Phenomenological Phase–Parameter Fits to N–N Data at Intermediate Energies*

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New p–p and n–p phase-parameter multiple energy fits (Y-IV) intended for the energy range from ~8 MeV to ~340 MeV employing 947 measurements for p–p and 1023 for n–p are discussed. Improvements over older work include: (a) more careful joining to low-energy data involving phase searches beginning at 1.397 MeV for p–p and 3.205 MeV for n–p; (b) inclusion of triple scattering and polarization data at 430 MeV; (c) data lumping in smaller groups than previously; (d) increased accuracy of one-pion-exchange (OPE) phases; (e) graded introduction of OPE treatment. Related matters include: (1) effects of Coulomb field corrections and of corrections for the apparent violation of short-range charge independence for the new fits and the older YRB1(K_0) – YLAN4M; (2) effects of varying the difference between the p–p and n–p effective ranges within its uncertainty limits; (3) analyses of mockups of experiments designed to improve knowledge of the difference just mentioned; (4) comparisons of values of the pion–nucleon coupling constant g_0^2 derived from p–p and n–p data in relation to the long-range charge independence of N–N interactions.

I. INTRODUCTION

This report is based primarily on work done in collaboration with K. A. Friedman, R. D. Haracz, John M. Holt, A. Prakash, and R. E. Seamon. Seamon and Friedman have been concerned with the phenomenological fits the longest among those cited.

The general knowledge of phase parameters (phase shifts and coupling coefficients) is useful in applications to theories of nuclear matter and of nuclear structure and may eventually be quantitatively explicable in terms of fundamental theory although it does not appear likely that the latter type of advance will take place in a convincing manner in the near future. A knowledge of phase parameters is also useful in studies of the effect of one-pion exchange (OPE) employing the contributions of phase parameters with high orbital angular momenta $L\hbar$ and total angular momenta $J\hbar$, which offer a way of making tests of "long-range" charge independence and of making comparisons between the pion–nucleon coupling constant derivable from nucleon–nucleon (N–N) scattering with supposedly the same pion–nucleon coupling constant derivable from pion-physics phenomena such as pion–nucleon scattering and the photodisintegration of the nucleon. Charge independence in N–N interactions taken literally is well known to be only approximate. It is generally believed that in some sense the content of charge independence can be stated more appropriately as a symmetry in isotopic spin space in a description of the pion–nucleon interaction. But since the masses of charged and neutral pions are different and since it is not known to what extent the pion is an elementary rather than composite particle, the exact statement of the symmetry involved is still uncertain. One may hope that N–N scattering research will contribute to the solution of these questions and considerable attention is being paid by the Yale group therefore to tests of the long-range charge independence in N–N interactions.

The YRB1(K_0) and YLAN4M fits were reported at the Dubna conference in 1964. Some revisions in these fits were finished in 1965. A listing of the phase parameters and an account of the data used in these fits is to appear in a chapter of a book on High-Energy Physics.[1] Since the completion of the fits just mentioned, a new set of multiple energy fits was undertaken. The arguments for rejecting data differing from the expected value by more than two standard deviations appeared unjustifiable except possibly as a matter of economy of effort. The Chauvenet criterion used by some may be shown to be in contradiction with the statistical assumptions leading to normal error distributions on which it is based. An entirely satisfactory data rejection criterion is, however, impossible to obtain. The following procedure was used for the new fits. Empirical fits of the differential cross section $\sigma(\theta)$ and of the polarization parameter $P(\theta)$ were made by means of polynomials in the energy E and the angle θ by least squares with weights corresponding to experimental errors. The frequency of deviations of individual measurements from the most probable values corresponding to the polynomial fit was represented by means of histograms and showed on the whole reasonably good agreement with expectation for a normal error distribution. Some data had been ascertained to have been definitely in error either on account of the renormalization of the p–C polarization or a revision such as that of the Harwell $\sigma(\theta)$. This experience with $P(\theta)$ and $\sigma(\theta)$ was interpreted in terms of a dangerous multiple of a standard deviation beyond which a nucleon–nucleon scattering is not trustworthy. This evidence pointed to three standard deviations from the probable value furnishing a reasonable dividing line between the data to be used and those to be rejected. The measurements rejected usually corresponded to giving an anomalous spread to the distribution of deviations. Evidence from YRB1(K_0) and YLAN4M often supported that from

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[1] G. Breit and R. D. Haracz, "Nucleon-Nucleon Scattering", in *High Energy Physics* (Academic Press Inc., New York, 1967), Vol. 1.

the empirical fits. The experimental papers were gone over with more care than previously regarding indications concerning assignments of statistical and normalization errors. In some cases these had to be changed on the basis of such detail regarding the origin of uncertainties as could be found. Among the n–p data rejected were the Harwell (1952) $\sigma(\theta)$ at 153.0 MeV; the Rochester (1952) $\sigma(\theta)$ at 215 MeV; the Carnegie Tech (1956) $P(\theta)$ at 350 MeV. In all there were 16 rejected n–p data and 8 of the p–p data. The rejection criterion applied to the YRB1(K_0) and YLAN4M data decks and data that became available after about June 1965. Such weeding out of data as took place in older data collections is not under discussion. The new carbon-scattering polarization measurements were used throughout the new N–N fits. In n–p phase parameter searches the large-angle data obtained by p–d scattering were not used because of the doubt expressed by Cromer[2] regarding the applicability of his corrections for the effect of the spectator proton at such angles. An exception was made for $R_t(\theta)$ of Thorndike at 203 MeV because of its sensitivity to differences between different n–p multiple energy fits. Other p–d data acceptance criteria are being investigated. The rejection of large-angle p–d data was not made in fit YLAN4M. The new fits have used more careful joining to data from 0 to a few MeV and took into account in the case of p–p some data at energies above 350 MeV. The accuracy of the OPE phases was raised and the transition energies from OPE to "searched" were introduced in a graded manner starting from low values of the orbital angular momentum $L\hbar$. Phase parameters for $L=0$, 1, and 2 were in the searched group throughout. The transition energies were 46 MeV for $L=3$, 110 MeV for $L=4$, 180 MeV for $L=5$; the coupling parameters ρ_J between states with $L=J\pm1$ had transition energies of $L=J$. Thus ρ_3 coupling $^3D_3$ to $^3G_3$ has the same transition energy as K_3, the phase shift for $^1F_3$. As in YRB1(K_0) and YLAN4M magnetic moment corrections were made throughout and the effect of electrostatic corrections was reexamined. The phase parameters were subjected to *causality tests* employing the logarithmic derivative method.

