Experimental Aspects of Nucleon–Nucleon Scattering near 210 MeV—A Critical Review*

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Data bearing on the nucleon–nucleon interaction near 210 MeV are reviewed. Proton–proton elastic scattering, neutron–proton elastic scattering (both free and from neutrons bound in deuterium), nucleon–nucleon bremsstrahlung, and tests of invariance principles are discussed. Three-nucleon experiments are mentioned briefly. Some revisions to published N–N scattering data are suggested, and complete tables of data are given.

I. INTRODUCTION

Prior to 1953, the only aspects of nucleon–nucleon (N–N) scattering that had received any experimental attention were the differential and total cross sections. These measurements were realized to be inadequate to determine the N–N scattering matrix. At that time, Oxley[1] attempted to measure the polarization parameter in p–p scattering. In the course of this experiment, performed at the Rochester 130-in. cyclotron, he discovered that the polarization parameter in *p-carbon* scattering was large. This discovery had great "engineering" significance, in that it enabled beams of polarized protons to be produced readily, and enabled the polarization of a beam of protons to be measured readily. Chamberlain and Segré were quick to apply this engineering information, and in a series of classic experiments[2] at the Berkeley 184-in. cyclotron, showed how the p–p scattering matrix could be determined, by measuring polarization and triple scattering parameters. Other laboratories followed, and soon N–N scattering programs were underway at many places.

The program at the Rochester 130-in. cyclotron lab has included p–p elastic scattering, n–p elastic scattering (both free and using bound neutron targets), N–N bremsstrahlung, and some related few-nucleon studies. Incident laboratory energies have been in the range 195–215 MeV. As the program is nearing completion, and the cyclotron will soon be shut down, this seems an appropriate occasion to review the entire program, and also measurements from other laboratories in the same energy range. Actually, a few months from now would be an even more appropriate time, in that several experiments are now nearing completion. These parts of the Rochester program that are not yet finished will be mentioned.

The review is not encyclopedic, in that measurements that have been superceded by significantly more accurate ones will often be omitted. It will be critical, in that opinions will occasionally be expressed on the correctness of measurements and error estimates. p–p and n–p elastic scattering are discussed in Secs. II and III, respectively. p–p and n–p bremsstrahlung are covered in Sec. IV, while checks of invariance principles, and 3-nucleon studies are included in Sec. V.

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[1] C. Oxley, W. Cartwright, J. Rouvina, E. Baskir, D. Klein, J. Ring, and W. Skillman, Phys. Rev. **91**, 419 (1953).

[2] O. Chamberlain, E. Segre, R. D. Tripp, C. Wiegand, and T. Ypsilantis, Phys. Rev. **105**, 288 (1957).

II. PROTON–PROTON ELASTIC SCATTERING

A. Differential and Total Cross Section

Measurements of the differential cross section $\sigma(\theta)$ have been made by Konradi and Tinlot,[3] and by Marshall, Brown, and Lobkowicz.[4]

Marshall *et al.*[4] measured the absolute differential cross section at 13 angles between 8.9° and 38.7° c.m. Angular resolution (rms) varied between ±0.60° and ±0.73° c.m.; cross sections and scattering angles have been corrected for this finite angular resolution. The mean energy of scattering as determined by range[5] in copper was 213±2 MeV, and the energy spread, 11 MeV, full width, half-maximum. The low-energy tail of the beam (below 190 MeV) was less than 0.2% of the total intensity.

Errors of importance to the *relative* cross section were counting statistics (1%–2%), alignment (0.3%), uncertainty in absorber correction (0.0–1%), and uncertainty in background subtraction. To a reasonable approximation, these may be combined as random errors, and this has been done in Table I. Errors of importance to the absolute cross section were uncertainties in target thickness (1%), geometry (0.6%), and monitoring (0.5%). These combined to give an uncertainty in the cross section normalization of ±1.3%.

The results are shown in Table I. Errors are the relative errors only. In addition, all points should be multiplied by a constant normalization factor, of 1.0±0.013.

Konradi and Tinlot[3] measured the *relative* differential cross section from 30° to 90° c.m. in 10° steps. Angular resolution (half-width, half-maximum) varied smoothly

[3] A. Konradi, Ph.D. thesis, Univ. of Rochester, 1961 (unpublished).

[4] J. F. Marshall, C. N. Brown, and F. Lobkowicz, Phys. Rev. **150**, 1119 (1966).

[5] The energy has been computed from the range-energy curves of M. Rich and R. Madey, UCRL-2301 (unpublished). A correction has been applied to adjust to a mean excitation potential of $I=314$ eV.

TABLE I. Proton–proton differential cross section as measured by Marshall, Brown, and Lobkowicz (Ref. 4), and by Konradi and Tinlot (Ref. 3). Marshall's numbers are absolute, and have an over-all normalization uncertainty of $\pm 1.3\%$, in addition to the random errors shown. Konradi's numbers are relative. In addition to the $\pm 1\%$ random error shown, they have a $\pm 1\%$ systematic error (see text). For convenience, Konradi's data have been normalized to Marshall's by the overlapping 2 pairs of points.

Marshall		Konradi	
$\theta_{c.m.}$ (deg)	σ (mb/sr)	$\theta_{c.m.}$ (deg)	σ (mb/sr)
8.9	4.86±0.12	30	3.70±0.04
9.8	4.12±0.10	40	3.73±0.04
10.4	3.67±0.09	50	3.62±0.04
12.2	3.49±0.06	60	3.55±0.04
13.2	3.47±0.07	70	3.57±0.04
14.8	3.50±0.04	80	3.57±0.04
17.2	3.55±0.06	90	3.52±0.04
18.5	3.59±0.04		
19.4	3.55±0.04		
21.7	3.77±0.06		
24.2	3.72±0.03		
29.0	3.76±0.03		
38.7	3.67±0.05		

from $\pm 2.4°$ c.m. at 30°, to $\pm 2.6°$ at 60°, to $\pm 3.3°$ at 90°. The mean energy of scattering was 210±2 MeV, as determined by range[5] in copper. The beam had a spread (including target thickness) of 10 MeV, full width, half-maximum, and a low-energy tail extending down to 150 MeV, which contained 8% of the beam. Thresholds were set so that this low-energy tail *was* detected, when scattered.

