Possibilities of Observation of Nonlinear Quantum Electrodynamic Effects in Vacuum

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Experiments that can reveal the nature of the second- and the fourth-order terms in the interaction between charged particles and electromagnetic fields in vacuum have been discussed here. An attempt has been made to emphasize the successes of the experiments already completed and the promising lines of future investigation. The theoretical considerations relevant to the experiments under review have also been outlined. In all cases studied so far, the experimental results agree with the predictions of conventional quantum electrodynamics.

I. INTRODUCTION

A long time has elapsed since the original suggestion of Delbrück[1] that the scattering of light by a nuclear Coulomb field is a particularly striking manifestation of the nonlinear electromagnetic effects in the vacuum. Bremsstrahlung radiation[2] from accelerated electrons and proton- or neutron-capture gamma rays[3,4] have been used in recent years as very intense sources of photons for the successful observation of Delbrück scattering. Different types of experiments[5–7] have been suggested for an experimental investigation of the closely related phenomenon of photon–photon scattering, originally considered by Vavilov[8] and Halpern,[9] and calculated in detail in the early days by various authors.[10–12] Important developments have also occurred on the theoretical side. By including in the field Lagrangian terms biquadratic in the fields, Weisskopf[13] showed early that classical electromagnetic theory can be modified to include these nonlinear effects. In terms of the S-matrix approach and with the help of the then recently discovered techniques of Feynman, photon–photon scattering was calculated in great detail, although not in completely closed form, by Karplus and Neumann.[14] At about this time, Schwinger[15] developed the gauge-invariant Lagrangian treatment for the calculation of higher-order electromagnetic effects including vacuum polarization, which can be extended to different problems. Since there is a great variety of experiments which in principle can be done in this field and since the theory, though basic, is rather involved, it was felt that a concise report on the comparative merits of the different approaches would be fruitful in highlighting the extent of the progress already achieved and the really promising lines of future investigation.

II. NONLINEAR ELECTROMAGNETIC EFFECTS OF THE LOWEST ORDER AND IN THE VACUUM

Effects involving the nonlinear electromagnetic behavior of the vacuum in the lowest order are: (a) the energy-level shifts of hydrogenic and muonic atoms,[16–19] arising from radiative corrections such as vacuum polarization and self-energy effects of the lowest order, (b) the lowest-order term in the anomalous magnetic moments[20–23] of the electron and the muon, (c) double

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Compton scattering of gamma rays,[24–29] (d) photon-splitting[30,31] and (e) the intensity-dependent frequency shift in and corrections to the cross sections for Compton scattering by free electrons.

The presence of the lowest-order radiative corrections has been confirmed to an impressive over-all accuracy of about 1 part in 10^{10} in the case of atomic hydrogen[17] and about 1 part in 10^5 in the case of muonic calcium and titanium.[18] The self-energy effect decreases the binding energy and is the dominating contribution in the case of ordinary atoms. But for muonic atoms, the vacuum polarization term is the dominant one. The latter increases the binding energy and is larger by several orders of magnitude in the case of muonic atoms on account of the greater mass of the muon and the smaller size of muonic atoms. However, the experimental accuracy for the determination of energies of muonic atom states is as yet considerably poorer than that possible in the case of ordinary atoms. The magnetic moments of the free electron and the muon have been determined[21–23] to accuracies of about 2 parts in 10^5 and 5 parts in 10^6, respectively. Thus, the $\alpha/2\pi$ term (where α is the fine structure constant and $1/\alpha$ is equal to 137.0391 ± 0.0012)[32,33] in the theoretical expression for the g-factor anomalies of free point electrons and muons has been confirmed to about 3 parts in 10^5 and 1 part in 10^5, respectively.

