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Some Work on the Theoretical Interpretation of the Reaction $D+P\rightarrow P+P+N$

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Enhancements in the differential cross section for the reactions $P+D\rightarrow P+P+N$ and $D+P\rightarrow P+P+N$ due to the final-state interactions near the deuteron breakup threshold energy were studied in the Born approximation. In order to obtain a simple qualitative picture as a first step, the zero-range theory of Frank and Gammel[1] was used. In this theory the inelastic scattering differential cross section is related to the elastic scattering cross section and n–p s-wave phase shifts calculated from the effective range theory. Calculations based on this theory were made for a number of values of the scattering parameters and the results of these calculations have been compared with recent experimental data.[2] It can be easily shown[3] that the zero-range theory of Frank–Gammel yields the following expression for the inelastic proton–deuteron scattering cross section for the reactions $P+D$ or $D+P\rightarrow P+P+N$.[4]

$$d\sigma/dT_3\,d\Omega_3\,d\Omega_4=\mathrm{PS}\times|M|^2, \tag{1}$$

where

$$|M|^2=\frac{9}{8\pi^2}(2E_bT_1)^{-\frac{1}{2}}\sum_{i=3,4}\left[{}^tD_i^{-1}+\frac{1}{3}\left(\frac{V_{\mathrm{SE}}}{V_{\mathrm{TE}}}\right)^2{}^sD_i^{-1}\right]\left(\frac{d\sigma}{d\Omega}\right)_i^{\mathrm{el}}$$

and PS is the phase-space factor given by

$$\mathrm{PS}=(T_3)^{\frac{1}{2}}(T_4)^{\frac{1}{2}}\,|\,1-(\bar{v}_4\cdot\bar{v}_5/v_4^2)\,|^{-1}.$$

The symbol ${}^tD_i$ is related to the triplet phase shift by

$${}^tD_i=E_i''\csc^2\delta_t=\left[\frac{\hbar^2}{ma_t^2}+\left(1-\frac{r_{0t}}{a_t}\right)E_i''+\frac{1}{4}\frac{r_{0t}^2m}{\hbar^2}(E_i'')^2\right],$$

where E_i'' is the excitation energy of the pair of outgoing particles other than the particle i in their own center of mass system, and δ's, a's, and r_0's stand for n–p s-wave phase shifts, scattering lengths, and the effective ranges, respectively. ${}^sD_i$ is related to the singlet phase shift in the same way. The ratio $V_{\mathrm{SE}}/V_{\mathrm{TE}}$ is the singlet to triplet ratio, and $(d\sigma/d\Omega)_i^{\mathrm{el}}$ is the experimentally measured elastic cross section in the center-of-mass system. The symbols E_b and T's refer to the deuteron binding energy and kinetic energies, respectively.

In the above expression, a summation appears in $|M|^2$, because the detectors are unable to distinguish the scattered proton from the ejected proton. The last factor in the phase space factor comes from the Jacobian of transformation for the delta function.

The quantity measured experimentally is the relative cross section $\sigma(T_3, T_4, \theta_3, \theta_4, |\phi_3-\phi_4|)$ where the last three variables are fixed in any given experimental run. In this case the differential cross section lies along a line dictated by kinematics.

For this case the cross section may be expressed as

$$d\sigma/dS\,d\Omega_3\,d\Omega_4=\mathrm{PS}\times|M|^2, \tag{2}$$

where

$$\mathrm{PS}=(T_3)^{\frac{1}{2}}(T_4)^{\frac{1}{2}}\left[\left(1-\frac{\bar{v}_3\cdot\bar{v}_5}{v_3^2}\right)^2+\left(1-\frac{\bar{v}_4\cdot\bar{v}_5}{v_4^2}\right)^2\right]^{-\frac{1}{2}}.$$

The last factor is the Jacobian of the transformation from $d\sigma/dT_3\,dT_4\,d\Omega_3\,d\Omega_4$ to $d\sigma/dS\,d\Omega_3\,d\Omega_4$, where dS is the line element dictated by the kinematic relationship[5] $T_4=f(T_3)$. This form of the phase space is used in Fig. 1 where two-dimensional plots of theory and experiment are compared.