II. PROTON–PROTON FITS

In the p–p searches the total number of measurements used was 947 but on account of lumping of some of the data the number of data in the deck was only 886. Most of the lumping took place for $\sigma(90°)$. In presenting χ^2 values track was kept of a small and often insignificant addition necessitated by the lumping procedure,

$$\Delta(\chi^2)=\sum_i w_i y_i^2-\langle y\rangle^2\sum_i w_i,$$

where the y_i are the measured values being lumped, the w_i are their weights, and $\langle y\rangle$ is the weighted mean of the y_i. The correction takes account of the existence of a mean-square deviation of the values being lumped from the fit and applies in the lumping procedure used. The low-energy end of the searched region of p–p data made use of the data of Knecht, Dahl, and Messelt[3] (KDM) who have ascertained the necessary vacuum polarization corrections. Employing the values of these corrections as in KDM the effect of vacuum polarization was removed from the data leaving the supposedly pure p–p interaction effect. The values of K_0 and of γ, the centroid of the ^3P phases were then adjusted by least square searches (grid method procedure) for a minimum at 1.397, 1.855, 2.425, and 3.037 MeV. The experimental uncertainties were increased in the ratio $(\tau\sigma)^{1/2}/\sigma$, where σ is the uncertainty in K_0 arrived at using the criterion of unit increase in χ^2 and $\tau=(4\chi^2N_\theta)^{1/4}\sigma$. The number of angles at which measurements were made is here denoted by N_θ. This criterion has been arrived at by KDM through "a somewhat subjective assessment of the data and the methods." Since in this case the experimenters themselves have studied the question of error assignments, their judgment has been given preference over more formal approaches. The KDM data have not been combined with other data in this energy region because of the presence of many systematic sources of error which have been studied by their predecessors at the same laboratory with guidance from Professor R. G. Herb. An exception to the "safety in numbers" policy has been made in this case by the Yale group. The resultant $K_0(E)$ graphs agree quite well with those for the "best meson fit (IE)" Eq. (8) of Yovits *et al.*[4] The final fit represents the KDM data better than statistically expected. The primary object of the fit Y–IV is however to ascertain the phase parameters at energies above 9 MeV for which the troubles associated with high-precision requirements at the lower E values are less acute. At the high-energy end the $(\text{Y–IV})_{p-p}$ search made use of the data of Roth *et al.*[5] on $D(\theta)$, $R(\theta)$, $A(\theta)$, $A'(\theta)$, and $P(\theta)$ at 430 MeV. These data were transferred to 330 MeV making use of the Scotti–Wong phases.[6] These gave a reproduction of the Scotti–Wong fits to the Roth *et al.* data given by the latter. Employing the same set of phases, an approximate reproduction of curves in Reference[7] was obtained. The data points were then placed around calculated curves at 330 MeV employing the same source[6] of phases in approximately the same positions as the data points had at 430 MeV with respect to the calculated curve at that energy.

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[2] A. H. Cromer and E. H. Thorndike, Phys. Rev. **131**, 1680 (1963); P. F. M. Koehler, E. H. Thorndike, and A. H. Cromer, *ibid.* **134B**, 1030 (1964).

[3] David J. Knecht, Per F. Dahl, and S. Messelt, Phys. Rev. **148**, 1031 (1966).

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[5] R. Roth, E. Engels, Jr., S. C. Wright, P. Kloeppel, R. Handler, and L. G. Pondrom, Phys. Rev. **140B**, 1533 (1965).

[6] A. Scotti and D. Y. Wong, Phys. Rev. Letters **10**, 142 (1963).

[7] A. Scotti and D. Y. Wong, Phys. Rev. **138B**, 145 (1965). (Used as in earlier abridged preprint form.)

TABLE I. Comparison of D values for some p–p data fits with Y–IV data deck ($E \leq 345$ MeV).

Phases	Y–IV	Y–IV	Y–IV	AM–IV[a]	YRB1(K_0)	Y–IV
$\Delta\chi^2$	No	Yes	No	No	No	No
Energies	All	All	>24 MeV	>24 MeV	≧9.69 MeV	≧9.69 MeV
MAG	Yes	Yes	Yes	No	Yes	Yes
D	1.269[b]	1.281[c]	1.413[b]	1.574[b]	1.760[b]	1.341[b]
Number of data in deck	886	886	754	754	835	835
Number of measurements	947	947	812	812	896	896

[a] Same pion–nucleon coupling constant, $g_0^2=13.0$, as in AM work.
[b] For data-deck entries.
[c] For data.

For each of the experimental quantities the deviations from the calculated curve were adjusted by application of a common factor to give the same χ^2 at 330 MeV as the points had originally at 430 MeV. The Scotti–Wong phases were preferred over other possibilities for this data transfer because their theoretical expressions appear suitable for continuing the phase parameters into the region of positive meson energies. The experimental uncertainties of these data do not appear to make an exact treatment necessary. It was felt, however, that the information regarding triple scattering parameters was too valuable to be disregarded. The over-all values of D—the weighted mean of squares of deviations—for the new fit $(\text{Y–IV})_{p-p}$ are compared with some other fits in Table I. The comparison between the second and third columns in this table shows the slight effect of $\Delta\chi^2$ and serves as a justification for its omission in succeeding columns. The effect is slight however only if the number of measurements replaces the number of data deck entries when $\Delta\chi^2$ is applied. In order to compare fit Y–IV with the best Livermore multiple energy fit AM–IV, it was necessary to omit from the Y–IV data deck the entries with $E<24$ MeV. The D value for the Y–IV phases increased as a result to 1.413 as in the fourth column. The AM–IV phases were fit by means of polynomials in E and the observables present in the Y–IV deck were computed either directly from the AM–IV phases or from the polynomial fits. The resultant fit gave $D=1.574$ as in the fifth column. In the sixth column is shown $D=1.760$ for the June 1965 edition of the Yale fit YRB1(K_0), and in the seventh $D=1.341$ for the Y–IV phases. In comparisons with AM–IV it may be relevant that the value $g_0^2=13.0$ is rather low. However a change to $g_0^2=16.0$ is not likely to increase D by more than 0.03.

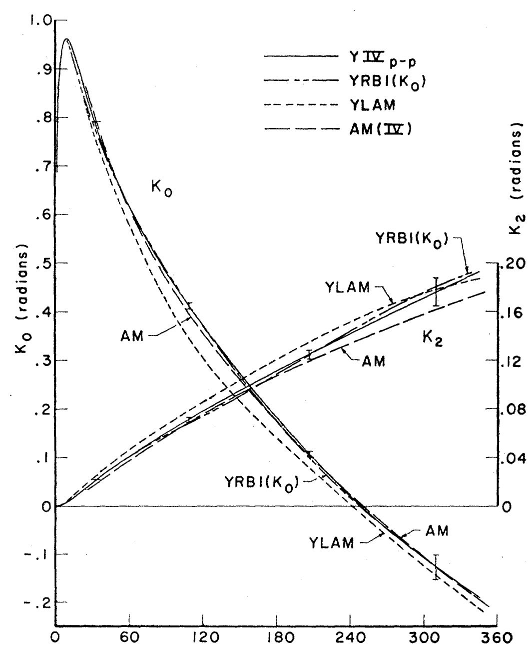

FIG. 1. Comparison of p–p phase shifts K_0 and K_2 for ${}^1S_0$ and ${}^1D_2$ states, respectively, as obtained in Yale fits Y–IV, YRB1(K_0), YLAM and the Livermore multiple energy fit AM–IV, AM standing for Arndt and MacGregor. The error bars are the parallel shift uncertainties corresponding to Y–IV with factor $D^{1/2}$ included as determined for energy intervals (0, 70), (70, 150), (150, 270), and (270, 345) MeV.