Errors of importance were counting statistics (±0.35%), monitor fluctuation (±0.7%), uncertainty in nuclear absorption correction (±0.5% to ±1.3%), and uncertainty in background subtraction (±0.2% to 0.8%). The first two are completely random from one angle to the next, while the last two are partially random, but have a part which is systematic, in that it varies smoothly with angle. The errors combine to a random error of ±1.0% and a systematic error of ±1.0%. [The systematic error can be viewed as an additive correction $Af(\theta)\sigma(\theta)$, where $f(\theta)$ varies smoothly with angle, has a mean value of zero, and an rms value of 1. The constant $A=0.0\pm 0.01$.]

Konradi's results are shown in Table I. They have been normalized by setting the sum of the 30° and 40° points equal to the sum of the 29.0° and 38.7° points of Marshall *et al.* Hence, their absolute normalization is uncertain by the ±1.3% of the latter results, plus ±1.0% due to the random errors of the overlapping points.

By integrating the results in Table I, one finds $\sigma_{tot}(\theta>20°)=21.3\pm 0.4$ mb. This is in agreement with, and of higher accuracy than, the total cross-section measurements of Chamberlain *et al.*,[6] which gave $\sigma_t(\theta>20°)=21.0\pm 0.9$ mb at 225 MeV.

[6] O. Chamberlain, G. Pettengill, E. Segre, and C. Wiegand, Phys. Rev. **93**, 1424 (1954).

B. Polarization

Measurements of the polarization parameter $P(\theta)$ have been made by Tinlot and Warner,[7] and by Marshall, Brown, and Lobkowicz.[4]

Marshall *et al.*[4] measured $P(\theta)$ simultaneously with the cross section by using a polarized incident beam; angle and energy information is summarized in Sec. IIA above. Except at the smallest two angles, the only error of importance is counting statistics; at the smallest two angles, background subtraction and misalignment are also important. The incident beam polarization was measured by scattering it from carbon, and using the p-carbon polarization measurements of Chestnut, Hafner, and Roberts.[8] Measurements were made at a lab angle of 14°, with detection thresholds of 185 and 150 MeV, and at 10°, with a threshold of 150 MeV. Since all measurements agreed the authors concluded that inelastic scattering effects could be neglected. A ±2.5% error in beam polarization is quoted, with the ±2.2% uncertainty in Chestnut *et al.*'s values as the major source.

Results are shown in Table II. Errors are the relative errors only. In addition all points should be multiplied by a constant normalization factor given by the authors as 1.0±0.025, due to uncertainty in beam polarization.

Tinlot and Warner[7] report polarization measurements by two methods. At 217 MeV, they used a CH_2–C subtraction, detecting both scattered and

TABLE II. Proton–proton polarization, as measured by Marshall, Brown, and Lobkowicz (Ref. 4) and by Tinlot and Warner (Ref. 7). Errors listed are relative only. In addition the polarization values should be multiplied by a constant normalization factor. Marshall *et al.* quoted their factor as 1.0±0.025, while the present author (E. H. T.) suggests 0.970±0.030. Tinlot and Warner quoted 1.0±0.022, while the present author suggests 0.985±0.035.

Marshall 213 MeV		Tinlot 210 MeV	
$\theta_{c.m.}$ (deg)	P	$\theta_{c.m.}$ (deg)	P
8.9	0.061±0.035		
9.8	0.120±0.027	30	0.312±0.008
10.4	0.133±0.019	40	0.319±0.010
12.2	0.173±0.013	50	0.303±0.009
13.2	0.215±0.015	60	0.240±0.009
14.8	0.218±0.012	70	0.163±0.008
17.2	0.255±0.012	80	0.084±0.008
18.5	0.269±0.020	90	−0.002±0.008
19.4	0.255±0.010		
21.7	0.299±0.015		
24.2	0.277±0.011		
29.0	0.321±0.010		
38.7	0.340±0.006		

[7] J. H. Tinlot and R. E. Warner, Phys. Rev. **124**, 890 (1961).
[8] W. G. Chestnut, E. M. Hafner, and A. Roberts, Phys. Rev. **104**, 449 (1956).

recoil protons in coincidence. They measured from 60° to 120°, getting good agreement between measurements at θ and $\pi-\theta$. They encountered some difficulty from a position-dependent low-energy tail to the beam, which they surmounted by setting energy thresholds low.

In the second method, at 210 MeV, they used a liquid-hydrogen target, and detected only single protons. They measured from 30° to 90°, in 10° steps. The position dependence of the low-energy tail was eliminated by a multiple scatterer.

Tinlot and Warner do not discuss how they determined the beam polarization. I believe they used essentially the same procedure as Marshall *et al.*, i.e., scattered from carbon and relied on Chestnut *et al.*'s[8] p-carbon polarization data.

Tinlot and Warner's two sets of measurements are in good agreement. As the 210-MeV set covers a larger angular range, and as it is less troubled by the "low-energy tail" problem, it is given in Table II, in preference to the 217-MeV set. Errors shown are relative only.[9] Beam polarization uncertainty is allowed for by a constant normalization factor, given by the authors as 1.0 ± 0.022.

There are earlier measurements at 210 MeV, by Baskir, Hafner, Roberts, and Tinlot,[10] which disagree with those of Tinlot and Warner in the angular region 60°–70°. The earlier measurements have two points that are high by 0.04, well outside the quoted errors. Although I can cite no technical error in the earlier experiment, I prefer the latter one. This is partly just because it was done later (when people had acquired more experience with polarization experiments), but also because its two sets of measurements (at 210 and 217 MeV) used different methods and agreed well with each other.

In determining their beam polarization, both Tinlot and Warner, and Marshall *et al.* rely on the early p-carbon polarization measurements of Chestnut *et al.*,[8] and neither group gives careful attention to inelastic scattering. Recent measurements by Adelberger[11] show that inelastic processes in the second scattering lower the carbon analyzing power significantly, in Marshall's case probably by 3–6%. By taking the square root of his measured asymmetry, Adelberger finds a beam polarization of 0.94 for his beam, which was very similar to Marshall *et al.*'s, though not to Tinlot and Warner's. In the light of Adelberger's measurements,[11] I would suggest normalization factors of 0.985 ± 0.035 for Tinlot and Warner's data, and 0.970 ± 0.030 for Marshall *et al.* data.

[9] Tinlot and Warner included the errors from uncertainty in beam polarization in the error on each point, but omitted a random alignment error of ±0.004; hence the listed errors in their Table I differ from those of Table II here.

