REVIEWS OF MODERN PHYSICS VOLUME 39, NUMBER 3 JULY 1967

Nucleon–Nucleon Interaction at Small Distances

RYOZO TAMAGAKI
Department of Physics, Hokkaido University, Sapporo, Japan

In 1956, the one-pion-exchange potential (OPEP) at large internucleon distances was established and this first success in substantial understanding of nuclear forces has become the keystone in studying nuclear forces since then. As was reported in the review article in 1956, this success was obtained by our Japanese group following the Taketani theory, which proposes to approach nuclear forces from the outside of the nucleon by combining meson-theoretical predictions at large distances with phenomenological descriptions at small distances. In this article we intend to report basic thoughts of approaches and main results recently obtained, in the theoretical works in Japan concerning nuclear forces at small distances.

I. INTRODUCTION

In 1956, the one-pion-exchange potential (OPEP) at large internucleon distances was established and this success in understanding of nuclear forces has become the keystone in studying nuclear forces since then. As reported in the review article in 1956,[1] this success was obtained by our Japanese group following the Taketani theory,[2] which proposes to approach nuclear forces from the outside of the nucleon by combining meson-theoretical predictions at large distances with phenomenological descriptions at small distances. The modified phase-shift analysis taking into account the effect of the OPEP tail now provides experimental information, which enables us to discuss nuclear forces at small distances. In this article we report basic thoughts of approaches and main results recently obtained, in the theoretical works in Japan concerning nuclear forces at small distances.[3]

The Taketani theory emphasizes the particular importance of adopting different approaches according to the extent of theoretical reliability in each region. It is appropriate to divide internucleon distances r into three regions as follows: (I) the well-established outermost region ($x=\mu r\gtrsim 1.5$, μ being the pion mass) where the OPEP is dominant, (II) the intermediate region ($x\cong 0.7\sim 1.5$) where the effects of two-pion-exchange, heavy meson exchange, etc. are to be investigated by a meson-theoretical treatment, and (III) the innermost region ($x\lesssim 0.7$) where by a phenomenological treatment we attempt to extract characteristic features to obtain clues to a future theory.

Therefore, it is important to see what characteristic phenomena or quantities each region of nuclear forces are associated with. In this respect, the validity of utilizing the impact parameter b_L, found first by Matsumoto and Watari[4] in nucleon–nucleon scattering, is to be noted in any treatment (potential model, momentum space calculation, or dispersion-theoretic treatment). The phase shift with an orbital angular momentum L is affected only slightly by nuclear forces at distances smaller than one half of the impact parameter $b_L=[L(L+1)]^{1/2}/k$, k being the barycentric momentum.

The first complete experiment at 315 MeV[5] showed the important feature in the intermediate region that there is a strong LS force in the triplet odd state, although the static potential is similar to the sum of the OPEP and the two-pion-exchange potential. The LS potential calculated by the pion theory, even if recoil effects are fully taken into account,[6] is weaker by one order of magnitude than experimentally needed. This gave rise to the conjecture that the strong LS force is mostly confined within the innermost region, and indicates some dynamics which cannot be explained pion-theoretically. Many attempts have been made to clarify the intermediate region. In Sec. II we show the results concerning nuclear forces in this region, obtained by the one-boson-exchange model and in the dispersion-theoretic approach.

The hard core proposed by Jastrow[7] is the most characteristic feature in the innermost region. Since the baryon itself is composed of some fundamental units, the problem of the repulsive core should be treated in connection with the study of the structure of elementary particles and that of high-energy physics. The phase-shift analysis in the inelastic region has been performed by Hoshizaki, Machida, and Hama in order to obtain information on the innermost region. This work has been reported by Hoshizaki at this Conference. Several proposals have been presented to explain the origin of the repulsive core, as is discussed in Sec. III. A somewhat detailed account of our proposal is presented in Sec. IV, where the repulsive core is regarded as a manifestation of the internal structure of the baryon. The calculated results are compared with the solutions of the phase-shift analysis.

---

[1] Progr. Theoret. Phys. (Kyoto) Suppl. No. 3 (1956).

[2] M. Taketani, S. Nakamura, and M. Sasaki, Progr. Theoret. Phys. (Kyoto) **6,** 581 (1951).

[3] For details and relation to other works, see a review article to be published in Progr. Theoret. Phys. (Kyoto) Suppl.

