Two-Nucleon Interaction as a Research Tool in Particle Physics*

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Knowledge of the two-nucleon interaction can be used as a proving ground for general theories of elementary particles. Within this context three aspects of the nucleon–nucleon problem are examined: (1) the tests of some group theoretical schemes through the nucleon–nucleon interaction; (2) a summary of the status of test of general conservation laws; and (3) an attempt to investigate the general analyticity structure of the S matrix through the two-nucleon interaction.

I. INTRODUCTION

The two-nucleon interaction is a central problem in modern physics for at least three reasons. First, it is the oldest and best understood strong interaction studied by man and hence it is of inherent interest. Second, it serves as a basis for a fundamental understanding of nuclear structure and hence, in a sense, is the key to a vast area of physics. Third, we can use our extensive knowledge of the two-nucleon interaction as a proving ground for general theories of elementary particles.

My talk is devoted to some aspects of this third facet of the two-nucleon interaction. To some extent supplementing the lecture by Elliot Leader, I address myself in particular to three topics, which serve as examples for the variety of uses of our knowledge of the two-nucleon interaction. The list is not supposed to be exhaustive, and neither can the discussion of the three topics be exhaustive. First, I talk about the tests of some group theoretical schemes through the nucleon–nucleon interaction, then I summarize the status of tests of general conservation laws, and finally I discuss an attempt to investigate the general analyticity structure of the S-matrix through the two-nucleon interaction.

II. GROUP THEORETICAL SCHEMES

The two-nucleon interaction has been used recently[1–4] to test the validity of the $SU(12)$ scheme and its relatives, such as $U(12)$, $SU(12)\mathcal{L}$, and the coplanar $U(3)\times U(3)$. These tests are based on the result that, instead of the customary ten scattering amplitudes for nucleon–nucleon scattering at a given energy and angle (five each in the two isotopic spin states), in case of $SU(12)$ invariance, under certain assumptions, there are only four independent amplitudes. The restrictive assumption under which this result holds is that the baryons belong to the 364 representation of $SU(12)$. In terms of $SU(6)$ and $SU(2)$ representations, we have

$$SU(12)_{364}=SU(6)_{56}\otimes SU(2)_4\oplus SU(6)_{70}\otimes SU(2)_2 \tag{2.1}$$

and the two $SU(6)$ representations appearing here can, in turn, be thought of in terms of $SU(3)$ and $SU(2)$ representations as

$$SU(6)_{56}=SU(3)_8\otimes SU(2)_2\oplus SU(3)_{10}\otimes SU(2)_4 \tag{2.2}$$

and

$$SU(6)_{70}=SU(3)_1\otimes SU(2)_2\oplus SU(3)_8\otimes SU(2)_2\oplus SU(3)_{10}\otimes SU(2)_2\oplus SU(3)_8\otimes SU(2)_4. \tag{2.3}$$

Even assuming that the assignment of the ordinary baryons to the $SU(3)_8$ is experimentally well established, the assignment of these particles to $SU(12)_{364}$ cannot be regarded as unambiguously known. Assuming, nevertheless, this assignment, we can write[5–8] the scattering matrix of baryon–baryon scattering as

$$\begin{aligned}T_{fi}&=\langle p'_1\alpha'_1 j'_1 p'_2\alpha'_2 j'_2 \mid T \mid p_1\alpha_1 j_1 p_2\alpha_2 j_2\rangle\\&=\bar{B}_{A'B'C'}(p'_1\alpha'_1 j'_1)\bar{B}_{D'E'F'}(p'_2\alpha'_2 j'_2)M_{A'\cdots F';A\cdots F}(p'_1p'_2;\,p_1p_2)B_{ABC}(p_1\alpha_1 j_1)B_{DEF}(p_2\alpha_2 j_2).\end{aligned} \tag{2.4}$$

Here p_1, p_2, p'_1, and p'_2, are the initial and final momenta of the baryons, α_1, α_2, α'_1, and α'_2 their Dirac spin indices, and j_1, j_2, j'_1, and j'_2 their $SU(3)$ quantum numbers. The quantity $B_{ABC}(p_1\alpha_1 j_1)$ is the "spinor tensor"

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[1] P. Freund and S. Lo, Phys. Rev. **140,** B927 (1965).
[2] P. B. Kantor, T. K. Kuo, R. F. Peierls, and T. L. Trueman, Phys. Rev. **140,** B1008 (1965).
[3] D. A. Akyeampong and R. Delbourgo, Phys. Rev. **140,** B1013 (1965).
[4] R. A. Arndt, M. J. Moravcsik, and R. Wright, Nuovo Cimento (to be published).
[5] M. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters **13,** 514 (1964).
[6] M. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters **14,** 267 (1965).
[7] S. Lo, Phys. Rev. **140,** B94 (1965).
[8] B. Sakita and K. C. Wali, Phys. Rev. **139,** B1355 (1965).

of the 364 representation of $SU(12)$, and its indices A, B, and C go over 1, 2, $\cdots$, 12, since each represents a set (μ, w), where μ has four values and w has three. The quantity $M_{A'\cdots F';A\cdots F}(p'_1p'_2;\, p_1p_2)$ is the M matrix for the reaction in $SU(12)$ space.

Equation (2.4) is quite analogous to

$$T_{fi}=\langle p'_1\alpha'_1p'_2\alpha'_2 \mid T \mid p_1\alpha_1p_2\alpha_2\rangle$$
$$=U_{\mu'_1}(p'_1\alpha'_1)\bar{U}_{\mu'_2}(p'_2\alpha'_2)M_{\mu'_1\mu'_2;\mu_1\mu_2}(p'_1p'_2p_1p_2)U_{\mu_1}(p_1\alpha_1)U_{\mu_2}(p_2\alpha_2) \tag{2.5}$$

which is the form of the scattering matrix for nucleon–nucleon scattering in ordinary Dirac space, with $U_\mu(p,\alpha)$ being the Dirac spinor, $\mu=1, 2, 3, 4$. The customary way to rewrite Eq. (2.5) in a manifestly three-dimensional way is to use the Pauli spinor $\chi_m(\alpha)$ which describes the spin state of the particle in its rest frame, and observe that

$$U_\mu(p,\alpha)=\sum_{m=1}^{2} D_{\mu m}(p)\chi_m(\alpha), \tag{2.6}$$

where

$$D(p)=\left(\frac{p_0+m}{2m}\right)^{1/2}\begin{pmatrix}1+(\boldsymbol{\sigma}\cdot\mathbf{p}/p_0+m)\\ 1-(\boldsymbol{\sigma}\cdot\mathbf{p}/p_0+m)\end{pmatrix}. \tag{2.7}$$

