SESSION E—TOPICS RELATED TO N–N INTERACTION

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Pion-Production Amplitudes and Nucleon–Nucleon Interaction*

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The present knowledge of the partial-wave proton–proton absorptions due to pion production below 1.5 GeV of laboratory kinetic energy is summarized. The three models which have been used for obtaining this kind of information from inelastic data are discussed: the Mandelstam model, the one-pion-exchange model, and the K matrix, or damping, model. Although there is some disagreement, in the near future one can hope to obtain reasonably trustworthy values for partial wave absorptions as a function of energy from the threshold of pion production to about 1.5 GeV. This information will be useful not only to reduce the number of parameters in high-energy phase-shift analysis, but also to be introduced as known inelasticities in partial wave dispersion relations.

I. INTRODUCTION

This paper is concerned with single pion production in nucleon–nucleon collisions below 1.5 GeV of laboratory kinetic energy. The connections between pion production, elastic nucleon–nucleon scattering and phase-shift analysis will be stressed. Due to the lack of data about pion production in neutron–proton collisions, this paper concentrates on the proton–proton system; i.e., on the isospin $I=1$ state.

The relevant elastic and inelastic reactions to be considered are

$$p+p\rightarrow p+p, \tag{A}$$

$$p+p\rightarrow p+n+\pi^+, \tag{B}$$

$$p+p\rightarrow p+p+\pi^0. \tag{C}$$

Double pion production can be neglected in the chosen energy range. On the other hand, deuteron production; i.e.,

$$p+p\rightarrow d+\pi^+, \tag{D}$$

is of importance since its total cross section peaks at about 600 MeV of laboratory kinetic energy.

In Sec. II we briefly discuss the general relations linking the inelastic amplitudes and absorptions with the elastic nucleon–nucleon matrix elements. Secs. III, IV, and V are devoted to presentations of the various models used for describing pion production and connect the data on inelastic processes with nucleon–nucleon interaction and phase shift analysis. In Sec. VI possible future developments are briefly outlined.

II. THE ABSORPTIONS AND THE ELASTIC MATRIX ELEMENTS

The matrix $S(J)$, describing the transitions between all open channels in states of angular momentum J at an energy $W\gtrless W_0$, where W_0 is the threshold for inelastic processes, is usually written in the form

$$S(J)=1+R(J). \tag{1}$$

The elastic part $S_{\rm el}(J)$ of $S(J)$ is a 4×4 matrix which must satisfy the unitarity condition

$$S_{\rm el}(J)^\dagger S_{\rm el}(J)=\mathbf{1}-{}^\circ R(J), \tag{2}$$

where the absorption matrix ${}^\circ R(J)$ is determined by the inelastic processes and vanishes below W_0. The elastic part of the R matrix can be written in the well-known form[1]

$$R_{\rm el}(J)=\begin{pmatrix} R_J & 0 & 0 & 0 \\ 0 & R_{JJ} & 0 & 0 \\ 0 & 0 & R_{J-1,J} & R^J \\ 0 & 0 & R^J & R_{J+1,J} \end{pmatrix} \tag{3}$$

and below W_0 it is usually parametrized by introducing

[1] M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Ann. Rev. Nucl. Sci. **10**, 291 (1960).

the "bar" phase shifts δ_J, δ_{JJ}, δ_+, and δ_- and the mixing parameter ϵ[1]:

$$R_{\rm el}(J)=\begin{pmatrix} \exp(2i\delta_J) & 0 & 0 & 0 \\ 0 & \exp(2i\delta_{JJ}) & 0 & 0 \\ 0 & 0 & \cos 2\epsilon \exp(2i\delta_-) & i\sin 2\epsilon \exp[i(\delta_-+\delta_+)] \\ 0 & 0 & i\sin 2\epsilon \exp[i(\delta_-+\delta_+)] & \cos 2\epsilon \exp(2i\delta_+) \end{pmatrix} -1, \qquad (W<W_0). \tag{4}$$

The Hermitian absorption matrix $^\circ R(J)$ has the same form as the matrix (3) and is defined by six real numbers: $^\circ R_J$, $^\circ R_{JJ}$, $^\circ R_{J\pm1,J}$, and $^\circ R=|\,^\circ R_J\,|\exp(i\phi_J)$. Four of these parameters are determined by the inelastic cross sections in the states of given orbital angular momentum l and total spin S through the well-known relations

$$^\circ R_J=(2k^2/\pi)[\sigma_J/(2J+1)] \qquad (l=J,\ S=0); \tag{5a}$$

$$^\circ R_{JJ}=(2k^2/\pi)[\sigma_{JJ}/(2J+1)] \qquad (l=J,\ S=1),$$

$$^\circ R_{J\mp1,J}=(2k^2/\pi)[\sigma_{J\mp1,J}/(2J+1)] \qquad (l=J\mp1,\ S=1). \tag{5b}$$

The nondiagonal absorption $^\circ R^J$ can be obtained from a knowledge of the inelastic amplitudes $S_{k,J\mp1}{}^{(\rm in)}$ describing the transitions to the inelastic channels which can be reached from the $S=1$ and $l=J\mp1$ nucleon–nucleon states:

$$|\,^\circ R^J\,|\exp(i\phi_J)=\sum_k S_{k,J-1}{}^{(\rm in)*}S_{k,J+1}{}^{(\rm in)}. \tag{6}$$

Given a model for the inelastic processes the absorption matrix can be computed. The absorptions of the 2×2 diagonal part of $^\circ R(J)$ must satisfy the well-known unitarity conditions

$$0\le{}^\circ R_J\le1, \qquad 0\le{}^\circ R_{JJ}\le1, \tag{7}$$

and the moduli of the elastic S matrix, sometimes called reflection coefficients, are immediately obtained:

$$|\,S_J\,|=\exp(-2\beta_J)=(1-{}^\circ R_J)^{1/2};$$

$$|\,S_{JJ}\,|=\exp(-2\beta_{JJ})=(1-{}^\circ R_{JJ})^{1/2}. \tag{8}$$

In these equations the imaginary parts of the phase shifts have been introduced.

