Study of Leptonic Three-Body Decays of Baryons

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Assuming isospin, the current–current form of the weak interactions of hadrons, and the most general matrix element for the hadron current, expressions are obtained for the decay rate, the final particle energy spectra, and the final baryon–lepton angular correlation. The results are presented in a manner that allows easier comparison between theories and significant data. In particular, a table is given which will allow comparisons between theory and experiment for decay widths. It also allows one to determine which of the many interactions are important in the energy spectra for the various decay modes.

1. INTRODUCTION

With the accumulation of more and more experimental data, it will become possible in the future to determine some of the finer aspects of the hadronic part of the semileptonic decay matrix elements. For such phenomenological analysis, it is desirable to have explicit formulas for the decay rates, energy spectra, angular correlation, etc., where due consideration is given to the total structure of the matrix elements, and which can be easily understood by all physicists.

This work was undertaken with the hope that it would form a bridge between experimental and theoretical physicists.[1] Some of the results have been obtained earlier by Belov *et al.*,[2] Harrington,[3] and others.[4–7] We have reduced all the functions to only two kinematic scalars, and these two are the physically meaningful momentum–transfer squared and one scalar involving energy or angle variables.

We are assuming the current–current form of the hadronic weak interaction and isospin. We *are not* assuming $SU(3)$, Cabibbo theory, or mass degeneracy. As more experimental data becomes available, this will allow rigorous tests of Cabibbo theory or other new models that may be proposed. (We keep the form factors real, which means we assume time-reversal invariance. However, since we assume isospin, a slight modification allows a test of T or CP violation. See Sec. 2.)

We use much of the notation of Cabibbo and Veltman.[8–10] The metric and form of the Dirac equation is given in Appendix A. In Sec. 2 we set up the problem for the decay width, and evaluate the matrix elements. Then, in Secs. 3 and 4, we obtain the formulas for the energy spectra and the angular correlation, respectively. In the final section we discuss the pertinence of our results to experiments. A table is given which allows an easy comparison between theory and experiment for decay widths and lets one determine which spectral terms are important for a given decay process, as shown by an example of current interest.

2. DECAY MATRIX ELEMENT FOR $B\to B'+l+\bar{\nu}$

We are considering the semileptonic decay of a baryon into a final baryon, a lepton, and an antineutrino, i.e.,

$$B\to B'+l+\bar{\nu}. \qquad (2.1)$$

(See Fig. 1.) The mass and 4-momenta of the initial baryon, final baryon, and final lepton are (M, p), (M', k), and (m, q'). The 4-momenta of the antineutrino is (q). We often use the quantities Q and K defined by

$$Q=q+q'=p-k,$$
$$K=p+k. \qquad (2.2)$$

The weak interaction Hamiltonian is assumed to be of the current–current type:

$$\mathcal{H}(x)=(G/\sqrt{2})[J^{\mu}(x)\mathcal{J}^{\mu}(x)+\text{h.c.}], \qquad (2.3)$$

where J and $\mathcal{J}$ are, respectively, the hadronic current and the leptonic current, G is the weak coupling constant[11]

$$G=(1.0232)10^{-5}/M_p^2,$$
$$G^2/2=0.6755\times10^{-22}\ \text{MeV}^{-4}, \qquad (2.4)$$

and M_p is the mass of the proton.

The matrix element of the leptonic current is

$$L^{\mu}=\langle l(q'),\bar{\nu}(q)\mid \mathcal{J}^{\mu}(0)\mid 0\rangle,$$
$$=(2\pi)^{-3}\bar{u}(q')[\gamma^{\mu}(1+\gamma^5)]v(q). \qquad (2.5)$$

The matrix element of the hadronic current between

[1] Many of the results of this investigation were reported earlier. See M. M. Nieto and H. T. Nieh, Bull. Am. Phys. Soc. **12**, 595 (1967).

[2] V. P. Belov, B. S. Mingalev, and V. M. Shekhter, Zh. Eksperim. i Teor. Fiz. **38**, 541 (1960) [English transl.: Soviet Phys.—JETP **11**, 392 (1960)].

[3] D. R. Harrington, Phys. Rev. **120**, 1482 (1960).

[4] L. Egardt, Nuovo Cimento **27**, 368 (1963).

[5] W. Drechsler, Nuovo Cimento **38**, 345 (1965).

[6] N. Brene, L. Veje, M. Roos, and C. Cronström, Phys. Rev. **149**, 1288 (1966).

[7] C. E. Carlson, Phys. Rev. **152**, 1433 (1966).

[8] M. Veltman in the *Proceedings of the 1964 Easter School at Herceg-Novi*, Volume III, CERN, Report 64-13, 1964 (unpublished).

[9] N. Cabibbo and M. Veltman, *Weak Interactions*, CERN, Report 65-30, 1965 (unpublished).

[10] N. Cabibbo, *1965 Brandeis University Summer Institute in Theoretical Physics, Volume II, Particle Symmetries*, M. Chretien and S. Deser, Eds. (Gordon and Breach Science Publishers, Inc., New York, 1966), p. 3.

[11] This value is taken from Ref. 6. If the determination of the coupling constant changes in the future, our numerical results can be changed by simple multiplication. All masses used in this work are taken from A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

two baryons is given by

$$H^\mu \equiv \langle B'(k) \mid J^\mu(0) \mid B(p) \rangle,$$
$$= (2\pi)^{-3}\bar{u}(k)[F_1\gamma^\mu + (F_2/M)\sigma^{\mu\nu}Q^\nu + i(F_3/M)Q^\mu$$
$$+G_1\gamma^\mu\gamma^5 + (G_2/M)\sigma^{\mu\nu}\gamma^5 Q^\nu + i(G_3/M)\gamma^5 Q^\mu]u(p), \quad (2.6)$$

where the F's and G's are the hadronic form factors, assumed to be functions of the momentum transfer squared (Q^2). In the terminology of Weinberg,[12] F_1, F_2, G_1, and G_3 are associated with "first-class" currents (proper Lorentz vectors with G parity $+1$, and axial Lorentz vectors with G parity -1). F_3 and G_2 are associated with "second-class" currents (G parity opposite that of the first-class currents).

If the lepton and antilepton currents are coupled to hadron currents that are members of the same isospin multiplet, then time-reversal violation occurs only by means of the second-class currents.[9,12] Thus, with this added assumption, we just make the form factors of the second-class currents imaginary; and if such terms exist, they violate time-reversal invariance. If one assumes $SU(3)$, then the result holds with the isospin multiplet becoming an octet.[13]

The terms in (2.6) are called

γ^μ	vector,
$\sigma^{\mu\nu}Q^\nu$	weak magnetism,
Q^μ	scalar,
$\gamma^\mu\gamma^5$	axial vector,
$\sigma^{\mu\nu}\gamma^5 Q^\nu$	axial magnetism,
$\gamma^5 Q^\mu$	induced pseudoscalar.

Equation (2.6) can also be written as

$$H^\mu = (2\pi)^{-3}\bar{u}(k)[f_1\gamma^\mu + i(f_2/M)K^\mu + i(f_3/M)Q^\mu$$
$$+g_1\gamma^\mu\gamma^5 + i(g_2/M)\gamma^5 K^\mu + i(g_3/M)\gamma^5 Q^\mu]u(p), \quad (2.7)$$

where

$$f_1 = F_1 + [1 + (M'/M)]F_2$$
$$f_2 = F_2$$
$$f_3 = F_3$$
$$g_1 = G_1 - [1 - (M'/M)]G_2$$
$$g_2 = G_2$$
$$g_3 = G_3. \quad (2.8)$$

The (2.6) form of H^μ has more physical content, since each term has a definite G parity and a definite unitary parity. However, using (2.7) allows great simplification in computation, and the results can trivially be converted by means of (2.8). For these reasons we use the (2.7) form of H^μ to calculate functions, but most of our numerical results are given in terms of the more physically meaningful (2.6) form of the hadron current.

[12] S. Weinberg, Phys. Rev. **112**, 1375 (1958). Also, Reference 8 has a good discussion on this point.

[13] N. Cabibbo, Phys. Letters **12**, 137 (1964).

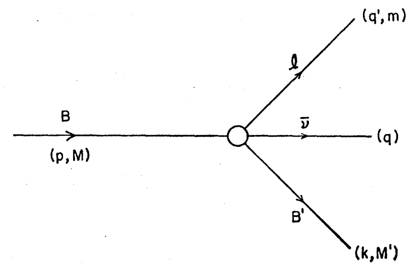

FIG. 1. The semileptonic decay process $B \to B' + l + \bar{\nu}$. The 4-momenta and mass of the initial baryon, final baryon, and final lepton are (p,M), (k,M'), and (q',m). The 4-momenta of the antineutrino is (q).