In Figs. 2 and 3 both experiment and theory have been projected on the T_3 axis. The cross section is then (as before)

$$d\sigma/dT_3\,d\Omega_3\,d\Omega_4=\mathrm{PS}\times|M|^2,$$

but now the proper phase-space factor is different, and has the form given in Eq. (1).

The figures and their captions are self-explanatory. The following conclusions can be drawn. Figure 1 shows that the observed cross section in the region shown is in qualitative agreement with the Frank–Gammel theory. It is seen that the calculated curve for the case where the scattered particle enters counter No. 4 (particle No. 3 resonating with particle No. 5) gives an appreciably larger contribution to the cross section in the region of the peak seen in the 30°–75° case than

[1] R. M. Frank and J. L. Gammel, Phys. Rev. **93,** 463 (1954).
[2] J. F. Mollenauer, P. F. Donovan, and J. V. Kane, Bull. Am. Phys. Soc. **9,** 389 (1964), Abstract BA3.
[3] R. E. Warner, Phys. Rev. **132,** 2621 (1963).
[4] The ordering of particle numbers in the reaction is $1+2\rightarrow 3+4+5$ and subscripts are used to designate these numbers.
[5] Č. Zupančič (private communication).

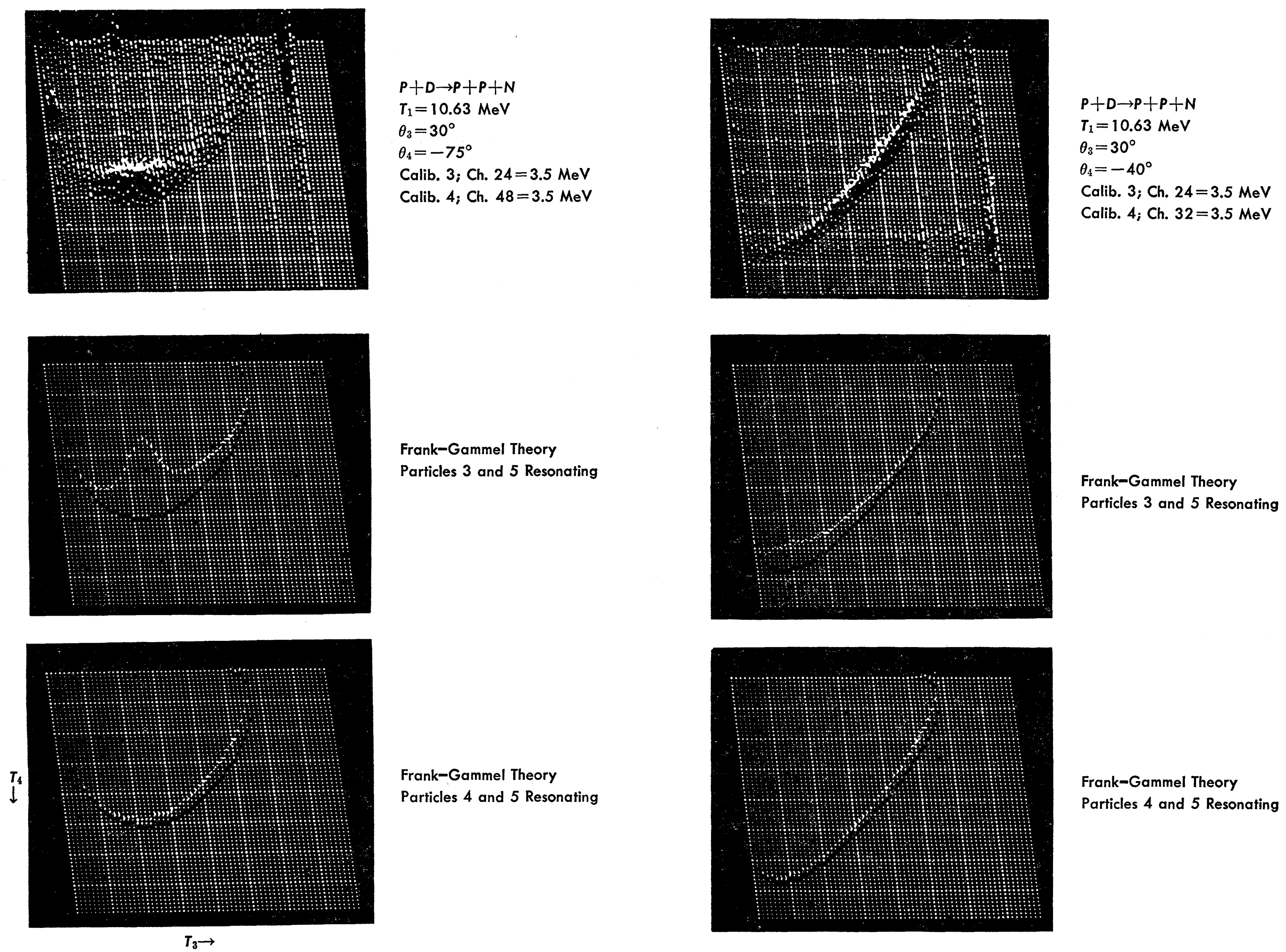

Fig. 1. Two measurements of the reaction $P+D \rightarrow P+P+N$ (top) and components of the cross section predicted by the Frank–Gammel theory (center and bottom).