[4] M. Matsumoto and W. Watari, Progr. Theoret. Phys. (Kyoto) **11,** 63 (1954).

[5] O. Chamberlain, E. Segre, R. D. Tripp, C. Wiegand, and T. Ypsilantis, Phys. Rev. **105,** 288 (1957).

[6] N. Hoshizaki and S. Machida, Progr. Theoret. Phys. (Kyoto) **27,** 288 (1962).

[7] R. Jastrow, Phys. Rev. **81,** 165 (1951).

II. INTERMEDIATE REGION

The Sakata model proposed in 1955[8] has presented a new viewpoint on the study of elementary-particle physics. The works developed on the basis of the full symmetry theory of Ikeda, Ogawa, and Ohnuki,[9] showed that the resonance itself should be considered as a substance of the same level as mesons and baryons. The idea of the one-boson-exchange (OBE) model was proposed in 1961 by Ogawa, Sawada, Ueda, Watari, and Yonezawa.[10–12] Their basic viewpoint represented in the OBE model is as follows[13]: In the composite theory, the strong interaction should be derived from the fundamental interaction between the fundamental particles, and the Yukawa interaction observed between mesons and nucleons is regarded as a "model" Hamiltonian (effective Hamiltonian) which already contains important correlations of the fundamental interaction. Thus the elementary units in the strong interaction are composite particles such as the nucleon, π, ρ, ω, the $(\frac{3}{2}, \frac{3}{2})$-resonance etc., and the lowest-order effects of the model Hamiltonian described in terms of these units predominate the higher-order effects of the model Hamiltonian. This model Hamiltonian has its validity in the region where the structure of the elementary units is not important (e.g., $r \gtrsim 0.5\mu^{-1}$), and therefore the OBE model has been applied to the problems except that of the S wave. Similar models have been presented by many authors on a somewhat different basis.[14]

The first work of the OBE model on nuclear forces was performed referring to the potential model by Hoshizaki, Otsuki, Watari, and Yonezawa.[11] This is called the one-boson-exchange potential (OBEP) model. The following two approaches have been taken in this work:

$$\text{(I)} \qquad \text{OPEP} + \sum \text{OBEP},$$

$$\text{(II)} \qquad \text{OPEP} + \text{TPEP} + \sum \text{OBEP}.$$

An essential difference between the two approaches lies in the effect of the $(\frac{3}{2}, \frac{3}{2})$-resonance of the two-pion-exchange potential (TPEP), which produces the strong attraction in every state. When this effect is included, the ($I=0$, scalar) meson is not needed in the OBEP(II) approach. The approach (I) has been further developed by Sawada, Ueda, Watari, and Yonezawa into the one-boson-exchange contribution (OBEC) model[12] which takes into account only the lowest-order effect of the Yukawa (model) Hamiltonian and has been applied to a variety of the strong interaction.[15] These works have shown that, as already pointed out in the OBEP (I) approach, we need indispensably the (I=0, vector) meson ω, the ($I=1$, vector) meson ρ, and the ($I=0$, scalar) meson. Apart from details, the conclusions of other works have converged at least concerning this point.[14,16]

The success of the OBE model has given rise to the question whether the two-pion-exchange effect plays only a minor role. Although the importance of this effect was recognized already in the OBEP (II) approach, we were aware of overcounting.

Furuichi, Watari, and Yonezawa and others[17,18] have investigated the problem to clarify the foundation of the OBE model by use of the partial-wave dispersion relation, concentrating their study on the two-pion-exchange effect in the intermediate region defined by $1.5\mu^{-1} \gtrsim b_L \gtrsim 0.7\mu^{-1}$. Separating the iterated one-pion effect from the two-pion effect is important in understanding the interrelation between the OBE contribution and the two-pion-exchange contribution. Similarity between the two contributions exists only for the "proper" two-pion contribution, the remaining part in the left-hand cut.

From the investigations performed by the OBE model and the dispersion theory, a fairly realistic understanding has been obtained about nuclear forces in the intermediate region.[13,18]

The uncorrelated two-pion ($I=J_t=0$) contributions (J_t being the angular momentum of exchanged two pions) provides the almost state-independent attractive central force, where the contribution of the $(\frac{3}{2}, \frac{3}{2})$ resonance is large and similar to that of the $I=0$ scalar meson exchange. The $I=0$ scalar meson exchange in the OBE model stands for this effect in addition to the correlated two-pion ($I=J_t=0$) contribution.