Later in this paper we integrate over the Q^2 variable. This can be done if we parameterize the form factors by a polynomial expansion. However, one does not expect a wide variation of the form factors because the maximum Q^2 is small. Because of this and the fact that F_1 and G_1 have lowest-order symmetry breaking, we keep the other form factors constant and express F_1 and G_1 by

$$F_1 = \tilde{F}[1 - (A/M^2)Q^2], \qquad G_1 = \tilde{G}[1 - (B/M^2)Q^2],$$
$$f_1 = \tilde{f}[1 - (\alpha/M^2)Q^2], \qquad g_1 = \tilde{g}[1 - (\beta/M^2)Q^2], \quad (2.9)$$

where A, α, B, β are parameter constants. This implies that

$$\tilde{F}A = \tilde{f}\alpha, \qquad \tilde{G}B = \tilde{g}\beta. \quad (2.10)$$

Any of the other form factors can be similarly expanded and integrated.

The total decay rate (inverse lifetime) is now given by

$$\Gamma = \tau^{-1} = (2\pi\hbar)^{-1} \int d^3q\, d^3q'\, d^3k\, \delta^4(p - k - q - q') \times \sum_{\text{spins}} \tfrac{1}{2}(\mid T \mid^2), \quad (2.11)$$

where

$$T = (2\pi)^4 (G/\sqrt{2}) H^\mu L^\mu. \quad (2.12)$$

To calculate the square of the matrix element we combine (2.5), (2.7), and (2.12) to obtain

$$\sum_{\text{spins}} \tfrac{1}{2}(\,|\,T\,|^2)=\frac{G^2}{4(2\pi)^4}\,\mathrm{Tr}\left[\frac{(-ik+M')}{2k_0}\left(f_1\gamma^\mu+i\frac{f_2}{M}K^\mu+i\frac{f_3}{M}Q^\mu+g_1\gamma^\mu\gamma^5+i\frac{g_2}{M}K^\mu\gamma^5+i\frac{g_3}{M}Q^\mu\gamma^5\right)\frac{-ip+M}{2p_0}\right.$$

$$\left.\times\left(f_1\gamma^\lambda+i\frac{f_2}{M}K^\lambda+i\frac{f_3}{M}Q^\lambda+g_1\gamma^\lambda\gamma^5-i\frac{g_2}{M}K^\lambda\gamma^5-i\frac{g_3}{M}Q^\lambda\gamma^5\right)\right]\mathrm{Tr}\left(\frac{-iq'+m}{2q_0'}\gamma^\mu(1+\gamma^5)\,\frac{(-iq)}{2q_0}\gamma^\lambda(1+\gamma^5)\right)$$

$$\equiv\frac{G^2}{2^6(2\pi)^4p_0k_0q_0'q_0}[32F^{\mathrm{I}}(Q^2,p\cdot q)]. \tag{2.13}$$

In the last line of (2.13) we use the fact that all invariants can be expressed in terms of the masses, and two scalars, here Q^2 and $(p\cdot q)$. It is useful to note that

$$Q^2=(p-k)^2=-M^2-M'^2-2(p\cdot k), \tag{2.14}$$

$$p\cdot k=-\tfrac{1}{2}(Q^2+M^2+M'^2), \tag{2.15}$$

$$p\cdot q'=\tfrac{1}{2}(Q^2+M'^2-M^2)-(p\cdot q). \tag{2.16}$$

A long trace calculation (see Appendix B) leads to the result

$$\begin{aligned}F^{\mathrm{I}}(Q^2,\,p\cdot q)=&(f_1^2+g_1^2)2(p\cdot q)[M'^2-M^2+m^2-2(p\cdot q)]+(f_1-g_1)^2[2Q^2(p\cdot q)-\tfrac{1}{2}(Q^2+m^2)(Q^2+M'^2-M^2)]\\&-[(f_2^2+g_2^2)/M^2][8(p\cdot q)^2-4(p\cdot q)(M'^2-M^2+m^2+Q^2)+\tfrac{1}{2}(Q^2+m^2)(m^2-4M^2)]\tfrac{1}{2}(Q^2+M^2+M'^2)\\&-[(f_3^2+g_3^2)/M^2]m^2\tfrac{1}{2}(Q^2+m^2)\tfrac{1}{2}(Q^2+M^2+M'^2)-[(f_2f_3+g_2g_3)/M^2][4m^2(p\cdot q)-m^2(Q^2+m^2)]\tfrac{1}{2}(Q^2+M^2+M'^2)\\&+MM'(f_1^2-g_1^2)(Q^2+m^2)-(M'/M)(f_2^2-g_2^2)[8(p\cdot q)^2-4(p\cdot q)(M'^2-M^2+m^2+Q^2)+\tfrac{1}{2}(Q^2+m^2)(m^2-4M^2)]\\&-(M'/M)(f_3^2-g_3^2)m^2\tfrac{1}{2}(Q^2+m^2)-(M'/M)(f_2f_3-g_2g_3)[4m^2(p\cdot q)-m^2(Q^2+m^2)]\\&+(M'/M)(f_1f_2+g_1g_2)[8(p\cdot q)^2-2(p\cdot q)(2Q^2+m^2+2M'^2-2M^2)-2M^2(Q^2+m^2)]\\&+2(M'/M)(f_1f_3+g_1g_3)m^2(p\cdot q)+(f_1f_2-g_1g_2)[8(p\cdot q)^2-2(p\cdot q)(2Q^2+3m^2+2M'^2-2M^2)-(Q^2+m^2)(2M^2-m^2)]\\&+(f_1f_3-g_1g_3)m^2[2(p\cdot q)-(Q^2+m^2)].\end{aligned} \tag{2.17}$$

Inserting (2.16) into (2.17) yields

$$\begin{aligned}F^{\mathrm{II}}(Q^2,\,p\cdot q')=&(f_1^2+g_1^2)(Q^2+M'^2-M^2-2p\cdot q')(m^2-Q^2+2p\cdot q')\\&+\tfrac{1}{2}(f_1-g_1)^2[Q^2(Q^2+M'^2-M^2-m^2-4p\cdot q')+m^2(M^2-M'^2)]\\&+[(f_2^2+g_2^2)/M^2][2(Q^2+M'^2-M^2-2p\cdot q')(m^2+2p\cdot q')+\tfrac{1}{2}(Q^2+m^2)(4M^2-m^2)]\tfrac{1}{2}(Q^2+M^2+M'^2)\\&-\tfrac{1}{2}[(f_3^2+g_3^2)/M^2]m^2(Q^2+m^2)\tfrac{1}{2}(Q^2+M^2+M'^2)\\&-[(f_2f_3+g_2g_3)/M^2]m^2(Q^2+2M'^2-2M^2-m^2-4p\cdot q')\tfrac{1}{2}(Q^2+M^2+M'^2)\\&+MM'(f_1^2-g_1^2)(Q^2+m^2)+(M'/M)(f_2^2-g_2^2)[2(Q^2+M'^2-M^2-2p\cdot q')(m^2+2p\cdot q')+\tfrac{1}{2}(Q^2+m^2)(4M^2-m^2)]\\&-\tfrac{1}{2}(M'/M)(f_3^2-g_3^2)m^2(Q^2+m^2)-(M'/M)(f_2f_3-g_2g_3)m^2(Q^2+2M'^2-2M^2-m^2-4p\cdot q')\\&-(M'/M)(f_1f_2+g_1g_2)[(Q^2+M'^2-M^2-2p\cdot q')(m^2+4p\cdot q')+2M^2(Q^2+m^2)]\\&+(M'/M)(f_1f_3+g_1g_3)m^2(Q^2+M'^2-M^2-2p\cdot q')\\&-(f_1f_2-g_1g_2)[(Q^2+M'^2-M^2-2p\cdot q')(3m^2+4p\cdot q')+(2M^2-m^2)(Q^2+m^2)]\\&-(f_1f_3-g_1g_3)m^2[M^2-M'^2+m^2+2p\cdot q'].\end{aligned} \tag{2.18}$$

3. ENERGY SPECTRA

To obtain the energy spectra we use the method described in Appendix C of Ref. 9.

Combining (2.13) and (2.11) yields

$$\Gamma=\frac{G^2}{\hbar(2\pi)^5 2}\int\frac{d^3qd^3q'd^3k}{p_0k_0q_0'q_0}\,\delta^4(p-k-q-q')\,F^{\text{I}}(Q^2,\,p\cdot q). \tag{3.1}$$

We work in the center-of-mass system, so that $p_0=M$, $(p\cdot q)=-Mq_0$, $(pq')=-Mq_0'$, and

$$Q^2=-M^2-M'^2+2Mk_0. \tag{3.2}$$

F^{I} and F^{II} are then independent of angle.