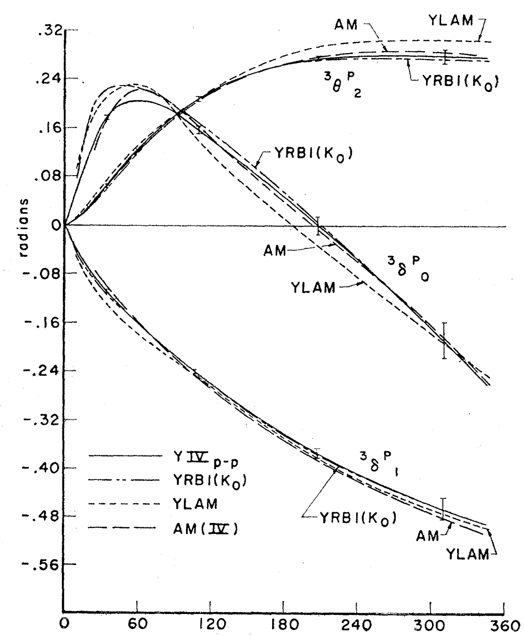

FIG. 2. Comparison of phase parameters for ${}^3P_0$, ${}^3P_1$, and ${}^3P_2$ states as obtained in Yale fits Y–IV, YRB1(K_0), YLAM and the Livermore multiple energy fit AM–IV. Conventions as in Fig. 1.

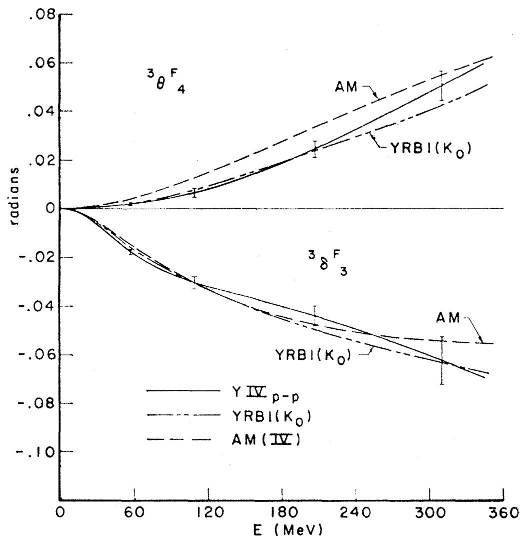

FIG. 3. Comparison of phase parameters for $^3F_3$ and $^3F_4$ states as obtained in fits Y-IV, YRB1(K_0), and AM-IV. Conventions as in Fig. 1.

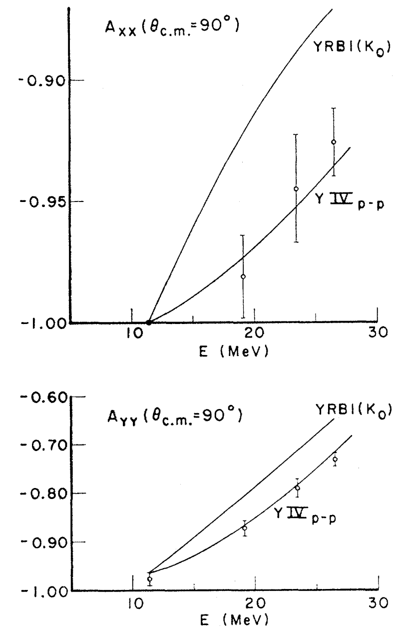

FIG. 5. Fits Y-IV and YRB1(K_0) compared with correlation parameters A_{xx} and A_{yy} (alias C_{nn}) as measured in p-p scattering by Catillon *et al.* (Gatlinburg Conference, fall 1966).

Although D is lower for $(\text{Y-IV})_{p-p}$ than for the other fits in corresponding energy regions employing the new Yale data collection, it is somewhat higher than AM-IV in a comparison with the AM-IV data table. Since the Yale data are believed to contain more information than those at Livermore the comparisons are presented as in Table I.

In Fig. 1 phase shifts K_0 and K_2 for Y-IV are compared with those from Arndt-MacGregor-IV (AM-IV), YRB1(K_0), and the old (1960) fit YLAM. The vertical lines indicate the uncertainties of the Y-IV determination obtained by the parallel shift procedure with $D^{1/2}$ factor included. The energy intervals used for these uncertainty estimates were (0, 70), (70, 150), (150, 270), (270, 345) MeV. For K_0 the agreement of the

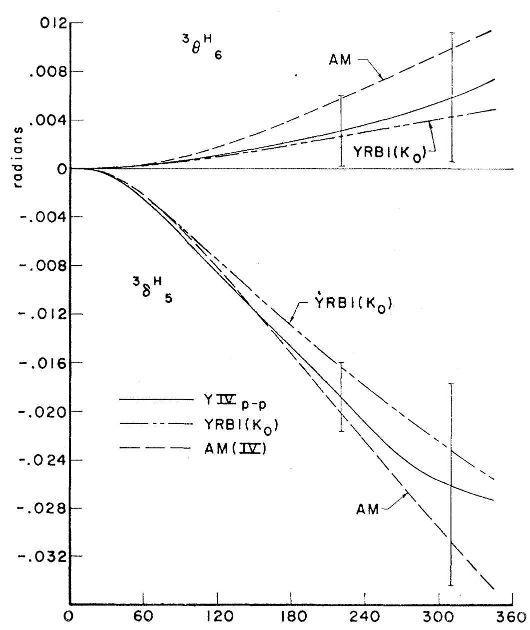

FIG. 4. Comparison of phase parameters for $^3H_5$ and $^3H_6$ states as obtained in fits Y-IV, YRB1(K_0), and AM-IV. Conventions as in Fig. 1.

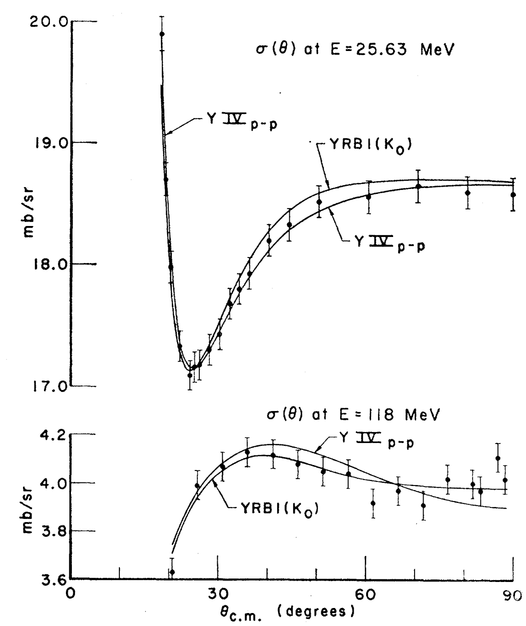

FIG. 6. Fits Y-IV and YRB1(K_0) compared with $\sigma(\theta)$ as measured in p-p scattering at 25.63 MeV by Jeong *et al.* and at 118 MeV by Palmieri *et al.*

