Meson Theory of Nucleon–Nucleon Interaction*

DAVID Y. WONG
University of California, San Diego, La Jolla, California

Exchange of π, η, ρ, ω, ϕ mesons and a pair of pions in the relative s state can account for all the important features of the nucleon–nucleon interaction. The method of dispersion relation is employed to calculate nucleon–nucleon phase shifts starting from the relativistic single-particle-exchange Feynman diagrams. Validity of the method is tested in the nonrelativistic limit where the solution of the dispersion relation is compared to the solution of the Schrödinger equation for a superposition of Yukawa potentials. For potentials that contain a long-range attraction and a strong short-range repulsion there is little difference between the two solutions. In the meson model, the attraction comes primarily from the π and the s-wave two-pion exchange. The exchange of ω and ϕ produces the required short-range replusive force.

Two approaches to the nucleon–nucleon scattering problem were considered. (a) The meson model was taken to be the basis of a detailed fit to the scattering data. A total of 12 adjustable parameters were introduced and a fit to the pp and np data was obtained. The accuracy is comparable to that of pure phenomenological potentials with considerably more parameters. (b) The number of parameters in the meson model is minimized using approximate symmetry considerations to clarify the physical significance of the model. With only four adjustable parameters, all the important features of the phase shift are reproduced. These results lend support to the idea that the strongly correlated multimeson systems ρ, ω, and ϕ play a dominant role in the inner part ($\lesssim 0.3$ F) of the nucleon–nucleon potential while the exchange of two pions in a relative s-state accounts for the gross feature of the medium-range attraction.

The discovery of strongly correlated multimeson systems in high-energy experiments[1] has led to a meson theory of nuclear forces that is qualitatively different from the meson theories of the past decade. On the one hand, the situation is now more complicated because the meson–meson interaction must be taken into account in addition to the meson–nucleon interaction. On the other hand, certain simplifications can be realized if the contribution of these strongly correlated states in some sense dominates the nuclear forces. For example, the exchange of a strongly correlated pair of pions may be replaced by the exchange of a single meson. The quantum numbers and the energy of such a meson can be determined by various high-energy reactions, in particular, the nucleon–antinucleon annihilation process.[2] Here, one examines annihilation into three or more pions and plots the number of events versus the invariant mass of various two-pion combinations. The result shows a spectrum with a distinct peak at $\sim$750 MeV and a width of $\sim$100 MeV. Further analysis of angular correlations determines the quantum number of the two-pion system to be $I=1$, $J=1$ (the ρ meson). In addition to the quantum numbers and the average mass, one also acquires the important information that a substantial fraction of the total events does fall within the peak of the spectrum. This shows that not only are the pions themselves correlated to form a ρ meson but this meson also couples strongly to the nucleon. Therefore, it must play a significant role in the nucleon–nucleon potential. One is then led to construct models which include the exchange of a single ρ meson with a given mass in the nucleon–nucleon potential. The coupling constant, however, cannot be obtained directly from the annihilation experiment because physical nucleons can only exchange virtual ρ mesons whose coupling strength need not be the same as that of a real meson. On the other hand, the phenomenological coupling constant can also absorb the uncorrelated part of the $I=1$ two-pion spectrum to a first approximation. Thereby, one obtains a very simple $I=1$ two-pion potential.[3]

In addition to the π and the ρ, the ω (780 MeV) and the ϕ (1020 MeV) mesons [both $I=0$, $J=1$, $P=(-1)$] are also found in nucleon–antinucleon annihilation. The decay of ω results in three pions while the decay mode of the ϕ is almost entirely $K\bar{K}$. In the same manner as one treats the ρ meson, the exchange of ω and ϕ between two nucleons can be used to replace the three-pion and the $K\bar{K}$ spectra, respectively. Here, the ω and ϕ contribution to the nucleon–nucleon potential is repulsive in all states (similar to the Coulomb potential between like charges). As shown later, the strength of the repulsion is sufficiently great so that the shielding effect is very strong at internucleon distances smaller than the Compton wavelength of the ϕ ($\sim$0.2 F). Therefore, it is plausible that higher mesonic resonances such as f_0, A_1, and A_2 can be neglected in the problem of nuclear forces.[4] Since uncorrelated three-pion and $K\bar{K}$ systems are probably unimportant at energies below 1 BeV, they can be neglected for the same reason.[5]

Returning now to the two-pion system, we have discussed the contribution to the nucleon–nucleon

* Work supported in part by the U.S. Atomic Energy Commission.

