Interaction of Two Nucleons at Low Energies. $I=1$*

LEON HELLER
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico

A summary of the knowledge of the low-energy interaction of two nucleons with one unit of isobaric spin is presented. The single energy analyses of accurate low-energy proton–proton scattering data are discussed with emphasis upon the electromagnetic effects which have been included and those which should be. The phase shifts which result are presented and discussed in terms of the effective range expansion and potentials. A brief discussion of charge symmetry and independence in terms of potentials and dispersion relations is given. The equality of the neutron–neutron and proton–proton nuclear interactions, which seems to be established, may not be sufficient for understanding the problem of mirror nuclei. The question of charge independence is much more difficult.

I. OUTLINE

Only the states of two nucleons having one unit of isobaric spin will be included in this discussion. Thus the deuteron and those other states of the neutron–proton system which are not available to the proton–proton system will not be considered. Restricting the energy to $E_{\text{LAB}} \lesssim 10$ MeV, $k\mu^{-1}$ = center-of-mass wave number $\times$ pion Compton wavelength $<\frac{1}{2}$, and consequently the most important states are singlet-s and triplet-p.

While bremsstrahlung experiments are being planned in this energy range,[1] none exist at the present time. The discussion will therefore be limited to elastic scattering.

II. PROTON–PROTON SCATTERING

A. Data

The accurate low-energy p–p data which have been analyzed in recent years are the following. Relative cross sections have been measured[2] to $\sim 1\%$ accuracy at six energies from 0.33766 to 0.40517 MeV, at a center of mass scattering angle of 90°. This is the energy region in which destructive interference between the Coulomb and nuclear amplitudes produces a very striking minimum in the plot of cross section vs energy, shown on Fig. 1. The s-wave phase shift obtained from the analysis (see below) is accurate to 0.1%, the enhancement resulting from the interference.

The most recent angular distributions measured at Wisconsin[3] to an accuracy of 0.1% to 0.3% have been slightly revised and reanalyzed.[4] Differential cross sections were measured at $\sim$13 angles at the laboratory energies 1.397, 1.855, 2.425, and 3.037 MeV.

The accuracy of some earlier Wisconsin data[5] which extends up to 4.2 MeV, has been questioned.[3,4] See, however, Ref. 6.

An angular distribution[7] at 9.69 MeV has an accuracy of 0.7%.

Using a polarized beam and target a measurement of the ratio A_{yy}/A_{xx} of two spin correlation parameters for 90° c.m. has been performed[8] at four energies from 11.4 to 26.5 MeV. Only the lowest of these energies will be considered here.

B. Analysis

There does not yet exist a simultaneous analysis of all the low-energy data,[9] either by itself or in conjunction with higher-energy data. Part of the reason for this situation is that the accuracy of the low-energy data requires that consideration be given to some small

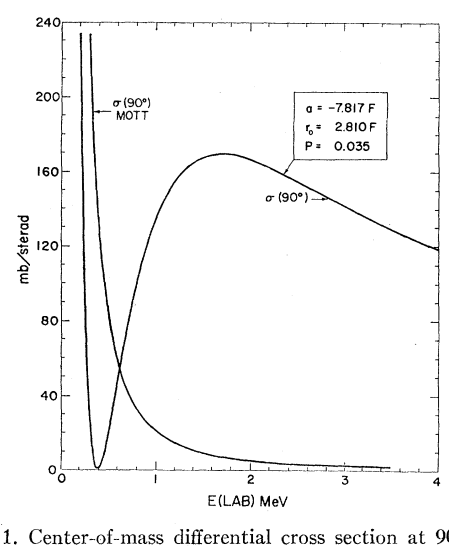

FIG. 1. Center-of-mass differential cross section at 90° versus the laboratory energy of the incident proton. The Mott cross section is shown and the actual cross section computed from the parameters which are found in Sec. IID to give the best quadratic fit to the effective range function.

* Work performed under the auspices of the U.S. Atomic Energy Commission.

[1] J. E. Brolley, Jr. (private communication).
[2] J. E. Brolley, Jr., J. D. Seagrave, and J. G. Beery, Phys. Rev. **135**, B1119 (1964).
[3] D. J. Knecht, S. Messelt, E. D. Berners, and L. C. Northcliffe, Phys. Rev. **114**, 550 (1959).
[4] D. J. Knecht, P. F. Dahl, and S. Messelt, Phys. Rev. **148**, 1031 (1966).
[5] H. R. Worthington, J. N. McGruer, and D. E. Findley, Phys. Rev. **90**, 899 (1953).
[6] R. J. Slobodrian, Nucl. Phys. **85**, 33 (1966).
[7] L. H. Johnston and D. E. Young, Phys. Rev. **116**, 989 (1959).
[8] P. Catillon, M. Chapellier, and D. Garreta, Report CEA-R 3193, Saclay.
[9] Two such analyses are now in progress. H. P. Noyes, P. Signell, and M. Sher (private communications).

electromagnetic effects which are not included in standard treatments. These effects are discussed below.