In order to rewrite Eq. (2.4) in manifestly three-dimensional notation, one uses an analog of Eq. (2.6). In particular, assuming that the baryon spin-tensor is that of the 364 representation of $SU(12)$ [or any other representation containing the $SU(6)_{56}\otimes SU(2)_4$ product] we can write

$$B(p,\alpha,j)_{\mu_1w_1\mu_2w_2\mu_3w_3}=\sum_{m_1m_2m_3} D_{\mu_1m_1}(p)D_{\mu_2m_2}(p)D_{\mu_3m_3}(p)B(\alpha,j)_{m_1w_1m_2w_2m_3w_3}, \tag{2.8}$$

where $B(\alpha,j)$ is the 56 representation baryon tensor of $SU(6)$. This, in turn, can be written as a product of Pauli spinor and $SU(3)$ tensors as follows:

$$B_{m_1w_1m_2w_2m_3w_3}=\chi_{m_1m_2m_3}d_{w_1w_2w_3}+(3\sqrt{2})^{-1}[(2\epsilon_{m_1m_2}\chi_{m_3}+\epsilon_{m_2m_3}\chi_{m_1})\epsilon_{w_1w_2w_4}b_{w_3}{}^{w_4}+(\epsilon_{m_1m_2}\chi_{m_3}+2\epsilon_{m_2m_3}\chi_{m_1})\epsilon_{w_2w_3w_4}b_{w_1}{}^{w_4}]. \tag{2.9}$$

Here $\chi_{m_1m_2m_3}$ is the spin-$\frac{3}{2}$ spinor in ordinary, three-dimensional space, totally symmetric in the indices; i.e., a spin-$\frac{3}{2}$ analog of χ_{m_1}. The tensor $b_{w_2}{}^{w_1}$ and $d_{w_1w_2w_3}$ are the $SU(3)$ octet and decuplet tensors, respectively, and $\epsilon_{m_1m_2}$ and $\epsilon_{m_1m_2m_3}$ are the two- and three-dimensional Levi-Civita (totally antisymmetric) tensors.

Substituting Eq. (2.9) into Eq. (2.8), and then Eq. (2.8) into Eq. (2.4) permits us to write the last of these in terms of only Pauli spinors as far as the spin space part of the ordinary (spin-$\frac{1}{2}$) baryon space is concerned.

If we assume invariance under $SU(12)$, we can write the M of Eq. (2.4) in a diagonal form

$$M_{A'\cdots F';A\cdots F}=\sum_{(p)} \mathfrak{J}^{(p)}(s,t,u)\delta_{AA'}\delta_{B'B}\cdots\delta_{F'F}, \tag{2.10}$$

where $\mathfrak{J}^{(p)}(s,t,u)$ is a function only of the kinematic variables s, t, u in ordinary momentum space and contains all the dynamical information about the reaction, and the sum is over all 6! permutations (p) of $A, \cdots F$.

The key observation consists now of noting that the B_{ABC} denoting the 364 representation of $SU(12)$ is completely symmetric in its indices. As a result, of the 720 permutations only those will give different terms in the contraction of Eq. (2.4) in which the *number* of indices contracted between, say $\bar{B}(p'_1\alpha'_1j'_1)$ and $B(p_1\alpha_1j_1)$ is different. There are clearly only four such different possibilities, and hence we get

$$\begin{aligned}T_{fi}=&\mathfrak{F}_1\bar{B}_{ABC}(p'_1\alpha'_1j'_1)\bar{B}_{DEF}(p'_2\alpha'_2j'_2)B_{ABC}(p_1\alpha_1j_1)B_{DEF}(p_2\alpha_2j_2)\\&+\mathfrak{F}_2\bar{B}_{ABC}(p'_1\alpha'_1j'_1)\bar{B}_{DEC}(p'_2\alpha'_2j'_2)B_{ABF}(p_1\alpha_1j_1)B_{DEF}(p_2\alpha_2j_2)\\&+\mathfrak{F}_3\bar{B}_{ABC}(p'_1\alpha'_1j'_1)\bar{B}_{DBC}(p'_2\alpha'_2j'_2)B_{AEF}(p_1\alpha_1j_1)B_{DEF}(p_2\alpha_2j_2)\\&+\mathfrak{F}_4\bar{B}_{ABC}(p'_1\alpha'_1j'_1)\bar{B}_{ABC}(p'_2\alpha'_2j'_2)B_{DEF}(p_1\alpha_1j_1)B_{DEF}(p_2\alpha_2j_2).\end{aligned} \tag{2.11}$$

That only four different amplitudes exists can also be seen from the decomposition

$$364\otimes 364=1\oplus 143\oplus 5940\oplus 126412. \tag{2.12}$$

In fact, $\mathfrak{F}_1$, $\mathfrak{F}_2$, $\mathfrak{F}_3$, and $\mathfrak{F}_4$ are the amplitudes pertaining to the 1-, 143-, 5940-, and 126 412-dimensional representations of $SU(12)$, respectively.

Having obtained Eq. (2.11), we can use Eqs. (2.9) and (2.8) to rewrite it in terms of Pauli spinors, which then allows us to calculate the (linear) relations between the four $\mathfrak{F}$'s and the five Wolfenstein parameters which are also amplitudes of nucleon–nucleon scattering in the three-dimensional notation, and hence must be some linear combinations of the $\mathfrak{F}$'s. In expressing the five Wolfenstein parameters in terms of the four $\mathfrak{F}$'s, we in fact impose a linear constraint on the former. This constraint turns out to be extremely simple, namely,

$$H=0, \tag{2.13}$$

where H is the amplitude of the term

$$\frac{\boldsymbol{\sigma}_1\cdot(\mathbf{k}-\mathbf{k}')}{|\mathbf{k}-\mathbf{k}'|}\frac{\boldsymbol{\sigma}_2\cdot(\mathbf{k}-\mathbf{k}')}{|\mathbf{k}-\mathbf{k}'|}-\frac{\boldsymbol{\sigma}_1\cdot(\mathbf{k}+\mathbf{k}')}{|\mathbf{k}+\mathbf{k}'|}\frac{\boldsymbol{\sigma}_2\cdot(\mathbf{k}+\mathbf{k}')}{|\mathbf{k}+\mathbf{k}'|}.$$

In addition to Eq. (2.13), which is supposed to hold for both proton–proton and neutron–proton scattering, we also get a connection between the amplitudes of the isotopic singlet and isotopic triplet part of nucleon–nucleon scattering, since the *four* amplitudes of Eq. (2.11) are supposed to describe *any kind* of nucleon–nucleon scattering. In fact, they are supposed to describe any kind of baryon–baryon scattering, but this prediction cannot be verified at the present since experimental information is available only on nucleon–nucleon, and to some extent, nucleon–antinucleon scattering. On the other hand, nucleon–nucleon scattering is in any case the most favorable testing ground, because of the very small mass difference between the proton and the neutron. If other baryon–baryon scattering data did not relate to the nucleon–nucleon data according to Eq. (2.11), one could always claim reprieve on account of the large hyperon–nucleon mass difference, which is supposed to break all the assumed group theoretical symmetries. In the comparison of proton–proton and neutron–proton scattering data, however, this excuse is much less plausible.