[10] E. Baskir, E. M. Hafner, A. Roberts, and J. H. Tinlot, Phys. Rev. **106**, 564 (1957).

[11] R. E. Adelberger, Ph.D. thesis, University of Rochester (in preparation), and private communication.

C. Triple Scattering Parameters R, A, D, and R'

The measurements of the triple scattering parameters at 213 MeV were beset with a systematic error, as an historical review will show.

All experiments were done at a mean energy of 213 ± 2 MeV (43.2 ± 0.7-g/cm^2 copper[5]), and measured from 30° to 90° in 10° intervals.

The first parameters to be measured were R and A, as reported by England, Gibson, Gotow, Heer, and Tinlot.[12] The R experiment was done with a horizontal beam, polarized vertically. The beam was scattered from hydrogen upwards in a vertical plane, and analyzed by scattering from carbon in a plane perpendicular to the vertical plane. No spin precession magnets were used. For the A experiment, the beam was bent upwards by a magnet, through an angle near 28 3/4°. This gave a longitudinal polarization of $P_1 \cos\chi$, and a transverse polarization of $P_1 \sin\chi$, where $\chi=26.7\pm0.5°$. The beam was then scattered from hydrogen, downwards in a vertical plane, and analyzed by scattering from carbon in a plane perpendicular to the vertical plane. Hence the parameter actually measured was $A\cos\chi+R\sin\chi$. While the R results were in agreement with phase shift predictions, the 80° and 90° A measurements could not be fit by phase shift searches.

The parameter D was measured next, by Gotow, Lobkowicz, and Heer.[13] A horizontal beam, with vertical polarization, was scattered from hydrogen, in a horizontal plane, and then analyzed by scattering from carbon in the horizontal plane. Measurements were made for both senses of the second (hydrogen) scattering angle, thus yielding measurements of both D and P_2' (or alternatively, if P_2' was set equal to previous measurements of P_2, yielding two measurements of D). At 80° and 90°, P_2' was found to disagree with P_2, and hence the two measurements of D disagreed. At smaller angles there was good agreement.

Gotow and Lobkowicz[14] then measured R'. A horizontal beam, with vertical polarization, was scattered from hydrogen, upward, in a vertical plane. It was then bent, up or down, through 29°, precessing the spin through an angle χ relative to the direction of motion. The beam was then analyzed by scattering from carbon, in a plane perpendicular to the vertical plane. Hence the parameter actually measured was $R\cos\chi-R'\sin\chi$, where χ was near $\pm60°$. The energy characteristics of the beam were improved for the large-angle measurements (70°–90°), as an energy-position correlation was by now the suspected source of trouble.

[12] A. England, W. Gibson, K. Gotow, E. Heer, and J. Tinlot, Phys. Rev. **124**, 561 (1961).

[13] K. Gotow, F. Lobkowicz, and E. Heer, Phys. Rev. **127**, 2206 (1962); K. Gotow and E. Heer, Phys. Rev. Letters **5**, 111 (1960).

[14] K. Gotow and F. Lobkowicz, Phys. Rev. **136**, B1345 (1964).

TABLE III. Triple scattering parameter R in p–p scattering at 213 MeV. The original measurements are the results of England *et al.* (Ref. 12) and the remeasurement is due to Gotow and Lobkowicz (unpublished). The recommended values are the present author's evaluation of best values and errors.

θ_2 (c.m.)	Original measurements	Remeasurements	Recommended values
30°	-0.203 ± 0.012	···	-0.203 ± 0.012
40°	-0.133 ± 0.017	···	-0.133 ± 0.017
50°	-0.041 ± 0.018	···	-0.041 ± 0.018
60°	$+0.071\pm0.026$	···	$+0.071\pm0.026$
70°	$+0.147\pm0.029$	···	$+0.147\pm0.029$
80°	$+0.248\pm0.042$	···	$+0.248\pm0.084$
90°	$+0.223\pm0.055$	$+0.257\pm0.125$	$+0.257\pm0.125$

Finally, Gotow and Lobkowicz[15] remeasured R at 90° and A at 80° and 90°. Aside from improving the energy characteristics of the beam, the procedure for the R remeasurement was the same as the earlier measurement. For the A remeasurement, the beam was bent upward more steeply than previously, so that $\chi=0°$, and A was measured directly. Within its poorer statistics, the remeasured R value agreed with the earlier value; the A measurements disagreed with earlier measurements.

In summary, a systematic error in the D and A measurements at 80° and 90° has been demonstrated; there is no evidence to suggest errors in the other measurements (except, of course, guilt by association). The detailed nature of the systematic error(s) is still not understood. It is strongly suspected that it involved a low-energy tail known to have been present in the beam.

Results for R are given in Table III. Three columns of results are given: the original measurement, the remeasurement, and my estimate of a "recommended" value and error to use in any analysis. At 90°, where there is a remeasurement, it is given as the "recommended" value. At 80°, the original measurement, with its error doubled, is suggested. At smaller angles, the original measurements with their normal errors are kept.

Results for A are given in Table IV. Here there are more columns, because A was not always measured directly. At 90° and 80°, the remeasurement is taken as the "recommended" value. Because measurement and remeasurement disagree as badly at 80° as at 90°, it is unclear what to do at 70°. I suggest the original measurement, with its error tripled. At smaller angles, the original measurements with their normal errors are kept. As has been mentioned often before, where the combination $A\cos\chi+R\sin\chi$ has been measured, it should be used directly in analyses, (rather than eliminating R) so that the error treatment will be correct.

Results for D are given in Table V. Only the originally quoted values are given, because I feel the authors have properly allowed for the presence of a systematic error in their quoted errors. Note that averaging over both senses of scattering angle cancels many errors, quite possibly including the one affecting the results.

Results for R' are given in Table VI. Again only the originally quoted values are given, because the energy characteristics of the beam were improved for the large-angle measurement, probably eliminating the systematic error. While values for both $R\cos\chi-R'\sin\chi$ and R' are listed, the *former* should be used directly in any analyses (rather than eliminating R) so that the error treatment will be correct.