For calculating the baryon spectrum the q' integration is done immediately, so that the integral part of (3.1) is

$$Z\equiv\int\frac{d^3k}{k_0}\int\frac{d^3q}{q_0q_0'}\,\delta(p_0-k_0-q_0-q_0')\,F^{\text{I}}(Q^2,\,p\cdot q). \tag{3.3}$$

Changing the $\mathbf{k}$ and $\mathbf{q}$ integrations to polar coordinates yields

$$Z=8\pi^2\int\frac{|\mathbf{k}|^2d|\mathbf{k}|}{k_0}\int\frac{|\mathbf{q}|^2d|\mathbf{q}|}{q_0q_0'}\int_{-1}^{1}dz\delta[M-k_0-q_0-(|\mathbf{q}|^2+|\mathbf{k}|^2+2z|\mathbf{q}||\mathbf{k}|)^{1/2}]\cdot F^{\text{I}}(Q^2,\,p\cdot q), \tag{3.4}$$

where z is the cosine of the angle between $\mathbf{q}$ and $\mathbf{k}$. We now use

$$\int dz\delta[f(z)]=1/|f'(z_0)|, \tag{3.5}$$

where z_0 is defined by $f(z_0)\equiv 0$, i.e.,

$$z_0=[(M-k_0-q_0)^2-|\mathbf{q}|^2-|\mathbf{k}|^2]/2|\mathbf{k}||\mathbf{q}|. \tag{3.6}$$

For our case

$$1/|f'(z_0)|=q_0'(z_0)/|\mathbf{k}||\mathbf{q}|, \tag{3.7}$$

so that

$$Z=8\pi^2\int\frac{|\mathbf{k}|d|\mathbf{k}|}{k_0}\int\frac{|\mathbf{q}|d|\mathbf{q}|}{q_0}\,F^{\text{I}}(Q^2,\,p\cdot q)\theta(1-z_0^2), \tag{3.8}$$

where the θ step function insures that $-1\leq z_0\leq 1$.

By realizing that

$$|\mathbf{k}|d|\mathbf{k}|=k_0dk_0, \tag{3.9}$$

we can change the momentum spectrum to an energy spectrum,[14] so that

$$Z=8\pi^2\int k_0dk_0\int q_0dq_0F^{\text{I}}(Q^2,\,p\cdot q)\theta(1-z_0^2). \tag{3.10}$$

The condition that $z_0^2\leq 1$ means that

$$a_-\leq q_0\leq a_+,$$

$$a_\pm=\frac{(M^2+M'^2+m_\nu^2-m^2-2Mk_0)(M-k_0)\pm|\mathbf{k}|R}{2(M^2+M'^2-2Mk_0)},$$

$$R=[(M^2+M'^2-m_\nu^2-m^2-2Mk_0)^2-4m_\nu^2m^2]^{1/2}, \tag{3.11}$$

[14] By making use of (3.9), our results can be converted at once to momentum spectra, if the reader so desires. Where it will cause no confusion, we shall henceforth use the word neutrino instead of antineutrino.

where we have temporarily left in a neutrino mass. The necessity for the factor in the square root to be positive implies that

$$a_0=(2M)^{-1}[M^2+M'^2-(m+m_\nu)^2]\geq k_0. \tag{3.12}$$

Now setting $m_\nu=0$ gives

$$a_\pm=\frac{(M^2+M'^2-m^2-2Mk_0)}{2(M^2+M'^2-2Mk_0)}\cdot(M-k_0\pm|\mathbf{k}|),$$

$$=\frac{Q^2+m^2}{2Q^2}\left[\frac{M^2-M'^2-Q^2}{2M}\pm|\mathbf{k}|\right]. \tag{3.13}$$

This finally yields the decay width Γ in terms of the baryon energy-spectrum function, S_B:

$$\Gamma=\frac{G^2}{(2\pi)^3M\hbar}\int_{M'}^{a_0}dk_0S_B(k_0), \tag{3.14}$$

$$S_B(k_0)=\int_{a-}^{a+}dq_0F^{\mathrm{I}}(Q^2,\,-Mq_0). \tag{3.15}$$

An explicit integration gives

$$\begin{aligned}S_B(k_0)=&(f_1^2+g_1^2)[2(M'^2-M^2+m^2)A_2-4A_3]+(f_1-g_1)^2[2Q^2A_2-\tfrac{1}{2}(Q^2+m^2)(Q^2+M'^2-M^2)A_1]\\&-[(f_2^2+g_2^2)/M^2][8A_3-4(M'^2-M^2+Q^2+m^2)A_2+\tfrac{1}{2}(Q^2+m^2)(m^2-4M^2)A_1]\tfrac{1}{2}(Q^2+M^2+M'^2)\\&-\tfrac{1}{4}[(f_3^2+g_3^2)/M^2]m^2(Q^2+m^2)(Q^2+M^2+M'^2)A_1-[(f_2f_3+g_2g_3)/M^2]m^2[2A_2-\tfrac{1}{2}(Q^2+m^2)A_1](Q^2+M^2+M'^2)\\&+MM'(f_1^2-g_1^2)(Q^2+m^2)A_1-(M'/M)(f_2^2-g_2^2)[8A_3-4(M'^2-M^2+Q^2+m^2)A_2+\tfrac{1}{2}(Q^2+m^2)(m^2-4M^2)A_1]\\&-\tfrac{1}{2}(M'/M)(f_3^2-g_3^2)m^2(Q^2+m^2)A_1-(M'/M)(f_2f_3-g_2g_3)m^2[4A_2-(Q^2+m^2)A_1]\\&-2(M'/M)(f_1f_2+g_1g_2)[-4A_3+(2Q^2+2M'^2-2M^2+m^2)A_2+M^2(Q^2+m^2)A_1]\\&+2(M'/M)(f_1f_3+g_1g_3)m^2A_2-(f_1f_2-g_1g_2)[-8A_3+2(2Q^2+2M'^2-2M^2+3m^2)A_2+(Q^2+m^2)(2M^2-m^2)A_1]\\&+(f_1f_3-g_1g_3)m^2[2A_2-(Q^2+m^2)A_1],\end{aligned} \tag{3.16}$$

$$A_n=[(-M)^{n-1}/n](a_+^n-a_-^n),\qquad n=1,2,3\cdots. \tag{3.17}$$

Use of (3.13) shows that the $(f_1-g_1)^2$ term is identically zero, so that there is no f_1g_1 interference in the baryon spectrum. However, this is not true for the other spectra as is commonly thought. We come back to this point later.

In a similar manner, one can obtain the lepton and neutrino spectra. For the lepton spectrum the result is

$$\Gamma=\frac{G^2}{(2\pi)^3M\hbar}\int_m^{A_0}dq_0'S_L(q_0'), \tag{3.18}$$

$$S_L(q_0')=\int_{A-}^{A+}dk_0F^{\mathrm{II}}(Q^2,\,-Mq_0'), \tag{3.19}$$

$$=(2M)^{-1}\int_{d-}^{d+}dQ^2F^{\mathrm{II}}(Q^2,\,-Mq_0'), \tag{3.20}$$

$$A_0=(M^2+m^2-M'^2)/2M, \tag{3.21}$$

$$A_\pm=\frac{(M-q_0')(M^2+M'^2+m^2-2Mq_0')\pm|\mathbf{q}'|(M^2-M'^2+m^2-2Mq_0')}{2(M^2+m^2-2Mq_0')}, \tag{3.22}$$

$$d_\pm=\frac{-M^2m^2+M(-q_0'\pm|\mathbf{q}'|)(M^2-M'^2+m^2-2Mq_0')}{(M^2+m^2-2Mq_0')}. \tag{3.23}$$

The integration of the spectral functions to obtain the lepton and neutrino spectra requires use of the form factor expansions (2.9). Using them in (3.20) yields