[1] For a brief summary, see S. Gasiorowicz, *Elementary Particle Physics* (John Wiley & Sons, Inc., New York, 1966), pp. 313–334.

[2] In addition to Ref. 1, some relevant data on nuclear–antinucleon annihilation are given by C. Baltay *et al.*, Phys. Rev. Letters **15**, 532 (1965).

[3] The I spin refers to that of the two-pion system, the projection on the nucleon–nucleon system gives a ratio of -3 to 1 for $I=0$ and $I=1$, respectively.

[4] There is also no experimental evidence of a strong coupling between the nucleon and f_0, A_1, or A_2.

[5] Aside from the energy spectrum, the three-pion and the $K\bar{K}$ systems can have a great variety of quantum numbers, but no strong correlation is found.

potential due to two pions with a total I-spin of one. For $I=0$, the experimental situation on the two-pion correlation is somewhat dubious. In the annihilation experiment, there is no evidence of a peak in the $I=0$ two-pion spectrum. However, the two pions at moderate energies are likely to be in a relative s state (the angular momentum must be even for $I=0$ and odd for $I=1$) and s-wave correlations often do not show a peak. In any event, the coupling of a nucleon to an $I=0$ $\pi\pi$ system has a spectrum starting from twice the pion rest mass, and the low-energy part of this spectrum is predominantly s wave; that is, the long-range part of the two-pion contribution to the nuclear force comes primarily from the $I=0$ s-wave $\pi\pi$ pair. We consider the simple model which parameterizes the entire $I=0$ $\pi\pi$ spectrum by taking a single scalar meson σ whose mass and coupling constant are to be determined phenomenologically by the nucleon–nucleon scattering data. Such a model does not rely essentially on the existence of the scalar meson.

A somewhat more complex way of parameterizing the $I=0$ $\pi\pi$ contribution had been considered by Scotti and Wong.[6] They took a continuous two-pion spectrum including a $\pi\pi$ scattering length and a cutoff as adjustable parameters and obtained results essentially the same as those given by the σ meson model. In the treatment of the continuous spectrum, it was important to normalize the coupling strength of the pion pair to the nucleon by making use of experimental data on pion–nucleon scattering. This constitutes an independent source of information on the coupling strength and can be regarded as a supporting evidence on the necessity of an $I=0$ $\pi\pi$ term in the nucleon–nucleon potential.

Finally, there is another meson, the η (548 MeV), which has the same quantum numbers as the pion except for $I=0$. The production of η is quite prominent in pion–nucleon reactions but is very rare in nucleon–antinucleon annihilation. There are also theoretical arguments that the η coupling to the nucleon should be weak compared to that of the pion. In practice, one finds that the η contribution to the nucleon–nucleon potential is small even if the coupling strength is comparable to that of the pion. Therefore, one may choose to neglect it altogether.

To summarize, we can construct a nucleon–nucleon potential in terms of the exchange of six mesons: π, σ, η, ρ, ω, and ϕ. The masses of all these mesons are known except the σ which is introduced purely as a way of parameterizing the exchange of two pions in a relative s state. As for the coupling constants, they are all undetermined except the pion. However, as shown later, one can employ the approximate SU_3 symmetry to relate some of the coupling constants.

Let us now consider the calculation of the potential and the methods of solving for the scattering amplitudes. Since we are replacing multimeson spectrums by single mesons of various quantum numbers and masses, the potential is simply the sum of the one-meson exchange diagrams calculated according to the Feynman rules. One can substitute this potential into the Bethe–Salpeter equation and try to solve for the scattering amplitudes. The solution would be the sum of ladder graphs which are probably the most important diagrams for nucleon–nucleon scattering in the elastic region. Unfortunately the Bethe–Salpeter equation has no solution because there is a divergence due to the spin of the vector mesons. Therefore, one must use some kind of an approximate scheme through the introduction of a cutoff.

Aside from the question of a cutoff, the Bethe–Salpeter equation for nucleon–nucleon scattering is so complex that it is not practical for obtaining a numerical solution. A further approximation can be made by reducing the Bethe–Salpeter equation into a nonrelativistic Lippman–Schwinger equation. However, the reduction can be justified only if both the momentum of the nucleon and the mass of the meson are small compared to the rest mass of the nucleon. This is certainly not the case for the vector mesons. An alternative approximation is to keep the relativistic single-meson exchange terms exactly and solve for the scattering amplitude by the approximate on-shell dispersion relation as follows.