The discussion of the unitarity limitations on the 2×2 nondiagonal part of $^\circ R(J)$ is more involved; here we quote only the final results.[2] The diagonal absorptions must satisfy the usual relations

$$0\le{}^\circ R_{J\mp1,J}\le1. \tag{9}$$

Moreover, if the modulus of the nondiagonal absorption is such that

$$0\le|\,^\circ R^J\,|^2\le(1-{}^\circ R_{J\mp1,J})\,|\,^\circ R_{J-1,J}\,|-{}^\circ R_{J+1,J}\,|, \tag{10}$$

$$\text{for } {}^\circ R_{J-1,J}\gtrless{}^\circ R_{J+1,J},$$

then the unitarity condition can be solved for the elastic amplitudes $S_{J\mp1,J}$ and S^J and a *single* solution exists. For this reason in the following pages Eq. (10) will be called the "one-solution unitarity limit." The moduli of the three elastic amplitudes.

$$S_{J\mp1,J}=|\,S_{J\mp1,J}\,|\exp(2i\alpha_{J\mp1,J});$$

$$S^J=|\,S^J\,|\exp(2i\alpha^J) \tag{11}$$

are functions not only of the three real numbers $^\circ R_{J\pm1,J}$ and $|\,^\circ R^J\,|$ but also of the angle

$$\chi_J=2(\alpha_{J-1,J}-\alpha_{J+1,J}-\phi_J). \tag{12}$$

We can thus write

$$|\,S_{J\pm1,J}\,|=f_\pm(^\circ R_{J-1,J};\,^\circ R_{J+1,J};\,|\,^\circ R^J\,|;\chi_J)$$

$$|\,S^J\,|=f_J(^\circ R_{J-1,J};R_{J+1,J};\,|\,^\circ R^J\,|;\chi_J). \tag{13}$$

The explicit expressions for the functions $f_\pm$ and f_J are given in Ref. 2. The phase α^J of the nondiagonal element can be expressed as a function of $\alpha_{J\pm1,J}$ and of the absorptions. Thus, given the absorption matrix $^\circ R(J)$, the real parameters which are left in the elastic matrix $R(J)$ are the *four* real phases of the diagonal elements, while below the threshold W_0 the free parameters are five [see Eq. (4)]. This reduction of the number of parameters is always encountered when the absorption matrix is nondiagonal[2,3] and is not very surprising as the hypothesis that the nondiagonal elements of $^\circ R(J)$ are known is quite stringent since they contain information about the inelastic matrix elements and not only about partial cross sections.

The unitarity condition and this reduction of the number of parameters affect the parametrization of $R(J)$ above W_0. The most natural choice would be to introduce the imaginary parts of the phases in the matrix of Eq. (4). The moduli of the elements of the nondiagonal 2×2 submatrix of $R(J)$ would then split into two factors: a mixing parameter factor, of the

[2] U. Amaldi, Jr., R. Biancastelli, and S. Francaviglia, Nuovo Cimento **43**, 641 (1966).

[3] R. L. Warnock, Phys. Rev. **146**, 1109 (1966). Other possible parametrizations have been given by N. Hoshizaki and R. A. Arndt in contributions to this conference.

type $\cos 2\epsilon$, and a reflection factor, of the type $\epsilon^{-2\beta}$. Since knowledge of ${}^{\circ}R(J)$ and of the real phases determines these moduli, the splitting is to some extent arbitrary. In the next section the parametrizations used in actual phase-shift calculations will be presented. A suggestion about the way to parametrize a matrix of the type we are discussing for use in N/D calculations, has been given by Warnock.[3]

Returning to the effect of the unitarity condition on the nondiagonal absorptions, it must be said that if $|\,{}^{\circ}R^J\,|^2$ is bigger than the upper limit of Eq. (10) but still such as to satisfy the condition

$$|\,{}^{\circ}R^J\,|^2 \leq (1-{}^{\circ}R_{J-1,J})\,(1-{}^{\circ}R_{J+1,J}) \qquad (14)$$

then *two* solutions to the unitarity conditions for $|\,S_{J\pm1,J}\,|$ and $|\,S^J\,|$ exist. In this case very complicated limitations on the real phases $\alpha_{J\pm1,J}$ are also obtained.[2] In the following pages we call Eq. (14) the "two-solution unitarity limit."

The previous discussion only refers to the consequences of the unitarity condition of Eq. (2); i.e., the condition for the "elastic" submatrix $S_{\mathrm{el}}(J)$. The inelastic two-body–three-body amplitudes have to satisfy other unitarity conditions which contain three-body–three-body amplitudes and which we do not write down explicitly.