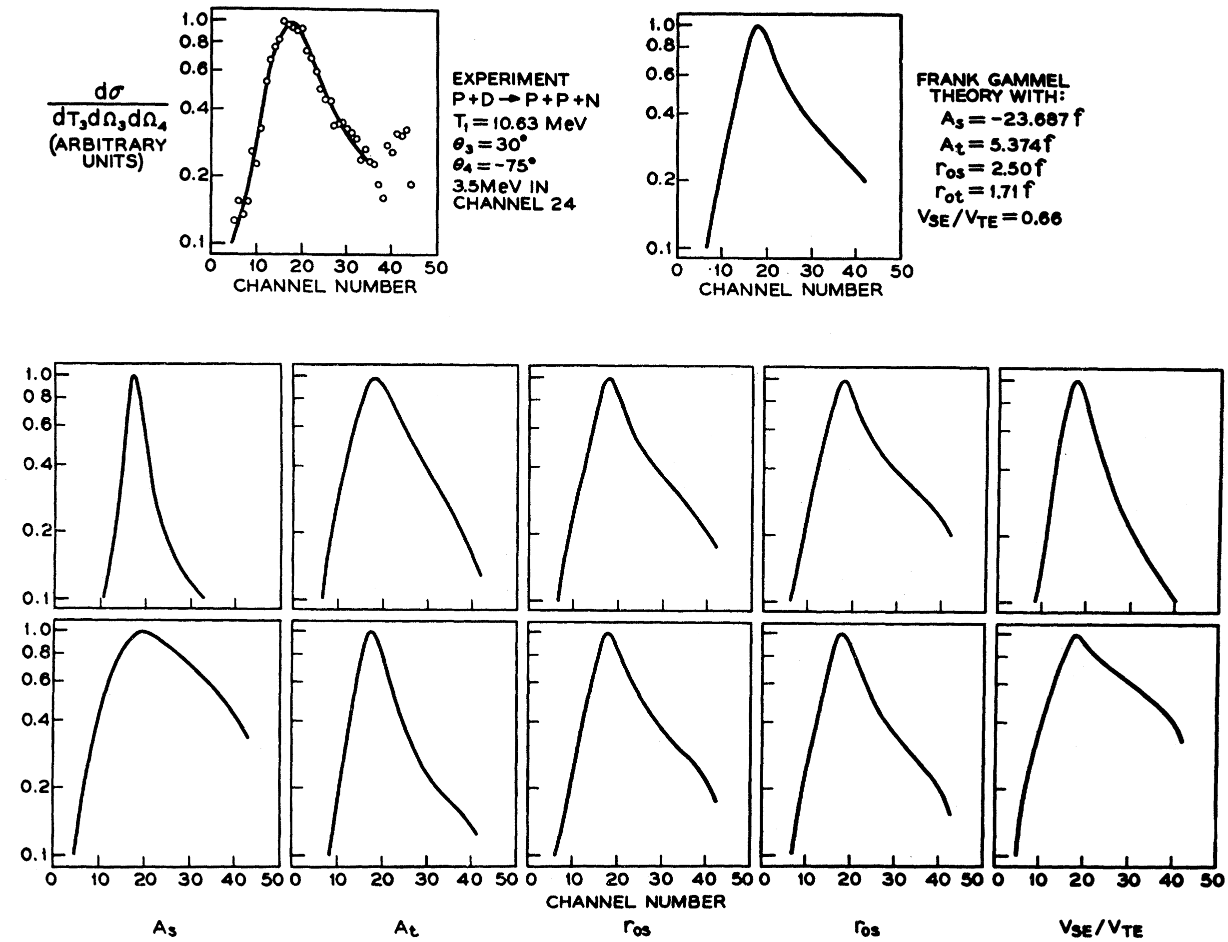

FIG. 2. The measured cross section for reaction $P+D\rightarrow P+P+N$ (top left) and the calculated cross section (top right) for the parameters given. Note that A_s in figure means a_s in text. The double row of figures shows the effect on the calculated cross section caused by doubling (upper row) and halving (lower row) the parameter designated at the bottom.

the other matrix element does. Because of this we feel justified in neglecting the smaller matrix element. Figures 2 and 3 (which are intended to compare peak shapes not absolute cross sections) show that the theory is most sensitive to the parameter a_s and the sensitivities to a_t, r_{0s}, $r_{0t}T$, and V_{SE}/V_{TE} are all lower. If we assume that a_t, $r_{0s}r_{0t}$, and V_{SE}/V_{TE} all have the accepted values,[6,1] then we estimate that this experiment could be used to determine a_s to within about $\pm 20\%$. Since a_s is given[6] a value with an accuracy of about one part in one thousand it is clear that this experiment and its analysis is not a practical method to determine these particular two-body scattering parameters. Nevertheless the method can be applied to other nuclei (such as unstable nuclei) where two-body data are difficult to measure and the qualitative results found in Fig. 1 allow a measure of confidence in the use of three-body reactions to measure two-body parameters for such cases.

[6] H. P. Noyes, Phys. Rev. **130**, 2025 (1963).

Discussion

EISBERG: Can you also, with increased accuracy, measure scattering parameters for two particles in the presence of a third particle?

KANE: Yes, but as far as accuracy is concerned, because of experimental errors, one is going to have to decide each situation as a particular case.

My point is, though, that almost any value of scattering length and effective range, or rather quite a large latitude in scattering length and effective range, is apt to give an agreement with experiment.

Maybe I didn't understand your question.