First we consider the spin singlet partial wave amplitude $A_J(p^2)$ (the I-spin index is suppressed) normalized so that

$$\operatorname{Im} A_J(p^2) = (mp/E) \mid A_J(p^2) \mid^2; \qquad p^2 > 0. \quad (1)$$

The quantity (mp/E) is the relativistic phase space factor. Let us denote the contribution of the sum of single-meson exchange terms by $B_J(p^2)$. The dispersion relation is the following integral equation for $A_J(p^2)$:

$$A_J(p^2) = B_J(p^2) + \pi^{-1} \int_0^\Lambda dp'^2 \left(\frac{mp'}{E'}\right) \frac{\mid A_J(p'^2) \mid^2}{p'^2 - p^2 - i\epsilon}. \quad (2)$$

The integral on the right-hand side is constructed so that the imaginary part of the amplitude satisfies the unitarity condition (reality of phase shift) given by (1). A cutoff Λ is introduced because of the logarithmic divergence in $B_J(p^2)$ due to the exchange of vector mesons.

In terms of analytic structures, one sees that the only singularities of the integral in (2) are two branch points at $p^2=0$ and $p^2=\Lambda$ joined by a branch cut along the positive real axis. All other singularities of $A_J(p^2)$ are contained in $B_J(p^2)$. From the analysis of the Yukawa-type potentials, one finds that the potential for the exchange of a meson with mass μ gives rise to branch points in the scattering amplitude at $p^2 = -(n\mu)^2/4$; $n=1, 2, \cdots$. The index n corresponds to

[6] A. Scotti and D. Y. Wong, Phys. Rev. Letters **10**, 142 (1963); and Phys. Rev. **138**, 145 (1965).

the number of times the potential acts, in other words, the exchange of n mesons. For a given potential, it is implicitly assumed that the mesons themselves do not interact.

Our model of $B_J(p^2)$ clearly does not contain the exchange of any meson more than once. While we can argue that the exchange of two mesons are replaced by ρ and σ, we have no provision in $B_J(p^2)$ for the rescattering of ρ, σ, etc. The justification for dropping such terms is that they correspond to very short range forces which may be effectively shielded by the repulsion due to ω and ϕ exchanges. Although the rescattering terms in $B_J(p^2)$ have been dropped, a certain part of the rescattering process is included in the solution $A_J(p^2)$ through the nonlinearity of the equation.

For a given potential $B_J(p^2)$, the integral equation (2) can be solved by the N/D method.[6] If the potential is sufficiently strong and attractive, the solution $A_J(p^2)$ will contain a bound state pole. The energy and residue of the pole are entirely determined by the potential $B_J(p^2)$. Formally, this pole must be added back into equation (2) as an extra singularity of $A_J(p^2)$.

For the triplet $L=J$ amplitude, the integral equation is formally identical to (2). For the triplet amplitudes with $L=J\pm1$, $A_J(p^2)$ and the potential are to be replaced by 2×2 matrices, but the method of solution is essentially the same as the uncoupled cases.

Among the parameters of the potential, we have predetermined values for the mass and the coupling constant of the pion and the masses of η, ρ, ω, and ϕ. The adjustable parameters are m_σ^2, g_σ^2, g_η^2, $g_{\rho1}^2$, $g_{\rho2}^2$, $g_{\omega1}^2$, $g_{\phi1}^2$. Here, $g_{\rho1}^2$ and $g_{\rho2}^2$ are the vector and the tensor coupling constants of the ρ meson. For the ω and ϕ mesons, the tensor couplings must be negligible, otherwise they would contribute to a large isoscalar anomalous magnetic moment of the nucleon. In addition to the above parameters, the cutoff Λ must also be taken as adjustable.

The threshold behavior of a partial wave amplitude of orbital angular momentum L is of the order p^{2L}. This behavior is satisfied by the potential, but in general not satisfied by the integral term of the dispersion relation. Since the threshold behavior must be the result of the cancellation between the dispersion integral and the rescattering terms in $B_J(p^2)$, we can add a term, say, an Lth-order pole, to the single-meson potential and adjust the residues to achieve the proper threshold behavior. Although this procedure is somewhat arbitrary, the dispersion integral is quadratic in the scattering amplitude and therefore small for higher partial waves. Consequently the added term in $B_J(p^2)$ is, in general, small compared to the single-meson potential, and the effect on the scattering amplitude is also small except for restoring the threshold behavior. In any event, the procedure described above introduces a pole whose position is an addition parameter of the model.

