SESSION B—PHENOMENOLOGICAL ANALYSIS OF ELASTIC SCATTERING

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Phase-Shift Analysis at Harwell

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Results of recent analyses of proton–proton and neutron–proton data in the 140-MeV region and of proton–proton data at 50 MeV are reported. The existing neutron–proton data in the 25-MeV region are examined and suggestions for experiments are made. It is argued that in fact the available data in this region tell one very little about the interaction.

I. INTRODUCTION

Analysis of nucleon–nucleon scattering has been carried out at Harwell for several years. Most of the work has been done in close collaboration with experimenters, both on the Harwell synchrocyclotron and on the proton linear accelerator at the nearby Rutherford Laboratory. The chief areas of interest have therefore naturally been those accessible with the accelerators—up to 150 MeV with the synchrocyclotron and up to 50 MeV with the linear accelerator. Recent work has included analysis at 140, 50, and 25 MeV. The $T=1$ phase shifts at these energies were already known with reasonable accuracy, but this has been substantially improved; the $T=0$ phases, obtained from the n–p data, are as usual considerably less certain.

II. ANALYSIS AT 140 MeV

The phase shifts in this energy region have been investigated many times now; the excuse for adding yet another analysis to the list is the recent remeasurements[1] of the differential cross section and polarization at Harwell to high accuracy (about $\frac{1}{2}\%$ for the differential cross section and about 0.0035 for the polarization). The selection of the data for the analysis has been considered carefully by Rose, and the resulting set appears to be the most reliable obtainable. Briefly, the p–p data set consists of all available measurements except the old cross-section measurements of Taylor, Wood, and Bird,[2] the n–p set of all the available measurements, excluding those obtained from p–d scattering. The reason for this is that the impulse approximation corrections required are so large as to cast some doubt over the final results. In fact, there are sufficient data left to determine the $T=0$ phases satisfactorily, and the unused data are predicted well.

The details of the analysis will be reported elsewhere. The phase shifts were assumed to vary linearly with energy, the variation being given by the Livermore energy-dependent analysis.[3] The final results are shown in Tables I and II, compared with the analysis of Arndt and MacGregor[3] (using the older data) and of Janout, Kazarinov, and Lehar.[4] It can be seen that the agreement is satisfactory. In this analysis the pion coupling constant has been kept fixed at $g^2=14$, but the data can be used for an attempt to determine g^2. The value found is $g^2=11\pm2$. This seems rather low, and since it depends on the very high partial waves only, it cannot be considered reliable.

The data obtained from p–d scattering are P, R, and A. The polarization does not agree well with the direct n–p measurements (see, for example, Rose[5]), and consequently does not agree with the predictions of this analysis either. The triple-scattering measurements,[6] however, are in excellent agreement with the predictions, as Fig. 1 shows. This can be regarded as a check of the impulse approximation for R and A, although it is not possible to say what is the reason for the discrepancy in P.

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[1] F. G. Cox, J. H. Eaton, O. N. Jarvis, and B. Rose (to be published).

[2] A. E. Taylor, E. Wood, and L. Bird, Nucl. Phys. **16**, 320 (1960).

[3] R. Arndt and M. H. MacGregor, Phys. Rev. **141**, 873 (1966).

[4] Z. Janout, Yu. M. Kazarinov, and F. Lehar, Dubna preprint E1-2953.

[5] B. Rose, Conference on Intermediate Energy Physics, Williamsburg, 1966.

[6] J. Lefrancois, R. A. Hoffman, E. H. Thorndike, and R. Wilson, Phys. Rev. **131**, 1660 (1963); A. Cromer and E. H. Thorndike, *ibid.* **131**, 1680 (1963).

Table I. $T=1$ phase shifts at 140 MeV.

