Comparison of Excited State of Helium 4 from Two Different Reactions*

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Evidence for an excited state of helium 4 has been found by Werntz[1] in an analysis of neutron spectra from the $D+T\rightarrow p+T+n$ reaction[2,3] and in a phase shift analysis[4] of elastic $p+T$ cross sections[5-7] as well as related data. The data were shown to be consistent with a 0^+ resonance located at 0.5 ± 0.1 MeV in the $p+T$ center-of-mass system, which corresponds to 20.3 ± 0.1-MeV excitation energy in He^4. The empirical width-at-half-height of the resonance is ~0.5 MeV (c.m.) which, depending on the choice of channel radius, corresponds closely to the Wigner limit[8] for the total width (~6 MeV).

Recently, Young and Ohlsen[9] and Donovan *et al.*[10] have analyzed the three-body breakup of the $D+He^3$ reaction into $p+T+p$ and $n+He^3+p$. Whereas the former authors[9] measured the single proton energy spectra at various angles with respect to the incident direction, Donovan *et al.* determined the proton spectra in coincidence with tritons and helium-3 particles, respectively. Both groups interpreted their results in terms of a He^4 state peaked near 20 MeV, i.e., at 20.08 ± 0.05 MeV (0.25 ± 0.06 MeV wide)[9] and 19.938 ± 0.025 MeV (0.175 ± 0.025 MeV wide),[10] respectively. In addition, Donovan *et al.*[10] suggested that there is another state in He^4 at 21.2 ± 0.2 MeV (1.1 ± 0.2 MeV wide). The purpose of this note is to show that very likely the state near 20 MeV is identical with the 0^+ s-wave resonance found by Werntz[1] and to elaborate on his model[1] for the $D+T$ breakup reaction, which is quite analogous to the $D+He^3$ breakup reactions. On the basis of this model we show also that the feature in the proton spectra interpreted[10] as a He^4 state near 21.2 MeV can be explained as a p-wave final state interaction in the $p+T$ system.

Independent of the reaction mechanism proposed for the $D+He^3$ breakup reaction, we can write the cross section for the detection of proton spectra in coincidence with tritons in the laboratory system

$$d^3\sigma/(dE_p\,d\Omega_p\,d\Omega_T)=[2\pi/(\hbar v)]\rho_p\,|\,M\,|^2, \qquad (1)$$

where E_p is the kinetic energy of the proton detected in the solid angle element $d\Omega_p$ coincident with a triton detected in the solid angle element $d\Omega_T$ and v is the relative velocity of D and He^3. The phase-space factor ρ_p can be evaluated using standard formulas.[11] The matrix element $|\,M\,|^2$ is discussed further below. The cross section for the detection of triton spectra in coincidence with protons can be written in similar notation.

$$d^3\sigma/(dE_T\,d\Omega_T\,d\Omega_p)=[2\pi/(\hbar v)]\rho_T\,|\,M\,|^2. \qquad (2)$$

If single proton spectra are detected from this breakup, the cross section is

$$d^2\sigma/(dE_p\,d\Omega_p)=[2\pi/(\hbar v)]\left(\int\rho_p\,d\Omega_T\right)\langle\,|\,M\,|^2\rangle$$
$$\equiv[2\pi/(\hbar v)]R_p\langle\,|\,M\,|^2\rangle. \qquad (3)$$

(The integral $R_p=\int\rho_p\,d\Omega_T$ is actually more easily evaluated in the center of mass system of the triton and undetected proton, than in the laboratory system.)