Finally, when we deal with baryon–baryon scattering involving identical particles, Eq. (2.11) must change sign under the exchange of the labels 1 and 2, which results in

$$\mathfrak{F}_1(s,t,u)=-\mathfrak{F}_4(s,u,t) \tag{2.14}$$

and

$$\mathfrak{F}_2(s,t,u)=-\mathfrak{F}_3(s,u,t). \tag{2.15}$$

This imposes two further constraints among the five Wolfenstein parameters for proton–proton scattering. These constraints are

$$\begin{aligned}(x_5y_3+x_7y_1)B+(x_5y_2-x_7y_1)G+(x_7y_1-x_5y_2)N-2iC&=0\\(x_4y_3+x_6y_1)B+(x_4y_2-x_6y_1)G+(x_6y_1-x_4y_2-1)N&=0,\end{aligned} \tag{2.16}$$

where B, G, N, and C are the Wolfenstein parameters defined by

$$\begin{aligned}M(k',k)=&B\mathcal{P}_s+C(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{k}\times\mathbf{k}'+N\boldsymbol{\sigma}_1\cdot\mathbf{k}\times\mathbf{k}'\boldsymbol{\sigma}_2\cdot\mathbf{k}'\times\mathbf{k}\mathcal{P}_T\\&+\tfrac{1}{2}G[\boldsymbol{\sigma}_1\cdot(\mathbf{k}-\mathbf{k}')\boldsymbol{\sigma}_2\cdot(\mathbf{k}-\mathbf{k}')+\boldsymbol{\sigma}_1\cdot(\mathbf{k}+\mathbf{k}')\boldsymbol{\sigma}_2\cdot(\mathbf{k}+\mathbf{k}')]+\tfrac{1}{2}H[\boldsymbol{\sigma}_1\cdot(\mathbf{k}-\mathbf{k}')\boldsymbol{\sigma}_2\cdot(\mathbf{k}-\mathbf{k}')-\boldsymbol{\sigma}_1\cdot(\mathbf{k}+\mathbf{k}')\boldsymbol{\sigma}_2\cdot(\mathbf{k}+\mathbf{k}')]\mathcal{P}_T,\end{aligned}$$

($\mathcal{P}_s$ and $\mathcal{P}_T$ are the spin singlet and triplet projection operators), and

$$\begin{aligned}y_1&\equiv(x_2-x_3)^{-1},\\y_2&\equiv x_2/x_1(x_2+x_3),\\y_3&\equiv x_3/x_1(x_2+x_3),\\x_1&\equiv(1-\gamma^3),\\x_2&\equiv(1+\gamma)(14\cos(\phi_a-\phi_b)-17),\\x_3&\equiv(1-\gamma)(14\cos(\phi_a-\phi_b)+17),\\x_4&\equiv\cos\phi_a-\gamma^3\cos\phi_b,\\x_5&\equiv\sin\phi_a-\gamma^3\sin\phi_b,\\x_6&\equiv10\cos(\phi_a-\phi_b)[\cos\phi_a-\gamma\cos\phi_b]+21(\cos\phi_b-\gamma\cos\phi_a),\\x_7&\equiv10\cos(\phi_a-\phi_b)[\sin\phi_a-\gamma\sin\phi_b]+21(\sin\phi_b-\gamma\sin\phi_a),\\\gamma&\equiv b^2/a^2,\\a\exp(i\tfrac{1}{2}\phi_a)&\equiv[m(E+m)]^{-1}[(E+m)^2-k^2e^{i\theta}],\\b\exp(i\tfrac{1}{2}\phi_b)&\equiv[m(E+m)]^{-1}[(E+m)^2+k^2e^{i\theta}].\end{aligned}$$

Similar constraints also hold for neutron–proton scattering at 90° in the center of mass system.

Equation (2.13) has a simple interpretation, namely the term whose coefficient is H is the only term in the M matrix whose matrix elements between two states of different total spin component in the direction perpendicular to the scattering plane are nonzero. Thus $H=0$ can be interpreted as $SU(12)$ symmetry demanding conservation of total spin component perpendicular to the scattering plane.

It might be mentioned that for S waves only, at low energies, the restriction discussed above follows already from $SU(6)$ symmetry. This is plausible in view of the fact that $SU(6)$ can be considered as a nonrelativistic, in fact static limiting case of $SU(12)$.

Let us investigate now the experimental information accumulated for nucleon–nucleon scattering agrees with the restrictions imposed by $SU(12)$ on the scattering amplitudes. In doing so one can proceed in two ways.

One way is to impose the restrictions (2.13) and (2.16) on the amplitudes and then calculate[1–3] from them the experimental observables. The constraints, (2.13) and (2.16) would then generate constraints on these observables, which could be tested against experimental data.

The other possibility is to determine the amplitudes themselves[4] from the experimental data, and then check on the amplitudes directly whether the constraints (2.13) and (2.16) hold or not.

In principle, there is no difference between the two methods, and they should give the same results. In practice, however, the second method is much preferable. This is so because the constraints (2.13) and (2.16) will produce simple restrictions only for a few observables, only some of which can be measured with existing techniques, and most of them are difficult experiments even at that. Thus, the actual comparison of the restrictions on the *observables* with experimental information will be based only on a few pieces of relatively poorly measured data. The results of such comparison, therefore, are not very conclusive.

In contrast, a study of the amplitudes determined from experimental information, as to whether Eqs. (2.13) and (2.16) are satisfied or not, utilizes *all* experimental information on nucleon–nucleon scattering, including datum points at isolated energies and angles. As a result, the constraints can be studied with greater precision and at all energies and angles.

In the first method, for proton–proton scattering, Eq. (2.13) will result in $5^2-4^2=9$ relations among the observables, while for neutron–proton scattering, the number is $6^2-5^2=11$. We will use the notation $L(a,\, b;\, c,\, d)$ for an experimental observable in which the spin states of the first initial, second initial, first final, and second final particles are described by $a, b, c,$ and d, respectively. The tags, $a, b, c,$ and d can be 0, l, m, and n, where 0 denotes an unpolarized state, l polarized in the $\hat{l}$ direction, etc. The three directions l, $\hat{m}$, and $\hat{n}$ are defined by

$$\hat{l}\equiv \mathbf{q}'-\mathbf{q}/|\,\mathbf{q}'-\mathbf{q}\,|, \qquad \hat{m}\equiv \mathbf{q}'\times\mathbf{q}/|\,\mathbf{q}'\times\mathbf{q}\,|, \qquad \hat{n}\equiv l\times\hat{m}, \tag{2.17}$$

where $\mathbf{q}$ and $\mathbf{q}'$ are the initial and final momenta in the center of mass system. We then have for proton–proton scattering

$$L(l,0;l,0)=L(n,0;n,0), \tag{2.18}$$

$$L(0,0;0,0)-L(m,0;m,0) =L(0,m;m,0)-L(0,0;m,m), \tag{2.19}$$

$$L(m,n;0,l)=-L(m,l;0,n), \tag{2.20}$$

$$L(n,n;0,0)=L(l,l;0,0), \tag{2.21}$$

$$L(l,0;0,l)=L(n,0;0,n), \tag{2.22}$$

$$L(l,m;0,n)=-L(n,m;0,l), \tag{2.23}$$

$$L(n,l;0,m)=-L(l,n;0,m), \tag{2.24}$$

$$L(n,l;0,0)=0, \tag{2.25}$$

$$L(n,n;0,m)=0. \tag{2.26}$$

For neutron–proton scattering, the last two relations are replaced by

$$L(n,l;0,0)=-L(l,n;0,0), \tag{2.25a}$$

$$L(n,n;0,m)=L(l,l;0,m), \tag{2.26a}$$

and the following two more relations are added

$$L(l,0;0,n)=-L(n,0;0,l), \tag{2.27}$$

$$L(l,m;0,l)=L(n,m;0,n). \tag{2.28}$$

If we use the second method, Eq. (2.13) can be checked directly against the experimental H. An example of this is shown in Fig. 1, where the real and imaginary parts of H are plotted at some sample energies, for proton–proton and neutron–proton scattering, as a function of angle, indicating also the limits of error as obtained from an energy-dependent modified-phase-shift analysis of the experimental data. It is clear from that figure that Eq. (2.13) is not satisfied, not even in the forward direction, where the *a priori* somewhat more reliable versions of $SU(12)$ are supposed to hold.