The strong LS force is provided by the net effect of

---

[8] S. Sakata, Progr. Theoret. Phys. (Kyoto) **16**, 686 (1956).

[9] S. Ogawa, Progr. Theoret. Phys. (Kyoto) **21**, 209 (1959); M. Ikeda, S. Ogawa, and Y. Ohnuki, *ibid.* **22**, 715 (1959); **23**, 1073 (1960).

[10] M. Yonezawa, talk at the meeting in Nagoya University, held on 14 November 1961.

[11] N. Hoshizaki, S. Otsuki, W. Watari, and M. Yonezawa, Progr. Theoret. Phys. (Kyoto) **27**, 1199 (1962).

[12] S. Sawada, T. Ueda, W. Watari, and M. Yonezawa, Progr. Theoret. Phys. **28**, 991 (1962).

[13] S. Ogawa, S. Sawada, T. Ueda, W. Watari, and M. Yonezawa, Progr. Theoret. Phys. (Kyoto) Suppl. (to be published).

[14] R. S. McKean, Phys. Rev. **125**, 1399 (1962); D. B. Lichtenberg, Nuovo Cimento **25**, 1106 (1962); R. A. Bryan, C. R. Dismukes, and W. Ramsey, Nucl. Phys. **45**, 353 (1963); A. Scotti and D. Y. Wong, Phys. Rev. Letters **10**, 142 (1963); Phys. Rev. **138**, B145 (1964).

[15] S. Sawada, T. Ueda, W. Watari, and M. Yonezawa, Progr. Theoret. Phys. (Kyoto) **32**, 380 (1964); M. Kikugawa, *ibid.* **31**, 654 (1964); T. Ueda, *ibid.* **29**, 829 (1963); M. Kikugawa, S. Sawada, T. Ueda, W. Watari, and M. Yonezawa, *ibid.* Suppl. Extra Number 564 (1965); and *ibid.* **37**, 88 (1967).

[16] V. V. Babikov, Progr. Theoret. Phys. (Kyoto) **29**, 712 (1963); R. A. Bryan and B. L. Scott, Phys. Rev. **135**, B434 (1964); R. A. Arndt, R. A. Bryan, and M. H. MacGregor, Phys. Letters **21**, 314 (1966); P. B. Kantor, Phys. Rev. Letters **12**, 52 (1964).

[17] S. Furuichi and S. Machida, Nuovo Cimento **19**, 396 (1961); S. Furuichi, Progr. Theoret. Phys. (Kyoto) **27**, 51 (1962); **29**, 235 (1963); S. Furuichi and M. Yonezawa, *ibid.* **33**, 238 (1965); S. Furuichi and W. Watari, *ibid.* **34**, 594 (1965); **36**, 348 (1966).

[18] S. Furuichi, Progr. Theoret. Phys. (Kyoto) Suppl. (to be published).

the following contributions: (a) the uncorrelated two-pion ($I=J_t=1$) contribution which is produced largely by the nucleon part and similar to the ρ contribution for the ^3P waves, (b) the correlated two-pion ($I=J_t=1$) contribution (ρ exchange), (c) the ω-meson exchange, and (d) the two-pion ($I=J_t=0$) or the $I=0$ scalar meson exchange. To the triplet odd LS force, all these effects contribute additively, and there remains a future study to clarify in what proportion each term contributes to the triplet LS force.

As for the triplet even LS potential, the value of the vector–tensor coupling term ($g_\rho f_\rho/4\pi$) of the ρ meson is closely related to the feature of this potential, which is quite uncertain in the potential model due to the competition with the other nonstatic terms.[19,20]

For the tensor part, the large tensor coupling for the ($I=J_t=1$) and ρ contributions gives the desired feature, suppressing the OPEP tensor potential in the triplet even and triplet odd states.