Two standard techniques for eliminating systematic errors, which have been applied in other experiments, are the reversal of the sign of polarization of the incident beam (with a solenoid magnet), and the reversal of the sense of the second (hydrogen) scattering angle. In *none* of the p–p triple scattering measurements has a

TABLE IV. Triple scattering parameter A in p–p scattering at 213 MeV. In the original measurements by England *et al.*[12] the directly measured quantity was $A\cos\chi+R\sin\chi$, where $\chi=26.7°\pm0.5°$; A has been obtained by using the R values of England *et al.* The remeasurement is due to Gotow and Lobkowicz (unpublished). The recommended values are the present author's evaluation of the best values and errors.

θ_2 (c.m.)	Original measurements $A\cos\chi+R\sin\chi$	A	Remeasurements A	Recommended values $A\cos\chi+R\sin\chi$	A
30°	$+0.449\pm0.016$	-0.400 ± 0.019	···	$+0.449\pm0.016$	···
40°	$+0.343\pm0.015$	-0.317 ± 0.019	···	$+0.343\pm0.015$	···
50°	$+0.202\pm0.017$	-0.205 ± 0.021	···	$+0.202\pm0.017$	···
60°	$+0.059\pm0.018$	-0.102 ± 0.025	···	$+0.059\pm0.018$	···
70°	-0.053 ± 0.029	-0.012 ± 0.036	···	-0.053 ± 0.087	···
80°	-0.032 ± 0.036	-0.090 ± 0.046	$+0.167\pm0.095$	···	$+0.167\pm0.095$
90°	$+0.060\pm0.064$	-0.180 ± 0.077	$+0.085\pm0.135$	···	$+0.085\pm0.135$

[15] K. Gotow and F. Lobkowicz (private communication).

solenoid been used; *only* in the D experiment have measurements been made for both senses of scattering angle. (A systematic error was revealed thereby.) Because these standard techniques have not been used in the 213-MeV p–p triple scattering measurements, the results are more apt to contain systematic errors than those of experiments utilizing these techniques.

III. NEUTRON–PROTON ELASTIC SCATTERING

n–p scattering has been studied in two ways: by bombarding hydrogen targets with neutron beams (free n–p scattering); and by bombarding deuterium targets (serving as an approximation to neutron targets) with proton beams. In the latter case, a theoretical argument is needed to relate the measurement to free n–p scattering amplitudes. The most appropriate at present is the impulse approximation of Chew.[16] Using this approach, Cromer,[17] and Cromer and Thorndike,[18] include the s-wave final-state interaction of two of the nucleons, getting a more accurate result than the simple spectator model,[19] which neglects final-state interactions. For the polarization and triple scattering parameters measured by using deuterium targets, described in Secs. IIIB and IIIC below, the formalism of Cromer and Thorndike has been used.

A. Differential and Total Cross Section

The n–p total cross section has been measured by Kazarinov and Simonov[20] at Dubna by a "good geometry" neutron beam absorption experiment. The beam, created by deuteron stripping, had an effective mean energy of 200 MeV, and a spread of 40 MeV, full width, half-maximum. (The energy dependence of the cross section over the beam spectrum was taken into consideration in calculating the mean energy.) They obtained $\sigma_{\text{tot}}=42.7\pm0.9$ mb, consistent with and of greater accuracy than the older measurements of Mott *et al.*,[21] at 180 and 220 MeV.

Using the same neutron beam, Kazarinov and Simonov[20] also measured the relative differential cross section. Recoil protons were detected over the range $67\frac{1}{2}° \lesssim \theta_{\text{c.m.}} \lesssim 180°$. Scattered neutrons were detected at smaller center of mass scattering angles, so that the full angular range was covered. Accuracy was typically $\pm3\%$ when protons were detected, and $\pm15\%$ when neutrons were detected. The results disagree with the older, nominally less accurate measurements of Guernsey *et al.*[22] by more than the claimed combined errors, at angles near 100°.

Thomas, Spalding, and Thorndike[23] have also measured the relative differential cross section, but the analysis is not yet complete. They produced a neutron beam by charge exchange scattering of a polarized proton beam on deuterium at 10° lab in the plane containing the polarization. The beam had a polarization of 73%, a mean energy[24] of 199 MeV, and an rms spread of ±12 MeV. (The price paid to get this high polarization and narrow energy spread was intensity, which was typically 1000 neutrons/sec over an area of 20 sq in.) The neutron beam was passed through a spin precession magnet which, when turned on, rotated the polarization through 180°. The neutron beam was incident on a liquid-hydrogen target, and protons recoiling in the plane normal to the polarization were

TABLE V. Triple scattering parameter D in p–p scattering at 213 MeV, as given by Gotow, Lobkowicz, and Heer.[13]

θ_2 (c.m.)	$D(\pm\theta_2)$ Left	$D(\pm\theta_2)$ Right	$D(\theta_2)$[a] Final value
30°	0.210±0.035	0.215±0.031	0.200±0.016
40°	0.206±0.028	0.258±0.031	0.232±0.026
50°	0.224±0.019	0.255±0.028	0.240±0.018
60°	0.325±0.035	0.286±0.048	0.319±0.021
70°	0.311±0.038	0.283±0.034	0.297±0.030
80°	0.427±0.046	0.290±0.044	0.36 ±0.07
90°	0.675±0.083	0.317±0.090	0.50 ±0.18

[a] Results obtained in an earlier run, and not included in the "left" and "right" columns, have been included at 30° and 60°.

TABLE VI. Triple scattering parameter R' in p–p scattering at 213 MeV, as given by Gotow and Lobkowicz.[14] The precessions angle χ, and the directly measured quantity $F(\theta_2,\chi)=R\cos\chi-R'\sin\chi$ are also listed. R' has been obtained from F by using the R measurements of England *et al.* (Ref. 12). Note that F, not R', should be used in any analysis of these data.