A special kind of test can be carried out by looking at the singlet and triplet scattering lengths of neutron–proton scattering. This can be done, for instance, by using Table II and Eq. (14) of Ref. 2, and taking the zero energy limit. In that case α and β of Ref. 2 are real, and equal to each other. Furthermore, for S waves, $\mathfrak{F}_1=-\mathfrak{F}_4$ and $\mathfrak{F}_2=-\mathfrak{F}_3$ hold at all angles. Thus we get from Table II of Ref. 2

$$B=64\alpha^6[f_0-\tfrac{1}{27}f_2]=N=G-N$$

$$C=0=H. \tag{2.29}$$

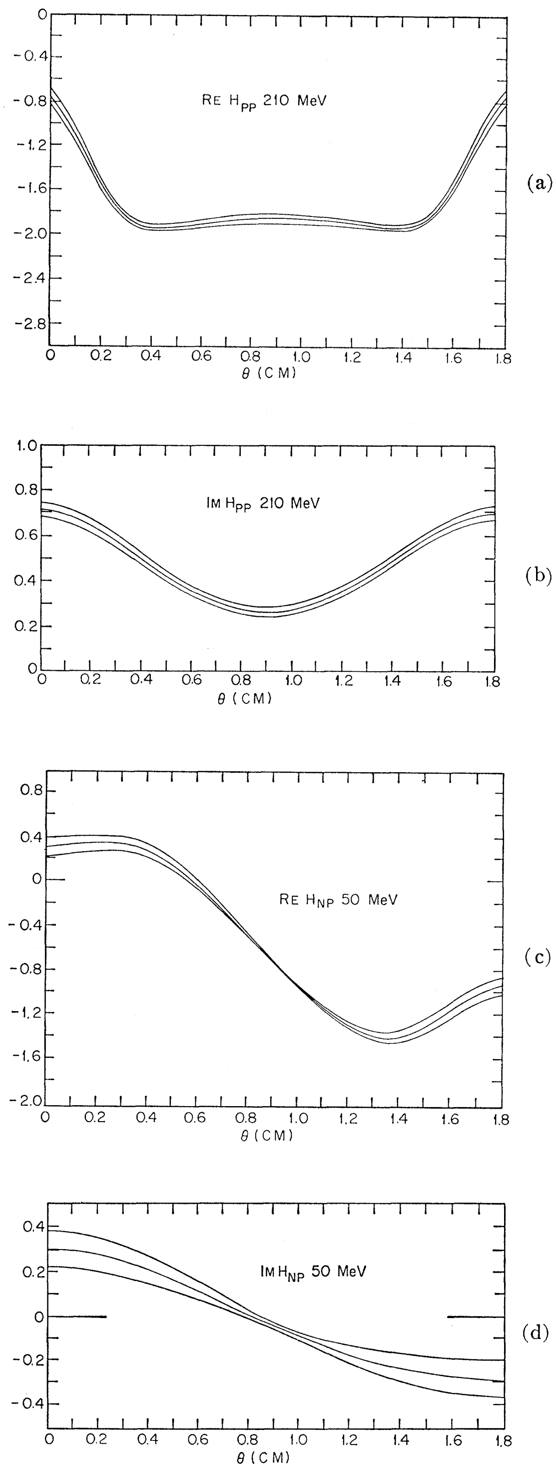

Fig. 1. The nucleon–nucleon scattering amplitude H at sample energies, as obtained from an energy-dependent modified-phase-shift analysis of experimental data. The center lines are the values obtained in the analysis, while the two lines on each side of the values indicate the error limits. The ordinate is in arbitrary units. The abscissa is in units of 100°.

Since for S waves only, we have

$$B = M_{ss} = (2ik)^{-1} P_0(\theta)[\exp(2i\delta_s) - 1]$$

$$N = G - N = (2ik)^{-1} P_0(\theta)[\exp(2i\delta_T) - 1] \quad (2.30)$$

and, in the zero energy limit

$$k \cot \delta = -1/a \quad (2.31)$$

we obtain

$$a_s = a_T \quad (2.32)$$

for neutron–proton scattering. In actuality, we have

$$a_s \approx -4a_T \quad (2.33)$$

which hardly can be called in agreement with Eq. (2.32). It might be mentioned again that Eq. (2.32) actually follows already from $SU(6)$ invariance.

The above conclusions concerning the validity of the predictions of $SU(12)$ invariance are quite discouraging indeed. It is possible, however, to be somewhat more charitable in the comparison of theory and experiment and make explicit allowance for those facts of life which $SU(12)$ is *a priori* known to fail in explaining. For example, $SU(12)$ is known not to be unitary, so that perhaps a comparison with scattering parameters determined within a formalism which does satisfy unitarity is unfair. Similarly, $SU(12)$ is known to be broken by mass differences, and in as much as nuclear forces are due to the exchange of a variety of particles whose masses are far from being degenerate as $SU(12)$ would have it, perhaps the above comparison is unfair on that count also.

One can object, of course, that making all these allowances is like interviewing, for the position of a bank teller, a drug addict kleptomaniac with a previous conviction for sex murder, to see whether otherwise he qualified for the job. But then again, on a desert island, being the only applicant, our kleptomaniac friend might in fact qualify. There might be some point, therefore, to investigate whether $SU(12)$ contains some partial, qualified elements of the truth.

To proceed along this line, one can attempt to eliminate the effect of the mass differences.[8,9] This can be tried, for example, as follows. As we have heard earlier at this conference, one of the very promising approaches to a fundamental theory of the two-nucleon interaction is the one-particle exchange scheme. By fitting such a theory against experimental information one can determine the masses and coupling constants of the various particles contributing to nucleon–nucleon scattering. These masses and coupling constants can then be compared with those predicted by group theoretical symmetries. In the original, untampered form of these theories, the masses would of course not agree at all with the experimental values, since these theories have no *a priori* way of resolving mass degeneracies. It has

[9] C. S. Lai, Phys. Rev **147**, 1136 (1966).

been possible, however, to use more or less *ad hoc* assumptions to build mass splitting into group theoretical schemes. If one uses a concrete special model for the interaction, these mass differences can then be used to generate corresponding differences in the interaction strengths of the various particles.

It would go beyond the purpose of this talk to explore the details of such a procedure. It might be useful, nevertheless, to outline briefly an example[8] of it.