III. THE MANDELSTAM MODEL

Until very recently nucleon–nucleon phase-shift analysis above the pion-production threshold have been performed by describing the inelastic processes by means of the well-known Mandelstam model.[4] This model is based on the following main hypotheses: (i) pion production takes place in a few angular momentum states of the nucleon–nucleon system; (ii) the outgoing pion is in a resonant N_{33}^* state with one of the nucleons; (iii) the production matrix elements are constant with respect to energy, apart from factors which take into account the pion–nucleon and nucleon–nucleon interactions; (iv) the nucleon–nucleon interaction is felt in the final state after the production of the N_{33}^* resonance. To ensure that inelastic cross sections do not exceed the unitarity limit, hypothesis (iii) is applied to the reaction matrix rather than the transition matrix elements.

In this model only nucleons which are in s- and p-wave states with respect to the N_{33}^* resonance are considered, so that one speaks of s-wave and p-wave production. In the first column of Table I the elastic proton–proton states of low angular momentum are indicated and the N^*N inelastic states to which they can be connected are shown in the second column. The notation used is as follows: $P_{3/2}$ represents the N_{33}^* state; the following small letter gives the orbital angular momentum in the center-of-mass system (c.m.s.) of the nucleon which does not interact with the pion; the subscript to this letter (1 or 2) is the combination of the spins of the N_{33}^* resonance and of the noninteracting nucleon and, finally, the index to the bracket is the total angular momentum of the system. In order to introduce the effect of the final nucleon–nucleon interaction it is necessary to decompose the total angular momentum into the sum of the relative nucleon–nucleon momentum and the angular momentum of the produced pion with respect to the c.m.s. The possible lower states are given in the third column using a notation which parallels the one used for the N^*N system. They are valid for π^+ production; for π^0 production the symmetric ${}^3S_1$ and ${}^1P_1$ proton–proton states are not allowed.

TABLE I. Angular momentum decomposition of the elastic proton–proton and of the inelastic nucleon–nucleon–pion channels (for the notation see text).

pp	N_{33}^*N	$(NN)\pi$
${}^1S_0$	...	$({}^3S_1p)_0$
${}^1D_2$	$({}^2P_{3/2}s_2)_2$	$({}^3S_1p)_2$; $({}^3P_2s)_2$
${}^3P_0$	$({}^2P_{3/2}p_1)_0$	$({}^1S_0s)_0$; $({}^1P_1p)_0$; $({}^3P_1p_0)$
${}^3P_1$	$({}^2P_{3/2}p_1)_1$; $({}^2P_{3/2}p_2)_1$	$({}^3S_1s)_1$; $({}^1P_1p)_1$; $({}^3P_5p)_1$; $({}^3P_1p)_1$
${}^3P_2$, ${}^3F_2$	$({}^2P_{3/2}p_1)_2$; $({}^2P_{3/2}p_2)_2$	$({}^1P_1p)_2$; $({}^3P_1p)_2$; $({}^3P_2p)_2$
${}^3F_3$	$({}^2P_{3/2}p_2)_3$	$({}^3P_2p)_3$

Table I shows some of the main properties of any model in which only N_{33}^* production is considered. First, π^+ production in the ${}^1D_2$ proton–proton state has the unique feature of being favored both by the existence of the N_{33}^* and of the bound ${}^3S_1$ proton–neutron state. Second, not only the ${}^1D_2$ but also the ${}^3P_1$ state should contribute to the cross section for deuteron production; the two processes can be distinguished by observing the angular distribution of the pions and this shows that there is little deuteron formation in the ${}^3P_1$ state. Third, the ${}^1S_0$ cross section is predicted as being negligible. Fourth, the small number of possible final states and the high centrifugal barriers make π^0 production less abundant than π^+ production; of course the argument should be substantiated by isospin considerations.

In the model proposed by Mandelstam the production in the ${}^3F_2$ and ${}^3F_3$ states was neglected. The model thus contained the parameter for s-state production and five complex parameters for p-state production. Through a careful choice some of the p parameters were made equal, so that with three real parameters it was possible to fit the experimental data.

The first use of the Mandelstam model to derive the modulus of an elastic matrix element as a function of the energy was made by Soroko, who considered the ${}^1D_2$ wave.[5] The behaviour that he obtained of the ${}^1D_2$ absorption, ${}^{\circ}R_2$, due to reaction (D) as a function of

[4] S. Mandelstam, Proc. Roy. Soc. (London) **A244**, 491 (1958).

[5] L. Soroko, Zh. Eksperim. i Teor. Fiz. **35**, 276 (1958) [English transl.: Soviet Phys.—JETP **8**, 190 (1959)].

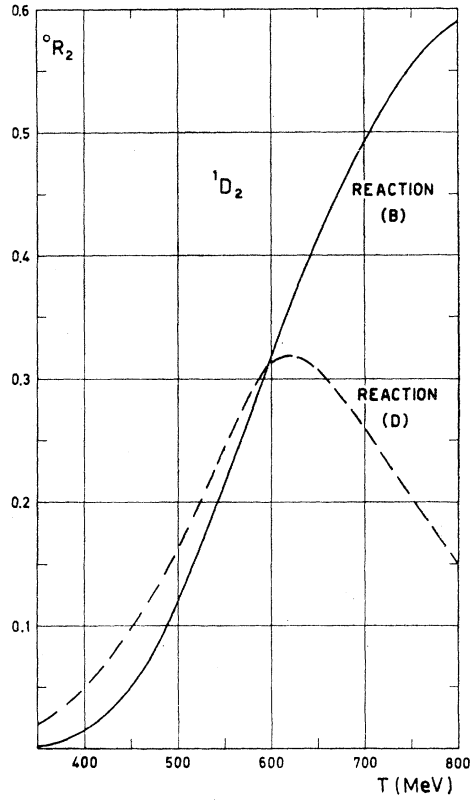

FIG. 1. Energy dependence of the $^1D_2$ absorption of the Mandelstam model computed by Soroko in Ref. 5. Two contributions are separated: deuteron production [reaction (D)] and π^+ production [reaction (B)].

the laboratory kinetic energy T is shown as a dashed line in Fig. 1. As mentioned, according to the pion angular distribution and thus also in the model, almost all deuteron formation occurs in the $^1D_2$ state, as confirmed by an experiment in which polarized protons were used.[6] The full line represents the absorptions due to reaction (B). The contribution of reaction (C) being small has been neglected.