$$
\begin{aligned}
2M S_L(q_0') ={}& (\tilde{f}^2+\tilde{g}^2)[-D_3+(-4Mq_0'-M'^2+M^2+m^2)D_2+(M'^2-M^2+2Mq_0')(m^2-2Mq_0')D_1] \\
&-2[(\tilde{f}^2\alpha+\tilde{g}^2\beta)/M^2][-D_4+(-4Mq_0'-M'^2+M^2+m^2)D_3+(M'^2-M^2+2Mq_0')(m^2-2Mq_0')D_2] \\
&+[(\tilde{f}^2\alpha^2+\tilde{g}^2\beta^2)/M^4][-D_5+(-4Mq_0'-M'^2+M^2+m^2)D_4+(M'^2-M^2+2Mq_0')(m^2-2Mq_0')D_3] \\
&+(\tilde{f}-\tilde{g})^2\{\tfrac{1}{2}D_3-[-2Mq_0'+\tfrac{1}{2}(M^2-M'^2+m^2)]D_2+\tfrac{1}{2}m^2(M^2-M'^2)D_1\} \\
&-2(\tilde{f}-\tilde{g})[(\tilde{f}\alpha-\tilde{g}\beta)/M^2]\{\tfrac{1}{2}D_4-[-2Mq_0'+\tfrac{1}{2}(M^2-M'^2+m^2)]D_3+\tfrac{1}{2}m^2(M^2-M'^2)D_2\} \\
&+[(\tilde{f}\alpha-\tilde{g}\beta)^2/M^4]\{\tfrac{1}{2}D_5-[-2Mq_0'+\tfrac{1}{2}(M^2-M'^2+m^2)]D_4+\tfrac{1}{2}m^2(M^2-M'^2)D_3\} \\
&+[(f_2^2+g_2^2)/M^2]\{(-2Mq_0'+M^2+\tfrac{3}{4}m^2)D_3 \\
&+D_2[-4M^2q_0'^2-2Mq_0'(2M'^2-m^2)+2m^2M'^2+(M^2-\tfrac{1}{4}m^2)(M^2+M'^2+m^2)] \\
&+D_1\,\tfrac{1}{2}(M^2+M'^2)[-8M^2q_0'^2-4Mq_0'(M'^2-M^2-m^2)+2m^2(M'^2-M^2)+\tfrac{1}{2}m^2(4M^2-m^2)]\} \\
&-\tfrac{1}{4}m^2[(f_3^2+g_3^2)/M^2][D_3+(M^2+M'^2+m^2)D_2+m^2(M^2+M'^2)D_1] \\
&-[(f_2f_3+g_2g_3)/M^2]m^2[\tfrac{1}{2}D_3+(2Mq_0'+\tfrac{3}{2}M'^2-\tfrac{1}{2}M^2-\tfrac{1}{2}m^2)D_2+(M^2+M'^2)(2Mq_0'+M'^2-M^2-\tfrac{1}{2}m^2)D_1] \\
&+MM'(\tilde{f}^2-\tilde{g}^2)[D_2+m^2D_1]-2MM'[(\tilde{f}^2\alpha-\tilde{g}^2\beta)/M^2][D_3+m^2D_2]+MM'[(\tilde{f}^2\alpha^2-\tilde{g}^2\beta^2)/M^4][D_4+m^2D_3] \\
&-(M'/M)(f_2^2-g_2^2)[(4Mq_0'-\tfrac{3}{2}m^2-2M^2)D_2+\{8M^2q_0'^2+4Mq_0'(M'^2-M^2-m^2)-m^2(2M'^2-\tfrac{1}{2}m^2)\}D_1] \\
&-(M'/M)(f_3^2-g_3^2)(\tfrac{1}{2}m^2)[D_2+m^2D_1]-(M'/M)(f_2f_3-g_2g_3)m^2[D_2+(+4Mq_0'+2M'^2-2M^2-m^2)D_1] \\
&-(M'/M)(\tilde{f}f_2+\tilde{g}g_2)\{(-4Mq_0'+m^2+2M^2)D_2+[-8M^2q_0'^2-2Mq_0'(2M'^2-2M^2-m^2)+m^2(M'^2+M^2)]D_1\} \\
&+(M'/M)[(\tilde{f}f_2\alpha+\tilde{g}g_2\beta)/M^2]\{(-4Mq_0'+m^2+2M^2)D_3 \\
&+[-8M^2q_0'^2-2Mq_0'(2M'^2-2M^2-m^2)+m^2(M'^2+M^2)]D_2\} \\
&+(M'/M)(\tilde{f}f_3+\tilde{g}g_3)m^2[D_2+(M'^2-M^2+2Mq_0')D_1]-(M'/M)[(\alpha\tilde{f}f_3+\beta\tilde{g}g_3)/M^2]m^2[D_3+(M'^2-M^2+2Mq_0')D_2] \\
&-(\tilde{f}f_2-\tilde{g}g_2)\{(-4Mq_0'+2m^2+2M^2)D_2+[-8M^2q_0'^2-2Mq_0'(2M'^2-2M^2-3m^2)+m^2(3M'^2-M^2-m^2)]D_1\} \\
&+[(\alpha\tilde{f}f_2-\beta\tilde{g}g_2)/M^2]\{(-4Mq_0'+2m^2+2M^2)D_3+[-8M^2q_0'^2-2Mq_0'(2M'^2-2M^2-3m^2)+m^2(3M'^2-M^2-m^2)]D_2\} \\
&-(\tilde{f}f_3-\tilde{g}g_3)m^2(-2Mq_0'-M'^2+M^2+m^2)D_1+[(\alpha\tilde{f}f_3-\beta\tilde{g}g_3)/M^2]m^2(-2Mq_0'-M'^2+M^2+m^2)D_2, \qquad (3.24)
\end{aligned}
$$

$$D_n=(d_+^n-d_-^n)/n, \qquad n=1,\ 2,\ 3\cdots. \qquad (3.25)$$

Lastly, we give the result for the neutrino spectrum $S_\nu(q_0)$:

$$\Gamma=\frac{G^2}{(2\pi)^3M\hbar}\int_0^{c_0} dq_0 S_\nu(q_0), \qquad (3.26)$$

$$
\begin{aligned}
S_\nu(q_0) &= \int_{c-}^{c+} dk_0 F^{\mathrm{I}}(Q^2,\ -Mq_0), \\
&= (2M)^{-1}\int_{b-}^{b+} dQ^2 F^{\mathrm{I}}(Q^2,\ -Mq_0), \qquad (3.27)
\end{aligned}
$$

$$c_0=(2M)^{-1}[M^2-(m+M')^2], \qquad (3.28)$$

$$c_\pm=\frac{(M^2+M'^2-m^2-2Mq_0)(M-q_0)\pm|\mathbf{q}|R'}{2(M^2-2Mq_0)}, \qquad (3.29)$$

$$R'=[(M^2-M'^2-m^2-2Mq_0)^2-4M'^2m^2], \qquad (3.30)$$

$$b_\pm=\frac{2M^2q_0^2+q_0M(M'^2-M^2+m^2)-m^2M^2\pm Mq_0R'}{(M^2-2Mq_0)}. \qquad (3.31)$$

Again using (2.9), this explicitly yields

$$
\begin{aligned}
2MS_\nu(q_0)=&-2Mq_0[M'^2-M^2+m^2+2Mq_0]\{(\tilde f^2+\tilde g^2)B_1[-2(\tilde f^2\alpha+\tilde g^2\beta)/M^2]B_2+[(\tilde f^2\alpha^2+\tilde g^2\beta^2)/M^4]B_3\}\\
&+(\tilde f-\tilde g)^2J_1-2(\tilde f-\tilde g)[(\tilde f\alpha-\tilde g\beta)/M^2]J_2+[(\tilde f\alpha-\tilde g\beta)/M^2]^2J_3\\
&+[(f_2^2+g_2^2)/M^2]\{N_1[8M^2q_0^2+4Mq_0(M'^2-M^2+m^2)+\tfrac{1}{2}m^2(m^2-4M^2)]+N_2[4Mq_0+\tfrac{1}{2}(m^2-4M^2)]\}\\
&+[(f_3^2+g_3^2)/M^2]\tfrac{1}{2}m^2(m^2N_1+N_2)-[(f_2f_3+g_2g_3)/M^2]m^2[(4Mq_0+m^2)N_1+N_2]\\
&+MM'(\tilde f^2-\tilde g^2)(m^2B_1+B_2)-(2M'/M)(\tilde f^2\alpha-\tilde g^2\beta)(m^2B_2+B_3)+(M'/M^3)(\tilde f^2\alpha^2-\tilde g^2\beta^2)(m^2B_3+B_4)\\
&-(M'/M)(f_2^2-g_2^2)\{B_1[8M^2q_0^2+4Mq_0(M'^2-M^2+m^2)+\tfrac{1}{2}m^2(m^2-4M^2)]+B_2[4Mq_0+\tfrac{1}{2}(m^2-4M^2)]\}\\
&-(M'/M)(f_3^2-g_3^2)\tfrac{1}{2}m^2[m^2B_1+B_2]+(M'/M)(f_2f_3-g_2g_3)m^2[(4Mq_0+m^2)B_1+B_2]\\
&-2(M'/M)\{B_1(\tilde f f_2+\tilde g g_2)-B_2[(\tilde f f_2\alpha+\tilde g g_2\beta)/M^2]\}[-4M^2q_0^2+Mq_0(2M^2-2M'^2-m^2)+m^2M^2]\\
&-2(M'/M)\{B_2(\tilde f f_2+\tilde g g_2)-B_3[(\tilde f f_2\alpha+\tilde g g_2\beta)/M^2]\}(M^2-2Mq_0)-2M'm^2q_0\{B_1(\tilde f f_3+\tilde g g_3)-B_2[(\tilde f f_3\alpha+\tilde g g_3\beta)/M^2]\}\\
&-2\{B_1(\tilde f f_2-\tilde g g_2)-B_2[(\tilde f f_2\alpha-\tilde g g_2\beta)/M^2]\}[-4M^2q_0^2+Mq_0(2M^2-2M'^2-3m^2)+\tfrac{1}{2}m^2(2M^2-m^2)]\\
&-2\{B_2(\tilde f f_2-\tilde g g_2)-B_3[(\tilde f f_2\alpha-\tilde g g_2\beta)/M^2]\}[M^2-2Mq_0-\tfrac{1}{2}m^2]-(\tilde f f_3-\tilde g g_3)m^2[(2Mq_0+m^2)B_1+B_2]\\
&+[(\tilde f f_3\alpha-\tilde g g_3\beta)/M^2]m^2[(2Mq_0+m^2)B_2+B_3],
\end{aligned}
\tag{3.32}
$$

where

$$
\begin{aligned}
B_n&=(b_+^{\,n}-b_-^{\,n})/n,\\
J_n&=-\tfrac{1}{2}B_{n+2}+\tfrac{1}{2}(M^2-M'^2-m^2-4Mq_0)^2B_{n+1}+\tfrac{1}{2}m^2(M^2-M'^2)B_n,\\
N_n&=-\tfrac{1}{2}B_{n+1}-\tfrac{1}{2}(M^2+M'^2)B_n, \qquad n=1,2,3,\cdots.
\end{aligned}
\tag{3.33}
$$

4. ANGULAR CORRELATION

The derivation of the angular correlation between the final baryon and lepton is more complicated than that of the energy spectra, but is done in a similar manner. Starting from (3.4) and changing to energy integrations, we have

$$Z=8\pi^2\int_{-1}^{1}dx\int dk_0\int dq_0'\,\frac{|\mathbf{k}||\mathbf{q}'|}{k_0q_0}\,\delta(M-k_0-q_0'-q_0)F^{\mathrm{II}}(Q^2,-Mq_0'),\tag{4.1}$$

$$
\begin{aligned}
q_0&\equiv(|\mathbf{q}'|^2+|\mathbf{k}|^2+2x|\mathbf{q}'||\mathbf{k}|)^{1/2}\\
&=M-k_0-q_0',
\end{aligned}
\tag{4.2}
$$

where x is the cosine of the angle between $\mathbf{q}'$ and $\mathbf{k}$.