Consider, for example, the meson field Φ in an $SU(12)$ scheme. In terms of transformation properties in Lorentz space, one can write such a field in general as

$$\Phi = 1\times\mathcal{S} + \gamma_\mu\otimes\mathcal{V}_\mu + \tfrac{1}{2}\sigma_{\mu\nu}\otimes\mathcal{T}_{\mu\nu} + \gamma_\mu\gamma_5\otimes\mathcal{A}_\mu + \gamma_5\otimes\mathcal{P}, \quad (2.34)$$

where the $1, \gamma_\mu, \cdots, \gamma_5$ refer to the Lorentz space and the capital script letters to $SU(3)$ space. This field is assumed to satisfy the equation

$$\tfrac{1}{2}[\gamma\cdot\partial\otimes 1'\otimes\mathcal{I}\otimes\mathcal{I}' - 1\otimes\tilde{\gamma}\cdot\partial\otimes\mathcal{I}\otimes\mathcal{I}']\Phi + m\Phi = 0, \quad (2.35)$$

where $\mathcal{I}$ and $\mathcal{I}'$ are identity operators in $SU(3)$ space. According to strict $SU(12)$ theory m is simply a constant, the common mass. We can, however, modify this equation by explicitly introducing differences in masses, for example, by writing the ansatz

$$\begin{aligned} m = m_0(1\otimes 1'\otimes\mathcal{I}\otimes\mathcal{I}') &+ \tfrac{1}{2}m_1(\gamma_\lambda\otimes\tilde{\gamma}_\lambda + \gamma_\lambda\gamma_5\otimes\tilde{\gamma}_\lambda\tilde{\gamma}_5)\otimes(\mathcal{I}\otimes\mathcal{I}') \\ &+ \tfrac{1}{2}m'(1\otimes 1' + \gamma_5\otimes\tilde{\gamma}_5)\otimes(\mathcal{D}\otimes\mathcal{I}' + \mathcal{I}\otimes\mathcal{D}') \\ &+ m_s\Big(\sum_{A=1}^{16}\gamma_A\otimes\tilde{\gamma}_5\tilde{\gamma}_A\tilde{\gamma}_5\Big)\otimes\Big(\sum_{i=1}^{8}\mathcal{L}_i\otimes\mathcal{L}_i\Big), \end{aligned} \quad (2.36)$$

where

$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.37)$$

and $\mathcal{L}_i$ are the $SU(3)$ generators.

If one solves (2.35) with (2.36), one gets of course a set of different equations for each of the pieces, in (2.34). Correspondingly the actual physical particles will also have different equations, and their masses will be related to the fudge factors m_0, m_1, m', and m_s as

$$\begin{aligned} m_\rho^2 &= m_\omega^2 = m_0^2, \\ m_{K^*}^2 &= m_0(m_0+m'), \\ m_\phi^2 &= m_0(m_0+2m') \\ m_\pi^2 &= m_0(m_0-4m_1), \\ m_K^2 &= m_0(m_0-4m_1+m'), \\ m_\eta^2 &= m_0(m_0-4m_1+\tfrac{4}{3}m'). \end{aligned} \quad (2.38)$$

The meson function (2.34) can be written in an explicit form containing only $\mathcal{V}$ and $\mathcal{P}$, since $\mathcal{S}$, $\mathcal{A}$, and $\mathcal{T}$ can be written in terms of them using the field equations. The result is

$$\begin{aligned} \Phi = \gamma_\mu\otimes\mathcal{V}_\mu - \sigma_{\mu\nu}\otimes(\mathcal{F}_{\mu\nu}/\mathcal{M}) + \gamma_5\times[\mathcal{P} + (X^0/\sqrt{3})\mathcal{D}] \\ - \gamma_\mu\gamma_5\partial_\mu\otimes[(\mathcal{P}/m_\rho) + (X^0/\sqrt{3}m_s)\mathcal{D}], \end{aligned} \quad (2.39)$$

where

$$m_\rho\mathcal{M} = \begin{pmatrix} m_\rho^2 & m_\rho^2 & m_{K^*}^2 \\ m_\rho^2 & m_\rho^2 & m_{K^*}^2 \\ m_{K^*}^2 & m_{K^*}^2 & m_\phi^2 \end{pmatrix} \quad (2.40)$$

and X^0 is the singlet pseudoscalar state, which is split from the octet pseudoscalar state by the last term in Eq. (2.36). We wrote (2.39) in terms of m_s, m_{K^*}, m_ϕ, and m_s as independent mass parameters, instead of in terms of the original m_0, m', m_1, and m_s.

A similar procedure can be followed for the baryons Ψ.

We can then insert Φ and Ψ into some over-all interaction Lagrangian describing vertex functions. For a baryon–baryon–meson vertex, for example, we can choose

$$L_{\text{int}} = iG\Psi_{ADC}\Phi_{AB}\Psi_{BDC}. \quad (2.41)$$

Then, due to the various factors appearing in the pieces of (2.39), we will also get various factors, multiplying the over-all coupling constant G, for the specific vertices of the various types of baryons and mesons, so that the effective coupling constants of these various vertices will also be related. For example, one gets

$$\begin{aligned} g_{NN\pi^0} &= \tfrac{5}{3}g_{NN\eta} \\ g_{\Lambda NK^0} &= -3\sqrt{3}g_{\Sigma NK^0} \text{ etc.} \end{aligned} \quad (2.42)$$

The above relations are, of course, dependent on the values of our four fudge factors m_0, m_1, m', and m_s. The last one is related only to the mass of the X^0 particles and hence is arbitrary. The other three, m_0, m_1, and m', however, are related to the physical masses of m_ρ, m_ω, m_k, m_{K^*}, m_π, m_η, and m_ϕ, through Eq. (2.38). The first question therefore is whether there is a set of m_0, m_1, and m' that reproduces the measured masses of these seven particles. The answer is in the affirmative to within a 5% accuracy. (For example, using $m_\pi = 1$, a satisfactory set is $m_0 = 5.50$, $m' = 1.88$, and $m_1 = 1.33$.) This is a considerable achievement for the theory which encourages a look at the coupling constants.

In an actual fitting[10] of a one-particle exchange model to experimental data on nucleon–nucleon scattering, the authors used exchanges of π, η, ω, ρ, s, and σ particles, where the last two are somewhat hypothetical scalar–isoscalar objects with masses of 700 and 400 MeV, respectively. Since vector mesons have two coupling constants, one has the following ten parameters: g_π, g_η, $g_{\omega 1}$, $g_{\omega 2}$, $g_{\rho 1}$, $g_{\rho 2}$, $g_{\phi 1}$, $g_{\phi 2}$, g_s, and g. This list is reduced to seven, using nucleon form-factor

[10] G. Köpp and G. Kramer, Phys. Letters 19, 593 (1965).

information to determine the ratios $g_{\omega 1}/g_{\omega 2}$, $g_{\rho 1}/g_{\rho 2}$, and $g_{\phi 1}/g_{\phi 2}$. Furthermore, g_π^2 was taken fixed from other determinations. Then one can determine the remaining six coupling constants from the best fit to experimental data. On the other hand, one can also take the g_ω's, g_ρ's, g_ϕ's, and the g_η from the $SU(12)$ predictions and fit only g_s and g_σ to the experimental data.

The comparison of the two sets of coupling constants thus obtained leads to the following conclusions.

(a) If one does not assume the presence of the s particle, the fits are all poor.

(b) With the inclusion of the s, both the phenomenological and the $SU(12)$ sets give good fits to the data, at least, as far as the $'D_2$, $'G_4$, ${}^3P_2$, ${}^3F_3$, ${}^3F_4$, ${}^3H_4$, and E_4 scattering parameters are concerned.

(5) The phenomenological and $SU(12)$ sets of coupling constants are only qualitatively similar, and there are considerable quantitative differences between them. This, in conjunction with b, is only another expression of the fact that for the time being the values of the coupling constants are only poorly determined by the experimental data, and hence such a test of $SU(12)$ can be at the present, at most qualitative. The general approach, however, is an interesting one and there is obviously much room for improvement here both experimentally and theoretically.