	$^1S_0$	$^1D_2$	$^1G_4$	$^3P_0$	$^3P_1$	$^3P_2$	$\bar{\epsilon}_2$
Harwell	17.58±0.62	5.22±0.22	0.58±0.09	6.74±0.52	−16.61±0.18	13.71±0.12	−2.77±0.09
Dubna	18.04±0.50	5.10±0.17	0.65±0.09	6.51±0.49	−16.80±0.15	13.62±0.10	−2.82±0.08
Livermore	16.78±0.74	4.84±0.27	0.62±0.13	6.66±0.58	−16.85±0.43	13.57±0.22	−2.88±0.16
	$^3F_2$	$^3F_3$	$^3F_4$	$\bar{\epsilon}$	$^3H_4$	$^3H_5$	$^3H_6$
Harwell	0.40±0.31	−1.91±0.20	0.52±0.18	−0.70±0.14	0.18±0.14	−0.38±0.16	0.11±0.09
Dubna	−0.04±0.26	−1.82±0.20	0.40±0.16	−0.66±0.16	0.13±0.12	−0.47±0.13	0.16±0.08
Livermore	0.67±0.32	−2.13±0.22	0.87±0.18	−0.66±0.07	0.44±0.18	−0.62±0.17	0.25±0.11

III. ANALYSIS AT 50 MeV

The p–p data in this region have also been analyzed several times recently, but have always lacked good differential cross-section data. This defect has now been remedied in the recent measurement at the Rutherford Laboratory,[7] which is an order of magnitude more accurate than the Tokyo data.[8] Using these data, one can determine the $T=1$ phase shifts at least as precisely as at 140 MeV. The results are shown in Table III, compared with previous results, using the older data.[3]

The S and P phase shifts are not greatly changed from the previous values, although the errors are considerably reduced. With the higher phase shifts however, there is some significant change. In the early analysis the $^1D_2$ phase and the mixing parameter ϵ_2 altered considerably when ϵ_2 was allowed to vary; the change in ϵ_2 moreover was in the opposite direction to what one would expect from results at higher energies, and the $^1D_2$ phase was also not easily compatible with other results. With the new data, there is no such shift; ϵ_2 and all higher parameters are given satisfactorily by one pion exchange, and the $^1D_2$ phase is in line with neighboring points. These phase shifts agree well with those reported by Hoshizaki at this conference. Some small differences which exist can be removed by a slight change in the normalization of the cross section.

The phase shifts are now so accurately known that there seems to be no motive for further p–p experiments at 50 MeV.

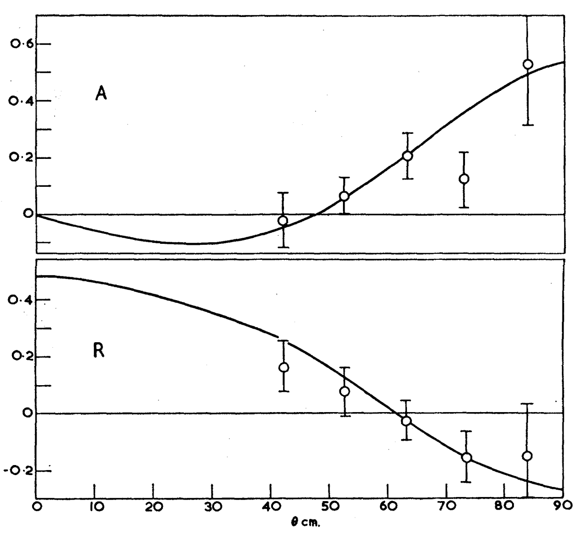

Fig. 1. Neutron–proton triple-scattering parameters near 140 MeV. Comparison of experiment and predictions.