In later papers the Mandelstam model has been used in various ways to deduce, from some knowledge of the inelastic processes, information about the moduli of the S matrix, i.e., the reflection coefficients, and thus reduce the number of parameters to be determined through phase-shift analysis of the elastic data. All of these analyses above the pion-production threshold have been performed by using elastic and inelastic data mostly obtained at the Joint Institute for Nuclear Research at about 650 MeV of laboratory kinetic energy.[7] We concentrate more on the use made and the information obtained on the inelastic processes than on the real phase shifts and their implications for the nucleon–nucleon interaction.

[6] Iu. K. Akimov, O. V. Savchenko, and L. M. Soroko, Zh. Eksperim. i Teor. Fiz. **35**, 89 (1959) [English transl.: Soviet Phys.—JETP **8**, 64 (1959)].

[7] References to the relevant experimental papers are found in the quoted theoretical literature.

The first phase-shift analyses of elastic proton–proton data around 650 MeV have been independently performed by Hoshizaki and Machida[8] and Zul'karneev and Silin.[9]

In the first paper the Mandelstam model has been extended to account also for $^3F_2$ and $^3F_3$ absorption, but reflection coefficients of fixed orbital angular momentum averaged over J have been introduced. This amounts to choosing the same reflection coefficients for the various waves of equal orbital momentum

$$[\beta(^3F_2)=\beta(^3F_3);\qquad \beta(^3P_0)=\beta(^3P_1)=\beta(^3P_2)],$$

as well as vanishing nondiagonal elements of the matrix $R(J)$ for $J=2$.[10] According to the discussions of Sec. II and of Ref. 2, the latter condition implies a vanishing nondiagonal absorption $^\circ R^2$, which in general is not true. Fortunately, in all the work quoted in this section, $R(2)$ came out to be almost diagonal around 650 MeV and probably the neglect of the nondiagonal absorption does not influence the results very much.

More detailed analyses using the same techniques have been published by Azhgirei *et al.*,[11,12] who found an almost unique solution indicating more absorption in $l=2$, 3 than in $l=1$ waves ($^\circ R\simeq 0.54$, 0.77, and 0.11, respectively) which confirmed the peripheral nature of the production processes. An attempt to separate the contributions due to the various total angular momenta J was performed later by Hama and Hoshizaki[13] under the conditions $\beta(^3P_0)=\beta(^3P_1)$ and $\beta(^3P_2)=\beta(^3F_2)$, which are less restrictive than the ones used in Ref. 8. An indication of high absorption in the $^3F_3$ state was obtained ($^\circ R\simeq 0.55$). A similar method has also been applied at 970 MeV[14] by considering s-, p-, and d-state production but reducing the number of inelastic parameters to five and by using measured instead than computed inelastic cross section. By limiting the ranges of variability of the elastic phase shifts so as to continue lower energy results, two solutions were found: one indicates peripheral absorptions and the other central absorption. The latter gives strong pion production in the $^1S_0$ state. In our opinion the second solution has to be discarded because it does not agree with the many results which show that pion production

[8] N. Hoshizaki and S. Machida, Progr. Theoret. Phys. (Kyoto) **29**, 49 (1963).

[9] R. Ya. Zul'karneev and I. N. Silin, Zh. Eksperim. i Teor. Fiz. **44**, 1106 (1963) [English transl.: Soviet Phys.—JETP **17**, 745 (1963)]; R. Ya. Zul'karneev and I. N. Silin, Phys. Letters **3**, 265 (1963).

[10] This point is discussed in Ref. 16.

[11] L. S. Azhgirei, N. P. Klepikov, Yu. P. Kumekin, M. G. Mescheryakov, S. B. Nurushev, and G. D. Stoletov, Zh. Eksperim. i Teor. Fiz. **45**, 1174 (1963) [English transl.: Soviet Phys.—JETP **18**, 810 (1964)]. L. S. Azhgirei, N. P. Klepikov, Yu. P. Kumekin, M. G. Mescheryakov, S. B. Nurushev, and G. D. Stoletov, Phys. Letters **6**, 196 (1963).

[12] L. S. Azhgirei, N. P. Klepikov, Yu. P. Kumekin, M. G. Mescheryakov, S. B. Nurushev, and G. D. Stoletov, Zh. Eksperim. i Teor. Fiz. **46**, 1074 (1964) [English transl.: Soviet Phys.—JETP **19**, 728 (1964)].

[13] Y. Hama and N. Hoshizaki, Progr. Theoret. Phys. (Kyoto) **31**, 609 (1964).

