SESSION C—THE NEUTRON–NEUTRON INTERACTION, CHARGE INDEPENDENCE, *N–N* INTERACTION IN NUCLEI

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Neutron-Neutron Interaction

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The present status of the problem of charge dependence is discussed. The information about the neutron–neutron interaction derived from the two-neutron system, three-nucleon systems, final-state interactions in multiparticle reactions, and peripheral processes is critically evaluated. The experimental data indicate the breakdown of charge independence by about 3–5%. Evidence concerning the violation of charge symmetry is inconclusive, but it seems that most of the data are consistent with the assumption that charge symmetry is satisfied within 0.5–1%. The most suitable studies which might improve the knowledge of the neutron–neutron forces are indicated.

I. INTRODUCTION

The overwhelming evidence: nucleon–nucleon and pion–nucleon scattering data, charge symmetric processes, and nuclear structure information, indicate that strong interactions are to a large extent charge-independent. In fact, neutron–proton, proton–proton, and neutron–neutron forces, when effects due to electromagnetic interactions are subtracted, are considered identical in equal space and spin states. The equality of proton–proton and neutron–neutron processes, or more generally, the equivalence of a system of nucleons and pions and its charge symmetric counterpart, is referred to as charge symmetry.

A small departure from charge symmetry and charge independence is expected. This breakdown, which can be originated by electromagnetic forces as discussed by Henley and others[1] or caused by hitherto unknown reasons, is related to the basic understanding of the nuclear interaction and is ultimately connected with the fundamental principles of elementary particle physics. Since departures are very small it is imperative to acquire precise data on the nucleon–nucleon interaction.

The investigation of the proton–proton scattering has nowadays reached a stage characterized by remarkable accuracy. Contrasted with the fact that neutron–neutron scattering experiments are only barely feasible one is forced to ask:

Why is it important to study neutron–neutron interaction?

One could argue that for the investigation of charge independence (though not for a charge symmetry check) it is sufficient to compare *np* and *pp* interactions. The most accurate information about these forces comes from nucleon–nucleon scattering and the greatest sensitivity is achieved if one studies the scattering at very low energies. Under such conditions only S waves are important and phase shifts could be obtained with a high degree of accuracy. Since the $^1S_0$ state is almost bound, the scattering length is a magnifying glass for a nuclear potential. A relative change in depth of a potential V is related to a relative change in the $^1S_0$ scattering length a and effective range r_0 through the relation[2]

$$(\Delta V/V) = A(\Delta a/a) + B(\Delta r_0/r_0), \qquad (1)$$

where A depends upon the shape of the potential but is of an order of 0.1. B is rather shape-independent. Thus, if one could determine deviations from charge independence from some other sources, the value of $\Delta a/a$ would, with the help of Eq. (1), give information on the shape of the nuclear potential.

Once the parameters describing the physical interaction between two nucleons are determined, in order to obtain "pure" nuclear forces it is necessary to correct for the effects produced by electromagnetic forces. These direct electromagnetic effects are: Coulomb interaction between two point charges, magnetic interaction, effects due to the finite charge and magnetic moment

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[1] For an excellent discussion about charge dependence of nuclear forces see: E. M. Henley, in *Isobaric Spin in Nuclear Physics*, J. D. Fox and D. Robson, Eds. (Academic Press Inc., New York, 1966), p. 3, and R. J. Blin Stoyle, *Selected Topics in Nuclear Spectroscopy* (North-Holland Publ. Co., Amsterdam, 1964), p. 213.

[2] M. J. Moravscik, Phys. Rev. **136**, B624 (1964).

distribution of nucleons, and vacuum polarization. Though it seems that these corrections can be performed quite accurately, their magnitude depends on the nature of the nuclear potential used in the calculation.

As a test of charge symmetry the parameters for various nuclear potentials have been determined by fitting p–p scattering data. The $^1S_0$ nn scattering length a_{nn} was then computed turning off the Coulomb interaction between two point charges. Thus, assuming the exact charge symmetry, the use of potentials with the hard core gives[3] a_{nn} between -16.4 and -16.9 F. The velocity-dependent potential yields[4] $a_{nn}=-19.3$ F, whereas the use of the nonlocal S wave separable potential of Yamaguchi type gives[5] $a_{nn}=-18.0$ F.

Although the n–p and p–p data cannot distinguish in favor of the hard core, soft core, or velocity-dependent potential, the nn scattering length could shed some light on the problem. Of course, the sole determination of a_{nn} is not sufficient. Other data sensitive either to the potential core and/or the charge symmetry violation are required to unambiguously interpret the meaning of the value of a_{nn}.

The corrections due to magnetic interaction and to the extended electromagnetic structure of the nucleons also depend on the nuclear potential. As early as 1950 Schwinger showed[6] that the interaction between point magnetic moments of two nucleons can change the $^1S_0$ scattering length by several fermis provided the nuclear potential behaves at small distances like the Yukawa potential. A less singular potential at small distances gave considerably smaller change in the scattering length. This dependence of corrections due to the finite charge and magnetic moment distribution upon the nuclear potential was demonstrated very recently by Slobodrian, who investigated[7] the hard core, soft core, and velocity-dependent potential and found that the corrections for these forces differ in such a way as to produce the difference in the predicted a_n of the order of 1 F.

Uncertainties in electromagnetic corrections due to the form of the nuclear potential are much larger than experimental uncertainties of np and pp interaction parameters. If the information about the nn force could be obtained with reasonable accuracy; e.g., a_{nn} within 5%, the study of nn interaction would provide a valuable insight into fine details of nuclear interaction.

