

REVIEWS

OF

MODERN PHYSICS

Optical Dispersion

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I. EARLY WORK

THE earliest observers of the phenomena of dispersion on record are Marcus in 1648 and Grimaldi in 1665. These were followed by Newton in 1666 who studied systematically the splitting of white light into colors employing a glass prism. He interpreted his results, however, on his corpuscular theory, which demanded that the corpuscles speed up as they enter a medium. Later it was shown that light in a medium travels more slowly than in a vacuum, thus invalidating his explanation. It was found that the ratio c/v (where c is the velocity of light in a vacuum and v is its velocity in the medium) could be set equal to a constant n , the *refractive index*, characteristic of the medium, and a function of the frequency.

For a considerable period nothing of importance was done in this field; then Fraunhofer in 1814, found that the spectrum of sunlight, when a slit was placed before the prism, was crossed by the dark lines which now bear his name. This interesting and at the time inexplicable discovery led to a great deal of investigation. Le Roux in 1862, noted that dispersion shown by a prism of iodine vapor was in the neighborhood of absorption lines different from the dispersion shown by a glass prism. This anomalous phenomenon was soon found to occur in other substances: Christiansen (in 1870) found it in aqueous solution of the dye fuchsin. Kundt in 1871 devised the method of

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crossed prisms for studying anomalous dispersion. This consisted in arranging the base of the prism of the substance to be studied at right angles to the base of the analyzing prism in the spectroscope. The procedure yields directly the familiar anomalous dispersion curves, as in Fig. 1, which will be discussed in detail later. This method was used by many investigators. In particular Becquerel,¹ Julius² and Wood³ studied the anomalous dispersion in sodium vapor in the neighborhood of the *D* lines.

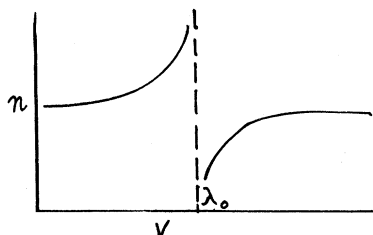


Fig. 1. Anomalous dispersion.

II. FIRST THEORETICAL TREATMENTS

Soon after Fraunhofer's work, Cauchy in 1836 deduced an equation giving the variation of refractive index with wave-length on the *elastic solid* theory of the ether. The restriction governing the deduction was that the wave-length of light was large compared with the size of and distance between the particles in the dispersing medium. He deduced the equation:

$$n = a + b/\lambda^2 + d/\lambda^4 + f/\lambda^6 + \dots \quad (1)$$

which gives the refractive index n in terms of the wave-length and four or more constants, $a, b, d, f \dots$ slowly increasing as the wave-length decreases and having no discontinuities, as in Fig. 2. This expression is found to give correct values in the case of *normal* dispersion, i.e., the dispersion in a spectral region far from any absorption lines. It has been shown, for instance, to give correct values for the refractive index of hydrogen in the visible.

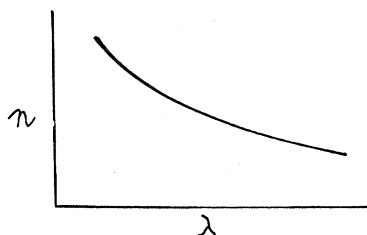


Fig. 2. Normal dispersion.

The discovery of anomalous dispersion put the matter in a different light. Eq. (1) is obviously inadequate to account for a sudden rise in the refractive index near absorption lines, as in Fig. 1. It was obvious that the sudden up-

¹ H. Becquerel, *Comptes Rendus* **127**, 899 (1898).

² W. H. Julius, *Phys. Zeits.* **2**, 349 (1901).

³ R. W. Wood, *Phil. Mag.* **3**, 128 (1902); **6**, 96 (1903); **8**, 293 (1904).

swing of the refractive index curve was due to some sort of resonance, and closely connected with the absorption line itself. It was assumed that the molecules or atoms possess natural periods of vibration, and that these are set into vibration by the incident light wave. Maxwell⁴ was the first to suggest this idea, but did not investigate the matter in detail. Sellmeier⁵ independently deduced the equation:

$$n^2 = 1 + a\lambda^2/(\lambda^2 - \lambda_0^2) \quad (2)$$

which represents the simple case of anomalous dispersion associated with an absorption line of wave-length λ_0 . Since most absorption spectra contain many absorption lines, the equation becomes

$$n^2 = 1 + \sum a\lambda^2/(\lambda^2 - \lambda_0^2) \quad (3)$$

where the summation is taken over all the absorption lines. This equation gives correctly the form of the refractive index curves not too near resonance; at resonance ($\lambda = \lambda_0$) the formula yields an infinite refractive index. Ketteler⁶ and Helmholtz⁷ pointed out that if a *frictional* term were introduced into the equation, the refractive index would remain finite at all points. They gave the expression

$$n^2 = 1 + \sum a\lambda^2/(\lambda^2 - \lambda_0^2 + b^2\lambda^2) \quad (4)$$

where owing to the new *damping* term in the denominator, the refractive index will always remain finite in value and not experience a discontinuity at λ_0 .

On the experimental side, it was found that this equation represented the observations well. In particular Wood⁸ working on cyanine and nitrosodimethyl aniline found this expression to give correct results, and later checked it in a series of experiments on the anomalous dispersion of sodium vapor in the neighborhood of the *D* lines.

III. THE CLASSICAL THEORY OF DISPERSION

Eq. (4) above was soon found not to be sufficiently general, nor was its deduction rigorous in the light of the more developed electron theory of matter. Drude and Voigt led the way to a new deduction, followed by Lorentz who finally put the dispersion equations into the form which now bears his name. We will consider this deduction in detail.

(1) The refractive index and the dielectric constant

It has been discovered by Maxwell that the refractive index n is connected with the dielectric constant K and the magnetic permeability μ of a medium by

$$n^2 = \mu K.$$

⁴ Clerk Maxwell, Math. Tripos. Exam., Cambridge Calendar, 1869.

⁵ W. Sellmeier, Pogg. Ann. **143**, 271 (1871); **145**, 399, 520 (1872); **147**, 386, 525 (1872).

⁶ E. Ketteler, *Theoretic Optik*, Braunschweig, 1885.

⁷ H. v. Helmholtz, Pogg. Ann. **154**, 582 (1875); Wied. Ann. **48**, 389, 723 (1893).

⁸ R. W. Wood, Phil. Mag. **46**, 380 (1898); **1**, 664 (1909).

Let \mathcal{E} , \mathcal{H} be the electric and magnetic intensity vectors measured respectively in electrostatic units and in gauss. Let \mathbf{D} be the displacement vector connected with \mathcal{E} by

$$\mathbf{D} = K\mathcal{E}/(4\pi) \quad (5)$$

K being the dielectric constant of the medium. For a medium containing no free charges Maxwell's equations are:

$$\text{curl } \mathcal{H} = (4\pi/c)\partial\mathbf{D}/\partial t \quad (6)$$

$$\text{curl } \mathcal{E} = - (1/c)\partial(\mu\mathcal{H})/\partial t. \quad (7)$$

Also

$$\text{div } (\mu\mathcal{H}) = 0, \text{div } \mathbf{D} = 0 \quad (7')$$

where c is the ratio of the electrostatic to the electromagnetic units of electric charge.

We suppose μ and K to be constant in the medium and not to vary with time. Taking the curl of (7) and substituting for curl \mathcal{H} the value given by (6) we have

$$\text{curl curl } \mathcal{E} = - (\mu K/c^2) \frac{\partial^2 \mathcal{E}}{\partial t^2}. \quad (8)$$

But by a well-known theorem in vector analysis

$$\text{curl curl } \mathcal{E} = \text{grad div } \mathcal{E} - \nabla^2 \mathcal{E}. \quad (9)$$

In the absence of free charges $\text{div } \mathcal{E} = 0$ so that

$$\nabla^2 \mathcal{E} - (\mu K/c^2) \partial^2 \mathcal{E} / \partial t^2 = 0. \quad (10)$$

Similarly it may be shown that

$$\nabla^2 \mathcal{H} - (\mu K/c^2) \partial^2 \mathcal{H} / \partial t^2 = 0. \quad (11)$$

The general solutions of (9) and (10) may be compounded of waves traveling in arbitrary directions with the velocity

$$v = c/(\mu K)^{1/2}. \quad (12)$$

If $\mu = K = 1$ as is the case for free space $v = c$. The constant c has the dimensions of velocity. We owe to *Maxwell* the hypothesis that the *electromagnetic waves* are of the same nature as *light waves* and that therefore the constant c is the velocity of light in empty space. A detailed discussion of the polarization properties of the electromagnetic waves is found in any comprehensive text book on electricity and magnetism.

TABLE I. *Maxwell's relation between index of refraction and dielectric constant.*

| Substance | K | n_{∞}^2 | Substance | K | n_{∞}^2 |
|-----------|----------|----------------|-----------------|----------|----------------|
| Hydrogen | 1.000264 | 1.000272 | Carbon dioxide | 1.000976 | 1.000880 |
| Oxygen | 1.000547 | 1.000533 | Carbon monoxide | 1.000693 | 1.000653 |
| Nitrogen | 1.000606 | 1.000581 | Methane | 1.000948 | 1.000863 |
| Air | 1.000572 | 1.000574 | Ethylene | 1.00138 | 1.00140 |

R. Minkowski, *Mueller-Pouillet's Lehrbuch der Physik II, Optik 2*, From St. Loria. Die Lichtbrechung in Gasen Braunschweig, 1914. Newer data found in Tausz and Hornung, *Zeits. f. tech. Physik* **8**, 351 (1927).

By the wave theory of light (Huygens wave-front construction) it follows that the refractive index is

$$n = c/v = (\mu K)^{1/2}. \quad (13)$$

In almost all but the ferromagnetic substances μ is very nearly 1. The relation (12) can be confirmed experimentally using infrared measurements for n and electrical measurements for K [see Table I].

It will be noted that the values are the same within experimental error over this frequency range. When n is measured with higher frequencies a departure sets in. The refractive index is thus not a constant but depends on the frequency. In order to find how it depends on the frequency we must form some model of the medium through which the light waves propagate.

(2) Polarization

The medium may be supposed to consist of neutral molecules each of which contains positive and negative charges. Under the action of the electric field of the light wave the charges are displaced with respect to each other (see Fig. 3) so that an electric doublet is formed. The strength of the doublet (electric moment) for each molecule will be denoted by p .

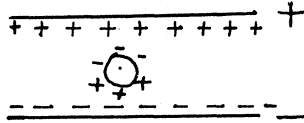


Fig. 3. Polarization.

The product of p and the number of molecules per unit volume N is called the polarization and will be denoted by P . Thus

$$P = Np. \quad (14)$$

In addition to the molecules there may be charges imbedded in the medium. Let their volume density be ρ . Then (Lorentz, Theory of Electrons pp. 132–136)

$$\text{div } \mathcal{E} = 4\pi(\rho - \text{div } P).$$

We define the electric displacement vector D by

$$4\pi D = \mathcal{E} + 4\pi P \quad (15)$$

so that

$$\text{div } D = \rho.$$

Maxwell's equations in a vacuum lead by the well-known method of averaging (Lorentz, Theory of Electrons) to Maxwell's equations in a medium [(6), (7) above] with $\mu = 1$. Just as for (13) we have now by (15)

$$n^2 = K = 4\pi D/\mathcal{E} = 1 + 4\pi P/\mathcal{E}. \quad (16)$$

The refractive index can be thus found from a knowledge of the polarization per unit volume produced by a unit electric field.

(3) Quasielastically bound electrons

It is customary to suppose that the electrons are bound to their position of equilibrium by elastic restoring forces obeying Hooke's law. The combined effect of the restoring force and the mass inertia of the electron makes the polarization per unit electric field highest when the frequency of the field is approximately equal to the frequency of free vibrations of the electrons. On account of frictional forces there is in general also a phase difference between the vibrations of \mathbf{P} and \mathcal{E} . The dielectric constant and the refractive index thus do not have a general meaning except for a constant frequency.

We discuss therefore the propagation of a plane monochromatic light wave of frequency $\omega/(2\pi)$ traveling along the x axis. The electric intensity is then

$$\mathcal{E} = \text{Re}[\mathcal{E}_0 e^{i\omega(t-x/v)}] \quad (17)$$

where $\text{Re}[\quad]$ denotes the real part of the quantity in $[\quad]$, \mathcal{E}_0 is a constant vector having for components $(\mathcal{E}_{0x}, \mathcal{E}_{0y}, \mathcal{E}_{0z})$ which may be in general complex and v is a constant which may also be complex. Similarly we set

$$\mathbf{D} = \text{Re}[\mathbf{D}_0 e^{i\omega(t-x/v)}], \quad \mathfrak{C} = \text{Re}[\mathfrak{C}_0 e^{i\omega(t-x/v)}]. \quad (17')$$

Substitution into (7') shows that

$$\mathfrak{C}_{0x} = \mathbf{D}_{0x} = \mathcal{E}_{0x} = 0$$

so that the waves are transverse. The elastic forces binding the electron to its position of equilibrium and the frictional forces determine the ratio

$$4\pi\mathbf{D}_0/\mathcal{E}_0 = K(\omega) = 1 + 4\pi\mathbf{P}_0/\mathcal{E}_0. \quad (18)$$

Substituting $4\pi\mathbf{D}_0 = K(\omega)\mathcal{E}_0$ into (6) we can satisfy both (6) and (7) by requiring that the equations should hold for

$$(1/(4\pi))K(\omega)\mathcal{E}_0 e^{i\omega(t-x/v)}, \quad \mathfrak{C}_0 e^{i\omega(t-x/v)}.$$

They will then hold automatically for \mathbf{D} and \mathfrak{C} . This substitution gives

$$c^2/v^2 = K(\omega) = \bar{n}^2(\omega). \quad (19)$$

The quantity $\bar{n}(\omega)$ is called the *complex refractive index* and $K(\omega)$ is called the *complex dielectric constant*. Their physical significance is given entirely by Eq. (17). Thus the real part of \bar{n} determines the phase of the wave as a function of x while the imaginary part of \bar{n} determines the amplitude. We write

$$\bar{n} = n(1 - i\kappa) \quad (20)$$

and suppose n, κ real. Then the exponential factor in (17) becomes

$$e^{-\omega n \kappa x/c} e^{i\omega(t-xn/c)}.$$

The phase velocity of the wave is c/n , the wave-length is $2\pi c/n\omega$. In one wave-length the amplitude of the wave decreases by the factor

$$e^{-2\pi\kappa}.$$

The energy of the wave decreases as

$$\exp [-(2\omega n\kappa/c)x].$$

Thus:

$$\begin{aligned} \alpha &= \text{absorption coefficient} = 2\omega n\kappa/c & (21) \\ \text{refractive index} &= n \\ \text{index of absorption (Lorentz, Th. of El. p. 159)} &= \omega n\kappa/c. \end{aligned}$$

The optical properties of the medium are thus determined completely as soon as we know the complex refractive index $K(\omega)$. This is obtained in the following way.