Double Compton scattering, a third-order process in which a photon interacts with an electron and produces two scattered photons, has been experimentally established by Boekelheide and Cavanagh independently. For about one-MeV incident photon energy, energy distribution of the scattered gamma rays was determined by Bracci *et al.* With the two scattered photons making an angle of 90° with the incident gamma ray and with each other, the differential cross section turned out to be about 4×10^{-30} cm^2/sr^2. Theoretically, the double Compton scattering cross sections are expected to be about α times the single scattering cross sections. The photon splitting effect in the nuclear Coulomb field is similar except that, in this case, an electron is replaced by a heavy nucleus of charge Ze, where Z is the atomic number, and the cross section becomes of order $Z^2\alpha^3$. Since the energy loss to the nucleus is much smaller than the corresponding loss to the electron in double Compton scattering, it should be possible to select the $\gamma-2\gamma$ events by means of energy discrimination applied to the outgoing gamma rays. Further, since the nuclear $\gamma-2\gamma$ cross section is expected to increase roughly as the sixth power of the energy, even a modest twofold increase in the incident gamma energy to about 2.6 MeV (such as that of ThC″ gamma rays) should lead to a substantial increase in the possibility of detection of the $\gamma-2\gamma$ effect.

The lowest-order frequency shift[34–36] in the Compton scattering of photons by free electrons is given by Eq. (1).

$$\frac{\lambda'-\lambda}{\lambda}=\frac{v}{c}\frac{\cos\delta-\cos\beta}{1-(v/c)\cos\delta}+\frac{\lambda_c}{\lambda}\frac{2[1-(v^2/c^2)]\sin^2\frac{1}{2}\theta}{1-(v/c)\cos\delta}+\frac{\mu^2[1-(v^2/c^2)]\sin^2\frac{1}{2}\theta}{[1-(v/c)\cos\delta]^2},\qquad(1)$$

where λ' is the wavelength of the radiation after scattering through the angle θ, λ is the wavelength of the incident radiation, δ and β are the angles between the direction of motion of the electron of speed v before the collision and the directions of the incident and the scattered photons, respectively, λ_c is the Compton wavelength h/mc, m is the mass of the electron, h is Planck's constant, c is the velocity of light in vacuum, and μ^2 is a parameter related to the intensity I and wavelength λ of the incident radiation through the relation $\mu^2=r_0\lambda^2I/\pi mc^3$, where r_0 is the classical radius e^2/mc^2 of the electron and e is the charge of the electron. The relations between the various angles are exhibited in Fig. 1. In Eq. (1), the first term represents the Doppler effect, the second term the Compton effect, and the third term is the intensity-dependent shift. This equation can be reduced to[37,38] simpler forms for electrons initially at rest. If electron kinetic energies can be kept down to fractions of an electron volt, then μ^2 has to be $\simeq10^{-3}$ at least for the effect to be observed

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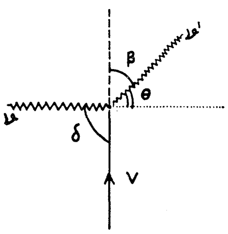

FIG. 1. Compton scattering of an incident photon of frequency ν by an electron moving with a velocity V before the scattering ν' is the frequency of the photon scattered through the angle θ. δ and β are the angles between the direction of motion of the electron before scattering and the directions of the incident and the scattered photons, respectively. The direction of electron motion after scattering is not shown.

with x rays of a few keV energy and $\simeq 10^{-6}$ at least for observations with optical radiation. Further, since μ^2 is proportional to λ^2, the effect is best sought with radiation of as long a wavelength as practicable. With lasers, extremely high intensity photon beams are available in the optical range of wavelengths. Therefore, I should then be of the order of 10^{13} W/cm^2 or about 10^{31} photons/cm^2 sec. Such intensities may become obtainable only at the focal spots of pulsed lasers. However, in the case of focused beams, a large range in the values of δ has to be considered, resulting in a considerable uncertainty in the Doppler shift. The intensity-dependent shift is always positive, a fact which may be utilized in its identification. However, in any case, μ^2 will have to be of the order of or greater than v/c for the Doppler shifts to be small in comparison with the intensity-dependent shifts. Thus, it will be necessary to restrict the permissible values of velocities of the electrons. The experiment will have to be done with genuinely free electrons in vacuum. Bound electrons may not be able to acquire the nonzero average velocities in the incident photon direction, that are necessary from the point of view of a classical explanation of the effect.[35] Secondly, the almost inevitable presence of positive ions in any feasible experiment will complicate the interpretation through the production of additional frequency-shifted components.[39] There is the additional difficulty that theoretical expressions really valid in the case of an experiment involving focused optical beams have not been developed as yet. A modification of the details of this experiment along the lines of the Kapitza–Dirac experiment[40–42] has also been proposed, in which a 10- to 1000-eV electron beam will be directed through a switched laser cavity. Such an experiment is also very difficult on account of the extreme requirements regarding collimation and monochromaticity of the electrons. Similarly, before the intensity-dependent corrections[43] to the scattering cross sections of free electrons for electromagnetic radiation can be established, the experimental accuracy will have to improve by several orders of magnitude over that presently available.