Integrating with respect to the q_0' variable yields a factor, as in (3.7), of

$$|f'(q_0')|^{-1}=\left|\frac{q_0|\mathbf{q}'|}{|\mathbf{q}'|(q_0'+q_0)+x|\mathbf{k}|q_0'}\right|.\tag{4.3}$$

q_0' must be such that the quantity in the δ function of (4.1) is zero. The solution of this is

$$
\begin{aligned}
y_0\equiv q_0'&=\frac{(M-k_0)(M^2+M'^2+m^2-2k_0M)-x|\mathbf{k}|(W)^{1/2}}{2[(M-k_0)^2-x^2(k_0^2-M'^2)]},\\
W&=(M^2+M'^2-m^2-2k_0M)^2-4m^2|\mathbf{k}|^2(1-x^2).
\end{aligned}
\tag{4.4}
$$

In obtaining (4.4) there is an ambiguity of sign in front of the square root. The listed $(-)$ sign can be seen to be correct by taking the limit

$$q_0'\simeq|\mathbf{q}'|\gg|\mathbf{k}|\gg m\simeq 0\tag{4.5}$$

and expanding (4.2) and (4.4) in powers of k.

We also need the quantity in the square root of (4.4) to be positive. The solution of this condition is

$$W=(k_0-K_+)(k_0-K_-)\geq 0,\tag{4.6}$$

$$
\begin{aligned}
K_\pm&=\frac{M(M^2+M'^2-m^2)\pm m(K_0)^{1/2}}{2[M^2-m^2(1-x^2)]},\\
K_0&=(1-x^2)[(M^2+M'^2+m^2)^2-4(m^2M^2+M^2M'^2+m^2M'^2x^2)].
\end{aligned}
\tag{4.7}
$$

The meaning of (4.6) is that the k_0 energy domain is restricted for certain values of x—specifically, between K_- and K_+. This introduces the factor

$$\Theta(k_0, x) = 1-\theta(k_0-K_-)+\theta(k_0-K_+). \tag{4.8}$$

Combining these results and using the kinematic limits for the k_0 integration, we have for the angular correlation $Y(x)$

$$\Gamma = \frac{G^2}{(2\pi)^3 M\hbar}\int_{-1}^{1} dx\, Y(x), \tag{4.9}$$

$$Y(x) = \int_{M'}^{(M^2+M'^2-m^2)/2M} \frac{dk_0\,|\,\mathbf{k}\,|\,|\,\mathbf{q}'\,|^2}{|\,|\,\mathbf{q}'\,|\,(M-k_0)+xq_0'\,|\,\mathbf{k}\,|\,|}\,\Theta(k_0, x)\,F^{\mathrm{II}}(Q^2, -Mq_0'),$$

$$q_0' \equiv y_0. \tag{4.10}$$

Remembering that

$$Q^2 = -M^2-M'^2+2Mk_0,$$

we can write

$$Y(x) = (2M)^{-1}\int_{-(M-M')^2}^{-m^2} \frac{dQ^2\,|\,\mathbf{k}\,|\,|\,\mathbf{q}'\,|^2}{|\,|\,\mathbf{q}'\,|\,(M-k_0)+xq_0'\,|\,\mathbf{k}\,|\,|}\,\Phi(Q^2, x)\,F^{\mathrm{II}}(Q^2, -Mq_0'), \tag{4.11}$$

$$\Phi(Q^2, x) = 1-\theta(Q^2-M^2-M'^2+2MK_-)+\theta(Q^2-M^2-M'^2+2MK_+). \tag{4.12}$$

5. DISCUSSION

The results obtained in the last two sections in principle allow a comparison between theory and experiment. However, at first glance the lines of equations seem too complex to be understood. Fortunately, this is not the case.

The first quantity sought is the decay width. From Sec. 3 we see that, given the masses of the particles involved in any particular decay mode, we can integrate the various terms of, say, the neutrino spectrum. Then by multiplying the results by the appropriate form factor constants for a given theoretical model, one could add the terms and have a predicted decay width.

In addition we wish to know which terms are significant for the spectra or angular correlation. To plot the curves of all the terms for all the decay modes would quickly get out of hand and leave one with an unintelligible mass of graphs. However, integrating the absolute value of the spectral terms would give a "feeling" for the term's possible importance in a given process. It is necessary to integrate the absolute value of the spectral terms because some terms might almost integrate to zero, (meaning that they gave negligible contribution to the decay width) but yet still be important in determining the shape of the spectra or angular correlation. (We see an example of this later.)

Indeed, some important terms integrate exactly to zero for the decay width, but are significant in determining the spectral shapes. An especially important example of this is the vector–axial vector interference term, $(\tilde{F}-\tilde{G})^2$, which comes from $(\tilde{F}, \tilde{G})$ cross terms with $(1+\gamma^5)$. As was shown in Sec. 3, this term is zero

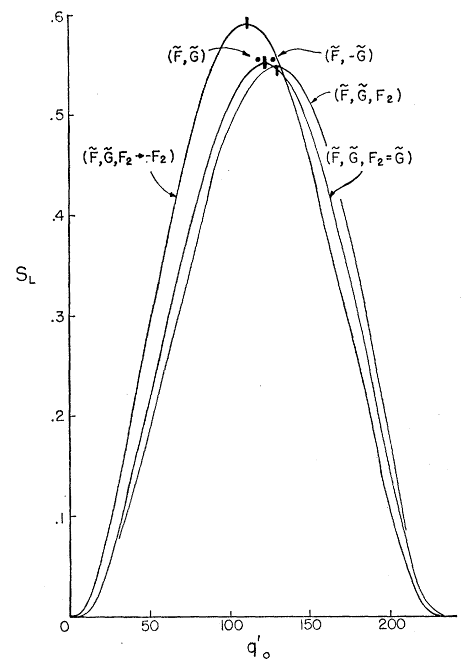

FIG. 2. This shows a graph of the lepton energy spectrum in absolute units of 10^5 (MeV·sec)$^{-1}$ as a function of the lepton energy in MeV for the process $\Sigma^- \rightarrow ne^- \bar{\nu}$ with vector, axial vector, and weak magnetism couplings given by Cabibbo theory [see Eq. (5.4)]. The vertical line indicates the spectrum maximum. The positions of the maxima for spectra with only vector and axial vector couplings, and for vector and negative axial vector couplings are also indicated. A second curve shows the spectrum that would exist if the weak magnetism form factor had the same value as the axial vector form factor. The maxima (123.5 MeV) would be in between the two maxima from the pure vector and axial vector couplings. A third curve shows the spectra if the weak magnetism form factor changes sign, $(F_2 \rightarrow -F_2)$.

Table I. This table gives information on the decay widths and spectra of the processes listing each column. For rows 1–18, the top one number the two are the same) gives the integral of the absolute value of the neutrino spectrum for that form factor term. The same in Eq. (2.4).