III. TESTS OF GENERAL CONSERVATION LAWS

The validity of general symmetries such as parity conservation, time-reversal invariance, PT invariance, charge conjugation invariance, and the PTC theorem have been under intense investigation for the last decade. In as much as we want to investigate those symmetries in strong interaction, nucleon–nucleon (and nucleon–antinucleon) scattering is a good reaction to concentrate on because of its relative simplicity and the wealth of information that is available.

In order to list the tests of conservation laws in nucleon–nucleon scattering, it might be useful to use the general nondynamical formalism[11–13] for particle reactions. According to that, the reaction

$$\tfrac{1}{2}+\tfrac{1}{2}\rightarrow\tfrac{1}{2}+\tfrac{1}{2} \tag{3.1}$$

can be factorized into

$$\tfrac{1}{2}+0\rightarrow\tfrac{1}{2}+0 \tag{3.2}$$

and

$$0+\tfrac{1}{2}\rightarrow 0+\tfrac{1}{2} \tag{3.3}$$

which are, in fact, identical. Denoting the M matrices of these three reactions by M_1, M_2, and M_3, we have

$$M_1(=)M_2\otimes M_3, \tag{3.4}$$

and

$$M_1^+(=)M_2^+\otimes M_3^+ + M_2^-\otimes M_3^-,$$
$$M_1^-(=)M_2^+\otimes M_3^- + M_2^-\otimes M_3^+, \tag{3.5}$$

where $(=)$ denotes nondynamical equality; i.e., equality except for the values of the invariant amplitudes. Furthermore, $\otimes$ means the outer product in spin space, and the superscripts $-$ and $+$ denote those parts of the M matrix which do and do not change sign, respectively, under a given transformation.

The M matrix of the constituent reaction (3.2) is[14]

$$M_2=a_0+a_1\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{l}+a_2\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{m}+a_3\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{n}, \tag{3.6}$$

where the a_i's are the invariant amplitudes, and l, and m, and n are defined in Eq. (2.17). Similarly

$$M_3=b_0+b_1\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{l}+b_2\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{m}+b_3\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{n}. \tag{3.7}$$

The observables for the constituent reactions are given in Ref. 14, Table II. Of the usual eight subclasses, one (the ooo one) is empty.

From these observables we can build up the observables for Eq. (3.1). Again one of the expected sixteen subclasses will be empty. The number of observables in the fifteen remaining subclasses is eight, except for the first subclass (composed of the products of two eee-type constituent subclasses) which contains 16 observables.

The M matrix of reaction (3.1) can be written under rotation invariance only as[15]

$$\begin{aligned}M_1=A_{00}&+A_{01}\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{l}+A_{02}\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{m}+A_{03}\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{n}+A_{10}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{l}+A_{11}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{l}\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{l}+A_{12}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{l}\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{m}+A_{13}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{l}\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{n}\\&+A_{20}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{m}+A_{21}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{m}\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{l}+A_{22}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{m}\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{m}+A_{23}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{m}\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{n}\\&+A_{30}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{n}+A_{22}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{n}\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{l}+A_{32}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{n}\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{m}+A_{33}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{n}\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{n}\end{aligned} \tag{3.8}$$

containing sixteen amplitudes, which correspond to the constituent amplitudes according to

$$A_{ij}\rightarrow a_ib_j. \tag{3.9}$$

In case of rotation invariance alone, therefore, the 256 observables depend on 256 bilinear products of amplitudes and hence all observables are linearly independent.

We will now list the restrictions on these observables under the various conservation laws.

[11] P. L. Csonka, M. J. Moravcsik, and M. D. Scadron, Ann. Phys. (N.Y.) **40**, 100 (1966).

[12] P. L. Csonka, M. J. Moravcsik, and M. D. Scadron, Ann. Phys. (N.Y.) **41**, 1 (1967).

[13] M. J. Moravcsik, Lectures on Non-Dynamical Test of Conservation Laws in Particle Reactions, Williamsburg, 1966 (to be published).

[14] P. L. Csonka, M. J. Moravcsik, and M. D. Scadron, Rev. Mod. Phys. **39**, 178 (1967).

[15] M. J. Moravcsik, in *Proceedings of the 2nd International Conference on Polarization Phenomena of Nucleons, Karlsruhe, 1965*, P. Huber and H. Schöpper, Eds. (Burkhäuser Verlag, Basel and Stuttgart, 1966).

A. Parity Conservation

The number of invariant amplitudes reduces from sixteen to eight. In particular we have

$$A_{ij}=0 \quad \text{for} \quad i+j=\text{odd} \tag{3.10}$$

if the product of the intrinsic parities of the four particles in Eq. (3.1) is +1 (which is the case for elastic nucleon–nucleon and nucleon–antinucleon scatterings) or

$$A_{ij}=0 \quad \text{for} \quad i+j=\text{even} \tag{3.11}$$

if the above-mentioned product is −1.

In either case, the number of nonzero bilinear combinations of A's will now be 64 instead of 256. Thus there will now be 192 linear relations among the 256 observables. 128 of these relations consist of

$$L(x, y; z, w)=0 \tag{3.12}$$

for those L's in whose argument the number of l's plus the number of n's is odd. Equation (3.12) holds whenever parity is conserved, whether the product of intrinsic parities is +1 or −1. A measurement of any of such L's is, therefore, a test of parity conservation.

The remaining 64 relations among the 256 observables are different depending on the product of the intrinsic parities. The relations, however, are all of the form

$$L(x, y; z, w)=\eta L(x', y'; z', w'), \tag{3.13}$$

where $a \rightarrow a'$ denotes the transformation

$$0 \leftrightarrow m \qquad l \leftrightarrow n \tag{3.14}$$

and η is +1 or −1, depending on the particular observable and on whether the product of intrinsic parities is +1 or −1. In particular, if the product of intrinsic parities is +1, then

$$\eta=(-1)^{\lambda+\nu+\beta/2}, \tag{3.15}$$

where λ is the number of l's appearing in the last two arguments of L, ν is the number of n's appearing in the first two arguments of L, β is the number of l's plus n's appearing in all four arguments of L. If the product of the intrinsic parities is −1, we get

$$\eta=-(-1)^{\lambda+\nu+\beta/2}. \tag{3.16}$$

These 64 relations therefore can be used not only for checking the validity of parity conservation, but also to establish the product of intrinsic parities of the participating particles.

This ends a complete discussion of all general nondynamic tests of parity conservation in nucleon–nucleon scattering. It might of course be possible to find some other tests which apply either in special dynamical situations (such as very low energy scattering), or under the assumption of specific dynamical models.

B. Time Reversal Invariance[16] (for the Reaction $a+b \rightarrow a+b$)

The number of independent invariant amplitudes reduces from sixteen to ten. In particular, we have

$$A_{01}=A_{10}=A_{12}=A_{21}=A_{13}=A_{31}=0. \tag{3.17}$$

As a result, we will have $256-10^2=156$ relations among the observables. Of these, $\frac{1}{2}256-\frac{1}{2}16=120$ will be so-called mirror relations, while $\frac{1}{4}16\times 9=36$ will be nonmirror-type relations. The mirror relations are

$$L(x, y; z, w)=\epsilon L(z, w; x, y), \tag{3.18}$$

where ϵ is +1 for the *eee*, *eoe*, *eoo*, and *eeo* subclasses, and −1 for the other four subclasses. These relations are tests of whether time reversal is *partially* violated or not, but they cannot distinguish between time-reversal invariance and total time-reversal violation (symmetric or antisymmetric M matrix).