As for the differences in coupling constants in various OBE analyses, we remark that the mass of the ($I=0$, scalar) meson m_s is closely related to those differences. For the larger m_s, we need the larger values of $g_s^2/4\pi$ and $g_\omega^2/4\pi$, since the contributions of the ($I=0$, scalar) and the ω mesons are apt to cancel each other.[13]

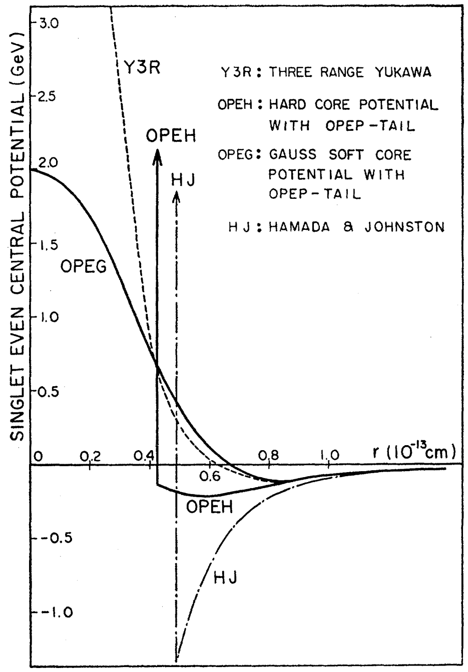

FIG. 1. Several examples for the repulsive core in the $^1S_0$ state. Y3R; the three-range superposition of the Yukawa potential with $f^2/4\pi=0.08$, $\mu=135.1$ MeV, $g_a^2/4\pi=16.5$, and $g_c^2/4\pi=32.72$ in Eq. (3). OPEH; the hard-core potential with the OPEP tail in Eq. (1). OPEG; the Gauss soft-core potential with the OPEP tail in Eq. (2). HJ; Hamada and Johnston's.[21]

III. PROPOSED MODELS FOR THE ORIGIN OF THE REPULSIVE CORE

Among various strong interactions, the nucleon–nucleon interaction has such a noticeable feature that it gives no bound state except the loosely bound deuteron and shows no resonance over a very wide energy range. The main reason for this feature is the existence of the repulsive core.

In the potentials currently used,[21,22] the repulsive core is represented by a hard core with a state-independent radius. However, various representations are possible, as we can show several examples of the $^1S_0$ repulsive core (OPEH, OPEG, Y3R, HJ) in Fig. 1. In the OPEH, the hard-core radius r_c is taken to be $2/M$, smaller than the HJ value, by cutting off the singular attraction just outside the hard core:

$$V(^1S_0)=-\mu(f^2/4\pi)Y(1+a_cY+b_cY^2)F^4(r/d)+U_c(r), \qquad (1)$$

where $Y=\exp(-\mu r)/\mu r$, $\mu=135.1$ MeV, $f^2/4\pi=0.08$, $a_c=10.0$, $b_c=7.3$, $F(r/d)=1-\exp[-(r/d^2]$ with $d=0.5\times10^{-13}$ cm and $U_c(r)=+\infty$ for $r<r_c=0.42\times10^{-13}$ cm. In the OPEG, a Gaussian soft core is adopted:

$$U_c(r)=U_c^0\exp[-(r/\eta_c)^2], \qquad (2)$$

with $U_c^0=2$ GeV, $\eta_c=0.5\times10^{-13}$ cm, $a_c=7.6$, $b_c=19.0$, and with the same values for the other parameters as in the OPEH. For such two representations (OPEH, OPEG), we have obtained reasonable potentials in all the two-nucleon states[23] which reproduce the recent solutions of the phase-shift analysis[24] quite well.

If we represent the repulsive core by a finite potential, the steepness of the soft core is indispensable. For the Gaussian soft core, we need the core height $U_c^0\gtrsim2$ GeV because $\eta_c\lesssim0.5\times10^{-13}$ cm. For the three-range superposition of the Yukawa potential (Y3R),[25] we need the steep gradient due to a rather drastic cancellation between the last two terms:

$$V(^1S_0)=-\mu(f^2/4\pi)e^{-\mu r}/r-(g_a^2/4\pi)e^{-4\mu r}/r$$
$$+(g_c^2/4\pi)e^{-5.5\mu r}/r, \qquad (3)$$

with $g_a^2/4\pi=16\sim19$ and $g_c^2/4\pi=32\sim38$.

---

[19] R. Tamagaki, M. Wada, and W. Watari, Progr. Theoret. Phys. (Kyoto) **31**, 623 (1964).

[20] R. Tamagaki and W. Watari, Progr. Theoret. Phys. (Kyoto) Suppl. (to be published).

