing ratios for the 3.95→0 and 7.03→0 transitions are given in Table X. The experimental results for the decay of the 2.31- and 3.95-MeV levels are from data presented in Sec. 2, while those relating to the decay of the 7.03-MeV level are taken from the results given in Ref. 38.

The experimental and theoretical phases as well as the magnitudes of the $E2/M1$ mixing ratios given in Table X are to be compared since they were calculated from a phase-consistent theory.[17] The phase of the $E2/M1$ mixing ratio for the 7.03→0 transition is insensitive to the wave functions used and agrees with experiment, as discussed previously.[41,46]

We discuss the $E2$ rates first since they appear less sensitive to the details of the calculations. In fact, Table IX shows that all the calculations save that for intermediate coupling give predictions for the four $E2$ rates considered which are in adequate agreement with experiment if collective enhancement is included, and the predictions of intermediate coupling are not in severe disagreement with experiment. The limit on the experimental $E2$ transition strength for the 7.03→3.95 transition is due to the fact that $\Gamma_\gamma(M1)+\Gamma_\gamma(E2)$ was measured for this transition and the $E2/M1$ mixing ratio is not known. Tables VIII and IX show that, with the exception of intermediate coupling, the prediction is that $\Gamma_\gamma(E2)$ should dominate this transition so that

TABLE IX. Electric-quadrupole radiative widths (10^{-3} eV) connecting the four lowest s^4p^{10} states in ^{14}N.

Calculation	3.95→0 a	3.95→0 b[a]	7.03→0 a	7.03→0 b	7.03→3.95 a	7.03→3.95 b	7.03→2.31
Soper	1.47	5.89	18.46	73.84	0.16	0.62	1.33
Elliott	1.14	4.58	10.45	41.80	0.37	1.47	0.78
Visscher and Ferrell	1.33	5.32	6.58	26.30	0.20	0.81	0.94
Cohen and Kurath I	1.21	4.83	9.70	38.80	0.34	1.35	0.57
Cohen and Kurath II	1.11	4.43	8.35	33.40	0.37	1.47	0.50
Experiment	4.81±0.33		33±9		≤(1.1±0.3)		0.62±0.14

[a] The columns headed (a) have no collective enhancement of E2 rates, while those designated (b) have collective enhancement with $\alpha=0.5$. The origin of the calculations is stated in Table III.

[45] E. K. Warburton, Phys. Rev. Letters **1**, 68 (1958).
[46] O. Häusser, H. J. Rose, J. S. Lopes, and R. D. Gill, Phys. Letters **22**, 604 (1966).

Table X. $E2/M1$ amplitude ratios for two s^4p^{10} transitions in ^{14}N.

Calculation	Transition 3.95→0 a[a]	b	7.03→0 a	b
Soper	+0.53	+1.06	+0.42	+0.84
Elliott	−1.52	−3.05	+0.30	+0.59
Visscher and Ferrell	−1.54	−3.08	+0.25	+0.50
Cohen and Kurath I	−1.72	−3.43	+0.29	+0.58
Cohen and Kurath II	−0.91	−1.83	+0.27	+0.54
Experiment	−(2.87±0.27)		+(0.6±0.1)	

[a] The columns headed (a) have no collective enhancement of E2 rates, while those designated (b) have collective enhancement with $\alpha=0.5$.

with this exception the predictions for the $E2$ rate are to be compared directly to the measured radiative width of the 7.03→3.95 transition.

The $E2$ results presented in Table IX provide rather conclusive evidence that collective enhancement of the $E2$ rates is needed if a pure s^4p^{10} configuration is assumed and that the weak surface coupling approximation gives an adequate representation of this enhancement although it is known to fail in other cases.[41,44] This is not surprising. For $\Delta T=0$ transitions the $E2$ matrix element can be written in the form $\lambda_0+\lambda_1T_3$, where λ_0 and λ_1 are the isoscalar and isovector components of the matrix element and T_3 is the z component of isobaric spin of the nucleus in question. In the weak-surface coupling approximation λ_0 is enhanced by a factor $(1+2\alpha)$ and λ_1 is unaffected; while more realistic approximations predict some enhancement (or retardation) of λ_1 also.[41] Thus the weak surface coupling approximation can fail badly in those cases for which λ_0 and λ_1T_3 interfere destructively, e.g., odd nuclei.[41,44] However, in a self-conjugate nucleus, i.e., $T_3=0$, the collective enhancement enters as a simple multiplicative factor in any model which preserves isotopic spin and thus the weak surface coupling approximation should be quite adequate.