θ_2 (c.m.)	χ	$F\pm\Delta F$	$R'\pm\Delta R'$
30°	−61°13′	0.331±0.021	0.491±0.025
40°	−61°08′	0.277±0.019	0.390±0.024
50°	−61°04′	0.135±0.017	0.177±0.022
60°	+60°30′	−0.070±0.018	0.120±0.025
70°	+59°12′	0.313±0.036	−0.277±0.045
80°	+58°09′	0.307±0.053	−0.208±0.068
90°	+57°11′	0.406±0.082	−0.340±0.104

[16] G. F. Chew, Phys. Rev. **80**, 196 (1950).
[17] A. H. Cromer, Phys. Rev. **129**, 1680 (1963).
[18] A. H. Cromer and E. H. Thorndike, Phys. Rev. **131**, 1680 (1963).
[19] A. Kuckes, R. Wilson, and P. Cooper, Ann. Phys. (N.Y.) **15**, 193 (1961).
[20] Yu. M. Kazarinov and Yu. N. Simonov, Zh. Eksperim. i Teor. Fiz. **43**, 35 (1962) [English transl.: Soviet Phys.—JETP **16**, 24 (1963)].
[21] G. Mott, G. Guernsey, and B. Nelson, Phys. Rev. **88**, 9 (1952).
[22] G. Guernsey, G. Mott, and B. Nelson, Phys. Rev. **88**, 15 (1952).
[23] A. Thomas, D. Spalding, and E. Thorndike (unpublished); A. Thomas, Ph.D. thesis, University of Rochester, 1967 (unpublished).
[24] The energy has been computed from the copper range energy curve of M. Rich and R. Madey, UCRL-2301 (unpublished).

Table VII. n–p polarization results. Shown are the quasifree values of Tinlot and Warner (Ref. 7) at 217 MeV, (P_{pn}^{QF}), the corrections (ΔP) due to Koehler *et al.* (Ref. 25). The inferred free n–p values (P_{pn}^{free}), and the free n–p values of Thomas *et al.* (Ref. 23) at 199 MeV, (P_{np}). In addition to the random errors shown, there is a systematic error of $[(0.04)^2+(1/4\Delta P)^2]^{1/2}$ in P_{pn}^{free}, and a $\pm10\%$ normalization uncertainty in P_{np} (see text).

$\theta_{\mathrm{c.m.}}$	P_{pn}^{QF}	ΔP	P_{pn}^{free}	P_{np}
40	0.468±0.029	0.033±0.019	0.501±0.035	
50	0.460±0.031	0.006±0.021	0.466±0.038	
60	0.372±0.041	−0.010±0.016	0.362±0.044	
70	0.258±0.033	−0.018±0.013	0.240±0.035	
76.9				0.132±0.028
80	0.032±0.036	−0.020±0.013	0.012±0.038	
86.6				0.029±0.017
90	−0.069±0.032	−0.018±0.011	−0.087±0.034	
96.3				−0.075±0.014
100	−0.124±0.029			
110	−0.184±0.029			
117.1				−0.111±0.011
120	−0.170±0.030			
127.4				−0.125±0.010
137.8				−0.125±0.009
148.1				−0.117±0.011
158.1				−0.071±0.012

detected. Six recoil proton telescopes were used to enable left and right scattering to be measured simultaneously at three angles. Half the data were taken with the spin precession magnet on, the other half with it off, giving zero beam polarization for the cross-section measurement. Measurements were made at eight angles between 77° and 158° c.m. Statistical accuracy is typically ±1%; various other sources of error increase the final errors to 2–3%. It is anticipated that an article[23] describing this experiment will be submitted to *The Physical Review* within a few months. This article will compare the new measurements with those of Kazarinov and Simonov,[20] and Guernsey *et al.*[22]

B. Polarization

Measurements of the polarization parameter $P(\theta)$ have been made by Tinlot and Warner,[7] and by Thomas, Spalding, and Thorndike.[23]

Tinlot and Warner[7] measured $P(\theta)$ in quasifree p–n scattering in deuterium, at 217 MeV,[5] from 40° to 120° c.m. Both scattered proton and recoiling neutron were detected. A CD_2–C subtraction was employed. Results are shown in Table VII. The dominant error is counting statistics. For angles from 40° to 90°, the calculated[25] "correction" relating these measurements to free n–p polarization is also shown, along with the inferred value of $P(\theta)$ in free n–p scattering. In addition to the errors shown for the correction, a systematic error of $[(0.04)^2+(1/4\Delta P)^2]^{1/2}$ is suggested[25] to allow for theoretical uncertainties. This error *is comparable with* the random error in the corrected points.

Thomas *et al.*[23] measured $P(\theta)$ in free n–p scattering at 199 MeV, from 77° to 158° c.m. They used the highly polarized (73%), fairly monoenergetic (±12 MeV, rms) neutron beam described in Sec. A above, performing cross section and polarization measurements simultaneously. By measuring the spin precession magnet off–on asymmetry, simultaneously in counters to the left and right of the beam line, most systematic errors were eliminated. Beam polarization was determined to be 0.73±0.08 by comparing asymmetries measured in the reaction $n+d\rightarrow p+2n$ (proton detected only) with those measured[26] with a polarized proton beam in the charge symmetric reaction $p+d\rightarrow n+2p$ (neutron detected only). (The polarization of the proton beam was determined by scattering it from carbon.) The uncertainty in neutron beam polarization is largely statistical, but includes a ±0.035 contribution from uncertainty in proton beam polarization.

Final analysis of the experiment has not been completed. *Preliminary* results are shown in Table VII. In addition to the random errors shown, there is a systematic error of ±10%, due to beam polarization uncertainty, which will shift all points up and down together.

The results of Thomas *et al.*[23] differ from those of Tinlot and Warner[7] in the region of overlap (80°–120° c.m.). The difference is in the direction to be expected

[25] P. Koehler, E. Thorndike, and A. Cromer, Phys. Rev. **134**, B1030 (1964).

[26] D. Spalding, A. Thomas, N. W. Reay, and E. Thorndike, Phys. Rev. **150**, 806 (1966); D. Spalding, Ph.D. thesis, University of Rochester, 1966 (unpublished).

from the difference in incident energy. When corrected to an energy of 199 MeV, P_{pn}^{free} agrees with P_{np} at 80°–100° c.m., but still disagrees with P_{np} at 110° and 120° c.m. by $\sim$2 standard deviations. The corrections from quasifree to free scattering[25] at 80° and 90° are in a direction to *increase* the discrepancy. The corrections are not expected to be particularly reliable at these large angles. Note, however, that the discrepancy does not exceed the suggested uncertainty in the corrections, $[(0.04)^2+(1/4\Delta P)^2]^{1/2}$.

In any analysis, the results of Thomas *et al.* should be used for center of mass angles greater than 70°, because of their significantly smaller random errors, and because their systematic error ($\pm 10\%$, due to beam polarization uncertainty) is smaller and more reliably determined, than the systematic error $[(0.04)^2+(1/4\Delta P)^2]^{1/2}$ in the quasifree to free correction required by the results of Tinlot and Warner. It must be reiterated that the results of Thomas *et al.* are *preliminary*; final results will be published[23] shortly.