[21] T. Hamada and I. D. Johnston, Nucl. Phys. **34**, 382 (1962).

[22] K. E. Lassila, M. H. Hull, H. M. Ruppel, F. A. McDonald, and G. Breit, Phys. Rev. **126**, 881 (1962).

[23] R. Tamagaki, Progr. Theoret. Phys. (Kyoto) (to be published).

[24] R. A. Arndt and M. H. MacGregor, Phys. Rev. **141**, 873 (1966).

[25] S. Otsuki, S. Sawada, R. Tamagaki, and W. Watari, Progr. Theoret. Phys. (Kyoto) **35**, 181 (1966).

We can present no definite evidence for the strong short-ranged repulsive core except on that of the $^1S_0$, and therefore the repulsive core may be considerably state-dependent. In fact, for the Gaussian shape a much softer core than the $^1S_0$ core is allowable for the other states; $U_c^0 \sim 500$ MeV with $\eta_c = 0.6 \times 10^{-13}$ cm.

The absorption in high-energy p–p scattering changes from the peripheral type to the central type when the energy goes very high.[26] This suggests a possibility that the repulsive core disappears at very high energies, although the coexistence of the repulsive core and the central absorption cannot be excluded completely at present.

The proposed models for the origin of the repulsive core presented up to now differ with respect to whether the interaction in the core region and that in the outer region are homogeneous or heterogeneous. The understanding of the repulsive core in the light of the neutral-vector meson exchange is based on the former viewpoint. The other proposals are based on the latter viewpoint. Also it is interesting to notice at what level the origin of the repulsive core is considered in each proposal.

Nambu proposed in 1957[27] that the repulsive core originates from the exchange of a neutral vector meson introduced to explain the isoscalar part of electromagnetic structure of the nucleon. Breit[28] and Sakurai[29] considered the neutral-vector meson exchange as the possibility of a common origin of the repulsive core and LS force. The results of an extensive study using the OBE model have led to a definite conclusion on the problem: To what extent does the vector meson exchange contribute to the repulsive core in the $^1S_0$ state? In the OBE potential we obtain the effective central coupling constant for the core potential defined in Eq. (3) as

$$\begin{aligned} g_c^2/4\pi &= g_\omega^2/4\pi - 2(f_\omega^2/4\pi) + g_\rho^2/4\pi - 2f_\rho^2/4\pi \\ &\cong g_\omega^2/4\pi - 2f_\rho^2/4\pi \lesssim 4,^{13} \end{aligned}$$

where the definition of the coupling constants is the same as in Ref. 11. This value is much smaller than $g_c^2/4\pi = 32 \sim 38$ obtained by the three-range Yukawa superposion (Y3R). Thus the contribution from the vector meson exchange plays only a partial role in producing the repulsive core.

Various heavy mesons contribute to nuclear forces in the innermost region, and we cannot deny a possibility that the accumulation of such various contributions adds up to form the repulsive core. However, it can hardly be expected that the accumulation results only in the repulsive effect because of complicated cancellations. Therefore, if we consider that the meson exchange is responsible for the repulsive core, we must look for some special condition which enables us to transform such complicated accumulation to a characteristic feature; i.e., the repulsive core.

In 1961, Machida[30] indicated a possibility to identify the repulsive core with a part of constructive forces characteristic of the Sakata model. He called this constructive force "the fundamental force," since the nucleon was regarded as one of the fundamental particles at that time and he considered that the N–N repulsive core is the inverted repulsion of the attractive fundamental force which forms a meson; i.e., an N–$\bar{N}$ bound pair. At present, since the baryon itself is considered composed of the fundamental triplet urbaryon t, a natural extension of this viewpoint may be to regard the repulsive core as originating from the t–t fundamental repulsion opposite to the t–$\bar{t}$ attraction which binds a meson. Thus the origin of the repulsive core is attributed to that of the more fundamental interaction. If the repulsive core results from the fundamental t–t repulsion, we have to find reasons why the core absorption becomes strong at super-high energies and why the coexistence of this core absorption and the repulsive core does not contradict the experimental data up to 30 GeV.