	(1) $n\rightarrow pe^-\bar{\nu}$	(2) $\Lambda\rightarrow pe^-\bar{\nu}$	(3) $\Lambda\rightarrow p\mu^-\bar{\nu}$	(4) $\Sigma^-\rightarrow ne^-\bar{\nu}$
(1) $\tilde{F}^2$	0.1889×10^{-3}	0.1514×10^{8}	0.2435×10^{7}	0.8997×10^{8}
(2) $\tilde{F}^2A$	0.3684×10^{-9}	0.2189×10^{6}	0.8450×10^{5}	0.2393×10^{7}
(3) $\tilde{F}^2A^2$	0.2029×10^{-15}	0.1231×10^{4}	0.7647×10^{3}	0.2472×10^{5}
(4) $\tilde{F}F_2$	-0.5453×10^{-8}	0.3544×10^{6}	0.4067×10^{5}	0.3971×10^{7}
	0.2650×10^{-6}	0.2998×10^{7}	0.2773×10^{6}	0.2407×10^{8}
(5) $\tilde{F}F_2A$	-0.6277×10^{-14}	0.3978×10^{4}	0.6961×10^{3}	0.8180×10^{5}
	0.3157×10^{-12}	0.3782×10^{5}	0.5209×10^{4}	0.5567×10^{6}
(6) $\tilde{F}F_3$	$\mp0.1344\times10^{-6}$	$\mp0.9911\times10^{2}$	$\mp0.3730\times10^{6}$	$\mp0.3747\times10^{3}$
(7) $\tilde{F}F_3A$	$\mp0.1539\times10^{-12}$	$\mp0.1000\times10^{1}$	$\mp0.6906\times10^{4}$	$\mp0.6927\times10^{1}$
(8) F_2^2	-0.1126×10^{-7}	0.2178×10^{6}	0.2502×10^{5}	0.2370×10^{7}
	0.2660×10^{-6}	0.2751×10^{7}	0.2565×10^{6}	0.2135×10^{8}
(9) F_3^2	0.5589×10^{-10}	0.3146×10^{1}	0.2172×10^{5}	0.1608×10^{2}
(10) $\tilde{G}^2$	0.5668×10^{-3}	0.4502×10^{8}	0.7270×10^{7}	0.2655×10^{9}
(11) $\tilde{G}^2B$	0.1325×10^{-8}	0.1082×10^{7}	0.2703×10^{6}	0.1169×10^{8}
(12) $\tilde{G}^2B^2$	0.8550×10^{-15}	0.8494×10^{4}	0.2607×10^{4}	0.1684×10^{6}
(13) $\tilde{G}G_2$	$\mp0.1175\times10^{-5}$	$\mp0.9532\times10^{7}$	$\mp0.1913\times10^{7}$	$\mp0.7607\times10^{8}$
(14) $\tilde{G}G_2B$	$\mp0.1472\times10^{-11}$	$\mp0.1375\times10^{6}$	$\mp0.3649\times10^{5}$	$\mp0.2012\times10^{7}$
(15) $\tilde{G}G_3$	0.3667×10^{-10}	0.5151×10^{1}	0.8647×10^{4}	0.2731×10^{2}
(16) $\tilde{G}G_3B$	0.3528×10^{-16}	0.3718×10^{-1}	0.1436×10^{3}	0.3616
	0.3534×10^{-16}	0.3742×10^{-1}		0.3632
(17) G_2^2	0.7133×10^{-9}	0.6492×10^{6}	0.1396×10^{6}	0.7018×10^{7}
(18) G_3^2	0.8231×10^{-17}	0.1010×10^{-1}	0.3901×10^{2}	0.1013
(19) $(\tilde{f}-\tilde{g})^2$	0.6600×10^{-7}	0.8101×10^{6}	0.7370×10^{5}	0.6679×10^{7}
(20) $(\tilde{f}-\tilde{g})(\tilde{F}A-\tilde{G}B)$	0.1573×10^{-12}	0.2046×10^{5}	0.2781×10^{4}	0.3094×10^{6}
(21) $(\tilde{F}A-\tilde{G}B)^2$	0.1006×10^{-18}	0.1550×10^{3}	0.2703×10^{2}	0.4302×10^{4}
(22) F_2F_3	0.2000×10^{-13}	0.1700	0.6611×10^{3}	0.1216×10^{1}
(23) G_2G_3	0.2035×10^{-13}	0.1700	0.6611×10^{3}	0.1216×10^{1}

for the final baryon spectra as all leptonic variables have been integrated out. In the static limit ($|\mathbf{k}|=m^2=0$) the interference term is also zero for the neutrino and lepton spectra [see (2.17)]. However, although the static approximation is justified for beta decay,[15] it is not universally valid for hadronic semileptonic decays. Thus, for large enough Q^2 the interference term can be non-negligible and be seen,[16] and should be taken into account.

The results of the type of numerical calculations described above for seventeen semileptonic decay

[15] J. D. Jackson, *Elementary Particle Physics and Field Theory, 1962 Brandeis Lectures*, K. W. Ford, Ed. (W. A. Benjamin, Inc., New York, 1963), Vol. I, p. 263.

[16] The possibility of the lepton spectrum having an interference term was brought to the attention of the author by P. Franzini and J. Cole. P. Franzini and N. Yeh had performed calculations for the lepton spectrum which showed that the terms existed, contrary to what is often believed. J. Cole had performed a Monte-Carlo integration for the process $\Sigma^-\rightarrow n+e^-+\bar{\nu}$, with imaginary data taken from a Dalitz plot. His result gave a shift of a few MeV in the lepton spectrum when one changed the sign of $\tilde{F}/\tilde{G}$.

number times the form factors labeling the row is the contribution to the decay width in sec^{-1}. The bottom number (if there is only
is true of rows 19–23, which integrate to zero for the decay width contribution. The value of the weak coupling constant used is given

(5) $\Sigma^- \to n\mu^-\bar{\nu}$	(6) $\Sigma^- \to \Lambda e^-\bar{\nu}$	(7) $\Xi^- \to \Lambda e^-\bar{\nu}$	(8) $\Xi^- \to \Lambda\mu^-\bar{\nu}$	(9) $\Xi^0 \to \Sigma^+ e^-\bar{\nu}$	(10) $\Xi^0 \to \Sigma^+\mu^-\bar{\nu}$
0.3985×10^{8}	0.3655×10^{6}	0.3190×10^{8}	0.8615×10^{7}	0.2938×10^{7}	0.2387×10^{5}
0.1946×10^{7}	0.9767×10^{3}	0.4423×10^{6}	0.2571×10^{6}	0.1525×10^{5}	0.3848×10^{3}
0.2660×10^{5}	0.1015×10^{1}	0.2383×10^{4}	0.2053×10^{4}	0.3075×10^{2}	0.1558×10^{1}
0.1566×10^{7}	0.1515×10^{4}	0.7148×10^{6}	0.1547×10^{6}	0.2396×10^{5}	0.6429×10^{2}
0.8373×10^{7}	0.3122×10^{5}	0.6187×10^{7}	0.1127×10^{7}	0.3495×10^{6}	0.6728×10^{3}
0.4101×10^{5}	0.3146×10^{1}	0.7692×10^{4}	0.2340×10^{4}	0.9659×10^{2}	0.5041
0.2467×10^{6}	0.7293×10^{2}	0.7482×10^{5}	0.1898×10^{5}	0.1585×10^{4}	0.5481×10^{1}
$\mp0.4637\times10^{7}$	$\mp0.4858\times10^{1}$	$\mp0.1522\times10^{3}$	$\mp0.1036\times10^{7}$	$\mp0.2323\times10^{2}$	$\mp0.3609\times10^{4}$
$\mp0.1318\times10^{6}$	$\mp0.9086\times10^{-2}$	$\mp0.1472\times10^{1}$	$\mp0.1705\times10^{5}$	$\mp0.8430\times10^{-1}$	$\mp0.2934\times10^{2}$
0.9353×10^{6}	0.9752×10^{3}	0.4402×10^{6}	0.9533×10^{5}	0.1522×10^{5}	0.4085×10^{2}
0.7471×10^{7}	0.3013×10^{5}	0.5689×10^{7}	0.1043×10^{7}	0.3325×10^{6}	0.6452×10^{3}
0.3059×10^{6}	0.6646×10^{-1}	0.4729×10^{1}	0.5477×10^{5}	0.4425	0.1540×10^{3}
0.1181×10^{9}	0.1095×10^{7}	0.9492×10^{8}	0.2571×10^{8}	0.8790×10^{7}	0.7155×10^{5}
0.6830×10^{7}	0.4873×10^{4}	0.2186×10^{7}	0.8533×10^{6}	0.7592×10^{5}	0.1164×10^{4}
0.1084×10^{6}	0.7084×10^{1}	0.1646×10^{5}	0.7501×10^{4}	0.2142×10^{3}	0.4752×10^{1}
$\mp0.3848\times10^{8}$	$\mp0.9979\times10^{5}$	$\mp0.1968\times10^{8}$	$\mp0.6367\times10^{7}$	$\mp0.1116\times10^{7}$	$\mp0.1270\times10^{5}$
$\mp0.1188\times10^{7}$	$\mp0.2665\times10^{3}$	$\mp0.2721\times10^{6}$	$\mp0.1100\times10^{6}$	$\mp0.5784\times10^{4}$	$\mp0.1036\times10^{3}$
0.2185×10^{6}	0.1031	0.7728×10^{1}	0.2812×10^{5}	0.6976	0.1880×10^{2}
0.5245×10^{4}	0.1379×10^{-3}	0.5347×10^{-1}	0.4048×10^{3}	0.1809×10^{-2}	0.1469
	0.1393×10^{-3}	0.5382×10^{-1}		0.1826×10^{-2}	
0.3682×10^{7}	0.2924×10^{4}	0.1312×10^{7}	0.4481×10^{6}	0.4555×10^{5}	0.5843×10^{3}
0.1470×10^{4}	0.3570×10^{-4}	0.1450×10^{-1}	0.1097×10^{3}	0.4750×10^{-3}	0.3857×10^{-1}
0.2292×10^{7}	0.8073×10^{4}	0.1669×10^{7}	0.3003×10^{6}	0.9158×10^{5}	0.1737×10^{3}
0.1360×10^{6}	0.3773×10^{2}	0.4041×10^{5}	0.1016×10^{5}	0.8308×10^{3}	0.2833×10^{1}
0.2157×10^{4}	0.5290×10^{-1}	0.2935×10^{3}	0.8962×10^{2}	0.2261×10^{1}	0.1161×10^{-1}
0.1784×10^{5}	0.1470×10^{-2}	0.2497	0.1921×10^{4}	0.1384×10^{-1}	0.1122×10^{1}
0.1784×10^{5}	0.1470×10^{-2}	0.2497	0.1921×10^{4}	0.1384×10^{-1}	0.1122×10^{1}

processes are listed in Table I. The numbers were obtained by integrating the neutrino spectrum (3.32), using the formula for the decay width (3.26). Before integration the spectra were converted to the F and G form factors by means of (2.8)–(2.10), and the similar form factor terms were combined. The weak coupling constant of (2.4) was used, the integrals were done to one part in 10^4, and the results are in units of sec^{-1}.