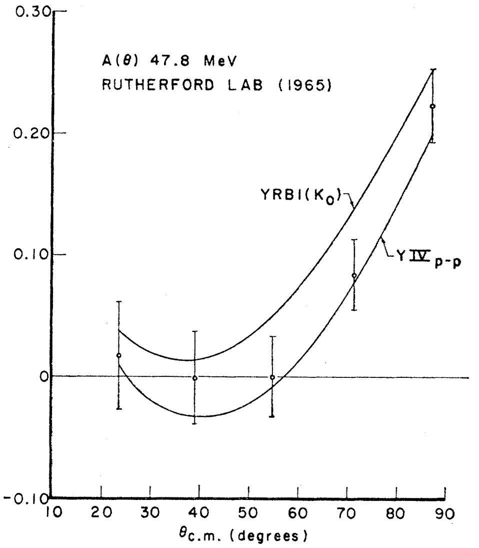

FIG. 7. Fits Y–IV and YRB1(K_0) compared with $A(\theta)$ as measured in p–p scattering at 47.8 MeV by Ashmore *et al.*

three newer fits is very close. For K_2 the agreement is not as good and the parallel shift uncertainties are larger. This trend continues as L increases. In Fig. 2 a similar comparison is made for the ^3P phases. The displacement of the maximum of the $^3P_0$ phase towards small energies for YRB1(K_0) as compared with the other fits is noteworthy. The Y–IV searches in this energy region were more thorough than those for YRB1(K_0). In Fig. 3 the comparison is made for $^3F_4$ and $^3F_3$. The uncertainties and differences are now relatively larger. The comparisons for $^3H_5$ and $^3H_6$ are shown in Fig. 4. These phases are obviously not well known as yet.

Figure 5 shows a comparison of the correlation parameters A_{xx} and A_{yy} (alias C_{nn}) at 90° of Catillon *et al.*[8] with YRB1(K_0) and $(\text{Y–IV})_{p-p}$. The superiority of the latter over the former is obvious here. In Fig. 6 are displayed the measurements of $\sigma(\theta)$ by Jeong *et al.*[9] at 25.63 MeV and of Palmieri *et al.*[10] at 118 together with the same fits. Figure 7 shows a similar comparison with $A(\theta)$ measurements of Ashmore *et al.*[11] at 47.8 MeV and Fig. 8 with measurements of $\sigma(\theta)$ at 144.1 and $P(\theta)$ at 140.7 MeV by Cox *et al.*[12]

III. NEUTRON–PROTON FITS

The low-energy end of the new $(\text{Y–IV})_{n-p}$ search is complicated by the uncertainties regarding the value of $(^1r_0)_{n-p}$, the effective range of the phase shift K_0 of the $^1S_0$ state. From an analysis of total cross section n–p scattering data, Noyes[13,14] concluded that $(^1r_0)_{n-p}$ is about 10% or more smaller than $(^1r_0)_{p-p}$. The analysis of the data used by him was confirmed at Yale.[15] At the same time the possibility of systematic errors in the theory of the experiments was reconsidered. Among such sources of error are dynamic effects of molecular electrons in n–p experiments at energies above the epithermal region as well as in p–p scattering in the low-energy region; the participation of molecular structure of the hydrocarbons used in the measurement of the coherent scattering scattering length by the "mirror reflection" method; some uncertainties in the estimate of the standard error, the total cross section at epithermal energies suggested by apparent inconsistencies of the dependence of σ^{tot} on E; the unusual smallness of the uncertainties of some total cross section measurements at about 3 and 5 MeV used in the

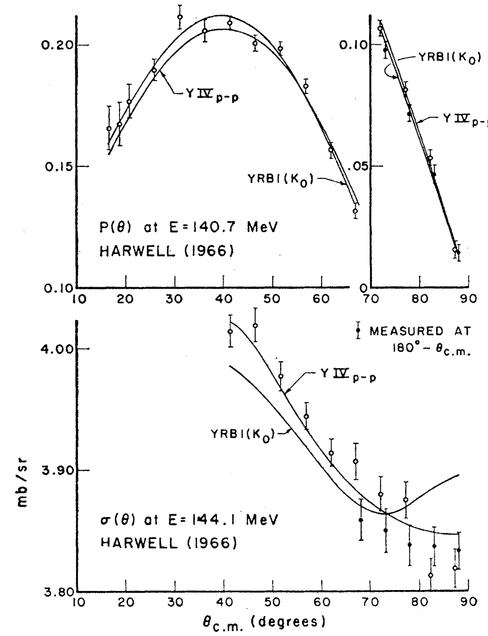

FIG. 8. Fits Y–IV and YRB1(K_0) compared with measurements of $P(\theta)$ at 140.7 MeV and of $\sigma(\theta)$ at 144.1 MeV by Cox *et al.* For $P(\theta)$ the AM–IV fit (not shown to avoid confusion) is slightly better for $\sigma(\theta)$ slightly worse than Y–IV.

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TABLE II. Uncertainties of $T=1$ phase parameters in radians as derived from mock-up of experiments.

Run designation	Measured quantities	ΔK_0	$\Delta^3\delta^P{}_0$	$\Delta^3\delta^P{}_1$	$\Delta^3\theta^P{}_2$	E (MeV)	Δ^1r_0 (%)
20_{I}	σ, D, R, A	0.0108	0.030	0.016	0.012	20	2.5
20_{II}	σ, D, R, A	0.0091	0.023	0.014	0.010	20	2.1
10_{I}	σ, D, R, A	0.0060	0.024	0.015	0.010	10	1.5
10_{II}	σ, D, R, A	0.0046	0.018	0.012	0.0078	10	1.2
20_1	σ, D, R, A	0.0016	0.0024	0.0017	0.0011	20	0.36
20_2	σ, D, A	0.0022	0.0025	0.0026	0.0015	20	0.50
20_3	σ, R, A	0.0019	0.0027	0.0020	0.0011	20	0.43
$20_{3'}$	σ, D, R	0.0017	0.0048	0.0022	0.0019	20	0.39
20_4	σ, D, R, A, R'	0.0017	0.0029	0.0019	0.0013	20	0.39
20_5	σ, D, R, A, A'	0.0017	0.0026	0.0020	0.0014	20	0.39

analysis. In addition there are the uncertainties in the values of the shape parameters. As a result of the consideration of such effects it appeared fair to conclude that although $({}^1r_0)_{n-p}$ is probably somewhat smaller than $({}^1r_0)_{p-p}$ there is an appreciable probability that it is not as small as 2.5 F and that a value as high as 2.70 F is hard to exclude. In the main part of the $(\text{Y-IV})_{n-p}$ work the value of $({}^1r_0)_{n-p}$ was used as 2.70 F. The value of $({}^1r_0)_{p-p}$ is not quite certain either but is around 2.75 F.