II. SOURCES OF INFORMATION ABOUT *nn* INTERACTION

Any system containing neutrons can in principle be a source of information about the nn force. However, the studies of a complex nuclear system are complicated. First, there is no adequate treatment for a many-body problem and one is always forced to use some model. Second, the problem of the off-the-energy-shell interaction has only recently been subject both to experimental and to theoretical investigations, but it is still not quite solved. Third, there is a possibility that nuclear forces contain many-body forces.

Whenever the study is concerned with complex systems, the information one obtains is always model-dependent and the force one extracts is only an "effective" force.

The study of complex nuclear systems has indeed revealed some information about charge independence and charge symmetry. In particular, the study of $T=1$ triplet in $A=14$ presented[8] evidence that the np force is more attractive than the pp or nn interaction, while the latter two do not differ very much. The quantitative statement about the precise departure from charge independence is hampered by uncertainties in the nuclear wave function. For example, admixing other configurations modifies the amount of possible charge dependence by an order of magnitude.[9] Lovitch investigates[10] the isotriplet in $A=6$ and again finds evidence for violation of charge independence though charge symmetry is preserved. The analysis of isomultiplets throughout the $1p$ shell leads[11] to the conclusion that there is evidence for a 2% charge dependence in the singlet state. Wilkinson[12] has used the wave functions derived from the study of electron scattering and $(p, 2p)$ reactions in the investigation of $1p$ shell nuclei and concludes that there is a weak indication that the np force is by about 2% stronger than the force between like nucleons. Fairbairn compares[13] the energy levels in mirror $1p$ shell nuclei and finds no evidence for any departure from charge symmetry.

The general conclusion that may be drawn from the study of complex nuclei is that charge independence is probably violated by about 2%, and the departure from charge symmetry is much smaller and could not be established.

We regard this evidence to be only of qualitative value and we maintain that the information about the nn force should be obtained from simple nuclear systems: neutron–neutron systems and few-body systems.

The study of the two-neutron system is by far the best and the only completely reliable way of studying the nn force.

A. Two-Neutron System

The idea to investigate the nn force by two colliding neutron beams is quite old, but it could not materialize earlier because even high flux reactors did not produce

[3] L. Heller, P. Signell, and N. R. Yoder, Phys. Rev. Letters 13, 577 (1964).
[4] P. Signell (private communication).
[5] D. R. Harrington, Phys. Rev. 139, B691 (1965).
[6] J. Schwinger, Phys. Rev. 78, 135 (1950).
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[8] A. Altman and W. M. MacDonald, Nucl. Phys. 35, 593 (1962).
[9] R. J. Blin Stoyle and S. C. Nair, Phys. Letters 7, 161 (1963).
[10] L. Lovitch, Nucl. Phys. 62, 653 (1965).
[11] S. Sengupta, Nucl. Phys. 30, 300 (1962).
[12] See D. H. Wilkinson in Ref. 15.
[13] W. M. Fairbairn, Nucl. Phys. A90, 135 (1967).

sufficiently intense beams. Nuclear explosions yield[14] adequately intense neutron beams (about 10^{24} neutrons). The progress made in the utilization of underground nuclear explosions for the study of neutron scattering is such that it seems[14] feasible to measure the scattering cross sections in a colliding beam experiment to an accuracy of 10% in the region from 20 keV to 2 MeV. The measurements performed with the indicated accuracy would enable one to determine the ${}^1S_0$ scattering length: its magnitude to an accuracy of 2.8–3.7%, and the sign of a_{nn} with a certainty of 99.99%. The effective range r_{nn} would be determined to an accuracy of about 50%.[2]

B. Three-Nucleon System

The equality between the Coulomb energy of He^3 E_c and the difference Δ in the binding energies of H^3 and He^3 nuclei have been regarded among the strongest evidences in favor of charge symmetry. This evidence has to be taken *cum grano salis* in view of the uncertainties in nuclear wave functions.

A contradictory conclusion reached in various analyses[15–22] only proves the complexity of the problem and it seems to us that the present stage of the discussion of various mirror nuclei, particularly the He^3–H^3 pair, does not allow any quantitative statements concerning the possible violation of charge symmetry.

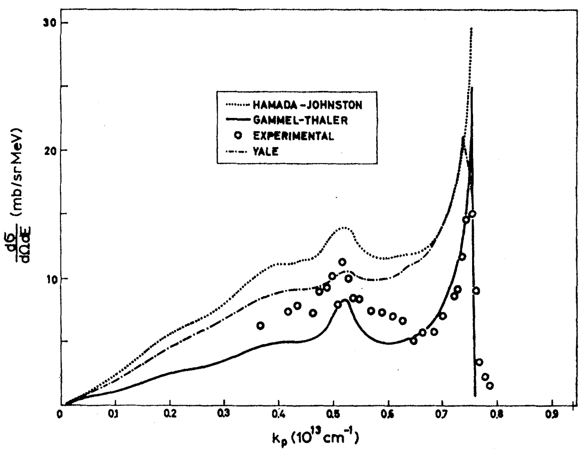

FIG. 1. The proton spectrum from the $D(n, p)2n$ reaction at $E_n = 14.4$ MeV (data from Ref. 23). The curves represent the predictions of the Born approximation calculation using the Hamada–Johnston (dotted curve), the Gammel–Thaler (solid curve), and the Yale potential (dashed dotted curve). (*Note:* The results for the Yale potential have been reduced by a factor of $\frac{1}{4}$ in this figure.)

C. Final-State Interaction in Multiparticle Reactions

In 1961 Ilakovac *et al.* found[23] a pronounced peak in a proton spectrum from the deuteron breakup induced by 14.4-MeV neutrons. The peak was kinematically associated with the low relative energy of two neutrons in the final state and it was assumed that its shape and magnitude reflect the influence of the nn force.