Consider an atom with a single electron of charge ϵ and mass m . Denote the coordinates of the electron with respect to its equilibrium position by

$$(\xi, \eta, \zeta).$$

The force on the electron in the y direction due to the light wave may be taken to be

$$\epsilon(\mathcal{E}_y + aP_y).$$

The first part $\epsilon\mathcal{E}_y$ is simply the force due to the electric intensity of the light wave. The second part represents the influence of neighboring atoms (Lorentz, Theory of Electrons, p. 138). For atoms arranged in a cubical lattice and some other symmetrical arrangements

$$a = 4\pi/3.$$

The force $(4\pi/3)P\epsilon$ is called the Lorentz-Lorenz force. The force of restitution we take to be $-m\omega_0^2\eta$ i.e., always opposite to the displacement η . ω_0 is a suitably chosen constant having the dimensions of frequency. We will identify $\omega_0/(2\pi)$ with the resonance frequency of the electron. The frictional force is usually taken to be proportional to the velocity and equal to $-m\omega' d\eta/dt$. Thus

$$m(d^2\eta/dt^2 + \omega'd\eta/dt + \omega_0^2\eta) = \epsilon(\mathcal{E}_y + aP_y). \quad (22)$$

Also by (14)

$$P_y = F\epsilon\eta \quad (22')$$

where F is the number of atoms per unit volume. For \mathcal{E} of the form (17) we can set

$$\eta = \text{Re}[\eta_0 e^{i\omega(t-x/v)}]; \quad P = \text{Re}[P_0 e^{i\omega(t-x/v)}]$$

so that

$$P_{0y} = F\epsilon\eta_0$$

and by (22), (18), (19)

$$\bar{n}^2(\omega) = K(\omega) = 1 + \frac{(4\pi F\epsilon^2/m)}{\omega_0^2 - \omega^2 + i\omega\omega' - Fa\epsilon^2/m} \quad (23)$$

If there are several kinds of particles with charges ϵ_s , masses m_s , natural frequency $\omega_{0s}/(2\pi)$ and if the number of each kind s per unit volume is F_s we obtain similarly

$$\bar{n}^2(\omega) = K(\omega) = 1 + \frac{\sum_s (4\pi F_s \epsilon_s^2 / m_s) / [\omega_{0s}^2 - \omega^2 + i\omega_s' \omega]}{1 - a \sum_s (F_s \epsilon_s^2 / m_s) / [\omega_{0s}^2 - \omega^2 + i\omega_s' \omega]}$$

which may also be written as

$$\frac{\bar{n}^2 - 1}{1 + a(\bar{n}^2 - 1)/4\pi} = \sum_s \frac{4\pi F_s \epsilon_s^2 / m_s}{\omega_{0s}^2 - \omega^2 + i\omega_s' \omega}. \quad (24)$$

If $a = 4\pi/3$ the left side of (24) is

$$\frac{3(\bar{n}^2 - 1)}{\bar{n}^2 + 2}$$

which is the Clausius-Mossotti form for the dependence of the dielectric constant on the density. It has been found by L. Lorenz to hold also for the refractive index and was derived by H. A. Lorentz from the electromagnetic theory as above.

For *low densities* the correction due to a may be neglected so that

$$\bar{n}^2 = 1 + \sum_s \frac{4\pi F_s \epsilon_s^2 / m_s}{\omega_{0s}^2 - \omega^2 + i\omega_s' \omega}. \quad (24')$$

Substituting (20) and identifying real and imaginary parts we have

$$n^2(1 - \kappa^2) = 1 + \sum_s \frac{4\pi F_s \epsilon_s^2}{m_s} \frac{\omega_{0s}^2 - \omega^2}{(\omega_{0s}^2 - \omega^2)^2 + \omega_s'^2 \omega^2} \equiv A \quad (25)$$

$$2n^2\kappa = \sum_s \frac{4\pi F_s \epsilon_s^2}{m_s} \frac{\omega_s' \omega}{(\omega_{0s}^2 - \omega^2)^2 + \omega_s'^2 \omega^2} \equiv B. \quad (26)$$

Solving these for n and κ we have

$$n^2 = \left(\frac{1}{2}\right) [A + (A^2 + B^2)^{1/2}] \cong A \quad (25')$$

$$\alpha^2 = (2\omega n \kappa / c)^2 = (2\omega^2 / c^2) [(A^2 + B^2)^{1/2} - A] \cong B^2 \omega^2 / (Ac^2). \quad (26')$$

Usually the approximations indicated in (25'), (26') suffice for the calculation of the refractive index and the absorption coefficient.

If ω is close to a particular ω_{0s} only one term in (26) is of importance. Supposing that $A \cong 1$ the maximum value of the absorption coefficient is

$$\alpha_{\max} = 4\pi F_s \epsilon_s^2 / (m_s \omega_s' c) \quad (27)$$

which is largest for small ω_s' . The width of the curve representing the absorption coefficient as a function of ω is proportional to ω_s' which is called the half width of the absorption line because the absorption coefficient is one half of its maximum value for

$$\omega = \omega_{0s} \pm (\omega_s' / 2).$$

The relation of the absorption to the refractive index is shown in Fig. 4. It will be noted that the maximum and minimum of n correspond to such wave-lengths that the absorption coefficient is $1/2$ of its maximum value. The integral of the area under the absorption curve is independent of ω_s' as long as ω_s' is small. This is easily verified by integrating B over ω .

By (21), (26) a thin layer of thickness l of the material containing a small number F_s of dispersing electrons per cm^3 absorbs the amount

$$(\pi e^2 / (mc)) F_s l \rho(\nu) \quad (27')$$

from radiation having an energy density $\rho(\nu)d\nu$ in the frequency range $d\nu$.

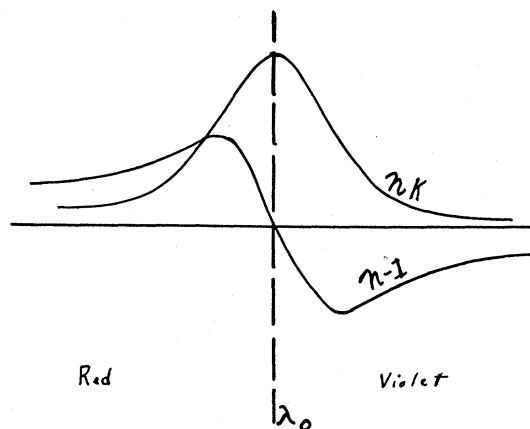


Fig. 4. Absorption ($n\kappa$) and dispersion ($n-1$).

IV. MODIFICATIONS DUE TO THE QUANTUM THEORY

The model of a dispersing medium used in the classical theory of dispersion is, of course, only schematic. Experience shows, however, that the results are nearly right. The refractive index of materials may be represented by means of the formulas (25), (26). The experimental evidence showing this is presented in the sections that follow.

It is very striking that the frequencies

$$\nu_{0s} = \omega_{0s} / (2\pi)$$

which must be used in the classical formulas are the absorption frequencies corresponding to jumps between stationary energy levels and related to the energies of these levels by Bohr's frequency condition

$$E_j - E_i = h\nu_{0s} = h\nu_{ji}. \quad (28)$$

Here E_i is the energy of the normal level of the atom and E_j is the energy of an upper level to which a transition may take place. Ladenburg⁹ advanced the hypothesis that if there are N atoms per unit volume in their normal

⁹ R. Ladenburg, *Zeits. f. Physik* **4**, 451 (1921); See also R. Ladenburg and F. Reiche, *Naturw.* **11**, 596 (1923). R. C. Tolman, *Phys. Rev.* **23**, 609 (1924).

state with a possible transition frequency ν_{0s} then the number F_s to be used in the dispersion formulas is

$$F_s = Nf_s = Nf_{ji} \quad (29)$$

where f_s is a number characteristic of the atom and the transition $i \rightarrow j$ but independent of N as long as N is not so large that the collisions among the atoms become important. He showed¹⁰ that it was very reasonable to expect the following connection between f_{ji} and Einstein's absorption probability from i to j

$$B_{ij} \cdot h\nu_{ji} = (\pi\epsilon^2/m)f_{ji} \quad (30)$$

f_{ji} may be called the number of dispersion electrons of frequency ν_{ji} per atom. As a consequence of Einstein's relation between the spontaneous emission probability A_{ji} and B_{ij} it follows as has been shown by Ladenburg that

$$f_{ji} = A_{ji}(g_i/g_j)(\tau_{cl}/3) \quad (31)$$

where τ_{cl} is the time constant¹¹ of a classical oscillator of electronic charge and mass (ϵ, m) and of resonant frequency ν_{ji} . The value of τ_{cl} is given by

$$\tau_{cl} = \frac{3mc^3}{8\pi^2\epsilon^2\nu_{ji}^2} = 1.131 \times 10^{-8}(\lambda/5000)^2 \text{ sec.} \quad (32)$$

where λ is the wave-length in A and the value of m ($=0.903 \times 10^{-27}$ gr) has been used.

Ladenburg's hypothesis has been fully confirmed by further developments of the quantum theory. The relation of the f numbers to the absorption and emission probabilities is exactly the same as has been postulated originally. The value of the emission probability is now known to be given by

$$A_{ji} = \frac{64\pi^4\epsilon^2\nu_{ji}^3}{3hc^3} \sum_{m_i} \{ |x_{m_j m_i}|^2 + |y_{m_j m_i}|^2 + |z_{m_j m_i}|^2 \} \quad (32')$$

where m_i, m_j are the magnetic sublevels of the states i, j , each sublevel being supposed to be defined by a definite wave function, the summation being performed by keeping m_j fixed and varying m_i . The numbers $x_{m_j m_i}, y_{m_j m_i}, z_{m_j m_i}$ are the matrix elements of the sums of the x, y, z coordinates of the electrons respectively. The summation over m_i is independent of m_j for free atoms satisfying the conservation of angular momentum (spectroscopic stability). Similarly if the summation is performed over m_j keeping m_i fixed it is independent of m_i . Thus

$$g_j \sum_{m_i} = g_i \sum_{m_j} = \sum_{m_i, m_j} \quad (32'')$$

and the explicit expression for f in terms of the matrix elements is using (31)

$$f_{ji} = \frac{8\pi^2 m \nu_{ji}}{3h} \sum_{m_j} \{ |x_{m_j m_i}|^2 + |y_{m_j m_i}|^2 + |z_{m_j m_i}|^2 \}. \quad (33)$$

¹⁰ Cf. also Ch. Führtbauer, Phys. Zeits. **21**, 322 (1920).

¹¹ i.e., the time required for the energy of the oscillator to drop to $(2.71828 \dots)^{-1}$ of its initial value on account of radiation.

So far we had the atom in its normal state i . Previously to the invention of quantum-mechanics Kramers and Heisenberg¹² have shown that the correspondence principle and Ladenburg's f number hypothesis can be unified by a modification of the classical dispersion formula applying to the case of excited atoms. This modification is also fully confirmed by the modern quantum theory. The result is that if the number of atoms per unit volume in the state i is N_i the classical dispersion formula is correct provided one sets

$$F_s = N_i f_{ji} + N_j f_{ij} \quad (34)$$

where

$$f_{ij} = -g_j f_{ji} / g_i. \quad (35)$$

The quantity f_{ij} is also given by (33). In fact interchanging i and j in (33) and using (32') we obtain (35) interpreting ν_{ji} algebraically in agreement with (28). If $E_j > E_i$, $f_{ji} > 0$ and $f_{ij} < 0$. The existence of negative terms in the dispersion formula was not realized before the work of Kramers and Heisenberg. The experimental proof of these terms has been performed in recent years by Ladenburg and his collaborators and the evidence will be reviewed in a special article by Ladenburg in these Reviews.

By (34) and (35) and by Einstein's relation

$$g_i B_{ij} = g_j B_{ji}$$

between the induced emission and the absorption probabilities we also have [cf. (27')]

$$(\pi\epsilon^2/m)F_s = h\nu_{ji}(N_i B_{ij} - N_j B_{ji}). \quad (36)$$

In the neighborhood of an absorption frequency but outside the region where the absorption is large ($|\omega_{0s} - \omega| \gg \omega_s'$) the refractive index is given by

$$n - 1 = \frac{\epsilon^2}{2\pi m} \frac{F_s}{\nu_s'^2 - \nu^2} = \frac{\epsilon^2}{4\pi m c^2} \frac{\lambda_s^3}{\lambda - \lambda_s} F_s. \quad (37)$$

This formula is used in experimental investigations of the refractive index of gases in the neighborhood of λ_s .

Instead of (34) and (36) one may also write

$$F_s = F_{ji} = N_i A_{ji} \frac{g_j}{g_i} \frac{mc^3}{8\pi^2 \epsilon^2 \nu^2} \left(1 - \frac{N_j g_i}{N_i g_j} \right) \quad (38)$$

which is used in the work of Ladenburg on negative dispersion.¹³ The *mean life* of an atom in an excited state j is

$$\tau_j = \frac{1}{\sum_{E_i < E_j} A_{ji}} = \frac{1}{\sum_{E_i < E_j} (1/\tau_{ji})} \quad (39)$$

¹² H. A. Kramers, Nature **113**, 673 (1924); **114**, 310 (1924); H. A. Kramers and W. Heisenberg, Zeits. f. Physik **31**, 681 (1925).

¹³ R. Ladenburg, Zeits. f. Physik **48**, 15 (1928).

where

$$\tau_{ji} = \frac{1}{A_{ji}} = \frac{g_i \tau_{ci}}{3g_i f_{ji}} \quad (40)$$

For a state j situated so that there is only one lower state to which a transition may take place τ_{ji} is the mean life. Formula (40) is useful in comparing the results of determinations of τ_{ji} by direct means with dispersion measurements of f_{ji} .

V. EXPERIMENTS AND RESULTS

We list below (Tables II–XVII) in the form of tables some of the most important results on the dispersion of gases, liquids, and solids. The tables are self explanatory. It will be noted that the refraction of gases (Tables II, III) and of liquids (Table XI) as well as solids (Table XIV) obeys the theoretical formulas. The methods used in obtaining dispersion data are familiar in principle to everyone. For a technical account of the experimental technique we may refer to Glazebrooks' Dictionary of Applied Physics¹⁴ to the article by Minkowski¹⁵ in the new edition of the Mueller-Pouillet Lehrbuch der Physik and that of Jaffe¹⁶ in the Wien Harms Handbuch der Experimental Physik.