One can take different views of the meson model. One possible viewpoint is that there are already six mesons in the model; one should see whether they contain enough degrees of freedom to form a basis for the phenomenological analysis of the nucleon–nucleon scattering data. This approach was taken by Scotti and Wong during 1963–65. In addition to m_σ^2, g_σ^2, g_η^2, $g_{\rho1}^2$, $g_{\rho2}^2$, $g_{\omega1}^2$, $g_{\phi1}^2$, they also took m_ρ^2 as an adjustable parameter because the spread of the ρ-meson spectrum is quite large ($\sim$100 MeV). Instead of using a sharp cutoff on the dispersion integral, they multiplied each vector meson potential by an exponential convergent factor, thereby introducing three cutoff parameters in the place of Λ. Together with the position of the pole for the threshold behavior, they used a total of twelve adjustable parameters.

By varying the twelve parameters, a fit to the pp and np scattering data up to 400 MeV was obtained. The results are shown in Figs. 1–12. The accuracy of the fit is comparable to those using purely phenomenological potentials. The number of parameters in the meson model is, however, considerably smaller.

Instead of using the meson model to obtain a detailed phenomenological fit to the data one can also take a somewhat different point of view. Since the replacement of multi-meson spectra by single mesons is an essential part of the model, one might wish to test its validity first on a more qualitative basis. To this end, one would want to reduce the number of adjustable parameters to a minimum, to see what is the role of each meson and what features of the nucleon–nucleon interaction can be reproduced by the model. This direction was pursued by Ball, Scotti, and Wong.[7]

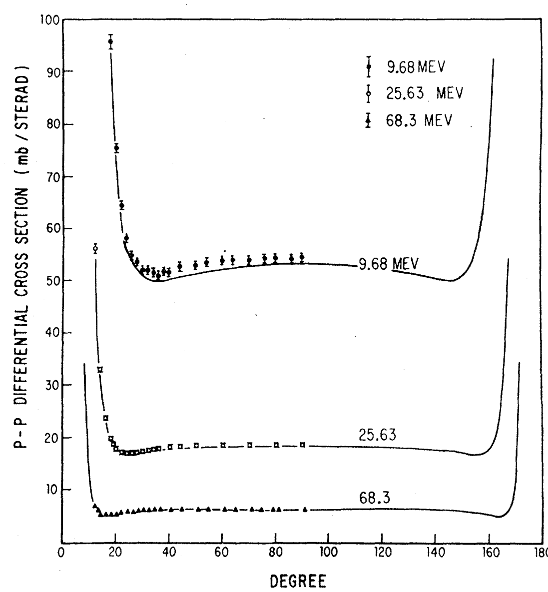

FIG. 1. P–P differential cross sections at 9.68, 25.63, and 68.3 MeV.

[7] J. S. Ball, A. Scotti, and D. Y. Wong, Phys. Rev. **142**, 1000 (1966).

TABLE I. Comparison of theoretical and experimental nuclear bar phase shifts in degrees.[a]

		Lab kinetic energy in MeV				
		50	96	142	210	310
$pp\ ^1s_0$	Exptl.	37.7±0.6	25.1±2.4	16.6±0.7	5.1±0.6	−6.9±1.6
	Theor.	42.0	28.6	17.4	3.6	−15.1
$^3p_0$	Exptl.	12.0	12.8±1.9	6.3±0.6	−0.7±0.6	−11.3±1.7
	Theor.	12.6	12.0	7.9	−0.1	−12.8
$^3p_1$	Exptl.	−8.0±0.3	−13.0±0.5	−17.1±0.4	−21.6±0.6	−28.5±1.3
	Theor.	−7.8	−11.4	−14.4	−18.4	−24.8
$^2p_2$	Exptl.	6.1±0.2	10.6±0.5	13.7±0.2	15.9±0.3	16.4±0.7
	Theor.	5.8	11.3	15.4	18.7	20.2
$np\ ^3s_1$	Exptl.	60.8±2.7	44.5±1.7	29.6±0.9	17.6±2.4	−1.0±5.2
	Theor.	76.4	56.4	42.0	25.9	5.8

[a] Parameters: $m_\sigma = 3.9\ m_\pi$, $g_\sigma^2 = 5.15$, $g_v^2 = 1.36$, cutoff = 600 MeV (Lab kinetic energy).