An attempt to extract the information about the nn interaction from this spectrum led to the first value for the scattering length, $a_{nn} = -22 \pm 2$ F.[24]

With improved experimental techniques the $D(n, p)2n$ experiment has been repeated[25–27] giving remarkably good agreement with the original result. Similar measurements[28,29] have been made for the ${}^3H(n, d)2n$ reaction.

1. Analyses of Neutron-Induced Breakup Processes

The proton spectrum from the $D(n, p)2n$ reaction was analyzed using the Born approximation and describing two neutrons in the final state with the ${}^1S_0$ wave function. The perturbing potentials $V_{np}+V_{nn}$ were taken to consist of a delta interaction for V_{np}, and a square well for V_{nn}. The same square-well potential was used to calculate the two-neutron wave function. Various modifications of this model were investigated: (i) inclusion[30] of the np final state interaction; (ii) consideration of possible interferences between the term describing the nn interaction in the final state and the nonresonant term,[31,32] and (iii) the use of realistic potentials, such as the Gammel–Thaler, the Yale, and the Hamada–Johnston potential.[33]

Figure 1 shows the proton spectrum from the $D(n, p)2n$ reaction and the results of the theoretical analysis using the realistic nucleon–nucleon potential. The Hamada–Johnston and the Gammel–Thaler potentials give a fair qualitative description of the spectrum, but no attempt has been made to investigate the

[14] B. C. Diven, in *Nuclear Structure Study with Neutrons*, M. Nève de Mévergnies, P. Van Assche, and J. Vervier, Eds. (North-Holland Publ. Co., Amsterdam, 1965), p. 441; C. D. Bowan and W. C. Dichinson, UCLRL, Report No. UCRL-7859 (1964) (unpublished).
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[23] K. Ilakovac, L. G. Kuo, M. Petravić, I. Slaus, and P. Tomaš, Phys. Rev. Letters **6**, 356 (1961).
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[30] K. Ilakovac, L. G. Kuo, M. Petravić, I. Slaus, and P. Tomaš, Nucl. Phys. **43**, 254 (1963).
[31] R. N. J. Phillips, Nucl. Phys. **31**, 643 (1962).
[32] D. Rendić, M. Cerineo, I. Slaus, and P. Tomaš, Glasnik Mat. Fiz. Astron. **19**, 276 (1964).
[33] D. R. Koehler and R. A. Mann, Phys. Rev. **135**, B91 (1964); D. R. Koehler, *ibid.* **138**, B607 (1965).

TABLE I. Results of the analyses of proton and deuteron spectra from the $D(n, p)2n$ and $H^3(n, d)2n$ reactions.

Reaction	Extracted a_{nn} (in F)	Remarks	References
$D(n, p)2n$	-22 ± 2	only *nn* FSI	24
$D(n, p)2n$	-21.7 ± 2	only *nn* FSI	25
$D(n, p)2n$	-22	*nn* and *np* FSI	30
$D(n, p)2n$	-22.9 ± 1	*nn* FSI+nonresonant	32
$D(n, p)2n$	$-23.6-1.6+2.0$	*nn* FSI+nonresonant	26
$D(n, p)2n$	$-14.\pm3$	only *nn* FSI	27
$H^3(n, d)2n$	-18 ± 3	only *nn* FSI	28

sensitivity of the model to a_{nn} and to subsequently extract a_{nn} from the comparison with the data. It is of interest to emphasize that the Born approximation model together with the Hamada–Johnston and the Gammel–Thaler potentials predicts the absolute cross section comparable with the experimental value. On the contrary, the models employing the delta and square-well potential with the depth adjusted to fit the nucleon–nucleon scattering are inconsistent with the data by an order of magnitude.

The $H^3(n, d)2n$ reaction was treated assuming a simple pickup process and using a simplified wave function for H^3.

The results of these analyses are given in Table I. The analyses have been subjected to strong criticism since a complex three-body problem is treated in a simplified version employing the Born approximation which is most likely not valid at the energies considered. Also, in most of the cases the analysis was restricted to a region of a spectrum. This region was determined, e.g., in Ref. 25, requiring that the value obtained for a_{nn} should not depend upon a small change in the investigated proton energy domain. The energy interval considered was from 11.2 to 12.6 MeV. Increasing or decreasing the investigated domain by 0.4 MeV or more would lead to a value of $|a_{nn}|$ which is smaller by 1–2 F.

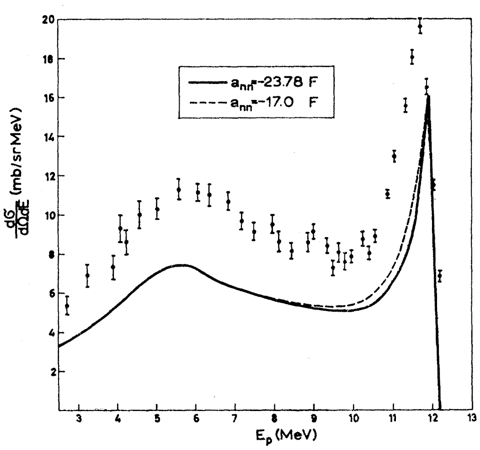

FIG. 2. The proton spectrum from the $D(n, p)2n$ reaction at 14.4 MeV (Ref. 25). The curves represent the prediction of the Amado model. The solid curve is for $a_{nn}=-23.78$ F, the dashed curve for $a_{nn}=-17$ F.

2. *The Analysis Based on the Exact Treatment of the Three-Body Problem*

The exact treatment of the three-body problem based on the work of Faddeev,[34] Amado,[35] Lovelace,[36] and others should at least in principle provide a possibility for extracting the *nn* force from the study of the $D(n, p)2n$ reaction. In order to carry out such an ambitious task, one should use the nucleon–nucleon interaction in its full complexity, possess an adequate knowledge of nuclear off-the-energy-shell interaction, and estimate the effects of possible three-body forces. Such a program is impossible to perform at present. A more modest analysis has been undertaken by Aaron and Amado[37] using a nonlocal separable *S*-wave spin-dependent nuclear force. In such a treatment one sacrifices the sophistication in the two-body force for a careful treatment of a three-body problem. The model of Amado and co-workers gives a good fit to the elastic neutron–deuteron scattering, the triton binding energy, and the total inelastic neutron–deuteron scattering cross section.