The present difficulty in ascertaining the difference in the ${}^1S_0$ effective ranges is caused largely by the necessity of working in a narrow energy region. The determination of ${}^1r_0$ depends in fact on that of the slope of a certain function of the phase shift plotted against the energy and therefore a large accuracy in the phase-shift determination is essential. On the other hand, expansions similar to the usual effective range expansion can be carried out expanding about $E=E_0$ for any finite value of E_0 rather than about $E=0$. The essential physical phenomena under test are much the same whether $E_0=0$ or not. The special advantage of $E_0=0$ is that the effects of phases with $L>0$ are very small. This advantage is somewhat illusory however because vacuum polarization corrections matter close to $E=0$ and because of the other low-energy complications already discussed. Making E_0 too large would not be practical because many phases become then important. But at 10 or 20 MeV the values of the phases of the P waves are still small enough to make their determination with an accuracy sufficient to correct for their presence possible. To determine the possibilities of carrying out a useful comparison, calculations intended to determine the accuracy obtainable in the effective range determination have been made by Seamon and the writer in connection with a proposal to the AEC in 1965.

A partial account of their description is as follows. For both p–p and n–p scattering the values of the experimental quantities were first calculated employing phases from phenomenological fits. It was then supposed that a set of experiments will furnish values agreeing with such a fit subject to errors estimated partly on the basis of those quoted in the literature and partly on the basis of attainable counting statistics with the planned equipment. The estimated errors in the measurements were used to calculate the uncertainties in the phase parameters that would be obtained if the experiments were performed. The method of the error matrix was used. Since this method presupposes that a least squares fit to the data has been made and that the sensitivities of the experimental quantities to the phase parameters are used for a least-squares fit to the data and since the latter is likely to be close to that of the phenomenological fits, values of the sensitivities corresponding to the latter were used. The effect of including K_2 in the searches was tried in some machine runs and did not seriously affect the main conclusions. On account of the necessity of eliminating the effect of partial waves with $L>0$ the mock experiments concerned with the p–p interaction included measurements of quantities other than the differential cross section so as to separate the effect of the P waves.

Results of machine runs employing the phenomenological fit YRB1(K_0) are as in Table II. The input data and their errors in the standard deviation convention were as follows. The values of all quantities were those of YRB1(K_0). The differential cross section $\sigma(\theta)$ was supposed available at 10° intervals in the center of mass scattering angle θ from 10° to 90° inclusive. The standard error was taken to consist of two components representing, respectively, an error that is uncorrelated from angle to angle and an error arising from lack of perfect knowledge of a normalizing factor common to the values at all angles. Each of these errors was taken to be $0.5/2^{1/2}=0.353\%$. The compounded standard error of each $\sigma(\theta)$ value was thus 0.5%. In runs $(20)_{\mathrm{I}}$ and $(10)_{\mathrm{I}}$ the measurements of D, R, A were supposed available at 20°, 40°, and 90°, in runs $(20)_{\mathrm{II}}$ and $(10)_{\mathrm{II}}$ they were supposed available at 10°, 20°, 30°, 40°, 60°, and 90°. In these four runs the uncorrelated errors of D, R, A were 0.08 at 20 MeV and 0.05 at 10 MeV. These errors were used as standard errors of the quantities—not on a percentage basis.

The initially assumed accuracy of the D, R, and A experiments corresponds approximately to one-hour

"Emperor" accelerator runs with presently available ion currents according to estimates of Professor Charles W. Drake and Professor Lee C. Northcliffe. Since a noticeable improvement in the accuracy obtainable for ΔK_0, ΔP_0, ΔP_1, ΔP_2 was caused by increasing the number of angles used for D, R, A from 3 in $(20)_{\mathrm{I}}$ and $(10)_{\mathrm{I}}$ to 6 in $(20)_{\mathrm{II}}$ and $(10)_{\mathrm{II}}$, the additional 6 computing machine runs $20_1, \cdots, 20_5$ were made in order to see how the accuracy improves with added measurements of the "triple scattering" parameters. The accuracy of the differential cross section measurements was kept the same as for $(20)_{\mathrm{I}}$, $(20)_{\mathrm{II}}$, $(10)_{\mathrm{I}}$, $(10)_{\mathrm{II}}$, but the standard error of D, R, and A was decreased by a factor 8 and the quantities were supposed to be measured at the 9 angles used for $\sigma(\theta)$. The Emperor machine time required for such determinations of D, R, A would be of the order of $9\times64\times3=1728$ h but the estimated accuracy obtainable is larger than necessary. For the II-type run the corresponding time would be $6\times1\times3=18$ h. In this case the accuracy for the effective range determination is marginal especially if the absence of K_2 in the searches is considered.

The six runs $20_1, \cdots, 20_5$ contain information regarding the relative value of measurements of D, R, and A. The omission of R spoils the accuracy of K_0 most, the omission of A affects it least. Since A requires a relatively elaborate instrumentation this circumstance is fortunate. Runs 20_4 and 20_5 indicate that R' and A' produce little effect that cannot be obtained by means of D, R, and A.

The last column in Table II shows the accuracy that would be obtainable for the mean effective range from the runs in Table II as if the shape parameter were known and the overall slope of the f function in the energy region from 0 to the measured energy were used. The employment of the chord rather than the tangent to the f function of effective range theory can subordinate the importance of the corrections at very low energies. The existence of other good measurements at a few MeV is disregarded in this estimate of accuracy. It is probable that with their inclusion a reliable and appreciably more accurate value of ${}^1r_0$ than that indicated by the last column in Table II could be obtained. Even 20_{II} and 10_{II} may be sufficiently accurate to settle the question of the presence of a 10% difference in p–p and n–p effective ranges but a higher accuracy would ease the requirements on n–p measurements.

In the comparison of p–p with n–p fits in the region from about 7 MeV to 20 MeV the Coulombian effects are much less important than in the region from 0–7 MeV. If there were no Coulombian and other electromagnetic effects the direct comparison of $k \cot K_0$, where $k/(2\pi)$ is the wave number, would suffice for the essential physical question of charge independence. The determination of the so-called "shape parameter" would be unnecessary therefore in such an idealized situation. In first approximation the comparison would be that of slopes of chords of the f function rather than of tangents. It is accordingly much less important to determine the shape parameter accurately in the 7- to 20-MeV energy range than in the lower energy region. Nevertheless, an approximate value is desirable to ascertain so as to be able to apply corrections for the effect of differences in its value between the p–p and n–p cases. For this reason it is desirable to perform measurements at several energies such as 5, 10, 15, and 20 MeV.

Calculations similar to those for Table II have also been made for n–p scattering for a variety of conditions. In recording the results the following notation is used.

Input Data

N for $\sigma(\theta)$, $P(\theta)$, $D(\theta)$, $R(\theta)$, $A(\theta)$;

$N-P$ for $\sigma(\theta)$, $D(\theta)$, $R(\theta)$, $A(\theta)$;

$N-D$ for $\sigma(\theta)$, $P(\theta)$, $R(\theta)$, $A(\theta)$;

etc.

$N+C_{nn}$ for $\sigma(\theta)$, $P(\theta)$, $D(\theta)$, $R(\theta)$, $A(\theta)$, $C_{nn}(\theta)$;

etc.