[14] Y. Hama and N. Hoshizaki, Progr. Theoret. Phys. (Kyoto) **31**, 615 (1964); **31**, 1162 (1964).

goes via N_{33}^* production, which in low angular momentum states does not contribute to the $^1S_0$ absorption (see Table I).

In parametrizing, for $W>W_0$, the matrix $R(J)$ of Eq. (4), Zul'karneev and Silin introduced the imaginary parts of the four phases and of the mixing parameter.[9,15] In the 2×2 nondiagonal submatrix of $R(J)$ there were thus 6 real parameters which, according to the discussion of Sec. II, is the correct total number of parameters if no knowledge of the absorption matrix $^\circ R(J)$ is supposed. This actually was the case, since the authors only used the Mandelstam model to obtain the value of the $^1D_2$ reflection coefficient[5] and to predict which were the other absorbed waves without introducing into the analysis other computed absorptions. While in the first analysis, by neglecting F-wave absorption, one good solution was found,[15] the introduction of F absorption gave more solutions[16] one of which coincides with the solution of Ref. 12, which is also preferred by an analysis of Kazarinov and Kiselev.[17] As a result of this comparison it is not possible to give numerical values for the absorptions at 650 MeV because of the large statistical errors, but a strong indication is obtained that F-wave absorption cannot be neglected, due to the peripheral character of the production process, and that among the $l=1$ states the $^3P_2$ wave is more strongly absorbed.

The problems of the values of the absorptions and, in particular, of which of the $l=1$ absorptions is larger could not be definitely solved by the above analyses because all the information contained in the Mandelstam model was not used. More recently Vovchenko extended the model to describe F-wave production, redetermined the model parameters using new experimental data on π^+ production at 655 MeV, and computed the absorptions without any use of the elastic data; the results are given in Table II.[18] A large $^3P_1$ absorption was obtained, contradicting many of the results of previous analyses, together with different absorptions in the $^3F_2$ and $^3P_2$ states. An explanation of this high $^3P_1$ absorption could possibly be found in the fact that this is the only $l=1$ state where N_{33}^* production is enhanced by the final state $^3S_1$ neutron-proton interaction (see Table I).

TABLE II. Proton–proton absorptions at $T=655$ MeV obtained by Vovchenko in Ref. 18.

State	$^\circ R$
$^1D_2$	0.49 ± 0.04
$^3P_0$	0.21 ± 0.11
$^3P_1$	0.70 ± 0.11
$^3P_2$	0.41 ± 0.05
$^3F_2$	0.21 ± 0.02
$^3F_3$	0.23 ± 0.05

As far as we know no use has yet been made in phase-shift analysis of these results which, being more complete than the ones previously mentioned, should allow searches containing a smaller number of free parameters.

IV. THE ONE-PION-EXCHANGE MODEL

Above 700–800 MeV the Mandelstam model very probably fails to describe pion production, since more partial waves are involved and the constancy of the reaction matrix amplitudes cannot be valid for too wide an energy range. However, another model adequately describes one-pion production between 800 and 1500 MeV; this is the one-pion-exchange model modified by the introduction of form factors.[19,20]

The theoretical justification for form factors to be introduced into the one-particle-exchange model has been greatly discussed in the literature, and it is known that at incoming energies greater than ~2 GeV the introduction of absorptive effects without form factors has explained many features of quasi-two-body reactions.[21] On the other hand, the main hypotheses on which the absorptive model rests are not valid for nucleon–nucleon collision around 1 GeV, since there are only two and not many open channels; in addition the elastic scattering is mainly potential scattering and not only the shadow of the inelastic processes. We thus believe that there is some physical meaning in a product of propagator and vertex form factors which fits the inelastic cross sections and energy distributions over a wide energy range, and that the existence of form factors does not contradict the presence of absorptive effects at higher energies.[22] However, one can also consider that the amplitudes used for describing one pion production are nothing more than phenomenological matrix elements which have the properties of reproducing the experimental data. This attitude has to be taken towards the high momentum transfer part of the matrix elements, where the dominance of the one-pion-exchange graph is certainly theoretically unjustified.

At variance with the Mandelstam model, the peripheral production amplitudes describe only the pro-

[15] R. Ya. Zul'karneev and I. N. Silin, Zh. Eksperim. i Teor. Fiz. **45**, 664 (1963) [English transl.: Soviet Phys.—JETP **18**, 456 (1964)].

[16] I. Bystriskii and R. Ya. Zul'karneev, Zh. Eksperim. i Teor. Fiz. **45**, 1169 (1963) [English transl.: Soviet Phys.—JETP **18**, 806 (1964)].

[17] Yu. M. Kazarinov and V. S. Kiselev, Zh. Eksperim. i Teor. Fiz. **46**, 797 (1964) [English transl.: Soviet Phys.—JETP **19**, 542 (1964)].

[18] V. G. Vovchenko, Doklady Akad. Nauk. SSSR **163**, 1348 (1965) [English transl.: Soviet Phys.—Doklady **10**, 761 (1966)]. V. G. Vovchenko, Yadern. Fiz. **3**, 1101 (1966) [English transl.: Soviet J. Nucl. Phys. **3**, 803 (1966)].

[19] E. Ferrari and F. Selleri, Nuovo Cimento **21**, 1020 (1961). E. Ferrari and F. Selleri, Suppl. Nuovo Cimento **24**, 453 (1962).

[20] Recently the peripheral model has been shown to be valid also for describing π^0 production at 610 MeV: W. Busza, D. G. Davis, B. G. Duff, R. E. Jennings, F. F. Heymann, D. T. Walton, E. H. Bellamy, T. F. Buckley, P. V. March, A. Stefanini, and J. A. Strong, Nuovo Cimento **42A**, 871 (1966).