Figures 2 and 3 represent the comparison of the

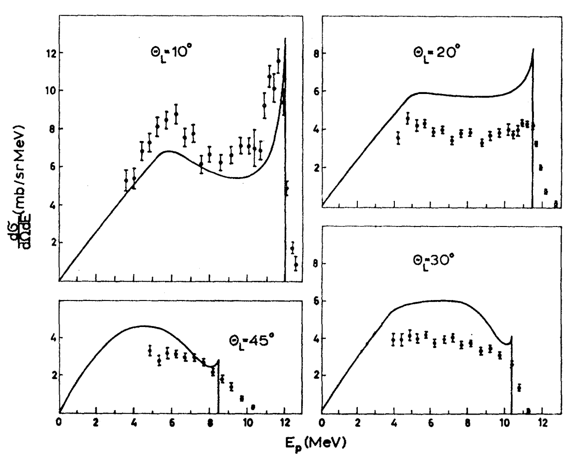

FIG. 3. The proton spectra from the reaction $D(n, p)2n$ at 14.4 MeV; $\theta_L=10°$, 20°, 30°, and 45° (Ref. 30). The solid curves are predictions of the Amado model.

[34] L. D. Faddeev, Zh. Eksperim. i Teor. Fiz. **39**, 1459 (1960) [English transl.: Soviet Phys.—JETP **12**, 1014 (1961)].
[35] R. D. Amado, Phys. Rev. **132**, 485 (1963); R. Aaron, R. D. Amado, and Y. Y. Yam, *ibid.* **136**, B650 (1964); Phys. Rev. Letters **13**, 574 (1964); Phys. Rev. **140**, B1291 (1965).
[36] C. A. Lovelace, Phys. Rev. **135**, B1225 (1964).
[37] R. Aaron and R. D. Amado, Phys. Rev. **150**, 857 (1966).

theoretical calculation of Aaron and Amado with the data[25,30] of the Zagreb group. The theory qualitatively reproduces the proton spectra at all the angles investigated. However, the calculations yield a too low cross section at 4.8° and a too high cross section at 20°, 30°, etc. The total breakup cross section is predicted correctly. The inability of the model to produce a correct cross section at 4.8° presumably arises from the neglect of higher partial waves in the nucleon–nucleon interaction.

In view of the fact that the Amado model is incapable of predicting a correct absolute cross section and it produces a wrong angular distribution, it is questionable whether there is much sense in trying to extract a_{nn} through the analysis of the shape of the spectrum obtained at some angle. Aaron and Amado have not attempted to extract a_{nn} and consider their model in its present form to be inadequate for that purpose.

3. The Comparison Procedure[38]

A comparison procedure is defined by the following conditions:

(1) Processes leading to two-neutron, two-proton, and neutron–proton final-state interactions should be measured and analyzed under equivalent conditions. If a reasonable model could be constructed giving good agreement with the data using known parameters derived from *pp* and *np* scattering, one would have confidence in the extracted *nn* parameters using the same model.

(2) The $^1S_0$ nucleon–nucleon enhancement should be the dominant feature of the spectrum. This condition rules out processes such as $H^3(H^3, \alpha)2n$, where the α–n interaction essentially determines the spectrum.

(3) All members of a group under investigation should have equivalent final states. This indicates that the $He^3(d, t)2p$ and $H^3(d, He^3)2n$ reactions are not quite appropriate because the p–t system has one more resonance (the 20.1-MeV level in He^4) as compared to the n-He^3 system.

(4) The comparison procedure should be made in a range of angles which correspond to the identical reaction mechanism.

Recently the procedure was applied[39] to the analysis of two groups: $D(n, p)2n$ and $D(p, n)2p$ reactions and $H^3(n, d)2n$, $He^3(p, d)2p$, and $He^3(n, d)np$ reactions.

The nucleon spectra at small angles (see Fig. 4) from deuteron breakup processes at incident energies around 14 MeV reveal two strong final-state interactions, i.e., *nn* (or *pp*) and *np*. At higher incident energies the *np* enhancement is kinematically removed and the main feature of the spectrum is the interaction between two undetected particles.