The total cross section with unpolarized beam and target is denoted by σ_T. In the listing just made the θ-dependent quantities are supposed to be available every 10° in θ in the following ranges:

$\sigma(\theta)$ in $10°\leq\theta\leq170°$;

$C_{nn}(\theta)$, $P(\theta)$ in $50°\leq\theta\leq150°$;

$D(\theta)$, $R(\theta)$, $A(\theta)$, $R'(\theta)$, $A'(\theta)$ in $10°\leq\theta\leq170°$.

TABLE III. Uncertainties in K_0 and $T=0$ phase parameters, expressed in radians, in mock-up of experiments. Neutron energy $=20$ MeV; phase search—type A.

Input data	Accuracies	ΔK_0	$\Delta^3\theta^S{}_1$	$\Delta\rho_1$	$\Delta^3\theta^D{}_1$	ΔK_1	Δ^1r_0 %	Row
N	N	0.0276	0.038	0.087	0.0029	0.016	7.0	1
$N+C_{nn}$	N	0.0272	0.032	0.015	0.0029	0.010	6.9	2
$N-A$	N	0.0306	0.043	0.128	0.0029	0.017	7.8	3
$N+\sigma_T$	N	0.0191	0.031	0.087	0.0029	0.015	4.9	4
$N+\sigma_T-P$	N	0.0207	0.045	0.090	0.059	0.019	5.3	5
$N+\sigma_T-R$	N	0.0203	0.034	0.093	0.0029	0.016	5.2	6
$N+\sigma_T-D$	N	0.0278	0.050	0.130	0.0029	0.017	7.1	7
$N+\sigma_T-A$	N	0.0231	0.037	0.128	0.0029	0.017	5.9	8
$N+\sigma_T+R'$	N	0.0189	0.031	0.085	0.0029	0.015	4.8	9
$N+\sigma_T+A'$	N	0.0190	0.031	0.080	0.0029	0.015	4.8	10
$N+\sigma_T+C_{nn}$	N	0.0190	0.027	0.011	0.0029	0.010	4.8	11
$N+\sigma_T$	$\frac{1}{3}(P, D, R, A)$	0.0143	0.042	0.036	0.0011	0.014	3.6	12

Input Accuracies

N for $\Delta\sigma(\theta)=4\%$, $\Delta D(\theta)=\Delta R(\theta)=\Delta A(\theta)=0.070$, $\Delta P(\theta)=0.009$, $\Delta R'(\theta)=\Delta A'(\theta)=0.14$, $\Delta C_{nn}(\theta)=0.02$, $\Delta\sigma_T=0.81\%$ at 10 MeV, 0.61% at 20 MeV;

$\frac{1}{3}(P, D, R, A)$ for $\Delta D(\theta)$, $\Delta P(\theta)$, $\Delta R(\theta)$, $\Delta A(\theta)$ at $\frac{1}{3}$ of above values etc.

Phases Supposed Searched

A for ${}^3\theta^S{}_1$, ρ_1, K_1, ${}^3\theta^D{}_1$, K_0;

B for ${}^3\theta^S{}_1$, ρ_1, K_1, ${}^3\theta^D{}_1$, K_0, ${}^3\delta^P{}_0$, ${}^3\delta^P{}_1$, ${}^3\theta^P{}_2$, ρ_2, ${}^3\theta^F{}_2$.

The input inaccuracies are mostly the result of discussions with Professor C. W. Drake and Professor L. C. Northcliffe and partly that of consulting the literature. They are believed to correspond to experimental runs of reasonably normal length. Correspondence with Dr. Roger B. Perkins of the Los Alamos Scientific Laboratory indicates that in his opinion (1965) the standard errors used above for $P(\theta)$ and $D(\theta)$ in n–p scattering may be difficult of attainment, and that measurements with twice that standard error would be much easier. Since measurement techniques are constantly improving, since the plan outlined does not depend exclusively on these two observables and since polarized targets are offering new possibilities, it appeared worthwhile to describe the results described in the tables that follow.

In Table III are collected values of phase parameter uncertainties obtained for type A phase parameter searches for measurements at 20 MeV. The phase parameters used in the calculations were those for YLAN4M. The table shows that by varying the input data the accuracy of the determination of one or another phase can be improved. In particular row 5 of the table shows that $P(\theta)$ is important for ${}^3\theta^D{}_1$; rows 2 and 11 that C_{nn} is helpful in determining the coupling parameter ρ_1; comparison of rows 1, 2, 3 with the other shows the value of σ_T for the determination of K_0. The column marked Δ^1r_0 gives an approximate estimate of the inaccuracy in the effective range. This is made on the same basis as in Table II but going to the limit of $\eta=e^2/\hbar v=0$. Table III shows that it is difficult to decrease the inaccuracy in ${}^1r_0$ below 5% with the assumed input inaccuracies. If, however, the inaccuracies in P, D, R, and A are all decreased by a factor 3, the inaccuracy in ${}^1r_0$ decreases to 3.6%, as seen in row 12. A further decrease in the inaccuracy can be probably obtained by the addition of R', A', and C_{nn} as indicated by rows 9, 10, and 11.

Since $\sigma(\theta)$ is an easier quantity to measure, the effect of varying $\Delta\sigma(\theta)$ is illustrated in Table IV which has been computed for the type B phase-parameter search. This type of search corresponds to a mistrust of charge independence not only for the ${}^1S_0$ state but also for the ${}^3P_0$, ${}^3P_1$, ${}^3P_2$, and ${}^3F_2$ states. The accuracy of the $T=0$ parameter suffers, of course, as a result of an increase in the number of adjustable parameters. The over-all decrease in the inaccuracies by the factor 3 in going from row 1 to row 8 of the last table is a consequence of the decrease of all of the input errors by that factor. The more instructive information in the table is the value of accurate $\sigma(\theta)$ measurements shown by it. Thus the gain in going from row 1 to row 4 is greater than that in going from row 1 to row 5. It is probable that the former of these is easier to achieve than the latter. Comparison with Table III shows that further increases in accuracy may be expected if σ_T, C_{nn} and the other triple scattering parameters are measured as well.

All of the Δ^1r_0 for p–p and n–p listed above do not

TABLE IV. Uncertainties in K_0 and $T=0$ phase parameters, expressed in radians, in mock-up of experiments. Neutron energy = 20 MeV; phase search—type B. Input accuracies type N, expect for items indicated.

Input data	Accuracies	ΔK_0	$\Delta^3\theta^{S}_1$	$\Delta\rho_1$	$\Delta^3\theta^{D}_1$	ΔK_1	Δ^1r_0 %	Row
N	N	0.0290	0.105	0.108	0.003	0.028	7.4	1
N	$\Delta\sigma(\theta)=2.8\%$	0.0234	0.094	0.107	0.003	0.025	6.0	2
N	$\Delta\sigma(\theta)=1.7\%$	0.0191	0.083	0.106	0.003	0.023	4.9	3
N	$\Delta\sigma(\theta)=1.33\%$	0.0179	0.079	0.105	0.003	0.022	4.6	4
N	$\frac{1}{3}(P, D, R, A)\Delta\sigma(\theta)=4\%$	0.0243	0.049	0.036	0.001	0.014	6.2	5
N	$\frac{1}{3}(P, D, R, A)\Delta\sigma(\theta)=2.8\%$	0.0176	0.044	0.036	0.001	0.012	4.5	6
N	$\frac{1}{3}(P, D, R, A)\Delta\sigma(\theta)=1.7\%$	0.0115	0.038	0.036	0.001	0.010	2.9	7
N	$\frac{1}{3}(P, D, R, A)\Delta\sigma(\theta)=1.33\%$	0.00965	0.035	0.036	0.001	0.009	2.5	8

have an absolute meaning but are meant only for the comparison of n–p with p–p values of $^1r_0$.