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III. NONLINEARITY EFFECTS OF THE NEXT HIGHER ORDER

Nonvanishing effects of this type but of the next higher order, that is fourth-order effects, are (f) fourth-order radiative corrections[44] to the energy levels of hydrogenic and muonic atoms, (g) fourth-order terms in the anomalous magnetic moments of the electron and the muon,[45–47] (h) scattering of light by light,[5–14] (i) scattering of light by the static electric field of the nucleus (Delbruck scattering),[1–4] (j) pair production in photon–photon collisions,[48,49] (k) dichroism and birefringence[50] of the vacuum, and (l) double photon decay.[51,52]

Fourth-order corrections are absolutely necessary to explain the total observed difference in energy (expressed in units of h) between $2\,^2S_{\frac{1}{2}}$ and $2\,^2P_{\frac{1}{2}}$ levels of hydrogen of 1057.77±0.10 Mc/sec and contribute terms of the order of 1 Mc/sec. However, on account of the complexity of the calculations and the neglect of the detailed structure of the nuclei, the calculations are uncertain[53] to terms of order 0.1 Mc/sec in any case. There is an over-all discrepancy of about 0.2 Mc/sec between theoretical calculations and the experimental results, for which there is as yet no definitive explanation. The latest measurements of Robiscoe *et al.* actually increase the discrepancy to about 0.4 Mc/sec. Similar measurements[54] of the $3\,^2S_{\frac{1}{2}}-3\,^2P_{\frac{1}{2}}$ energy difference for singly charged helium confirm the over-all correctness of the theory, although in this case the experimental accuracy is so far about two orders of magnitude poorer. The accuracies presently attainable with lithium-drifted germanium detectors[19] and bent crystal spectrographs[18] for muonic x-ray energy determinations will have to improve by at least an order of magnitude before fourth order corrections to the energy levels of muonic atoms can be experimentally established. The presence

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of the $-0.328\ \alpha^2/\pi^2$ term in the expression for the g-factor anomalies of the free point electron and the muon is confirmed to an accuracy of about 1 percent.[21–23]

Effects under consideration here involve contributions from diagrams having closed electron loops and are thus particularly suitable for the verification of certain specific features of relativistic quantum electrodynamics. Various proposals have been made for an experimental study of photon–photon scattering. Mckenna and Platzman[5] conclude that it will be difficult to observe photon–photon scattering with the help of only two laser sources.[55] So they consider, in a low-energy approximation, the problem of photon–photon scattering in the presence of a large external static electric field, which can act as a source of virtual photons. They checked the accuracy of their calculation by considering the limit of vanishing external field intensity, in which case the cross sections are well-known through the extensive work of Karplus and Neumann. They find that the transition rates are still too small to be measurable. Rosen and Whitmore[7] consider the possibility of producing two properly directed and time-synchronized bursts of x-ray flash bulbs so that about 10 to 20 photon–photon scattering events may take place per flash. However, the experimental requirements for the unambiguous identification of these events are so severe that it is difficult to see how the experiment will succeed at the present time. Harutyunian *et al.*[6] recognize that lasers provide the largest available photon densities and that the cross section for photon–photon scattering in the low-energy limit increases as the sixth power of the energy in the center of mass system. So they propose an experiment in which bremsstrahlung photons of about 6-BeV energy from a future 10 or more BeV electron accelerator are to be scattered by the 1.78-eV photons from an intense pulsed ruby laser, the energy in the center of mass system amounting to about 100 keV. However, the cross section, which will be strongly peaked around the incident high-energy photon direction, is still expected to approach only about 10^{-35} cm^2/sr^2. Thus, with the available photon sources, only small counting rates of the order of 2 or 3 per day will result.

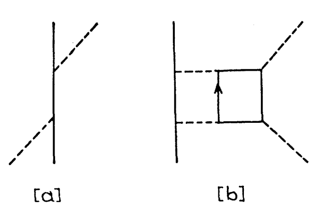

FIG. 2. Feynman diagrams for (a) Compton scattering by a nucleus and (b) one of the radiative corrections.