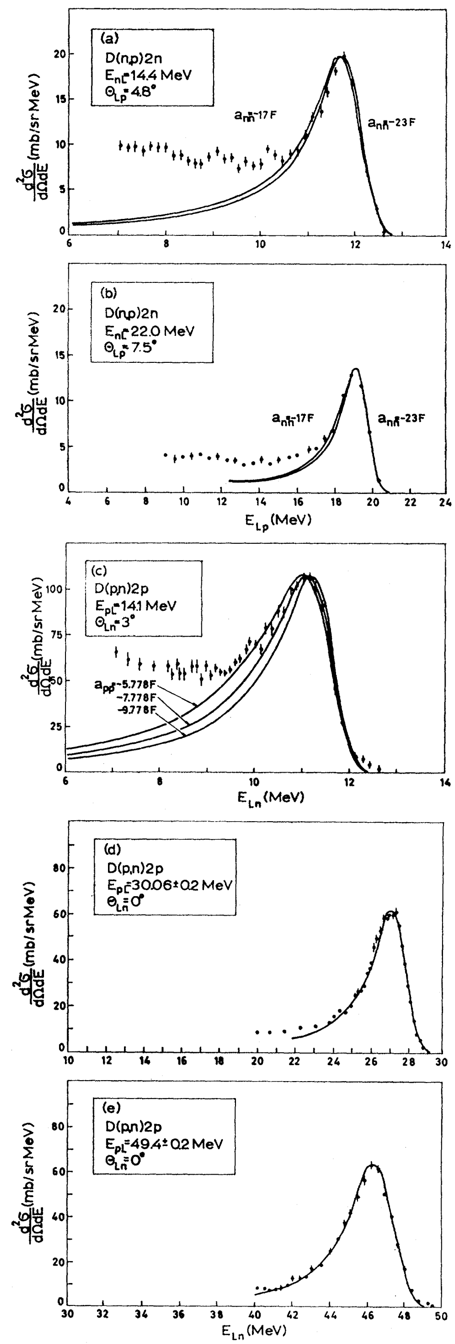

FIG. 4. Analysis of the nucleon spectra from nucleon + deuteron breakup reactions. (a) Proton spectrum from the reaction $D(n, p)2n$ at 14.4 MeV and a mean laboratory scattering angle of 4.8°. (b) Proton spectrum from the reaction $D(n, p)2n$ at 22.0 MeV and a mean laboratory scattering angle of 7.5°. (c) Neutron spectrum from the reaction $D(p, n)2p$ at 14.1 MeV and a laboratory scattering angle of 3°. (d) Neutron spectrum from the reaction $D(p, n)2p$ at 30.06 MeV and a laboratory scattering angle of 0°. (e) Neutron spectrum from the reaction $D(p, n)2p$ at 49.4 MeV and a laboratory scattering angle of 0°. The experimental data are given by the dots and error bars. The solid lines give the predictions for the spectra according to the calculations outlined in the text. Where more than one solid curve is shown, the respective values for the scattering lengths used, are indicated in the figure. All curves have the experimental energy resoltuion folded in.

[38] The comparative procedure was proposed by W. T. H. van Oers, I. Šlaus, and T. A. Tombrello, Bull. Am. Phys. Soc. **10**, 693 (1965), and by Baumgartner *et al.* (Ref. 43).

[39] W. T. H. van Oers and I. Šlaus (to be published).

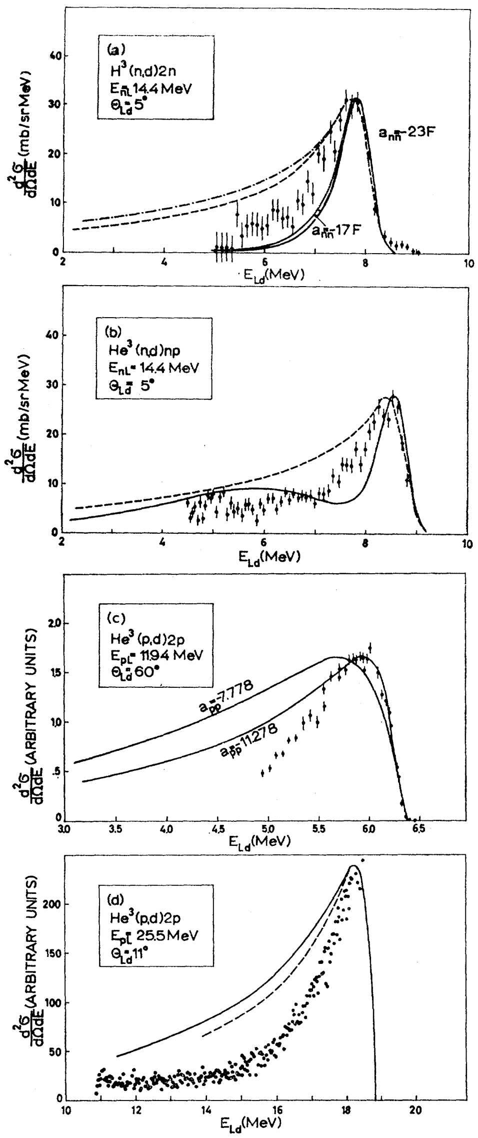

FIG. 5. Analysis of the deuteron spectra from nucleon + trion breakup reactions. (a) Deuteron spectrum from the reaction $H^3(n, d)2n$ at 14.4 MeV and a mean laboratory scattering angle of 5°. (b) Deuteron spectrum from the reaction $He^3(n, d)np$ at 14.4 MeV and a mean laboratory scattering angle of 5°. (c) Deuteron spectrum from the reaction $He^3(p, d)2p$ at 11.94 MeV and a laboratory scattering angle of 6°. (d) Deuteron spectrum from the reaction $He^3(p, d)2p$ at 25.5 MeV and a laboratory scattering angle of 11°.

The experimental data are given by the dots and error bars. The dotted and dashed curve of figure (a) correspond to the prediction of the spectrum according to the Watson–Migdal expression. The dashed curves result from calculation II. The solid curves are the results of calculation I, retaining only the contribution of the dominant pickup process. Where two solid curves are shown the respective values of the scattering lengths used are indicated in the figure. All curves shown have the experimental energy resolution folded in.

The deuteron spectra at small angles from the trion breakup (see Fig. 5) exhibit one prominent peak corresponding to the nucleon–nucleon final-state interaction. The nucleon–deuteron system does not exhibit a strong resonance and the behavior of n–d phase shifts explains why one does not expect any particular enhancement due to the n–d interaction in the final state.