We now consider the $M1$ rates. Table VIII shows that the predictions for the 3.95→2.31 and 7.03→0 $M1$ rates are relatively insensitive to the variations between the calculations and are in good accord with experiment. This is as expected from the form of Eqs. (51) and (59a) and the fact that these two transitions are both strong ones of their respective types (i.e., $\Delta T=1$ and 0, respectively). For pure s^4p^{10} states, the 7.03→3.95 $M1$ transition gives a measure of the C_D^{10} coefficient of Table III [see Eq. (59a)] and is quite weak and sensitive to the calculation. The presently available experimental information on this transition is not adequate to test the predictions: in particular a measure of the $E2/M1$ mixing ratio is necessary for this purpose. The 3.95→0 $M1$ rate has already been discussed.

Two predictions are given in Table VIII for the ^{14}N 2.31→0 transition. The first (1) is calculated directly from Eq. (43b) and thus displays the results as obtained from pure p-shell models. The second, third, and fourth values of column (1) correspond to p^{-2} wave functions which make the Gamow–Teller matrix element vanish. All three values agree closely amongst themselves and approximate the experimental width within about a factor of 2. The first value (intermediate coupling prediction) differs by a factor of eight from the experimental value and the corresponding eigenfunctions yield a large beta decay matrix element. To make the Gamow–Teller matrix element vanish, one must in this case construct a destructive interplay of p-shell and $(2s, 1d)$-shell contributions as described in Sec. 2.3. To reproduce simultaneously the experimental value of the $M1$ width either the 2.31 MeV or the ground state would have to arise mainly from the $(2s, 1d)$ configuration. If we consider the prediction of the intermediate coupling model regardless of the Gamow–Teller matrix element and take the effects of admixtures from the $(2s, 1d)$ configuration in accord with True's model calculation, we arrive at the first number given in column (2). Clearly the inclusion of admixtures of reasonable strength does not improve the agreement between experiment and the prediction of a model which is based on conventional central and spin–orbit interactions only. Finally, we consider the predictions which are obtained when the effects of $(2s, 1d)$ admixtures are included in the more sophisticated models. Since the $(2s, 1d)$ admixtures as taken from True's model make a nonvanishing contribution to the total Gamow–Teller matrix element $\langle \mathbf{G}\rangle$, we must then compensate for this contribution by inserting a nonvanishing p-shell Gamow–Teller matrix element so as to keep $\langle \mathbf{G}\rangle=0$. Because the contribution from the $(2s, 1d)$ configuration is small only a slight "overcancellation" of $\langle \mathbf{G}(p^{-2})\rangle$ is needed to reinforce $\langle \mathbf{G}\rangle=0$. The calculation was carried out in the Cohen and Kurath case as described in Sec. 4 and led to the set of wave functions labelled Cohen and Kurath II. The pure p-shell prediction (1) for this case is meaningless and prediction (2) presents the meaningful value arrived at from this model when $(2s, 1d)$ admixtures are included. After the bulk of cancellation of the Gamow–Teller matrix element has been achieved in the p-shell itself to the extent of a slight "overcancellation", both $\langle \mathbf{G}\rangle=0$ and the 2.31→0 $M1$ width are reproduced. It is obvious from the internal agreement of the predictions (1) for the Elliott, Visscher–Ferrell, and Cohen–Kurath I cases that a relaxation of $\langle \mathbf{G}(p^{-2})\rangle=0$ which is imposed when the eigenfunctions are extracted from the Elliott and Visscher–Ferrell models would yield predictions for the width which are very similar to that obtained when proceeding from case I of Cohen and Kurath to case II. Thus the two models which include a tensor interaction and the sets of wave functions extracted from the Cohen and Kurath empirical matrix elements give reasonable agreement with experiment. When, in addition, the effects of configuration mixing are included, the agreement is almost perfect.

Since the Cohen and Kurath empirical matrix elements naturally incorporate the effects of the tensor force (though they need not to be interpreted in these terms) we conclude that all the models considered which explicitly or implicitly invoke a tensor interaction can at the same time explain the vanishing of the Gamow–Teller matrix element and reproduce the correct value for the $M1$ width of the $2.31 \rightarrow 0$ transition. The "interplay" model which does not use the tensor force cannot achieve this agreement.