The general principles underlying the analysis of nucleon–nucleon scattering have been described in many places. For the most recent review article, which includes a detailed bibliography, see Ref. 10. The point of view which we adopt is this: although a complete separation cannot be made between electromagnetic and nuclear interactions [two examples: (i) the nuclear force depends upon the masses of the mesons which are affected by electromagnetic forces, (ii) the anomalous magnetic moment results from nuclear forces], the following two-step procedure can be employed.[11] First write down the electromagnetic interaction between two nucleons which one believes must be present on the basis of the well-studied interaction between electrons and nucleons. Included here are the Coulomb potential, its modification due to vacuum polarization, and magnetic terms of the spin–orbit and moment–moment types. To determine the actual form of these terms appeal must be made to some nonrelativistic reduction of an approximate relativistic theory[12] of two spin-$\frac{1}{2}$ particles with charge and anomalous magnetic moment. Now solve the Schrödinger equation in partial waves with just these electromagnetic potentials and obtain regular, $S_L(r)$, and irregular, $T_L(r)$, solutions as well as the complete scattering matrix. (If only the Coulomb potential were included—this is the standard practice—this procedure would yield the Coulomb wave functions and the Coulomb scattering amplitude.) This completes step one.

Step two begins by turning on the nuclear interaction between the nucleons.[13] When the nucleons are separated by distances which are sufficiently great compared to the range of the nuclear force, the (partial) wave function $R_L(r)$ *must* have the form of a linear combination of the functions $S_L(r)$ and $T_L(r)$ which solve the purely electromagnetic potential (Step 1). Provided $S_L(r)$ and $T_L(r)$ have the same normalization at infinity, one writes

$$R_L(r) \underset{(r\gg b)}{=} \cos\delta_L{}^E S_L(r) + \sin\delta_L{}^E T_L(r),$$

where b is the range of the nuclear force and the superscript E on the phase shift indicates that it is defined with respect to the electromagnetic functions S and T. This phase shift includes *all* nuclear effects between the two nucleons, including such things as modification of their anomalous magnetic moments when they are in strong interaction with each other. Going out to still larger values of r where all potentials except Coulomb are negligible,

$$R_L(r) = \cos K_L F_L(r) + \sin K_L G_L(r),$$

where F_L and G_L are Coulomb functions, and

$$K_L = \delta_L{}^E + \tau_L$$

with τ_L being the phase shift produced by the electromagnetic potential with respect to Coulomb functions, i.e.,

$$S_L(r) \to \cos\tau_L F_L(r) + \sin\tau_L G_L(r).$$

Several comments follow concerning the procedure outlined above.

(i) In the standard approximation which involves only Coulomb and nuclear interactions, one always solves the Coulomb problem first and then defines nuclear phase shifts with respect to Coulomb functions. It is sometimes stated that the special nature of the Coulomb potential at infinity (falling off like r^{-1} and the associated logarithmic phase factor) forces a two-step solution. We claim that the essential point is *not* that the Coulomb potential has *infinite* range (one could put in screening), but rather that it has a *large* range and therefore produces scattering in many angular momentum states. For this reason it is convenient to write the scattering amplitude as the sum of the Coulomb amplitude and a "nuclear" amplitude, the latter being expressed in terms of phase shifts defined with respect to Coulomb functions and receiving contributions from only a few angular momentum states.

There is another, practical, reason why it is convenient to have solved the Coulomb problem first. When attempts are made to find nuclear potentials which fit the elastic data, it is necessary to solve the differential equation for a given partial wave each time the parameters of the potential are varied. If the Coulomb functions have been separately computed it is only necessary to integrate the equation beyond the range of the nuclear force and match on to Coulomb functions. Otherwise, it would be necessary to integrate out to the place where the Coulomb functions become asymptotic. Especially for low energies this can be a much greater distance.

(ii) Although the vacuum polarization and magnetic interactions do not go like r^{-1}, they both produce scattering in many angular momentum states.[14] The vacuum polarization potential falls off exponentially

[10] G. Breit and R. D. Haracz, "Nucleon-Nucleon Scattering," in *High Energy Physics* (Academic Press Inc., New York, to be published), Vol. I, Chap. II.