IV. MEAN P-WAVE PHASE SHIFTS

It has been known for a long time that the shape of the Coulomb interference dip in the p–p cross section at low energies can be used to determine the average P-wave phase shift. This is also true at higher energies, where there are often accurate cross-section data avail-

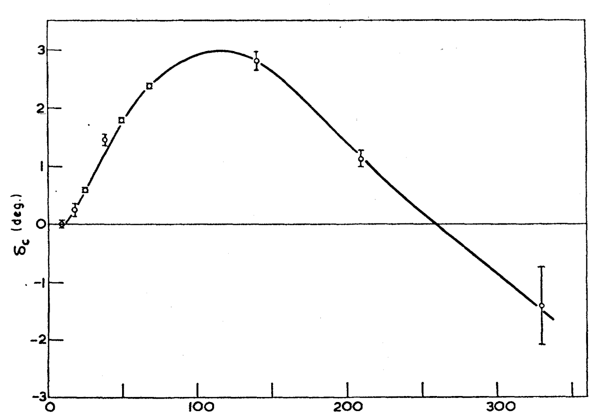

Fig. 2. Mean P-wave phase shift as a function of energy. Below 10 MeV it is negative, but too small to show. The curve is a guide to the eye only.

[7] C. J. Batty, T. C. Griffith, D. C. Imrie, G. J. Lusch, and L. A. Robbins (to be published).
[8] K. Nisimura *et al.*, Tokyo Report INSJ—45 (1961).

Table II. $T=0$ phase shifts at 140 MeV.

	$^1P_1$	$^1F_3$	$^3S_1$	$\bar{\epsilon}_1$	$^3D_1$	$^3D_2$	$^3D_3$	$\bar{\epsilon}_3$	$^3G_3$	$^3G_4$	$^3G_5$
Harwell (9)	−15.36±2.66	−1.76±1.05	30.07±1.12	1.83±1.06	−14.11±0.90	25.11±0.90	1.33±0.73	3.10±0.51	−2.38±0.42	3.23	−0.46
Harwell (11)	−10.73±4.06	−1.16±1.39	31.60±1.16	0.07±0.12	−15.90±1.03	24.37±1.30	−0.25±1.08	2.48±0.54	−2.69±0.63	5.74±1.01	0.13±0.30
Dubna	−11.60±2.35	−3.21±1.15	31.05±1.00	0.78±1.57	−14.45±1.06	26.19±1.01	−0.36±1.30	3.62±1.01	−2.94±0.89	4.98±0.93	−0.19±0.32
Livermore	−15.36±2.14	−0.85±0.77	29.98±0.97	0.92±0.93	−15.00±0.74	23.72±1.20	1.02±0.82	3.31±0.59	−1.99±0.55	4.13±0.83	0.12±0.32

able, but not sufficient other data to make a complete analysis possible. Accurate values have been obtained for

$$\delta_c=(\delta_{10}+3\delta_{11}+5\delta_{12})/9$$

at 10, 18.2, 39.4, and 68.3 MeV in addition to the energies at which complete analyses have been made. The data used include, as well as the cross sections, only a limited amount of polarization and spin correlation data.

The results of the analyses are shown in Fig. 2. It is apparent that δ_c can be determined very accurately. It is interesting to notice how the three regions of nuclear force show up. At low energies there is the weakly repulsive long-range one-pion interaction, then a much stronger shorter-range attraction, followed by the even shorter-range repulsion at higher energies.

V. ANALYSIS AT 25 MeV

Both p–p and n–p data have accumulated rapidly in this region recently. The latest analyses are from Dubna[9] and Livermore.[10] Both these suffer from some defects. In the Dubna analysis data were shifted to a common energy by renormalization, a procedure which is not adequate at this energy. This defect was corrected in the Livermore analysis, but this omitted some useful n–p data, and assumed too high an accuracy for the n–p polarization. The most important data omitted were the recent total cross sections of Grace, Sowerby, and Morris,[11] which are accurate to nearly $\frac{1}{2}\%$ and the differential cross section of Brolley, Coon, and Fowler.[12] This experiment, although published in 1951, is still the most accurate for the backward quadrant.

The present analysis includes these data; the only significant difference in the p–p data is the inclusion of the latest spin correlation parameters from Saclay.[13] Table IV shows that the $T=1$ phases are well-determined, and in good agreement with the Livermore values. There are some differences in the $T=0$ phases, the most important being the difference in the $^1P_1$ phase. This is directly related to the rise in the backward angle cross section, which is only evident in the results of Brolley, Coon, and Fowler. A great deal of significance must be attached to this.