[21] See the review paper by J. D. Jackson, Rev. Mod. Phys. **37**, 484 (1965), and the invited paper by the same author to the Intern. Conference on High Energy Phys., 31 August–7 September (1966), Berkeley, California.

[22] This point is better discussed in Ref. 24.

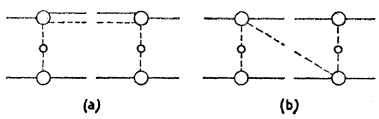

FIG. 2. Graphical representation of the unitarity integrals computed in the peripheral model approach to the calculation of the partial wave absorptions.

cesses (B) and (C)[23] and are given as functions of the momenta of the particles and are not decomposed into angular momentum states. The technique which has been used to obtain the absorption matrices $^{\circ}R(J)$ consists of two successive steps. First, using as input the amplitudes for one-pion production in momentum space, the usual unitarity integrals, graphically represented in Fig. 2, have been computed.[24] In nucleon–nucleon scattering five "overlap functions," which depend upon the energy W and the proton–proton scattering angle θ, are thus obtained. Second, the contributions to the different partial waves have been obtained by applying the same unitarity transformation, which connects the five elastic nucleon–nucleon amplitudes to the matrix elements in states of fixed total angular momentum and spin.[25]

The only approximation consists in neglecting the overlap integrals of Fig. 2(b), since they are known to be small, less than a few percent, compared with the integrals of Fig. 2(a).[24] In the published numerical computation only the N_{33}^* resonance has been retained for the upper vertex. The form factor of the virtual pion–nucleon scattering was taken from Ref. 26. A more recent analysis based on the Mandelstam representation does not disagree with this result.[27] The fit to the inelastic processes over a very wide range of energies was obtained with a single parameter. Below about 800 MeV the fit to one-pion production is not very good, probably because the final state nucleon–nucleon interaction has been neglected; this sets a lower limit to the validity of the model. The upper limit is around 1500 MeV, where production of two pions and of higher pion–nucleon resonances becomes important.

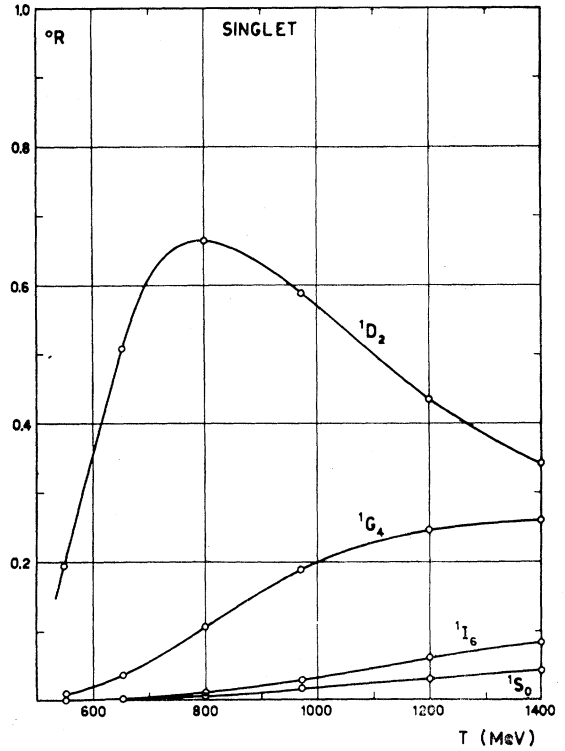

FIG. 3. The singlet absorptions predicted by the peripheral model.

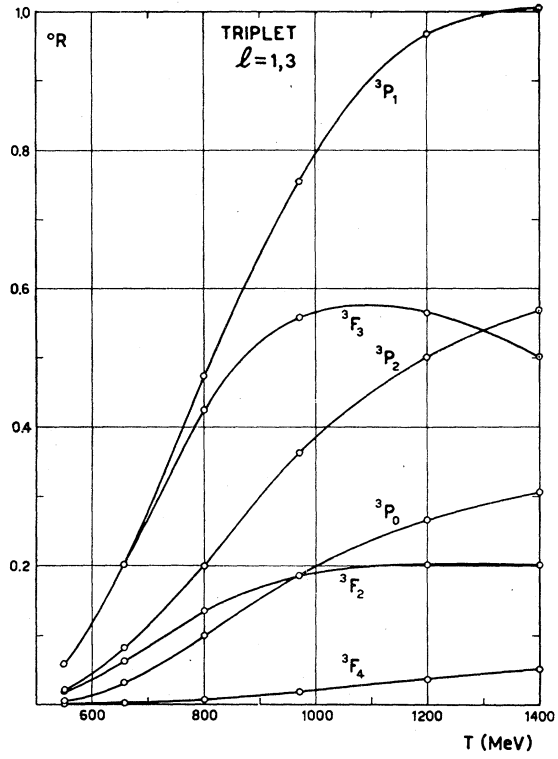

FIG. 4. The triplet $l=1$ and $l=3$ absorptions predicted by the peripheral model.