C. Triple Scattering Parameters D, R_t, and D_t

Three n–p triple scattering parameters have been measured near 210 MeV. In all three cases, neutrons bound in deuterium were used as targets, and an impulse approximation calculation used to relate the measurements to the parameters in free n–p scattering. It is my opinion that the theoretical uncertainties in the impulse approximation calculation are small compared to the experimental errors quoted for *these* experiments. However, in the absence of a rigorous and detailed theory of p–d inelastic scattering, this point is somewhat conjectural.

Warner and Tinlot[27] have measured the parameter D, in quasifree p–n scattering, at 212 MeV.[5] A deuterium target was bombarded by polarized protons. Recoiling neutrons and scattered protons were detected in coincidence, and the protons were spin analyzed.

TABLE VIII. Depolarization in quasifree p–n scattering (Ref. 27), the correction to it (Ref. 25), and the inferred free n–p scattering depolarization parameter. There is a systematic error of $[(0.04)^2+(1/4\Delta D)^2]^{1/2}$ in the inferred free parameter (see text) in addition to the listed error.

θ_p(lab)	D_{np} (quasifree)	ΔD	D_{np}(free)	θ_p(c.m.)
19.2°	0.71±0.07	0.08±0.05	0.79±0.09	40°
24.1°	0.85±0.08	0.05±0.03	0.90±0.09	50°
28.8°	0.79±0.08	0.03±0.03	0.82±0.08	60°
33.6°	0.99±0.14	0.02±0.02	1.01±0.14	70°
38.4°	1.05±0.45	0.01±0.01	1.06±0.45	80°

[27] R. E. Warner and J. H. Tinlot, Phys. Rev. **125**, 1028 (1962).

TABLE IX. R_t in p–d charge exchange scattering at 203 MeV (Ref. 28). Note that it is the measured parameter R_t^{pd} that is given, and *not* the free n–p R_t parameter. In addition to the random errors shown, there is a systematic error which moves all values of R_t together by $\pm 14\%$ of their value. Also listed is the value of f to be used in Eq. (1) to relate R_t^{pd} to free N–N amplitudes.

θ_{lab} (deg)	$\theta_{\text{c.m.}}$ (deg)	R_t^{pd}	f
0	179.2	−0.252±0.089	66.7
5	169.2	−0.505±0.090	3.33
10	158.9	−0.870±0.066	0.92
15	148.8	−0.892±0.057	0.40
20	139.0	−0.568±0.116	0.17

The procedure was quite similar to that used for the p–p D measurement,[13] mentioned in Sec. IIC. Measurements were made for both senses of second scattering angle, except at $\theta_{\text{c.m.}}=50°$. The two sets of measurements agreed at 60° and 70°, and disagreed by 1.9 standard deviations at 40°, and 1.4 standard deviations at 80°. Because of the evidence for a systematic error at large angles found in the p–p D measurement, Warner and Tinlot doubled the error on their 80° point, but did not enlarge the error on their 40° point. I concur in this treatment. Results are given in Table VIII, where the impulse approximation corrections of Koehler, Thorndike, and Cromer[25] are also given, as are the inferred free n–p D values. A systematic error of $[(0.04)^2+(1/4\Delta D)^2]^{1/2}$ is suggested to allow for theoretical uncertainty in the correction.

Reay, Thorndike, Spalding, and Thomas[28] have measured the parameter R_t in (p,n) scattering from deuterium, at 203 MeV.[24] A deuterium target was bombarded with 90% polarized protons, and high energy neutrons recoiling into forward angles were detected and spin analyzed. (Only the neutron was detected.) The incident beam had a horizontal transverse polarization. The sign of the polarization could be reversed by reversing the current through a solenoid magnet. Neutrons recoiling in a horizontal plane were spin analyzed by charge-exchange scattering from hydrogen in a vertical plane. At two angles (10° and 15° lab) measurements were made for both senses of second scattering angle, and good agreement obtained. At 0°, 5°, and 20° only one measurement was made.

The "analyzing power" P_3 of the third scattering is the n–p polarization parameter at a "charge exchange" angle of 25° lab. By using the recent polarization measurement of Thomas *et al.*[23] of P(25° lab, 199 MeV) $=0.125\pm0.016$ the results of Reay *et al.*[28] have been reanalyzed to give slightly different values, with a reduced systematic error.

Results are given in Table IX. In addition to the

[28] N. W. Reay, E. Thorndike, D. Spalding, and A. Thomas, Phys. Rev. **150**, 801 (1966).

TABLE X. D_t in p–d charge exchange scattering at 197 MeV (Ref. 29). Given are the measured parameter D_t^{pd}, the value of f used in Eq. (1), the "correction" ΔD_t relating p–d and n–p D_t values, and the inferred value of D_t in free n–p scattering, D_t^{np}.

$\theta_{\rm lab}$ (deg)	$\theta_{\rm c.m.}$ (deg)	D_t^{pd}	f	ΔD_t	D_t^{np}
15.5	147.4	$+0.087\pm0.068$	0.39	$+0.008$	$+0.095\pm0.068$
19.7	138.6	-0.018 ± 0.071	0.19	$+0.004$	-0.014 ± 0.071
25.3	126.9	$+0.058\pm0.103$	0.00	0.000	$+0.058\pm0.103$

random errors listed, there is a systematic error which moves all values of R_t together by $\pm14\%$ of their value. Note that it is the measured parameter R_t^{pd}, and *not* the free n–p R_t parameter that is listed. The measured parameter was related to free N–N scattering amplitudes by an impulse approximation calculation that included the s-wave final-state interaction of the two protons:

$$R_t^{pd}=(R_t^{np}+fR_t^{ces})/(1+f). \qquad (1)$$

The quantity f is given in Table IX. Since it is sometimes very large, it is *not* appropriate to "correct" R_t^{pd} and quote a value for R_t^{np}. R_t^{ces} is a definite function of the N–N scattering amplitudes. The (somewhat complicated) expression is given by Reay *et al.*[28] For many phase shift solutions $R_t^{ces}\approx R_t^{np}$; however, I have been unable to find a simple expression for their difference.

Spalding, Thomas, and Thorndike[29] have measured the parameter D_t in (p, n) scattering from deuterium, at 197 MeV,[24] under conditions quite similar to the R_t experiment just described. The incident proton beam had a horizontal, transverse polarization, whose sign could be reversed with a solenoid. Neutrons recoiling in a vertical plane (both up and down) were spin analyzed by charge exchange scattering from CH_2 in a vertical plane.