Noyes[14] has argued that the $^1P_1$ phase shift, being repulsive should be close to the one-pion-exchange value, and indeed this is shown by the Hamada–

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[9] Z. Janout, Yu. M. Kazavinov, and F. Lehar, Dubna preprint E1-2952.

[10] R. Arndt and M. H. MacGregor, UCRL 70075.

[11] D. E. Groce, B. D. Sowerby and J. M. Morris, Phys. Letters **17,** 40 (1965).

[12] J. E. Brolley, J. H. Coon, and J. L. Fowler, Phys. Rev. **82,** 190 (1951).

[13] P. Catillon, M. Chapellier, and D. Garreta, International Conference on Nuclear Physics, Gatlingburg, 1966.

[14] H. P. Noyes, Phys. Rev. **130,** 2025 (1963).

TABLE III. $T=1$ phase shifts at 50 MeV.

	$^1S_0$	$^1D_2$	$^3P_0$	$^3P_1$	$^3P_2$	$\bar{\epsilon}_2$	$^3P_2$
New	38.12 ± 0.47	1.81 ± 0.11	11.46 ± 0.36	-8.16 ± 0.21	5.85 ± 0.10	-1.89 ± 0.14	0.35 ± 0.13
Old	37.50 ± 0.78	2.16 ± 0.27	12.08 ± 0.79	-7.98 ± 0.31	6.06 ± 0.02	-2.27 ± 0.36	0.55 ± 0.32
OPE		1.12				-1.82	0.37

Johnston potential[15] (see Fig. 3[3,10,16,17]). The Livermore results and the results obtained at 50 MeV suggest behavior completely unlike this. As it is important to know whether the OPE effect does dominate at low energies, a confirmation of the shape of the backward angle cross section would be a worthwhile experiment.

The comparison between theory and experiment[12,18] is shown in Fig. 4, together with the uncertainty in the predicted curve. In the forward direction the cross section is predicted very accurately, the uncertainty being too small to show. This is because the $T=0$ amplitudes are dominated by the $^3S_1$ phase, and are therefore almost entirely imaginary. The value is therefore closely controlled by the accurately known total cross section. The essential datum to be obtained is the ratio $\sigma(180°)/\sigma(90°)$, which is given as 1.13 ± 0.03 by this analysis.

Of the other parameters, the $^3S_1$ phase, as has been noted, is determined primarily by the total cross section, and so is given with improved accuracy. The comparison of the $^3D_1$ phase is somewhat misleading; in this analysis the $^3D_2$ and $^3D_3$ phases were not varied. In fact only one ^3D parameter can be determined by the present data. This is the spin–orbit splitting, which is obtained from the shape of the n–p polarization.

The polarization is approximately fitted by the expression

$$P d\sigma/d\Omega = \sin\theta(\alpha+\beta\cos\theta).$$

The two parameters are known to about 10%. In terms of the Wolfenstein parameters

$$P d\sigma/d\Omega = 2\,\mathrm{Re}\,C^*N.$$

The phase shifts appearing in C are small, so that C is almost pure imaginary, and the imaginary part of N is again dominated by the $^3S_1$ phase. Hence α will be determined by the spin-orbit splitting of the ^3P waves, and β by the spin-orbit splitting of the ^3D waves. In fact

$$C \propto \sin\theta(\Delta_p+5\Delta_D\cos\theta),$$

where Δ_p and Δ_D are expectation values of the spin–orbit interaction in P and D states. Any spin–orbit effect in the D waves is thus greatly magnified.