From the results presented in Tables VIII–X we conclude that the intermediate-coupling predictions are definitely inferior to the other four for the electromagnetic transitions as well as for the ^{14}C beta decay. We have not included the effects of configuration mixing in any of the $E2$ rates or in the $M1$ rates from the $(2^+, 0)$ state because we do not know how to do so in a quantitative manner. However, these empirical rates are all strong ones of their respective types and are in good agreement with the p^{-2} predictions. It is unlikely that the effects of configuration mixing would be strong enough to seriously perturb this good agreement.

4.2. Magnetic Moment and Other Properties of ^{14}N

The magnetic moment of ^{14}N on our model [Eq. (1)] is described by

$$\mu = \tfrac{1}{2} + (\mu_+ - \tfrac{1}{2})[A^2\{C_S{}^2 - \tfrac{1}{2}C_D{}^2\} + B^2\{C(s_{1/2}s_{1/2})^2 + \tfrac{1}{5}C(d_{5/2}\,d_{5/2})^2 - \tfrac{1}{5}C(d_{3/2}\,d_{3/2})^2 - \tfrac{1}{2}C(s_{1/2}\,d_{3/2})^2 + \tfrac{1}{2}C(d_{3/2}\,d_{5/2})^2 + \tfrac{2}{5}7^{1/2}C(d_{5/2}\,d_{5/2})C(d_{3/2}\,d_{5/2}) - \tfrac{3}{5}\sqrt{2}C(d_{3/2}\,d_{5/2})C(d_{3/2}\,d_{3/2})\}], \quad (61)$$

with $\mu_+ = \tfrac{1}{2}g^{(+)}$ and μ in nuclear magnetons. All coefficients, of course, are those of the $(1^+, 0)$ ground state.

The pure p^{-2} model predictions for μ of the five sets of p^{-2} wave functions of Table III are given in the first column of Table XI, the second column (a) lists the predictions of Eq. (61) using True's $p^8(2s, 1d)$ wave functions his values of A_0^{10} and B_0^{10} and the p^{-2} wave functions of Table III, while the third column (b) is similar to (a) but uses the wave functions of Elliott and Flowers (Table I) for the $p^8(2s, 1d)$ wave functions instead of those of True.

Table XI shows that all the models under consideration give predictions for the magnetic moment which are close to the experimental value. In the first column of Table XI the predictions of the pure p-shell models have been collected. On the average these are approximately 20% below the experimental value and the variations within this column are in the order of 10%; that is the magnetic moment as predicted by the pure p-shell models is insensitive to the specific model employed. This is obvious from an inspection of Eq.

TABLE XI. Magnetic moment of ^{14}N in nuclear magnetons.

Calculation	s^4p^{10} [a]	$s^4p^{10}+p^8(2s, 1d)$ (a)	(b)
Soper	0.358	0.370	0.392
Elliott	0.320	0.334	0.356
Visscher and Ferrell	0.351	0.363	0.385
Cohen and Kurath I	0.327	0.341	0.363
Cohen and Kurath II	0.331	0.345	0.367
Experiment		0.404	

[a] In column 1 are the predictions of the pure p^{-2} model. In column 2 the model (a) is that of column 1 for the p^{-2} wave functions, True's values of A_0^{10} and B_0^{10}, and his wave function (Table I) for the $p^8(2s, 1d)$ admixtures. The last column (b) is similar to (a) but with $p^8(2s, 1d)$ wave functions replaced by those of Elliott and Flowers (Table I).

(61). In any of the p-shell models (cf. Table III) the $^{13}D_1$ term is the dominating one in the ^{14}N ground-state wave function. Even when changing the intermediate-coupling parameter so as to vary the coupling from pure LS (i.e., $C_D=1$) to pure jj, the $^{13}D_1$ term changes in intensity by only 25%, this being still a much larger effect than the one caused by including a tensor force in the interaction, of which the major effect was the change of the sign of the (small) $^{13}S_1$ term. Because only the intensities of these two quantities enter the expression for the magnetic moment and because the model-insensitive $^{13}D_1$ intensity is dominant in any case the magnetic moment cannot provide evidence to distinguish between the p-shell models. However, intermediate coupling has been ruled out already on the basis of other evidence (cf. the previous sections).