Tables XVIII–XXVI give a summary of information obtained on the constants f for different elements. The f values are obtained by the following methods.

1. Methods of determining f values

(a) *Anomalous dispersion.* The anomalous dispersion is measured in the neighborhood of an absorption line. The number of molecules is determined

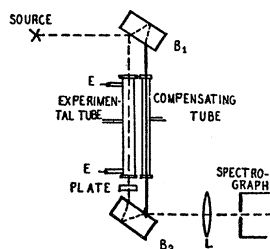


Fig. 5. Jamin interferometer experimental arrangement. Mirror blocks $B_1 B_2$, electrodes, E .

by measurements of the vapor pressure. For low gas densities and close to the absorption line but not within it one may use the approximate formula

$$n - 1 = (\epsilon^2 N / 4\pi m c^2) \lambda_1^3 f_1 / (\lambda - \lambda_1) \quad (37)$$

where N = number of atoms per cm^3 ; ϵ = electronic charge; n = electronic mass; c = velocity of light; λ_1 = wave-length of the absorption line (in cm).

¹⁴ Vol. 4, p. 772, Macmillan 1923.

¹⁵ R. Minkowski, Mueller-Pouillet, Lehrbuch der Physik II, Optik 2 (Vieweg und Sohn 1929). Referred to below as Min. (M-P).

¹⁶ G. Jaffe; Wien-Harms Handbuch der Experimental Physik XIX Akademische Verlags Gesellschaft, Leipzig 1928. Referred to below as G. J. (W-H).

The most widely used and the most accurate way of obtaining n by means of anomalous dispersion is the *hook method* of Roschdestvenski.¹⁷ The principle of the method is as follows. The column of the gas to be studied is inserted into the path of one of the interfering rays of a Jamin interferometer. [Fig. 5] A compensating plate is inserted into the path of the other ray. The fringes are thrown on to the slit of a spectrograph by means of the lens L . Without the compensating plate the fringes appear in the spectrograph as shown in Fig. 6 A and with the compensating plate as in Fig. 6 B.

The points of maximum or minimum displacement of a given fringe i.e., up and down in Fig. 6 B are the hooks. The measurement is made by determining the distance between the two hooks on each side of the absorption line in a direction perpendicular to the slit i.e., horizontally in Fig. 6 B. To every distance in the horizontal direction there corresponds a definite wave-length determined by the dispersion of the spectrograph used. A knowledge of the dispersion of the instrument and the measurement of the distance between hooks thus leads at once to a knowledge of the wave-length difference between the hooks to the right and to the left of the absorption line. This wave-length difference we call

$$\lambda' - \lambda''.$$

To every fringe there corresponds a definite phase difference between the two interfering rays. It depends on the material, size, and alignment of the interferometer plates, on the refractive index n' and the thickness l' of the compensating plate and on the refractive index n and the thickness l of the column of gas. In the absence of the gas column the phase difference varies uniformly along the slit, and is nearly independent of the wave-length as follows from the usual theory of the interferometer and as is obvious from the fact that the distance between fringes is the same on the right and on the left side of Fig. 6 A, B. The distance y along the slit may be thus taken to be proportional to the optical path difference produced by the interferometer plates in the absence of the compensating plate and of the column of gas. Let the order of interference for a given fringe be k . Then for this fringe

$$Cy - (n(\lambda) - 1)l + (n'(\lambda) - 1)l' = k\lambda$$

where C is the constant of proportionality between the path difference and y . For the hooks y is either a maximum or a minimum. Using the above equation, solving for y as a function of λ , determining the values of λ for the maximum and minimum, using (37) for $n(\lambda) - 1$ and treating $dn'(\lambda)/d\lambda$ as a constant it is found that

$$f_1 = \frac{\pi mc^2}{\epsilon^2 \lambda_1^3 N} \frac{K(\lambda' - \lambda'')^2}{l} \quad (41)$$

¹⁷ D. Roschdestvensky, Ann. d. Physik **39**, 307 (1912); Trans. Opt. Inst. Leningrad **2**, Nr. 13 (1921). See also Jamin, Ann. d. Chem. et de Phys. **52**, 163 (1858); Puccianti, Il Nuovo Cim. **2**, 257, 1901; R. Ladenburg and St. Loria, Phys. Zeits. **9**, 875 (1908); for previous applications of the interferometer, and S. A. Korff and J. Q. Stewart, Rev. Sci. Inst. **1**, 341 (1930); S. A. Korff and J. Q. Stewart, Phys. Rev. **32**, 676 (1928); S. A. Korff, Phys. Rev. **33**, 584 (1929); **34**, 457 (1929) for similar applications of the Michelson interferometer.

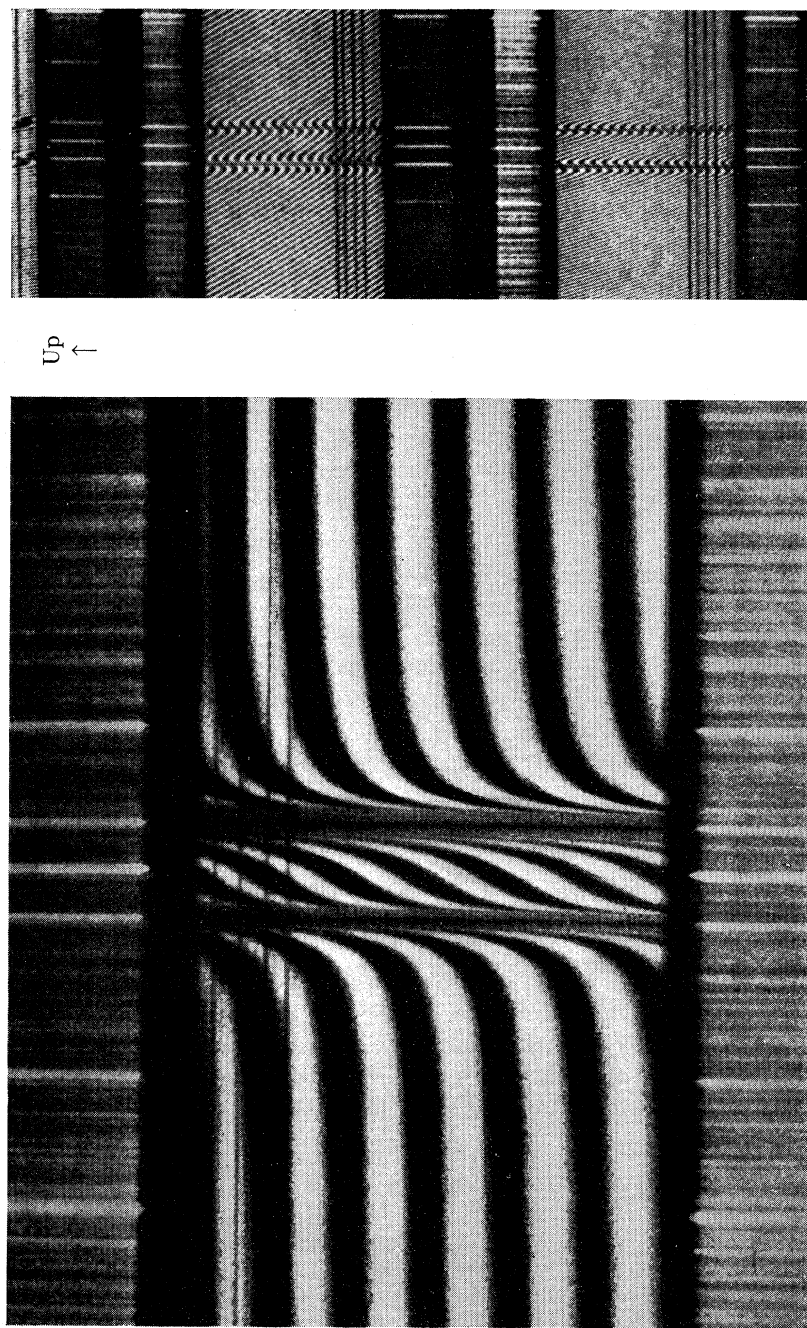


Fig. 6. Interference patterns. Obtained with Jamin type interferometer. A, without addition of plane parallel plate. B, various orders with plate inserted, showing hooks.

where

$$K = k - l' \frac{dn'}{d\lambda}. \quad (41')$$

The number K is obtained most readily experimentally by removing the column of gas and leaving the optical system undisturbed. In this case one obtains fringes in the form of straight oblique lines as on the right and left sides of Fig. 6 B. For these fringes

$$Cy + (n'(\lambda) - 1)l' = k\lambda$$

so that if y is kept fixed, k and λ vary in such a way that

$$K\Delta\lambda = -\lambda\Delta k$$

or

$$K \cong -\lambda_1 \Delta k / \Delta\lambda. \quad (41'')$$

One therefore simply counts the number of fringes crossing a horizontal line in a given wave-length interval on the photographic plate. This number of fringes Δk divided by the wave-length interval $\Delta\lambda$ and multiplied by the wave-length λ_1 of the absorption line gives the dimensionless constant K .

In some cases it is necessary to consider the effects of neighboring lines on the hooks belonging to a given absorption line. These points are discussed by Ladenburg and Wolfsohn.¹⁸

(b) *Magnetic rotation.* Linearly polarized light is passed through the dispersing material. When a magnetic field is applied along the direction of propagation of the light the plane of polarization is rotated (Faraday effect) through an angle which is proportional to the field strength, the number of atoms per unit volume, the length of path, and depends besides on the nature of the atoms in the path of the light ray. In the neighborhood of an absorption line there is a quantitative relationship between the angle of rotation of the plane of polarization, the conditions of the experiment just mentioned and the Ladenburg f numbers. This relationship is¹⁹ to a sufficient approximation:

$$\chi = \frac{Nf_1\epsilon^3 l \mathcal{H}}{8\pi m^2 c^4} \frac{\lambda^2 \lambda_1^2}{(\lambda - \lambda_1)^2} \sum_i g_i I_i \quad (42)$$

where λ_1 is the wave-length of the absorption line, \mathcal{H} is the magnetic field, g_i is the Landé g factor for the i th circularly polarized (σ) Zeeman component, and I_i is the ratio of the intensity of this i th Zeeman component to the sum of all circularly polarized Zeeman components and the other quantities

¹⁸ R. Ladenburg and G. Wolfsohn, *Zeits. f. Physik* **63**, 616 (1930). See p. 619 also pp. 620–21 for details of tube construction for high temperature work.

¹⁹ H. Senftleben, *Ann. d. Physik* **47**, 977 (1916) (Dissertation Breslau, 1915); R. Ladenburg, *Ann. d. Physik* **38**, 249 (1912); W. Kuhn, *Math. Phys. Communications of the Danish Academy* **VII**, 12, 11 (1926).

have the same meaning as in (41). In the case of the classical Zeeman effect the $\sum_i = 1$ and the formula becomes equivalent to that of Becquerel²⁰

$$\chi = \frac{l\epsilon}{2mc^2} \mathcal{H}\lambda \frac{\partial n}{\partial \lambda} \quad (42')$$

which is immediately interpretable in terms of the Larmor precession. The relation of the result of Kuhn to Becquerel's formula is clear if one recalls that the rotation of the plane of polarization may be attributed to the difference of the refractive indices of the two oppositely rotating circularly polarized rays. We thus refrain from giving a more detailed derivation of the quantum formula (42).

It should be noted that the formula applies outside the region of true absorption and that it is supposed that the wave-length is outside the Zeeman pattern formed by the magnetic field.

The measurement of the angle of rotation χ is made by various methods. The Savart plate appears to be capable of giving the more accurate results.²¹

(c) *Absorption methods.* The absorption methods of determining the line strengths (f values) are those of overall absorption (German "Total absorption") line absorption (German "Linienabsorption") and of natural breadth.

In the method of *overall absorption* the light from a source of a continuous spectrum is passed through a monochromator and a suitably chosen wave-length interval of the continuous spectrum is then led through the material to be investigated and the amount of light absorbed is measured. By taking into account the length of column of the material, the theoretical intensity distribution due to Doppler broadening and to radiation damping it is possible to compute from the measured absorption the f value of the line. In general the calculations are somewhat involved because the absorption coefficient varies throughout the wave-length interval.²²

In order to obtain sufficiently broad lines to allow the absorption coefficient to be measured within the lines the absorption lines must be broadened. Füchtbauer and his collaborators²³ have made measurements on this principle, the broadening being produced by the addition of foreign gases. This has also the apparent advantage of allowing one to measure the intensity distribution in the spectrum of the transmitted light with instruments of moderate

²⁰ H. Becquerel, C. R. 125, 679 (1897); J. J. Larmor—Aether and Matter (Cambridge 1900) p. 352. See also R. Ladenburg, Zeits. f. Physik 34, 898 (1925).

²¹ For further details see R. Ladenburg, Magneto und Elektrooptik Mueller-Pouillet. Lehrbuch der Physik Vol. II part 2, p. 2120.

²² R. Ladenburg, Verh. d. Deut. Phys. Ges. 16, 765 (1914), see p. 769; R. Ladenburg and F. Reiche, Ann. d. Physik 42, 181 (1931).

²³ Chr. Füchtbauer, Phys. Zeits. 14, 1164 (1913); Chr. Füchtbauer and C. Schell, Verh. d. Deut. Phys. Ges. 15, 974 (1913); Chr. Füchtbauer and W. Hoffman, Ann. d. Physik 43, 96 (1914); Chr. Füchtbauer and G. Joos, Phys. Zeits. 21, 694 (1920); Chr. Füchtbauer and Bartels, Zeits. f. Physik 4, 337 (1921); Chr. Füchtbauer and G. Joos, Phys. Zeits. 23, 73 (1922); Chr. Füchtbauer, G. Joos and Dinckelacker, Ann. d. Phys. 71, 222 (1923); Chr. Füchtbauer and Wolf, Ann. d. Physik 3, 359 (1929).

resolving power. Nevertheless it is questionable whether the interior of the atom is sufficiently unaffected by the collisions to make the conclusions about f values entirely trustworthy. In the tables below we give therefore preference to measurements on sharp rather than broadened lines.

In the method of *line absorption* the material to be investigated is exposed to light from a source having the same length and vapor density as the absorbing tube.²⁴ This method appears to be capable of giving very accurate results. It should be remembered however that it is subject to corrections on account of the fine (and hyperfine) structure of spectral lines.