[11] For more discussion of relativistic and electromagnetic effects in nucleon–nucleon scattering see G. Breit, Rev. Mod. Phys. **34**, 766 (1962); and Ref. 10.

[12] See, for example, A. Garren, Phys. Rev. **101**, 419 (1956); H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Springer-Verlag, Berlin, 1957), and the references in Ref. 11.

[13] The discussion presented here follows L. Heller, Phys. Rev. **120**, 627 (1960). In this reference the electromagnetic interaction consisted of Coulomb and vacuum polarization potentials only, and was called the "electric" potential.

[14] The importance of scattering in high L states due to vacuum polarization was first pointed out by L. Durand, III, Phys. Rev. **108**, 1597 (1957).

at large distances with a range corresponding to the Compton wavelength of an electron–positron pair (193 F). The magnetic interactions, falling off like r^{-3}, do not involve very large distances but nevertheless $(2L+1)P_L(x)$ times the spin–orbit phase shifts in the states $L=J\pm1$ is of order[11,15] $L^{-1/2}$ for large L and consequently many states are required to obtain an accurate scattering amplitude. For the reasons given in point (i), therefore, it is convenient to group these potentials with the Coulomb potential, solve the electromagnetic problem first, and then define the "nuclear" phase shifts δ_L^E.

(iii) In the high L states where there is negligible nuclear scattering one merely solves the electromagnetic problem. In the low L states it is a matter of taste whether the scattering amplitude is expressed in terms of δ_L^E phase shifts or in terms of K_L phase shifts, remembering that

$$[\exp(2i\tau_L)-1]+\exp(2i\tau_L)[\exp(2i\delta_L^E)-1] = [\exp(2iK_L)-1].$$

When one comes to consider the energy dependence of the phase shifts, a preference for δ_L^E is indicated. The reason is that it is possible to write down a function of $\cot\delta_L^E$, called the effective range function, involving *only quantities gotten from the solution of the electromagnetic problem*,[13] which is an analytic function of k at $k=0$. Furthermore, if the nuclear interaction can be written as a superposition of Yukawa potentials with maximum range μ^{-1}, then the singularities[16] of the effective range function closest to $k=0$ are located at $k=\pm(\frac{1}{2})i\mu$.

These analytic properties enable one to introduce a physically sensible parameterization of the phase shift at low energies in terms of a low-order polynomial representation of the effective range function.[17] One must realize, however, that the singularities at $k=\pm\frac{1}{2}i\mu$ and other more distant ones are present, and this limits the physical basis for a low-order polynomial (but not necessarily the accuracy of its fit) to laboratory energies much less than 10 MeV, corresponding to μ^{-1} being the pion Compton wavelength.

(iv) Since all the electromagnetic interactions other than Coulomb are weak, it is permissible to solve the electromagnetic problem in the form of perturbations on the Coulomb functions. Note that the two-step approach to the problem outlined above completely obviates the matter of "wave function distortion."[10,11] This concept arises only when one tries to include some of the small electromagnetic interactions *after* the Coulomb *plus* nuclear problem has been solved.[18]

(v) To compare proton–proton scattering in a given state with neutron–proton and neutron–neutron scattering, in connection with the study of charge independence and charge symmetry of the nuclear force, it is desirable to isolate as many electromagnetic effects as possible. While the procedure which we have been discussing tries to do exactly this, it must be remembered that a clean separation is not possible at small distances. One example which has already been mentioned is a possible alteration of the nucleons' anomalous magnetic moments when they are at small separations, and the resultant change in their electromagnetic interaction *at small distances*. Another possibility is that the charge and magnetic structure of the nucleons is altered when they are in strong interaction. [In the discussion given above some additional small electromagnetic effects which were not mentioned are the difference between the Coulomb interaction of point nucleons and the Coulomb interactions of finite-sized nucleons, and similarly for the magnetic interactions. Although these differences are of short range (the size of the nucleons), they should still be included with the electromagnetic interaction. From the earliest days people have in fact tried to separate these effects from the nuclear interaction.[19–23]]

The conclusion to this section is that it is both desirable and convenient to separate electromagnetic from nuclear interactions, but that lacking a fundamental theory there are some small effects at small distances for which it is not yet possible to achieve this. This program has so far been carried out only to a limited extent, and this is discussed next.

C. Results

Magnetic and finite size effects on low-energy data[24] have to date been considered only for the purpose of determining their effects upon the scattering length

[15] J. T. Holdeman, Jr., Technical Report No. COO-1573-1, *Electromagnetic Scattering of Charged Particles with Spin*, Case Institute of Technology (1966). I want to thank Dr. Holdeman for some discussions about the magnetic interactions.