By using the approximate SU_3 symmetry and the theory of ω–ϕ mixing, they introduce a single vector-meson coupling constant g_v^2 with

$$g_{\rho 1}^2 = g_{\omega 1}^2 = \tfrac{1}{2} g_{\phi 1}^2 = g_v^2. \tag{3}$$

Furthermore, assuming the vector-meson-dominant model of the low momentum transfer nucleon form factor, they relate $g_{\rho 2}^2$ to $g_{\rho 1}^2$ with the ratio given by the isovector anomalous magnetic moment:

$$g_{\rho 2}^2 = 13.4 g_{\rho 1}^2 = 13.4 g_v^2. \tag{4}$$

Since the contribution of the η meson is small, they use the even cruder SU_6 symmetry to relate the η-coupling constant to that of the pion. This gives

$$g_\eta^2 \simeq (m_\pi/m_\eta)^2 (1-1.6)^2 g_\pi^2.$$

Finally, a cutoff parameter for the total energy squared, s_c, was used to terminate the dispersion integral $(\Lambda = \frac{1}{4}s_c - m^2)$ and the pole for the threshold behavior was taken to be at the total energy squared equal to $-s_c(p^2 = -\frac{1}{4}s_c - m^2)$. In this way, the total number of adjustable parameters was reduced to four, namely, m_σ^2, g_σ^2, g_v^2, and s_c.

With the four-parameter model described above, it was found that all the important features of the pp and np phase shifts can be reproduced. The results of a typical fit is shown in Table I.

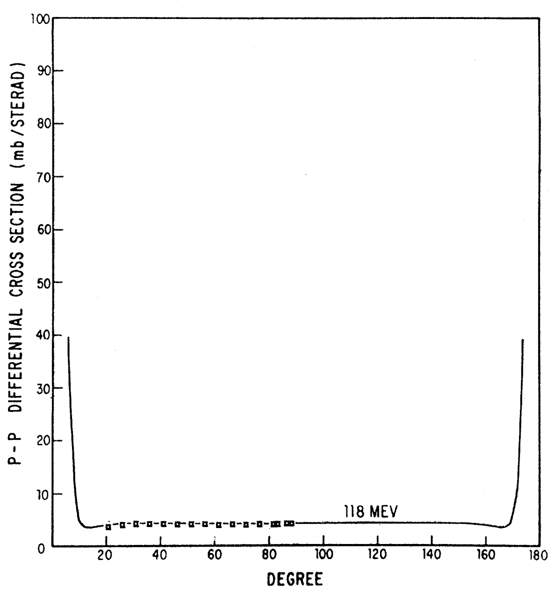

FIG. 2. P–P differential cross sections at 118 MeV.

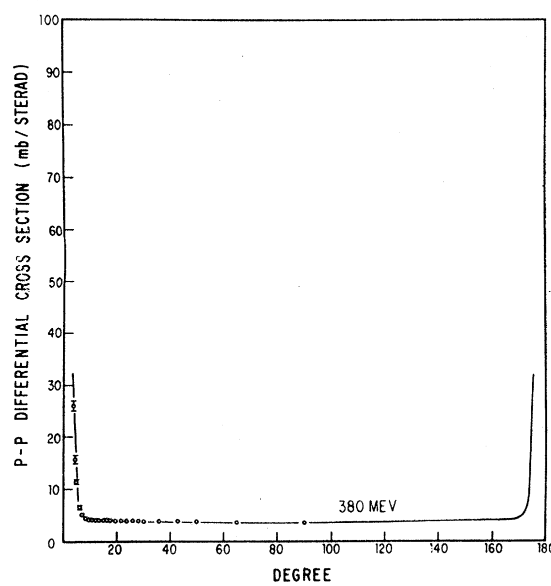

FIG. 3. P–P differential cross sections at 380 MeV.

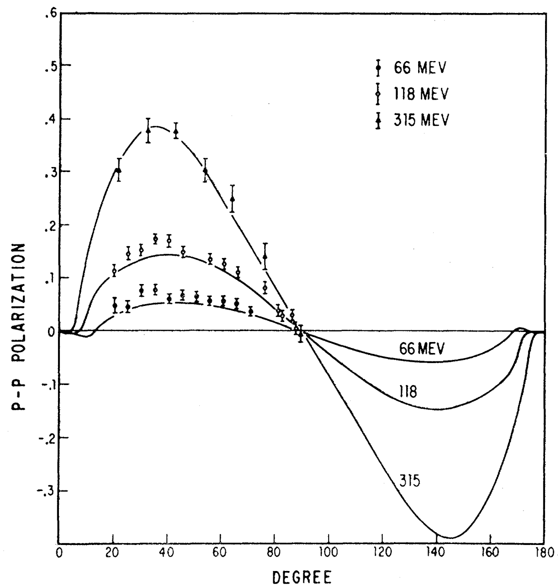

FIG. 4. *P–P* polarization at 66, 118, and 315 MeV.