The absorptions obtained in Ref. 25 are reproduced in Figs. 3–5. The $^1D_2$ absorption can be compared with that computed for reaction (B) by Soroko[5] by means of the Mandelstam model and represented by the solid line of Fig. 1; they agree surprisingly well both in behavior and in absolute value around 800 MeV, which is at the border of validity of both models. This example, which refers to one of the most interesting

[23] Process (D), which has a very small cross section above 800 MeV so that can be neglected, has been recently described using the peripheral model with final-state interaction: J. N. Chahoud, G. Russo, and F. Selleri, Nuovo Cimento **45A**, 38 (1966).

[24] U. Amaldi, Jr., and F. Selleri, Phys. Rev. **128**, 2772 (1962).

[25] U. Amaldi, Jr., R. Biancastelli, and S. Francaviglia, Nuovo Cimento **47**, 85 (1967).

[26] F. Selleri, Nuovo Cimento **40**, 236 (1965).

[27] F. Gutbrod, Nuovo Cimento **45**, 830 (1966).

waves for reasons discussed in Sec. III, shows that the two models give complementary and probably reliable information.

Figure 4 shows that the OPE model predicts higher absorption in the $^3P_1$ than in the $^3P_2$ state; this conclusion is however valid only if the final nucleon–nucleon interaction is neglected, which is reasonable only at high relative energies of the final nucleon–nucleon system, as appears from the angular momentum decomposition of Table I. This indication of the peripheral model agrees with the results of the Vovchenko calculation reported in Table II and disagrees with most phase-shift analysis discussed in Sec. III. It must however be remarked that the $l=1$ absorptions of Table II are much higher at 650 MeV than the peripheral ones, which is very probably due to final-state interaction. Recently, in addition to N_{33}^* peripheral production we have introduced[28] the production of the S_{11} and S_{31} pion–nucleon waves, which a dispersion treatment shows to be the main nonresonating waves in virtual-pion–nucleon scattering.[26] As expected they influence almost uniquely the $l=1$ waves increasing the $^3P_1$ and $^3P_0$ absorptions, which are anyway the less reliable predictions of the model since only one-pion exchange has been considered.

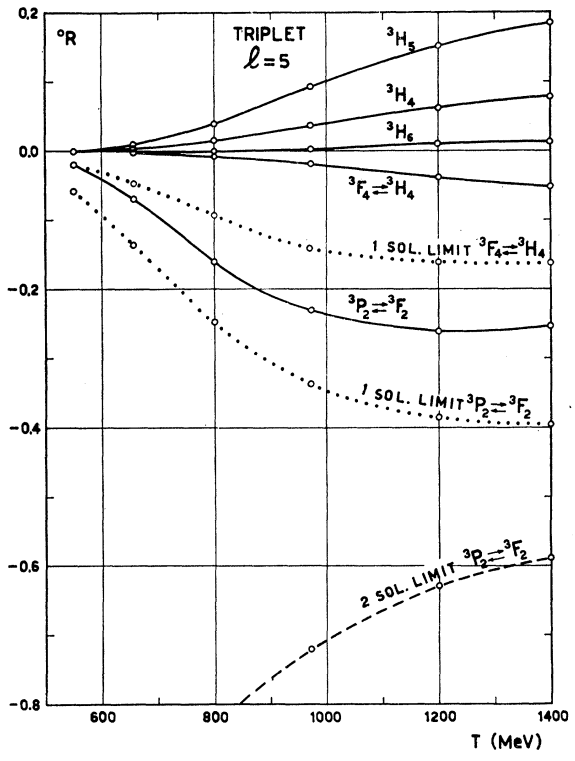

FIG. 5. The triplet $l=5$ absorptions and the nondiagonal absorptions for $J=2$ and $J=4$ predicted by the peripheral model. The "one solution" and the "two solution" unitarity limits of Eqs. (10) and (14) for the two nondiagonal absorptions are also shown.

[28] U. Amaldi, Jr., R. Biancastelli, and S. Francaviglia (to be published).

TABLE III. Proton–proton absorption at $T=970$ MeV obtained by Kikugawa *et al.* in Ref. 31.

State	$^\circ R$
$^1S_0$	0–0.19
$^1D_2$	0.24–0.28
$^3P_0$	0–0.04
$^3P_1$	0.22–0.51
$^3P_2$	0.22–0.32
$^3F_2$	0.54–0.68
$^3F_3$	0.03
$^3F_4$	0.09
$^3G_4$	0–0.39

Absorption in the F waves are as high as in the P waves (see Fig. 4) due to the supposed peripheral nature of the production process. The peripheral model also predicts nonnegligible H absorptions and nondiagonal $J=2$ and $J=4$ absorptions which are smaller than the "one-solution unitarity limit" of Eq. (10) (see Fig. 5). As already mentioned these quantities cannot be computed using the present version of the Mandelstam model.

A detailed discussion of the reliability of the computed absorptions[25] shows that while the diagonal elements of $^\circ R(J)$ are not expected to be too far from the true values, the nondiagonal elements $^\circ R^2$ and $^\circ R^4$ can be considered good approximations to the moduli of these quantities if the phases of the three-body–three-body elements of the $S(J)$ matrix are almost randomly distributed, so that the well known random phase approximation can be used.

At 970 MeV the OPE absorptions differ sensibly from the absorptions obtained by Hama and Hoshizaki in the phase-shift analysis quoted in Sec. III,[14] and suggest a different, and probably better, starting point for further work.[29]

V. THE *K*-MATRIX MODEL

Very recently a study of proton–proton absorptions at 970 MeV has appeared, which is based on a simultaneous treatment of elastic and inelastic processes.[30,31] In this model the elastic and inelastic processes

$$p+p \rightarrow p+p, \qquad \text{(A)}$$

$$p+p \rightarrow N+N^*, \qquad \text{(E)}$$

$$N+N^* \rightarrow N+N^* \qquad \text{(F)}$$

[29] This kind of work is in progress: R. A. Arndt, paper presented to this conference.