For rows 1–18, there are two numbers in each column (one number is printed if the two numbers are the same). The top number is the decay width figure. This figure times the value of the form factors that label the row will give that term's contribution to the decay width.

Take, for example, beta decay using only the form factors $\tilde{F}=1$ and $\tilde{G}=1.18$. From column 1 we then have

$$\begin{aligned} \Gamma &= (1.0)^2(0.1889\ 10^{-3})+(1.18)^2(0.5668\ 10^{-3}), \\ &= (0.98)\ 10^{-3}\ \text{sec}^{-1}, \\ \tau &= (1.02)\ 10^{3}\ \text{sec}. \end{aligned} \tag{5.1}$$

TABLE I. (*Continued*)

	(11) $\Sigma^+\to\Lambda e^+\bar\nu$	(12) $\Xi^-\to\Sigma^0 e^-\bar\nu$	(13) $\Xi^-\to\Sigma^0\mu^-\bar\nu$	(14) $\Xi^-\to\Xi^0 e^-\bar\nu$	(15) $\Sigma^0\to\Sigma^+ e^-\bar\nu$	(16) $\Sigma^-\to\Sigma^0 e^-\bar\nu$	(17) $\Sigma^0\to p e^-\bar\nu$
(1) $\tilde F^2$	0.2211×10^{6}	0.3350×10^{7}	0.4227×10^{5}	0.1233×10^{1}	0.2666×10^{-1}	0.2906	0.8416×10^{8}
(2) $\tilde F^2A$	0.4879×10^{3}	0.1816×10^{5}	0.6984×10^{3}	0.1789×10^{-4}	0.1214×10^{-6}	0.3002×10^{-5}	0.2194×10^{-7}
(3) $\tilde F^2A^2$	0.4185	0.3827×10^{2}	0.2903×10^{1}	0.9702×10^{-10}	0.1889×10^{-12}	0.1131×10^{-10}	0.2222×10^{5}
(4) $\tilde F F_2$	0.7542×10^{3} 0.1716×10^{5}	0.2856×10^{5} 0.4072×10^{6}	0.1341×10^{3} 0.1386×10^{4}	0.1795×10^{-4} 0.7532×10^{-2}	-0.196×10^{-6} 0.8346×10^{-4}	0.1572×10^{-5} 0.1467×10^{-2}	0.3638×10^{7} 0.2230×10^{-8}
(5) $\tilde F F_2A$	0.1294×10^{1} 0.3311×10^{2}	0.1203×10^{3} 0.1929×10^{4}	0.1075×10^{1} 0.1161×10^{2}	0.2005×10^{-9} 0.9227×10^{-7}	-0.6096×10^{-12} 0.2930×10^{-9}	0.1268×10^{-10} 0.1250×10^{-7}	0.7346×10^{5} 0.5055×10^{6}
(6) $\tilde F F_3$	$\mp0.3277\times10^{1}$	$\mp0.2565\times10^{2}$	$\mp0.6296\times10^{4}$	$\mp0.1771\times10^{-3}$	$\mp0.8118\times10^{-5}$	$\mp0.5957\times10^{-4}$	$\mp0.3570\times10^{8}$
(7) $\tilde F F_3A$	$\mp0.5063\times10^{-2}$	$\mp0.9725\times10^{-1}$	$\mp0.5260\times10^{2}$	$\mp0.1814\times10^{-8}$	$\mp0.2528\times10^{-10}$	$\mp0.4322\times10^{-9}$	$\mp0.6471\times10^{1}$
(8) F_2^2	0.4871×10^{3} 0.1662×10^{5}	0.1813×10^{5} 0.3870×10^{6}	0.8511×10^{2} 0.1328×10^{4}	0.187×10^{-5} 0.7516×10^{-2}	-0.5923×10^{-6} 0.8345×10^{-4}	-0.2368×10^{-5} 0.1465×10^{-2}	0.2173×10^{7} 0.1980×10^{8}
(9) F_3^2	0.4075×10^{-1}	0.4994	0.2701×10^{3}	0.1844×10^{-6}	0.4859×10^{-8}	0.5292×10^{-7}	0.1517×10^{2}
(10) $\tilde G^2$	0.6625×10^{6}	0.1002×10^{8}	0.1267×10^{6}	0.3698×10^{1}	0.7997×10^{-1}	0.8718	0.2484×10^{9}
(11) $\tilde G^2B$	0.2435×10^{4}	0.9041×10^{5}	0.2118×10^{4}	0.8667×10^{-4}	0.5426×10^{-6}	0.1425×10^{-4}	0.1072×10^{8}
(12) $\tilde G^2B^2$	0.2923×10^{1}	0.2666×10^{3}	0.8901×10^{1}	0.6537×10^{-9}	0.1138×10^{-11}	0.7417×10^{-10}	9.1514×10^{6}
(13) $\tilde G G_2$	$\mp0.5487\times10^{5}$	$\mp0.1300\times10^{7}$	$\mp0.2274\times10^{5}$	$\mp0.2443\times10^{-1}$	$\mp0.2835\times10^{-3}$	$\mp0.4806\times10^{-2}$	$\mp0.7048\times10^{8}$
(14) $\tilde G G_2B$	$\mp0.1210\times10^{3}$	$\mp0.7041\times10^{4}$	$\mp0.1908\times10^{3}$	$\mp0.3403\times10^{-6}$	$\mp0.1113\times10^{-8}$	$\mp0.4636\times10^{-7}$	$\mp0.1828\times10^{7}$
(15) $\tilde G G_3$	0.6302×10^{-1}	0.7884	0.3834×10^{2}	0.2519×10^{-6}	0.5596×10^{-8}	0.6885×10^{-7}	0.2573×10^{2}
(16) $\tilde G G_3B$	0.6960×10^{-4} 0.7031×10^{4}	0.2136×10^{-2} 0.2155×10^{-2}	0.3062	0.1895×10^{-11} 0.1917×10^{-11}	0.1333×10^{-13} 0.1346×10^{-13}	0.3713×10^{-12} 0.3755×10^{-12}	0.3340 0.3355
(17) G_2^2	0.1461×10^{4}	0.5425×10^{5}	0.1065×10^{4}	0.5157×10^{-4}	0.3155×10^{-6}	0.8431×10^{-5}	0.6438×10^{7}
(18) G_3^2	0.1796×10^{-4}	0.5614×10^{-3}	0.8046×10^{-1}	0.4737×10^{-12}	0.3315×10^{-14}	0.9258×10^{-13}	0.9346×10^{-1}
(19) $(\tilde f-\tilde g)^2$	0.4425×10^{4}	0.1068×10^{6}	0.3583×10^{3}	0.1887×10^{-2}	0.2087×10^{-4}	0.3672×10^{-3}	0.6180×10^{7}
(20) $(\tilde f-\tilde g)(\tilde F A-\tilde G B)$	0.1708×10^{2}	0.1013×10^{4}	0.6011×10^{1}	0.4624×10^{-7}	0.1465×10^{-9}	0.6255×10^{-8}	0.2807×10^{6}
(21) $(\tilde F A-\tilde G B)^2$	0.1977×10^{-1}	0.2879×10^{1}	0.2536×10^{-1}	0.3371×10^{-12}	0.2990×10^{-15}	0.8153×10^{-13}	0.3826×10^{4}
(22) F_2F_3	0.8167×10^{-3}	0.1598×10^{-1}	0.2292×10^{1}	0.2835×10^{-9}	0.3820×10^{-11}	0.6720×10^{-10}	0.1135×10^{1}
(23) G_2G_3	0.8167×10^{-3}	0.1598×10^{-1}	0.2292×10^{1}	0.2837×10^{-9}	0.3824×10^{-11}	0.6703×10^{-10}	0.1135×10^{1}

TABLE II. Using the form factors of Eq. (5.4) with the labeled sign changes, in the table we give the coordinates of the lepton energy spectrum maxima for the decay $\Sigma^- \to n e^- \bar{\nu}$.