By passing white light through the material under investigation and then focussing it on the slit of a spectrograph the intensity distribution of the transmitted light may be obtained. In the core of the line the shape of the absorption line is determined to a large extent by the Doppler effect broadening. In the wings the shape is determined mainly by the radiation damping because the absorption due to radiation damping falls off less rapidly with the distance from the line center. For the details of this way of using the natural breadth we refer to the investigation of Minkowski on the line breadth of the sodium D lines.²⁵ It will be noted that in practice the use of very low pressures, although ideal from the theoretical point of view, is not very advisable on account of errors introduced by the finite resolving power of the apparatus. Too high pressures (higher than 0.01 mm) bring in false effects due to pressure broadening. Uniformity of temperature throughout the absorption tube must also be carefully maintained.

A similar method has been used previously by Harrison and Slater.²⁶ Their results agree fairly well with those obtained by other methods so far as relative values of transition probabilities are concerned. The absolute values are however wrong by a factor of about 50. The reason for this disagreement not being very clear we must refrain, from listing their results in the tables below.

(*d*) *Emission methods.* By observing the relative intensities of spectral lines emitted under known conditions of excitation i.e., with known populations of the different levels it is possible to obtain relative values of the transition probabilities and therefore also of the f numbers. Absolute values may perhaps be also obtainable if absolute energy measurements could be made with sufficient certainty.

Conclusions from emission intensities are usually subject to the error of not taking into account the absorption which takes place in the source itself. This exists even if the ordinary self reversal due to cold layers of the excited gas are absent.²⁷ In the measurements of Jacob on K given below this effect

²⁴ See also R. Ladenburg and F. Reiche, *Ann. d. Physik* **42**, 181 (1913); *Jahresbericht der Schlesiischen Ges. f. vaterl. Cultur* 1912: P. Kunze, *Ann. d. Physik* **85**, 1013 (1928); H. Kopfermann and W. Tietze, *Zeits. f. Physik* **56**, 604 (1929); R. Ladenburg and S. Levy, *Zeits. f. Physik* **65**, 200 (1930).—The latter for numerical tables.

²⁵ R. Minkowski, *Zeits. f. Physik* **36**, 839 (1926).

²⁶ G. R. Harrison, *Phys. Rev.* **25**, 768 (1925); J. C. Slater, *Phys. Rev.* **25**, 783 (1925); G. R. Harrison and J. C. Slater, *Phys. Rev.* **26**, 176 (1925).

²⁷ R. Minkowski, *Zeits. f. Physik* **63**, 188 (1930).

was corrected for by measuring the absorption of the source for the lines in question.

(e) *Polarization of resonance radiation.* By observing the effect of a magnetic field on the polarization of resonance radiation it is possible to deduce the mean life τ of the atom in the upper level and hence the transition probability and the f number of the resonance line. There are at present two reliable methods of determining τ by means of resonance radiation. The first consists in the measurement of the depolarization produced by a given magnetic field. Polarized light is sent into the resonance bulb and the resonance radiation is observed at right angles to the incident light. Magnetic fields of various strengths are applied along the direction of observation. If the polarization in the absence of a magnetic field is P_0 then theoretically the polarization in a magnetic field of strength \mathcal{H} is

$$P = \frac{P_0}{1 + \left(\frac{\epsilon \mathcal{H}}{mc} g \tau \right)^2} \quad (43)$$

where g is the Landé g factor for the upper level of the resonance line.²⁸ From the experimental values of the polarization P and from the knowledge of g , τ is deduced. Hence by Eq. (31) f follows. By P in (43) is meant the difference in the intensities of the amounts of light polarized with electric vector parallel and perpendicular to the direction of the electric vector of the incident light divided by the sum of these intensities.

Another method is to determine the direction of maximum polarization in a magnetic field of given strength. For no magnetic field this direction is the same as that of the incident light. In a field of strength \mathcal{H} it is turned through an angle ϕ given by²⁹

$$\tan 2\phi = \frac{\epsilon \mathcal{H}}{mc} g \tau \cdot \quad (44)$$

The experimental development of the use of the depolarization is due to Wood and Ellett³⁰ and to Hanle. The most accurate measurements appear to

²⁸ This formula may be derived by means of Dirac's theory of radiation [J. R. Oppenheimer, *Zeits. f. Physik* **43**, 27 (1927)]. The qualitative connection of the Larmor precession frequency with the mean life has been pointed out by Bohr [*Naturw.* **12**, 1115 (1924)] and by G. Breit [*Phil. Mag.* **47**, 832 (1924)]. Formula (43) has been first derived by J. A. Eldridge [*Phys. Rev.* **24**, 234 (1924)]. A more rigorous treatment is given by G. Breit [*Journ. Opt. Soc. Amer.* **10**, 439 (1925)].

²⁹ V. Weisskopf, *Ann. d. Physik* **9**, 23 (1931); see p. 55. See also G. Breit, *Journ. Opt. Soc. Amer.* **10**, 439 (1925). The quantum treatment of Weisskopf can be made to include the general case of a multiple lower level. See VII §4 of following review.

³⁰ R. W. Wood and A. Ellett, *Proc. Roy. Soc.* **103**, 396 (1923); *Phys. Rev.* **24**, 243 (1924). A. Ellett, *Journ. Opt. Soc. Amer.* **10**, 427 (1925).

have been made by v. Keussler³¹ for 2537A of Hg. The angle of maximum polarization has been used by Hanle³² and Ellett.³³

In addition to these methods it is also possible to have a comparison of τ with a directly measurable time interval by applying alternating magnetic fields³⁴ of frequencies comparable to $1/\tau$. This method has not yielded so far results of accuracy comparable to that of the two just described.

(f) *Direct measurements of τ .* The earliest are the well known canal ray experiments of Wien. They give misleading results except in some particularly favorable cases. If the τ for the level for which the mean life is measured is long in comparison with that of the levels lying above it and connected to it by transitions the number of atoms in the level is determined essentially by τ and is given by

$$e^{-t/\tau}.$$

If, on the other hand, there are upper levels of comparable or longer τ they go on feeding any level below them even after a time sufficient to depopulate this level when left to itself. Thus for the 3P_1 state of mercury Wien's canal rays give an approximately correct result while for H_α , H_β , H_γ the results are not nearly right.³⁵

By using optical excitation of atomic beams Koenig and Ellett³⁶ have recently made a fairly accurate determination of τ for the 3P_1 state of Cd.

A method similar in principle has been developed by Webb³⁷ and has been used by him, Garrett and Randall (see Table XXVII) for the determination of mean lives of levels in Hg. The experimental arrangement involves the use of alternating current excitation for the source and a synchronized detection by means of a photoelectric cell made to operate at the proper time by an electrical arrangement.

The results will be seen to be in very good agreement with the calculations of Zehnden and Zemansky by the method of line absorption (Table XXVII) using the measurements of Kopfermann and Tietze for the 2537 line of Hg. There are still some outstanding discrepancies inasmuch as the life of the same level appears to be different when measured by means of different lines.

³⁴ E. Fermi and F. Rasetti, *Zeits. f. Physik* **33**, 246 (1925); G. Breit and A. Ellett, *Phys. Rev.* **25**, 888 (1925).

³¹ F. v. Keussler, *Ann. d. Physik* **82**, 810 (1927).

³² W. Hanle, *Zeits. f. Physik* **30**, 92 (1924); **35**, 346 (1925); **41**, 164 (1927).

³³ A. Ellett, *Journ. Opt. Soc.* **10**, 427 (1925); See pp. 432-433.

³⁵ R. Ladenburg and R. Minkowski, *Ann. d. Physik* **87**, 298 (1928).

³⁶ Koenig and Ellett, *Phys. Rev.* **39**, 576 (1932); For older similar experiments see R. W. Wood, *Proc. Roy. Soc.* **99**, 362 (1921); Stern and Vollmer, *Phys. Zeits.* **20**, 183 (1919).

³⁷ H. W. Webb, *Phys. Rev.* **24**, 113 (1924); F. G. Slack, *Phys. Rev.* **28**, 1 (1926); H. W. Webb and H. A. Messenger, *Phys. Rev.* **33**, 319 (1929); R. H. Randall, *Phys. Rev.* **35**, 1161 (1930).

2. Tables of refractive indices

TABLE II. *Refractive index of argon.*¹

| in Å | $(n-1) \times 10^6$ Observed by Quarder ² | Computed by I | Computed by Ia |
|------|---------------------------------------------------------|------------------|-------------------|
| 2441 | 303.78 | 303.97 | 303.97 |
| 2492 | 302.80 | | 302.86 |
| 2618 | 300.38 | 300.37 | 300.37 |
| 2766 | 298.11 | | 297.97 |
| 2824 | 297.14 | | 297.17 |
| 2961 | 295.50 | 295.28 | 295.29 |
| 3349 | 291.62 | 291.43 | 291.41 |
| 4275 | 286.34 | 286.25 | 286.26 |
| 4651 | 284.99 | | 285.01 |
| 5105 | 283.79 | | 283.87 |
| 5153 | 283.67 | | 283.77 |
| 5218 | 283.50 | 283.03 | 283.62 |
| 5700 | 282.55 | 282.77 | 282.77 |
| 5782 | 282.47 | 282.65 | 282.65 |

$$n - 1 = \frac{4.9981 \times 10^{27}}{17953 \times 10^{27} - \nu^2}, \quad \lambda_0 = 708.0\text{Å}, F = 4.58N \quad (\text{I})$$

N = number of atoms per cm^3 [See Eq. (37)]

$$n - 1 = \frac{0.02715 \times 10^{27}}{8499.6 \times 10^{27} - \nu^2} + \frac{5.0131 \times 10^{27}}{18215 \times 10^{27} - \nu^2}, \quad \lambda_1 = 1029\text{Å}, F_1 = 0.025N \quad (\text{Ia})$$

$$\lambda_2 = 702.9\text{Å}, F_2 = 4.59N$$

¹ Wolf and Herzfeld (Geiger-Scheel Handbuch XX p. 490).

² B. Quarder, Ann. d. Physik 74, 255 (1924).

TABLE III. *Refractive index of hydrogen.*¹

| in Å | $(n-1) \times 10^7$ Observed by Kirn ² | $(n-1) \times 10^7$ Computed ³ | in Å | $(n-1) \times 10^7$ Observed by Kirn ² | $(n-1) \times 10^7$ Computed ³ |
|------|---------------------------------------------------------|----------------------------------------------|------|---------------------------------------------------------|----------------------------------------------|
| 1854 | 1759.96 | 1760.29 | 2753 | 1515.00 | 1515.15 |
| 1862 | 1755.41 | 1755.74 | 2894 | 1498.59 | 1498.83 |
| 1935 | 1718.24 | 1718.18 | 2968 | 1491.01 | 1491.33 |
| 1990 | 1693.95 | 1693.73 | 3342 | 1461.33 | 1461.39 |
| 2302 | 1594.18 | 1593.93 | 4047 | 1427.41 | 1427.45 |
| 2379 | 1576.81 | 1576.68 | 4078 | 1426.32 | 1426.37 |
| 2535 | 1546.90 | 1547.01 | 4359 | 1417.73 | 1417.72 |
| | | | 5462 | 1396.50 | 1396.20 |

(2) Formula used:

$$n - 1 = \frac{0.75379 \times 10^{27}}{16681.3 \times 10^{27} - \nu^2} + \frac{0.919974 \times 10^{27}}{10130.5 \times 10^{27} - \nu^2}, \quad \lambda_1 = 734.5\text{Å}, F_1 = 0.69N$$

$$\lambda_2 = 942.6\text{Å}, F_2 = 0.84N$$

N = number of molecules per cm^3 [See Eq. (37)]

¹ Wolf and Herzfeld (Geiger-Scheel Handbuch XX p. 492).

² M. Kirn, Ann. d. Physik 64, 566 (1921).

³ H. Schüler and K. L. Wolf, Zeits. f. Physik 34, 343 (1925).

TABLE IV. Dispersion formulas for air.

| Observer | Dispersion formula | Observed region in μ |
|------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|--------------------------|
| Kayser und Runge 1893 | $(n_0 - 1) \cdot 10^7 = 2878.7 + 13.16\lambda^{-2} + 0.316\lambda^{-4}$ | 0.236-0.563 |
| Scheel 1907..... | $(n_0 - 1) \cdot 10^7 = 2870.5 + 16.23\lambda^{-2}$ | 0.436-0.706 |
| Rentschler 1908..... | $(n_0 - 1) \cdot 10^7 = 2903.1 + 3.80\lambda^{-2} + 1.23\lambda^{-4}$ | 0.334-0.577 |
| Koch ¹ 1909..... | $n_0^2 = 1.00057642 + \frac{3.277 \cdot 10^{-6}}{\Lambda^2 - 5685.00 \cdot 10^{-6}}$ | 0.436-8.678 |
| C. and M. Cuthbertson 1909..... | $(n_0 - 1) \cdot 10^7 = 2885.4 + 13.38\lambda^{-2} + 0.50\lambda^{-4}$ | 0.486-0.656 |
| Koch ¹ 1912..... | $\frac{3}{2} \cdot \frac{n_0^2 - 1}{n_0^2 + 2} \cdot 10^7 = \frac{571119.0}{211.146 - \Lambda^{-2}} + \frac{8700.3}{49.3608 - \Lambda^{-2}}$ | 0.238-0.546 |
| Howell 1915..... | $(n_0 - 1) \cdot 10^7 = 2881.7 + 11.83\lambda^{-2} + 0.46\lambda^{-4}$ | 0.265-0.668 |
| Dickey 1917..... | $(n_0 - 1) \cdot 10^7 = 2878.12 + 11.59\lambda^{-2} + 0.551\lambda^{-4}$ | 0.249-0.640 |
| Meggers and Peters 1919 | $(n_0 - 1) \cdot 10^7 = 2875.66 + 13.412\lambda^{-2} + 0.3777\lambda^{-4}$ | 0.222-0.900 |
| Traub ¹ 1920..... | $(n_0 - 1) \cdot 10^7 = \frac{551716.0}{199.5534 - \Lambda^{-2}} + \frac{4880.3}{43.8193 - \Lambda^{-2}}$ | 0.185-0.546 |
| Stoll 1922..... | $(n_0 - 1) \cdot 10^7 = 2871.87 + 16.170 \cdot \lambda^{-2}$ | 0.439-0.922 |
| Rusch ¹ 1923..... | $(n_0 - 1) \cdot 10^7 = \frac{595911}{205.97 - \Lambda^{-2}}$ | 0.546-6.763 |
| Quarder ¹ 1924..... | $(n_0 - 1) \cdot 10^7 = 2879.87 + 18.04\lambda^{-2}$ | 0.262-0.578 |
| Pérard ¹ 1925..... | $(n_0 - 1) \cdot 10^7 = 2880.2 + 14.78\lambda^{-2} + 0.316\lambda^{-4}$ | 0.436-0.644 |

TABLE V. Values of $(n_0 - 1) \cdot 10^7$ for air.

| λ in μ | 0.300 | 0.450 | 0.550 | 0.650 | $\lambda = \infty$ |
|--------------------------------|-----------|----------|----------|-----------|--------------------|
| Kayser and Runge 1893..... | 3063.9 | 2954.3 | 2928.7 | (2911.6) | (2878.7) |
| Scheel 1907..... | (3050.8) | 2950.6 | 2924.1 | 2908.9 | (2870.5) |
| Rentschler 1908..... | (3097.2) | 2951.9 | 2929.1 | 2919.0 | (2903.1) |
| Koch ¹ 1909..... | (3075.9) | 2964.9 | 2936.9 | 2921.0 | (2881.7) |
| C. and M. Cuthbertson 1909.. | (3095.8) | 2963.7 | 2935.09 | 2919.9 | (2885.4) |
| Koch ¹ 1912..... | 3081.5 | 2964.7 | (2936.6) | (2920.0) | (2880.5) |
| Howell 1915..... | 3069.9 | 2951.3 | 2925.8 | 2912.3 | (2881.7) |
| Dickey 1917..... | 3074.92 | 2948.79 | 2922.46 | 2908.64 | (2878.12) |
| Meggers and Peters 1919..... | 3071.31 | 2951.10 | 2924.12 | 2909.52 | (2875.66) |
| Traub ¹ 1920..... | 3076.98 | 2960.38 | 2931.76 | (2915.67) | (2876.13) |
| Stoll 1922..... | (3051.53) | 2951.72 | 2925.32 | 2910.14 | (2871.87) |
| Rusch ¹ 1923..... | (3058.1) | (2964.1) | 2940.8 | 2926.7 | (2893.2) |
| Quarder ¹ 1924..... | 3080.31 | 2968.96 | 2939.51 | (2922.57) | (2879.87) |
| Pérard ¹ 1925..... | (3083.4) | 2960.9 | 2932.6 | 2917.0 | (2880.2) |

¹ Air purified of carbon dioxide.
Ja. (W.-H.)