We now assume with Werntz[1] that the reaction $D+He^3\rightarrow p+T+p$ proceeds by a stripping process $D+He^3\rightarrow(p+T)+p$ and that the $(p+T)$ system has final state interactions in various states characterized by their channel spin S (singlet or triplet) and relative orbital angular momentum L. Spin–orbit interaction is ignored. This type of model will be credible[12] if the $(p+T)$ system interacts strongly at low relative momentum $|\,\mathbf{k}\,|$, which fortunately is the case here. One can then show[13] (for one channel spin)

$$M=\sum_L a_L m_L, \qquad (4)$$

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[12] K. M. Watson, Phys. Rev. **88**, 1163 (1952).
[13] J. R. Gillespie, University of California, Lawrence Radiation Laboratory Report UCRL-10762, (9 April 1963) (unpublished).

where $a_L(\mathbf{k}_p, \mathbf{k})$ is a function of the direction $\mathbf{k}_p$ of the detected proton and of the direction $\hat{\mathbf{k}}$. As long as the energy of the $D+\mathrm{He}^3$ system is below the threshold for emission of neutrons one expects a_L to be proportional to the ordinary stripping amplitude for capture of a neutron with angular momentum L. The matrix element m_L can be estimated by the factored wave-function method,[13] first proposed by Watson.[12] If we write the scattering matrix element for $T+p$ scattering as $D_L \exp(2i\delta_L)$, where D_L and $\delta_L=\omega_L-\Phi_L+\beta_L$ are real quantities, with ω_L equal to the Coulomb phase shift ($\omega_0=0$), $-\Phi_L$ equal to the hard sphere phase shift and β_L equal to the nuclear phase shift, one finds

$$m_L \propto -\exp(-i\delta_L)(A_L/k)[(D_L-1)\cos\beta_L - i(D_L+1)\sin\beta_L], \quad (5)$$

which, for pure elastic scattering ($D_L=1$), reduces to the well-known form[1,12,14]

$$m_L \propto 2i \exp(-i\delta_L)(A_L/k)\sin\beta_L. \quad (6)$$

In these expressions $A_L^2=F_L^2+G_L^2$, where F_L and G_L are the usual Coulomb functions.

In using Eq. (4) to evaluate $|M|^2$ one should note that whereas in stripping to bound states no interference terms exist between terms of opposite parity, here the breakup of the $(T+p)$ system allows such terms. Restricting ourselves to s-wave and p-wave stripping and taking into account the singlet and triplet channel spins, we have in obvious notation

$$|M|^2=\tfrac{1}{4}[|{}^1a_0{}^1m_0|^2+2\,\mathrm{Re}({}^1a_0{}^1m_0{}^1a_1{}^{*1}m_1{}^*)+|{}^1a_1{}^1m_1|^2] + \tfrac{3}{4}[|{}^3a_0{}^3m_0|^2+2\,\mathrm{Re}({}^3a_0{}^3m_0{}^3a_1{}^{*3}m_1{}^*)+|{}^3a_1{}^3m_1|^2]. \quad (7)$$

On the other hand, in the matrix element $\langle|M|^2\rangle$, which is averaged over all orientations of $\hat{\mathbf{k}}$, the interference terms drop out.

Leaving a_1/a_0 as adjustable parameters, the above expressions were applied to the experimental cross section[9,10] of the $D+\mathrm{He}^3\rightarrow p+T+p$ breakup reaction. For this purpose a least-squares fitted phase-shift analysis[15] of the $T(p,p)$, $T(p,n)$, and $\mathrm{He}^3(n,n)$ reactions was used, based on a method rather similar to that of Werntz[4]: for the ${}^1S$ phase shift a Breit–Wigner energy dependence was assumed; for the ${}^3S$, ${}^1P$, and ${}^3P$ phases an effective range approximation was used.[16] The resulting real parts of the phase shifts are shown in Fig. 1. A typical fit to the data at hand is indicated in Fig. 2, which gives the $p+T$ elastic scattering cross section at 120° (c.m.) of Jarmie *et al.*[6] (symbol ○)