In the analyses of deuteron breakup processes the np final-state interaction was neglected, and only one perturbing potential V_{np} was considered. The zero-range approximation was used and V_{np} was a delta potential. Figure 4(a) shows the experimental data from Ref. 25 compared with such a calculation using $a_{nn}=-17$ and -23 F, and the statistical analysis favors $a_{nn}=-21.5+3, -1$F and thus reproduces the results of Refs. 24 and 25, as it should since the calculation is essentially identical. The large value +3 for the uncertainty is here introduced to emphasize the fact mentioned earlier that the extension or reduction of the region of the spectrum under investigation leads to a_{nn} around -20 F. Exactly the same conclusion is drawn by analyzing the data from Ref. 26. The 22-MeV data[40] could favor a_{nn} between -15 and -18 F, though in view of experimental uncertainties even $a_{nn}=-21$ F would not be definitely excluded [Fig. 4(b)].

The proton spectra from the $D(p, n)2p$ reaction at 14.1 MeV,[41] 30.06 MeV, and 49.4 MeV[42] are shown in Fig. 4(c), (d), (e) and compared with the present model. The 14.1-MeV data favor $a_{pp}=-7.0$ F, when one considers any reasonable portion of the spectrum. The extension of the investigated spectrum below $E_p=10$ MeV would yield $a_{pp}=-6$ F. This has to be contrasted with the analysis at higher incident energies at which the data definitely favor $a_{pp}=-7.78$ F, the value identical with the one determined from the pp-scattering data. This situation can be understood because at higher energies the np enhancement is kinematically removed and it is reasonable to expect that the influence due to the third particle (neutron) is reduced.

The analysis of trion breakup processes was performed by representing the two-nucleon interaction by a spin-dependent central potential with a Yukawa radial dependence. The exchange mixture chosen was of the Serber type. The initial state contained a trion wave function of Gaussian form, whereas the final state was calculated with a Hulthén wave function for the deuteron and an S-state wave function for the two strongly interacting nucleons. The transition matrix elements contained four spatial integrals corresponding to a pickup process, a heavy-particle stripping, and a knockout of a bound or unbound nucleon–nucleon pair. Another Born approximation calculation was also done assuming that the trion consisted of a nucleon plus a deuteron, and that the deuteron was ejected by a delta-type interaction with the incident nucleon. We shall further refer to these calculations as I and II, respectively.

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[40] K. Debertin, K. Hofmann, and E. Rössle, Nucl. Phys. **81**, 220 (1966).

[41] J. D. Anderson, C. Wong, J. W. McClure, and B. A. Pohl, Phys. Rev. Letters **15**, 66 (1965).

[42] C. J. Batty, R. S. Gilmore, and G. H. Stafford, Phys. Letters **16**, 137 (1965).

TABLE II. The results of comparative analyses.

Reaction	Incident energy (MeV)	Scattering angle (degrees)	FWHM Gaussian resolution function (MeV)	Energy shift (MeV)	Scattering length	Reference
D$(n, p)2n$	13.9	4.5	0.65	+0.22		39
D$(n, p)2n$	14.4	4.8	0.70	+0.02	−21.5+3−1	39
D$(n, p)2n$	22	7.5	1.20	−0.40		39
D$(p, n)2p$	14.1	3	···	+0.02	−7.0	39
D$(p, n)2p$	30.1	0	···	−0.10	−7.78	39
D$(p, n)2p$	49.4	0	···	···	−7.78	39
$H^3(d, He^3)2n$	32.5	6	0.240	···	−16.1±1.0	43
$He^3(d, t)2p$	29.8	8	0.140	···	7.41+0.61, −0.67	43

From the results shown in Fig. 5 we conclude that the comparative analysis of the trion breakup processes is not successful in the energy range considered. Consequently, these processes cannot be presently explored to extract *nn* scattering parameters. The deuteron spectra are mainly determined by the first step of a sequential process. The fits to the $He^3(p, d)2p$ data will certainly not improve using a modified trion wave function.

The comparison procedure has also been applied by Baumgartner *et al.*[43] to the $He^3(d, t)2p$ reaction studied at an incident energy of 29.8 MeV and to the $H^3(d, He^3)2n$ reaction studied at incident energies of 32.5 and 40.2 MeV (laboratory angles between 6° and 25°). As emphasized earlier this pair of reactions does not satisfy all conditions necessary for the application of a sound comparison. However, both reactions can be experimentally investigated with a considerably higher accuracy than neutron-induced reactions.

The authors apply the Watson–Migdal model which provides a good fit to the data and the justification is found in the peripheral nature of both processes which implies a small overlap of the outgoing trion wave function with the wave function of two nucleons. Since the analysis of $He^3(d, t)2p$ yields a_{pp}

$$a_{pp} = -7.41+0.39,\ -0.49\ \text{F}$$

in good agreement with the p–p scattering value, it is argued that the model is good and that its application to $H^3(d, He^3)2n$ would give *nn* scattering parameters. The analysis yields

$$a_{nn} = -16.1 \pm 1.0\ \text{F},$$

which varies less than 0.2 F when different portions of the spectrum are employed for the fit. The value of the effective range r_{nn} employed in extracting a_{nn} was 2.65 F. For the best value $a_{nn} = -16.1$ F, variation in r_{nn} gave the best fit for

$$r_{nn} = (3.2 \pm 1.6)\ \text{F}.$$

In spite of the apparent success the difficulties in the application of the comparison procedure to the nucleon-induced trion breakup indicate that the comparative method, while certainly potentially very powerful, must be used cautiously. Thus, in our opinion, the conclusion reached in Ref. 43 that the comparative analysis of the $H^3(d, He^3)2n$ and $He^3(d, t)2p$ reactions gives a_{nn} up to ±1 F is premature.

Table II summarizes the results of comparative analyses.

D. The D$(\pi^-, \gamma)2n$ Reaction

The use of the D$(\pi^-, \gamma)2n$ reaction to determine the *nn* scattering length was suggested by Watson and Stuart[44] in 1951. This process is particularly suitable since all three particles in the final state can be detected and yet there is only one strong interaction in the final state. The presence of only one pair of particles in the final state which are subjected to strong interaction makes this reaction much "cleaner" than, e.g., the D$(n, p)2n$ reaction. The pion capture occurs from the lowest pi-mesonic Bohr orbit, i.e., practically at rest in the laboratory system. The available reaction energy is 136.07 MeV and it is known up to ±0.03%.