Our knowledge of the P-wave splitting could obviously be improved by an improvement in our knowledge of the normalization of the polarization. The error on this is about 10%, which is about the same as the error on Δ_p determined from the phase shifts. The

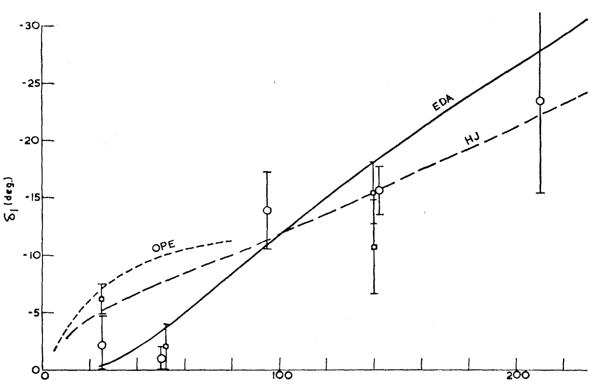

FIG. 3. $^1P_1$ phase shift as a function of energy. Circles—points from Refs. 3, 10, 16; squares—points from this paper and Ref. 17. EDA—Livermore energy-dependent analysis.[3] HJ—Hamada–Johnston potential.

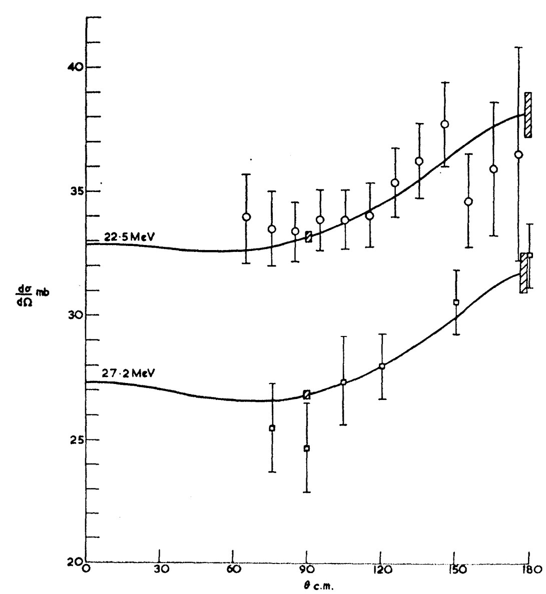

FIG. 4. Neutron–proton differential cross section. Experimental points from Ref. 18 (22.5 MeV and Ref. 12 (27.2 MeV). Shaded rectangles denote errors on predicted curves.

[15] T. Hamada and I. D. Johnston, Nucl. Phys. **34**, 382 (1962).
[16] R. M. Wright, M. H. MacGregor, and R. A. Arndt, UCRL-70075 (Part 6).
[17] C. J. Batty and J. K. Perring, Nucl. Phys. **59**, 141 (1964).
[18] E. R. Flynn and P. G. Bendt, Phys. Rev. **128**, 1268 (1962).

TABLE IV. phase shifts at 25 MeV.

	$^1S_0$	$^1D_2$	$^3P_0$	$^3P_1$	$^3P_2$	$^1P_1$	$^3S_1$	$\bar{\epsilon}_1$	$^3D_1$
Harwell	48.58±0.22	0.64±0.03	8.07±0.40	−4.94±0.17	2.43±0.10	−6.21±1.28	84.03±1.57	0.98±1.12	−3.05±0.22
Livermore	48.68±0.28	0.87±0.09	7.83±0.59	−4.80±0.28	2.38±0.12	−2.06±2.66	79.48±4.61	−1.58±2.35	−3.53±4.26

TABLE V. N–P phase shifts at 25 MeV.

	$^1S_0$	$^1D_2$	$^3P_0$	$^3P_1$	$^3P_2$	$^1P_1$	$^3S_1$	$\bar{\epsilon}_1$	$^3D_1$
C.I.	48.58±0.22	0.64±0.03	8.07±0.40	−4.94±0.17	2.43±0.10	−6.21±1.28	84.03±1.57	0.98±1.12	=3.05±0.22
P–P corrected	50.53	0.76	9.57	−5.57	2.70	−5.82±1.36	80.84±1.09	1.48±1.16	−3.11±0.19

requirement here is for a better knowledge of the n–He^4 polarization which is used to measure the neutron beam polarization.