“Recently Tausz and Görlacher (27) have made an accurate determination of the refractive indices of hydrogen, oxygen, nitrogen, argon and air. They employed precision interferometer methods, enabling a variation in fringe position as small as 1/30th of a fringe to be observed. They also measured accurately the temperature, pressure, and length of dispersing column. The measurements gave the following means, reduced to standard temperature and pressure:

| wave-length | n for H | for A | for O | for N | for Air |
|-------------|------------|------------|------------|------------|------------|
| 6564 | 1.00013869 | 1.00028110 | 1.00027008 | 1.00029729 | 1.00029188 |
| 5876 | 13963 | 28237 | 27151 | 29842 | 29317 |
| 5461 | 14018 | 28314 | 27227 | 29914 | 29391 |
| 4358 | 14188 | 28611 | 27627 | 30226 | 29713 |

TABLE VI. Dispersion formulas for oxygen, nitrogen and hydrogen.

| Gas | Observer | λ in air, or Λ in vacuo, in μ | Region in μ |
|----------|---------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------|
| Oxygen | Koch 1909..... | $n_0^2 = 1.00052842 + \frac{3.699 \cdot 10^{-6}}{\Lambda^2 - 7000.00 \cdot 10^{-6}}$ | 0.436-8.678 |
| | Howell 1915..... | $(n_0 - 1) \cdot 10^7 = 2674 + 12 \cdot \lambda^{-2} + 1.06 \cdot \lambda^{-4}$ | 0.275-0.668 |
| | Stoll 1922..... | $(n_0 - 1) \cdot 10^7 = 2650.86 + 20.074 \cdot \lambda^{-2}$ | 0.439-0.922 |
| | Ladenburg and Wolfsohn 1932 ¹ | $n_0 - 1 = \frac{N e^2 \lambda^2}{2 \pi m c^2} \left[\frac{f' \lambda'^2}{\lambda^2 - \lambda'^2} + \frac{f'' \lambda''^2}{\lambda^2 - \lambda''^2} \right]$ $\lambda' = 1.46860 \mu; f' = 0.202$ $\lambda'' = 0.54436 \mu; f'' = 5.925$ $N e^2 / 2 \pi m c^2 = 1.2098 \cdot 10^6$ | 0.590-0.192 |
| Nitrogen | Koch 1913..... | $\frac{3}{2} \cdot \frac{n_0^2 - 1}{n_0^2 + 2} \cdot 10^7 = \frac{395345.0}{152.294 - \Lambda^{-2}} + \frac{83734.0}{240.651 + \Lambda^{-2}}$ | 0.238-0.546 |
| | Stoll 1922..... | $(n_0 - 1) \cdot 10^7 = 2907.27 + 22.65 \lambda^{-2}$ | 0.439-0.922 |
| Hydrogen | Koch 1909..... | $n_0^2 = 1.00027216 + \frac{2.112 \cdot 10^{-6}}{\Lambda^2 - 7760.00 \cdot 10^{-6}}$ | 0.436-8.678 |
| | Koch 1912..... | $\frac{3}{2} \cdot \frac{n_0^2 - 1}{n_0^2 + 2} \cdot 10^7 = \frac{142858.2}{116.3126 - \Lambda^{-2}} + \frac{75449.4}{568.6707 + \Lambda^{-2}}$ | 0.230-0.456 |
| | Howell 1915..... | $(n_0 - 1) \cdot 10^7 = 1387.6 + 8.0 \lambda^{-2} + 0.136 \lambda^{-4}$ | 0.275-0.588 |
| | Kirn 1921..... | $(n_0 - 1) \cdot 10^7 = \frac{122007.0}{118.6371 - \Lambda^{-2}} + \frac{58119.81}{175.3273 - \Lambda^{-2}}$ | 0.185-0.546 |

¹ Kindly supplied by the authors before publication.
Ja. (W.-H.)

O₂

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TABLE VII. Constants of the dispersion formula. $n_0 - 1 = C/(\nu_0^2 - \nu^2)$ for inert gases.

| Gas | Observer | $C \cdot 10^{-27}$ | $\nu_0^2 \cdot 10^{-27}$ | λ_0 | $(n_0 - 1)_\infty \cdot 10^7$ | Region in μ |
|---------|-----------------------|--------------------|--------------------------|-------------|-------------------------------|-----------------|
| Helium | C. and M. Cuthbertson | 1.21238 | 34992 | 507 | 346.48 | 0.480-0.671 |
| Helium | Koch ¹ | 1.3314 | 38423 | 484 | 346.5 | 0.230-0.644 |
| Neon | C. and M. Cuthbertson | 2.59826 | 38916 | 481 | 667.67 | 0.480-0.671 |
| Argon | C. and M. Cuthbertson | 4.71632 | 17009 | 726 | 2759.7 | 0.480-0.671 |
| Argon | Rusch I ² | 4.5471 | 18348 | 700 | 2781 | 0.546-0.656 |
| Argon | Rusch II | 4.216 | 15177 | 770 | 2778 | 0.546-6.763 |
| Argon | Quarder ¹ | 4.9981 | 17953 | 708 | 2784.0 | 0.244-0.578 |
| Krypton | C. and M. Cuthbertson | 5.3446 | 12768 | 840 | 4185.9 | 0.480-0.671 |
| Xenon | C. and M. Cuthbertson | 6.1209 | 8978 | 1001 | 6817.7 | 0.480-0.671 |

¹ Computed by Herzfeld and Wolf.² Computed without consideration of the infrared value.

Ja. (W.-H.)

K. F. Herzfeld and K. L. Wolf, Ann. d. Physik **76**, 71, 567 (1925).C. and M. Cuthbertson, Proc. Roy. Soc. **84**, 13 (1910).C. Cuthbertson and E. P. Metcalfe, Proc. Roy. Soc. **80**, 411 (1908).W. Burton, Proc. Roy. Soc. **81**, 440 (1910).K. Herrmann, Verh. d. Deut. Phys. Ges. **10**, 476 (1908).

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W. Burton, Proc. Roy. Soc. **80**, 390 (1908).M. Rusch, Ann. d. Phys. **70**, 373 (1923).B. Quarder, Ann. d. Phys. **74**, 255 (1924).TABLE VIIa. Constants of the dispersion formula $(n_0 - 1) = C/(\nu_0^2 - \nu^2)$ for inert gases according to C. and M. Cuthbertson (1932).

| Gas | $C \cdot 10^{-27}$ | $\nu_0^2 \cdot 10^{-27}$ |
|-------------|--------------------|--------------------------|
| Helium..... | 1.32614 | 38313.7 |
| Neon..... | 2.61303 | 39160 |

C. and M. Cuthbertson, Proc. Roy. Soc. **135**, 40 (1932).TABLE VIII. Constants of the dispersion formula $n_0 - 1 = C/(\nu_0^2 - \nu^2)$ for elements and compounds, according to Cuthbertson.

| Substance | $C \cdot 10^{27}$ | $\nu_0^2 \cdot 10^{27}$ | λ_0 in A | $(n_0 - 1)_\infty \cdot 10^6$ | Additive value computed | % error in additive value |
|-------------------------|-------------------|-------------------------|--------------------|-------------------------------|-------------------------|---------------------------|
| <i>Elements</i> | | | | | | |
| (H ₂)..... | 1.692 | 12409 | 851.6 | 136.35 | | |
| (N ₂)..... | 5.0345 | 17095 | 725.6 | 294.5 | | |
| (O ₂)..... | 3.3970 | 12804 | 838.4 | 265.3 | | |
| (S ₂)..... | 14.424 | 4600 | 1389.8 | 3135.6 | | |
| (P ₄)..... | 15.22 | 6534 | 1173.6 | 2329.4 | | |
| (F ₂)..... | — | — | — | 195 | | |
| <i>(D-line)</i> | | | | | | |
| (Cl ₂)..... | 7.3131 | 9629.4 | 966.8 | 759.46 | | |
| (Br ₂)..... | 4.2838 | 3919.2 | 1515.4 | 1093 | | |
| (I ₂)..... | — | — | — $\lambda = 6438$ | 2100 | | |
| | | | $\lambda = 6100$ | 2160 | | |
| <i>Compounds</i> | | | | | | |
| (H ₂ O)..... | 2.6270 | 10697 | 917.27 | 245.6 | 269 | - 8.1 |
| (H ₂ S)..... | 4.834 | 7808 | 1073.6 | 619.1 | 659 | - 6.1 |
| (SO ₂)..... | 5.728 | 8929 | 1004 | 641.5 | 788 | -18.6 |
| (HCl)..... | 4.6425 | 10664 | 918.67 | 435.34 | 448 | - 2.8 |
| (HBr)..... | 5.1446 | 8668.4 | 1018.9 | 593.5 | 615 | - 3.5 |
| (HI)..... | 5.7900 | 6556.4 | 1372.7 | 883.1 | 918? | - 3.8 |
| (NH ₃)..... | 2.9658 | 8135.3 | 1051.8 | 364.6 | 352 | + 3.6 |
| (NO)..... | 3.5210 | 12216 | 858.3 | 288.2 | 280 | + 3.0 |
| (N ₂ O)..... | 5.6685 | 11416 | 887.9 | 496.5 | 427 | +16.3 |
| (O ₃)..... | 2.0703 | 4281.4 | 1449.9 | 483.6 | 398 | +21.5 |
| (CO ₂)..... | 6.2144 | 14097 | 799 | | | |
| (CO)..... | 3.64067 | 11124 | 899 | | | |
| (CH ₄)..... | 5.02763 | 11689.3 | 877 | | | |

Ja. (W.-H.)

C. Cuthbertson, Phil. Mag. (6) **25**, 592 (1913).

TABLE IX. Constants of dispersion formula $n^2 = a + D \cdot \lambda^2 / (\lambda^2 - \lambda_v^2) - \kappa_1 \lambda^2$ for various liquids.

| Substance | Observer | Region in μ | a | D | λ_v in μ | κ_1 |
|------------------------------------|----------------------|-----------------|----------|----------|----------------------|------------|
| Water $t = 19.8^\circ$ | Flatow..... | 0.214-0.589 | 1.379185 | 0.383782 | 0.12703 | 0.01383 |
| | Seegert..... | 0.808-2.327 | | | | |
| Sulphur dioxide $t = 18^\circ$ | Martens..... | 0.260-0.361 | 1.63403 | 0.88113 | 0.21753 | 0.000300 |
| | v.d. Willigen..... | 0.397-0.760 | | | | |
| | Rubens..... | 0.777-1.998 | | | | |
| Methyl alcohol $t = 15.5^\circ$ | Seegert..... | 0.204-0.394 | 1.47249 | 0.28310 | 0.13468 | 0.006889 |
| | Landolt u. Jahn..... | 0.486-0.651 | | | | |
| | Seegert..... | 0.808-2.327 | | | | |
| Ethyl alcohol $t = 18^\circ$ | Martens..... | 0.186-0.394 | 1.335626 | 0.497725 | 0.12444 | 0.004496 |
| | Landolt u. Jahn..... | 0.397-0.760 | | | | |
| | Seegert..... | 0.808-2.327 | | | | |
| Propyl alcohol $t = 19.5^\circ$ | Seegert..... | 0.204-0.394 | 1.46655 | 0.43044 | 0.13031 | 0.004244 |
| | Landolt u. Jahn..... | 0.486-0.656 | | | | |
| | Seegert..... | 0.808-2.327 | | | | |
| Acetone | Seegert..... | 0.202-0.394 | 1.49257 | 0.35021 | 0.1437 | 0.002497 |
| | Landolt u. Jahn..... | 0.486-0.671 | | | | |
| | Seegert..... | 0.808-2.327 | | | | |
| Ethyl nitrate | Seegert..... | 0.243-0.394 | 1.627335 | 0.265175 | 0.1772 | 0.00337 |
| | Brühl..... | 0.434-0.670 | | | | |
| | Seegert..... | 0.808-2.327 | | | | |
| Benzol | Simon..... | 0.284-0.769 | 1.4528 | 0.7264 | 0.17450 | — |
| | Rubens..... | 0.810-1.850 | | | | |
| Toluol | Wood..... | 0.226-0.479 | 1.37 | 0.77 | 0.182 | — |
| Xylol | Rubens..... | 0.434-1.881 | 1.0000 | 1.1724 | 0.13660 | — |

Ja. (W.-H.)

Landolt-Börnstein Physikalische-chemische Tabellen 5 Aufl. Berlin 1923 und Ergänzungsband. Berlin 1927.