The 36 nonmirror-type relations, on the other hand, can also be used to distinguish between invariance and total violation, and generally involve more than two observables. They will not be listed here, since they will be discussed in the more relevant case when both parity conservation and time-reversal invariance are assumed to hold.

C. PT Invariance[17] (for the Reaction $a+b \rightarrow a+b$)

The number of independent invariant amplitudes is again reduced from sixteen to ten. The restrictions now are as follows

$$A_{03}=A_{13}=A_{23}=A_{30}=A_{31}=A_{32}=0. \tag{3.19}$$

As a result, we will again have 156 relations among the 256 observables, of which again 120 are mirror relations and 36 nonmirror relations. For the mirror relations ϵ is now +1 for the subclasses *eee*, *ooe*, *eoe*, *oee*, and −1 for the other four.

D. Parity Conservation and Time-Reversal Invariance (for the Reaction $a+b \rightarrow a+b$)

Under $P+T$ the number of invariant amplitudes is reduced from sixteen to six. In particular, we have

$$A_{01}=A_{03}=A_{10}=A_{12}=A_{21}=A_{23}=A_{30}=A_{32}=A_{13}=A_{31}=0. \tag{3.20}$$

As a result, we will have $256-6^2=220$ linear relations among the observables. Of these, 128 will be the ones given by Eq. (3.12). This leaves 92 more relations. Of these, $\frac{1}{2}128-\frac{1}{2}16=56$ will be mirror relations, and 64 will be of the type (3.12), with 32 overlaps, so that Eq. (3.13) and the mirror relations together give $56+64-32=88$ relations. The remaining four are of

[16] P. L. Csonka and M. J. Moravcsik, Phys. Rev. **152**, 1310 (1966).

[17] P. L. Csonka and M. J. Moravcsik, UCRL-70076; and (to be published).

somewhat more complicated type

$$L(0,0;0,0)-L(0,m;0,m)=L(l,l;l,l)-L(l,n;l,n), \quad (3.21)$$

$$L(0,l;0,l)-L(0,n;0,n)=L(l,0;l,0)-L(n,0;n,0), \quad (3.22)$$

$$L(m,0;0,m)=L(m,m;0,0)+L(n,n;l,l)+L(n,l;l,n), \quad (3.23)$$

$$L(0,n;m,l)+L(0,l;m,n)=L(l,0;n,m)+L(n,0;l,m). \quad (3.24)$$

E. Tests of the *TCP* Theorem

For the previous four transformations, elastic nucleon–nucleon scattering was a self-transforming[18] reaction, and hence tests could be derived from considering that one reaction by itself. For the transformation of *PTC*, the type of self-transforming reaction is

$$a+b\rightarrow\bar{a}+\bar{b} \quad (3.25)$$

and hence to test the *PTC* theorem we can consider nucleon–antinucleon scattering such as

$$p+\bar{p}\rightarrow p+\bar{p} \quad (3.26)$$

or

$$n+\bar{p}\rightarrow p+\bar{n}. \quad (3.27)$$

Apart from this difference in the reaction itself, the observables involved in the tests and the relations among them will be the same as for *PT* invariance.

It might be mentioned that *one* mirror relation by itself is not enough to test a certain conservation law. For example, for reaction (3.26), when we establish that

$$L(m,0;0,0)=L(0,0;m,0) \quad (3.28)$$

we do not know whether what we have proven is *TC* invariance or *TPC* invariance. If in addition, we also establish, for example, that

$$L(l,0;0,0)=L(0,0;l,0) \quad (3.29)$$

then we know that we are testing *TCP*, since for *TC* Eq. (3.29) would have a − sign on the right-hand side. Of course, if *P* is conserved in itself, then both sides of Eq. (3.29) are zero.

In practice, nucleon–nucleon scattering has so far hardly been used to test conservation laws. An investigation[19] of the status of this problem about two years ago indicated that joint test of parity conservation and time-reversal invariance in proton–proton scattering are at best good to about 7%. It is evident that much remains to be done in this field.

[18] P. L. Csonka and M. J. Moravcsik, UCRL-70052; and (to be published).
[19] E. H. Thorndike, Phys. Rev. **138**, B586 (1965).

IV. THE ANALYTICITY STRUCTURE OF THE *S* MATRIX

Another, quite different type of application of our knowledge of the two-nucleon interaction is in connection with the *experimental* exploration of the analyticity structure of the *S* matrix. In *S*-matrix theory certain assumptions are made about the locations of singularities in the complex plane, and about the residues and discontinuities associated with these singularities. Some of these assumptions may be derivable from some basic principles, while others may be based on less firm considerations. All of them, however, deserve experimental testing.

One of the most amenable features of the hypotesized structure of the *S* matrix is the pole or cut (depending on the variable) associated with the exchange of one particle of the lightest type. Such a singularity is usually well separated from other singularities and is located close to the physical region so that is effect can be well discerned for instance in terms of data near 0° or 180° when viewed in the momentum transfer variable. In nucleon–nucleon scattering this closest singularity corresponds to the one-pion exchange and it has been utilized in the interpretation of experiments for some years with considerable amount of success so that one can be quite confident in considering one-pion exchange an experimentally verified feature of the two-nucleon interaction.

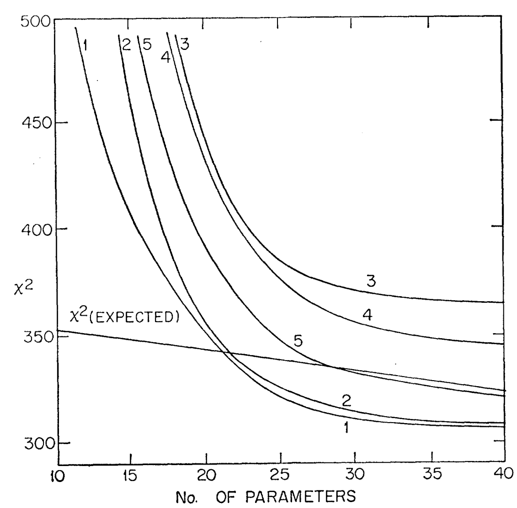

FIG. 2. χ^2 vs number of search parameters. The curves depicted are for the following terms:

Curve	Form	β's
1	1	2, 3, 5, 9
2	4	50, 0
3	4	50, 90°
4	5	···
5	6	···.

Curves 1 and 2 are "proper" while the remaining 3 curves are "improper".

After such initial success it is natural to wonder whether it would be possible to verify some of the other, experimentally less conspicuous postulates of S-matrix theory. Let us consider, in particular, partial wave scattering amplitudes for elastic nucleon–nucleon scattering. In S-matrix theory such an amplitude is assumed to have a series of partially overlapping cuts along the negative real axis of the energy plane, starting at a sequence of negative values and reaching out to $-\infty$. The cut which starts nearest to the origin corresponds to one-pion exchange, while the others represent the various multi-pion exchanges. There is also supposed to be a cut along the positive real axis, reaching from the threshold of the inelastic channel to $+\infty$. Beside cuts along the real axis, there are supposed to be no singularities anywhere in the complex plane.