	$\tilde{F}, \tilde{G}$	$\tilde{F}, -\tilde{G}$	$\tilde{F}, \tilde{G}, F_2$	$\tilde{F}, \tilde{G}, -F_2$	$\tilde{F}, -\tilde{G}, F_2$	$\tilde{F}, -\tilde{G}, -F_2$
Energy of spectrum maxima (MeV)	120	128	$129\frac{1}{4}$	$111\frac{1}{2}$	$118\frac{1}{4}$	137
Flux of spectrum maxima $[10^5(\text{MeV}\cdot\text{sec})^{-1}]$	0.556	0.555	0.550	0.592	0.551	0.591

The second numbers of each column for rows 1–18 are the integrals of the absolute values of the spectral terms. Rows 19–23 are terms that integrate to zero and so just the absolute value integrals of these terms are given. (These rows are partially labeled by $\tilde{f}$ and $\tilde{g}$ to save unnecessary, essentially equivalent lines.) Note the almost complete equality of rows 22 and 23.

An example of the worth of integrating the absolute value terms is seen in column 4, the process[17]

$$\Sigma^- \to n + e^- + \bar{\nu}. \qquad (5.2)$$

Now weak magnetism is known to exist. We see from row 8, column 4 that, although the weak magnetism terms are not important for this decya width, the large Q^2 of this process should make them important for the spectra.

F_2 is given by[18–20]

$$F_2 = +V(\mu_p - \mu_n)(M/2M_p) = +V(3.70)(M/2M_p),$$
$$= +2.36V, \qquad (5.3)$$

where μ_n and μ_p are the magnetic moments of the neutron and proton and V is a constant. If one accepts Cabibbo theory, then for process (5.2) we have[6,20]

$$V = 0.116,$$
$$F_2 = +0.274,$$
$$\tilde{F} = -0.211,$$
$$\tilde{G} = 0.103. \qquad (5.4)$$

In Fig. 2 we have plotted the electron spectra for process (5.2) using the form factors of (5.4). The position of the maxima (129.25 MeV) is indicated by a vertical line. The positions of the maxima of the curves obtained by using $(\tilde{F}, \tilde{G})$ or $(\tilde{F}, -\tilde{G})$ are indicated, as are the curves for $F_2 = \tilde{G} = 0.103$, and $F_2 \to -F_2$.

The results are significant. If just vector and axial vector couplings exist (and we can ignore the Q^2 dependence of the form factors), then a determination of the electron spectrum to 8 MeV would allow an unambiguous test of Cabibbo theory. The sign of $\tilde{F}/\tilde{G}$ could be determined in addition to the magnitude. Unfortunately, we see that if weak magnetism exists here, then a test requires greater accuracy. Switching form factor signs moves the spectral maxima (see Table II); there is even more uncertainty if the magnitude of F_2 changes (see Fig. 2).

The above examples should show how a judicious combination of Table I and the results of Secs. 3 and 4 can facilitate a phenomenological understanding of experimental results and their relationship to theoretical models.

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APPENDIX A

This appendix gives the conventions used in the paper. Two dotted 4-vectors are written as $a\cdot b$, with the space part of a 4-vector in boldface $\mathbf{a}$. The Majorana gauge is used for the γ matrices, meaning that they have the simple properties

$$\{\gamma^\mu, \gamma^\lambda\} = \delta^{\mu\lambda},$$
$$\sigma^{\mu\lambda} \equiv (\gamma^\mu\gamma^\lambda - \gamma^\lambda\gamma^\mu)/2i, \qquad u, \lambda = 1, 2, 3, 4,$$
$$\gamma^{\lambda\dagger} = \gamma^\lambda,$$
$$\gamma^5 = \gamma^1\gamma^2\gamma^3\gamma^4 = \gamma^{5\dagger}. \qquad (A1)$$

[17] An investigation of this process is being done by a Columbia–Rutgers–Stony Brook collaboration. Preliminary results were reported earlier. See C. Baltay *et al.*, Bull. Am. Phys. Soc. **12**, 568 (1967).
[18] C. S. Wu, Rev. Mod. Phys. **36**, 618 (1964).
[19] H. F. Schopper, *Weak Interactions and Nuclear Beta Decay* (North-Holland Publ. Co., Amsterdam, 1966).
[20] In Ref. 6, the notation of N. Brene, B. Hellesen, and M. Roos [Phys. Letters **11**, 344 (1964)] has been used. The q in that paper is equal to $-Q$ in our notation, so an extra minus sign is needed in the weak magnetism term.

The 4-vectors have an imaginary fourth component, so that

$$p_4=-p_4^*\equiv ip_0=i(|\mathbf{p}|^2+M^2)^{1/2}. \tag{A2}$$

The Dirac equation is

$$(ip\!\!\!/+M)u(\mathbf{p})=0, \tag{A3}$$

$$\bar{u}(\mathbf{p})(ip\!\!\!/+M)=u^\dagger(\mathbf{p})\gamma^4(ip\!\!\!/+M)=0, \tag{A4}$$

where

$$p\!\!\!/=p^\mu\gamma^\mu. \tag{A5}$$

The spin summations for particles and antiparticles are

$$\sum_{i=1}^{2} u_\beta{}^i(\mathbf{p})\bar{u}_\alpha{}^i(\mathbf{p})=(2p_0)^{-1}(-ip\!\!\!/+M)_{\beta\alpha}, \tag{A6}$$

$$\sum_{i=1}^{2} v_\beta{}^i(\mathbf{p})\bar{v}_\alpha{}^i(\mathbf{p})=(2p_0)^{-1}(-ip\!\!\!/-M)_{\beta\alpha}. \tag{A7}$$

APPENDIX B

For those who want to follow the trace calculation in detail, an outline is given here. By taking the traces in (2.13), $F^{\rm I}$ can be written as

$$32F^{\rm I}(Q^2,\,p\cdot q)=(T_1{}^{\mu\nu}+T_2{}^{\mu\nu}+T_3{}^{\mu\nu}+T_4{}^{\mu\nu})T_0{}^{\mu\nu}, \tag{B1}$$

where the T's in (B1) are given by

$$T_1{}^{\mu\lambda}=-4[(f_1{}^2+g_1{}^2)(k^\mu p^\lambda+k^\lambda p^\mu-k\cdot p\delta^{\mu\lambda})-2f_1g_1\epsilon^{\mu\lambda\pi\sigma}k^\pi p^\sigma]+[4(k\cdot p)/M^2][(f_2K^\mu+f_3Q^\mu)(f_2K^\lambda+f_3Q^\lambda)+(g_2K^\mu+g_3Q^\mu)(g_2K^\lambda+g_3Q^\lambda)], \tag{B2}$$

$$T_2{}^{\mu\lambda}=4MM'[\delta^{\mu\lambda}(f_1{}^2-g_1{}^2)-(1/M^2)(f_2K^\mu+f_3Q^\mu)(f_2K^\lambda+f_3Q^\lambda)+(1/M^2)(g_2K^\mu+g_3Q^\mu)(g_2K^\lambda+g_3Q^\lambda)], \tag{B3}$$

$$T_3{}^{\mu\lambda}=(4M'/M)f_1[p^\mu(f_2K^\lambda+f_3Q^\lambda)+p^\lambda(f_2K^\mu+f_3Q^\mu)]+(4M'/M)g_1[p^\mu(g_2K^\lambda+g_3Q^\lambda)+p^\lambda(g_2K^\mu+g_3Q^\mu)], \tag{B4}$$

$$T_4{}^{\mu\lambda}=4f_1[k^\mu(f_2K^\lambda+f_3Q^\lambda)+k^\lambda(f_2K^\mu+f_3Q^\mu)]-4g_1[k^\mu(g_2K^\lambda+g_3Q^\lambda)+k^\lambda(g_2K^\mu+g_3Q^\mu)], \tag{B5}$$

$$T_0{}^{\mu\lambda}=-8[q'^\mu q^\lambda+q'^\lambda q^\mu-\delta^{\mu\lambda}(q'\cdot q)-\epsilon^{\mu\lambda\alpha\beta}q'^\alpha q^\beta]. \tag{B6}$$

T_0 is the lepton trace. The baryon spin sums give factors of $(-ik\!\!\!/+M')$ and $(-ip\!\!\!/+M)$. T_1, T_2, T_3, and T_4 come from the traces involving $k\!\!\!/p\!\!\!/$, MM', $M'p\!\!\!/$, and $k\!\!\!/M$.

In the contraction in (B1), many terms cancel because of the complete antisymmetry of $\epsilon^{\mu\lambda\alpha\beta}$. By then using

$$\sum_{\mu\lambda}\epsilon^{\mu\lambda\alpha\beta}\epsilon^{\mu\lambda\pi\sigma}=2(\delta^{\alpha\pi}\delta^{\beta\sigma}-\delta^{\alpha\sigma}\delta^{\beta\pi}), \tag{B7}$$

the contraction can be completed. The result is in the form of dot products of p, k, q', q, K, and Q. By using Eqs. (2.14)–(2.16), all the dot products can be expressed in terms of Q^2, $(p\cdot q)$, and the masses of the particles. When this is done, equation (2.17) for $F^{\rm I}(Q^2,\,p\cdot q)$ is obtained.