The results are given in Table X. The errors include an allowance for suspected systematic errors, which were comparable with the statistical error. The measured parameter D_t^{pd} is related to free N–N amplitudes by an expression identical to Eq. (1), but with R_t replaced everywhere with D_t. The expression for D_t^{ces} is given by Reay *et al.*[28] As f is small at these angles, it is possible to "correct" the measured values, and list the free n–p scattering parameter D_t^{np}.

IV. NUCLEON–NUCLEON BREMSSTRAHLUNG

A. p–p Bremsstrahlung

The reaction $p+p\rightarrow p+p+\gamma$ has been studied at 204 MeV[24] by Rothe, Koehler, and Thorndike,[30] using a 90% polarized proton beam. They detected all three final-state particles in coincidence: the γ-ray in a small solid-angle threshold counter, and both protons in a large-solid-angle spark chamber array. Directions of all particles, and energies of the protons were thus determined. Cross sections, and gamma-ray and proton asymmetries due to the polarized beam, were measured.

The reaction can be described by the following 5 variables, all defined in the over-all center-of-mass system: the direction and energy of the γ ray (θ_γ, ϕ_γ, E_γ) and the direction of the difference in momentum ($\mathbf{P}_1-\mathbf{P}_2$) of the two protons ($\theta_{\rm c.m.}$, $\phi_{\rm c.m.}$). Polar angles are defined with respect to the incident polarized proton direction, azimuthal angles, with respect to the horizontal plane. (The beam polarization was vertical.) Averaging over polarization, one may fold all angular variables into the region 0°–90°.

The results show a peaking at $\cos\theta_{\rm c.m.}=0$, having dropped roughly 30% by $\cos\theta_{\rm c.m.}=0.5$. The results show a peaking at $\phi_{\rm c.m.}-\phi_\gamma=0$, being described roughly by $1+\frac{3}{2}\cos^2(\phi_{\rm c.m.}-\phi_\gamma)$. (Experimental detection efficiency falls rapidly beyond $\cos\theta_{\rm c.m.}=0.6$ and beyond $\phi_{\rm c.m.}-\phi_\gamma=60°$.) Distribution in E_γ is essentially flat. The integrated cross section $d\sigma/d\Omega_\gamma$ ($E_\gamma>35$ MeV) is 73.0 ± 7.4, 42.0 ± 6.2, and 48.0 ± 2.7 μb/sr, at $\theta_\gamma=34°$, $59\frac{1}{2}°$, and 72°, respectively. (In addition to the random error shown, there is a systematic error of $\pm22\%$, affecting the 3 angles equally.) The total cross section $\sigma(E_\gamma>35$ MeV) is 0.70 ± 0.15 μb, where the 22% systematic error *has* been included. The proton asymmetries agree in sign and magnitude with elastic p–p polarization at 210 MeV. γ-ray asymmetries are of the same sign and magnitude as those predicted[31] and measured[32] for n–p radiative capture with a polarized proton beam. Through the appropriate Jacobian transformation the cross section $d^2\sigma/d\Omega_1d\Omega_2$ has been obtained for comparison with other experiments and theory. (Here Ω_1 and Ω_2 are the *laboratory* solid angles of the two protons.) For coplanar events, with protons at equal laboratory angles, $d^2\sigma/d\Omega_1d\Omega_2$ equals 13.0 ± 2.4, 14.0 ± 2.7 and 29.0 ± 6.0 μb/(sr)^2 at $\theta_{\rm lab}=30°$, 35°, and 40°, respectively.

This experiment is described in fullest detail in Ref.

[29] D. Spalding, A. Thomas, and E. Thorndike, Phys. Rev. **158,** 1338 (1967); D. Spalding, Ph.D. thesis, University of Rochester, 1966 (unpublished).

[30] K. Rothe, P. Koehler, and E. Thorndike, Phys. Rev. **157,** 1247 (1967).

[31] A. Donnachie and P. J. O'Donnell, Nucl. Phys. **53,** 128 (1964).

[32] P. Koehler, K. Rothe, and E. Thorndike, Bull. Am. Phys. Soc. **11,** 303 (1966).

30. Earlier reports[33] contain some errors, and should *not* be used.

B. n–p Bremsstrahlung

Koehler, Rothe, and Thorndike[34] have studied radiative p–d interactions at 197 MeV[24] as a means of learning about n–p bremsstrahlung. The impulse approximation suggests the following processes will be important:

$$p+d\rightarrow n_s+p+p+\gamma \quad \text{(quasifree } pp \text{ bremsstrahlung)}, \tag{1}$$

$$p+d\rightarrow p_s+n+p+\gamma \quad \text{(quasifree } pn \text{ bremsstrahlung)}, \tag{2A}$$

$$p+d\rightarrow p_s+d+\gamma \quad \text{(quasifree } pn \text{ radiative capture)}. \tag{2B}$$

(The subscript s implies the particle is a "spectator", with momentum characteristic of the deuteron wave function.) Final-state interactions between the spectator particle and the other nucleons will modify the above reactions. In particular, the spectator particle and one of the other nucleons will on occasions bind to form a deuteron, giving rise to the reaction:

$$p+d\rightarrow d+p+\gamma \quad (pd \text{ bremsstrahlung}). \tag{3}$$

In the experimental procedure used, a liquid-deuterium target was bombarded with a 90% polarized, 199-MeV proton beam. The γ-ray direction, and the charged-particle directions and ranges were measured. Spectator particles were *not* detected, nor were neutrons. Thus reaction (3) was 3 times overdetermined, reactions (1) and (2B) were just determined, and reaction (2A) was underdetermined.

The impulse approximation was found to give a reasonable description of the data. The momentum spectra of the spectator particles in reactions (1) and (2B) are in agreement with a Hulthèn wave function. The quasifree p–p bremsstrahlung cross section is $(50\pm10)\%$ of the free p–p bremsstrahlung cross section,[30] and the energy and angular distributions are in agreement. The quasifree p–n radiative capture cross section is $(75\pm15)\%$ of the free n–p radiative capture cross section,[31] and the measured right-left γ-ray asymmetries due to the use of a polarized proton beam are in agreement with calculations[31] for the free process.