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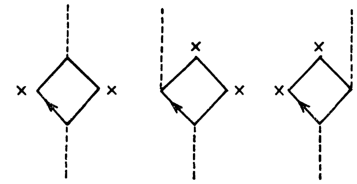

FIG. 3. Feynman diagrams for nonvanishing contributions of the lowest order to Delbrück scattering. The symbol × indicates the static electric field of a nucleus of charge Ze. Three similar diagrams with the reversed sense of the arrow are not shown.

With Feynman diagram techniques, it can be easily shown[56] (Figs. 2 and 3) that Delbrück scattering may be regarded as a radiative correction to the Compton scattering of an incident photon by the nucleus. The lowest-order nonvanishing contribution to Delbrück scattering is given by the three diagrams of Fig. 3 and three similar ones with the sense of the arrow reversed. The similarity between these diagrams and the diagram in Fig. 2(b) for radiative correction to the Compton scattering is evident. Since each diagram has two vertices at each of which the effective charge is Ze, one gets an enhancement of the cross sections through the factor $Z^4\alpha^2$. Thus, for a heavy nucleus, such as lead, the Delbrück scattering cross section is about 1600 times larger than that for photon–photon scattering in the MeV region of photon energies and therefore becomes measurable. The Delbrück amplitude consists of a real, that is dispersive, and an imaginary, that is absorptive, part, the former arising from purely virtual intermediate electron states in the nuclear field and therefore being closely related to the interesting effect of polarization of the vacuum. The imaginary part is related to the corresponding inelastic process, namely pair production. After the initial suggestion of Delbrück, high- and low-energy-limit calculations were done by several authors.[57,58] Forward scattering cross sections were evaluated by Rohrlich and Gluckstern[59] in an analytically closed form valid to the lowest order of the Born approximation. Estimates of the contributions of the higher-order Born terms[60] and of the screening[61] of the nuclear Coulomb field by the atomic electrons have been made. For gamma-ray energies of a few MeV, the first effect is about 10 percent and the second negligible. The imaginary Delbrück amplitude has been shown,[62,63] in the lowest-order Born approximation, to be quite small in comparison with the real part

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up to a few MeV energy. However, just above the threshold of 2 mc^2, it increases roughly as the cube of the energy and has been verified at 9 MeV,[4] 17 MeV,[3] and 87 MeV.[2] The latter two measurements, made at angles considerably less than a degree, are in agreement with the results of the theoretical calculations.[64,65] Calculations of Sannikov[66] indicate an appreciable real part at large scattering angles even for energies of the order of 10 MeV but are in contradiction with the experimental results at 9 MeV. The angular distribution of the real part for large angles and energy below the pair-production threshold is expected[67] to have the characteristic dipole form. In the work of Ehlotzky and Sheppey,[68] which is based on a fixed angle dispersion relation, the Delbrück amplitude is evaluated for energies between 1 and 20 MeV and angles between 0° and 120°. The resulting cross sections at 9 MeV are, if the real part is completely neglected, slightly smaller than the corresponding experimental values between 20° and 30°. However, this small disagreement cannot be considered a decisive verification of the real part of the Delbrück amplitude, especially since inelastic scattering events and bremsstrahlung from secondary electrons influence the interpretation of experimental data in a marked way. Contrary to some of the earlier conclusions, recent experiments in the large angle region at 0.662 MeV,[69–71] about 1.11 MeV,[72–74] 1.17 MeV and 1.33 MeV,[70,72,75,76] and 2.62 MeV [70,75,77,78] do not lead to any definite conclusion regarding the presence of Delbrück scattering. Of course at 0.662 MeV, the imaginary Delbrück amplitude is zero and as remarked earlier, at the other energies, it is expected to be quite negligible. One of the major difficulties in the identification of Delbrück scattering is that it is coherent with other elastic processes such as Rayleigh scattering from the bound electrons in an atom. The latter has been calculated exactly[79] only for large scattering angles and for the K-shell electrons of mercury at 0.32 mc^2, 0.64 mc^2, 1.28 mc^2, and 2.56 mc^2 energies. Rayleigh scattering cross sections for large angles in the case of gamma rays of energies around 1 MeV are expected to be at least an order of magnitude larger than the corresponding Delbrück cross sections. Therefore, in addition to exact theoretical calculations, experiments with an absolute accuracy of a percent or better are absolutely necessary for a clear demonstration of the dispersive Delbrück amplitude at large angles. The total elastic scattering cross sections are quite large and easy to measure in the small angle region. However, the momentum transfer to the electron during scattering is then small and so $L,M\cdots$shell binding effects in Rayleigh scattering become important. The best theoretical estimate of these effects, valid in the small angle region,[80] is based on nonrelativistic form-factor calculations and has been shown to be quite inadequate[81,82,83] for an unambiguous interpretation of the experimental results. Thus elaborate relativistic calculations incorporating the effects of electrons in the less bound $L,M\cdots$shells will have to be done before the small angle data at energies of a few MeV can reveal the presence of the dispersive term in the Delbrück amplitude. The analytic properties of scattering amplitudes in general imply a relation between the real and the imaginary parts of the Delbrück amplitude. Since the imaginary part has been demonstrated through the high-energy photon experiments, the real part may also be considered as having been indirectly established. However, an independent and direct check will be very desirable.