All the discussion hitherto has assumed charge independence in its simplest form; i.e., that the same $T=1$ phase shifts can be used for both p–p and n–p scattering. This is well-known to be untrue at zero energy, but there is no information whatever on how accurate it is at 25 MeV on higher energies, except that the data are consistent with the assumption. One can gain some idea of how big the discrepancy might be by looking at the predictions of effective range theory for the $^1S_0$ phase shift. Assuming that only the scattering lengths differ, one finds

$$\delta_{np}-\delta_{pp}\sim 2°.$$

If the n–p effective range is smaller than the p–p, as is suggested by Noyes,[14] the difference is larger.

For the higher phase shifts one can only suppose that

$$\delta_{pp}\simeq C^2\delta_{np},$$

where $C^2=2\pi\eta/(e^{2\pi\eta}-1)$ is the usual Coulomb penetration factor. This is 0.90 at 25 MeV, and so again makes a noticeable difference.

An attempt has been made to take these corrections into account in the analysis. The p–p data can be analyzed alone, and the resulting phase shifts corrected by the methods above for use in the n–p analysis. The results are shown in Table V. It can be seen that only the $^3S_1$ phase is affected appreciably. Presumably this is a result of the increase in the $^1S_0$ phase; the $^3S_1$ phase must be decreased in order to keep the total cross-section constant. Apart from this, it will make little difference to an analysis what assumption is made about charge independence.

As a final point it is interesting to ask how much we are actually learning from the analysis of the n–p data. If there were no n–p data, it would be necessary to make some assumptions about the phase shifts. Naturally one would take the $^3S_1$ phase from effective range theory, ϵ_1 from the Wong theory,[19] and all the other phases from one-pion exchange. The fit to the data with these assumptions is quite good, giving a χ^2 of 72 for 50 data. This can be reduced substantially by searching on the $^3D_1$ phase, in order to fit the polarization (see Table VI). The resulting χ^2 of 45 is perfectly satisfactory statistically. However, about 10 of this comes from one of the total cross-section points; varying the $^3S_1$ phase reduces this. The $^1P_1$ phase has been varied throughout, since the search procedure does not work with only one parameter. However, it does not move significantly from the one-pion value. Allowing ϵ_1 to vary gives no appreciable improvement.

Thus the data really only give two useful pieces of

[19] D. Y. Wong, Phys. Rev. Letters **2**, 406 (1959).

Table VI. $T=0$ phase shifts at 25 MeV.

Varying	$^1P_1$[a]	$^3S_1$	$\bar{\epsilon}_1$	$^3D_1$[b]	χ^2 (50 points)
$^1P_1$, $^3D_1$	-7.19 ± 0.45	77.95	2.35	-3.16 ± 0.19	45.2
$^1P_1$, $^3S_1$, $^3D_1$	-6.75 ± 0.47	80.20 ± 0.72	2.35	-3.10 ± 0.19	33.5
$^1P_1$, $^3S_1$, $\bar{\epsilon}_1$, $^3D_1$	-5.82 ± 1.36	80.84 ± 1.09	1.48 ± 1.16	-3.11 ± 0.19	32.9

[a] $^1P_1$ (OPE) $=-7.02$.

[b] $^3D_1$ (OPE) $=-2.19$.

information, that there is a significant spin–orbit effect in the D waves, and that the $^3S_1$ phase is slightly different from the effective range value. The conclusion to be drawn is that one must be careful not to infer too much from n–p data; this is especially true at higher energies, when one cannot reasonably avoid using more parameters than are justified by the data. The values of χ^2 are often very much smaller than the expected statistical value, so that it may be possible to change a parameter by several times its standard deviation and still obtain a good fit.

Finally, it should be emphasized that a great deal of more accurate n–p data will be required if good values of the phase shifts are to be obtained.