[16] The argument (unpublished) follows H. Cornille and A. Martin, Nuovo Cimento **26**, 298 (1962), where the Coulomb plus nuclear case was considered.

[17] The fact that the phase shift K_L is not suitable for use in an effective range function was first pointed out by L. L. Foldy and E. Erikson, Phys Rev. **98**, 775 (1955), in connection with the vacuum polarization potential. For more details see Ref. 13.

[18] This procedure was employed for the vacuum polarization potential by Foldy and Eriksen, Ref. 17. It requires the assumption of a particular nuclear potential.

[19] J. Schwinger, Phys. Rev. **78**, 135 (1950).

[20] E. E. Salpeter, Phys. Rev. **91**, 994 (1953).

[21] Riazuddin, Nucl. Phys. **7**, 217 (1964).

[22] R. E. Schneider and R. M. Thaler, Phys. Rev. **137**, B874 (1965).

[23] R. J. Slobodrian, Phys. Rev. **145**, 766 (1966).

[24] Magnetic effects of higher energies have been discussed by A. Garren, Phys. Rev. **101**, 419 (1956); and G. Breit and H. M. Ruppel, Phys. Rev. **127**, 2123 (1962). The most recent analysis by the Yale group included magnetic effects at low (and higher) energies, in the high angular momentum states, using plane wave Born approximation. One must check to see if there is an intermediate energy region in which magnetic and Coulomb effects are simultaneously important where such a procedure would not be satisfactory. I want to thank Professor Breit for a discussion of this point.

and effective range in the singlet s state, and in particular to compare these quantities in the various charge states of the nucleon–nucleon system. We return to this question in Sec. III.

An analysis of the Los Alamos data in the region of the Coulomb-nuclear interference minimum[2] has been performed[25] including vacuum polarization explicitly in the scattering amplitude. Furthermore, since these data consist of 90° cross sections at six energies, the energy dependence of the s-wave phase shift δ_0^E was parameterized by using the effective range formula which itself includes the effect of vacuum polarization.[13] It was found that the energy of the minimum of the 90° cross section is at[25,26] 0.38243 ± 0.00020 MeV, and furthermore $\delta_0^E=14.611°\pm0.011°$ at the precise energy 0.38243 MeV. This point is shown on Fig. 2.

To obtain this result it is necessary to make an assumption about the three p-wave and higher wave phase shifts which cannot all be determined from differential cross-section data. While the assumption which shall be discussed has the effect of making these higher waves of negligible consequence below 0.4 MeV, this is not true at the other experimental energies.

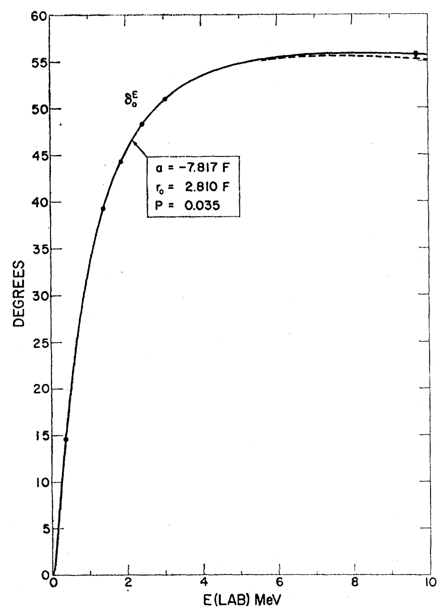

FIG. 2. The s-wave phase shift versus the laboratory energy of the incident proton. δ_0^E is computed from the parameters which are found in Sec. IID to give the best quadratic fit to the effective range function. The points are the result of phase-shift analyses at single energies. The error bars are all smaller than the points except at 9.69 MeV. The broken curve is the prediction of the Hamada–Johnston potential for δ_0^C (no vacuum polarization). Below 5 MeV it is too close (within 0.2°) to the solid curve to be distinguished on this plot.

At any rate, if no restriction is placed upon the higher waves, the s-wave phase shift becomes very uncertain.[27]

The argument was given by Noyes[28] in connection with an analysis of the Wisconsin data,[3] and repeated[29] for a combined analysis of the Minnesota cross section data at 9.69 MeV[7] with the 11.4-MeV Saclay spin correlation data,[8] that the spin–orbit force is of short range compared to the tensor force, the latter having a one-pion exchange (OPE) contribution. This makes it plausible that Δ_{LS} is small compared to Δ_T at low energies, where the three linear combinations (of the three p-wave phase shifts $\delta_{1,J}$)

$$\Delta_C=\tfrac{1}{9}(\delta_{1,0}+3\delta_{1,1}+5\delta_{1,2}),$$

$$\Delta_T=\tfrac{5}{72}(2\delta_{1,0}-3\delta_{1,1}+\delta_{1,2}),$$

$$\Delta_{LS}=\tfrac{1}{12}(-2\delta_{1,0}-3\delta_{1,1}+5\delta_{1,2})$$

were shown[30] to be proportional, *in Born approximation*, to the matrix elements of the central, tensor, and spin–orbit potentials, respectively, assuming no other forces. Examination[29] of four models which fit higher-energy data shows that they predict Δ_{LS}/Δ_T to be in the range 0.07–0.15 at 9.69 MeV, and furthermore that Δ_T is very close to the OPEC result (with the Coulomb effect accounted for in an approximate way).