[30] M. Kikugawa, S. Sawada, T. Ueda, W. Watari, and M. Yonezawa, Suppl. Progr. Theoret. Phys. (Kyoto), Extra number 548 (1965).

[31] M. Kikugawa, S. Sawada, T. Ueda, W. Watari, and M. Yonezawa, Progr. Theoret. Phys. (Kyoto) **37**, 88 (1967).

have been treated simultaneously as two-body processes. The Born amplitudes for the reactions (A) and (E) have been computed taking into account in the first case one π, one ρ, one ω, and one scalar meson exchanges, while in the second reaction one π and one ρ exchanges have been considered.[32] The amplitudes for process (F) have been introduced by means of two parameters for each state of given angular momentum and parity. The Born amplitudes have been unitarized by the hypothesis that they equal the elements of the reaction matrix K. As is well known this hypothesis has been widely used in various treatments of elastic and inelastic processes.[33]

In the present case, the unitarization procedure has given a good fit to single pion production at 970 MeV when both π and ρ exchanges are considered. Moreover the cross-section for process (E) is not much influenced by the values of the parameters which describe reaction (F). On the other hand, some of the reflection coefficients, and even more the real elastic phases, depend upon these parameters. Table III shows the values of the absorptions computed from the reflection coefficients of Ref. 31 on the hypothesis that the $S(J)$ matrices are diagonal; i.e., using a simple formula of the type given in Eq. (8). The indicated ranges are due to the sensitivity of the absorption to the parameters of reaction (F).

A comparison of this table with the values obtained for 970 MeV from the curves of Fig. 3 and 4 shows that for the $l=1$ waves the K-matrix absorptions are systematically lower than the OPE values. For the F waves the peripheral and the K-matrix models strongly disagree since they give high $^3F_3$ and $^3F_2$ absorptions respectively. The very important $^1D_2$ absorption, for which around 800 MeV the Mandelstam and the peripheral models predict similar values, is certainly too small and in the K-matrix model its value strongly depends upon the value of the ununitarized amplitudes of reaction (E). At this stage it is difficult to assess the real meaning of the disagreement between the OPE and the K-matrix models.

In general the K-matrix model has the advantage that the total S-matrix satisfies unitarity, but this is obtained by introducing parameters for the reaction (F) which influence the inelastic and, even more, the elastic amplitudes.

VI. CONCLUSIONS

We are now able to obtain from various models the energy dependence of the diagonal and, maybe, also the nondiagonal absorptions in proton–proton collision below about 1500 MeV; i.e., in the energy range in which single pion production dominates. In our opinion the Mandelstam model, extended to take into account F-wave production, is a very good starting point for obtaining the absorptions between the threshold for N_{33}^* production and about 700 MeV. The models to be used in the higher energy range should give absorptions which smoothly continue the behavior at lower energies; at present it is difficult to decide which is the best model amongst the two already proposed and the others which, possibly, could be used.

The information about the absorptions should make it possible to carry out high-energy phase-shift analyses.[34] There is another point of great interest arising from our knowing of the behavior of the absorptions over a reasonably wide energy range. Recently progress has been made in applying the N/D method to many-channel problems with arbitrary inelasticity.[3] Of course, the nucleon–nucleon absorptions are the input data to be used in any study of the influence of the inelastic channels upon the low-energy behavior of the phase shifts. When we are reasonably sure about the energy dependence of the partial wave absorptions, the well-known work by Coulter, Scotti, and Shaw will be extended using the new technique to cases other than that of the $^1D_2$ partial wave.[35]

ACKNOWLEDGMENTS

I am grateful to Dr. R. Biancastelli and Dr. S. Francaviglia for careful readings of the manuscript and useful observations.

[32] Meson exchange models for the elastic process (A) below the threshold of pion production have been widely considered. See for instance: S. Sawada, T. Ueda, W. Watari, and M. Yonezawa, Progr. Theoret. Phys. (Kyoto) **28,** 991 (1962); **32,** 380 (1964); **33,** 489 (1965); R. A. Bryan, C. R. Dismushes, and M. Ramsay, Nucl. Phys. **45,** 353 (1963); A. Scotti and D. Y. Wong, Phys. Rev. Letters **10,** 142 (1963); R. A. Bryan and R. A. Arndt, Phys. Rev. **135,** B434 (1964); A. Scotti and D. Y. Wong, *ibid.* **138,** B145 (1965).

[33] See for instance: R. C. Arnold, Phys. Rev. **136,** B1388 (1964); K. Dietz and H. Pilkuhn, Nuovo Cimento **37,** 1561 (1965); **39,** 928 (1965); E. J. Squires, *ibid.* **34,** 1328 (1964); **39,** 300 (1965). A review of the subject has recently appeared: A. C. Hearn and S. D. Drell, "Peripheral Processes" in *High Energy Physics* (Academic Press Inc., New York, 1966), Chap. 9.

[34] A method for extending this kind of work to even higher energies has been proposed: Y. Hama and N. Hoshizaki, Progr. Theoret. Phys. (Kyoto) **34,** 584 (1965).

[35] P. W. Coulter, A. Scotti, and G. L. Shaw, Phys. Rev. **136,** B1399 (1964).