TABLE X. Dependence on temperature of constants for water in formula

$$n^2 = a + D \cdot \lambda^2 / (\lambda^2 - \lambda_v^2) - \kappa_1 \lambda^2 \text{ according to Flatow.}$$

| Temp. | a | D | λ_v | κ_1 |
|-------|---------|---------|-------------|------------|
| 0° | 1.36763 | 0.39802 | 0.12613 | 0.013414 |
| 20° | 1.37512 | 0.38850 | 0.12604 | 0.013414 |
| 40° | 1.37584 | 0.38174 | 0.12744 | 0.013414 |
| 60° | 1.37282 | 0.37558 | 0.12780 | 0.013414 |
| 80° | 1.37292 | 0.36463 | 0.12864 | 0.013414 |

Ja. (W.-H.)

E. Flatow, Diss. Berlin 1903. Ann. d. Physik 12, 85 (1903).

TABLE XI. Dispersion of xylol.

| Wave-length | n observed | n computed | Wave-length | n observed | n computed |
|--------------|-----------------|-----------------|-------------|-----------------|-----------------|
| 0.4361 μ | 1.5170 | 1.5170 | 1.012 | 1.4822 | 1.4813 |
| .4862 | 1.5075 | 1.5076 | 1.096 | 1.4808 | 1.4802 |
| .5893 | 1.4965 | 1.4963 | 1.195 | 1.4795 | 1.4792 |
| .6563 | 1.4922 | 1.4918 | 1.316 | 1.4784 | 1.4782 |
| .823 | 1.4857 | 1.4851 | 1.461 | 1.4775 | 1.4774 |
| .878 | 1.4845 | 1.4837 | 1.645 | 1.4768 | 1.4769 |
| .940 | 1.4834 | 1.4825 | 1.881 | 1.4760 | 1.4760 |
| | | | ∞ | 1.5 | 1.4739 |

Min. (M.-P.)

This is a test of the formula

$$n^2 = 1 + \frac{1.1724\lambda^2}{\lambda^2 - (1366)^2}$$

for xylol by Martens Ann. d. Physik, [4] ser. 6, 603 (1901).

TABLE XII.

Comparison of reflecting power as obtained by the method of residual rays with the dielectric constant.

| | Reflecting power for residual rays of | | | | | | | | Dielectric constant K | R_{∞} |
|--------------------------------------------------|---------------------------------------|-------------------------------|------------------|-----------------|-----------------|----------------|-------------------|--------------------------|-------------------------|--------------|
| | CaFl ₂ 23 μ | CaFl ₂ 33 μ | NaCl 52 μ | KCl 63 μ | KBr 83 μ | KI 94 μ | TlBr 117 μ | Hg vapor ca 300 μ | | |
| Calcite | 4.9 | 61.0 | 19.6 | 12.7 | 31.6 | 65.6 | 39.6 | 25.0 | 8.3 | 23.5 |
| Marble | 5.2 | 53.3 | 18.0 | 11.8 | 25.1 | 54.4 | 36.0 | 24.5 | 8.2 | 23.5 |
| Gypsum | 13.2 | 18.5 | 33.0 | 28.5 | 30.9 | 35.4 | 23.8 | 22.1 | 7.5 | 21.2 |
| Fluorspar | 55.3 | 83.0 | 30.0 | 25.2 | 21.6 | 20.4 | 20.6 | 19.9 | 6.82 | 19.9 |
| Rock salt | 2.1 | 1.7 | 80.2 | 64.5 | 27.5 | 24.3 | 19.9 | 17.9 | 5.82 | 17.2 |
| Sylvine | 2.3 | 1.6 | 39.9 | 80.0 | 37.5 | 24.4 | 17.4 | 14.0 | 4.75 | 13.8 |
| KBr | 3.8 | 3.2 | 2.2 | 18.0 | 82.8 | 56.3 | 21.9 | 14.9 | 4.60 | 13.5 |
| KI | 5.6 | 4.4 | 2.2 | 2.3 | 30.2 | 75.0 | 23.0 | 15.8 | 5.10 | 14.9 |
| NH ₄ Cl | 3.8 | 3.1 | 79.5 | 54.7 | 32.3 | 26.3 | 23.4 | 19.8 | 6.85 | 20.0 |
| NH ₄ Br | 6.0 | 3.8 | 55.1 | 66.4 | 41.5 | 28.8 | 24.4 | 21.1 | 6.98 | 20.3 |
| AgCl | 9.4 | 6.9 | 19.9 | 38.8 | 48.9 | 49.3 | 39.1 | 31.4 | 10.9 | 28.5 |
| AgBr | 12.9 | 11.7 | 6.1 | 4.3 | 25.9 | 40.0 | 42.7 | 35.6 | 12.1 | 30.7 |
| AgCN | 8.3 | 7.0 | 4.0 | 3.2 | 32.4 | 48.8 | 22.0 | 17.3 | 5.57 | 16.4 |
| Hg(Cl) ₂ | — | — | — | — | — | — | 22.6 | 19.3 | 6.52 | 19.1 |
| HgCl ₂ | 9.8 | 8.3 | 6.8 | 4.5 | 35.0 | 56.5 | 51.3 | 33.5 | 9.36 | 25.7 |
| TlCl | 13.9 | 10.5 | 8.1 | 38.0 | 74.0 | 80.6 | 76.0 | 56.7 | 35 | 51 |
| TlBr | 15.1 | 13.1 | 9.6 | 5.5 | 8.2 | 28.0 | 76.8 | 59.7 | 42 | 54 |
| TlI | 17.4 | 16.7 | 16.3 | 12.1 | 7.2 | 6.9 | 48.6 | 50.3 | 30 | 47 |
| Pb(Cl) ₂ | 10.2 | 6.9 | 23.3 | 43.2 | 61.8 | 71.4 | 58.9 | 51.8 | 42 | 54 |
| Water | 6.5 | 7.2 | 9.3 | 10.6 | 10.9 | 11.1 | 12.7 | 15.1 | 81 | 64 |
| H ₂ SO ₄ | 8.8 | 7.9 | 16.9 | 18.7 | 17.7 | 17.7 | 18.4 | 21.7 | — | — |
| 2H ₂ O+H ₂ O | 9.2 | 10.9 | 19.4 | 20.6 | 22.3 | — | — | 28.8 | — | — |
| 2H ₂ O+H ₂ SO ₄ | 8.1 | 9.6 | 14.2 | 17.0 | 19.0 | — | — | 26.5 | — | — |
| Glycerin | 5.8 | 5.5 | 5.2 | 6.3 | 7.5 | — | 8.5 | 9.4 | 56.2 | 58.1 |
| Castor oil | 4.0 | 4.0 | 4.1 | 4.5 | 4.4 | — | 4.3 | 4.8 | 4.78 | 13.9 |

Min. (M.-P.)
 Maxwell's formula, $R = (K^{1/2} - 1)^2 / (K^{1/2} + 1)^2$ as investigated by Rubens, Preuss. akad. Phys.-math. Kl. Sitz. ber. 1915 p. 4 and 1916 p. 1280.
 Rubens and Liebisch (ibid. 1919. p. 198 and 876).

TABLE XIII. Dispersion constants of alcohols.

| λ (cm) | Methyl alcohol | | |
|----------------|----------------|----------|-------|
| | n | κ | K |
| 33.30 | 6.04 | 0.38 | 31.1 |
| 40.20 | 5.40 | 0.23 | 30.8 |
| 48.88 | 5.74 | 0.12 | 32.5 |
| 49.00 | 5.71 | 0.12 | 32.3 |
| 55.88 | 5.52 | 0.09 | 30.2 |
| 65.34 | 5.16 | 0.08 | 31.8 |
| 72.00 | 5.69 | 0.07 | 32.2 |
| 75.44 | 5.87 | 0.06 | 34.4 |
| 79.92 | 5.43 | 0.06 | 29.4 |
| 84.74 | 5.64 | 0.06 | 3.18 |
| Ethyl alcohol | | | |
| 29.40 | 2.97 | 0.705 | 4.44 |
| 38.00 | 3.45 | 0.524 | 8.62 |
| 45.00 | 3.74 | 0.347 | 12.30 |
| 50.94 | 3.71 | 0.289 | 12.6 |
| 57.96 | 4.00 | 0.286 | 14.7 |
| 65.34 | 4.29 | 0.217 | 17.5 |
| 72.00 | 4.47 | 0.226 | 14.0 |
| 74.80 | 4.37 | 0.179 | 18.5 |
| 79.70 | 4.31 | 0.171 | 18.0 |
| 84.70 | 4.44 | 0.161 | 19.2 |
| 89.70 | 4.60 | 0.131 | 20.9 |
| Propyl alcohol | | | |
| 29.38 | 1.22 | 2.04 | -4.70 |
| 31.20 | 1.31 | 1.63 | -2.84 |
| 37.16 | 1.64 | 0.839 | +0.79 |
| 44.86 | 3.00 | 0.444 | +7.22 |

Obtained by means of Maxwell's formula $R = [(K^{1/2} - 1) / (K^{1/2} + 1)]^2 \cdot K = n^2(1 - \kappa^2)$;
 $\bar{n} = n(1 - i\kappa)$; See Eq. (20) above] C. Potapenko, Zeits. f. Physik 20, 21 (1923).

TABLE XIV. Ordinary ray in quartz.

| <i>a</i> | | | | | |
|-----------------------------------|----------|----------|---------|---------|-------------|
| Wave-length λ in μ | Observed | Computed | I | II | $\delta\pi$ |
| 0.198 | 1.65070 | 1.65077 | + 7 | 1.65077 | +7 |
| 0.274 | 1.5850 | 1.58757 | + 7 | 1.58757 | +7 |
| 0.358 | 1.56400 | 1.56395 | - 5 | 1.56395 | -5 |
| 0.434 | 1.553869 | 1.5539 | + 3 | 1.5539 | +3 |
| 0.534 | 1.54663 | 1.5466 | + 3 | 1.5466 | +3 |
| 0.656 | 1.541807 | 1.5419 | + 9 | 1.5419 | +9 |
| 1.160 | 1.5329 | 1.5330 | + 1 | 1.5329 | ± 0 |
| 1.617 | 1.5271 | 1.5271 | ± 0 | 1.5269 | -2 |
| 1.969 | 1.5216 | 1.5221 | + 5 | 1.5216 | ± 0 |
| 2.32 | 1.5156 | 1.5162 | + 6 | 1.5152 | -4 |
| 2.60 | 1.5099 | 1.5111 | + 12 | 1.5096 | -3 |
| 2.86 | 1.5039 | 1.5057 | + 18 | 1.5034 | -5 |
| 3.06 | 1.4985 | 1.5013 | + 28 | 1.4983 | -2 |
| 3.21 | 1.4942 | 1.4980 | + 38 | 1.4944 | +2 |
| 3.42 | 1.4877 | 1.4926 | + 49 | 1.4879 | +2 |
| 3.67 | 1.4790 | 1.4861 | + 63 | 1.4799 | +1 |
| 3.84 | 1.4739 | 1.4812 | + 73 | 1.4738 | -1 |
| 4.01 | 1.4678 | 1.4761 | + 83 | 1.4683 | +5 |
| 4.15 | 1.4619 | 1.4717 | + 98 | 1.4616 | -3 |
| 4.26 | 1.4567 | 1.4682 | +115 | 1.4570 | +3 |

| <i>b</i> | | | | | | | |
|--------------------|-----------------|-----------------|---------------------|--------------------|-----------------|-----------------|---------------------|
| λ in μ | n Observed | n Computed | $\delta \cdot 10^4$ | λ in μ | n Observed | n Computed | $\delta \cdot 10^4$ |
| 0.19881 | 1.65070 | 1.65077 | +0.7 | 1.617 | 1.5272 | 1.5270 | -2 |
| 0.23125 | 1.61402 | 1.61396 | -0.6 | 1.969 | 1.5216 | 1.5216 | 0 |
| 0.27467 | 1.58750 | 1.58757 | +0.7 | 2.327 | 1.5156 | 1.5153 | -3 |
| 0.31798 | 1.57290 | 1.57273 | -1.7 | 2.59 | 1.5101 | 1.5098 | -3 |
| 0.35818 | 1.56400 | 1.56395 | -0.5 | 2.84 | 1.5039 | 1.5039 | 0 |
| 0.40458 | 1.557059 | 1.5571 | 0 | 3.03 | 1.4987 | 1.4986 | -1 |
| 0.43409 | 1.553869 | 1.5539 | 0 | 3.18 | 1.4944 | 1.4946 | +2 |
| 0.48616 | 1.549606 | 1.5496 | 0 | 3.40 | 1.4879 | 1.4880 | +1 |
| 0.53496 | 1.546633 | 1.5466 | 0 | 3.63 | 1.4799 | 1.4801 | +2 |
| 0.58932 | 1.544147 | 1.5442 | +1 | 3.80 | 1.4740 | 1.4739 | -1 |
| 0.65633 | 1.541807 | 1.5419 | +1 | 3.96 | 1.4679 | 1.4676 | -3 |
| 0.76824 | 1.538930 | 1.5390 | +1 | 4.09 | 1.4620 | 1.4619 | -1 |
| 1.160 | 1.5329 | 1.5329 | 0 | 4.20 | 1.4569 | 1.4569 | 0 |

Min. (M.-P.)

Table XIVa according to

$$n^2 = 2.35681 + 0.010654/(\lambda^2 - 0.010627) - 0.01113\lambda^2 \quad (\text{I})$$

$$n^2 = 2.35681 + 0.010654/(\lambda^2 - 0.010657) - 0.01113\lambda^2 - 0.0001023\lambda^4. \quad (\text{II})$$

Table XIVb according to

$$n^2 = 3.4629 + 0.010654/(\lambda^2 - 0.010627) + 111.47/(\lambda^2 - 100.8).$$

H. Rubens, Wied. Ann. 53, 278 (1894) and 54, 476 (1895).

TABLE XV. Ordinary ray in quartz.

| λ in μ | Obs. n | Comp. n | λ in μ | Obs. n | Comp. n |
|--------------------|----------|-----------|--------------------|----------|-----------|
| 33 | 2.53 | 2.39 | 83 | 2.15 | 2.16 |
| 52 | 2.23 | 2.22 | 110 | 2.13 | 2.15 |
| 63 | 2.19 | 2.19 | 313 | 2.11 | 2.14 |
| | | | ∞ | 2.11 | 2.14 |

Min. (M.-P.)

This is a formula for ordinary ray in quartz. (Cf. Table XIVa and XIVb.)

$$n^2 = 4.57877 + \frac{0.010654}{0.010627 - \lambda^2} + \frac{44.224}{78.22 - \lambda^2} + \frac{773.55}{430.56 - \lambda^2}$$

revised by Minkowski using frequencies found by Nichols, Ann. d. Physik [3] 60, 401 (1897) and Rubens and Nichols (ibid. p. 418). It applies to longer wave-lengths than XIVa and XIVb and applies at short wave-lengths as well as XIVb.