The small value of Δ_{LS} coupled with the fourfold sign ambiguity[31] of the p-wave shifts is said to be the explanation for the impossibility of obtaining an unambiguous phase-shift analysis[29] of the combined data near 10 MeV. On the other hand, if Δ_{LS}/Δ_T is *assigned* a value within the range stated above, well-defined values for δ_0^E, Δ_C^E, and Δ_T^E emerge with very little additional uncertainty due to the spread in Δ_{LS}/Δ_T. For the details of this analysis—which includes vacuum polarization—see Ref. 29. The results are that at 9.69 MeV $\delta_0^E=55.69°\pm0.28°$, $\Delta_C^E=-0.020°\pm0.029°$, and $\Delta_T^E=0.91°\pm0.28°$. The singlet d-wave phase shift and Δ_T are both found to be within 30% of their OPEC values. The s-wave phase shift is shown on Fig. 2. An interpretation of the small value of Δ_C (consistent with zero) is given in Refs. 28 and 29 in terms of a cancellation between the weak repulsive (in the triplet state) OPE, and a strong medium-range spin-independent attraction which is due to an s-state π–π interaction.

Since there is only cross section data at the Los Alamos[2] and Wisconsin[4] energies, it is not sufficient to specify just the ratio Δ_{LS}/Δ_T; both values must be given. On the basis of the discussion given above, one is

[25] M. L. Gursky and L. Heller, Phys. Rev. **136**, B1693 (1964).

[26] This results is consistent with an earlier less accurate measurement by D. I. Cooper, D. H. Frisch, and R. L. Zimmerman, Phys. Rev. **94**, 1209 (1954).

[27] M. H. MacGregor, Phys. Rev. **113**, 1559 (1959).

[28] H. P. Noyes, Phys. Rev. Letters **12**, 171 (1964).

[29] H. P. Noyes and H. M. Lipinski, SLAC preprint, "Unique Determination of Three Proton–Proton Scattering Parameters at 9.69 MeV" (1967).

[30] J. L. Gammel and R. M. Thaler, Progr. Cosmic Ray Phys. **5**, 99 (1960).

[31] E. Clementel and C. Villi, Nuovo Cimento **2**, 1165 (1955).

TABLE I. S-wave phase shifts obtained from the low-energy p–p data.

E(MeV)	δ_0^E	τ_0^a	K_0
0.38243	14.611°±0.011°	−0.105°	14.506°
1.397	39.321°±0.029°	−0.085°	39.236°
1.855	44.329°±0.025°	−0.080°	44.249°
2.425	48.355°±0.026°	−0.075°	48.280°
3.037	51.023°±0.041°	−0.071°	50.952°
9.69	55.69° ±0.28°	−0.052°	55.64°

[a] These values of τ_0, the vacuum polarization phase shift, can be obtained from formulas in Refs. 14 and 25, and from graphs in Ref. 13.

confident that it is a good approximation to set all phases other than δ_0 and Δ_C equal to their OPEC values at these low energies. This procedure has been followed in the analysis[25] of the Los Alamos data, and in the recent analyses[28,4] of the Wisconsin experiments. Variation of Δ_T by 40% from its OPEC value changes the values of δ_0^E at 3.037 MeV by less than its uncertainty, and has a still smaller effect at the lower energies.[4] Even larger changes in Δ_{LS} (away from zero) are unimportant.

Before stating the values of δ_0^E which are obtained at the Wisconsin energies, two remarks are needed. The vacuum polarization was handled in a somewhat different manner from that described above, following the procedure in Ref. 14 where the s-wave part of the amplitude is expressed in terms of K_0 and all terms in the cross section involving vacuum polarization, including interference terms, are pre-evaluated [called $(\Delta\sigma)_{vp}$]. This requires assuming an approximate value for K_0, but one can go back after the analysis and check the adequacy of the assumption, or else iterate the result. At any rate it is K_0 which was directly gotten from the analysis, and one uses $\delta_0^E=K_0-\tau_0$ to obtain the phase shift which we have been discussing.