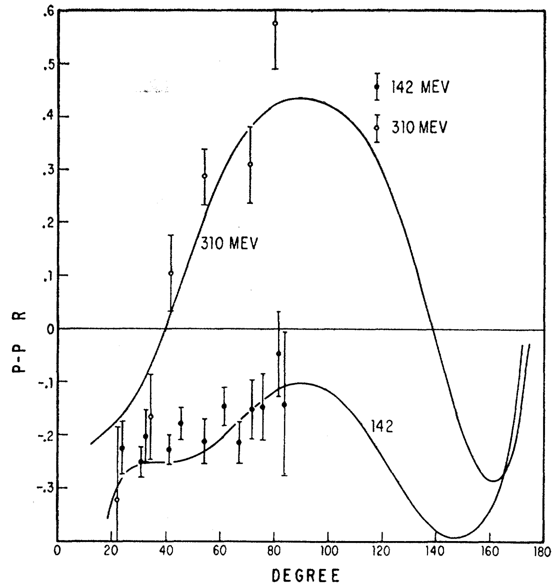

FIG. 6. *P–P R* parameter at 142 and 310 MeV.

Aside from getting a moderately good fit to the data, the numerical values of the parameters are also physically reasonable. The value of $m_\sigma = 3.9 m_\pi$ corresponds to a two-pion spectrum with approximately one-pion unit of kinetic energy for each pion. The coupling constant $g_\sigma^2 = 5.15$ is quite large. It accounts for the bulk of the medium range attraction which is not highly spin or *I*-spin-dependent. The value $g_v^2 = 1.36$ is comparable to that deduced from the vector dominant model of the electromagnetic form factor. The amount of repulsion due to ω and ϕ is roughly four times g_v^2 which is probably high enough to give an effective shielding for the very short range forces. The ρ-exchange apparently provides enough additional spin and *I*-spin dependence to account for the qualitative features of the *p* and higher partial waves. As for the cutoff, 600 MeV in lab kinetic energy is roughly where the elastic unitarity condition begins to fail substantially, and our dispersion relation is no longer valid beyond that region. With these results, one can feel fairly confident that the correlated multimeson systems do play an important role in the nuclear forces and that the gross features of the nucleon–nucleon interaction can be reproduced by a simple model where these multimeson

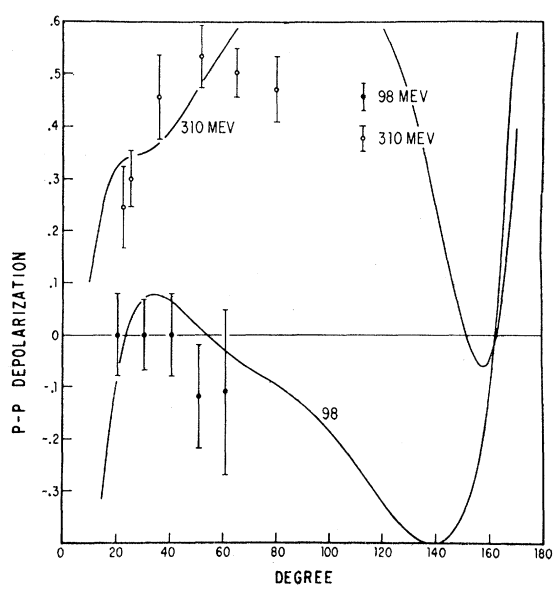

FIG. 5. *P–P* depolarization at 98 and 310 MeV.

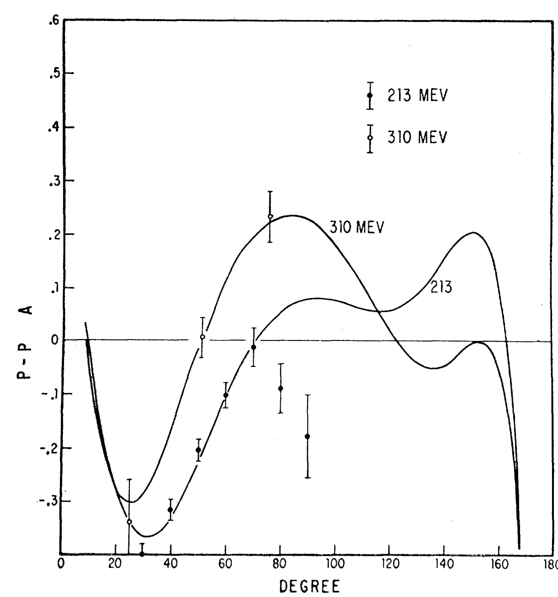

FIG. 7. *P–P A* parameter at 213 and 310 MeV.