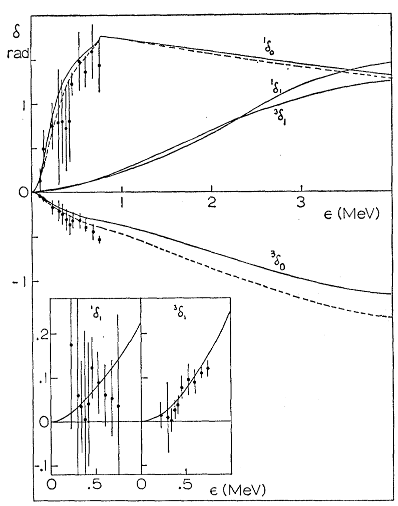

FIG. 1. Phase shifts for the proton scattering on tritium. The assumed energy dependences of the phase shifts are described in the text. The solid lines result from the best over-all fit to $T(p,p)$, $T(p,n)$ and $\mathrm{He}^3(n,n)$ data from various sources. The dotted lines produce the best fit to the elastic $p+T$ scattering data of Jarmie *et al.* (Ref. 5) shown in Fig. 2. A point by point phase shift analysis made by Balashko *et al.* (Ref. 17) is shown by the solid circles with appropriate error bars. For $\epsilon>0.764$ MeV, the $T(p,n)$ threshold, only the real part of the phase shift is shown.

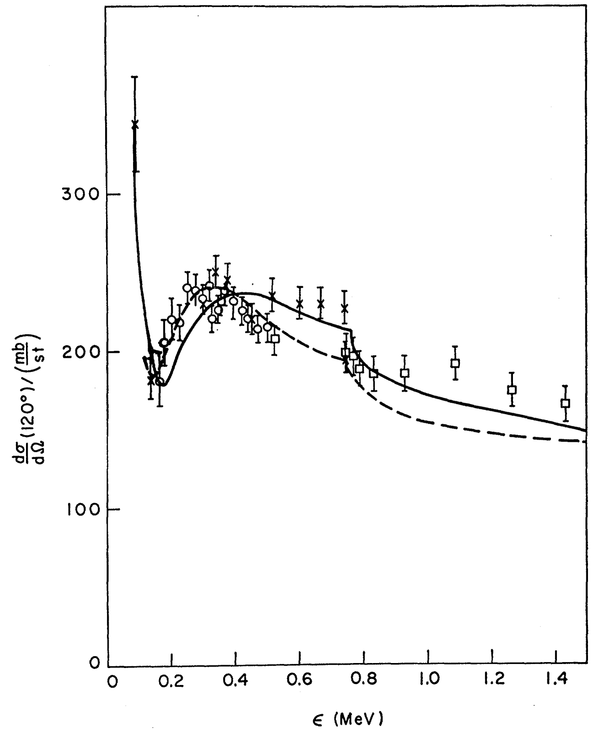

FIG. 2. Differential elastic scattering cross section in c.m. system for $p+T$ scattering at 120° vs proton c.m. energy. The data are taken from the following references: Symbol ○, Ref. 5; symbol □, Ref. 7; symbol ×, Ref. 17. The solid and dashed lines are from phase-shift fits described in the text and shown in Fig. 1.

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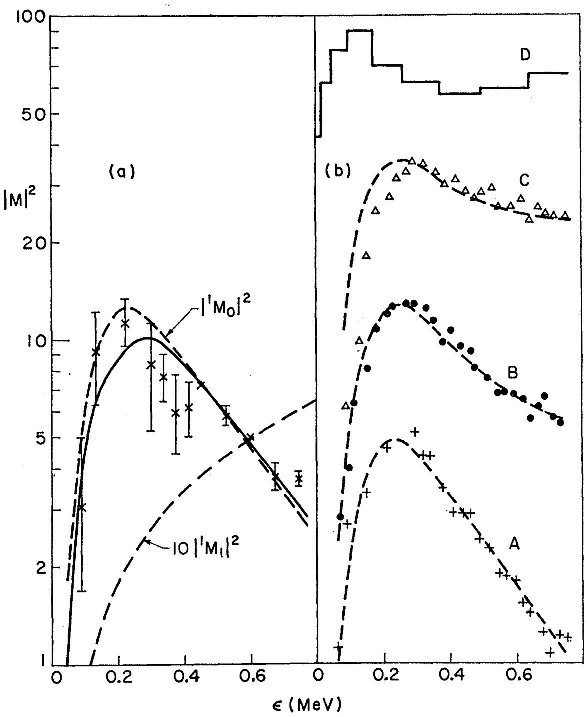