The second remark concerns the uncertainties in the phase shifts. In the analyses of the Los Alamos data and the combined Minnesota–Saclay data it was assumed that the experimental errors are purely random, and the uncertainties in the phase shifts were obtained from the customary analysis involving the matrix of second derivatives of χ^2 with respect to the searched parameters. In Ref. 4 it is stated that the uncertainties assigned to the Wisconsin cross sections largely reflect the presence of *systematic* errors and it is argued that the uncertainties in the phase shifts could be significantly larger (by a factor of $\sim$4) than those obtained from the random error assumption. The final proposed uncertainties[4] are only larger by a factor of $\sim$2. These s-wave phase shifts and errors and the ones discussed earlier are collected in Table I, and are shown on Fig. 2.

D. Discussion

Since there is no fundamental theoretical calculation of the s-wave phase shift available, the best one can do in a review article is to describe the attempts which have been made to fit the phase shift. These are of two types, a direct parameterization, and calculations from models with adjustable constants, both of which are also used at higher energies. The first method employs the effective range function,[13,32] mentioned in Sec. IIB,

$$X(k^2)\equiv[C^2k/(1-\phi_0)][(1+\chi_0)\cot\delta_0^E-\tan\tau_0]+2\eta k[h(\eta)+l_0(\eta)].$$

χ_0, ϕ_0, τ_0, and l_0 arise in the solution of the electromagnetic problem, and are all small compared to unity because of the weakness of the non-Coulomb parts of the electromagnetic interaction. They are calculated in Refs. 13 and 25 for the Coulomb plus vacuum polarization problem. η, C^2, and h are parameters which arise in the pure Coulomb problem.

Since one knows that $X(k^2)$ is analytic at $k^2=0$, it has become customary to express it as a low order polynomial in k^2, e.g.,

$$X(k^2)=(-1/a)+\tfrac{1}{2}r_0k^2-Pr_0^3k^4+Qr_0^5k^6,$$

where a, r_0, and P and Q are called the scattering length, effective range, and (two) shape parameters, respectively. It is important to remember that although this is a convenient way to parameterize the phase shift, there is no fundamental significance to these effective range parameters. Furthermore, while there exist radial integrals involving the s-wave function and its energy derivatives at zero energy[33] which define the coefficients in the power series expansion of $X(k^2)$, these are not exactly the coefficients which one obtains by fitting to phase shifts at finite energies.[34] This is especially clear if one recalls that the power series expansion itself has a radius of convergence of only 10 MeV, and we are including a phase shift at 9.69 MeV.

We have, nevertheless, made linear, quadratic, and cubic fits to the phase shifts δ_0^E in Table I, and obtain the parameters, χ^2 values and probabilities p shown in Table II. These parameters are consistent with an earlier set,[25] but the inclusion of the 9.69 MeV phase shift ties them down more firmly. It is clear that the linear fit is unacceptable and the quadratic fit is quite satisfactory. Using the parameters of the latter, δ_0^E is shown as a function of energy on Fig. 2.

[32] The conventional effective range function, not involving χ, φ, τ, and l, was used in Ref. 17 for the phase shift δ_0^C which the same nuclear potential would produce if the only electromagnetic interaction were Coulomb. $\delta_0^C=K_0-\Delta_0$ and Δ_0 must first be computed by assuming a nuclear potential.

[33] J. D. Jackson and J. M. Blatt, Rev. Mod. Phys. 22, 77 (1950).

[34] See the discussion in Ref. 25.

TABLE II. Effective range parameters obtained from three different polynomial fits to six low-energy phase shifts.

Fit	a(F)	r_0(F)	P	Q	$\chi_{\rm fit}^2$	$p(\chi^2>\chi_{\rm fit}^2)$
Linear	-7.804 ± 0.006	2.748 ± 0.008	0	0	16.8	0.003
Quadratic	-7.817 ± 0.007	2.810 ± 0.018	0.035 ± 0.009	0	1.11	0.77
Cubic	-7.819 ± 0.009	2.820 ± 0.044	0.043 ± 0.034	0.008 ± 0.031	1.05	0.59

If the smaller Wisconsin uncertainties[4] are used, $\chi^2=4.35$ and the parameters of the quadratic fit are shifted very slightly.

None of the phenomenological models of the proton–proton interaction have taken the precision low energy data into account except to get the scattering length and effective range approximately correct. We arbitrarily choose one potential, Hamada–Johnston,[35] and show on Fig. 2 its prediction for δ_0^C, the phase shift with respect to Coulomb functions obtained by neglecting all other electromagnetic effects.[36] It appears from Fig. 2 that slight alterations in the parameters of the potential (as well as inclusion of vacuum polarization) will easily bring its predictions into agreement with the δ_0^E curve. We suggest that these alterations be performed with all electromagnetic effects included and by fitting directly to the data.