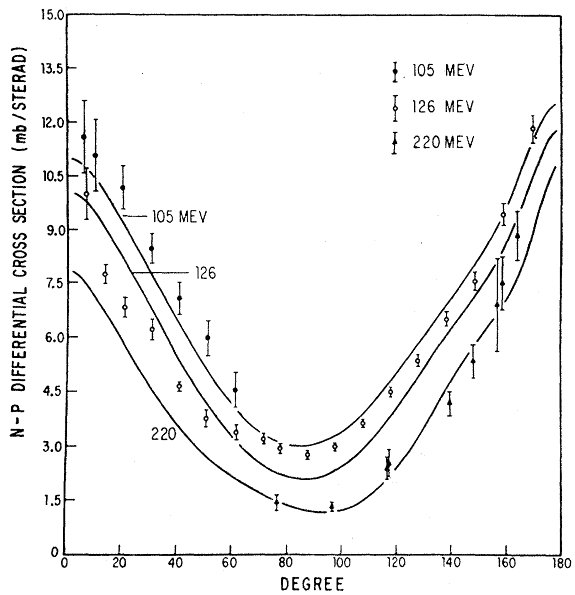

FIG. 8. N–P differential cross sections at 105, 126, and 220 MeV.

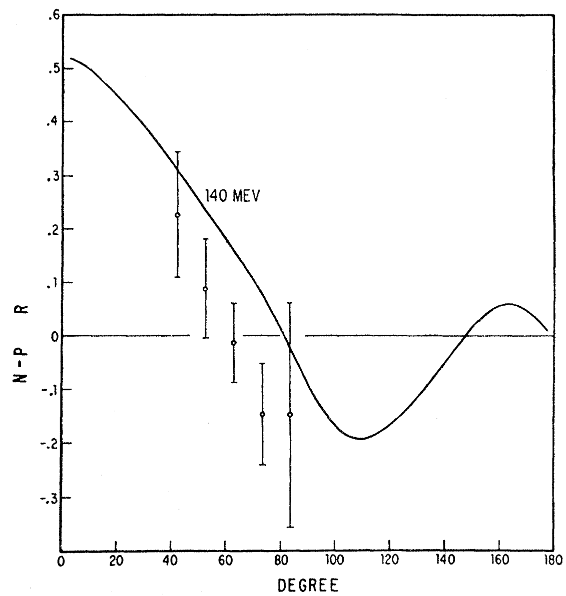

FIG. 10. N–P depolarization at 128 MeV.

systems are replaced by single mesons of given masses and a given set of quantum numbers. The solution given above does contain a triplet $J=1$ np bound state, but the binding energy is greater than the observed deuteron by $\sim$8 MeV.

The major weakness of the dispersion theory is the neglect of rescattering terms in $B_J(p^2)$. To concentrate on this point, we construct a simple model in which the rescattering corrections can be calculated exactly, and compare the solution with that of the dispersion relation. We pose the following problem: Take the superposition of a medium-range attractive Yukawa potential plus a short-range repulsive Yukawa potential. Choose a fixed value for the s-wave scattering length and for the effective range. Adjust the strengths of the attraction and the repulsion to fit the predetermined scattering length and effective range using (a) the dispersion relation and (b) the Lippman–Schwinger equation. Having done this, we compare the difference of the required coupling constants for (a) and (b) and also compare the energy dependence of the s-wave phase shift in the two solutions.

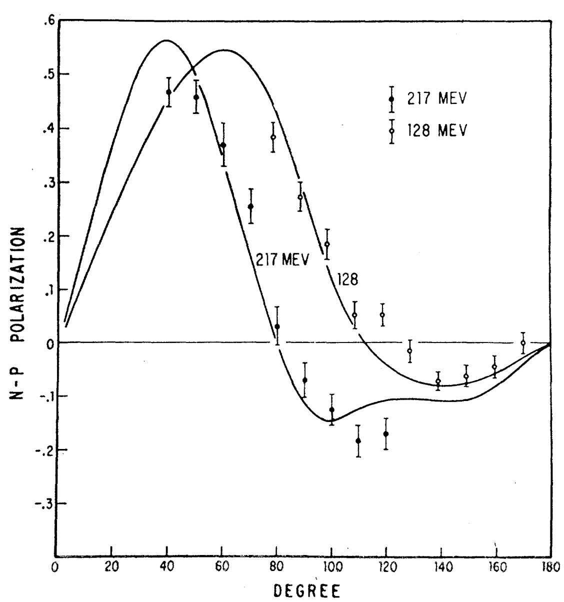

FIG. 9. N–P polarization at 128 and 217 MeV.