The cross section for pair production in photon–photon collisions is actually about $(1/\alpha)^2 \simeq 10^4$ times larger than the photon–photon scattering cross section and is thus in the low-energy limit comparable to the single Compton scattering cross section per electron. The cross section increases slowly from the threshold at an energy of mc^2, reaches a maximum and then decreases as $(mc^2/w)^2[\log(2w/mc^2)-1]$, where w is the photon energy. Therefore, this process is quite likely to be observed in the synchronized x-ray flash bulb experiment[7] metnioned earlier. From the unitarity of the S matrix, it follows that the total cross section $\sigma(w)$ in photon–photon collisions is given by $(4\pi\hbar c/w)a_2(w, 0)$, where $a_2(w, 0)$ is the imaginary part of the photon–photon scattering amplitude at energy w in the forward direction. Since the cross section $\sigma(w)$ may be approximated by the pair-production cross section alone and the real part $a_1(w, 0)$ of the photon–photon scattering amplitude in the forward direction can be obtained from $a_2(w, 0)$ by analytic

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continuation, the forward photon–photon scattering cross section can be derived from the total pair-production cross section. Thus an independent check of the accuracy of the theoretical work of Karplus and Neuman pertaining to photon–photon scattering becomes available. There is a possibility of an indirect astrophysical check of the phenomenon of photon–photon pair production. Recently, intense microwave cosmic radiation of 7.3- and 3.2-cm wavelength has been detected[84,85] and attributed to extraterrestrial sources. This has been interpreted[86,87] as the expansion red-shifted remnant of blackbody emission from a very early stage of the universe corresponding to an optically thick gas of electrons, positrons, photons and nucleons at 10^{10} °K. During the expansion of the universe, the radiation is believed to retain its blackbody character while being adiabatically cooled to its present value of 3.5°K. At this temperature, the expected average number density of photons in the universe is around 10^3 per cc. If there are any high energy photons in the primary cosmic radiation, they will interact with this target of low energy photons of cosmic dimensions. It has been estimated[49] that, on account of the pair-production in photon–photon collisions, photons of energy greater than about 10^{14}eV are unlikely to be present in the primary cosmic rays. If such a prediction is borne out by future experiments, it will be an indirect test of the general validity of these ideas.

The electric fields required for the observation of the birefringence and the dichroism of the vacuum, that is differences in refractive indices and absorptions per unit path in the vacuum for light beams polarized parallel and perpendicular, respectively, to an external electric field, are larger by a factor of a million or so than the very intense fields necessary for the breaking of bonds in crystals. So these phenomena are extremely unlikely to be observable.

Double photon decay is most likely to be observed when there is no competing single photon decay probability as for example between a first excited 0^+ state and a 0^+ ground state of an even–even nucleus such as ^{16}O, ^{40}Ca, ^{72}Ge, or ^{90}Zr. In the last-mentioned case, the two-photon decay probability from the first excited 0^+ state at 1.762 MeV to the 0^+ ground state has been determined to be about 10^{-3} of that due to pair production and E^0 electron conversion processes.