The presence of one-pion exchange as the longest-range part of the nuclear interaction was first confirmed in the high angular momentum states. For its role in the p-states see Sec. IIB. Noyes has argued[37] that the curvature of the effective range function (the fact that P is positive) is strong evidence for the presence of OPE in the s state, since various models which include OPE predict $P>0$. While there is no reason to doubt this conclusion, we propose that a definitive test would be to include the coupling constant g_s^2 as a separate adjustable parameter in the s-wave portion of these models when fitting them to the data, just as was originally done in the high angular momentum states.

One may ask a similar question concerning the vacuum polarization (VP) potential. It was first claimed in Ref. 17 that a better fit to the energy dependence of the s-wave phase shift was obtained with the inclusion of VP than with its omission. In the original analysis[3] of the 1959 Wisconsin angular distributions the same conclusion was reached more forcefully. An unpublished set of single energy analyses[38] of the same data, which treated the experimental errors as statistical, yielded best values for the ratio $\lambda_{\rm VP}$ of the strength of the VP potential to its theoretical value in the neighborhood of unity, but showed a systematic variation with energy. The discussion of the systematic errors in these data given in Sec. IIC is capable of removing the energy dependence of $\lambda_{\rm VP}$, but the best that can be said[38] is that $\lambda_{\rm VP}$ is within $\sim$0.5 of unity.

An indication of the importance which the magnetic interaction will have[39] in the analysis of the low-energy data can be gotten by noting that the p-wave magnetic phase shift is of order $A\alpha kM^{-1}$ with α the fine structure constant, k the center of mass wave number, and M^{-1} the nucleon Compton wavelength. A is a numerical factor involving the nucleons' magnetic moments and depending upon the value of J. Somewhere in the 5–10-MeV region this phase shift will equal that due to VP. Since, furthermore, the falloff with increasing angular momentum is slow, the complete magnetic amplitude may be comparable with the VP amplitude which is known to make a significant contribution in the low-energy experiments. We urge that magnetic effects be included in future analyses.

III. CHARGE SYMMETRY AND INDEPENDENCE

A great deal has been written about this question recently[40] and other contributions to this conference will bring the subject up to date. We merely mention a few points and make no attempt to give a complete presentation.

There are two different definitions of charge symmetry in use,[41] one stronger than the other. The weak statement is that (the nuclear part of) the neutron–neutron interaction is the same as the proton–proton interaction; this version of charge symmetry is the one which is tested by comparing proton–proton scattering with neutron–neutron scattering (the latter only as a final-state interaction). The stronger statement (which implies the weaker one) is that one may simultaneously exchange all neutrons for protons and all protons for neutrons in any experiment and obtain identical results. This version leads to the conclusion that there cannot be a term $(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{n}$ in the neutron–proton scattering amplitude.[42] Clearly the weak version implies nothing whatsoever about the neutron–proton system. The evidence favoring weak charge sym-

[35] T. Hamada and I. D. Johnston, Nucl. Phys. **34**, 382 (1962).
[36] I want to thank Dr. M. Rich for supplying these phase shifts.
[37] For the most recent statement see Ref. 29.
[38] I am grateful to Dr. H. P. Noyes for sending me these results.
[39] See Ref. 24 for effects at higher energies.
[40] For a review, see E. M. Henley, in *Isobaric Spin in Nuclear Physics*, J. D. Fox and D. Robson, Eds. (Academic Press Inc., New York, 1966), p. 3.
[41] I want to thank Dr. J. E. Simmons for raising this question.
[42] L. Wolfenstein, Ann. Rev. Nucl. Sci. **6**, 43 (1956).

metry[43] appears to be much firmer than that for the strong version, and it may be necessary to keep this in mind when discussing mirror nuclei where it is still unclear whether or not the Coulomb effect accounts for the energy difference. In this latter connection the importance of using accurate wave functions has recently been stressed.[44]

The discussion of charge symmetry in terms of potentials is carried out by simply adding to any proposed nuclear potential whatever electromagnetic potentials are present in the system under consideration.[43,22]