TABLE XVI. Dispersion formula constants for fluorspar, quartz, rocksalt and sylvine.

| | | $n^2 = n_\infty^2 + \sum_i \frac{M_{vi}}{\lambda^2 - \lambda_{vi}^2} - \frac{M_r}{\lambda^2 - \lambda_r^2} \text{ and } n^2 = b + \sum_i \frac{M_{vi}}{\lambda^2 - \lambda_{vi}^2} - K_1 \lambda^2 - K_2 \lambda^4$ | | | | | |
|----------------|----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------|-------------------------|-------------------------------|----------------------|---------------------------|
| | Computed by | Region in μ | n_∞^2 or b | M_{vi} | λ_{vi}^2 | M_r or K_1 | λ_r^2 or K_2 |
| Fluorspar | Paschen 1895 | 0.185-9.42 | $n_\infty^2 = 6.09651$ | $M_{v1} = 0.0061386$ | $\lambda_{v1}^2 = 0.008840$ | $M_r = 5114.65$ | $\lambda_r^2 = 1260.56$ |
| | Paschen 1895 | 0.185-9.42 | $b = 2.03888$ | 0.006166 | 0.0086959 | $K_1 = 0.003200$ | $K_2 = 0.0_529195$ |
| | Paschen 1901 | 0.325-9.42 | $n_\infty^2 = 6.09104$ | 0.00612093 | 0.008884 | $M_r = 5102.07$ | $\lambda_r^2 = 1259.19$ |
| | Paschen 1901 | 0.325-9.42 | $b = 2.03913$ | 0.006125 | 0.008884 | $K_1 = 0.003255$ | $K_2 = 0.0_52894$ |
| | Schönrock 1920 | 0.404-0.71 | $n_\infty^2 = 2.0388463$ | 0.00614797 | 0.0086716 | $K_1 = 0.00326424$ | $K_2 = 0.0_5289$ |
| Schönrock 1920 | 0.71-2.6 | $b = 2.0388303$ | 0.00616369 | 0.0069932 | $K_1 = 0.00321287$ | $K_2 = 0.0_5289$ | |
| Quartz | Rubens 1895 | 0.198-4.20 | $n_\infty^2 = 3.4629$ | 0.010654 | 0.010627 | $M_r = 111.47$ | $\lambda_r^2 = 100.77$ |
| | Schönrock 1927 | 0.4-0.71 | $n_\infty^2 = 3.4918838$ | 0.01067085 | 0.0106291 | $M_r = 114.37807$ | $\lambda_r^2 = 100.77$ |
| | Schönrock 1927 | 0.71-2.4 | $n_\infty^2 = 3.5968913$ | 0.01064379 | 0.0106291 | $M_r = 138.20519$ | $\lambda_r^2 = 111.45202$ |
| Rocksalt | Paschen 1908 | 0.185-22.3 | $n_\infty^2 = 5.680137$ | $M_{v1} = 0.01278685$ | $\lambda_{v1}^2 = 0.0148500$ | $M_r = 12059.95$ | $\lambda_r^2 = 3600$ |
| | Paschen 1908 | | $b = 2.330165$ | $M_{v2} = 0.0053433924$ | $\lambda_{v2}^2 = 0.02547414$ | $K_1 = 0.0_39285837$ | $K_2 = 0.0_6286086$ |
| Sylvine | Paschen 1908 | 0.185-22.5 | $n_\infty^2 = 3.86619$ | $M_{v1} = 0.008344206$ | $\lambda_{v1}^2 = 0.0119082$ | $M_r = 5569.715$ | $\lambda_r^2 = 3292.47$ |
| | Paschen 1908 | | $b = 2.174967$ | $M_{v2} = 0.00698382$ | $\lambda_{v2}^2 = 0.0255550$ | $K_1 = 0.0_3513495$ | $K_2 = 0.0_6167587$ |

| | | $\text{Constants of the formula } \frac{n^2-1}{n^2+2} = \sum_i \frac{C_i}{v_i^2 - v^2} \text{ according to Herzfeld and Wolf}$ | | |
|----|------------------------------|--------------------------------------------------------------------------------------------------------------------------------|----------------------------|-----------------------------|
| | | C_1 | C_2 | C_3 |
| 14 | Rocksalt | $C_1 = 3.83 \cdot 10^{31}$ | $C_2 = 7.19 \cdot 10^{31}$ | $C_3 = 0.30 \cdot 10^{31}$ |
| 15 | 0.185-0.656 μ Sylvine | $\lambda_{v1} = 0.03419\mu$ | | $\lambda_{v2} = 0.09359\mu$ |
| | 0.185-0.656 μ | $\lambda_{v1} = 0.05150\mu$ | $C_2 = 6.86 \cdot 10^{31}$ | $\lambda_{v2} = 0.09720\mu$ |

Wien-Harms Handbuch p. 176.

TABLE XVII. *Dispersion of metals.*
Comparison of computed and observed conductivities.

| Metal | Observer | C | λ_0' in μ | $\sigma \cdot 10^{-15}$ from optical data | $\sigma \cdot 10^{-15}$ Obs. |
|----------|------------------------------|-------|-----------------------|-------------------------------------------------|---------------------------------|
| Silver | Försterling and Fréedericksz | 50 | 17.6 | 140 | 570 |
| Gold | " | 44 | 37 | 250 | 420 |
| Platinum | " | 49 | 1.6 | 100 | 530 |
| Copper | " | 27 | 25 | 12 | 85 |
| Iridium | " | 100 | 0.83 | 13 | 17 |
| Mercury | Meier | 82.5 | 0.748 | 9.25 | 9.58 |
| Bismuth | Kent | 132 | 0.354 | 7.01 | 6.71 |
| Lead | " | 142.5 | 0.448 | 9.58 | 9.18 |
| Cadmium | " | 92.5 | 1.953 | 27.0 | 26.5 |
| Zinc | " | 129.5 | 0.854 | 16.7 | 17.3 |

$$2n^2\kappa = c\lambda'_0\lambda^3/(\lambda^2 + \lambda'_0{}^2), \quad n^2(1 - \kappa^2) = 1 - c\lambda^2\lambda'_0{}^2/(\lambda^2 + \lambda'_0{}^2)$$

Ja. (W.-H.)

K. Försterling and V. Fréedericksz, Ann. d. Physik **43**, 1227 (1914).

W. Meier, Diss. Göttingen 1910; Ann. d. Physik **31**, 1017 (1910).

C. V. Kent, Phys. Rev. (2) **14**, 459 (1919); See also H. A. Wilson, Phys. Rev. (2) **15**, 317 (1920).

Brian O'Brien, Phys. Rev. (2) **27**, 93 (1926).

3. Tables of f values

TABLE XVIII. *H and He⁺.*

| | H | | | Snoek ⁴ 1928-29 |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|---------------------------------------------|----------------------------------------------------------|--------------------------------------------|
| | Theory ¹ | Carst and Ladenburg ² 1928 | Experiment Snoek and Ornstein ³ 1928 | |
| $f_{H\alpha}/f_{H\beta}$ | 5.37 | 5.2 | 5.25 ± 0.1 | 5.13 |
| $f_{H\beta}/f_{H\gamma}$ | 2.66 | | | 2.44 |
| Method: | | Anomalous dispersion | Total absorption | Total absorption |
| ¹ E. Schroedinger, Ann. d. Physik 80 , 437 (1926); See p. 478. ² A. Carst and Ladenburg, Zeits. f. Physik 48 , 192 (1928). ³ J. L. Snoek and L. S. Ornstein, Zeits. f. Physik 50 , 600 (1928); See p. 604. ⁴ J. L. Snoek, Zeits. f. Physik 52 , 654 (1928-29). | | | | |
| | He ⁺ 6th quantum state | | Exper. | |
| | Theory | | | |
| | $\tau = 1.17 \times 10^{-8}$ sec. L. R. Maxwell, Phys. Rev. 38 , 1664 (1931). | | | $\tau = (1.1 \pm 0.2) \times 10^{-8}$ sec. |

TABLE XIX. *Li f values of lines in principal series.*

| Number of line in series | λ in A | 100f | |
|-----------------------------------------------------|----------------|----------------------------------------------|---------------------------------------|
| | | Experiment Anomalous dispersion ¹ | Theory ² |
| 1 | 6708 | (75.00) | 75. |
| 2 | 3233 | $0.549 \pm 3\%$ | 0.551 |
| 3 | 2741 | .478 | .471 |
| 4 | 2563 | .314 | .253 |
| 5 | 2475 | .192 | |
| 6 | 2426 | .128 | |
| 7 | 2395 | .091(3) | |
| 8 | 2374 | .0676 | |
| 9 | 2359 | .0520 | |
| 10 | 2348 | .0406 | |
| 11 | 2340 | .0327 | |
| 12 | 2334 | $.0266 \pm 5\%$ | |
| Continuous spectrum Σf : | | | 0.24 ³ .26 ⁴ |
| $\sum_1^{\infty} f_i + (\Sigma f)_{\text{cont.}} =$ | | | 1.01 |

¹ A. Filipov, Zeits. f. Physik **69**, 526 (1931). The column given above was calculated from Filipov's measured ratios of f values and Trumpy's calculated value for the first line. Accuracy 3 percent for the first lines and 5 percent for the last.

² B. Trumpy, Zeits. f. Physik **61**, 54 (1930); see p. 59. Zeits. f. Physik **66**, 720 (1930).

³ B. Trumpy, Zeits. f. Physik **54**, 372 (1929).

⁴ J. Hargreaves, Proc. Camb. Phil. Soc. **25**, 75 (1929).

TABLE XX. *Na. f values of lines in principal series.*

| Number of line in series | λ in A | Experiment anomalous dispersion ¹ Filipov and Prokofjew (1929) | Theory ² | Theory ³ | Experiment magneto-rotation (Ladenburg-Minkowski) ^{4, 5} 1931 | Experiment line breadth (Minkowski) ⁶ 1926 | Experiment magnetorotation of resonance radiation (Ellett) 1925 ⁶ | Duration of resonance radiation Hupfeld ⁷ 1929 |
|--------------------------|----------------|---------------------------------------------------------------------------|---------------------|---------------------|------------------------------------------------------------------------|-------------------------------------------------------|------------------------------------------------------------------------------|-----------------------------------------------------------|
| | | | | | | | | |
| | | | | | λ in A | Designation of line | f | |
| 1 | 5893 | (97.55) | 97.96 | 97.55 | 107 ± 5 | 97 ± 5 | 116 | 105 |
| 2 | 3303 | $1.403 \pm 3\%$ | 1.426 | 1.44 | | | $\tau =$ | $\tau =$ |
| 3 | 2853 | 0.205 | .221 | .241 | | | $= 1.35 \times 10^{-8}$ | $= (1.5 \pm .3) \times 10^{-8}$ |
| 4 | 2680 | 0.0631 | .073 | .098 | | | | |
| 5 | 2594 | .0256 | | | | | | |
| 6 | 2544 | .0134 | | | | | | |
| 7 | 2512 | .00811 | | | | | | |
| 8 | 2491 | .00537 | | | | | | |
| 9 | 2476 | .00384 | | | | | | |
| 10 | 2464 | .00284 | | | | | | |
| 11 | 2456 | .00217 | | | | | | |
| 12 | 2449 | .00173 | | | | | | |
| 13 | 2444 | .00140 | | | | | | |
| 14 | 2440 | .00116 | | | | | | |
| 15 | 2437 | .00092 (5) | | | | | | |
| 16 | 2434 | .00075 (2) | | | | | | |
| | | | | | 11398 | $2p-2s$ | 0.163 | |
| | | | | | 8189 | $2p-3d$ | 0.832 | |
| | | | | | 5688 | $2p-4d$ | 0.108 | |
| | | | | | 22084 | $2s-3p$ | 1.35 | |

¹ A. Filipov and W. K. Prokofjew, Zeits. f. Physik **56**, 458 (1929); See p. 470. The column given above was calculated from the measured ratios of f values and Trumpy's calculated value for the first line (97.55). Accuracy 3 percent for the first lines and 5 percent for the 12th and 13th.

² W. K. Prokofjew, Zeits. f. Physik **58**, 255 (1929); See p. 264.

³ B. Trumpy, Zeits. f. Physik **61**, 54 (1929); See p. 57.

⁴ R. Ladenburg and E. Thiele, Zeits. f. Physik **72**, 699 (1931); R. Ladenburg and R. Minkowski, Zeits. f. Physik **6**, 153 (1921).

⁵ R. Minkowski, Zeits. f. Physik **36**, 839 (1926).

⁶ A. Ellett, Jour. Opt. Soc. Am. **10**, 427 (1925).

⁷ H. H. Hupfeld, Zeits. f. Physik **54**, 484 (1929); See here also for data on τ for J_3, K_3, Na_2 .

⁸ See also W. Schütz, Zeits. f. Physik **45**, 30 (1927); M. Weingeroff, Zeits. f. Physik **67**, 679 (1931).