An attempt has been made[20] recently to test these assumptions about the singularities of partial wave amplitudes in nucleon–nucleon scattering. Energy-dependent modified-phase-shift analyses have been performed on the existing experimental data, using a number of different energy dependences for that part of the partial wave amplitudes which are not given by

TABLE I. Basis functions for phenomenological fit:

$$\delta_l^n = \delta_l^\pi + \sum_{i=1}^{i\max} \alpha_j^l f_{lj}^n,$$

δ_l^π = one-pion-exchange contribution (OPEC) phase shift, M = nucleon mass (938 MeV), μ = pion mass (135 MeV), T = lab kinetic energy in MeV, $(K/E) = 1/(1+2M/T)^{1/2}$ = kinematic threshold factor, $X_j = 1+(\beta_j^2\mu^2/MT)$.

Form (n)	f_{li}^n	
1	$\frac{1}{2}(K/E)(X_i-1)Q_l(X_i)$	
2	$\frac{1}{2}(K/E)(X_1-1)Q_{l+i-1}(X_i)$	
3	$(K/E)(X_0-1)\int_{-1}^{+1} Q_l[(X_0-X)/(1-X)](X+1)^{3/2}dx$	$X_0 = 1+(4\mu^2/MT)$
4	$(K/E)[T'^l/(T'+\cos\beta_2)^2+\sin^2\beta_2]$	$T' = T/i\beta_1$
5	$(K/E)(T/140)^{l+i-1}$	
6	$(K/E)(T/140)^{l+1/2(i-1)}$	
7	$(K/E)T'^{l+i-1}e^{-T'}$	$T' = T/\beta_i$
8	$(K/E)\{1-\exp[\beta_2(T/\beta_1)^l]\}[1+(T/\beta_1)^{1/2(i-1)}]$	

the one-pion exchange. In all of the functions describing these energy dependences of phase shifts, certain general properties have been built in, such as the proper threshold and asymptotic behavior. Beyond that, however, these functions can be, *a priori*, anything, and therefore, it is possible, at least in principle, to investigate whether functions which have the "proper" analytic properties according to S-matrix theory are more successful in describing the experimental information than functions with "random" analytic properties.

The quantitative gauge of success in such an analysis in the goodness of fit as a function of the number of free parameters. Such curves are given in Figs. 2 and 3, for functional dependences as shown in Table I. The set of experimental data used in this process was selected one containing 363 individual data points. The details of the process of comparison were described in Ref. 20.

It is of some interest to mention that most of the "improper" functions used in the comparison were, in the *physical* region, of the same type, as the "proper" ones, so that a casual glance could not distinguish the two types from each other. In particular, the "improper" functions also varied smoothly with energy, and peaked at about the same energy as the "proper" ones.

By looking at Figs. 1 and 2, as well as some similar results not plotted here, one can deduce the following conclusions.

(1) Forms with "proper" analytic behavior give slightly better fits to the data than the "improper" forms. The difference in the goodness of the fits is significant if judged by strictly mathematical criteria, but perhaps such a too literal interpretation of such mathematical test is unwise.

(2) In particular, and to a more pronounced extent, polynomials (which are "improper") give a worse fit

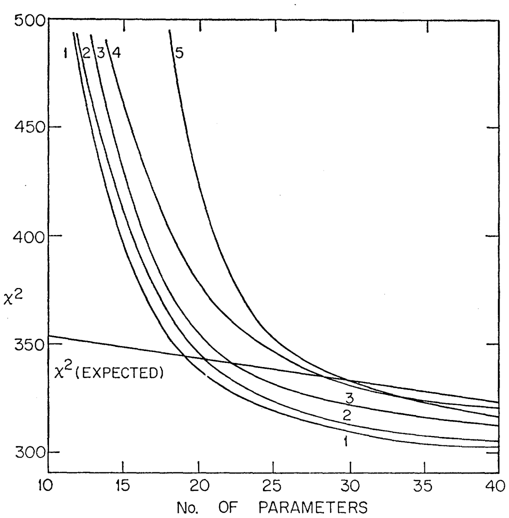

FIG. 3. χ^2 vs number of search parameters. The curves depicted are for the following forms:

Curve	Form	β's
1	3	...
2	2	3
3	8	140, 0.5
4	7	100, 150, 200, 250
5	7	50, 100, 200, 400.

The β's appearing in the basis functions are "unsearched" parameters while the α's are varied to fit the data. The "OPEC" term used for ${}^1S_0(\delta_0^\pi)$ was, in fact, the phase shift given by effective range parameters and the phenomenological sum for ${}^1S_0$ was over basis functions having an $l=1$ threshold dependence. This kept the low-energy ${}^1S_0$ phase shift at its effective range value.

[20] R. A. Arndt and M. J. Moravcsik, UCRL-70040; and Nuovo Cimento (to be published).

than "proper" forms. Thus the practice of representing left hand cuts by polynomials does not appear to be a sound one.

(3) A series of poles along the negative real axis gives almost as good a fit to the data as the cuts. Thus the practice of replacing cuts by appropriately constructed poles is pragmatically justified, at least at the present accuracy of experimental information.

(4) If we replace the cuts along the negative real axis by pairs of poles off the negative real axis, arranged symmetrically on the two sides of the axis, the fit gradually worsens as the poles get farther off the real axis.

It is hardly necessary to stress the preliminary nature of this investigation. For one thing, the procedure consists of trying a substantial but not infinite number of specific examples, and not of utilizing general considerations. Second, the present work tries to test only some rather rough features of the analyticity structure of the S matrix (such as whether all singularities of partial wave amplitudes lie on the real axis or not). Third, even with these limited objectives, the superiority of the analytically "proper" functions is only marginal. Nevertheless, the present work may be considered a feasibility study of the method, yielding positive results. As such, it provides another incentive for continuing the systematic acquisition of precision information on nucleon–nucleon scattering.

For the present time, the results seem to indicate that theories with the "proper" analyticity behavior are only barely if at all superior to theories with "wrong" analyticity behavior in explaining the information on the two-nucleon interaction. In other words, although nobody has undertaken such a task, it would presumably be possible to construct a theory of the two-nucleon interaction which, in addition to the one-pion exchange (whose necessity is established beyond doubt) would consist of postulates which are, from the S-matrix point of view, quite inadmissible, and such a theory could be essentially just as successful from a practical point of view as the S-matrix theory itself. If this can be done for the two-nucleon interaction which is the experimentally best known strong interaction, it could most likely be done, *a fortiori*, for other, worse known, strong interactions. One could therefore conclude that apart from one-particle exchange terms, there is no hard experimental evidence today that the structure of reaction amplitudes is as prescribed by current S-matrix theory. One could also infer from this result that there is no hard experimental evidence today that analyticity is in fact an important guide for the construction of reaction amplitudes. To make such a claim on a firm basis, one would have to undertake a critical study of other consequences of analyticity also, such as some of the forward dispersion relations which appear to have some direct experimental basis. Furthermore, we may, of course, have other aesthetic considerations that might make analyticity an appealing principle for us, and we might be able to support it also through more or less rigorous connections established between it and other generally held principles such as causality. But physics, in the final analysis, is an experimental science, and therefore until we have solid experimental evidence that consequences of analyticity provide a theory which can correlate the experimental information much more simply and economically than other possible theories, it is perhaps worth maintaining an open mind.

V. CONCLUSION

As mentioned at the beginning, the three quite different applications of the two-nucleon interaction to general problems of particle physics are meant to serve only as examples of what can be done. In an age when there might be a temptation to proliferate dynamical theories of elementary particle processes, my purpose was to emphasize that the two-nucleon interaction might, in many cases, be very suitable to provide an "instant test" of such theories against experimental information.