Although the highest accuracies listed in the simulated experiments are probably not obtainable for some time, some of the more moderate accuracies such as 3 or 5% for $^1r_0$ do not appear out of reach with sufficient attention to the development of experimental techniques. The estimates reported do not take into account the possible presence of systematic errors.

The determination of K_0 for the main Y–IV n–p searches involved making a least-squares adjustment of the singlet and triplet scattering lengths and of the triplet effective range to low energy n–p measurements from 0–5 MeV keeping $^1r_0$ at 2.70 F and excluding the total cross section at 3.204 MeV. The K_0 from the Y–IV p–p searches was corrected for Coulomb and apparent charge independence violation effects by calculating these effects for a modification of the Yale potential. This modification took into account the low-energy p–p measurements used for $(\text{Y–IV})_{p-p}$ and also the Brolley *et al.*[16] measurements at the interference minimum. The $(K_0)_{n-p}$ obtained by applying the two corrections was smoothed below 20 MeV into the low E n–p anchor with osculating contacts at 20 and at 3 MeV. This $K_0(E)$ provided a starting point for preliminary n–p searches with K_0 allowed to vary from 3 to 20 MeV and the $T=0$ phases up to 46 MeV, still maintaining a smooth join at 20 MeV. In succeeding $T=0$ searches K_0 was kept fixed.

TABLE V. Comparison of D values for some n–p data fits with Y–IV data deck.

Phases	Y–IV	Y–IV	Y–IV	AM–IV[a]
$\Delta\chi^2$	No	Yes	No	No
Energies	All	All	>24 MeV	>24 MeV
MAG	Yes	Yes	Yes	No
D	1.253[b]	1.236[c]	1.284[b]	1.414[b]
Number of data in deck	921	921	828	828
Number of measurements	1023	1023	900	900

[a] Same pion–nucleon coupling constant, $g_0^2=13.0$ as in AM work.
[b] For data-deck entries.
[c] For data.

[16] J. E. Brolley, Jr., J. D. Seagrave, and J. G. Beery, Phys. Rev. **135B**, 1119 (1964).

The low E anchor for $^3\theta^{S}_1$ at 3 MeV was obtained by modifying the Yale potential to reproduce modern values of the binding energy of the deuteron and of the triplet scattering length. In the regular $T=0$ searches $^3\theta^{S}_1$ was made to join smoothly the modified Yale potential $^3\theta^{S}_1$ in $0<E<3$ MeV. A number of total cross-section measurements was used at energies below those employed in search YLAN4M.

The searches were made with $T=0$ correction functions applied for $E>3$ MeV. In a number of cases lumped total cross sections employed measurements through a smaller energy interval than in search YLAN4M. The number of measurements used in the new searches was 1023, the number of data in the search deck 921, the number of high-angle p–d measurements

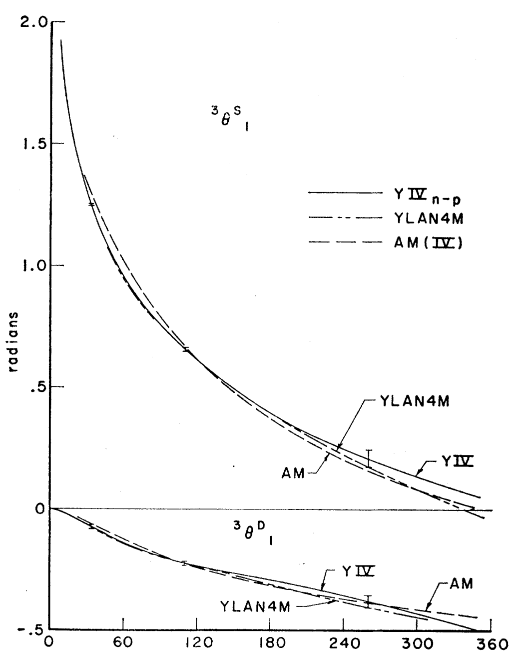

FIG. 9. Comparison of phase parameters $^3\theta^{S}_1$ and $^3\theta^{D}_1$ for $^3S_1$ and $^3D_1$ states, respectively, as obtained in Yale fits Y–IV, YLAN4M and Livermore multiple energy fit AM–IV, AM standing for Arndt and MacGregor. The error bars are parallel shift uncertainties corresponding to Y–IV with factor $D^{1/2}$ included determined for energy intervals (0, 65), (65, 160), and (160, 350) MeV.

that have not been used 29. The p–d measurements were discarded for the searches performed for $\theta > 70°$. It is planned to vary the acceptance criterion for p–d data since the rôle of the spectator proton is not quite clear. The Cromer corrections[2] were applied usually as in the experimental papers. The comparison of D values with previous searches is as in Table V.

In Fig. 9 are shown values of ${}^3\theta^{S}{}_1$ and ${}^3\theta^{D}{}_1$ for YLAN4M, AM–IV, and Y–IV together with parallel shift uncertainties made in the intervals (0, 65), (65, 160), (160, 350) MeV. The differences between fits are not very great. Fig. 10 illustrates the situation for ${}^3D_2$. The differences and uncertainties are larger. In Fig. 11 a similar comparison is shown for ${}^3D_3$ and ${}^3G_3$ with the same general trend regarding increasing lack of definiteness of the phases.

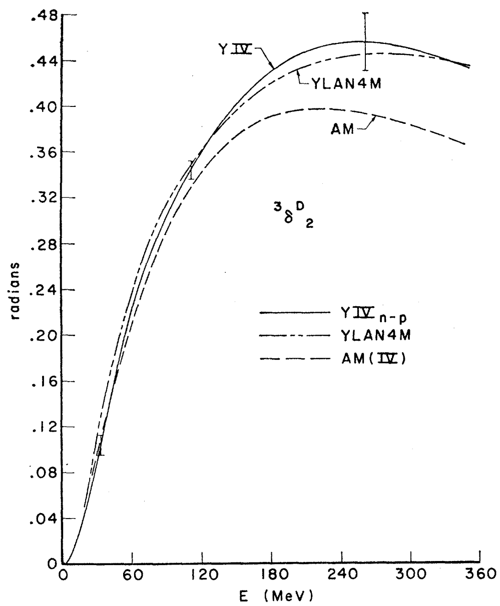

Fig. 10. Comparison of phase shift ${}^3\delta^{D}{}_2$ for state ${}^3D_2$ as obtained in fits Y–IV, YLAN4M, and AM–IV. Conventions as in Fig. 9.