Taketani and Fujimoto[31] presented in 1965 a possibility that the repulsive core can be a manifestation of the internal structure of mesons. According to this idea, the meson is composed of a baryon and an antibaryon and spreads within

$$\lambda\!\!\!^{-}_\pi > \gamma_\pi \sim \lambda\!\!\!^{-}_B \gg \gamma_B,$$

where $\lambda\!\!\!^{-}_\pi(\lambda\!\!\!^{-}_B)$ and $\gamma_\pi(\gamma_B)$ are the Compton wavelength and the size of the pion (baryon), respectively. The level revealing the internal structure of a baryon is considered to lie at a one step deeper place than the level concerning the composite character of a meson in the structure of nature. They considered that a new mechanism of the baryon pair excitation is of more fundamental importance concerning the repulsive core. They emphasized that the problem of the repulsive core is to be investigated from a wide viewpoint without preferring one of the various possibilities.

In 1964, Otsuki, Tamagaki, and Wada proposed another possible origin based on the analogy with the α–α repulsive core[26]: The repulsive core is a manifestation of the many-urfermion structure of the nucleon core which shows up at low energies through the antisymmetrization, and it turns absorptive at very high energies where a number of inelastic channels

---

[26] S. Otsuki, R. Tamagaki, and M. Wada, Progr. Theoret. Phys. (Kyoto) **32**, 220 (1964).

[27] Y. Nambu, Phys. Rev. **106**, 1366 (1957).

[28] G. Breit, Proc. Natl. Acad. Sci. (U.S.) **46**, 746 (1960); Phys. Rev. **120**, 287 (1960).

[29] J. J. Sakurai, Nuovo Cimento **16**, 388 (1960); Ann. Phys. (N.Y.) **11**, 1 (1960); Phys. Rev. **119**, 1784 (1960).

[30] S. Machida, Soryusiron-Kenkyu (mimeographed circular in Japanese) **24**, 53 (1961).

[31] M. Taketani and Y. Fujimoto, Progr. Theoret. Phys. (Kyoto) Suppl. Extra number 651 (1965).

open. This model is developed to explain the energy dependence of p–p scattering phase shifts, by representing such a feature by a strong exchange kernel, as is shown in the next section.

The proposals mentioned here are speculative, and further investigations are needed for each possibility without preferring one of the above-mentioned possibilities.

IV. STRONG EXCHANGE KERNEL EQUIVALENT TO THE REPULSIVE CORE

Owing to the hard internal structure of the α particle as shown up in its large binding and high first excited state, the freedom of its internal motion is frozen at low energies. Thus the freedom of motion of the whole system is concentrated on the α–α relative motion. Since only the relative motion which is comparible with the Pauli principle takes place, the relative radial wave functions $U_L(R)$ have two nodes for the S wave $(L=0)$, one node for the D wave $(L=2)$, and no node for the waves higher than $G(L\geqq 4)$, where R is the distance between two α particles. The inside wave functions $(R\lesssim 2.5\times 10^{-13}$ cm) are almost energy-independent mainly due to the strong exchange kernels and the outermost node at $R\sim 2\times 10^{-13}$ cm plays a role equivalent to the hard core.[32] Provided that the nucleon has a central part of such a hard internal structure of urfermions which are spreading in the range of $1/M$, a similar situation can be expected.

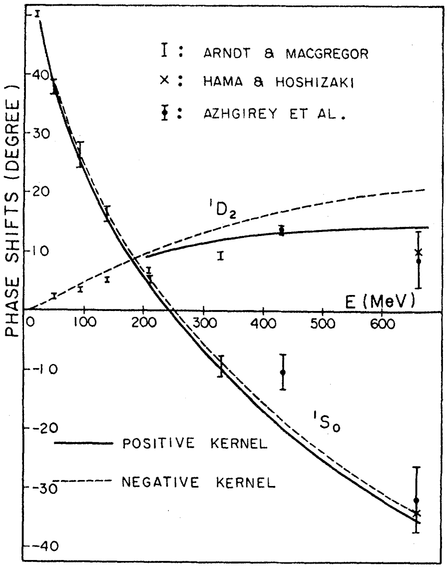

FIG. 2. Phase shifts in the singlet even state calculated by using the exchange kernels $W_{SL}(x,x')$ in Eq. (5) with $\eta=\eta'=30$, $\xi=40$, and $W_0^{(0)}=5\times 10^4$ for the positive kernel and $W_0^{(0)}=-6.8\times 10^4$ for the negative kernel. As for the direct potentials, see SE-1 and SE-2 in Table 1 in Ref. 36). The solutions of the phase-shift analysis are: Below 330 MeV (Ref. 24); 660 MeV [Y. Hama and N. Hoshizaki, Progr. Theoret. Phys. (Kyoto) **31,** 609 (1964); L. S. Azhgirey *et al.*, Phys. Letters **6,** 196 (1963); L. S. Azhgirey, J. Nucl. Phys. **1,** 867 (1965)]; 435 MeV [L. S. Azhgirey *et al.*, Phys. Letters **6,** 196 (1963); L. S. Azhgirey, J. Nucl. Phys. **1,** 867 (1965)].