Column (a) of Table XI shows the predictions which are obtained when a $(2s, 1d)$ admixture of the strength according to True's model [cf. Eq. (26a)] is taken into account and when for the $(2s, 1d)$ part the eigenfunctions of True (cf. Table III) are used. The approximately 6% admixture has the effect of increasing the magnetic moment on the average by approximately 10%. Column (b) is included to show that the contribution to the magnetic moment from $(2s, 1d)$ is quite sensitive to the specific set of wave functions used to present the $(2s, 1d)$ part. In the average the variation of μ which is introduced when replacing True's wave function for $(2s, 1d)$ by the mass 18 wave functions of Elliott and Flowers[33] is approximately as large ($\approx 10\%$) as the change produced when proceeding from the pure p-shell models to those which include admixtures from $(2s, 1d)$ in accord with True's model. In as much as it is meaningful to attach any significance to the deviations of the model predictions of Table XI from the experimental value of the magnetic moment we can conclude that a small contamination of the ^{14}N ground state arising from the $(2s, 1d)$ configuration is not in contradiction to the indications which we have obtained in the previous sections for the presence of such contaminations.

Other measured quantities which bear on the wave functions of the p^{-2} states of mass 14 are the electric-quadrupole moment of ^{14}N and the spectroscopic factors for single nucleon transfer reactions connecting mass 14 with masses 13 and 15. The electric-quadrupole moment is insensitive to the wave functions, has a high degree of experimental uncertainty, and is sensitive to admixtures of configurations other than $p^8(2s, 1d)$. Furthermore, it is described satisfactorily by all of the models we have considered and thus does not distinguish between them. The direct-reaction spectroscopic factors contain the usual uncertainties associated with our inexact knowledge of the reaction mechanisms. For instance, the ratio of the spectroscopic factors for the $^{14}N(d, p)^{15}N$ reaction leading to the $p_{1/2}^{-1}$ and $p_{3/2}^{-1}$ states of ^{15}N is capable, in principle, of giving specific information concerning the ^{14}N ground-state wave function[16]; but, in actual fact, the uncertainties in our knowledge of the Q dependence of (d, p) reduced widths drastically reduces the usefulness of this information. A recent study[15] of the $^{13}C(He^3, d)^{14}N$ reaction has resulted in spectroscopic factors which are in satisfactory agreement with the predictions of the five sets of wave functions given in Table III. However, these results are insensitive to the differences between these wave functions and also to configuration mixing.

5. SUMMARY

Our conclusions can be briefly summarized as follows:

(1) Wave functions generated by a central plus spin–orbit interaction and a pure s^4p^{10} configuration are incapable of explaining either the gamma or beta matrix elements of mass 14.

(2) Wave functions generated by admixing the $s^4p^8(2s, 1d)$ configuration with the predominant s^4p^{10} configuration are also incapable of explaining simultaneously the gamma and beta matrix elements of mass 14 if the interaction for the latter is retained as central plus spin orbit in character.

(3) Previous calculations have shown that the inclusion of a tensor component in the (largely) central plus spin–orbit interaction can generate pure s^4p^{10} wave functions which largely account for the Gamow–Teller matrix element. We have shown that, at the same time, this modification largely removes the discrepancies with the gamma matrix elements. This is true for those wave functions derived[7,21,42] by "first principles" from two-nucleon scattering data[18] and those extracted from a least-squares fit to a large number of binding energies[19] as well as those arrived at by forcing a fit to the beta matrix elements.[2,3] Thus, inclusion of a tensor part to the interaction is well-founded theoretically and, as well, is a sufficient condition for a rather successful accounting of both the gamma and beta data.

(4) Admixing small amounts of the $s^4p^{10}(2s, 1d)$ configuration to the "tensor-including" s^4p^{10} configuration gives even better agreement with experiment. All data is accounted for adequately (or better) except for the $\Delta T=0$ $E2$ rates.

(5) The predicted $\Delta T=0$ $E2$ rates are about $\frac{1}{4}$ of the measured rates. This is presumably due to collective enhancement via configurational admixtures of higher orbitals—such as s^4p^9f—which do not have an important influence on the Gamow–Teller or $M1$ rates. The effective charge description gives a reasonable account of this collective enhancement.

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