TABLE XI. Inferred free n–p bremsstrahlung cross sections. The differential cross section $d\sigma/d\Omega_\gamma(\theta_\gamma, E_\gamma>40$ MeV) and the γ-ray direction θ_γ are expressed in the $np\gamma$ center of mass. E_γ is the γ-ray laboratory energy. Cross sections inferred from quasifree p–n bremsstrahlung include in their errors an estimate of the uncertainty due to theory; cross sections inferred from p–d bremsstrahlung do not, as there is no easy way of estimating them.

Source of result	$(d\sigma/d\Omega_\gamma)(\theta_\gamma, E_\gamma>40$ MeV) (μb/sr) $\theta_\gamma=60°$	$\theta_\gamma=108°$	$\theta_\gamma=147°$	$\sigma_{tot}(E_\gamma>40$ MeV) (μb)
Quasifree p–n bremsstrahlung	3.4 ± 1.0	2.5 ± 0.8	1.8 ± 0.5	35 ± 12
p–d bremsstrahlung		2.7 ± 0.4	3.2 ± 0.5	

Free n–p bremsstrahlung cross sections have been inferred both from quasifree pn bremsstrahlung, and from p–d bremsstrahlung; the former are expected to be theoretically more reliable. The results are shown in Table XI. The differential cross sections inferred from quasifree p–n bremsstrahlung have been integrated over the γ-ray angles to obtain the total cross section quoted. It is believed that the quoted errors allow for the uncertainty in the theoretical treatment. However, should there be an important p–d radiative process which is *not* due to an N–N radiative interaction, then this treatment is incorrect. Further theoretical work in this area is highly desirable.

The cross sections found are rather high, being 30 to 70 times the p–p bremsstrahlung cross section[30] at the same energy.

V. RELATED TOPICS

A. Time Reversal and Parity Tests

The most sensitive test of time reversal invariance in N–N scattering performed to date is the comparison of polarization parameter $P(\theta)$ and asymmetry parameter $\mathcal{A}n(\theta)$ in p–p scattering. Such comparisons have been made at 142 MeV by Hwang *et al.*,[35] at 178 MeV by Hillman, Johansson, and Tibell,[36] and at 210 MeV by Abashian and Hafner.[37] The three sets of measurements have been analyzed by Thorndike,[38] and found to imply that the time reversal noninvariant mixing of $^3P_2$ and $^3F_2$ states is consistent with zero, and does not exceed 7% of its maximum possible value, at the 95% confidence level.

The best tests of parity conservation in N–N scattering are indirect ones, from nuclear physics. These tests are very sensitive indeed, and are on the borderline of seeing effects of parity nonconservation from the

[33] K. Rothe, P. Koehler, and E. Thorndike, Phys. Rev. Letters **16**, 1118 (1966); K. Rothe, P. Koehler, and E. Thorndike, Proc. Williamsburg Conf. Intermediate Energy Physics, Vol. II, 677 (1966).

[34] P. F. M. Koehler, Ph.D. thesis, University of Rochester, March 1967 (unpublished); P. F. M. Koehler, K. W. Rothe, and E. H. Thorndike, Phys. Rev. Letters **18**, 933 (1967); P. Koehler, K. Rothe, and E. Thorndike (unpublished).

[35] C. H. Hwang, T. R. Ophel, E. H. Thorndike, and R. Wilson, Phys. Rev. **119**, 352 (1960).

[36] P. Hillman, A. Johansson, and G. Tibell, Phys. Rev. **110**, 1218 (1958).

[37] A. Abashian and E. M. Hafner, Phys. Rev. Letters **1**, 255 (1958).

[38] E. H. Thorndike, Phys. Rev. **138**, B586 (1965).

weak interactions. An extensive list of references to these experiments is given by Michel.[39]

Direct tests of parity conservation in N–N scattering are several orders of magnitude less sensitive. Existing experiments have been analyzed by Thorndike,[38] and are seen to provide no useful limit on the size of the parity nonconserving mixing of ${}^1S_0$ and ${}^3P_0$ states.

An experiment to look for a (parity nonconserving) dependence of differential cross section on longitudinal polarization of the incident beam is being performed at Rochester by Gucker. This experiment should be an order of magnitude more sensitive than previous direct tests, but still several orders of magnitude less sensitive than the indirect, nuclear physics tests.

B. Three-Nucleon Experiments

Several p–d inelastic scattering experiments have already been mentioned, in Secs. IIIB and C, and in Sec. IVB. Those experiments were aimed primarily at studying the n–p interaction. Here we mention some experiments aimed at learning about nucleon–deuteron phenomena per se.

Tinlot and Warner[7] compared the polarization parameter $P(\theta)$ in free pp scattering and quasifree p–p scattering in deuterium. They detected both protons, and restricted the angular range of the recoil proton to the quasifree peak. The differences between free and quasifree parameters do not exceed 0.02, while the statistical errors are typically ± 0.017. Hence, the differences are consistent with zero. They are also consistent with an impulse approximation calculation of Koehler, Thorndike, and Cromer,[25] which allows for the s-wave final state interaction of the neutron with either proton.

Thomas, Spalding, and Thorndike[23] have compared polarization parameters in free n–p scattering and n–p charge exchange scattering in deuterium, using the experimental setup described in Secs. IIIA and B. Only the recoiling proton was detected. They found good agreement in the angular range 127° to 158° c.m., which became even better when the deuterium data had impulse approximation corrections applied. Statistical accuracy in the difference in polarization in this region was ± 0.017. At 77° and 96°, where accuracy was ± 0.030, agreement was worse, approaching 2 standard deviations.

Adelberger has[40] just recently completed a measurement of p–d elastic scattering at 200 MeV. Differential cross section has been measured from 90° to 170° c.m., in 5° steps, to an (anticipated) accuracy of 5%. Polarization has been measured from 80° to 170°, to an (anticipated) accuracy of ± 0.01.

Brown is currently carrying out an extensive set of measurements of cross section and polarization in inelastic p–d scattering. Both final state protons are detected, and their energy measured. A broad range of kinematic conditions is being covered, both near to and far from the quasifree peak.

[39] F. C. Michel, Phys. Rev. **133,** B329 (1964).

[40] R. E. Adelberger and C. N. Brown, Bull. Am. Phys. Soc. **12,** 466 (1967); R. E. Adelberger, Ph.D. thesis, University of Rochester (in preparation).