If instead, one chooses to study charge symmetry from the standpoint of partial-wave dispersion relations the treatment becomes somewhat more complicated. It was originally shown[45] that the Coulomb effect enters the dispersion relation in three distinct ways. First, one considers the amplitude $\exp(i\delta)\times \sin\delta/C^2q$, where C^2 is the Coulomb penetration factor. The reason for this choice is that it has the right analyticity and symmetry properties.[46–48] Second, the kinematic part of the kernel of the integral equation for the D function (in the N/D method) is altered. And third, the left hand discontinuity which enters the integral equation is altered. As an illustration of this last point, if the left-hand singularities in the neutron–neutron problem are represented by a single pole, then the proper statement of charge symmetry is that there will be a series of branch points in the proton–proton problem.[48] It was the failure to recognize this which led to the very poor prediction for the neutron–neutron scattering length in Ref. 45. If the correct singularities are used,[48] agreement with the experimental value of a_{nn} is again obtained.[49,50]

Another example of the Coulomb modification of the left-hand cut arises in the many boson-exchange models. Each boson's contribution to the discontinuity is altered when the Coulomb interaction is present.[51] One of these models[52] included the Coulomb modification of the OPE portion of the cut, but omitted it from all the other bosons.[53] Since OPE is only a minor portion of the interaction in the singlet s state, there could be a significant error involved. An estimate of this error[54] shows that it produces a difference between the neutron–neutron and proton–proton phase shifts at 10 MeV of 5.3° instead of 2.5°. This is out of a total of approximately 55°.

While there are some small indirect electromagnetic effects which are capable of destroying both charge symmetry and charge independence, electromagnetic mass splittings of the mesons (which maintain charge symmetry) appear to have the major effect upon charge dependence. For a review of the mechanisms which might account for the difference between $a_{nn}=-16.4\pm 1.9$ F[49] and $a_{np}=-23.7$ F,[55] see Ref. 40. Only about $\frac{1}{3}$ of the difference is directly due to the splitting of the mass of the π-mesons.

There may also be a problem with the neutron–proton effective range. Noyes has analyzed the low energy neutron–proton data[56,57] and states that on the basis of charge independence the value of r_{np} should be 2.73 ± 0.03 F, but an analysis of all the low energy neutron–proton data yields $r_{np}=2.52\pm 0.10$ F. One portion of these data[58] favors an even smaller value[57] of r_{np}. It has been pointed out,[59] however, that systematic errors might reduce the discrepancy in the effective ranges to much less than 10%.

It is clear that the questions of charge symmetry and independence require an understanding of some of the fundamental features of the strong and electromagnetic interactions. Phenomenological analyses of the nucleon–nucleon data which employ as much theoretical information as possible, including the cleanest possible separation of direct electromagnetic effects, will undoubtedly contribute to this basic understanding.

ACKNOWLEDGMENTS

I wank to thank M. Rich, P. Signell, H. P. Noyes, and J. T. Holdeman, Jr., for valuable discussions and correspondence on various aspects of this material.

[43] See, for example, L. Heller, P. Signell, and N. R. Yoder, Phys. Rev. Letters **13**, 577 (1964), where the experimental value of the neutron-neutron scattering length, $a_{nn}\simeq -16.4$, is found to be consistent with proton–proton nuclear potentials.

[44] V. K. Gupta and A. N. Mitra, Phys. Letters **24B**, 27 (1967). Also see the contribution to this conference by K. Okamoto and C. Lucas. They conclude that charge symmetry is violated.

[45] D. Y. Wong and H. P. Noyes, Phys. Rev. **126**, 1866 (1962).

[46] See Ref. 16.

[47] J. R. Rix, thesis, Harvard University (1965).

[48] L. Heller and M. Rich, Phys. Rev. **144**, 1324 (1966).

[49] R. P. Haddock, R. M. Salter, Jr., M. Zeller, J. B. Czirr, and D. R. Nygren, Phys. Rev. Letters **14**, 318 (1965).

[50] For a discussion of the various experiments which give information about the neutron–neutron interaction, see the contribution to this conference by I. Šlaus.

[51] See Fig. 2 in Ref. 48 for single boson exchange. This effect is repeated for each boson.

[52] A. Scotti and D. Y. Wong, Phys. Rev. **138**, B145 (1965).

[53] I want to thank Dr. H. P. Noyes for a clarification of this point.

[54] I want to thank Dr. M. Rich for these numbers obtained from a one-boson-exchange model fitted to the low-energy data.

[55] R. Wilson, *The Nucleon–Nucleon Interaction* (Interscience Publishers, Inc., New York, 1963), p. 37.

[56] H. P. Noyes, Phys. Rev. **130**, 2025 (1963).

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[58] C. E. Engelke, R. E. Benenson, E. Melkonian, and J. M. Lebowitz, Phys. Rev. **129**, 324 (1963).

[59] G. Breit, K. A. Friedman, and R. E. Seamon, Suppl. Progr. Theoret. Phys., Extra Number, 449 (1965).