Effects of order higher than the fourth have not been discussed here, since the prospects of an experimental verification of the same in the immediate future are not very bright especially in view of the theoretical uncertainties concerning the rule of the strong and the weak interactions in the modification of conventional quantum electrodynamics. Further, nonlinear electromagnetic effects such as second and third harmonic generation,[88–90] multiple photon absorption[91–94] by atoms of certain materials such as CaF_2:Eu^{2+}, atomic cesium and anthracene, stimulated Raman scattering in certain liquids[95] and solids,[96] stimulated inverse Raman spectra,[97] and two-quantum photoelectric emission[98] from sodium have been excluded from the purview of this article, since they depend in a sensitive way on the detailed properties of the materials concerned.

IV. CONCLUSIONS

A large number of different types of experiments that are likely to elucidate the nature of nonlinear electromagnetic phenomena in vacuum have been considered. The main purpose of this work is to see whether certain specific features of conventional quantum electrodynamics are verified through experimental observations. The over-all agreement is excellent. From among the experiments considered, there is no clear-cut case so far in which a definite contradiction with the predictions of quantum electrodynamics has been detected. However, within the limits of experimental accuracy, no definite statement can be made from these results regarding many interesting questions such as the possible finite size of the electron and the muon, the possible existence of electric dipole moments in the case of these particles and the possibility of a heavy electron with a tensor coupling to the ordinary electron.[99] Experiments of the type discussed here may

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in future provide sensitive tests of the validity of field theories with an indefinite metric,[100–105] that have been considered off and on as attractive alternatives from the point of view of removing divergence difficulties associated with conventional quantum electrodynamics.

Notes added in proof.

(A) W. K. Roberts and D. C. Liu, Bull. Am. Phys. Soc. **11,** 368 (1966), have put an experimental upper limit of 1×10^{-30} cm^2/sr^2 for the photon-splitting cross section for lead at 1.33 MeV and backward angles. Such an effect has been claimed to have been observed since at 1.11 MeV by A. W. Adler and S. G. Cohen, Phys. Rev. **146,** 1001 (1966). In the case of copper and cobalt and for photon pairs produced at average angles of 105° with respect to the incident photon direction and of 130° with respect to each other, the cross section turns out to be

$$(3\pm1)Z^2(\Delta w/mc^2)^2\times10^{-35}\ \mathrm{cm}^2/\mathrm{sr}^2,$$

where Δw is the energy interval corresponding to one component of the pair and Z is the atomic number of the target nucleus.

(B) T. W. B. Kibble, Phys. Rev. Letters **16,** 1054 (1966), considers the expressions, for the intensity-dependent frequency shift, valid for focused beams and suggests a partial explanation of the observations of Ramsden and Davies through the large accelerations suffered by electrons in passing through strong intensity gradients in the neighborhood of focal spots.

(C) L. S. Bartell, H. B. Thomson, and R. R. Roskos, Phys. Rev. Letters **14,** 851 (1965). This is a report on a successful experiment, along the lines of the Kapitza–Dirac experiment, involving the use of a collimated beam of 1.65-keV electrons incident perpendicularly on a ruby laser beam and the observation of stimulated Compton scattering of the electrons.

(D) An unusual method of testing the validity of conventional quantum electrodynamics at short distances has been suggested by S. D. Drell, Phys. Rev. Letters **13,** 257 (1964). He considers the photoproduction of muon pairs in hydrogen with the negative muons emerging at small angles to the photon beam and possessing kinetic energies very near the kinematic limit. Additional calculations along these lines have been reported by R. D. Parsons, Bull. Am. Phys. Soc. **11,** 397 (1966). Further, systematic discrepancies had existed between the calculated rates of photoproduction of electron pairs in the neighborhood of light nuclei and the observed singles rates of electron production in the relevant experiments, e.g., R. B. Blumenthal, D. C. Elm, W. L. Faissler, P. M. Joseph, L. J. Lanzerotti, F. M. Pipkin, and D. G. Stairs, Phys. Rev. Letters **14,** 660 (1965). However, further experimental and theoretical work in the BeV region of photon energies has removed these apparent discrepancies, e.g., J. K. Walker, M. Wong, R. Fessel, R. Little, and H. Winick, Phys. Rev. **144,** B1126 (1966).

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