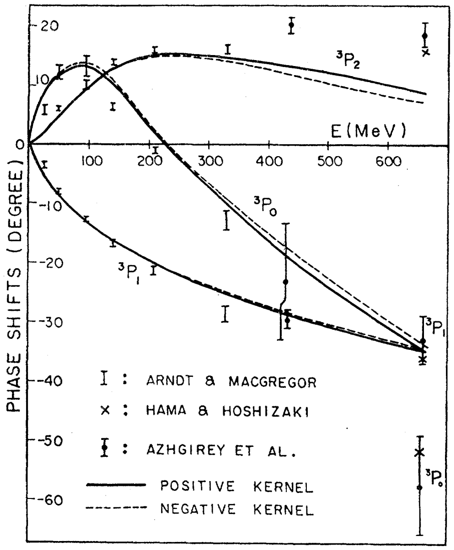

FIG. 3. Phase shifts in the ^3P states calculated by using the exchange kernels $W_{SL}(x,x')$ in Eq. (5) with $\eta=\eta'=30$, $\xi=40$, and $W_1^{(0)}=6\times 10^4$ for the positive kernel and $W_1^{(0)}=-3.3\times 10^5$ for the negative kernel. As for the direct potentials, see TO-1 and TO-3 in Table 1 in Ref. 36.

In connection with the existing models of baryons,

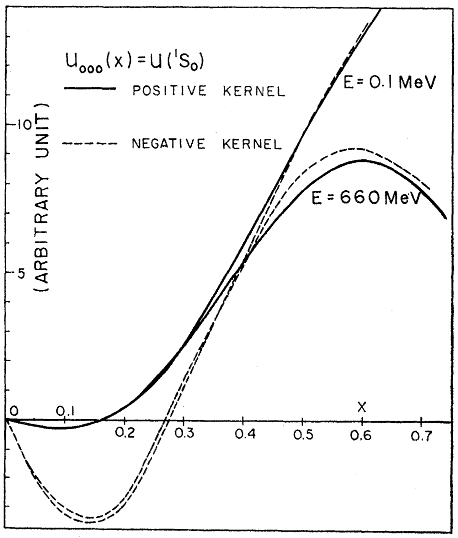

FIG. 4. Radial wave functions $u_{000}(x)$ in the $^1S_0$ state calculated by using the exchange kernels. The parameters are the same as those used in calculating the phase shifts in Fig. 1.

---

[32] R. Tamagaki and H. Tanaka, Progr. Theoret. Phys. (Kyoto) **34,** 191 (1965); S. Okai and S. C. Park, Phys. Rev. **145,** 787 (1966).

Machida and Namiki[33] pursued this viewpoint, representing the existence of the repulsive core by the condition that the wave function vanishes at the origin. They found that the *ttt*-type configuration for the baryon core as in the quark model[34] is possible, and they have developed such attempts for various phenomena.

This model called the "structural core" model has been studied further by Otsuki, Yasuno, Sato, and myself.[35,36] The best way to represent the many-urfermion structure, irrespective of details, is to employ a strong exchange kernel which characterizes the core region and makes at least an energy-independent node near the core surface. The radial equation for the two-nucleon relative motion is written as

$$\left\{-\frac{\mu}{M}\frac{d^2}{dx^2}+\frac{\mu}{M}\frac{L(L+1)}{x^2}+\mu^{-1}V_D(SLJ;x)-\frac{k^2}{\mu M}\right\}U_{SLJ}(x)+\mu^{-1}\int W_{SL}(x,x')U_{SLJ}(x')dx'=0, \quad (4)$$