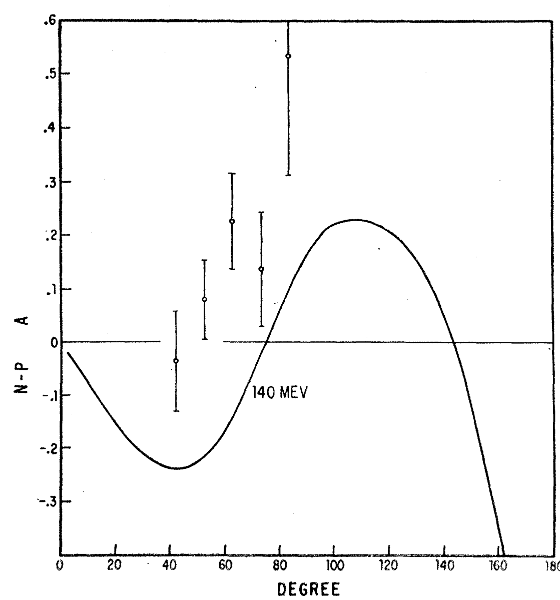

FIG. 11. N–P R parameter at 140 MeV.

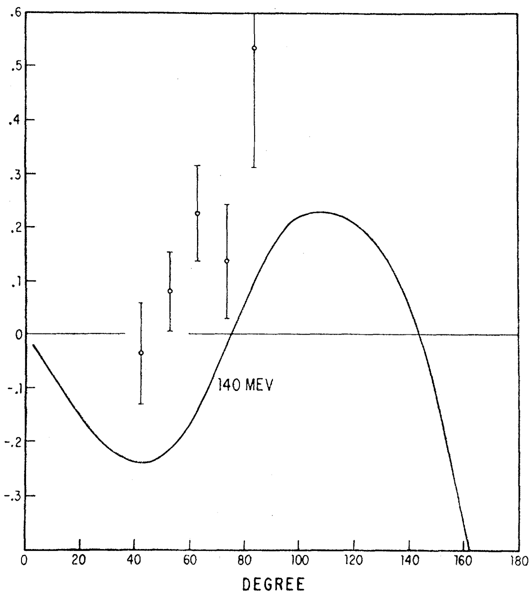

Fig. 12. *N–P A* parameter at 140 MeV.

A typical example is as follows. We take an attractive potential of inverse range $\mu_A = 3m_\pi$ and a repulsive potential with $\mu_R = 6m_\pi$. For a scattering length of $a = -4.07\lambda_\pi$ and an effective range of $r = 1.95\lambda_\pi$, the required coupling constants in (a) and (b) are[8]

(a) Dispersion relation	(b) Lippman–Schwinger equation
$g_A^2 = 10.5$	$g_A^2 = 9.5$
$g_R^2 = 15.0$	$g_R^2 = 16.0$.

[8] G. Marchesini and D. Y. Wong, *Proceedings of the XIII International Conference on High Energy Physics* (to be published).

The difference in the two sets of coupling constants is only of the order of 10%. Furthermore, the energy dependences of the two solutions are in fact very similar. For example, the s-wave phase shift passes through zero at $p^2 = 6.4\lambda_\pi^{-2}$ in case (a) and shifts slightly to $p^2 = 6.6\lambda_\pi^{-2}$ in case (b). These results show that for the type of potential considered the missing rescattering terms in the dispersion model can be simulated quite well by the readjustment of the coupling constants to the extent of $\sim 10\%$. In other words, the neglect of the rescattering terms can account for a 10% uncertainty in the physical interpretation of the coupling constants, but the semi-phenomenological fit of the scattering amplitude is not significantly affected. Since this simple model gives an s-wave phase shift quite similar to the $pp\ {}^1s_0$ phase shift, it is probably true that the neglect of rescattering terms in our meson model can be justified to the same extent.

Similar comparisons for purely attractive potentials, show that the difference between the dispersion theory and the Lippman–Schwinger equation can be very large. For example, a single Yukawa potential with the range of λ_π and $g^2 = 3.5$ has a bound state of $\sim$10-MeV binding energy. The same binding energy would require a coupling constant about twice as large if one uses the dispersion relations. This result further emphasizes the importance of the inner repulsive force in the application of the dispersion relation to the meson model.

To summarize, a simple model of nuclear forces based on the exchange of correlated mesonic systems is in good agreement with experimental data on nucleon–nucleon scattering. Such a model can be used as a basis for the detailed phenomenological analysis of the data, but perhaps more significantly, it also leads to a qualitative understanding of the nuclear forces in terms of meson–nucleon and meson–meson interactions.