In 1954 Phillips and Crowe[45] investigated the gamma-ray spectrum. Their measurements gave the first quantitative information about the *nn* scattering length. The analysis showed that

$$a_{nn} = -15.9\ \text{F with the errors extending from}$$

$$-8.9\ \text{F to } -\infty\ \text{F}.$$

The bound state of two neutrons with the binding energy of more than 50 keV turned out to be less probable than 0.1%.

This investigation was repeated by Ryan[46] in 1964 using a gamma-ray pair spectrometer with a resolution of 1%. The results of this measurement are given in Table III.

McVoy[47] has realized that an essential improvement can be obtained if one measured the energy and/or the angular distribution of the two neutrons in coincidence. The superiority of this procedure is obvious when one

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[44] K. M. Watson and R. N. Stuart, Phys. Rev. **82**, 738 (1951).
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[46] J. W. Ryan, Phys. Rev. Letters **12**, 564 (1964).
[47] K. W. McVoy, Phys. Rev. **121**, 140 (1961).

Table III. Measurements of nn scattering length using the reaction $D(\pi^-, \gamma)2n$.

	a_{nn}	Probable error	Standard deviation	Reference
	−15.9 F	+7.4 F −∞		45
E_γ: 120–131.5	−15.2 F	−3.6 F +2.6 F		46
E_γ: 122–131.5	−18.8 F	−5.9 F +3.6 F		46
E_γ: 124–131.5	−18.8 F	−6.8 F +4.0 F		46
E_γ: 126–131.5	−16.9	−7.8 F +4.4 F		46
	<−10 F			a
	12 F		±5 F	b
	−16.47 F		±1.27 F	49

[a] P. K. Kloepel (unpublished).
[b] T. Sloan (unpublished).

realizes that the $^1S_0$ enhancement is related to the low relative energy of two neutrons and to the high energy of a gamma ray. The analysis which is based on the expression of the Watson–Migdal type is sound only if the gamma-ray energy is larger than 130.9 MeV. The corresponding neutron energies are smaller than 5 MeV.

A very accurate study of the $D(\pi^-, \gamma)2n$ reaction was performed by Haddock *et al.*[48,49] The measurement of the neutron energy was performed with the time-of-flight technique having the time resolution of 5 nsec. Using a distance of 10 ft, this implies about $2\frac{1}{2}\%$ time resolution in the relevant energy region. Such an arrangement made it possible to determine a_{nn} which is equivalent to the use of a gamma-ray spectrometer of a resolution of $\leq 0.05\%$. The time-of-flight of one neutron (t_1) at a particular angle (θ) between two neutrons was used to compare with the theoretical model. A complementary condition, i.e., the θ spectrum at fixed t_1, was examined for the consistency of a_{nn} obtained. Figure 6 shows the experimental spectra in neutron time-of-flight for four values of the angle (θ) between two neutrons. The phase space prediction and the theoretical curves for $a_{nn}=-16$ F and $a_{nn}=-27$ F are shown in Fig. 6(a).

Assuming that the sign of a_{nn} is negative, that $r_{nn}=$ 2.65 F and using in the analysis based on the theoretical treatment of McVoy[47] and Bander[50] only the data corresponding to $E_\gamma \geq 130.9$ MeV, the following value of a_{nn} was obtained

$$a_{nn} = -16.47 \pm 1.27 \text{ F}. \quad (3)$$

From a re-examination of the calculation and the approximations made by McVoy, Bander[50] concluded that the analysis of the reaction $D(\pi^-, \gamma)2n$ can be performed with an uncertainty not larger than ±1 F. This conclusion might be somewhat too optimistic.[2]

[48] R. P. Haddock, R. M. Salter, Jr., M. Zeller, J. B. Czirr, and D. R. Nygren, Phys. Rev. Letters **14**, 9 (1965).
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III. CHARGE DEPENDENCE OF NUCLEAR FORCES

The present knowledge of the nn force is quite insufficient and in some cases evidences are contradictory. The data mainly concern the $^1S_0$ state and are restricted to rather low energies. The only parameter which can now be stated with some confidence is the $^1S_0$ nn scattering length. Even in this case the situation is far from being quite satisfactory. The analyses of the $D(n, p)2n$ and $H^3(n, d)2n$ reactions at 14.4 MeV and of the $H^3(d, He^3)2n$ reaction at 32.5 and 40.2 MeV are presently unreliable. In the same way the value quoted in Ref. 43 for the nn effective range is unreliable. Thus, the present value of a_{nn} is derived from the study of the $D(\pi^-, \gamma)2n$ reaction, mainly from the measurement of Haddock *et al.*[48]

The value $a_{nn}=-16.5\pm1.3$ F is in good agreement with the values obtained from the analysis of pp scattering for the potential with the hard core. The neutron–neutron and proton–proton scattering lengths should be corrected for the effects of finite charge and magnetic moment distributions. According to Riazuddin[51] these corrections for a_{nn} amount to about −0.8 F bringing a_{nn} to about −17.3 F. Schneider and Thaler[52] estimate that the magnetic correction changes a_{nn} by −0.6 F, whereas the inclusion of distributed charge and magnetic moment produces a negligible correction. The vacuum polarization correction for a_{pp} is about −0.2 F.[53]

The experimental uncertainties together with an additional theoretical uncertainly of ±1 F lead to the conclusion that the present knowledge of a_{nn} is consistent with the assumption of charge symmetry within ±1.5%[54] provided that the nucleon–nucleon interaction has a hard core.