TABLE XXI. *Cs f values of lines in principal series.*

| Number of line in series | λ in Å | 100f Experiment ³ | Method |
|--------------------------|----------------|------------------------------|-------------------------------|
| 1 | 8943 | 32 ± 10% | Magnetic rotation |
| | 8521 | 66 " | |
| 2 | 4593 | 0.269 " | " |
| | 4555 | 1.15 " | |
| 5 | 3478 | 0.00383 | Total ² absorption |
| 6 | 3399 | .00230 | " |
| 7 | 3348 | .00179 | " |
| 8 | 3313 | .00141 | " |
| 9 | 3289 | .00102 | " |
| 10 | 3270 | .00081 | " |
| 11 | 3257 | .00062 | " |
| 12 | 3246 | .00049 | " |
| 13 | 3237 | .00039 | " |
| 14 | 3231 | .00030 | " |

¹ R. Minkowski and W. Mühlenbruch, *Zeits. f. Physik* **63**, 198 (1930).² F. Waibel, *Zeits. f. Physik* **53**, 459 (1929); See p. 480.³ For calculated values see T. Mutō *Pro. Phys.—Math. Soc. Japan* 3rd Ser. **12**, 93, (1930).TABLE XXII. *Doublet intensity ratios in principal series of alkalis.*

| Element | Number of lines in series | Wave-length of components | Absolute measured values of <i>f</i> | Ratio of <i>f</i> values | Observer and date | Method | Theory ¹⁰ Fermi |
|---------|---------------------------|---------------------------|--------------------------------------|--------------------------|----------------------------------------------|-----------------------|----------------------------|
| Na | 1 | 5896A 5890 | 0.36 0.71 | 2.0 ± 0.1 | Ladenburg and Minkowski ¹ 1921 | Magnetic rotation | 2.00 |
| | 1 | 5896 5890 | 0.32 0.64 | 2.0 ± (0.05) | Minkowski ² 1926 | Natural breadth | |
| K | 1 | 7699 7645 | | 2.0 ± 0.08 | Weiler ³ 1929 | Magnetic rotation | 2.0 |
| | 2 | 4047 4044 | | 2.0 ± 0.08 | " " | " | |
| | 2 | " | | 2.10 | Rasetti ⁴ 1924 | Anomalous dispersion | 2.2 |
| | 2 | " | | 1.91 ± 0.09 | Jacob ⁵ 1928 | Intensity measurement | |
| Rb | 2 | 4216 4202 | | > 2.3 | Jacob ⁵ 1928 | Intensity measurement | |
| | 2 | " | | 2.57 | Roschdestvenski ⁶ 1921 | Anomalous dispersion | |
| Cs | 1 | 8943 8521 | 0.32 0.66 ± 10% | 2.06 | Minkowski and Mühlenbruch ⁷ 1930 | Magnetic rotation | 2.02 |
| | 2 | 4593 4555 | 0.00269 0.0115 ± 10% | 4.3 > 3.6 | " " " Schütz ⁸ 1930 | " Total absorption | |
| | 2 | " | | 4.0 | Jacob ⁵ 1928 | Intensity measurement | 4.3 |
| | 2 | " | | 4.07 | Roschdestvenski ⁶ 1921 | Anomalous dispersion | |
| | 2 | " | | 3.85 | Rasetti ⁴ 1924 | | |
| | 2 | " | | 3.3 | Füchtbauer and Wolff ⁹ 1929 | | |
| | 3 | 3889 3876 | | 4.7 | | | |

¹ R. Ladenburg and R. Minkowski, *Zeits. f. Physik* **6**, 153 (1921).² R. Minkowski, *Zeits. f. Physik* **36**, 839 (1926).³ I. Weiler, *Ann. d. Physik* **1**, 361 (1929).⁴ F. Rasetti, *Il Nuovo Cim.* **1**, 115 (1924).⁵ H. Jacob, *Ann. d. Physik* **86**, 449 (1928).⁶ D. S. Roschdestvenski, *Trans. Opt. Inst. Leningrad* **2**, 13 (1921).⁷ R. Minkowski and W. Mühlenbruch, *Zeits. f. Physik* **63**, 198 (1930).⁸ W. Schütz, *Zeits. f. Physik* **64**, 687 (1930); p. 694.⁹ C. Füchtbauer and H. W. Wolff, *Ann. d. Physik* **3**, 359 (1929).¹⁰ E. Fermi, *Zeits. f. Physik* **59**, 680 (1930). The theoretical values are not very exact and it is difficult to estimate the accuracy of the calculation. The importance of Fermi's work lies in supplying an explanation for the deviations of doublet intensity ratios from 2.See also S. Sambursky, *Zeits. f. Physik* **49**, 731 (1928).

TABLE XXIII. Ratio of f values of the first to the second line of the principal series of the alkalis.

| Element | Ratio of f values | Ratio of emission probabilities | Method | Observer |
|---------|---------------------|---------------------------------|----------------------|--------------------------------------------------------------------------|
| Li | 136.5 | 31.4 | Anomalous dispersion | Filipov; Zeits. f. Physik 69 , 526 (1931). |
| Na | 69.5 | 21.8 | " " | Fil. and Prokofjew; Zeits. f. Physik 56 , 458 (1929). |
| K | 111.5 | 30.3 | " " | Prokofjew and Gamow; Zeits. f. Physik 44 , 887 (1927). |
| | 98.5 | 26.8 | Mag. rotation | J. Weiler; Ann. d. Physik 1 , 361 (1929). |
| Rb | 70.3 | 20.3 | Anomalous dispersion | D. S. Roschdestvenski; Trans. Opt. Inst. Leningrad 2 , 13 (1921). |
| Cs | 69.0 | 19.2 | Mag. rotation | Minkowski and Mühlenbruch; Zeits. f. Physik 63 , 198 (1930). |

TABLE XXIV. f and A values for forbidden lines.

| Element | Line | Spectroscopic notation | $100f$ | A | Method | Observer | |
|---------|--------|----------------------------------------------------------------|---------|--------------|----------------------|---------------------------------------------------------------|-------------------------------------------------------------------------------------------------|
| Na | 3426.7 | $\{1^2S_{1/2} - 3^2D_{3/2}\}$ $\{1^2S_{1/2} - 3^2D_{5/2}\}$ | 0.00025 | 280 | Anomalous dispersion | Prokofjew, Zeits. f. Physik 57 , 387 (1929). | |
| K | 4642.2 | $1^2S_{1/2} - 3^2D_{3/2}$ | 0.00037 | 230 | " | " | |
| | 4641.6 | $1^2S_{1/2} - 3^2D_{5/2}$ | | | | | |
| Rb | 5165.3 | $1^2S_{1/2} - 3^2D_{3/2}$ | 0.00032 | 400 | " | " | |
| Cs | 6894.7 | $1^2S_{1/2} - 3^2D_{3/2}$ | 0.00023 | 65 | " | The line $1^2S_{1/2} - 3^2D_{5/2}$ did not appear. | |
| | 6848.5 | $1^2S_{1/2} - 3^2D_{5/2}$ | | | | | |
| | 4425.8 | $1^2S_{1/2} - 4^2D_{3/2}$ | 0.00011 | 80 | | | Values opposite } refer to a sum of f values for two lines. For Rb only one line is observed. |
| | 4417.7 | $1^2S_{1/2} - 4^2D_{5/2}$ | | | | | |
| Hg | 2656 | $1^1S_0 - 3^1P_0$ | | 0.1 | Emission intensity | Gaviola, Est. de la Ciencias Físicas etc. LaPlata, Dec. 1929. | |
| | 2270 | $1^1S_0 - 3^1P_2$ | | $1 < A < 10$ | Absorption intensity | Lord Rayleigh, Proc. Roy. Soc. 117 , 294 (1930). | |

See also R. Ladenburg, Zeits. f. Elektrochemie, p. 631 (1930) and for theory A. Rubinowicz, Zeits. f. Physik **61**, 338 (1930). L. D. Huff and W. V. Houston, Phys. Rev. **36**, 842 (1930).

TABLE XXV. Ratio of f values for $1^1S_0 - 2^1P_1$ and $1^1S_0 - 2^3P_1$ of Ca, Sr, Ba.

| | f_1/f_2 | A_2/A_1 | λ_1 | λ_2 | Method |
|----|-----------|------------------------|-------------|-------------|----------------------|
| Ca | 33000 | 1.253×10^{-5} | 4227 | 6573 | Anomalous dispersion |
| Sr | 1660 | 26.92×10^{-5} | 4608 | 6893 | |
| Ba | 146 | 335.4×10^{-5} | 5536 | 7911 | |

Observed by W. Prokofjew, Zeits. f. Physik **50**, 701 (1928). See p. 708.

TABLE XXVI. *f* values for *Tl*.

$f_{2768} = .20; f_{3776} = 0.08$; Det. Kgl. Danske Videnskab. Selskab. Mathem fysiske Meddelelser 7, 12 (1926)

$f_{5350}/f_{3776} = A_{5350}/A_{3776} = 0.95$, Prokofjew and Solovjev, Zeits. f. Physik 48, 276, (1928.)

Other approximate values given in Kuhn's paper.

TABLE XXVII. *f* values for lines of *Cd*, *Hg*, and *Zn*.

| Element | Wave-length in Å | Spectroscopic notation | <i>f</i> | $\tau = 1/A$ in sec. | Method | Observer |
|---------|-----------------------------------------------|--------------------------|----------------------------------------------------------------------|----------------------------------------------------------------------|-------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------|
| Cd | 2288 ($\tau_{cl} = 2.37 \times 10^{-9}$) | $1^1S_0 - 2^1P_1$ | 1.20 1.19 | 1.98×10^{-9} 1.99×10^{-9} | Mag. rot. Line absor. | Kuhn ¹ 1926 Zemansky ² 1931 |
| | 3261 ($\tau_{cl} = 4.8 \times 10^{-9}$) | $1^1S_0 - 2^3P_1$ " " | 1.9×10^{-3} 2.1×10^{-3} 1.9×10^{-3} | 2.5×10^{-4} 2.3×10^{-4} 2.5×10^{-4} | Mag. rot. Pol. res. rad. Atomic beam | Kuhn ¹ 1926 Ellett ³ 1929 Koenig and Ellett ⁴ 1932 |
| Hg | 1849 ($\tau_{cl} = 1.55 \times 10^{-9}$) | $1^1S_0 - 2^1P_1$ | 0.96 | 1.6×10^{-9} | Indirect dispersion evidence | Ladenburg and Wolfsohn ⁵ 1930 |
| | 2537 | $1^1S_0 - 2^3P_1$ | 0.0285 | 1.02×10^{-7} | Canal ray | Wien ⁶ 1924 |
| | " | " " | 0.0257 | 1.13×10^{-7} | Pol. res. rad. | v. Keussler ⁷ 1927 |
| | " | " " | 0.0297 | $.98 \times 10^{-7}$ | " " | Olson ⁸ 1928 |
| | " | " " | 0.0255 ± 0.0005 0.0270 | 1.14×10^{-7} 1.08×10^{-7} | An. disp. Alternating voltage Line absor. | Ladenburg and Wolfsohn ⁵ 1930 Garrett and Webb ⁹ 1931 Zehnden and Zemansky ¹⁰ 1931 |
| Zn | 3076 ($\tau_{cl} = 2.91 \times 10^{-9}$) | $1^1S_0 - 2^3P_1$ | 0.00043 | 1×10^{-5} | Pol. res. rad. | Solleillet ¹¹ 1928 |

¹ W. Kuhn, Naturwiss. 13, 725 (1925); 14, 48 (1926); Det. Kgl. Danske Videnskab. Selskab. Mathem.-fysiske Meddelelser 7, 12 (1926).

² M. W. Zemansky, Zeits. f. Physik 72, 587 (1931).

³ A. Ellett, Phys. Rev. 33 (124A) (1929).

⁴ H. D. Koenig and A. Ellett, Phys. Rev. 39, 576 (1932).

⁵ R. Ladenburg and G. Wolfsohn, Zeits. f. Physik 65, 207 (1930).

⁶ W. Wien, Ann. d. Phys. 73, 483 (1924); cf. discussion in (5) and in R. Ladenburg and K. Minkowski, Ann. d. Physik 87, 298 (1928).

⁷ F. v. Keussler, Ann. d. Physik 82, 810 (1927).

⁸ Olson, Phys. Rev. 32, 443 (1928).

⁹ P. H. Garrett and H. W. Webb, Phys. Rev. 37, 1686 (1931).

¹⁰ W. Zehnden and M. W. Zemansky, Zeits. f. Physik 72, 442 (1931).

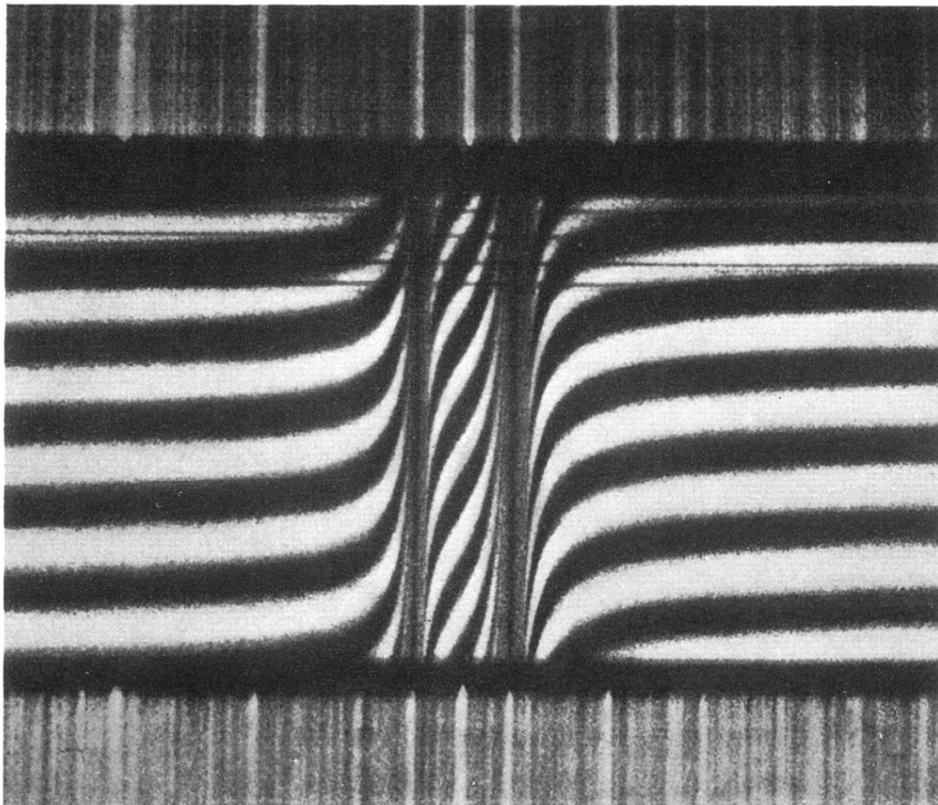
¹¹ P. Solleillet, Comptes Rendus 187, 723 (1928).

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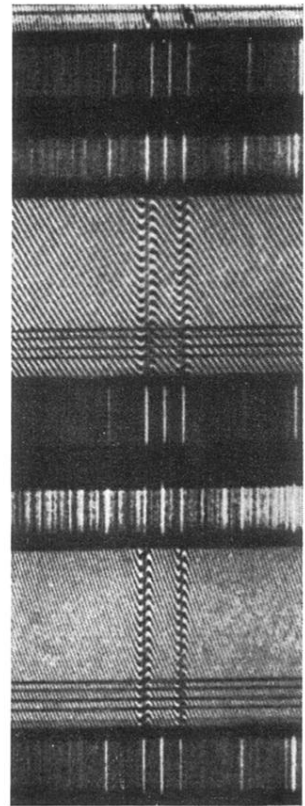
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G. Jaffe; Wien-Harms *Handbuch der Experimental Physik XIX Akademische Verlags Gessellschaft*, Leipzig, 1928. Referred to in text as Ja. (W-H).



A

U_p
↑



B

Fig. 6. Interference patterns. Obtained with Jamin type interferometer. A, without addition of plane parallel plate, B, various orders with plate inserted, showing hooks.