FIG. 3. (a) Calculated shapes of the matrix elements characterizing the $(p+T)$ final state interaction in the $D+He^3 \rightarrow (p+T)+p$ breakup reaction. The singlet s- and p-wave matrix elements $|{}^1m_0|^2$ and $|{}^1m_1|^2$ are shown. In the case of the s-wave matrix element, the three sets of phase shifts ${}^1\delta_0$, shown in Fig. 1, have been used, in order to give an idea about the uncertainty of this type of analysis. (b) Curves A, B, and C use the data of Young and Ohlsen (Ref. 9) described in Table I. The dotted curve uses the (dotted) matrix elements shown in (a) with expression (8) of the text. Curve D gives the data of Donovan *et al.* (Ref. 10) which is shown once more as crosses in Fig. 4.

and Balashko *et al.*[17] (symbol ×). The solid lines show the phase shifts which give the best over-all fit to all the data used; the dotted lines result from a fit which was forced to follow mainly the 120° data of Jarmie *et al.* The difference of these two fits is indicative of the uncertainty of our phase shift analysis. In general, our results[15] are very similar to those of Werntz.[4]

Balashko *et al.*[17] have also made a point by point phase shift fit mainly of their own extensive $p+T$ elastic scattering data[17] (between 50- and 990-keV proton lab. energy). Their results are shown as solid circles on Fig. 1. It is remarkable that these different analyses yield such similar results.

To compare the calculated expressions for $\langle|M|^2\rangle$ with the single proton spectra of Young and Ohlsen,[9] experimental cross sections were divided by the phase-space factor R_p indicated in Eq. (3). Since it turned out that the phase shifts β_L were such that ${}^3m_0 \cong 0$ and ${}^1m_1 \cong {}^3m_1$, the experimental matrix element $\langle|M|^2\rangle$ was fitted by the following expression which follows immediately from Eq. (7)

$$\langle|M|^2\rangle \propto |{}^1m_0|^2 + 4\alpha|{}^1m_1|^2, \tag{8}$$

where α was treated as an adjustable parameter. The separate forms of $|{}^1m_0|^2$ and $|{}^1m_1|^2$ are shown in Fig. 3(a) as a function of the c.m. energy ϵ of the $(p+T)$ system. [$\epsilon = \hbar^2k^2/(2\mu)$, where μ is the reduced mass of the proton in the $(p+T)$ system.] To indicate the accuracy of these shapes, we give the element $|{}^1m_0|^2$ as calculated from the two different phase shift fits shown in Fig. 1, as well as from the results of Balashko *et al.*[17] For $|{}^1m_1|^2$ we show only the "dotted-line" fit of Fig. 1. In Fig. 3(b), curves A to C give Young and Ohlsen's data[9] (with the phase-space factor removed) and our fit by expression (8). The variation of α with the energy available in the $p+T+p$ c.m. system and with the c.m. angle between the detected proton and the entering deuteron is shown in Table I. It is clear that α increases rapidly with increasing c.m. angle of the detected proton. This is what one would expect if the $D+He^3$ reaction proceeds by stripping, since the ratio of p-wave to s-wave stripping should increase with increasing proton angle.

To analyze the data of Donovan *et al.*,[10] Eqs. (1) and (2) were used. The resulting experimental matrix elements $|M|^2$ were matched in magnitude. Figure 3(b)D and Fig. 4 show the results. Some uncertainty in our analysis of the c.m. energy ϵ is apparent in Fig. 4, but the shapes of the element $|M|^2$ calculated from the proton and the triton spectra are quite similar. Expression (7) was evaluated with the same approximations as indicated in Eq. (8) assuming all coefficients a to be real. The dotted curve in Fig. 4 gives the resultant fit. The interference term in the matrix element has only a minor influence, mainly between 0.5 and 1.0 MeV. The effect of the p-wave final state inter-

TABLE I. Empirical values of parameter α in expression (8).