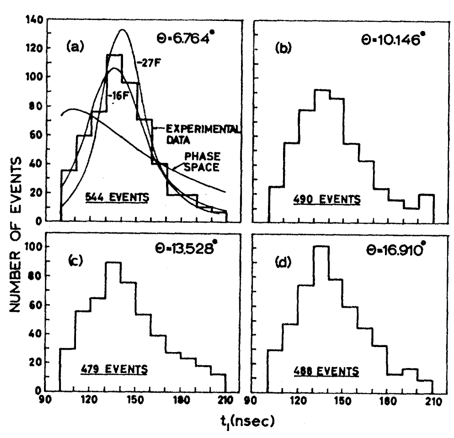

Fig. 6. Histograms of time-of-flight spectra from the reaction $D(\pi^-, \gamma)2n$. (Refs. 48 and 49). The phase space prediction and the theoretical curves are shown in Fig. 6(a).

[51] Riazuddin, Nucl. Phys. **4**, 217, 223 (1958).
[52] R. E. Schneider and R. M. Thaler, Phys. Rev. **137**, B874 (1965).
[53] L. Heller, Phys. Rev. **120**, 672 (1960).
[54] The experimental error limits would yield[55] $-0.5 \leq (\Delta V/V)_{cs} \leq +1.1$ using the parameter A for the Yukawa potential.

The present knowledge of the *nn* interaction does not shed light on the problem of the violation of charge independence of nuclear forces. The evidence concerning the breakdown of charge independence is[1]:

$^1S_0$ scattering length:

$a_{np}-a_{pp}$ (Coulomb corrections included)

$$=-(6.83\pm0.15)\ \text{F} \quad (2)$$

which for the Yukawa potential implies

$$(\Delta V/V)=(4.8\pm0.1)\%.$$

Thus, the *np* potential is by about 5% more attractive than the *pp* potential. The corrections due to vacuum polarization and the finite charge and magnetic moment distribution reduces this difference to

$$a_{np}(\text{nuclear})-a_{pp}(\text{nuclear})=-(6.3\pm0.2)\ \text{F}$$

and

$$(\Delta V/V)=(4.1\pm0.1)\%. \quad (3)$$

The *np* effective range determined using also the recent data of Engelke *et al.*[55] is

$$r_{np}=2.5166\pm0.1036\ \text{F} \quad (4a)$$

and it is in serious disagreement[56] with the prediction of charge independence

$$r_{np}(\text{C.I. value})=2.73\pm0.03\ \text{F}. \quad (4b)$$

The analysis based on *np* data excluding the data of Ref. 55 would be consistent with charge independence. Noyes emphasizes[56] that in order to obtain the value (4a) for r_{np} one has to reduce the attraction at $r=2$–4 F by almost 30% and increase it near the core by the same amount.

Nuclear structure information indicates that nuclear forces are by about 1–3% charge-dependent.

The explanation of charge dependence can be searched for in the electromagnetic characteristics of nucleons and mesons. The effect of the pion mass difference was discussed by Sugie[57] in 1954 and by Riazuddin[51] in 1958. Lin[58] calculated nuclear potentials of Brueckner–Watson type up to the fourth order in the perturbation expansion taking into account the pion mass difference. Three coupling constants G_{π^0nn}, G_{π^0pp}, and $G_{\pi^\pm np}$ and the radius of the hard core are treated as parameters to be determined by fitting the $a_{np}-a_{pp}$ difference and the $T=1$ first excited state in Li.[6] Lin obtains small differences in coupling constants (up to 1%) and the predicted values of a_{nn} range between -19.5 and -22.2 F. Thus, this analysis leads to the violation of charge symmetry. Heller *et al.*[3] and Noyes[56] also consider the pion mass difference and allow the splitting of coupling constants, but maintain charge symmetry. The result of their analyses is that the use of $G^2_{\pi^\pm np}$ which is larger by about 3.5% than $G^2_{\pi^0nn}=G^2_{\pi^0pp}$ would account for the difference between $^1S_0$ scattering lengths. Henley and Morrison[59] investigated the mass splitting in OPE, TPE, and in one-ρ-meson-exchange potentials using a charge-independent boundary conditions to replace the short-range behavior. If $m_{\rho^\pm}-m_{\rho^0}$ is taken to be 2 MeV and using equal coupling constants, they obtain $a_{np}=-18.27$ F. The remaining difference between a_{np} and a_{pp} (nuclear) can be obtained in several ways, e.g., by introducing a 2% charge dependence of neutral to charged pion coupling constants.

The influence of different lifetimes of charged and neutral pions on the effective range parameters is negligible[60]: $\Delta a_{pp}\leq0.02$ F, $\Delta r_{pp}=-0.001$ F.

Charge symmetry breaking effects, e.g., radiation correction and isospin mixing, could amount[51,61,62] to as much as 1%, making the *nn* interactions stronger than the *pp* interaction.

IV. CONCLUSION

The discussion in Secs. II and III demonstrated that the present knowledge of neutron–neutron interactions is not adequate to solve problems raised in the Introduction. Indeed, very precise *nn* data are required and we will point out which processes seem to be most appropriate to yield such data.

First, the study of direct *nn* scattering, probably using underground nuclear explosions.

Second, the comparative study of $D(n,p)2n$ and $D(p,n)2p$ reactions in the energy region between 20 and 200 MeV.

Third, the investigation of the processes $A(a,b)2n$ in the framework of the Faddeev approach using realistic nucleon–nucleon forces.

Fourth, an adequate understanding of peripheral processes will open a wide field for the study of the *nn* interaction.

Fifth, the study of the $D(\pi^-,\gamma)2n$ reaction, especially in view of the development of meson factories might prove to be a very promising method for studying the *nn* interaction at higher relative energies.

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