where $x=\mu r$ and $x'=\mu r'$. S, L, and J mean the spin, orbital, and total angular momentum, respectively. $V_D(SLJ;\, x)$ tends to the usual meson exchange potential outside the core region $(x\gtrsim 0.5)$. The nonlocal exchange kernel $W_{SL}(x,\, x')$ vanishing rapidly for $x,\, x'\gtrsim 0.5$ dominates the main feature of the inside wave function. The gross property of such an exchange kernel is represented by

$$W_{SL}(x,x')=(\mu^2/M)W_s^{(0)}xx' \times\{\exp[-(\eta x^2+\eta' x'^2)]+x\leftrightarrow x'\}\mathcal{I}_L(\xi xx'), \quad (5)$$

where $\mathcal{I}_L(\rho)\equiv(\pi/2\rho)^{1/2}I_{L+1/2}(\rho)$ and the order of magnitude of the parameters is estimated from the spatial size of the nucleon core $(\sim 1/M)$; $\eta\sim\eta'\sim\xi=30\sim 60$ and $|W_s^{(0)}|\sim 10^{5\sim 7}$.

Some examples illustrating the equivalent role of the strong exchange kernel to that of the repulsive core are shown in Figs. 2 and 3 for the singlet even and the triplet odd states, where as the direct potential $V_D(SLJ;\, x)$ we employ the usual potential form which is smoothly cut off in the core region.[35] Both positive and negative kernels are possible. The nodal behavior of the radial wave functions $U_{SLJ}(x)$ is as follows:

One node for the ${}^1S_0$ and ${}^3P_J$ states.
No node for the higher states than the D wave.

Figure 4 shows the nodal behavior of the ${}^1S_0$ wave $U_{000}(x)$, and the same behavior appears in the ${}^3P_J$ waves $U_{11J}(x)$. The inside wave functions are almost energy-independent and the energy dependence appears at the region $(x>0.4)$.

We discuss briefly the physical interpretation of these results.

(1) The wave with nodes almost energy-independent at short distances seems to be in some excited configuration. However, we can rule out this possibility by considering the Pauli principle, which excludes some low angular momentum states without a node, similarly to the α–α case. A most simple model by which we can visualize such a situation is that the nucleon core is in the *ttt* configuration with the space symmetry.

(2) This exclusion occurs in a way which is dependent on the angular momentum. This feature corresponds to the disappearance of the repulsive core in the high angular momentum states, and is one of the characteristic features of this model to be checked in the future.

(3) If we want to reproduce only the scattering phase shifts by the hard-core model, it is sufficient to make a node near the core radius[28]; i.e., to assume a superstrong interaction in the core region. However, in order to exclude the lower bound states with the baryon number=2 than the deuteron, the exclusion principle is indispensable. This thought (the lowest configuration with nodes) characterizes our "structural core" model.

(4) The essentially energy-independent feature of the inside wave functions distinguishes our model from the other attempts to reproduce the ${}^1S_0$-phase shifts by using a nonlocal kernel as an alternative expression for a velocity-dependent potential.[37] This energy-independent feature probably corresponds to the success of the modified boundary condition model proposed by Saylor, Bryan, and Marshak.[38]

In this model, the repulsive core has no literal meaning and rather is a phenomenological substitute for a character of the many-body system which is hardly excited at low energies.

---

[33] S. Machida and M. Namiki, Progr. Theoret. Phys. (Kyoto) **33**, 125 (1965).

[34] M. Gell-Mann, Phys. Letters **8**, 214 (1964); G. Zweig, CERN preprint (1964).

[35] S. Otsuki, R. Tamagaki, and M. Yasuno, Progr. Theoret Phys. (Kyoto) Extra Number 578 (1965).

[36] M. Sato and R. Tamagaki, Progr. Theoret. Phys. (Kyoto) **37**, No. 6 (1967).

[37] For example, R. E. Peierls, *Proceeding of the International Conference on Nuclear Structure* (University of Toronto Press, Toronto, Canada, 1960), p. 7; M. Razavy, G. Field, and J. S. Levinger, Phys. Rev. **125**, 269 (1962); O. Rojo and L. M. Simmons, *ibid.* **125**, 273 (1962); A. M. Green, Nucl. Phys. **33**, 218 (1962).

[38] D. P. Saylor, R. A. Bryan, and R. E. Marshak, Phys. Rev. Letters **5**, 266 (1960); H. Feshbach, E. Lomon, and A. Tubis, *ibid.* **6**, 635 (1961).