Curve in Fig. 3(b)	Energy in $p+T+p$ c.m. system (MeV)	C.m. angle of detected proton w.r. to deuteron	α
A	4.5	21°	~0
Not shown	3.9	21°	1.0
B	3.3	21°	1.3
Not shown	3.3	29°	1.3
C	3.3	36°	3
Not shown	3.3	45°	5

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action is apparent. The curve shown uses

$$[\tfrac{1}{4}({}^1a_1)^2+\tfrac{3}{4}({}^3a_1)^2]/({}^1a_0)^2=4.2.$$

This is quite consistent with the values of α given in Table I, because the experiment[10] analyzed here corresponded to a c.m. energy of the $p+T+p$ system of 11.3 MeV and a c.m. angle of the detected proton with respect to the deuteron of 85°. The deviation of the curve in Fig. 4 from the experimental points for $\epsilon>3$ MeV may be due to a breakdown of the theory or neglect of higher L phase shifts, which are certainly important in this energy range.

In summary it is clear that the main features of the $D+\mathrm{He}^3\rightarrow p+T+p$ breakup reaction can be understood in terms of the ${}^1S_0$ state of He^4 proposed by Werntz[1] and in terms of a p-wave final state interaction in the $p+T$ system. Since the phase shift ${}^1\delta_0$ passes through 90° near $\epsilon=0.6$ MeV [see Fig. (1)] one can say that this state is located near 20.4 MeV. On the other hand, from a practical point of view it might be more useful to speak about the peak location of the $p+T$ cross section ($\epsilon\sim0.3$ MeV) or the peak location of the proton spectrum ($\epsilon\sim0.2$ MeV) in the $D+\mathrm{He}^3\rightarrow(p+T)+p$ breakup reaction, even though these do not occur at exactly identical energies, because of different energy dependence[18] on the relative momentum k and the penetration factor A_0^2.

The p-wave interaction proposed here would also aid the analysis of the $D+T\rightarrow(p+T)+n$ breakup reaction made by Werntz,[1] as can be seen from the inspection of his phase shift fits to the experimental data,[2,3] which did not take into account this interaction.

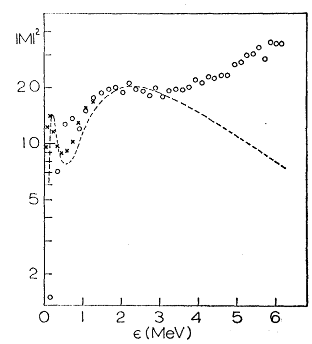

FIG. 4. Matrix elements characterizing the proton and triton spectra in the $D+\mathrm{He}^3\rightarrow(p+T)+p$ breakup reaction, calculated from the data of Donovan *et al.* (Ref. 10). The crosses are calculated from a proton spectrum in coincidence with tritons, the circles are from a triton spectrum in coincidence with protons. The curve is a fit assuming s-wave and p-wave final-state interactions in the $(p+T)$ system. Expression (7) was used with the simplifications mentioned in the text.

Finally we wish to remark that the same model can be applied to the $D+\mathrm{He}^3\rightarrow(n+\mathrm{He}^3)+p$ reaction. As soon as experimental data are available, we can use our neutron phase[15] shifts to attempt a fit by means of the $(n+\mathrm{He}^3)$ final state interaction. Unfortunately here the matrix elements $|{}^3m_0|^2$ and $|{}^1m_0|^2$ are of comparable magnitude, so that in expression (7) there are a considerable number of adjustable parameters, which means that no simplification analogous to expression (8) can be made.

The furnishing of yet unpublished data by Balashko *et al.*[17] and by Donovan *et al.*[10] is most gratefully acknowledged. Discussions with Dr. T. A. Griffy and Dr. G. G. Ohlsen have been extremely helpful. Details of the theory will be published elsewhere.[19]

[18] The functional energy dependence of the ${}^1S$ part of the $T-p$ cross section without Coulomb scattering is of course $[\sin^2({}^1\delta_0)]/\epsilon$.

[19] W. E. Meyerhof and D. U. L. Yu (to be published).