Since the arguments for using $({}^1r_0)_{n-p} = 2.70$ F rather than 2.50 F are concerned only with possibilities and are not binding, the n–p searches for $T=0$ phases were repeated employing ${}^1r_0 = 2.50$ F. The values of K_0 for this search involved the following steps: determination of differences in K_0 that would be caused in the effective range approximation to k cot K_0 by the change in ${}^1r_0$. Smoothing a relatively small wiggle in the plot of the difference of the two K_0 against E at high E. Employing this $K_0(E)$ as a starting point, K_0 was searched together with the $T=0$ phases against n–p data taking care not to change ${}^1r_0$. A slightly lower D was obtained (by about 0.005) in comparison with D for the original K_0. This improvement in quality of fit is not in itself significant. However this search shows that available n–p data in the 3- to 300-MeV region do not speak against the

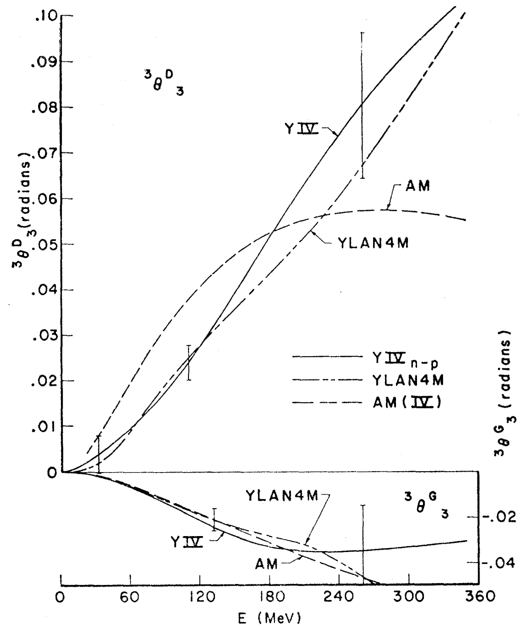

Fig. 11. Comparison of phase parameters for ${}^3D_3$ and ${}^3G_3$ states as obtained in fits Y–IV, YLAN4M, and AM–IV. Conventions as in Fig. 9.

smaller ${}^1r_0$. Searches against n–p data were also carried out keeping ${}^1r_0 = 2.70$ F for variations of $T=0$ phases and of K_0. The readjustment of K_0 in the 100- to 300-MeV region was much the same as when ${}^1r_0$ was kept at 2.50 F. Slight readjustments of $T=0$ phases resulted in both cases. For ${}^1r_0 = 2.50$ F the larger changes expressed in radians were -0.01 for ${}^3\theta^{S}{}_1$ at 100 MeV, 0.05 and 0.03 for ρ_1 and ${}^3\theta^{G}{}_4$, respectively, at 350 MeV. The change for ${}^3S_1$ is about twice the parallel shift uncertainty; those for ρ_1 and ${}^3\theta^{G}{}_4$ are comparable with their respective parallel shift uncertainties.

IV. CHARGE INDEPENDENCE

The electrostatic effects and those of the violation of charge independence for ${}^1S_0$ on N–N scattering analysis which have been investigated by Seamon, Friedman, and Breit[17] made use of searches YRB1(K_0) and YLAN4M. One of the results of applying these corrections was to decrease the difference $(g_0{}^2)_{p-p} - (g_0{}^2)_{p-n}$ to about 15.1−14.8. As previously mentioned[18]

Table VI. Recent values of $g_0{}^2$ from p–p data.

Search	$g_0{}^2$	$g_0{}^2$ for $f^2 = 0.08$
YRB1(K_0)	15.1±0.6	15.5[a] ($m_\pi = m_{\pi^0}$)
Y–IV	15.99±0.62	

[a] 14.5 for $m_\pi = m_{\pi\pm}$; both from $f^2 = g_0{}^2(m_\pi/2M)^2$.

[17] R. E. Seamon, K. A. Friedman, and G. Breit, Phys. Rev. **145**, 779 (1966).

[18] G. Breit, "Two Nucleon Interaction", in *Perspectives in Modern Physics* (Interscience Publishers, Inc., New York, 1966).

Table VII. Recent values of g_0^2 from n–p data.

Corrections[a]	YLAN4M December 1965	YLAN4M April 1966	Y–IV February 1967
α	14.69±1.12	···	13.76±0.74
β	14.83±1.11	14.86±1.12	13.82±0.74
γ	13.98±1.14	14.75±1.1	13.84±0.74

[a] Combined application of Coulomb and charge independence corrections designated by α; of Coulomb corrections alone by β; of no corrections by γ.

this work was vitiated by a programming error which affected the Coulomb uncorrected n–p value. The effect on long-range charge independence was thus too large in Ref. 17. The corrected value of the effect for the YRB1(K_0), YLAN4M fits is 14.9–14.8. The "Coulomb corrected" value was not affected by the error. The changes in the $T=0$ phases are also smaller than in Ref. 17. The largest combined effect was for ${}^3\theta^S{}_1$ near 100 MeV with a change of -0.005 rad which is only slightly smaller than the parallel shift uncertainty. About two thirds of this effect came from the charge independence violation effect. The Coulomb correction alone had an effect of 0.01 rad on K_1 near 350 MeV, about one fourth of the parallel shift uncertainty. The single energy mock searches in Ref. 17 were not affected by the programming error.

Coulomb corrections and short-range charge independence were also used in the Y–IV work. As in Ref. 17 the short-range charge independence correction mainly takes into account the difference in the scattering lengths. The Coulomb corrections arise from the effect of the Coulomb field on the translation of $T=1$ phases from p–p to n–p cases. The Yale potential was used for estimates of both of these primary effects. The changes in $T=1$ phases cause a readjustment of $T=0$ searched phases. The combined effect of the changes in all phases gives rise to an effect on g_0^2.

Table VI shows values of g_0^2 obtained from p–p data employing fits YRB1(K_0) and Y–IV. The uncertainty estimate includes the factor $D^{1/2}$. The two values agree well within the uncertainty of the determination. For comparison there are also shown values of g_0^2 corresponding to the usual $f^2=0.08$. In Table VII are listed the values of g_0^2 obtained from n–p data by means of fits YLAN4M and Y–IV. For the former the published (1965) and the corrected (1966) values are shown. For both YLAN4M versus YRB1(K_0) and for $(\text{Y–IV})_{n-p}$ versus $(\text{Y–IV})_{p-p}$ the most probable values of g_0^2 are smaller as derived from n–p rather than from p–p interactions, but in the older work the difference was within the uncertainty of the determination while in the newer the difference (2.2) is larger than the sum of the estimated standard errors (1.4). Such a difference appears unlikely on purely statistical grounds. It may be an indication of a fundamental limitation in the employment of the pseudoscalar coupling theory. That such effects might be obtainable is suggested by the difference between the values of g_0^2 that correspond to the same f^2 of the pseudovector form of theory with neutral and charged pion masses. But a more prosaic explanation might lie in the presence of effects of two-pion exchange and of vector meson exchange effects on the OPE group of phases used in the g_0^2 determinations. Work is in progress on improved g_0^2 determinations taking these effects into account.

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