EISBERG: My question is, is there any potentiality, when one has very much better data, to obtain information about three-

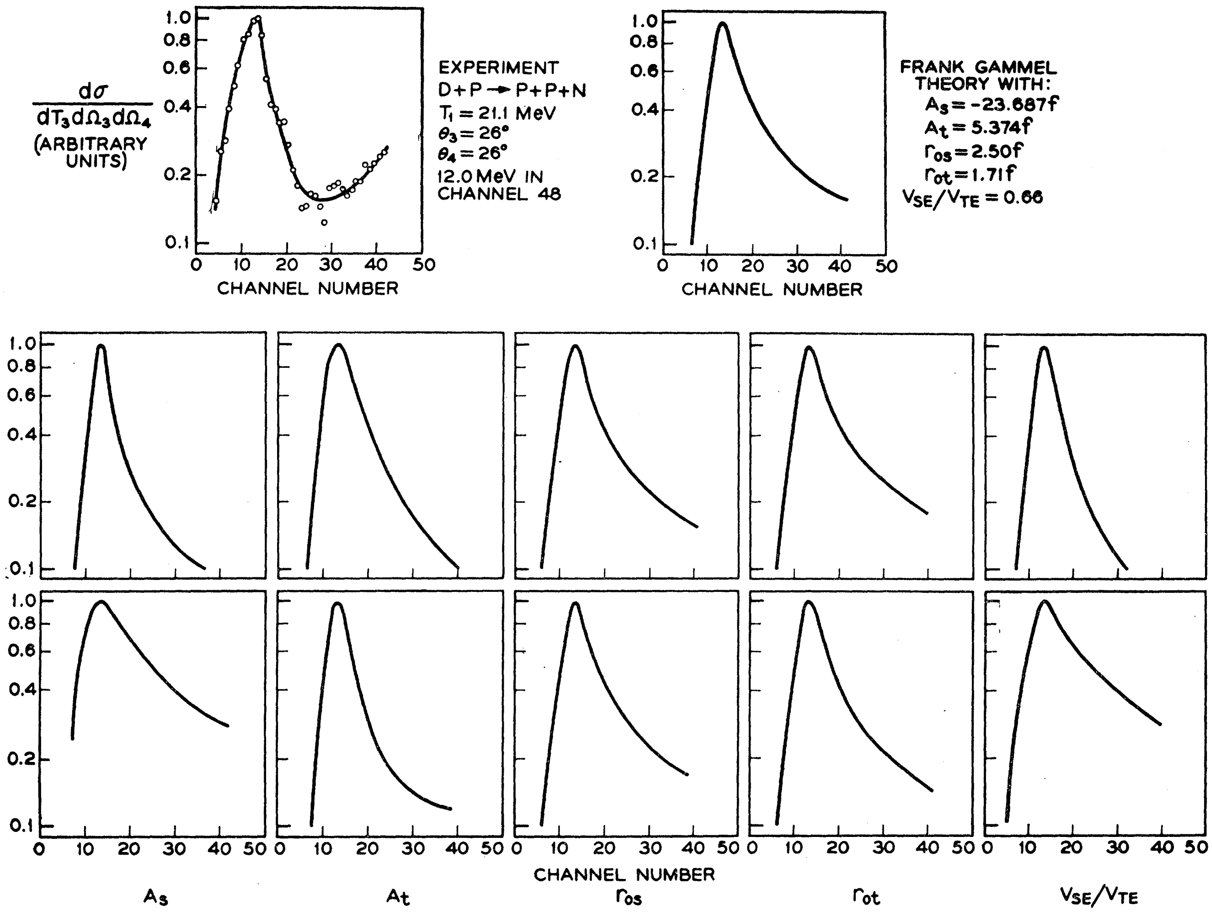

FIG. 3. Same as Fig. 2 except that the reaction is $D+P\rightarrow P+P+N$. The target and projectile are reversed in this case and the angles are changed. This case has the same c.m. energy as the case shown in Fig. 2.

body forces? Seeing to what extent the scattering lengths obtained from two free nucleons interacting are different in order to explain a reaction such as this?

KANE: I would say this is the direction we are working in; namely, that we want to get really good theory and see if it agrees with good experimental data.

This theory certainly can stand improvement, and certainly so can the experimental data. Now with really good theory and good experimental data, then the deviations should reveal three-body force effects, or other effects that we haven't thought of yet, I guess. Does that answer your question?

EISBERG: Yes.